Computer Algebra
Independent Integration Tests

Summer 2023 edition with Rubi V 4.17.3

4-Trig-functions/4.5-Secant/120-4.5.1.4-d-tan-^n-a+b-sec-^m

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CHAPTER 1

Introduction

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This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [365]. This is test number [120].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 13.3.1 (August 16, 2023) on windows 10.
2. Rubi 4.17.3 (Sept 25, 2023) on Mathematica 13.3.1 on windows 10
3. Maple 2023.1 (July, 12, 2023) on windows 10.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
5. FriCAS 1.3.9 (July 8, 2023) based on sbcl 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
6. Giac/Xcas 1.9.0-57 (June 26, 2023) on Linux via sagemath 10.1 (Aug 20, 2023).
7. Sympy 1.12 (May 10, 2023) Using Python 3.11.3 on Linux.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	98.36 (359)	1.64 (6)
Mathematica	94.52 (345)	5.48 (20)
Maple	90.68 (331)	9.32 (34)
Fricas	71.23 (260)	28.77 (105)
Giac	60.27 (220)	39.73 (145)
Maxima	51.78 (189)	48.22 (176)
Mupad	49.59 (181)	50.41 (184)
Sympy	10.96 (40)	89.04 (325)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

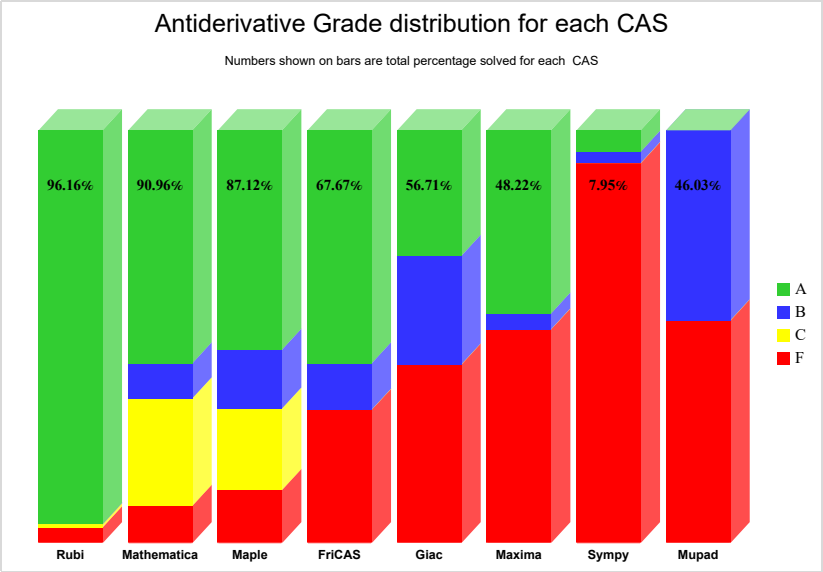
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

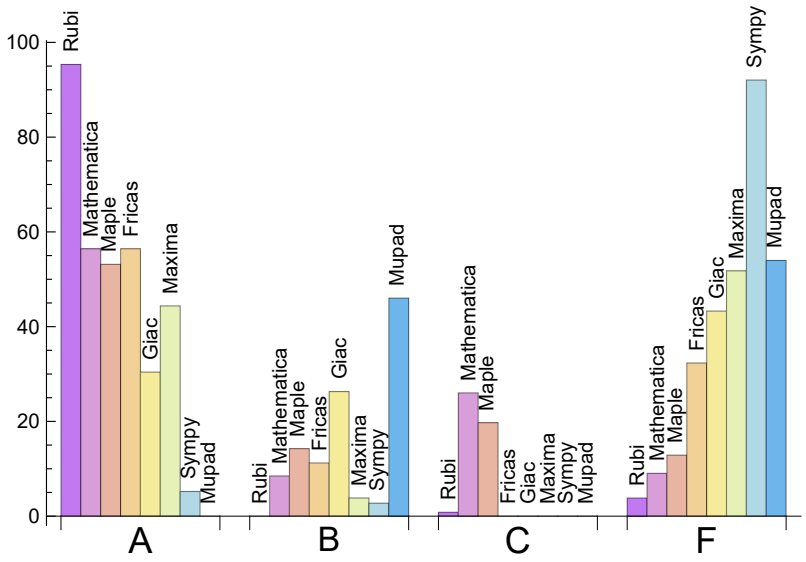
System	% A grade	% B grade	% C grade	% F grade
Rubi	94.795	0.000	0.000	5.205
Mathematica	56.438	8.493	26.027	9.041
Fricas	56.438	11.233	0.000	32.329
Maple	53.151	14.247	19.726	12.877
Maxima	44.384	3.836	0.000	51.781
Giac	30.411	26.301	0.000	43.288
Sympy	5.205	2.740	0.000	92.055
Mupad	0.000	46.027	0.000	53.973

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima

and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

System	Number failed	Percentage normal failure	Percentage time-out failure	Percentage exception failure
Rubi	6	100.00	0.00	0.00
Mathematica	20	100.00	0.00	0.00
Maple	34	100.00	0.00	0.00
Fricas	105	41.90	58.10	0.00
Giac	145	97.93	0.00	2.07
Maxima	176	67.05	13.64	19.32
Mupad	184	0.00	100.00	0.00
Sympy	325	86.46	13.54	0.00

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)
Rubi	0.46
Maxima	0.58
Fricas	0.71
Giac	1.54
Mathematica	4.02
Maple	5.54
Sympy	7.51
Mupad	15.50

Table 1.5: Time performance for each CAS

System	Mean size	Normalized mean	Median size	Normalized median
Maxima	109.20	1.15	94.00	1.00
Sympy	137.72	1.95	97.00	1.36
Rubi	154.55	0.97	122.00	1.00
Giac	224.35	2.01	164.00	1.62
Mathematica	229.78	1.45	125.00	0.97
Fricas	336.38	2.45	178.00	1.60
Maple	366.87	1.72	143.00	1.19
Mupad	418.38	2.89	118.00	1.45

Table 1.6: Leaf size performance for each CAS

1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the y axis is the percentage solved which Rubi itself needed the number of rules given the x axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

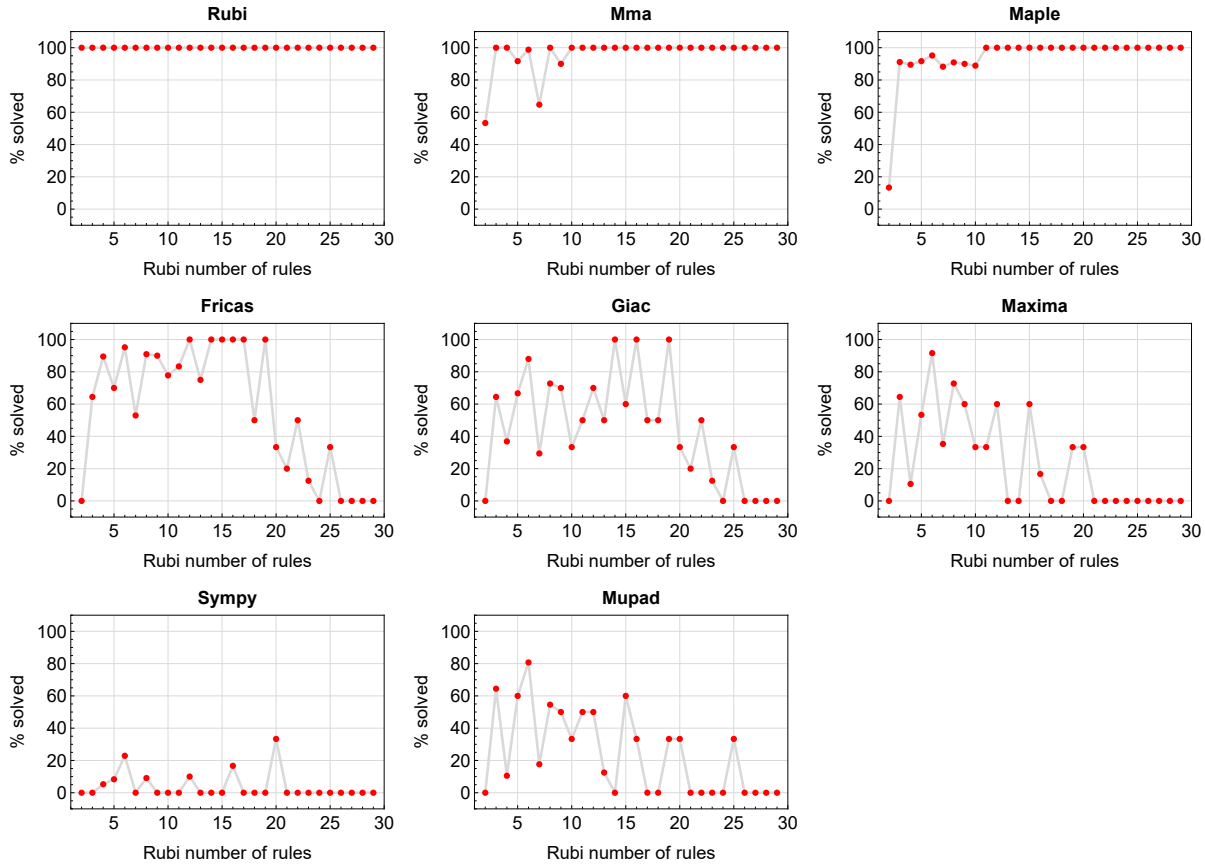


Figure 1.1: Solving statistics per number of Rubi rules used

1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

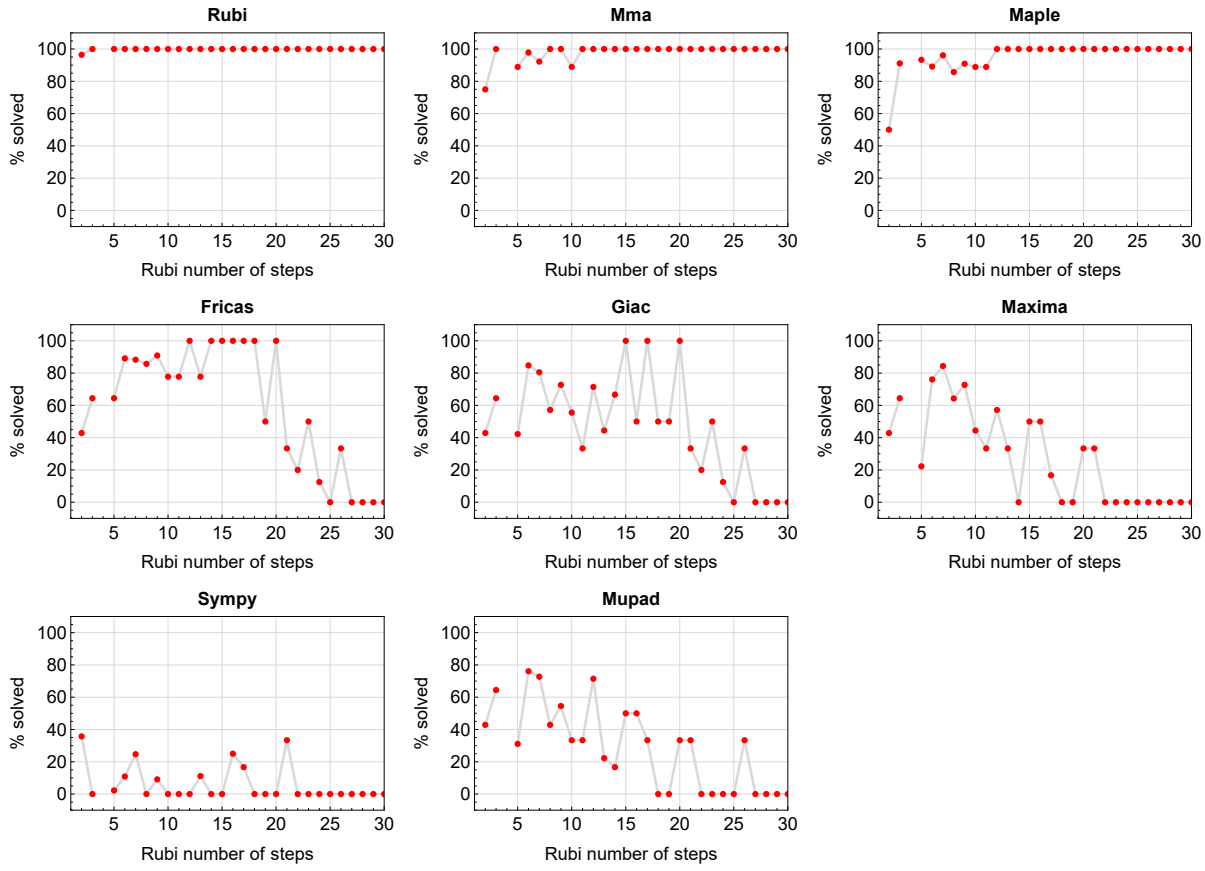


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

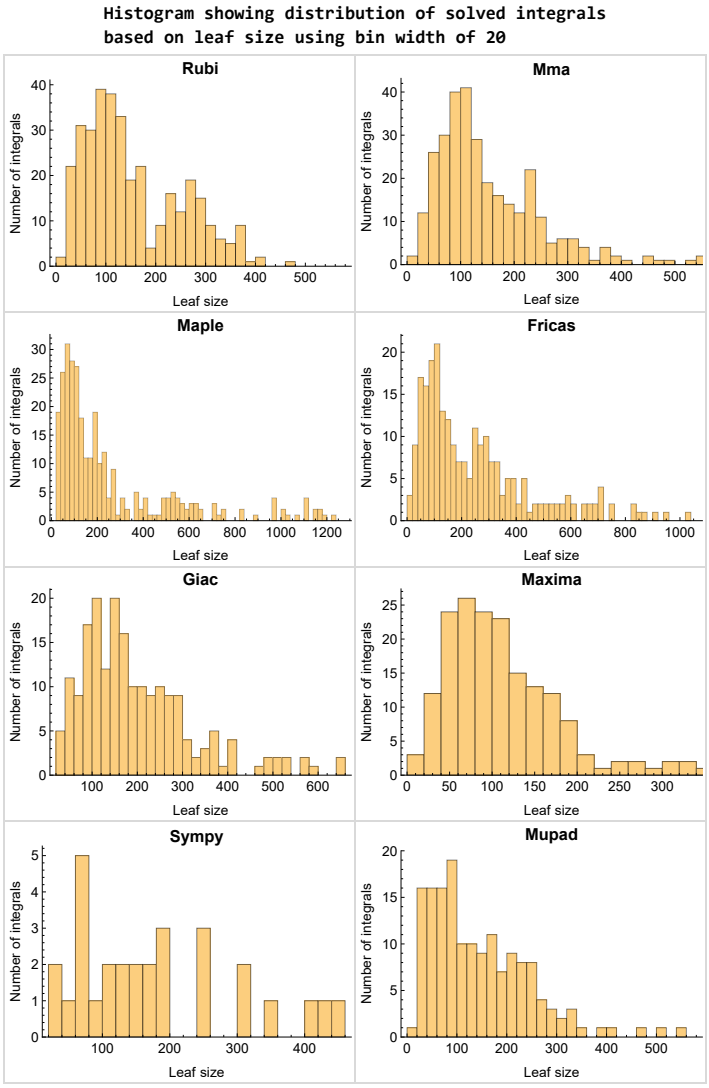


Figure 1.3: Solved integrals based on leaf size distribution

1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

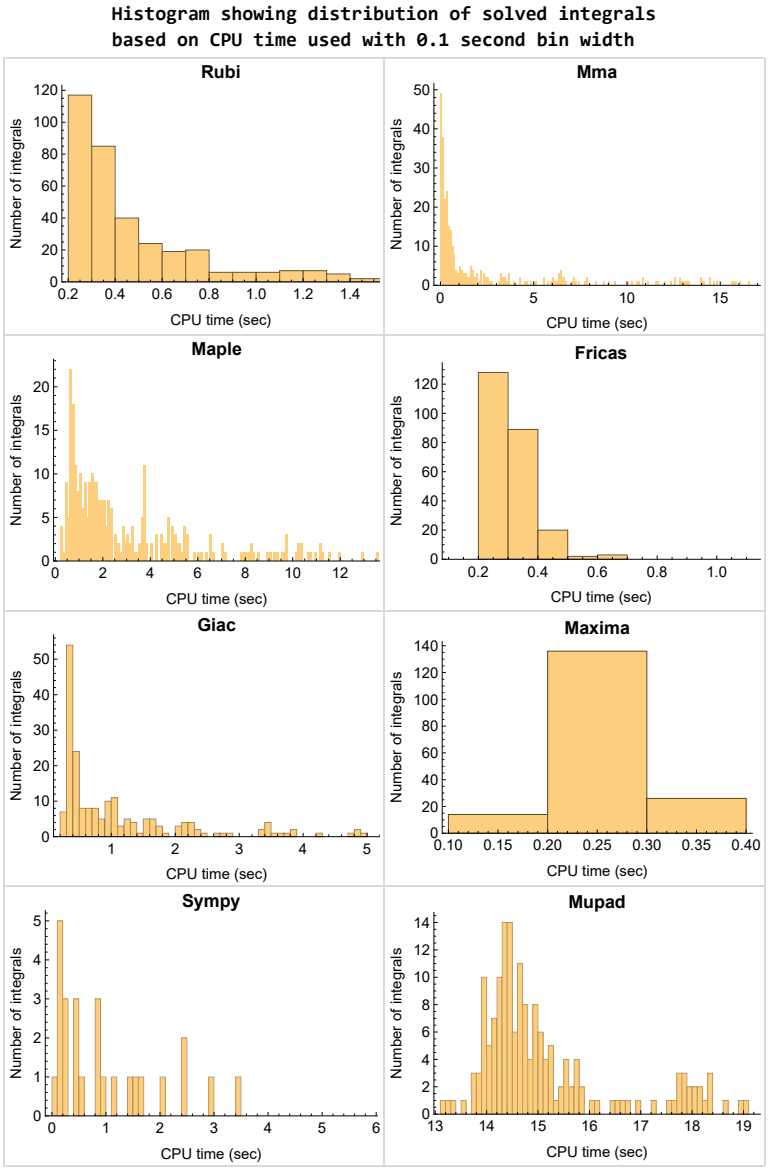


Figure 1.4: Solved integrals histogram based on CPU time used

1.8 Leaf size vs. CPU time used

The following gives the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time.

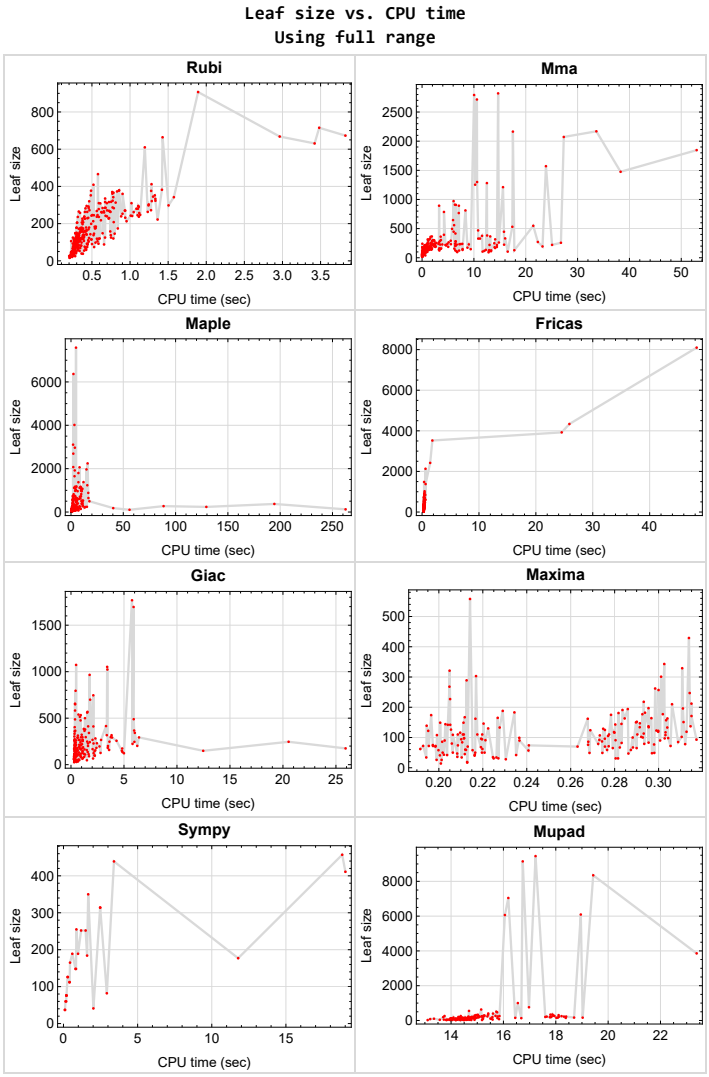


Figure 1.5: Leaf size vs. CPU time. Full range

1.9 list of integrals with no known antiderivative

{348, 349, 350, 351, 352, 358, 359, 360, 361, 362, 363, 364, 365}

1.10 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

Rubi {104, 105, 106, 107, 108, 109, 110, 118, 119, 120, 121, 122, 123, 124, 125, 126, 314, 315, 318, 319, 322, 326, 327, 330, 335, 336, 338, 339}

Mathematica {107, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 131, 133, 134, 143, 145, 156, 157, 169, 177, 178, 179, 189, 190, 192, 193, 201, 204, 205, 207, 226, 227, 229, 230, 231, 232, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 312, 313, 314, 315, 316, 317, 332, 340, 341, 347}

Maple {104, 111, 118, 119, 120, 121, 122, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 155, 167, 189, 192, 201, 204, 207, 243, 247, 248, 249, 250, 251, 252, 253, 254, 255, 312, 313, 314, 315, 316, 317, 322, 329, 330, 338, 339, 342}

Maxima Verification phase not currently implemented.

Fricas Verification phase not currently implemented.

Sympy Verification phase not currently implemented.

Giac Verification phase not currently implemented.

Mupad Verification phase not currently implemented.

1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.13 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.14 Important notes about some of the results

1.14.1 Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

1.14.2 Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

1.14.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

1.14.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

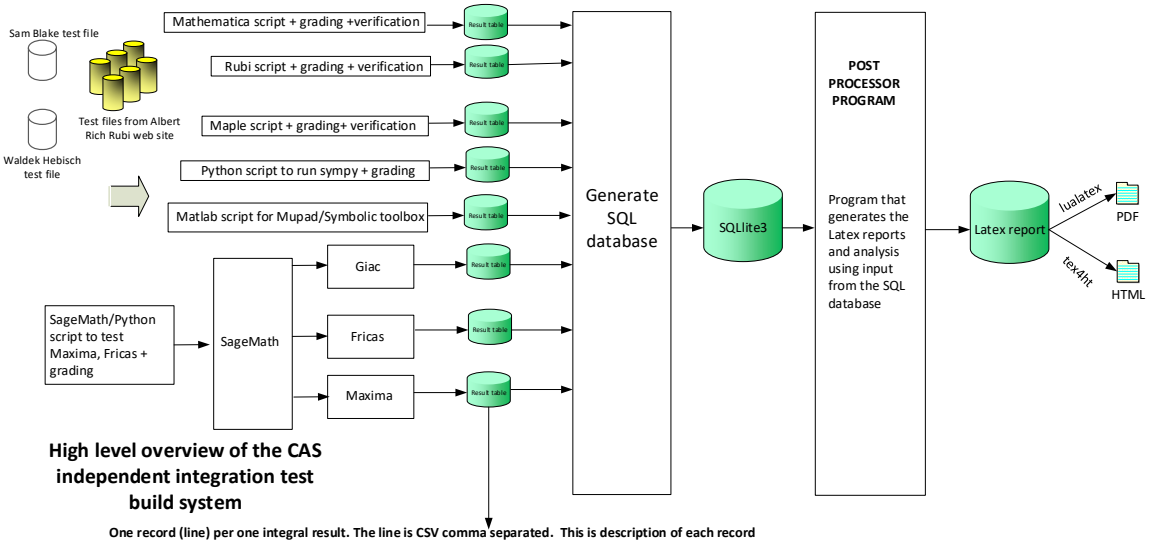
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.15 Design of the test system

The following diagram gives a high level view of the current test build system.



High level overview of the CAS independent integration test build system

One record (line) per one integral result. The line is CSV comma separated. This is description of each record

1. integer, the problem number.
2. integer. 0 for failed, 1 for passed, -1 for timeout, -2 for CAS specific exception. (this is not the grade field)
3. integer. Leaf size of result.
4. integer. Leaf size of the optimal antiderivative.
5. number. CPU time used to solve this integral. 0 if failed.
6. string. The integral in Latex format
7. string. The input used in CAS own syntax.
8. string. The result (antiderivative) produced by CAS in Latex format
9. string. The optimal antiderivative in Latex format.
10. integer. 0 or 1. Indicates if problem has known antiderivative or not
11. String. The result (antiderivative) in CAS own syntax.
12. String. The grade of the antiderivative. Can be "A", "B", "C", or "F"
13. String. Small string description of why the grade was given.
14. integer. 1 if result was verified or 0 if not verified. (For mma, rubi and maple only)

*The following fields are present only in **Rubi** Table file*

15. integer. Number of steps used.
16. integer. Number of rules used.
17. integer. Integrand leaf size.
18. real number. Ratio. Field 16 over field 17
19. String of form "{n,n,..}" which is list of the rules used by Rubi
20. String. The optimal antiderivative in Mathematica syntx

Nasser M. Abbasi
June 27, 2023
Design.vsdx

CHAPTER 2

DETAILED SUMMARY TABLES OF RESULTS

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2.1 List of integrals sorted by grade for each CAS

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2.1.4 Fricas 23
2.1.5 Maxima 24
2.1.6 Giac 24
2.1.7 Mupad 25
2.1.8 Sympy 26

2.1.1 Rubi

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 314, 315, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 353, 354, 355, 356, 357 }

B grade { }

C grade { }

F normal fail { 243, 312, 313, 316, 317, 347 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.1.2 Mma

A grade { 1, 2, 3, 4, 5, 7, 8, 9, 10, 11, 12, 13, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 56, 57, 58, 59, 60, 61, 62, 63, 68, 71, 72, 73, 74, 75, 76, 77, 78, 82, 83, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 101, 103, 135, 136, 137, 138, 141, 142, 144, 145, 146, 147, 148, 149, 150, 151, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 170, 171, 172, 173, 177, 179, 180, 181, 182, 183, 184, 190, 191, 192, 193, 194, 195, 196, 202, 203, 204, 205, 206, 207, 209, 210, 211, 219, 220, 221, 222, 223, 224, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 286, 287, 288, 289, 290, 291, 292, 293, 296, 297, 299, 300, 301, 302, 303, 304, 305, 306, 309, 310, 311, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 340, 341, 343, 344, 345, 346, 353, 354, 355, 356, 357 }

B grade { 6, 50, 64, 65, 66, 67, 69, 70, 79, 80, 81, 84, 97, 98, 99, 100, 102, 201, 226, 227, 229, 230, 231, 232, 294, 295, 298, 307, 308, 342, 347 }

C grade { 14, 15, 16, 17, 18, 32, 33, 34, 35, 36, 51, 52, 53, 54, 55, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 131, 133, 134, 139, 140, 143, 152, 169, 174, 175, 176, 178, 185, 186, 187, 188, 189, 197, 198, 199, 200, 213, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 267, 268, 269, 270, 283, 284, 285, 312, 313, 314, 315, 316, 317, 335, 336, 337, 338, 339 }

F normal fail { 127, 128, 129, 130, 132, 208, 212, 214, 215, 216, 217, 218, 225, 228, 250, 251, 252, 253, 254, 255 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.1.3 Maple

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 18, 19, 20, 21, 22, 23, 24, 28, 29, 30, 31, 37, 38, 39, 40, 41, 42, 56, 57, 58, 59, 60, 61, 62, 63, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 99, 100, 101, 102, 103, 105, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 135, 136, 137, 138, 139, 140, 141, 142, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 165, 166, 167, 168, 171, 172, 173, 176, 177, 178, 179, 181, 182, 183, 184, 185, 189, 191, 194, 195, 196, 197, 201, 203, 206, 234, 236, 237, 239, 240, 241, 242, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 320, 324, 328, 333, 337 }

B grade { 104, 106, 143, 163, 164, 169, 170, 174, 175, 180, 186, 187, 188, 190, 192, 193, 198, 199, 200, 202, 204, 205, 207, 294, 295, 312, 313, 314, 315, 316, 317, 318, 319, 321, 322, 323, 325, 326, 327, 329, 330, 331, 332, 334, 335, 336, 338, 339, 340, 341, 342, 343 }

C grade { 14, 15, 16, 17, 25, 26, 27, 32, 33, 34, 35, 36, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 64, 65, 66, 67, 79, 80, 81, 82, 95, 96, 97, 98, 107, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 233, 235, 238, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 282 }

F normal fail { 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 344, 345, 346, 347, 353, 354, 355, 356, 357 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.1.4 Fricas

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 10, 11, 12, 14, 15, 16, 19, 20, 21, 22, 23, 24, 25, 26, 27, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 153, 154, 155, 157, 158, 159, 160, 161, 162, 165, 166, 167, 169, 170, 171, 172, 176, 177, 178, 179, 180, 181, 182, 183, 184, 189, 190, 191, 192, 193, 194, 195, 201, 202, 203, 204, 205, 206, 207, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 294, 295, 296, 297, 299, 300, 301, 302, 303, 304, 308, 310, 318, 319, 320, 326, 327, 335, 336 }

B grade { 9, 13, 17, 18, 28, 46, 150, 151, 152, 156, 163, 164, 168, 173, 174, 175, 185, 186, 187, 188, 196, 197, 198, 199, 200, 266, 293, 298, 305, 306, 307, 309, 311, 321, 322, 328, 329, 330, 337, 338, 339 }

C grade { }

F normal fail { 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 323, 324, 325, 331, 332, 333, 334, 340, 341, 342, 344, 345, 346, 347, 353, 354, 355, 356, 357 }

F(-1) timedout fail { 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 312, 313, 314, 315, 316, 317, 343 }

F(-2) exception fail { }

2.1.5 Maxima

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 100, 101, 102, 103, 135, 136, 137, 147, 148, 149, 159, 160, 161, 171, 172, 173, 183, 184, 185, 197, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 299, 300, 301, 302, 303, 304, 305, 318, 319, 320, 326, 327, 328, 335, 336, 337 }

B grade { 64, 65, 66, 67, 79, 80, 81, 96, 97, 98, 99, 195, 196, 306 }

C grade { }

F normal fail { 118, 119, 120, 121, 122, 123, 124, 125, 126, 129, 131, 133, 134, 138, 139, 141, 142, 143, 144, 145, 150, 151, 155, 156, 162, 163, 167, 168, 169, 174, 175, 176, 177, 178, 179, 180, 181, 182, 186, 187, 188, 189, 190, 191, 192, 193, 198, 199, 201, 202, 203, 204, 205, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 244, 245, 246, 247, 248, 249, 250, 252, 254, 312, 313, 314, 315, 316, 317, 321, 322, 323, 324, 325, 329, 330, 331, 332, 333, 334, 338, 341, 342, 343, 344, 345, 346, 347, 353, 354, 355, 356, 357 }

F(-1) timedout fail { 127, 128, 130, 132, 140, 146, 152, 153, 154, 157, 158, 164, 165, 166, 170, 194, 200, 206, 243, 251, 253, 255, 339, 340 }

F(-2) exception fail { 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 294, 295, 296, 297, 298, 307, 308, 309, 310, 311 }

2.1.6 Giac

A grade { 8, 9, 10, 11, 12, 14, 15, 16, 17, 18, 19, 24, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 60, 61, 62, 63, 64, 65, 68, 69, 70, 74, 75, 76, 77, 78, 79, 80, 82, 83, 84, 85, 86, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 171, 172, 173, 174, 175, 176, 177, 180, 181, 182, 183, 184, 185, 186, 187, 188, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 202, 203, 204, 205, 206, 207, 260, 267, 276, 279, 282, 309, 310, 311 }

B grade { 1, 2, 3, 4, 5, 6, 7, 13, 20, 21, 22, 23, 25, 38, 39, 40, 41, 42, 43, 44, 57, 58, 59, 66, 67, 71, 72, 73, 81, 87, 88, 89, 178, 179, 189, 190, 201, 256, 257, 258, 259, 261, 262, 263, 264, 265, 266, 268, 269, 270, 271, 272, 273, 274, 275, 277, 278, 280, 281, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 318, 319, 320, 321, 322, 326, 327, 328, 330, 336, 337 }

C grade { }

F normal fail { 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 312, 313, 314, 315, 316, 317, 323, 324, 325, 331, 332, 333, 334, 335, 340, 341, 342, 343, 344, 345, 346, 347, 353, 354, 355, 356, 357 }

F(-1) timedout fail { }

F(-2) exception fail { 329, 338, 339 }

2.1.7 Mupad

A grade { }

B grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 137, 149, 161, 173, 185, 197, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 320, 328, 337 }

C grade { }

F normal fail { }

F(-1) timedout fail { 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 312, 313, 314, 315, 316, 317, 318, 319, 321, 322, 323, 324, 325, 326, 327, 329, 330, 331, 332, 333, 334, 335, 336, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 353, 354, 355, 356, 357 }

F(-2) exception fail { }

2.1.8 Sympy

A grade { 1, 2, 3, 4, 5, 19, 21, 23, 40, 41, 256, 257, 258, 259, 271, 272, 273, 274, 275 }

B grade { 20, 22, 37, 38, 39, 60, 75, 90, 91, 290 }

C grade { }

F normal fail { 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 24, 25, 26, 27, 29, 30, 31, 32, 33, 34, 42, 43, 44, 47, 48, 49, 50, 51, 52, 56, 57, 58, 59, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 112, 113, 114, 115, 116, 121, 122, 123, 124, 125, 126, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 153, 154, 155, 156, 160, 161, 167, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 230, 231, 232, 234, 235, 236, 237, 240, 241, 242, 243, 244, 245, 246, 247, 250, 251, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 276, 277, 278, 279, 280, 281, 282, 283, 284, 286, 287, 288, 289, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 353, 354, 355, 356, 357 }

F(-1) timedout fail { 18, 28, 35, 36, 45, 46, 53, 54, 55, 111, 117, 118, 119, 120, 127, 128, 129, 130, 152, 157, 158, 159, 162, 163, 164, 165, 166, 168, 169, 170, 207, 229, 233, 238, 239, 248, 249, 252, 253, 254, 255, 285, 361, 362 }

F(-2) exception fail { }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column `N.S.` means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	151	116	134	100	116	123	184	293	259
N.S.	1	0.77	0.89	0.66	0.77	0.81	1.22	1.94	1.72
time (sec)	N/A	0.269	0.570	1.458	0.210	0.291	1.592	6.418	17.949

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	118	92	106	82	94	101	148	247	204
N.S.	1	0.78	0.90	0.69	0.80	0.86	1.25	2.09	1.73
time (sec)	N/A	0.260	0.507	1.030	0.220	0.297	0.844	3.630	17.600

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	66	82	60	72	79	112	201	151
N.S.	1	0.76	0.94	0.69	0.83	0.91	1.29	2.31	1.74
time (sec)	N/A	0.243	0.392	0.862	0.197	0.279	0.414	1.604	18.381

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	46	55	44	50	57	76	155	96
N.S.	1	0.81	0.96	0.77	0.88	1.00	1.33	2.72	1.68
time (sec)	N/A	0.238	0.141	0.898	0.202	0.281	0.204	0.637	15.219

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	22	25	20	26	34	37	106	40
N.S.	1	0.88	1.00	0.80	1.04	1.36	1.48	4.24	1.60
time (sec)	N/A	0.211	0.014	0.292	0.199	0.298	0.098	0.324	14.700

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	38	33	14	16	0	58	34
N.S.	1	1.00	2.38	2.06	0.88	1.00	0.00	3.62	2.12
time (sec)	N/A	0.202	0.018	0.444	0.201	0.278	0.000	0.279	14.579

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	51	114	75	42	69	0	103	46
N.S.	1	0.89	2.00	1.32	0.74	1.21	0.00	1.81	0.81
time (sec)	N/A	0.242	0.521	0.498	0.235	0.279	0.000	0.314	13.950

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	81	175	111	86	150	0	149	88
N.S.	1	0.85	1.84	1.17	0.91	1.58	0.00	1.57	0.93
time (sec)	N/A	0.262	0.162	0.697	0.202	0.272	0.000	0.361	13.980

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	133	111	165	151	126	241	0	197	118
N.S.	1	0.83	1.24	1.14	0.95	1.81	0.00	1.48	0.89
time (sec)	N/A	0.283	0.441	0.831	0.206	0.285	0.000	0.386	14.184

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	129	148	241	180	164	156	0	174	242
N.S.	1	1.15	1.87	1.40	1.27	1.21	0.00	1.35	1.88
time (sec)	N/A	0.632	0.052	1.674	0.297	0.319	0.000	4.832	15.177

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	115	179	143	134	134	0	146	188
N.S.	1	1.13	1.75	1.40	1.31	1.31	0.00	1.43	1.84
time (sec)	N/A	0.480	0.052	1.460	0.291	0.296	0.000	2.217	14.919

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	82	117	104	102	112	0	118	134
N.S.	1	1.12	1.60	1.42	1.40	1.53	0.00	1.62	1.84
time (sec)	N/A	0.348	0.035	1.537	0.311	0.284	0.000	0.940	14.871

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	48	60	67	65	87	0	88	80
N.S.	1	1.07	1.33	1.49	1.44	1.93	0.00	1.96	1.78
time (sec)	N/A	0.231	0.023	0.698	0.290	0.285	0.000	0.504	14.215

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	43	26	31	33	0	26	19
N.S.	1	1.00	1.65	1.00	1.19	1.27	0.00	1.00	0.73
time (sec)	N/A	0.206	0.017	0.443	0.282	0.253	0.000	0.299	14.177

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	58	62	62	59	72	0	56	53
N.S.	1	1.05	1.13	1.13	1.07	1.31	0.00	1.02	0.96
time (sec)	N/A	0.310	0.029	0.912	0.287	0.293	0.000	0.328	14.331

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	92	79	118	79	139	0	83	156
N.S.	1	1.10	0.94	1.40	0.94	1.65	0.00	0.99	1.86
time (sec)	N/A	0.441	0.035	1.069	0.293	0.261	0.000	0.347	14.441

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	111	119	92	150	100	210	0	113	204
N.S.	1	1.07	0.83	1.35	0.90	1.89	0.00	1.02	1.84
time (sec)	N/A	0.583	0.040	1.409	0.293	0.276	0.000	0.393	14.787

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	140	158	111	205	119	279	0	140	252
N.S.	1	1.13	0.79	1.46	0.85	1.99	0.00	1.00	1.80
time (sec)	N/A	0.737	0.049	1.777	0.300	0.293	0.000	0.429	15.269

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	192	132	193	114	149	156	314	342	308
N.S.	1	0.69	1.01	0.59	0.78	0.81	1.64	1.78	1.60
time (sec)	N/A	0.283	1.193	2.273	0.211	0.291	2.473	6.023	18.108

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	132	94	159	84	110	117	252	292	249
N.S.	1	0.71	1.20	0.64	0.83	0.89	1.91	2.21	1.89
time (sec)	N/A	0.275	0.743	1.724	0.209	0.299	1.187	3.875	18.220

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	120	84	125	74	97	104	189	242	192
N.S.	1	0.70	1.04	0.62	0.81	0.87	1.58	2.02	1.60
time (sec)	N/A	0.262	0.326	1.364	0.211	0.277	0.595	1.775	17.792

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	50	83	46	58	65	126	192	133
N.S.	1	0.77	1.28	0.71	0.89	1.00	1.94	2.95	2.05
time (sec)	N/A	0.250	0.137	1.078	0.200	0.281	0.297	0.772	16.683

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	36	51	34	43	52	60	142	76
N.S.	1	0.75	1.06	0.71	0.90	1.08	1.25	2.96	1.58
time (sec)	N/A	0.235	0.082	0.440	0.216	0.279	0.137	0.403	14.193

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	29	29	49	31	35	0	64	36
N.S.	1	0.83	0.83	1.40	0.89	1.00	0.00	1.83	1.03
time (sec)	N/A	0.234	0.032	0.497	0.225	0.295	0.000	0.299	14.059

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	32	56	69	34	48	0	111	50
N.S.	1	0.80	1.40	1.72	0.85	1.20	0.00	2.78	1.25
time (sec)	N/A	0.244	0.060	0.750	0.226	0.271	0.000	0.345	13.861

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	69	86	114	72	122	0	138	62
N.S.	1	0.81	1.01	1.34	0.85	1.44	0.00	1.62	0.73
time (sec)	N/A	0.270	0.216	1.019	0.199	0.282	0.000	0.388	14.630

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	127	99	114	171	109	191	0	186	113
N.S.	1	0.78	0.90	1.35	0.86	1.50	0.00	1.46	0.89
time (sec)	N/A	0.286	0.170	1.595	0.206	0.295	0.000	0.413	13.990

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	169	129	146	213	165	322	0	238	149
N.S.	1	0.76	0.86	1.26	0.98	1.91	0.00	1.41	0.88
time (sec)	N/A	0.318	0.272	2.887	0.227	0.291	0.000	0.442	14.456

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	161	161	218	168	151	165	0	180	234
N.S.	1	1.00	1.35	1.04	0.94	1.02	0.00	1.12	1.45
time (sec)	N/A	0.408	0.702	2.176	0.311	0.296	0.000	2.398	15.388

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	119	150	130	119	139	0	148	174
N.S.	1	1.00	1.26	1.09	1.00	1.17	0.00	1.24	1.46
time (sec)	N/A	0.356	0.341	1.630	0.313	0.300	0.000	1.090	14.769

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	81	92	83	111	0	99	101
N.S.	1	1.00	1.12	1.28	1.15	1.54	0.00	1.38	1.40
time (sec)	N/A	0.309	0.219	0.797	0.295	0.297	0.000	0.623	13.347

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	46	30	48	42	0	31	24
N.S.	1	1.00	1.31	0.86	1.37	1.20	0.00	0.89	0.69
time (sec)	N/A	0.251	0.035	1.106	0.297	0.279	0.000	0.324	13.106

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	87	53	77	81	0	50	39
N.S.	1	1.00	1.26	0.77	1.12	1.17	0.00	0.72	0.57
time (sec)	N/A	0.313	0.351	1.319	0.298	0.280	0.000	0.352	13.936

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	107	105	100	97	118	0	80	78
N.S.	1	1.00	0.98	0.93	0.91	1.10	0.00	0.75	0.73
time (sec)	N/A	0.324	0.336	1.625	0.303	0.288	0.000	0.386	13.984

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	139	139	121	143	117	173	0	112	182
N.S.	1	1.00	0.87	1.03	0.84	1.24	0.00	0.81	1.31
time (sec)	N/A	0.372	0.581	2.565	0.303	0.292	0.000	0.414	14.325

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	179	179	141	188	137	274	0	145	230
N.S.	1	1.00	0.79	1.05	0.77	1.53	0.00	0.81	1.28
time (sec)	N/A	0.408	1.766	4.477	0.295	0.285	0.000	0.484	15.173

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	210	144	214	124	162	169	439	367	337
N.S.	1	0.69	1.02	0.59	0.77	0.80	2.09	1.75	1.60
time (sec)	N/A	0.288	0.783	3.026	0.217	0.313	3.405	5.959	17.795

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	98	167	86	110	117	350	317	278
N.S.	1	0.72	1.22	0.63	0.80	0.85	2.55	2.31	2.03
time (sec)	N/A	0.272	0.464	2.221	0.211	0.304	1.667	3.829	17.913

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	138	96	140	84	110	117	255	267	221
N.S.	1	0.70	1.01	0.61	0.80	0.85	1.85	1.93	1.60
time (sec)	N/A	0.275	0.294	1.659	0.211	0.288	0.871	2.031	17.653

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	99	70	92	66	84	91	165	217	162
N.S.	1	0.71	0.93	0.67	0.85	0.92	1.67	2.19	1.64
time (sec)	N/A	0.265	0.235	1.475	0.211	0.302	0.442	0.929	18.074

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	48	64	44	58	65	76	167	105
N.S.	1	0.73	0.97	0.67	0.88	0.98	1.15	2.53	1.59
time (sec)	N/A	0.235	0.115	0.694	0.215	0.274	0.193	0.434	14.589

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	39	36	79	43	61	0	145	86
N.S.	1	0.81	0.75	1.65	0.90	1.27	0.00	3.02	1.79
time (sec)	N/A	0.246	0.064	0.681	0.202	0.289	0.000	0.353	13.956

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	34	46	69	34	50	0	109	48
N.S.	1	0.85	1.15	1.72	0.85	1.25	0.00	2.72	1.20
time (sec)	N/A	0.252	0.103	1.410	0.210	0.289	0.000	0.350	14.326

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	52	72	94	59	82	0	138	78
N.S.	1	0.85	1.18	1.54	0.97	1.34	0.00	2.26	1.28
time (sec)	N/A	0.257	0.132	1.575	0.212	0.283	0.000	0.406	14.257

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	85	102	136	96	178	0	165	94
N.S.	1	0.79	0.95	1.27	0.90	1.66	0.00	1.54	0.88
time (sec)	N/A	0.273	0.503	2.846	0.219	0.303	0.000	0.427	14.407

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	149	115	130	193	142	272	0	213	130
N.S.	1	0.77	0.87	1.30	0.95	1.83	0.00	1.43	0.87
time (sec)	N/A	0.301	0.271	5.226	0.205	0.289	0.000	0.484	14.025

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	237	237	292	228	262	178	0	196	263
N.S.	1	1.00	1.23	0.96	1.11	0.75	0.00	0.83	1.11
time (sec)	N/A	0.536	1.189	2.927	0.298	0.317	0.000	2.690	15.036

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	169	169	200	194	210	152	0	164	203
N.S.	1	1.00	1.18	1.15	1.24	0.90	0.00	0.97	1.20
time (sec)	N/A	0.445	0.457	2.261	0.306	0.285	0.000	1.278	14.832

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	98	107	139	147	113	0	132	146
N.S.	1	1.00	1.09	1.42	1.50	1.15	0.00	1.35	1.49
time (sec)	N/A	0.353	0.374	1.000	0.291	0.302	0.000	0.738	14.309

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	109	71	85	84	0	66	35
N.S.	1	1.00	2.22	1.45	1.73	1.71	0.00	1.35	0.71
time (sec)	N/A	0.284	0.626	1.302	0.309	0.289	0.000	0.368	13.978

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	85	53	90	82	0	50	39
N.S.	1	1.00	1.23	0.77	1.30	1.19	0.00	0.72	0.57
time (sec)	N/A	0.329	0.260	1.729	0.301	0.279	0.000	0.350	13.568

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	107	105	76	122	118	0	66	62
N.S.	1	1.00	0.98	0.71	1.14	1.10	0.00	0.62	0.58
time (sec)	N/A	0.367	0.473	2.892	0.300	0.271	0.000	0.397	14.239

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	141	141	123	121	152	160	0	96	91
N.S.	1	1.00	0.87	0.86	1.08	1.13	0.00	0.68	0.65
time (sec)	N/A	0.413	0.758	4.797	0.290	0.269	0.000	0.439	14.505

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	179	179	141	166	182	235	0	128	206
N.S.	1	1.00	0.79	0.93	1.02	1.31	0.00	0.72	1.15
time (sec)	N/A	0.445	1.793	7.813	0.294	0.282	0.000	0.500	15.082

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	213	213	159	209	212	314	0	161	254
N.S.	1	1.00	0.75	0.98	1.00	1.47	0.00	0.76	1.19
time (sec)	N/A	0.509	3.675	12.938	0.315	0.308	0.000	0.571	15.849

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	135	92	137	86	90	95	0	245	208
N.S.	1	0.68	1.01	0.64	0.67	0.70	0.00	1.81	1.54
time (sec)	N/A	0.268	0.414	1.239	0.237	0.293	0.000	5.983	18.113

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	72	103	62	70	75	0	201	153
N.S.	1	0.74	1.06	0.64	0.72	0.77	0.00	2.07	1.58
time (sec)	N/A	0.265	0.194	1.113	0.213	0.307	0.000	3.487	18.022

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	46	65	48	50	55	0	157	99
N.S.	1	0.70	0.98	0.73	0.76	0.83	0.00	2.38	1.50
time (sec)	N/A	0.250	0.159	1.018	0.220	0.280	0.000	1.488	14.395

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	26	21	24	28	33	0	111	44
N.S.	1	0.93	0.75	0.86	1.00	1.18	0.00	3.96	1.57
time (sec)	N/A	0.239	0.054	0.617	0.230	0.274	0.000	0.598	13.941

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	19	27	17	19	41	31	21
N.S.	1	1.00	1.12	1.59	1.00	1.12	2.41	1.82	1.24
time (sec)	N/A	0.201	0.015	0.241	0.213	0.289	2.017	0.322	14.494

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	51	67	43	47	60	0	86	49
N.S.	1	0.84	1.10	0.70	0.77	0.98	0.00	1.41	0.80
time (sec)	N/A	0.255	0.088	0.522	0.208	0.279	0.000	0.292	14.418

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	81	97	67	91	139	0	157	76
N.S.	1	0.79	0.94	0.65	0.88	1.35	0.00	1.52	0.74
time (sec)	N/A	0.279	0.421	0.687	0.221	0.282	0.000	0.338	15.834

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	145	111	135	91	130	217	0	211	132
N.S.	1	0.77	0.93	0.63	0.90	1.50	0.00	1.46	0.91
time (sec)	N/A	0.293	0.382	0.649	0.222	0.283	0.000	0.356	14.283

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	B	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	105	120	301	194	329	127	0	149	193
N.S.	1	1.14	2.87	1.85	3.13	1.21	0.00	1.42	1.84
time (sec)	N/A	0.592	2.327	1.193	0.311	0.289	0.000	4.799	15.578

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	B	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	87	893	151	247	107	0	123	139
N.S.	1	1.12	11.45	1.94	3.17	1.37	0.00	1.58	1.78
time (sec)	N/A	0.434	7.103	0.954	0.314	0.277	0.000	2.134	15.212

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	B	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	53	241	104	163	86	0	96	83
N.S.	1	1.08	4.92	2.12	3.33	1.76	0.00	1.96	1.69
time (sec)	N/A	0.328	1.200	0.662	0.304	0.274	0.000	0.886	14.402

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	B	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	60	49	78	35	0	50	25
N.S.	1	1.00	2.86	2.33	3.71	1.67	0.00	2.38	1.19
time (sec)	N/A	0.232	0.372	0.530	0.312	0.271	0.000	0.455	13.829

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	64	100	59	93	64	0	66	65
N.S.	1	1.05	1.64	0.97	1.52	1.05	0.00	1.08	1.07
time (sec)	N/A	0.397	1.021	0.616	0.317	0.286	0.000	0.326	14.217

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	98	254	85	137	134	0	98	158
N.S.	1	1.11	2.89	0.97	1.56	1.52	0.00	1.11	1.80
time (sec)	N/A	0.528	1.074	0.665	0.315	0.282	0.000	0.351	14.415

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	125	359	111	177	198	0	127	206
N.S.	1	1.07	3.07	0.95	1.51	1.69	0.00	1.09	1.76
time (sec)	N/A	0.652	1.632	0.733	0.302	0.279	0.000	0.393	14.903

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	120	84	125	78	80	85	0	223	193
N.S.	1	0.70	1.04	0.65	0.67	0.71	0.00	1.86	1.61
time (sec)	N/A	0.268	0.378	1.679	0.206	0.297	0.000	5.808	17.773

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	50	83	46	50	55	0	180	135
N.S.	1	0.77	1.28	0.71	0.77	0.85	0.00	2.77	2.08
time (sec)	N/A	0.253	0.170	1.334	0.221	0.287	0.000	3.471	15.771

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	36	51	38	40	45	0	136	77
N.S.	1	0.75	1.06	0.79	0.83	0.94	0.00	2.83	1.60
time (sec)	N/A	0.241	0.090	0.847	0.216	0.287	0.000	1.580	14.186

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	27	30	29	31	31	0	33	22
N.S.	1	0.82	0.91	0.88	0.94	0.94	0.00	1.00	0.67
time (sec)	N/A	0.243	0.047	0.692	0.227	0.303	0.000	0.647	14.354

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	28	56	37	35	43	177	57	35
N.S.	1	0.78	1.56	1.03	0.97	1.19	4.92	1.58	0.97
time (sec)	N/A	0.225	0.106	0.244	0.225	0.286	11.780	0.367	14.186

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	65	83	55	74	106	0	117	62
N.S.	1	0.80	1.02	0.68	0.91	1.31	0.00	1.44	0.77
time (sec)	N/A	0.262	0.141	0.590	0.241	0.300	0.000	0.319	14.246

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	123	95	121	79	110	162	0	186	89
N.S.	1	0.77	0.98	0.64	0.89	1.32	0.00	1.51	0.72
time (sec)	N/A	0.279	0.262	0.704	0.220	0.310	0.000	0.369	14.182

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	165	125	154	103	167	283	0	236	151
N.S.	1	0.76	0.93	0.62	1.01	1.72	0.00	1.43	0.92
time (sec)	N/A	0.301	0.610	0.717	0.212	0.297	0.000	0.420	14.320

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	B	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	123	495	161	301	117	0	136	179
N.S.	1	1.03	4.16	1.35	2.53	0.98	0.00	1.14	1.50
time (sec)	N/A	0.453	5.790	1.423	0.301	0.287	0.000	4.858	15.047

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	B	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	76	767	106	196	97	0	99	111
N.S.	1	1.06	10.65	1.47	2.72	1.35	0.00	1.38	1.54
time (sec)	N/A	0.401	7.081	1.057	0.311	0.296	0.000	2.220	14.478

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	B	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	38	177	71	123	66	0	79	61
N.S.	1	1.12	5.21	2.09	3.62	1.94	0.00	2.32	1.79
time (sec)	N/A	0.374	1.050	0.717	0.299	0.272	0.000	0.949	14.212

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	39	42	30	49	42	0	29	22
N.S.	1	1.18	1.27	0.91	1.48	1.27	0.00	0.88	0.67
time (sec)	N/A	0.349	0.020	0.623	0.298	0.283	0.000	0.508	13.988

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	111	149	72	113	106	0	84	78
N.S.	1	1.04	1.39	0.67	1.06	0.99	0.00	0.79	0.73
time (sec)	N/A	0.437	1.483	0.658	0.293	0.274	0.000	0.337	14.336

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	139	143	314	98	157	154	0	114	182
N.S.	1	1.03	2.26	0.71	1.13	1.11	0.00	0.82	1.31
time (sec)	N/A	0.458	1.365	0.619	0.303	0.279	0.000	0.398	14.935

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	179	183	276	124	197	250	0	144	230
N.S.	1	1.02	1.54	0.69	1.10	1.40	0.00	0.80	1.28
time (sec)	N/A	0.528	6.498	0.670	0.295	0.292	0.000	0.417	15.781

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	98	140	86	90	95	0	246	225
N.S.	1	0.72	1.02	0.63	0.66	0.69	0.00	1.80	1.64
time (sec)	N/A	0.274	0.455	4.210	0.203	0.286	0.000	20.535	18.396

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	99	70	93	66	70	75	0	202	167
N.S.	1	0.71	0.94	0.67	0.71	0.76	0.00	2.04	1.69
time (sec)	N/A	0.261	0.293	2.391	0.198	0.287	0.000	6.252	18.699

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	50	64	46	50	55	0	158	109
N.S.	1	0.77	0.98	0.71	0.77	0.85	0.00	2.43	1.68
time (sec)	N/A	0.252	0.140	1.622	0.206	0.275	0.000	3.598	15.504

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	37	36	37	45	53	0	112	72
N.S.	1	0.80	0.78	0.80	0.98	1.15	0.00	2.43	1.57
time (sec)	N/A	0.251	0.140	1.035	0.212	0.292	0.000	1.666	14.651

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	32	33	30	36	42	457	56	36
N.S.	1	0.91	0.94	0.86	1.03	1.20	13.06	1.60	1.03
time (sec)	N/A	0.249	0.051	0.807	0.208	0.293	18.795	0.736	14.508

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	44	79	49	60	76	411	87	48
N.S.	1	0.79	1.41	0.88	1.07	1.36	7.34	1.55	0.86
time (sec)	N/A	0.238	0.113	0.322	0.212	0.296	19.012	0.438	14.629

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	79	97	67	98	151	0	143	75
N.S.	1	0.78	0.96	0.66	0.97	1.50	0.00	1.42	0.74
time (sec)	N/A	0.279	0.230	0.578	0.211	0.308	0.000	0.354	14.260

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	143	109	139	91	146	240	0	212	102
N.S.	1	0.76	0.97	0.64	1.02	1.68	0.00	1.48	0.71
time (sec)	N/A	0.291	0.486	0.749	0.220	0.286	0.000	0.409	14.658

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	185	139	169	115	188	317	0	261	170
N.S.	1	0.75	0.91	0.62	1.02	1.71	0.00	1.41	0.92
time (sec)	N/A	0.317	0.891	0.743	0.229	0.291	0.000	0.490	14.537

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	237	242	362	227	429	147	0	175	265
N.S.	1	1.02	1.53	0.96	1.81	0.62	0.00	0.74	1.12
time (sec)	N/A	0.648	3.588	5.024	0.314	0.306	0.000	25.908	15.698

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	B	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	169	174	303	195	343	127	0	149	208
N.S.	1	1.03	1.79	1.15	2.03	0.75	0.00	0.88	1.23
time (sec)	N/A	0.521	3.629	3.214	0.303	0.276	0.000	12.476	15.036

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	B	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	99	103	230	138	257	97	0	123	148
N.S.	1	1.04	2.32	1.39	2.60	0.98	0.00	1.24	1.49
time (sec)	N/A	0.446	2.075	2.007	0.300	0.304	0.000	4.996	14.976

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	B	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	74	241	105	171	87	0	97	92
N.S.	1	1.12	3.65	1.59	2.59	1.32	0.00	1.47	1.39
time (sec)	N/A	0.505	1.727	1.282	0.315	0.300	0.000	2.249	14.727

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	55	117	61	98	83	0	63	37
N.S.	1	1.20	2.54	1.33	2.13	1.80	0.00	1.37	0.80
time (sec)	N/A	0.377	0.853	0.768	0.304	0.300	0.000	1.073	14.480

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	74	125	45	72	80	0	50	35
N.S.	1	1.23	2.08	0.75	1.20	1.33	0.00	0.83	0.58
time (sec)	N/A	0.416	0.324	0.680	0.305	0.269	0.000	0.608	13.810

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	143	146	252	85	133	142	0	99	91
N.S.	1	1.02	1.76	0.59	0.93	0.99	0.00	0.69	0.64
time (sec)	N/A	0.502	1.638	0.637	0.304	0.275	0.000	0.396	14.088

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	177	184	366	111	177	216	0	131	205
N.S.	1	1.04	2.07	0.63	1.00	1.22	0.00	0.74	1.16
time (sec)	N/A	0.532	1.885	0.640	0.293	0.283	0.000	0.475	14.279

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	215	218	394	137	218	282	0	160	254
N.S.	1	1.01	1.83	0.64	1.01	1.31	0.00	0.74	1.18
time (sec)	N/A	0.609	4.220	0.901	0.293	0.291	0.000	0.533	14.919

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F(-2)	F(-1)	F	F	F(-1)
verified	N/A	No	Yes	No	TBD	TBD	TBD	TBD	TBD
size	310	295	186	580	0	0	0	0	0
N.S.	1	0.95	0.60	1.87	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.092	2.575	6.000	0.000	0.000	0.000	0.000	0.000

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-2)	F(-1)	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	282	270	214	372	0	0	0	0	0
N.S.	1	0.96	0.76	1.32	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.939	2.153	5.588	0.000	0.000	0.000	0.000	0.000

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F(-2)	F(-1)	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	272	253	182	505	0	0	0	0	0
N.S.	1	0.93	0.67	1.86	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.881	1.381	5.040	0.000	0.000	0.000	0.000	0.000

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F(-2)	F(-1)	F	F	F(-1)
verified	N/A	No	No	Yes	TBD	TBD	TBD	TBD	TBD
size	244	230	220	141	0	0	0	0	0
N.S.	1	0.94	0.90	0.58	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.765	0.870	4.422	0.000	0.000	0.000	0.000	0.000

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-2)	F(-1)	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	305	290	196	511	0	0	0	0	0
N.S.	1	0.95	0.64	1.68	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.124	2.632	4.696	0.000	0.000	0.000	0.000	0.000

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-2)	F(-1)	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	282	270	200	458	0	0	0	0	0
N.S.	1	0.96	0.71	1.62	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.932	1.638	4.784	0.000	0.000	0.000	0.000	0.000

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-2)	F(-1)	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	346	333	254	546	0	0	0	0	0
N.S.	1	0.96	0.73	1.58	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.296	2.748	4.590	0.000	0.000	0.000	0.000	0.000

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-2)	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	366	366	117	605	0	0	0	0	0
N.S.	1	1.00	0.32	1.65	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.807	12.893	6.596	0.000	0.000	0.000	0.000	0.000

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-2)	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	335	335	257	397	0	0	0	0	0
N.S.	1	1.00	0.77	1.19	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.684	26.787	6.591	0.000	0.000	0.000	0.000	0.000

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-2)	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	309	309	106	530	0	0	0	0	0
N.S.	1	1.00	0.34	1.72	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.625	11.676	5.431	0.000	0.000	0.000	0.000	0.000

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-2)	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	278	278	220	269	0	0	0	0	0
N.S.	1	1.00	0.79	0.97	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.585	13.012	5.459	0.000	0.000	0.000	0.000	0.000

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-2)	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	310	310	238	536	0	0	0	0	0
N.S.	1	1.00	0.77	1.73	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.672	7.763	4.951	0.000	0.000	0.000	0.000	0.000

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-2)	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	316	316	224	483	0	0	0	0	0
N.S.	1	1.00	0.71	1.53	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.685	15.823	5.554	0.000	0.000	0.000	0.000	0.000

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-2)	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	370	370	2820	571	0	0	0	0	0
N.S.	1	1.00	7.62	1.54	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.774	14.671	4.913	0.000	0.000	0.000	0.000	0.000

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	No	No	No	TBD	TBD	TBD	TBD	TBD
size	330	321	332	1026	0	0	0	0	0
N.S.	1	0.97	1.01	3.11	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.297	16.028	4.637	0.000	0.000	0.000	0.000	0.000

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	No	No	No	TBD	TBD	TBD	TBD	TBD
size	326	303	129	1166	0	0	0	0	0
N.S.	1	0.93	0.40	3.58	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.255	17.800	3.858	0.000	0.000	0.000	0.000	0.000

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	No	No	No	TBD	TBD	TBD	TBD	TBD
size	295	280	271	976	0	0	0	0	0
N.S.	1	0.95	0.92	3.31	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.086	22.333	4.027	0.000	0.000	0.000	0.000	0.000

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F(-1)	F	F	F(-1)
verified	N/A	No	No	No	TBD	TBD	TBD	TBD	TBD
size	285	262	105	1100	0	0	0	0	0
N.S.	1	0.92	0.37	3.86	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.032	16.519	3.637	0.000	0.000	0.000	0.000	0.000

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F(-1)	F	F	F(-1)
verified	N/A	No	No	No	TBD	TBD	TBD	TBD	TBD
size	257	239	1211	205	0	0	0	0	0
N.S.	1	0.93	4.71	0.80	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.872	15.615	3.749	0.000	0.000	0.000	0.000	0.000

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F(-1)	F	F	F(-1)
verified	N/A	No	No	Yes	TBD	TBD	TBD	TBD	TBD
size	315	298	2715	256	0	0	0	0	0
N.S.	1	0.95	8.62	0.81	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.287	10.581	2.362	0.000	0.000	0.000	0.000	0.000

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F(-1)	F	F	F(-1)
verified	N/A	No	No	Yes	TBD	TBD	TBD	TBD	TBD
size	290	280	1253	611	0	0	0	0	0
N.S.	1	0.97	4.32	2.11	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.100	10.239	3.750	0.000	0.000	0.000	0.000	0.000

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F(-1)	F	F	F(-1)
verified	N/A	No	No	No	TBD	TBD	TBD	TBD	TBD
size	359	342	180	1116	0	0	0	0	0
N.S.	1	0.95	0.50	3.11	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.593	13.994	2.988	0.000	0.000	0.000	0.000	0.000

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F(-1)	F	F	F(-1)
verified	N/A	No	No	No	TBD	TBD	TBD	TBD	TBD
size	328	323	1299	744	0	0	0	0	0
N.S.	1	0.98	3.96	2.27	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.326	10.617	3.870	0.000	0.000	0.000	0.000	0.000

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	C	F(-1)	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	N/A	No	TBD	TBD	TBD	TBD	TBD
size	372	373	0	1181	0	0	0	0	0
N.S.	1	1.00	0.00	3.17	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.842	0.000	5.192	0.000	0.000	0.000	0.000	0.000

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	C	F(-1)	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	N/A	No	TBD	TBD	TBD	TBD	TBD
size	339	342	0	1007	0	0	0	0	0
N.S.	1	1.01	0.00	2.97	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.774	0.000	4.508	0.000	0.000	0.000	0.000	0.000

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	C	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	N/A	No	TBD	TBD	TBD	TBD	TBD
size	312	316	0	1141	0	0	0	0	0
N.S.	1	1.01	0.00	3.66	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.735	0.000	4.916	0.000	0.000	0.000	0.000	0.000

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	C	F(-1)	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	N/A	No	TBD	TBD	TBD	TBD	TBD
size	281	285	0	961	0	0	0	0	0
N.S.	1	1.01	0.00	3.42	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.676	0.000	6.340	0.000	0.000	0.000	0.000	0.000

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	310	317	812	261	0	0	0	0	0
N.S.	1	1.02	2.62	0.84	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.757	8.364	4.490	0.000	0.000	0.000	0.000	0.000

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	C	F(-1)	F(-1)	F	F	F(-1)
verified	N/A	Yes	N/A	No	TBD	TBD	TBD	TBD	TBD
size	316	323	0	638	0	0	0	0	0
N.S.	1	1.02	0.00	2.02	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.783	0.000	5.271	0.000	0.000	0.000	0.000	0.000

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	363	377	2792	705	0	0	0	0	0
N.S.	1	1.04	7.69	1.94	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.843	10.039	4.229	0.000	0.000	0.000	0.000	0.000

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	365	379	1281	633	0	0	0	0	0
N.S.	1	1.04	3.51	1.73	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.852	12.508	5.598	0.000	0.000	0.000	0.000	0.000

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	147	134	102	101	145	299	0	0	0
N.S.	1	0.91	0.69	0.69	0.99	2.03	0.00	0.00	0.00
time (sec)	N/A	0.325	0.625	5.028	0.283	0.402	0.000	0.000	0.000

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	99	95	80	81	107	259	0	0	0
N.S.	1	0.96	0.81	0.82	1.08	2.62	0.00	0.00	0.00
time (sec)	N/A	0.280	0.275	4.745	0.273	0.332	0.000	0.000	0.000

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	49	60	42	67	184	0	0	47
N.S.	1	0.96	1.18	0.82	1.31	3.61	0.00	0.00	0.92
time (sec)	N/A	0.234	0.101	0.934	0.288	0.312	0.000	0.000	14.576

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	72	72	92	0	242	0	0	0
N.S.	1	0.99	0.99	1.26	0.00	3.32	0.00	0.00	0.00
time (sec)	N/A	0.281	0.078	2.374	0.000	0.332	0.000	0.000	0.000

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	131	138	87	137	0	426	0	0	0
N.S.	1	1.05	0.66	1.05	0.00	3.25	0.00	0.00	0.00
time (sec)	N/A	0.334	0.551	2.620	0.000	0.378	0.000	0.000	0.000

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-1)	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	193	207	102	173	0	529	0	0	0
N.S.	1	1.07	0.53	0.90	0.00	2.74	0.00	0.00	0.00
time (sec)	N/A	0.386	0.585	3.038	0.000	0.373	0.000	0.000	0.000

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	222	205	134	233	0	371	0	0	0
N.S.	1	0.92	0.60	1.05	0.00	1.67	0.00	0.00	0.00
time (sec)	N/A	0.305	7.811	5.352	0.000	0.342	0.000	0.000	0.000

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	160	149	110	201	0	331	0	0	0
N.S.	1	0.93	0.69	1.26	0.00	2.07	0.00	0.00	0.00
time (sec)	N/A	0.284	6.658	5.176	0.000	0.342	0.000	0.000	0.000

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	A	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	96	93	226	186	0	283	0	0	0
N.S.	1	0.97	2.35	1.94	0.00	2.95	0.00	0.00	0.00
time (sec)	N/A	0.254	3.484	4.731	0.000	0.297	0.000	0.000	0.000

Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	113	124	161	0	422	0	0	0
N.S.	1	1.04	1.14	1.48	0.00	3.87	0.00	0.00	0.00
time (sec)	N/A	0.273	3.344	2.809	0.000	0.390	0.000	0.000	0.000

Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	196	206	232	196	0	547	0	0	0
N.S.	1	1.05	1.18	1.00	0.00	2.79	0.00	0.00	0.00
time (sec)	N/A	0.346	7.207	2.781	0.000	0.408	0.000	0.000	0.000

Problem 146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-1)	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	280	295	210	226	0	646	0	0	0
N.S.	1	1.05	0.75	0.81	0.00	2.31	0.00	0.00	0.00
time (sec)	N/A	0.429	4.657	2.653	0.000	0.418	0.000	0.000	0.000

Problem 147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	169	155	112	112	162	334	0	0	0
N.S.	1	0.92	0.66	0.66	0.96	1.98	0.00	0.00	0.00
time (sec)	N/A	0.350	0.430	4.931	0.268	0.362	0.000	0.000	0.000

Problem 148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	116	92	92	124	290	0	0	0
N.S.	1	0.96	0.76	0.76	1.02	2.40	0.00	0.00	0.00
time (sec)	N/A	0.292	0.217	4.260	0.269	0.346	0.000	0.000	0.000

Problem 149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	70	70	57	86	238	0	0	67
N.S.	1	0.96	0.96	0.78	1.18	3.26	0.00	0.00	0.92
time (sec)	N/A	0.250	0.122	0.749	0.268	0.355	0.000	0.000	14.334

Problem 150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	72	91	0	243	0	0	0
N.S.	1	1.00	0.99	1.25	0.00	3.33	0.00	0.00	0.00
time (sec)	N/A	0.279	0.082	1.528	0.000	0.332	0.000	0.000	0.000

Problem 151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	112	100	138	0	378	0	0	0
N.S.	1	1.03	0.92	1.27	0.00	3.47	0.00	0.00	0.00
time (sec)	N/A	0.314	0.313	1.939	0.000	0.329	0.000	0.000	0.000

Problem 152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-1)	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	171	178	104	174	0	589	0	0	0
N.S.	1	1.04	0.61	1.02	0.00	3.44	0.00	0.00	0.00
time (sec)	N/A	0.369	0.721	1.963	0.000	0.422	0.000	0.000	0.000

Problem 153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-1)	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	258	236	147	250	0	415	0	0	0
N.S.	1	0.91	0.57	0.97	0.00	1.61	0.00	0.00	0.00
time (sec)	N/A	0.304	9.384	4.029	0.000	0.358	0.000	0.000	0.000

Problem 154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-1)	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	194	178	123	218	0	371	0	0	0
N.S.	1	0.92	0.63	1.12	0.00	1.91	0.00	0.00	0.00
time (sec)	N/A	0.300	7.148	3.724	0.000	0.310	0.000	0.000	0.000

Problem 155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	128	118	97	187	0	321	0	0	0
N.S.	1	0.92	0.76	1.46	0.00	2.51	0.00	0.00	0.00
time (sec)	N/A	0.290	6.787	3.482	0.000	0.326	0.000	0.000	0.000

Problem 156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	64	61	102	85	0	264	0	0	0
N.S.	1	0.95	1.59	1.33	0.00	4.12	0.00	0.00	0.00
time (sec)	N/A	0.251	0.381	1.939	0.000	0.344	0.000	0.000	0.000

Problem 157	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-1)	A	F(-1)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	144	145	226	197	0	531	0	0	0
N.S.	1	1.01	1.57	1.37	0.00	3.69	0.00	0.00	0.00
time (sec)	N/A	0.314	6.687	1.739	0.000	0.404	0.000	0.000	0.000

Problem 158	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-1)	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	226	238	235	227	0	708	0	0	0
N.S.	1	1.05	1.04	1.00	0.00	3.13	0.00	0.00	0.00
time (sec)	N/A	0.400	7.457	1.846	0.000	0.405	0.000	0.000	0.000

Problem 159	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	193	176	156	124	181	386	0	0	0
N.S.	1	0.91	0.81	0.64	0.94	2.00	0.00	0.00	0.00
time (sec)	N/A	0.342	0.547	262.509	0.282	0.375	0.000	0.000	0.000

Problem 160	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	145	137	102	104	143	334	0	0	0
N.S.	1	0.94	0.70	0.72	0.99	2.30	0.00	0.00	0.00
time (sec)	N/A	0.337	0.435	55.967	0.280	0.368	0.000	0.000	0.000

Problem 161	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	91	82	74	105	282	0	0	92
N.S.	1	0.94	0.85	0.76	1.08	2.91	0.00	0.00	0.95
time (sec)	N/A	0.253	0.160	7.177	0.277	0.336	0.000	0.000	13.961

Problem 162	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	83	116	0	300	0	0	0
N.S.	1	1.00	0.87	1.22	0.00	3.16	0.00	0.00	0.00
time (sec)	N/A	0.299	0.077	5.423	0.000	0.349	0.000	0.000	0.000

Problem 163	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	106	107	96	240	0	398	0	0	0
N.S.	1	1.01	0.91	2.26	0.00	3.75	0.00	0.00	0.00
time (sec)	N/A	0.314	0.334	15.210	0.000	0.339	0.000	0.000	0.000

Problem 164	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-1)	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	147	151	109	272	0	503	0	0	0
N.S.	1	1.03	0.74	1.85	0.00	3.42	0.00	0.00	0.00
time (sec)	N/A	0.343	0.693	88.517	0.000	0.331	0.000	0.000	0.000

Problem 165	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-1)	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	290	265	173	268	0	477	0	0	0
N.S.	1	0.91	0.60	0.92	0.00	1.64	0.00	0.00	0.00
time (sec)	N/A	0.322	8.758	1.253	0.000	0.359	0.000	0.000	0.000

Problem 166	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-1)	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	224	205	149	236	0	425	0	0	0
N.S.	1	0.92	0.67	1.05	0.00	1.90	0.00	0.00	0.00
time (sec)	N/A	0.293	7.813	129.408	0.000	0.366	0.000	0.000	0.000

Problem 167	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	160	147	125	233	0	374	0	0	0
N.S.	1	0.92	0.78	1.46	0.00	2.34	0.00	0.00	0.00
time (sec)	N/A	0.284	6.417	13.545	0.000	0.344	0.000	0.000	0.000

Problem 168	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	64	124	89	0	270	0	0	0
N.S.	1	0.97	1.88	1.35	0.00	4.09	0.00	0.00	0.00
time (sec)	N/A	0.247	0.584	6.621	0.000	0.352	0.000	0.000	0.000

Problem 169	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	96	91	81	180	0	355	0	0	0
N.S.	1	0.95	0.84	1.88	0.00	3.70	0.00	0.00	0.00
time (sec)	N/A	0.252	0.196	40.391	0.000	0.343	0.000	0.000	0.000

Problem 170	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-1)	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	176	177	203	372	0	650	0	0	0
N.S.	1	1.01	1.15	2.11	0.00	3.69	0.00	0.00	0.00
time (sec)	N/A	0.346	6.396	194.437	0.000	0.400	0.000	0.000	0.000

Problem 171	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	126	113	88	94	129	285	0	167	0
N.S.	1	0.90	0.70	0.75	1.02	2.26	0.00	1.33	0.00
time (sec)	N/A	0.305	0.155	3.783	0.275	0.359	0.000	2.037	0.000

Problem 172	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	74	66	74	91	241	0	126	0
N.S.	1	0.95	0.85	0.95	1.17	3.09	0.00	1.62	0.00
time (sec)	N/A	0.277	0.075	3.142	0.280	0.374	0.000	1.087	0.000

Problem 173	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	44	26	49	137	0	48	27
N.S.	1	1.00	1.42	0.84	1.58	4.42	0.00	1.55	0.87
time (sec)	N/A	0.228	0.033	0.693	0.274	0.318	0.000	0.784	15.183

Problem 174	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	98	57	229	0	384	0	111	0
N.S.	1	1.07	0.62	2.49	0.00	4.17	0.00	1.21	0.00
time (sec)	N/A	0.298	0.062	1.802	0.000	0.347	0.000	0.752	0.000

Problem 175	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	152	168	90	272	0	546	0	174	0
N.S.	1	1.11	0.59	1.79	0.00	3.59	0.00	1.14	0.00
time (sec)	N/A	0.357	0.160	1.752	0.000	0.353	0.000	0.843	0.000

Problem 176	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	214	236	102	308	0	705	0	241	0
N.S.	1	1.10	0.48	1.44	0.00	3.29	0.00	1.13	0.00
time (sec)	N/A	0.404	0.257	1.577	0.000	0.394	0.000	0.890	0.000

Problem 177	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	189	178	167	220	0	355	0	266	0
N.S.	1	0.94	0.88	1.16	0.00	1.88	0.00	1.41	0.00
time (sec)	N/A	0.296	14.085	3.719	0.000	0.304	0.000	2.839	0.000

Problem 178	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	B	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	125	122	238	188	0	311	0	228	0
N.S.	1	0.98	1.90	1.50	0.00	2.49	0.00	1.82	0.00
time (sec)	N/A	0.270	2.465	3.689	0.000	0.292	0.000	1.601	0.000

Problem 179	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	B	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	63	65	119	95	0	235	0	187	0
N.S.	1	1.03	1.89	1.51	0.00	3.73	0.00	2.97	0.00
time (sec)	N/A	0.248	0.586	3.159	0.000	0.272	0.000	1.142	0.000

Problem 180	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	165	172	187	336	0	503	0	269	0
N.S.	1	1.04	1.13	2.04	0.00	3.05	0.00	1.63	0.00
time (sec)	N/A	0.322	5.046	2.013	0.000	0.343	0.000	1.095	0.000

Problem 181	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	251	263	229	372	0	666	0	385	0
N.S.	1	1.05	0.91	1.48	0.00	2.65	0.00	1.53	0.00
time (sec)	N/A	0.399	1.656	2.104	0.000	0.360	0.000	1.206	0.000

Problem 182	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	335	352	263	408	0	823	0	500	0
N.S.	1	1.05	0.79	1.22	0.00	2.46	0.00	1.49	0.00
time (sec)	N/A	0.482	3.251	1.949	0.000	0.391	0.000	1.322	0.000

Problem 183	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	92	79	82	110	261	0	154	0
N.S.	1	0.92	0.79	0.82	1.10	2.61	0.00	1.54	0.00
time (sec)	N/A	0.302	0.108	3.593	0.290	0.312	0.000	2.290	0.000

Problem 184	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	56	56	65	71	191	0	85	0
N.S.	1	1.04	1.04	1.20	1.31	3.54	0.00	1.57	0.00
time (sec)	N/A	0.275	0.052	3.292	0.277	0.305	0.000	1.252	0.000

Problem 185	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	52	38	45	70	244	0	87	50
N.S.	1	0.96	0.70	0.83	1.30	4.52	0.00	1.61	0.93
time (sec)	N/A	0.241	0.029	0.872	0.276	0.312	0.000	0.927	14.466

Problem 186	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	120	128	60	304	0	485	0	164	0
N.S.	1	1.07	0.50	2.53	0.00	4.04	0.00	1.37	0.00
time (sec)	N/A	0.323	0.048	1.579	0.000	0.360	0.000	0.922	0.000

Problem 187	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	176	197	90	400	0	592	0	253	0
N.S.	1	1.12	0.51	2.27	0.00	3.36	0.00	1.44	0.00
time (sec)	N/A	0.381	0.128	1.915	0.000	0.376	0.000	1.092	0.000

Problem 188	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	238	263	99	436	0	837	0	322	0
N.S.	1	1.11	0.42	1.83	0.00	3.52	0.00	1.35	0.00
time (sec)	N/A	0.434	0.207	1.817	0.000	0.396	0.000	1.010	0.000

Problem 189	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	B	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	157	151	248	233	0	343	0	296	0
N.S.	1	0.96	1.58	1.48	0.00	2.18	0.00	1.89	0.00
time (sec)	N/A	0.293	1.928	3.740	0.000	0.299	0.000	3.748	0.000

Problem 190	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F	B	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	95	94	162	174	0	295	0	230	0
N.S.	1	0.99	1.71	1.83	0.00	3.11	0.00	2.42	0.00
time (sec)	N/A	0.272	3.316	3.756	0.000	0.291	0.000	2.444	0.000

Problem 191	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	83	132	136	0	295	0	72	0
N.S.	1	0.98	1.55	1.60	0.00	3.47	0.00	0.85	0.00
time (sec)	N/A	0.260	0.780	1.821	0.000	0.294	0.000	1.141	0.000

Problem 192	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F	A	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	215	227	210	497	0	603	0	143	0
N.S.	1	1.06	0.98	2.31	0.00	2.80	0.00	0.67	0.00
time (sec)	N/A	0.362	6.510	2.004	0.000	0.344	0.000	1.091	0.000

Problem 193	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F	A	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	303	320	240	541	0	712	0	261	0
N.S.	1	1.06	0.79	1.79	0.00	2.35	0.00	0.86	0.00
time (sec)	N/A	0.455	2.524	1.870	0.000	0.367	0.000	1.089	0.000

Problem 194	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-1)	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	387	409	264	577	0	955	0	379	0
N.S.	1	1.06	0.68	1.49	0.00	2.47	0.00	0.98	0.00
time (sec)	N/A	0.534	5.129	1.587	0.000	0.439	0.000	1.333	0.000

Problem 195	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	76	69	72	163	241	0	126	0
N.S.	1	0.97	0.88	0.92	2.09	3.09	0.00	1.62	0.00
time (sec)	N/A	0.294	0.232	3.747	0.285	0.310	0.000	2.788	0.000

Problem 196	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	56	50	87	125	245	0	90	0
N.S.	1	1.04	0.93	1.61	2.31	4.54	0.00	1.67	0.00
time (sec)	N/A	0.272	0.056	1.784	0.276	0.308	0.000	1.706	0.000

Problem 197	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	40	62	87	321	0	116	69
N.S.	1	1.00	0.51	0.79	1.12	4.12	0.00	1.49	0.88
time (sec)	N/A	0.254	0.084	0.876	0.274	0.304	0.000	1.227	14.759

Problem 198	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	144	154	60	378	0	573	0	206	0
N.S.	1	1.07	0.42	2.62	0.00	3.98	0.00	1.43	0.00
time (sec)	N/A	0.342	0.143	1.732	0.000	0.356	0.000	0.928	0.000

Problem 199	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	200	226	90	528	0	748	0	295	0
N.S.	1	1.13	0.45	2.64	0.00	3.74	0.00	1.48	0.00
time (sec)	N/A	0.400	0.286	1.758	0.000	0.376	0.000	1.081	0.000

Problem 200	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F(-1)	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	262	292	99	564	0	905	0	364	0
N.S.	1	1.11	0.38	2.15	0.00	3.45	0.00	1.39	0.00
time (sec)	N/A	0.461	0.347	1.675	0.000	0.395	0.000	1.273	0.000

Problem 201	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	F	A	F	B	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	127	123	447	211	0	323	0	257	0
N.S.	1	0.97	3.52	1.66	0.00	2.54	0.00	2.02	0.00
time (sec)	N/A	0.281	6.090	3.700	0.000	0.293	0.000	4.295	0.000

Problem 202	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	114	188	318	0	414	0	66	0
N.S.	1	1.01	1.66	2.81	0.00	3.66	0.00	0.58	0.00
time (sec)	N/A	0.278	0.982	3.727	0.000	0.331	0.000	2.084	0.000

Problem 203	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	127	148	193	199	0	492	0	47	0
N.S.	1	1.17	1.52	1.57	0.00	3.87	0.00	0.37	0.00
time (sec)	N/A	0.289	2.348	2.095	0.000	0.325	0.000	1.503	0.000

Problem 204	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F	A	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	265	282	232	589	0	691	0	177	0
N.S.	1	1.06	0.88	2.22	0.00	2.61	0.00	0.67	0.00
time (sec)	N/A	0.426	5.565	1.883	0.000	0.379	0.000	1.085	0.000

Problem 205	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F	A	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	355	377	287	710	0	868	0	296	0
N.S.	1	1.06	0.81	2.00	0.00	2.45	0.00	0.83	0.00
time (sec)	N/A	0.493	3.438	1.929	0.000	0.392	0.000	1.391	0.000

Problem 206	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-1)	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	439	466	292	746	0	1023	0	412	0
N.S.	1	1.06	0.67	1.70	0.00	2.33	0.00	0.94	0.00
time (sec)	N/A	0.575	5.529	1.744	0.000	0.441	0.000	1.620	0.000

Problem 207	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F(-1)	A	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	177	258	241	331	0	674	0	109	0
N.S.	1	1.46	1.36	1.87	0.00	3.81	0.00	0.62	0.00
time (sec)	N/A	0.345	6.358	3.069	0.000	0.425	0.000	1.336	0.000

Problem 208	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	125	125	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.306	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 209	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	243	243	226	0	0	0	0	0	0
N.S.	1	1.00	0.93	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.509	2.103	0.000	0.000	0.000	0.000	0.000	0.000

Problem 210	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	161	161	171	0	0	0	0	0	0
N.S.	1	1.00	1.06	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.406	1.227	0.000	0.000	0.000	0.000	0.000	0.000

Problem 211	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	129	129	105	0	0	0	0	0	0
N.S.	1	1.00	0.81	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.383	0.432	0.000	0.000	0.000	0.000	0.000	0.000

Problem 212	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	130	138	0	0	0	0	0	0	0
N.S.	1	1.06	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.511	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 213	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	169	176	329	0	0	0	0	0	0
N.S.	1	1.04	1.95	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.545	10.842	0.000	0.000	0.000	0.000	0.000	0.000

Problem 214	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	252	260	0	0	0	0	0	0	0
N.S.	1	1.03	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.678	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 215	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	131	131	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.318	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 216	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	131	131	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.298	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 217	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	131	131	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.299	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 218	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	131	131	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.315	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 219	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	123	121	87	0	0	0	0	0	0
N.S.	1	0.98	0.71	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.317	0.315	0.000	0.000	0.000	0.000	0.000	0.000

Problem 220	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	97	95	72	0	0	0	0	0	0
N.S.	1	0.98	0.74	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.296	0.115	0.000	0.000	0.000	0.000	0.000	0.000

Problem 221	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	69	73	49	0	0	0	0	0	0
N.S.	1	1.06	0.71	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.261	0.039	0.000	0.000	0.000	0.000	0.000	0.000

Problem 222	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	43	43	43	0	0	0	0	0	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.221	0.033	0.000	0.000	0.000	0.000	0.000	0.000

Problem 223	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	74	81	57	0	0	0	0	0	0
N.S.	1	1.09	0.77	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.264	0.029	0.000	0.000	0.000	0.000	0.000	0.000

Problem 224	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	127	137	96	0	0	0	0	0	0
N.S.	1	1.08	0.76	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.313	0.198	0.000	0.000	0.000	0.000	0.000	0.000

Problem 225	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	106	106	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.267	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 226	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	106	106	910	0	0	0	0	0	0
N.S.	1	1.00	8.58	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.264	6.330	0.000	0.000	0.000	0.000	0.000	0.000

Problem 227	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	102	102	893	0	0	0	0	0	0
N.S.	1	1.00	8.75	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.261	3.283	0.000	0.000	0.000	0.000	0.000	0.000

Problem 228	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	106	106	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.272	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 229	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	114	114	2072	0	0	0	0	0	0
N.S.	1	1.00	18.18	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.276	27.339	0.000	0.000	0.000	0.000	0.000	0.000

Problem 230	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	114	114	238	0	0	0	0	0	0
N.S.	1	1.00	2.09	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.261	4.876	0.000	0.000	0.000	0.000	0.000	0.000

Problem 231	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	111	111	229	0	0	0	0	0	0
N.S.	1	1.00	2.06	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.261	9.022	0.000	0.000	0.000	0.000	0.000	0.000

Problem 232	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	112	112	2164	0	0	0	0	0	0
N.S.	1	1.00	19.32	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.267	17.523	0.000	0.000	0.000	0.000	0.000	0.000

Problem 233	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F(-2)	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	320	241	185	494	0	0	0	0	0
N.S.	1	0.75	0.58	1.54	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.013	14.429	10.215	0.000	0.000	0.000	0.000	0.000

Problem 234	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-2)	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	346	257	191	501	0	0	0	0	0
N.S.	1	0.74	0.55	1.45	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.117	1.244	10.760	0.000	0.000	0.000	0.000	0.000

Problem 235	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F(-2)	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	274	208	169	141	0	0	0	0	0
N.S.	1	0.76	0.62	0.51	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.773	1.832	9.347	0.000	0.000	0.000	0.000	0.000

Problem 236	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-2)	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	299	232	189	507	0	0	0	0	0
N.S.	1	0.78	0.63	1.70	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.928	1.423	10.382	0.000	0.000	0.000	0.000	0.000

Problem 237	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-2)	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	320	243	224	382	0	0	0	0	0
N.S.	1	0.76	0.70	1.19	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.986	2.130	10.122	0.000	0.000	0.000	0.000	0.000

Problem 238	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F(-2)	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	357	266	93	496	0	0	0	0	0
N.S.	1	0.75	0.26	1.39	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.650	12.830	17.606	0.000	0.000	0.000	0.000	0.000

Problem 239	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-2)	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	343	260	220	524	0	0	0	0	0
N.S.	1	0.76	0.64	1.53	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.618	14.443	10.332	0.000	0.000	0.000	0.000	0.000

Problem 240	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-2)	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	311	236	118	265	0	0	0	0	0
N.S.	1	0.76	0.38	0.85	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.585	12.012	11.140	0.000	0.000	0.000	0.000	0.000

Problem 241	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-2)	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	339	262	221	531	0	0	0	0	0
N.S.	1	0.77	0.65	1.57	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.622	25.080	11.511	0.000	0.000	0.000	0.000	0.000

Problem 242	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-2)	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	375	284	127	405	0	0	0	0	0
N.S.	1	0.76	0.34	1.08	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.647	13.324	11.109	0.000	0.000	0.000	0.000	0.000

Problem 243	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	F	C	C	F(-1)	F(-1)	F	F	F(-1)
verified	N/A	N/A	No	No	TBD	TBD	TBD	TBD	TBD
size	405	0	316	1116	0	0	0	0	0
N.S.	1	0.00	0.78	2.76	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.000	14.104	8.227	0.000	0.000	0.000	0.000	0.000

Problem 244	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	325	245	135	611	0	0	0	0	0
N.S.	1	0.75	0.42	1.88	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.082	12.308	8.163	0.000	0.000	0.000	0.000	0.000

Problem 245	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	347	263	249	252	0	0	0	0	0
N.S.	1	0.76	0.72	0.73	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.199	13.937	8.317	0.000	0.000	0.000	0.000	0.000

Problem 246	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	290	213	112	207	0	0	0	0	0
N.S.	1	0.73	0.39	0.71	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.934	13.295	7.973	0.000	0.000	0.000	0.000	0.000

Problem 247	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	325	237	194	1100	0	0	0	0	0
N.S.	1	0.73	0.60	3.38	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.085	23.234	9.772	0.000	0.000	0.000	0.000	0.000

Problem 248	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	335	247	130	976	0	0	0	0	0
N.S.	1	0.74	0.39	2.91	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.107	12.942	9.795	0.000	0.000	0.000	0.000	0.000

Problem 249	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	371	272	261	1166	0	0	0	0	0
N.S.	1	0.73	0.70	3.14	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.266	14.815	9.232	0.000	0.000	0.000	0.000	0.000

Problem 250	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	C	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	N/A	No	TBD	TBD	TBD	TBD	TBD
size	413	313	0	705	0	0	0	0	0
N.S.	1	0.76	0.00	1.71	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.811	0.000	8.546	0.000	0.000	0.000	0.000	0.000

Problem 251	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	C	F(-1)	F(-1)	F	F	F(-1)
verified	N/A	Yes	N/A	No	TBD	TBD	TBD	TBD	TBD
size	359	269	0	638	0	0	0	0	0
N.S.	1	0.75	0.00	1.78	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.744	0.000	9.009	0.000	0.000	0.000	0.000	0.000

Problem 252	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	C	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	N/A	No	TBD	TBD	TBD	TBD	TBD
size	355	263	0	262	0	0	0	0	0
N.S.	1	0.74	0.00	0.74	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.742	0.000	8.095	0.000	0.000	0.000	0.000	0.000

Problem 253	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	C	F(-1)	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	N/A	No	TBD	TBD	TBD	TBD	TBD
size	321	239	0	961	0	0	0	0	0
N.S.	1	0.74	0.00	2.99	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.686	0.000	10.284	0.000	0.000	0.000	0.000	0.000

Problem 254	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	C	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	N/A	No	TBD	TBD	TBD	TBD	TBD
size	357	265	0	1141	0	0	0	0	0
N.S.	1	0.74	0.00	3.20	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.730	0.000	8.912	0.000	0.000	0.000	0.000	0.000

Problem 255	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	C	F(-1)	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	N/A	No	TBD	TBD	TBD	TBD	TBD
size	389	287	0	1007	0	0	0	0	0
N.S.	1	0.74	0.00	2.59	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.755	0.000	9.694	0.000	0.000	0.000	0.000	0.000

Problem 256	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	111	120	106	90	94	101	148	317	221
N.S.	1	1.08	0.95	0.81	0.85	0.91	1.33	2.86	1.99
time (sec)	N/A	0.768	0.448	1.407	0.210	0.265	0.812	3.379	18.314

Problem 257	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	92	82	67	72	79	112	248	162
N.S.	1	1.10	0.98	0.80	0.86	0.94	1.33	2.95	1.93
time (sec)	N/A	0.572	0.203	1.097	0.193	0.285	0.404	1.531	19.026

Problem 258	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	59	55	47	50	57	76	179	102
N.S.	1	1.07	1.00	0.85	0.91	1.04	1.38	3.25	1.85
time (sec)	N/A	0.410	0.143	0.845	0.217	0.289	0.191	0.617	15.538

Problem 259	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	25	23	26	34	37	107	40
N.S.	1	1.00	1.00	0.92	1.04	1.36	1.48	4.28	1.60
time (sec)	N/A	0.273	0.010	0.412	0.201	0.269	0.102	0.318	14.897

Problem 260	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	41	64	33	34	38	0	61	51
N.S.	1	0.95	1.49	0.77	0.79	0.88	0.00	1.42	1.19
time (sec)	N/A	0.296	0.018	0.471	0.194	0.261	0.000	0.276	14.476

Problem 261	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	71	114	75	62	99	0	170	86
N.S.	1	0.99	1.58	1.04	0.86	1.38	0.00	2.36	1.19
time (sec)	N/A	0.422	0.793	0.548	0.220	0.285	0.000	0.295	14.468

Problem 262	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	105	175	111	99	168	0	266	128
N.S.	1	1.03	1.72	1.09	0.97	1.65	0.00	2.61	1.25
time (sec)	N/A	0.561	0.103	0.716	0.237	0.270	0.000	0.337	14.492

Problem 263	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	130	138	216	151	133	237	0	358	170
N.S.	1	1.06	1.66	1.16	1.02	1.82	0.00	2.75	1.31
time (sec)	N/A	0.737	0.490	0.907	0.228	0.271	0.000	0.362	14.093

Problem 264	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	115	179	143	134	134	0	228	331
N.S.	1	1.13	1.75	1.40	1.31	1.31	0.00	2.24	3.25
time (sec)	N/A	0.483	0.035	1.913	0.295	0.278	0.000	2.127	15.023

Problem 265	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	82	117	104	102	112	0	172	267
N.S.	1	1.12	1.60	1.42	1.40	1.53	0.00	2.36	3.66
time (sec)	N/A	0.346	0.023	1.505	0.292	0.278	0.000	0.928	15.238

Problem 266	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	48	60	67	65	87	0	115	96
N.S.	1	1.07	1.33	1.49	1.44	1.93	0.00	2.56	2.13
time (sec)	N/A	0.232	0.007	0.939	0.279	0.285	0.000	0.476	14.610

Problem 267	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	43	35	31	33	0	52	48
N.S.	1	1.00	1.65	1.35	1.19	1.27	0.00	2.00	1.85
time (sec)	N/A	0.199	0.013	0.460	0.280	0.266	0.000	0.302	14.341

Problem 268	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	58	62	86	59	87	0	112	90
N.S.	1	1.05	1.13	1.56	1.07	1.58	0.00	2.04	1.64
time (sec)	N/A	0.309	0.017	1.232	0.277	0.260	0.000	0.333	14.615

Problem 269	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	92	79	129	79	130	0	170	132
N.S.	1	1.10	0.94	1.54	0.94	1.55	0.00	2.02	1.57
time (sec)	N/A	0.426	0.024	1.215	0.273	0.267	0.000	0.335	14.831

Problem 270	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	111	119	92	162	100	179	0	225	174
N.S.	1	1.07	0.83	1.46	0.90	1.61	0.00	2.03	1.57
time (sec)	N/A	0.561	0.028	1.694	0.288	0.283	0.000	0.370	14.660

Problem 271	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	185	217	173	133	174	181	314	489	344
N.S.	1	1.17	0.94	0.72	0.94	0.98	1.70	2.64	1.86
time (sec)	N/A	0.361	1.166	6.163	0.196	0.309	2.475	5.918	17.871

Problem 272	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	149	172	138	113	139	146	252	415	280
N.S.	1	1.15	0.93	0.76	0.93	0.98	1.69	2.79	1.88
time (sec)	N/A	0.332	0.675	3.736	0.194	0.291	1.493	3.306	17.893

Problem 273	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	133	105	93	108	115	189	341	215
N.S.	1	1.16	0.91	0.81	0.94	1.00	1.64	2.97	1.87
time (sec)	N/A	0.301	0.326	2.283	0.199	0.292	0.983	1.693	17.834

Problem 274	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	92	74	75	75	82	126	267	151
N.S.	1	1.06	0.85	0.86	0.86	0.94	1.45	3.07	1.74
time (sec)	N/A	0.281	0.385	1.513	0.197	0.304	0.275	0.742	16.451

Problem 275	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	43	42	40	42	51	60	191	81
N.S.	1	0.91	0.89	0.85	0.89	1.09	1.28	4.06	1.72
time (sec)	N/A	0.229	0.046	0.720	0.204	0.269	0.183	0.357	13.203

Problem 276	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	70	54	48	62	68	0	101	96
N.S.	1	1.15	0.89	0.79	1.02	1.11	0.00	1.66	1.57
time (sec)	N/A	0.262	0.147	0.692	0.192	0.277	0.000	0.302	14.380

Problem 277	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	116	89	92	72	113	0	209	98
N.S.	1	1.26	0.97	1.00	0.78	1.23	0.00	2.27	1.07
time (sec)	N/A	0.324	0.561	0.753	0.195	0.282	0.000	0.330	14.267

Problem 278	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	126	174	139	136	122	203	0	360	164
N.S.	1	1.38	1.10	1.08	0.97	1.61	0.00	2.86	1.30
time (sec)	N/A	0.374	3.247	1.136	0.195	0.280	0.000	0.367	14.336

Problem 279	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	157	157	132	167	150	184	0	282	403
N.S.	1	1.00	0.84	1.06	0.96	1.17	0.00	1.80	2.57
time (sec)	N/A	0.407	1.259	3.656	0.290	0.278	0.000	2.302	15.420

Problem 280	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	116	107	129	118	151	0	220	332
N.S.	1	1.00	0.92	1.11	1.02	1.30	0.00	1.90	2.86
time (sec)	N/A	0.353	0.618	2.356	0.280	0.294	0.000	1.045	15.413

Problem 281	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	66	91	82	115	0	158	227
N.S.	1	1.00	0.94	1.30	1.17	1.64	0.00	2.26	3.24
time (sec)	N/A	0.327	0.224	1.267	0.283	0.275	0.000	0.571	14.663

Problem 282	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	39	47	47	44	0	80	58
N.S.	1	1.00	0.81	0.98	0.98	0.92	0.00	1.67	1.21
time (sec)	N/A	0.269	0.622	1.126	0.286	0.269	0.000	0.295	14.139

Problem 283	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	70	111	76	102	0	176	118
N.S.	1	1.00	0.82	1.31	0.89	1.20	0.00	2.07	1.39
time (sec)	N/A	0.321	0.262	1.317	0.268	0.285	0.000	0.319	14.001

Problem 284	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	122	122	84	154	96	152	0	273	191
N.S.	1	1.00	0.69	1.26	0.79	1.25	0.00	2.24	1.57
time (sec)	N/A	0.348	0.375	2.064	0.276	0.272	0.000	0.356	13.700

Problem 285	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	153	153	94	187	116	206	0	366	258
N.S.	1	1.00	0.61	1.22	0.76	1.35	0.00	2.39	1.69
time (sec)	N/A	0.391	0.399	3.218	0.280	0.274	0.000	0.397	13.750

Problem 286	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	250	229	228	273	268	293	0	1768	631
N.S.	1	0.92	0.91	1.09	1.07	1.17	0.00	7.07	2.52
time (sec)	N/A	0.421	3.910	2.182	0.205	0.344	0.000	5.760	15.150

Problem 287	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	170	157	156	184	183	205	0	1052	395
N.S.	1	0.92	0.92	1.08	1.08	1.21	0.00	6.19	2.32
time (sec)	N/A	0.356	6.228	1.683	0.234	0.342	0.000	3.414	15.602

Problem 288	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	108	99	98	115	110	129	0	560	227
N.S.	1	0.92	0.91	1.06	1.02	1.19	0.00	5.19	2.10
time (sec)	N/A	0.308	0.362	1.239	0.218	0.321	0.000	1.527	15.085

Problem 289	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	58	52	57	57	69	0	289	115
N.S.	1	0.98	0.88	0.97	0.97	1.17	0.00	4.90	1.95
time (sec)	N/A	0.277	0.143	0.722	0.241	0.277	0.000	0.614	14.914

Problem 290	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	34	19	33	19	19	82	114	71
N.S.	1	0.97	0.54	0.94	0.54	0.54	2.34	3.26	2.03
time (sec)	N/A	0.220	0.109	0.279	0.213	0.266	2.924	0.348	14.699

Problem 291	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	100	82	75	68	75	0	257	93
N.S.	1	1.06	0.87	0.80	0.72	0.80	0.00	2.73	0.99
time (sec)	N/A	0.316	0.150	0.617	0.219	0.286	0.000	0.311	14.753

Problem 292	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	157	165	151	126	144	263	0	403	174
N.S.	1	1.05	0.96	0.80	0.92	1.68	0.00	2.57	1.11
time (sec)	N/A	0.402	0.981	0.937	0.203	0.349	0.000	0.351	14.726

Problem 293	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	234	246	220	193	289	576	0	649	290
N.S.	1	1.05	0.94	0.82	1.24	2.46	0.00	2.77	1.24
time (sec)	N/A	0.526	4.503	1.216	0.213	0.428	0.000	0.392	15.273

Problem 294	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	F(-2)	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	198	271	907	411	0	603	0	746	9148
N.S.	1	1.37	4.58	2.08	0.00	3.05	0.00	3.77	46.20
time (sec)	N/A	0.687	6.529	1.540	0.000	0.612	0.000	2.107	16.738

Problem 295	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	F(-2)	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	126	146	287	234	0	444	0	476	6062
N.S.	1	1.16	2.28	1.86	0.00	3.52	0.00	3.78	48.11
time (sec)	N/A	0.788	1.911	0.880	0.000	0.408	0.000	0.902	16.056

Problem 296	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	88	115	108	0	253	0	140	121
N.S.	1	1.16	1.51	1.42	0.00	3.33	0.00	1.84	1.59
time (sec)	N/A	0.650	0.329	0.808	0.000	0.321	0.000	0.486	14.406

Problem 297	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	106	125	147	115	0	362	0	582	1002
N.S.	1	1.18	1.39	1.08	0.00	3.42	0.00	5.49	9.45
time (sec)	N/A	0.739	0.608	0.755	0.000	0.302	0.000	0.384	16.551

Problem 298	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	F(-2)	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	177	222	416	184	0	742	0	1073	3859
N.S.	1	1.25	2.35	1.04	0.00	4.19	0.00	6.06	21.80
time (sec)	N/A	1.342	6.439	0.905	0.000	0.315	0.000	0.483	23.381

Problem 299	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	255	231	230	287	321	423	0	1696	760
N.S.	1	0.91	0.90	1.13	1.26	1.66	0.00	6.65	2.98
time (sec)	N/A	0.421	0.915	3.602	0.205	0.365	0.000	5.906	16.976

Problem 300	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	179	165	163	200	227	312	0	1023	505
N.S.	1	0.92	0.91	1.12	1.27	1.74	0.00	5.72	2.82
time (sec)	N/A	0.367	1.510	2.539	0.205	0.331	0.000	3.435	15.729

Problem 301	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	109	107	127	149	219	0	568	286
N.S.	1	0.90	0.88	1.05	1.23	1.81	0.00	4.69	2.36
time (sec)	N/A	0.316	0.338	1.268	0.201	0.319	0.000	1.557	14.901

Problem 302	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	62	72	74	102	0	313	124
N.S.	1	1.00	0.84	0.97	1.00	1.38	0.00	4.23	1.68
time (sec)	N/A	0.293	0.183	0.836	0.209	0.295	0.000	0.649	14.307

Problem 303	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	50	54	49	41	46	0	238	257
N.S.	1	0.93	1.00	0.91	0.76	0.85	0.00	4.41	4.76
time (sec)	N/A	0.250	0.030	0.496	0.200	0.289	0.000	0.368	14.368

Problem 304	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	138	139	136	114	142	234	0	303	160
N.S.	1	1.01	0.99	0.83	1.03	1.70	0.00	2.20	1.16
time (sec)	N/A	0.367	0.431	1.048	0.204	0.345	0.000	0.321	14.617

Problem 305	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	197	199	187	164	303	693	0	656	313
N.S.	1	1.01	0.95	0.83	1.54	3.52	0.00	3.33	1.59
time (sec)	N/A	0.465	1.770	1.468	0.217	0.411	0.000	0.379	14.957

Problem 306	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	278	285	283	230	558	1378	0	795	471
N.S.	1	1.03	1.02	0.83	2.01	4.96	0.00	2.86	1.69
time (sec)	N/A	0.603	6.451	2.361	0.214	0.618	0.000	0.434	15.760

Problem 307	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	F(-2)	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	200	283	559	365	0	843	0	411	9452
N.S.	1	1.42	2.80	1.82	0.00	4.22	0.00	2.06	47.26
time (sec)	N/A	0.781	6.180	2.076	0.000	0.560	0.000	2.113	17.227

Problem 308	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	F(-2)	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	150	174	327	219	0	584	0	294	7044
N.S.	1	1.16	2.18	1.46	0.00	3.89	0.00	1.96	46.96
time (sec)	N/A	0.824	1.534	1.152	0.000	0.343	0.000	0.936	16.187

Problem 309	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	90	80	115	0	383	0	144	551
N.S.	1	1.06	0.94	1.35	0.00	4.51	0.00	1.69	6.48
time (sec)	N/A	0.485	0.311	0.789	0.000	0.301	0.000	0.535	14.685

Problem 310	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	227	227	209	184	0	705	0	332	6093
N.S.	1	1.00	0.92	0.81	0.00	3.11	0.00	1.46	26.84
time (sec)	N/A	0.712	1.625	1.440	0.000	0.324	0.000	0.335	18.955

Problem 311	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	360	360	303	260	0	1481	0	487	8348
N.S.	1	1.00	0.84	0.72	0.00	4.11	0.00	1.35	23.19
time (sec)	N/A	0.914	2.111	1.815	0.000	0.358	0.000	0.383	19.430

Problem 312	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	F	C	B	F	F(-1)	F	F	F(-1)
verified	N/A	N/A	No	No	TBD	TBD	TBD	TBD	TBD
size	761	0	1846	2967	0	0	0	0	0
N.S.	1	0.00	2.43	3.90	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.000	52.967	3.689	0.000	0.000	0.000	0.000	0.000

Problem 313	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	F	C	B	F	F(-1)	F	F	F(-1)
verified	N/A	N/A	No	No	TBD	TBD	TBD	TBD	TBD
size	740	0	548	1651	0	0	0	0	0
N.S.	1	0.00	0.74	2.23	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.000	21.452	3.363	0.000	0.000	0.000	0.000	0.000

Problem 314	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F(-1)	F	F	F(-1)
verified	N/A	No	No	No	TBD	TBD	TBD	TBD	TBD
size	415	352	646	820	0	0	0	0	0
N.S.	1	0.85	1.56	1.98	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.353	5.978	2.538	0.000	0.000	0.000	0.000	0.000

Problem 315	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F(-1)	F	F	F(-1)
verified	N/A	No	No	No	TBD	TBD	TBD	TBD	TBD
size	422	412	1475	1919	0	0	0	0	0
N.S.	1	0.98	3.50	4.55	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.237	38.297	3.578	0.000	0.000	0.000	0.000	0.000

Problem 316	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	F	C	B	F	F(-1)	F	F	F(-1)
verified	N/A	N/A	No	No	TBD	TBD	TBD	TBD	TBD
size	863	0	1571	4016	0	0	0	0	0
N.S.	1	0.00	1.82	4.65	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.000	23.898	3.274	0.000	0.000	0.000	0.000	0.000

Problem 317	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	F	C	B	F	F(-1)	F	F	F(-1)
verified	N/A	N/A	No	No	TBD	TBD	TBD	TBD	TBD
size	836	0	2169	7581	0	0	0	0	0
N.S.	1	0.00	2.59	9.07	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.000	33.622	4.790	0.000	0.000	0.000	0.000	0.000

Problem 318	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	F	B	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	169	132	152	883	191	425	0	966	0
N.S.	1	0.78	0.90	5.22	1.13	2.51	0.00	5.72	0.00
time (sec)	N/A	0.363	1.908	16.530	0.284	0.438	0.000	1.758	0.000

Problem 319	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	F	B	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	81	87	465	108	311	0	539	0
N.S.	1	0.81	0.87	4.65	1.08	3.11	0.00	5.39	0.00
time (sec)	N/A	0.299	0.700	10.670	0.303	0.441	0.000	0.705	0.000

Problem 320	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	49	49	42	67	192	0	185	47
N.S.	1	0.96	0.96	0.82	1.31	3.76	0.00	3.63	0.92
time (sec)	N/A	0.239	0.136	0.659	0.282	0.427	0.000	0.405	13.724

Problem 321	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	106	118	100	543	0	2132	0	293	0
N.S.	1	1.11	0.94	5.12	0.00	20.11	0.00	2.76	0.00
time (sec)	N/A	0.365	0.234	2.245	0.000	0.604	0.000	0.630	0.000

Problem 322	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	B	F(-1)
verified	N/A	No	Yes	No	TBD	TBD	TBD	TBD	TBD
size	215	268	212	2072	0	3523	0	514	0
N.S.	1	1.25	0.99	9.64	0.00	16.39	0.00	2.39	0.00
time (sec)	N/A	0.552	2.452	2.199	0.000	1.810	0.000	0.837	0.000

Problem 323	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	344	345	376	1380	0	0	0	0	0
N.S.	1	1.00	1.09	4.01	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.252	11.573	11.952	0.000	0.000	0.000	0.000	0.000

Problem 324	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	125	125	151	182	0	0	0	0	0
N.S.	1	1.00	1.21	1.46	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.234	0.133	6.573	0.000	0.000	0.000	0.000	0.000

Problem 325	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	246	246	154	546	0	0	0	0	0
N.S.	1	1.00	0.63	2.22	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.485	3.607	5.809	0.000	0.000	0.000	0.000	0.000

Problem 326	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	F	B	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	148	123	131	646	175	381	0	699	0
N.S.	1	0.83	0.89	4.36	1.18	2.57	0.00	4.72	0.00
time (sec)	N/A	0.337	0.871	16.651	0.278	0.426	0.000	1.807	0.000

Problem 327	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	F	B	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	71	66	312	92	273	0	265	0
N.S.	1	0.90	0.84	3.95	1.16	3.46	0.00	3.35	0.00
time (sec)	N/A	0.301	0.412	11.293	0.273	0.406	0.000	0.819	0.000

Problem 328	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	31	26	49	145	0	102	27
N.S.	1	1.00	1.00	0.84	1.58	4.68	0.00	3.29	0.87
time (sec)	N/A	0.235	0.091	0.742	0.269	0.392	0.000	0.493	14.771

Problem 329	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	106	118	100	658	0	2420	0	0	0
N.S.	1	1.11	0.94	6.21	0.00	22.83	0.00	0.00	0.00
time (sec)	N/A	0.361	0.292	2.292	0.000	1.423	0.000	0.000	0.000

Problem 330	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	B	F(-1)
verified	N/A	No	Yes	No	TBD	TBD	TBD	TBD	TBD
size	260	272	280	3107	0	4336	0	533	0
N.S.	1	1.05	1.08	11.95	0.00	16.68	0.00	2.05	0.00
time (sec)	N/A	0.473	2.382	2.011	0.000	25.895	0.000	0.920	0.000

Problem 331	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	404	610	446	2243	0	0	0	0	0
N.S.	1	1.51	1.10	5.55	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.227	15.799	15.874	0.000	0.000	0.000	0.000	0.000

Problem 332	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	310	310	382	1069	0	0	0	0	0
N.S.	1	1.00	1.23	3.45	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.013	12.550	10.915	0.000	0.000	0.000	0.000	0.000

Problem 333	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	106	106	138	147	0	0	0	0	0
N.S.	1	1.00	1.30	1.39	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.222	0.124	5.436	0.000	0.000	0.000	0.000	0.000

Problem 334	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	361	361	366	1374	0	0	0	0	0
N.S.	1	1.00	1.01	3.81	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.759	13.148	7.097	0.000	0.000	0.000	0.000	0.000

Problem 335	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	A	A	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	148	119	117	1234	194	467	0	0	0
N.S.	1	0.80	0.79	8.34	1.31	3.16	0.00	0.00	0.00
time (sec)	N/A	0.374	0.560	15.602	0.286	0.503	0.000	0.000	0.000

Problem 336	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	A	A	F	B	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	79	66	731	110	317	0	258	0
N.S.	1	0.90	0.75	8.31	1.25	3.60	0.00	2.93	0.00
time (sec)	N/A	0.329	0.258	9.564	0.277	0.443	0.000	1.105	0.000

Problem 337	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	52	43	45	70	260	0	214	50
N.S.	1	0.96	0.80	0.83	1.30	4.81	0.00	3.96	0.93
time (sec)	N/A	0.249	0.104	0.789	0.263	0.421	0.000	0.778	15.587

Problem 338	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	B	F	F(-2)	F(-1)
verified	N/A	No	Yes	No	TBD	TBD	TBD	TBD	TBD
size	142	143	212	2692	0	3924	0	0	0
N.S.	1	1.01	1.49	18.96	0.00	27.63	0.00	0.00	0.00
time (sec)	N/A	0.416	0.672	2.217	0.000	24.540	0.000	0.000	0.000

Problem 339	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F(-1)	B	F	F(-2)	F(-1)
verified	N/A	No	Yes	No	TBD	TBD	TBD	TBD	TBD
size	236	310	315	6367	0	8098	0	0	0
N.S.	1	1.31	1.33	26.98	0.00	34.31	0.00	0.00	0.00
time (sec)	N/A	0.593	1.092	2.335	0.000	48.259	0.000	0.000	0.000

Problem 340	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	530	907	531	1965	0	0	0	0	0
N.S.	1	1.71	1.00	3.71	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.898	17.412	14.836	0.000	0.000	0.000	0.000	0.000

Problem 341	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	344	382	329	828	0	0	0	0	0
N.S.	1	1.11	0.96	2.41	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.379	11.177	9.790	0.000	0.000	0.000	0.000	0.000

Problem 342	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	347	376	972	1799	0	0	0	0	0
N.S.	1	1.08	2.80	5.18	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.259	6.080	7.080	0.000	0.000	0.000	0.000	0.000

Problem 343	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	449	664	469	2062	0	0	0	0	0
N.S.	1	1.48	1.04	4.59	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.434	10.812	8.203	0.000	0.000	0.000	0.000	0.000

Problem 344	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	245	245	238	0	0	0	0	0	0
N.S.	1	1.00	0.97	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.524	3.161	0.000	0.000	0.000	0.000	0.000	0.000

Problem 345	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	160	160	178	0	0	0	0	0	0
N.S.	1	1.00	1.11	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.400	1.013	0.000	0.000	0.000	0.000	0.000	0.000

Problem 346	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	129	129	106	0	0	0	0	0	0
N.S.	1	1.00	0.82	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.381	0.584	0.000	0.000	0.000	0.000	0.000	0.000

Problem 347	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	F	B	F	F	F	F	F	F(-1)
verified	N/A	N/A	No	N/A	TBD	TBD	TBD	TBD	TBD
size	266	0	786	0	0	0	0	0	0
N.S.	1	0.00	2.95	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.000	4.223	0.000	0.000	0.000	0.000	0.000	0.000

Problem 348	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	25	27	23	25	25	24	25	27
N.S.	1	1.00	1.08	0.92	1.00	1.00	0.96	1.00	1.08
time (sec)	N/A	0.242	29.433	1.182	1.427	0.387	69.961	1.874	18.998

Problem 349	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	25	27	23	25	25	24	25	27
N.S.	1	1.00	1.08	0.92	1.00	1.00	0.96	1.00	1.08
time (sec)	N/A	0.236	0.888	1.102	1.331	0.378	1.412	1.040	14.985

Problem 350	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	25	27	23	25	25	24	25	27
N.S.	1	1.00	1.08	0.92	1.00	1.00	0.96	1.00	1.08
time (sec)	N/A	0.239	15.729	1.148	1.334	0.385	0.719	0.836	20.955

Problem 351	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	25	27	23	25	53	24	25	27
N.S.	1	1.00	1.08	0.92	1.00	2.12	0.96	1.00	1.08
time (sec)	N/A	0.246	19.967	1.130	1.255	0.538	4.317	1.425	28.130

Problem 352	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	25	23	25	25	22	25	27
N.S.	1	1.00	1.09	1.00	1.09	1.09	0.96	1.09	1.17
time (sec)	N/A	0.221	7.759	1.869	1.886	0.464	33.982	0.829	15.760

Problem 353	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	177	160	137	0	0	0	0	0	0
N.S.	1	0.90	0.77	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.347	1.374	0.000	0.000	0.000	0.000	0.000	0.000

Problem 354	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	102	99	80	0	0	0	0	0	0
N.S.	1	0.97	0.78	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.289	0.562	0.000	0.000	0.000	0.000	0.000	0.000

Problem 355	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	48	48	48	0	0	0	0	0	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.220	0.171	0.000	0.000	0.000	0.000	0.000	0.000

Problem 356	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	162	171	132	0	0	0	0	0	0
N.S.	1	1.06	0.81	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.370	0.386	0.000	0.000	0.000	0.000	0.000	0.000

Problem 357	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	279	284	186	0	0	0	0	0	0
N.S.	1	1.02	0.67	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.444	0.709	0.000	0.000	0.000	0.000	0.000	0.000

Problem 358	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	21	21	23	21	23	23	20	23	25
N.S.	1	1.00	1.10	1.00	1.10	1.10	0.95	1.10	1.19
time (sec)	N/A	0.212	13.790	1.375	19.761	0.300	29.679	1.547	17.989

Problem 359	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	21	21	23	21	23	23	20	23	25
N.S.	1	1.00	1.10	1.00	1.10	1.10	0.95	1.10	1.19
time (sec)	N/A	0.960	12.262	0.983	5.804	0.274	3.984	0.710	15.068

Problem 360	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	21	21	23	21	23	23	20	23	25
N.S.	1	1.00	1.10	1.00	1.10	1.10	0.95	1.10	1.19
time (sec)	N/A	0.211	8.653	1.199	5.851	0.317	19.226	0.568	18.059

Problem 361	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	21	21	23	21	23	23	0	23	25
N.S.	1	1.00	1.10	1.00	1.10	1.10	0.00	1.10	1.19
time (sec)	N/A	0.223	13.642	1.167	18.680	0.606	0.000	0.679	28.690

Problem 362	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	25	21	23	23	0	23	25
N.S.	1	1.00	1.09	0.91	1.00	1.00	0.00	1.00	1.09
time (sec)	N/A	0.226	7.605	1.146	1.918	0.355	0.000	0.429	19.280

Problem 363	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	25	21	23	23	22	23	25
N.S.	1	1.00	1.09	0.91	1.00	1.00	0.96	1.00	1.09
time (sec)	N/A	0.221	10.142	1.515	1.747	0.350	6.022	0.583	14.741

Problem 364	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	25	21	23	23	22	23	25
N.S.	1	1.00	1.09	0.91	1.00	1.00	0.96	1.00	1.09
time (sec)	N/A	0.218	23.219	1.334	1.934	0.354	2.489	0.752	18.949

Problem 365	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	25	21	23	23	22	23	25
N.S.	1	1.00	1.09	0.91	1.00	1.00	0.96	1.00	1.09
time (sec)	N/A	0.224	10.638	1.118	1.969	0.439	52.603	0.565	20.456

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [298] had the largest ratio of [1.19047999999999998]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	7	6	0.77	19	0.316
2	A	7	6	0.78	19	0.316
3	A	7	6	0.76	19	0.316
4	A	7	6	0.81	19	0.316
5	A	7	6	0.88	17	0.353
6	A	5	4	1.00	17	0.235
7	A	7	6	0.89	19	0.316
8	A	7	6	0.85	19	0.316
9	A	7	6	0.83	19	0.316
10	A	10	10	1.15	19	0.526
11	A	8	8	1.13	19	0.421
12	A	5	5	1.12	19	0.263
13	A	3	3	1.07	19	0.158
14	A	3	3	1.00	19	0.158
15	A	6	6	1.05	19	0.316
16	A	9	9	1.10	19	0.474
17	A	12	12	1.07	19	0.632
18	A	15	15	1.13	19	0.789
19	A	7	6	0.69	21	0.286
20	A	7	6	0.71	21	0.286
21	A	7	6	0.70	21	0.286
22	A	7	6	0.77	21	0.286

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
23	A	7	6	0.75	19	0.316
24	A	6	5	0.83	19	0.263
25	A	7	6	0.80	21	0.286
26	A	7	6	0.81	21	0.286
27	A	7	6	0.78	21	0.286
28	A	7	6	0.76	21	0.286
29	A	3	3	1.00	21	0.143
30	A	3	3	1.00	21	0.143
31	A	3	3	1.00	21	0.143
32	A	3	3	1.00	21	0.143
33	A	3	3	1.00	21	0.143
34	A	3	3	1.00	21	0.143
35	A	3	3	1.00	21	0.143
36	A	3	3	1.00	21	0.143
37	A	7	6	0.69	21	0.286
38	A	7	6	0.72	21	0.286
39	A	7	6	0.70	21	0.286
40	A	7	6	0.71	21	0.286
41	A	7	6	0.73	19	0.316
42	A	7	6	0.81	19	0.316
43	A	7	6	0.85	21	0.286
44	A	7	6	0.85	21	0.286
45	A	7	6	0.79	21	0.286
46	A	7	6	0.77	21	0.286
47	A	3	3	1.00	21	0.143
48	A	3	3	1.00	21	0.143
49	A	3	3	1.00	21	0.143
50	A	3	3	1.00	21	0.143
51	A	3	3	1.00	21	0.143
52	A	3	3	1.00	21	0.143
53	A	3	3	1.00	21	0.143
54	A	3	3	1.00	21	0.143
55	A	3	3	1.00	21	0.143
56	A	7	6	0.68	21	0.286

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
57	A	7	6	0.74	21	0.286
58	A	7	6	0.70	21	0.286
59	A	7	6	0.93	21	0.286
60	A	5	4	1.00	19	0.211
61	A	7	6	0.84	19	0.316
62	A	7	6	0.79	21	0.286
63	A	7	6	0.77	21	0.286
64	A	11	11	1.14	21	0.524
65	A	8	8	1.12	21	0.381
66	A	6	6	1.08	21	0.286
67	A	3	3	1.00	21	0.143
68	A	9	9	1.05	21	0.429
69	A	12	12	1.11	21	0.571
70	A	15	15	1.07	21	0.714
71	A	7	6	0.70	21	0.286
72	A	7	6	0.77	21	0.286
73	A	7	6	0.75	21	0.286
74	A	6	5	0.82	21	0.238
75	A	7	6	0.78	19	0.316
76	A	7	6	0.80	19	0.316
77	A	7	6	0.77	21	0.286
78	A	7	6	0.76	21	0.286
79	A	5	5	1.03	21	0.238
80	A	5	5	1.06	21	0.238
81	A	9	8	1.12	21	0.381
82	A	5	5	1.18	21	0.238
83	A	5	5	1.04	21	0.238
84	A	5	5	1.03	21	0.238
85	A	5	5	1.02	21	0.238
86	A	7	6	0.72	21	0.286
87	A	7	6	0.71	21	0.286
88	A	7	6	0.77	21	0.286
89	A	7	6	0.80	21	0.286
90	A	7	6	0.91	21	0.286

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
91	A	7	6	0.79	19	0.316
92	A	7	6	0.78	19	0.316
93	A	7	6	0.76	21	0.286
94	A	7	6	0.75	21	0.286
95	A	6	6	1.02	21	0.286
96	A	6	6	1.03	21	0.286
97	A	6	6	1.04	21	0.286
98	A	11	10	1.12	21	0.476
99	A	6	6	1.20	21	0.286
100	A	6	6	1.23	21	0.286
101	A	6	6	1.02	21	0.286
102	A	6	6	1.04	21	0.286
103	A	6	6	1.01	21	0.286
104	A	24	23	0.95	23	1.000
105	A	22	21	0.96	23	0.913
106	A	21	20	0.93	23	0.870
107	A	19	18	0.94	23	0.783
108	A	24	23	0.95	23	1.000
109	A	22	21	0.96	23	0.913
110	A	27	26	0.96	23	1.130
111	A	3	3	1.00	25	0.120
112	A	3	3	1.00	25	0.120
113	A	3	3	1.00	25	0.120
114	A	3	3	1.00	25	0.120
115	A	3	3	1.00	25	0.120
116	A	3	3	1.00	25	0.120
117	A	3	3	1.00	25	0.120
118	A	28	27	0.97	25	1.080
119	A	27	26	0.93	25	1.040
120	A	25	24	0.95	25	0.960
121	A	24	23	0.92	25	0.920
122	A	22	21	0.93	25	0.840
123	A	27	26	0.95	25	1.040
124	A	25	24	0.97	25	0.960

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
125	A	30	29	0.95	25	1.160
126	A	28	27	0.98	25	1.080
127	A	5	5	1.00	25	0.200
128	A	5	5	1.01	25	0.200
129	A	5	5	1.01	25	0.200
130	A	5	5	1.01	25	0.200
131	A	5	5	1.02	25	0.200
132	A	5	5	1.02	25	0.200
133	A	5	5	1.04	25	0.200
134	A	5	5	1.04	25	0.200
135	A	7	6	0.91	23	0.261
136	A	11	10	0.96	23	0.435
137	A	7	6	0.96	21	0.286
138	A	10	9	0.99	21	0.429
139	A	13	12	1.05	23	0.522
140	A	18	17	1.07	23	0.739
141	A	5	4	0.92	23	0.174
142	A	5	4	0.93	23	0.174
143	A	6	5	0.97	23	0.217
144	A	8	7	1.04	23	0.304
145	A	12	11	1.05	23	0.478
146	A	16	15	1.05	23	0.652
147	A	7	6	0.92	23	0.261
148	A	12	11	0.96	23	0.478
149	A	8	7	0.96	21	0.333
150	A	10	9	1.00	21	0.429
151	A	11	10	1.03	23	0.435
152	A	16	15	1.04	23	0.652
153	A	5	4	0.91	23	0.174
154	A	5	4	0.92	23	0.174
155	A	5	4	0.92	23	0.174
156	A	5	4	0.95	23	0.174
157	A	9	8	1.01	23	0.348
158	A	14	13	1.05	23	0.565

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
159	A	7	6	0.91	23	0.261
160	A	13	12	0.94	23	0.522
161	A	9	8	0.94	21	0.381
162	A	13	12	1.00	21	0.571
163	A	11	10	1.01	23	0.435
164	A	14	13	1.03	23	0.565
165	A	5	4	0.91	23	0.174
166	A	5	4	0.92	23	0.174
167	A	5	4	0.92	23	0.174
168	A	5	4	0.97	23	0.174
169	A	6	5	0.95	23	0.217
170	A	11	10	1.01	23	0.435
171	A	7	6	0.90	23	0.261
172	A	10	9	0.95	23	0.391
173	A	6	5	1.00	21	0.238
174	A	13	12	1.07	21	0.571
175	A	15	14	1.11	23	0.609
176	A	20	19	1.10	23	0.826
177	A	5	4	0.94	23	0.174
178	A	6	5	0.98	23	0.217
179	A	5	4	1.03	23	0.174
180	A	9	8	1.04	23	0.348
181	A	14	13	1.05	23	0.565
182	A	17	16	1.05	23	0.696
183	A	7	6	0.92	23	0.261
184	A	9	8	1.04	23	0.348
185	A	7	6	0.96	21	0.286
186	A	15	14	1.07	21	0.667
187	A	17	16	1.12	23	0.696
188	A	22	21	1.11	23	0.913
189	A	6	5	0.96	23	0.217
190	A	5	4	0.99	23	0.174
191	A	5	4	0.98	23	0.174
192	A	13	12	1.06	23	0.522

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
193	A	17	16	1.06	23	0.696
194	A	20	19	1.06	23	0.826
195	A	7	6	0.97	23	0.261
196	A	9	8	1.04	23	0.348
197	A	8	7	1.00	21	0.333
198	A	17	16	1.07	21	0.762
199	A	19	18	1.13	23	0.783
200	A	24	23	1.11	23	1.000
201	A	5	4	0.97	23	0.174
202	A	6	5	1.01	23	0.217
203	A	6	5	1.17	23	0.217
204	A	14	13	1.06	23	0.565
205	A	18	17	1.06	23	0.739
206	A	23	22	1.06	23	0.957
207	A	10	9	1.46	23	0.391
208	A	2	2	1.00	23	0.087
209	A	3	3	1.00	23	0.130
210	A	3	3	1.00	23	0.130
211	A	7	6	1.00	21	0.286
212	A	10	9	1.06	23	0.391
213	A	5	5	1.04	23	0.217
214	A	6	6	1.03	23	0.261
215	A	2	2	1.00	25	0.080
216	A	2	2	1.00	25	0.080
217	A	2	2	1.00	25	0.080
218	A	2	2	1.00	25	0.080
219	A	8	7	0.98	21	0.333
220	A	7	6	0.98	21	0.286
221	A	8	7	1.06	21	0.333
222	A	5	4	1.00	19	0.211
223	A	9	8	1.09	19	0.421
224	A	11	10	1.08	21	0.476
225	A	2	2	1.00	21	0.095
226	A	2	2	1.00	21	0.095

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
227	A	2	2	1.00	21	0.095
228	A	2	2	1.00	21	0.095
229	A	2	2	1.00	23	0.087
230	A	2	2	1.00	23	0.087
231	A	2	2	1.00	23	0.087
232	A	2	2	1.00	23	0.087
233	A	24	23	0.75	23	1.000
234	A	26	25	0.74	23	1.087
235	A	21	20	0.76	23	0.870
236	A	23	22	0.78	23	0.957
237	A	24	23	0.76	23	1.000
238	A	5	5	0.75	25	0.200
239	A	5	5	0.76	25	0.200
240	A	5	5	0.76	25	0.200
241	A	5	5	0.77	25	0.200
242	A	5	5	0.76	25	0.200
243	F	0	0	N/A	0.000	N/A
244	A	27	26	0.75	25	1.040
245	A	29	28	0.76	25	1.120
246	A	24	23	0.73	25	0.920
247	A	26	25	0.73	25	1.000
248	A	27	26	0.74	25	1.040
249	A	29	28	0.73	25	1.120
250	A	7	7	0.76	25	0.280
251	A	7	7	0.75	25	0.280
252	A	7	7	0.74	25	0.280
253	A	7	7	0.74	25	0.280
254	A	7	7	0.74	25	0.280
255	A	7	7	0.74	25	0.280
256	A	21	20	1.08	19	1.053
257	A	17	16	1.10	19	0.842
258	A	13	12	1.07	19	0.632
259	A	9	8	1.00	17	0.471

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
260	A	10	9	0.95	17	0.529
261	A	13	12	0.99	19	0.632
262	A	16	15	1.03	19	0.789
263	A	20	19	1.06	19	1.000
264	A	8	8	1.13	19	0.421
265	A	5	5	1.12	19	0.263
266	A	3	3	1.07	19	0.158
267	A	3	3	1.00	19	0.158
268	A	6	6	1.05	19	0.316
269	A	9	9	1.10	19	0.474
270	A	12	12	1.07	19	0.632
271	A	6	5	1.17	21	0.238
272	A	6	5	1.15	21	0.238
273	A	6	5	1.16	21	0.238
274	A	6	5	1.06	21	0.238
275	A	6	5	0.91	19	0.263
276	A	7	6	1.15	19	0.316
277	A	8	7	1.26	21	0.333
278	A	10	9	1.38	21	0.429
279	A	3	3	1.00	21	0.143
280	A	3	3	1.00	21	0.143
281	A	3	3	1.00	21	0.143
282	A	3	3	1.00	21	0.143
283	A	3	3	1.00	21	0.143
284	A	3	3	1.00	21	0.143
285	A	3	3	1.00	21	0.143
286	A	6	5	0.92	21	0.238
287	A	6	5	0.92	21	0.238
288	A	6	5	0.92	21	0.238
289	A	6	5	0.98	21	0.238
290	A	7	6	0.97	19	0.316
291	A	6	5	1.06	19	0.263
292	A	6	5	1.05	21	0.238
293	A	6	5	1.05	21	0.238

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
294	A	5	5	1.37	21	0.238
295	A	12	11	1.16	21	0.524
296	A	14	13	1.16	21	0.619
297	A	17	16	1.18	21	0.762
298	A	26	25	1.25	21	1.190
299	A	6	5	0.91	21	0.238
300	A	6	5	0.92	21	0.238
301	A	6	5	0.90	21	0.238
302	A	6	5	1.00	21	0.238
303	A	6	5	0.93	19	0.263
304	A	6	5	1.01	19	0.263
305	A	6	5	1.01	21	0.238
306	A	6	5	1.03	21	0.238
307	A	5	5	1.42	21	0.238
308	A	12	11	1.16	21	0.524
309	A	11	10	1.06	21	0.476
310	A	5	5	1.00	21	0.238
311	A	5	5	1.00	21	0.238
312	F	0	0	N/A	0.000	N/A
313	F	0	0	N/A	0.000	N/A
314	A	24	23	0.85	25	0.920
315	A	22	21	0.98	25	0.840
316	F	0	0	N/A	0.000	N/A
317	F	0	0	N/A	0.000	N/A
318	A	8	7	0.78	23	0.304
319	A	7	6	0.81	23	0.261
320	A	7	6	0.96	21	0.286
321	A	7	6	1.11	21	0.286
322	A	14	13	1.25	23	0.565
323	A	13	13	1.00	23	0.565
324	A	2	2	1.00	14	0.143
325	A	3	3	1.00	23	0.130
326	A	8	7	0.83	23	0.304

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
327	A	7	6	0.90	23	0.261
328	A	6	5	1.00	21	0.238
329	A	7	6	1.11	21	0.286
330	A	8	7	1.05	23	0.304
331	A	3	3	1.51	23	0.130
332	A	10	10	1.00	23	0.435
333	A	2	2	1.00	14	0.143
334	A	3	3	1.00	23	0.130
335	A	8	7	0.80	23	0.304
336	A	7	6	0.90	23	0.261
337	A	7	6	0.96	21	0.286
338	A	7	6	1.01	21	0.286
339	A	8	7	1.31	23	0.304
340	A	3	3	1.71	23	0.130
341	A	13	13	1.11	23	0.565
342	A	11	11	1.08	14	0.786
343	A	3	3	1.48	23	0.130
344	A	3	3	1.00	23	0.130
345	A	3	3	1.00	23	0.130
346	A	7	6	1.00	21	0.286
347	F	0	0	N/A	0.000	N/A
348	N/A	2	0	1.00	25	0.000
349	N/A	2	0	1.00	25	0.000
350	N/A	2	0	1.00	25	0.000
351	N/A	2	0	1.00	25	0.000
352	N/A	2	0	1.00	23	0.000
353	A	6	5	0.90	21	0.238
354	A	6	5	0.97	21	0.238
355	A	5	4	1.00	19	0.211
356	A	6	5	1.06	19	0.263
357	A	6	5	1.02	21	0.238
358	N/A	2	0	1.00	21	0.000
359	N/A	16	0	1.00	21	0.000

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
360	N/A	2	0	1.00	21	0.000
361	N/A	2	0	1.00	21	0.000
362	N/A	2	0	1.00	23	0.000
363	N/A	2	0	1.00	23	0.000
364	N/A	2	0	1.00	23	0.000
365	N/A	2	0	1.00	23	0.000

CHAPTER 3

Listing of integrals

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3.156 $\int \cot^2(c+dx)(a+a\sec(c+dx))^{3/2}\,dx$ . . . 1171
3.157 $\int \cot^4(c+dx)(a+a\sec(c+dx))^{3/2}\,dx$ . . . 1176
3.158 $\int \cot^6(c+dx)(a+a\sec(c+dx))^{3/2}\,dx$ . . . 1183
3.159 $\int (a+a\sec(c+dx))^{5/2}\tan^5(c+dx)\,dx$ . . . 1191
3.160 $\int (a+a\sec(c+dx))^{5/2}\tan^3(c+dx)\,dx$ . . . 1197
3.161 $\int (a+a\sec(c+dx))^{5/2}\tan(c+dx)\,dx$ . . . 1204
3.162 $\int \cot(c+dx)(a+a\sec(c+dx))^{5/2}\,dx$ . . . 1210
3.163 $\int \cot^3(c+dx)(a+a\sec(c+dx))^{5/2}\,dx$ . . . 1217
3.164 $\int \cot^5(c+dx)(a+a\sec(c+dx))^{5/2}\,dx$ . . . 1224
3.165 $\int (a+a\sec(c+dx))^{5/2}\tan^6(c+dx)\,dx$ . . . 1232
3.166 $\int (a+a\sec(c+dx))^{5/2}\tan^4(c+dx)\,dx$ . . . 1238
3.167 $\int (a+a\sec(c+dx))^{5/2}\tan^2(c+dx)\,dx$ . . . 1244
3.168 $\int \cot^2(c+dx)(a+a\sec(c+dx))^{5/2}\,dx$ . . . 1250
3.169 $\int \cot^4(c+dx)(a+a\sec(c+dx))^{5/2}\,dx$ . . . 1255
3.170 $\int \cot^6(c+dx)(a+a\sec(c+dx))^{5/2}\,dx$ . . . 1261
3.171 $\int \frac{\tan^5(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx$ . . . 1269
3.172 $\int \frac{\tan^3(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx$ . . . 1275

3.173 $\int \frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx$ . . . 1282
3.174 $\int \frac{\cot(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx$ . . . 1287
3.175 $\int \frac{\cot^3(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx$ . . . 1294
3.176 $\int \frac{\cot^5(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx$ . . . 1303
3.177 $\int \frac{\tan^6(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx$ . . . 1314
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3.183 $\int \frac{\tan^5(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx$ . . . 1359
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3.194 $\int \frac{\cot^6(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx$ . . . 1440
3.195 $\int \frac{\tan^5(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx$ . . . 1451
3.196 $\int \frac{\tan^3(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx$ . . . 1457
3.197 $\int \frac{\tan(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx$ . . . 1463
3.198 $\int \frac{\cot(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx$ . . . 1469
3.199 $\int \frac{\cot^3(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx$ . . . 1477
3.200 $\int \frac{\cot^5(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx$ . . . 1488
3.201 $\int \frac{\tan^6(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx$ . . . 1500
3.202 $\int \frac{\tan^4(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx$ . . . 1506
3.203 $\int \frac{\tan^2(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx$ . . . 1513
3.204 $\int \frac{\cot^2(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx$ . . . 1519

3.205 $\int \frac{\cot^4(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx$ . . . 1528
3.206 $\int \frac{\cot^6(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx$ . . . 1539
3.207 $\int \frac{\tan^2(e+fx)}{(a+a\sec(e+fx))^{9/2}}\,dx$ . . . 1551
3.208 $\int (a+a\sec(c+dx))^n (e\tan(c+dx))^m\,dx$ . . . 1559
3.209 $\int (a+a\sec(c+dx))^3 (e\tan(c+dx))^m\,dx$ . . . 1563
3.210 $\int (a+a\sec(c+dx))^2 (e\tan(c+dx))^m\,dx$ . . . 1568
3.211 $\int (a+a\sec(c+dx))(e\tan(c+dx))^m\,dx$ . . . 1573
3.212 $\int \frac{(e\tan(c+dx))^m}{a+a\sec(c+dx)}\,dx$ . . . 1578
3.213 $\int \frac{(e\tan(c+dx))^m}{(a+a\sec(c+dx))^2}\,dx$ . . . 1584
3.214 $\int \frac{(e\tan(c+dx))^m}{(a+a\sec(c+dx))^3}\,dx$ . . . 1589
3.215 $\int (a+a\sec(c+dx))^{3/2} (e\tan(c+dx))^m\,dx$ . . . 1594
3.216 $\int \sqrt{a+a\sec(c+dx)}\,(e\tan(c+dx))^m\,dx$ . . . 1598
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3.218 $\int \frac{(e\tan(c+dx))^m}{(a+a\sec(c+dx))^{3/2}}\,dx$ . . . 1607
3.219 $\int (a+a\sec(c+dx))^n \tan^7(c+dx)\,dx$ . . . 1611
3.220 $\int (a+a\sec(c+dx))^n \tan^5(c+dx)\,dx$ . . . 1616
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3.226 $\int (a+a\sec(c+dx))^n \tan^2(c+dx)\,dx$ . . . 1645
3.227 $\int \cot^2(c+dx)(a+a\sec(c+dx))^n\,dx$ . . . 1650
3.228 $\int \cot^4(c+dx)(a+a\sec(c+dx))^n\,dx$ . . . 1655
3.229 $\int (a+a\sec(c+dx))^n \tan^{\frac{3}{2}}(c+dx)\,dx$ . . . 1659
3.230 $\int (a+a\sec(c+dx))^n \sqrt{\tan(c+dx)}\,dx$ . . . 1664
3.231 $\int \frac{(a+a\sec(c+dx))^n}{\sqrt{\tan(c+dx)}}\,dx$ . . . 1669
3.232 $\int \frac{(a+a\sec(c+dx))^n}{\tan^{\frac{3}{2}}(c+dx)}\,dx$ . . . 1674
3.233 $\int (e\cot(c+dx))^{5/2}(a+a\sec(c+dx))\,dx$ . . . 1679
3.234 $\int (e\cot(c+dx))^{3/2}(a+a\sec(c+dx))\,dx$ . . . 1690
3.235 $\int \sqrt{e\cot(c+dx)}(a+a\sec(c+dx))\,dx$ . . . 1701
3.236 $\int \frac{a+a\sec(c+dx)}{\sqrt{e\cot(c+dx)}}\,dx$ . . . 1711
3.237 $\int \frac{a+a\sec(c+dx)}{(e\cot(c+dx))^{3/2}}\,dx$ . . . 1721
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3.239 $\int (e\cot(c+dx))^{3/2}(a+a\sec(c+dx))^2\,dx$ . . . 1737
3.240 $\int \sqrt{e\cot(c+dx)}(a+a\sec(c+dx))^2\,dx$ . . . 1743
3.241 $\int \frac{(a+a\sec(c+dx))^2}{\sqrt{e\cot(c+dx)}}\,dx$ . . . 1749

3.242 $\int \frac{(a+a\sec(c+dx))^2}{(e\cot(c+dx))^{3/2}}\,dx$ . . . 1755
3.243 $\int \frac{(e\cot(c+dx))^{3/2}}{a+a\sec(c+dx)}\,dx$ . . . 1761
3.244 $\int \frac{\sqrt{e\cot(c+dx)}}{a+a\sec(c+dx)}\,dx$ . . . 1773
3.245 $\int \frac{1}{\sqrt{e\cot(c+dx)}(a+a\sec(c+dx))}\,dx$ . . . 1784
3.246 $\int \frac{1}{(e\cot(c+dx))^{3/2}(a+a\sec(c+dx))}\,dx$ . . . 1796
3.247 $\int \frac{1}{(e\cot(c+dx))^{5/2}(a+a\sec(c+dx))}\,dx$ . . . 1806
3.248 $\int \frac{1}{(e\cot(c+dx))^{7/2}(a+a\sec(c+dx))}\,dx$ . . . 1817
3.249 $\int \frac{1}{(e\cot(c+dx))^{9/2}(a+a\sec(c+dx))}\,dx$ . . . 1828
3.250 $\int \frac{1}{\sqrt{e\cot(c+dx)}(a+a\sec(c+dx))^2}\,dx$ . . . 1840
3.251 $\int \frac{1}{(e\cot(c+dx))^{3/2}(a+a\sec(c+dx))^2}\,dx$ . . . 1846
3.252 $\int \frac{1}{(e\cot(c+dx))^{5/2}(a+a\sec(c+dx))^2}\,dx$ . . . 1852
3.253 $\int \frac{1}{(e\cot(c+dx))^{7/2}(a+a\sec(c+dx))^2}\,dx$ . . . 1858
3.254 $\int \frac{1}{(e\cot(c+dx))^{9/2}(a+a\sec(c+dx))^2}\,dx$ . . . 1865
3.255 $\int \frac{1}{(e\cot(c+dx))^{11/2}(a+a\sec(c+dx))^2}\,dx$ . . . 1872
3.256 $\int (a+b\sec(c+dx))\tan^7(c+dx)\,dx$ . . . 1879
3.257 $\int (a+b\sec(c+dx))\tan^5(c+dx)\,dx$ . . . 1888
3.258 $\int (a+b\sec(c+dx))\tan^3(c+dx)\,dx$ . . . 1896
3.259 $\int (a+b\sec(c+dx))\tan(c+dx)\,dx$ . . . 1903
3.260 $\int \cot(c+dx)(a+b\sec(c+dx))\,dx$ . . . 1908
3.261 $\int \cot^3(c+dx)(a+b\sec(c+dx))\,dx$ . . . 1913
3.262 $\int \cot^5(c+dx)(a+b\sec(c+dx))\,dx$ . . . 1920
3.263 $\int \cot^7(c+dx)(a+b\sec(c+dx))\,dx$ . . . 1928
3.264 $\int (a+b\sec(c+dx))\tan^6(c+dx)\,dx$ . . . 1937
3.265 $\int (a+b\sec(c+dx))\tan^4(c+dx)\,dx$ . . . 1944
3.266 $\int (a+b\sec(c+dx))\tan^2(c+dx)\,dx$ . . . 1950
3.267 $\int \cot^2(c+dx)(a+b\sec(c+dx))\,dx$ . . . 1955
3.268 $\int \cot^4(c+dx)(a+b\sec(c+dx))\,dx$ . . . 1960
3.269 $\int \cot^6(c+dx)(a+b\sec(c+dx))\,dx$ . . . 1965
3.270 $\int \cot^8(c+dx)(a+b\sec(c+dx))\,dx$ . . . 1971
3.271 $\int (a+b\sec(c+dx))^2\tan^9(c+dx)\,dx$ . . . 1978
3.272 $\int (a+b\sec(c+dx))^2\tan^7(c+dx)\,dx$ . . . 1985
3.273 $\int (a+b\sec(c+dx))^2\tan^5(c+dx)\,dx$ . . . 1992
3.274 $\int (a+b\sec(c+dx))^2\tan^3(c+dx)\,dx$ . . . 1998
3.275 $\int (a+b\sec(c+dx))^2\tan(c+dx)\,dx$ . . . 2004
3.276 $\int \cot(c+dx)(a+b\sec(c+dx))^2\,dx$ . . . 2010
3.277 $\int \cot^3(c+dx)(a+b\sec(c+dx))^2\,dx$ . . . 2016
3.278 $\int \cot^5(c+dx)(a+b\sec(c+dx))^2\,dx$ . . . 2022
3.279 $\int (a+b\sec(c+dx))^2\tan^6(c+dx)\,dx$ . . . 2030

3.280 $\int (a+b\sec(c+dx))^2 \tan^4(c+dx)\,dx$ . . . 2036
3.281 $\int (a+b\sec(c+dx))^2 \tan^2(c+dx)\,dx$ . . . 2042
3.282 $\int \cot^2(c+dx)(a+b\sec(c+dx))^2\,dx$ . . . 2047
3.283 $\int \cot^4(c+dx)(a+b\sec(c+dx))^2\,dx$ . . . 2051
3.284 $\int \cot^6(c+dx)(a+b\sec(c+dx))^2\,dx$ . . . 2056
3.285 $\int \cot^8(c+dx)(a+b\sec(c+dx))^2\,dx$ . . . 2062
3.286 $\int \frac{\tan^9(c+dx)}{a+b\sec(c+dx)}\,dx$ . . . 2068
3.287 $\int \frac{\tan^7(c+dx)}{a+b\sec(c+dx)}\,dx$ . . . 2075
3.288 $\int \frac{\tan^5(c+dx)}{a+b\sec(c+dx)}\,dx$ . . . 2082
3.289 $\int \frac{\tan^3(c+dx)}{a+b\sec(c+dx)}\,dx$ . . . 2088
3.290 $\int \frac{\tan(c+dx)}{a+b\sec(c+dx)}\,dx$ . . . 2094
3.291 $\int \frac{\cot(c+dx)}{a+b\sec(c+dx)}\,dx$ . . . 2099
3.292 $\int \frac{\cot^3(c+dx)}{a+b\sec(c+dx)}\,dx$ . . . 2104
3.293 $\int \frac{\cot^5(c+dx)}{a+b\sec(c+dx)}\,dx$ . . . 2110
3.294 $\int \frac{\tan^6(c+dx)}{a+b\sec(c+dx)}\,dx$ . . . 2117
3.295 $\int \frac{\tan^4(c+dx)}{a+b\sec(c+dx)}\,dx$ . . . 2126
3.296 $\int \frac{\tan^2(c+dx)}{a+b\sec(c+dx)}\,dx$ . . . 2135
3.297 $\int \frac{\cot^2(c+dx)}{a+b\sec(c+dx)}\,dx$ . . . 2142
3.298 $\int \frac{\cot^4(c+dx)}{a+b\sec(c+dx)}\,dx$ . . . 2151
3.299 $\int \frac{\tan^9(c+dx)}{(a+b\sec(c+dx))^2}\,dx$ . . . 2164
3.300 $\int \frac{\tan^7(c+dx)}{(a+b\sec(c+dx))^2}\,dx$ . . . 2171
3.301 $\int \frac{\tan^5(c+dx)}{(a+b\sec(c+dx))^2}\,dx$ . . . 2178
3.302 $\int \frac{\tan^3(c+dx)}{(a+b\sec(c+dx))^2}\,dx$ . . . 2185
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3.304 $\int \frac{\cot(c+dx)}{(a+b\sec(c+dx))^2}\,dx$ . . . 2196
3.305 $\int \frac{\cot^3(c+dx)}{(a+b\sec(c+dx))^2}\,dx$ . . . 2202
3.306 $\int \frac{\cot^5(c+dx)}{(a+b\sec(c+dx))^2}\,dx$ . . . 2209
3.307 $\int \frac{\tan^6(c+dx)}{(a+b\sec(c+dx))^2}\,dx$ . . . 2217
3.308 $\int \frac{\tan^4(c+dx)}{(a+b\sec(c+dx))^2}\,dx$ . . . 2225
3.309 $\int \frac{\tan^2(c+dx)}{(a+b\sec(c+dx))^2}\,dx$ . . . 2233
3.310 $\int \frac{\cot^2(c+dx)}{(a+b\sec(c+dx))^2}\,dx$ . . . 2240
3.311 $\int \frac{\cot^4(c+dx)}{(a+b\sec(c+dx))^2}\,dx$ . . . 2247
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3.313 $\int \frac{(e\tan(c+dx))^{3/2}}{a+b\sec(c+dx)}\,dx$ . . . 2271

3.314 $\int \frac{\sqrt{e\tan(c+dx)}}{a+b\sec(c+dx)}\,dx$ . . . 2289
3.315 $\int \frac{1}{(a+b\sec(c+dx))\sqrt{e\tan(c+dx)}}\,dx$ . . . 2302
3.316 $\int \frac{1}{(a+b\sec(c+dx))(e\tan(c+dx))^{3/2}}\,dx$ . . . 2314
3.317 $\int \frac{1}{(a+b\sec(c+dx))(e\tan(c+dx))^{5/2}}\,dx$ . . . 2329
3.318 $\int \sqrt{a+b\sec(c+dx)}\tan^5(c+dx)\,dx$ . . . 2344
3.319 $\int \sqrt{a+b\sec(c+dx)}\tan^3(c+dx)\,dx$ . . . 2352
3.320 $\int \sqrt{a+b\sec(c+dx)}\tan(c+dx)\,dx$ . . . 2358
3.321 $\int \cot(c+dx)\sqrt{a+b\sec(c+dx)}\,dx$ . . . 2364
3.322 $\int \cot^3(c+dx)\sqrt{a+b\sec(c+dx)}\,dx$ . . . 2371
3.323 $\int \sqrt{a+b\sec(c+dx)}\tan^2(c+dx)\,dx$ . . . 2381
3.324 $\int \sqrt{a+b\sec(c+dx)}\,dx$ . . . 2390
3.325 $\int \cot^2(c+dx)\sqrt{a+b\sec(c+dx)}\,dx$ . . . 2395
3.326 $\int \frac{\tan^5(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$ . . . 2401
3.327 $\int \frac{\tan^3(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$ . . . 2408
3.328 $\int \frac{\tan(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$ . . . 2414
3.329 $\int \frac{\cot(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$ . . . 2419
3.330 $\int \frac{\cot^3(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$ . . . 2425
3.331 $\int \frac{\tan^4(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$ . . . 2433
3.332 $\int \frac{\tan^2(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$ . . . 2440
3.333 $\int \frac{1}{\sqrt{a+b\sec(c+dx)}}\,dx$ . . . 2448
3.334 $\int \frac{\cot^2(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$ . . . 2453
3.335 $\int \frac{\tan^5(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx$ . . . 2459
3.336 $\int \frac{\tan^3(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx$ . . . 2465
3.337 $\int \frac{\tan(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx$ . . . 2472
3.338 $\int \frac{\cot(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx$ . . . 2478
3.339 $\int \frac{\cot^3(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx$ . . . 2484
3.340 $\int \frac{\tan^4(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx$ . . . 2490
3.341 $\int \frac{\tan^2(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx$ . . . 2498
3.342 $\int \frac{1}{(a+b\sec(c+dx))^{3/2}}\,dx$ . . . 2507
3.343 $\int \frac{\cot^2(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx$ . . . 2516
3.344 $\int (a+b\sec(e+fx))^3(d\tan(e+fx))^n\,dx$ . . . 2523
3.345 $\int (a+b\sec(e+fx))^2(d\tan(e+fx))^n\,dx$ . . . 2528
3.346 $\int (a+b\sec(e+fx))(d\tan(e+fx))^n\,dx$ . . . 2533
3.347 $\int \frac{(d\tan(e+fx))^n}{a+b\sec(e+fx)}\,dx$ . . . 2538
3.348 $\int (a+b\sec(c+dx))^{3/2}(e\tan(c+dx))^m\,dx$ . . . 2543

3.349 $\int \sqrt{a+b\sec(c+dx)}(e\tan(c+dx))^m\,dx$ . . . 2547
3.350 $\int \frac{(e\tan(c+dx))^m}{\sqrt{a+b\sec(c+dx)}}\,dx$ . . . 2551
3.351 $\int \frac{(e\tan(c+dx))^m}{(a+b\sec(c+dx))^{3/2}}\,dx$ . . . 2555
3.352 $\int (a+b\sec(c+dx))^n(e\tan(c+dx))^m\,dx$ . . . 2559
3.353 $\int (a+b\sec(c+dx))^n\tan^5(c+dx)\,dx$ . . . 2563
3.354 $\int (a+b\sec(c+dx))^n\tan^3(c+dx)\,dx$ . . . 2568
3.355 $\int (a+b\sec(c+dx))^n\tan(c+dx)\,dx$ . . . 2573
3.356 $\int \cot(c+dx)(a+b\sec(c+dx))^n\,dx$ . . . 2578
3.357 $\int \cot^3(c+dx)(a+b\sec(c+dx))^n\,dx$ . . . 2583
3.358 $\int (a+b\sec(c+dx))^n\tan^4(c+dx)\,dx$ . . . 2588
3.359 $\int (a+b\sec(c+dx))^n\tan^2(c+dx)\,dx$ . . . 2592
3.360 $\int \cot^2(c+dx)(a+b\sec(c+dx))^n\,dx$ . . . 2600
3.361 $\int \cot^4(c+dx)(a+b\sec(c+dx))^n\,dx$ . . . 2604
3.362 $\int (a+b\sec(c+dx))^n\tan^{\frac{3}{2}}(c+dx)\,dx$ . . . 2608
3.363 $\int (a+b\sec(c+dx))^n\sqrt{\tan(c+dx)}\,dx$ . . . 2612
3.364 $\int \frac{(a+b\sec(c+dx))^n}{\sqrt{\tan(c+dx)}}\,dx$ . . . 2616
3.365 $\int \frac{(a+b\sec(c+dx))^n}{\tan^{\frac{3}{2}}(c+dx)}\,dx$ . . . 2620

3.1 $\int (a + a\sec(c+dx))\tan^9(c+dx)\,dx$

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3.1.1 Optimal result

Integrand size = 19, antiderivative size = 151

$$\begin{aligned}\int (a + a\sec(c+dx))\tan^9(c+dx)\,dx = &-\frac{a\log(\cos(c+dx))}{d} + \frac{a\sec(c+dx)}{d} \\ &- \frac{2a\sec^2(c+dx)}{d} - \frac{4a\sec^3(c+dx)}{3d} \\ &+ \frac{3a\sec^4(c+dx)}{2d} + \frac{6a\sec^5(c+dx)}{5d} \\ &- \frac{2a\sec^6(c+dx)}{3d} - \frac{4a\sec^7(c+dx)}{7d} \\ &+ \frac{a\sec^8(c+dx)}{8d} + \frac{a\sec^9(c+dx)}{9d}\end{aligned}$$

output

```
-a*ln(cos(d*x+c))/d+a*sec(d*x+c)/d-2*a*sec(d*x+c)^2/d-4/3*a*sec(d*x+c)^3/d
+3/2*a*sec(d*x+c)^4/d+6/5*a*sec(d*x+c)^5/d-2/3*a*sec(d*x+c)^6/d-4/7*a*sec(
d*x+c)^7/d+1/8*a*sec(d*x+c)^8/d+1/9*a*sec(d*x+c)^9/d
```

3.1.2 Mathematica [A] (verified)

Time = 0.57 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.89

$$\int (a + a\sec(c+dx))\tan^9(c+dx)\,dx$$
$$= \frac{a\sec(c+dx)}{d} - \frac{4a\sec^3(c+dx)}{3d} + \frac{6a\sec^5(c+dx)}{5d} - \frac{4a\sec^7(c+dx)}{7d} + \frac{a\sec^9(c+dx)}{9d}$$
$$- \frac{a(24\log(\cos(c+dx)) + 12\tan^2(c+dx) - 6\tan^4(c+dx) + 4\tan^6(c+dx) - 3\tan^8(c+dx))}{24d}$$

input
```
Integrate[(a + a*Sec[c + d*x])*Tan[c + d*x]^9,x]
```

output
```
(a*Sec[c + d*x])/d - (4*a*Sec[c + d*x]^3)/(3*d) + (6*a*Sec[c + d*x]^5)/(5*
d) - (4*a*Sec[c + d*x]^7)/(7*d) + (a*Sec[c + d*x]^9)/(9*d) - (a*(24*Log[Co
s[c + d*x]] + 12*Tan[c + d*x]^2 - 6*Tan[c + d*x]^4 + 4*Tan[c + d*x]^6 - 3*
Tan[c + d*x]^8))/(24*d)
```

3.1.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.77, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {3042, 25, 4367, 27, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^9(c+dx)(a\sec(c+dx)+a)\,dx$$
$$\downarrow 3042$$
$$\int -\cot\left(c+dx+\frac{\pi}{2}\right)^9\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)dx$$
$$\downarrow 25$$
$$-\int \cot\left(\frac{1}{2}(2c+\pi)+dx\right)^9\left(\csc\left(\frac{1}{2}(2c+\pi)+dx\right)a+a\right)dx$$
$$\downarrow 4367$$
$$-\frac{\int a^9(1-\cos(c+dx))^4(\cos(c+dx)+1)^5\sec^{10}(c+dx)d\cos(c+dx)}{a^8 d}$$

$$\downarrow 27$$

$$-\frac{a\int(1-\cos(c+dx))^4(\cos(c+dx)+1)^5\sec^{10}(c+dx)d\cos(c+dx)}{d}$$

$$\downarrow 99$$

$$-\frac{a\int\left(\sec^{10}(c+dx)+\sec^9(c+dx)-4\sec^8(c+dx)-4\sec^7(c+dx)+6\sec^6(c+dx)+6\sec^5(c+dx)-4\sec^4(c+\right.}{d}$$

$$\downarrow 2009$$

$$-\frac{a(-\frac{1}{9}\sec^9(c+dx)-\frac{1}{8}\sec^8(c+dx)+\frac{4}{7}\sec^7(c+dx)+\frac{2}{3}\sec^6(c+dx)-\frac{6}{5}\sec^5(c+dx)-\frac{3}{2}\sec^4(c+dx)+\frac{4}{3}\sec^{3}}{d}$$

input
```
Int[(a + a*Sec[c + d*x])*Tan[c + d*x]^9,x]
```

output
```
-((a*(Log[Cos[c + d*x]] - Sec[c + d*x] + 2*Sec[c + d*x]^2 + (4*Sec[c + d*x
]^3)/3 - (3*Sec[c + d*x]^4)/2 - (6*Sec[c + d*x]^5)/5 + (2*Sec[c + d*x]^6)/
3 + (4*Sec[c + d*x]^7)/7 - Sec[c + d*x]^8/8 - Sec[c + d*x]^9/9))/d)
```

3.1.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4367

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Simp[1/(a^(m - n - 1)*b^n*d)   Subst[Int[(a - b*x)^((m -
1)/2)*((a + b*x)^((m - 1)/2 + n)/x^(m + n)), x], x, Sin[c + d*x]], x] /; Fr
eeQ[{a, b, c, d}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && Integer
Q[n]
```

3.1.4 Maple [A] (verified)

Time = 1.46 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.66

method	result
derivativedivides	$\frac{a\left(\frac{\sec(dx+c)^9}{9}+\frac{\sec(dx+c)^8}{8}-\frac{4\sec(dx+c)^7}{7}-\frac{2\sec(dx+c)^6}{3}+\frac{6\sec(dx+c)^5}{5}+\frac{3\sec(dx+c)^4}{2}-\frac{4\sec(dx+c)^3}{3}-2\sec(dx+c)^2+\sec(dx$
default	$\frac{a\left(\frac{\sec(dx+c)^9}{9}+\frac{\sec(dx+c)^8}{8}-\frac{4\sec(dx+c)^7}{7}-\frac{2\sec(dx+c)^6}{3}+\frac{6\sec(dx+c)^5}{5}+\frac{3\sec(dx+c)^4}{2}-\frac{4\sec(dx+c)^3}{3}-2\sec(dx+c)^2+\sec(dx$
parts	$\frac{a\left(\frac{\tan(dx+c)^8}{8}-\frac{\tan(dx+c)^6}{6}+\frac{\tan(dx+c)^4}{4}-\frac{\tan(dx+c)^2}{2}+\frac{\ln\left(1+\tan(dx+c)^2\right)}{2}\right)}{d}+\frac{a\left(\frac{\sec(dx+c)^9}{9}-\frac{4\sec(dx+c)^7}{7}+\frac{6\sec(dx+c)}{5}\right.}{d}$
risch	$iax+\frac{2iac}{d}+\frac{2a\left(315\,e^{17i(dx+c)}-1260\,e^{16i(dx+c)}+840\,e^{15i(dx+c)}-5040\,e^{14i(dx+c)}+4788\,e^{13i(dx+c)}-14280\,e^{12i(dx+$

input

```
int((a+a*sec(d*x+c))*tan(d*x+c)^9,x,method=_RETURNVERBOSE)
```

output

```
a/d*(1/9*sec(d*x+c)^9+1/8*sec(d*x+c)^8-4/7*sec(d*x+c)^7-2/3*sec(d*x+c)^6+6
/5*sec(d*x+c)^5+3/2*sec(d*x+c)^4-4/3*sec(d*x+c)^3-2*sec(d*x+c)^2+sec(d*x+c
)+ln(sec(d*x+c)))
```

3.1.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.81

$$\int (a+a\sec(c+dx))\tan^9(c+dx)\,dx =$$

$$-\frac{2520\,a\cos(dx+c)^9\log(-\cos(dx+c))-2520\,a\cos(dx+c)^8+5040\,a\cos(dx+c)^7+3360\,a\cos(dx+$$

input

```
integrate((a+a*sec(d*x+c))*tan(d*x+c)^9,x, algorithm="fricas")
```

output
```
-1/2520*(2520*a*cos(d*x + c)^9*log(-cos(d*x + c)) - 2520*a*cos(d*x + c)^8
+ 5040*a*cos(d*x + c)^7 + 3360*a*cos(d*x + c)^6 - 3780*a*cos(d*x + c)^5 -
3024*a*cos(d*x + c)^4 + 1680*a*cos(d*x + c)^3 + 1440*a*cos(d*x + c)^2 - 31
5*a*cos(d*x + c) - 280*a)/(d*cos(d*x + c)^9)
```

3.1.6 Sympy [A] (verification not implemented)

Time = 1.59 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.22

$$\int (a + a\sec(c+dx))\tan^9(c+dx)\,dx$$

$$= \begin{cases} \frac{a\log\left(\tan^2(c+dx)+1\right)}{2d} + \frac{a\tan^8(c+dx)\sec(c+dx)}{9d} + \frac{a\tan^8(c+dx)}{8d} - \frac{8a\tan^6(c+dx)\sec(c+dx)}{63d} - \frac{a\tan^6(c+dx)}{6d} + \frac{16a\tan^4(c+dx)\,\mathrm{s}}{105d} \\ x(a\sec(c)+a)\tan^9(c) \end{cases}$$

input
```
integrate((a+a*sec(d*x+c))*tan(d*x+c)**9,x)
```

output
```
Piecewise((a*log(tan(c + d*x)**2 + 1)/(2*d) + a*tan(c + d*x)**8*sec(c + d*
x)/(9*d) + a*tan(c + d*x)**8/(8*d) - 8*a*tan(c + d*x)**6*sec(c + d*x)/(63*
d) - a*tan(c + d*x)**6/(6*d) + 16*a*tan(c + d*x)**4*sec(c + d*x)/(105*d) +
 a*tan(c + d*x)**4/(4*d) - 64*a*tan(c + d*x)**2*sec(c + d*x)/(315*d) - a*t
an(c + d*x)**2/(2*d) + 128*a*sec(c + d*x)/(315*d), Ne(d, 0)), (x*(a*sec(c)
 + a)*tan(c)**9, True))
```

3.1.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.77

$$\int (a + a\sec(c+dx))\tan^9(c+dx)\,dx =$$

$$-\frac{2520\,a\log\left(\cos\left(dx+c\right)\right) - \frac{2520\,a\cos(dx+c)^8 - 5040\,a\cos(dx+c)^7 - 3360\,a\cos(dx+c)^6 + 3780\,a\cos(dx+c)^5 + 3024\,a\cos(dx+c)^4 - 168}{\cos(dx+c)^9}}{2520\,d}$$

input
```
integrate((a+a*sec(d*x+c))*tan(d*x+c)^9,x, algorithm="maxima")
```

output
```
-1/2520*(2520*a*log(cos(d*x + c)) - (2520*a*cos(d*x + c)^8 - 5040*a*cos(d*
x + c)^7 - 3360*a*cos(d*x + c)^6 + 3780*a*cos(d*x + c)^5 + 3024*a*cos(d*x
+ c)^4 - 1680*a*cos(d*x + c)^3 - 1440*a*cos(d*x + c)^2 + 315*a*cos(d*x + c
) + 280*a)/cos(d*x + c)^9)/d
```

3.1.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 293 vs. $2(137) = 274$.

Time = 6.42 (sec) , antiderivative size = 293, normalized size of antiderivative = 1.94

$$\int (a + a\sec(c+dx))\tan^9(c+dx)\,dx$$

$$= \frac{2520\,a\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right) - 2520\,a\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right) + \frac{9177\,a+\frac{87633\,a(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{375732\,a(\cos(dx+c)-1)}{(\cos(dx+c)+1)^2}}{}}{}$$

input
```
integrate((a+a*sec(d*x+c))*tan(d*x+c)^9,x, algorithm="giac")
```

output
```
1/2520*(2520*a*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)) - 2520
*a*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)) + (9177*a + 87633*
a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 375732*a*(cos(d*x + c) - 1)^2/(c
os(d*x + c) + 1)^2 + 953988*a*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 +
1594782*a*(cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4 + 1336734*a*(cos(d*x +
 c) - 1)^5/(cos(d*x + c) + 1)^5 + 781956*a*(cos(d*x + c) - 1)^6/(cos(d*x +
 c) + 1)^6 + 302004*a*(cos(d*x + c) - 1)^7/(cos(d*x + c) + 1)^7 + 69201*a*
(cos(d*x + c) - 1)^8/(cos(d*x + c) + 1)^8 + 7129*a*(cos(d*x + c) - 1)^9/(c
os(d*x + c) + 1)^9)/((cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)^9)/d
```

3.1.9 Mupad [B] (verification not implemented)

Time = 17.95 (sec) , antiderivative size = 259, normalized size of antiderivative = 1.72

$$\int (a + a\sec(c+dx))\tan^9(c+dx)\,dx = \frac{2\,a\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2\right)}{d}$$

$$- \frac{2\,a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{16} - 18\,a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{14} + \frac{218\,a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{12}}{3} - 174\,a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{10} + \frac{1382\,a\tan(}{5}}{d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{18} - 9\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{16} + 36\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{14} - 84\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{12} + 126\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{10} -}$$

input
```
int(tan(c + d*x)^9*(a + a/cos(c + d*x)),x)
```

output

```
(2*a*atanh(tan(c/2 + (d*x)/2)^2))/d - ((256*a)/315 - (326*a*tan(c/2 + (d*x
)/2)^2)/35 + (1654*a*tan(c/2 + (d*x)/2)^4)/35 - (2114*a*tan(c/2 + (d*x)/2)
^6)/15 + (1382*a*tan(c/2 + (d*x)/2)^8)/5 - 174*a*tan(c/2 + (d*x)/2)^10 + (
218*a*tan(c/2 + (d*x)/2)^12)/3 - 18*a*tan(c/2 + (d*x)/2)^14 + 2*a*tan(c/2
+ (d*x)/2)^16)/(d*(9*tan(c/2 + (d*x)/2)^2 - 36*tan(c/2 + (d*x)/2)^4 + 84*t
an(c/2 + (d*x)/2)^6 - 126*tan(c/2 + (d*x)/2)^8 + 126*tan(c/2 + (d*x)/2)^10
 - 84*tan(c/2 + (d*x)/2)^12 + 36*tan(c/2 + (d*x)/2)^14 - 9*tan(c/2 + (d*x)
/2)^16 + tan(c/2 + (d*x)/2)^18 - 1))
```

3.2 $\int (a + a\sec(c+dx))\tan^7(c+dx)\,dx$

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3.2.1 Optimal result

Integrand size = 19, antiderivative size = 118

$$\int (a + a\sec(c+dx))\tan^7(c+dx)\,dx = \frac{a\log(\cos(c+dx))}{d} - \frac{a\sec(c+dx)}{d} + \frac{3a\sec^2(c+dx)}{2d} + \frac{a\sec^3(c+dx)}{d} - \frac{3a\sec^4(c+dx)}{4d} - \frac{3a\sec^5(c+dx)}{5d} + \frac{a\sec^6(c+dx)}{6d} + \frac{a\sec^7(c+dx)}{7d}$$

output
```
a*ln(cos(d*x+c))/d-a*sec(d*x+c)/d+3/2*a*sec(d*x+c)^2/d+a*sec(d*x+c)^3/d-3/
4*a*sec(d*x+c)^4/d-3/5*a*sec(d*x+c)^5/d+1/6*a*sec(d*x+c)^6/d+1/7*a*sec(d*x
+c)^7/d
```

3.2.2 Mathematica [A] (verified)

Time = 0.51 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.90

$$\begin{aligned}&\int (a + a\sec(c+dx))\tan^7(c+dx)\,dx \\ &= -\frac{a\sec(c+dx)}{d} + \frac{a\sec^3(c+dx)}{d} - \frac{3a\sec^5(c+dx)}{5d} + \frac{a\sec^7(c+dx)}{7d} \\ &\quad + \frac{a(12\log(\cos(c+dx)) + 6\tan^2(c+dx) - 3\tan^4(c+dx) + 2\tan^6(c+dx))}{12d}\end{aligned}$$

input
```
Integrate[(a + a*Sec[c + d*x])*Tan[c + d*x]^7,x]
```

output
```
-((a*Sec[c + d*x])/d) + (a*Sec[c + d*x]^3)/d - (3*a*Sec[c + d*x]^5)/(5*d)
+ (a*Sec[c + d*x]^7)/(7*d) + (a*(12*Log[Cos[c + d*x]] + 6*Tan[c + d*x]^2 -
 3*Tan[c + d*x]^4 + 2*Tan[c + d*x]^6))/(12*d)
```

3.2.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.78, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {3042, 25, 4367, 27, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^7(c+dx)(a\sec(c+dx)+a)\,dx$$

$$\downarrow 3042$$

$$\int -\cot\left(c+dx+\frac{\pi}{2}\right)^7\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)dx$$

$$\downarrow 25$$

$$-\int \cot\left(\frac{1}{2}(2c+\pi)+dx\right)^7\left(\csc\left(\frac{1}{2}(2c+\pi)+dx\right)a+a\right)dx$$

$$\downarrow 4367$$

$$-\frac{\int a^7(1-\cos(c+dx))^3(\cos(c+dx)+1)^4\sec^8(c+dx)d\cos(c+dx)}{a^6 d}$$

$$\downarrow 27$$

$$-\frac{a\int(1-\cos(c+dx))^3(\cos(c+dx)+1)^4\sec^8(c+dx)d\cos(c+dx)}{d}$$

$$\downarrow 99$$

$$-\frac{a\int\left(\sec^8(c+dx)+\sec^7(c+dx)-3\sec^6(c+dx)-3\sec^5(c+dx)+3\sec^4(c+dx)+3\sec^3(c+dx)-\sec^2(c+d\right.}{d}$$

$$\downarrow 2009$$

$$-\frac{a\left(-\frac{1}{7}\sec^7(c+dx)-\frac{1}{6}\sec^6(c+dx)+\frac{3}{5}\sec^5(c+dx)+\frac{3}{4}\sec^4(c+dx)-\sec^3(c+dx)-\frac{3}{2}\sec^2(c+dx)+\sec(c+\right.}{d}$$

input
```
Int[(a + a*Sec[c + d*x])*Tan[c + d*x]^7,x]
```

output
```
-((a*(-Log[Cos[c + d*x]] + Sec[c + d*x] - (3*Sec[c + d*x]^2)/2 - Sec[c + d
*x]^3 + (3*Sec[c + d*x]^4)/4 + (3*Sec[c + d*x]^5)/5 - Sec[c + d*x]^6/6 - S
ec[c + d*x]^7/7))/d)
```

3.2.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4367
```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Simp[1/(a^(m - n - 1)*b^n*d)   Subst[Int[(a - b*x)^((m -
1)/2)*((a + b*x)^((m - 1)/2 + n)/x^(m + n)), x], x, Sin[c + d*x]], x] /; Fr
eeQ[{a, b, c, d}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && Integer
Q[n]
```

3.2.4 Maple [A] (verified)

Time = 1.03 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.69

method	result
derivativedivides	$\frac{a\left(\frac{\sec(dx+c)^7}{7}+\frac{\sec(dx+c)^6}{6}-\frac{3\sec(dx+c)^5}{5}-\frac{3\sec(dx+c)^4}{4}+\sec(dx+c)^3+\frac{3\sec(dx+c)^2}{2}-\sec(dx+c)-\ln(\sec(dx+c))\right)}{d}$
default	$\frac{a\left(\frac{\sec(dx+c)^7}{7}+\frac{\sec(dx+c)^6}{6}-\frac{3\sec(dx+c)^5}{5}-\frac{3\sec(dx+c)^4}{4}+\sec(dx+c)^3+\frac{3\sec(dx+c)^2}{2}-\sec(dx+c)-\ln(\sec(dx+c))\right)}{d}$
parts	$\frac{a\left(\frac{\tan(dx+c)^6}{6}-\frac{\tan(dx+c)^4}{4}+\frac{\tan(dx+c)^2}{2}-\frac{\ln\left(1+\tan(dx+c)^2\right)}{2}\right)}{d}+\frac{a\left(\frac{\sec(dx+c)^7}{7}-\frac{3\sec(dx+c)^5}{5}+\sec(dx+c)^3-\sec(dx+c)\right.}{d}$
risch	$-iax-\frac{2iac}{d}-\frac{2a(105\,e^{13i(dx+c)}-315\,e^{12i(dx+c)}+210\,e^{11i(dx+c)}-945\,e^{10i(dx+c)}+903\,e^{9i(dx+c)}-1820\,e^{8i(dx+c)}+}{105d(e^$

input
```
int((a+a*sec(d*x+c))*tan(d*x+c)^7,x,method=_RETURNVERBOSE)
```

output
```
a/d*(1/7*sec(d*x+c)^7+1/6*sec(d*x+c)^6-3/5*sec(d*x+c)^5-3/4*sec(d*x+c)^4+s
ec(d*x+c)^3+3/2*sec(d*x+c)^2-sec(d*x+c)-ln(sec(d*x+c)))
```

3.2.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.86

$$\int (a + a\sec(c+dx))\tan^7(c+dx)\,dx$$
$$= \frac{420\,a\cos(dx+c)^7\log(-\cos(dx+c)) - 420\,a\cos(dx+c)^6 + 630\,a\cos(dx+c)^5 + 420\,a\cos(dx+c)^4 -}{420\,d\cos(dx+c)^7}$$

input
```
integrate((a+a*sec(d*x+c))*tan(d*x+c)^7,x, algorithm="fricas")
```

output
```
1/420*(420*a*cos(d*x + c)^7*log(-cos(d*x + c)) - 420*a*cos(d*x + c)^6 + 63
0*a*cos(d*x + c)^5 + 420*a*cos(d*x + c)^4 - 315*a*cos(d*x + c)^3 - 252*a*c
os(d*x + c)^2 + 70*a*cos(d*x + c) + 60*a)/(d*cos(d*x + c)^7)
```

3.2.6 Sympy [A] (verification not implemented)

Time = 0.84 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.25

$$\int (a + a\sec(c+dx))\tan^7(c+dx)\,dx$$
$$= \begin{cases} -\frac{a\log\left(\tan^2(c+dx)+1\right)}{2d} + \frac{a\tan^6(c+dx)\sec(c+dx)}{7d} + \frac{a\tan^6(c+dx)}{6d} - \frac{6a\tan^4(c+dx)\sec(c+dx)}{35d} - \frac{a\tan^4(c+dx)}{4d} + \frac{8a\tan^2(c+dx)}{35d} \\ x(a\sec(c)+a)\tan^7(c) \end{cases}$$

input
```
integrate((a+a*sec(d*x+c))*tan(d*x+c)**7,x)
```

output
```
Piecewise((-a*log(tan(c + d*x)**2 + 1)/(2*d) + a*tan(c + d*x)**6*sec(c + d
*x)/(7*d) + a*tan(c + d*x)**6/(6*d) - 6*a*tan(c + d*x)**4*sec(c + d*x)/(35
*d) - a*tan(c + d*x)**4/(4*d) + 8*a*tan(c + d*x)**2*sec(c + d*x)/(35*d) +
a*tan(c + d*x)**2/(2*d) - 16*a*sec(c + d*x)/(35*d), Ne(d, 0)), (x*(a*sec(c
) + a)*tan(c)**7, True))
```

3.2.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.80

$$\int (a + a\sec(c+dx))\tan^7(c+dx)\,dx$$
$$= \frac{420\,a\log\left(\cos(dx+c)\right) - \frac{420\,a\cos(dx+c)^6 - 630\,a\cos(dx+c)^5 - 420\,a\cos(dx+c)^4 + 315\,a\cos(dx+c)^3 + 252\,a\cos(dx+c)^2 - 70\,a\cos(dx+c)}{\cos(dx+c)^7}}{420\,d}$$

input
```
integrate((a+a*sec(d*x+c))*tan(d*x+c)^7,x, algorithm="maxima")
```

output
```
1/420*(420*a*log(cos(d*x + c)) - (420*a*cos(d*x + c)^6 - 630*a*cos(d*x + c
)^5 - 420*a*cos(d*x + c)^4 + 315*a*cos(d*x + c)^3 + 252*a*cos(d*x + c)^2 -
 70*a*cos(d*x + c) - 60*a)/cos(d*x + c)^7)/d
```

3.2.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 247 vs. $2(108) = 216$.

Time $= 3.63$ (sec) , antiderivative size $= 247$, normalized size of antiderivative $= 2.09$

$$\int (a + a\sec(c+dx))\tan^7(c+dx)\,dx =$$

$$-\frac{420\,a\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right)-420\,a\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right)+\frac{1473\,a+\frac{11151\,a(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{36813\,a(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}}{420\,d}$$

input
```
integrate((a+a*sec(d*x+c))*tan(d*x+c)^7,x, algorithm="giac")
```

output
```
-1/420*(420*a*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)) - 420*a
*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)) + (1473*a + 11151*a*
(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 36813*a*(cos(d*x + c) - 1)^2/(cos(
d*x + c) + 1)^2 + 69475*a*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 + 5603
5*a*(cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4 + 28749*a*(cos(d*x + c) - 1)
^5/(cos(d*x + c) + 1)^5 + 8463*a*(cos(d*x + c) - 1)^6/(cos(d*x + c) + 1)^6
 + 1089*a*(cos(d*x + c) - 1)^7/(cos(d*x + c) + 1)^7)/((cos(d*x + c) - 1)/(
cos(d*x + c) + 1) + 1)^7)/d
```

3.2.9 Mupad [B] (verification not implemented)

Time $= 17.60$ (sec) , antiderivative size $= 204$, normalized size of antiderivative $= 1.73$

$$\int (a + a\sec(c+dx))\tan^7(c+dx)\,dx$$

$$=\frac{2\,a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{12}-14\,a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}+\frac{128\,a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^8}{3}-\frac{224\,a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6}{3}+\frac{166\,a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4}{5}}{d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{14}-7\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{12}+21\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}-35\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^8+35\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6-21\tan\right.}$$

$$-\frac{2\,a\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2\right)}{d}$$

input
```
int(tan(c + d*x)^7*(a + a/cos(c + d*x)),x)
```

output
```
((32*a)/35 - (42*a*tan(c/2 + (d*x)/2)^2)/5 + (166*a*tan(c/2 + (d*x)/2)^4)/
5 - (224*a*tan(c/2 + (d*x)/2)^6)/3 + (128*a*tan(c/2 + (d*x)/2)^8)/3 - 14*a
*tan(c/2 + (d*x)/2)^10 + 2*a*tan(c/2 + (d*x)/2)^12)/(d*(7*tan(c/2 + (d*x)/
2)^2 - 21*tan(c/2 + (d*x)/2)^4 + 35*tan(c/2 + (d*x)/2)^6 - 35*tan(c/2 + (d
*x)/2)^8 + 21*tan(c/2 + (d*x)/2)^10 - 7*tan(c/2 + (d*x)/2)^12 + tan(c/2 +
(d*x)/2)^14 - 1)) - (2*a*atanh(tan(c/2 + (d*x)/2)^2))/d
```

3.3 $\int (a + a\sec(c+dx))\tan^5(c+dx)\,dx$

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3.3.1 Optimal result

Integrand size = 19, antiderivative size = 87

$$\int (a + a\sec(c+dx))\tan^5(c+dx)\,dx = -\frac{a\log(\cos(c+dx))}{d} + \frac{a\sec(c+dx)}{d} - \frac{a\sec^2(c+dx)}{d} - \frac{2a\sec^3(c+dx)}{3d} + \frac{a\sec^4(c+dx)}{4d} + \frac{a\sec^5(c+dx)}{5d}$$

output
```
-a*ln(cos(d*x+c))/d+a*sec(d*x+c)/d-a*sec(d*x+c)^2/d-2/3*a*sec(d*x+c)^3/d+1
/4*a*sec(d*x+c)^4/d+1/5*a*sec(d*x+c)^5/d
```

3.3.2 Mathematica [A] (verified)

Time = 0.39 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.94

$$\int (a + a\sec(c+dx))\tan^5(c+dx)\,dx = \frac{a\sec(c+dx)}{d} - \frac{2a\sec^3(c+dx)}{3d} + \frac{a\sec^5(c+dx)}{5d} - \frac{a(4\log(\cos(c+dx)) + 2\tan^2(c+dx) - \tan^4(c+dx))}{4d}$$

input
```
Integrate[(a + a*Sec[c + d*x])*Tan[c + d*x]^5,x]
```

output
```
(a*Sec[c + d*x])/d - (2*a*Sec[c + d*x]^3)/(3*d) + (a*Sec[c + d*x]^5)/(5*d)
 - (a*(4*Log[Cos[c + d*x]] + 2*Tan[c + d*x]^2 - Tan[c + d*x]^4))/(4*d)
```

3.3.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.76, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {3042, 25, 4367, 27, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^5(c+dx)(a\sec(c+dx)+a)\,dx$$

$$\downarrow 3042$$

$$\int -\cot\left(c+dx+\frac{\pi}{2}\right)^5\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)dx$$

$$\downarrow 25$$

$$-\int \cot\left(\frac{1}{2}(2c+\pi)+dx\right)^5\left(\csc\left(\frac{1}{2}(2c+\pi)+dx\right)a+a\right)dx$$

$$\downarrow 4367$$

$$-\frac{\int a^5(1-\cos(c+dx))^2(\cos(c+dx)+1)^3\sec^6(c+dx)d\cos(c+dx)}{a^4 d}$$

$$\downarrow 27$$

$$-\frac{a\int(1-\cos(c+dx))^2(\cos(c+dx)+1)^3\sec^6(c+dx)d\cos(c+dx)}{d}$$

$$\downarrow 99$$

$$-\frac{a\int\left(\sec^6(c+dx)+\sec^5(c+dx)-2\sec^4(c+dx)-2\sec^3(c+dx)+\sec^2(c+dx)+\sec(c+dx)\right)d\cos(c+dx)}{d}$$

$$\downarrow 2009$$

$$-\frac{a\left(-\frac{1}{5}\sec^5(c+dx)-\frac{1}{4}\sec^4(c+dx)+\frac{2}{3}\sec^3(c+dx)+\sec^2(c+dx)-\sec(c+dx)+\log(\cos(c+dx))\right)}{d}$$

input
```
Int[(a + a*Sec[c + d*x])*Tan[c + d*x]^5,x]
```

output
```
-((a*(Log[Cos[c + d*x]] - Sec[c + d*x] + Sec[c + d*x]^2 + (2*Sec[c + d*x]^
3)/3 - Sec[c + d*x]^4/4 - Sec[c + d*x]^5/5))/d)
```

3.3.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4367
```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Simp[1/(a^(m - n - 1)*b^n*d)   Subst[Int[(a - b*x)^((m -
1)/2)*((a + b*x)^((m - 1)/2 + n)/x^(m + n)), x], x, Sin[c + d*x]], x] /; Fr
eeQ[{a, b, c, d}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && Integer
Q[n]
```

3.3.4 Maple [A] (verified)

Time = 0.86 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.69

method	result
derivativedivides	$\frac{a\left(\frac{\sec(dx+c)^5}{5}+\frac{\sec(dx+c)^4}{4}-\frac{2\sec(dx+c)^3}{3}-\sec(dx+c)^2+\sec(dx+c)+\ln(\sec(dx+c))\right)}{d}$
default	$\frac{a\left(\frac{\sec(dx+c)^5}{5}+\frac{\sec(dx+c)^4}{4}-\frac{2\sec(dx+c)^3}{3}-\sec(dx+c)^2+\sec(dx+c)+\ln(\sec(dx+c))\right)}{d}$
parts	$\frac{a\left(\frac{\tan(dx+c)^4}{4}-\frac{\tan(dx+c)^2}{2}+\frac{\ln\left(1+\tan(dx+c)^2\right)}{2}\right)}{d}+\frac{a\left(\frac{\sec(dx+c)^5}{5}-\frac{2\sec(dx+c)^3}{3}+\sec(dx+c)\right)}{d}$
risch	$iax+\frac{2iac}{d}+\frac{2a\left(15\,\mathrm{e}^{9i(dx+c)}-30\,\mathrm{e}^{8i(dx+c)}+20\,\mathrm{e}^{7i(dx+c)}-60\,\mathrm{e}^{6i(dx+c)}+58\,\mathrm{e}^{5i(dx+c)}-60\,\mathrm{e}^{4i(dx+c)}+20\,\mathrm{e}^{3i(dx+c)}-3\right.}{15d\left(\mathrm{e}^{2i(dx+c)}+1\right)^5}$

input
```
int((a+a*sec(d*x+c))*tan(d*x+c)^5,x,method=_RETURNVERBOSE)
```

output
```
a/d*(1/5*sec(d*x+c)^5+1/4*sec(d*x+c)^4-2/3*sec(d*x+c)^3-sec(d*x+c)^2+sec(d
*x+c)+ln(sec(d*x+c)))
```

3.3.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.91

$$\int (a + a\sec(c+dx))\tan^5(c+dx)\,dx =$$
$$-\frac{60\,a\cos(dx+c)^5\log(-\cos(dx+c)) - 60\,a\cos(dx+c)^4 + 60\,a\cos(dx+c)^3 + 40\,a\cos(dx+c)^2 - 15}{60\,d\cos(dx+c)^5}$$

input
```
integrate((a+a*sec(d*x+c))*tan(d*x+c)^5,x, algorithm="fricas")
```

output
```
-1/60*(60*a*cos(d*x + c)^5*log(-cos(d*x + c)) - 60*a*cos(d*x + c)^4 + 60*a
*cos(d*x + c)^3 + 40*a*cos(d*x + c)^2 - 15*a*cos(d*x + c) - 12*a)/(d*cos(d
*x + c)^5)
```

3.3.6 Sympy [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.29

$$\int (a + a\sec(c+dx))\tan^5(c+dx)\,dx$$
$$= \begin{cases} \frac{a\log(\tan^2(c+dx)+1)}{2d} + \frac{a\tan^4(c+dx)\sec(c+dx)}{5d} + \frac{a\tan^4(c+dx)}{4d} - \frac{4a\tan^2(c+dx)\sec(c+dx)}{15d} - \frac{a\tan^2(c+dx)}{2d} + \frac{8a\sec(c+dx)}{15d} & \\ x(a\sec(c)+a)\tan^5(c) & \end{cases}$$

input
```
integrate((a+a*sec(d*x+c))*tan(d*x+c)**5,x)
```

output
```
Piecewise((a*log(tan(c + d*x)**2 + 1)/(2*d) + a*tan(c + d*x)**4*sec(c + d*
x)/(5*d) + a*tan(c + d*x)**4/(4*d) - 4*a*tan(c + d*x)**2*sec(c + d*x)/(15*
d) - a*tan(c + d*x)**2/(2*d) + 8*a*sec(c + d*x)/(15*d), Ne(d, 0)), (x*(a*s
ec(c) + a)*tan(c)**5, True))
```

3.3.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.83

$$\int (a + a\sec(c+dx))\tan^5(c+dx)\,dx$$
$$= -\frac{60\,a\log(\cos(dx+c)) - \frac{60\,a\cos(dx+c)^4 - 60\,a\cos(dx+c)^3 - 40\,a\cos(dx+c)^2 + 15\,a\cos(dx+c) + 12\,a}{\cos(dx+c)^5}}{60\,d}$$

input
```
integrate((a+a*sec(d*x+c))*tan(d*x+c)^5,x, algorithm="maxima")
```

output
```
-1/60*(60*a*log(cos(d*x + c)) - (60*a*cos(d*x + c)^4 - 60*a*cos(d*x + c)^3
 - 40*a*cos(d*x + c)^2 + 15*a*cos(d*x + c) + 12*a)/cos(d*x + c)^5)/d
```

3.3.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 201 vs. 2(81) = 162.

Time = 1.60 (sec) , antiderivative size = 201, normalized size of antiderivative = 2.31

$$\int (a + a\sec(c+dx))\tan^5(c+dx)\,dx$$
$$= \frac{60\,a\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right) - 60\,a\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right) + \frac{201\,a + \frac{1125\,a(\cos(dx+c)-1)}{\cos(dx+c)+1} + \frac{2610\,a(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2} + \frac{1970\,a(\cos}{(\cos}}{\left(\frac{\cos(dx+c)}{\cos(dx+c)}\right.}}{60\,d}$$

input
```
integrate((a+a*sec(d*x+c))*tan(d*x+c)^5,x, algorithm="giac")
```

output
```
1/60*(60*a*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)) - 60*a*log
(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)) + (201*a + 1125*a*(cos(d
*x + c) - 1)/(cos(d*x + c) + 1) + 2610*a*(cos(d*x + c) - 1)^2/(cos(d*x + c
) + 1)^2 + 1970*a*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 + 805*a*(cos(d
*x + c) - 1)^4/(cos(d*x + c) + 1)^4 + 137*a*(cos(d*x + c) - 1)^5/(cos(d*x
+ c) + 1)^5)/((cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)^5)/d
```

3.3.9 Mupad [B] (verification not implemented)

Time = 18.38 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.74

$$\int (a + a\sec(c+dx))\tan^5(c+dx)\,dx = \frac{2\,a\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2\right)}{d} - \frac{2\,a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^8 - 10\,a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6 + \frac{62\,a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4}{3} - \frac{22\,a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2}{3} + \frac{16\,a}{15}}{d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{10} - 5\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^8 + 10\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6 - 10\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4 + 5\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2 - 1\right)}$$

input
```
int(tan(c + d*x)^5*(a + a/cos(c + d*x)),x)
```

output
```
(2*a*atanh(tan(c/2 + (d*x)/2)^2))/d - ((16*a)/15 - (22*a*tan(c/2 + (d*x)/2
)^2)/3 + (62*a*tan(c/2 + (d*x)/2)^4)/3 - 10*a*tan(c/2 + (d*x)/2)^6 + 2*a*t
an(c/2 + (d*x)/2)^8)/(d*(5*tan(c/2 + (d*x)/2)^2 - 10*tan(c/2 + (d*x)/2)^4
+ 10*tan(c/2 + (d*x)/2)^6 - 5*tan(c/2 + (d*x)/2)^8 + tan(c/2 + (d*x)/2)^10
 - 1))
```

3.4 $\int (a + a\sec(c+dx))\tan^3(c+dx)\,dx$

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3.4.1 Optimal result

Integrand size = 19, antiderivative size = 57

$$\int (a + a\sec(c+dx))\tan^3(c+dx)\,dx = \frac{a\log(\cos(c+dx))}{d} - \frac{a\sec(c+dx)}{d} + \frac{a\sec^2(c+dx)}{2d} + \frac{a\sec^3(c+dx)}{3d}$$

output
```
a*ln(cos(d*x+c))/d-a*sec(d*x+c)/d+1/2*a*sec(d*x+c)^2/d+1/3*a*sec(d*x+c)^3/
d
```

3.4.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.96

$$\int (a + a\sec(c+dx))\tan^3(c+dx)\,dx = -\frac{a\sec(c+dx)}{d} + \frac{a\sec^3(c+dx)}{3d} + \frac{a(2\log(\cos(c+dx)) + \tan^2(c+dx))}{2d}$$

input
```
Integrate[(a + a*Sec[c + d*x])*Tan[c + d*x]^3,x]
```

output
```
-((a*Sec[c + d*x])/d) + (a*Sec[c + d*x]^3)/(3*d) + (a*(2*Log[Cos[c + d*x]]
 + Tan[c + d*x]^2))/(2*d)
```

3.4.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.81, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {3042, 25, 4367, 27, 84, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^3(c+dx)(a\sec(c+dx)+a)\,dx$$

$$\downarrow 3042$$

$$\int -\cot\left(c+dx+\frac{\pi}{2}\right)^3\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)dx$$

$$\downarrow 25$$

$$-\int \cot\left(\frac{1}{2}(2c+\pi)+dx\right)^3\left(\csc\left(\frac{1}{2}(2c+\pi)+dx\right)a+a\right)dx$$

$$\downarrow 4367$$

$$-\frac{\int a^3(1-\cos(c+dx))(\cos(c+dx)+1)^2\sec^4(c+dx)d\cos(c+dx)}{a^2d}$$

$$\downarrow 27$$

$$-\frac{a\int(1-\cos(c+dx))(\cos(c+dx)+1)^2\sec^4(c+dx)d\cos(c+dx)}{d}$$

$$\downarrow 84$$

$$-\frac{a\int\left(\sec^4(c+dx)+\sec^3(c+dx)-\sec^2(c+dx)-\sec(c+dx)\right)d\cos(c+dx)}{d}$$

$$\downarrow 2009$$

$$-\frac{a\left(-\frac{1}{3}\sec^3(c+dx)-\frac{1}{2}\sec^2(c+dx)+\sec(c+dx)-\log(\cos(c+dx))\right)}{d}$$

input
```
Int[(a + a*Sec[c + d*x])*Tan[c + d*x]^3,x]
```

output
```
-((a*(-Log[Cos[c + d*x]] + Sec[c + d*x] - Sec[c + d*x]^2/2 - Sec[c + d*x]^
3/3))/d)
```

3.4.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 84
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && EqQ[b*e + a*f, 0] &&  !(ILtQ[n + p + 2, 0
] && GtQ[n + 2*p, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4367
```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Simp[1/(a^(m - n - 1)*b^n*d)   Subst[Int[(a - b*x)^((m -
1)/2)*((a + b*x)^((m - 1)/2 + n)/x^(m + n)), x], x, Sin[c + d*x]], x] /; Fr
eeQ[{a, b, c, d}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && Integer
Q[n]
```

3.4.4 Maple [A] (verified)

Time = 0.90 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.77

method	result	size
derivativedivides	$\frac{a\left(\frac{\sec(dx+c)^3}{3}+\frac{\sec(dx+c)^2}{2}-\sec(dx+c)-\ln(\sec(dx+c))\right)}{d}$	44
default	$\frac{a\left(\frac{\sec(dx+c)^3}{3}+\frac{\sec(dx+c)^2}{2}-\sec(dx+c)-\ln(\sec(dx+c))\right)}{d}$	44
parts	$\frac{a\left(\frac{\tan(dx+c)^2}{2}-\frac{\ln\left(1+\tan(dx+c)^2\right)}{2}\right)}{d}+\frac{a\left(\frac{\sec(dx+c)^3}{3}-\sec(dx+c)\right)}{d}$	55
risch	$-iax-\frac{2iac}{d}-\frac{2a\left(3\,e^{5i(dx+c)}-3\,e^{4i(dx+c)}+2\,e^{3i(dx+c)}-3\,e^{2i(dx+c)}+3\,e^{i(dx+c)}\right)}{3d\left(e^{2i(dx+c)}+1\right)^3}+\frac{a\ln\left(e^{2i(dx+c)}+1\right)}{d}$	107

input
```
int((a+a*sec(d*x+c))*tan(d*x+c)^3,x,method=_RETURNVERBOSE)
```

output
```
a/d*(1/3*sec(d*x+c)^3+1/2*sec(d*x+c)^2-sec(d*x+c)-ln(sec(d*x+c)))
```

3.4.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00

$$\int (a + a\sec(c + dx))\tan^3(c + dx)\,dx$$
$$= \frac{6\,a\cos(dx+c)^3\log(-\cos(dx+c)) - 6\,a\cos(dx+c)^2 + 3\,a\cos(dx+c) + 2\,a}{6\,d\cos(dx+c)^3}$$

input
```
integrate((a+a*sec(d*x+c))*tan(d*x+c)^3,x, algorithm="fricas")
```

output
```
1/6*(6*a*cos(d*x + c)^3*log(-cos(d*x + c)) - 6*a*cos(d*x + c)^2 + 3*a*cos(
d*x + c) + 2*a)/(d*cos(d*x + c)^3)
```

3.4.6 Sympy [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.33

$$\int (a + a\sec(c + dx))\tan^3(c + dx)\,dx$$
$$= \begin{cases} -\frac{a\log(\tan^2(c+dx)+1)}{2d} + \frac{a\tan^2(c+dx)\sec(c+dx)}{3d} + \frac{a\tan^2(c+dx)}{2d} - \frac{2a\sec(c+dx)}{3d} & \text{for } d \neq 0 \\ x(a\sec(c)+a)\tan^3(c) & \text{otherwise} \end{cases}$$

input
```
integrate((a+a*sec(d*x+c))*tan(d*x+c)**3,x)
```

output
```
Piecewise((-a*log(tan(c + d*x)**2 + 1)/(2*d) + a*tan(c + d*x)**2*sec(c + d
*x)/(3*d) + a*tan(c + d*x)**2/(2*d) - 2*a*sec(c + d*x)/(3*d), Ne(d, 0)), (
x*(a*sec(c) + a)*tan(c)**3, True))
```

3.4.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.88

$$\int (a + a\sec(c+dx))\tan^3(c+dx)\,dx = \frac{6\,a\log\left(\cos\left(dx+c\right)\right) - \frac{6\,a\cos(dx+c)^2 - 3\,a\cos(dx+c) - 2\,a}{\cos(dx+c)^3}}{6\,d}$$

input
```
integrate((a+a*sec(d*x+c))*tan(d*x+c)^3,x, algorithm="maxima")
```

output
```
1/6*(6*a*log(cos(d*x + c)) - (6*a*cos(d*x + c)^2 - 3*a*cos(d*x + c) - 2*a)
/cos(d*x + c)^3)/d
```

3.4.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 155 vs. 2(53) = 106.

Time = 0.64 (sec) , antiderivative size = 155, normalized size of antiderivative = 2.72

$$\int (a + a\sec(c+dx))\tan^3(c+dx)\,dx =$$

$$-\frac{6\,a\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right) - 6\,a\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right) + \frac{19\,a+\frac{69\,a(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{45\,a(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}+\frac{11\,a(\cos(dx+c)}{}}{\left(\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right)^3}}{6\,d}$$

input
```
integrate((a+a*sec(d*x+c))*tan(d*x+c)^3,x, algorithm="giac")
```

output
```
-1/6*(6*a*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)) - 6*a*log(a
bs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)) + (19*a + 69*a*(cos(d*x +
c) - 1)/(cos(d*x + c) + 1) + 45*a*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^
2 + 11*a*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3)/((cos(d*x + c) - 1)/(c
os(d*x + c) + 1) + 1)^3)/d
```

3.4.9 Mupad [B] (verification not implemented)

Time = 15.22 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.68

$$\int (a + a\sec(c+dx))\tan^3(c+dx)\,dx$$

$$= \frac{2a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4 - 6a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2 + \frac{4a}{3}}{d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6 - 3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4 + 3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2 - 1\right)} - \frac{2a\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2\right)}{d}$$

input
```
int(tan(c + d*x)^3*(a + a/cos(c + d*x)),x)
```

output
```
((4*a)/3 - 6*a*tan(c/2 + (d*x)/2)^2 + 2*a*tan(c/2 + (d*x)/2)^4)/(d*(3*tan(
c/2 + (d*x)/2)^2 - 3*tan(c/2 + (d*x)/2)^4 + tan(c/2 + (d*x)/2)^6 - 1)) - (
2*a*atanh(tan(c/2 + (d*x)/2)^2))/d
```

3.5 $\int (a + a\sec(c + dx))\tan(c + dx)\,dx$

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3.5.1 Optimal result

Integrand size = 17, antiderivative size = 25

$$\int (a + a\sec(c + dx))\tan(c + dx)\,dx = -\frac{a\log(\cos(c + dx))}{d} + \frac{a\sec(c + dx)}{d}$$

output
```
-a*ln(cos(d*x+c))/d+a*sec(d*x+c)/d
```

3.5.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int (a + a\sec(c + dx))\tan(c + dx)\,dx = -\frac{a\log(\cos(c + dx))}{d} + \frac{a\sec(c + dx)}{d}$$

input
```
Integrate[(a + a*Sec[c + d*x])*Tan[c + d*x],x]
```

output
```
-((a*Log[Cos[c + d*x]])/d) + (a*Sec[c + d*x])/d
```

3.5.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.88, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.353$, Rules used = {3042, 25, 4367, 27, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan(c+dx)(a\sec(c+dx)+a)\,dx$$

$$\downarrow 3042$$

$$\int -\cot\left(c+dx+\frac{\pi}{2}\right)\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)dx$$

$$\downarrow 25$$

$$-\int \cot\left(\frac{1}{2}(2c+\pi)+dx\right)\left(\csc\left(\frac{1}{2}(2c+\pi)+dx\right)a+a\right)dx$$

$$\downarrow 4367$$

$$-\frac{\int a(\cos(c+dx)+1)\sec^2(c+dx)d\cos(c+dx)}{d}$$

$$\downarrow 27$$

$$-\frac{a\int(\cos(c+dx)+1)\sec^2(c+dx)d\cos(c+dx)}{d}$$

$$\downarrow 49$$

$$-\frac{a\int\left(\sec^2(c+dx)+\sec(c+dx)\right)d\cos(c+dx)}{d}$$

$$\downarrow 2009$$

$$-\frac{a(\log(\cos(c+dx))-\sec(c+dx))}{d}$$

input
```
Int[(a + a*Sec[c + d*x])*Tan[c + d*x],x]
```

output
```
-((a*(Log[Cos[c + d*x]] - Sec[c + d*x]))/d)
```

3.5.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4367
```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Simp[1/(a^(m - n - 1)*b^n*d)   Subst[Int[(a - b*x)^((m -
1)/2)*((a + b*x)^((m - 1)/2 + n)/x^(m + n)), x], x, Sin[c + d*x]], x] /; Fr
eeQ[{a, b, c, d}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && Integer
Q[n]
```

3.5.4 Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.80

method	result	size
derivativedivides	$\frac{a(\sec(dx+c)+\ln(\sec(dx+c)))}{d}$	20
default	$\frac{a(\sec(dx+c)+\ln(\sec(dx+c)))}{d}$	20
parts	$\frac{a\ln\left(1+\tan(dx+c)^2\right)}{2d}+\frac{a\sec(dx+c)}{d}$	30
risch	$iax+\frac{2iac}{d}+\frac{2a\,e^{i(dx+c)}}{d\left(e^{2i(dx+c)}+1\right)}-\frac{a\ln\left(e^{2i(dx+c)}+1\right)}{d}$	61

input
```
int((a+a*sec(d*x+c))*tan(d*x+c),x,method=_RETURNVERBOSE)
```

output
```
1/d*a*(sec(d*x+c)+ln(sec(d*x+c)))
```

3.5.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.36

$$\int (a + a\sec(c+dx))\tan(c+dx)\,dx = -\frac{a\cos(dx+c)\log(-\cos(dx+c)) - a}{d\cos(dx+c)}$$

input
```
integrate((a+a*sec(d*x+c))*tan(d*x+c),x, algorithm="fricas")
```

output
```
-(a*cos(d*x + c)*log(-cos(d*x + c)) - a)/(d*cos(d*x + c))
```

3.5.6 Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.48

$$\int (a + a\sec(c+dx))\tan(c+dx)\,dx = \begin{cases} \frac{a\log\left(\tan^2(c+dx)+1\right)}{2d} + \frac{a\sec(c+dx)}{d} & \text{for } d \neq 0 \\ x(a\sec(c)+a)\tan(c) & \text{otherwise} \end{cases}$$

input
```
integrate((a+a*sec(d*x+c))*tan(d*x+c),x)
```

output
```
Piecewise((a*log(tan(c + d*x)**2 + 1)/(2*d) + a*sec(c + d*x)/d, Ne(d, 0)),
 (x*(a*sec(c) + a)*tan(c), True))
```

3.5.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.04

$$\int (a + a\sec(c+dx))\tan(c+dx)\,dx = -\frac{a\log(\cos(dx+c)) - \frac{a}{\cos(dx+c)}}{d}$$

input
```
integrate((a+a*sec(d*x+c))*tan(d*x+c),x, algorithm="maxima")
```

output
```
-(a*log(cos(d*x + c)) - a/cos(d*x + c))/d
```

3.5.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 106 vs. $2(25) = 50$.

Time $= 0.32$ (sec) , antiderivative size $= 106$, normalized size of antiderivative $= 4.24$

$$\int (a + a\sec(c+dx))\tan(c+dx)\,dx = \frac{a\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right) - a\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right) + \frac{3\,a+\frac{a(\cos(dx+c)-1)}{\cos(dx+c)+1}}{\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1}}{d}$$

input
```
integrate((a+a*sec(d*x+c))*tan(d*x+c),x, algorithm="giac")
```

output
```
(a*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)) - a*log(abs(-(cos(
d*x + c) - 1)/(cos(d*x + c) + 1) - 1)) + (3*a + a*(cos(d*x + c) - 1)/(cos(
d*x + c) + 1))/((cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1))/d
```

3.5.9 Mupad [B] (verification not implemented)

Time $= 14.70$ (sec) , antiderivative size $= 40$, normalized size of antiderivative $= 1.60$

$$\int (a + a\sec(c+dx))\tan(c+dx)\,dx = \frac{2\,a\,\operatorname{atanh}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2\right)}{d} - \frac{2\,a}{d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2-1\right)}$$

input
```
int(tan(c + d*x)*(a + a/cos(c + d*x)),x)
```

output
```
(2*a*atanh(tan(c/2 + (d*x)/2)^2))/d - (2*a)/(d*(tan(c/2 + (d*x)/2)^2 - 1))
```

3.6 $\int \cot(c+dx)(a+a\sec(c+dx))\,dx$

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3.6.1 Optimal result

Integrand size = 17, antiderivative size = 16

$$\int \cot(c+dx)(a+a\sec(c+dx))\,dx = \frac{a\log(1-\cos(c+dx))}{d}$$

output
```
a*ln(1-cos(d*x+c))/d
```

3.6.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 38 vs. 2(16) = 32.

Time = 0.02 (sec) , antiderivative size = 38, normalized size of antiderivative = 2.38

$$\int \cot(c+dx)(a+a\sec(c+dx))\,dx = a\left(\frac{2\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{d} + \frac{2\log\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{d}\right)$$

input
```
Integrate[Cot[c + d*x]*(a + a*Sec[c + d*x]),x]
```

output
```
a*((2*Log[Cos[(c + d*x)/2]])/d + (2*Log[Tan[c/2 + (d*x)/2]])/d)
```

3.6.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {3042, 25, 4367, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot(c+dx)(a\sec(c+dx)+a)\,dx$$

$$\downarrow 3042$$

$$\int -\frac{a\csc\left(c+dx+\frac{\pi}{2}\right)+a}{\cot\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\downarrow 25$$

$$-\int \frac{\csc\left(\frac{1}{2}(2c+\pi)+dx\right)a+a}{\cot\left(\frac{1}{2}(2c+\pi)+dx\right)}dx$$

$$\downarrow 4367$$

$$-\frac{a^2\int \frac{1}{a-a\cos(c+dx)}d\cos(c+dx)}{d}$$

$$\downarrow 16$$

$$\frac{a\log(1-\cos(c+dx))}{d}$$

input
```
Int[Cot[c + d*x]*(a + a*Sec[c + d*x]),x]
```

output
```
(a*Log[1 - Cos[c + d*x]])/d
```

3.6.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4367
```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Simp[1/(a^(m - n - 1)*b^n*d)   Subst[Int[(a - b*x)^((m -
1)/2)*((a + b*x)^((m - 1)/2 + n)/x^(m + n)), x], x, Sin[c + d*x]], x] /; Fr
eeQ[{a, b, c, d}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && Integer
Q[n]
```

3.6.4 Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 33, normalized size of antiderivative = 2.06

method	result	size
derivativedivides	$\frac{a\ln(-\cot(dx+c)+\csc(dx+c))+a\ln(\sin(dx+c))}{d}$	33
default	$\frac{a\ln(-\cot(dx+c)+\csc(dx+c))+a\ln(\sin(dx+c))}{d}$	33
risch	$-iax-\frac{2iac}{d}+\frac{2a\ln\left(e^{i(dx+c)}-1\right)}{d}$	33

input
```
int(cot(d*x+c)*(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(a*ln(-cot(d*x+c)+csc(d*x+c))+a*ln(sin(d*x+c)))
```

3.6.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \cot(c+dx)(a+a\sec(c+dx))\,dx = \frac{a\log\left(-\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)}{d}$$

input
```
integrate(cot(d*x+c)*(a+a*sec(d*x+c)),x, algorithm="fricas")
```

output
```
a*log(-1/2*cos(d*x + c) + 1/2)/d
```

3.6.6 Sympy [F]

$$\int \cot(c+dx)(a+a\sec(c+dx))\,dx = a\left(\int \cot(c+dx)\sec(c+dx)\,dx + \int \cot(c+dx)\,dx\right)$$

input
```
integrate(cot(d*x+c)*(a+a*sec(d*x+c)),x)
```

output
```
a*(Integral(cot(c + d*x)*sec(c + d*x), x) + Integral(cot(c + d*x), x))
```

3.6.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \cot(c+dx)(a+a\sec(c+dx))\,dx = \frac{a\log(\cos(dx+c)-1)}{d}$$

input
```
integrate(cot(d*x+c)*(a+a*sec(d*x+c)),x, algorithm="maxima")
```

output
```
a*log(cos(d*x + c) - 1)/d
```

3.6.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 58 vs. $2(16) = 32$.

Time = 0.28 (sec) , antiderivative size = 58, normalized size of antiderivative = 3.62

$$\int \cot(c+dx)(a+a\sec(c+dx))\,dx = \frac{a\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right) - a\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right)}{d}$$

input
```
integrate(cot(d*x+c)*(a+a*sec(d*x+c)),x, algorithm="giac")
```

output
```
(a*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1)) - a*log(abs(-(cos(d*x
 + c) - 1)/(cos(d*x + c) + 1) + 1)))/d
```

3.6.9 Mupad [B] (verification not implemented)

Time = 14.58 (sec) , antiderivative size = 34, normalized size of antiderivative = 2.12

$$\int \cot(c+dx)(a+a\sec(c+dx))\,dx = \frac{a\left(2\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)-\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+1\right)\right)}{d}$$

input
```
int(cot(c + d*x)*(a + a/cos(c + d*x)),x)
```

output
```
(a*(2*log(tan(c/2 + (d*x)/2)) - log(tan(c/2 + (d*x)/2)^2 + 1)))/d
```

3.7 $\int \cot^3(c+dx)(a+a\sec(c+dx))\,dx$

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3.7.1 Optimal result

Integrand size = 19, antiderivative size = 57

$$\int \cot^3(c+dx)(a+a\sec(c+dx))\,dx = -\frac{a}{2d(1-\cos(c+dx))} - \frac{3a\log(1-\cos(c+dx))}{4d} - \frac{a\log(1+\cos(c+dx))}{4d}$$

output
```
-1/2*a/d/(1-cos(d*x+c))-3/4*a*ln(1-cos(d*x+c))/d-1/4*a*ln(1+cos(d*x+c))/d
```

3.7.2 Mathematica [A] (verified)

Time = 0.52 (sec) , antiderivative size = 114, normalized size of antiderivative = 2.00

$$\begin{aligned}&\int \cot^3(c+dx)(a+a\sec(c+dx))\,dx \\ &= -\frac{a\csc^2\left(\frac{1}{2}(c+dx)\right)}{8d} + \frac{a\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{2d} - \frac{a\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{2d} \\ &\quad - \frac{a(\cot^2(c+dx)+2\log(\cos(c+dx))+2\log(\tan(c+dx)))}{2d} + \frac{a\sec^2\left(\frac{1}{2}(c+dx)\right)}{8d}\end{aligned}$$

input
```
Integrate[Cot[c + d*x]^3*(a + a*Sec[c + d*x]),x]
```

output
```
-1/8*(a*Csc[(c + d*x)/2]^2)/d + (a*Log[Cos[(c + d*x)/2]])/(2*d) - (a*Log[S
in[(c + d*x)/2]])/(2*d) - (a*(Cot[c + d*x]^2 + 2*Log[Cos[c + d*x]] + 2*Log
[Tan[c + d*x]]))/(2*d) + (a*Sec[(c + d*x)/2]^2)/(8*d)
```

3.7.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.89, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {3042, 25, 4367, 27, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^3(c+dx)(a\sec(c+dx)+a)\,dx$$

$$\downarrow 3042$$

$$\int -\frac{a\csc\left(c+dx+\frac{\pi}{2}\right)+a}{\cot\left(c+dx+\frac{\pi}{2}\right)^3}dx$$

$$\downarrow 25$$

$$-\int \frac{\csc\left(\frac{1}{2}(2c+\pi)+dx\right)a+a}{\cot\left(\frac{1}{2}(2c+\pi)+dx\right)^3}dx$$

$$\downarrow 4367$$

$$-\frac{a^4\int \frac{\cos^2(c+dx)}{a^3(1-\cos(c+dx))^2(\cos(c+dx)+1)}d\cos(c+dx)}{d}$$

$$\downarrow 27$$

$$-\frac{a\int \frac{\cos^2(c+dx)}{(1-\cos(c+dx))^2(\cos(c+dx)+1)}d\cos(c+dx)}{d}$$

$$\downarrow 99$$

$$-\frac{a\int \left(\frac{1}{4(\cos(c+dx)+1)}+\frac{3}{4(\cos(c+dx)-1)}+\frac{1}{2(\cos(c+dx)-1)^2}\right)d\cos(c+dx)}{d}$$

$$\downarrow 2009$$

$$-\frac{a\left(\frac{1}{2(1-\cos(c+dx))}+\frac{3}{4}\log(1-\cos(c+dx))+\frac{1}{4}\log(\cos(c+dx)+1)\right)}{d}$$

input
```
Int[Cot[c + d*x]^3*(a + a*Sec[c + d*x]),x]
```

output
```
-((a*(1/(2*(1 - Cos[c + d*x])) + (3*Log[1 - Cos[c + d*x]])/4 + Log[1 + Cos
[c + d*x]]/4))/d)
```

3.7.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4367
```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Simp[1/(a^(m - n - 1)*b^n*d)   Subst[Int[(a - b*x)^((m -
1)/2)*((a + b*x)^((m - 1)/2 + n)/x^(m + n)), x], x, Sin[c + d*x]], x] /; Fr
eeQ[{a, b, c, d}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && Integer
Q[n]
```

3.7.4 Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.32

method	result	size
derivativedivides	$\frac{a\left(-\frac{\cos(dx+c)^3}{2\sin(dx+c)^2}-\frac{\cos(dx+c)}{2}-\frac{\ln(-\cot(dx+c)+\csc(dx+c))}{2}\right)+a\left(-\frac{\cot(dx+c)^2}{2}-\ln(\sin(dx+c))\right)}{d}$	75
default	$\frac{a\left(-\frac{\cos(dx+c)^3}{2\sin(dx+c)^2}-\frac{\cos(dx+c)}{2}-\frac{\ln(-\cot(dx+c)+\csc(dx+c))}{2}\right)+a\left(-\frac{\cot(dx+c)^2}{2}-\ln(\sin(dx+c))\right)}{d}$	75
risch	$iax+\frac{2iac}{d}+\frac{a\,e^{i(dx+c)}}{d\left(e^{i(dx+c)}-1\right)^2}-\frac{3a\ln\left(e^{i(dx+c)}-1\right)}{2d}-\frac{a\ln\left(e^{i(dx+c)}+1\right)}{2d}$	78

input
```
int(cot(d*x+c)^3*(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(a*(-1/2/sin(d*x+c)^2*cos(d*x+c)^3-1/2*cos(d*x+c)-1/2*ln(-cot(d*x+c)+c
sc(d*x+c)))+a*(-1/2*cot(d*x+c)^2-ln(sin(d*x+c))))
```

3.7.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.21

$$\int \cot^3(c+dx)(a+a\sec(c+dx))\,dx =$$
$$-\frac{(a\cos(dx+c)-a)\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)+3\,(a\cos(dx+c)-a)\log\left(-\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)-2\,a}{4\,(d\cos(dx+c)-d)}$$

input
```
integrate(cot(d*x+c)^3*(a+a*sec(d*x+c)),x, algorithm="fricas")
```

output
```
-1/4*((a*cos(d*x + c) - a)*log(1/2*cos(d*x + c) + 1/2) + 3*(a*cos(d*x + c)
 - a)*log(-1/2*cos(d*x + c) + 1/2) - 2*a)/(d*cos(d*x + c) - d)
```

3.7.6 Sympy [F]

$$\int \cot^3(c+dx)(a+a\sec(c+dx))\,dx = a\left(\int \cot^3(c+dx)\sec(c+dx)\,dx + \int \cot^3(c+dx)\,dx\right)$$

input
```
integrate(cot(d*x+c)**3*(a+a*sec(d*x+c)),x)
```

output
```
a*(Integral(cot(c + d*x)**3*sec(c + d*x), x) + Integral(cot(c + d*x)**3, x
))
```

3.7.7 Maxima [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.74

$$\int \cot^3(c+dx)(a+a\sec(c+dx))\,dx = -\frac{a\log(\cos(dx+c)+1)+3\,a\log(\cos(dx+c)-1)-\frac{2\,a}{\cos(dx+c)-1}}{4\,d}$$

input
```
integrate(cot(d*x+c)^3*(a+a*sec(d*x+c)),x, algorithm="maxima")
```

output
```
-1/4*(a*log(cos(d*x + c) + 1) + 3*a*log(cos(d*x + c) - 1) - 2*a/(cos(d*x +
 c) - 1))/d
```

3.7.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 103 vs. 2(49) = 98.

Time = 0.31 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.81

$$\int \cot^3(c+dx)(a+a\sec(c+dx))\,dx = -\frac{3\,a\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)-4\,a\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right)-\frac{\left(a+\frac{3\,a(\cos(dx+c)-1)}{\cos(dx+c)+1}\right)(\cos(dx+c)+1)}{\cos(dx+c)-1}}{4\,d}$$

input
```
integrate(cot(d*x+c)^3*(a+a*sec(d*x+c)),x, algorithm="giac")
```

output
```
-1/4*(3*a*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1)) - 4*a*log(abs(
-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)) - (a + 3*a*(cos(d*x + c) - 1)
/(cos(d*x + c) + 1))*(cos(d*x + c) + 1)/(cos(d*x + c) - 1))/d
```

3.7.9 Mupad [B] (verification not implemented)

Time = 13.95 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.81

$$\int \cot^3(c+dx)(a+a\sec(c+dx))\,dx = -\frac{a\left(\cot\left(\frac{c}{2}+\frac{dx}{2}\right)^2+6\,\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)-4\,\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+1\right)\right)}{4\,d}$$

input
```
int(cot(c + d*x)^3*(a + a/cos(c + d*x)),x)
```

output
```
-(a*(6*log(tan(c/2 + (d*x)/2)) - 4*log(tan(c/2 + (d*x)/2)^2 + 1) + cot(c/2
 + (d*x)/2)^2))/(4*d)
```

3.8 $\int \cot^5(c+dx)(a+a\sec(c+dx))\,dx$

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3.8.1 Optimal result

Integrand size = 19, antiderivative size = 95

$$\int \cot^5(c+dx)(a+a\sec(c+dx))\,dx = -\frac{a}{8d(1-\cos(c+dx))^2} + \frac{3a}{4d(1-\cos(c+dx))} + \frac{a}{8d(1+\cos(c+dx))} + \frac{11a\log(1-\cos(c+dx))}{16d} + \frac{5a\log(1+\cos(c+dx))}{16d}$$

output
```
-1/8*a/d/(1-cos(d*x+c))^2+3/4*a/d/(1-cos(d*x+c))+1/8*a/d/(1+cos(d*x+c))+11
/16*a*ln(1-cos(d*x+c))/d+5/16*a*ln(1+cos(d*x+c))/d
```

3.8.2 Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.84

$$\int \cot^5(c+dx)(a+a\sec(c+dx))\,dx = \frac{a\cot^2(c+dx)}{2d} - \frac{a\cot^4(c+dx)}{4d} + \frac{5a\csc^2\left(\frac{1}{2}(c+dx)\right)}{32d} - \frac{a\csc^4\left(\frac{1}{2}(c+dx)\right)}{64d} - \frac{3a\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{8d} + \frac{a\log(\cos(c+dx))}{d} + \frac{3a\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{8d} + \frac{a\log(\tan(c+dx))}{d} - \frac{5a\sec^2\left(\frac{1}{2}(c+dx)\right)}{32d} + \frac{a\sec^4\left(\frac{1}{2}(c+dx)\right)}{64d}$$

input
```
Integrate[Cot[c + d*x]^5*(a + a*Sec[c + d*x]),x]
```

output
```
(a*Cot[c + d*x]^2)/(2*d) - (a*Cot[c + d*x]^4)/(4*d) + (5*a*Csc[(c + d*x)/2
]^2)/(32*d) - (a*Csc[(c + d*x)/2]^4)/(64*d) - (3*a*Log[Cos[(c + d*x)/2]])/
(8*d) + (a*Log[Cos[c + d*x]])/d + (3*a*Log[Sin[(c + d*x)/2]])/(8*d) + (a*L
og[Tan[c + d*x]])/d - (5*a*Sec[(c + d*x)/2]^2)/(32*d) + (a*Sec[(c + d*x)/2
]^4)/(64*d)
```

3.8.3 Rubi [A] (verified)

Time $= 0.26$ (sec) , antiderivative size $= 81$, normalized size of antiderivative $= 0.85$, number of steps used $= 7$, number of rules used $= 6$, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used $= \{3042, 25, 4367, 27, 99, 2009\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^5(c+dx)(a\sec(c+dx)+a)\,dx$$

$$\downarrow 3042$$

$$\int -\frac{a\csc\left(c+dx+\frac{\pi}{2}\right)+a}{\cot\left(c+dx+\frac{\pi}{2}\right)^5}dx$$

$$\downarrow 25$$

$$-\int \frac{\csc\left(\frac{1}{2}(2c+\pi)+dx\right)a+a}{\cot\left(\frac{1}{2}(2c+\pi)+dx\right)^5}dx$$

$$\downarrow 4367$$

$$-\frac{a^6\int \frac{\cos^4(c+dx)}{a^5(1-\cos(c+dx))^3(\cos(c+dx)+1)^2}d\cos(c+dx)}{d}$$

$$\downarrow 27$$

$$-\frac{a\int \frac{\cos^4(c+dx)}{(1-\cos(c+dx))^3(\cos(c+dx)+1)^2}d\cos(c+dx)}{d}$$

$$\downarrow 99$$

$$-\frac{a\int \left(-\frac{5}{16(\cos(c+dx)+1)}+\frac{1}{8(\cos(c+dx)+1)^2}-\frac{11}{16(\cos(c+dx)-1)}-\frac{3}{4(\cos(c+dx)-1)^2}-\frac{1}{4(\cos(c+dx)-1)^3}\right)d\cos(c+dx)}{d}$$

$$\downarrow 2009$$

$$-\frac{a\left(-\frac{3}{4(1-\cos(c+dx))}-\frac{1}{8(\cos(c+dx)+1)}+\frac{1}{8(1-\cos(c+dx))^2}-\frac{11}{16}\log(1-\cos(c+dx))-\frac{5}{16}\log(\cos(c+dx)+1)\right)}{d}$$

input
```
Int[Cot[c + d*x]^5*(a + a*Sec[c + d*x]),x]
```

output
```
-((a*(1/(8*(1 - Cos[c + d*x])^2) - 3/(4*(1 - Cos[c + d*x])) - 1/(8*(1 + Co
s[c + d*x])) - (11*Log[1 - Cos[c + d*x]])/16 - (5*Log[1 + Cos[c + d*x]])/1
6))/d)
```

3.8.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4367
```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Simp[1/(a^(m - n - 1)*b^n*d)   Subst[Int[(a - b*x)^((m -
1)/2)*((a + b*x)^((m - 1)/2 + n)/x^(m + n)), x], x, Sin[c + d*x]], x] /; Fr
eeQ[{a, b, c, d}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && Integer
Q[n]
```

3.8.4 Maple [A] (verified)

Time = 0.70 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.17

method	result
derivativedivides	$\frac{a\left(-\frac{\cos(dx+c)^5}{4\sin(dx+c)^4}+\frac{\cos(dx+c)^5}{8\sin(dx+c)^2}+\frac{\cos(dx+c)^3}{8}+\frac{3\cos(dx+c)}{8}+\frac{3\ln(-\cot(dx+c)+\csc(dx+c))}{8}\right)+a\left(-\frac{\cot(dx+c)^4}{4}+\frac{\cot(dx+c)^2}{2}+\right.}{d}$
default	$\frac{a\left(-\frac{\cos(dx+c)^5}{4\sin(dx+c)^4}+\frac{\cos(dx+c)^5}{8\sin(dx+c)^2}+\frac{\cos(dx+c)^3}{8}+\frac{3\cos(dx+c)}{8}+\frac{3\ln(-\cot(dx+c)+\csc(dx+c))}{8}\right)+a\left(-\frac{\cot(dx+c)^4}{4}+\frac{\cot(dx+c)^2}{2}+\right.}{d}$
risch	$-iax-\frac{2iac}{d}-\frac{a\left(5\,e^{5i(dx+c)}+6\,e^{4i(dx+c)}-14\,e^{3i(dx+c)}+6\,e^{2i(dx+c)}+5\,e^{i(dx+c)}\right)}{4d\left(e^{i(dx+c)}-1\right)^4\left(e^{i(dx+c)}+1\right)^2}+\frac{5a\ln\left(e^{i(dx+c)}+1\right)}{8d}+\frac{11a\ln}{}$

input
```
int(cot(d*x+c)^5*(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(a*(-1/4/sin(d*x+c)^4*cos(d*x+c)^5+1/8/sin(d*x+c)^2*cos(d*x+c)^5+1/8*c
os(d*x+c)^3+3/8*cos(d*x+c)+3/8*ln(-cot(d*x+c)+csc(d*x+c)))+a*(-1/4*cot(d*x
+c)^4+1/2*cot(d*x+c)^2+ln(sin(d*x+c))))
```

3.8.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.58

$$\int \cot^5(c+dx)(a+a\sec(c+dx))\,dx =$$

$$-\frac{10\,a\cos(dx+c)^2+6\,a\cos(dx+c)-5\left(a\cos(dx+c)^3-a\cos(dx+c)^2-a\cos(dx+c)+a\right)\log\left(\frac{1}{2}\,c\right.}{16\left(d\cos(dx+c)^3-d\,co\right.}$$

input
```
integrate(cot(d*x+c)^5*(a+a*sec(d*x+c)),x, algorithm="fricas")
```

output
```
-1/16*(10*a*cos(d*x + c)^2 + 6*a*cos(d*x + c) - 5*(a*cos(d*x + c)^3 - a*co
s(d*x + c)^2 - a*cos(d*x + c) + a)*log(1/2*cos(d*x + c) + 1/2) - 11*(a*cos
(d*x + c)^3 - a*cos(d*x + c)^2 - a*cos(d*x + c) + a)*log(-1/2*cos(d*x + c)
 + 1/2) - 12*a)/(d*cos(d*x + c)^3 - d*cos(d*x + c)^2 - d*cos(d*x + c) + d)
```

3.8.6 Sympy [F]

$$\int \cot^5(c+dx)(a+a\sec(c+dx))\,dx = a\left(\int \cot^5(c+dx)\sec(c+dx)\,dx + \int \cot^5(c+dx)\,dx\right)$$

input
```
integrate(cot(d*x+c)**5*(a+a*sec(d*x+c)),x)
```

output
```
a*(Integral(cot(c + d*x)**5*sec(c + d*x), x) + Integral(cot(c + d*x)**5, x
))
```

3.8.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.91

$$\int \cot^5(c+dx)(a+a\sec(c+dx))\,dx$$
$$= \frac{5\,a\log(\cos(dx+c)+1) + 11\,a\log(\cos(dx+c)-1) - \frac{2\left(5\,a\cos(dx+c)^2+3\,a\cos(dx+c)-6\,a\right)}{\cos(dx+c)^3-\cos(dx+c)^2-\cos(dx+c)+1}}{16\,d}$$

input
```
integrate(cot(d*x+c)^5*(a+a*sec(d*x+c)),x, algorithm="maxima")
```

output
```
1/16*(5*a*log(cos(d*x + c) + 1) + 11*a*log(cos(d*x + c) - 1) - 2*(5*a*cos(
d*x + c)^2 + 3*a*cos(d*x + c) - 6*a)/(cos(d*x + c)^3 - cos(d*x + c)^2 - co
s(d*x + c) + 1))/d
```

3.8.8 Giac [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.57

$$\int \cot^5(c+dx)(a+a\sec(c+dx))\,dx$$
$$= \frac{22\,a\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right) - 32\,a\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right) - \frac{\left(a+\frac{10\,a(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{33\,a(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}\right)(\cos(dx+c)+1)^2}{(\cos(dx+c)-1)^2} -}{32\,d}$$

input
```
integrate(cot(d*x+c)^5*(a+a*sec(d*x+c)),x, algorithm="giac")
```

output
```
1/32*(22*a*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1)) - 32*a*log(ab
s(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)) - (a + 10*a*(cos(d*x + c) -
 1)/(cos(d*x + c) + 1) + 33*a*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2)*(
cos(d*x + c) + 1)^2/(cos(d*x + c) - 1)^2 - 2*a*(cos(d*x + c) - 1)/(cos(d*x
 + c) + 1))/d
```

3.8.9 Mupad [B] (verification not implemented)

Time = 13.98 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.93

$$\int \cot^5(c+dx)(a+a\sec(c+dx))\,dx = \frac{a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2}{16\,d} - \frac{a\,\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+1\right)}{d} - \frac{\cot\left(\frac{c}{2}+\frac{dx}{2}\right)^4\left(\frac{a}{4}-\frac{5\,a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2}{2}\right)}{8\,d} + \frac{11\,a\,\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{8\,d}$$

input
```
int(cot(c + d*x)^5*(a + a/cos(c + d*x)),x)
```

output
```
(a*tan(c/2 + (d*x)/2)^2)/(16*d) - (a*log(tan(c/2 + (d*x)/2)^2 + 1))/d - (c
ot(c/2 + (d*x)/2)^4*(a/4 - (5*a*tan(c/2 + (d*x)/2)^2)/2))/(8*d) + (11*a*lo
g(tan(c/2 + (d*x)/2)))/(8*d)
```

3.9 $\int \cot^7(c+dx)(a+a\sec(c+dx))\,dx$

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3.9.1 Optimal result

Integrand size = 19, antiderivative size = 133

$$\int \cot^7(c+dx)(a+a\sec(c+dx))\,dx = -\frac{a}{24d(1-\cos(c+dx))^3} + \frac{9a}{32d(1-\cos(c+dx))^2} - \frac{15a}{16d(1-\cos(c+dx))} + \frac{a}{32d(1+\cos(c+dx))^2} - \frac{a}{4d(1+\cos(c+dx))} - \frac{21a\log(1-\cos(c+dx))}{32d} - \frac{11a\log(1+\cos(c+dx))}{32d}$$

output
```
-1/24*a/d/(1-cos(d*x+c))^3+9/32*a/d/(1-cos(d*x+c))^2-15/16*a/d/(1-cos(d*x+
c))+1/32*a/d/(1+cos(d*x+c))^2-1/4*a/d/(1+cos(d*x+c))-21/32*a*ln(1-cos(d*x+
c))/d-11/32*a*ln(1+cos(d*x+c))/d
```

3.9.2 Mathematica [A] (verified)

Time = 0.44 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.24

$$\int \cot^7(c+dx)(a+a\sec(c+dx))\,dx = -\frac{a\left(192\cot^2(c+dx) - 96\cot^4(c+dx) + 64\cot^6(c+dx) + 66\csc^2\left(\frac{1}{2}(c+dx)\right) - 12\csc^4\left(\frac{1}{2}(c+dx)\right) + \text{c}\right.}{}$$

input
```
Integrate[Cot[c + d*x]^7*(a + a*Sec[c + d*x]),x]
```

output
```
-1/384*(a*(192*Cot[c + d*x]^2 - 96*Cot[c + d*x]^4 + 64*Cot[c + d*x]^6 + 66
*Csc[(c + d*x)/2]^2 - 12*Csc[(c + d*x)/2]^4 + Csc[(c + d*x)/2]^6 - 120*Log
[Cos[(c + d*x)/2]] + 384*Log[Cos[c + d*x]] + 120*Log[Sin[(c + d*x)/2]] + 3
84*Log[Tan[c + d*x]] - 66*Sec[(c + d*x)/2]^2 + 12*Sec[(c + d*x)/2]^4 - Sec
[(c + d*x)/2]^6))/d
```

3.9.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.83, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {3042, 25, 4367, 27, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^7(c+dx)(a\sec(c+dx)+a)\,dx$$

$$\downarrow 3042$$

$$\int -\frac{a\csc\left(c+dx+\frac{\pi}{2}\right)+a}{\cot\left(c+dx+\frac{\pi}{2}\right)^7}dx$$

$$\downarrow 25$$

$$-\int \frac{\csc\left(\frac{1}{2}(2c+\pi)+dx\right)a+a}{\cot\left(\frac{1}{2}(2c+\pi)+dx\right)^7}dx$$

$$\downarrow 4367$$

$$-\frac{a^8\int \frac{\cos^6(c+dx)}{a^7(1-\cos(c+dx))^4(\cos(c+dx)+1)^3}d\cos(c+dx)}{d}$$

$$\downarrow 27$$

$$-\frac{a\int \frac{\cos^6(c+dx)}{(1-\cos(c+dx))^4(\cos(c+dx)+1)^3}d\cos(c+dx)}{d}$$

$$\downarrow 99$$

$$-\frac{a\int \left(\frac{11}{32(\cos(c+dx)+1)}-\frac{1}{4(\cos(c+dx)+1)^2}+\frac{1}{16(\cos(c+dx)+1)^3}+\frac{21}{32(\cos(c+dx)-1)}+\frac{15}{16(\cos(c+dx)-1)^2}+\frac{9}{16(\cos(c+dx)-1)^3}+\frac{}{8(}\right.}{d}$$

$$\downarrow 2009$$

$$-\frac{a\left(\frac{15}{16(1-\cos(c+dx))}+\frac{1}{4(\cos(c+dx)+1)}-\frac{9}{32(1-\cos(c+dx))^2}-\frac{1}{32(\cos(c+dx)+1)^2}+\frac{1}{24(1-\cos(c+dx))^3}+\frac{21}{32}\log(1-\cos(c+dx)\right.}{d}$$

input
```
Int[Cot[c + d*x]^7*(a + a*Sec[c + d*x]),x]
```

output
```
-((a*(1/(24*(1 - Cos[c + d*x])^3) - 9/(32*(1 - Cos[c + d*x])^2) + 15/(16*(
1 - Cos[c + d*x])) - 1/(32*(1 + Cos[c + d*x])^2) + 1/(4*(1 + Cos[c + d*x])
) + (21*Log[1 - Cos[c + d*x]])/32 + (11*Log[1 + Cos[c + d*x]])/32))/d)
```

3.9.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4367
```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Simp[1/(a^(m - n - 1)*b^n*d)   Subst[Int[(a - b*x)^((m -
1)/2)*((a + b*x)^((m - 1)/2 + n)/x^(m + n)), x], x, Sin[c + d*x]], x] /; Fr
eeQ[{a, b, c, d}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && Integer
Q[n]
```

3.9.4 Maple [A] (verified)

Time = 0.83 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.14

method	result
derivativedivides	$\frac{a\left(-\frac{\cos(dx+c)^7}{6\sin(dx+c)^6}+\frac{\cos(dx+c)^7}{24\sin(dx+c)^4}-\frac{\cos(dx+c)^7}{16\sin(dx+c)^2}-\frac{\cos(dx+c)^5}{16}-\frac{5\cos(dx+c)^3}{48}-\frac{5\cos(dx+c)}{16}-\frac{5\ln(-\cot(dx+c)+\csc(dx+c))}{16}\right)+c}{d}$
default	$\frac{a\left(-\frac{\cos(dx+c)^7}{6\sin(dx+c)^6}+\frac{\cos(dx+c)^7}{24\sin(dx+c)^4}-\frac{\cos(dx+c)^7}{16\sin(dx+c)^2}-\frac{\cos(dx+c)^5}{16}-\frac{5\cos(dx+c)^3}{48}-\frac{5\cos(dx+c)}{16}-\frac{5\ln(-\cot(dx+c)+\csc(dx+c))}{16}\right)+c}{d}$
risch	$iax+\frac{2iac}{d}+\frac{a\left(33\,e^{9i(dx+c)}+78\,e^{8i(dx+c)}-184\,e^{7i(dx+c)}+2\,e^{6i(dx+c)}+270\,e^{5i(dx+c)}+2\,e^{4i(dx+c)}-184\,e^{3i(dx+c)}+7\right.}{24d\left(e^{i(dx+c)}-1\right)^6\left(e^{i(dx+c)}+1\right)^4}$

input
```
int(cot(d*x+c)^7*(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(a*(-1/6/sin(d*x+c)^6*cos(d*x+c)^7+1/24/sin(d*x+c)^4*cos(d*x+c)^7-1/16
/sin(d*x+c)^2*cos(d*x+c)^7-1/16*cos(d*x+c)^5-5/48*cos(d*x+c)^3-5/16*cos(d*
x+c)-5/16*ln(-cot(d*x+c)+csc(d*x+c)))+a*(-1/6*cot(d*x+c)^6+1/4*cot(d*x+c)^
4-1/2*cot(d*x+c)^2-ln(sin(d*x+c))))
```

3.9.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 241 vs. 2(113) = 226.

Time = 0.28 (sec) , antiderivative size = 241, normalized size of antiderivative = 1.81

$$\int \cot^7(c+dx)(a+a\sec(c+dx))\,dx$$
$$=\frac{66\,a\cos(dx+c)^4+78\,a\cos(dx+c)^3-158\,a\cos(dx+c)^2-58\,a\cos(dx+c)-33\left(a\cos(dx+c)^5-a\,c\right.}{}$$

input
```
integrate(cot(d*x+c)^7*(a+a*sec(d*x+c)),x, algorithm="fricas")
```

output
```
1/96*(66*a*cos(d*x + c)^4 + 78*a*cos(d*x + c)^3 - 158*a*cos(d*x + c)^2 - 5
8*a*cos(d*x + c) - 33*(a*cos(d*x + c)^5 - a*cos(d*x + c)^4 - 2*a*cos(d*x +
 c)^3 + 2*a*cos(d*x + c)^2 + a*cos(d*x + c) - a)*log(1/2*cos(d*x + c) + 1/
2) - 63*(a*cos(d*x + c)^5 - a*cos(d*x + c)^4 - 2*a*cos(d*x + c)^3 + 2*a*co
s(d*x + c)^2 + a*cos(d*x + c) - a)*log(-1/2*cos(d*x + c) + 1/2) + 88*a)/(d
*cos(d*x + c)^5 - d*cos(d*x + c)^4 - 2*d*cos(d*x + c)^3 + 2*d*cos(d*x + c)
^2 + d*cos(d*x + c) - d)
```

3.9.6 Sympy [F]

$$\int \cot^7(c+dx)(a+a\sec(c+dx))\,dx = a\left(\int \cot^7(c+dx)\sec(c+dx)\,dx + \int \cot^7(c+dx)\,dx\right)$$

input
```
integrate(cot(d*x+c)**7*(a+a*sec(d*x+c)),x)
```

output
```
a*(Integral(cot(c + d*x)**7*sec(c + d*x), x) + Integral(cot(c + d*x)**7, x
))
```

3.9.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.95

$$\int \cot^7(c+dx)(a+a\sec(c+dx))\,dx =$$

$$-\frac{33\,a\log(\cos(dx+c)+1)+63\,a\log(\cos(dx+c)-1)-\frac{2\left(33\,a\cos(dx+c)^4+39\,a\cos(dx+c)^3-79\,a\cos(dx+c)^2-29\,a\,c\right.}{\cos(dx+c)^5-\cos(dx+c)^4-2\cos(dx+c)^3+2\cos(dx+c)^2+c}}{96\,d}$$

input
```
integrate(cot(d*x+c)^7*(a+a*sec(d*x+c)),x, algorithm="maxima")
```

output
```
-1/96*(33*a*log(cos(d*x + c) + 1) + 63*a*log(cos(d*x + c) - 1) - 2*(33*a*c
os(d*x + c)^4 + 39*a*cos(d*x + c)^3 - 79*a*cos(d*x + c)^2 - 29*a*cos(d*x +
 c) + 44*a)/(cos(d*x + c)^5 - cos(d*x + c)^4 - 2*cos(d*x + c)^3 + 2*cos(d*
x + c)^2 + cos(d*x + c) - 1))/d
```

3.9.8 Giac [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.48

$$\int \cot^7(c+dx)(a+a\sec(c+dx))\,dx =$$

$$-\frac{252\,a\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)-384\,a\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right)-\frac{\left(2\,a+\frac{21\,a(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{132\,a(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}+\frac{462\,a(\cos(dx+c)}{(\cos(dx+c)-1)^3}\right.}{384\,d}$$

input
```
integrate(cot(d*x+c)^7*(a+a*sec(d*x+c)),x, algorithm="giac")
```

output
```
-1/384*(252*a*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1)) - 384*a*lo
g(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)) - (2*a + 21*a*(cos(d*x
+ c) - 1)/(cos(d*x + c) + 1) + 132*a*(cos(d*x + c) - 1)^2/(cos(d*x + c) +
1)^2 + 462*a*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3)*(cos(d*x + c) + 1)
^3/(cos(d*x + c) - 1)^3 - 42*a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 3*a
*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2)/d
```

3.9.9 Mupad [B] (verification not implemented)

Time = 14.18 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.89

$$\int \cot^7(c+dx)(a+a\sec(c+dx))\,dx$$
$$= \frac{a\,\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+1\right)}{d} - \frac{\cot\left(\frac{c}{2}+\frac{dx}{2}\right)^6\left(11\,a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4 - \frac{7\,a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2}{4}+\frac{a}{6}\right)}{32\,d}$$
$$- \frac{7\,a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2}{64\,d} + \frac{a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4}{128\,d} - \frac{21\,a\,\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{16\,d}$$

input
```
int(cot(c + d*x)^7*(a + a/cos(c + d*x)),x)
```

output
```
(a*log(tan(c/2 + (d*x)/2)^2 + 1))/d - (cot(c/2 + (d*x)/2)^6*(a/6 - (7*a*ta
n(c/2 + (d*x)/2)^2)/4 + 11*a*tan(c/2 + (d*x)/2)^4))/(32*d) - (7*a*tan(c/2
+ (d*x)/2)^2)/(64*d) + (a*tan(c/2 + (d*x)/2)^4)/(128*d) - (21*a*log(tan(c/
2 + (d*x)/2)))/(16*d)
```

3.10 $\int (a + a\sec(c+dx))\tan^8(c+dx)\,dx$

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3.10.1 Optimal result

Integrand size = 19, antiderivative size = 129

$$\begin{aligned}\int (a + a\sec(c+dx))\tan^8(c+dx)\,dx = ax &+ \frac{35a\operatorname{arctanh}(\sin(c+dx))}{128d} \\ &- \frac{(128a + 35a\sec(c+dx))\tan(c+dx)}{128d} \\ &+ \frac{(64a + 35a\sec(c+dx))\tan^3(c+dx)}{192d} \\ &- \frac{(48a + 35a\sec(c+dx))\tan^5(c+dx)}{240d} \\ &+ \frac{(8a + 7a\sec(c+dx))\tan^7(c+dx)}{56d}\end{aligned}$$

output

```
a*x+35/128*a*arctanh(sin(d*x+c))/d-1/128*(128*a+35*a*sec(d*x+c))*tan(d*x+c
)/d+1/192*(64*a+35*a*sec(d*x+c))*tan(d*x+c)^3/d-1/240*(48*a+35*a*sec(d*x+c
))*tan(d*x+c)^5/d+1/56*(8*a+7*a*sec(d*x+c))*tan(d*x+c)^7/d
```

3.10.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 241, normalized size of antiderivative = 1.87

$$\begin{aligned}\int (a + a\sec(c+dx))\tan^8(c+dx)\,dx = {} & \frac{a\arctan(\tan(c+dx))}{d} + \frac{35a\operatorname{arctanh}(\sin(c+dx))}{128d} \\ & - \frac{a\tan(c+dx)}{d} + \frac{35a\sec(c+dx)\tan(c+dx)}{128d} \\ & + \frac{35a\sec^3(c+dx)\tan(c+dx)}{192d} \\ & + \frac{7a\sec^5(c+dx)\tan(c+dx)}{48d} \\ & - \frac{7a\sec^7(c+dx)\tan(c+dx)}{8d} + \frac{a\tan^3(c+dx)}{3d} \\ & + \frac{7a\sec^5(c+dx)\tan^3(c+dx)}{3d} \\ & - \frac{a\tan^5(c+dx)}{5d} - \frac{7a\sec^3(c+dx)\tan^5(c+dx)}{3d} \\ & + \frac{a\tan^7(c+dx)}{7d} + \frac{a\sec(c+dx)\tan^7(c+dx)}{d}\end{aligned}$$

input
```
Integrate[(a + a*Sec[c + d*x])*Tan[c + d*x]^8,x]
```

output
```
(a*ArcTan[Tan[c + d*x]])/d + (35*a*ArcTanh[Sin[c + d*x]])/(128*d) - (a*Tan
[c + d*x])/d + (35*a*Sec[c + d*x]*Tan[c + d*x])/(128*d) + (35*a*Sec[c + d*
x]^3*Tan[c + d*x])/(192*d) + (7*a*Sec[c + d*x]^5*Tan[c + d*x])/(48*d) - (7
*a*Sec[c + d*x]^7*Tan[c + d*x])/(8*d) + (a*Tan[c + d*x]^3)/(3*d) + (7*a*Se
c[c + d*x]^5*Tan[c + d*x]^3)/(3*d) - (a*Tan[c + d*x]^5)/(5*d) - (7*a*Sec[c
 + d*x]^3*Tan[c + d*x]^5)/(3*d) + (a*Tan[c + d*x]^7)/(7*d) + (a*Sec[c + d*
x]*Tan[c + d*x]^7)/d
```

3.10.3 Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.15, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.526$, Rules used = {3042, 4369, 3042, 4369, 3042, 4369, 27, 3042, 4369, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^8(c+dx)(a\sec(c+dx)+a)\,dx$$

$$\downarrow 3042$$

$$\int \cot\left(c+dx+\frac{\pi}{2}\right)^8\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)dx$$

$$\downarrow 4369$$

$$\frac{\tan^7(c+dx)(7a\sec(c+dx)+8a)}{56d}-\frac{1}{8}\int(7\sec(c+dx)a+8a)\tan^6(c+dx)dx$$

$$\downarrow 3042$$

$$\frac{\tan^7(c+dx)(7a\sec(c+dx)+8a)}{56d}-\frac{1}{8}\int\cot\left(c+dx+\frac{\pi}{2}\right)^6\left(7\csc\left(c+dx+\frac{\pi}{2}\right)a+8a\right)dx$$

$$\downarrow 4369$$

$$\frac{1}{8}\left(\frac{1}{6}\int(35\sec(c+dx)a+48a)\tan^4(c+dx)dx-\frac{\tan^5(c+dx)(35a\sec(c+dx)+48a)}{30d}\right)+\frac{\tan^7(c+dx)(7a\sec(c+dx)+8a)}{56d}$$

$$\downarrow 3042$$

$$\frac{1}{8}\left(\frac{1}{6}\int\cot\left(c+dx+\frac{\pi}{2}\right)^4\left(35\csc\left(c+dx+\frac{\pi}{2}\right)a+48a\right)dx-\frac{\tan^5(c+dx)(35a\sec(c+dx)+48a)}{30d}\right)+\frac{\tan^7(c+dx)(7a\sec(c+dx)+8a)}{56d}$$

$$\downarrow 4369$$

$$\frac{1}{8}\left(\frac{1}{6}\left(\frac{\tan^3(c+dx)(35a\sec(c+dx)+64a)}{4d}-\frac{1}{4}\int 3(35\sec(c+dx)a+64a)\tan^2(c+dx)dx\right)-\frac{\tan^5(c+dx)(35a}{3}\right.$$
$$\left.+\frac{\tan^7(c+dx)(7a\sec(c+dx)+8a)}{56d}\right.$$

$$\downarrow 27$$

$$\frac{1}{8}\left(\frac{1}{6}\left(\frac{\tan^3(c+dx)(35a\sec(c+dx)+64a)}{4d}-\frac{3}{4}\int(35\sec(c+dx)a+64a)\tan^2(c+dx)dx\right)-\frac{\tan^5(c+dx)(35a\,s}{30}\right.$$
$$\left.+\frac{\tan^7(c+dx)(7a\sec(c+dx)+8a)}{56d}\right.$$

$$\downarrow 3042$$

$$\frac{1}{8}\left(\frac{1}{6}\left(\frac{\tan^3(c+dx)(35a\sec(c+dx)+64a)}{4d}-\frac{3}{4}\int\cot\left(c+dx+\frac{\pi}{2}\right)^2\left(35\csc\left(c+dx+\frac{\pi}{2}\right)a+64a\right)dx\right)-\frac{\tan^5}{}$$

$$\frac{\tan^7(c+dx)(7a\sec(c+dx)+8a)}{56d}$$

$$\downarrow 4369$$

$$\frac{1}{8}\left(\frac{1}{6}\left(\frac{\tan^3(c+dx)(35a\sec(c+dx)+64a)}{4d}-\frac{3}{4}\left(\frac{\tan(c+dx)(35a\sec(c+dx)+128a)}{2d}-\frac{1}{2}\int(35\sec(c+dx)a+\right.\right.\right.$$

$$\frac{\tan^7(c+dx)(7a\sec(c+dx)+8a)}{56d}$$

$$\downarrow 2009$$

$$\frac{1}{8}\left(\frac{1}{6}\left(\frac{\tan^3(c+dx)(35a\sec(c+dx)+64a)}{4d}-\frac{3}{4}\left(\frac{1}{2}\left(-\frac{35a\operatorname{arctanh}(\sin(c+dx))}{d}-128ax\right)+\frac{\tan(c+dx)(35a\sec}{2d}\right.\right.\right.$$

$$\frac{\tan^7(c+dx)(7a\sec(c+dx)+8a)}{56d}$$

input
```
Int[(a + a*Sec[c + d*x])*Tan[c + d*x]^8,x]
```

output
```
((8*a + 7*a*Sec[c + d*x])*Tan[c + d*x]^7)/(56*d) + (-1/30*((48*a + 35*a*Se
c[c + d*x])*Tan[c + d*x]^5)/d + (((64*a + 35*a*Sec[c + d*x])*Tan[c + d*x]^
3)/(4*d) - (3*((-128*a*x - (35*a*ArcTanh[Sin[c + d*x]])/d)/2 + ((128*a + 3
5*a*Sec[c + d*x])*Tan[c + d*x])/(2*d)))/4)/6)/8
```

3.10.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4369

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(-e)*(e*Cot[c + d*x])^(m - 1)*((a*m + b*(m - 1)*Csc
[c + d*x])/(d*m*(m - 1))), x] - Simp[e^2/m   Int[(e*Cot[c + d*x])^(m - 2)*(
a*m + b*(m - 1)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && GtQ[m
, 1]
```

3.10.4 Maple [A] (verified)

Time = 1.67 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.40

method	result
derivativedivides	$\frac{a\left(\frac{\sin(dx+c)^9}{8\cos(dx+c)^8}-\frac{\sin(dx+c)^9}{48\cos(dx+c)^6}+\frac{\sin(dx+c)^9}{64\cos(dx+c)^4}-\frac{5\sin(dx+c)^9}{128\cos(dx+c)^2}-\frac{5\sin(dx+c)^7}{128}-\frac{7\sin(dx+c)^5}{128}-\frac{35\sin(dx+c)^3}{384}-\frac{35\sin(dx+c)}{128}+$ … d
default	$\frac{a\left(\frac{\sin(dx+c)^9}{8\cos(dx+c)^8}-\frac{\sin(dx+c)^9}{48\cos(dx+c)^6}+\frac{\sin(dx+c)^9}{64\cos(dx+c)^4}-\frac{5\sin(dx+c)^9}{128\cos(dx+c)^2}-\frac{5\sin(dx+c)^7}{128}-\frac{7\sin(dx+c)^5}{128}-\frac{35\sin(dx+c)^3}{384}-\frac{35\sin(dx+c)}{128}+$ … d
parts	$\frac{a\left(\frac{\tan(dx+c)^7}{7}-\frac{\tan(dx+c)^5}{5}+\frac{\tan(dx+c)^3}{3}-\tan(dx+c)+\arctan(\tan(dx+c))\right)}{d}+\frac{a\left(\frac{\sin(dx+c)^9}{8\cos(dx+c)^8}-\frac{\sin(dx+c)^9}{48\cos(dx+c)^6}+\frac{\sin(dx}{64\cos(d}\right.$
risch	$ax+\frac{ia\left(9765\,e^{15i(dx+c)}-53760\,e^{14i(dx+c)}+3185\,e^{13i(dx+c)}-215040\,e^{12i(dx+c)}+62965\,e^{11i(dx+c)}-555520\,e^{10i(dx+c)}\right.$ …

input

```
int((a+a*sec(d*x+c))*tan(d*x+c)^8,x,method=_RETURNVERBOSE)
```

output

```
1/d*(a*(1/8*sin(d*x+c)^9/cos(d*x+c)^8-1/48*sin(d*x+c)^9/cos(d*x+c)^6+1/64*
sin(d*x+c)^9/cos(d*x+c)^4-5/128*sin(d*x+c)^9/cos(d*x+c)^2-5/128*sin(d*x+c)
^7-7/128*sin(d*x+c)^5-35/384*sin(d*x+c)^3-35/128*sin(d*x+c)+35/128*ln(sec(
d*x+c)+tan(d*x+c)))+a*(1/7*tan(d*x+c)^7-1/5*tan(d*x+c)^5+1/3*tan(d*x+c)^3-
tan(d*x+c)+d*x+c))
```

3.10.5 Fricas [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.21

$$\int (a+a\sec(c+dx))\tan^8(c+dx)\,dx$$

$$=\frac{26880\,adx\cos(dx+c)^8+3675\,a\cos(dx+c)^8\log(\sin(dx+c)+1)-3675\,a\cos(dx+c)^8\log(-\sin(dx+$$

input

```
integrate((a+a*sec(d*x+c))*tan(d*x+c)^8,x, algorithm="fricas")
```

output
```
1/26880*(26880*a*d*x*cos(d*x + c)^8 + 3675*a*cos(d*x + c)^8*log(sin(d*x +
c) + 1) - 3675*a*cos(d*x + c)^8*log(-sin(d*x + c) + 1) - 2*(22528*a*cos(d*
x + c)^7 + 9765*a*cos(d*x + c)^6 - 15616*a*cos(d*x + c)^5 - 11410*a*cos(d*
x + c)^4 + 8448*a*cos(d*x + c)^3 + 7000*a*cos(d*x + c)^2 - 1920*a*cos(d*x
+ c) - 1680*a)*sin(d*x + c))/(d*cos(d*x + c)^8)
```

3.10.6 Sympy [F]

$$\int (a + a\sec(c+dx))\tan^8(c+dx)\,dx = a\left(\int \tan^8(c+dx)\sec(c+dx)\,dx + \int \tan^8(c+dx)\,dx\right)$$

input
```
integrate((a+a*sec(d*x+c))*tan(d*x+c)**8,x)
```

output
```
a*(Integral(tan(c + d*x)**8*sec(c + d*x), x) + Integral(tan(c + d*x)**8, x
))
```

3.10.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.27

$$\int (a + a\sec(c+dx))\tan^8(c+dx)\,dx$$

$$= \frac{256\left(15\tan(dx+c)^7 - 21\tan(dx+c)^5 + 35\tan(dx+c)^3 + 105\,dx + 105\,c - 105\tan(dx+c)\right)a + 35\,a}{}$$

input
```
integrate((a+a*sec(d*x+c))*tan(d*x+c)^8,x, algorithm="maxima")
```

output
```
1/26880*(256*(15*tan(d*x + c)^7 - 21*tan(d*x + c)^5 + 35*tan(d*x + c)^3 +
105*d*x + 105*c - 105*tan(d*x + c))*a + 35*a*(2*(279*sin(d*x + c)^7 - 511*
sin(d*x + c)^5 + 385*sin(d*x + c)^3 - 105*sin(d*x + c))/(sin(d*x + c)^8 -
4*sin(d*x + c)^6 + 6*sin(d*x + c)^4 - 4*sin(d*x + c)^2 + 1) + 105*log(sin(
d*x + c) + 1) - 105*log(sin(d*x + c) - 1)))/d
```

3.10.8 Giac [A] (verification not implemented)

Time = 4.83 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.35

$$\int (a + a\sec(c + dx))\tan^8(c + dx)\,dx$$

$$= \frac{13440\,(dx + c)a + 3675\,a\log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) + 1\right|\right) - 3675\,a\log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) - 1\right|\right) + \frac{2\left(9765\,a\tan\left(\frac{1}{2}\right.\right.}{}$$

input
```
integrate((a+a*sec(d*x+c))*tan(d*x+c)^8,x, algorithm="giac")
```

output
```
1/13440*(13440*(d*x + c)*a + 3675*a*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 3
675*a*log(abs(tan(1/2*d*x + 1/2*c) - 1)) + 2*(9765*a*tan(1/2*d*x + 1/2*c)^
15 - 83825*a*tan(1/2*d*x + 1/2*c)^13 + 321013*a*tan(1/2*d*x + 1/2*c)^11 -
724649*a*tan(1/2*d*x + 1/2*c)^9 + 1078359*a*tan(1/2*d*x + 1/2*c)^7 - 50868
3*a*tan(1/2*d*x + 1/2*c)^5 + 140175*a*tan(1/2*d*x + 1/2*c)^3 - 17115*a*tan
(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^8)/d
```

3.10.9 Mupad [B] (verification not implemented)

Time = 15.18 (sec) , antiderivative size = 242, normalized size of antiderivative = 1.88

$$\int (a + a\sec(c + dx))\tan^8(c + dx)\,dx = a\,x$$

$$- \frac{-\frac{93\,a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{15}}{64} + \frac{2395\,a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{13}}{192} - \frac{45859\,a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{11}}{960} + \frac{724649\,a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{9}}{6720} - \frac{359453\,a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{7}}{2240} +}{d\left(\tan\left(\frac{c}{2} + \frac{dx}{2}\right)^{16} - 8\tan\left(\frac{c}{2} + \frac{dx}{2}\right)^{14} + 28\tan\left(\frac{c}{2} + \frac{dx}{2}\right)^{12} - 56\tan\left(\frac{c}{2} + \frac{dx}{2}\right)^{10} + 70\tan\left(\frac{c}{2} + \frac{dx}{2}\right)^{8} - 56\right.}$$

$$+ \frac{35\,a\,\mathrm{atanh}\left(\tan\left(\frac{c}{2} + \frac{dx}{2}\right)\right)}{64\,d}$$

input
```
int(tan(c + d*x)^8*(a + a/cos(c + d*x)),x)
```

output

```
a*x - ((163*a*tan(c/2 + (d*x)/2))/64 - (1335*a*tan(c/2 + (d*x)/2)^3)/64 +
(24223*a*tan(c/2 + (d*x)/2)^5)/320 - (359453*a*tan(c/2 + (d*x)/2)^7)/2240
+ (724649*a*tan(c/2 + (d*x)/2)^9)/6720 - (45859*a*tan(c/2 + (d*x)/2)^11)/9
60 + (2395*a*tan(c/2 + (d*x)/2)^13)/192 - (93*a*tan(c/2 + (d*x)/2)^15)/64)
/(d*(28*tan(c/2 + (d*x)/2)^4 - 8*tan(c/2 + (d*x)/2)^2 - 56*tan(c/2 + (d*x)
/2)^6 + 70*tan(c/2 + (d*x)/2)^8 - 56*tan(c/2 + (d*x)/2)^10 + 28*tan(c/2 +
(d*x)/2)^12 - 8*tan(c/2 + (d*x)/2)^14 + tan(c/2 + (d*x)/2)^16 + 1)) + (35*
a*atanh(tan(c/2 + (d*x)/2)))/(64*d)
```

3.11 $\int (a + a\sec(c+dx))\tan^6(c+dx)\,dx$

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3.11.1 Optimal result

Integrand size = 19, antiderivative size = 102

$$\begin{aligned}\int (a + a\sec(c+dx))\tan^6(c+dx)\,dx = &-ax - \frac{5a\operatorname{arctanh}(\sin(c+dx))}{16d} \\ &+ \frac{(16a + 5a\sec(c+dx))\tan(c+dx)}{16d} \\ &- \frac{(8a + 5a\sec(c+dx))\tan^3(c+dx)}{24d} \\ &+ \frac{(6a + 5a\sec(c+dx))\tan^5(c+dx)}{30d}\end{aligned}$$

output

```
-a*x-5/16*a*arctanh(sin(d*x+c))/d+1/16*(16*a+5*a*sec(d*x+c))*tan(d*x+c)/d-
1/24*(8*a+5*a*sec(d*x+c))*tan(d*x+c)^3/d+1/30*(6*a+5*a*sec(d*x+c))*tan(d*x
+c)^5/d
```

3.11.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.75

$$\begin{aligned}\int (a + a\sec(c+dx))\tan^6(c+dx)\,dx = &-\frac{a\arctan(\tan(c+dx))}{d} - \frac{5a\operatorname{arctanh}(\sin(c+dx))}{16d} \\ &+ \frac{a\tan(c+dx)}{d} - \frac{5a\sec(c+dx)\tan(c+dx)}{16d} \\ &- \frac{5a\sec^3(c+dx)\tan(c+dx)}{24d} \\ &+ \frac{5a\sec^5(c+dx)\tan(c+dx)}{6d} - \frac{a\tan^3(c+dx)}{3d} \\ &- \frac{5a\sec^3(c+dx)\tan^3(c+dx)}{3d} \\ &+ \frac{a\tan^5(c+dx)}{5d} + \frac{a\sec(c+dx)\tan^5(c+dx)}{d}\end{aligned}$$

input
```
Integrate[(a + a*Sec[c + d*x])*Tan[c + d*x]^6,x]
```

output
```
-((a*ArcTan[Tan[c + d*x]])/d) - (5*a*ArcTanh[Sin[c + d*x]])/(16*d) + (a*Ta
n[c + d*x])/d - (5*a*Sec[c + d*x]*Tan[c + d*x])/(16*d) - (5*a*Sec[c + d*x]
^3*Tan[c + d*x])/(24*d) + (5*a*Sec[c + d*x]^5*Tan[c + d*x])/(6*d) - (a*Tan
[c + d*x]^3)/(3*d) - (5*a*Sec[c + d*x]^3*Tan[c + d*x]^3)/(3*d) + (a*Tan[c
+ d*x]^5)/(5*d) + (a*Sec[c + d*x]*Tan[c + d*x]^5)/d
```

3.11.3 Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.13, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$, Rules used = {3042, 4369, 3042, 4369, 27, 3042, 4369, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^6(c+dx)(a\sec(c+dx)+a)\,dx$$

$$\downarrow 3042$$

$$\int \cot\left(c+dx+\frac{\pi}{2}\right)^6\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)dx$$

$$\downarrow 4369$$

$$\frac{\tan^5(c+dx)(5a\sec(c+dx)+6a)}{30d}-\frac{1}{6}\int(5\sec(c+dx)a+6a)\tan^4(c+dx)dx$$

$$\downarrow 3042$$

$$\frac{\tan^5(c+dx)(5a\sec(c+dx)+6a)}{30d}-\frac{1}{6}\int\cot\left(c+dx+\frac{\pi}{2}\right)^4\left(5\csc\left(c+dx+\frac{\pi}{2}\right)a+6a\right)dx$$

$$\downarrow 4369$$

$$\frac{1}{6}\left(\frac{1}{4}\int 3(5\sec(c+dx)a+8a)\tan^2(c+dx)dx-\frac{\tan^3(c+dx)(5a\sec(c+dx)+8a)}{4d}\right)+\frac{\tan^5(c+dx)(5a\sec(c+dx)+6a)}{30d}$$

$$\downarrow 27$$

$$\frac{1}{6}\left(\frac{3}{4}\int(5\sec(c+dx)a+8a)\tan^2(c+dx)dx-\frac{\tan^3(c+dx)(5a\sec(c+dx)+8a)}{4d}\right)+\frac{\tan^5(c+dx)(5a\sec(c+dx)+6a)}{30d}$$

$$\downarrow 3042$$

$$\frac{1}{6}\left(\frac{3}{4}\int\cot\left(c+dx+\frac{\pi}{2}\right)^2\left(5\csc\left(c+dx+\frac{\pi}{2}\right)a+8a\right)dx-\frac{\tan^3(c+dx)(5a\sec(c+dx)+8a)}{4d}\right)+\frac{\tan^5(c+dx)(5a\sec(c+dx)+6a)}{30d}$$

$$\downarrow 4369$$

$$\frac{1}{6}\left(\frac{3}{4}\left(\frac{\tan(c+dx)(5a\sec(c+dx)+16a)}{2d}-\frac{1}{2}\int(5\sec(c+dx)a+16a)dx\right)-\frac{\tan^3(c+dx)(5a\sec(c+dx)+8a)}{4d}\right)+\frac{\tan^5(c+dx)(5a\sec(c+dx)+6a)}{30d}$$

$$\downarrow 2009$$

$$\frac{1}{6}\left(\frac{3}{4}\left(\frac{1}{2}\left(-\frac{5a\operatorname{arctanh}(\sin(c+dx))}{d}-16ax\right)+\frac{\tan(c+dx)(5a\sec(c+dx)+16a)}{2d}\right)-\frac{\tan^3(c+dx)(5a\sec(c+d\right.$$

$$\left.\frac{\tan^5(c+dx)(5a\sec(c+dx)+6a)}{30d}\right.$$

input
```
Int[(a + a*Sec[c + d*x])*Tan[c + d*x]^6,x]
```

output
```
((6*a + 5*a*Sec[c + d*x])*Tan[c + d*x]^5)/(30*d) + (-1/4*((8*a + 5*a*Sec[c
 + d*x])*Tan[c + d*x]^3)/d + (3*((-16*a*x - (5*a*ArcTanh[Sin[c + d*x]])/d)
/2 + ((16*a + 5*a*Sec[c + d*x])*Tan[c + d*x])/(2*d)))/4)/6
```

3.11.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4369
```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(-e)*(e*Cot[c + d*x])^(m - 1)*((a*m + b*(m - 1)*Csc
[c + d*x])/(d*m*(m - 1))), x] - Simp[e^2/m   Int[(e*Cot[c + d*x])^(m - 2)*(
a*m + b*(m - 1)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && GtQ[m
, 1]
```

3.11.4 Maple [A] (verified)

Time = 1.46 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.40

method	result
derivativedivides	$\frac{a\left(\frac{\sin(dx+c)^7}{6\cos(dx+c)^6}-\frac{\sin(dx+c)^7}{24\cos(dx+c)^4}+\frac{\sin(dx+c)^7}{16\cos(dx+c)^2}+\frac{\sin(dx+c)^5}{16}+\frac{5\sin(dx+c)^3}{48}+\frac{5\sin(dx+c)}{16}-\frac{5\ln(\sec(dx+c)+\tan(dx+c))}{16}\right)+a\left(\frac{t}{d}\right.$
default	$\frac{a\left(\frac{\sin(dx+c)^7}{6\cos(dx+c)^6}-\frac{\sin(dx+c)^7}{24\cos(dx+c)^4}+\frac{\sin(dx+c)^7}{16\cos(dx+c)^2}+\frac{\sin(dx+c)^5}{16}+\frac{5\sin(dx+c)^3}{48}+\frac{5\sin(dx+c)}{16}-\frac{5\ln(\sec(dx+c)+\tan(dx+c))}{16}\right)+a\left(\frac{t}{d}\right.$
parts	$\frac{a\left(\frac{\tan(dx+c)^5}{5}-\frac{\tan(dx+c)^3}{3}+\tan(dx+c)-\arctan(\tan(dx+c))\right)}{d}+\frac{a\left(\frac{\sin(dx+c)^7}{6\cos(dx+c)^6}-\frac{\sin(dx+c)^7}{24\cos(dx+c)^4}+\frac{\sin(dx+c)^7}{16\cos(dx+c)^2}+\frac{\sin(d}{1}\right.$
risch	$-ax-\frac{ia\left(165\,e^{11i(dx+c)}-720\,e^{10i(dx+c)}-25\,e^{9i(dx+c)}-2160\,e^{8i(dx+c)}+450\,e^{7i(dx+c)}-3680\,e^{6i(dx+c)}-450\,e^{5i(dx+c)}\right.$

input
```
int((a+a*sec(d*x+c))*tan(d*x+c)^6,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a*(1/6*sin(d*x+c)^7/cos(d*x+c)^6-1/24*sin(d*x+c)^7/cos(d*x+c)^4+1/16*
sin(d*x+c)^7/cos(d*x+c)^2+1/16*sin(d*x+c)^5+5/48*sin(d*x+c)^3+5/16*sin(d*x
+c)-5/16*ln(sec(d*x+c)+tan(d*x+c)))+a*(1/5*tan(d*x+c)^5-1/3*tan(d*x+c)^3+t
an(d*x+c)-d*x-c))
```

3.11.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.31

$$\int (a + a\sec(c+dx))\tan^6(c+dx)\,dx =$$
$$-\frac{480\,adx\cos(dx+c)^6 + 75\,a\cos(dx+c)^6\log(\sin(dx+c)+1) - 75\,a\cos(dx+c)^6\log(-\sin(dx+c)}{}$$

input
```
integrate((a+a*sec(d*x+c))*tan(d*x+c)^6,x, algorithm="fricas")
```

output
```
-1/480*(480*a*d*x*cos(d*x + c)^6 + 75*a*cos(d*x + c)^6*log(sin(d*x + c) +
1) - 75*a*cos(d*x + c)^6*log(-sin(d*x + c) + 1) - 2*(368*a*cos(d*x + c)^5
+ 165*a*cos(d*x + c)^4 - 176*a*cos(d*x + c)^3 - 130*a*cos(d*x + c)^2 + 48*
a*cos(d*x + c) + 40*a)*sin(d*x + c))/(d*cos(d*x + c)^6)
```

3.11.6 Sympy [F]

$$\int (a + a\sec(c+dx))\tan^6(c+dx)\,dx = a\left(\int \tan^6(c+dx)\sec(c+dx)\,dx + \int \tan^6(c+dx)\,dx\right)$$

input
```
integrate((a+a*sec(d*x+c))*tan(d*x+c)**6,x)
```

output
```
a*(Integral(tan(c + d*x)**6*sec(c + d*x), x) + Integral(tan(c + d*x)**6, x
))
```

3.11.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.31

$$\int (a + a\sec(c+dx))\tan^6(c+dx)\,dx$$

$$= \frac{32\left(3\tan(dx+c)^5 - 5\tan(dx+c)^3 - 15\,dx - 15\,c + 15\tan(dx+c)\right)a - 5\,a\left(\frac{2\left(33\sin(dx+c)^5 - 40\sin(dx+c)^3 + 15\sin(dx+c)\right)}{\sin(dx+c)^6 - 3\sin(dx+c)^4 + 3\sin(dx+c)^2 - 1} + 15\log(\sin(dx+c)+1) - 15\log(\sin(dx+c)-1)\right)}{480\,d}$$

input
```
integrate((a+a*sec(d*x+c))*tan(d*x+c)^6,x, algorithm="maxima")
```

output
```
1/480*(32*(3*tan(d*x + c)^5 - 5*tan(d*x + c)^3 - 15*d*x - 15*c + 15*tan(d*
x + c))*a - 5*a*(2*(33*sin(d*x + c)^5 - 40*sin(d*x + c)^3 + 15*sin(d*x + c
))/(sin(d*x + c)^6 - 3*sin(d*x + c)^4 + 3*sin(d*x + c)^2 - 1) + 15*log(sin
(d*x + c) + 1) - 15*log(sin(d*x + c) - 1)))/d
```

3.11.8 Giac [A] (verification not implemented)

Time = 2.22 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.43

$$\int (a + a\sec(c+dx))\tan^6(c+dx)\,dx =$$

$$-\frac{240\,(dx+c)a + 75\,a\log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) + 1\right|\right) - 75\,a\log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) - 1\right|\right) + \frac{2\left(165\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{11} - 1095\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{9} + 3138\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7} - 5118\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5} + 1945\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3} - 315\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2 - 1\right)^6}}{240\,d}$$

input
```
integrate((a+a*sec(d*x+c))*tan(d*x+c)^6,x, algorithm="giac")
```

output
```
-1/240*(240*(d*x + c)*a + 75*a*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 75*a*l
og(abs(tan(1/2*d*x + 1/2*c) - 1)) + 2*(165*a*tan(1/2*d*x + 1/2*c)^11 - 109
5*a*tan(1/2*d*x + 1/2*c)^9 + 3138*a*tan(1/2*d*x + 1/2*c)^7 - 5118*a*tan(1/
2*d*x + 1/2*c)^5 + 1945*a*tan(1/2*d*x + 1/2*c)^3 - 315*a*tan(1/2*d*x + 1/2
*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^6)/d
```

3.11.9 Mupad [B] (verification not implemented)

Time = 14.92 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.84

$$\int (a + a\sec(c+dx))\tan^6(c+dx)\,dx$$

$$= \frac{-\frac{11\,a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{11}}{8} + \frac{73\,a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{9}}{8} - \frac{523\,a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{7}}{20} + \frac{853\,a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{5}}{20} - \frac{389\,a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{3}}{24} + \frac{21\,a\tan\left(}{8}}{d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{12} - 6\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{10} + 15\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{8} - 20\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{6} + 15\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{4} - 6\tan\left(\right.\right.}$$

$$- a\,x - \frac{5\,a\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{8\,d}$$

input
```
int(tan(c + d*x)^6*(a + a/cos(c + d*x)),x)
```

output
```
((21*a*tan(c/2 + (d*x)/2))/8 - (389*a*tan(c/2 + (d*x)/2)^3)/24 + (853*a*ta
n(c/2 + (d*x)/2)^5)/20 - (523*a*tan(c/2 + (d*x)/2)^7)/20 + (73*a*tan(c/2 +
 (d*x)/2)^9)/8 - (11*a*tan(c/2 + (d*x)/2)^11)/8)/(d*(15*tan(c/2 + (d*x)/2)
^4 - 6*tan(c/2 + (d*x)/2)^2 - 20*tan(c/2 + (d*x)/2)^6 + 15*tan(c/2 + (d*x)
/2)^8 - 6*tan(c/2 + (d*x)/2)^10 + tan(c/2 + (d*x)/2)^12 + 1)) - a*x - (5*a
*atanh(tan(c/2 + (d*x)/2)))/(8*d)
```

3.12 $\int (a + a\sec(c+dx))\tan^4(c+dx)\,dx$

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3.12.1 Optimal result

Integrand size = 19, antiderivative size = 73

$$\int (a + a\sec(c+dx))\tan^4(c+dx)\,dx = ax + \frac{3a\operatorname{arctanh}(\sin(c+dx))}{8d} - \frac{(8a+3a\sec(c+dx))\tan(c+dx)}{8d} + \frac{(4a+3a\sec(c+dx))\tan^3(c+dx)}{12d}$$

output
```
a*x+3/8*a*arctanh(sin(d*x+c))/d-1/8*(8*a+3*a*sec(d*x+c))*tan(d*x+c)/d+1/12
*(4*a+3*a*sec(d*x+c))*tan(d*x+c)^3/d
```

3.12.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.60

$$\int (a + a\sec(c+dx))\tan^4(c+dx)\,dx = \frac{a\arctan(\tan(c+dx))}{d} + \frac{3a\operatorname{arctanh}(\sin(c+dx))}{8d} - \frac{a\tan(c+dx)}{d} + \frac{3a\sec(c+dx)\tan(c+dx)}{8d} - \frac{3a\sec^3(c+dx)\tan(c+dx)}{4d} + \frac{a\tan^3(c+dx)}{3d} + \frac{a\sec(c+dx)\tan^3(c+dx)}{d}$$

input
```
Integrate[(a + a*Sec[c + d*x])*Tan[c + d*x]^4,x]
```

output
```
(a*ArcTan[Tan[c + d*x]])/d + (3*a*ArcTanh[Sin[c + d*x]])/(8*d) - (a*Tan[c
+ d*x])/d + (3*a*Sec[c + d*x]*Tan[c + d*x])/(8*d) - (3*a*Sec[c + d*x]^3*Ta
n[c + d*x])/(4*d) + (a*Tan[c + d*x]^3)/(3*d) + (a*Sec[c + d*x]*Tan[c + d*x
]^3)/d
```

3.12.3 Rubi [A] (verified)

Time $= 0.35$ (sec) , antiderivative size $= 82$, normalized size of antiderivative $= 1.12$, number of steps used $= 5$, number of rules used $= 5$, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used $= \{3042, 4369, 3042, 4369, 2009\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^4(c+dx)(a\sec(c+dx)+a)\,dx$$

$$\downarrow 3042$$

$$\int \cot\left(c+dx+\frac{\pi}{2}\right)^4\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)dx$$

$$\downarrow 4369$$

$$\frac{\tan^3(c+dx)(3a\sec(c+dx)+4a)}{12d}-\frac{1}{4}\int(3\sec(c+dx)a+4a)\tan^2(c+dx)dx$$

$$\downarrow 3042$$

$$\frac{\tan^3(c+dx)(3a\sec(c+dx)+4a)}{12d}-\frac{1}{4}\int\cot\left(c+dx+\frac{\pi}{2}\right)^2\left(3\csc\left(c+dx+\frac{\pi}{2}\right)a+4a\right)dx$$

$$\downarrow 4369$$

$$\frac{1}{4}\left(\frac{1}{2}\int(3\sec(c+dx)a+8a)dx-\frac{\tan(c+dx)(3a\sec(c+dx)+8a)}{2d}\right)+\frac{\tan^3(c+dx)(3a\sec(c+dx)+4a)}{12d}$$

$$\downarrow 2009$$

$$\frac{1}{4}\left(\frac{1}{2}\left(\frac{3a\operatorname{arctanh}(\sin(c+dx))}{d}+8ax\right)-\frac{\tan(c+dx)(3a\sec(c+dx)+8a)}{2d}\right)+\frac{\tan^3(c+dx)(3a\sec(c+dx)+4a)}{12d}$$

input
```
Int[(a + a*Sec[c + d*x])*Tan[c + d*x]^4,x]
```

output
```
((4*a + 3*a*Sec[c + d*x])*Tan[c + d*x]^3)/(12*d) + ((8*a*x + (3*a*ArcTanh[
Sin[c + d*x]])/d)/2 - ((8*a + 3*a*Sec[c + d*x])*Tan[c + d*x])/(2*d))/4
```

3.12.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4369
```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(-e)*(e*Cot[c + d*x])^(m - 1)*((a*m + b*(m - 1)*Csc
[c + d*x])/(d*m*(m - 1))), x] - Simp[e^2/m   Int[(e*Cot[c + d*x])^(m - 2)*(
a*m + b*(m - 1)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && GtQ[m
, 1]
```

3.12.4 Maple [A] (verified)

Time = 1.54 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.42

method	result
derivativedivides	$\frac{a\left(\frac{\sin(dx+c)^5}{4\cos(dx+c)^4}-\frac{\sin(dx+c)^5}{8\cos(dx+c)^2}-\frac{\sin(dx+c)^3}{8}-\frac{3\sin(dx+c)}{8}+\frac{3\ln(\sec(dx+c)+\tan(dx+c))}{8}\right)+a\left(\frac{\tan(dx+c)^3}{3}-\tan(dx+c)+dx+c\right.}{d}$
default	$\frac{a\left(\frac{\sin(dx+c)^5}{4\cos(dx+c)^4}-\frac{\sin(dx+c)^5}{8\cos(dx+c)^2}-\frac{\sin(dx+c)^3}{8}-\frac{3\sin(dx+c)}{8}+\frac{3\ln(\sec(dx+c)+\tan(dx+c))}{8}\right)+a\left(\frac{\tan(dx+c)^3}{3}-\tan(dx+c)+dx+c\right.}{d}$
parts	$\frac{a\left(\frac{\tan(dx+c)^3}{3}-\tan(dx+c)+\arctan(\tan(dx+c))\right)}{d}+\frac{a\left(\frac{\sin(dx+c)^5}{4\cos(dx+c)^4}-\frac{\sin(dx+c)^5}{8\cos(dx+c)^2}-\frac{\sin(dx+c)^3}{8}-\frac{3\sin(dx+c)}{8}+\frac{3\ln(\sec(d}{d}$
risch	$ax+\frac{ia\left(15\,e^{7i(dx+c)}-48\,e^{6i(dx+c)}-9\,e^{5i(dx+c)}-96\,e^{4i(dx+c)}+9\,e^{3i(dx+c)}-80\,e^{2i(dx+c)}-15\,e^{i(dx+c)}-32\right)}{12d\left(e^{2i(dx+c)}+1\right)^4}-\frac{3a\ln(e}$

input
```
int((a+a*sec(d*x+c))*tan(d*x+c)^4,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a*(1/4*sin(d*x+c)^5/cos(d*x+c)^4-1/8*sin(d*x+c)^5/cos(d*x+c)^2-1/8*si
n(d*x+c)^3-3/8*sin(d*x+c)+3/8*ln(sec(d*x+c)+tan(d*x+c)))+a*(1/3*tan(d*x+c)
^3-tan(d*x+c)+d*x+c))
```

3.12.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.53

$$\int (a + a\sec(c+dx))\tan^4(c+dx)\,dx$$
$$= \frac{48\,adx\cos(dx+c)^4 + 9\,a\cos(dx+c)^4\log(\sin(dx+c)+1) - 9\,a\cos(dx+c)^4\log(-\sin(dx+c)+1) -}{48\,d\cos(dx+c)^4}$$

input
```
integrate((a+a*sec(d*x+c))*tan(d*x+c)^4,x, algorithm="fricas")
```

output
```
1/48*(48*a*d*x*cos(d*x + c)^4 + 9*a*cos(d*x + c)^4*log(sin(d*x + c) + 1) -
 9*a*cos(d*x + c)^4*log(-sin(d*x + c) + 1) - 2*(32*a*cos(d*x + c)^3 + 15*a
*cos(d*x + c)^2 - 8*a*cos(d*x + c) - 6*a)*sin(d*x + c))/(d*cos(d*x + c)^4)
```

3.12.6 Sympy [F]

$$\int (a + a\sec(c+dx))\tan^4(c+dx)\,dx = a\left(\int \tan^4(c+dx)\sec(c+dx)\,dx + \int \tan^4(c+dx)\,dx\right)$$

input
```
integrate((a+a*sec(d*x+c))*tan(d*x+c)**4,x)
```

output
```
a*(Integral(tan(c + d*x)**4*sec(c + d*x), x) + Integral(tan(c + d*x)**4, x
))
```

3.12.7 Maxima [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.40

$$\int (a + a\sec(c+dx))\tan^4(c+dx)\,dx$$

$$= \frac{16\left(\tan(dx+c)^3 + 3\,dx + 3\,c - 3\,\tan(dx+c)\right)a + 3\,a\left(\frac{2\left(5\sin(dx+c)^3 - 3\sin(dx+c)\right)}{\sin(dx+c)^4 - 2\sin(dx+c)^2 + 1} + 3\log(\sin(dx+c)+1)\right.}{48\,d}$$

input
```
integrate((a+a*sec(d*x+c))*tan(d*x+c)^4,x, algorithm="maxima")
```

output
```
1/48*(16*(tan(d*x + c)^3 + 3*d*x + 3*c - 3*tan(d*x + c))*a + 3*a*(2*(5*sin
(d*x + c)^3 - 3*sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) + 3*
log(sin(d*x + c) + 1) - 3*log(sin(d*x + c) - 1)))/d
```

3.12.8 Giac [A] (verification not implemented)

Time = 0.94 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.62

$$\int (a + a\sec(c+dx))\tan^4(c+dx)\,dx$$

$$= \frac{24\,(dx+c)a + 9\,a\log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) + 1\right|\right) - 9\,a\log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) - 1\right|\right) + \frac{2\left(15\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7 - 71\,a\,t\right.}{24\,d}$$

input
```
integrate((a+a*sec(d*x+c))*tan(d*x+c)^4,x, algorithm="giac")
```

output
```
1/24*(24*(d*x + c)*a + 9*a*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 9*a*log(ab
s(tan(1/2*d*x + 1/2*c) - 1)) + 2*(15*a*tan(1/2*d*x + 1/2*c)^7 - 71*a*tan(1
/2*d*x + 1/2*c)^5 + 137*a*tan(1/2*d*x + 1/2*c)^3 - 33*a*tan(1/2*d*x + 1/2*
c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^4)/d
```

3.12.9 Mupad [B] (verification not implemented)

Time = 14.87 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.84

$$\int (a + a\sec(c+dx))\tan^4(c+dx)\,dx$$

$$= a\,x - \frac{-\frac{5\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}{4} + \frac{71\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{12} - \frac{137\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{12} + \frac{11\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{4}}{d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8 - 4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6 + 6\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4 - 4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2 + 1\right)} + \frac{3\,a\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{4\,d}$$

input
```
int(tan(c + d*x)^4*(a + a/cos(c + d*x)),x)
```

output
```
a*x - ((11*a*tan(c/2 + (d*x)/2))/4 - (137*a*tan(c/2 + (d*x)/2)^3)/12 + (71
*a*tan(c/2 + (d*x)/2)^5)/12 - (5*a*tan(c/2 + (d*x)/2)^7)/4)/(d*(6*tan(c/2
+ (d*x)/2)^4 - 4*tan(c/2 + (d*x)/2)^2 - 4*tan(c/2 + (d*x)/2)^6 + tan(c/2 +
 (d*x)/2)^8 + 1)) + (3*a*atanh(tan(c/2 + (d*x)/2)))/(4*d)
```

3.13 $\int (a + a\sec(c+dx))\tan^2(c+dx)\,dx$

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3.13.1 Optimal result

Integrand size = 19, antiderivative size = 45

$$\int (a + a\sec(c+dx))\tan^2(c+dx)\,dx = -ax - \frac{a\operatorname{arctanh}(\sin(c+dx))}{2d} + \frac{(2a + a\sec(c+dx))\tan(c+dx)}{2d}$$

output
```
-a*x-1/2*a*arctanh(sin(d*x+c))/d+1/2*(2*a+a*sec(d*x+c))*tan(d*x+c)/d
```

3.13.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.33

$$\int (a + a\sec(c+dx))\tan^2(c+dx)\,dx = -\frac{a\arctan(\tan(c+dx))}{d} - \frac{a\operatorname{arctanh}(\sin(c+dx))}{2d} + \frac{a\tan(c+dx)}{d} + \frac{a\sec(c+dx)\tan(c+dx)}{2d}$$

input
```
Integrate[(a + a*Sec[c + d*x])*Tan[c + d*x]^2,x]
```

output
```
-((a*ArcTan[Tan[c + d*x]])/d) - (a*ArcTanh[Sin[c + d*x]])/(2*d) + (a*Tan[c
 + d*x])/d + (a*Sec[c + d*x]*Tan[c + d*x])/(2*d)
```

3.13.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.07, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {3042, 4369, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^2(c+dx)(a\sec(c+dx)+a)\,dx$$

$$\downarrow 3042$$

$$\int \cot\left(c+dx+\frac{\pi}{2}\right)^2\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)dx$$

$$\downarrow 4369$$

$$\frac{\tan(c+dx)(a\sec(c+dx)+2a)}{2d}-\frac{1}{2}\int(\sec(c+dx)a+2a)dx$$

$$\downarrow 2009$$

$$\frac{1}{2}\left(-\frac{a\operatorname{arctanh}(\sin(c+dx))}{d}-2ax\right)+\frac{\tan(c+dx)(a\sec(c+dx)+2a)}{2d}$$

input
```
Int[(a + a*Sec[c + d*x])*Tan[c + d*x]^2,x]
```

output
```
(-2*a*x - (a*ArcTanh[Sin[c + d*x]])/d)/2 + ((2*a + a*Sec[c + d*x])*Tan[c +
 d*x])/(2*d)
```

3.13.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4369

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(-e)*(e*Cot[c + d*x])^(m - 1)*((a*m + b*(m - 1)*Csc
[c + d*x])/(d*m*(m - 1))), x] - Simp[e^2/m   Int[(e*Cot[c + d*x])^(m - 2)*(
a*m + b*(m - 1)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && GtQ[m
, 1]
```

3.13.4 Maple [A] (verified)

Time = 0.70 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.49

method	result	size
derivativedivides	$\frac{a\left(\frac{\sin(dx+c)^3}{2\cos(dx+c)^2}+\frac{\sin(dx+c)}{2}-\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+a(\tan(dx+c)-dx-c)}{d}$	67
default	$\frac{a\left(\frac{\sin(dx+c)^3}{2\cos(dx+c)^2}+\frac{\sin(dx+c)}{2}-\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+a(\tan(dx+c)-dx-c)}{d}$	67
parts	$\frac{a(\tan(dx+c)-\arctan(\tan(dx+c)))}{d}+\frac{a\left(\frac{\sin(dx+c)^3}{2\cos(dx+c)^2}+\frac{\sin(dx+c)}{2}-\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)}{d}$	71
risch	$-ax-\frac{ia\left(e^{3i(dx+c)}-2e^{2i(dx+c)}-e^{i(dx+c)}-2\right)}{d\left(e^{2i(dx+c)}+1\right)^2}-\frac{a\ln\left(e^{i(dx+c)}+i\right)}{2d}+\frac{a\ln\left(e^{i(dx+c)}-i\right)}{2d}$	97

input

```
int((a+a*sec(d*x+c))*tan(d*x+c)^2,x,method=_RETURNVERBOSE)
```

output

```
1/d*(a*(1/2*sin(d*x+c)^3/cos(d*x+c)^2+1/2*sin(d*x+c)-1/2*ln(sec(d*x+c)+tan
(d*x+c)))+a*(tan(d*x+c)-d*x-c))
```

3.13.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 87 vs. $2(41) = 82$.

Time = 0.28 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.93

$$\int (a + a\sec(c + dx))\tan^2(c + dx)\,dx =$$

$$-\frac{4\,adx\cos(dx+c)^2 + a\cos(dx+c)^2\log(\sin(dx+c)+1) - a\cos(dx+c)^2\log(-\sin(dx+c)+1) - 2}{4\,d\cos(dx+c)^2}$$

input

```
integrate((a+a*sec(d*x+c))*tan(d*x+c)^2,x, algorithm="fricas")
```

output
```
-1/4*(4*a*d*x*cos(d*x + c)^2 + a*cos(d*x + c)^2*log(sin(d*x + c) + 1) - a*
cos(d*x + c)^2*log(-sin(d*x + c) + 1) - 2*(2*a*cos(d*x + c) + a)*sin(d*x +
 c))/(d*cos(d*x + c)^2)
```

3.13.6 Sympy [F]

$$\int (a + a\sec(c+dx))\tan^2(c+dx)\,dx = a\left(\int \tan^2(c+dx)\sec(c+dx)\,dx + \int \tan^2(c+dx)\,dx\right)$$

input
```
integrate((a+a*sec(d*x+c))*tan(d*x+c)**2,x)
```

output
```
a*(Integral(tan(c + d*x)**2*sec(c + d*x), x) + Integral(tan(c + d*x)**2, x
))
```

3.13.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.44

$$\int (a + a\sec(c+dx))\tan^2(c+dx)\,dx = -\frac{4\,(dx + c - \tan(dx+c))a + a\left(\frac{2\sin(dx+c)}{\sin(dx+c)^2-1} + \log(\sin(dx+c)+1) - \log(\sin(dx+c)-1)\right)}{4\,d}$$

input
```
integrate((a+a*sec(d*x+c))*tan(d*x+c)^2,x, algorithm="maxima")
```

output
```
-1/4*(4*(d*x + c - tan(d*x + c))*a + a*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1
) + log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)))/d
```

3.13.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 88 vs. $2(41) = 82$.

Time $= 0.50$ (sec) , antiderivative size $= 88$, normalized size of antiderivative $= 1.96$

$$\int (a + a\sec(c+dx))\tan^2(c+dx)\,dx =$$

$$-\frac{2\,(dx+c)a + a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right) - a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right) + \frac{2\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3 - 3\,a\tan\left(\frac{1}{2}\,d\right.\right.}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)^2}}{2\,d}$$

input
```
integrate((a+a*sec(d*x+c))*tan(d*x+c)^2,x, algorithm="giac")
```

output
```
-1/2*(2*(d*x + c)*a + a*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - a*log(abs(tan
(1/2*d*x + 1/2*c) - 1)) + 2*(a*tan(1/2*d*x + 1/2*c)^3 - 3*a*tan(1/2*d*x +
1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^2)/d
```

3.13.9 Mupad [B] (verification not implemented)

Time $= 14.22$ (sec) , antiderivative size $= 80$, normalized size of antiderivative $= 1.78$

$$\int (a + a\sec(c+dx))\tan^2(c+dx)\,dx = \frac{3\,a\tan\left(\frac{c}{2}+\frac{dx}{2}\right) - a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3}{d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4 - 2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2 + 1\right)} - a\,x - \frac{a\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{d}$$

input
```
int(tan(c + d*x)^2*(a + a/cos(c + d*x)),x)
```

output
```
(3*a*tan(c/2 + (d*x)/2) - a*tan(c/2 + (d*x)/2)^3)/(d*(tan(c/2 + (d*x)/2)^4
 - 2*tan(c/2 + (d*x)/2)^2 + 1)) - a*x - (a*atanh(tan(c/2 + (d*x)/2)))/d
```

3.14 $\int \cot^2(c+dx)(a+a\sec(c+dx))\,dx$

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3.14.1 Optimal result

Integrand size = 19, antiderivative size = 26

$$\int \cot^2(c+dx)(a+a\sec(c+dx))\,dx = -ax - \frac{\cot(c+dx)(a+a\sec(c+dx))}{d}$$

output
```
-a*x-cot(d*x+c)*(a+a*sec(d*x+c))/d
```

3.14.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.65

$$\int \cot^2(c+dx)(a+a\sec(c+dx))\,dx$$
$$= -\frac{a\csc(c+dx)}{d} - \frac{a\cot(c+dx)\,\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},-\tan^2(c+dx)\right)}{d}$$

input
```
Integrate[Cot[c + d*x]^2*(a + a*Sec[c + d*x]),x]
```

output
```
-((a*Csc[c + d*x])/d) - (a*Cot[c + d*x]*Hypergeometric2F1[-1/2, 1, 1/2, -T
an[c + d*x]^2])/d
```

3.14.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {3042, 4370, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^2(c+dx)(a\sec(c+dx)+a)\,dx$$

$$\downarrow 3042$$

$$\int \frac{a\csc\left(c+dx+\frac{\pi}{2}\right)+a}{\cot\left(c+dx+\frac{\pi}{2}\right)^2}dx$$

$$\downarrow 4370$$

$$\int -a dx - \frac{\cot(c+dx)(a\sec(c+dx)+a)}{d}$$

$$\downarrow 24$$

$$-\frac{\cot(c+dx)(a\sec(c+dx)+a)}{d} - ax$$

input
```
Int[Cot[c + d*x]^2*(a + a*Sec[c + d*x]),x]
```

output
```
-(a*x) - (Cot[c + d*x]*(a + a*Sec[c + d*x]))/d
```

3.14.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4370
```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(-(e*Cot[c + d*x])^(m + 1))*((a + b*Csc[c + d*x])/(
d*e*(m + 1))), x] - Simp[1/(e^2*(m + 1))   Int[(e*Cot[c + d*x])^(m + 2)*(a*
(m + 1) + b*(m + 2)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && L
tQ[m, -1]
```

3.14.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.44 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

method	result	size
risch	$-ax-\frac{2ia}{d\left(\mathrm{e}^{i(dx+c)}-1\right)}$	26
derivativedivides	$\frac{-\frac{a}{\sin(dx+c)}+a(-\cot(dx+c)-dx-c)}{d}$	35
default	$\frac{-\frac{a}{\sin(dx+c)}+a(-\cot(dx+c)-dx-c)}{d}$	35

input
```
int(cot(d*x+c)^2*(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
-a*x-2*I*a/d/(exp(I*(d*x+c))-1)
```

3.14.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.27

$$\int \cot^2(c+dx)(a+a\sec(c+dx))\,dx = -\frac{adx\sin(dx+c)+a\cos(dx+c)+a}{d\sin(dx+c)}$$

input
```
integrate(cot(d*x+c)^2*(a+a*sec(d*x+c)),x, algorithm="fricas")
```

output
```
-(a*d*x*sin(d*x + c) + a*cos(d*x + c) + a)/(d*sin(d*x + c))
```

3.14.6 Sympy [F]

$$\int \cot^2(c+dx)(a+a\sec(c+dx))\,dx = a\left(\int \cot^2(c+dx)\sec(c+dx)\,dx + \int \cot^2(c+dx)\,dx\right)$$

input
```
integrate(cot(d*x+c)**2*(a+a*sec(d*x+c)),x)
```

output
```
a*(Integral(cot(c + d*x)**2*sec(c + d*x), x) + Integral(cot(c + d*x)**2, x
))
```

3.14.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.19

$$\int \cot^2(c+dx)(a+a\sec(c+dx))\,dx = -\frac{\left(dx+c+\frac{1}{\tan(dx+c)}\right)a + \frac{a}{\sin(dx+c)}}{d}$$

input
```
integrate(cot(d*x+c)^2*(a+a*sec(d*x+c)),x, algorithm="maxima")
```

output
```
-((d*x + c + 1/tan(d*x + c))*a + a/sin(d*x + c))/d
```

3.14.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \cot^2(c+dx)(a+a\sec(c+dx))\,dx = -\frac{(dx+c)a + \frac{a}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}}{d}$$

input
```
integrate(cot(d*x+c)^2*(a+a*sec(d*x+c)),x, algorithm="giac")
```

output
```
-((d*x + c)*a + a/tan(1/2*d*x + 1/2*c))/d
```

3.14.9 Mupad [B] (verification not implemented)

Time = 14.18 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.73

$$\int \cot^2(c+dx)(a+a\sec(c+dx))\,dx = -\frac{a\left(\cot\left(\frac{c}{2}+\frac{dx}{2}\right)+d\,x\right)}{d}$$

input
```
int(cot(c + d*x)^2*(a + a/cos(c + d*x)),x)
```

output
```
-(a*(cot(c/2 + (d*x)/2) + d*x))/d
```

3.15 $\int \cot^4(c+dx)(a+a\sec(c+dx))\,dx$

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3.15.1 Optimal result

Integrand size = 19, antiderivative size = 55

$$\int \cot^4(c+dx)(a+a\sec(c+dx))\,dx = ax - \frac{\cot^3(c+dx)(a+a\sec(c+dx))}{3d} + \frac{\cot(c+dx)(3a+2a\sec(c+dx))}{3d}$$

output
```
a*x-1/3*cot(d*x+c)^3*(a+a*sec(d*x+c))/d+1/3*cot(d*x+c)*(3*a+2*a*sec(d*x+c)
)/d
```

3.15.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.03 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.13

$$\int \cot^4(c+dx)(a+a\sec(c+dx))\,dx = \frac{a\csc(c+dx)}{d} - \frac{a\csc^3(c+dx)}{3d} - \frac{a\cot^3(c+dx)\,\text{Hypergeometric2F1}\left(-\frac{3}{2},1,-\frac{1}{2},-\tan^2(c+dx)\right)}{3d}$$

input
```
Integrate[Cot[c + d*x]^4*(a + a*Sec[c + d*x]),x]
```

output
```
(a*Csc[c + d*x])/d - (a*Csc[c + d*x]^3)/(3*d) - (a*Cot[c + d*x]^3*Hypergeo
metric2F1[-3/2, 1, -1/2, -Tan[c + d*x]^2])/(3*d)
```

3.15.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.05, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {3042, 4370, 25, 3042, 4370, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^4(c+dx)(a\sec(c+dx)+a)\,dx$$

$$\downarrow 3042$$

$$\int \frac{a\csc\left(c+dx+\frac{\pi}{2}\right)+a}{\cot\left(c+dx+\frac{\pi}{2}\right)^4}dx$$

$$\downarrow 4370$$

$$\frac{1}{3}\int -\cot^2(c+dx)(2\sec(c+dx)a+3a)dx - \frac{\cot^3(c+dx)(a\sec(c+dx)+a)}{3d}$$

$$\downarrow 25$$

$$-\frac{1}{3}\int \cot^2(c+dx)(2\sec(c+dx)a+3a)dx - \frac{\cot^3(c+dx)(a\sec(c+dx)+a)}{3d}$$

$$\downarrow 3042$$

$$-\frac{1}{3}\int \frac{2\csc\left(c+dx+\frac{\pi}{2}\right)a+3a}{\cot\left(c+dx+\frac{\pi}{2}\right)^2}dx - \frac{\cot^3(c+dx)(a\sec(c+dx)+a)}{3d}$$

$$\downarrow 4370$$

$$\frac{1}{3}\left(\frac{\cot(c+dx)(2a\sec(c+dx)+3a)}{d} - \int -3a dx\right) - \frac{\cot^3(c+dx)(a\sec(c+dx)+a)}{3d}$$

$$\downarrow 24$$

$$\frac{1}{3}\left(\frac{\cot(c+dx)(2a\sec(c+dx)+3a)}{d} + 3ax\right) - \frac{\cot^3(c+dx)(a\sec(c+dx)+a)}{3d}$$

input
```
Int[Cot[c + d*x]^4*(a + a*Sec[c + d*x]),x]
```

output
```
-1/3*(Cot[c + d*x]^3*(a + a*Sec[c + d*x]))/d + (3*a*x + (Cot[c + d*x]*(3*a
 + 2*a*Sec[c + d*x]))/d)/3
```

3.15.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4370
```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(-(e*Cot[c + d*x])^(m + 1))*((a + b*Csc[c + d*x])/(
d*e*(m + 1))), x] - Simp[1/(e^2*(m + 1))   Int[(e*Cot[c + d*x])^(m + 2)*(a*
(m + 1) + b*(m + 2)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && L
tQ[m, -1]
```

3.15.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.91 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.13

method	result	size
risch	$ax+\frac{2ia\left(3\,\mathrm{e}^{3i(dx+c)}-5\,\mathrm{e}^{i(dx+c)}+4\right)}{3d\left(\mathrm{e}^{i(dx+c)}-1\right)^3\left(\mathrm{e}^{i(dx+c)}+1\right)}$	62
derivativedivides	$\frac{a\left(-\frac{\cos(dx+c)^4}{3\sin(dx+c)^3}+\frac{\cos(dx+c)^4}{3\sin(dx+c)}+\frac{\left(2+\cos(dx+c)^2\right)\sin(dx+c)}{3}\right)+a\left(-\frac{\cot(dx+c)^3}{3}+\cot(dx+c)+dx+c\right)}{d}$	86
default	$\frac{a\left(-\frac{\cos(dx+c)^4}{3\sin(dx+c)^3}+\frac{\cos(dx+c)^4}{3\sin(dx+c)}+\frac{\left(2+\cos(dx+c)^2\right)\sin(dx+c)}{3}\right)+a\left(-\frac{\cot(dx+c)^3}{3}+\cot(dx+c)+dx+c\right)}{d}$	86

input
```
int(cot(d*x+c)^4*(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
a*x+2/3*I*a*(3*exp(3*I*(d*x+c))-5*exp(I*(d*x+c))+4)/d/(exp(I*(d*x+c))-1)^3
/(exp(I*(d*x+c))+1)
```

3.15.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.31

$$\int \cot^4(c+dx)(a+a\sec(c+dx))\,dx$$
$$= \frac{4\,a\cos(dx+c)^2 - a\cos(dx+c) + 3\,(adx\cos(dx+c) - adx)\sin(dx+c) - 2\,a}{3\,(d\cos(dx+c) - d)\sin(dx+c)}$$

input
```
integrate(cot(d*x+c)^4*(a+a*sec(d*x+c)),x, algorithm="fricas")
```

output
```
1/3*(4*a*cos(d*x + c)^2 - a*cos(d*x + c) + 3*(a*d*x*cos(d*x + c) - a*d*x)*
sin(d*x + c) - 2*a)/((d*cos(d*x + c) - d)*sin(d*x + c))
```

3.15.6 Sympy [F]

$$\int \cot^4(c+dx)(a+a\sec(c+dx))\,dx = a\left(\int \cot^4(c+dx)\sec(c+dx)\,dx + \int \cot^4(c+dx)\,dx\right)$$

input
```
integrate(cot(d*x+c)**4*(a+a*sec(d*x+c)),x)
```

output
```
a*(Integral(cot(c + d*x)**4*sec(c + d*x), x) + Integral(cot(c + d*x)**4, x
))
```

3.15.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.07

$$\int \cot^4(c+dx)(a+a\sec(c+dx))\,dx = \frac{\left(3\,dx + 3\,c + \frac{3\tan(dx+c)^2-1}{\tan(dx+c)^3}\right)a + \frac{\left(3\sin(dx+c)^2-1\right)a}{\sin(dx+c)^3}}{3\,d}$$

input
```
integrate(cot(d*x+c)^4*(a+a*sec(d*x+c)),x, algorithm="maxima")
```

output
```
1/3*((3*d*x + 3*c + (3*tan(d*x + c)^2 - 1)/tan(d*x + c)^3)*a + (3*sin(d*x
+ c)^2 - 1)*a/sin(d*x + c)^3)/d
```

3.15.8 Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.02

$$\int \cot^4(c+dx)(a+a\sec(c+dx))\,dx = \frac{12\,(dx+c)a - 3\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right) + \frac{12\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-a}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3}}{12\,d}$$

input
```
integrate(cot(d*x+c)^4*(a+a*sec(d*x+c)),x, algorithm="giac")
```

output
```
1/12*(12*(d*x + c)*a - 3*a*tan(1/2*d*x + 1/2*c) + (12*a*tan(1/2*d*x + 1/2*
c)^2 - a)/tan(1/2*d*x + 1/2*c)^3)/d
```

3.15.9 Mupad [B] (verification not implemented)

Time = 14.33 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.96

$$\int \cot^4(c+dx)(a+a\sec(c+dx))\,dx = a\,x - \frac{a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{4\,d} - \frac{\frac{a}{12} - a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2}{d\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3}$$

input
```
int(cot(c + d*x)^4*(a + a/cos(c + d*x)),x)
```

output
```
a*x - (a*tan(c/2 + (d*x)/2))/(4*d) - (a/12 - a*tan(c/2 + (d*x)/2)^2)/(d*ta
n(c/2 + (d*x)/2)^3)
```

3.16 $\int \cot^6(c+dx)(a+a\sec(c+dx))\,dx$

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3.16.1 Optimal result

Integrand size = 19, antiderivative size = 84

$$\int \cot^6(c+dx)(a+a\sec(c+dx))\,dx = -ax - \frac{\cot^5(c+dx)(a+a\sec(c+dx))}{5d} + \frac{\cot^3(c+dx)(5a+4a\sec(c+dx))}{15d} - \frac{\cot(c+dx)(15a+8a\sec(c+dx))}{15d}$$

output
```
-a*x-1/5*cot(d*x+c)^5*(a+a*sec(d*x+c))/d+1/15*cot(d*x+c)^3*(5*a+4*a*sec(d*
x+c))/d-1/15*cot(d*x+c)*(15*a+8*a*sec(d*x+c))/d
```

3.16.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.04 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.94

$$\int \cot^6(c+dx)(a+a\sec(c+dx))\,dx = -\frac{a\csc(c+dx)}{d} + \frac{2a\csc^3(c+dx)}{3d} - \frac{a\csc^5(c+dx)}{5d} - \frac{a\cot^5(c+dx)\,\text{Hypergeometric2F1}\left(-\frac{5}{2},1,-\frac{3}{2},-\tan^2(c+dx)\right)}{5d}$$

input
```
Integrate[Cot[c + d*x]^6*(a + a*Sec[c + d*x]),x]
```

output
```
-((a*Csc[c + d*x])/d) + (2*a*Csc[c + d*x]^3)/(3*d) - (a*Csc[c + d*x]^5)/(5
*d) - (a*Cot[c + d*x]^5*Hypergeometric2F1[-5/2, 1, -3/2, -Tan[c + d*x]^2])
/(5*d)
```

3.16.3 Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.10, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.474$, Rules used = {3042, 4370, 25, 3042, 4370, 25, 3042, 4370, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^6(c+dx)(a\sec(c+dx)+a)\,dx$$

$$\downarrow 3042$$

$$\int \frac{a\csc\left(c+dx+\frac{\pi}{2}\right)+a}{\cot\left(c+dx+\frac{\pi}{2}\right)^6}dx$$

$$\downarrow 4370$$

$$\frac{1}{5}\int -\cot^4(c+dx)(4\sec(c+dx)a+5a)dx - \frac{\cot^5(c+dx)(a\sec(c+dx)+a)}{5d}$$

$$\downarrow 25$$

$$-\frac{1}{5}\int \cot^4(c+dx)(4\sec(c+dx)a+5a)dx - \frac{\cot^5(c+dx)(a\sec(c+dx)+a)}{5d}$$

$$\downarrow 3042$$

$$-\frac{1}{5}\int \frac{4\csc\left(c+dx+\frac{\pi}{2}\right)a+5a}{\cot\left(c+dx+\frac{\pi}{2}\right)^4}dx - \frac{\cot^5(c+dx)(a\sec(c+dx)+a)}{5d}$$

$$\downarrow 4370$$

$$\frac{1}{5}\left(\frac{\cot^3(c+dx)(4a\sec(c+dx)+5a)}{3d} - \frac{1}{3}\int -\cot^2(c+dx)(8\sec(c+dx)a+15a)dx\right) - \frac{\cot^5(c+dx)(a\sec(c+dx)+a)}{5d}$$

$$\downarrow 25$$

$$\frac{1}{5}\left(\frac{1}{3}\int \cot^2(c+dx)(8\sec(c+dx)a+15a)dx+\frac{\cot^3(c+dx)(4a\sec(c+dx)+5a)}{3d}\right)-\frac{\cot^5(c+dx)(a\sec(c+dx)+a)}{5d}$$

$$\downarrow 3042$$

$$\frac{1}{5}\left(\frac{1}{3}\int \frac{8\csc\left(c+dx+\frac{\pi}{2}\right)a+15a}{\cot\left(c+dx+\frac{\pi}{2}\right)^2}dx+\frac{\cot^3(c+dx)(4a\sec(c+dx)+5a)}{3d}\right)-\frac{\cot^5(c+dx)(a\sec(c+dx)+a)}{5d}$$

$$\downarrow 4370$$

$$\frac{1}{5}\left(\frac{1}{3}\left(\int -15adx-\frac{\cot(c+dx)(8a\sec(c+dx)+15a)}{d}\right)+\frac{\cot^3(c+dx)(4a\sec(c+dx)+5a)}{3d}\right)-\frac{\cot^5(c+dx)(a\sec(c+dx)+a)}{5d}$$

$$\downarrow 24$$

$$\frac{1}{5}\left(\frac{\cot^3(c+dx)(4a\sec(c+dx)+5a)}{3d}+\frac{1}{3}\left(-\frac{\cot(c+dx)(8a\sec(c+dx)+15a)}{d}-15ax\right)\right)-\frac{\cot^5(c+dx)(a\sec(c+dx)+a)}{5d}$$

input
```
Int[Cot[c + d*x]^6*(a + a*Sec[c + d*x]),x]
```

output
```
-1/5*(Cot[c + d*x]^5*(a + a*Sec[c + d*x]))/d + ((Cot[c + d*x]^3*(5*a + 4*a
*Sec[c + d*x]))/(3*d) + (-15*a*x - (Cot[c + d*x]*(15*a + 8*a*Sec[c + d*x])
)/d)/3)/5
```

3.16.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4370

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(-(e*Cot[c + d*x])^(m + 1))*((a + b*Csc[c + d*x])/(
d*e*(m + 1))), x] - Simp[1/(e^2*(m + 1))   Int[(e*Cot[c + d*x])^(m + 2)*(a*
(m + 1) + b*(m + 2)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && L
tQ[m, -1]
```

3.16.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.07 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.40

method	result
risch	$-ax-\frac{2ia\left(15\,\mathrm{e}^{7i(dx+c)}+15\,\mathrm{e}^{6i(dx+c)}-65\,\mathrm{e}^{5i(dx+c)}+25\,\mathrm{e}^{4i(dx+c)}+73\,\mathrm{e}^{3i(dx+c)}-31\,\mathrm{e}^{2i(dx+c)}-31\,\mathrm{e}^{i(dx+c)}+23\right)}{15d\left(\mathrm{e}^{i(dx+c)}-1\right)^5\left(\mathrm{e}^{i(dx+c)}+1\right)^3}$
derivativedivides	$\frac{a\left(-\frac{\cos(dx+c)^6}{5\sin(dx+c)^5}+\frac{\cos(dx+c)^6}{15\sin(dx+c)^3}-\frac{\cos(dx+c)^6}{5\sin(dx+c)}-\frac{\left(\frac{8}{3}+\cos(dx+c)^4+\frac{4\cos(dx+c)^2}{3}\right)\sin(dx+c)}{5}\right)+a\left(-\frac{\cot(dx+c)^5}{5}+\frac{\cot(dx+c)^3}{3}\right.}{d}$
default	$\frac{a\left(-\frac{\cos(dx+c)^6}{5\sin(dx+c)^5}+\frac{\cos(dx+c)^6}{15\sin(dx+c)^3}-\frac{\cos(dx+c)^6}{5\sin(dx+c)}-\frac{\left(\frac{8}{3}+\cos(dx+c)^4+\frac{4\cos(dx+c)^2}{3}\right)\sin(dx+c)}{5}\right)+a\left(-\frac{\cot(dx+c)^5}{5}+\frac{\cot(dx+c)^3}{3}\right.}{d}$

input

```
int(cot(d*x+c)^6*(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

output

```
-a*x-2/15*I*a*(15*exp(7*I*(d*x+c))+15*exp(6*I*(d*x+c))-65*exp(5*I*(d*x+c))
+25*exp(4*I*(d*x+c))+73*exp(3*I*(d*x+c))-31*exp(2*I*(d*x+c))-31*exp(I*(d*x
+c))+23)/d/(exp(I*(d*x+c))-1)^5/(exp(I*(d*x+c))+1)^3
```

3.16.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.65

$$\int \cot^6(c+dx)(a+a\sec(c+dx))\,dx =$$

$$-\frac{23\,a\cos(dx+c)^4-8\,a\cos(dx+c)^3-27\,a\cos(dx+c)^2+7\,a\cos(dx+c)+15\left(adx\cos(dx+c)^3-a\right.}{15\left(d\cos(dx+c)^3-d\cos(dx+c)^2-d\cos(dx+c)+d\right.}$$

input

```
integrate(cot(d*x+c)^6*(a+a*sec(d*x+c)),x, algorithm="fricas")
```

output
```
-1/15*(23*a*cos(d*x + c)^4 - 8*a*cos(d*x + c)^3 - 27*a*cos(d*x + c)^2 + 7*
a*cos(d*x + c) + 15*(a*d*x*cos(d*x + c)^3 - a*d*x*cos(d*x + c)^2 - a*d*x*c
os(d*x + c) + a*d*x)*sin(d*x + c) + 8*a)/((d*cos(d*x + c)^3 - d*cos(d*x +
c)^2 - d*cos(d*x + c) + d)*sin(d*x + c))
```

3.16.6 Sympy [F]

$$\int \cot^6(c+dx)(a+a\sec(c+dx))\,dx = a\left(\int \cot^6(c+dx)\sec(c+dx)\,dx + \int \cot^6(c+dx)\,dx\right)$$

input
```
integrate(cot(d*x+c)**6*(a+a*sec(d*x+c)),x)
```

output
```
a*(Integral(cot(c + d*x)**6*sec(c + d*x), x) + Integral(cot(c + d*x)**6, x
))
```

3.16.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.94

$$\int \cot^6(c+dx)(a+a\sec(c+dx))\,dx$$
$$= -\frac{\left(15\,dx + 15\,c + \frac{15\tan(dx+c)^4 - 5\tan(dx+c)^2 + 3}{\tan(dx+c)^5}\right)a + \frac{\left(15\sin(dx+c)^4 - 10\sin(dx+c)^2 + 3\right)a}{\sin(dx+c)^5}}{15\,d}$$

input
```
integrate(cot(d*x+c)^6*(a+a*sec(d*x+c)),x, algorithm="maxima")
```

output
```
-1/15*((15*d*x + 15*c + (15*tan(d*x + c)^4 - 5*tan(d*x + c)^2 + 3)/tan(d*x
 + c)^5)*a + (15*sin(d*x + c)^4 - 10*sin(d*x + c)^2 + 3)*a/sin(d*x + c)^5)
/d
```

3.16.8 Giac [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.99

$$\int \cot^6(c+dx)(a+a\sec(c+dx))\,dx =$$

$$-\frac{5\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+240\,(dx+c)a-90\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+\frac{3\left(80\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-10\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5}}{240\,d}$$

input
```
integrate(cot(d*x+c)^6*(a+a*sec(d*x+c)),x, algorithm="giac")
```

output
```
-1/240*(5*a*tan(1/2*d*x + 1/2*c)^3 + 240*(d*x + c)*a - 90*a*tan(1/2*d*x +
1/2*c) + 3*(80*a*tan(1/2*d*x + 1/2*c)^4 - 10*a*tan(1/2*d*x + 1/2*c)^2 + a)
/tan(1/2*d*x + 1/2*c)^5)/d
```

3.16.9 Mupad [B] (verification not implemented)

Time = 14.44 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.86

$$\int \cot^6(c+dx)(a+a\sec(c+dx))\,dx =$$

$$-\frac{a\left(3\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^8+5\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^8-90\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^2\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^6+240\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^4\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{}}{240\,d\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^3\sin\left(\frac{c}{2}+\right.}$$

input
```
int(cot(c + d*x)^6*(a + a/cos(c + d*x)),x)
```

output
```
-(a*(3*cos(c/2 + (d*x)/2)^8 + 5*sin(c/2 + (d*x)/2)^8 - 90*cos(c/2 + (d*x)/
2)^2*sin(c/2 + (d*x)/2)^6 + 240*cos(c/2 + (d*x)/2)^4*sin(c/2 + (d*x)/2)^4
- 30*cos(c/2 + (d*x)/2)^6*sin(c/2 + (d*x)/2)^2 + 240*cos(c/2 + (d*x)/2)^3*
sin(c/2 + (d*x)/2)^5*(c + d*x)))/(240*d*cos(c/2 + (d*x)/2)^3*sin(c/2 + (d*
x)/2)^5)
```

3.17 $\int \cot^8(c+dx)(a+a\sec(c+dx))\,dx$

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3.17.1 Optimal result

Integrand size = 19, antiderivative size = 111

$$\begin{aligned}\int \cot^8(c+dx)(a+a\sec(c+dx))\,dx = ax &- \frac{\cot^7(c+dx)(a+a\sec(c+dx))}{7d} \\ &+ \frac{\cot^5(c+dx)(7a+6a\sec(c+dx))}{35d} \\ &+ \frac{\cot(c+dx)(35a+16a\sec(c+dx))}{35d} \\ &- \frac{\cot^3(c+dx)(35a+24a\sec(c+dx))}{105d}\end{aligned}$$

output
```
a*x-1/7*cot(d*x+c)^7*(a+a*sec(d*x+c))/d+1/35*cot(d*x+c)^5*(7*a+6*a*sec(d*x
+c))/d+1/35*cot(d*x+c)*(35*a+16*a*sec(d*x+c))/d-1/105*cot(d*x+c)^3*(35*a+2
4*a*sec(d*x+c))/d
```

3.17.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.04 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.83

$$\begin{aligned}&\int \cot^8(c+dx)(a+a\sec(c+dx))\,dx \\ &= \frac{a\csc(c+dx)}{d} - \frac{a\csc^3(c+dx)}{d} + \frac{3a\csc^5(c+dx)}{5d} - \frac{a\csc^7(c+dx)}{7d} \\ &\quad - \frac{a\cot^7(c+dx)\,\text{Hypergeometric2F1}\left(-\frac{7}{2},1,-\frac{5}{2},-\tan^2(c+dx)\right)}{7d}\end{aligned}$$

input
```
Integrate[Cot[c + d*x]^8*(a + a*Sec[c + d*x]),x]
```

output
```
(a*Csc[c + d*x])/d - (a*Csc[c + d*x]^3)/d + (3*a*Csc[c + d*x]^5)/(5*d) - (
a*Csc[c + d*x]^7)/(7*d) - (a*Cot[c + d*x]^7*Hypergeometric2F1[-7/2, 1, -5/
2, -Tan[c + d*x]^2])/(7*d)
```

3.17.3 Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.07, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.632$, Rules used = {3042, 4370, 25, 3042, 4370, 25, 3042, 4370, 27, 3042, 4370, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^8(c+dx)(a\sec(c+dx)+a)\,dx$$

$$\downarrow 3042$$

$$\int \frac{a\csc\left(c+dx+\frac{\pi}{2}\right)+a}{\cot\left(c+dx+\frac{\pi}{2}\right)^8}dx$$

$$\downarrow 4370$$

$$\frac{1}{7}\int -\cot^6(c+dx)(6\sec(c+dx)a+7a)dx - \frac{\cot^7(c+dx)(a\sec(c+dx)+a)}{7d}$$

$$\downarrow 25$$

$$-\frac{1}{7}\int \cot^6(c+dx)(6\sec(c+dx)a+7a)dx - \frac{\cot^7(c+dx)(a\sec(c+dx)+a)}{7d}$$

$$\downarrow 3042$$

$$-\frac{1}{7}\int \frac{6\csc\left(c+dx+\frac{\pi}{2}\right)a+7a}{\cot\left(c+dx+\frac{\pi}{2}\right)^6}dx - \frac{\cot^7(c+dx)(a\sec(c+dx)+a)}{7d}$$

$$\downarrow 4370$$

$$\frac{1}{7}\left(\frac{\cot^5(c+dx)(6a\sec(c+dx)+7a)}{5d} - \frac{1}{5}\int -\cot^4(c+dx)(24\sec(c+dx)a+35a)dx\right) - \frac{\cot^7(c+dx)(a\sec(c+dx)+a)}{7d}$$

$$\downarrow 25$$

$$\frac{1}{7}\left(\frac{1}{5}\int \cot^4(c+dx)(24\sec(c+dx)a+35a)dx+\frac{\cot^5(c+dx)(6a\sec(c+dx)+7a)}{5d}\right)-\frac{\cot^7(c+dx)(a\sec(c+dx)+a)}{7d}$$

$$\downarrow 3042$$

$$\frac{1}{7}\left(\frac{1}{5}\int \frac{24\csc\left(c+dx+\frac{\pi}{2}\right)a+35a}{\cot\left(c+dx+\frac{\pi}{2}\right)^4}dx+\frac{\cot^5(c+dx)(6a\sec(c+dx)+7a)}{5d}\right)-\frac{\cot^7(c+dx)(a\sec(c+dx)+a)}{7d}$$

$$\downarrow 4370$$

$$\frac{1}{7}\left(\frac{1}{5}\left(\frac{1}{3}\int -3\cot^2(c+dx)(16\sec(c+dx)a+35a)dx-\frac{\cot^3(c+dx)(24a\sec(c+dx)+35a)}{3d}\right)+\frac{\cot^5(c+dx)(6a\right.$$
$$\left.-\frac{\cot^7(c+dx)(a\sec(c+dx)+a)}{7d}\right.$$

$$\downarrow 27$$

$$\frac{1}{7}\left(\frac{1}{5}\left(-\int \cot^2(c+dx)(16\sec(c+dx)a+35a)dx-\frac{\cot^3(c+dx)(24a\sec(c+dx)+35a)}{3d}\right)+\frac{\cot^5(c+dx)(6a\sec\right.$$
$$\left.-\frac{\cot^7(c+dx)(a\sec(c+dx)+a)}{7d}\right.$$

$$\downarrow 3042$$

$$\frac{1}{7}\left(\frac{1}{5}\left(-\int \frac{16\csc\left(c+dx+\frac{\pi}{2}\right)a+35a}{\cot\left(c+dx+\frac{\pi}{2}\right)^2}dx-\frac{\cot^3(c+dx)(24a\sec(c+dx)+35a)}{3d}\right)+\frac{\cot^5(c+dx)(6a\sec(c+dx\right.$$
$$\left.-\frac{\cot^7(c+dx)(a\sec(c+dx)+a)}{7d}\right.$$

$$\downarrow 4370$$

$$\frac{1}{7}\left(\frac{1}{5}\left(-\int -35adx-\frac{\cot^3(c+dx)(24a\sec(c+dx)+35a)}{3d}+\frac{\cot(c+dx)(16a\sec(c+dx)+35a)}{d}\right)+\cot^5(c+dx\right.$$
$$\left.-\frac{\cot^7(c+dx)(a\sec(c+dx)+a)}{7d}\right.$$

$$\downarrow 24$$

$$\frac{1}{7}\left(\frac{\cot^5(c+dx)(6a\sec(c+dx)+7a)}{5d}+\frac{1}{5}\left(-\frac{\cot^3(c+dx)(24a\sec(c+dx)+35a)}{3d}+\frac{\cot(c+dx)(16a\sec(c+dx)\right.\right.$$
$$\left.\left.-\frac{\cot^7(c+dx)(a\sec(c+dx)+a)}{7d}\right.\right.$$

input
```
Int[Cot[c + d*x]^8*(a + a*Sec[c + d*x]),x]
```

output
```
-1/7*(Cot[c + d*x]^7*(a + a*Sec[c + d*x]))/d + ((Cot[c + d*x]^5*(7*a + 6*a
*Sec[c + d*x]))/(5*d) + (35*a*x + (Cot[c + d*x]*(35*a + 16*a*Sec[c + d*x])
)/d - (Cot[c + d*x]^3*(35*a + 24*a*Sec[c + d*x]))/(3*d))/5)/7
```

3.17.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4370
```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(-(e*Cot[c + d*x])^(m + 1))*((a + b*Csc[c + d*x])/(
d*e*(m + 1))), x] - Simp[1/(e^2*(m + 1))   Int[(e*Cot[c + d*x])^(m + 2)*(a*
(m + 1) + b*(m + 2)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && L
tQ[m, -1]
```

3.17.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.41 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.35

method	result
risch	$ax+\frac{2ia\left(105\,e^{11i(dx+c)}+210\,e^{10i(dx+c)}-735\,e^{9i(dx+c)}+1638\,e^{7i(dx+c)}-196\,e^{6i(dx+c)}-1882\,e^{5i(dx+c)}+880\,e^{4i(dx+c)}\right.}{105d\left(e^{i(dx+c)}-1\right)^7\left(e^{i(dx+c)}+1\right)^5}$
derivativedivides	$\frac{a\left(-\frac{\cos(dx+c)^8}{7\sin(dx+c)^7}+\frac{\cos(dx+c)^8}{35\sin(dx+c)^5}-\frac{\cos(dx+c)^8}{35\sin(dx+c)^3}+\frac{\cos(dx+c)^8}{7\sin(dx+c)}+\frac{\left(\frac{16}{5}+\cos(dx+c)^6+\frac{6\cos(dx+c)^4}{5}+\frac{8\cos(dx+c)^2}{5}\right)\sin(dx+c)}{7}\right)}{d}$
default	$\frac{a\left(-\frac{\cos(dx+c)^8}{7\sin(dx+c)^7}+\frac{\cos(dx+c)^8}{35\sin(dx+c)^5}-\frac{\cos(dx+c)^8}{35\sin(dx+c)^3}+\frac{\cos(dx+c)^8}{7\sin(dx+c)}+\frac{\left(\frac{16}{5}+\cos(dx+c)^6+\frac{6\cos(dx+c)^4}{5}+\frac{8\cos(dx+c)^2}{5}\right)\sin(dx+c)}{7}\right)}{d}$

input
```
int(cot(d*x+c)^8*(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
a*x+2/105*I*a*(105*exp(11*I*(d*x+c))+210*exp(10*I*(d*x+c))-735*exp(9*I*(d*
x+c))+1638*exp(7*I*(d*x+c))-196*exp(6*I*(d*x+c))-1882*exp(5*I*(d*x+c))+880
*exp(4*I*(d*x+c))+1025*exp(3*I*(d*x+c))-494*exp(2*I*(d*x+c))-247*exp(I*(d*
x+c))+176)/d/(exp(I*(d*x+c))-1)^7/(exp(I*(d*x+c))+1)^5
```

3.17.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 210 vs. 2(103) = 206.

Time = 0.28 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.89

$$\int \cot^8(c+dx)(a+a\sec(c+dx))\,dx$$
$$=\frac{176\,a\cos(dx+c)^6-71\,a\cos(dx+c)^5-335\,a\cos(dx+c)^4+125\,a\cos(dx+c)^3+225\,a\cos(dx+c)^2-}{105\left(d\cos(dx+c)^5-d\cos(dx\right.}$$

input
```
integrate(cot(d*x+c)^8*(a+a*sec(d*x+c)),x, algorithm="fricas")
```

output
```
1/105*(176*a*cos(d*x + c)^6 - 71*a*cos(d*x + c)^5 - 335*a*cos(d*x + c)^4 +
 125*a*cos(d*x + c)^3 + 225*a*cos(d*x + c)^2 - 57*a*cos(d*x + c) + 105*(a*
d*x*cos(d*x + c)^5 - a*d*x*cos(d*x + c)^4 - 2*a*d*x*cos(d*x + c)^3 + 2*a*d
*x*cos(d*x + c)^2 + a*d*x*cos(d*x + c) - a*d*x)*sin(d*x + c) - 48*a)/((d*c
os(d*x + c)^5 - d*cos(d*x + c)^4 - 2*d*cos(d*x + c)^3 + 2*d*cos(d*x + c)^2
 + d*cos(d*x + c) - d)*sin(d*x + c))
```

3.17.6 Sympy [F]

$$\int \cot^8(c+dx)(a+a\sec(c+dx))\,dx = a\left(\int \cot^8(c+dx)\sec(c+dx)\,dx + \int \cot^8(c+dx)\,dx\right)$$

input
```
integrate(cot(d*x+c)**8*(a+a*sec(d*x+c)),x)
```

output
```
a*(Integral(cot(c + d*x)**8*sec(c + d*x), x) + Integral(cot(c + d*x)**8, x
))
```

3.17.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.90

$$\int \cot^8(c+dx)(a+a\sec(c+dx))\,dx$$
$$= \frac{\left(105\,dx + 105\,c + \frac{105\tan(dx+c)^6 - 35\tan(dx+c)^4 + 21\tan(dx+c)^2 - 15}{\tan(dx+c)^7}\right)a + \frac{3\left(35\sin(dx+c)^6 - 35\sin(dx+c)^4 + 21\sin(dx+c)^2 - 5\right)a}{\sin(dx+c)^7}}{105\,d}$$

input
```
integrate(cot(d*x+c)^8*(a+a*sec(d*x+c)),x, algorithm="maxima")
```

output
```
1/105*((105*d*x + 105*c + (105*tan(d*x + c)^6 - 35*tan(d*x + c)^4 + 21*tan
(d*x + c)^2 - 15)/tan(d*x + c)^7)*a + 3*(35*sin(d*x + c)^6 - 35*sin(d*x +
c)^4 + 21*sin(d*x + c)^2 - 5)*a/sin(d*x + c)^7)/d
```

3.17.8 Giac [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.02

$$\int \cot^8(c+dx)(a+a\sec(c+dx))\,dx =$$

$$-\frac{21\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-280\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-6720\,(dx+c)a+3045\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\frac{6720\,a\tan\left(\frac{1}{2}\right.}{6720\,d}$$

input
```
integrate(cot(d*x+c)^8*(a+a*sec(d*x+c)),x, algorithm="giac")
```

output
```
-1/6720*(21*a*tan(1/2*d*x + 1/2*c)^5 - 280*a*tan(1/2*d*x + 1/2*c)^3 - 6720
*(d*x + c)*a + 3045*a*tan(1/2*d*x + 1/2*c) - (6720*a*tan(1/2*d*x + 1/2*c)^
6 - 1015*a*tan(1/2*d*x + 1/2*c)^4 + 168*a*tan(1/2*d*x + 1/2*c)^2 - 15*a)/t
an(1/2*d*x + 1/2*c)^7)/d
```

3.17.9 Mupad [B] (verification not implemented)

Time = 14.79 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.84

$$\int \cot^8(c+dx)(a+a\sec(c+dx))\,dx =$$

$$-\frac{a\left(15\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{12}+21\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{12}-280\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^2\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}+3045\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^4\sin\left(\frac{c}{2}\right.\right.}{}$$

input
```
int(cot(c + d*x)^8*(a + a/cos(c + d*x)),x)
```

output
```
-(a*(15*cos(c/2 + (d*x)/2)^12 + 21*sin(c/2 + (d*x)/2)^12 - 280*cos(c/2 + (
d*x)/2)^2*sin(c/2 + (d*x)/2)^10 + 3045*cos(c/2 + (d*x)/2)^4*sin(c/2 + (d*x
)/2)^8 - 6720*cos(c/2 + (d*x)/2)^6*sin(c/2 + (d*x)/2)^6 + 1015*cos(c/2 + (
d*x)/2)^8*sin(c/2 + (d*x)/2)^4 - 168*cos(c/2 + (d*x)/2)^10*sin(c/2 + (d*x)
/2)^2 - 6720*cos(c/2 + (d*x)/2)^5*sin(c/2 + (d*x)/2)^7*(c + d*x)))/(6720*d
*cos(c/2 + (d*x)/2)^5*sin(c/2 + (d*x)/2)^7)
```

3.18 $\int \cot^{10}(c+dx)(a+a\sec(c+dx))\,dx$

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3.18.1 Optimal result

Integrand size = 19, antiderivative size = 140

$$\begin{aligned}\int \cot^{10}(c+dx)(a+a\sec(c+dx))\,dx = &-ax-\frac{\cot^9(c+dx)(a+a\sec(c+dx))}{9d}\\&+\frac{\cot^7(c+dx)(9a+8a\sec(c+dx))}{63d}\\&-\frac{\cot^5(c+dx)(21a+16a\sec(c+dx))}{105d}\\&+\frac{\cot^3(c+dx)(105a+64a\sec(c+dx))}{315d}\\&-\frac{\cot(c+dx)(315a+128a\sec(c+dx))}{315d}\end{aligned}$$

output

```
-a*x-1/9*cot(d*x+c)^9*(a+a*sec(d*x+c))/d+1/63*cot(d*x+c)^7*(9*a+8*a*sec(d*
x+c))/d-1/105*cot(d*x+c)^5*(21*a+16*a*sec(d*x+c))/d+1/315*cot(d*x+c)^3*(10
5*a+64*a*sec(d*x+c))/d-1/315*cot(d*x+c)*(315*a+128*a*sec(d*x+c))/d
```

3.18.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.05 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.79

$$\int \cot^{10}(c+dx)(a+a\sec(c+dx))\,dx$$
$$= -\frac{a\csc(c+dx)}{d} + \frac{4a\csc^3(c+dx)}{3d} - \frac{6a\csc^5(c+dx)}{5d} + \frac{4a\csc^7(c+dx)}{7d}$$
$$- \frac{a\csc^9(c+dx)}{9d} - \frac{a\cot^9(c+dx)\,\text{Hypergeometric2F1}\left(-\frac{9}{2},1,-\frac{7}{2},-\tan^2(c+dx)\right)}{9d}$$

input
```
Integrate[Cot[c + d*x]^10*(a + a*Sec[c + d*x]),x]
```

output
```
-((a*Csc[c + d*x])/d) + (4*a*Csc[c + d*x]^3)/(3*d) - (6*a*Csc[c + d*x]^5)/
(5*d) + (4*a*Csc[c + d*x]^7)/(7*d) - (a*Csc[c + d*x]^9)/(9*d) - (a*Cot[c +
 d*x]^9*Hypergeometric2F1[-9/2, 1, -7/2, -Tan[c + d*x]^2])/(9*d)
```

3.18.3 Rubi [A] (verified)

Time = 0.74 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.13, number of steps used = 15, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.789$, Rules used = {3042, 4370, 25, 3042, 4370, 27, 3042, 4370, 25, 3042, 4370, 25, 3042, 4370, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^{10}(c+dx)(a\sec(c+dx)+a)\,dx$$
$$\downarrow 3042$$
$$\int \frac{a\csc\left(c+dx+\frac{\pi}{2}\right)+a}{\cot\left(c+dx+\frac{\pi}{2}\right)^{10}}dx$$
$$\downarrow 4370$$
$$\frac{1}{9}\int -\cot^8(c+dx)(8\sec(c+dx)a+9a)dx - \frac{\cot^9(c+dx)(a\sec(c+dx)+a)}{9d}$$
$$\downarrow 25$$
$$-\frac{1}{9}\int \cot^8(c+dx)(8\sec(c+dx)a+9a)dx - \frac{\cot^9(c+dx)(a\sec(c+dx)+a)}{9d}$$

$$\downarrow 3042$$

$$-\frac{1}{9}\int \frac{8\csc\left(c+dx+\frac{\pi}{2}\right)a+9a}{\cot\left(c+dx+\frac{\pi}{2}\right)^8}dx-\frac{\cot^9(c+dx)(a\sec(c+dx)+a)}{9d}$$

$$\downarrow 4370$$

$$\frac{1}{9}\left(\frac{\cot^7(c+dx)(8a\sec(c+dx)+9a)}{7d}-\frac{1}{7}\int -3\cot^6(c+dx)(16\sec(c+dx)a+21a)dx\right)-\frac{\cot^9(c+dx)(a\sec(c+dx)+a)}{9d}$$

$$\downarrow 27$$

$$\frac{1}{9}\left(\frac{3}{7}\int \cot^6(c+dx)(16\sec(c+dx)a+21a)dx+\frac{\cot^7(c+dx)(8a\sec(c+dx)+9a)}{7d}\right)-\frac{\cot^9(c+dx)(a\sec(c+dx)+a)}{9d}$$

$$\downarrow 3042$$

$$\frac{1}{9}\left(\frac{3}{7}\int \frac{16\csc\left(c+dx+\frac{\pi}{2}\right)a+21a}{\cot\left(c+dx+\frac{\pi}{2}\right)^6}dx+\frac{\cot^7(c+dx)(8a\sec(c+dx)+9a)}{7d}\right)-\frac{\cot^9(c+dx)(a\sec(c+dx)+a)}{9d}$$

$$\downarrow 4370$$

$$\frac{1}{9}\left(\frac{3}{7}\left(\frac{1}{5}\int -\cot^4(c+dx)(64\sec(c+dx)a+105a)dx-\frac{\cot^5(c+dx)(16a\sec(c+dx)+21a)}{5d}\right)+\frac{\cot^7(c+dx)(8a}{}\right.$$
$$\left.\frac{\cot^9(c+dx)(a\sec(c+dx)+a)}{9d}\right)$$

$$\downarrow 25$$

$$\frac{1}{9}\left(\frac{3}{7}\left(-\frac{1}{5}\int \cot^4(c+dx)(64\sec(c+dx)a+105a)dx-\frac{\cot^5(c+dx)(16a\sec(c+dx)+21a)}{5d}\right)+\frac{\cot^7(c+dx)(8a}{}\right.$$
$$\left.\frac{\cot^9(c+dx)(a\sec(c+dx)+a)}{9d}\right)$$

$$\downarrow 3042$$

$$\frac{1}{9}\left(\frac{3}{7}\left(-\frac{1}{5}\int \frac{64\csc\left(c+dx+\frac{\pi}{2}\right)a+105a}{\cot\left(c+dx+\frac{\pi}{2}\right)^4}dx-\frac{\cot^5(c+dx)(16a\sec(c+dx)+21a)}{5d}\right)+\frac{\cot^7(c+dx)(8a\sec(c+}{7d}\right.$$
$$\left.\frac{\cot^9(c+dx)(a\sec(c+dx)+a)}{9d}\right)$$

$$\downarrow 4370$$

$$\frac{1}{9}\left(\frac{3}{7}\left(\frac{1}{5}\left(\frac{\cot^3(c+dx)(64a\sec(c+dx)+105a)}{3d}-\frac{1}{3}\int-\cot^2(c+dx)(128\sec(c+dx)a+315a)dx\right)-\frac{\cot^5(c+d}{}\right.\right.$$

$$\frac{\cot^9(c+dx)(a\sec(c+dx)+a)}{9d}$$

$$\downarrow 25$$

$$\frac{1}{9}\left(\frac{3}{7}\left(\frac{1}{5}\left(\frac{1}{3}\int\cot^2(c+dx)(128\sec(c+dx)a+315a)dx+\frac{\cot^3(c+dx)(64a\sec(c+dx)+105a)}{3d}\right)-\frac{\cot^5(c+dx)}{}\right.\right.$$

$$\frac{\cot^9(c+dx)(a\sec(c+dx)+a)}{9d}$$

$$\downarrow 3042$$

$$\frac{1}{9}\left(\frac{3}{7}\left(\frac{1}{5}\left(\frac{1}{3}\int\frac{128\csc\left(c+dx+\frac{\pi}{2}\right)a+315a}{\cot\left(c+dx+\frac{\pi}{2}\right)^2}dx+\frac{\cot^3(c+dx)(64a\sec(c+dx)+105a)}{3d}\right)-\frac{\cot^5(c+dx)(16a\sec}{5d}\right.\right.$$

$$\frac{\cot^9(c+dx)(a\sec(c+dx)+a)}{9d}$$

$$\downarrow 4370$$

$$\frac{1}{9}\left(\frac{3}{7}\left(\frac{1}{5}\left(\frac{1}{3}\left(\int-315adx-\frac{\cot(c+dx)(128a\sec(c+dx)+315a)}{d}\right)+\frac{\cot^3(c+dx)(64a\sec(c+dx)+105a)}{3d}\right)-\right.\right.$$

$$\frac{\cot^9(c+dx)(a\sec(c+dx)+a)}{9d}$$

$$\downarrow 24$$

$$\frac{1}{9}\left(\frac{\cot^7(c+dx)(8a\sec(c+dx)+9a)}{7d}+\frac{3}{7}\left(\frac{1}{5}\left(\frac{\cot^3(c+dx)(64a\sec(c+dx)+105a)}{3d}+\frac{1}{3}\left(-\frac{\cot(c+dx)(128a\sec}{d}\right.\right.\right.\right.$$

$$\frac{\cot^9(c+dx)(a\sec(c+dx)+a)}{9d}$$

input
```
Int[Cot[c + d*x]^10*(a + a*Sec[c + d*x]),x]
```

output
```
-1/9*(Cot[c + d*x]^9*(a + a*Sec[c + d*x]))/d + ((Cot[c + d*x]^7*(9*a + 8*a
*Sec[c + d*x]))/(7*d) + (3*(-1/5*(Cot[c + d*x]^5*(21*a + 16*a*Sec[c + d*x]
))/d + ((Cot[c + d*x]^3*(105*a + 64*a*Sec[c + d*x]))/(3*d) + (-315*a*x - (
Cot[c + d*x]*(315*a + 128*a*Sec[c + d*x]))/d)/3)/5))/7)/9
```

3.18.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4370
```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(-(e*Cot[c + d*x])^(m + 1))*((a + b*Csc[c + d*x])/(
d*e*(m + 1))), x] - Simp[1/(e^2*(m + 1))   Int[(e*Cot[c + d*x])^(m + 2)*(a*
(m + 1) + b*(m + 2)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && L
tQ[m, -1]
```

3.18.4 Maple [A] (verified)

Time = 1.78 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.46

method	result
derivativedivides	$\frac{a\left(-\frac{\cos(dx+c)^{10}}{9\sin(dx+c)^9}+\frac{\cos(dx+c)^{10}}{63\sin(dx+c)^7}-\frac{\cos(dx+c)^{10}}{105\sin(dx+c)^5}+\frac{\cos(dx+c)^{10}}{63\sin(dx+c)^3}-\frac{\cos(dx+c)^{10}}{9\sin(dx+c)}-\frac{\left(\frac{128}{35}+\cos(dx+c)^8+\frac{8\cos(dx+c)^6}{7}+\frac{48\cos(dx+c)}{35}\right.}{9}\right.}{d}$
default	$\frac{a\left(-\frac{\cos(dx+c)^{10}}{9\sin(dx+c)^9}+\frac{\cos(dx+c)^{10}}{63\sin(dx+c)^7}-\frac{\cos(dx+c)^{10}}{105\sin(dx+c)^5}+\frac{\cos(dx+c)^{10}}{63\sin(dx+c)^3}-\frac{\cos(dx+c)^{10}}{9\sin(dx+c)}-\frac{\left(\frac{128}{35}+\cos(dx+c)^8+\frac{8\cos(dx+c)^6}{7}+\frac{48\cos(dx+c)}{35}\right.}{9}\right.}{d}$
risch	$-ax-\frac{2ia\left(315\,e^{15i(dx+c)}+945\,e^{14i(dx+c)}-3045\,e^{13i(dx+c)}-1155\,e^{12i(dx+c)}+10143\,e^{11i(dx+c)}+1869\,e^{10i(dx+c)}-18\right.}{}$

input
```
int(cot(d*x+c)^10*(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(a*(-1/9/sin(d*x+c)^9*cos(d*x+c)^10+1/63/sin(d*x+c)^7*cos(d*x+c)^10-1/
105/sin(d*x+c)^5*cos(d*x+c)^10+1/63/sin(d*x+c)^3*cos(d*x+c)^10-1/9/sin(d*x
+c)*cos(d*x+c)^10-1/9*(128/35+cos(d*x+c)^8+8/7*cos(d*x+c)^6+48/35*cos(d*x+
c)^4+64/35*cos(d*x+c)^2)*sin(d*x+c))+a*(-1/9*cot(d*x+c)^9+1/7*cot(d*x+c)^7
-1/5*cot(d*x+c)^5+1/3*cot(d*x+c)^3-cot(d*x+c)-d*x-c))
```

3.18.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 279 vs. $2(130) = 260$.

Time $= 0.29$ (sec) , antiderivative size $= 279$, normalized size of antiderivative $= 1.99$

$$\int \cot^{10}(c+dx)(a+a\sec(c+dx))\,dx =$$
$$-\frac{563\,a\cos(dx+c)^8 - 248\,a\cos(dx+c)^7 - 1498\,a\cos(dx+c)^6 + 658\,a\cos(dx+c)^5 + 1610\,a\cos(dx+}{31}$$

input
```
integrate(cot(d*x+c)^10*(a+a*sec(d*x+c)),x, algorithm="fricas")
```

output
```
-1/315*(563*a*cos(d*x + c)^8 - 248*a*cos(d*x + c)^7 - 1498*a*cos(d*x + c)^
6 + 658*a*cos(d*x + c)^5 + 1610*a*cos(d*x + c)^4 - 602*a*cos(d*x + c)^3 -
763*a*cos(d*x + c)^2 + 187*a*cos(d*x + c) + 315*(a*d*x*cos(d*x + c)^7 - a*
d*x*cos(d*x + c)^6 - 3*a*d*x*cos(d*x + c)^5 + 3*a*d*x*cos(d*x + c)^4 + 3*a
*d*x*cos(d*x + c)^3 - 3*a*d*x*cos(d*x + c)^2 - a*d*x*cos(d*x + c) + a*d*x)
*sin(d*x + c) + 128*a)/((d*cos(d*x + c)^7 - d*cos(d*x + c)^6 - 3*d*cos(d*x
 + c)^5 + 3*d*cos(d*x + c)^4 + 3*d*cos(d*x + c)^3 - 3*d*cos(d*x + c)^2 - d
*cos(d*x + c) + d)*sin(d*x + c))
```

3.18.6 Sympy [F(-1)]

Timed out.

$$\int \cot^{10}(c+dx)(a+a\sec(c+dx))\,dx = \text{Timed out}$$

input
```
integrate(cot(d*x+c)**10*(a+a*sec(d*x+c)),x)
```

output
```
Timed out
```

3.18.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.85

$$\int \cot^{10}(c+dx)(a+a\sec(c+dx))\,dx =$$

$$-\frac{\left(315\,dx+315\,c+\frac{315\tan(dx+c)^8-105\tan(dx+c)^6+63\tan(dx+c)^4-45\tan(dx+c)^2+35}{\tan(dx+c)^9}\right)a+\frac{\left(315\sin(dx+c)^8-420\sin(dx+c)^6-}{\sin}}{315\,d}$$

input
```
integrate(cot(d*x+c)^10*(a+a*sec(d*x+c)),x, algorithm="maxima")
```

output
```
-1/315*((315*d*x + 315*c + (315*tan(d*x + c)^8 - 105*tan(d*x + c)^6 + 63*t
an(d*x + c)^4 - 45*tan(d*x + c)^2 + 35)/tan(d*x + c)^9)*a + (315*sin(d*x +
 c)^8 - 420*sin(d*x + c)^6 + 378*sin(d*x + c)^4 - 180*sin(d*x + c)^2 + 35)
*a/sin(d*x + c)^9)/d
```

3.18.8 Giac [A] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.00

$$\int \cot^{10}(c+dx)(a+a\sec(c+dx))\,dx =$$

$$-\frac{45\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-630\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+4830\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+80640\,(dx+c)a-40950\,a}{80640\,d}$$

input
```
integrate(cot(d*x+c)^10*(a+a*sec(d*x+c)),x, algorithm="giac")
```

output
```
-1/80640*(45*a*tan(1/2*d*x + 1/2*c)^7 - 630*a*tan(1/2*d*x + 1/2*c)^5 + 483
0*a*tan(1/2*d*x + 1/2*c)^3 + 80640*(d*x + c)*a - 40950*a*tan(1/2*d*x + 1/2
*c) + (80640*a*tan(1/2*d*x + 1/2*c)^8 - 13650*a*tan(1/2*d*x + 1/2*c)^6 + 2
898*a*tan(1/2*d*x + 1/2*c)^4 - 450*a*tan(1/2*d*x + 1/2*c)^2 + 35*a)/tan(1/
2*d*x + 1/2*c)^9)/d
```

3.18.9 Mupad [B] (verification not implemented)

Time = 15.27 (sec) , antiderivative size = 252, normalized size of antiderivative = 1.80

$$\int \cot^{10}(c+dx)(a+a\sec(c+dx))\,dx =$$

$$-\frac{a\left(35\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{16}+45\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{16}-630\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{2}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{14}+4830\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{4}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{12}-40950\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{6}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}+80640\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{8}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{8}-13650\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{6}+2898\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{12}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{4}-450\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{14}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{2}+80640\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{7}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{9}(c+dx)\right)}{80640\,d\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{7}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{9}}$$

input
```
int(cot(c + d*x)^10*(a + a/cos(c + d*x)),x)
```

output
```
-(a*(35*cos(c/2 + (d*x)/2)^16 + 45*sin(c/2 + (d*x)/2)^16 - 630*cos(c/2 + (
d*x)/2)^2*sin(c/2 + (d*x)/2)^14 + 4830*cos(c/2 + (d*x)/2)^4*sin(c/2 + (d*x
)/2)^12 - 40950*cos(c/2 + (d*x)/2)^6*sin(c/2 + (d*x)/2)^10 + 80640*cos(c/2
 + (d*x)/2)^8*sin(c/2 + (d*x)/2)^8 - 13650*cos(c/2 + (d*x)/2)^10*sin(c/2 +
 (d*x)/2)^6 + 2898*cos(c/2 + (d*x)/2)^12*sin(c/2 + (d*x)/2)^4 - 450*cos(c/
2 + (d*x)/2)^14*sin(c/2 + (d*x)/2)^2 + 80640*cos(c/2 + (d*x)/2)^7*sin(c/2
+ (d*x)/2)^9*(c + d*x)))/(80640*d*cos(c/2 + (d*x)/2)^7*sin(c/2 + (d*x)/2)^
9)
```

3.19 $\int (a + a\sec(c+dx))^2 \tan^9(c+dx)\,dx$

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3.19.1 Optimal result

Integrand size = 21, antiderivative size = 192

$$\begin{aligned}\int (a + a\sec(c+dx))^2 \tan^9(c+dx)\,dx = &-\frac{a^2\log(\cos(c+dx))}{d} \\ &+ \frac{2a^2\sec(c+dx)}{d} - \frac{3a^2\sec^2(c+dx)}{2d} \\ &- \frac{8a^2\sec^3(c+dx)}{3d} + \frac{a^2\sec^4(c+dx)}{2d} \\ &+ \frac{12a^2\sec^5(c+dx)}{5d} + \frac{a^2\sec^6(c+dx)}{3d} \\ &- \frac{8a^2\sec^7(c+dx)}{7d} - \frac{3a^2\sec^8(c+dx)}{8d} \\ &+ \frac{2a^2\sec^9(c+dx)}{9d} + \frac{a^2\sec^{10}(c+dx)}{10d}\end{aligned}$$

output
```
-a^2*ln(cos(d*x+c))/d+2*a^2*sec(d*x+c)/d-3/2*a^2*sec(d*x+c)^2/d-8/3*a^2*se
c(d*x+c)^3/d+1/2*a^2*sec(d*x+c)^4/d+12/5*a^2*sec(d*x+c)^5/d+1/3*a^2*sec(d*
x+c)^6/d-8/7*a^2*sec(d*x+c)^7/d-3/8*a^2*sec(d*x+c)^8/d+2/9*a^2*sec(d*x+c)^
9/d+1/10*a^2*sec(d*x+c)^10/d
```

3.19.2 Mathematica [A] (verified)

Time = 1.19 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.01

$$\int (a + a\sec(c+dx))^2 \tan^9(c+dx)\,dx$$
$$= \frac{a^2(63080\cos(c+dx) + 39600\cos(3(c+dx)) - 1050\cos(2(c+dx))(68 + 63\log(\cos(c+dx))) - 21(1662$$

input
```
Integrate[(a + a*Sec[c + d*x])^2*Tan[c + d*x]^9,x]
```

output
```
(a^2*(63080*Cos[c + d*x] + 39600*Cos[3*(c + d*x)] - 1050*Cos[2*(c + d*x)]*
(68 + 63*Log[Cos[c + d*x]]) - 21*(1662 - 1072*Cos[5*(c + d*x)] + 600*Cos[6
*(c + d*x)] - 220*Cos[7*(c + d*x)] + 90*Cos[8*(c + d*x)] - 60*Cos[9*(c + d
*x)] + 1890*Log[Cos[c + d*x]] + 675*Cos[6*(c + d*x)]*Log[Cos[c + d*x]] + 1
50*Cos[8*(c + d*x)]*Log[Cos[c + d*x]] + 15*Cos[10*(c + d*x)]*Log[Cos[c + d
*x]] + 40*Cos[4*(c + d*x)]*(37 + 45*Log[Cos[c + d*x]])))*Sec[c + d*x]^10)/
(161280*d)
```

3.19.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.69, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 25, 4367, 27, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^9(c+dx)(a\sec(c+dx)+a)^2\,dx$$
$$\downarrow 3042$$
$$\int -\cot\left(c+dx+\frac{\pi}{2}\right)^9 \left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^2 dx$$
$$\downarrow 25$$
$$-\int \cot\left(\frac{1}{2}(2c+\pi)+dx\right)^9 \left(\csc\left(\frac{1}{2}(2c+\pi)+dx\right)a+a\right)^2 dx$$
$$\downarrow 4367$$
$$-\frac{\int a^{10}(1-\cos(c+dx))^4(\cos(c+dx)+1)^6\sec^{11}(c+dx)d\cos(c+dx)}{a^8 d}$$

$$\downarrow 27$$

$$-\frac{a^2\int(1-\cos(c+dx))^4(\cos(c+dx)+1)^6\sec^{11}(c+dx)d\cos(c+dx)}{d}$$

$$\downarrow 99$$

$$-\frac{a^2\int\left(\sec^{11}(c+dx)+2\sec^{10}(c+dx)-3\sec^9(c+dx)-8\sec^8(c+dx)+2\sec^7(c+dx)+12\sec^6(c+dx)+2\sec\right.}{d}$$

$$\downarrow 2009$$

$$-\frac{a^2(-\frac{1}{10}\sec^{10}(c+dx)-\frac{2}{9}\sec^9(c+dx)+\frac{3}{8}\sec^8(c+dx)+\frac{8}{7}\sec^7(c+dx)-\frac{1}{3}\sec^6(c+dx)-\frac{12}{5}\sec^5(c+dx)-\frac{1}{2}}{d}$$

input
```
Int[(a + a*Sec[c + d*x])^2*Tan[c + d*x]^9,x]
```

output
```
-((a^2*(Log[Cos[c + d*x]] - 2*Sec[c + d*x] + (3*Sec[c + d*x]^2)/2 + (8*Sec
[c + d*x]^3)/3 - Sec[c + d*x]^4/2 - (12*Sec[c + d*x]^5)/5 - Sec[c + d*x]^6
/3 + (8*Sec[c + d*x]^7)/7 + (3*Sec[c + d*x]^8)/8 - (2*Sec[c + d*x]^9)/9 -
Sec[c + d*x]^10/10))/d)
```

3.19.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4367
```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Simp[1/(a^(m - n - 1)*b^n*d)   Subst[Int[(a - b*x)^((m -
1)/2)*((a + b*x)^((m - 1)/2 + n)/x^(m + n)), x], x, Sin[c + d*x]], x] /; Fr
eeQ[{a, b, c, d}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && Integer
Q[n]
```

3.19.4 Maple [A] (verified)

Time = 2.27 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.59

method	result
derivativedivides	$\frac{a^2\left(\frac{\sec(dx+c)^{10}}{10}+\frac{2\sec(dx+c)^9}{9}-\frac{3\sec(dx+c)^8}{8}-\frac{8\sec(dx+c)^7}{7}+\frac{\sec(dx+c)^6}{3}+\frac{12\sec(dx+c)^5}{5}+\frac{\sec(dx+c)^4}{2}-\frac{8\sec(dx+c)^3}{3}-\frac{3\sec(}{}\right.}{d}$
default	$\frac{a^2\left(\frac{\sec(dx+c)^{10}}{10}+\frac{2\sec(dx+c)^9}{9}-\frac{3\sec(dx+c)^8}{8}-\frac{8\sec(dx+c)^7}{7}+\frac{\sec(dx+c)^6}{3}+\frac{12\sec(dx+c)^5}{5}+\frac{\sec(dx+c)^4}{2}-\frac{8\sec(dx+c)^3}{3}-\frac{3\sec(}{}\right.}{d}$
parts	$\frac{a^2\left(\frac{\tan(dx+c)^8}{8}-\frac{\tan(dx+c)^6}{6}+\frac{\tan(dx+c)^4}{4}-\frac{\tan(dx+c)^2}{2}+\frac{\ln\left(1+\tan(dx+c)^2\right)}{2}\right)}{d}+\frac{a^2\tan(dx+c)^{10}}{10d}+\frac{2a^2\left(\frac{\sec(dx+c)^9}{9}-\right.}{}$
risch	$ia^2x+\frac{2ia^2c}{d}+\frac{2a^2\left(630\,e^{19i(dx+c)}-945\,e^{18i(dx+c)}+2310\,e^{17i(dx+c)}-6300\,e^{16i(dx+c)}+11256\,e^{15i(dx+c)}-15540\,e^{14}\right.}{}$

input
```
int((a+a*sec(d*x+c))^2*tan(d*x+c)^9,x,method=_RETURNVERBOSE)
```

output
```
a^2/d*(1/10*sec(d*x+c)^10+2/9*sec(d*x+c)^9-3/8*sec(d*x+c)^8-8/7*sec(d*x+c)
^7+1/3*sec(d*x+c)^6+12/5*sec(d*x+c)^5+1/2*sec(d*x+c)^4-8/3*sec(d*x+c)^3-3/
2*sec(d*x+c)^2+2*sec(d*x+c)+ln(sec(d*x+c)))
```

3.19.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.81

$$\int (a + a\sec(c+dx))^2 \tan^9(c+dx)\,dx =$$

$$-\frac{2520\,a^2\cos(dx+c)^{10}\log(-\cos(dx+c)) - 5040\,a^2\cos(dx+c)^9 + 3780\,a^2\cos(dx+c)^8 + 6720\,a^2\cos$$

input
```
integrate((a+a*sec(d*x+c))^2*tan(d*x+c)^9,x, algorithm="fricas")
```

output
```
-1/2520*(2520*a^2*cos(d*x + c)^10*log(-cos(d*x + c)) - 5040*a^2*cos(d*x +
c)^9 + 3780*a^2*cos(d*x + c)^8 + 6720*a^2*cos(d*x + c)^7 - 1260*a^2*cos(d*
x + c)^6 - 6048*a^2*cos(d*x + c)^5 - 840*a^2*cos(d*x + c)^4 + 2880*a^2*cos
(d*x + c)^3 + 945*a^2*cos(d*x + c)^2 - 560*a^2*cos(d*x + c) - 252*a^2)/(d*
cos(d*x + c)^10)
```

3.19.6 Sympy [A] (verification not implemented)

Time = 2.47 (sec) , antiderivative size = 314, normalized size of antiderivative = 1.64

$$\int (a + a\sec(c+dx))^2 \tan^9(c+dx)\,dx$$

$$= \begin{cases} \frac{a^2\log(\tan^2(c+dx)+1)}{2d} + \frac{a^2\tan^8(c+dx)\sec^2(c+dx)}{10d} + \frac{2a^2\tan^8(c+dx)\sec(c+dx)}{9d} + \frac{a^2\tan^8(c+dx)}{8d} - \frac{a^2\tan^6(c+dx)\sec^2(c+dx)}{10d} - \\ x(a\sec(c)+a)^2\tan^9(c) \end{cases}$$

input
```
integrate((a+a*sec(d*x+c))**2*tan(d*x+c)**9,x)
```

output
```
Piecewise((a**2*log(tan(c + d*x)**2 + 1)/(2*d) + a**2*tan(c + d*x)**8*sec(
c + d*x)**2/(10*d) + 2*a**2*tan(c + d*x)**8*sec(c + d*x)/(9*d) + a**2*tan(
c + d*x)**8/(8*d) - a**2*tan(c + d*x)**6*sec(c + d*x)**2/(10*d) - 16*a**2*
tan(c + d*x)**6*sec(c + d*x)/(63*d) - a**2*tan(c + d*x)**6/(6*d) + a**2*ta
n(c + d*x)**4*sec(c + d*x)**2/(10*d) + 32*a**2*tan(c + d*x)**4*sec(c + d*x
)/(105*d) + a**2*tan(c + d*x)**4/(4*d) - a**2*tan(c + d*x)**2*sec(c + d*x)
**2/(10*d) - 128*a**2*tan(c + d*x)**2*sec(c + d*x)/(315*d) - a**2*tan(c +
d*x)**2/(2*d) + a**2*sec(c + d*x)**2/(10*d) + 256*a**2*sec(c + d*x)/(315*d
), Ne(d, 0)), (x*(a*sec(c) + a)**2*tan(c)**9, True))
```

3.19.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.78

$$\int (a + a\sec(c+dx))^2 \tan^9(c+dx)\,dx =$$

$$-\frac{2520\,a^2\log(\cos(dx+c)) - \frac{5040\,a^2\cos(dx+c)^9 - 3780\,a^2\cos(dx+c)^8 - 6720\,a^2\cos(dx+c)^7 + 1260\,a^2\cos(dx+c)^6 + 6048\,a^2\cos(dx+c)}{\cos(dx+c)^{10}}}{2520\,d}$$

input
```
integrate((a+a*sec(d*x+c))^2*tan(d*x+c)^9,x, algorithm="maxima")
```

output
```
-1/2520*(2520*a^2*log(cos(d*x + c)) - (5040*a^2*cos(d*x + c)^9 - 3780*a^2*
cos(d*x + c)^8 - 6720*a^2*cos(d*x + c)^7 + 1260*a^2*cos(d*x + c)^6 + 6048*
a^2*cos(d*x + c)^5 + 840*a^2*cos(d*x + c)^4 - 2880*a^2*cos(d*x + c)^3 - 94
5*a^2*cos(d*x + c)^2 + 560*a^2*cos(d*x + c) + 252*a^2)/cos(d*x + c)^10)/d
```

3.19.8 Giac [A] (verification not implemented)

Time = 6.02 (sec) , antiderivative size = 342, normalized size of antiderivative = 1.78

$$\int (a + a\sec(c+dx))^2 \tan^9(c+dx)\,dx$$

$$= \frac{2520\,a^2\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right) - 2520\,a^2\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right) + \frac{11477\,a^2 + \frac{119810\,a^2(\cos(dx+c)-1)}{\cos(dx+c)+1} + \frac{566865\,a^2(\cos(}{(\cos(dx+c}}$$

input
```
integrate((a+a*sec(d*x+c))^2*tan(d*x+c)^9,x, algorithm="giac")
```

output
```
1/2520*(2520*a^2*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)) - 25
20*a^2*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)) + (11477*a^2 +
 119810*a^2*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 566865*a^2*(cos(d*x +
c) - 1)^2/(cos(d*x + c) + 1)^2 + 1605720*a^2*(cos(d*x + c) - 1)^3/(cos(d*x
 + c) + 1)^3 + 3031770*a^2*(cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4 + 299
5020*a^2*(cos(d*x + c) - 1)^5/(cos(d*x + c) + 1)^5 + 2171610*a^2*(cos(d*x
+ c) - 1)^6/(cos(d*x + c) + 1)^6 + 1114200*a^2*(cos(d*x + c) - 1)^7/(cos(d
*x + c) + 1)^7 + 382545*a^2*(cos(d*x + c) - 1)^8/(cos(d*x + c) + 1)^8 + 78
850*a^2*(cos(d*x + c) - 1)^9/(cos(d*x + c) + 1)^9 + 7381*a^2*(cos(d*x + c)
 - 1)^10/(cos(d*x + c) + 1)^10)/((cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1
)^10)/d
```

3.19.9 Mupad [B] (verification not implemented)

Time = 18.11 (sec) , antiderivative size = 308, normalized size of antiderivative = 1.60

$$\int (a+a\sec(c+dx))^2\tan^9(c+dx)\,dx = \frac{2a^2\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2\right)}{d}$$
$$-\frac{2a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{18}-20a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{16}+\frac{272a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{14}}{3}-\frac{740a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{12}}{3}+\frac{2252a^2\tan\left(}{5}}{d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{20}-10\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{18}+45\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{16}-120\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{14}+210\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{12}\right.}$$

input
```
int(tan(c + d*x)^9*(a + a/cos(c + d*x))^2,x)
```

output
```
(2*a^2*atanh(tan(c/2 + (d*x)/2)^2))/d - ((1150*a^2*tan(c/2 + (d*x)/2)^2)/6
3 - (652*a^2*tan(c/2 + (d*x)/2)^4)/7 + (2000*a^2*tan(c/2 + (d*x)/2)^6)/7 -
 588*a^2*tan(c/2 + (d*x)/2)^8 + (2252*a^2*tan(c/2 + (d*x)/2)^10)/5 - (740*
a^2*tan(c/2 + (d*x)/2)^12)/3 + (272*a^2*tan(c/2 + (d*x)/2)^14)/3 - 20*a^2*
tan(c/2 + (d*x)/2)^16 + 2*a^2*tan(c/2 + (d*x)/2)^18 - (512*a^2)/315)/(d*(4
5*tan(c/2 + (d*x)/2)^4 - 10*tan(c/2 + (d*x)/2)^2 - 120*tan(c/2 + (d*x)/2)^
6 + 210*tan(c/2 + (d*x)/2)^8 - 252*tan(c/2 + (d*x)/2)^10 + 210*tan(c/2 + (
d*x)/2)^12 - 120*tan(c/2 + (d*x)/2)^14 + 45*tan(c/2 + (d*x)/2)^16 - 10*tan
(c/2 + (d*x)/2)^18 + tan(c/2 + (d*x)/2)^20 + 1))
```

3.20 $\int (a + a\sec(c+dx))^2 \tan^7(c+dx)\,dx$

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3.20.1 Optimal result

Integrand size = 21, antiderivative size = 132

$$\begin{aligned}\int (a + a\sec(c+dx))^2 \tan^7(c+dx)\,dx = {} & \frac{a^2\log(\cos(c+dx))}{d} - \frac{2a^2\sec(c+dx)}{d} \\ & + \frac{a^2\sec^2(c+dx)}{d} + \frac{2a^2\sec^3(c+dx)}{d} \\ & - \frac{6a^2\sec^5(c+dx)}{5d} - \frac{a^2\sec^6(c+dx)}{3d} \\ & + \frac{2a^2\sec^7(c+dx)}{7d} + \frac{a^2\sec^8(c+dx)}{8d}\end{aligned}$$

output
```
a^2*ln(cos(d*x+c))/d-2*a^2*sec(d*x+c)/d+a^2*sec(d*x+c)^2/d+2*a^2*sec(d*x+c
)^3/d-6/5*a^2*sec(d*x+c)^5/d-1/3*a^2*sec(d*x+c)^6/d+2/7*a^2*sec(d*x+c)^7/d
+1/8*a^2*sec(d*x+c)^8/d
```

3.20.2 Mathematica [A] (verified)

Time = 0.74 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.20

$$\int (a + a\sec(c+dx))^2 \tan^7(c+dx)\,dx$$
$$= \frac{a^2(-6156\cos(c+dx) + 7(-636\cos(3(c+dx)) + 20\cos(2(c+dx))(29 + 42\log(\cos(c+dx))) + 5(104 -$$

input
```
Integrate[(a + a*Sec[c + d*x])^2*Tan[c + d*x]^7,x]
```

output
```
(a^2*(-6156*Cos[c + d*x] + 7*(-636*Cos[3*(c + d*x)] + 20*Cos[2*(c + d*x)]*
(29 + 42*Log[Cos[c + d*x]]) + 5*(104 - 36*Cos[5*(c + d*x)] + 12*Cos[6*(c +
 d*x)] - 12*Cos[7*(c + d*x)] + 105*Log[Cos[c + d*x]] + 24*Cos[6*(c + d*x)]
*Log[Cos[c + d*x]] + 3*Cos[8*(c + d*x)]*Log[Cos[c + d*x]] + 12*Cos[4*(c +
d*x)]*(6 + 7*Log[Cos[c + d*x]])))*Sec[c + d*x]^8)/(13440*d)
```

3.20.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.71, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 25, 4367, 27, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^7(c+dx)(a\sec(c+dx)+a)^2\,dx$$

$$\downarrow 3042$$

$$\int -\cot\left(c+dx+\frac{\pi}{2}\right)^7\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^2 dx$$

$$\downarrow 25$$

$$-\int \cot\left(\frac{1}{2}(2c+\pi)+dx\right)^7\left(\csc\left(\frac{1}{2}(2c+\pi)+dx\right)a+a\right)^2 dx$$

$$\downarrow 4367$$

$$-\frac{\int a^8(1-\cos(c+dx))^3(\cos(c+dx)+1)^5\sec^9(c+dx)d\cos(c+dx)}{a^6 d}$$

$$\downarrow 27$$

$$-\frac{a^2\int(1-\cos(c+dx))^3(\cos(c+dx)+1)^5\sec^9(c+dx)d\cos(c+dx)}{d}$$

$$\downarrow 99$$

$$-\frac{a^2\int\left(\sec^9(c+dx)+2\sec^8(c+dx)-2\sec^7(c+dx)-6\sec^6(c+dx)+6\sec^4(c+dx)+2\sec^3(c+dx)-2\sec^2(c\right.}{d}$$

$$\downarrow 2009$$

$$-\frac{a^2\left(-\frac{1}{8}\sec^8(c+dx)-\frac{2}{7}\sec^7(c+dx)+\frac{1}{3}\sec^6(c+dx)+\frac{6}{5}\sec^5(c+dx)-2\sec^3(c+dx)-\sec^2(c+dx)+2\sec(c\right.}{d}$$

input
```
Int[(a + a*Sec[c + d*x])^2*Tan[c + d*x]^7,x]
```

output
```
-((a^2*(-Log[Cos[c + d*x]] + 2*Sec[c + d*x] - Sec[c + d*x]^2 - 2*Sec[c + d
*x]^3 + (6*Sec[c + d*x]^5)/5 + Sec[c + d*x]^6/3 - (2*Sec[c + d*x]^7)/7 - S
ec[c + d*x]^8/8))/d)
```

3.20.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4367
```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Simp[1/(a^(m - n - 1)*b^n*d)   Subst[Int[(a - b*x)^((m -
1)/2)*((a + b*x)^((m - 1)/2 + n)/x^(m + n)), x], x, Sin[c + d*x]], x] /; Fr
eeQ[{a, b, c, d}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && Integer
Q[n]
```

3.20.4 Maple [A] (verified)

Time = 1.72 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.64

method	result
derivativedivides	$\frac{a^2\left(\frac{\sec(dx+c)^8}{8}+\frac{2\sec(dx+c)^7}{7}-\frac{\sec(dx+c)^6}{3}-\frac{6\sec(dx+c)^5}{5}+2\sec(dx+c)^3+\sec(dx+c)^2-2\sec(dx+c)-\ln(\sec(dx+c))\right)}{d}$
default	$\frac{a^2\left(\frac{\sec(dx+c)^8}{8}+\frac{2\sec(dx+c)^7}{7}-\frac{\sec(dx+c)^6}{3}-\frac{6\sec(dx+c)^5}{5}+2\sec(dx+c)^3+\sec(dx+c)^2-2\sec(dx+c)-\ln(\sec(dx+c))\right)}{d}$
parts	$\frac{a^2\left(\frac{\tan(dx+c)^6}{6}-\frac{\tan(dx+c)^4}{4}+\frac{\tan(dx+c)^2}{2}-\frac{\ln\left(1+\tan(dx+c)^2\right)}{2}\right)}{d}+\frac{a^2\tan(dx+c)^8}{8d}+\frac{2a^2\left(\frac{\sec(dx+c)^7}{7}-\frac{3\sec(dx+c)^5}{5}+s}{d}$
risch	$-ia^2x-\frac{2ia^2c}{d}-\frac{4a^2\left(105\,e^{15i(dx+c)}-105\,e^{14i(dx+c)}+315\,e^{13i(dx+c)}-630\,e^{12i(dx+c)}+1113\,e^{11i(dx+c)}-1015\,e^{10i(}$

input
```
int((a+a*sec(d*x+c))^2*tan(d*x+c)^7,x,method=_RETURNVERBOSE)
```

output
```
a^2/d*(1/8*sec(d*x+c)^8+2/7*sec(d*x+c)^7-1/3*sec(d*x+c)^6-6/5*sec(d*x+c)^5
+2*sec(d*x+c)^3+sec(d*x+c)^2-2*sec(d*x+c)-ln(sec(d*x+c)))
```

3.20.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.89

$$\int (a + a\sec(c+dx))^2 \tan^7(c+dx)\,dx$$
$$= \frac{840\,a^2\cos(dx+c)^8\log(-\cos(dx+c)) - 1680\,a^2\cos(dx+c)^7 + 840\,a^2\cos(dx+c)^6 + 1680\,a^2\cos(dx+}{840\,d\cos(dx+c)^8}$$

input
```
integrate((a+a*sec(d*x+c))^2*tan(d*x+c)^7,x, algorithm="fricas")
```

output
```
1/840*(840*a^2*cos(d*x + c)^8*log(-cos(d*x + c)) - 1680*a^2*cos(d*x + c)^7
 + 840*a^2*cos(d*x + c)^6 + 1680*a^2*cos(d*x + c)^5 - 1008*a^2*cos(d*x + c
)^3 - 280*a^2*cos(d*x + c)^2 + 240*a^2*cos(d*x + c) + 105*a^2)/(d*cos(d*x
+ c)^8)
```

3.20.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 252 vs. $2(119) = 238$.

Time $= 1.19$ (sec) , antiderivative size $= 252$, normalized size of antiderivative $= 1.91$

$$\int (a + a\sec(c+dx))^2 \tan^7(c+dx)\,dx$$

$$= \begin{cases} -\frac{a^2 \log\left(\tan^2(c+dx)+1\right)}{2d} + \frac{a^2 \tan^6(c+dx)\sec^2(c+dx)}{8d} + \frac{2a^2 \tan^6(c+dx)\sec(c+dx)}{7d} + \frac{a^2 \tan^6(c+dx)}{6d} - \frac{a^2 \tan^4(c+dx)\sec^2(c+dx)}{8d} \\ x(a\sec(c)+a)^2 \tan^7(c) \end{cases}$$

input
```
integrate((a+a*sec(d*x+c))**2*tan(d*x+c)**7,x)
```

output
```
Piecewise((-a**2*log(tan(c + d*x)**2 + 1)/(2*d) + a**2*tan(c + d*x)**6*sec
(c + d*x)**2/(8*d) + 2*a**2*tan(c + d*x)**6*sec(c + d*x)/(7*d) + a**2*tan(
c + d*x)**6/(6*d) - a**2*tan(c + d*x)**4*sec(c + d*x)**2/(8*d) - 12*a**2*t
an(c + d*x)**4*sec(c + d*x)/(35*d) - a**2*tan(c + d*x)**4/(4*d) + a**2*tan
(c + d*x)**2*sec(c + d*x)**2/(8*d) + 16*a**2*tan(c + d*x)**2*sec(c + d*x)/
(35*d) + a**2*tan(c + d*x)**2/(2*d) - a**2*sec(c + d*x)**2/(8*d) - 32*a**2
*sec(c + d*x)/(35*d), Ne(d, 0)), (x*(a*sec(c) + a)**2*tan(c)**7, True))
```

3.20.7 Maxima [A] (verification not implemented)

Time $= 0.21$ (sec) , antiderivative size $= 110$, normalized size of antiderivative $= 0.83$

$$\int (a + a\sec(c+dx))^2 \tan^7(c+dx)\,dx$$

$$= \frac{840\,a^2 \log\left(\cos(dx+c)\right) - \frac{1680\,a^2\cos(dx+c)^7 - 840\,a^2\cos(dx+c)^6 - 1680\,a^2\cos(dx+c)^5 + 1008\,a^2\cos(dx+c)^3 + 280\,a^2\cos(dx+c)^2 - 240}{\cos(dx+c)^8}}{840\,d}$$

input
```
integrate((a+a*sec(d*x+c))^2*tan(d*x+c)^7,x, algorithm="maxima")
```

output
```
1/840*(840*a^2*log(cos(d*x + c)) - (1680*a^2*cos(d*x + c)^7 - 840*a^2*cos(
d*x + c)^6 - 1680*a^2*cos(d*x + c)^5 + 1008*a^2*cos(d*x + c)^3 + 280*a^2*c
os(d*x + c)^2 - 240*a^2*cos(d*x + c) - 105*a^2)/cos(d*x + c)^8)/d
```

3.20.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 292 vs. $2(124) = 248$.

Time = 3.88 (sec) , antiderivative size = 292, normalized size of antiderivative = 2.21

$$\int (a + a\sec(c+dx))^2 \tan^7(c+dx)\,dx =$$

$$-\frac{840\,a^2 \log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right) - 840\,a^2 \log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right) + \frac{3819\,a^2+\frac{32232\,a^2(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{120372\,a^2(\cos(dx}{(\cos(dx+c)+}}{}$$

input
```
integrate((a+a*sec(d*x+c))^2*tan(d*x+c)^7,x, algorithm="giac")
```

output
```
-1/840*(840*a^2*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)) - 840
*a^2*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)) + (3819*a^2 + 32
232*a^2*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 120372*a^2*(cos(d*x + c) -
 1)^2/(cos(d*x + c) + 1)^2 + 261464*a^2*(cos(d*x + c) - 1)^3/(cos(d*x + c)
 + 1)^3 + 258370*a^2*(cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4 + 175448*a^
2*(cos(d*x + c) - 1)^5/(cos(d*x + c) + 1)^5 + 77364*a^2*(cos(d*x + c) - 1)
^6/(cos(d*x + c) + 1)^6 + 19944*a^2*(cos(d*x + c) - 1)^7/(cos(d*x + c) + 1
)^7 + 2283*a^2*(cos(d*x + c) - 1)^8/(cos(d*x + c) + 1)^8)/((cos(d*x + c) -
 1)/(cos(d*x + c) + 1) + 1)^8)/d
```

3.20.9 Mupad [B] (verification not implemented)

Time = 18.22 (sec) , antiderivative size = 249, normalized size of antiderivative = 1.89

$$\int (a + a\sec(c+dx))^2 \tan^7(c+dx)\,dx$$

$$= \frac{2\,a^2 \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{14} - 16\,a^2 \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{12} + \frac{170\,a^2 \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}}{3} - \frac{352\,a^2 \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{8}}{3} + \frac{2386\,a^2 \tan\left(\frac{c}{2}+\frac{dx}{2}\right.}{15}}{d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{16} - 8\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{14} + 28\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{12} - 56\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{10} + 70\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{8} - 56\,\mathrm{ta}\right.}$$

$$- \frac{2\,a^2\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2\right)}{d}$$

input
```
int(tan(c + d*x)^7*(a + a/cos(c + d*x))^2,x)
```

output
```
((582*a^2*tan(c/2 + (d*x)/2)^2)/35 - (336*a^2*tan(c/2 + (d*x)/2)^4)/5 + (2
386*a^2*tan(c/2 + (d*x)/2)^6)/15 - (352*a^2*tan(c/2 + (d*x)/2)^8)/3 + (170
*a^2*tan(c/2 + (d*x)/2)^10)/3 - 16*a^2*tan(c/2 + (d*x)/2)^12 + 2*a^2*tan(c
/2 + (d*x)/2)^14 - (64*a^2)/35)/(d*(28*tan(c/2 + (d*x)/2)^4 - 8*tan(c/2 +
(d*x)/2)^2 - 56*tan(c/2 + (d*x)/2)^6 + 70*tan(c/2 + (d*x)/2)^8 - 56*tan(c/
2 + (d*x)/2)^10 + 28*tan(c/2 + (d*x)/2)^12 - 8*tan(c/2 + (d*x)/2)^14 + tan
(c/2 + (d*x)/2)^16 + 1)) - (2*a^2*atanh(tan(c/2 + (d*x)/2)^2))/d
```

3.21 $\int (a + a\sec(c+dx))^2 \tan^5(c+dx)\,dx$

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3.21.1 Optimal result

Integrand size = 21, antiderivative size = 120

$$\begin{aligned}\int (a + a\sec(c+dx))^2 \tan^5(c+dx)\,dx = &-\frac{a^2\log(\cos(c+dx))}{d} \\ &+ \frac{2a^2\sec(c+dx)}{d} - \frac{a^2\sec^2(c+dx)}{2d} \\ &- \frac{4a^2\sec^3(c+dx)}{3d} - \frac{a^2\sec^4(c+dx)}{4d} \\ &+ \frac{2a^2\sec^5(c+dx)}{5d} + \frac{a^2\sec^6(c+dx)}{6d}\end{aligned}$$

output

```
-a^2*ln(cos(d*x+c))/d+2*a^2*sec(d*x+c)/d-1/2*a^2*sec(d*x+c)^2/d-4/3*a^2*se
c(d*x+c)^3/d-1/4*a^2*sec(d*x+c)^4/d+2/5*a^2*sec(d*x+c)^5/d+1/6*a^2*sec(d*x
+c)^6/d
```

3.21.2 Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.04

$$\int (a + a\sec(c+dx))^2 \tan^5(c+dx)\,dx$$
$$= \frac{a^2(312\cos(c+dx) - 5(14 - 28\cos(3(c+dx)) + 6\cos(4(c+dx)) - 12\cos(5(c+dx)) + 30\log(\cos(c+dx$$

input
```
Integrate[(a + a*Sec[c + d*x])^2*Tan[c + d*x]^5,x]
```

output
```
(a^2*(312*Cos[c + d*x] - 5*(14 - 28*Cos[3*(c + d*x)] + 6*Cos[4*(c + d*x)]
- 12*Cos[5*(c + d*x)] + 30*Log[Cos[c + d*x]] + 18*Cos[4*(c + d*x)]*Log[Cos
[c + d*x]] + 3*Cos[6*(c + d*x)]*Log[Cos[c + d*x]] + 9*Cos[2*(c + d*x)]*(4
+ 5*Log[Cos[c + d*x]])))*Sec[c + d*x]^6)/(480*d)
```

3.21.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.70, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 25, 4367, 27, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^5(c+dx)(a\sec(c+dx)+a)^2\,dx$$

$$\downarrow 3042$$

$$\int -\cot\left(c+dx+\frac{\pi}{2}\right)^5\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^2 dx$$

$$\downarrow 25$$

$$-\int \cot\left(\frac{1}{2}(2c+\pi)+dx\right)^5\left(\csc\left(\frac{1}{2}(2c+\pi)+dx\right)a+a\right)^2 dx$$

$$\downarrow 4367$$

$$-\frac{\int a^6(1-\cos(c+dx))^2(\cos(c+dx)+1)^4\sec^7(c+dx)d\cos(c+dx)}{a^4 d}$$

$$\downarrow 27$$

$$-\frac{a^2\int(1-\cos(c+dx))^2(\cos(c+dx)+1)^4\sec^7(c+dx)d\cos(c+dx)}{d}$$

$$\downarrow 99$$

$$-\frac{a^2\int\left(\sec^7(c+dx)+2\sec^6(c+dx)-\sec^5(c+dx)-4\sec^4(c+dx)-\sec^3(c+dx)+2\sec^2(c+dx)+\sec(c+dx\right.}{d}$$

$$\downarrow 2009$$

$$-\frac{a^2\left(-\frac{1}{6}\sec^6(c+dx)-\frac{2}{5}\sec^5(c+dx)+\frac{1}{4}\sec^4(c+dx)+\frac{4}{3}\sec^3(c+dx)+\frac{1}{2}\sec^2(c+dx)-2\sec(c+dx)+\log(co\right.}{d}$$

input
```
Int[(a + a*Sec[c + d*x])^2*Tan[c + d*x]^5,x]
```

output
```
-((a^2*(Log[Cos[c + d*x]] - 2*Sec[c + d*x] + Sec[c + d*x]^2/2 + (4*Sec[c +
 d*x]^3)/3 + Sec[c + d*x]^4/4 - (2*Sec[c + d*x]^5)/5 - Sec[c + d*x]^6/6))/
d)
```

3.21.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4367
```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Simp[1/(a^(m - n - 1)*b^n*d)   Subst[Int[(a - b*x)^((m -
1)/2)*((a + b*x)^((m - 1)/2 + n)/x^(m + n)), x], x, Sin[c + d*x]], x] /; Fr
eeQ[{a, b, c, d}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && Integer
Q[n]
```

3.21.4 Maple [A] (verified)

Time = 1.36 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.62

method	result
derivativedivides	$\frac{a^2\left(\frac{\sec(dx+c)^6}{6}+\frac{2\sec(dx+c)^5}{5}-\frac{\sec(dx+c)^4}{4}-\frac{4\sec(dx+c)^3}{3}-\frac{\sec(dx+c)^2}{2}+2\sec(dx+c)+\ln(\sec(dx+c))\right)}{d}$
default	$\frac{a^2\left(\frac{\sec(dx+c)^6}{6}+\frac{2\sec(dx+c)^5}{5}-\frac{\sec(dx+c)^4}{4}-\frac{4\sec(dx+c)^3}{3}-\frac{\sec(dx+c)^2}{2}+2\sec(dx+c)+\ln(\sec(dx+c))\right)}{d}$
parts	$\frac{a^2\left(\frac{\tan(dx+c)^4}{4}-\frac{\tan(dx+c)^2}{2}+\frac{\ln\left(1+\tan(dx+c)^2\right)}{2}\right)}{d}+\frac{a^2\tan(dx+c)^6}{6d}+\frac{2a^2\left(\frac{\sec(dx+c)^5}{5}-\frac{2\sec(dx+c)^3}{3}+\sec(dx+c)\right)}{d}$
risch	$ia^2x+\frac{2ia^2c}{d}+\frac{2a^2\left(30\,e^{11i(dx+c)}-15\,e^{10i(dx+c)}+70\,e^{9i(dx+c)}-90\,e^{8i(dx+c)}+156\,e^{7i(dx+c)}-70\,e^{6i(dx+c)}+156\,e^{5i(}\right.}{15d\left(e^{2i(dx+c)}+1\right)^6}$

input
```
int((a+a*sec(d*x+c))^2*tan(d*x+c)^5,x,method=_RETURNVERBOSE)
```

output
```
a^2/d*(1/6*sec(d*x+c)^6+2/5*sec(d*x+c)^5-1/4*sec(d*x+c)^4-4/3*sec(d*x+c)^3
-1/2*sec(d*x+c)^2+2*sec(d*x+c)+ln(sec(d*x+c)))
```

3.21.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.87

$$\int (a+a\sec(c+dx))^2\tan^5(c+dx)\,dx =$$
$$-\frac{60\,a^2\cos(dx+c)^6\log(-\cos(dx+c))-120\,a^2\cos(dx+c)^5+30\,a^2\cos(dx+c)^4+80\,a^2\cos(dx+c)^3}{60\,d\cos(dx+c)^6}$$

input
```
integrate((a+a*sec(d*x+c))^2*tan(d*x+c)^5,x, algorithm="fricas")
```

output
```
-1/60*(60*a^2*cos(d*x + c)^6*log(-cos(d*x + c)) - 120*a^2*cos(d*x + c)^5 +
 30*a^2*cos(d*x + c)^4 + 80*a^2*cos(d*x + c)^3 + 15*a^2*cos(d*x + c)^2 - 2
4*a^2*cos(d*x + c) - 10*a^2)/(d*cos(d*x + c)^6)
```

3.21.6 Sympy [A] (verification not implemented)

Time = 0.59 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.58

$$\int (a + a\sec(c+dx))^2 \tan^5(c+dx)\,dx$$

$$= \begin{cases} \frac{a^2 \log\left(\tan^2(c+dx)+1\right)}{2d} + \frac{a^2 \tan^4(c+dx)\sec^2(c+dx)}{6d} + \frac{2a^2 \tan^4(c+dx)\sec(c+dx)}{5d} + \frac{a^2 \tan^4(c+dx)}{4d} - \frac{a^2 \tan^2(c+dx)\sec^2(c+dx)}{6d} - \\ x(a\sec(c)+a)^2 \tan^5(c) \end{cases}$$

input
```
integrate((a+a*sec(d*x+c))**2*tan(d*x+c)**5,x)
```

output
```
Piecewise((a**2*log(tan(c + d*x)**2 + 1)/(2*d) + a**2*tan(c + d*x)**4*sec(
c + d*x)**2/(6*d) + 2*a**2*tan(c + d*x)**4*sec(c + d*x)/(5*d) + a**2*tan(c
 + d*x)**4/(4*d) - a**2*tan(c + d*x)**2*sec(c + d*x)**2/(6*d) - 8*a**2*tan
(c + d*x)**2*sec(c + d*x)/(15*d) - a**2*tan(c + d*x)**2/(2*d) + a**2*sec(c
 + d*x)**2/(6*d) + 16*a**2*sec(c + d*x)/(15*d), Ne(d, 0)), (x*(a*sec(c) + 
a)**2*tan(c)**5, True))
```

3.21.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.81

$$\int (a + a\sec(c+dx))^2 \tan^5(c+dx)\,dx =$$

$$-\frac{60\,a^2 \log\left(\cos(dx+c)\right) - \frac{120\,a^2\cos(dx+c)^5 - 30\,a^2\cos(dx+c)^4 - 80\,a^2\cos(dx+c)^3 - 15\,a^2\cos(dx+c)^2 + 24\,a^2\cos(dx+c) + 10\,a^2}{\cos(dx+c)^6}}{60\,d}$$

input
```
integrate((a+a*sec(d*x+c))^2*tan(d*x+c)^5,x, algorithm="maxima")
```

output
```
-1/60*(60*a^2*log(cos(d*x + c)) - (120*a^2*cos(d*x + c)^5 - 30*a^2*cos(d*x
 + c)^4 - 80*a^2*cos(d*x + c)^3 - 15*a^2*cos(d*x + c)^2 + 24*a^2*cos(d*x + 
c) + 10*a^2)/cos(d*x + c)^6)/d
```

3.21.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 242 vs. 2(110) = 220.

Time = 1.77 (sec) , antiderivative size = 242, normalized size of antiderivative = 2.02

$$\int (a + a\sec(c+dx))^2 \tan^5(c+dx)\,dx$$

$$= \frac{60\,a^2 \log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right) - 60\,a^2 \log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right) + \frac{275\,a^2+\frac{1770\,a^2(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{4845\,a^2(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}+\frac{}{}}{60\,d}$$

input
```
integrate((a+a*sec(d*x+c))^2*tan(d*x+c)^5,x, algorithm="giac")
```

output
```
1/60*(60*a^2*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)) - 60*a^2
*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)) + (275*a^2 + 1770*a^
2*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 4845*a^2*(cos(d*x + c) - 1)^2/(c
os(d*x + c) + 1)^2 + 4780*a^2*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 +
2925*a^2*(cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4 + 1002*a^2*(cos(d*x + c
) - 1)^5/(cos(d*x + c) + 1)^5 + 147*a^2*(cos(d*x + c) - 1)^6/(cos(d*x + c)
 + 1)^6)/((cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)^6)/d
```

3.21.9 Mupad [B] (verification not implemented)

Time = 17.79 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.60

$$\int (a + a\sec(c+dx))^2 \tan^5(c+dx)\,dx = \frac{2\,a^2\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2\right)}{d}$$

$$- \frac{2\,a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{10} - 12\,a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^8 + \frac{92\,a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6}{3} - 44\,a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4 + \frac{74\,a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right.}{5}}{d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{12} - 6\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{10} + 15\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^8 - 20\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6 + 15\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4 - 6\,\mathrm{ta}\right.}$$

input
```
int(tan(c + d*x)^5*(a + a/cos(c + d*x))^2,x)
```

output
```
(2*a^2*atanh(tan(c/2 + (d*x)/2)^2))/d - ((74*a^2*tan(c/2 + (d*x)/2)^2)/5 -
 44*a^2*tan(c/2 + (d*x)/2)^4 + (92*a^2*tan(c/2 + (d*x)/2)^6)/3 - 12*a^2*ta
n(c/2 + (d*x)/2)^8 + 2*a^2*tan(c/2 + (d*x)/2)^10 - (32*a^2)/15)/(d*(15*tan
(c/2 + (d*x)/2)^4 - 6*tan(c/2 + (d*x)/2)^2 - 20*tan(c/2 + (d*x)/2)^6 + 15*
tan(c/2 + (d*x)/2)^8 - 6*tan(c/2 + (d*x)/2)^10 + tan(c/2 + (d*x)/2)^12 + 1
))
```

3.22 $\int (a + a\sec(c+dx))^2 \tan^3(c+dx)\,dx$

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3.22.1 Optimal result

Integrand size = 21, antiderivative size = 65

$$\int (a + a\sec(c+dx))^2 \tan^3(c+dx)\,dx = \frac{a^2\log(\cos(c+dx))}{d} - \frac{2a^2\sec(c+dx)}{d} + \frac{2a^2\sec^3(c+dx)}{3d} + \frac{a^2\sec^4(c+dx)}{4d}$$

output
```
a^2*ln(cos(d*x+c))/d-2*a^2*sec(d*x+c)/d+2/3*a^2*sec(d*x+c)^3/d+1/4*a^2*sec
(d*x+c)^4/d
```

3.22.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.28

$$\int (a + a\sec(c+dx))^2 \tan^3(c+dx)\,dx = \frac{a^2(-20\cos(c+dx) + 3(2 - 4\cos(3(c+dx)) + 3\log(\cos(c+dx)) + 4\cos(2(c+dx))\log(\cos(c+dx)) + c}{24d}$$

input
```
Integrate[(a + a*Sec[c + d*x])^2*Tan[c + d*x]^3,x]
```

output
```
(a^2*(-20*Cos[c + d*x] + 3*(2 - 4*Cos[3*(c + d*x)] + 3*Log[Cos[c + d*x]] +
 4*Cos[2*(c + d*x)]*Log[Cos[c + d*x]] + Cos[4*(c + d*x)]*Log[Cos[c + d*x]]
))*Sec[c + d*x]^4)/(24*d)
```

3.22.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.77, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 25, 4367, 27, 84, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^3(c+dx)(a\sec(c+dx)+a)^2\,dx$$

$$\downarrow 3042$$

$$\int -\cot\left(c+dx+\frac{\pi}{2}\right)^3\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^2 dx$$

$$\downarrow 25$$

$$-\int \cot\left(\frac{1}{2}(2c+\pi)+dx\right)^3\left(\csc\left(\frac{1}{2}(2c+\pi)+dx\right)a+a\right)^2 dx$$

$$\downarrow 4367$$

$$-\frac{\int a^4(1-\cos(c+dx))(\cos(c+dx)+1)^3\sec^5(c+dx)d\cos(c+dx)}{a^2 d}$$

$$\downarrow 27$$

$$-\frac{a^2\int(1-\cos(c+dx))(\cos(c+dx)+1)^3\sec^5(c+dx)d\cos(c+dx)}{d}$$

$$\downarrow 84$$

$$-\frac{a^2\int\left(\sec^5(c+dx)+2\sec^4(c+dx)-2\sec^2(c+dx)-\sec(c+dx)\right)d\cos(c+dx)}{d}$$

$$\downarrow 2009$$

$$-\frac{a^2\left(-\frac{1}{4}\sec^4(c+dx)-\frac{2}{3}\sec^3(c+dx)+2\sec(c+dx)-\log(\cos(c+dx))\right)}{d}$$

input
```
Int[(a + a*Sec[c + d*x])^2*Tan[c + d*x]^3,x]
```

output
```
-((a^2*(-Log[Cos[c + d*x]] + 2*Sec[c + d*x] - (2*Sec[c + d*x]^3)/3 - Sec[c
 + d*x]^4/4))/d)
```

3.22.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 84
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && EqQ[b*e + a*f, 0] &&  !(ILtQ[n + p + 2, 0
] && GtQ[n + 2*p, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4367
```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Simp[1/(a^(m - n - 1)*b^n*d)   Subst[Int[(a - b*x)^((m -
1)/2)*((a + b*x)^((m - 1)/2 + n)/x^(m + n)), x], x, Sin[c + d*x]], x] /; Fr
eeQ[{a, b, c, d}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && Integer
Q[n]
```

3.22.4 Maple [A] (verified)

Time = 1.08 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.71

method	result	si
derivativedivides	$\frac{a^2\left(\frac{\sec(dx+c)^4}{4}+\frac{2\sec(dx+c)^3}{3}-2\sec(dx+c)-\ln(\sec(dx+c))\right)}{d}$	46
default	$\frac{a^2\left(\frac{\sec(dx+c)^4}{4}+\frac{2\sec(dx+c)^3}{3}-2\sec(dx+c)-\ln(\sec(dx+c))\right)}{d}$	46
parts	$\frac{a^2\left(\frac{\tan(dx+c)^2}{2}-\frac{\ln\left(1+\tan(dx+c)^2\right)}{2}\right)}{d}+\frac{a^2\tan(dx+c)^4}{4d}+\frac{2a^2\left(\frac{\sec(dx+c)^3}{3}-\sec(dx+c)\right)}{d}$	76
risch	$-ia^2x-\frac{2ia^2c}{d}-\frac{4a^2\left(3\,e^{7i(dx+c)}+5\,e^{5i(dx+c)}-3\,e^{4i(dx+c)}+5\,e^{3i(dx+c)}+3\,e^{i(dx+c)}\right)}{3d\left(e^{2i(dx+c)}+1\right)^4}+\frac{a^2\ln\left(e^{2i(dx+c)}+1\right)}{d}$	11

input
```
int((a+a*sec(d*x+c))^2*tan(d*x+c)^3,x,method=_RETURNVERBOSE)
```

output
```
a^2/d*(1/4*sec(d*x+c)^4+2/3*sec(d*x+c)^3-2*sec(d*x+c)-ln(sec(d*x+c)))
```

3.22.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00

$$\int (a + a\sec(c+dx))^2 \tan^3(c+dx)\,dx$$
$$= \frac{12\,a^2\cos(dx+c)^4\log(-\cos(dx+c)) - 24\,a^2\cos(dx+c)^3 + 8\,a^2\cos(dx+c) + 3\,a^2}{12\,d\cos(dx+c)^4}$$

input
```
integrate((a+a*sec(d*x+c))^2*tan(d*x+c)^3,x, algorithm="fricas")
```

output
```
1/12*(12*a^2*cos(d*x + c)^4*log(-cos(d*x + c)) - 24*a^2*cos(d*x + c)^3 + 8
*a^2*cos(d*x + c) + 3*a^2)/(d*cos(d*x + c)^4)
```

3.22.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 126 vs. 2(58) = 116.

Time = 0.30 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.94

$$\int (a + a\sec(c+dx))^2 \tan^3(c+dx)\,dx$$
$$= \begin{cases} -\frac{a^2\log(\tan^2(c+dx)+1)}{2d} + \frac{a^2\tan^2(c+dx)\sec^2(c+dx)}{4d} + \frac{2a^2\tan^2(c+dx)\sec(c+dx)}{3d} + \frac{a^2\tan^2(c+dx)}{2d} - \frac{a^2\sec^2(c+dx)}{4d} - \frac{4a^2\sec($ \\ x(a\sec(c)+a)^2\tan^3(c) \end{cases}$$

input
```
integrate((a+a*sec(d*x+c))**2*tan(d*x+c)**3,x)
```

output
```
Piecewise((-a**2*log(tan(c + d*x)**2 + 1)/(2*d) + a**2*tan(c + d*x)**2*sec
(c + d*x)**2/(4*d) + 2*a**2*tan(c + d*x)**2*sec(c + d*x)/(3*d) + a**2*tan(
c + d*x)**2/(2*d) - a**2*sec(c + d*x)**2/(4*d) - 4*a**2*sec(c + d*x)/(3*d)
, Ne(d, 0)), (x*(a*sec(c) + a)**2*tan(c)**3, True))
```

3.22.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.89

$$\int (a+a\sec(c+dx))^2\tan^3(c+dx)\,dx = \frac{12\,a^2\log\left(\cos\left(dx+c\right)\right) - \frac{24\,a^2\cos(dx+c)^3 - 8\,a^2\cos(dx+c) - 3\,a^2}{\cos(dx+c)^4}}{12\,d}$$

input
```
integrate((a+a*sec(d*x+c))^2*tan(d*x+c)^3,x, algorithm="maxima")
```

output
```
1/12*(12*a^2*log(cos(d*x + c)) - (24*a^2*cos(d*x + c)^3 - 8*a^2*cos(d*x +
c) - 3*a^2)/cos(d*x + c)^4)/d
```

3.22.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 192 vs. 2(61) = 122.

Time = 0.77 (sec) , antiderivative size = 192, normalized size of antiderivative = 2.95

$$\int (a + a\sec(c+dx))^2\tan^3(c+dx)\,dx =$$

$$-\frac{12\,a^2\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right) - 12\,a^2\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right) + \frac{57\,a^2+\frac{252\,a^2(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{246\,a^2(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}+}{\left(\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+}}{12\,d}$$

input
```
integrate((a+a*sec(d*x+c))^2*tan(d*x+c)^3,x, algorithm="giac")
```

output
```
-1/12*(12*a^2*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)) - 12*a^
2*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)) + (57*a^2 + 252*a^2
*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 246*a^2*(cos(d*x + c) - 1)^2/(cos
(d*x + c) + 1)^2 + 124*a^2*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 + 25*
a^2*(cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4)/((cos(d*x + c) - 1)/(cos(d*
x + c) + 1) + 1)^4)/d
```

3.22.9 Mupad [B] (verification not implemented)

Time = 16.68 (sec) , antiderivative size = 133, normalized size of antiderivative = 2.05

$$\int (a + a\sec(c+dx))^2 \tan^3(c+dx)\,dx$$

$$= \frac{2a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6 - 8a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4 + \frac{38a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2}{3} - \frac{8a^2}{3}}{d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^8 - 4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6 + 6\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4 - 4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2 + 1\right)} - \frac{2a^2\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2\right)}{d}$$

input
```
int(tan(c + d*x)^3*(a + a/cos(c + d*x))^2,x)
```

output
```
((38*a^2*tan(c/2 + (d*x)/2)^2)/3 - 8*a^2*tan(c/2 + (d*x)/2)^4 + 2*a^2*tan(
c/2 + (d*x)/2)^6 - (8*a^2)/3)/(d*(6*tan(c/2 + (d*x)/2)^4 - 4*tan(c/2 + (d*
x)/2)^2 - 4*tan(c/2 + (d*x)/2)^6 + tan(c/2 + (d*x)/2)^8 + 1)) - (2*a^2*ata
nh(tan(c/2 + (d*x)/2)^2))/d
```

3.23 $\int (a + a\sec(c+dx))^2 \tan(c+dx)\,dx$

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3.23.1 Optimal result

Integrand size = 19, antiderivative size = 48

$$\int (a + a\sec(c+dx))^2 \tan(c+dx)\,dx = -\frac{a^2 \log(\cos(c+dx))}{d} + \frac{2a^2 \sec(c+dx)}{d} + \frac{a^2 \sec^2(c+dx)}{2d}$$

output
```
-a^2*ln(cos(d*x+c))/d+2*a^2*sec(d*x+c)/d+1/2*a^2*sec(d*x+c)^2/d
```

3.23.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.06

$$\int (a + a\sec(c+dx))^2 \tan(c+dx)\,dx = -\frac{a^2(-1 - 4\cos(c+dx) + \log(\cos(c+dx)) + \cos(2(c+dx))\log(\cos(c+dx)))\sec^2(c+dx)}{2d}$$

input
```
Integrate[(a + a*Sec[c + d*x])^2*Tan[c + d*x],x]
```

output
```
-1/2*(a^2*(-1 - 4*Cos[c + d*x] + Log[Cos[c + d*x]] + Cos[2*(c + d*x)]*Log[
Cos[c + d*x]])*Sec[c + d*x]^2)/d
```

3.23.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.75, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {3042, 25, 4367, 27, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan(c+dx)(a\sec(c+dx)+a)^2\,dx$$

$$\downarrow 3042$$

$$\int -\cot\left(c+dx+\frac{\pi}{2}\right)\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^2 dx$$

$$\downarrow 25$$

$$-\int \cot\left(\frac{1}{2}(2c+\pi)+dx\right)\left(\csc\left(\frac{1}{2}(2c+\pi)+dx\right)a+a\right)^2 dx$$

$$\downarrow 4367$$

$$-\frac{\int a^2(\cos(c+dx)+1)^2\sec^3(c+dx)d\cos(c+dx)}{d}$$

$$\downarrow 27$$

$$-\frac{a^2\int(\cos(c+dx)+1)^2\sec^3(c+dx)d\cos(c+dx)}{d}$$

$$\downarrow 49$$

$$-\frac{a^2\int\left(\sec^3(c+dx)+2\sec^2(c+dx)+\sec(c+dx)\right)d\cos(c+dx)}{d}$$

$$\downarrow 2009$$

$$-\frac{a^2\left(-\frac{1}{2}\sec^2(c+dx)-2\sec(c+dx)+\log(\cos(c+dx))\right)}{d}$$

input
```
Int[(a + a*Sec[c + d*x])^2*Tan[c + d*x],x]
```

output
```
-((a^2*(Log[Cos[c + d*x]] - 2*Sec[c + d*x] - Sec[c + d*x]^2/2))/d)
```

3.23.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4367
```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Simp[1/(a^(m - n - 1)*b^n*d)   Subst[Int[(a - b*x)^((m -
1)/2)*((a + b*x)^((m - 1)/2 + n)/x^(m + n)), x], x, Sin[c + d*x]], x] /; Fr
eeQ[{a, b, c, d}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && Integer
Q[n]
```

3.23.4 Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.71

method	result	size
derivativedivides	$\frac{a^2\left(\frac{\sec(dx+c)^2}{2}+2\sec(dx+c)+\ln(\sec(dx+c))\right)}{d}$	34
default	$\frac{a^2\left(\frac{\sec(dx+c)^2}{2}+2\sec(dx+c)+\ln(\sec(dx+c))\right)}{d}$	34
parts	$\frac{a^2\ln\left(1+\tan(dx+c)^2\right)}{2d}+\frac{a^2\sec(dx+c)^2}{2d}+\frac{2a^2\sec(dx+c)}{d}$	51
risch	$ia^2x+\frac{2ia^2c}{d}+\frac{2a^2\left(2\,e^{3i(dx+c)}+e^{2i(dx+c)}+2\,e^{i(dx+c)}\right)}{d\left(e^{2i(dx+c)}+1\right)^2}-\frac{a^2\ln\left(e^{2i(dx+c)}+1\right)}{d}$	92

input
```
int((a+a*sec(d*x+c))^2*tan(d*x+c),x,method=_RETURNVERBOSE)
```

output
```
1/d*a^2*(1/2*sec(d*x+c)^2+2*sec(d*x+c)+ln(sec(d*x+c)))
```

3.23.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.08

$$\int (a + a\sec(c+dx))^2 \tan(c+dx)\,dx = -\frac{2\,a^2\cos(dx+c)^2\log(-\cos(dx+c)) - 4\,a^2\cos(dx+c) - a^2}{2\,d\cos(dx+c)^2}$$

input
```
integrate((a+a*sec(d*x+c))^2*tan(d*x+c),x, algorithm="fricas")
```

output
```
-1/2*(2*a^2*cos(d*x + c)^2*log(-cos(d*x + c)) - 4*a^2*cos(d*x + c) - a^2)/
(d*cos(d*x + c)^2)
```

3.23.6 Sympy [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.25

$$\int (a + a\sec(c+dx))^2 \tan(c+dx)\,dx = \begin{cases} \frac{a^2\log(\tan^2(c+dx)+1)}{2d} + \frac{a^2\sec^2(c+dx)}{2d} + \frac{2a^2\sec(c+dx)}{d} & \text{for } d \neq 0 \\ x(a\sec(c)+a)^2\tan(c) & \text{otherwise} \end{cases}$$

input
```
integrate((a+a*sec(d*x+c))**2*tan(d*x+c),x)
```

output
```
Piecewise((a**2*log(tan(c + d*x)**2 + 1)/(2*d) + a**2*sec(c + d*x)**2/(2*d
) + 2*a**2*sec(c + d*x)/d, Ne(d, 0)), (x*(a*sec(c) + a)**2*tan(c), True))
```

3.23.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.90

$$\int (a + a\sec(c+dx))^2 \tan(c+dx)\,dx = -\frac{2\,a^2 \log\left(\cos\left(dx+c\right)\right) - \frac{4\,a^2\cos(dx+c)+a^2}{\cos(dx+c)^2}}{2\,d}$$

input
```
integrate((a+a*sec(d*x+c))^2*tan(d*x+c),x, algorithm="maxima")
```

output
```
-1/2*(2*a^2*log(cos(d*x + c)) - (4*a^2*cos(d*x + c) + a^2)/cos(d*x + c)^2)
/d
```

3.23.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 142 vs. $2(46) = 92$.

Time = 0.40 (sec) , antiderivative size = 142, normalized size of antiderivative = 2.96

$$\int (a + a\sec(c+dx))^2 \tan(c+dx)\,dx$$
$$= \frac{2\,a^2 \log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right) - 2\,a^2 \log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right) + \frac{11\,a^2 + \frac{10\,a^2(\cos(dx+c)-1)}{\cos(dx+c)+1} + \frac{3\,a^2(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}}{\left(\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right)^2}}{2\,d}$$

input
```
integrate((a+a*sec(d*x+c))^2*tan(d*x+c),x, algorithm="giac")
```

output
```
1/2*(2*a^2*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)) - 2*a^2*lo
g(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)) + (11*a^2 + 10*a^2*(cos
(d*x + c) - 1)/(cos(d*x + c) + 1) + 3*a^2*(cos(d*x + c) - 1)^2/(cos(d*x +
c) + 1)^2)/((cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)^2)/d
```

3.23.9 Mupad [B] (verification not implemented)

Time = 14.19 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.58

$$\int (a + a\sec(c+dx))^2 \tan(c+dx)\,dx = \frac{2a^2 \operatorname{atanh}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2\right)}{d} - \frac{2a^2 \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2 - 4a^2}{d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4 - 2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2 + 1\right)}$$

input
```
int(tan(c + d*x)*(a + a/cos(c + d*x))^2,x)
```

output
```
(2*a^2*atanh(tan(c/2 + (d*x)/2)^2))/d - (2*a^2*tan(c/2 + (d*x)/2)^2 - 4*a^
2)/(d*(tan(c/2 + (d*x)/2)^4 - 2*tan(c/2 + (d*x)/2)^2 + 1))
```

3.24 $\int \cot(c+dx)(a+a\sec(c+dx))^2\,dx$

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3.24.1 Optimal result

Integrand size = 19, antiderivative size = 35

$$\int \cot(c+dx)(a+a\sec(c+dx))^2\,dx = \frac{2a^2\log(1-\cos(c+dx))}{d} - \frac{a^2\log(\cos(c+dx))}{d}$$

output
```
2*a^2*ln(1-cos(d*x+c))/d-a^2*ln(cos(d*x+c))/d
```

3.24.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.83

$$\int \cot(c+dx)(a+a\sec(c+dx))^2\,dx = -\frac{a^2\left(\log(\cos(c+dx)) - 4\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)}{d}$$

input
```
Integrate[Cot[c + d*x]*(a + a*Sec[c + d*x])^2,x]
```

output
```
-((a^2*(Log[Cos[c + d*x]] - 4*Log[Sin[(c + d*x)/2]]))/d)
```

3.24.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.83, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {3042, 25, 4367, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot(c+dx)(a\sec(c+dx)+a)^2\,dx$$

$$\downarrow 3042$$

$$\int -\frac{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^2}{\cot\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\downarrow 25$$

$$-\int \frac{\left(\csc\left(\frac{1}{2}(2c+\pi)+dx\right)a+a\right)^2}{\cot\left(\frac{1}{2}(2c+\pi)+dx\right)}dx$$

$$\downarrow 4367$$

$$-\frac{a^2\int \frac{(\cos(c+dx)+1)\sec(c+dx)}{1-\cos(c+dx)}d\cos(c+dx)}{d}$$

$$\downarrow 86$$

$$-\frac{a^2\int \left(\sec(c+dx)-\frac{2}{\cos(c+dx)-1}\right)d\cos(c+dx)}{d}$$

$$\downarrow 2009$$

$$-\frac{a^2(\log(\cos(c+dx))-2\log(1-\cos(c+dx)))}{d}$$

input
```
Int[Cot[c + d*x]*(a + a*Sec[c + d*x])^2,x]
```

output
```
-((a^2*(-2*Log[1 - Cos[c + d*x]] + Log[Cos[c + d*x]]))/d)
```

3.24.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4367
```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Simp[1/(a^(m - n - 1)*b^n*d)   Subst[Int[(a - b*x)^((m -
1)/2)*((a + b*x)^((m - 1)/2 + n)/x^(m + n)), x], x, Sin[c + d*x]], x] /; Fr
eeQ[{a, b, c, d}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && Integer
Q[n]
```

3.24.4 Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.40

method	result	size
derivativedivides	$\frac{a^2\ln(\tan(dx+c))+2a^2\ln(-\cot(dx+c)+\csc(dx+c))+a^2\ln(\sin(dx+c))}{d}$	49
default	$\frac{a^2\ln(\tan(dx+c))+2a^2\ln(-\cot(dx+c)+\csc(dx+c))+a^2\ln(\sin(dx+c))}{d}$	49
risch	$-ia^2x-\frac{2ia^2c}{d}+\frac{4a^2\ln\left(e^{i(dx+c)}-1\right)}{d}-\frac{a^2\ln\left(e^{2i(dx+c)}+1\right)}{d}$	59

input
```
int(cot(d*x+c)*(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^2*ln(tan(d*x+c))+2*a^2*ln(-cot(d*x+c)+csc(d*x+c))+a^2*ln(sin(d*x+c)
))
```

3.24.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00

$$\int \cot(c+dx)(a+a\sec(c+dx))^2\,dx = -\frac{a^2\log\left(-\cos\left(dx+c\right)\right)-2\,a^2\log\left(-\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)}{d}$$

input
```
integrate(cot(d*x+c)*(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

output
```
-(a^2*log(-cos(d*x + c)) - 2*a^2*log(-1/2*cos(d*x + c) + 1/2))/d
```

3.24.6 Sympy [F]

$$\int \cot(c+dx)(a+a\sec(c+dx))^2\,dx = a^2\left(\int 2\cot\left(c+dx\right)\sec\left(c+dx\right)dx + \int \cot\left(c+dx\right)\sec^2\left(c+dx\right)dx + \int \cot\left(c+dx\right)dx\right)$$

input
```
integrate(cot(d*x+c)*(a+a*sec(d*x+c))**2,x)
```

output
```
a**2*(Integral(2*cot(c + d*x)*sec(c + d*x), x) + Integral(cot(c + d*x)*sec
(c + d*x)**2, x) + Integral(cot(c + d*x), x))
```

3.24.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.89

$$\int \cot(c+dx)(a+a\sec(c+dx))^2\,dx = \frac{2\,a^2\log\left(\cos\left(dx+c\right)-1\right)-a^2\log\left(\cos\left(dx+c\right)\right)}{d}$$

input
```
integrate(cot(d*x+c)*(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

output
```
(2*a^2*log(cos(d*x + c) - 1) - a^2*log(cos(d*x + c)))/d
```

3.24.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.83

$$\int \cot(c+dx)(a+a\sec(c+dx))^2\,dx = \frac{2\,a^2\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right) - a^2\log\left(\left|\frac{(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}-1\right|\right)}{d}$$

input
```
integrate(cot(d*x+c)*(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

output
```
(2*a^2*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1)) - a^2*log(abs((co
s(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 1)))/d
```

3.24.9 Mupad [B] (verification not implemented)

Time = 14.06 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.03

$$\int \cot(c+dx)(a+a\sec(c+dx))^2\,dx = \frac{a^2\left(4\,\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right) - \ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4-1\right)\right)}{d}$$

input
```
int(cot(c + d*x)*(a + a/cos(c + d*x))^2,x)
```

output
```
(a^2*(4*log(tan(c/2 + (d*x)/2)) - log(tan(c/2 + (d*x)/2)^4 - 1)))/d
```

3.25 $\int \cot^3(c+dx)(a+a\sec(c+dx))^2\,dx$

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3.25.1 Optimal result

Integrand size = 21, antiderivative size = 40

$$\int \cot^3(c+dx)(a+a\sec(c+dx))^2\,dx = -\frac{a^2}{d(1-\cos(c+dx))} - \frac{a^2\log(1-\cos(c+dx))}{d}$$

output
```
-a^2/d/(1-cos(d*x+c))-a^2*ln(1-cos(d*x+c))/d
```

3.25.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.40

$$\int \cot^3(c+dx)(a+a\sec(c+dx))^2\,dx = \frac{a^2\csc^2\left(\frac{1}{2}(c+dx)\right)\left(-1-2\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)+2\cos(c+dx)\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)}{2d}$$

input
```
Integrate[Cot[c + d*x]^3*(a + a*Sec[c + d*x])^2,x]
```

output
```
(a^2*Csc[(c + d*x)/2]^2*(-1 - 2*Log[Sin[(c + d*x)/2]] + 2*Cos[c + d*x]*Log
[Sin[(c + d*x)/2]]))/(2*d)
```

3.25.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.80, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 25, 4367, 27, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^3(c+dx)(a\sec(c+dx)+a)^2\,dx$$

$$\downarrow 3042$$

$$\int -\frac{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^2}{\cot\left(c+dx+\frac{\pi}{2}\right)^3}dx$$

$$\downarrow 25$$

$$-\int \frac{\left(\csc\left(\frac{1}{2}(2c+\pi)+dx\right)a+a\right)^2}{\cot\left(\frac{1}{2}(2c+\pi)+dx\right)^3}dx$$

$$\downarrow 4367$$

$$-\frac{a^4\int \frac{\cos(c+dx)}{a^2(1-\cos(c+dx))^2}d\cos(c+dx)}{d}$$

$$\downarrow 27$$

$$-\frac{a^2\int \frac{\cos(c+dx)}{(1-\cos(c+dx))^2}d\cos(c+dx)}{d}$$

$$\downarrow 49$$

$$-\frac{a^2\int \left(\frac{1}{\cos(c+dx)-1}+\frac{1}{(\cos(c+dx)-1)^2}\right)d\cos(c+dx)}{d}$$

$$\downarrow 2009$$

$$-\frac{a^2\left(\frac{1}{1-\cos(c+dx)}+\log(1-\cos(c+dx))\right)}{d}$$

input
```
Int[Cot[c + d*x]^3*(a + a*Sec[c + d*x])^2,x]
```

output
```
-((a^2*((1 - Cos[c + d*x])^(-1) + Log[1 - Cos[c + d*x]]))/d)
```

3.25.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4367
```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Simp[1/(a^(m - n - 1)*b^n*d)   Subst[Int[(a - b*x)^((m -
1)/2)*((a + b*x)^((m - 1)/2 + n)/x^(m + n)), x], x, Sin[c + d*x]], x] /; Fr
eeQ[{a, b, c, d}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && Integer
Q[n]
```

3.25.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.75 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.72

method	result	siz
risch	$ia^2x + \frac{2ia^2c}{d} + \frac{2a^2e^{i(dx+c)}}{d\left(e^{i(dx+c)}-1\right)^2} - \frac{2a^2\ln\left(e^{i(dx+c)}-1\right)}{d}$	69
derivativedivides	$\frac{-\frac{a^2}{2\sin(dx+c)^2}+2a^2\left(-\frac{\cos(dx+c)^3}{2\sin(dx+c)^2}-\frac{\cos(dx+c)}{2}-\frac{\ln(-\cot(dx+c)+\csc(dx+c))}{2}\right)+a^2\left(-\frac{\cot(dx+c)^2}{2}-\ln(\sin(dx+c))\right)}{d}$	93
default	$\frac{-\frac{a^2}{2\sin(dx+c)^2}+2a^2\left(-\frac{\cos(dx+c)^3}{2\sin(dx+c)^2}-\frac{\cos(dx+c)}{2}-\frac{\ln(-\cot(dx+c)+\csc(dx+c))}{2}\right)+a^2\left(-\frac{\cot(dx+c)^2}{2}-\ln(\sin(dx+c))\right)}{d}$	93

input
```
int(cot(d*x+c)^3*(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
I*a^2*x+2*I/d*a^2*c+2*a^2*exp(I*(d*x+c))/d/(exp(I*(d*x+c))-1)^2-2/d*a^2*ln
(exp(I*(d*x+c))-1)
```

3.25.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.20

$$\int \cot^3(c+dx)(a+a\sec(c+dx))^2\,dx = \frac{a^2-(a^2\cos(dx+c)-a^2)\log\left(-\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)}{d\cos(dx+c)-d}$$

input
```
integrate(cot(d*x+c)^3*(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

output
```
(a^2 - (a^2*cos(d*x + c) - a^2)*log(-1/2*cos(d*x + c) + 1/2))/(d*cos(d*x +
 c) - d)
```

3.25.6 Sympy [F]

$$\int \cot^3(c+dx)(a+a\sec(c+dx))^2\,dx = a^2\left(\int 2\cot^3(c+dx)\sec(c+dx)\,dx + \int \cot^3(c+dx)\sec^2(c+dx)\,dx + \int \cot^3(c+dx)\,dx\right)$$

input
```
integrate(cot(d*x+c)**3*(a+a*sec(d*x+c))**2,x)
```

output
```
a**2*(Integral(2*cot(c + d*x)**3*sec(c + d*x), x) + Integral(cot(c + d*x)*
*3*sec(c + d*x)**2, x) + Integral(cot(c + d*x)**3, x))
```

3.25.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.85

$$\int \cot^3(c+dx)(a+a\sec(c+dx))^2\,dx = -\frac{a^2\log\left(\cos\left(dx+c\right)-1\right)-\frac{a^2}{\cos(dx+c)-1}}{d}$$

input
```
integrate(cot(d*x+c)^3*(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

output
```
-(a^2*log(cos(d*x + c) - 1) - a^2/(cos(d*x + c) - 1))/d
```

3.25.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 111 vs. 2(37) = 74.

Time = 0.35 (sec) , antiderivative size = 111, normalized size of antiderivative = 2.78

$$\int \cot^3(c+dx)(a+a\sec(c+dx))^2\,dx =$$
$$-\frac{2\,a^2\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)-2\,a^2\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right)-\frac{\left(a^2+\frac{2\,a^2(\cos(dx+c)-1)}{\cos(dx+c)+1}\right)(\cos(dx+c)+1)}{\cos(dx+c)-1}}{2\,d}$$

input
```
integrate(cot(d*x+c)^3*(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

output
```
-1/2*(2*a^2*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1)) - 2*a^2*log(
abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)) - (a^2 + 2*a^2*(cos(d*x +
 c) - 1)/(cos(d*x + c) + 1))*(cos(d*x + c) + 1)/(cos(d*x + c) - 1))/d
```

3.25.9 Mupad [B] (verification not implemented)

Time = 13.86 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.25

$$\int \cot^3(c+dx)(a+a\sec(c+dx))^2\,dx$$
$$=-\frac{a^2\left(\frac{\cot\left(\frac{c}{2}+\frac{dx}{2}\right)^2}{2}+2\,\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)-\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+1\right)\right)}{d}$$

input
```
int(cot(c + d*x)^3*(a + a/cos(c + d*x))^2,x)
```

output
```
-(a^2*(2*log(tan(c/2 + (d*x)/2)) - log(tan(c/2 + (d*x)/2)^2 + 1) + cot(c/2
 + (d*x)/2)^2/2))/d
```

3.26 $\int \cot^5(c+dx)(a+a\sec(c+dx))^2\,dx$

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3.26.1 Optimal result

Integrand size = 21, antiderivative size = 85

$$\int \cot^5(c+dx)(a+a\sec(c+dx))^2\,dx = -\frac{a^2}{4d(1-\cos(c+dx))^2}+\frac{5a^2}{4d(1-\cos(c+dx))} +\frac{7a^2\log(1-\cos(c+dx))}{8d}+\frac{a^2\log(1+\cos(c+dx))}{8d}$$

output
```
-1/4*a^2/d/(1-cos(d*x+c))^2+5/4*a^2/d/(1-cos(d*x+c))+7/8*a^2*ln(1-cos(d*x+
c))/d+1/8*a^2*ln(1+cos(d*x+c))/d
```

3.26.2 Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.01

$$\int \cot^5(c+dx)(a+a\sec(c+dx))^2\,dx = -\frac{a^2(1+\cos(c+dx))^2\left(-10\csc^2\left(\frac{1}{2}(c+dx)\right)+\csc^4\left(\frac{1}{2}(c+dx)\right)-4\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)+7\log\left(\sin\left(\frac{1}{2}\right.\right.\right.\right.}{64d}$$

input
```
Integrate[Cot[c + d*x]^5*(a + a*Sec[c + d*x])^2,x]
```

output
```
-1/64*(a^2*(1 + Cos[c + d*x])^2*(-10*Csc[(c + d*x)/2]^2 + Csc[(c + d*x)/2]
^4 - 4*(Log[Cos[(c + d*x)/2]] + 7*Log[Sin[(c + d*x)/2]]))*Sec[(c + d*x)/2]
^4)/d
```

3.26.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.81, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 25, 4367, 27, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^5(c+dx)(a\sec(c+dx)+a)^2\,dx$$

$$\downarrow 3042$$

$$\int -\frac{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^2}{\cot\left(c+dx+\frac{\pi}{2}\right)^5}dx$$

$$\downarrow 25$$

$$-\int \frac{\left(\csc\left(\frac{1}{2}(2c+\pi)+dx\right)a+a\right)^2}{\cot\left(\frac{1}{2}(2c+\pi)+dx\right)^5}dx$$

$$\downarrow 4367$$

$$-\frac{a^6\int \frac{\cos^3(c+dx)}{a^4(1-\cos(c+dx))^3(\cos(c+dx)+1)}d\cos(c+dx)}{d}$$

$$\downarrow 27$$

$$-\frac{a^2\int \frac{\cos^3(c+dx)}{(1-\cos(c+dx))^3(\cos(c+dx)+1)}d\cos(c+dx)}{d}$$

$$\downarrow 99$$

$$-\frac{a^2\int \left(-\frac{1}{8(\cos(c+dx)+1)}-\frac{7}{8(\cos(c+dx)-1)}-\frac{5}{4(\cos(c+dx)-1)^2}-\frac{1}{2(\cos(c+dx)-1)^3}\right)d\cos(c+dx)}{d}$$

$$\downarrow 2009$$

$$-\frac{a^2\left(-\frac{5}{4(1-\cos(c+dx))}+\frac{1}{4(1-\cos(c+dx))^2}-\frac{7}{8}\log(1-\cos(c+dx))-\frac{1}{8}\log(\cos(c+dx)+1)\right)}{d}$$

input
```
Int[Cot[c + d*x]^5*(a + a*Sec[c + d*x])^2,x]
```

output
```
-((a^2*(1/(4*(1 - Cos[c + d*x])^2) - 5/(4*(1 - Cos[c + d*x])) - (7*Log[1 -
 Cos[c + d*x]])/8 - Log[1 + Cos[c + d*x]]/8))/d)
```

3.26.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4367
```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Simp[1/(a^(m - n - 1)*b^n*d)   Subst[Int[(a - b*x)^((m -
1)/2)*((a + b*x)^((m - 1)/2 + n)/x^(m + n)), x], x, Sin[c + d*x]], x] /; Fr
eeQ[{a, b, c, d}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && Integer
Q[n]
```

3.26.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.02 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.34

method	result
risch	$-ia^2x - \frac{2ia^2c}{d} - \frac{a^2\left(5\,e^{3i(dx+c)}-8\,e^{2i(dx+c)}+5\,e^{i(dx+c)}\right)}{2d\left(e^{i(dx+c)}-1\right)^4} + \frac{a^2\ln\left(e^{i(dx+c)}+1\right)}{4d} + \frac{7a^2\ln\left(e^{i(dx+c)}-1\right)}{4d}$
derivativedivides	$\frac{-\frac{a^2\cos(dx+c)^4}{4\sin(dx+c)^4}+2a^2\left(-\frac{\cos(dx+c)^5}{4\sin(dx+c)^4}+\frac{\cos(dx+c)^5}{8\sin(dx+c)^2}+\frac{\cos(dx+c)^3}{8}+\frac{3\cos(dx+c)}{8}+\frac{3\ln(-\cot(dx+c)+\csc(dx+c))}{8}\right)+a^2\left(-\frac{\cot(d}{d}\right.$
default	$\frac{-\frac{a^2\cos(dx+c)^4}{4\sin(dx+c)^4}+2a^2\left(-\frac{\cos(dx+c)^5}{4\sin(dx+c)^4}+\frac{\cos(dx+c)^5}{8\sin(dx+c)^2}+\frac{\cos(dx+c)^3}{8}+\frac{3\cos(dx+c)}{8}+\frac{3\ln(-\cot(dx+c)+\csc(dx+c))}{8}\right)+a^2\left(-\frac{\cot(d}{d}\right.$

input
```
int(cot(d*x+c)^5*(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
-I*a^2*x-2*I/d*a^2*c-1/2*a^2/d/(exp(I*(d*x+c))-1)^4*(5*exp(3*I*(d*x+c))-8*
exp(2*I*(d*x+c))+5*exp(I*(d*x+c)))+1/4/d*a^2*ln(exp(I*(d*x+c))+1)+7/4/d*a^
2*ln(exp(I*(d*x+c))-1)
```

3.26.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.44

$$\int \cot^5(c+dx)(a+a\sec(c+dx))^2\,dx =$$
$$-\frac{10\,a^2\cos(dx+c) - 8\,a^2 - \left(a^2\cos(dx+c)^2 - 2\,a^2\cos(dx+c) + a^2\right)\log\left(\frac{1}{2}\cos(dx+c) + \frac{1}{2}\right) - 7\left(a^2 c}{8\left(d\cos(dx+c)^2 - 2\,d\cos(dx+c) + d\right)}$$

input
```
integrate(cot(d*x+c)^5*(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

output
```
-1/8*(10*a^2*cos(d*x + c) - 8*a^2 - (a^2*cos(d*x + c)^2 - 2*a^2*cos(d*x +
c) + a^2)*log(1/2*cos(d*x + c) + 1/2) - 7*(a^2*cos(d*x + c)^2 - 2*a^2*cos(
d*x + c) + a^2)*log(-1/2*cos(d*x + c) + 1/2))/(d*cos(d*x + c)^2 - 2*d*cos(
d*x + c) + d)
```

3.26.6 Sympy [F]

$$\int \cot^5(c+dx)(a+a\sec(c+dx))^2\,dx = a^2\left(\int 2\cot^5(c+dx)\sec(c+dx)\,dx\right.$$
$$+\int \cot^5(c+dx)\sec^2(c+dx)\,dx$$
$$\left.+\int \cot^5(c+dx)\,dx\right)$$

input
```
integrate(cot(d*x+c)**5*(a+a*sec(d*x+c))**2,x)
```

output
```
a**2*(Integral(2*cot(c + d*x)**5*sec(c + d*x), x) + Integral(cot(c + d*x)*
*5*sec(c + d*x)**2, x) + Integral(cot(c + d*x)**5, x))
```

3.26.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.85

$$\int \cot^5(c+dx)(a+a\sec(c+dx))^2\,dx$$
$$= \frac{a^2\log(\cos(dx+c)+1)+7\,a^2\log(\cos(dx+c)-1)-\frac{2\,(5\,a^2\cos(dx+c)-4\,a^2)}{\cos(dx+c)^2-2\,\cos(dx+c)+1}}{8\,d}$$

input
```
integrate(cot(d*x+c)^5*(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

output
```
1/8*(a^2*log(cos(d*x + c) + 1) + 7*a^2*log(cos(d*x + c) - 1) - 2*(5*a^2*co
s(d*x + c) - 4*a^2)/(cos(d*x + c)^2 - 2*cos(d*x + c) + 1))/d
```

3.26.8 Giac [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.62

$$\int \cot^5(c+dx)(a+a\sec(c+dx))^2\,dx$$
$$= \frac{14\,a^2\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)-16\,a^2\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right)-\frac{\left(a^2+\frac{8\,a^2(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{21\,a^2(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}\right)(\cos(dx+c)+1)}{(\cos(dx+c)-1)^2}}{16\,d}$$

input
```
integrate(cot(d*x+c)^5*(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

output
```
1/16*(14*a^2*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1)) - 16*a^2*lo
g(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)) - (a^2 + 8*a^2*(cos(d*x
 + c) - 1)/(cos(d*x + c) + 1) + 21*a^2*(cos(d*x + c) - 1)^2/(cos(d*x + c)
+ 1)^2)*(cos(d*x + c) + 1)^2/(cos(d*x + c) - 1)^2)/d
```

3.26.9 Mupad [B] (verification not implemented)

Time = 14.63 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.73

$$\int \cot^5(c+dx)(a+a\sec(c+dx))^2\,dx = \frac{a^2\left(-\frac{\cot\left(\frac{c}{2}+\frac{dx}{2}\right)^4}{16}+\frac{\cot\left(\frac{c}{2}+\frac{dx}{2}\right)^2}{2}+\frac{7\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{4}-\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+1\right)\right)}{d}$$

input
```
int(cot(c + d*x)^5*(a + a/cos(c + d*x))^2,x)
```

output
```
(a^2*((7*log(tan(c/2 + (d*x)/2)))/4 - log(tan(c/2 + (d*x)/2)^2 + 1) + cot(
c/2 + (d*x)/2)^2/2 - cot(c/2 + (d*x)/2)^4/16))/d
```

3.27 $\int \cot^7(c+dx)(a+a\sec(c+dx))^2\,dx$

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3.27.1 Optimal result

Integrand size = 21, antiderivative size = 127

$$\int \cot^7(c+dx)(a+a\sec(c+dx))^2\,dx = -\frac{a^2}{12d(1-\cos(c+dx))^3} + \frac{a^2}{2d(1-\cos(c+dx))^2} - \frac{23a^2}{16d(1-\cos(c+dx))} - \frac{a^2}{16d(1+\cos(c+dx))} - \frac{13a^2\log(1-\cos(c+dx))}{16d} - \frac{3a^2\log(1+\cos(c+dx))}{16d}$$

output
```
-1/12*a^2/d/(1-cos(d*x+c))^3+1/2*a^2/d/(1-cos(d*x+c))^2-23/16*a^2/d/(1-cos
(d*x+c))-1/16*a^2/d/(1+cos(d*x+c))-13/16*a^2*ln(1-cos(d*x+c))/d-3/16*a^2*l
n(1+cos(d*x+c))/d
```

3.27.2 Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.90

$$\int \cot^7(c+dx)(a+a\sec(c+dx))^2\,dx = -\frac{a^2(1+\cos(c+dx))^2\sec^4\left(\frac{1}{2}(c+dx)\right)\left(69\csc^2\left(\frac{1}{2}(c+dx)\right) - 12\csc^4\left(\frac{1}{2}(c+dx)\right) + \csc^6\left(\frac{1}{2}(c+dx)\right) + 3\right.}{384d}$$

input
```
Integrate[Cot[c + d*x]^7*(a + a*Sec[c + d*x])^2,x]
```

output
```
-1/384*(a^2*(1 + Cos[c + d*x])^2*Sec[(c + d*x)/2]^4*(69*Csc[(c + d*x)/2]^2
 - 12*Csc[(c + d*x)/2]^4 + Csc[(c + d*x)/2]^6 + 3*(12*Log[Cos[(c + d*x)/2]
] + 52*Log[Sin[(c + d*x)/2]] + Sec[(c + d*x)/2]^2)))/d
```

3.27.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.78, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 25, 4367, 27, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^7(c+dx)(a\sec(c+dx)+a)^2\,dx$$

$$\downarrow 3042$$

$$\int -\frac{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^2}{\cot\left(c+dx+\frac{\pi}{2}\right)^7}dx$$

$$\downarrow 25$$

$$-\int \frac{\left(\csc\left(\frac{1}{2}(2c+\pi)+dx\right)a+a\right)^2}{\cot\left(\frac{1}{2}(2c+\pi)+dx\right)^7}dx$$

$$\downarrow 4367$$

$$-\frac{a^8\int \frac{\cos^5(c+dx)}{a^6(1-\cos(c+dx))^4(\cos(c+dx)+1)^2}d\cos(c+dx)}{d}$$

$$\downarrow 27$$

$$-\frac{a^2\int \frac{\cos^5(c+dx)}{(1-\cos(c+dx))^4(\cos(c+dx)+1)^2}d\cos(c+dx)}{d}$$

$$\downarrow 99$$

$$-\frac{a^2\int \left(\frac{3}{16(\cos(c+dx)+1)}-\frac{1}{16(\cos(c+dx)+1)^2}+\frac{13}{16(\cos(c+dx)-1)}+\frac{23}{16(\cos(c+dx)-1)^2}+\frac{1}{(\cos(c+dx)-1)^3}+\frac{1}{4(\cos(c+dx)-1)^4}\right)d\cos$$

$$\downarrow 2009$$

$$-\frac{a^2\left(\frac{23}{16(1-\cos(c+dx))}+\frac{1}{16(\cos(c+dx)+1)}-\frac{1}{2(1-\cos(c+dx))^2}+\frac{1}{12(1-\cos(c+dx))^3}+\frac{13}{16}\log(1-\cos(c+dx))+\frac{3}{16}\log(\cos(c+\right.}{d}$$

input
```
Int[Cot[c + d*x]^7*(a + a*Sec[c + d*x])^2,x]
```

output
```
-((a^2*(1/(12*(1 - Cos[c + d*x])^3) - 1/(2*(1 - Cos[c + d*x])^2) + 23/(16*
(1 - Cos[c + d*x])) + 1/(16*(1 + Cos[c + d*x])) + (13*Log[1 - Cos[c + d*x]
])/16 + (3*Log[1 + Cos[c + d*x]])/16))/d)
```

3.27.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4367
```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Simp[1/(a^(m - n - 1)*b^n*d)   Subst[Int[(a - b*x)^((m -
1)/2)*((a + b*x)^((m - 1)/2 + n)/x^(m + n)), x], x, Sin[c + d*x]], x] /; Fr
eeQ[{a, b, c, d}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && Integer
Q[n]
```

3.27.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.60 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.35

method	result
risch	$ia^2x + \frac{2ia^2c}{d} + \frac{a^2\left(33\,e^{7i(dx+c)} - 36\,e^{6i(dx+c)} - 49\,e^{5i(dx+c)} + 136\,e^{4i(dx+c)} - 49\,e^{3i(dx+c)} - 36\,e^{2i(dx+c)} + 33\,e^{i(dx+c)}\right.}{12d\left(e^{i(dx+c)}-1\right)^6\left(e^{i(dx+c)}+1\right)^2}$
derivativedivides	$\frac{-\frac{a^2\cos(dx+c)^6}{6\sin(dx+c)^6} + 2a^2\left(-\frac{\cos(dx+c)^7}{6\sin(dx+c)^6} + \frac{\cos(dx+c)^7}{24\sin(dx+c)^4} - \frac{\cos(dx+c)^7}{16\sin(dx+c)^2} - \frac{\cos(dx+c)^5}{16} - \frac{5\cos(dx+c)^3}{48} - \frac{5\cos(dx+c)}{16} - \frac{5\ln(-\cot(dx}{d}$
default	$\frac{-\frac{a^2\cos(dx+c)^6}{6\sin(dx+c)^6} + 2a^2\left(-\frac{\cos(dx+c)^7}{6\sin(dx+c)^6} + \frac{\cos(dx+c)^7}{24\sin(dx+c)^4} - \frac{\cos(dx+c)^7}{16\sin(dx+c)^2} - \frac{\cos(dx+c)^5}{16} - \frac{5\cos(dx+c)^3}{48} - \frac{5\cos(dx+c)}{16} - \frac{5\ln(-\cot(dx}{d}$

input
```
int(cot(d*x+c)^7*(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
I*a^2*x+2*I/d*a^2*c+1/12*a^2/d/(exp(I*(d*x+c))-1)^6/(exp(I*(d*x+c))+1)^2*(
33*exp(7*I*(d*x+c))-36*exp(6*I*(d*x+c))-49*exp(5*I*(d*x+c))+136*exp(4*I*(d
*x+c))-49*exp(3*I*(d*x+c))-36*exp(2*I*(d*x+c))+33*exp(I*(d*x+c)))-13/8/d*a
^2*ln(exp(I*(d*x+c))-1)-3/8/d*a^2*ln(exp(I*(d*x+c))+1)
```

3.27.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.50

$$\int \cot^7(c+dx)(a+a\sec(c+dx))^2\,dx$$
$$= \frac{66\,a^2\cos(dx+c)^3 - 36\,a^2\cos(dx+c)^2 - 74\,a^2\cos(dx+c) + 52\,a^2 - 9\left(a^2\cos(dx+c)^4 - 2\,a^2\cos(dx+\right.}{48\left(d\cos(\right.}$$

input
```
integrate(cot(d*x+c)^7*(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

output
```
1/48*(66*a^2*cos(d*x + c)^3 - 36*a^2*cos(d*x + c)^2 - 74*a^2*cos(d*x + c)
+ 52*a^2 - 9*(a^2*cos(d*x + c)^4 - 2*a^2*cos(d*x + c)^3 + 2*a^2*cos(d*x +
c) - a^2)*log(1/2*cos(d*x + c) + 1/2) - 39*(a^2*cos(d*x + c)^4 - 2*a^2*cos
(d*x + c)^3 + 2*a^2*cos(d*x + c) - a^2)*log(-1/2*cos(d*x + c) + 1/2))/(d*c
os(d*x + c)^4 - 2*d*cos(d*x + c)^3 + 2*d*cos(d*x + c) - d)
```

3.27.6 Sympy [F]

$$\int \cot^7(c+dx)(a+a\sec(c+dx))^2\,dx = a^2\left(\int 2\cot^7(c+dx)\sec(c+dx)\,dx + \int \cot^7(c+dx)\sec^2(c+dx)\,dx + \int \cot^7(c+dx)\,dx\right)$$

input
```
integrate(cot(d*x+c)**7*(a+a*sec(d*x+c))**2,x)
```

output
```
a**2*(Integral(2*cot(c + d*x)**7*sec(c + d*x), x) + Integral(cot(c + d*x)*
*7*sec(c + d*x)**2, x) + Integral(cot(c + d*x)**7, x))
```

3.27.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.86

$$\int \cot^7(c+dx)(a+a\sec(c+dx))^2\,dx = -\frac{9a^2\log(\cos(dx+c)+1)+39a^2\log(\cos(dx+c)-1)-\frac{2\left(33a^2\cos(dx+c)^3-18a^2\cos(dx+c)^2-37a^2\cos(dx+c)+26\right.}{\cos(dx+c)^4-2\cos(dx+c)^3+2\cos(dx+c)-1}}{48d}$$

input
```
integrate(cot(d*x+c)^7*(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

output
```
-1/48*(9*a^2*log(cos(d*x + c) + 1) + 39*a^2*log(cos(d*x + c) - 1) - 2*(33*
a^2*cos(d*x + c)^3 - 18*a^2*cos(d*x + c)^2 - 37*a^2*cos(d*x + c) + 26*a^2)
/(cos(d*x + c)^4 - 2*cos(d*x + c)^3 + 2*cos(d*x + c) - 1))/d
```

3.27.8 Giac [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.46

$$\int \cot^7(c+dx)(a+a\sec(c+dx))^2\,dx =$$

$$-\frac{78\,a^2\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)-96\,a^2\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right)-\frac{3\,a^2(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{\left(a^2+\frac{9\,a^2(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{48\,a^2(\cos($$
$$96\,d$$

input
```
integrate(cot(d*x+c)^7*(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

output
```
-1/96*(78*a^2*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1)) - 96*a^2*l
og(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)) - 3*a^2*(cos(d*x + c)
 - 1)/(cos(d*x + c) + 1) - (a^2 + 9*a^2*(cos(d*x + c) - 1)/(cos(d*x + c) +
 1) + 48*a^2*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 143*a^2*(cos(d*x +
 c) - 1)^3/(cos(d*x + c) + 1)^3)*(cos(d*x + c) + 1)^3/(cos(d*x + c) - 1)^3
)/d
```

3.27.9 Mupad [B] (verification not implemented)

Time = 13.99 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.89

$$\int \cot^7(c+dx)(a+a\sec(c+dx))^2\,dx$$
$$=\frac{a^2\,\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+1\right)}{d}-\frac{13\,a^2\,\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{8\,d}$$
$$-\frac{\cot\left(\frac{c}{2}+\frac{dx}{2}\right)^6\left(8\,a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4-\frac{3\,a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2}{2}+\frac{a^2}{6}\right)}{16\,d}-\frac{a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2}{32\,d}$$

input
```
int(cot(c + d*x)^7*(a + a/cos(c + d*x))^2,x)
```

output
```
(a^2*log(tan(c/2 + (d*x)/2)^2 + 1))/d - (13*a^2*log(tan(c/2 + (d*x)/2)))/(
8*d) - (cot(c/2 + (d*x)/2)^6*(8*a^2*tan(c/2 + (d*x)/2)^4 - (3*a^2*tan(c/2
 + (d*x)/2)^2)/2 + a^2/6))/(16*d) - (a^2*tan(c/2 + (d*x)/2)^2)/(32*d)
```

3.28 $\int \cot^9(c+dx)(a+a\sec(c+dx))^2\,dx$

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3.28.1 Optimal result

Integrand size = 21, antiderivative size = 169

$$\begin{aligned}\int \cot^9(c+dx)(a+a\sec(c+dx))^2\,dx = &-\frac{a^2}{32d(1-\cos(c+dx))^4}+\frac{11a^2}{48d(1-\cos(c+dx))^3}\\ &-\frac{3a^2}{4d(1-\cos(c+dx))^2}+\frac{51a^2}{32d(1-\cos(c+dx))}\\ &-\frac{a^2}{64d(1+\cos(c+dx))^2}+\frac{9a^2}{64d(1+\cos(c+dx))}\\ &+\frac{99a^2\log(1-\cos(c+dx))}{128d}\\ &+\frac{29a^2\log(1+\cos(c+dx))}{128d}\end{aligned}$$

output

```
-1/32*a^2/d/(1-cos(d*x+c))^4+11/48*a^2/d/(1-cos(d*x+c))^3-3/4*a^2/d/(1-cos
(d*x+c))^2+51/32*a^2/d/(1-cos(d*x+c))-1/64*a^2/d/(1+cos(d*x+c))^2+9/64*a^2
/d/(1+cos(d*x+c))+99/128*a^2*ln(1-cos(d*x+c))/d+29/128*a^2*ln(1+cos(d*x+c)
)/d
```

3.28.2 Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.86

$$\int \cot^9(c+dx)(a+a\sec(c+dx))^2\,dx =$$
$$-\frac{a^2(1+\cos(c+dx))^2\sec^4\left(\frac{1}{2}(c+dx)\right)\left(-1224\csc^2\left(\frac{1}{2}(c+dx)\right)+288\csc^4\left(\frac{1}{2}(c+dx)\right)-44\csc^6\left(\frac{1}{2}(c+\right.\right.}{}$$

input
```
Integrate[Cot[c + d*x]^9*(a + a*Sec[c + d*x])^2,x]
```

output
```
-1/6144*(a^2*(1 + Cos[c + d*x])^2*Sec[(c + d*x)/2]^4*(-1224*Csc[(c + d*x)/
2]^2 + 288*Csc[(c + d*x)/2]^4 - 44*Csc[(c + d*x)/2]^6 + 3*Csc[(c + d*x)/2]
^8 - 6*(116*Log[Cos[(c + d*x)/2]] + 396*Log[Sin[(c + d*x)/2]] + 18*Sec[(c
+ d*x)/2]^2 - Sec[(c + d*x)/2]^4)))/d
```

3.28.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.76, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 25, 4367, 27, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^9(c+dx)(a\sec(c+dx)+a)^2\,dx$$
$$\downarrow 3042$$
$$\int -\frac{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^2}{\cot\left(c+dx+\frac{\pi}{2}\right)^9}dx$$
$$\downarrow 25$$
$$-\int \frac{\left(\csc\left(\frac{1}{2}(2c+\pi)+dx\right)a+a\right)^2}{\cot\left(\frac{1}{2}(2c+\pi)+dx\right)^9}dx$$
$$\downarrow 4367$$
$$-\frac{a^{10}\int \frac{\cos^7(c+dx)}{a^8(1-\cos(c+dx))^5(\cos(c+dx)+1)^3}d\cos(c+dx)}{d}$$
$$\downarrow 27$$

$$-\frac{a^2\int \frac{\cos^7(c+dx)}{(1-\cos(c+dx))^5(\cos(c+dx)+1)^3}d\cos(c+dx)}{d}$$

$$\downarrow 99$$

$$-\frac{a^2\int \left(-\frac{29}{128(\cos(c+dx)+1)}+\frac{9}{64(\cos(c+dx)+1)^2}-\frac{1}{32(\cos(c+dx)+1)^3}-\frac{99}{128(\cos(c+dx)-1)}-\frac{51}{32(\cos(c+dx)-1)^2}-\frac{3}{2(\cos(c+dx)-1)^3}\right.}{d}$$

$$\downarrow 2009$$

$$-\frac{a^2\left(-\frac{51}{32(1-\cos(c+dx))}-\frac{9}{64(\cos(c+dx)+1)}+\frac{3}{4(1-\cos(c+dx))^2}+\frac{1}{64(\cos(c+dx)+1)^2}-\frac{11}{48(1-\cos(c+dx))^3}+\frac{1}{32(1-\cos(c+dx))^4}-\frac{9}{1}\right.}{d}$$

input
```
Int[Cot[c + d*x]^9*(a + a*Sec[c + d*x])^2,x]
```

output
```
-((a^2*(1/(32*(1 - Cos[c + d*x])^4) - 11/(48*(1 - Cos[c + d*x])^3) + 3/(4*
(1 - Cos[c + d*x])^2) - 51/(32*(1 - Cos[c + d*x])) + 1/(64*(1 + Cos[c + d*
x])^2) - 9/(64*(1 + Cos[c + d*x])) - (99*Log[1 - Cos[c + d*x]])/128 - (29*
Log[1 + Cos[c + d*x]])/128))/d)
```

3.28.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4367
```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Simp[1/(a^(m - n - 1)*b^n*d)   Subst[Int[(a - b*x)^((m -
1)/2)*((a + b*x)^((m - 1)/2 + n)/x^(m + n)), x], x, Sin[c + d*x]], x] /; Fr
eeQ[{a, b, c, d}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && Integer
Q[n]
```

3.28.4 Maple [A] (verified)

Time = 2.89 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.26

method	result
derivativedivides	$-\frac{a^2\cos(dx+c)^8}{8\sin(dx+c)^8}+2a^2\left(-\frac{\cos(dx+c)^9}{8\sin(dx+c)^8}+\frac{\cos(dx+c)^9}{48\sin(dx+c)^6}-\frac{\cos(dx+c)^9}{64\sin(dx+c)^4}+\frac{5\cos(dx+c)^9}{128\sin(dx+c)^2}+\frac{5\cos(dx+c)^7}{128}+\frac{7\cos(dx+c)^5}{128}+\frac{35\cos(d}{38}$
default	$-\frac{a^2\cos(dx+c)^8}{8\sin(dx+c)^8}+2a^2\left(-\frac{\cos(dx+c)^9}{8\sin(dx+c)^8}+\frac{\cos(dx+c)^9}{48\sin(dx+c)^6}-\frac{\cos(dx+c)^9}{64\sin(dx+c)^4}+\frac{5\cos(dx+c)^9}{128\sin(dx+c)^2}+\frac{5\cos(dx+c)^7}{128}+\frac{7\cos(dx+c)^5}{128}+\frac{35\cos(d}{38}$
risch	$-ia^2x-\frac{2ia^2c}{d}-\frac{a^2\left(279\,e^{11i(dx+c)}-156\,e^{10i(dx+c)}-1141\,e^{9i(dx+c)}+2080\,e^{8i(dx+c)}+670\,e^{7i(dx+c)}-2696\,e^{6i(dx+c}\right.}{96d\left(e^{i(dx+c)}-1\right)^8\left(e^{i(dx+}\right.}$

input
```
int(cot(d*x+c)^9*(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-1/8*a^2/sin(d*x+c)^8*cos(d*x+c)^8+2*a^2*(-1/8/sin(d*x+c)^8*cos(d*x+c
)^9+1/48/sin(d*x+c)^6*cos(d*x+c)^9-1/64/sin(d*x+c)^4*cos(d*x+c)^9+5/128/si
n(d*x+c)^2*cos(d*x+c)^9+5/128*cos(d*x+c)^7+7/128*cos(d*x+c)^5+35/384*cos(d
*x+c)^3+35/128*cos(d*x+c)+35/128*ln(-cot(d*x+c)+csc(d*x+c)))+a^2*(-1/8*cot
(d*x+c)^8+1/6*cot(d*x+c)^6-1/4*cot(d*x+c)^4+1/2*cot(d*x+c)^2+ln(sin(d*x+c)
)))
```

3.28.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 322 vs. 2(145) = 290.

Time = 0.29 (sec) , antiderivative size = 322, normalized size of antiderivative = 1.91

$$\int \cot^9(c+dx)(a+a\sec(c+dx))^2\,dx =$$

$$-\frac{558\,a^2\cos(dx+c)^5-156\,a^2\cos(dx+c)^4-1268\,a^2\cos(dx+c)^3+676\,a^2\cos(dx+c)^2+686\,a^2\cos(d}{}$$

input
```
integrate(cot(d*x+c)^9*(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

output
```
-1/384*(558*a^2*cos(d*x + c)^5 - 156*a^2*cos(d*x + c)^4 - 1268*a^2*cos(d*x
 + c)^3 + 676*a^2*cos(d*x + c)^2 + 686*a^2*cos(d*x + c) - 448*a^2 - 87*(a^
2*cos(d*x + c)^6 - 2*a^2*cos(d*x + c)^5 - a^2*cos(d*x + c)^4 + 4*a^2*cos(d
*x + c)^3 - a^2*cos(d*x + c)^2 - 2*a^2*cos(d*x + c) + a^2)*log(1/2*cos(d*x
 + c) + 1/2) - 297*(a^2*cos(d*x + c)^6 - 2*a^2*cos(d*x + c)^5 - a^2*cos(d*
x + c)^4 + 4*a^2*cos(d*x + c)^3 - a^2*cos(d*x + c)^2 - 2*a^2*cos(d*x + c)
+ a^2)*log(-1/2*cos(d*x + c) + 1/2))/(d*cos(d*x + c)^6 - 2*d*cos(d*x + c)^
5 - d*cos(d*x + c)^4 + 4*d*cos(d*x + c)^3 - d*cos(d*x + c)^2 - 2*d*cos(d*x
 + c) + d)
```

3.28.6 Sympy [F(-1)]

Timed out.

$$\int \cot^9(c+dx)(a+a\sec(c+dx))^2\,dx = \text{Timed out}$$

input
```
integrate(cot(d*x+c)**9*(a+a*sec(d*x+c))**2,x)
```

output
```
Timed out
```

3.28.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.98

$$\int \cot^9(c+dx)(a+a\sec(c+dx))^2\,dx$$

$$= \frac{87\,a^2\log\left(\cos\left(dx+c\right)+1\right)+297\,a^2\log\left(\cos\left(dx+c\right)-1\right)-\frac{2\left(279\,a^2\cos(dx+c)^5-78\,a^2\cos(dx+c)^4-634\,a^2\cos(dx+c)^3+\right.}{\cos(dx+c)^6-2\,\cos(dx+c)^5-\cos(dx+c)^4+4\,\cos(dx}}{384\,d}$$

input
```
integrate(cot(d*x+c)^9*(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

output
```
1/384*(87*a^2*log(cos(d*x + c) + 1) + 297*a^2*log(cos(d*x + c) - 1) - 2*(2
79*a^2*cos(d*x + c)^5 - 78*a^2*cos(d*x + c)^4 - 634*a^2*cos(d*x + c)^3 + 3
38*a^2*cos(d*x + c)^2 + 343*a^2*cos(d*x + c) - 224*a^2)/(cos(d*x + c)^6 -
2*cos(d*x + c)^5 - cos(d*x + c)^4 + 4*cos(d*x + c)^3 - cos(d*x + c)^2 - 2*
cos(d*x + c) + 1))/d
```

3.28.8 Giac [A] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 238, normalized size of antiderivative = 1.41

$$\int \cot^9(c+dx)(a+a\sec(c+dx))^2\,dx$$

$$= \frac{1188\,a^2\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right) - 1536\,a^2\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right) - \frac{96\,a^2(\cos(dx+c)-1)}{\cos(dx+c)+1} - \frac{6\,a^2(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2} - \frac{\left(3\,a^2}{}}{1536\,d}$$

input
```
integrate(cot(d*x+c)^9*(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

output
```
1/1536*(1188*a^2*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1)) - 1536*
a^2*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)) - 96*a^2*(cos(d*x
 + c) - 1)/(cos(d*x + c) + 1) - 6*a^2*(cos(d*x + c) - 1)^2/(cos(d*x + c) +
 1)^2 - (3*a^2 + 32*a^2*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 174*a^2*(c
os(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 768*a^2*(cos(d*x + c) - 1)^3/(co
s(d*x + c) + 1)^3 + 2475*a^2*(cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4)*(c
os(d*x + c) + 1)^4/(cos(d*x + c) - 1)^4)/d
```

3.28.9 Mupad [B] (verification not implemented)

Time = 14.46 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.88

$$\int \cot^9(c+dx)(a+a\sec(c+dx))^2\,dx$$

$$= \frac{a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2}{16\,d} - \frac{a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4}{256\,d} + \frac{99\,a^2\,\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{64\,d} + \frac{\cot\left(\frac{c}{2}+\frac{dx}{2}\right)^8\left(32\,a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6 - \frac{29\,a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4}{4} + \frac{4\,a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2}{3} - \frac{a^2}{8}\right)}{64\,d} - \frac{a^2\,\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+1\right)}{d}$$

input
```
int(cot(c + d*x)^9*(a + a/cos(c + d*x))^2,x)
```

output
```
(a^2*tan(c/2 + (d*x)/2)^2)/(16*d) - (a^2*tan(c/2 + (d*x)/2)^4)/(256*d) + (
99*a^2*log(tan(c/2 + (d*x)/2)))/(64*d) + (cot(c/2 + (d*x)/2)^8*((4*a^2*tan
(c/2 + (d*x)/2)^2)/3 - (29*a^2*tan(c/2 + (d*x)/2)^4)/4 + 32*a^2*tan(c/2 +
(d*x)/2)^6 - a^2/8))/(64*d) - (a^2*log(tan(c/2 + (d*x)/2)^2 + 1))/d
```

3.29 $\int (a + a\sec(c+dx))^2 \tan^6(c+dx)\,dx$

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3.29.1 Optimal result

Integrand size = 21, antiderivative size = 161

$$\begin{aligned}\int (a + a\sec(c+dx))^2 \tan^6(c+dx)\,dx = {} & -a^2x - \frac{5a^2\operatorname{arctanh}(\sin(c+dx))}{8d} + \frac{a^2\tan(c+dx)}{d} \\ & + \frac{5a^2\sec(c+dx)\tan(c+dx)}{8d} - \frac{a^2\tan^3(c+dx)}{3d} \\ & - \frac{5a^2\sec(c+dx)\tan^3(c+dx)}{12d} + \frac{a^2\tan^5(c+dx)}{5d} \\ & + \frac{a^2\sec(c+dx)\tan^5(c+dx)}{3d} + \frac{a^2\tan^7(c+dx)}{7d}\end{aligned}$$

output

```
-a^2*x-5/8*a^2*arctanh(sin(d*x+c))/d+a^2*tan(d*x+c)/d+5/8*a^2*sec(d*x+c)*t
an(d*x+c)/d-1/3*a^2*tan(d*x+c)^3/d-5/12*a^2*sec(d*x+c)*tan(d*x+c)^3/d+1/5*
a^2*tan(d*x+c)^5/d+1/3*a^2*sec(d*x+c)*tan(d*x+c)^5/d+1/7*a^2*tan(d*x+c)^7/
d
```

3.29.2 Mathematica [A] (verified)

Time = 0.70 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.35

$$\begin{aligned}\int (a+a\sec(c+dx))^2\tan^6(c+dx)\,dx = &-\frac{a^2\arctan(\tan(c+dx))}{d}\\ &-\frac{5a^2\text{arctanh}(\sin(c+dx))}{8d}+\frac{a^2\tan(c+dx)}{d}\\ &-\frac{5a^2\sec(c+dx)\tan(c+dx)}{8d}\\ &-\frac{5a^2\sec^3(c+dx)\tan(c+dx)}{12d}\\ &+\frac{5a^2\sec^5(c+dx)\tan(c+dx)}{3d}-\frac{a^2\tan^3(c+dx)}{3d}\\ &-\frac{10a^2\sec^3(c+dx)\tan^3(c+dx)}{3d}+\frac{a^2\tan^5(c+dx)}{5d}\\ &+\frac{2a^2\sec(c+dx)\tan^5(c+dx)}{d}+\frac{a^2\tan^7(c+dx)}{7d}\end{aligned}$$

input
```
Integrate[(a + a*Sec[c + d*x])^2*Tan[c + d*x]^6,x]
```

output
```
-((a^2*ArcTan[Tan[c + d*x]])/d) - (5*a^2*ArcTanh[Sin[c + d*x]])/(8*d) + (a
^2*Tan[c + d*x])/d - (5*a^2*Sec[c + d*x]*Tan[c + d*x])/(8*d) - (5*a^2*Sec[
c + d*x]^3*Tan[c + d*x])/(12*d) + (5*a^2*Sec[c + d*x]^5*Tan[c + d*x])/(3*d
) - (a^2*Tan[c + d*x]^3)/(3*d) - (10*a^2*Sec[c + d*x]^3*Tan[c + d*x]^3)/(3
*d) + (a^2*Tan[c + d*x]^5)/(5*d) + (2*a^2*Sec[c + d*x]*Tan[c + d*x]^5)/d +
 (a^2*Tan[c + d*x]^7)/(7*d)
```

3.29.3 Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3042, 4374, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^6(c+dx)(a\sec(c+dx)+a)^2\,dx$$

$$\downarrow 3042$$

$$\int \cot\left(c+dx+\frac{\pi}{2}\right)^6 \left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^2 dx$$

$$\downarrow 4374$$

$$\int \left(a^2\tan^6(c+dx)+a^2\tan^6(c+dx)\sec^2(c+dx)+2a^2\tan^6(c+dx)\sec(c+dx)\right) dx$$

$$\downarrow 2009$$

$$-\frac{5a^2\text{arctanh}(\sin(c+dx))}{8d}+\frac{a^2\tan^7(c+dx)}{7d}+\frac{a^2\tan^5(c+dx)}{5d}-\frac{a^2\tan^3(c+dx)}{3d}+\frac{a^2\tan(c+dx)}{d}+\frac{a^2\tan^5(c+dx)\sec(c+dx)}{3d}-\frac{5a^2\tan^3(c+dx)\sec(c+dx)}{12d}+\frac{5a^2\tan(c+dx)\sec(c+dx)}{8d}-a^2x$$

input
```
Int[(a + a*Sec[c + d*x])^2*Tan[c + d*x]^6,x]
```

output
```
-(a^2*x) - (5*a^2*ArcTanh[Sin[c + d*x]])/(8*d) + (a^2*Tan[c + d*x])/d + (5
*a^2*Sec[c + d*x]*Tan[c + d*x])/(8*d) - (a^2*Tan[c + d*x]^3)/(3*d) - (5*a^
2*Sec[c + d*x]*Tan[c + d*x]^3)/(12*d) + (a^2*Tan[c + d*x]^5)/(5*d) + (a^2*
Sec[c + d*x]*Tan[c + d*x]^5)/(3*d) + (a^2*Tan[c + d*x]^7)/(7*d)
```

3.29.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4374
```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

3.29.4 Maple [A] (verified)

Time = 2.18 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.04

method	result
parts	$\frac{a^2\left(\frac{\tan(dx+c)^5}{5}-\frac{\tan(dx+c)^3}{3}+\tan(dx+c)-\arctan(\tan(dx+c))\right)}{d}+\frac{a^2\tan(dx+c)^7}{7d}+\frac{2a^2\left(\frac{\sin(dx+c)^7}{6\cos(dx+c)^6}-\frac{\sin(dx+c)^7}{24\cos(dx+c)^4}$
derivativedivides	$\frac{\frac{a^2\sin(dx+c)^7}{7\cos(dx+c)^7}+2a^2\left(\frac{\sin(dx+c)^7}{6\cos(dx+c)^6}-\frac{\sin(dx+c)^7}{24\cos(dx+c)^4}+\frac{\sin(dx+c)^7}{16\cos(dx+c)^2}+\frac{\sin(dx+c)^5}{16}+\frac{5\sin(dx+c)^3}{48}+\frac{5\sin(dx+c)}{16}-\frac{5\ln(\sec(dx+c)+t}{16}$
default	$\frac{\frac{a^2\sin(dx+c)^7}{7\cos(dx+c)^7}+2a^2\left(\frac{\sin(dx+c)^7}{6\cos(dx+c)^6}-\frac{\sin(dx+c)^7}{24\cos(dx+c)^4}+\frac{\sin(dx+c)^7}{16\cos(dx+c)^2}+\frac{\sin(dx+c)^5}{16}+\frac{5\sin(dx+c)^3}{48}+\frac{5\sin(dx+c)}{16}-\frac{5\ln(\sec(dx+c)+t}{16}$
risch	$-a^2x-\frac{ia^2\left(1155\,e^{13i(dx+c)}-1680\,e^{12i(dx+c)}+980\,e^{11i(dx+c)}-10080\,e^{10i(dx+c)}+2975\,e^{9i(dx+c)}-16240\,e^{8i(dx+c)}-2\right.}{420d\left(e^{2i(dx+}\right.$

input
```
int((a+a*sec(d*x+c))^2*tan(d*x+c)^6,x,method=_RETURNVERBOSE)
```

output
```
a^2/d*(1/5*tan(d*x+c)^5-1/3*tan(d*x+c)^3+tan(d*x+c)-arctan(tan(d*x+c)))+1/
7*a^2*tan(d*x+c)^7/d+2*a^2/d*(1/6*sin(d*x+c)^7/cos(d*x+c)^6-1/24*sin(d*x+c
)^7/cos(d*x+c)^4+1/16*sin(d*x+c)^7/cos(d*x+c)^2+1/16*sin(d*x+c)^5+5/48*sin
(d*x+c)^3+5/16*sin(d*x+c)-5/16*ln(sec(d*x+c)+tan(d*x+c)))
```

3.29.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.02

$$\int (a+a\sec(c+dx))^2\tan^6(c+dx)\,dx =$$
$$-\frac{1680\,a^2dx\cos(dx+c)^7+525\,a^2\cos(dx+c)^7\log(\sin(dx+c)+1)-525\,a^2\cos(dx+c)^7\log(-\sin(dx$$

input
```
integrate((a+a*sec(d*x+c))^2*tan(d*x+c)^6,x, algorithm="fricas")
```

output
```
-1/1680*(1680*a^2*d*x*cos(d*x + c)^7 + 525*a^2*cos(d*x + c)^7*log(sin(d*x
+ c) + 1) - 525*a^2*cos(d*x + c)^7*log(-sin(d*x + c) + 1) - 2*(1168*a^2*co
s(d*x + c)^6 + 1155*a^2*cos(d*x + c)^5 - 256*a^2*cos(d*x + c)^4 - 910*a^2*
cos(d*x + c)^3 - 192*a^2*cos(d*x + c)^2 + 280*a^2*cos(d*x + c) + 120*a^2)*
sin(d*x + c))/(d*cos(d*x + c)^7)
```

3.29.6 Sympy [F]

$$\int (a + a\sec(c+dx))^2 \tan^6(c+dx)\,dx = a^2\left(\int 2\tan^6(c+dx)\sec(c+dx)\,dx + \int \tan^6(c+dx)\sec^2(c+dx)\,dx + \int \tan^6(c+dx)\,dx\right)$$

input
```
integrate((a+a*sec(d*x+c))**2*tan(d*x+c)**6,x)
```

output
```
a**2*(Integral(2*tan(c + d*x)**6*sec(c + d*x), x) + Integral(tan(c + d*x)*
*6*sec(c + d*x)**2, x) + Integral(tan(c + d*x)**6, x))
```

3.29.7 Maxima [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.94

$$\int (a + a\sec(c+dx))^2 \tan^6(c+dx)\,dx$$

$$= \frac{240\,a^2\tan(dx+c)^7 + 112\left(3\tan(dx+c)^5 - 5\tan(dx+c)^3 - 15\,dx - 15\,c + 15\tan(dx+c)\right)a^2 - 35\,a^2}{1680\,d}$$

input
```
integrate((a+a*sec(d*x+c))^2*tan(d*x+c)^6,x, algorithm="maxima")
```

output
```
1/1680*(240*a^2*tan(d*x + c)^7 + 112*(3*tan(d*x + c)^5 - 5*tan(d*x + c)^3
- 15*d*x - 15*c + 15*tan(d*x + c))*a^2 - 35*a^2*(2*(33*sin(d*x + c)^5 - 40
*sin(d*x + c)^3 + 15*sin(d*x + c))/(sin(d*x + c)^6 - 3*sin(d*x + c)^4 + 3*
sin(d*x + c)^2 - 1) + 15*log(sin(d*x + c) + 1) - 15*log(sin(d*x + c) - 1))
)/d
```

3.29.8 Giac [A] (verification not implemented)

Time = 2.40 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.12

$$\int (a + a\sec(c+dx))^2 \tan^6(c+dx)\,dx =$$

$$-\frac{840\,(dx+c)a^2 + 525\,a^2\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right) - 525\,a^2\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right) + \frac{2\left(315\,a^2\tan(\frac{1}{2}\right.}{}$$

input
```
integrate((a+a*sec(d*x+c))^2*tan(d*x+c)^6,x, algorithm="giac")
```

output
```
-1/840*(840*(d*x + c)*a^2 + 525*a^2*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 5
25*a^2*log(abs(tan(1/2*d*x + 1/2*c) - 1)) + 2*(315*a^2*tan(1/2*d*x + 1/2*c
)^13 - 2660*a^2*tan(1/2*d*x + 1/2*c)^11 + 9863*a^2*tan(1/2*d*x + 1/2*c)^9
- 21216*a^2*tan(1/2*d*x + 1/2*c)^7 + 29673*a^2*tan(1/2*d*x + 1/2*c)^5 - 96
60*a^2*tan(1/2*d*x + 1/2*c)^3 + 1365*a^2*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*
x + 1/2*c)^2 - 1)^7)/d
```

3.29.9 Mupad [B] (verification not implemented)

Time = 15.39 (sec) , antiderivative size = 234, normalized size of antiderivative = 1.45

$$\int (a + a\sec(c+dx))^2 \tan^6(c+dx)\,dx = -a^2\,x - \frac{5\,a^2\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{4\,d}$$

$$-\frac{\frac{3\,a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{13}}{4} - \frac{19\,a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{11}}{3} + \frac{1409\,a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{9}}{60} - \frac{1768\,a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{7}}{35} + \frac{1413\,a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{5}}{20} - 23\,a}{d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{14} - 7\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{12} + 21\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{10} - 35\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{8} + 35\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{6} - 21\right.}$$

input
```
int(tan(c + d*x)^6*(a + a/cos(c + d*x))^2,x)
```

output
```
- a^2*x - (5*a^2*atanh(tan(c/2 + (d*x)/2)))/(4*d) - ((1413*a^2*tan(c/2 + (
d*x)/2)^5)/20 - 23*a^2*tan(c/2 + (d*x)/2)^3 - (1768*a^2*tan(c/2 + (d*x)/2)
^7)/35 + (1409*a^2*tan(c/2 + (d*x)/2)^9)/60 - (19*a^2*tan(c/2 + (d*x)/2)^1
1)/3 + (3*a^2*tan(c/2 + (d*x)/2)^13)/4 + (13*a^2*tan(c/2 + (d*x)/2))/4)/(d
*(7*tan(c/2 + (d*x)/2)^2 - 21*tan(c/2 + (d*x)/2)^4 + 35*tan(c/2 + (d*x)/2)
^6 - 35*tan(c/2 + (d*x)/2)^8 + 21*tan(c/2 + (d*x)/2)^10 - 7*tan(c/2 + (d*x
)/2)^12 + tan(c/2 + (d*x)/2)^14 - 1))
```

3.30 $\int (a + a\sec(c+dx))^2 \tan^4(c+dx)\,dx$

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3.30.1 Optimal result

Integrand size = 21, antiderivative size = 119

$$\int (a + a\sec(c+dx))^2 \tan^4(c+dx)\,dx = a^2 x + \frac{3a^2 \operatorname{arctanh}(\sin(c+dx))}{4d} - \frac{a^2 \tan(c+dx)}{d} - \frac{3a^2 \sec(c+dx)\tan(c+dx)}{4d} + \frac{a^2 \tan^3(c+dx)}{3d} + \frac{a^2 \sec(c+dx)\tan^3(c+dx)}{2d} + \frac{a^2 \tan^5(c+dx)}{5d}$$

output
```
a^2*x+3/4*a^2*arctanh(sin(d*x+c))/d-a^2*tan(d*x+c)/d-3/4*a^2*sec(d*x+c)*ta
n(d*x+c)/d+1/3*a^2*tan(d*x+c)^3/d+1/2*a^2*sec(d*x+c)*tan(d*x+c)^3/d+1/5*a^
2*tan(d*x+c)^5/d
```

3.30.2 Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.26

$$\int (a + a\sec(c+dx))^2 \tan^4(c+dx)\,dx = \frac{a^2 \arctan(\tan(c+dx))}{d} + \frac{3a^2 \operatorname{arctanh}(\sin(c+dx))}{4d} - \frac{a^2 \tan(c+dx)}{d} + \frac{3a^2 \sec(c+dx)\tan(c+dx)}{4d} - \frac{3a^2 \sec^3(c+dx)\tan(c+dx)}{2d} + \frac{a^2 \tan^3(c+dx)}{3d} + \frac{2a^2 \sec(c+dx)\tan^3(c+dx)}{d} + \frac{a^2 \tan^5(c+dx)}{5d}$$

input
```
Integrate[(a + a*Sec[c + d*x])^2*Tan[c + d*x]^4,x]
```

output
```
(a^2*ArcTan[Tan[c + d*x]])/d + (3*a^2*ArcTanh[Sin[c + d*x]])/(4*d) - (a^2*
Tan[c + d*x])/d + (3*a^2*Sec[c + d*x]*Tan[c + d*x])/(4*d) - (3*a^2*Sec[c +
 d*x]^3*Tan[c + d*x])/(2*d) + (a^2*Tan[c + d*x]^3)/(3*d) + (2*a^2*Sec[c +
d*x]*Tan[c + d*x]^3)/d + (a^2*Tan[c + d*x]^5)/(5*d)
```

3.30.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3042, 4374, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^4(c+dx)(a\sec(c+dx)+a)^2\,dx$$

$$\downarrow 3042$$

$$\int \cot\left(c+dx+\frac{\pi}{2}\right)^4\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^2 dx$$

$$\downarrow 4374$$

$$\int \left(a^2\tan^4(c+dx)+a^2\tan^4(c+dx)\sec^2(c+dx)+2a^2\tan^4(c+dx)\sec(c+dx)\right)dx$$

$$\downarrow 2009$$

$$\frac{3a^2\text{arctanh}(\sin(c+dx))}{4d}+\frac{a^2\tan^5(c+dx)}{5d}+\frac{a^2\tan^3(c+dx)}{3d}-\frac{a^2\tan(c+dx)}{d}+\frac{a^2\tan^3(c+dx)\sec(c+dx)}{2d}-\frac{3a^2\tan(c+dx)\sec(c+dx)}{4d}+a^2x$$

input
```
Int[(a + a*Sec[c + d*x])^2*Tan[c + d*x]^4,x]
```

output
```
a^2*x + (3*a^2*ArcTanh[Sin[c + d*x]])/(4*d) - (a^2*Tan[c + d*x])/d - (3*a^
2*Sec[c + d*x]*Tan[c + d*x])/(4*d) + (a^2*Tan[c + d*x]^3)/(3*d) + (a^2*Sec
[c + d*x]*Tan[c + d*x]^3)/(2*d) + (a^2*Tan[c + d*x]^5)/(5*d)
```

3.30.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4374
```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

3.30.4 Maple [A] (verified)

Time = 1.63 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.09

method	result
derivativedivides	$\frac{\frac{a^2\sin(dx+c)^5}{5\cos(dx+c)^5}+2a^2\left(\frac{\sin(dx+c)^5}{4\cos(dx+c)^4}-\frac{\sin(dx+c)^5}{8\cos(dx+c)^2}-\frac{\sin(dx+c)^3}{8}-\frac{3\sin(dx+c)}{8}+\frac{3\ln(\sec(dx+c)+\tan(dx+c))}{8}\right)+a^2\left(\frac{\tan(dx+c)^3}{3}-$
default	$\frac{\frac{a^2\sin(dx+c)^5}{5\cos(dx+c)^5}+2a^2\left(\frac{\sin(dx+c)^5}{4\cos(dx+c)^4}-\frac{\sin(dx+c)^5}{8\cos(dx+c)^2}-\frac{\sin(dx+c)^3}{8}-\frac{3\sin(dx+c)}{8}+\frac{3\ln(\sec(dx+c)+\tan(dx+c))}{8}\right)+a^2\left(\frac{\tan(dx+c)^3}{3}-$
parts	$\frac{a^2\left(\frac{\tan(dx+c)^3}{3}-\tan(dx+c)+\arctan(\tan(dx+c))\right)}{d}+\frac{a^2\tan(dx+c)^5}{5d}+\frac{2a^2\left(\frac{\sin(dx+c)^5}{4\cos(dx+c)^4}-\frac{\sin(dx+c)^5}{8\cos(dx+c)^2}-\frac{\sin(dx+c)^3}{8}-$
risch	$a^2x+\frac{ia^2\left(75\,e^{9i(dx+c)}-60\,e^{8i(dx+c)}+30\,e^{7i(dx+c)}-360\,e^{6i(dx+c)}-320\,e^{4i(dx+c)}-30\,e^{3i(dx+c)}-280\,e^{2i(dx+c)}-75\,e^{i($

input
```
int((a+a*sec(d*x+c))^2*tan(d*x+c)^4,x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/5*a^2*sin(d*x+c)^5/cos(d*x+c)^5+2*a^2*(1/4*sin(d*x+c)^5/cos(d*x+c)^
4-1/8*sin(d*x+c)^5/cos(d*x+c)^2-1/8*sin(d*x+c)^3-3/8*sin(d*x+c)+3/8*ln(sec
(d*x+c)+tan(d*x+c)))+a^2*(1/3*tan(d*x+c)^3-tan(d*x+c)+d*x+c))
```

3.30.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.17

$$\int (a + a\sec(c+dx))^2 \tan^4(c+dx)\,dx$$
$$= \frac{120\,a^2 dx\cos(dx+c)^5 + 45\,a^2\cos(dx+c)^5\log(\sin(dx+c)+1) - 45\,a^2\cos(dx+c)^5\log(-\sin(dx+c)}{120\,d\,c}$$

input
```
integrate((a+a*sec(d*x+c))^2*tan(d*x+c)^4,x, algorithm="fricas")
```

output
```
1/120*(120*a^2*d*x*cos(d*x + c)^5 + 45*a^2*cos(d*x + c)^5*log(sin(d*x + c)
 + 1) - 45*a^2*cos(d*x + c)^5*log(-sin(d*x + c) + 1) - 2*(68*a^2*cos(d*x +
 c)^4 + 75*a^2*cos(d*x + c)^3 + 4*a^2*cos(d*x + c)^2 - 30*a^2*cos(d*x + c)
 - 12*a^2)*sin(d*x + c))/(d*cos(d*x + c)^5)
```

3.30.6 Sympy [F]

$$\int (a + a\sec(c+dx))^2 \tan^4(c+dx)\,dx = a^2\left(\int 2\tan^4(c+dx)\sec(c+dx)\,dx + \int \tan^4(c+dx)\sec^2(c+dx)\,dx + \int \tan^4(c+dx)\,dx\right)$$

input
```
integrate((a+a*sec(d*x+c))**2*tan(d*x+c)**4,x)
```

output
```
a**2*(Integral(2*tan(c + d*x)**4*sec(c + d*x), x) + Integral(tan(c + d*x)*
*4*sec(c + d*x)**2, x) + Integral(tan(c + d*x)**4, x))
```

3.30.7 Maxima [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.00

$$\int (a + a\sec(c+dx))^2 \tan^4(c+dx)\,dx$$

$$= \frac{24\,a^2 \tan(dx+c)^5 + 40\left(\tan(dx+c)^3 + 3\,dx + 3\,c - 3\,\tan(dx+c)\right)a^2 + 15\,a^2\left(\frac{2\left(5\,\sin(dx+c)^3 - 3\,\sin(dx+c)\right)}{\sin(dx+c)^4 - 2\,\sin(dx+c)^2 + 1}\right.}{120\,d}$$

input
```
integrate((a+a*sec(d*x+c))^2*tan(d*x+c)^4,x, algorithm="maxima")
```

output
```
1/120*(24*a^2*tan(d*x + c)^5 + 40*(tan(d*x + c)^3 + 3*d*x + 3*c - 3*tan(d*
x + c))*a^2 + 15*a^2*(2*(5*sin(d*x + c)^3 - 3*sin(d*x + c))/(sin(d*x + c)^
4 - 2*sin(d*x + c)^2 + 1) + 3*log(sin(d*x + c) + 1) - 3*log(sin(d*x + c) -
 1)))/d
```

3.30.8 Giac [A] (verification not implemented)

Time = 1.09 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.24

$$\int (a + a\sec(c+dx))^2 \tan^4(c+dx)\,dx$$

$$= \frac{60\,(dx+c)a^2 + 45\,a^2\log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) + 1\right|\right) - 45\,a^2\log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) - 1\right|\right) + \frac{2\left(15\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right.}{60\,d}$$

input
```
integrate((a+a*sec(d*x+c))^2*tan(d*x+c)^4,x, algorithm="giac")
```

output
```
1/60*(60*(d*x + c)*a^2 + 45*a^2*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 45*a^
2*log(abs(tan(1/2*d*x + 1/2*c) - 1)) + 2*(15*a^2*tan(1/2*d*x + 1/2*c)^9 -
110*a^2*tan(1/2*d*x + 1/2*c)^7 + 328*a^2*tan(1/2*d*x + 1/2*c)^5 - 530*a^2*
tan(1/2*d*x + 1/2*c)^3 + 105*a^2*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*
c)^2 - 1)^5)/d
```

3.30.9 Mupad [B] (verification not implemented)

Time = 14.77 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.46

$$\int (a + a\sec(c+dx))^2 \tan^4(c+dx)\,dx = a^2 x + \frac{3a^2 \operatorname{atanh}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{2d} + \frac{\frac{a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^9}{2} - \frac{11a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^7}{3} + \frac{164a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5}{15} - \frac{53a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3}{3} + \frac{7a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{2}}{d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{10} - 5\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^8 + 10\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6 - 10\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4 + 5\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2 - 1\right)}$$

input
```
int(tan(c + d*x)^4*(a + a/cos(c + d*x))^2,x)
```

output
```
a^2*x + (3*a^2*atanh(tan(c/2 + (d*x)/2)))/(2*d) + ((164*a^2*tan(c/2 + (d*x
)/2)^5)/15 - (53*a^2*tan(c/2 + (d*x)/2)^3)/3 - (11*a^2*tan(c/2 + (d*x)/2)^
7)/3 + (a^2*tan(c/2 + (d*x)/2)^9)/2 + (7*a^2*tan(c/2 + (d*x)/2))/2)/(d*(5*
tan(c/2 + (d*x)/2)^2 - 10*tan(c/2 + (d*x)/2)^4 + 10*tan(c/2 + (d*x)/2)^6 -
 5*tan(c/2 + (d*x)/2)^8 + tan(c/2 + (d*x)/2)^10 - 1))
```

3.31 $\int (a + a\sec(c+dx))^2 \tan^2(c+dx)\,dx$

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3.31.1 Optimal result

Integrand size = 21, antiderivative size = 72

$$\int (a + a\sec(c+dx))^2 \tan^2(c+dx)\,dx = -a^2x - \frac{a^2\operatorname{arctanh}(\sin(c+dx))}{d} + \frac{a^2\tan(c+dx)}{d} + \frac{a^2\sec(c+dx)\tan(c+dx)}{d} + \frac{a^2\tan^3(c+dx)}{3d}$$

output
```
-a^2*x-a^2*arctanh(sin(d*x+c))/d+a^2*tan(d*x+c)/d+a^2*sec(d*x+c)*tan(d*x+c
)/d+1/3*a^2*tan(d*x+c)^3/d
```

3.31.2 Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.12

$$\int (a + a\sec(c+dx))^2 \tan^2(c+dx)\,dx = -\frac{a^2\arctan(\tan(c+dx))}{d} - \frac{a^2\operatorname{arctanh}(\sin(c+dx))}{d} + \frac{a^2\tan(c+dx)}{d} + \frac{a^2\sec(c+dx)\tan(c+dx)}{d} + \frac{a^2\tan^3(c+dx)}{3d}$$

input
```
Integrate[(a + a*Sec[c + d*x])^2*Tan[c + d*x]^2,x]
```

output
```
-((a^2*ArcTan[Tan[c + d*x]])/d) - (a^2*ArcTanh[Sin[c + d*x]])/d + (a^2*Tan
[c + d*x])/d + (a^2*Sec[c + d*x]*Tan[c + d*x])/d + (a^2*Tan[c + d*x]^3)/(3
*d)
```

3.31.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3042, 4374, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^2(c+dx)(a\sec(c+dx)+a)^2\,dx$$

$$\downarrow 3042$$

$$\int \cot\left(c+dx+\frac{\pi}{2}\right)^2\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^2 dx$$

$$\downarrow 4374$$

$$\int \left(a^2\tan^2(c+dx)+a^2\tan^2(c+dx)\sec^2(c+dx)+2a^2\tan^2(c+dx)\sec(c+dx)\right)dx$$

$$\downarrow 2009$$

$$-\frac{a^2\text{arctanh}(\sin(c+dx))}{d}+\frac{a^2\tan^3(c+dx)}{3d}+\frac{a^2\tan(c+dx)}{d}+\frac{a^2\tan(c+dx)\sec(c+dx)}{d}-a^2x$$

input
```
Int[(a + a*Sec[c + d*x])^2*Tan[c + d*x]^2,x]
```

output
```
-(a^2*x) - (a^2*ArcTanh[Sin[c + d*x]])/d + (a^2*Tan[c + d*x])/d + (a^2*Sec
[c + d*x]*Tan[c + d*x])/d + (a^2*Tan[c + d*x]^3)/(3*d)
```

3.31.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4374
```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

3.31.4 Maple [A] (verified)

Time = 0.80 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.28

method	result
parts	$\frac{a^2(\tan(dx+c)-\arctan(\tan(dx+c)))}{d}+\frac{a^2\tan(dx+c)^3}{3d}+\frac{2a^2\left(\frac{\sin(dx+c)^3}{2\cos(dx+c)^2}+\frac{\sin(dx+c)}{2}-\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)}{d}$
derivativedivides	$\frac{\frac{a^2\sin(dx+c)^3}{3\cos(dx+c)^3}+2a^2\left(\frac{\sin(dx+c)^3}{2\cos(dx+c)^2}+\frac{\sin(dx+c)}{2}-\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+a^2(\tan(dx+c)-dx-c)}{d}$
default	$\frac{\frac{a^2\sin(dx+c)^3}{3\cos(dx+c)^3}+2a^2\left(\frac{\sin(dx+c)^3}{2\cos(dx+c)^2}+\frac{\sin(dx+c)}{2}-\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+a^2(\tan(dx+c)-dx-c)}{d}$
risch	$-a^2x-\frac{2ia^2\left(3\,e^{5i(dx+c)}-6\,e^{2i(dx+c)}-3\,e^{i(dx+c)}-2\right)}{3d\left(e^{2i(dx+c)}+1\right)^3}-\frac{a^2\ln\left(e^{i(dx+c)}+i\right)}{d}+\frac{a^2\ln\left(e^{i(dx+c)}-i\right)}{d}$

input
```
int((a+a*sec(d*x+c))^2*tan(d*x+c)^2,x,method=_RETURNVERBOSE)
```

output
```
a^2/d*(tan(d*x+c)-arctan(tan(d*x+c)))+1/3*a^2*tan(d*x+c)^3/d+2*a^2/d*(1/2*
sin(d*x+c)^3/cos(d*x+c)^2+1/2*sin(d*x+c)-1/2*ln(sec(d*x+c)+tan(d*x+c)))
```

3.31.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.54

$$\int (a + a\sec(c+dx))^2 \tan^2(c+dx)\,dx =$$
$$-\frac{6\,a^2 dx\cos(dx+c)^3 + 3\,a^2\cos(dx+c)^3\log(\sin(dx+c)+1) - 3\,a^2\cos(dx+c)^3\log(-\sin(dx+c)+}{6\,d\cos(dx+c)^3}$$

input
```
integrate((a+a*sec(d*x+c))^2*tan(d*x+c)^2,x, algorithm="fricas")
```

output
```
-1/6*(6*a^2*d*x*cos(d*x + c)^3 + 3*a^2*cos(d*x + c)^3*log(sin(d*x + c) + 1
) - 3*a^2*cos(d*x + c)^3*log(-sin(d*x + c) + 1) - 2*(2*a^2*cos(d*x + c)^2
+ 3*a^2*cos(d*x + c) + a^2)*sin(d*x + c))/(d*cos(d*x + c)^3)
```

3.31.6 Sympy [F]

$$\int (a + a\sec(c+dx))^2 \tan^2(c+dx)\,dx = a^2\left(\int 2\tan^2(c+dx)\sec(c+dx)\,dx + \int \tan^2(c+dx)\sec^2(c+dx)\,dx + \int \tan^2(c+dx)\,dx\right)$$

input
```
integrate((a+a*sec(d*x+c))**2*tan(d*x+c)**2,x)
```

output
```
a**2*(Integral(2*tan(c + d*x)**2*sec(c + d*x), x) + Integral(tan(c + d*x)*
*2*sec(c + d*x)**2, x) + Integral(tan(c + d*x)**2, x))
```

3.31.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.15

$$\int (a + a\sec(c+dx))^2 \tan^2(c+dx)\,dx$$
$$= \frac{2\,a^2\tan(dx+c)^3 - 6\,(dx+c-\tan(dx+c))a^2 - 3\,a^2\left(\frac{2\sin(dx+c)}{\sin(dx+c)^2-1} + \log(\sin(dx+c)+1) - \log(\sin(dx\right.}{6\,d}$$

input
```
integrate((a+a*sec(d*x+c))^2*tan(d*x+c)^2,x, algorithm="maxima")
```

output
```
1/6*(2*a^2*tan(d*x + c)^3 - 6*(d*x + c - tan(d*x + c))*a^2 - 3*a^2*(2*sin(
d*x + c)/(sin(d*x + c)^2 - 1) + log(sin(d*x + c) + 1) - log(sin(d*x + c) -
 1)))/d
```

3.31.8 Giac [A] (verification not implemented)

Time = 0.62 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.38

$$\int (a + a\sec(c+dx))^2 \tan^2(c+dx)\,dx =$$
$$-\frac{3\,(dx+c)a^2 + 3\,a^2\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right) - 3\,a^2\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right) - \frac{4\left(a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-3\right.}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\right.\right.}}{3\,d}$$

input
```
integrate((a+a*sec(d*x+c))^2*tan(d*x+c)^2,x, algorithm="giac")
```

output
```
-1/3*(3*(d*x + c)*a^2 + 3*a^2*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 3*a^2*l
og(abs(tan(1/2*d*x + 1/2*c) - 1)) - 4*(a^2*tan(1/2*d*x + 1/2*c)^3 - 3*a^2*
tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^3)/d
```

3.31.9 Mupad [B] (verification not implemented)

Time = 13.35 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.40

$$\int (a + a\sec(c+dx))^2 \tan^2(c+dx)\,dx$$
$$= \frac{\frac{4a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3}{3} - 4a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6 - 3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4 + 3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2 - 1\right)} - \frac{2a^2\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{d} - a^2 x$$

input
```
int(tan(c + d*x)^2*(a + a/cos(c + d*x))^2,x)
```

output
```
((4*a^2*tan(c/2 + (d*x)/2)^3)/3 - 4*a^2*tan(c/2 + (d*x)/2))/(d*(3*tan(c/2
+ (d*x)/2)^2 - 3*tan(c/2 + (d*x)/2)^4 + tan(c/2 + (d*x)/2)^6 - 1)) - (2*a^
2*atanh(tan(c/2 + (d*x)/2)))/d - a^2*x
```

3.32 $\int \cot^2(c+dx)(a+a\sec(c+dx))^2\,dx$

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3.32.1 Optimal result

Integrand size = 21, antiderivative size = 35

$$\int \cot^2(c+dx)(a+a\sec(c+dx))^2\,dx = -a^2x - \frac{2a^2\cot(c+dx)}{d} - \frac{2a^2\csc(c+dx)}{d}$$

output
```
-a^2*x-2*a^2*cot(d*x+c)/d-2*a^2*csc(d*x+c)/d
```

3.32.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.03 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.31

$$\int \cot^2(c+dx)(a+a\sec(c+dx))^2\,dx = -\frac{2a^2\cot\left(\frac{c}{2}+\frac{dx}{2}\right)\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},-\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{d}$$

input
```
Integrate[Cot[c + d*x]^2*(a + a*Sec[c + d*x])^2,x]
```

output
```
(-2*a^2*Cot[c/2 + (d*x)/2]*Hypergeometric2F1[-1/2, 1, 1/2, -Tan[c/2 + (d*x
)/2]^2])/d
```

3.32.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3042, 4374, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^2(c+dx)(a\sec(c+dx)+a)^2\,dx$$

$$\downarrow 3042$$

$$\int \frac{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^2}{\cot\left(c+dx+\frac{\pi}{2}\right)^2}dx$$

$$\downarrow 4374$$

$$\int \left(a^2\cot^2(c+dx)+a^2\csc^2(c+dx)+2a^2\cot(c+dx)\csc(c+dx)\right)dx$$

$$\downarrow 2009$$

$$-\frac{2a^2\cot(c+dx)}{d}-\frac{2a^2\csc(c+dx)}{d}+a^2(-x)$$

input
```
Int[Cot[c + d*x]^2*(a + a*Sec[c + d*x])^2,x]
```

output
```
-(a^2*x) - (2*a^2*Cot[c + d*x])/d - (2*a^2*Csc[c + d*x])/d
```

3.32.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4374
```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

3.32.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.11 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.86

method	result	size
risch	$-a^2 x - \frac{4ia^2}{d(e^{i(dx+c)}-1)}$	30
derivativedivides	$\frac{-a^2\cot(dx+c)-\frac{2a^2}{\sin(dx+c)}+a^2(-\cot(dx+c)-dx-c)}{d}$	50
default	$\frac{-a^2\cot(dx+c)-\frac{2a^2}{\sin(dx+c)}+a^2(-\cot(dx+c)-dx-c)}{d}$	50

input
```
int(cot(d*x+c)^2*(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
-a^2*x-4*I*a^2/d/(exp(I*(d*x+c))-1)
```

3.32.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.20

$$\int \cot^2(c+dx)(a+a\sec(c+dx))^2\,dx = -\frac{a^2 dx\sin(dx+c)+2\,a^2\cos(dx+c)+2\,a^2}{d\sin(dx+c)}$$

input
```
integrate(cot(d*x+c)^2*(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

output
```
-(a^2*d*x*sin(d*x + c) + 2*a^2*cos(d*x + c) + 2*a^2)/(d*sin(d*x + c))
```

3.32.6 Sympy [F]

$$\int \cot^2(c+dx)(a+a\sec(c+dx))^2\,dx = a^2\left(\int 2\cot^2(c+dx)\sec(c+dx)\,dx + \int \cot^2(c+dx)\sec^2(c+dx)\,dx + \int \cot^2(c+dx)\,dx\right)$$

input
```
integrate(cot(d*x+c)**2*(a+a*sec(d*x+c))**2,x)
```

output
```
a**2*(Integral(2*cot(c + d*x)**2*sec(c + d*x), x) + Integral(cot(c + d*x)*
*2*sec(c + d*x)**2, x) + Integral(cot(c + d*x)**2, x))
```

3.32.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.37

$$\int \cot^2(c+dx)(a+a\sec(c+dx))^2\,dx = -\frac{\left(dx+c+\frac{1}{\tan(dx+c)}\right)a^2+\frac{2\,a^2}{\sin(dx+c)}+\frac{a^2}{\tan(dx+c)}}{d}$$

input
```
integrate(cot(d*x+c)^2*(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

output
```
-((d*x + c + 1/tan(d*x + c))*a^2 + 2*a^2/sin(d*x + c) + a^2/tan(d*x + c))/
d
```

3.32.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.89

$$\int \cot^2(c+dx)(a+a\sec(c+dx))^2\,dx = -\frac{(dx+c)a^2+\frac{2\,a^2}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}}{d}$$

input
```
integrate(cot(d*x+c)^2*(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

output
```
-((d*x + c)*a^2 + 2*a^2/tan(1/2*d*x + 1/2*c))/d
```

3.32.9 Mupad [B] (verification not implemented)

Time = 13.11 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.69

$$\int \cot^2(c+dx)(a+a\sec(c+dx))^2\,dx = -a^2\,x - \frac{2\,a^2\cot\left(\frac{c}{2}+\frac{dx}{2}\right)}{d}$$

input
```
int(cot(c + d*x)^2*(a + a/cos(c + d*x))^2,x)
```

output
```
- a^2*x - (2*a^2*cot(c/2 + (d*x)/2))/d
```

3.33 $\int \cot^4(c+dx)(a+a\sec(c+dx))^2\,dx$

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3.33.1 Optimal result

Integrand size = 21, antiderivative size = 69

$$\int \cot^4(c+dx)(a+a\sec(c+dx))^2\,dx = a^2x + \frac{a^2\cot(c+dx)}{d} - \frac{2a^2\cot^3(c+dx)}{3d} + \frac{2a^2\csc(c+dx)}{d} - \frac{2a^2\csc^3(c+dx)}{3d}$$

output
```
a^2*x+a^2*cot(d*x+c)/d-2/3*a^2*cot(d*x+c)^3/d+2*a^2*csc(d*x+c)/d-2/3*a^2*c
sc(d*x+c)^3/d
```

3.33.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.35 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.26

$$\int \cot^4(c+dx)(a+a\sec(c+dx))^2\,dx = -\frac{a^2\cot^3(c+dx)}{3d} + \frac{2a^2\csc(c+dx)}{d} - \frac{2a^2\csc^3(c+dx)}{3d} - \frac{a^2\cot^3(c+dx)\,\text{Hypergeometric2F1}\left(-\frac{3}{2},1,-\frac{1}{2},-\tan^2(c+dx)\right)}{3d}$$

input
```
Integrate[Cot[c + d*x]^4*(a + a*Sec[c + d*x])^2,x]
```

output
```
-1/3*(a^2*Cot[c + d*x]^3)/d + (2*a^2*Csc[c + d*x])/d - (2*a^2*Csc[c + d*x]
^3)/(3*d) - (a^2*Cot[c + d*x]^3*Hypergeometric2F1[-3/2, 1, -1/2, -Tan[c +
d*x]^2])/(3*d)
```

3.33.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3042, 4374, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^4(c+dx)(a\sec(c+dx)+a)^2\,dx$$

$$\downarrow 3042$$

$$\int \frac{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^2}{\cot\left(c+dx+\frac{\pi}{2}\right)^4}dx$$

$$\downarrow 4374$$

$$\int \left(a^2\cot^4(c+dx)+2a^2\cot^3(c+dx)\csc(c+dx)+a^2\cot^2(c+dx)\csc^2(c+dx)\right)dx$$

$$\downarrow 2009$$

$$-\frac{2a^2\cot^3(c+dx)}{3d}+\frac{a^2\cot(c+dx)}{d}-\frac{2a^2\csc^3(c+dx)}{3d}+\frac{2a^2\csc(c+dx)}{d}+a^2x$$

input
```
Int[Cot[c + d*x]^4*(a + a*Sec[c + d*x])^2,x]
```

output
```
a^2*x + (a^2*Cot[c + d*x])/d - (2*a^2*Cot[c + d*x]^3)/(3*d) + (2*a^2*Csc[c
 + d*x])/d - (2*a^2*Csc[c + d*x]^3)/(3*d)
```

3.33.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4374
```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

3.33.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.32 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.77

method	result
risch	$a^2 x + \frac{2ia^2\left(6\,e^{2i(dx+c)} - 9\,e^{i(dx+c)} + 5\right)}{3d\left(e^{i(dx+c)} - 1\right)^3}$
derivativedivides	$\frac{-\frac{a^2\cos(dx+c)^3}{3\sin(dx+c)^3} + 2a^2\left(-\frac{\cos(dx+c)^4}{3\sin(dx+c)^3} + \frac{\cos(dx+c)^4}{3\sin(dx+c)} + \frac{\left(2+\cos(dx+c)^2\right)\sin(dx+c)}{3}\right) + a^2\left(-\frac{\cot(dx+c)^3}{3} + \cot(dx+c) + dx + c\right)}{d}$
default	$\frac{-\frac{a^2\cos(dx+c)^3}{3\sin(dx+c)^3} + 2a^2\left(-\frac{\cos(dx+c)^4}{3\sin(dx+c)^3} + \frac{\cos(dx+c)^4}{3\sin(dx+c)} + \frac{\left(2+\cos(dx+c)^2\right)\sin(dx+c)}{3}\right) + a^2\left(-\frac{\cot(dx+c)^3}{3} + \cot(dx+c) + dx + c\right)}{d}$

input
```
int(cot(d*x+c)^4*(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
a^2*x+2/3*I*a^2*(6*exp(2*I*(d*x+c))-9*exp(I*(d*x+c))+5)/d/(exp(I*(d*x+c))-
1)^3
```

3.33.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.17

$$\int \cot^4(c+dx)(a+a\sec(c+dx))^2\,dx$$
$$= \frac{5\,a^2\cos(dx+c)^2 + a^2\cos(dx+c) - 4\,a^2 + 3\,(a^2 dx\cos(dx+c) - a^2 dx)\sin(dx+c)}{3\,(d\cos(dx+c) - d)\sin(dx+c)}$$

input
```
integrate(cot(d*x+c)^4*(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

output
```
1/3*(5*a^2*cos(d*x + c)^2 + a^2*cos(d*x + c) - 4*a^2 + 3*(a^2*d*x*cos(d*x
+ c) - a^2*d*x)*sin(d*x + c))/((d*cos(d*x + c) - d)*sin(d*x + c))
```

3.33.6 Sympy [F]

$$\int \cot^4(c+dx)(a+a\sec(c+dx))^2\,dx = a^2\left(\int 2\cot^4(c+dx)\sec(c+dx)\,dx + \int \cot^4(c+dx)\sec^2(c+dx)\,dx + \int \cot^4(c+dx)\,dx\right)$$

input
```
integrate(cot(d*x+c)**4*(a+a*sec(d*x+c))**2,x)
```

output
```
a**2*(Integral(2*cot(c + d*x)**4*sec(c + d*x), x) + Integral(cot(c + d*x)*
*4*sec(c + d*x)**2, x) + Integral(cot(c + d*x)**4, x))
```

3.33.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.12

$$\int \cot^4(c+dx)(a+a\sec(c+dx))^2\,dx$$
$$= \frac{\left(3\,dx + 3\,c + \frac{3\tan(dx+c)^2-1}{\tan(dx+c)^3}\right)a^2 + \frac{2\left(3\sin(dx+c)^2-1\right)a^2}{\sin(dx+c)^3} - \frac{a^2}{\tan(dx+c)^3}}{3\,d}$$

input
```
integrate(cot(d*x+c)^4*(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

output
```
1/3*((3*d*x + 3*c + (3*tan(d*x + c)^2 - 1)/tan(d*x + c)^3)*a^2 + 2*(3*sin(
d*x + c)^2 - 1)*a^2/sin(d*x + c)^3 - a^2/tan(d*x + c)^3)/d
```

3.33.8 Giac [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.72

$$\int \cot^4(c+dx)(a+a\sec(c+dx))^2\,dx = \frac{6\,(dx+c)a^2 + \frac{9\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-a^2}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3}}{6\,d}$$

input
```
integrate(cot(d*x+c)^4*(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

output
```
1/6*(6*(d*x + c)*a^2 + (9*a^2*tan(1/2*d*x + 1/2*c)^2 - a^2)/tan(1/2*d*x +
1/2*c)^3)/d
```

3.33.9 Mupad [B] (verification not implemented)

Time = 13.94 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.57

$$\int \cot^4(c+dx)(a+a\sec(c+dx))^2\,dx = a^2\,x + \frac{a^2\left(9\cot\left(\frac{c}{2}+\frac{dx}{2}\right)-\cot\left(\frac{c}{2}+\frac{dx}{2}\right)^3\right)}{6\,d}$$

input
```
int(cot(c + d*x)^4*(a + a/cos(c + d*x))^2,x)
```

output
```
a^2*x + (a^2*(9*cot(c/2 + (d*x)/2) - cot(c/2 + (d*x)/2)^3))/(6*d)
```

3.34 $\int \cot^6(c+dx)(a+a\sec(c+dx))^2\,dx$

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3.34.1 Optimal result

Integrand size = 21, antiderivative size = 107

$$\int \cot^6(c+dx)(a+a\sec(c+dx))^2\,dx = -a^2x - \frac{a^2\cot(c+dx)}{d} + \frac{a^2\cot^3(c+dx)}{3d} - \frac{2a^2\cot^5(c+dx)}{5d} - \frac{2a^2\csc(c+dx)}{d} + \frac{4a^2\csc^3(c+dx)}{3d} - \frac{2a^2\csc^5(c+dx)}{5d}$$

output

```
-a^2*x-a^2*cot(d*x+c)/d+1/3*a^2*cot(d*x+c)^3/d-2/5*a^2*cot(d*x+c)^5/d-2*a^
2*csc(d*x+c)/d+4/3*a^2*csc(d*x+c)^3/d-2/5*a^2*csc(d*x+c)^5/d
```

3.34.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.34 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.98

$$\int \cot^6(c+dx)(a+a\sec(c+dx))^2\,dx = -\frac{a^2\cot^5(c+dx)}{5d} - \frac{2a^2\csc(c+dx)}{d} + \frac{4a^2\csc^3(c+dx)}{3d} - \frac{2a^2\csc^5(c+dx)}{5d} - \frac{a^2\cot^5(c+dx)\,\text{Hypergeometric2F1}\left(-\frac{5}{2},1,-\frac{3}{2},-\tan^2(c+dx)\right)}{5d}$$

input
```
Integrate[Cot[c + d*x]^6*(a + a*Sec[c + d*x])^2,x]
```

output
```
-1/5*(a^2*Cot[c + d*x]^5)/d - (2*a^2*Csc[c + d*x])/d + (4*a^2*Csc[c + d*x]
^3)/(3*d) - (2*a^2*Csc[c + d*x]^5)/(5*d) - (a^2*Cot[c + d*x]^5*Hypergeomet
ric2F1[-5/2, 1, -3/2, -Tan[c + d*x]^2])/(5*d)
```

3.34.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3042, 4374, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^6(c+dx)(a\sec(c+dx)+a)^2\,dx$$

$$\downarrow 3042$$

$$\int \frac{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^2}{\cot\left(c+dx+\frac{\pi}{2}\right)^6}dx$$

$$\downarrow 4374$$

$$\int \left(a^2\cot^6(c+dx)+2a^2\cot^5(c+dx)\csc(c+dx)+a^2\cot^4(c+dx)\csc^2(c+dx)\right)dx$$

$$\downarrow 2009$$

$$-\frac{2a^2\cot^5(c+dx)}{5d}+\frac{a^2\cot^3(c+dx)}{3d}-\frac{a^2\cot(c+dx)}{d}-\frac{2a^2\csc^5(c+dx)}{5d}+\frac{4a^2\csc^3(c+dx)}{3d}-\frac{2a^2\csc(c+dx)}{d}-a^2x$$

input
```
Int[Cot[c + d*x]^6*(a + a*Sec[c + d*x])^2,x]
```

output
```
-(a^2*x) - (a^2*Cot[c + d*x])/d + (a^2*Cot[c + d*x]^3)/(3*d) - (2*a^2*Cot[
c + d*x]^5)/(5*d) - (2*a^2*Csc[c + d*x])/d + (4*a^2*Csc[c + d*x]^3)/(3*d)
- (2*a^2*Csc[c + d*x]^5)/(5*d)
```

3.34.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4374
```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

3.34.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.62 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.93

method	result
risch	$-a^2x-\frac{4ia^2\left(15\,\mathrm{e}^{5i(dx+c)}-30\,\mathrm{e}^{4i(dx+c)}+10\,\mathrm{e}^{3i(dx+c)}+35\,\mathrm{e}^{2i(dx+c)}-37\,\mathrm{e}^{i(dx+c)}+13\right)}{15d\left(\mathrm{e}^{i(dx+c)}-1\right)^5\left(\mathrm{e}^{i(dx+c)}+1\right)}$
derivativedivides	$\frac{-\frac{a^2\cos(dx+c)^5}{5\sin(dx+c)^5}+2a^2\left(-\frac{\cos(dx+c)^6}{5\sin(dx+c)^5}+\frac{\cos(dx+c)^6}{15\sin(dx+c)^3}-\frac{\cos(dx+c)^6}{5\sin(dx+c)}-\frac{\left(\frac{8}{3}+\cos(dx+c)^4+\frac{4\cos(dx+c)^2}{3}\right)\sin(dx+c)}{5}\right)+a^2\left(-\cot$
default	$\frac{-\frac{a^2\cos(dx+c)^5}{5\sin(dx+c)^5}+2a^2\left(-\frac{\cos(dx+c)^6}{5\sin(dx+c)^5}+\frac{\cos(dx+c)^6}{15\sin(dx+c)^3}-\frac{\cos(dx+c)^6}{5\sin(dx+c)}-\frac{\left(\frac{8}{3}+\cos(dx+c)^4+\frac{4\cos(dx+c)^2}{3}\right)\sin(dx+c)}{5}\right)+a^2\left(-\cot$

input
```
int(cot(d*x+c)^6*(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
-a^2*x-4/15*I*a^2*(15*exp(5*I*(d*x+c))-30*exp(4*I*(d*x+c))+10*exp(3*I*(d*x
+c))+35*exp(2*I*(d*x+c))-37*exp(I*(d*x+c))+13)/d/(exp(I*(d*x+c))-1)^5/(exp
(I*(d*x+c))+1)
```

3.34.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.10

$$\int \cot^6(c+dx)(a+a\sec(c+dx))^2\,dx =$$
$$-\frac{26\,a^2\cos(dx+c)^3-22\,a^2\cos(dx+c)^2-17\,a^2\cos(dx+c)+16\,a^2+15\left(a^2dx\cos(dx+c)^2-2\,a^2dx\,c\right.}{15\left(d\cos(dx+c)^2-2\,d\cos(dx+c)+d\right)\sin(dx+c)}$$

input
```
integrate(cot(d*x+c)^6*(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

output
```
-1/15*(26*a^2*cos(d*x + c)^3 - 22*a^2*cos(d*x + c)^2 - 17*a^2*cos(d*x + c)
 + 16*a^2 + 15*(a^2*d*x*cos(d*x + c)^2 - 2*a^2*d*x*cos(d*x + c) + a^2*d*x)
*sin(d*x + c))/((d*cos(d*x + c)^2 - 2*d*cos(d*x + c) + d)*sin(d*x + c))
```

3.34.6 Sympy [F]

$$\int \cot^6(c+dx)(a+a\sec(c+dx))^2\,dx = a^2\left(\int 2\cot^6(c+dx)\sec(c+dx)\,dx\right.$$
$$+\int \cot^6(c+dx)\sec^2(c+dx)\,dx$$
$$\left.+\int \cot^6(c+dx)\,dx\right)$$

input
```
integrate(cot(d*x+c)**6*(a+a*sec(d*x+c))**2,x)
```

output
```
a**2*(Integral(2*cot(c + d*x)**6*sec(c + d*x), x) + Integral(cot(c + d*x)*
*6*sec(c + d*x)**2, x) + Integral(cot(c + d*x)**6, x))
```

3.34.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.91

$$\int \cot^6(c+dx)(a+a\sec(c+dx))^2\,dx =$$
$$-\frac{\left(15\,dx+15\,c+\frac{15\tan(dx+c)^4-5\tan(dx+c)^2+3}{\tan(dx+c)^5}\right)a^2+\frac{2\left(15\sin(dx+c)^4-10\sin(dx+c)^2+3\right)a^2}{\sin(dx+c)^5}+\frac{3a^2}{\tan(dx+c)^5}}{15\,d}$$

input
```
integrate(cot(d*x+c)^6*(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

output
```
-1/15*((15*d*x + 15*c + (15*tan(d*x + c)^4 - 5*tan(d*x + c)^2 + 3)/tan(d*x
 + c)^5)*a^2 + 2*(15*sin(d*x + c)^4 - 10*sin(d*x + c)^2 + 3)*a^2/sin(d*x +
 c)^5 + 3*a^2/tan(d*x + c)^5)/d
```

3.34.8 Giac [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.75

$$\int \cot^6(c+dx)(a+a\sec(c+dx))^2\,dx$$
$$=-\frac{120\,(dx+c)a^2-15\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+\frac{165\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-25\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+3\,a^2}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5}}{120\,d}$$

input
```
integrate(cot(d*x+c)^6*(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

output
```
-1/120*(120*(d*x + c)*a^2 - 15*a^2*tan(1/2*d*x + 1/2*c) + (165*a^2*tan(1/2
*d*x + 1/2*c)^4 - 25*a^2*tan(1/2*d*x + 1/2*c)^2 + 3*a^2)/tan(1/2*d*x + 1/2
*c)^5)/d
```

3.34.9 Mupad [B] (verification not implemented)

Time = 13.98 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.73

$$\int \cot^6(c+dx)(a+a\sec(c+dx))^2\,dx = \frac{a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{8d} - \frac{\frac{11a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4}{8} - \frac{5a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2}{24} + \frac{a^2}{40}}{d\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5} - a^2x$$

input
```
int(cot(c + d*x)^6*(a + a/cos(c + d*x))^2,x)
```

output
```
(a^2*tan(c/2 + (d*x)/2))/(8*d) - ((11*a^2*tan(c/2 + (d*x)/2)^4)/8 - (5*a^2
*tan(c/2 + (d*x)/2)^2)/24 + a^2/40)/(d*tan(c/2 + (d*x)/2)^5) - a^2*x
```

3.35 $\int \cot^8(c+dx)(a+a\sec(c+dx))^2\,dx$

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3.35.1 Optimal result

Integrand size = 21, antiderivative size = 139

$$\begin{aligned}\int \cot^8(c+dx)(a+a\sec(c+dx))^2\,dx = a^2x &+ \frac{a^2\cot(c+dx)}{d} - \frac{a^2\cot^3(c+dx)}{3d} \\ &+ \frac{a^2\cot^5(c+dx)}{5d} - \frac{2a^2\cot^7(c+dx)}{7d} \\ &+ \frac{2a^2\csc(c+dx)}{d} - \frac{2a^2\csc^3(c+dx)}{d} \\ &+ \frac{6a^2\csc^5(c+dx)}{5d} - \frac{2a^2\csc^7(c+dx)}{7d}\end{aligned}$$

output
```
a^2*x+a^2*cot(d*x+c)/d-1/3*a^2*cot(d*x+c)^3/d+1/5*a^2*cot(d*x+c)^5/d-2/7*a
^2*cot(d*x+c)^7/d+2*a^2*csc(d*x+c)/d-2*a^2*csc(d*x+c)^3/d+6/5*a^2*csc(d*x+
c)^5/d-2/7*a^2*csc(d*x+c)^7/d
```

3.35.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.58 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.87

$$\begin{aligned}&\int \cot^8(c+dx)(a+a\sec(c+dx))^2\,dx \\ &= -\frac{a^2\cot^7(c+dx)}{7d} + \frac{2a^2\csc(c+dx)}{d} - \frac{2a^2\csc^3(c+dx)}{d} + \frac{6a^2\csc^5(c+dx)}{5d} \\ &\quad - \frac{2a^2\csc^7(c+dx)}{7d} - \frac{a^2\cot^7(c+dx)\,\text{Hypergeometric2F1}\left(-\frac{7}{2},1,-\frac{5}{2},-\tan^2(c+dx)\right)}{7d}\end{aligned}$$

input
```
Integrate[Cot[c + d*x]^8*(a + a*Sec[c + d*x])^2,x]
```

output
```
-1/7*(a^2*Cot[c + d*x]^7)/d + (2*a^2*Csc[c + d*x])/d - (2*a^2*Csc[c + d*x]
^3)/d + (6*a^2*Csc[c + d*x]^5)/(5*d) - (2*a^2*Csc[c + d*x]^7)/(7*d) - (a^2
*Cot[c + d*x]^7*Hypergeometric2F1[-7/2, 1, -5/2, -Tan[c + d*x]^2])/(7*d)
```

3.35.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3042, 4374, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^8(c+dx)(a\sec(c+dx)+a)^2\,dx$$

$$\downarrow 3042$$

$$\int \frac{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^2}{\cot\left(c+dx+\frac{\pi}{2}\right)^8}dx$$

$$\downarrow 4374$$

$$\int \left(a^2\cot^8(c+dx)+2a^2\cot^7(c+dx)\csc(c+dx)+a^2\cot^6(c+dx)\csc^2(c+dx)\right)dx$$

$$\downarrow 2009$$

$$-\frac{2a^2\cot^7(c+dx)}{7d}+\frac{a^2\cot^5(c+dx)}{5d}-\frac{a^2\cot^3(c+dx)}{3d}+\frac{a^2\cot(c+dx)}{d}-\frac{2a^2\csc^7(c+dx)}{7d}+\frac{6a^2\csc^5(c+dx)}{5d}-\frac{2a^2\csc^3(c+dx)}{d}+\frac{2a^2\csc(c+dx)}{d}+a^2x$$

input
```
Int[Cot[c + d*x]^8*(a + a*Sec[c + d*x])^2,x]
```

output
```
a^2*x + (a^2*Cot[c + d*x])/d - (a^2*Cot[c + d*x]^3)/(3*d) + (a^2*Cot[c + d
*x]^5)/(5*d) - (2*a^2*Cot[c + d*x]^7)/(7*d) + (2*a^2*Csc[c + d*x])/d - (2*
a^2*Csc[c + d*x]^3)/d + (6*a^2*Csc[c + d*x]^5)/(5*d) - (2*a^2*Csc[c + d*x]
^7)/(7*d)
```

3.35.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4374
```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

3.35.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.56 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.03

method	result
risch	$a^2x+\frac{2ia^2\left(210\,e^{9i(dx+c)}-315\,e^{8i(dx+c)}-420\,e^{7i(dx+c)}+1470\,e^{6i(dx+c)}-504\,e^{5i(dx+c)}-1204\,e^{4i(dx+c)}+1108\,e^{3i(dx+}\right.}{105d\left(e^{i(dx+c)}-1\right)^7\left(e^{i(dx+c)}+1\right)^3}$
derivativedivides	$\frac{-\frac{a^2\cos(dx+c)^7}{7\sin(dx+c)^7}+2a^2\left(-\frac{\cos(dx+c)^8}{7\sin(dx+c)^7}+\frac{\cos(dx+c)^8}{35\sin(dx+c)^5}-\frac{\cos(dx+c)^8}{35\sin(dx+c)^3}+\frac{\cos(dx+c)^8}{7\sin(dx+c)}+\frac{\left(\frac{16}{5}+\cos(dx+c)^6+\frac{6\cos(dx+c)^4}{5}+\frac{8\cos(d}{5}\right.\right.}{7}}{d}$
default	$\frac{-\frac{a^2\cos(dx+c)^7}{7\sin(dx+c)^7}+2a^2\left(-\frac{\cos(dx+c)^8}{7\sin(dx+c)^7}+\frac{\cos(dx+c)^8}{35\sin(dx+c)^5}-\frac{\cos(dx+c)^8}{35\sin(dx+c)^3}+\frac{\cos(dx+c)^8}{7\sin(dx+c)}+\frac{\left(\frac{16}{5}+\cos(dx+c)^6+\frac{6\cos(dx+c)^4}{5}+\frac{8\cos(d}{5}\right.\right.}{7}}{d}$

input
```
int(cot(d*x+c)^8*(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
a^2*x+2/105*I*a^2*(210*exp(9*I*(d*x+c))-315*exp(8*I*(d*x+c))-420*exp(7*I*(
d*x+c))+1470*exp(6*I*(d*x+c))-504*exp(5*I*(d*x+c))-1204*exp(4*I*(d*x+c))+1
108*exp(3*I*(d*x+c))+258*exp(2*I*(d*x+c))-554*exp(I*(d*x+c))+191)/d/(exp(I
*(d*x+c))-1)^7/(exp(I*(d*x+c))+1)^3
```

3.35.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.24

$$\int \cot^8(c+dx)(a+a\sec(c+dx))^2\,dx$$
$$= \frac{191\,a^2\cos(dx+c)^5 - 172\,a^2\cos(dx+c)^4 - 253\,a^2\cos(dx+c)^3 + 258\,a^2\cos(dx+c)^2 + 87\,a^2\cos(dx+c)}{105\left(d\cos(dx+c)^4 - 2\,d\cos(dx+c)^3\right.}$$

input
```
integrate(cot(d*x+c)^8*(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

output
```
1/105*(191*a^2*cos(d*x + c)^5 - 172*a^2*cos(d*x + c)^4 - 253*a^2*cos(d*x +
 c)^3 + 258*a^2*cos(d*x + c)^2 + 87*a^2*cos(d*x + c) - 96*a^2 + 105*(a^2*d
*x*cos(d*x + c)^4 - 2*a^2*d*x*cos(d*x + c)^3 + 2*a^2*d*x*cos(d*x + c) - a^
2*d*x)*sin(d*x + c))/((d*cos(d*x + c)^4 - 2*d*cos(d*x + c)^3 + 2*d*cos(d*x
 + c) - d)*sin(d*x + c))
```

3.35.6 Sympy [F(-1)]

Timed out.

$$\int \cot^8(c+dx)(a+a\sec(c+dx))^2\,dx = \text{Timed out}$$

input
```
integrate(cot(d*x+c)**8*(a+a*sec(d*x+c))**2,x)
```

output
```
Timed out
```

3.35.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.84

$$\int \cot^8(c+dx)(a+a\sec(c+dx))^2\,dx$$
$$= \frac{\left(105\,dx + 105\,c + \frac{105\tan(dx+c)^6 - 35\tan(dx+c)^4 + 21\tan(dx+c)^2 - 15}{\tan(dx+c)^7}\right)a^2 + \frac{6\left(35\sin(dx+c)^6 - 35\sin(dx+c)^4 + 21\sin(dx+c)^2 - 5\right)a}{\sin(dx+c)^7}}{105\,d}$$

input
```
integrate(cot(d*x+c)^8*(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

output
```
1/105*((105*d*x + 105*c + (105*tan(d*x + c)^6 - 35*tan(d*x + c)^4 + 21*tan
(d*x + c)^2 - 15)/tan(d*x + c)^7)*a^2 + 6*(35*sin(d*x + c)^6 - 35*sin(d*x
+ c)^4 + 21*sin(d*x + c)^2 - 5)*a^2/sin(d*x + c)^7 - 15*a^2/tan(d*x + c)^7
)/d
```

3.35.8 Giac [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.81

$$\int \cot^8(c+dx)(a+a\sec(c+dx))^2\,dx$$
$$= \frac{35\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+3360\,(dx+c)a^2-735\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+\frac{4410\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6-770\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7}}{3360\,d}$$

input
```
integrate(cot(d*x+c)^8*(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

output
```
1/3360*(35*a^2*tan(1/2*d*x + 1/2*c)^3 + 3360*(d*x + c)*a^2 - 735*a^2*tan(1
/2*d*x + 1/2*c) + (4410*a^2*tan(1/2*d*x + 1/2*c)^6 - 770*a^2*tan(1/2*d*x +
 1/2*c)^4 + 147*a^2*tan(1/2*d*x + 1/2*c)^2 - 15*a^2)/tan(1/2*d*x + 1/2*c)^
7)/d
```

3.35.9 Mupad [B] (verification not implemented)

Time = 14.33 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.31

$$\int \cot^8(c+dx)(a+a\sec(c+dx))^2\,dx$$
$$= \frac{a^2\left(35\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}-15\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}-735\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^2\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^8+4410\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^4\sin\left(\frac{c}{2}+\right.\right.}{3360\,d\cos}$$

input
```
int(cot(c + d*x)^8*(a + a/cos(c + d*x))^2,x)
```

output
```
(a^2*(35*sin(c/2 + (d*x)/2)^10 - 15*cos(c/2 + (d*x)/2)^10 - 735*cos(c/2 +
(d*x)/2)^2*sin(c/2 + (d*x)/2)^8 + 4410*cos(c/2 + (d*x)/2)^4*sin(c/2 + (d*x
)/2)^6 - 770*cos(c/2 + (d*x)/2)^6*sin(c/2 + (d*x)/2)^4 + 147*cos(c/2 + (d*
x)/2)^8*sin(c/2 + (d*x)/2)^2 + 3360*cos(c/2 + (d*x)/2)^3*sin(c/2 + (d*x)/2
)^7*(c + d*x)))/(3360*d*cos(c/2 + (d*x)/2)^3*sin(c/2 + (d*x)/2)^7)
```

3.36 $\int \cot^{10}(c+dx)(a+a\sec(c+dx))^2\,dx$

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3.36.1 Optimal result

Integrand size = 21, antiderivative size = 179

$$\begin{aligned}\int \cot^{10}(c+dx)(a+a\sec(c+dx))^2\,dx = {} & -a^2x - \frac{a^2\cot(c+dx)}{d} + \frac{a^2\cot^3(c+dx)}{3d} \\ & - \frac{a^2\cot^5(c+dx)}{5d} + \frac{a^2\cot^7(c+dx)}{7d} \\ & - \frac{2a^2\cot^9(c+dx)}{9d} - \frac{2a^2\csc(c+dx)}{d} \\ & + \frac{8a^2\csc^3(c+dx)}{3d} - \frac{12a^2\csc^5(c+dx)}{5d} \\ & + \frac{8a^2\csc^7(c+dx)}{7d} - \frac{2a^2\csc^9(c+dx)}{9d}\end{aligned}$$

output

```
-a^2*x-a^2*cot(d*x+c)/d+1/3*a^2*cot(d*x+c)^3/d-1/5*a^2*cot(d*x+c)^5/d+1/7*
a^2*cot(d*x+c)^7/d-2/9*a^2*cot(d*x+c)^9/d-2*a^2*csc(d*x+c)/d+8/3*a^2*csc(d
*x+c)^3/d-12/5*a^2*csc(d*x+c)^5/d+8/7*a^2*csc(d*x+c)^7/d-2/9*a^2*csc(d*x+c
)^9/d
```

3.36.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 1.77 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.79

$$\begin{aligned}
&\int \cot^{10}(c+dx)(a+a\sec(c+dx))^2\,dx \\
&= -\frac{a^2\cot^9(c+dx)}{9d} - \frac{2a^2\csc(c+dx)}{d} + \frac{8a^2\csc^3(c+dx)}{3d} \\
&\quad - \frac{12a^2\csc^5(c+dx)}{5d} + \frac{8a^2\csc^7(c+dx)}{7d} - \frac{2a^2\csc^9(c+dx)}{9d} \\
&\quad - \frac{a^2\cot^9(c+dx)\,\text{Hypergeometric2F1}\left(-\frac{9}{2},1,-\frac{7}{2},-\tan^2(c+dx)\right)}{9d}
\end{aligned}$$

input
```
Integrate[Cot[c + d*x]^10*(a + a*Sec[c + d*x])^2,x]
```

output
```
-1/9*(a^2*Cot[c + d*x]^9)/d - (2*a^2*Csc[c + d*x])/d + (8*a^2*Csc[c + d*x]
^3)/(3*d) - (12*a^2*Csc[c + d*x]^5)/(5*d) + (8*a^2*Csc[c + d*x]^7)/(7*d) -
 (2*a^2*Csc[c + d*x]^9)/(9*d) - (a^2*Cot[c + d*x]^9*Hypergeometric2F1[-9/2
, 1, -7/2, -Tan[c + d*x]^2])/(9*d)
```

3.36.3 Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3042, 4374, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^{10}(c+dx)(a\sec(c+dx)+a)^2\,dx$$

$$\downarrow \text{3042}$$

$$\int \frac{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^2}{\cot\left(c+dx+\frac{\pi}{2}\right)^{10}}dx$$

$$\downarrow \text{4374}$$

$$\int \left(a^2\cot^{10}(c+dx)+2a^2\cot^9(c+dx)\csc(c+dx)+a^2\cot^8(c+dx)\csc^2(c+dx)\right)dx$$

$$\downarrow 2009$$

$$-\frac{2a^2\cot^9(c+dx)}{9d}+\frac{a^2\cot^7(c+dx)}{7d}-\frac{a^2\cot^5(c+dx)}{5d}+\frac{a^2\cot^3(c+dx)}{3d}-\frac{a^2\cot(c+dx)}{d}-\frac{2a^2\csc^9(c+dx)}{9d}+\frac{8a^2\csc^7(c+dx)}{7d}-\frac{12a^2\csc^5(c+dx)}{5d}+\frac{8a^2\csc^3(c+dx)}{3d}-\frac{2a^2\csc(c+dx)}{d}-a^2x$$

input
```
Int[Cot[c + d*x]^10*(a + a*Sec[c + d*x])^2,x]
```

output
```
-(a^2*x) - (a^2*Cot[c + d*x])/d + (a^2*Cot[c + d*x]^3)/(3*d) - (a^2*Cot[c
+ d*x]^5)/(5*d) + (a^2*Cot[c + d*x]^7)/(7*d) - (2*a^2*Cot[c + d*x]^9)/(9*d
) - (2*a^2*Csc[c + d*x])/d + (8*a^2*Csc[c + d*x]^3)/(3*d) - (12*a^2*Csc[c
+ d*x]^5)/(5*d) + (8*a^2*Csc[c + d*x]^7)/(7*d) - (2*a^2*Csc[c + d*x]^9)/(9
*d)
```

3.36.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4374
```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

3.36.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 4.48 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.05

method	result
risch	$-a^2x-\frac{4ia^2\left(315\,e^{13i(dx+c)}-315\,e^{12i(dx+c)}-1470\,e^{11i(dx+c)}+3360\,e^{10i(dx+c)}+1113\,e^{9i(dx+c)}-6447\,e^{8i(dx+c)}+202\right.}{315d\left(e^{i(dx+c)}-\right.}$
derivativedivides	$-\frac{a^2\cos(dx+c)^9}{9\sin(dx+c)^9}+2a^2\left(-\frac{\cos(dx+c)^{10}}{9\sin(dx+c)^9}+\frac{\cos(dx+c)^{10}}{63\sin(dx+c)^7}-\frac{\cos(dx+c)^{10}}{105\sin(dx+c)^5}+\frac{\cos(dx+c)^{10}}{63\sin(dx+c)^3}-\frac{\cos(dx+c)^{10}}{9\sin(dx+c)}-\frac{\left(\frac{128}{35}+\cos(dx+c)^8+\frac{8}{\right.}\right.$
default	$-\frac{a^2\cos(dx+c)^9}{9\sin(dx+c)^9}+2a^2\left(-\frac{\cos(dx+c)^{10}}{9\sin(dx+c)^9}+\frac{\cos(dx+c)^{10}}{63\sin(dx+c)^7}-\frac{\cos(dx+c)^{10}}{105\sin(dx+c)^5}+\frac{\cos(dx+c)^{10}}{63\sin(dx+c)^3}-\frac{\cos(dx+c)^{10}}{9\sin(dx+c)}-\frac{\left(\frac{128}{35}+\cos(dx+c)^8+\frac{8}{\right.}\right.$

input
```
int(cot(d*x+c)^10*(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
-a^2*x-4/315*I*a^2*(315*exp(13*I*(d*x+c))-315*exp(12*I*(d*x+c))-1470*exp(1
1*I*(d*x+c))+3360*exp(10*I*(d*x+c))+1113*exp(9*I*(d*x+c))-6447*exp(8*I*(d*
x+c))+2028*exp(7*I*(d*x+c))+7008*exp(6*I*(d*x+c))-4867*exp(5*I*(d*x+c))-23
21*exp(4*I*(d*x+c))+3314*exp(3*I*(d*x+c))-16*exp(2*I*(d*x+c))-881*exp(I*(d
*x+c))+299)/d/(exp(I*(d*x+c))-1)^9/(exp(I*(d*x+c))+1)^5
```

3.36.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 274, normalized size of antiderivative = 1.53

$$\int \cot^{10}(c+dx)(a+a\sec(c+dx))^2\,dx =$$
$$-\frac{598\,a^2\cos(dx+c)^7-566\,a^2\cos(dx+c)^6-1212\,a^2\cos(dx+c)^5+1310\,a^2\cos(dx+c)^4+860\,a^2\cos(}{315\,(d}$$

input
```
integrate(cot(d*x+c)^10*(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

output
```
-1/315*(598*a^2*cos(d*x + c)^7 - 566*a^2*cos(d*x + c)^6 - 1212*a^2*cos(d*x
 + c)^5 + 1310*a^2*cos(d*x + c)^4 + 860*a^2*cos(d*x + c)^3 - 1014*a^2*cos(
d*x + c)^2 - 197*a^2*cos(d*x + c) + 256*a^2 + 315*(a^2*d*x*cos(d*x + c)^6
 - 2*a^2*d*x*cos(d*x + c)^5 - a^2*d*x*cos(d*x + c)^4 + 4*a^2*d*x*cos(d*x +
c)^3 - a^2*d*x*cos(d*x + c)^2 - 2*a^2*d*x*cos(d*x + c) + a^2*d*x)*sin(d*x
+ c))/((d*cos(d*x + c)^6 - 2*d*cos(d*x + c)^5 - d*cos(d*x + c)^4 + 4*d*cos
(d*x + c)^3 - d*cos(d*x + c)^2 - 2*d*cos(d*x + c) + d)*sin(d*x + c))
```

3.36.6 Sympy [F(-1)]

Timed out.

$$\int \cot^{10}(c+dx)(a+a\sec(c+dx))^2\,dx = \text{Timed out}$$

input
```
integrate(cot(d*x+c)**10*(a+a*sec(d*x+c))**2,x)
```

output
```
Timed out
```

3.36.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.77

$$\int \cot^{10}(c+dx)(a+a\sec(c+dx))^2\,dx =$$

$$-\frac{\left(315\,dx+315\,c+\frac{315\tan(dx+c)^8-105\tan(dx+c)^6+63\tan(dx+c)^4-45\tan(dx+c)^2+35}{\tan(dx+c)^9}\right)a^2+\frac{2\left(315\sin(dx+c)^8-420\sin(dx+c\right.}{315\,d}$$

input
```
integrate(cot(d*x+c)^10*(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

output
```
-1/315*((315*d*x + 315*c + (315*tan(d*x + c)^8 - 105*tan(d*x + c)^6 + 63*t
an(d*x + c)^4 - 45*tan(d*x + c)^2 + 35)/tan(d*x + c)^9)*a^2 + 2*(315*sin(d
*x + c)^8 - 420*sin(d*x + c)^6 + 378*sin(d*x + c)^4 - 180*sin(d*x + c)^2 +
 35)*a^2/sin(d*x + c)^9 + 35*a^2/tan(d*x + c)^9)/d
```

3.36.8 Giac [A] (verification not implemented)

Time = 0.48 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.81

$$\int \cot^{10}(c+dx)(a+a\sec(c+dx))^2\,dx$$

$$= \frac{63\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5 - 945\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3 - 40320\,(dx+c)a^2 + 11655\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right) - \frac{51345\,a}{}}{40320\,d}$$

input
```
integrate(cot(d*x+c)^10*(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

output
```
1/40320*(63*a^2*tan(1/2*d*x + 1/2*c)^5 - 945*a^2*tan(1/2*d*x + 1/2*c)^3 -
40320*(d*x + c)*a^2 + 11655*a^2*tan(1/2*d*x + 1/2*c) - (51345*a^2*tan(1/2*
d*x + 1/2*c)^8 - 9765*a^2*tan(1/2*d*x + 1/2*c)^6 + 2331*a^2*tan(1/2*d*x +
1/2*c)^4 - 405*a^2*tan(1/2*d*x + 1/2*c)^2 + 35*a^2)/tan(1/2*d*x + 1/2*c)^9
)/d
```

3.36.9 Mupad [B] (verification not implemented)

Time = 15.17 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.28

$$\int \cot^{10}(c+dx)(a+a\sec(c+dx))^2\,dx =$$

$$-\frac{a^2\left(35\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{14} - 63\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{14} + 945\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^2\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{12} - 11655\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^4\sin\right.}{}$$

input
```
int(cot(c + d*x)^10*(a + a/cos(c + d*x))^2,x)
```

output
```
-(a^2*(35*cos(c/2 + (d*x)/2)^14 - 63*sin(c/2 + (d*x)/2)^14 + 945*cos(c/2 +
 (d*x)/2)^2*sin(c/2 + (d*x)/2)^12 - 11655*cos(c/2 + (d*x)/2)^4*sin(c/2 + (
d*x)/2)^10 + 51345*cos(c/2 + (d*x)/2)^6*sin(c/2 + (d*x)/2)^8 - 9765*cos(c/
2 + (d*x)/2)^8*sin(c/2 + (d*x)/2)^6 + 2331*cos(c/2 + (d*x)/2)^10*sin(c/2 +
 (d*x)/2)^4 - 405*cos(c/2 + (d*x)/2)^12*sin(c/2 + (d*x)/2)^2 + 40320*cos(c
/2 + (d*x)/2)^5*sin(c/2 + (d*x)/2)^9*(c + d*x)))/(40320*d*cos(c/2 + (d*x)/
2)^5*sin(c/2 + (d*x)/2)^9)
```

3.37 $\int (a + a\sec(c+dx))^3 \tan^9(c+dx)\,dx$

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3.37.1 Optimal result

Integrand size = 21, antiderivative size = 210

$$\begin{aligned}\int (a + a\sec(c+dx))^3 \tan^9(c+dx)\,dx = &-\frac{a^3\log(\cos(c+dx))}{d} + \frac{3a^3\sec(c+dx)}{d} \\ &- \frac{a^3\sec^2(c+dx)}{2d} - \frac{11a^3\sec^3(c+dx)}{3d} \\ &- \frac{3a^3\sec^4(c+dx)}{2d} + \frac{14a^3\sec^5(c+dx)}{5d} \\ &+ \frac{7a^3\sec^6(c+dx)}{3d} - \frac{6a^3\sec^7(c+dx)}{7d} \\ &- \frac{11a^3\sec^8(c+dx)}{8d} - \frac{a^3\sec^9(c+dx)}{9d} \\ &+ \frac{3a^3\sec^{10}(c+dx)}{10d} + \frac{a^3\sec^{11}(c+dx)}{11d}\end{aligned}$$

output

```
-a^3*ln(cos(d*x+c))/d+3*a^3*sec(d*x+c)/d-1/2*a^3*sec(d*x+c)^2/d-11/3*a^3*s
ec(d*x+c)^3/d-3/2*a^3*sec(d*x+c)^4/d+14/5*a^3*sec(d*x+c)^5/d+7/3*a^3*sec(d
*x+c)^6/d-6/7*a^3*sec(d*x+c)^7/d-11/8*a^3*sec(d*x+c)^8/d-1/9*a^3*sec(d*x+c
)^9/d+3/10*a^3*sec(d*x+c)^10/d+1/11*a^3*sec(d*x+c)^11/d
```

3.37.2 Mathematica [A] (verified)

Time = 0.78 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.02

$$\int (a + a\sec(c+dx))^3 \tan^9(c+dx)\,dx =$$
$$-\frac{a^3(-1151740 - 1613260\cos(2(c+dx)) + 960960\cos(3(c+dx)) - 1131504\cos(4(c+dx)) + 314160\cos$$

input
```
Integrate[(a + a*Sec[c + d*x])^3*Tan[c + d*x]^9,x]
```

output
```
-1/3548160*(a^3*(-1151740 - 1613260*Cos[2*(c + d*x)] + 960960*Cos[3*(c + d
*x)] - 1131504*Cos[4*(c + d*x)] + 314160*Cos[5*(c + d*x)] - 432894*Cos[6*(
c + d*x)] + 145530*Cos[7*(c + d*x)] - 106260*Cos[8*(c + d*x)] + 6930*Cos[9
*(c + d*x)] - 20790*Cos[10*(c + d*x)] + 1143450*Cos[3*(c + d*x)]*Log[Cos[c
 + d*x]] + 571725*Cos[5*(c + d*x)]*Log[Cos[c + d*x]] + 190575*Cos[7*(c + d
*x)]*Log[Cos[c + d*x]] + 38115*Cos[9*(c + d*x)]*Log[Cos[c + d*x]] + 3465*C
os[11*(c + d*x)]*Log[Cos[c + d*x]] + 462*Cos[c + d*x]*(2606 + 3465*Log[Cos
[c + d*x]]))*Sec[c + d*x]^11)/d
```

3.37.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.69, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 25, 4367, 27, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^9(c+dx)(a\sec(c+dx)+a)^3\,dx$$
$$\downarrow 3042$$
$$\int -\cot\left(c+dx+\frac{\pi}{2}\right)^9\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^3 dx$$
$$\downarrow 25$$
$$-\int \cot\left(\frac{1}{2}(2c+\pi)+dx\right)^9\left(\csc\left(\frac{1}{2}(2c+\pi)+dx\right)a+a\right)^3 dx$$
$$\downarrow 4367$$

$$-\frac{\int a^{11}(1-\cos(c+dx))^4(\cos(c+dx)+1)^7\sec^{12}(c+dx)d\cos(c+dx)}{a^8 d}$$

$$\downarrow 27$$

$$-\frac{a^3\int(1-\cos(c+dx))^4(\cos(c+dx)+1)^7\sec^{12}(c+dx)d\cos(c+dx)}{d}$$

$$\downarrow 99$$

$$-\frac{a^3\int\left(\sec^{12}(c+dx)+3\sec^{11}(c+dx)-\sec^{10}(c+dx)-11\sec^9(c+dx)-6\sec^8(c+dx)+14\sec^7(c+dx)+14\right.}{d}$$

$$\downarrow 2009$$

$$-\frac{a^3\left(-\frac{1}{11}\sec^{11}(c+dx)-\frac{3}{10}\sec^{10}(c+dx)+\frac{1}{9}\sec^9(c+dx)+\frac{11}{8}\sec^8(c+dx)+\frac{6}{7}\sec^7(c+dx)-\frac{7}{3}\sec^6(c+dx)-\right.}{d}$$

input
```
Int[(a + a*Sec[c + d*x])^3*Tan[c + d*x]^9,x]
```

output
```
-((a^3*(Log[Cos[c + d*x]] - 3*Sec[c + d*x] + Sec[c + d*x]^2/2 + (11*Sec[c
+ d*x]^3)/3 + (3*Sec[c + d*x]^4)/2 - (14*Sec[c + d*x]^5)/5 - (7*Sec[c + d*
x]^6)/3 + (6*Sec[c + d*x]^7)/7 + (11*Sec[c + d*x]^8)/8 + Sec[c + d*x]^9/9
- (3*Sec[c + d*x]^10)/10 - Sec[c + d*x]^11/11))/d)
```

3.37.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4367
```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Simp[1/(a^(m - n - 1)*b^n*d)   Subst[Int[(a - b*x)^((m -
1)/2)*((a + b*x)^((m - 1)/2 + n)/x^(m + n)), x], x, Sin[c + d*x]], x] /; Fr
eeQ[{a, b, c, d}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && Integer
Q[n]
```

3.37.4 Maple [A] (verified)

Time = 3.03 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.59

method	result
derivativedivides	$\frac{a^3\left(\frac{\sec(dx+c)^{11}}{11}+\frac{3\sec(dx+c)^{10}}{10}-\frac{\sec(dx+c)^9}{9}-\frac{11\sec(dx+c)^8}{8}-\frac{6\sec(dx+c)^7}{7}+\frac{7\sec(dx+c)^6}{3}+\frac{14\sec(dx+c)^5}{5}-\frac{3\sec(dx+c)^4}{2}-\frac{1}{d}\right.}{d}$
default	$\frac{a^3\left(\frac{\sec(dx+c)^{11}}{11}+\frac{3\sec(dx+c)^{10}}{10}-\frac{\sec(dx+c)^9}{9}-\frac{11\sec(dx+c)^8}{8}-\frac{6\sec(dx+c)^7}{7}+\frac{7\sec(dx+c)^6}{3}+\frac{14\sec(dx+c)^5}{5}-\frac{3\sec(dx+c)^4}{2}-\frac{1}{d}\right.}{d}$
parts	$\frac{a^3\left(\frac{\tan(dx+c)^8}{8}-\frac{\tan(dx+c)^6}{6}+\frac{\tan(dx+c)^4}{4}-\frac{\tan(dx+c)^2}{2}+\frac{\ln\left(1+\tan(dx+c)^2\right)}{2}\right)}{d}+\frac{a^3\left(\frac{\sec(dx+c)^{11}}{11}-\frac{4\sec(dx+c)^9}{9}+\frac{6\sec(dx}{d}\right.}{d}$
risch	$ia^3x+\frac{2ia^3c}{d}+\frac{2a^3\left(10395\,e^{21i(dx+c)}-3465\,e^{20i(dx+c)}+53130\,e^{19i(dx+c)}-72765\,e^{18i(dx+c)}+216447\,e^{17i(dx+c)}-15\right.}{}$

input
```
int((a+a*sec(d*x+c))^3*tan(d*x+c)^9,x,method=_RETURNVERBOSE)
```

output
```
a^3/d*(1/11*sec(d*x+c)^11+3/10*sec(d*x+c)^10-1/9*sec(d*x+c)^9-11/8*sec(d*x
+c)^8-6/7*sec(d*x+c)^7+7/3*sec(d*x+c)^6+14/5*sec(d*x+c)^5-3/2*sec(d*x+c)^4
-11/3*sec(d*x+c)^3-1/2*sec(d*x+c)^2+3*sec(d*x+c)+ln(sec(d*x+c)))
```

3.37.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.80

$$\int (a + a\sec(c+dx))^3 \tan^9(c+dx)\,dx =$$

$$-\frac{27720\,a^3\cos(dx+c)^{11}\log(-\cos(dx+c)) - 83160\,a^3\cos(dx+c)^{10} + 13860\,a^3\cos(dx+c)^9 + 101640}{}$$

input
```
integrate((a+a*sec(d*x+c))^3*tan(d*x+c)^9,x, algorithm="fricas")
```

output
```
-1/27720*(27720*a^3*cos(d*x + c)^11*log(-cos(d*x + c)) - 83160*a^3*cos(d*x
 + c)^10 + 13860*a^3*cos(d*x + c)^9 + 101640*a^3*cos(d*x + c)^8 + 41580*a^
3*cos(d*x + c)^7 - 77616*a^3*cos(d*x + c)^6 - 64680*a^3*cos(d*x + c)^5 + 2
3760*a^3*cos(d*x + c)^4 + 38115*a^3*cos(d*x + c)^3 + 3080*a^3*cos(d*x + c)
^2 - 8316*a^3*cos(d*x + c) - 2520*a^3)/(d*cos(d*x + c)^11)
```

3.37.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 439 vs. 2(190) = 380.

Time = 3.40 (sec) , antiderivative size = 439, normalized size of antiderivative = 2.09

$$\int (a + a\sec(c+dx))^3 \tan^9(c+dx)\,dx$$

$$= \begin{cases} \frac{a^3\log(\tan^2(c+dx)+1)}{2d} + \frac{a^3\tan^8(c+dx)\sec^3(c+dx)}{11d} + \frac{3a^3\tan^8(c+dx)\sec^2(c+dx)}{10d} + \frac{a^3\tan^8(c+dx)\sec(c+dx)}{3d} + \frac{a^3\tan^8(c+dx)}{8d} - \\ x(a\sec(c)+a)^3\tan^9(c) \end{cases}$$

input
```
integrate((a+a*sec(d*x+c))**3*tan(d*x+c)**9,x)
```

output
```
Piecewise((a**3*log(tan(c + d*x)**2 + 1)/(2*d) + a**3*tan(c + d*x)**8*sec(
c + d*x)**3/(11*d) + 3*a**3*tan(c + d*x)**8*sec(c + d*x)**2/(10*d) + a**3*
tan(c + d*x)**8*sec(c + d*x)/(3*d) + a**3*tan(c + d*x)**8/(8*d) - 8*a**3*t
an(c + d*x)**6*sec(c + d*x)**3/(99*d) - 3*a**3*tan(c + d*x)**6*sec(c + d*x
)**2/(10*d) - 8*a**3*tan(c + d*x)**6*sec(c + d*x)/(21*d) - a**3*tan(c + d*
x)**6/(6*d) + 16*a**3*tan(c + d*x)**4*sec(c + d*x)**3/(231*d) + 3*a**3*tan
(c + d*x)**4*sec(c + d*x)**2/(10*d) + 16*a**3*tan(c + d*x)**4*sec(c + d*x)
/(35*d) + a**3*tan(c + d*x)**4/(4*d) - 64*a**3*tan(c + d*x)**2*sec(c + d*x
)**3/(1155*d) - 3*a**3*tan(c + d*x)**2*sec(c + d*x)**2/(10*d) - 64*a**3*ta
n(c + d*x)**2*sec(c + d*x)/(105*d) - a**3*tan(c + d*x)**2/(2*d) + 128*a**3
*sec(c + d*x)**3/(3465*d) + 3*a**3*sec(c + d*x)**2/(10*d) + 128*a**3*sec(c
 + d*x)/(105*d), Ne(d, 0)), (x*(a*sec(c) + a)**3*tan(c)**9, True))
```

3.37.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 162, normalized size of antiderivative = 0.77

$$\int (a + a\sec(c+dx))^3 \tan^9(c+dx)\,dx =$$
$$-\frac{27720\,a^3\log(\cos(dx+c)) - \frac{83160\,a^3\cos(dx+c)^{10} - 13860\,a^3\cos(dx+c)^9 - 101640\,a^3\cos(dx+c)^8 - 41580\,a^3\cos(dx+c)^7 + 77616\,a^3}{}}{27720\,d}$$

input
```
integrate((a+a*sec(d*x+c))^3*tan(d*x+c)^9,x, algorithm="maxima")
```

output
```
-1/27720*(27720*a^3*log(cos(d*x + c)) - (83160*a^3*cos(d*x + c)^10 - 13860
*a^3*cos(d*x + c)^9 - 101640*a^3*cos(d*x + c)^8 - 41580*a^3*cos(d*x + c)^7
 + 77616*a^3*cos(d*x + c)^6 + 64680*a^3*cos(d*x + c)^5 - 23760*a^3*cos(d*x
 + c)^4 - 38115*a^3*cos(d*x + c)^3 - 3080*a^3*cos(d*x + c)^2 + 8316*a^3*co
s(d*x + c) + 2520*a^3)/cos(d*x + c)^11)/d
```

3.37.8 Giac [A] (verification not implemented)

Time = 5.96 (sec) , antiderivative size = 367, normalized size of antiderivative = 1.75

$$\int (a + a\sec(c+dx))^3 \tan^9(c+dx)\,dx$$
$$= \frac{27720\,a^3\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right) - 27720\,a^3\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right) + \frac{153343\,a^3 + \frac{1742213\,a^3(\cos(dx+c)-1)}{\cos(dx+c)+1} + \frac{9043705\,a}{(\cos}}{}$$

input
```
integrate((a+a*sec(d*x+c))^3*tan(d*x+c)^9,x, algorithm="giac")
```

output
```
1/27720*(27720*a^3*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)) -
27720*a^3*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)) + (153343*a
^3 + 1742213*a^3*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 9043705*a^3*(cos(
d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 28369275*a^3*(cos(d*x + c) - 1)^3/(
cos(d*x + c) + 1)^3 + 59954070*a^3*(cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)
^4 + 67458930*a^3*(cos(d*x + c) - 1)^5/(cos(d*x + c) + 1)^5 + 57997170*a^3
*(cos(d*x + c) - 1)^6/(cos(d*x + c) + 1)^6 + 36975510*a^3*(cos(d*x + c) -
1)^7/(cos(d*x + c) + 1)^7 + 16879995*a^3*(cos(d*x + c) - 1)^8/(cos(d*x + c
) + 1)^8 + 5213945*a^3*(cos(d*x + c) - 1)^9/(cos(d*x + c) + 1)^9 + 976261*
a^3*(cos(d*x + c) - 1)^10/(cos(d*x + c) + 1)^10 + 83711*a^3*(cos(d*x + c)
- 1)^11/(cos(d*x + c) + 1)^11)/((cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)
^11)/d
```

3.37.9 Mupad [B] (verification not implemented)

Time = 17.80 (sec) , antiderivative size = 337, normalized size of antiderivative = 1.60

$$\int (a + a\sec(c+dx))^3 \tan^9(c+dx)\,dx = \frac{2a^3 \operatorname{atanh}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2\right)}{d}$$
$$- \frac{2a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{20} - 22a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{18} + \frac{332a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{16}}{3} - \frac{1012a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{14}}{3} + \frac{10456a^3\,\text{ta}}{}}{d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{22} - 11\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{20} + 55\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{18} - 165\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{16} + 330\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{14}\right.}$$

input
```
int(tan(c + d*x)^9*(a + a/cos(c + d*x))^3,x)
```

output
```
(2*a^3*atanh(tan(c/2 + (d*x)/2)^2))/d - ((10090*a^3*tan(c/2 + (d*x)/2)^4)/
63 - (9334*a^3*tan(c/2 + (d*x)/2)^2)/315 - (3676*a^3*tan(c/2 + (d*x)/2)^6)
/7 + (8164*a^3*tan(c/2 + (d*x)/2)^8)/7 - (5192*a^3*tan(c/2 + (d*x)/2)^10)/
5 + (10456*a^3*tan(c/2 + (d*x)/2)^12)/15 - (1012*a^3*tan(c/2 + (d*x)/2)^14
)/3 + (332*a^3*tan(c/2 + (d*x)/2)^16)/3 - 22*a^3*tan(c/2 + (d*x)/2)^18 + 2
*a^3*tan(c/2 + (d*x)/2)^20 + (8704*a^3)/3465)/(d*(11*tan(c/2 + (d*x)/2)^2
- 55*tan(c/2 + (d*x)/2)^4 + 165*tan(c/2 + (d*x)/2)^6 - 330*tan(c/2 + (d*x)
/2)^8 + 462*tan(c/2 + (d*x)/2)^10 - 462*tan(c/2 + (d*x)/2)^12 + 330*tan(c/
2 + (d*x)/2)^14 - 165*tan(c/2 + (d*x)/2)^16 + 55*tan(c/2 + (d*x)/2)^18 - 1
1*tan(c/2 + (d*x)/2)^20 + tan(c/2 + (d*x)/2)^22 - 1))
```

3.38 $\int (a + a\sec(c+dx))^3 \tan^7(c+dx)\,dx$

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3.38.1 Optimal result

Integrand size = 21, antiderivative size = 137

$$\begin{aligned}\int (a + a\sec(c+dx))^3 \tan^7(c+dx)\,dx = {} & \frac{a^3 \log(\cos(c+dx))}{d} - \frac{3a^3 \sec(c+dx)}{d} \\ & + \frac{8a^3 \sec^3(c+dx)}{3d} + \frac{3a^3 \sec^4(c+dx)}{2d} \\ & - \frac{6a^3 \sec^5(c+dx)}{5d} - \frac{4a^3 \sec^6(c+dx)}{3d} \\ & + \frac{3a^3 \sec^8(c+dx)}{8d} + \frac{a^3 \sec^9(c+dx)}{9d}\end{aligned}$$

output
```
a^3*ln(cos(d*x+c))/d-3*a^3*sec(d*x+c)/d+8/3*a^3*sec(d*x+c)^3/d+3/2*a^3*sec
(d*x+c)^4/d-6/5*a^3*sec(d*x+c)^5/d-4/3*a^3*sec(d*x+c)^6/d+3/8*a^3*sec(d*x+
c)^8/d+1/9*a^3*sec(d*x+c)^9/d
```

3.38.2 Mathematica [A] (verified)

Time = 0.46 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.22

$$\int (a + a\sec(c+dx))^3 \tan^7(c+dx)\,dx$$
$$= \frac{a^3(-3754 - 7632\cos(2(c+dx)) + 1560\cos(3(c+dx)) - 3528\cos(4(c+dx)) + 1080\cos(5(c+dx)) - 12$$

input
```
Integrate[(a + a*Sec[c + d*x])^3*Tan[c + d*x]^7,x]
```

output
```
(a^3*(-3754 - 7632*Cos[2*(c + d*x)] + 1560*Cos[3*(c + d*x)] - 3528*Cos[4*(
c + d*x)] + 1080*Cos[5*(c + d*x)] - 1200*Cos[6*(c + d*x)] - 270*Cos[8*(c +
 d*x)] + 3780*Cos[3*(c + d*x)]*Log[Cos[c + d*x]] + 1620*Cos[5*(c + d*x)]*L
og[Cos[c + d*x]] + 405*Cos[7*(c + d*x)]*Log[Cos[c + d*x]] + 45*Cos[9*(c +
d*x)]*Log[Cos[c + d*x]] + 90*Cos[c + d*x]*(40 + 63*Log[Cos[c + d*x]]))*Sec
[c + d*x]^9)/(11520*d)
```

3.38.3 Rubi [A] (verified)

Time $= 0.27$ (sec) , antiderivative size $= 98$, normalized size of antiderivative $= 0.72$, number of steps used $= 7$, number of rules used $= 6$, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used $= \{3042, 25, 4367, 27, 99, 2009\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^7(c+dx)(a\sec(c+dx)+a)^3\,dx$$

$$\downarrow 3042$$

$$\int -\cot\left(c+dx+\frac{\pi}{2}\right)^7\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^3 dx$$

$$\downarrow 25$$

$$-\int \cot\left(\frac{1}{2}(2c+\pi)+dx\right)^7\left(\csc\left(\frac{1}{2}(2c+\pi)+dx\right)a+a\right)^3 dx$$

$$\downarrow 4367$$

$$-\frac{\int a^9(1-\cos(c+dx))^3(\cos(c+dx)+1)^6\sec^{10}(c+dx)d\cos(c+dx)}{a^6 d}$$

$$\downarrow 27$$

$$-\frac{a^3\int(1-\cos(c+dx))^3(\cos(c+dx)+1)^6\sec^{10}(c+dx)d\cos(c+dx)}{d}$$

$$\downarrow 99$$

$$-\frac{a^3\int\left(\sec^{10}(c+dx)+3\sec^9(c+dx)-8\sec^7(c+dx)-6\sec^6(c+dx)+6\sec^5(c+dx)+8\sec^4(c+dx)-3\sec^2\right.}{d}$$

$$\downarrow 2009$$

$$-\frac{a^3\left(-\frac{1}{9}\sec^9(c+dx)-\frac{3}{8}\sec^8(c+dx)+\frac{4}{3}\sec^6(c+dx)+\frac{6}{5}\sec^5(c+dx)-\frac{3}{2}\sec^4(c+dx)-\frac{8}{3}\sec^3(c+dx)+3\sec\right.}{d}$$

input
```
Int[(a + a*Sec[c + d*x])^3*Tan[c + d*x]^7,x]
```

output
```
-((a^3*(-Log[Cos[c + d*x]] + 3*Sec[c + d*x] - (8*Sec[c + d*x]^3)/3 - (3*Se
c[c + d*x]^4)/2 + (6*Sec[c + d*x]^5)/5 + (4*Sec[c + d*x]^6)/3 - (3*Sec[c +
 d*x]^8)/8 - Sec[c + d*x]^9/9))/d)
```

3.38.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4367
```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Simp[1/(a^(m - n - 1)*b^n*d)   Subst[Int[(a - b*x)^((m -
1)/2)*((a + b*x)^((m - 1)/2 + n)/x^(m + n)), x], x, Sin[c + d*x]], x] /; Fr
eeQ[{a, b, c, d}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && Integer
Q[n]
```

3.38.4 Maple [A] (verified)

Time = 2.22 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.63

method	result
derivativedivides	$\frac{a^3\left(\frac{\sec(dx+c)^9}{9}+\frac{3\sec(dx+c)^8}{8}-\frac{4\sec(dx+c)^6}{3}-\frac{6\sec(dx+c)^5}{5}+\frac{3\sec(dx+c)^4}{2}+\frac{8\sec(dx+c)^3}{3}-3\sec(dx+c)-\ln(\sec(dx+c))\right)}{d}$
default	$\frac{a^3\left(\frac{\sec(dx+c)^9}{9}+\frac{3\sec(dx+c)^8}{8}-\frac{4\sec(dx+c)^6}{3}-\frac{6\sec(dx+c)^5}{5}+\frac{3\sec(dx+c)^4}{2}+\frac{8\sec(dx+c)^3}{3}-3\sec(dx+c)-\ln(\sec(dx+c))\right)}{d}$
parts	$\frac{a^3\left(\frac{\tan(dx+c)^6}{6}-\frac{\tan(dx+c)^4}{4}+\frac{\tan(dx+c)^2}{2}-\frac{\ln\left(1+\tan(dx+c)^2\right)}{2}\right)}{d}+\frac{a^3\left(\frac{\sec(dx+c)^9}{9}-\frac{3\sec(dx+c)^7}{7}+\frac{3\sec(dx+c)^5}{5}-\frac{\sec(dx+$
risch	$-ia^3x-\frac{2ia^3c}{d}-\frac{2a^3\left(135\,e^{17i(dx+c)}+600\,e^{15i(dx+c)}-540\,e^{14i(dx+c)}+1764\,e^{13i(dx+c)}-780\,e^{12i(dx+c)}+3816\,e^{11i(}$

input
```
int((a+a*sec(d*x+c))^3*tan(d*x+c)^7,x,method=_RETURNVERBOSE)
```

output
```
a^3/d*(1/9*sec(d*x+c)^9+3/8*sec(d*x+c)^8-4/3*sec(d*x+c)^6-6/5*sec(d*x+c)^5
+3/2*sec(d*x+c)^4+8/3*sec(d*x+c)^3-3*sec(d*x+c)-ln(sec(d*x+c)))
```

3.38.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.85

$$\int (a+a\sec(c+dx))^3\tan^7(c+dx)\,dx$$
$$=\frac{360\,a^3\cos(dx+c)^9\log(-\cos(dx+c))-1080\,a^3\cos(dx+c)^8+960\,a^3\cos(dx+c)^6+540\,a^3\cos(dx+c}{360\,d\cos(dx+c)^9}$$

input
```
integrate((a+a*sec(d*x+c))^3*tan(d*x+c)^7,x, algorithm="fricas")
```

output
```
1/360*(360*a^3*cos(d*x + c)^9*log(-cos(d*x + c)) - 1080*a^3*cos(d*x + c)^8
 + 960*a^3*cos(d*x + c)^6 + 540*a^3*cos(d*x + c)^5 - 432*a^3*cos(d*x + c)^
4 - 480*a^3*cos(d*x + c)^3 + 135*a^3*cos(d*x + c) + 40*a^3)/(d*cos(d*x + c
)^9)
```

3.38.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 350 vs. $2(126) = 252$.

Time = 1.67 (sec) , antiderivative size = 350, normalized size of antiderivative = 2.55

$$\int (a + a\sec(c+dx))^3 \tan^7(c+dx)\,dx$$

$$= \begin{cases} -\frac{a^3 \log\left(\tan^2(c+dx)+1\right)}{2d} + \frac{a^3 \tan^6(c+dx)\sec^3(c+dx)}{9d} + \frac{3a^3 \tan^6(c+dx)\sec^2(c+dx)}{8d} + \frac{3a^3 \tan^6(c+dx)\sec(c+dx)}{7d} + \frac{a^3 \tan^6(c+dx)}{6d} \\ x(a\sec(c)+a)^3 \tan^7(c) \end{cases}$$

input
```
integrate((a+a*sec(d*x+c))**3*tan(d*x+c)**7,x)
```

output
```
Piecewise((-a**3*log(tan(c + d*x)**2 + 1)/(2*d) + a**3*tan(c + d*x)**6*sec
(c + d*x)**3/(9*d) + 3*a**3*tan(c + d*x)**6*sec(c + d*x)**2/(8*d) + 3*a**3
*tan(c + d*x)**6*sec(c + d*x)/(7*d) + a**3*tan(c + d*x)**6/(6*d) - 2*a**3*
tan(c + d*x)**4*sec(c + d*x)**3/(21*d) - 3*a**3*tan(c + d*x)**4*sec(c + d*
x)**2/(8*d) - 18*a**3*tan(c + d*x)**4*sec(c + d*x)/(35*d) - a**3*tan(c + d
*x)**4/(4*d) + 8*a**3*tan(c + d*x)**2*sec(c + d*x)**3/(105*d) + 3*a**3*tan
(c + d*x)**2*sec(c + d*x)**2/(8*d) + 24*a**3*tan(c + d*x)**2*sec(c + d*x)/
(35*d) + a**3*tan(c + d*x)**2/(2*d) - 16*a**3*sec(c + d*x)**3/(315*d) - 3*
a**3*sec(c + d*x)**2/(8*d) - 48*a**3*sec(c + d*x)/(35*d), Ne(d, 0)), (x*(a
*sec(c) + a)**3*tan(c)**7, True))
```

3.38.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.80

$$\int (a + a\sec(c+dx))^3 \tan^7(c+dx)\,dx$$

$$= \frac{360\,a^3 \log\left(\cos(dx+c)\right) - \frac{1080\,a^3\cos(dx+c)^8 - 960\,a^3\cos(dx+c)^6 - 540\,a^3\cos(dx+c)^5 + 432\,a^3\cos(dx+c)^4 + 480\,a^3\cos(dx+c)^3 - 135\,a^3}{\cos(dx+c)^9}}{360\,d}$$

input
```
integrate((a+a*sec(d*x+c))^3*tan(d*x+c)^7,x, algorithm="maxima")
```

output
```
1/360*(360*a^3*log(cos(d*x + c)) - (1080*a^3*cos(d*x + c)^8 - 960*a^3*cos(
d*x + c)^6 - 540*a^3*cos(d*x + c)^5 + 432*a^3*cos(d*x + c)^4 + 480*a^3*cos
(d*x + c)^3 - 135*a^3*cos(d*x + c) - 40*a^3)/cos(d*x + c)^9)/d
```

3.38.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 317 vs. $2(125) = 250$.

Time = 3.83 (sec) , antiderivative size = 317, normalized size of antiderivative = 2.31

$$\int (a + a\sec(c+dx))^3 \tan^7(c+dx)\,dx =$$

$$-\frac{2520\,a^3 \log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right) - 2520\,a^3 \log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right) + \frac{14297\,a^3+\frac{133713\,a^3(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{560052\,a^3(}{(\cos(d}}{}$$

input
```
integrate((a+a*sec(d*x+c))^3*tan(d*x+c)^7,x, algorithm="giac")
```

output
```
-1/2520*(2520*a^3*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)) - 2
520*a^3*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)) + (14297*a^3
+ 133713*a^3*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 560052*a^3*(cos(d*x +
 c) - 1)^2/(cos(d*x + c) + 1)^2 + 1384068*a^3*(cos(d*x + c) - 1)^3/(cos(d*
x + c) + 1)^3 + 1594782*a^3*(cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4 + 13
36734*a^3*(cos(d*x + c) - 1)^5/(cos(d*x + c) + 1)^5 + 781956*a^3*(cos(d*x
+ c) - 1)^6/(cos(d*x + c) + 1)^6 + 302004*a^3*(cos(d*x + c) - 1)^7/(cos(d*
x + c) + 1)^7 + 69201*a^3*(cos(d*x + c) - 1)^8/(cos(d*x + c) + 1)^8 + 7129
*a^3*(cos(d*x + c) - 1)^9/(cos(d*x + c) + 1)^9)/((cos(d*x + c) - 1)/(cos(d
*x + c) + 1) + 1)^9)/d
```

3.38.9 Mupad [B] (verification not implemented)

Time = 17.91 (sec) , antiderivative size = 278, normalized size of antiderivative = 2.03

$$\int (a + a\sec(c+dx))^3 \tan^7(c+dx)\,dx$$

$$= \frac{2\,a^3 \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{16} - 18\,a^3 \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{14} + \frac{218\,a^3 \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{12}}{3} - 174\,a^3 \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{10} + \frac{1382\,a^3 \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{}}{5}}{d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{18} - 9\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{16} + 36\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{14} - 84\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{12} + 126\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{10} - 12\right.}$$

$$-\frac{2\,a^3\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2\right)}{d}$$

input
```
int(tan(c + d*x)^7*(a + a/cos(c + d*x))^3,x)
```

output

```
((602*a^3*tan(c/2 + (d*x)/2)^4)/5 - (138*a^3*tan(c/2 + (d*x)/2)^2)/5 - (15
58*a^3*tan(c/2 + (d*x)/2)^6)/5 + (1382*a^3*tan(c/2 + (d*x)/2)^8)/5 - 174*a
^3*tan(c/2 + (d*x)/2)^10 + (218*a^3*tan(c/2 + (d*x)/2)^12)/3 - 18*a^3*tan(
c/2 + (d*x)/2)^14 + 2*a^3*tan(c/2 + (d*x)/2)^16 + (128*a^3)/45)/(d*(9*tan(
c/2 + (d*x)/2)^2 - 36*tan(c/2 + (d*x)/2)^4 + 84*tan(c/2 + (d*x)/2)^6 - 126
*tan(c/2 + (d*x)/2)^8 + 126*tan(c/2 + (d*x)/2)^10 - 84*tan(c/2 + (d*x)/2)^
12 + 36*tan(c/2 + (d*x)/2)^14 - 9*tan(c/2 + (d*x)/2)^16 + tan(c/2 + (d*x)/
2)^18 - 1)) - (2*a^3*atanh(tan(c/2 + (d*x)/2)^2))/d
```

3.39 $\int (a + a\sec(c+dx))^3 \tan^5(c+dx)\,dx$

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3.39.1 Optimal result

Integrand size = 21, antiderivative size = 138

$$\int (a + a\sec(c+dx))^3 \tan^5(c+dx)\,dx = -\frac{a^3 \log(\cos(c+dx))}{d} + \frac{3a^3 \sec(c+dx)}{d} + \frac{a^3 \sec^2(c+dx)}{2d} - \frac{5a^3 \sec^3(c+dx)}{3d} - \frac{5a^3 \sec^4(c+dx)}{4d} + \frac{a^3 \sec^5(c+dx)}{5d} + \frac{a^3 \sec^6(c+dx)}{2d} + \frac{a^3 \sec^7(c+dx)}{7d}$$

output
```
-a^3*ln(cos(d*x+c))/d+3*a^3*sec(d*x+c)/d+1/2*a^3*sec(d*x+c)^2/d-5/3*a^3*se
c(d*x+c)^3/d-5/4*a^3*sec(d*x+c)^4/d+1/5*a^3*sec(d*x+c)^5/d+1/2*a^3*sec(d*x
+c)^6/d+1/7*a^3*sec(d*x+c)^7/d
```

3.39.2 Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.01

$$\int (a + a\sec(c+dx))^3 \tan^5(c+dx)\,dx =$$
$$-\frac{a^3(-3732 - 4522\cos(2(c+dx)) + 1050\cos(3(c+dx)) - 2380\cos(4(c+dx)) - 210\cos(5(c+dx)) - 6$$

input
```
Integrate[(a + a*Sec[c + d*x])^3*Tan[c + d*x]^5,x]
```

output
```
-1/6720*(a^3*(-3732 - 4522*Cos[2*(c + d*x)] + 1050*Cos[3*(c + d*x)] - 2380
*Cos[4*(c + d*x)] - 210*Cos[5*(c + d*x)] - 630*Cos[6*(c + d*x)] + 2205*Cos
[3*(c + d*x)]*Log[Cos[c + d*x]] + 735*Cos[5*(c + d*x)]*Log[Cos[c + d*x]] +
 105*Cos[7*(c + d*x)]*Log[Cos[c + d*x]] + 105*Cos[c + d*x]*(8 + 35*Log[Cos
[c + d*x]]))*Sec[c + d*x]^7)/d
```

3.39.3 Rubi [A] (verified)

Time $= 0.28$ (sec) , antiderivative size $= 96$, normalized size of antiderivative $= 0.70$, number of steps used $= 7$, number of rules used $= 6$, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used $= \{3042, 25, 4367, 27, 99, 2009\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^5(c+dx)(a\sec(c+dx)+a)^3\,dx$$

$$\downarrow 3042$$

$$\int -\cot\left(c+dx+\frac{\pi}{2}\right)^5\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^3 dx$$

$$\downarrow 25$$

$$-\int \cot\left(\frac{1}{2}(2c+\pi)+dx\right)^5\left(\csc\left(\frac{1}{2}(2c+\pi)+dx\right)a+a\right)^3 dx$$

$$\downarrow 4367$$

$$-\frac{\int a^7(1-\cos(c+dx))^2(\cos(c+dx)+1)^5\sec^8(c+dx)d\cos(c+dx)}{a^4 d}$$

$$\downarrow 27$$

$$-\frac{a^3\int(1-\cos(c+dx))^2(\cos(c+dx)+1)^5\sec^8(c+dx)d\cos(c+dx)}{d}$$

$$\downarrow 99$$

$$-\frac{a^3\int\left(\sec^8(c+dx)+3\sec^7(c+dx)+\sec^6(c+dx)-5\sec^5(c+dx)-5\sec^4(c+dx)+\sec^3(c+dx)+3\sec^2(c+\right.}{d}$$

$$\downarrow 2009$$

$$-\frac{a^3\left(-\frac{1}{7}\sec^7(c+dx)-\frac{1}{2}\sec^6(c+dx)-\frac{1}{5}\sec^5(c+dx)+\frac{5}{4}\sec^4(c+dx)+\frac{5}{3}\sec^3(c+dx)-\frac{1}{2}\sec^2(c+dx)-3\sec\right.}{d}$$

input
```
Int[(a + a*Sec[c + d*x])^3*Tan[c + d*x]^5,x]
```

output
```
-((a^3*(Log[Cos[c + d*x]] - 3*Sec[c + d*x] - Sec[c + d*x]^2/2 + (5*Sec[c +
 d*x]^3)/3 + (5*Sec[c + d*x]^4)/4 - Sec[c + d*x]^5/5 - Sec[c + d*x]^6/2 -
Sec[c + d*x]^7/7))/d)
```

3.39.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4367
```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Simp[1/(a^(m - n - 1)*b^n*d)   Subst[Int[(a - b*x)^((m -
1)/2)*((a + b*x)^((m - 1)/2 + n)/x^(m + n)), x], x, Sin[c + d*x]], x] /; Fr
eeQ[{a, b, c, d}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && Integer
Q[n]
```

3.39.4 Maple [A] (verified)

Time = 1.66 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.61

method	result
derivativedivides	$\frac{a^3\left(\frac{\sec(dx+c)^7}{7}+\frac{\sec(dx+c)^6}{2}+\frac{\sec(dx+c)^5}{5}-\frac{5\sec(dx+c)^4}{4}-\frac{5\sec(dx+c)^3}{3}+\frac{\sec(dx+c)^2}{2}+3\sec(dx+c)+\ln(\sec(dx+c))\right)}{d}$
default	$\frac{a^3\left(\frac{\sec(dx+c)^7}{7}+\frac{\sec(dx+c)^6}{2}+\frac{\sec(dx+c)^5}{5}-\frac{5\sec(dx+c)^4}{4}-\frac{5\sec(dx+c)^3}{3}+\frac{\sec(dx+c)^2}{2}+3\sec(dx+c)+\ln(\sec(dx+c))\right)}{d}$
parts	$\frac{a^3\left(\frac{\tan(dx+c)^4}{4}-\frac{\tan(dx+c)^2}{2}+\frac{\ln\left(1+\tan(dx+c)^2\right)}{2}\right)}{d}+\frac{a^3\left(\frac{\sec(dx+c)^7}{7}-\frac{2\sec(dx+c)^5}{5}+\frac{\sec(dx+c)^3}{3}\right)}{d}+\frac{3a^3\left(\frac{\sec(dx+c)^5}{5}-$
risch	$ia^3x+\frac{2ia^3c}{d}+\frac{2a^3\left(315\,e^{13i(dx+c)}+105\,e^{12i(dx+c)}+1190\,e^{11i(dx+c)}-525\,e^{10i(dx+c)}+2261\,e^{9i(dx+c)}-420\,e^{8i(dx+c)}\right.}{105d}$

input
```
int((a+a*sec(d*x+c))^3*tan(d*x+c)^5,x,method=_RETURNVERBOSE)
```

output
```
a^3/d*(1/7*sec(d*x+c)^7+1/2*sec(d*x+c)^6+1/5*sec(d*x+c)^5-5/4*sec(d*x+c)^4
-5/3*sec(d*x+c)^3+1/2*sec(d*x+c)^2+3*sec(d*x+c)+ln(sec(d*x+c)))
```

3.39.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.85

$$\int (a+a\sec(c+dx))^3\tan^5(c+dx)\,dx =$$
$$-\frac{420\,a^3\cos(dx+c)^7\log(-\cos(dx+c))-1260\,a^3\cos(dx+c)^6-210\,a^3\cos(dx+c)^5+700\,a^3\cos(dx+c)^4+525\,a^3\cos(dx+c)^3-84\,a^3\cos(dx+c)^2-210\,a^3\cos(dx+c)-60\,a^3}{420\,d\cos(dx+c)^7}$$

input
```
integrate((a+a*sec(d*x+c))^3*tan(d*x+c)^5,x, algorithm="fricas")
```

output
```
-1/420*(420*a^3*cos(d*x + c)^7*log(-cos(d*x + c)) - 1260*a^3*cos(d*x + c)^
6 - 210*a^3*cos(d*x + c)^5 + 700*a^3*cos(d*x + c)^4 + 525*a^3*cos(d*x + c)
^3 - 84*a^3*cos(d*x + c)^2 - 210*a^3*cos(d*x + c) - 60*a^3)/(d*cos(d*x + c
)^7)
```

3.39.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 255 vs. $2(121) = 242$.

Time $= 0.87$ (sec) , antiderivative size $= 255$, normalized size of antiderivative $= 1.85$

$$\int (a + a\sec(c+dx))^3 \tan^5(c+dx)\,dx$$

$$= \begin{cases} \frac{a^3 \log\left(\tan^2(c+dx)+1\right)}{2d} + \frac{a^3 \tan^4(c+dx)\sec^3(c+dx)}{7d} + \frac{a^3 \tan^4(c+dx)\sec^2(c+dx)}{2d} + \frac{3a^3 \tan^4(c+dx)\sec(c+dx)}{5d} + \frac{a^3 \tan^4(c+dx)}{4d} - \\ x(a\sec(c)+a)^3 \tan^5(c) \end{cases}$$

input
```
integrate((a+a*sec(d*x+c))**3*tan(d*x+c)**5,x)
```

output
```
Piecewise((a**3*log(tan(c + d*x)**2 + 1)/(2*d) + a**3*tan(c + d*x)**4*sec(
c + d*x)**3/(7*d) + a**3*tan(c + d*x)**4*sec(c + d*x)**2/(2*d) + 3*a**3*ta
n(c + d*x)**4*sec(c + d*x)/(5*d) + a**3*tan(c + d*x)**4/(4*d) - 4*a**3*tan
(c + d*x)**2*sec(c + d*x)**3/(35*d) - a**3*tan(c + d*x)**2*sec(c + d*x)**2
/(2*d) - 4*a**3*tan(c + d*x)**2*sec(c + d*x)/(5*d) - a**3*tan(c + d*x)**2/
(2*d) + 8*a**3*sec(c + d*x)**3/(105*d) + a**3*sec(c + d*x)**2/(2*d) + 8*a*
*3*sec(c + d*x)/(5*d), Ne(d, 0)), (x*(a*sec(c) + a)**3*tan(c)**5, True))
```

3.39.7 Maxima [A] (verification not implemented)

Time $= 0.21$ (sec) , antiderivative size $= 110$, normalized size of antiderivative $= 0.80$

$$\int (a + a\sec(c+dx))^3 \tan^5(c+dx)\,dx =$$

$$-\frac{420\,a^3 \log\left(\cos(dx+c)\right) - \frac{1260\,a^3\cos(dx+c)^6 + 210\,a^3\cos(dx+c)^5 - 700\,a^3\cos(dx+c)^4 - 525\,a^3\cos(dx+c)^3 + 84\,a^3\cos(dx+c)^2 + 210}{\cos(dx+c)^7}}{420\,d}$$

input
```
integrate((a+a*sec(d*x+c))^3*tan(d*x+c)^5,x, algorithm="maxima")
```

output
```
-1/420*(420*a^3*log(cos(d*x + c)) - (1260*a^3*cos(d*x + c)^6 + 210*a^3*cos
(d*x + c)^5 - 700*a^3*cos(d*x + c)^4 - 525*a^3*cos(d*x + c)^3 + 84*a^3*cos
(d*x + c)^2 + 210*a^3*cos(d*x + c) + 60*a^3)/cos(d*x + c)^7)/d
```

3.39.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 267 vs. $2(126) = 252$.

Time $= 2.03$ (sec) , antiderivative size $= 267$, normalized size of antiderivative $= 1.93$

$$\int (a + a\sec(c+dx))^3 \tan^5(c+dx)\,dx$$

$$= \frac{420\,a^3 \log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right) - 420\,a^3 \log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right) + \frac{2497\,a^3 + \frac{18319\,a^3(\cos(dx+c)-1)}{\cos(dx+c)+1} + \frac{58317\,a^3(\cos(dx+c)-}{(\cos(dx+c)+1)^2}}{420\,d}$$

input
```
integrate((a+a*sec(d*x+c))^3*tan(d*x+c)^5,x, algorithm="giac")
```

output
```
1/420*(420*a^3*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)) - 420*
a^3*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)) + (2497*a^3 + 183
19*a^3*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 58317*a^3*(cos(d*x + c) - 1
)^2/(cos(d*x + c) + 1)^2 + 69475*a^3*(cos(d*x + c) - 1)^3/(cos(d*x + c) +
1)^3 + 56035*a^3*(cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4 + 28749*a^3*(co
s(d*x + c) - 1)^5/(cos(d*x + c) + 1)^5 + 8463*a^3*(cos(d*x + c) - 1)^6/(co
s(d*x + c) + 1)^6 + 1089*a^3*(cos(d*x + c) - 1)^7/(cos(d*x + c) + 1)^7)/((
cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)^7)/d
```

3.39.9 Mupad [B] (verification not implemented)

Time $= 17.65$ (sec) , antiderivative size $= 221$, normalized size of antiderivative $= 1.60$

$$\int (a + a\sec(c+dx))^3 \tan^5(c+dx)\,dx = \frac{2\,a^3\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2\right)}{d}$$

$$- \frac{2\,a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{12} - 14\,a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{10} + \frac{128\,a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^8}{3} - \frac{224\,a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6}{3} + \frac{422\,a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right.}{5}}{d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{14} - 7\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{12} + 21\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{10} - 35\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^8 + 35\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6 - 21\right.}$$

input
```
int(tan(c + d*x)^5*(a + a/cos(c + d*x))^3,x)
```

output

```
(2*a^3*atanh(tan(c/2 + (d*x)/2)^2))/d - ((422*a^3*tan(c/2 + (d*x)/2)^4)/5
- (382*a^3*tan(c/2 + (d*x)/2)^2)/15 - (224*a^3*tan(c/2 + (d*x)/2)^6)/3 + (
128*a^3*tan(c/2 + (d*x)/2)^8)/3 - 14*a^3*tan(c/2 + (d*x)/2)^10 + 2*a^3*tan
(c/2 + (d*x)/2)^12 + (352*a^3)/105)/(d*(7*tan(c/2 + (d*x)/2)^2 - 21*tan(c/
2 + (d*x)/2)^4 + 35*tan(c/2 + (d*x)/2)^6 - 35*tan(c/2 + (d*x)/2)^8 + 21*ta
n(c/2 + (d*x)/2)^10 - 7*tan(c/2 + (d*x)/2)^12 + tan(c/2 + (d*x)/2)^14 - 1)
)
```

3.40 $\int (a + a\sec(c+dx))^3 \tan^3(c+dx)\,dx$

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3.40.1 Optimal result

Integrand size = 21, antiderivative size = 99

$$\int (a + a\sec(c+dx))^3 \tan^3(c+dx)\,dx = \frac{a^3 \log(\cos(c+dx))}{d} - \frac{3a^3 \sec(c+dx)}{d} - \frac{a^3 \sec^2(c+dx)}{d} + \frac{2a^3 \sec^3(c+dx)}{3d} + \frac{3a^3 \sec^4(c+dx)}{4d} + \frac{a^3 \sec^5(c+dx)}{5d}$$

output
```
a^3*ln(cos(d*x+c))/d-3*a^3*sec(d*x+c)/d-a^3*sec(d*x+c)^2/d+2/3*a^3*sec(d*x
+c)^3/d+3/4*a^3*sec(d*x+c)^4/d+1/5*a^3*sec(d*x+c)^5/d
```

3.40.2 Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.93

$$\int (a + a\sec(c+dx))^3 \tan^3(c+dx)\,dx = -\frac{a^3(142 + 280\cos(2(c+dx)) + 90\cos(4(c+dx)) + \cos(3(c+dx))(60 - 75\log(\cos(c+dx))) - 150\cos(}{240d}$$

input
```
Integrate[(a + a*Sec[c + d*x])^3*Tan[c + d*x]^3,x]
```

output
```
-1/240*(a^3*(142 + 280*Cos[2*(c + d*x)] + 90*Cos[4*(c + d*x)] + Cos[3*(c +
 d*x)]*(60 - 75*Log[Cos[c + d*x]]) - 150*Cos[c + d*x]*Log[Cos[c + d*x]] -
15*Cos[5*(c + d*x)]*Log[Cos[c + d*x]])*Sec[c + d*x]^5)/d
```

3.40.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.71, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 25, 4367, 27, 84, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^3(c+dx)(a\sec(c+dx)+a)^3\,dx$$

$$\downarrow 3042$$

$$\int -\cot\left(c+dx+\frac{\pi}{2}\right)^3\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^3 dx$$

$$\downarrow 25$$

$$-\int \cot\left(\frac{1}{2}(2c+\pi)+dx\right)^3\left(\csc\left(\frac{1}{2}(2c+\pi)+dx\right)a+a\right)^3 dx$$

$$\downarrow 4367$$

$$-\frac{\int a^5(1-\cos(c+dx))(\cos(c+dx)+1)^4\sec^6(c+dx)d\cos(c+dx)}{a^2 d}$$

$$\downarrow 27$$

$$-\frac{a^3\int(1-\cos(c+dx))(\cos(c+dx)+1)^4\sec^6(c+dx)d\cos(c+dx)}{d}$$

$$\downarrow 84$$

$$-\frac{a^3\int\left(\sec^6(c+dx)+3\sec^5(c+dx)+2\sec^4(c+dx)-2\sec^3(c+dx)-3\sec^2(c+dx)-\sec(c+dx)\right)d\cos(c+d}{d}$$

$$\downarrow 2009$$

$$-\frac{a^3\left(-\frac{1}{5}\sec^5(c+dx)-\frac{3}{4}\sec^4(c+dx)-\frac{2}{3}\sec^3(c+dx)+\sec^2(c+dx)+3\sec(c+dx)-\log(\cos(c+dx))\right)}{d}$$

input
```
Int[(a + a*Sec[c + d*x])^3*Tan[c + d*x]^3,x]
```

output
```
-((a^3*(-Log[Cos[c + d*x]] + 3*Sec[c + d*x] + Sec[c + d*x]^2 - (2*Sec[c +
d*x]^3)/3 - (3*Sec[c + d*x]^4)/4 - Sec[c + d*x]^5/5))/d)
```

3.40.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 84
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && EqQ[b*e + a*f, 0] &&  !(ILtQ[n + p + 2, 0
] && GtQ[n + 2*p, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4367
```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Simp[1/(a^(m - n - 1)*b^n*d)   Subst[Int[(a - b*x)^((m -
1)/2)*((a + b*x)^((m - 1)/2 + n)/x^(m + n)), x], x, Sin[c + d*x]], x] /; Fr
eeQ[{a, b, c, d}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && Integer
Q[n]
```

3.40.4 Maple [A] (verified)

Time = 1.48 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.67

method	result
derivativedivides	$\frac{a^3\left(\frac{\sec(dx+c)^5}{5}+\frac{3\sec(dx+c)^4}{4}+\frac{2\sec(dx+c)^3}{3}-\sec(dx+c)^2-3\sec(dx+c)-\ln(\sec(dx+c))\right)}{d}$
default	$\frac{a^3\left(\frac{\sec(dx+c)^5}{5}+\frac{3\sec(dx+c)^4}{4}+\frac{2\sec(dx+c)^3}{3}-\sec(dx+c)^2-3\sec(dx+c)-\ln(\sec(dx+c))\right)}{d}$
parts	$\frac{a^3\left(\frac{\tan(dx+c)^2}{2}-\frac{\ln\left(1+\tan(dx+c)^2\right)}{2}\right)}{d}+\frac{a^3\left(\frac{\sec(dx+c)^5}{5}-\frac{\sec(dx+c)^3}{3}\right)}{d}+\frac{3a^3\left(\frac{\sec(dx+c)^3}{3}-\sec(dx+c)\right)}{d}+\frac{3a^3\tan(d}{4d}$
risch	$-ia^3x-\frac{2ia^3c}{d}-\frac{2a^3\left(45\,e^{9i(dx+c)}+30\,e^{8i(dx+c)}+140\,e^{7i(dx+c)}+142\,e^{5i(dx+c)}+140\,e^{3i(dx+c)}+30\,e^{2i(dx+c)}+45\,e^{i(}\right.}{15d\left(e^{2i(dx+c)}+1\right)^5}$

input
```
int((a+a*sec(d*x+c))^3*tan(d*x+c)^3,x,method=_RETURNVERBOSE)
```

output
```
a^3/d*(1/5*sec(d*x+c)^5+3/4*sec(d*x+c)^4+2/3*sec(d*x+c)^3-sec(d*x+c)^2-3*s
ec(d*x+c)-ln(sec(d*x+c)))
```

3.40.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.92

$$\int (a + a\sec(c + dx))^3 \tan^3(c + dx)\, dx$$
$$= \frac{60\, a^3 \cos(dx + c)^5 \log(-\cos(dx + c)) - 180\, a^3 \cos(dx + c)^4 - 60\, a^3 \cos(dx + c)^3 + 40\, a^3 \cos(dx + c)^2 +}{60\, d \cos(dx + c)^5}$$

input
```
integrate((a+a*sec(d*x+c))^3*tan(d*x+c)^3,x, algorithm="fricas")
```

output
```
1/60*(60*a^3*cos(d*x + c)^5*log(-cos(d*x + c)) - 180*a^3*cos(d*x + c)^4 -
60*a^3*cos(d*x + c)^3 + 40*a^3*cos(d*x + c)^2 + 45*a^3*cos(d*x + c) + 12*a
^3)/(d*cos(d*x + c)^5)
```

3.40.6 Sympy [A] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.67

$$\int (a + a\sec(c + dx))^3 \tan^3(c + dx)\, dx$$
$$= \begin{cases} -\frac{a^3 \log(\tan^2(c+dx)+1)}{2d} + \frac{a^3 \tan^2(c+dx)\sec^3(c+dx)}{5d} + \frac{3a^3 \tan^2(c+dx)\sec^2(c+dx)}{4d} + \frac{a^3 \tan^2(c+dx)\sec(c+dx)}{d} + \frac{a^3 \tan^2(c+dx)}{2d} \\ x(a\sec(c) + a)^3 \tan^3(c) \end{cases}$$

input
```
integrate((a+a*sec(d*x+c))**3*tan(d*x+c)**3,x)
```

output
```
Piecewise((-a**3*log(tan(c + d*x)**2 + 1)/(2*d) + a**3*tan(c + d*x)**2*sec
(c + d*x)**3/(5*d) + 3*a**3*tan(c + d*x)**2*sec(c + d*x)**2/(4*d) + a**3*t
an(c + d*x)**2*sec(c + d*x)/d + a**3*tan(c + d*x)**2/(2*d) - 2*a**3*sec(c
+ d*x)**3/(15*d) - 3*a**3*sec(c + d*x)**2/(4*d) - 2*a**3*sec(c + d*x)/d, N
e(d, 0)), (x*(a*sec(c) + a)**3*tan(c)**3, True))
```

3.40.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.85

$$\int (a + a\sec(c+dx))^3 \tan^3(c+dx)\,dx$$
$$= \frac{60\,a^3 \log(\cos(dx+c)) - \frac{180\,a^3\cos(dx+c)^4 + 60\,a^3\cos(dx+c)^3 - 40\,a^3\cos(dx+c)^2 - 45\,a^3\cos(dx+c) - 12\,a^3}{\cos(dx+c)^5}}{60\,d}$$

input
```
integrate((a+a*sec(d*x+c))^3*tan(d*x+c)^3,x, algorithm="maxima")
```

output
```
1/60*(60*a^3*log(cos(d*x + c)) - (180*a^3*cos(d*x + c)^4 + 60*a^3*cos(d*x
+ c)^3 - 40*a^3*cos(d*x + c)^2 - 45*a^3*cos(d*x + c) - 12*a^3)/cos(d*x + c
)^5)/d
```

3.40.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 217 vs. 2(93) = 186.

Time = 0.93 (sec) , antiderivative size = 217, normalized size of antiderivative = 2.19

$$\int (a + a\sec(c+dx))^3 \tan^3(c+dx)\,dx =$$
$$-\frac{60\,a^3 \log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right) - 60\,a^3 \log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right) + \frac{393\,a^3 + \frac{2085\,a^3(\cos(dx+c)-1)}{\cos(dx+c)+1} + \frac{2610\,a^3(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}}{(}}{60\,d}$$

input
```
integrate((a+a*sec(d*x+c))^3*tan(d*x+c)^3,x, algorithm="giac")
```

output
```
-1/60*(60*a^3*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)) - 60*a^
3*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)) + (393*a^3 + 2085*a
^3*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 2610*a^3*(cos(d*x + c) - 1)^2/(
cos(d*x + c) + 1)^2 + 1970*a^3*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 +
 805*a^3*(cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4 + 137*a^3*(cos(d*x + c)
 - 1)^5/(cos(d*x + c) + 1)^5)/((cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)^
5)/d
```

3.40.9 Mupad [B] (verification not implemented)

Time = 18.07 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.64

$$\int (a + a\sec(c + dx))^3 \tan^3(c + dx)\, dx$$

$$= \frac{2a^3 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^8 - 10a^3 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^6 + \frac{62a^3 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^4}{3} - \frac{70a^3 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^2}{3} + \frac{64a^3}{15}}{d\left(\tan\left(\frac{c}{2} + \frac{dx}{2}\right)^{10} - 5\tan\left(\frac{c}{2} + \frac{dx}{2}\right)^8 + 10\tan\left(\frac{c}{2} + \frac{dx}{2}\right)^6 - 10\tan\left(\frac{c}{2} + \frac{dx}{2}\right)^4 + 5\tan\left(\frac{c}{2} + \frac{dx}{2}\right)^2 - 1\right)} - \frac{2a^3 \operatorname{atanh}\left(\tan\left(\frac{c}{2} + \frac{dx}{2}\right)^2\right)}{d}$$

input
```
int(tan(c + d*x)^3*(a + a/cos(c + d*x))^3,x)
```

output
```
((62*a^3*tan(c/2 + (d*x)/2)^4)/3 - (70*a^3*tan(c/2 + (d*x)/2)^2)/3 - 10*a^
3*tan(c/2 + (d*x)/2)^6 + 2*a^3*tan(c/2 + (d*x)/2)^8 + (64*a^3)/15)/(d*(5*t
an(c/2 + (d*x)/2)^2 - 10*tan(c/2 + (d*x)/2)^4 + 10*tan(c/2 + (d*x)/2)^6 -
5*tan(c/2 + (d*x)/2)^8 + tan(c/2 + (d*x)/2)^10 - 1)) - (2*a^3*atanh(tan(c/
2 + (d*x)/2)^2))/d
```

3.41 $\int (a + a\sec(c+dx))^3 \tan(c+dx)\,dx$

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3.41.1 Optimal result

Integrand size = 19, antiderivative size = 66

$$\int (a + a\sec(c+dx))^3 \tan(c+dx)\,dx = -\frac{a^3 \log(\cos(c+dx))}{d} + \frac{3a^3 \sec(c+dx)}{d} + \frac{3a^3 \sec^2(c+dx)}{2d} + \frac{a^3 \sec^3(c+dx)}{3d}$$

output
```
-a^3*ln(cos(d*x+c))/d+3*a^3*sec(d*x+c)/d+3/2*a^3*sec(d*x+c)^2/d+1/3*a^3*se
c(d*x+c)^3/d
```

3.41.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.97

$$\int (a + a\sec(c+dx))^3 \tan(c+dx)\,dx = -\frac{a^3(-22 - 18\cos(2(c+dx)) + 9\cos(c+dx)(-2 + \log(\cos(c+dx))) + 3\cos(3(c+dx))\log(\cos(c+dx)}{12d}$$

input
```
Integrate[(a + a*Sec[c + d*x])^3*Tan[c + d*x],x]
```

output
```
-1/12*(a^3*(-22 - 18*Cos[2*(c + d*x)] + 9*Cos[c + d*x]*(-2 + Log[Cos[c + d
*x]]) + 3*Cos[3*(c + d*x)]*Log[Cos[c + d*x]])*Sec[c + d*x]^3)/d
```

3.41.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.73, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {3042, 25, 4367, 27, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan(c+dx)(a\sec(c+dx)+a)^3\,dx$$

$$\downarrow 3042$$

$$\int -\cot\left(c+dx+\frac{\pi}{2}\right)\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^3 dx$$

$$\downarrow 25$$

$$-\int \cot\left(\frac{1}{2}(2c+\pi)+dx\right)\left(\csc\left(\frac{1}{2}(2c+\pi)+dx\right)a+a\right)^3 dx$$

$$\downarrow 4367$$

$$-\frac{\int a^3(\cos(c+dx)+1)^3\sec^4(c+dx)d\cos(c+dx)}{d}$$

$$\downarrow 27$$

$$-\frac{a^3\int(\cos(c+dx)+1)^3\sec^4(c+dx)d\cos(c+dx)}{d}$$

$$\downarrow 49$$

$$-\frac{a^3\int\left(\sec^4(c+dx)+3\sec^3(c+dx)+3\sec^2(c+dx)+\sec(c+dx)\right)d\cos(c+dx)}{d}$$

$$\downarrow 2009$$

$$-\frac{a^3\left(-\frac{1}{3}\sec^3(c+dx)-\frac{3}{2}\sec^2(c+dx)-3\sec(c+dx)+\log(\cos(c+dx))\right)}{d}$$

input
```
Int[(a + a*Sec[c + d*x])^3*Tan[c + d*x],x]
```

output
```
-((a^3*(Log[Cos[c + d*x]] - 3*Sec[c + d*x] - (3*Sec[c + d*x]^2)/2 - Sec[c
+ d*x]^3/3))/d)
```

3.41.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4367
```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Simp[1/(a^(m - n - 1)*b^n*d)   Subst[Int[(a - b*x)^((m -
1)/2)*((a + b*x)^((m - 1)/2 + n)/x^(m + n)), x], x, Sin[c + d*x]], x] /; Fr
eeQ[{a, b, c, d}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && Integer
Q[n]
```

3.41.4 Maple [A] (verified)

Time = 0.69 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.67

method	result	size
derivativedivides	$\frac{a^3\left(\frac{\sec(dx+c)^3}{3}+\frac{3\sec(dx+c)^2}{2}+3\sec(dx+c)+\ln(\sec(dx+c))\right)}{d}$	44
default	$\frac{a^3\left(\frac{\sec(dx+c)^3}{3}+\frac{3\sec(dx+c)^2}{2}+3\sec(dx+c)+\ln(\sec(dx+c))\right)}{d}$	44
parts	$\frac{a^3\ln\left(1+\tan(dx+c)^2\right)}{2d}+\frac{a^3\sec(dx+c)^3}{3d}+\frac{3a^3\sec(dx+c)}{d}+\frac{3a^3\sec(dx+c)^2}{2d}$	67
risch	$ia^3x+\frac{2ia^3c}{d}+\frac{2a^3\left(9\,e^{5i(dx+c)}+9\,e^{4i(dx+c)}+22\,e^{3i(dx+c)}+9\,e^{2i(dx+c)}+9\,e^{i(dx+c)}\right)}{3d\left(e^{2i(dx+c)}+1\right)^3}-\frac{a^3\ln\left(e^{2i(dx+c)}+1\right)}{d}$	116

input
```
int((a+a*sec(d*x+c))^3*tan(d*x+c),x,method=_RETURNVERBOSE)
```

output
```
1/d*a^3*(1/3*sec(d*x+c)^3+3/2*sec(d*x+c)^2+3*sec(d*x+c)+ln(sec(d*x+c)))
```

3.41.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.98

$$\int (a + a\sec(c + dx))^3 \tan(c + dx)\, dx$$
$$= -\frac{6\,a^3 \cos(dx + c)^3 \log(-\cos(dx + c)) - 18\,a^3 \cos(dx + c)^2 - 9\,a^3 \cos(dx + c) - 2\,a^3}{6\,d \cos(dx + c)^3}$$

input
```
integrate((a+a*sec(d*x+c))^3*tan(d*x+c),x, algorithm="fricas")
```

output
```
-1/6*(6*a^3*cos(d*x + c)^3*log(-cos(d*x + c)) - 18*a^3*cos(d*x + c)^2 - 9*
a^3*cos(d*x + c) - 2*a^3)/(d*cos(d*x + c)^3)
```

3.41.6 Sympy [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.15

$$\int (a + a\sec(c + dx))^3 \tan(c + dx)\, dx$$
$$= \begin{cases} \frac{a^3 \log(\tan^2(c+dx)+1)}{2d} + \frac{a^3 \sec^3(c+dx)}{3d} + \frac{3a^3 \sec^2(c+dx)}{2d} + \frac{3a^3 \sec(c+dx)}{d} & \text{for } d \neq 0 \\ x(a\sec(c) + a)^3 \tan(c) & \text{otherwise} \end{cases}$$

input
```
integrate((a+a*sec(d*x+c))**3*tan(d*x+c),x)
```

output
```
Piecewise((a**3*log(tan(c + d*x)**2 + 1)/(2*d) + a**3*sec(c + d*x)**3/(3*d
) + 3*a**3*sec(c + d*x)**2/(2*d) + 3*a**3*sec(c + d*x)/d, Ne(d, 0)), (x*(a
*sec(c) + a)**3*tan(c), True))
```

3.41.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.88

$$\int (a+a\sec(c+dx))^3\tan(c+dx)\,dx = -\frac{6\,a^3\log\left(\cos\left(dx+c\right)\right) - \frac{18\,a^3\cos(dx+c)^2+9\,a^3\cos(dx+c)+2\,a^3}{\cos(dx+c)^3}}{6\,d}$$

input
```
integrate((a+a*sec(d*x+c))^3*tan(d*x+c),x, algorithm="maxima")
```

output
```
-1/6*(6*a^3*log(cos(d*x + c)) - (18*a^3*cos(d*x + c)^2 + 9*a^3*cos(d*x + c
) + 2*a^3)/cos(d*x + c)^3)/d
```

3.41.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 167 vs. 2(62) = 124.

Time = 0.43 (sec) , antiderivative size = 167, normalized size of antiderivative = 2.53

$$\int (a+a\sec(c+dx))^3\tan(c+dx)\,dx$$

$$= \frac{6\,a^3\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right) - 6\,a^3\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right) + \frac{51\,a^3+\frac{69\,a^3(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{45\,a^3(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}+\frac{11\,a^3(\cos(}{\left(\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right)^3}}{6\,d}$$

input
```
integrate((a+a*sec(d*x+c))^3*tan(d*x+c),x, algorithm="giac")
```

output
```
1/6*(6*a^3*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)) - 6*a^3*lo
g(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)) + (51*a^3 + 69*a^3*(cos
(d*x + c) - 1)/(cos(d*x + c) + 1) + 45*a^3*(cos(d*x + c) - 1)^2/(cos(d*x +
 c) + 1)^2 + 11*a^3*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3)/((cos(d*x +
 c) - 1)/(cos(d*x + c) + 1) + 1)^3)/d
```

3.41.9 Mupad [B] (verification not implemented)

Time = 14.59 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.59

$$\int (a + a\sec(c + dx))^3 \tan(c + dx)\, dx$$
$$= \frac{2a^3 \operatorname{atanh}\left(\tan\left(\frac{c}{2} + \frac{dx}{2}\right)^2\right)}{d}$$
$$- \frac{2a^3 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^4 - 6a^3 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^2 + \frac{20a^3}{3}}{d\left(\tan\left(\frac{c}{2} + \frac{dx}{2}\right)^6 - 3\tan\left(\frac{c}{2} + \frac{dx}{2}\right)^4 + 3\tan\left(\frac{c}{2} + \frac{dx}{2}\right)^2 - 1\right)}$$

input
```
int(tan(c + d*x)*(a + a/cos(c + d*x))^3,x)
```

output
```
(2*a^3*atanh(tan(c/2 + (d*x)/2)^2))/d - (2*a^3*tan(c/2 + (d*x)/2)^4 - 6*a^
3*tan(c/2 + (d*x)/2)^2 + (20*a^3)/3)/(d*(3*tan(c/2 + (d*x)/2)^2 - 3*tan(c/
2 + (d*x)/2)^4 + tan(c/2 + (d*x)/2)^6 - 1))
```

3.42 $\int \cot(c+dx)(a+a\sec(c+dx))^3\,dx$

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3.42.1 Optimal result

Integrand size = 19, antiderivative size = 48

$$\int \cot(c+dx)(a+a\sec(c+dx))^3\,dx = \frac{4a^3\log(1-\cos(c+dx))}{d} - \frac{3a^3\log(\cos(c+dx))}{d} + \frac{a^3\sec(c+dx)}{d}$$

output
```
4*a^3*ln(1-cos(d*x+c))/d-3*a^3*ln(cos(d*x+c))/d+a^3*sec(d*x+c)/d
```

3.42.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.75

$$\int \cot(c+dx)(a+a\sec(c+dx))^3\,dx = \frac{a^3\left(-3\log(\cos(c+dx)) + 8\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right) + \sec(c+dx)\right)}{d}$$

input
```
Integrate[Cot[c + d*x]*(a + a*Sec[c + d*x])^3,x]
```

output
```
(a^3*(-3*Log[Cos[c + d*x]] + 8*Log[Sin[(c + d*x)/2]] + Sec[c + d*x]))/d
```

3.42.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.81, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {3042, 25, 4367, 27, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot(c+dx)(a\sec(c+dx)+a)^3\,dx$$

$$\downarrow 3042$$

$$\int -\frac{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^3}{\cot\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\downarrow 25$$

$$-\int \frac{\left(\csc\left(\frac{1}{2}(2c+\pi)+dx\right)a+a\right)^3}{\cot\left(\frac{1}{2}(2c+\pi)+dx\right)}dx$$

$$\downarrow 4367$$

$$-\frac{a^2\int \frac{a(\cos(c+dx)+1)^2\sec^2(c+dx)}{1-\cos(c+dx)}d\cos(c+dx)}{d}$$

$$\downarrow 27$$

$$-\frac{a^3\int \frac{(\cos(c+dx)+1)^2\sec^2(c+dx)}{1-\cos(c+dx)}d\cos(c+dx)}{d}$$

$$\downarrow 99$$

$$-\frac{a^3\int \left(\sec^2(c+dx)+3\sec(c+dx)-\frac{4}{\cos(c+dx)-1}\right)d\cos(c+dx)}{d}$$

$$\downarrow 2009$$

$$-\frac{a^3(-\sec(c+dx)-4\log(1-\cos(c+dx))+3\log(\cos(c+dx)))}{d}$$

input
```
Int[Cot[c + d*x]*(a + a*Sec[c + d*x])^3,x]
```

output
```
-((a^3*(-4*Log[1 - Cos[c + d*x]] + 3*Log[Cos[c + d*x]] - Sec[c + d*x]))/d)
```

3.42.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4367
```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Simp[1/(a^(m - n - 1)*b^n*d)   Subst[Int[(a - b*x)^((m -
1)/2)*((a + b*x)^((m - 1)/2 + n)/x^(m + n)), x], x, Sin[c + d*x]], x] /; Fr
eeQ[{a, b, c, d}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && Integer
Q[n]
```

3.42.4 Maple [A] (verified)

Time = 0.68 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.65

method	result
derivativedivides	$\frac{a^3\left(\frac{1}{\cos(dx+c)}+\ln(-\cot(dx+c)+\csc(dx+c))\right)+3a^3\ln(\tan(dx+c))+3a^3\ln(-\cot(dx+c)+\csc(dx+c))+a^3\ln(\sin(dx+c))}{d}$
default	$\frac{a^3\left(\frac{1}{\cos(dx+c)}+\ln(-\cot(dx+c)+\csc(dx+c))\right)+3a^3\ln(\tan(dx+c))+3a^3\ln(-\cot(dx+c)+\csc(dx+c))+a^3\ln(\sin(dx+c))}{d}$
risch	$-ia^3x-\frac{2ia^3c}{d}+\frac{2a^3e^{i(dx+c)}}{d\left(e^{2i(dx+c)}+1\right)}+\frac{8a^3\ln\left(e^{i(dx+c)}-1\right)}{d}-\frac{3a^3\ln\left(e^{2i(dx+c)}+1\right)}{d}$

input
```
int(cot(d*x+c)*(a+a*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^3*(1/cos(d*x+c)+ln(-cot(d*x+c)+csc(d*x+c)))+3*a^3*ln(tan(d*x+c))+3*
a^3*ln(-cot(d*x+c)+csc(d*x+c))+a^3*ln(sin(d*x+c)))
```

3.42.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.27

$$\int \cot(c+dx)(a+a\sec(c+dx))^3\,dx$$
$$= -\frac{3\,a^3\cos(dx+c)\log(-\cos(dx+c)) - 4\,a^3\cos(dx+c)\log\left(-\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right) - a^3}{d\cos(dx+c)}$$

input
```
integrate(cot(d*x+c)*(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

output
```
-(3*a^3*cos(d*x + c)*log(-cos(d*x + c)) - 4*a^3*cos(d*x + c)*log(-1/2*cos(
d*x + c) + 1/2) - a^3)/(d*cos(d*x + c))
```

3.42.6 Sympy [F]

$$\int \cot(c+dx)(a+a\sec(c+dx))^3\,dx = a^3\left(\int 3\cot(c+dx)\sec(c+dx)\,dx + \int 3\cot(c+dx)\sec^2(c+dx)\,dx + \int \cot(c+dx)\sec^3(c+dx)\,dx + \int \cot(c+dx)\,dx\right)$$

input
```
integrate(cot(d*x+c)*(a+a*sec(d*x+c))**3,x)
```

output
```
a**3*(Integral(3*cot(c + d*x)*sec(c + d*x), x) + Integral(3*cot(c + d*x)*s
ec(c + d*x)**2, x) + Integral(cot(c + d*x)*sec(c + d*x)**3, x) + Integral(
cot(c + d*x), x))
```

3.42.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.90

$$\int \cot(c+dx)(a+a\sec(c+dx))^3\,dx = \frac{4\,a^3\log\left(\cos\left(dx+c\right)-1\right)-3\,a^3\log\left(\cos\left(dx+c\right)\right)+\frac{a^3}{\cos(dx+c)}}{d}$$

input
```
integrate(cot(d*x+c)*(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

output
```
(4*a^3*log(cos(d*x + c) - 1) - 3*a^3*log(cos(d*x + c)) + a^3/cos(d*x + c))
/d
```

3.42.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 145 vs. 2(48) = 96.

Time = 0.35 (sec) , antiderivative size = 145, normalized size of antiderivative = 3.02

$$\int \cot(c+dx)(a+a\sec(c+dx))^3\,dx = \frac{4\,a^3\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)-a^3\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right)-3\,a^3\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right)+\frac{5\,a^3+\frac{3\,a^3(\cos(dx+c)-1)}{\cos(dx+c)+1}}{\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1}}{d}$$

input
```
integrate(cot(d*x+c)*(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

output
```
(4*a^3*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1)) - a^3*log(abs(-(c
os(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)) - 3*a^3*log(abs(-(cos(d*x + c) -
 1)/(cos(d*x + c) + 1) - 1)) + (5*a^3 + 3*a^3*(cos(d*x + c) - 1)/(cos(d*x
+ c) + 1))/((cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1))/d
```

3.42.9 Mupad [B] (verification not implemented)

Time = 13.96 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.79

$$\int \cot(c+dx)(a+a\sec(c+dx))^3\,dx = \frac{8\,a^3\,\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{d} - \frac{2\,a^3}{d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2-1\right)} - \frac{3\,a^3\,\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2-1\right)}{d} - \frac{a^3\,\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+1\right)}{d}$$

input
```
int(cot(c + d*x)*(a + a/cos(c + d*x))^3,x)
```

output
```
(8*a^3*log(tan(c/2 + (d*x)/2)))/d - (2*a^3)/(d*(tan(c/2 + (d*x)/2)^2 - 1))
 - (3*a^3*log(tan(c/2 + (d*x)/2)^2 - 1))/d - (a^3*log(tan(c/2 + (d*x)/2)^2
 + 1))/d
```

3.43 $\int \cot^3(c+dx)(a+a\sec(c+dx))^3\,dx$

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3.43.1 Optimal result

Integrand size = 21, antiderivative size = 40

$$\int \cot^3(c+dx)(a+a\sec(c+dx))^3\,dx = -\frac{2a^3}{d(1-\cos(c+dx))} - \frac{a^3\log(1-\cos(c+dx))}{d}$$

output
```
-2*a^3/d/(1-cos(d*x+c))-a^3*ln(1-cos(d*x+c))/d
```

3.43.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.15

$$\int \cot^3(c+dx)(a+a\sec(c+dx))^3\,dx = -\frac{a^3\left(\cot^2\left(\frac{1}{2}(c+dx)\right)+2\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)+\log\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)\right)\right)}{d}$$

input
```
Integrate[Cot[c + d*x]^3*(a + a*Sec[c + d*x])^3,x]
```

output
```
-((a^3*(Cot[(c + d*x)/2]^2 + 2*(Log[Cos[(c + d*x)/2]] + Log[Tan[(c + d*x)/
2]])))/d)
```

3.43.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.85, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 25, 4367, 27, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^3(c+dx)(a\sec(c+dx)+a)^3\,dx$$

$$\downarrow 3042$$

$$\int -\frac{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^3}{\cot\left(c+dx+\frac{\pi}{2}\right)^3}dx$$

$$\downarrow 25$$

$$-\int \frac{\left(\csc\left(\frac{1}{2}(2c+\pi)+dx\right)a+a\right)^3}{\cot\left(\frac{1}{2}(2c+\pi)+dx\right)^3}dx$$

$$\downarrow 4367$$

$$-\frac{a^4\int \frac{\cos(c+dx)+1}{a(1-\cos(c+dx))^2}d\cos(c+dx)}{d}$$

$$\downarrow 27$$

$$-\frac{a^3\int \frac{\cos(c+dx)+1}{(1-\cos(c+dx))^2}d\cos(c+dx)}{d}$$

$$\downarrow 49$$

$$-\frac{a^3\int \left(\frac{1}{\cos(c+dx)-1}+\frac{2}{(\cos(c+dx)-1)^2}\right)d\cos(c+dx)}{d}$$

$$\downarrow 2009$$

$$-\frac{a^3\left(\frac{2}{1-\cos(c+dx)}+\log(1-\cos(c+dx))\right)}{d}$$

input
```
Int[Cot[c + d*x]^3*(a + a*Sec[c + d*x])^3,x]
```

output
```
-((a^3*(2/(1 - Cos[c + d*x]) + Log[1 - Cos[c + d*x]]))/d)
```

3.43.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]  Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a  Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4367
```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Simp[1/(a^(m - n - 1)*b^n*d)  Subst[Int[(a - b*x)^((m -
1)/2)*((a + b*x)^((m - 1)/2 + n)/x^(m + n)), x], x, Sin[c + d*x]], x] /; Fr
eeQ[{a, b, c, d}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && Integer
Q[n]
```

3.43.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.41 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.72

method	result
risch	$ia^3x + \frac{2ia^3c}{d} + \frac{4a^3e^{i(dx+c)}}{d\left(e^{i(dx+c)}-1\right)^2} - \frac{2a^3\ln\left(e^{i(dx+c)}-1\right)}{d}$
derivativedivides	$\frac{a^3\left(-\frac{\cot(dx+c)\csc(dx+c)}{2}+\frac{\ln(-\cot(dx+c)+\csc(dx+c))}{2}\right)-\frac{3a^3}{2\sin(dx+c)^2}+3a^3\left(-\frac{\cos(dx+c)^3}{2\sin(dx+c)^2}-\frac{\cos(dx+c)}{2}-\frac{\ln(-\cot(dx+c)+\csc}{2}\right.}{d}$
default	$\frac{a^3\left(-\frac{\cot(dx+c)\csc(dx+c)}{2}+\frac{\ln(-\cot(dx+c)+\csc(dx+c))}{2}\right)-\frac{3a^3}{2\sin(dx+c)^2}+3a^3\left(-\frac{\cos(dx+c)^3}{2\sin(dx+c)^2}-\frac{\cos(dx+c)}{2}-\frac{\ln(-\cot(dx+c)+\csc}{2}\right.}{d}$

input
```
int(cot(d*x+c)^3*(a+a*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
I*a^3*x+2*I/d*a^3*c+4*a^3*exp(I*(d*x+c))/d/(exp(I*(d*x+c))-1)^2-2/d*a^3*ln
(exp(I*(d*x+c))-1)
```

3.43.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.25

$$\int \cot^3(c+dx)(a+a\sec(c+dx))^3\,dx = \frac{2\,a^3 - (a^3\cos(dx+c) - a^3)\log\left(-\frac{1}{2}\cos(dx+c) + \frac{1}{2}\right)}{d\cos(dx+c) - d}$$

input
```
integrate(cot(d*x+c)^3*(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

output
```
(2*a^3 - (a^3*cos(d*x + c) - a^3)*log(-1/2*cos(d*x + c) + 1/2))/(d*cos(d*x
 + c) - d)
```

3.43.6 Sympy [F]

$$\int \cot^3(c+dx)(a+a\sec(c+dx))^3\,dx = a^3\left(\int 3\cot^3(c+dx)\sec(c+dx)\,dx + \int 3\cot^3(c+dx)\sec^2(c+dx)\,dx + \int \cot^3(c+dx)\sec^3(c+dx)\,dx + \int \cot^3(c+dx)\,dx\right)$$

input
```
integrate(cot(d*x+c)**3*(a+a*sec(d*x+c))**3,x)
```

output
```
a**3*(Integral(3*cot(c + d*x)**3*sec(c + d*x), x) + Integral(3*cot(c + d*x
)**3*sec(c + d*x)**2, x) + Integral(cot(c + d*x)**3*sec(c + d*x)**3, x) +
Integral(cot(c + d*x)**3, x))
```

3.43.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.85

$$\int \cot^3(c+dx)(a+a\sec(c+dx))^3\,dx = -\frac{a^3\log\left(\cos\left(dx+c\right)-1\right)-\frac{2\,a^3}{\cos(dx+c)-1}}{d}$$

input
```
integrate(cot(d*x+c)^3*(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

output
```
-(a^3*log(cos(d*x + c) - 1) - 2*a^3/(cos(d*x + c) - 1))/d
```

3.43.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 109 vs. 2(38) = 76.

Time = 0.35 (sec) , antiderivative size = 109, normalized size of antiderivative = 2.72

$$\int \cot^3(c+dx)(a+a\sec(c+dx))^3\,dx$$
$$= -\frac{a^3\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)-a^3\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right)-\frac{\left(a^3+\frac{a^3(\cos(dx+c)-1)}{\cos(dx+c)+1}\right)(\cos(dx+c)+1)}{\cos(dx+c)-1}}{d}$$

input
```
integrate(cot(d*x+c)^3*(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

output
```
-(a^3*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1)) - a^3*log(abs(-(co
s(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)) - (a^3 + a^3*(cos(d*x + c) - 1)/(
cos(d*x + c) + 1))*(cos(d*x + c) + 1)/(cos(d*x + c) - 1))/d
```

3.43.9 Mupad [B] (verification not implemented)

Time = 14.33 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.20

$$\int \cot^3(c+dx)(a+a\sec(c+dx))^3\,dx$$
$$= -\frac{a^3\left(\cot\left(\frac{c}{2}+\frac{dx}{2}\right)^2+2\,\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)-\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+1\right)\right)}{d}$$

input
```
int(cot(c + d*x)^3*(a + a/cos(c + d*x))^3,x)
```

output
```
-(a^3*(2*log(tan(c/2 + (d*x)/2)) - log(tan(c/2 + (d*x)/2)^2 + 1) + cot(c/2
 + (d*x)/2)^2))/d
```

3.44 $\int \cot^5(c+dx)(a+a\sec(c+dx))^3\,dx$

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3.44.1 Optimal result

Integrand size = 21, antiderivative size = 61

$$\int \cot^5(c+dx)(a+a\sec(c+dx))^3\,dx = -\frac{a^3}{2d(1-\cos(c+dx))^2}+\frac{2a^3}{d(1-\cos(c+dx))}+\frac{a^3\log(1-\cos(c+dx))}{d}$$

output
```
-1/2*a^3/d/(1-cos(d*x+c))^2+2*a^3/d/(1-cos(d*x+c))+a^3*ln(1-cos(d*x+c))/d
```

3.44.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.18

$$\int \cot^5(c+dx)(a+a\sec(c+dx))^3\,dx = -\frac{a^3(1+\cos(c+dx))^3\left(-8\csc^2\left(\frac{1}{2}(c+dx)\right)+\csc^4\left(\frac{1}{2}(c+dx)\right)-16\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)\sec^6\left(\frac{1}{2}(c+dx)\right)}{64d}$$

input
```
Integrate[Cot[c + d*x]^5*(a + a*Sec[c + d*x])^3,x]
```

output
```
-1/64*(a^3*(1 + Cos[c + d*x])^3*(-8*Csc[(c + d*x)/2]^2 + Csc[(c + d*x)/2]^
4 - 16*Log[Sin[(c + d*x)/2]])*Sec[(c + d*x)/2]^6)/d
```

3.44.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.85, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 25, 4367, 27, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^5(c+dx)(a\sec(c+dx)+a)^3\,dx$$

$$\downarrow 3042$$

$$\int -\frac{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^3}{\cot\left(c+dx+\frac{\pi}{2}\right)^5}dx$$

$$\downarrow 25$$

$$-\int \frac{\left(\csc\left(\frac{1}{2}(2c+\pi)+dx\right)a+a\right)^3}{\cot\left(\frac{1}{2}(2c+\pi)+dx\right)^5}dx$$

$$\downarrow 4367$$

$$-\frac{a^6\int \frac{\cos^2(c+dx)}{a^3(1-\cos(c+dx))^3}d\cos(c+dx)}{d}$$

$$\downarrow 27$$

$$-\frac{a^3\int \frac{\cos^2(c+dx)}{(1-\cos(c+dx))^3}d\cos(c+dx)}{d}$$

$$\downarrow 49$$

$$-\frac{a^3\int \left(-\frac{2}{(\cos(c+dx)-1)^2}-\frac{1}{(\cos(c+dx)-1)^3}+\frac{1}{1-\cos(c+dx)}\right)d\cos(c+dx)}{d}$$

$$\downarrow 2009$$

$$-\frac{a^3\left(-\frac{2}{1-\cos(c+dx)}+\frac{1}{2(1-\cos(c+dx))^2}-\log(1-\cos(c+dx))\right)}{d}$$

input
```
Int[Cot[c + d*x]^5*(a + a*Sec[c + d*x])^3,x]
```

output
```
-((a^3*(1/(2*(1 - Cos[c + d*x])^2) - 2/(1 - Cos[c + d*x]) - Log[1 - Cos[c
+ d*x]]))/d)
```

3.44.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4367
```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Simp[1/(a^(m - n - 1)*b^n*d)   Subst[Int[(a - b*x)^((m -
1)/2)*((a + b*x)^((m - 1)/2 + n)/x^(m + n)), x], x, Sin[c + d*x]], x] /; Fr
eeQ[{a, b, c, d}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && Integer
Q[n]
```

3.44.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.58 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.54

method	result
risch	$-ia^3x - \frac{2ia^3c}{d} - \frac{2a^3\left(2\,e^{3i(dx+c)}-3\,e^{2i(dx+c)}+2\,e^{i(dx+c)}\right)}{d\left(e^{i(dx+c)}-1\right)^4} + \frac{2a^3\ln\left(e^{i(dx+c)}-1\right)}{d}$
derivativedivides	$\frac{a^3\left(-\frac{\cos(dx+c)^3}{4\sin(dx+c)^4}-\frac{\cos(dx+c)^3}{8\sin(dx+c)^2}-\frac{\cos(dx+c)}{8}-\frac{\ln(-\cot(dx+c)+\csc(dx+c))}{8}\right)-\frac{3a^3\cos(dx+c)^4}{4\sin(dx+c)^4}+3a^3\left(-\frac{\cos(dx+c)^5}{4\sin(dx+c)^4}+\frac{\cos(dx+}{d}$
default	$\frac{a^3\left(-\frac{\cos(dx+c)^3}{4\sin(dx+c)^4}-\frac{\cos(dx+c)^3}{8\sin(dx+c)^2}-\frac{\cos(dx+c)}{8}-\frac{\ln(-\cot(dx+c)+\csc(dx+c))}{8}\right)-\frac{3a^3\cos(dx+c)^4}{4\sin(dx+c)^4}+3a^3\left(-\frac{\cos(dx+c)^5}{4\sin(dx+c)^4}+\frac{\cos(dx+}{d}$

input
```
int(cot(d*x+c)^5*(a+a*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
-I*a^3*x-2*I/d*a^3*c-2*a^3/d/(exp(I*(d*x+c))-1)^4*(2*exp(3*I*(d*x+c))-3*ex
p(2*I*(d*x+c))+2*exp(I*(d*x+c)))+2/d*a^3*ln(exp(I*(d*x+c))-1)
```

3.44.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.34

$$\int \cot^5(c+dx)(a+a\sec(c+dx))^3\,dx = -\frac{4\,a^3\cos(dx+c)-3\,a^3-2\left(a^3\cos(dx+c)^2-2\,a^3\cos(dx+c)+a^3\right)\log\left(-\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)}{2\left(d\cos(dx+c)^2-2\,d\cos(dx+c)+d\right)}$$

input
```
integrate(cot(d*x+c)^5*(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

output
```
-1/2*(4*a^3*cos(d*x + c) - 3*a^3 - 2*(a^3*cos(d*x + c)^2 - 2*a^3*cos(d*x +
 c) + a^3)*log(-1/2*cos(d*x + c) + 1/2))/(d*cos(d*x + c)^2 - 2*d*cos(d*x +
 c) + d)
```

3.44.6 Sympy [F]

$$\int \cot^5(c+dx)(a+a\sec(c+dx))^3\,dx = a^3\left(\int 3\cot^5(c+dx)\sec(c+dx)\,dx + \int 3\cot^5(c+dx)\sec^2(c+dx)\,dx + \int \cot^5(c+dx)\sec^3(c+dx)\,dx + \int \cot^5(c+dx)\,dx\right)$$

input
```
integrate(cot(d*x+c)**5*(a+a*sec(d*x+c))**3,x)
```

output
```
a**3*(Integral(3*cot(c + d*x)**5*sec(c + d*x), x) + Integral(3*cot(c + d*x
)**5*sec(c + d*x)**2, x) + Integral(cot(c + d*x)**5*sec(c + d*x)**3, x) +
Integral(cot(c + d*x)**5, x))
```

3.44.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.97

$$\int \cot^5(c+dx)(a+a\sec(c+dx))^3\,dx = \frac{2\,a^3\log\left(\cos\left(dx+c\right)-1\right)-\frac{4\,a^3\cos(dx+c)-3\,a^3}{\cos(dx+c)^2-2\,\cos(dx+c)+1}}{2\,d}$$

input
```
integrate(cot(d*x+c)^5*(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

output
```
1/2*(2*a^3*log(cos(d*x + c) - 1) - (4*a^3*cos(d*x + c) - 3*a^3)/(cos(d*x +
 c)^2 - 2*cos(d*x + c) + 1))/d
```

3.44.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 138 vs. 2(55) = 110.

Time = 0.41 (sec) , antiderivative size = 138, normalized size of antiderivative = 2.26

$$\int \cot^5(c+dx)(a+a\sec(c+dx))^3\,dx$$
$$= \frac{8\,a^3\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)-8\,a^3\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right)-\frac{\left(a^3+\frac{6\,a^3(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{12\,a^3(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}\right)(\cos(dx+c)+1)^2}{(\cos(dx+c)-1)^2}}{8\,d}$$

input
```
integrate(cot(d*x+c)^5*(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

output
```
1/8*(8*a^3*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1)) - 8*a^3*log(a
bs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)) - (a^3 + 6*a^3*(cos(d*x +
c) - 1)/(cos(d*x + c) + 1) + 12*a^3*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1
)^2)*(cos(d*x + c) + 1)^2/(cos(d*x + c) - 1)^2)/d
```

3.44.9 Mupad [B] (verification not implemented)

Time = 14.26 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.28

$$\int \cot^5(c+dx)(a+a\sec(c+dx))^3\,dx = \frac{2a^3\,\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{d} + \frac{\frac{3a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2}{4}-\frac{a^3}{8}}{d\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4} - \frac{a^3\,\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+1\right)}{d}$$

input
```
int(cot(c + d*x)^5*(a + a/cos(c + d*x))^3,x)
```

output
```
(2*a^3*log(tan(c/2 + (d*x)/2)))/d + ((3*a^3*tan(c/2 + (d*x)/2)^2)/4 - a^3/
8)/(d*tan(c/2 + (d*x)/2)^4) - (a^3*log(tan(c/2 + (d*x)/2)^2 + 1))/d
```

3.45 $\int \cot^7(c+dx)(a+a\sec(c+dx))^3\,dx$

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3.45.1 Optimal result

Integrand size = 21, antiderivative size = 107

$$\int \cot^7(c+dx)(a+a\sec(c+dx))^3\,dx = -\frac{a^3}{6d(1-\cos(c+dx))^3} + \frac{7a^3}{8d(1-\cos(c+dx))^2} - \frac{17a^3}{8d(1-\cos(c+dx))} - \frac{15a^3\log(1-\cos(c+dx))}{16d} - \frac{a^3\log(1+\cos(c+dx))}{16d}$$

output
```
-1/6*a^3/d/(1-cos(d*x+c))^3+7/8*a^3/d/(1-cos(d*x+c))^2-17/8*a^3/d/(1-cos(d
*x+c))-15/16*a^3*ln(1-cos(d*x+c))/d-1/16*a^3*ln(1+cos(d*x+c))/d
```

3.45.2 Mathematica [A] (verified)

Time = 0.50 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.95

$$\int \cot^7(c+dx)(a+a\sec(c+dx))^3\,dx = -\frac{a^3(1+\cos(c+dx))^3\left(102\csc^2\left(\frac{1}{2}(c+dx)\right) - 21\csc^4\left(\frac{1}{2}(c+dx)\right) + 2\csc^6\left(\frac{1}{2}(c+dx)\right) + 12\left(\log\left(\cos\left(\frac{1}{2}(\right.\right.\right.\right.}{768d}$$

input
```
Integrate[Cot[c + d*x]^7*(a + a*Sec[c + d*x])^3,x]
```

output
```
-1/768*(a^3*(1 + Cos[c + d*x])^3*(102*Csc[(c + d*x)/2]^2 - 21*Csc[(c + d*x
)/2]^4 + 2*Csc[(c + d*x)/2]^6 + 12*(Log[Cos[(c + d*x)/2]] + 15*Log[Sin[(c
+ d*x)/2]]))*Sec[(c + d*x)/2]^6)/d
```

3.45.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.79, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 25, 4367, 27, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^7(c+dx)(a\sec(c+dx)+a)^3\,dx$$

$$\downarrow 3042$$

$$\int -\frac{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^3}{\cot\left(c+dx+\frac{\pi}{2}\right)^7}dx$$

$$\downarrow 25$$

$$-\int \frac{\left(\csc\left(\frac{1}{2}(2c+\pi)+dx\right)a+a\right)^3}{\cot\left(\frac{1}{2}(2c+\pi)+dx\right)^7}dx$$

$$\downarrow 4367$$

$$-\frac{a^8\int \frac{\cos^4(c+dx)}{a^5(1-\cos(c+dx))^4(\cos(c+dx)+1)}d\cos(c+dx)}{d}$$

$$\downarrow 27$$

$$-\frac{a^3\int \frac{\cos^4(c+dx)}{(1-\cos(c+dx))^4(\cos(c+dx)+1)}d\cos(c+dx)}{d}$$

$$\downarrow 99$$

$$-\frac{a^3\int \left(\frac{1}{16(\cos(c+dx)+1)}+\frac{15}{16(\cos(c+dx)-1)}+\frac{17}{8(\cos(c+dx)-1)^2}+\frac{7}{4(\cos(c+dx)-1)^3}+\frac{1}{2(\cos(c+dx)-1)^4}\right)d\cos(c+dx)}{d}$$

$$\downarrow 2009$$

$$-\frac{a^3\left(\frac{17}{8(1-\cos(c+dx))}-\frac{7}{8(1-\cos(c+dx))^2}+\frac{1}{6(1-\cos(c+dx))^3}+\frac{15}{16}\log(1-\cos(c+dx))+\frac{1}{16}\log(\cos(c+dx)+1)\right)}{d}$$

input
```
Int[Cot[c + d*x]^7*(a + a*Sec[c + d*x])^3,x]
```

output
```
-((a^3*(1/(6*(1 - Cos[c + d*x])^3) - 7/(8*(1 - Cos[c + d*x])^2) + 17/(8*(1
 - Cos[c + d*x])) + (15*Log[1 - Cos[c + d*x]])/16 + Log[1 + Cos[c + d*x]]/
16))/d)
```

3.45.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4367
```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Simp[1/(a^(m - n - 1)*b^n*d)   Subst[Int[(a - b*x)^((m -
1)/2)*((a + b*x)^((m - 1)/2 + n)/x^(m + n)), x], x, Sin[c + d*x]], x] /; Fr
eeQ[{a, b, c, d}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && Integer
Q[n]
```

3.45.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.85 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.27

method	result
risch	$ia^3x + \frac{2ia^3c}{d} + \frac{a^3\left(51\,e^{5i(dx+c)} - 162\,e^{4i(dx+c)} + 238\,e^{3i(dx+c)} - 162\,e^{2i(dx+c)} + 51\,e^{i(dx+c)}\right)}{12d\left(e^{i(dx+c)}-1\right)^6} - \frac{15a^3\ln\left(e^{i(dx+c)}-1\right.}{8d}$
derivativedivides	$\frac{a^3\left(-\frac{\cos(dx+c)^5}{6\sin(dx+c)^6} - \frac{\cos(dx+c)^5}{24\sin(dx+c)^4} + \frac{\cos(dx+c)^5}{48\sin(dx+c)^2} + \frac{\cos(dx+c)^3}{48} + \frac{\cos(dx+c)}{16} + \frac{\ln(-\cot(dx+c)+\csc(dx+c))}{16}\right) - \frac{a^3\cos(dx+c)^6}{2\sin(dx+c)^6} + 3}{d}$
default	$\frac{a^3\left(-\frac{\cos(dx+c)^5}{6\sin(dx+c)^6} - \frac{\cos(dx+c)^5}{24\sin(dx+c)^4} + \frac{\cos(dx+c)^5}{48\sin(dx+c)^2} + \frac{\cos(dx+c)^3}{48} + \frac{\cos(dx+c)}{16} + \frac{\ln(-\cot(dx+c)+\csc(dx+c))}{16}\right) - \frac{a^3\cos(dx+c)^6}{2\sin(dx+c)^6} + 3}{d}$

input
```
int(cot(d*x+c)^7*(a+a*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
I*a^3*x+2*I/d*a^3*c+1/12*a^3/d/(exp(I*(d*x+c))-1)^6*(51*exp(5*I*(d*x+c))-1
62*exp(4*I*(d*x+c))+238*exp(3*I*(d*x+c))-162*exp(2*I*(d*x+c))+51*exp(I*(d*
x+c)))-15/8/d*a^3*ln(exp(I*(d*x+c))-1)-1/8/d*a^3*ln(exp(I*(d*x+c))+1)
```

3.45.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.66

$$\int \cot^7(c+dx)(a+a\sec(c+dx))^3\,dx$$
$$= \frac{102\,a^3\cos(dx+c)^2 - 162\,a^3\cos(dx+c) + 68\,a^3 - 3\left(a^3\cos(dx+c)^3 - 3\,a^3\cos(dx+c)^2 + 3\,a^3\cos(dx+c)\right.}{48\left(d\cos(dx+c)^3\right.}$$

input
```
integrate(cot(d*x+c)^7*(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

output
```
1/48*(102*a^3*cos(d*x + c)^2 - 162*a^3*cos(d*x + c) + 68*a^3 - 3*(a^3*cos(
d*x + c)^3 - 3*a^3*cos(d*x + c)^2 + 3*a^3*cos(d*x + c) - a^3)*log(1/2*cos(
d*x + c) + 1/2) - 45*(a^3*cos(d*x + c)^3 - 3*a^3*cos(d*x + c)^2 + 3*a^3*co
s(d*x + c) - a^3)*log(-1/2*cos(d*x + c) + 1/2))/(d*cos(d*x + c)^3 - 3*d*co
s(d*x + c)^2 + 3*d*cos(d*x + c) - d)
```

3.45.6 Sympy [F(-1)]

Timed out.

$$\int \cot^7(c+dx)(a+a\sec(c+dx))^3\,dx = \text{Timed out}$$

input
```
integrate(cot(d*x+c)**7*(a+a*sec(d*x+c))**3,x)
```

output
```
Timed out
```

3.45.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.90

$$\int \cot^7(c+dx)(a+a\sec(c+dx))^3\,dx =$$

$$-\frac{3\,a^3\log\left(\cos\left(dx+c\right)+1\right)+45\,a^3\log\left(\cos\left(dx+c\right)-1\right)-\frac{2\left(51\,a^3\cos(dx+c)^2-81\,a^3\cos(dx+c)+34\,a^3\right)}{\cos(dx+c)^3-3\,\cos(dx+c)^2+3\,\cos(dx+c)-1}}{48\,d}$$

input
```
integrate(cot(d*x+c)^7*(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

output
```
-1/48*(3*a^3*log(cos(d*x + c) + 1) + 45*a^3*log(cos(d*x + c) - 1) - 2*(51*
a^3*cos(d*x + c)^2 - 81*a^3*cos(d*x + c) + 34*a^3)/(cos(d*x + c)^3 - 3*cos
(d*x + c)^2 + 3*cos(d*x + c) - 1))/d
```

3.45.8 Giac [A] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.54

$$\int \cot^7(c+dx)(a+a\sec(c+dx))^3\,dx =$$

$$-\frac{90\,a^3\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)-96\,a^3\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right)-\frac{\left(2\,a^3+\frac{15\,a^3(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{66\,a^3(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}+\frac{165\,a^3(\cos}{(\cos(dx+c)-1)^3}}{96\,d}$$

input
```
integrate(cot(d*x+c)^7*(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

output
```
-1/96*(90*a^3*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1)) - 96*a^3*l
og(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)) - (2*a^3 + 15*a^3*(cos
(d*x + c) - 1)/(cos(d*x + c) + 1) + 66*a^3*(cos(d*x + c) - 1)^2/(cos(d*x +
 c) + 1)^2 + 165*a^3*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3)*(cos(d*x +
 c) + 1)^3/(cos(d*x + c) - 1)^3)/d
```

3.45.9 Mupad [B] (verification not implemented)

Time = 14.41 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.88

$$\int \cot^7(c+dx)(a+a\sec(c+dx))^3\,dx = \frac{a^3\,\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+1\right)}{d} - \frac{\frac{11\,a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4}{2} - \frac{5\,a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2}{4} + \frac{a^3}{6}}{8\,d\,\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6} - \frac{15\,a^3\,\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{8\,d}$$

input
```
int(cot(c + d*x)^7*(a + a/cos(c + d*x))^3,x)
```

output
```
(a^3*log(tan(c/2 + (d*x)/2)^2 + 1))/d - ((11*a^3*tan(c/2 + (d*x)/2)^4)/2 -
 (5*a^3*tan(c/2 + (d*x)/2)^2)/4 + a^3/6)/(8*d*tan(c/2 + (d*x)/2)^6) - (15*
a^3*log(tan(c/2 + (d*x)/2)))/(8*d)
```

3.46 $\int \cot^9(c+dx)(a+a\sec(c+dx))^3\,dx$

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3.46.1 Optimal result

Integrand size = 21, antiderivative size = 149

$$\begin{aligned}\int \cot^9(c+dx)(a+a\sec(c+dx))^3\,dx = &-\frac{a^3}{16d(1-\cos(c+dx))^4}+\frac{5a^3}{12d(1-\cos(c+dx))^3}\\ &-\frac{39a^3}{32d(1-\cos(c+dx))^2}+\frac{9a^3}{4d(1-\cos(c+dx))}\\ &+\frac{a^3}{32d(1+\cos(c+dx))}\\ &+\frac{57a^3\log(1-\cos(c+dx))}{64d}\\ &+\frac{7a^3\log(1+\cos(c+dx))}{64d}\end{aligned}$$

output

```
-1/16*a^3/d/(1-cos(d*x+c))^4+5/12*a^3/d/(1-cos(d*x+c))^3-39/32*a^3/d/(1-co
s(d*x+c))^2+9/4*a^3/d/(1-cos(d*x+c))+1/32*a^3/d/(1+cos(d*x+c))+57/64*a^3*l
n(1-cos(d*x+c))/d+7/64*a^3*ln(1+cos(d*x+c))/d
```

3.46.2 Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.87

$$\int \cot^9(c+dx)(a+a\sec(c+dx))^3\,dx$$
$$= \frac{a^3(1+\cos(c+dx))^3\sec^6\left(\frac{1}{2}(c+dx)\right)\left(864\csc^2\left(\frac{1}{2}(c+dx)\right)-234\csc^4\left(\frac{1}{2}(c+dx)\right)+40\csc^6\left(\frac{1}{2}(c+dx)\right)-$$
$$6144d$$

input
```
Integrate[Cot[c + d*x]^9*(a + a*Sec[c + d*x])^3,x]
```

output
```
(a^3*(1 + Cos[c + d*x])^3*Sec[(c + d*x)/2]^6*(864*Csc[(c + d*x)/2]^2 - 234
*Csc[(c + d*x)/2]^4 + 40*Csc[(c + d*x)/2]^6 - 3*Csc[(c + d*x)/2]^8 + 12*(1
4*Log[Cos[(c + d*x)/2]] + 114*Log[Sin[(c + d*x)/2]] + Sec[(c + d*x)/2]^2))
)/(6144*d)
```

3.46.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.77, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 25, 4367, 27, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^9(c+dx)(a\sec(c+dx)+a)^3\,dx$$
$$\downarrow 3042$$
$$\int -\frac{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^3}{\cot\left(c+dx+\frac{\pi}{2}\right)^9}dx$$
$$\downarrow 25$$
$$-\int \frac{\left(\csc\left(\frac{1}{2}(2c+\pi)+dx\right)a+a\right)^3}{\cot\left(\frac{1}{2}(2c+\pi)+dx\right)^9}dx$$
$$\downarrow 4367$$
$$-\frac{a^{10}\int \frac{\cos^6(c+dx)}{a^7(1-\cos(c+dx))^5(\cos(c+dx)+1)^2}d\cos(c+dx)}{d}$$
$$\downarrow 27$$

$$-\frac{a^3\int \frac{\cos^6(c+dx)}{(1-\cos(c+dx))^5(\cos(c+dx)+1)^2}\,d\cos(c+dx)}{d}$$

$$\downarrow 99$$

$$-\frac{a^3\int \left(-\frac{7}{64(\cos(c+dx)+1)}+\frac{1}{32(\cos(c+dx)+1)^2}-\frac{57}{64(\cos(c+dx)-1)}-\frac{9}{4(\cos(c+dx)-1)^2}-\frac{39}{16(\cos(c+dx)-1)^3}-\frac{5}{4(\cos(c+dx)-1)^4}-\right.}{d}$$

$$\downarrow 2009$$

$$-\frac{a^3\left(-\frac{9}{4(1-\cos(c+dx))}-\frac{1}{32(\cos(c+dx)+1)}+\frac{39}{32(1-\cos(c+dx))^2}-\frac{5}{12(1-\cos(c+dx))^3}+\frac{1}{16(1-\cos(c+dx))^4}-\frac{57}{64}\log(1-\cos(c+d\right.}{d}$$

input
```
Int[Cot[c + d*x]^9*(a + a*Sec[c + d*x])^3,x]
```

output
```
-((a^3*(1/(16*(1 - Cos[c + d*x])^4) - 5/(12*(1 - Cos[c + d*x])^3) + 39/(32
*(1 - Cos[c + d*x])^2) - 9/(4*(1 - Cos[c + d*x])) - 1/(32*(1 + Cos[c + d*x
])) - (57*Log[1 - Cos[c + d*x]])/64 - (7*Log[1 + Cos[c + d*x]])/64))/d)
```

3.46.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4367
```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Simp[1/(a^(m - n - 1)*b^n*d)   Subst[Int[(a - b*x)^((m -
1)/2)*((a + b*x)^((m - 1)/2 + n)/x^(m + n)), x], x, Sin[c + d*x]], x] /; Fr
eeQ[{a, b, c, d}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && Integer
Q[n]
```

3.46.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 5.23 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.30

method	result
risch	$-ia^3x - \frac{2ia^3c}{d} - \frac{a^3\left(213\,e^{9i(dx+c)} - 606\,e^{8i(dx+c)} + 472\,e^{7i(dx+c)} + 846\,e^{6i(dx+c)} - 1658\,e^{5i(dx+c)} + 846\,e^{4i(dx+c)} + 4\right.}{48d\left(e^{i(dx+c)}-1\right)^8\left(e^{i(dx+c)}+1\right)^2}$
derivativedivides	$a^3\left(-\frac{\cos(dx+c)^7}{8\sin(dx+c)^8} - \frac{\cos(dx+c)^7}{48\sin(dx+c)^6} + \frac{\cos(dx+c)^7}{192\sin(dx+c)^4} - \frac{\cos(dx+c)^7}{128\sin(dx+c)^2} - \frac{\cos(dx+c)^5}{128} - \frac{5\cos(dx+c)^3}{384} - \frac{5\cos(dx+c)}{128} - \frac{5\ln(-\cot(dx+}{1}\right.$
default	$a^3\left(-\frac{\cos(dx+c)^7}{8\sin(dx+c)^8} - \frac{\cos(dx+c)^7}{48\sin(dx+c)^6} + \frac{\cos(dx+c)^7}{192\sin(dx+c)^4} - \frac{\cos(dx+c)^7}{128\sin(dx+c)^2} - \frac{\cos(dx+c)^5}{128} - \frac{5\cos(dx+c)^3}{384} - \frac{5\cos(dx+c)}{128} - \frac{5\ln(-\cot(dx+}{1}\right.$

input
```
int(cot(d*x+c)^9*(a+a*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
-I*a^3*x-2*I/d*a^3*c-1/48*a^3/d/(exp(I*(d*x+c))-1)^8/(exp(I*(d*x+c))+1)^2*
(213*exp(9*I*(d*x+c))-606*exp(8*I*(d*x+c))+472*exp(7*I*(d*x+c))+846*exp(6*
I*(d*x+c))-1658*exp(5*I*(d*x+c))+846*exp(4*I*(d*x+c))+472*exp(3*I*(d*x+c))
-606*exp(2*I*(d*x+c))+213*exp(I*(d*x+c)))+7/32/d*a^3*ln(exp(I*(d*x+c))+1)+
57/32/d*a^3*ln(exp(I*(d*x+c))-1)
```

3.46.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 272 vs. $2(127) = 254$.

Time = 0.29 (sec) , antiderivative size = 272, normalized size of antiderivative = 1.83

$$\int \cot^9(c+dx)(a+a\sec(c+dx))^3\,dx =$$

$$-\frac{426\,a^3\cos(dx+c)^4 - 606\,a^3\cos(dx+c)^3 - 190\,a^3\cos(dx+c)^2 + 666\,a^3\cos(dx+c) - 272\,a^3 - 21\,(a}{}$$

input
```
integrate(cot(d*x+c)^9*(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

output
```
-1/192*(426*a^3*cos(d*x + c)^4 - 606*a^3*cos(d*x + c)^3 - 190*a^3*cos(d*x
+ c)^2 + 666*a^3*cos(d*x + c) - 272*a^3 - 21*(a^3*cos(d*x + c)^5 - 3*a^3*c
os(d*x + c)^4 + 2*a^3*cos(d*x + c)^3 + 2*a^3*cos(d*x + c)^2 - 3*a^3*cos(d*
x + c) + a^3)*log(1/2*cos(d*x + c) + 1/2) - 171*(a^3*cos(d*x + c)^5 - 3*a^
3*cos(d*x + c)^4 + 2*a^3*cos(d*x + c)^3 + 2*a^3*cos(d*x + c)^2 - 3*a^3*cos
(d*x + c) + a^3)*log(-1/2*cos(d*x + c) + 1/2))/(d*cos(d*x + c)^5 - 3*d*cos
(d*x + c)^4 + 2*d*cos(d*x + c)^3 + 2*d*cos(d*x + c)^2 - 3*d*cos(d*x + c) +
 d)
```

3.46.6 Sympy [F(-1)]

Timed out.

$$\int \cot^9(c+dx)(a+a\sec(c+dx))^3\,dx = \text{Timed out}$$

input
```
integrate(cot(d*x+c)**9*(a+a*sec(d*x+c))**3,x)
```

output
```
Timed out
```

3.46.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.95

$$\int \cot^9(c+dx)(a+a\sec(c+dx))^3\,dx$$

$$= \frac{21\,a^3\log\left(\cos\left(dx+c\right)+1\right)+171\,a^3\log\left(\cos\left(dx+c\right)-1\right)-\frac{2\left(213\,a^3\cos(dx+c)^4-303\,a^3\cos(dx+c)^3-95\,a^3\cos(dx+c)^2+\right.}{\cos(dx+c)^5-3\,\cos(dx+c)^4+2\,\cos(dx+c)^3+2\,\cos(dx+}}{192\,d}$$

input
```
integrate(cot(d*x+c)^9*(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

output
```
1/192*(21*a^3*log(cos(d*x + c) + 1) + 171*a^3*log(cos(d*x + c) - 1) - 2*(2
13*a^3*cos(d*x + c)^4 - 303*a^3*cos(d*x + c)^3 - 95*a^3*cos(d*x + c)^2 + 3
33*a^3*cos(d*x + c) - 136*a^3)/(cos(d*x + c)^5 - 3*cos(d*x + c)^4 + 2*cos(
d*x + c)^3 + 2*cos(d*x + c)^2 - 3*cos(d*x + c) + 1))/d
```

3.46.8 Giac [A] (verification not implemented)

Time = 0.48 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.43

$$\int \cot^9(c+dx)(a+a\sec(c+dx))^3\,dx$$

$$= \frac{684\,a^3\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right) - 768\,a^3\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right) - \frac{12\,a^3(\cos(dx+c)-1)}{\cos(dx+c)+1} - \frac{\left(3\,a^3+\frac{28\,a^3(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{132\,a^3}{(co}\right.}{768\,d}$$

input
```
integrate(cot(d*x+c)^9*(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

output
```
1/768*(684*a^3*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1)) - 768*a^3
*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)) - 12*a^3*(cos(d*x +
c) - 1)/(cos(d*x + c) + 1) - (3*a^3 + 28*a^3*(cos(d*x + c) - 1)/(cos(d*x +
 c) + 1) + 132*a^3*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 504*a^3*(co
s(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 + 1425*a^3*(cos(d*x + c) - 1)^4/(co
s(d*x + c) + 1)^4)*(cos(d*x + c) + 1)^4/(cos(d*x + c) - 1)^4)/d
```

3.46.9 Mupad [B] (verification not implemented)

Time = 14.03 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.87

$$\int \cot^9(c+dx)(a+a\sec(c+dx))^3\,dx$$

$$= \frac{a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2}{64\,d} + \frac{57\,a^3\,\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{32\,d} + \frac{21\,a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6 - \frac{11\,a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4}{2} + \frac{7\,a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2}{6} - \frac{a^3}{8}}{32\,d\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^8} - \frac{a^3\,\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+1\right)}{d}$$

input
```
int(cot(c + d*x)^9*(a + a/cos(c + d*x))^3,x)
```

output
```
(a^3*tan(c/2 + (d*x)/2)^2)/(64*d) + (57*a^3*log(tan(c/2 + (d*x)/2)))/(32*d
) + ((7*a^3*tan(c/2 + (d*x)/2)^2)/6 - (11*a^3*tan(c/2 + (d*x)/2)^4)/2 + 21
*a^3*tan(c/2 + (d*x)/2)^6 - a^3/8)/(32*d*tan(c/2 + (d*x)/2)^8) - (a^3*log(
tan(c/2 + (d*x)/2)^2 + 1))/d
```

3.47 $\int (a + a\sec(c+dx))^3 \tan^6(c+dx)\,dx$

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3.47.1 Optimal result

Integrand size = 21, antiderivative size = 237

$$\begin{aligned}\int (a + a\sec(c+dx))^3 \tan^6(c+dx)\,dx = &-a^3x - \frac{125a^3\operatorname{arctanh}(\sin(c+dx))}{128d} \\ &+ \frac{a^3\tan(c+dx)}{d} + \frac{115a^3\sec(c+dx)\tan(c+dx)}{128d} \\ &+ \frac{5a^3\sec^3(c+dx)\tan(c+dx)}{64d} \\ &- \frac{a^3\tan^3(c+dx)}{3d} - \frac{5a^3\sec(c+dx)\tan^3(c+dx)}{8d} \\ &- \frac{5a^3\sec^3(c+dx)\tan^3(c+dx)}{48d} \\ &+ \frac{a^3\tan^5(c+dx)}{5d} + \frac{a^3\sec(c+dx)\tan^5(c+dx)}{2d} \\ &+ \frac{a^3\sec^3(c+dx)\tan^5(c+dx)}{8d} + \frac{3a^3\tan^7(c+dx)}{7d}\end{aligned}$$

output

```
-a^3*x-125/128*a^3*arctanh(sin(d*x+c))/d+a^3*tan(d*x+c)/d+115/128*a^3*sec(
d*x+c)*tan(d*x+c)/d+5/64*a^3*sec(d*x+c)^3*tan(d*x+c)/d-1/3*a^3*tan(d*x+c)^
3/d-5/8*a^3*sec(d*x+c)*tan(d*x+c)^3/d-5/48*a^3*sec(d*x+c)^3*tan(d*x+c)^3/d
+1/5*a^3*tan(d*x+c)^5/d+1/2*a^3*sec(d*x+c)*tan(d*x+c)^5/d+1/8*a^3*sec(d*x+
c)^3*tan(d*x+c)^5/d+3/7*a^3*tan(d*x+c)^7/d
```

3.47.2 Mathematica [A] (verified)

Time = 1.19 (sec) , antiderivative size = 292, normalized size of antiderivative = 1.23

$$\begin{aligned}\int (a + a\sec(c+dx))^3 \tan^6(c+dx)\,dx = &-\frac{a^3 \arctan(\tan(c+dx))}{d} \\ &- \frac{125a^3 \text{arctanh}(\sin(c+dx))}{128d} + \frac{a^3\tan(c+dx)}{d} \\ &- \frac{125a^3\sec(c+dx)\tan(c+dx)}{128d} \\ &- \frac{125a^3\sec^3(c+dx)\tan(c+dx)}{192d} \\ &+ \frac{119a^3\sec^5(c+dx)\tan(c+dx)}{48d} \\ &+ \frac{a^3\sec^7(c+dx)\tan(c+dx)}{8d} - \frac{a^3\tan^3(c+dx)}{3d} \\ &- \frac{5a^3\sec^3(c+dx)\tan^3(c+dx)}{d} \\ &- \frac{a^3\sec^5(c+dx)\tan^3(c+dx)}{3d} + \frac{a^3\tan^5(c+dx)}{5d} \\ &+ \frac{3a^3\sec(c+dx)\tan^5(c+dx)}{d} \\ &+ \frac{a^3\sec^3(c+dx)\tan^5(c+dx)}{3d} + \frac{3a^3\tan^7(c+dx)}{7d}\end{aligned}$$

input
```
Integrate[(a + a*Sec[c + d*x])^3*Tan[c + d*x]^6,x]
```

output
```
-((a^3*ArcTan[Tan[c + d*x]])/d) - (125*a^3*ArcTanh[Sin[c + d*x]])/(128*d)
+ (a^3*Tan[c + d*x])/d - (125*a^3*Sec[c + d*x]*Tan[c + d*x])/(128*d) - (12
5*a^3*Sec[c + d*x]^3*Tan[c + d*x])/(192*d) + (119*a^3*Sec[c + d*x]^5*Tan[c
 + d*x])/(48*d) + (a^3*Sec[c + d*x]^7*Tan[c + d*x])/(8*d) - (a^3*Tan[c + d
*x]^3)/(3*d) - (5*a^3*Sec[c + d*x]^3*Tan[c + d*x]^3)/d - (a^3*Sec[c + d*x]
^5*Tan[c + d*x]^3)/(3*d) + (a^3*Tan[c + d*x]^5)/(5*d) + (3*a^3*Sec[c + d*x
]*Tan[c + d*x]^5)/d + (a^3*Sec[c + d*x]^3*Tan[c + d*x]^5)/(3*d) + (3*a^3*T
an[c + d*x]^7)/(7*d)
```

3.47.3 Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 237, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3042, 4374, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^6(c+dx)(a\sec(c+dx)+a)^3\,dx$$

$$\downarrow 3042$$

$$\int \cot\left(c+dx+\frac{\pi}{2}\right)^6\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^3 dx$$

$$\downarrow 4374$$

$$\int (a^3\tan^6(c+dx)+a^3\tan^6(c+dx)\sec^3(c+dx)+3a^3\tan^6(c+dx)\sec^2(c+dx)+3a^3\tan^6(c+dx)\sec(c+dx)$$

$$\downarrow 2009$$

$$-\frac{125a^3\operatorname{arctanh}(\sin(c+dx))}{128d}+\frac{3a^3\tan^7(c+dx)}{7d}+\frac{a^3\tan^5(c+dx)}{5d}-\frac{a^3\tan^3(c+dx)}{3d}+\frac{a^3\tan(c+dx)}{d}+\frac{a^3\tan^5(c+dx)\sec^3(c+dx)}{8d}-\frac{5a^3\tan^3(c+dx)\sec^3(c+dx)}{48d}+\frac{5a^3\tan(c+dx)\sec^3(c+dx)}{64d}+\frac{a^3\tan^5(c+dx)\sec(c+dx)}{2d}-\frac{5a^3\tan^3(c+dx)\sec(c+dx)}{8d}+\frac{115a^3\tan(c+dx)\sec(c+dx)}{128d}-a^3x$$

input
```
Int[(a + a*Sec[c + d*x])^3*Tan[c + d*x]^6,x]
```

output
```
-(a^3*x) - (125*a^3*ArcTanh[Sin[c + d*x]])/(128*d) + (a^3*Tan[c + d*x])/d
+ (115*a^3*Sec[c + d*x]*Tan[c + d*x])/(128*d) + (5*a^3*Sec[c + d*x]^3*Tan[
c + d*x])/(64*d) - (a^3*Tan[c + d*x]^3)/(3*d) - (5*a^3*Sec[c + d*x]*Tan[c
+ d*x]^3)/(8*d) - (5*a^3*Sec[c + d*x]^3*Tan[c + d*x]^3)/(48*d) + (a^3*Tan[
c + d*x]^5)/(5*d) + (a^3*Sec[c + d*x]*Tan[c + d*x]^5)/(2*d) + (a^3*Sec[c +
 d*x]^3*Tan[c + d*x]^5)/(8*d) + (3*a^3*Tan[c + d*x]^7)/(7*d)
```

3.47.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4374
```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

3.47.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.93 (sec) , antiderivative size = 228, normalized size of antiderivative = 0.96

method	result
risch	$-a^3x - \frac{ia^3(27195\,e^{15i(dx+c)}+65135\,e^{13i(dx+c)}-161280\,e^{12i(dx+c)}+63595\,e^{11i(dx+c)}-286720\,e^{10i(dx+c)}+133175\,e^{9}$
derivativedivides	$\frac{a^3\left(\frac{\sin(dx+c)^7}{8\cos(dx+c)^8}+\frac{\sin(dx+c)^7}{48\cos(dx+c)^6}-\frac{\sin(dx+c)^7}{192\cos(dx+c)^4}+\frac{\sin(dx+c)^7}{128\cos(dx+c)^2}+\frac{\sin(dx+c)^5}{128}+\frac{5\sin(dx+c)^3}{384}+\frac{5\sin(dx+c)}{128}-\frac{5\ln(\sec(dx+c)+}{128}$
default	$\frac{a^3\left(\frac{\sin(dx+c)^7}{8\cos(dx+c)^8}+\frac{\sin(dx+c)^7}{48\cos(dx+c)^6}-\frac{\sin(dx+c)^7}{192\cos(dx+c)^4}+\frac{\sin(dx+c)^7}{128\cos(dx+c)^2}+\frac{\sin(dx+c)^5}{128}+\frac{5\sin(dx+c)^3}{384}+\frac{5\sin(dx+c)}{128}-\frac{5\ln(\sec(dx+c)+}{128}$
parts	$\frac{a^3\left(\frac{\tan(dx+c)^5}{5}-\frac{\tan(dx+c)^3}{3}+\tan(dx+c)-\arctan(\tan(dx+c))\right)}{d}+\frac{a^3\left(\frac{\sin(dx+c)^7}{8\cos(dx+c)^8}+\frac{\sin(dx+c)^7}{48\cos(dx+c)^6}-\frac{\sin(dx+c)^7}{192\cos(dx+c)^4}+$

input
```
int((a+a*sec(d*x+c))^3*tan(d*x+c)^6,x,method=_RETURNVERBOSE)
```

output
```
-a^3*x-1/6720*I*a^3*(27195*exp(15*I*(d*x+c))+65135*exp(13*I*(d*x+c))-16128
0*exp(12*I*(d*x+c))+63595*exp(11*I*(d*x+c))-286720*exp(10*I*(d*x+c))+13317
5*exp(9*I*(d*x+c))-519680*exp(8*I*(d*x+c))-133175*exp(7*I*(d*x+c))-544768*
exp(6*I*(d*x+c))-63595*exp(5*I*(d*x+c))-254464*exp(4*I*(d*x+c))-65135*exp(
3*I*(d*x+c))-118784*exp(2*I*(d*x+c))-27195*exp(I*(d*x+c))-14848)/d/(exp(2*
I*(d*x+c))+1)^8-125/128*a^3/d*ln(exp(I*(d*x+c))+I)+125/128*a^3/d*ln(exp(I*
(d*x+c))-I)
```

3.47.5 Fricas [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 178, normalized size of antiderivative = 0.75

$$\int (a + a\sec(c+dx))^3 \tan^6(c+dx)\,dx =$$

$$-\frac{26880\,a^3 dx\cos(dx+c)^8 + 13125\,a^3\cos(dx+c)^8\log(\sin(dx+c)+1) - 13125\,a^3\cos(dx+c)^8\log(-s}{}$$

input
```
integrate((a+a*sec(d*x+c))^3*tan(d*x+c)^6,x, algorithm="fricas")
```

output
```
-1/26880*(26880*a^3*d*x*cos(d*x + c)^8 + 13125*a^3*cos(d*x + c)^8*log(sin(
d*x + c) + 1) - 13125*a^3*cos(d*x + c)^8*log(-sin(d*x + c) + 1) - 2*(14848
*a^3*cos(d*x + c)^7 + 27195*a^3*cos(d*x + c)^6 + 7424*a^3*cos(d*x + c)^5 -
 17710*a^3*cos(d*x + c)^4 - 14592*a^3*cos(d*x + c)^3 + 1960*a^3*cos(d*x +
c)^2 + 5760*a^3*cos(d*x + c) + 1680*a^3)*sin(d*x + c))/(d*cos(d*x + c)^8)
```

3.47.6 Sympy [F]

$$\int (a + a\sec(c+dx))^3 \tan^6(c+dx)\,dx = a^3\left(\int 3\tan^6(c+dx)\sec(c+dx)\,dx + \int 3\tan^6(c+dx)\sec^2(c+dx)\,dx + \int \tan^6(c+dx)\sec^3(c+dx)\,dx + \int \tan^6(c+dx)\,dx\right)$$

input
```
integrate((a+a*sec(d*x+c))**3*tan(d*x+c)**6,x)
```

output
```
a**3*(Integral(3*tan(c + d*x)**6*sec(c + d*x), x) + Integral(3*tan(c + d*x
)**6*sec(c + d*x)**2, x) + Integral(tan(c + d*x)**6*sec(c + d*x)**3, x) +
Integral(tan(c + d*x)**6, x))
```

3.47.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 262, normalized size of antiderivative = 1.11

$$\int (a + a\sec(c+dx))^3 \tan^6(c+dx)\,dx$$

$$= \frac{11520\,a^3 \tan(dx+c)^7 + 1792\left(3\tan(dx+c)^5 - 5\tan(dx+c)^3 - 15\,dx - 15\,c + 15\tan(dx+c)\right)a^3 + 3}{}$$

input
```
integrate((a+a*sec(d*x+c))^3*tan(d*x+c)^6,x, algorithm="maxima")
```

output
```
1/26880*(11520*a^3*tan(d*x + c)^7 + 1792*(3*tan(d*x + c)^5 - 5*tan(d*x + c
)^3 - 15*d*x - 15*c + 15*tan(d*x + c))*a^3 + 35*a^3*(2*(15*sin(d*x + c)^7
+ 73*sin(d*x + c)^5 - 55*sin(d*x + c)^3 + 15*sin(d*x + c))/(sin(d*x + c)^8
 - 4*sin(d*x + c)^6 + 6*sin(d*x + c)^4 - 4*sin(d*x + c)^2 + 1) - 15*log(si
n(d*x + c) + 1) + 15*log(sin(d*x + c) - 1)) - 840*a^3*(2*(33*sin(d*x + c)^
5 - 40*sin(d*x + c)^3 + 15*sin(d*x + c))/(sin(d*x + c)^6 - 3*sin(d*x + c)^
4 + 3*sin(d*x + c)^2 - 1) + 15*log(sin(d*x + c) + 1) - 15*log(sin(d*x + c)
 - 1)))/d
```

3.47.8 Giac [A] (verification not implemented)

Time = 2.69 (sec) , antiderivative size = 196, normalized size of antiderivative = 0.83

$$\int (a + a\sec(c+dx))^3 \tan^6(c+dx)\,dx =$$

$$-\frac{13440\,(dx+c)a^3 + 13125\,a^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right) - 13125\,a^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right) + \frac{2\left(315}{}$$

input
```
integrate((a+a*sec(d*x+c))^3*tan(d*x+c)^6,x, algorithm="giac")
```

output
```
-1/13440*(13440*(d*x + c)*a^3 + 13125*a^3*log(abs(tan(1/2*d*x + 1/2*c) + 1
)) - 13125*a^3*log(abs(tan(1/2*d*x + 1/2*c) - 1)) + 2*(315*a^3*tan(1/2*d*x
 + 1/2*c)^15 - 11375*a^3*tan(1/2*d*x + 1/2*c)^13 + 79723*a^3*tan(1/2*d*x +
 1/2*c)^11 - 269879*a^3*tan(1/2*d*x + 1/2*c)^9 + 550089*a^3*tan(1/2*d*x +
1/2*c)^7 - 749973*a^3*tan(1/2*d*x + 1/2*c)^5 + 212625*a^3*tan(1/2*d*x + 1/
2*c)^3 - 26565*a^3*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^8)/d
```

3.47.9 Mupad [B] (verification not implemented)

Time = 15.04 (sec) , antiderivative size = 263, normalized size of antiderivative = 1.11

$$\int (a + a\sec(c+dx))^3 \tan^6(c+dx)\,dx = -a^3 x - \frac{125\,a^3 \operatorname{atanh}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{64\,d}$$
$$- \frac{\frac{3\,a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{15}}{64} - \frac{325\,a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{13}}{192} + \frac{11389\,a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{11}}{960} - \frac{269879\,a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{9}}{6720} + \frac{183363\,a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{7}}{2240} -}{d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{16} - 8\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{14} + 28\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{12} - 56\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{10} + 70\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{8} - 56\right.}$$

input
```
int(tan(c + d*x)^6*(a + a/cos(c + d*x))^3,x)
```

output
```
- a^3*x - (125*a^3*atanh(tan(c/2 + (d*x)/2)))/(64*d) - ((2025*a^3*tan(c/2
+ (d*x)/2)^3)/64 - (35713*a^3*tan(c/2 + (d*x)/2)^5)/320 + (183363*a^3*tan(
c/2 + (d*x)/2)^7)/2240 - (269879*a^3*tan(c/2 + (d*x)/2)^9)/6720 + (11389*a
^3*tan(c/2 + (d*x)/2)^11)/960 - (325*a^3*tan(c/2 + (d*x)/2)^13)/192 + (3*a
^3*tan(c/2 + (d*x)/2)^15)/64 - (253*a^3*tan(c/2 + (d*x)/2))/64)/(d*(28*tan
(c/2 + (d*x)/2)^4 - 8*tan(c/2 + (d*x)/2)^2 - 56*tan(c/2 + (d*x)/2)^6 + 70*
tan(c/2 + (d*x)/2)^8 - 56*tan(c/2 + (d*x)/2)^10 + 28*tan(c/2 + (d*x)/2)^12
 - 8*tan(c/2 + (d*x)/2)^14 + tan(c/2 + (d*x)/2)^16 + 1))
```

3.48 $\int (a + a\sec(c+dx))^3 \tan^4(c+dx)\,dx$

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3.48.1 Optimal result

Integrand size = 21, antiderivative size = 169

$$\begin{aligned}\int (a + a\sec(c+dx))^3 \tan^4(c+dx)\,dx = {} & a^3 x + \frac{19a^3 \operatorname{arctanh}(\sin(c+dx))}{16d} \\ & - \frac{a^3 \tan(c+dx)}{d} - \frac{17a^3 \sec(c+dx)\tan(c+dx)}{16d} \\ & - \frac{a^3 \sec^3(c+dx)\tan(c+dx)}{8d} + \frac{a^3 \tan^3(c+dx)}{3d} \\ & + \frac{3a^3 \sec(c+dx)\tan^3(c+dx)}{4d} \\ & + \frac{a^3 \sec^3(c+dx)\tan^3(c+dx)}{6d} + \frac{3a^3 \tan^5(c+dx)}{5d}\end{aligned}$$

output

```
a^3*x+19/16*a^3*arctanh(sin(d*x+c))/d-a^3*tan(d*x+c)/d-17/16*a^3*sec(d*x+c
)*tan(d*x+c)/d-1/8*a^3*sec(d*x+c)^3*tan(d*x+c)/d+1/3*a^3*tan(d*x+c)^3/d+3/
4*a^3*sec(d*x+c)*tan(d*x+c)^3/d+1/6*a^3*sec(d*x+c)^3*tan(d*x+c)^3/d+3/5*a^
3*tan(d*x+c)^5/d
```

3.48.2 Mathematica [A] (verified)

Time = 0.46 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.18

$$\begin{aligned}\int (a + a\sec(c+dx))^3 \tan^4(c+dx)\,dx = {} & \frac{a^3 \arctan(\tan(c+dx))}{d} + \frac{19a^3 \operatorname{arctanh}(\sin(c+dx))}{16d} \\ & - \frac{a^3 \tan(c+dx)}{d} + \frac{19a^3 \sec(c+dx)\tan(c+dx)}{16d} \\ & - \frac{53a^3 \sec^3(c+dx)\tan(c+dx)}{24d} \\ & - \frac{a^3 \sec^5(c+dx)\tan(c+dx)}{6d} + \frac{a^3 \tan^3(c+dx)}{3d} \\ & + \frac{3a^3 \sec(c+dx)\tan^3(c+dx)}{d} \\ & + \frac{a^3 \sec^3(c+dx)\tan^3(c+dx)}{3d} + \frac{3a^3 \tan^5(c+dx)}{5d}\end{aligned}$$

input
```
Integrate[(a + a*Sec[c + d*x])^3*Tan[c + d*x]^4,x]
```

output
```
(a^3*ArcTan[Tan[c + d*x]])/d + (19*a^3*ArcTanh[Sin[c + d*x]])/(16*d) - (a^
3*Tan[c + d*x])/d + (19*a^3*Sec[c + d*x]*Tan[c + d*x])/(16*d) - (53*a^3*Se
c[c + d*x]^3*Tan[c + d*x])/(24*d) - (a^3*Sec[c + d*x]^5*Tan[c + d*x])/(6*d
) + (a^3*Tan[c + d*x]^3)/(3*d) + (3*a^3*Sec[c + d*x]*Tan[c + d*x]^3)/d + (
a^3*Sec[c + d*x]^3*Tan[c + d*x]^3)/(3*d) + (3*a^3*Tan[c + d*x]^5)/(5*d)
```

3.48.3 Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3042, 4374, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^4(c+dx)(a\sec(c+dx)+a)^3\,dx$$

$$\downarrow \text{3042}$$

$$\int \cot\left(c+dx+\frac{\pi}{2}\right)^4 \left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^3 dx$$

$$\downarrow \text{4374}$$

$$\int (a^3\tan^4(c+dx)+a^3\tan^4(c+dx)\sec^3(c+dx)+3a^3\tan^4(c+dx)\sec^2(c+dx)+3a^3\tan^4(c+dx)\sec(c+dx)$$

$$\downarrow 2009$$

$$\frac{19a^3\text{arctanh}(\sin(c+dx))}{16d}+\frac{3a^3\tan^5(c+dx)}{5d}+\frac{a^3\tan^3(c+dx)}{3d}-\frac{a^3\tan(c+dx)}{d}+\frac{a^3\tan^3(c+dx)\sec^3(c+dx)}{6d}-\frac{a^3\tan(c+dx)\sec^3(c+dx)}{8d}+\frac{3a^3\tan^3(c+dx)\sec(c+dx)}{4d}-\frac{17a^3\tan(c+dx)\sec(c+dx)}{16d}+a^3x$$

input
```
Int[(a + a*Sec[c + d*x])^3*Tan[c + d*x]^4,x]
```

output
```
a^3*x + (19*a^3*ArcTanh[Sin[c + d*x]])/(16*d) - (a^3*Tan[c + d*x])/d - (17
*a^3*Sec[c + d*x]*Tan[c + d*x])/(16*d) - (a^3*Sec[c + d*x]^3*Tan[c + d*x])
/(8*d) + (a^3*Tan[c + d*x]^3)/(3*d) + (3*a^3*Sec[c + d*x]*Tan[c + d*x]^3)/
(4*d) + (a^3*Sec[c + d*x]^3*Tan[c + d*x]^3)/(6*d) + (3*a^3*Tan[c + d*x]^5)
/(5*d)
```

3.48.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4374
```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

3.48.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.26 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.15

method	result
risch	$a^3x+\frac{ia^3\left(435\,e^{11i(dx+c)}+240\,e^{10i(dx+c)}+865\,e^{9i(dx+c)}-1200\,e^{8i(dx+c)}-210\,e^{7i(dx+c)}-1760\,e^{6i(dx+c)}+210\,e^{5i(dx+}\right.}{120d\left(e^{2i(dx+c)}+1\right)^6}$
derivativedivides	$\frac{a^3\left(\frac{\sin(dx+c)^5}{6\cos(dx+c)^6}+\frac{\sin(dx+c)^5}{24\cos(dx+c)^4}-\frac{\sin(dx+c)^5}{48\cos(dx+c)^2}-\frac{\sin(dx+c)^3}{48}-\frac{\sin(dx+c)}{16}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{16}\right)+\frac{3a^3\sin(dx+c)^5}{5\cos(dx+c)^5}+3a^3}$
default	$\frac{a^3\left(\frac{\sin(dx+c)^5}{6\cos(dx+c)^6}+\frac{\sin(dx+c)^5}{24\cos(dx+c)^4}-\frac{\sin(dx+c)^5}{48\cos(dx+c)^2}-\frac{\sin(dx+c)^3}{48}-\frac{\sin(dx+c)}{16}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{16}\right)+\frac{3a^3\sin(dx+c)^5}{5\cos(dx+c)^5}+3a^3}$
parts	$\frac{a^3\left(\frac{\tan(dx+c)^3}{3}-\tan(dx+c)+\arctan(\tan(dx+c))\right)}{d}+\frac{a^3\left(\frac{\sin(dx+c)^5}{6\cos(dx+c)^6}+\frac{\sin(dx+c)^5}{24\cos(dx+c)^4}-\frac{\sin(dx+c)^5}{48\cos(dx+c)^2}-\frac{\sin(dx+c)^3}{48}-\frac{\sin}{}\right.}{d}$

input
```
int((a+a*sec(d*x+c))^3*tan(d*x+c)^4,x,method=_RETURNVERBOSE)
```

output
```
a^3*x+1/120*I*a^3*(435*exp(11*I*(d*x+c))+240*exp(10*I*(d*x+c))+865*exp(9*I
*(d*x+c))-1200*exp(8*I*(d*x+c))-210*exp(7*I*(d*x+c))-1760*exp(6*I*(d*x+c))
+210*exp(5*I*(d*x+c))-1440*exp(4*I*(d*x+c))-865*exp(3*I*(d*x+c))-1296*exp(
2*I*(d*x+c))-435*exp(I*(d*x+c))-176)/d/(exp(2*I*(d*x+c))+1)^6+19/16*a^3/d*
ln(exp(I*(d*x+c))+I)-19/16*a^3/d*ln(exp(I*(d*x+c))-I)
```

3.48.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.90

$$\int (a+a\sec(c+dx))^3\tan^4(c+dx)\,dx$$
$$=\frac{480\,a^3dx\cos(dx+c)^6+285\,a^3\cos(dx+c)^6\log(\sin(dx+c)+1)-285\,a^3\cos(dx+c)^6\log(-\sin(dx+c}{}$$

input
```
integrate((a+a*sec(d*x+c))^3*tan(d*x+c)^4,x, algorithm="fricas")
```

output
```
1/480*(480*a^3*d*x*cos(d*x + c)^6 + 285*a^3*cos(d*x + c)^6*log(sin(d*x + c
) + 1) - 285*a^3*cos(d*x + c)^6*log(-sin(d*x + c) + 1) - 2*(176*a^3*cos(d*
x + c)^5 + 435*a^3*cos(d*x + c)^4 + 208*a^3*cos(d*x + c)^3 - 110*a^3*cos(d
*x + c)^2 - 144*a^3*cos(d*x + c) - 40*a^3)*sin(d*x + c))/(d*cos(d*x + c)^6
)
```

3.48.6 Sympy [F]

$$\int (a + a\sec(c+dx))^3 \tan^4(c+dx)\,dx = a^3\left(\int 3\tan^4(c+dx)\sec(c+dx)\,dx + \int 3\tan^4(c+dx)\sec^2(c+dx)\,dx + \int \tan^4(c+dx)\sec^3(c+dx)\,dx + \int \tan^4(c+dx)\,dx\right)$$

input
```
integrate((a+a*sec(d*x+c))**3*tan(d*x+c)**4,x)
```

output
```
a**3*(Integral(3*tan(c + d*x)**4*sec(c + d*x), x) + Integral(3*tan(c + d*x
)**4*sec(c + d*x)**2, x) + Integral(tan(c + d*x)**4*sec(c + d*x)**3, x) +
Integral(tan(c + d*x)**4, x))
```

3.48.7 Maxima [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.24

$$\int (a + a\sec(c+dx))^3 \tan^4(c+dx)\,dx = \frac{288\,a^3\tan(dx+c)^5 + 160\left(\tan(dx+c)^3 + 3\,dx + 3\,c - 3\tan(dx+c)\right)a^3 - 5\,a^3\left(\frac{2\left(3\sin(dx+c)^5+8\sin(dx+c)^3}{\sin(dx+c)^6-3\sin(dx+c)^4+3}\right.\right.}{}$$

input
```
integrate((a+a*sec(d*x+c))^3*tan(d*x+c)^4,x, algorithm="maxima")
```

output
```
1/480*(288*a^3*tan(d*x + c)^5 + 160*(tan(d*x + c)^3 + 3*d*x + 3*c - 3*tan(
d*x + c))*a^3 - 5*a^3*(2*(3*sin(d*x + c)^5 + 8*sin(d*x + c)^3 - 3*sin(d*x
+ c))/(sin(d*x + c)^6 - 3*sin(d*x + c)^4 + 3*sin(d*x + c)^2 - 1) - 3*log(s
in(d*x + c) + 1) + 3*log(sin(d*x + c) - 1)) + 90*a^3*(2*(5*sin(d*x + c)^3
- 3*sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) + 3*log(sin(d*x
+ c) + 1) - 3*log(sin(d*x + c) - 1)))/d
```

3.48.8 Giac [A] (verification not implemented)

Time = 1.28 (sec) , antiderivative size = 164, normalized size of antiderivative = 0.97

$$\int (a + a\sec(c+dx))^3 \tan^4(c+dx)\,dx$$

$$= \frac{240\,(dx+c)a^3 + 285\,a^3 \log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right) - 285\,a^3 \log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right) - \frac{2\left(45\,a^3\tan\left(\frac{1}{2}\,dx+\right.\right.}{240\,d}$$

input
```
integrate((a+a*sec(d*x+c))^3*tan(d*x+c)^4,x, algorithm="giac")
```

output
```
1/240*(240*(d*x + c)*a^3 + 285*a^3*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 28
5*a^3*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(45*a^3*tan(1/2*d*x + 1/2*c)^
11 - 95*a^3*tan(1/2*d*x + 1/2*c)^9 - 366*a^3*tan(1/2*d*x + 1/2*c)^7 + 1746
*a^3*tan(1/2*d*x + 1/2*c)^5 - 3135*a^3*tan(1/2*d*x + 1/2*c)^3 + 525*a^3*ta
n(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^6)/d
```

3.48.9 Mupad [B] (verification not implemented)

Time = 14.83 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.20

$$\int (a + a\sec(c+dx))^3 \tan^4(c+dx)\,dx = a^3 x + \frac{19\,a^3\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{8\,d}$$
$$+ \frac{-\frac{3\,a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{11}}{8} + \frac{19\,a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^9}{24} + \frac{61\,a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^7}{20} - \frac{291\,a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5}{20} + \frac{209\,a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3}{8} - \frac{35\,a^3}{}}{d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{12} - 6\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{10} + 15\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^8 - 20\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6 + 15\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4 - 6\,\mathrm{ta}\right.}$$

input
```
int(tan(c + d*x)^4*(a + a/cos(c + d*x))^3,x)
```

output
```
a^3*x + (19*a^3*atanh(tan(c/2 + (d*x)/2)))/(8*d) + ((209*a^3*tan(c/2 + (d*
x)/2)^3)/8 - (291*a^3*tan(c/2 + (d*x)/2)^5)/20 + (61*a^3*tan(c/2 + (d*x)/2
)^7)/20 + (19*a^3*tan(c/2 + (d*x)/2)^9)/24 - (3*a^3*tan(c/2 + (d*x)/2)^11)
/8 - (35*a^3*tan(c/2 + (d*x)/2))/8)/(d*(15*tan(c/2 + (d*x)/2)^4 - 6*tan(c/
2 + (d*x)/2)^2 - 20*tan(c/2 + (d*x)/2)^6 + 15*tan(c/2 + (d*x)/2)^8 - 6*tan
(c/2 + (d*x)/2)^10 + tan(c/2 + (d*x)/2)^12 + 1))
```

3.49 $\int (a + a\sec(c+dx))^3 \tan^2(c+dx)\,dx$

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3.49.1 Optimal result

Integrand size = 21, antiderivative size = 98

$$\int (a + a\sec(c+dx))^3 \tan^2(c+dx)\,dx = -a^3x - \frac{13a^3\operatorname{arctanh}(\sin(c+dx))}{8d} + \frac{a^3\tan(c+dx)}{d} + \frac{11a^3\sec(c+dx)\tan(c+dx)}{8d} + \frac{a^3\sec^3(c+dx)\tan(c+dx)}{4d} + \frac{a^3\tan^3(c+dx)}{d}$$

output
```
-a^3*x-13/8*a^3*arctanh(sin(d*x+c))/d+a^3*tan(d*x+c)/d+11/8*a^3*sec(d*x+c)
*tan(d*x+c)/d+1/4*a^3*sec(d*x+c)^3*tan(d*x+c)/d+a^3*tan(d*x+c)^3/d
```

3.49.2 Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.09

$$\int (a + a\sec(c+dx))^3 \tan^2(c+dx)\,dx = -\frac{a^3\arctan(\tan(c+dx))}{d} - \frac{13a^3\operatorname{arctanh}(\sin(c+dx))}{8d} + \frac{a^3\tan(c+dx)}{d} + \frac{11a^3\sec(c+dx)\tan(c+dx)}{8d} + \frac{a^3\sec^3(c+dx)\tan(c+dx)}{4d} + \frac{a^3\tan^3(c+dx)}{d}$$

input
```
Integrate[(a + a*Sec[c + d*x])^3*Tan[c + d*x]^2,x]
```

output
```
-((a^3*ArcTan[Tan[c + d*x]])/d) - (13*a^3*ArcTanh[Sin[c + d*x]])/(8*d) + (
a^3*Tan[c + d*x])/d + (11*a^3*Sec[c + d*x]*Tan[c + d*x])/(8*d) + (a^3*Sec[
c + d*x]^3*Tan[c + d*x])/(4*d) + (a^3*Tan[c + d*x]^3)/d
```

3.49.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3042, 4374, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^2(c+dx)(a\sec(c+dx)+a)^3\,dx$$

$$\downarrow 3042$$

$$\int \cot\left(c+dx+\frac{\pi}{2}\right)^2\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^3 dx$$

$$\downarrow 4374$$

$$\int (a^3\tan^2(c+dx)+a^3\tan^2(c+dx)\sec^3(c+dx)+3a^3\tan^2(c+dx)\sec^2(c+dx)+3a^3\tan^2(c+dx)\sec(c+dx)$$

$$\downarrow 2009$$

$$-\frac{13a^3\text{arctanh}(\sin(c+dx))}{8d}+\frac{a^3\tan^3(c+dx)}{d}+\frac{a^3\tan(c+dx)}{d}+\frac{a^3\tan(c+dx)\sec^3(c+dx)}{4d}+\frac{11a^3\tan(c+dx)\sec(c+dx)}{8d}-a^3x$$

input
```
Int[(a + a*Sec[c + d*x])^3*Tan[c + d*x]^2,x]
```

output
```
-(a^3*x) - (13*a^3*ArcTanh[Sin[c + d*x]])/(8*d) + (a^3*Tan[c + d*x])/d + (
11*a^3*Sec[c + d*x]*Tan[c + d*x])/(8*d) + (a^3*Sec[c + d*x]^3*Tan[c + d*x]
)/(4*d) + (a^3*Tan[c + d*x]^3)/d
```

3.49.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4374
```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

3.49.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.00 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.42

method	result
risch	$-a^3x-\frac{ia^3\left(11\,e^{7i(dx+c)}+16\,e^{6i(dx+c)}+19\,e^{5i(dx+c)}-19\,e^{3i(dx+c)}-16\,e^{2i(dx+c)}-11\,e^{i(dx+c)}\right)}{4d\left(e^{2i(dx+c)}+1\right)^4}-\frac{13a^3\ln\left(e^{i(dx+c)}-\right.}{8d}$
derivativedivides	$\frac{a^3\left(\frac{\sin(dx+c)^3}{4\cos(dx+c)^4}+\frac{\sin(dx+c)^3}{8\cos(dx+c)^2}+\frac{\sin(dx+c)}{8}-\frac{\ln(\sec(dx+c)+\tan(dx+c))}{8}\right)+\frac{a^3\sin(dx+c)^3}{\cos(dx+c)^3}+3a^3\left(\frac{\sin(dx+c)^3}{2\cos(dx+c)^2}+\frac{\sin(dx+c)}{2}-\frac{\ln}{}\right.}{d}$
default	$\frac{a^3\left(\frac{\sin(dx+c)^3}{4\cos(dx+c)^4}+\frac{\sin(dx+c)^3}{8\cos(dx+c)^2}+\frac{\sin(dx+c)}{8}-\frac{\ln(\sec(dx+c)+\tan(dx+c))}{8}\right)+\frac{a^3\sin(dx+c)^3}{\cos(dx+c)^3}+3a^3\left(\frac{\sin(dx+c)^3}{2\cos(dx+c)^2}+\frac{\sin(dx+c)}{2}-\frac{\ln}{}\right.}{d}$
parts	$\frac{a^3(\tan(dx+c)-\arctan(\tan(dx+c)))}{d}+\frac{a^3\left(\frac{\sin(dx+c)^3}{4\cos(dx+c)^4}+\frac{\sin(dx+c)^3}{8\cos(dx+c)^2}+\frac{\sin(dx+c)}{8}-\frac{\ln(\sec(dx+c)+\tan(dx+c))}{8}\right)}{d}+\frac{3a^3\left(\right.}{}$

input
```
int((a+a*sec(d*x+c))^3*tan(d*x+c)^2,x,method=_RETURNVERBOSE)
```

output
```
-a^3*x-1/4*I*a^3/d/(exp(2*I*(d*x+c))+1)^4*(11*exp(7*I*(d*x+c))+16*exp(6*I*
(d*x+c))+19*exp(5*I*(d*x+c))-19*exp(3*I*(d*x+c))-16*exp(2*I*(d*x+c))-11*ex
p(I*(d*x+c)))-13/8*a^3/d*ln(exp(I*(d*x+c))+I)+13/8*a^3/d*ln(exp(I*(d*x+c))
-I)
```

3.49.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.15

$$\int (a + a\sec(c+dx))^3 \tan^2(c+dx)\,dx =$$

$$-\frac{16\,a^3 dx\cos(dx+c)^4 + 13\,a^3\cos(dx+c)^4\log(\sin(dx+c)+1) - 13\,a^3\cos(dx+c)^4\log(-\sin(dx+c}{16\,d\cos(dx+c)^4}$$

input
```
integrate((a+a*sec(d*x+c))^3*tan(d*x+c)^2,x, algorithm="fricas")
```

output
```
-1/16*(16*a^3*d*x*cos(d*x + c)^4 + 13*a^3*cos(d*x + c)^4*log(sin(d*x + c)
+ 1) - 13*a^3*cos(d*x + c)^4*log(-sin(d*x + c) + 1) - 2*(11*a^3*cos(d*x +
c)^2 + 8*a^3*cos(d*x + c) + 2*a^3)*sin(d*x + c))/(d*cos(d*x + c)^4)
```

3.49.6 Sympy [F]

$$\int (a + a\sec(c+dx))^3 \tan^2(c+dx)\,dx = a^3\left(\int 3\tan^2(c+dx)\sec(c+dx)\,dx + \int 3\tan^2(c+dx)\sec^2(c+dx)\,dx + \int \tan^2(c+dx)\sec^3(c+dx)\,dx + \int \tan^2(c+dx)\,dx\right)$$

input
```
integrate((a+a*sec(d*x+c))**3*tan(d*x+c)**2,x)
```

output
```
a**3*(Integral(3*tan(c + d*x)**2*sec(c + d*x), x) + Integral(3*tan(c + d*x
)**2*sec(c + d*x)**2, x) + Integral(tan(c + d*x)**2*sec(c + d*x)**3, x) +
Integral(tan(c + d*x)**2, x))
```

3.49.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.50

$$\int (a + a\sec(c+dx))^3 \tan^2(c+dx)\,dx$$

$$= \frac{16\,a^3 \tan(dx+c)^3 - 16\,(dx + c - \tan(dx+c))a^3 + a^3\left(\frac{2\left(\sin(dx+c)^3+\sin(dx+c)\right)}{\sin(dx+c)^4-2\,\sin(dx+c)^2+1} - \log(\sin(dx+c)+1) + \right.}{16\,d}$$

input
```
integrate((a+a*sec(d*x+c))^3*tan(d*x+c)^2,x, algorithm="maxima")
```

output
```
1/16*(16*a^3*tan(d*x + c)^3 - 16*(d*x + c - tan(d*x + c))*a^3 + a^3*(2*(si
n(d*x + c)^3 + sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - log
(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) - 12*a^3*(2*sin(d*x + c)/(sin(
d*x + c)^2 - 1) + log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)))/d
```

3.49.8 Giac [A] (verification not implemented)

Time = 0.74 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.35

$$\int (a + a\sec(c+dx))^3 \tan^2(c+dx)\,dx =$$

$$-\frac{8\,(dx+c)a^3 + 13\,a^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right) - 13\,a^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right) - \frac{2\left(5\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right.}{8\,d}$$

input
```
integrate((a+a*sec(d*x+c))^3*tan(d*x+c)^2,x, algorithm="giac")
```

output
```
-1/8*(8*(d*x + c)*a^3 + 13*a^3*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 13*a^3
*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(5*a^3*tan(1/2*d*x + 1/2*c)^7 - 13
*a^3*tan(1/2*d*x + 1/2*c)^5 + 3*a^3*tan(1/2*d*x + 1/2*c)^3 + 21*a^3*tan(1/
2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^4)/d
```

3.49.9 Mupad [B] (verification not implemented)

Time = 14.31 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.49

$$\int (a + a\sec(c+dx))^3 \tan^2(c+dx)\,dx$$

$$= \frac{\frac{5a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^7}{4} - \frac{13a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5}{4} + \frac{3a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3}{4} + \frac{21a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{4}}{d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^8 - 4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6 + 6\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4 - 4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2 + 1\right)} - a^3 x - \frac{13a^3\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{4d}$$

input
```
int(tan(c + d*x)^2*(a + a/cos(c + d*x))^3,x)
```

output
```
((3*a^3*tan(c/2 + (d*x)/2)^3)/4 - (13*a^3*tan(c/2 + (d*x)/2)^5)/4 + (5*a^3
*tan(c/2 + (d*x)/2)^7)/4 + (21*a^3*tan(c/2 + (d*x)/2))/4)/(d*(6*tan(c/2 +
(d*x)/2)^4 - 4*tan(c/2 + (d*x)/2)^2 - 4*tan(c/2 + (d*x)/2)^6 + tan(c/2 + (
d*x)/2)^8 + 1)) - a^3*x - (13*a^3*atanh(tan(c/2 + (d*x)/2)))/(4*d)
```

3.50 $\int \cot^2(c+dx)(a+a\sec(c+dx))^3\,dx$

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3.50.1 Optimal result

Integrand size = 21, antiderivative size = 49

$$\int \cot^2(c+dx)(a+a\sec(c+dx))^3\,dx = -a^3x + \frac{a^3\operatorname{arctanh}(\sin(c+dx))}{d} - \frac{4a^3\cot(c+dx)}{d} - \frac{4a^3\csc(c+dx)}{d}$$

output
```
-a^3*x+a^3*arctanh(sin(d*x+c))/d-4*a^3*cot(d*x+c)/d-4*a^3*csc(d*x+c)/d
```

3.50.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 109 vs. 2(49) = 98.

Time = 0.63 (sec) , antiderivative size = 109, normalized size of antiderivative = 2.22

$$\int \cot^2(c+dx)(a+a\sec(c+dx))^3\,dx = -\frac{a^3(1+\cos(c+dx))^3\sec^6\left(\frac{1}{2}(c+dx)\right)\left(dx+\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(\cos\left(\frac{1}{2}(c+d\right.\right.\right.}{8d}$$

input
```
Integrate[Cot[c + d*x]^2*(a + a*Sec[c + d*x])^3,x]
```

output
```
-1/8*(a^3*(1 + Cos[c + d*x])^3*Sec[(c + d*x)/2]^6*(d*x + Log[Cos[(c + d*x)
/2] - Sin[(c + d*x)/2]] - Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] - 4*Csc
[c/2]*Csc[(c + d*x)/2]*Sin[(d*x)/2]))/d
```

3.50.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3042, 4374, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^2(c+dx)(a\sec(c+dx)+a)^3\,dx$$

$$\downarrow 3042$$

$$\int \frac{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^3}{\cot\left(c+dx+\frac{\pi}{2}\right)^2}dx$$

$$\downarrow 4374$$

$$\int \left(a^3\cot^2(c+dx)+3a^3\csc^2(c+dx)+3a^3\cot(c+dx)\csc(c+dx)+a^3\csc^2(c+dx)\sec(c+dx)\right)dx$$

$$\downarrow 2009$$

$$\frac{a^3\text{arctanh}(\sin(c+dx))}{d}-\frac{4a^3\cot(c+dx)}{d}-\frac{4a^3\csc(c+dx)}{d}+a^3(-x)$$

input
```
Int[Cot[c + d*x]^2*(a + a*Sec[c + d*x])^3,x]
```

output
```
-(a^3*x) + (a^3*ArcTanh[Sin[c + d*x]])/d - (4*a^3*Cot[c + d*x])/d - (4*a^3
*Csc[c + d*x])/d
```

3.50.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4374
```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

3.50.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.30 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.45

method	result	size
risch	$-a^3x-\frac{8ia^3}{d\left(e^{i(dx+c)}-1\right)}-\frac{a^3\ln\left(e^{i(dx+c)}-i\right)}{d}+\frac{a^3\ln\left(e^{i(dx+c)}+i\right)}{d}$	71
derivativedivides	$\frac{a^3\left(-\frac{1}{\sin(dx+c)}+\ln(\sec(dx+c)+\tan(dx+c))\right)-3a^3\cot(dx+c)-\frac{3a^3}{\sin(dx+c)}+a^3(-\cot(dx+c)-dx-c)}{d}$	79
default	$\frac{a^3\left(-\frac{1}{\sin(dx+c)}+\ln(\sec(dx+c)+\tan(dx+c))\right)-3a^3\cot(dx+c)-\frac{3a^3}{\sin(dx+c)}+a^3(-\cot(dx+c)-dx-c)}{d}$	79

input
```
int(cot(d*x+c)^2*(a+a*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
-a^3*x-8*I*a^3/d/(exp(I*(d*x+c))-1)-a^3/d*ln(exp(I*(d*x+c))-I)+a^3/d*ln(ex
p(I*(d*x+c))+I)
```

3.50.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.71

$$\int \cot^2(c+dx)(a+a\sec(c+dx))^3\,dx =$$
$$-\frac{2\,a^3 dx\sin(dx+c)-a^3\log(\sin(dx+c)+1)\sin(dx+c)+a^3\log(-\sin(dx+c)+1)\sin(dx+c)+8}{2\,d\sin(dx+c)}$$

input
```
integrate(cot(d*x+c)^2*(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

output
```
-1/2*(2*a^3*d*x*sin(d*x + c) - a^3*log(sin(d*x + c) + 1)*sin(d*x + c) + a^
3*log(-sin(d*x + c) + 1)*sin(d*x + c) + 8*a^3*cos(d*x + c) + 8*a^3)/(d*sin
(d*x + c))
```

3.50.6 Sympy [F]

$$\int \cot^2(c+dx)(a+a\sec(c+dx))^3\,dx = a^3\left(\int 3\cot^2(c+dx)\sec(c+dx)\,dx + \int 3\cot^2(c+dx)\sec^2(c+dx)\,dx + \int \cot^2(c+dx)\sec^3(c+dx)\,dx + \int \cot^2(c+dx)\,dx\right)$$

input
```
integrate(cot(d*x+c)**2*(a+a*sec(d*x+c))**3,x)
```

output
```
a**3*(Integral(3*cot(c + d*x)**2*sec(c + d*x), x) + Integral(3*cot(c + d*x
)**2*sec(c + d*x)**2, x) + Integral(cot(c + d*x)**2*sec(c + d*x)**3, x) +
Integral(cot(c + d*x)**2, x))
```

3.50.7 Maxima [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.73

$$\int \cot^2(c+dx)(a+a\sec(c+dx))^3\,dx = -\frac{2\left(dx+c+\frac{1}{\tan(dx+c)}\right)a^3 + a^3\left(\frac{2}{\sin(dx+c)} - \log\left(\sin(dx+c)+1\right) + \log\left(\sin(dx+c)-1\right)\right) + \frac{6a^3}{\sin(dx+c)} + \frac{}{}}{2d}$$

input
```
integrate(cot(d*x+c)^2*(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

output
```
-1/2*(2*(d*x + c + 1/tan(d*x + c))*a^3 + a^3*(2/sin(d*x + c) - log(sin(d*x
 + c) + 1) + log(sin(d*x + c) - 1)) + 6*a^3/sin(d*x + c) + 6*a^3/tan(d*x +
 c))/d
```

3.50.8 Giac [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.35

$$\int \cot^2(c+dx)(a+a\sec(c+dx))^3\,dx = -\frac{(dx+c)a^3 - a^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right) + a^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right) + \frac{4\,a^3}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}}{d}$$

input
```
integrate(cot(d*x+c)^2*(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

output
```
-((d*x + c)*a^3 - a^3*log(abs(tan(1/2*d*x + 1/2*c) + 1)) + a^3*log(abs(tan
(1/2*d*x + 1/2*c) - 1)) + 4*a^3/tan(1/2*d*x + 1/2*c))/d
```

3.50.9 Mupad [B] (verification not implemented)

Time = 13.98 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.71

$$\int \cot^2(c+dx)(a+a\sec(c+dx))^3\,dx = -\frac{a^3\left(4\cot\left(\frac{c}{2}+\frac{dx}{2}\right) - 2\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)+d\,x\right)}{d}$$

input
```
int(cot(c + d*x)^2*(a + a/cos(c + d*x))^3,x)
```

output
```
-(a^3*(4*cot(c/2 + (d*x)/2) - 2*atanh(tan(c/2 + (d*x)/2)) + d*x))/d
```

3.51 $\int \cot^4(c+dx)(a+a\sec(c+dx))^3\,dx$

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3.51.1 Optimal result

Integrand size = 21, antiderivative size = 69

$$\int \cot^4(c+dx)(a+a\sec(c+dx))^3\,dx = a^3x + \frac{a^3\cot(c+dx)}{d} - \frac{4a^3\cot^3(c+dx)}{3d} + \frac{3a^3\csc(c+dx)}{d} - \frac{4a^3\csc^3(c+dx)}{3d}$$

output
```
a^3*x+a^3*cot(d*x+c)/d-4/3*a^3*cot(d*x+c)^3/d+3*a^3*csc(d*x+c)/d-4/3*a^3*c
sc(d*x+c)^3/d
```

3.51.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.26 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.23

$$\int \cot^4(c+dx)(a+a\sec(c+dx))^3\,dx = -\frac{a^3\cot^3(c+dx)}{d} + \frac{3a^3\csc(c+dx)}{d} - \frac{4a^3\csc^3(c+dx)}{3d} - \frac{a^3\cot^3(c+dx)\,\text{Hypergeometric2F1}\left(-\frac{3}{2},1,-\frac{1}{2},-\tan^2(c+dx)\right)}{3d}$$

input
```
Integrate[Cot[c + d*x]^4*(a + a*Sec[c + d*x])^3,x]
```

output
```
-((a^3*Cot[c + d*x]^3)/d) + (3*a^3*Csc[c + d*x])/d - (4*a^3*Csc[c + d*x]^3
)/(3*d) - (a^3*Cot[c + d*x]^3*Hypergeometric2F1[-3/2, 1, -1/2, -Tan[c + d*
x]^2])/(3*d)
```

3.51.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3042, 4374, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^4(c+dx)(a\sec(c+dx)+a)^3\,dx$$

$$\downarrow 3042$$

$$\int \frac{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^3}{\cot\left(c+dx+\frac{\pi}{2}\right)^4}dx$$

$$\downarrow 4374$$

$$\int \left(a^3\cot^4(c+dx)+3a^3\cot^3(c+dx)\csc(c+dx)+3a^3\cot^2(c+dx)\csc^2(c+dx)+a^3\cot(c+dx)\csc^3(c+dx)\right)a$$

$$\downarrow 2009$$

$$-\frac{4a^3\cot^3(c+dx)}{3d}+\frac{a^3\cot(c+dx)}{d}-\frac{4a^3\csc^3(c+dx)}{3d}+\frac{3a^3\csc(c+dx)}{d}+a^3x$$

input
```
Int[Cot[c + d*x]^4*(a + a*Sec[c + d*x])^3,x]
```

output
```
a^3*x + (a^3*Cot[c + d*x])/d - (4*a^3*Cot[c + d*x]^3)/(3*d) + (3*a^3*Csc[c
 + d*x])/d - (4*a^3*Csc[c + d*x]^3)/(3*d)
```

3.51.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4374
```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

3.51.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.73 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.77

method	result
risch	$a^3 x + \frac{2ia^3\left(9\,\mathrm{e}^{2i(dx+c)} - 12\,\mathrm{e}^{i(dx+c)} + 7\right)}{3d\left(\mathrm{e}^{i(dx+c)} - 1\right)^3}$
derivativedivides	$\frac{-\frac{a^3}{3\sin(dx+c)^3} - \frac{a^3\cos(dx+c)^3}{\sin(dx+c)^3} + 3a^3\left(-\frac{\cos(dx+c)^4}{3\sin(dx+c)^3} + \frac{\cos(dx+c)^4}{3\sin(dx+c)} + \frac{\left(2+\cos(dx+c)^2\right)\sin(dx+c)}{3}\right) + a^3\left(-\frac{\cot(dx+c)^3}{3} + \cot(dx}{d}$
default	$\frac{-\frac{a^3}{3\sin(dx+c)^3} - \frac{a^3\cos(dx+c)^3}{\sin(dx+c)^3} + 3a^3\left(-\frac{\cos(dx+c)^4}{3\sin(dx+c)^3} + \frac{\cos(dx+c)^4}{3\sin(dx+c)} + \frac{\left(2+\cos(dx+c)^2\right)\sin(dx+c)}{3}\right) + a^3\left(-\frac{\cot(dx+c)^3}{3} + \cot(dx}{d}$

input
```
int(cot(d*x+c)^4*(a+a*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
a^3*x+2/3*I*a^3*(9*exp(2*I*(d*x+c))-12*exp(I*(d*x+c))+7)/d/(exp(I*(d*x+c))
-1)^3
```

3.51.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.19

$$\int \cot^4(c+dx)(a+a\sec(c+dx))^3\,dx$$
$$= \frac{7\,a^3\cos(dx+c)^2 + 2\,a^3\cos(dx+c) - 5\,a^3 + 3\,(a^3 dx\cos(dx+c) - a^3 dx)\sin(dx+c)}{3\,(d\cos(dx+c) - d)\sin(dx+c)}$$

input
```
integrate(cot(d*x+c)^4*(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

output
```
1/3*(7*a^3*cos(d*x + c)^2 + 2*a^3*cos(d*x + c) - 5*a^3 + 3*(a^3*d*x*cos(d*
x + c) - a^3*d*x)*sin(d*x + c))/((d*cos(d*x + c) - d)*sin(d*x + c))
```

3.51.6 Sympy [F]

$$\int \cot^4(c+dx)(a+a\sec(c+dx))^3\,dx = a^3\left(\int 3\cot^4(c+dx)\sec(c+dx)\,dx + \int 3\cot^4(c+dx)\sec^2(c+dx)\,dx + \int \cot^4(c+dx)\sec^3(c+dx)\,dx + \int \cot^4(c+dx)\,dx\right)$$

input
```
integrate(cot(d*x+c)**4*(a+a*sec(d*x+c))**3,x)
```

output
```
a**3*(Integral(3*cot(c + d*x)**4*sec(c + d*x), x) + Integral(3*cot(c + d*x
)**4*sec(c + d*x)**2, x) + Integral(cot(c + d*x)**4*sec(c + d*x)**3, x) +
Integral(cot(c + d*x)**4, x))
```

3.51.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.30

$$\int \cot^4(c+dx)(a+a\sec(c+dx))^3\,dx$$
$$= \frac{\left(3\,dx+3\,c+\frac{3\tan(dx+c)^2-1}{\tan(dx+c)^3}\right)a^3+\frac{3\left(3\sin(dx+c)^2-1\right)a^3}{\sin(dx+c)^3}-\frac{a^3}{\sin(dx+c)^3}-\frac{3a^3}{\tan(dx+c)^3}}{3\,d}$$

input
```
integrate(cot(d*x+c)^4*(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

output
```
1/3*((3*d*x + 3*c + (3*tan(d*x + c)^2 - 1)/tan(d*x + c)^3)*a^3 + 3*(3*sin(
d*x + c)^2 - 1)*a^3/sin(d*x + c)^3 - a^3/sin(d*x + c)^3 - 3*a^3/tan(d*x +
c)^3)/d
```

3.51.8 Giac [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.72

$$\int \cot^4(c+dx)(a+a\sec(c+dx))^3\,dx = \frac{3\,(dx+c)a^3+\frac{6\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-a^3}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3}}{3\,d}$$

input
```
integrate(cot(d*x+c)^4*(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

output
```
1/3*(3*(d*x + c)*a^3 + (6*a^3*tan(1/2*d*x + 1/2*c)^2 - a^3)/tan(1/2*d*x +
1/2*c)^3)/d
```

3.51.9 Mupad [B] (verification not implemented)

Time = 13.57 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.57

$$\int \cot^4(c+dx)(a+a\sec(c+dx))^3\,dx = a^3\,x+\frac{a^3\left(6\cot\left(\frac{c}{2}+\frac{dx}{2}\right)-\cot\left(\frac{c}{2}+\frac{dx}{2}\right)^3\right)}{3\,d}$$

input
```
int(cot(c + d*x)^4*(a + a/cos(c + d*x))^3,x)
```

output
```
a^3*x + (a^3*(6*cot(c/2 + (d*x)/2) - cot(c/2 + (d*x)/2)^3))/(3*d)
```

3.52 $\int \cot^6(c+dx)(a+a\sec(c+dx))^3\,dx$

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3.52.1 Optimal result

Integrand size = 21, antiderivative size = 107

$$\int \cot^6(c+dx)(a+a\sec(c+dx))^3\,dx = -a^3x - \frac{a^3\cot(c+dx)}{d} + \frac{a^3\cot^3(c+dx)}{3d} - \frac{4a^3\cot^5(c+dx)}{5d} - \frac{3a^3\csc(c+dx)}{d} + \frac{7a^3\csc^3(c+dx)}{3d} - \frac{4a^3\csc^5(c+dx)}{5d}$$

output
```
-a^3*x-a^3*cot(d*x+c)/d+1/3*a^3*cot(d*x+c)^3/d-4/5*a^3*cot(d*x+c)^5/d-3*a^
3*csc(d*x+c)/d+7/3*a^3*csc(d*x+c)^3/d-4/5*a^3*csc(d*x+c)^5/d
```

3.52.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.47 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.98

$$\int \cot^6(c+dx)(a+a\sec(c+dx))^3\,dx = -\frac{3a^3\cot^5(c+dx)}{5d} - \frac{3a^3\csc(c+dx)}{d} + \frac{7a^3\csc^3(c+dx)}{3d} - \frac{4a^3\csc^5(c+dx)}{5d} - \frac{a^3\cot^5(c+dx)\,\text{Hypergeometric2F1}\left(-\frac{5}{2},1,-\frac{3}{2},-\tan^2(c+dx)\right)}{5d}$$

input
```
Integrate[Cot[c + d*x]^6*(a + a*Sec[c + d*x])^3,x]
```

output
```
(-3*a^3*Cot[c + d*x]^5)/(5*d) - (3*a^3*Csc[c + d*x])/d + (7*a^3*Csc[c + d*
x]^3)/(3*d) - (4*a^3*Csc[c + d*x]^5)/(5*d) - (a^3*Cot[c + d*x]^5*Hypergeom
etric2F1[-5/2, 1, -3/2, -Tan[c + d*x]^2])/(5*d)
```

3.52.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3042, 4374, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^6(c+dx)(a\sec(c+dx)+a)^3\,dx$$

$$\downarrow 3042$$

$$\int \frac{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^3}{\cot\left(c+dx+\frac{\pi}{2}\right)^6}dx$$

$$\downarrow 4374$$

$$\int \left(a^3\cot^6(c+dx)+3a^3\cot^5(c+dx)\csc(c+dx)+3a^3\cot^4(c+dx)\csc^2(c+dx)+a^3\cot^3(c+dx)\csc^3(c+dx)\right)$$

$$\downarrow 2009$$

$$-\frac{4a^3\cot^5(c+dx)}{5d}+\frac{a^3\cot^3(c+dx)}{3d}-\frac{a^3\cot(c+dx)}{d}-\frac{4a^3\csc^5(c+dx)}{5d}+\frac{7a^3\csc^3(c+dx)}{3d}-\frac{3a^3\csc(c+dx)}{d}-a^3x$$

input
```
Int[Cot[c + d*x]^6*(a + a*Sec[c + d*x])^3,x]
```

output
```
-(a^3*x) - (a^3*Cot[c + d*x])/d + (a^3*Cot[c + d*x]^3)/(3*d) - (4*a^3*Cot[
c + d*x]^5)/(5*d) - (3*a^3*Csc[c + d*x])/d + (7*a^3*Csc[c + d*x]^3)/(3*d)
- (4*a^3*Csc[c + d*x]^5)/(5*d)
```

3.52.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4374
```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

3.52.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.89 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.71

method	result
risch	$-a^3x-\frac{2ia^3\left(45\,e^{4i(dx+c)}-135\,e^{3i(dx+c)}+185\,e^{2i(dx+c)}-115\,e^{i(dx+c)}+32\right)}{15d\left(e^{i(dx+c)}-1\right)^5}$
derivativedivides	$\frac{a^3\left(-\frac{\cos(dx+c)^4}{5\sin(dx+c)^5}-\frac{\cos(dx+c)^4}{15\sin(dx+c)^3}+\frac{\cos(dx+c)^4}{15\sin(dx+c)}+\frac{\left(2+\cos(dx+c)^2\right)\sin(dx+c)}{15}\right)-\frac{3a^3\cos(dx+c)^5}{5\sin(dx+c)^5}+3a^3\left(-\frac{\cos(dx+c)^6}{5\sin(dx+c)^5}+\frac{\cos}{15\,si}\right.}{d}$
default	$\frac{a^3\left(-\frac{\cos(dx+c)^4}{5\sin(dx+c)^5}-\frac{\cos(dx+c)^4}{15\sin(dx+c)^3}+\frac{\cos(dx+c)^4}{15\sin(dx+c)}+\frac{\left(2+\cos(dx+c)^2\right)\sin(dx+c)}{15}\right)-\frac{3a^3\cos(dx+c)^5}{5\sin(dx+c)^5}+3a^3\left(-\frac{\cos(dx+c)^6}{5\sin(dx+c)^5}+\frac{\cos}{15\,si}\right.}{d}$

input
```
int(cot(d*x+c)^6*(a+a*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
-a^3*x-2/15*I*a^3*(45*exp(4*I*(d*x+c))-135*exp(3*I*(d*x+c))+185*exp(2*I*(d
*x+c))-115*exp(I*(d*x+c))+32)/d/(exp(I*(d*x+c))-1)^5
```

3.52.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.10

$$\int \cot^6(c+dx)(a+a\sec(c+dx))^3\,dx =$$
$$-\frac{32\,a^3\cos(dx+c)^3 - 19\,a^3\cos(dx+c)^2 - 29\,a^3\cos(dx+c) + 22\,a^3 + 15\left(a^3 dx\cos(dx+c)^2 - 2\,a^3 dx\,c\right.}{15\left(d\cos(dx+c)^2 - 2\,d\cos(dx+c) + d\right)\sin(dx+c)}$$

input
```
integrate(cot(d*x+c)^6*(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

output
```
-1/15*(32*a^3*cos(d*x + c)^3 - 19*a^3*cos(d*x + c)^2 - 29*a^3*cos(d*x + c)
 + 22*a^3 + 15*(a^3*d*x*cos(d*x + c)^2 - 2*a^3*d*x*cos(d*x + c) + a^3*d*x)
*sin(d*x + c))/((d*cos(d*x + c)^2 - 2*d*cos(d*x + c) + d)*sin(d*x + c))
```

3.52.6 Sympy [F]

$$\int \cot^6(c+dx)(a+a\sec(c+dx))^3\,dx = a^3\left(\int 3\cot^6(c+dx)\sec(c+dx)\,dx\right.$$
$$+\int 3\cot^6(c+dx)\sec^2(c+dx)\,dx$$
$$+\int \cot^6(c+dx)\sec^3(c+dx)\,dx$$
$$\left.+\int \cot^6(c+dx)\,dx\right)$$

input
```
integrate(cot(d*x+c)**6*(a+a*sec(d*x+c))**3,x)
```

output
```
a**3*(Integral(3*cot(c + d*x)**6*sec(c + d*x), x) + Integral(3*cot(c + d*x
)**6*sec(c + d*x)**2, x) + Integral(cot(c + d*x)**6*sec(c + d*x)**3, x) +
Integral(cot(c + d*x)**6, x))
```

3.52.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.14

$$\int \cot^6(c+dx)(a+a\sec(c+dx))^3\,dx =$$

$$-\frac{\left(15\,dx+15\,c+\frac{15\tan(dx+c)^4-5\tan(dx+c)^2+3}{\tan(dx+c)^5}\right)a^3+\frac{3\left(15\sin(dx+c)^4-10\sin(dx+c)^2+3\right)a^3}{\sin(dx+c)^5}-\frac{\left(5\sin(dx+c)^2-3\right)a^3}{\sin(dx+c)^5}+\frac{}{\tan(}}{15\,d}$$

input
```
integrate(cot(d*x+c)^6*(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

output
```
-1/15*((15*d*x + 15*c + (15*tan(d*x + c)^4 - 5*tan(d*x + c)^2 + 3)/tan(d*x
 + c)^5)*a^3 + 3*(15*sin(d*x + c)^4 - 10*sin(d*x + c)^2 + 3)*a^3/sin(d*x +
 c)^5 - (5*sin(d*x + c)^2 - 3)*a^3/sin(d*x + c)^5 + 9*a^3/tan(d*x + c)^5)/
d
```

3.52.8 Giac [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.62

$$\int \cot^6(c+dx)(a+a\sec(c+dx))^3\,dx$$

$$=-\frac{60\,(dx+c)a^3+\frac{105\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-20\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+3\,a^3}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5}}{60\,d}$$

input
```
integrate(cot(d*x+c)^6*(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

output
```
-1/60*(60*(d*x + c)*a^3 + (105*a^3*tan(1/2*d*x + 1/2*c)^4 - 20*a^3*tan(1/2
*d*x + 1/2*c)^2 + 3*a^3)/tan(1/2*d*x + 1/2*c)^5)/d
```

3.52.9 Mupad [B] (verification not implemented)

Time = 14.24 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.58

$$\int \cot^6(c+dx)(a+a\sec(c+dx))^3\,dx = \frac{a^3\cot\left(\frac{c}{2}+\frac{dx}{2}\right)^3}{3\,d} - a^3\,x - \frac{a^3\cot\left(\frac{c}{2}+\frac{dx}{2}\right)^5}{20\,d} - \frac{7\,a^3\cot\left(\frac{c}{2}+\frac{dx}{2}\right)}{4\,d}$$

input
```
int(cot(c + d*x)^6*(a + a/cos(c + d*x))^3,x)
```

output
```
(a^3*cot(c/2 + (d*x)/2)^3)/(3*d) - a^3*x - (a^3*cot(c/2 + (d*x)/2)^5)/(20*
d) - (7*a^3*cot(c/2 + (d*x)/2))/(4*d)
```

3.53 $\int \cot^8(c+dx)(a+a\sec(c+dx))^3\,dx$

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3.53.1 Optimal result

Integrand size = 21, antiderivative size = 141

$$\begin{aligned}\int \cot^8(c+dx)(a+a\sec(c+dx))^3\,dx = a^3x &+ \frac{a^3\cot(c+dx)}{d} - \frac{a^3\cot^3(c+dx)}{3d} \\ &+ \frac{a^3\cot^5(c+dx)}{5d} - \frac{4a^3\cot^7(c+dx)}{7d} \\ &+ \frac{3a^3\csc(c+dx)}{d} - \frac{10a^3\csc^3(c+dx)}{3d} \\ &+ \frac{11a^3\csc^5(c+dx)}{5d} - \frac{4a^3\csc^7(c+dx)}{7d}\end{aligned}$$

output
```
a^3*x+a^3*cot(d*x+c)/d-1/3*a^3*cot(d*x+c)^3/d+1/5*a^3*cot(d*x+c)^5/d-4/7*a
^3*cot(d*x+c)^7/d+3*a^3*csc(d*x+c)/d-10/3*a^3*csc(d*x+c)^3/d+11/5*a^3*csc(
d*x+c)^5/d-4/7*a^3*csc(d*x+c)^7/d
```

3.53.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.76 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.87

$$\begin{aligned}&\int \cot^8(c+dx)(a+a\sec(c+dx))^3\,dx \\ &= -\frac{3a^3\cot^7(c+dx)}{7d} + \frac{3a^3\csc(c+dx)}{d} - \frac{10a^3\csc^3(c+dx)}{3d} + \frac{11a^3\csc^5(c+dx)}{5d} \\ &\quad - \frac{4a^3\csc^7(c+dx)}{7d} - \frac{a^3\cot^7(c+dx)\,\text{Hypergeometric2F1}\left(-\frac{7}{2},1,-\frac{5}{2},-\tan^2(c+dx)\right)}{7d}\end{aligned}$$

input
```
Integrate[Cot[c + d*x]^8*(a + a*Sec[c + d*x])^3,x]
```

output
```
(-3*a^3*Cot[c + d*x]^7)/(7*d) + (3*a^3*Csc[c + d*x])/d - (10*a^3*Csc[c + d
*x]^3)/(3*d) + (11*a^3*Csc[c + d*x]^5)/(5*d) - (4*a^3*Csc[c + d*x]^7)/(7*d
) - (a^3*Cot[c + d*x]^7*Hypergeometric2F1[-7/2, 1, -5/2, -Tan[c + d*x]^2])
/(7*d)
```

3.53.3 Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3042, 4374, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^8(c+dx)(a\sec(c+dx)+a)^3\,dx$$

$$\downarrow 3042$$

$$\int \frac{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^3}{\cot\left(c+dx+\frac{\pi}{2}\right)^8}dx$$

$$\downarrow 4374$$

$$\int \left(a^3\cot^8(c+dx)+3a^3\cot^7(c+dx)\csc(c+dx)+3a^3\cot^6(c+dx)\csc^2(c+dx)+a^3\cot^5(c+dx)\csc^3(c+dx)\right)$$

$$\downarrow 2009$$

$$-\frac{4a^3\cot^7(c+dx)}{7d}+\frac{a^3\cot^5(c+dx)}{5d}-\frac{a^3\cot^3(c+dx)}{3d}+\frac{a^3\cot(c+dx)}{d}-\frac{4a^3\csc^7(c+dx)}{7d}+\frac{11a^3\csc^5(c+dx)}{5d}-\frac{10a^3\csc^3(c+dx)}{3d}+\frac{3a^3\csc(c+dx)}{d}+a^3x$$

input
```
Int[Cot[c + d*x]^8*(a + a*Sec[c + d*x])^3,x]
```

output
```
a^3*x + (a^3*Cot[c + d*x])/d - (a^3*Cot[c + d*x]^3)/(3*d) + (a^3*Cot[c + d
*x]^5)/(5*d) - (4*a^3*Cot[c + d*x]^7)/(7*d) + (3*a^3*Csc[c + d*x])/d - (10
*a^3*Csc[c + d*x]^3)/(3*d) + (11*a^3*Csc[c + d*x]^5)/(5*d) - (4*a^3*Csc[c
+ d*x]^7)/(7*d)
```

3.53.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4374
```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

3.53.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 4.80 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.86

method	result
risch	$a^3 x + \frac{2ia^3\left(315\,e^{7i(dx+c)}-1155\,e^{6i(dx+c)}+1715\,e^{5i(dx+c)}-525\,e^{4i(dx+c)}-1379\,e^{3i(dx+c)}+1939\,e^{2i(dx+c)}-1011\,e^{i(dx}{105d\left(e^{i(dx+c)}-1\right)^7\left(e^{i(dx+c)}+1\right)}$
derivativedivides	$a^3\left(-\frac{\cos(dx+c)^6}{7\sin(dx+c)^7}-\frac{\cos(dx+c)^6}{35\sin(dx+c)^5}+\frac{\cos(dx+c)^6}{105\sin(dx+c)^3}-\frac{\cos(dx+c)^6}{35\sin(dx+c)}-\frac{\left(\frac{8}{3}+\cos(dx+c)^4+\frac{4\cos(dx+c)^2}{3}\right)\sin(dx+c)}{35}\right)-\frac{3a^3\cos(dx-}{7\sin(dx+}$
default	$a^3\left(-\frac{\cos(dx+c)^6}{7\sin(dx+c)^7}-\frac{\cos(dx+c)^6}{35\sin(dx+c)^5}+\frac{\cos(dx+c)^6}{105\sin(dx+c)^3}-\frac{\cos(dx+c)^6}{35\sin(dx+c)}-\frac{\left(\frac{8}{3}+\cos(dx+c)^4+\frac{4\cos(dx+c)^2}{3}\right)\sin(dx+c)}{35}\right)-\frac{3a^3\cos(dx-}{7\sin(dx+}$

input
```
int(cot(d*x+c)^8*(a+a*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
a^3*x+2/105*I*a^3*(315*exp(7*I*(d*x+c))-1155*exp(6*I*(d*x+c))+1715*exp(5*I
*(d*x+c))-525*exp(4*I*(d*x+c))-1379*exp(3*I*(d*x+c))+1939*exp(2*I*(d*x+c))
-1011*exp(I*(d*x+c))+221)/d/(exp(I*(d*x+c))-1)^7/(exp(I*(d*x+c))+1)
```

3.53.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.13

$$\int \cot^8(c+dx)(a+a\sec(c+dx))^3\,dx$$
$$= \frac{221\,a^3\cos(dx+c)^4 - 348\,a^3\cos(dx+c)^3 - 25\,a^3\cos(dx+c)^2 + 303\,a^3\cos(dx+c) - 136\,a^3 + 105\,(a^3d}{105\,(d\cos(dx+c)^3 - 3\,d\cos(dx+c)^2 + 3\,d\cos(}$$

input
```
integrate(cot(d*x+c)^8*(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

output
```
1/105*(221*a^3*cos(d*x + c)^4 - 348*a^3*cos(d*x + c)^3 - 25*a^3*cos(d*x +
c)^2 + 303*a^3*cos(d*x + c) - 136*a^3 + 105*(a^3*d*x*cos(d*x + c)^3 - 3*a^
3*d*x*cos(d*x + c)^2 + 3*a^3*d*x*cos(d*x + c) - a^3*d*x)*sin(d*x + c))/((d
*cos(d*x + c)^3 - 3*d*cos(d*x + c)^2 + 3*d*cos(d*x + c) - d)*sin(d*x + c))
```

3.53.6 Sympy [F(-1)]

Timed out.

$$\int \cot^8(c+dx)(a+a\sec(c+dx))^3\,dx = \text{Timed out}$$

input
```
integrate(cot(d*x+c)**8*(a+a*sec(d*x+c))**3,x)
```

output
```
Timed out
```

3.53.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.08

$$\int \cot^8(c+dx)(a+a\sec(c+dx))^3\,dx$$
$$= \frac{\left(105\,dx + 105\,c + \frac{105\tan(dx+c)^6 - 35\tan(dx+c)^4 + 21\tan(dx+c)^2 - 15}{\tan(dx+c)^7}\right)a^3 + \frac{9\left(35\sin(dx+c)^6 - 35\sin(dx+c)^4 + 21\sin(dx+c)^2 - 5\right)a}{\sin(dx+c)^7}}{105\,d}$$

input
```
integrate(cot(d*x+c)^8*(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

output
```
1/105*((105*d*x + 105*c + (105*tan(d*x + c)^6 - 35*tan(d*x + c)^4 + 21*tan
(d*x + c)^2 - 15)/tan(d*x + c)^7)*a^3 + 9*(35*sin(d*x + c)^6 - 35*sin(d*x
+ c)^4 + 21*sin(d*x + c)^2 - 5)*a^3/sin(d*x + c)^7 - (35*sin(d*x + c)^4 -
42*sin(d*x + c)^2 + 15)*a^3/sin(d*x + c)^7 - 45*a^3/tan(d*x + c)^7)/d
```

3.53.8 Giac [A] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.68

$$\int \cot^8(c+dx)(a+a\sec(c+dx))^3\,dx$$
$$= \frac{1680\,(dx+c)a^3 - 105\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right) + \frac{2730\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6 - 560\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4 + 126\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2 - 15\,a^3}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7}}{1680\,d}$$

input
```
integrate(cot(d*x+c)^8*(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

output
```
1/1680*(1680*(d*x + c)*a^3 - 105*a^3*tan(1/2*d*x + 1/2*c) + (2730*a^3*tan(
1/2*d*x + 1/2*c)^6 - 560*a^3*tan(1/2*d*x + 1/2*c)^4 + 126*a^3*tan(1/2*d*x
+ 1/2*c)^2 - 15*a^3)/tan(1/2*d*x + 1/2*c)^7)/d
```

3.53.9 Mupad [B] (verification not implemented)

Time = 14.51 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.65

$$\int \cot^8(c+dx)(a+a\sec(c+dx))^3\,dx$$
$$= \frac{a^3\left(126\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2 - 560\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4 + 2730\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6 - 105\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^8 + 1680\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^7\right.}{1680\,d\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^7}$$

input
```
int(cot(c + d*x)^8*(a + a/cos(c + d*x))^3,x)
```

output
```
(a^3*(126*tan(c/2 + (d*x)/2)^2 - 560*tan(c/2 + (d*x)/2)^4 + 2730*tan(c/2 +
 (d*x)/2)^6 - 105*tan(c/2 + (d*x)/2)^8 + 1680*tan(c/2 + (d*x)/2)^7*(c + d*
x) - 15))/(1680*d*tan(c/2 + (d*x)/2)^7)
```

3.54 $\int \cot^{10}(c+dx)(a+a\sec(c+dx))^3\,dx$

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3.54.1 Optimal result

Integrand size = 21, antiderivative size = 179

$$\begin{aligned}\int \cot^{10}(c+dx)(a+a\sec(c+dx))^3\,dx = {} & -a^3x - \frac{a^3\cot(c+dx)}{d} + \frac{a^3\cot^3(c+dx)}{3d} \\ & - \frac{a^3\cot^5(c+dx)}{5d} + \frac{a^3\cot^7(c+dx)}{7d} \\ & - \frac{4a^3\cot^9(c+dx)}{9d} - \frac{3a^3\csc(c+dx)}{d} \\ & + \frac{13a^3\csc^3(c+dx)}{3d} - \frac{21a^3\csc^5(c+dx)}{5d} \\ & + \frac{15a^3\csc^7(c+dx)}{7d} - \frac{4a^3\csc^9(c+dx)}{9d}\end{aligned}$$

output

```
-a^3*x-a^3*cot(d*x+c)/d+1/3*a^3*cot(d*x+c)^3/d-1/5*a^3*cot(d*x+c)^5/d+1/7*
a^3*cot(d*x+c)^7/d-4/9*a^3*cot(d*x+c)^9/d-3*a^3*csc(d*x+c)/d+13/3*a^3*csc(
d*x+c)^3/d-21/5*a^3*csc(d*x+c)^5/d+15/7*a^3*csc(d*x+c)^7/d-4/9*a^3*csc(d*x
+c)^9/d
```

3.54.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 1.79 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.79

$$\begin{aligned}&\int \cot^{10}(c+dx)(a+a\sec(c+dx))^3\,dx\\&=-\frac{a^3\cot^9(c+dx)}{3d}-\frac{3a^3\csc(c+dx)}{d}+\frac{13a^3\csc^3(c+dx)}{3d}\\&\quad-\frac{21a^3\csc^5(c+dx)}{5d}+\frac{15a^3\csc^7(c+dx)}{7d}-\frac{4a^3\csc^9(c+dx)}{9d}\\&\quad-\frac{a^3\cot^9(c+dx)\,\text{Hypergeometric2F1}\left(-\frac{9}{2},1,-\frac{7}{2},-\tan^2(c+dx)\right)}{9d}\end{aligned}$$

input
```
Integrate[Cot[c + d*x]^10*(a + a*Sec[c + d*x])^3,x]
```

output
```
-1/3*(a^3*Cot[c + d*x]^9)/d - (3*a^3*Csc[c + d*x])/d + (13*a^3*Csc[c + d*x
]^3)/(3*d) - (21*a^3*Csc[c + d*x]^5)/(5*d) + (15*a^3*Csc[c + d*x]^7)/(7*d)
 - (4*a^3*Csc[c + d*x]^9)/(9*d) - (a^3*Cot[c + d*x]^9*Hypergeometric2F1[-9
/2, 1, -7/2, -Tan[c + d*x]^2])/(9*d)
```

3.54.3 Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3042, 4374, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^{10}(c+dx)(a\sec(c+dx)+a)^3\,dx$$

$$\downarrow 3042$$

$$\int \frac{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^3}{\cot\left(c+dx+\frac{\pi}{2}\right)^{10}}dx$$

$$\downarrow 4374$$

$$\int \left(a^3\cot^{10}(c+dx)+3a^3\cot^9(c+dx)\csc(c+dx)+3a^3\cot^8(c+dx)\csc^2(c+dx)+a^3\cot^7(c+dx)\csc^3(c+dx)\right)$$

$$\downarrow 2009$$

$$-\frac{4a^3\cot^9(c+dx)}{9d}+\frac{a^3\cot^7(c+dx)}{7d}-\frac{a^3\cot^5(c+dx)}{5d}+\frac{a^3\cot^3(c+dx)}{3d}-\frac{a^3\cot(c+dx)}{d}-\frac{4a^3\csc^9(c+dx)}{9d}+\frac{15a^3\csc^7(c+dx)}{7d}-\frac{21a^3\csc^5(c+dx)}{5d}+\frac{13a^3\csc^3(c+dx)}{3d}-\frac{3a^3\csc(c+dx)}{d}-a^3x$$

input
```
Int[Cot[c + d*x]^10*(a + a*Sec[c + d*x])^3,x]
```

output
```
-(a^3*x) - (a^3*Cot[c + d*x])/d + (a^3*Cot[c + d*x]^3)/(3*d) - (a^3*Cot[c
+ d*x]^5)/(5*d) + (a^3*Cot[c + d*x]^7)/(7*d) - (4*a^3*Cot[c + d*x]^9)/(9*d
) - (3*a^3*Csc[c + d*x])/d + (13*a^3*Csc[c + d*x]^3)/(3*d) - (21*a^3*Csc[c
 + d*x]^5)/(5*d) + (15*a^3*Csc[c + d*x]^7)/(7*d) - (4*a^3*Csc[c + d*x]^9)/
(9*d)
```

3.54.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4374
```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

3.54.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 7.81 (sec) , antiderivative size = 166, normalized size of antiderivative = 0.93

method	result
risch	$-a^3x-\frac{2ia^3\left(945\,\mathrm{e}^{11i(dx+c)}-3150\,\mathrm{e}^{10i(dx+c)}+2625\,\mathrm{e}^{9i(dx+c)}+6300\,\mathrm{e}^{8i(dx+c)}-13482\,\mathrm{e}^{7i(dx+c)}+5292\,\mathrm{e}^{6i(dx+c)}+105\right.}{315d\left(\mathrm{e}^{i(dx+c)}-1\right)^9\left(\mathrm{e}^{i(dx+c)}\right.}$
derivativedivides	$a^3\left(-\frac{\cos(dx+c)^8}{9\sin(dx+c)^9}-\frac{\cos(dx+c)^8}{63\sin(dx+c)^7}+\frac{\cos(dx+c)^8}{315\sin(dx+c)^5}-\frac{\cos(dx+c)^8}{315\sin(dx+c)^3}+\frac{\cos(dx+c)^8}{63\sin(dx+c)}+\frac{\left(\frac{16}{5}+\cos(dx+c)^6+\frac{6\cos(dx+c)^4}{5}+\frac{8\cos(d\right.}{63}\right.$
default	$a^3\left(-\frac{\cos(dx+c)^8}{9\sin(dx+c)^9}-\frac{\cos(dx+c)^8}{63\sin(dx+c)^7}+\frac{\cos(dx+c)^8}{315\sin(dx+c)^5}-\frac{\cos(dx+c)^8}{315\sin(dx+c)^3}+\frac{\cos(dx+c)^8}{63\sin(dx+c)}+\frac{\left(\frac{16}{5}+\cos(dx+c)^6+\frac{6\cos(dx+c)^4}{5}+\frac{8\cos(d\right.}{63}\right.$

input
```
int(cot(d*x+c)^10*(a+a*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
-a^3*x-2/315*I*a^3*(945*exp(11*I*(d*x+c))-3150*exp(10*I*(d*x+c))+2625*exp(
9*I*(d*x+c))+6300*exp(8*I*(d*x+c))-13482*exp(7*I*(d*x+c))+5292*exp(6*I*(d*
x+c))+10566*exp(5*I*(d*x+c))-11736*exp(4*I*(d*x+c))+1289*exp(3*I*(d*x+c))+
4866*exp(2*I*(d*x+c))-3063*exp(I*(d*x+c))+668)/d/(exp(I*(d*x+c))-1)^9/(exp
(I*(d*x+c))+1)^3
```

3.54.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 235, normalized size of antiderivative = 1.31

$$\int \cot^{10}(c+dx)(a+a\sec(c+dx))^3\,dx =$$
$$-\frac{668\,a^3\cos(dx+c)^6-1059\,a^3\cos(dx+c)^5-573\,a^3\cos(dx+c)^4+1813\,a^3\cos(dx+c)^3-393\,a^3\cos($$
$$315\left(d\cos(dx+c)^5\right.$$

input
```
integrate(cot(d*x+c)^10*(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

output
```
-1/315*(668*a^3*cos(d*x + c)^6 - 1059*a^3*cos(d*x + c)^5 - 573*a^3*cos(d*x
 + c)^4 + 1813*a^3*cos(d*x + c)^3 - 393*a^3*cos(d*x + c)^2 - 789*a^3*cos(d
*x + c) + 368*a^3 + 315*(a^3*d*x*cos(d*x + c)^5 - 3*a^3*d*x*cos(d*x + c)^4
 + 2*a^3*d*x*cos(d*x + c)^3 + 2*a^3*d*x*cos(d*x + c)^2 - 3*a^3*d*x*cos(d*x
 + c) + a^3*d*x)*sin(d*x + c))/((d*cos(d*x + c)^5 - 3*d*cos(d*x + c)^4 + 2
*d*cos(d*x + c)^3 + 2*d*cos(d*x + c)^2 - 3*d*cos(d*x + c) + d)*sin(d*x + c
))
```

3.54.6 Sympy [F(-1)]

Timed out.

$$\int \cot^{10}(c+dx)(a+a\sec(c+dx))^3\,dx = \text{Timed out}$$

input
```
integrate(cot(d*x+c)**10*(a+a*sec(d*x+c))**3,x)
```

output
```
Timed out
```

3.54.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.02

$$\int \cot^{10}(c+dx)(a+a\sec(c+dx))^3\,dx =$$

$$-\frac{\left(315\,dx+315\,c+\frac{315\tan(dx+c)^8-105\tan(dx+c)^6+63\tan(dx+c)^4-45\tan(dx+c)^2+35}{\tan(dx+c)^9}\right)a^3+\frac{3\left(315\sin(dx+c)^8-420\sin(dx+c\right.}{315\,d}$$

input
```
integrate(cot(d*x+c)^10*(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

output
```
-1/315*((315*d*x + 315*c + (315*tan(d*x + c)^8 - 105*tan(d*x + c)^6 + 63*t
an(d*x + c)^4 - 45*tan(d*x + c)^2 + 35)/tan(d*x + c)^9)*a^3 + 3*(315*sin(d
*x + c)^8 - 420*sin(d*x + c)^6 + 378*sin(d*x + c)^4 - 180*sin(d*x + c)^2 +
 35)*a^3/sin(d*x + c)^9 - (105*sin(d*x + c)^6 - 189*sin(d*x + c)^4 + 135*s
in(d*x + c)^2 - 35)*a^3/sin(d*x + c)^9 + 105*a^3/tan(d*x + c)^9)/d
```

3.54.8 Giac [A] (verification not implemented)

Time = 0.50 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.72

$$\int \cot^{10}(c+dx)(a+a\sec(c+dx))^3\,dx =$$

$$-\frac{105\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+20160\,(dx+c)a^3-2520\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+\frac{31185\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^8-6720\,a^3\tan\left(\frac{1}{2}\right.}{20160\,d}$$

input
```
integrate(cot(d*x+c)^10*(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

output
```
-1/20160*(105*a^3*tan(1/2*d*x + 1/2*c)^3 + 20160*(d*x + c)*a^3 - 2520*a^3*
tan(1/2*d*x + 1/2*c) + (31185*a^3*tan(1/2*d*x + 1/2*c)^8 - 6720*a^3*tan(1/
2*d*x + 1/2*c)^6 + 1827*a^3*tan(1/2*d*x + 1/2*c)^4 - 360*a^3*tan(1/2*d*x +
 1/2*c)^2 + 35*a^3)/tan(1/2*d*x + 1/2*c)^9)/d
```

3.54.9 Mupad [B] (verification not implemented)

Time = 15.08 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.15

$$\int \cot^{10}(c+dx)(a+a\sec(c+dx))^3\,dx =$$

$$-\frac{a^3\left(35\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{12}+105\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{12}-2520\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^2\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}+31185\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^4\mathrm{s}\right.}{}$$

input
```
int(cot(c + d*x)^10*(a + a/cos(c + d*x))^3,x)
```

output
```
-(a^3*(35*cos(c/2 + (d*x)/2)^12 + 105*sin(c/2 + (d*x)/2)^12 - 2520*cos(c/2
 + (d*x)/2)^2*sin(c/2 + (d*x)/2)^10 + 31185*cos(c/2 + (d*x)/2)^4*sin(c/2 +
 (d*x)/2)^8 - 6720*cos(c/2 + (d*x)/2)^6*sin(c/2 + (d*x)/2)^6 + 1827*cos(c/
2 + (d*x)/2)^8*sin(c/2 + (d*x)/2)^4 - 360*cos(c/2 + (d*x)/2)^10*sin(c/2 +
(d*x)/2)^2 + 20160*cos(c/2 + (d*x)/2)^3*sin(c/2 + (d*x)/2)^9*(c + d*x)))/(
20160*d*cos(c/2 + (d*x)/2)^3*sin(c/2 + (d*x)/2)^9)
```

3.55 $\int \cot^{12}(c+dx)(a+a\sec(c+dx))^3\,dx$

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3.55.1 Optimal result

Integrand size = 21, antiderivative size = 213

$$\begin{aligned}\int \cot^{12}(c+dx)(a+a\sec(c+dx))^3\,dx = a^3x &+ \frac{a^3\cot(c+dx)}{d} - \frac{a^3\cot^3(c+dx)}{3d} \\ &+ \frac{a^3\cot^5(c+dx)}{5d} - \frac{a^3\cot^7(c+dx)}{7d} \\ &+ \frac{a^3\cot^9(c+dx)}{9d} - \frac{4a^3\cot^{11}(c+dx)}{11d} \\ &+ \frac{3a^3\csc(c+dx)}{d} - \frac{16a^3\csc^3(c+dx)}{3d} \\ &+ \frac{34a^3\csc^5(c+dx)}{5d} - \frac{36a^3\csc^7(c+dx)}{7d} \\ &+ \frac{19a^3\csc^9(c+dx)}{9d} - \frac{4a^3\csc^{11}(c+dx)}{11d}\end{aligned}$$

output

```
a^3*x+a^3*cot(d*x+c)/d-1/3*a^3*cot(d*x+c)^3/d+1/5*a^3*cot(d*x+c)^5/d-1/7*a
^3*cot(d*x+c)^7/d+1/9*a^3*cot(d*x+c)^9/d-4/11*a^3*cot(d*x+c)^11/d+3*a^3*cs
c(d*x+c)/d-16/3*a^3*csc(d*x+c)^3/d+34/5*a^3*csc(d*x+c)^5/d-36/7*a^3*csc(d*
x+c)^7/d+19/9*a^3*csc(d*x+c)^9/d-4/11*a^3*csc(d*x+c)^11/d
```

3.55.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 3.67 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.75

$$\begin{aligned}
&\int \cot^{12}(c+dx)(a+a\sec(c+dx))^3\,dx \\
&= -\frac{3a^3\cot^{11}(c+dx)}{11d} + \frac{3a^3\csc(c+dx)}{d} - \frac{16a^3\csc^3(c+dx)}{3d} + \frac{34a^3\csc^5(c+dx)}{5d} \\
&\quad - \frac{36a^3\csc^7(c+dx)}{7d} + \frac{19a^3\csc^9(c+dx)}{9d} - \frac{4a^3\csc^{11}(c+dx)}{11d} \\
&\quad - \frac{a^3\cot^{11}(c+dx)\,\text{Hypergeometric2F1}\left(-\frac{11}{2},1,-\frac{9}{2},-\tan^2(c+dx)\right)}{11d}
\end{aligned}$$

input
```
Integrate[Cot[c + d*x]^12*(a + a*Sec[c + d*x])^3,x]
```

output
```
(-3*a^3*Cot[c + d*x]^11)/(11*d) + (3*a^3*Csc[c + d*x])/d - (16*a^3*Csc[c +
 d*x]^3)/(3*d) + (34*a^3*Csc[c + d*x]^5)/(5*d) - (36*a^3*Csc[c + d*x]^7)/(
7*d) + (19*a^3*Csc[c + d*x]^9)/(9*d) - (4*a^3*Csc[c + d*x]^11)/(11*d) - (a
^3*Cot[c + d*x]^11*Hypergeometric2F1[-11/2, 1, -9/2, -Tan[c + d*x]^2])/(11
*d)
```

3.55.3 Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3042, 4374, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^{12}(c+dx)(a\sec(c+dx)+a)^3\,dx$$

$$\downarrow \text{3042}$$

$$\int \frac{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^3}{\cot\left(c+dx+\frac{\pi}{2}\right)^{12}}dx$$

$$\downarrow \text{4374}$$

$$\int (a^3 \cot^{12}(c+dx) + 3a^3 \cot^{11}(c+dx)\csc(c+dx) + 3a^3 \cot^{10}(c+dx)\csc^2(c+dx) + a^3 \cot^9(c+dx)\csc^3(c+dx)$$

$$\downarrow 2009$$

$$-\frac{4a^3 \cot^{11}(c+dx)}{11d} + \frac{a^3 \cot^9(c+dx)}{9d} - \frac{a^3 \cot^7(c+dx)}{7d} + \frac{a^3 \cot^5(c+dx)}{5d} - \frac{a^3 \cot^3(c+dx)}{3d} + \frac{a^3 \cot(c+dx)}{d} - \frac{4a^3 \csc^{11}(c+dx)}{11d} + \frac{19a^3 \csc^9(c+dx)}{9d} - \frac{36a^3 \csc^7(c+dx)}{7d} + \frac{34a^3 \csc^5(c+dx)}{5d} - \frac{16a^3 \csc^3(c+dx)}{3d} + \frac{3a^3 \csc(c+dx)}{d} + a^3 x$$

input
```
Int[Cot[c + d*x]^12*(a + a*Sec[c + d*x])^3,x]
```

output
```
a^3*x + (a^3*Cot[c + d*x])/d - (a^3*Cot[c + d*x]^3)/(3*d) + (a^3*Cot[c + d
*x]^5)/(5*d) - (a^3*Cot[c + d*x]^7)/(7*d) + (a^3*Cot[c + d*x]^9)/(9*d) - (
4*a^3*Cot[c + d*x]^11)/(11*d) + (3*a^3*Csc[c + d*x])/d - (16*a^3*Csc[c + d
*x]^3)/(3*d) + (34*a^3*Csc[c + d*x]^5)/(5*d) - (36*a^3*Csc[c + d*x]^7)/(7*
d) + (19*a^3*Csc[c + d*x]^9)/(9*d) - (4*a^3*Csc[c + d*x]^11)/(11*d)
```

3.55.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4374
```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

3.55.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 12.94 (sec) , antiderivative size = 209, normalized size of antiderivative = 0.98

method	result
risch	$a^3x+\frac{2ia^3\left(10395\,\mathrm{e}^{15i(dx+c)}-31185\,\mathrm{e}^{14i(dx+c)}+1155\,\mathrm{e}^{13i(dx+c)}+148995\,\mathrm{e}^{12i(dx+c)}-190113\,\mathrm{e}^{11i(dx+c)}-117117\,\mathrm{e}^{10i}\right.$
derivativedivides	$a^3\left(-\frac{\cos(dx+c)^{10}}{11\sin(dx+c)^{11}}-\frac{\cos(dx+c)^{10}}{99\sin(dx+c)^9}+\frac{\cos(dx+c)^{10}}{693\sin(dx+c)^7}-\frac{\cos(dx+c)^{10}}{1155\sin(dx+c)^5}+\frac{\cos(dx+c)^{10}}{693\sin(dx+c)^3}-\frac{\cos(dx+c)^{10}}{99\sin(dx+c)}-\frac{\left(\frac{128}{35}+\cos(dx+c)^8+\right.}{}\right.$
default	$a^3\left(-\frac{\cos(dx+c)^{10}}{11\sin(dx+c)^{11}}-\frac{\cos(dx+c)^{10}}{99\sin(dx+c)^9}+\frac{\cos(dx+c)^{10}}{693\sin(dx+c)^7}-\frac{\cos(dx+c)^{10}}{1155\sin(dx+c)^5}+\frac{\cos(dx+c)^{10}}{693\sin(dx+c)^3}-\frac{\cos(dx+c)^{10}}{99\sin(dx+c)}-\frac{\left(\frac{128}{35}+\cos(dx+c)^8+\right.}{}\right.$

input
```
int(cot(d*x+c)^12*(a+a*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
a^3*x+2/3465*I*a^3*(10395*exp(15*I*(d*x+c))-31185*exp(14*I*(d*x+c))+1155*e
xp(13*I*(d*x+c))+148995*exp(12*I*(d*x+c))-190113*exp(11*I*(d*x+c))-117117*
exp(10*I*(d*x+c))+434775*exp(9*I*(d*x+c))-138105*exp(8*I*(d*x+c))-385055*e
xp(7*I*(d*x+c))+374781*exp(6*I*(d*x+c))+63289*exp(5*I*(d*x+c))-223655*exp(
4*I*(d*x+c))+75685*exp(3*I*(d*x+c))+43345*exp(2*I*(d*x+c))-34323*exp(I*(d*
x+c))+7453)/d/(exp(I*(d*x+c))-1)^11/(exp(I*(d*x+c))+1)^5
```

3.55.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 314, normalized size of antiderivative = 1.47

$$\int \cot^{12}(c+dx)(a+a\sec(c+dx))^3\,dx$$
$$=\frac{7453\,a^3\cos(dx+c)^8-11964\,a^3\cos(dx+c)^7-11866\,a^3\cos(dx+c)^6+30542\,a^3\cos(dx+c)^5+90\,a^3\cos$$

input
```
integrate(cot(d*x+c)^12*(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

output
```
1/3465*(7453*a^3*cos(d*x + c)^8 - 11964*a^3*cos(d*x + c)^7 - 11866*a^3*cos
(d*x + c)^6 + 30542*a^3*cos(d*x + c)^5 + 90*a^3*cos(d*x + c)^4 - 26438*a^3
*cos(d*x + c)^3 + 8539*a^3*cos(d*x + c)^2 + 7671*a^3*cos(d*x + c) - 3712*a
^3 + 3465*(a^3*d*x*cos(d*x + c)^7 - 3*a^3*d*x*cos(d*x + c)^6 + a^3*d*x*cos
(d*x + c)^5 + 5*a^3*d*x*cos(d*x + c)^4 - 5*a^3*d*x*cos(d*x + c)^3 - a^3*d*
x*cos(d*x + c)^2 + 3*a^3*d*x*cos(d*x + c) - a^3*d*x)*sin(d*x + c))/((d*cos
(d*x + c)^7 - 3*d*cos(d*x + c)^6 + d*cos(d*x + c)^5 + 5*d*cos(d*x + c)^4 -
 5*d*cos(d*x + c)^3 - d*cos(d*x + c)^2 + 3*d*cos(d*x + c) - d)*sin(d*x + c
))
```

3.55.6 Sympy [F(-1)]

Timed out.

$$\int \cot^{12}(c+dx)(a+a\sec(c+dx))^3\,dx = \text{Timed out}$$

input
```
integrate(cot(d*x+c)**12*(a+a*sec(d*x+c))**3,x)
```

output
```
Timed out
```

3.55.7 Maxima [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.00

$$\int \cot^{12}(c+dx)(a+a\sec(c+dx))^3\,dx$$

$$= \frac{\left(3465\,dx + 3465\,c + \frac{3465\tan(dx+c)^{10} - 1155\tan(dx+c)^8 + 693\tan(dx+c)^6 - 495\tan(dx+c)^4 + 385\tan(dx+c)^2 - 315}{\tan(dx+c)^{11}}\right)a^3 + \frac{15\left(693\,s}{}$$

input
```
integrate(cot(d*x+c)^12*(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

output
```
1/3465*((3465*d*x + 3465*c + (3465*tan(d*x + c)^10 - 1155*tan(d*x + c)^8 +
 693*tan(d*x + c)^6 - 495*tan(d*x + c)^4 + 385*tan(d*x + c)^2 - 315)/tan(d
*x + c)^11)*a^3 + 15*(693*sin(d*x + c)^10 - 1155*sin(d*x + c)^8 + 1386*sin
(d*x + c)^6 - 990*sin(d*x + c)^4 + 385*sin(d*x + c)^2 - 63)*a^3/sin(d*x +
c)^11 - (1155*sin(d*x + c)^8 - 2772*sin(d*x + c)^6 + 2970*sin(d*x + c)^4 -
 1540*sin(d*x + c)^2 + 315)*a^3/sin(d*x + c)^11 - 945*a^3/tan(d*x + c)^11)
/d
```

3.55.8 Giac [A] (verification not implemented)

Time = 0.57 (sec) , antiderivative size = 161, normalized size of antiderivative = 0.76

$$\int \cot^{12}(c+dx)(a+a\sec(c+dx))^3\,dx =$$

$$-\frac{693\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-11550\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-887040\,(dx+c)a^3+159390\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{88}$$

input
```
integrate(cot(d*x+c)^12*(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

output
```
-1/887040*(693*a^3*tan(1/2*d*x + 1/2*c)^5 - 11550*a^3*tan(1/2*d*x + 1/2*c)
^3 - 887040*(d*x + c)*a^3 + 159390*a^3*tan(1/2*d*x + 1/2*c) - 5*(264726*a^
3*tan(1/2*d*x + 1/2*c)^10 - 59136*a^3*tan(1/2*d*x + 1/2*c)^8 + 18018*a^3*t
an(1/2*d*x + 1/2*c)^6 - 4554*a^3*tan(1/2*d*x + 1/2*c)^4 + 770*a^3*tan(1/2*
d*x + 1/2*c)^2 - 63*a^3)/tan(1/2*d*x + 1/2*c)^11)/d
```

3.55.9 Mupad [B] (verification not implemented)

Time = 15.85 (sec) , antiderivative size = 254, normalized size of antiderivative = 1.19

$$\int \cot^{12}(c+dx)(a+a\sec(c+dx))^3\,dx =$$

$$-\frac{a^3\left(315\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{16}+693\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{16}-11550\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^2\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{14}+159390\cos\left(\frac{c}{2}+\frac{dx}{2}\right.\right.}{}$$

input
```
int(cot(c + d*x)^12*(a + a/cos(c + d*x))^3,x)
```

output
```
-(a^3*(315*cos(c/2 + (d*x)/2)^16 + 693*sin(c/2 + (d*x)/2)^16 - 11550*cos(c
/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2)^14 + 159390*cos(c/2 + (d*x)/2)^4*sin(c/
2 + (d*x)/2)^12 - 1323630*cos(c/2 + (d*x)/2)^6*sin(c/2 + (d*x)/2)^10 + 295
680*cos(c/2 + (d*x)/2)^8*sin(c/2 + (d*x)/2)^8 - 90090*cos(c/2 + (d*x)/2)^1
0*sin(c/2 + (d*x)/2)^6 + 22770*cos(c/2 + (d*x)/2)^12*sin(c/2 + (d*x)/2)^4
- 3850*cos(c/2 + (d*x)/2)^14*sin(c/2 + (d*x)/2)^2 - 887040*cos(c/2 + (d*x)
/2)^5*sin(c/2 + (d*x)/2)^11*(c + d*x)))/(887040*d*cos(c/2 + (d*x)/2)^5*sin
(c/2 + (d*x)/2)^11)
```

3.56 $\int \frac{\tan^9(c+dx)}{a+a\sec(c+dx)}\,dx$

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3.56.1 Optimal result

Integrand size = 21, antiderivative size = 135

$$\int \frac{\tan^9(c+dx)}{a+a\sec(c+dx)}\,dx = -\frac{\log(\cos(c+dx))}{ad} - \frac{\sec(c+dx)}{ad} - \frac{3\sec^2(c+dx)}{2ad} + \frac{\sec^3(c+dx)}{ad} + \frac{3\sec^4(c+dx)}{4ad} - \frac{3\sec^5(c+dx)}{5ad} - \frac{\sec^6(c+dx)}{6ad} + \frac{\sec^7(c+dx)}{7ad}$$

output
```
-ln(cos(d*x+c))/a/d-sec(d*x+c)/a/d-3/2*sec(d*x+c)^2/a/d+sec(d*x+c)^3/a/d+3
/4*sec(d*x+c)^4/a/d-3/5*sec(d*x+c)^5/a/d-1/6*sec(d*x+c)^6/a/d+1/7*sec(d*x+
c)^7/a/d
```

3.56.2 Mathematica [A] (verified)

Time = 0.41 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.01

$$\int \frac{\tan^9(c+dx)}{a+a\sec(c+dx)}\,dx = -\frac{(35\cos(c+dx)(104+105\log(\cos(c+dx))) + 3(212+602\cos(2(c+dx)) + 140\cos(4(c+dx)) + 210\,c$$

input
```
Integrate[Tan[c + d*x]^9/(a + a*Sec[c + d*x]),x]
```

output
```
-1/6720*((35*Cos[c + d*x]*(104 + 105*Log[Cos[c + d*x]]) + 3*(212 + 602*Cos
[2*(c + d*x)] + 140*Cos[4*(c + d*x)] + 210*Cos[5*(c + d*x)] + 70*Cos[6*(c
+ d*x)] + 245*Cos[5*(c + d*x)]*Log[Cos[c + d*x]] + 35*Cos[7*(c + d*x)]*Log
[Cos[c + d*x]] + 105*Cos[3*(c + d*x)]*(6 + 7*Log[Cos[c + d*x]])))*Sec[c +
d*x]^7)/(a*d)
```

3.56.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.68, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 25, 4367, 27, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^9(c+dx)}{a\sec(c+dx)+a}\,dx$$

$$\downarrow 3042$$

$$\int -\frac{\cot\left(c+dx+\frac{\pi}{2}\right)^9}{a\csc\left(c+dx+\frac{\pi}{2}\right)+a}dx$$

$$\downarrow 25$$

$$-\int \frac{\cot\left(\frac{1}{2}(2c+\pi)+dx\right)^9}{\csc\left(\frac{1}{2}(2c+\pi)+dx\right)a+a}dx$$

$$\downarrow 4367$$

$$-\frac{\int a^7(1-\cos(c+dx))^4(\cos(c+dx)+1)^3\sec^8(c+dx)d\cos(c+dx)}{a^8d}$$

$$\downarrow 27$$

$$-\frac{\int(1-\cos(c+dx))^4(\cos(c+dx)+1)^3\sec^8(c+dx)d\cos(c+dx)}{ad}$$

$$\downarrow 99$$

$$-\frac{\int\left(\sec^8(c+dx)-\sec^7(c+dx)-3\sec^6(c+dx)+3\sec^5(c+dx)+3\sec^4(c+dx)-3\sec^3(c+dx)-\sec^2(c+dx\right.}{ad}$$

$$\downarrow 2009$$

$$-\frac{-\frac{1}{7}\sec^7(c+dx)+\frac{1}{6}\sec^6(c+dx)+\frac{3}{5}\sec^5(c+dx)-\frac{3}{4}\sec^4(c+dx)-\sec^3(c+dx)+\frac{3}{2}\sec^2(c+dx)+\sec(c+dx\right.}{ad}$$

input
```
Int[Tan[c + d*x]^9/(a + a*Sec[c + d*x]),x]
```

output
```
-((Log[Cos[c + d*x]] + Sec[c + d*x] + (3*Sec[c + d*x]^2)/2 - Sec[c + d*x]^
3 - (3*Sec[c + d*x]^4)/4 + (3*Sec[c + d*x]^5)/5 + Sec[c + d*x]^6/6 - Sec[c
 + d*x]^7/7)/(a*d))
```

3.56.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4367
```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Simp[1/(a^(m - n - 1)*b^n*d)   Subst[Int[(a - b*x)^((m -
1)/2)*((a + b*x)^((m - 1)/2 + n)/x^(m + n)), x], x, Sin[c + d*x]], x] /; Fr
eeQ[{a, b, c, d}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && Integer
Q[n]
```

3.56.4 Maple [A] (verified)

Time = 1.24 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.64

method	result
derivativedivides	$\frac{-\frac{3}{2\cos(dx+c)^2}-\ln(\cos(dx+c))+\frac{1}{\cos(dx+c)^3}+\frac{3}{4\cos(dx+c)^4}-\frac{3}{5\cos(dx+c)^5}-\frac{1}{6\cos(dx+c)^6}+\frac{1}{7\cos(dx+c)^7}-\frac{1}{\cos(dx+c)}}{da}$
default	$\frac{-\frac{3}{2\cos(dx+c)^2}-\ln(\cos(dx+c))+\frac{1}{\cos(dx+c)^3}+\frac{3}{4\cos(dx+c)^4}-\frac{3}{5\cos(dx+c)^5}-\frac{1}{6\cos(dx+c)^6}+\frac{1}{7\cos(dx+c)^7}-\frac{1}{\cos(dx+c)}}{da}$
risch	$\frac{ix}{a}+\frac{2ic}{da}-\frac{2\left(105\,e^{13i(dx+c)}+315\,e^{12i(dx+c)}+210\,e^{11i(dx+c)}+945\,e^{10i(dx+c)}+903\,e^{9i(dx+c)}+1820\,e^{8i(dx+c)}+636\,e^{7i}\right.}{105da\left(e^{2i(dx+}\right.}$

input
```
int(tan(d*x+c)^9/(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d/a*(-3/2/cos(d*x+c)^2-ln(cos(d*x+c))+1/cos(d*x+c)^3+3/4/cos(d*x+c)^4-3/
5/cos(d*x+c)^5-1/6/cos(d*x+c)^6+1/7/cos(d*x+c)^7-1/cos(d*x+c))
```

3.56.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.70

$$\int \frac{\tan^9(c+dx)}{a+a\sec(c+dx)}\,dx =$$
$$-\frac{420\,\cos(dx+c)^7\log(-\cos(dx+c))+420\,\cos(dx+c)^6+630\,\cos(dx+c)^5-420\,\cos(dx+c)^4-31}{420\,ad\cos(dx+c)^7}$$

input
```
integrate(tan(d*x+c)^9/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

output
```
-1/420*(420*cos(d*x + c)^7*log(-cos(d*x + c)) + 420*cos(d*x + c)^6 + 630*c
os(d*x + c)^5 - 420*cos(d*x + c)^4 - 315*cos(d*x + c)^3 + 252*cos(d*x + c)
^2 + 70*cos(d*x + c) - 60)/(a*d*cos(d*x + c)^7)
```

3.56.6 Sympy [F]

$$\int \frac{\tan^9(c+dx)}{a+a\sec(c+dx)}\,dx = \frac{\int \frac{\tan^9(c+dx)}{\sec(c+dx)+1}\,dx}{a}$$

input
```
integrate(tan(d*x+c)**9/(a+a*sec(d*x+c)),x)
```

output
```
Integral(tan(c + d*x)**9/(sec(c + d*x) + 1), x)/a
```

3.56.7 Maxima [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.67

$$\int \frac{\tan^9(c+dx)}{a+a\sec(c+dx)}\,dx = -\frac{\frac{420\log(\cos(dx+c))}{a} + \frac{420\cos(dx+c)^6+630\cos(dx+c)^5-420\cos(dx+c)^4-315\cos(dx+c)^3+252\cos(dx+c)^2+70\cos(dx+c)-60}{a\cos(dx+c)^7}}{420\,d}$$

input
```
integrate(tan(d*x+c)^9/(a+a*sec(d*x+c)),x, algorithm="maxima")
```

output
```
-1/420*(420*log(cos(d*x + c))/a + (420*cos(d*x + c)^6 + 630*cos(d*x + c)^5
 - 420*cos(d*x + c)^4 - 315*cos(d*x + c)^3 + 252*cos(d*x + c)^2 + 70*cos(d
*x + c) - 60)/(a*cos(d*x + c)^7))/d
```

3.56.8 Giac [A] (verification not implemented)

Time = 5.98 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.81

$$\int \frac{\tan^9(c+dx)}{a+a\sec(c+dx)}\,dx = \frac{\frac{420\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right)}{a} - \frac{420\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right)}{a} + \frac{\frac{5775(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{20685(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}+\frac{42595(\cos(dx+c)-1)^3}{(\cos(dx+c)+1)^3}+\frac{56035(\cos(dx}{a\left(\frac{\cos(dx+}{\cos(dx+}\right.}}{420\,d}$$

input
```
integrate(tan(d*x+c)^9/(a+a*sec(d*x+c)),x, algorithm="giac")
```

output
```
1/420*(420*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1))/a - 420*lo
g(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1))/a + (5775*(cos(d*x + c)
 - 1)/(cos(d*x + c) + 1) + 20685*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2
 + 42595*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 + 56035*(cos(d*x + c) -
 1)^4/(cos(d*x + c) + 1)^4 + 28749*(cos(d*x + c) - 1)^5/(cos(d*x + c) + 1)
^5 + 8463*(cos(d*x + c) - 1)^6/(cos(d*x + c) + 1)^6 + 1089*(cos(d*x + c) -
 1)^7/(cos(d*x + c) + 1)^7 + 705)/(a*((cos(d*x + c) - 1)/(cos(d*x + c) + 1
) + 1)^7))/d
```

3.56.9 Mupad [B] (verification not implemented)

Time = 18.11 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.54

$$\int \frac{\tan^9(c+dx)}{a+a\sec(c+dx)}\,dx = \frac{2\,\operatorname{atanh}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2\right)}{a\,d}$$
$$-\frac{-2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{12}+14\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}-\frac{128\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^8}{3}+\frac{32\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6}{3}+\frac{26\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4}{5}}{d\left(-a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{14}+7\,a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{12}-21\,a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}+35\,a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^8-35\,a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6+\cdots\right.}$$

input
```
int(tan(c + d*x)^9/(a + a/cos(c + d*x)),x)
```

output
```
(2*atanh(tan(c/2 + (d*x)/2)^2))/(a*d) - ((26*tan(c/2 + (d*x)/2)^4)/5 - (22
*tan(c/2 + (d*x)/2)^2)/5 + (32*tan(c/2 + (d*x)/2)^6)/3 - (128*tan(c/2 + (d
*x)/2)^8)/3 + 14*tan(c/2 + (d*x)/2)^10 - 2*tan(c/2 + (d*x)/2)^12 + 32/35)/
(d*(a - 7*a*tan(c/2 + (d*x)/2)^2 + 21*a*tan(c/2 + (d*x)/2)^4 - 35*a*tan(c/
2 + (d*x)/2)^6 + 35*a*tan(c/2 + (d*x)/2)^8 - 21*a*tan(c/2 + (d*x)/2)^10 +
7*a*tan(c/2 + (d*x)/2)^12 - a*tan(c/2 + (d*x)/2)^14))
```

3.57 $\int \frac{\tan^7(c+dx)}{a+a\sec(c+dx)}\,dx$

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3.57.1 Optimal result

Integrand size = 21, antiderivative size = 97

$$\int \frac{\tan^7(c+dx)}{a+a\sec(c+dx)}\,dx = \frac{\log(\cos(c+dx))}{ad} + \frac{\sec(c+dx)}{ad} + \frac{\sec^2(c+dx)}{ad} - \frac{2\sec^3(c+dx)}{3ad} - \frac{\sec^4(c+dx)}{4ad} + \frac{\sec^5(c+dx)}{5ad}$$

output
```
ln(cos(d*x+c))/a/d+sec(d*x+c)/a/d+sec(d*x+c)^2/a/d-2/3*sec(d*x+c)^3/a/d-1/
4*sec(d*x+c)^4/a/d+1/5*sec(d*x+c)^5/a/d
```

3.57.2 Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.06

$$\int \frac{\tan^7(c+dx)}{a+a\sec(c+dx)}\,dx = \frac{(58+40\cos(2(c+dx))+60\cos(3(c+dx))+30\cos(4(c+dx))+75\cos(3(c+dx))\log(\cos(c+dx))+1}{240ad}$$

input
```
Integrate[Tan[c + d*x]^7/(a + a*Sec[c + d*x]),x]
```

output
```
((58 + 40*Cos[2*(c + d*x)] + 60*Cos[3*(c + d*x)] + 30*Cos[4*(c + d*x)] + 7
5*Cos[3*(c + d*x)]*Log[Cos[c + d*x]] + 15*Cos[5*(c + d*x)]*Log[Cos[c + d*x
]] + 30*Cos[c + d*x]*(4 + 5*Log[Cos[c + d*x]]))*Sec[c + d*x]^5)/(240*a*d)
```

3.57.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.74, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 25, 4367, 27, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^7(c+dx)}{a\sec(c+dx)+a}\,dx$$

$$\downarrow 3042$$

$$\int -\frac{\cot\left(c+dx+\frac{\pi}{2}\right)^7}{a\csc\left(c+dx+\frac{\pi}{2}\right)+a}dx$$

$$\downarrow 25$$

$$-\int \frac{\cot\left(\frac{1}{2}(2c+\pi)+dx\right)^7}{\csc\left(\frac{1}{2}(2c+\pi)+dx\right)a+a}dx$$

$$\downarrow 4367$$

$$-\frac{\int a^5(1-\cos(c+dx))^3(\cos(c+dx)+1)^2\sec^6(c+dx)d\cos(c+dx)}{a^6 d}$$

$$\downarrow 27$$

$$-\frac{\int(1-\cos(c+dx))^3(\cos(c+dx)+1)^2\sec^6(c+dx)d\cos(c+dx)}{ad}$$

$$\downarrow 99$$

$$-\frac{\int\left(\sec^6(c+dx)-\sec^5(c+dx)-2\sec^4(c+dx)+2\sec^3(c+dx)+\sec^2(c+dx)-\sec(c+dx)\right)d\cos(c+dx)}{ad}$$

$$\downarrow 2009$$

$$-\frac{-\frac{1}{5}\sec^5(c+dx)+\frac{1}{4}\sec^4(c+dx)+\frac{2}{3}\sec^3(c+dx)-\sec^2(c+dx)-\sec(c+dx)-\log(\cos(c+dx))}{ad}$$

input
```
Int[Tan[c + d*x]^7/(a + a*Sec[c + d*x]),x]
```

output
```
-((-Log[Cos[c + d*x]] - Sec[c + d*x] - Sec[c + d*x]^2 + (2*Sec[c + d*x]^3)
/3 + Sec[c + d*x]^4/4 - Sec[c + d*x]^5/5)/(a*d))
```

3.57.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4367
```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Simp[1/(a^(m - n - 1)*b^n*d)   Subst[Int[(a - b*x)^((m -
1)/2)*((a + b*x)^((m - 1)/2 + n)/x^(m + n)), x], x, Sin[c + d*x]], x] /; Fr
eeQ[{a, b, c, d}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && Integer
Q[n]
```

3.57.4 Maple [A] (verified)

Time = 1.11 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.64

method	result
derivativedivides	$\frac{-\frac{2}{3\cos(dx+c)^3}+\ln(\cos(dx+c))+\frac{1}{\cos(dx+c)^2}+\frac{1}{5\cos(dx+c)^5}-\frac{1}{4\cos(dx+c)^4}+\frac{1}{\cos(dx+c)}}{da}$
default	$\frac{-\frac{2}{3\cos(dx+c)^3}+\ln(\cos(dx+c))+\frac{1}{\cos(dx+c)^2}+\frac{1}{5\cos(dx+c)^5}-\frac{1}{4\cos(dx+c)^4}+\frac{1}{\cos(dx+c)}}{da}$
risch	$-\frac{ix}{a}-\frac{2ic}{da}+\frac{2e^{9i(dx+c)}+4e^{8i(dx+c)}+\frac{8e^{7i(dx+c)}}{3}+8e^{6i(dx+c)}+\frac{116e^{5i(dx+c)}}{15}+8e^{4i(dx+c)}+\frac{8e^{3i(dx+c)}}{3}+4e^{2i(dx+c)}}{da\left(e^{2i(dx+c)}+1\right)^5}$

input
```
int(tan(d*x+c)^7/(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d/a*(-2/3/cos(d*x+c)^3+ln(cos(d*x+c))+1/cos(d*x+c)^2+1/5/cos(d*x+c)^5-1/
4/cos(d*x+c)^4+1/cos(d*x+c))
```

3.57.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.77

$$\int \frac{\tan^7(c+dx)}{a+a\sec(c+dx)}\,dx$$
$$= \frac{60\cos(dx+c)^5\log(-\cos(dx+c)) + 60\cos(dx+c)^4 + 60\cos(dx+c)^3 - 40\cos(dx+c)^2 - 15\cos(dx+c) + 12}{60\,ad\cos(dx+c)^5}$$

input
```
integrate(tan(d*x+c)^7/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

output
```
1/60*(60*cos(d*x + c)^5*log(-cos(d*x + c)) + 60*cos(d*x + c)^4 + 60*cos(d*
x + c)^3 - 40*cos(d*x + c)^2 - 15*cos(d*x + c) + 12)/(a*d*cos(d*x + c)^5)
```

3.57.6 Sympy [F]

$$\int \frac{\tan^7(c+dx)}{a+a\sec(c+dx)}\,dx = \frac{\int \frac{\tan^7(c+dx)}{\sec(c+dx)+1}\,dx}{a}$$

input
```
integrate(tan(d*x+c)**7/(a+a*sec(d*x+c)),x)
```

output
```
Integral(tan(c + d*x)**7/(sec(c + d*x) + 1), x)/a
```

3.57.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.72

$$\int \frac{\tan^7(c+dx)}{a+a\sec(c+dx)}\,dx = \frac{\frac{60\log(\cos(dx+c))}{a} + \frac{60\cos(dx+c)^4 + 60\cos(dx+c)^3 - 40\cos(dx+c)^2 - 15\cos(dx+c) + 12}{a\cos(dx+c)^5}}{60\,d}$$

input
```
integrate(tan(d*x+c)^7/(a+a*sec(d*x+c)),x, algorithm="maxima")
```

output
```
1/60*(60*log(cos(d*x + c))/a + (60*cos(d*x + c)^4 + 60*cos(d*x + c)^3 - 40
*cos(d*x + c)^2 - 15*cos(d*x + c) + 12)/(a*cos(d*x + c)^5))/d
```

3.57.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 201 vs. $2(91) = 182$.

Time = 3.49 (sec) , antiderivative size = 201, normalized size of antiderivative = 2.07

$$\int \frac{\tan^7(c+dx)}{a+a\sec(c+dx)}\,dx = -\frac{\frac{60\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right)}{a} - \frac{60\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right)}{a} + \frac{\frac{485\,(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{1330\,(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}+\frac{1970\,(\cos(dx+c)-1)^3}{(\cos(dx+c)+1)^3}+\frac{805\,(\cos(dx+c)-1)^4}{(\cos(dx+c)+1)^4}+\frac{137\,(\cos(dx+c)-1)^5}{(\cos(dx+c)+1)^5}+73}{a\left(\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right)^5}}{60\,d}$$

input
```
integrate(tan(d*x+c)^7/(a+a*sec(d*x+c)),x, algorithm="giac")
```

output
```
-1/60*(60*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1))/a - 60*log(
abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1))/a + (485*(cos(d*x + c) -
1)/(cos(d*x + c) + 1) + 1330*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 1
970*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 + 805*(cos(d*x + c) - 1)^4/(
cos(d*x + c) + 1)^4 + 137*(cos(d*x + c) - 1)^5/(cos(d*x + c) + 1)^5 + 73)/
(a*((cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)^5))/d
```

3.57.9 Mupad [B] (verification not implemented)

Time = 18.02 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.58

$$\int \frac{\tan^7(c+dx)}{a+a\sec(c+dx)}\,dx = \frac{-2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^8+10\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6+\frac{2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4}{3}-\frac{10\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2}{3}+\frac{16}{15}}{d\left(-a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}+5\,a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^8-10\,a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6+10\,a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4-5\,a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+\right.} - \frac{2\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2\right)}{a\,d}$$

input
```
int(tan(c + d*x)^7/(a + a/cos(c + d*x)),x)
```

output
```
((2*tan(c/2 + (d*x)/2)^4)/3 - (10*tan(c/2 + (d*x)/2)^2)/3 + 10*tan(c/2 + (
d*x)/2)^6 - 2*tan(c/2 + (d*x)/2)^8 + 16/15)/(d*(a - 5*a*tan(c/2 + (d*x)/2)
^2 + 10*a*tan(c/2 + (d*x)/2)^4 - 10*a*tan(c/2 + (d*x)/2)^6 + 5*a*tan(c/2 +
 (d*x)/2)^8 - a*tan(c/2 + (d*x)/2)^10)) - (2*atanh(tan(c/2 + (d*x)/2)^2))/
(a*d)
```

3.58 $\int \frac{\tan^5(c+dx)}{a+a\sec(c+dx)}\,dx$

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3.58.1 Optimal result

Integrand size = 21, antiderivative size = 66

$$\int \frac{\tan^5(c+dx)}{a+a\sec(c+dx)}\,dx = -\frac{\log(\cos(c+dx))}{ad} - \frac{\sec(c+dx)}{ad} - \frac{\sec^2(c+dx)}{2ad} + \frac{\sec^3(c+dx)}{3ad}$$

output
```
-ln(cos(d*x+c))/a/d-sec(d*x+c)/a/d-1/2*sec(d*x+c)^2/a/d+1/3*sec(d*x+c)^3/a
/d
```

3.58.2 Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.98

$$\int \frac{\tan^5(c+dx)}{a+a\sec(c+dx)}\,dx = -\frac{(2+6\cos(2(c+dx))+3\cos(3(c+dx))\log(\cos(c+dx))+\cos(c+dx)(6+9\log(\cos(c+dx))))\sec^3(c}{12ad}$$

input
```
Integrate[Tan[c + d*x]^5/(a + a*Sec[c + d*x]),x]
```

output
```
-1/12*((2 + 6*Cos[2*(c + d*x)] + 3*Cos[3*(c + d*x)]*Log[Cos[c + d*x]] + Co
s[c + d*x]*(6 + 9*Log[Cos[c + d*x]]))*Sec[c + d*x]^3)/(a*d)
```

3.58.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.70, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 25, 4367, 27, 84, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^5(c+dx)}{a\sec(c+dx)+a}\,dx$$

$$\downarrow 3042$$

$$\int -\frac{\cot\left(c+dx+\frac{\pi}{2}\right)^5}{a\csc\left(c+dx+\frac{\pi}{2}\right)+a}dx$$

$$\downarrow 25$$

$$-\int \frac{\cot\left(\frac{1}{2}(2c+\pi)+dx\right)^5}{\csc\left(\frac{1}{2}(2c+\pi)+dx\right)a+a}dx$$

$$\downarrow 4367$$

$$-\frac{\int a^3(1-\cos(c+dx))^2(\cos(c+dx)+1)\sec^4(c+dx)d\cos(c+dx)}{a^4d}$$

$$\downarrow 27$$

$$-\frac{\int (1-\cos(c+dx))^2(\cos(c+dx)+1)\sec^4(c+dx)d\cos(c+dx)}{ad}$$

$$\downarrow 84$$

$$-\frac{\int \left(\sec^4(c+dx)-\sec^3(c+dx)-\sec^2(c+dx)+\sec(c+dx)\right)d\cos(c+dx)}{ad}$$

$$\downarrow 2009$$

$$-\frac{-\frac{1}{3}\sec^3(c+dx)+\frac{1}{2}\sec^2(c+dx)+\sec(c+dx)+\log(\cos(c+dx))}{ad}$$

input
```
Int[Tan[c + d*x]^5/(a + a*Sec[c + d*x]),x]
```

output
```
-((Log[Cos[c + d*x]] + Sec[c + d*x] + Sec[c + d*x]^2/2 - Sec[c + d*x]^3/3)
/(a*d))
```

3.58.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 84
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && EqQ[b*e + a*f, 0] &&  !(ILtQ[n + p + 2, 0
] && GtQ[n + 2*p, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4367
```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Simp[1/(a^(m - n - 1)*b^n*d)   Subst[Int[(a - b*x)^((m -
1)/2)*((a + b*x)^((m - 1)/2 + n)/x^(m + n)), x], x, Sin[c + d*x]], x] /; Fr
eeQ[{a, b, c, d}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && Integer
Q[n]
```

3.58.4 Maple [A] (verified)

Time = 1.02 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.73

method	result	size
derivativedivides	$\frac{-\ln(\cos(dx+c))+\frac{1}{3\cos(dx+c)^3}-\frac{1}{2\cos(dx+c)^2}-\frac{1}{\cos(dx+c)}}{da}$	48
default	$\frac{-\ln(\cos(dx+c))+\frac{1}{3\cos(dx+c)^3}-\frac{1}{2\cos(dx+c)^2}-\frac{1}{\cos(dx+c)}}{da}$	48
risch	$\frac{ix}{a}+\frac{2ic}{da}-\frac{2\left(3\,e^{5i(dx+c)}+3\,e^{4i(dx+c)}+2\,e^{3i(dx+c)}+3\,e^{2i(dx+c)}+3\,e^{i(dx+c)}\right)}{3da\left(e^{2i(dx+c)}+1\right)^3}-\frac{\ln\left(e^{2i(dx+c)}+1\right)}{da}$	116

input
```
int(tan(d*x+c)^5/(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d/a*(-ln(cos(d*x+c))+1/3/cos(d*x+c)^3-1/2/cos(d*x+c)^2-1/cos(d*x+c))
```

3.58.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.83

$$\int \frac{\tan^5(c+dx)}{a+a\sec(c+dx)}\,dx = -\frac{6\cos(dx+c)^3\log(-\cos(dx+c)) + 6\cos(dx+c)^2 + 3\cos(dx+c) - 2}{6\,ad\cos(dx+c)^3}$$

input
```
integrate(tan(d*x+c)^5/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

output
```
-1/6*(6*cos(d*x + c)^3*log(-cos(d*x + c)) + 6*cos(d*x + c)^2 + 3*cos(d*x +
 c) - 2)/(a*d*cos(d*x + c)^3)
```

3.58.6 Sympy [F]

$$\int \frac{\tan^5(c+dx)}{a+a\sec(c+dx)}\,dx = \frac{\int \frac{\tan^5(c+dx)}{\sec(c+dx)+1}\,dx}{a}$$

input
```
integrate(tan(d*x+c)**5/(a+a*sec(d*x+c)),x)
```

output
```
Integral(tan(c + d*x)**5/(sec(c + d*x) + 1), x)/a
```

3.58.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.76

$$\int \frac{\tan^5(c+dx)}{a+a\sec(c+dx)}\,dx = -\frac{\frac{6\log(\cos(dx+c))}{a} + \frac{6\cos(dx+c)^2+3\cos(dx+c)-2}{a\cos(dx+c)^3}}{6\,d}$$

input
```
integrate(tan(d*x+c)^5/(a+a*sec(d*x+c)),x, algorithm="maxima")
```

output
```
-1/6*(6*log(cos(d*x + c))/a + (6*cos(d*x + c)^2 + 3*cos(d*x + c) - 2)/(a*c
os(d*x + c)^3))/d
```

3.58.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 157 vs. 2(62) = 124.

Time = 1.49 (sec) , antiderivative size = 157, normalized size of antiderivative = 2.38

$$\int \frac{\tan^5(c+dx)}{a+a\sec(c+dx)}\,dx$$
$$= \frac{\frac{6\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right)}{a} - \frac{6\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right)}{a} + \frac{\frac{21(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{45(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}+\frac{11(\cos(dx+c)-1)^3}{(\cos(dx+c)+1)^3}+3}{a\left(\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right)^3}}{6\,d}$$

input
```
integrate(tan(d*x+c)^5/(a+a*sec(d*x+c)),x, algorithm="giac")
```

output
```
1/6*(6*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1))/a - 6*log(abs(
-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1))/a + (21*(cos(d*x + c) - 1)/(c
os(d*x + c) + 1) + 45*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 11*(cos(
d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 + 3)/(a*((cos(d*x + c) - 1)/(cos(d*x
+ c) + 1) + 1)^3))/d
```

3.58.9 Mupad [B] (verification not implemented)

Time = 14.40 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.50

$$\int \frac{\tan^5(c+dx)}{a+a\sec(c+dx)}\,dx$$
$$= \frac{2\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2\right)}{a\,d} + \frac{2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4+2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2-\frac{4}{3}}{d\left(-a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6+3\,a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4-3\,a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+a\right)}$$

input
```
int(tan(c + d*x)^5/(a + a/cos(c + d*x)),x)
```

output
```
(2*atanh(tan(c/2 + (d*x)/2)^2))/(a*d) + (2*tan(c/2 + (d*x)/2)^2 + 2*tan(c/
2 + (d*x)/2)^4 - 4/3)/(d*(a - 3*a*tan(c/2 + (d*x)/2)^2 + 3*a*tan(c/2 + (d*
x)/2)^4 - a*tan(c/2 + (d*x)/2)^6))
```

3.59 $\int \frac{\tan^3(c+dx)}{a+a\sec(c+dx)}\,dx$

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3.59.9 Mupad [B] (verification not implemented) 496

3.59.1 Optimal result

Integrand size = 21, antiderivative size = 28

$$\int \frac{\tan^3(c+dx)}{a+a\sec(c+dx)}\,dx = \frac{\log(\cos(c+dx))}{ad} + \frac{\sec(c+dx)}{ad}$$

output
```
ln(cos(d*x+c))/a/d+sec(d*x+c)/a/d
```

3.59.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.75

$$\int \frac{\tan^3(c+dx)}{a+a\sec(c+dx)}\,dx = \frac{\log(\cos(c+dx)) + \sec(c+dx)}{ad}$$

input
```
Integrate[Tan[c + d*x]^3/(a + a*Sec[c + d*x]),x]
```

output
```
(Log[Cos[c + d*x]] + Sec[c + d*x])/(a*d)
```

3.59.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 25, 4367, 27, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^3(c+dx)}{a\sec(c+dx)+a}\,dx$$
$$\downarrow 3042$$
$$\int -\frac{\cot\left(c+dx+\frac{\pi}{2}\right)^3}{a\csc\left(c+dx+\frac{\pi}{2}\right)+a}dx$$
$$\downarrow 25$$
$$-\int \frac{\cot\left(\frac{1}{2}(2c+\pi)+dx\right)^3}{\csc\left(\frac{1}{2}(2c+\pi)+dx\right)a+a}dx$$
$$\downarrow 4367$$
$$-\frac{\int a(1-\cos(c+dx))\sec^2(c+dx)d\cos(c+dx)}{a^2d}$$
$$\downarrow 27$$
$$-\frac{\int (1-\cos(c+dx))\sec^2(c+dx)d\cos(c+dx)}{ad}$$
$$\downarrow 49$$
$$-\frac{\int \left(\sec^2(c+dx)-\sec(c+dx)\right)d\cos(c+dx)}{ad}$$
$$\downarrow 2009$$
$$-\frac{-\sec(c+dx)-\log(\cos(c+dx))}{ad}$$

input
```
Int[Tan[c + d*x]^3/(a + a*Sec[c + d*x]),x]
```

output
```
-((-Log[Cos[c + d*x]] - Sec[c + d*x])/(a*d))
```

3.59.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4367
```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Simp[1/(a^(m - n - 1)*b^n*d)   Subst[Int[(a - b*x)^((m -
1)/2)*((a + b*x)^((m - 1)/2 + n)/x^(m + n)), x], x, Sin[c + d*x]], x] /; Fr
eeQ[{a, b, c, d}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && Integer
Q[n]
```

3.59.4 Maple [A] (verified)

Time = 0.62 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.86

method	result	size
derivativedivides	$\frac{\ln(\cos(dx+c))+\frac{1}{\cos(dx+c)}}{da}$	24
default	$\frac{\ln(\cos(dx+c))+\frac{1}{\cos(dx+c)}}{da}$	24
risch	$-\frac{ix}{a}-\frac{2ic}{da}+\frac{2\,\mathrm{e}^{i(dx+c)}}{da\left(\mathrm{e}^{2i(dx+c)}+1\right)}+\frac{\ln\left(\mathrm{e}^{2i(dx+c)}+1\right)}{da}$	68

input
```
int(tan(d*x+c)^3/(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d/a*(ln(cos(d*x+c))+1/cos(d*x+c))
```

3.59.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.18

$$\int \frac{\tan^3(c+dx)}{a+a\sec(c+dx)}\,dx = \frac{\cos(dx+c)\log(-\cos(dx+c))+1}{ad\cos(dx+c)}$$

input
```
integrate(tan(d*x+c)^3/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

output
```
(cos(d*x + c)*log(-cos(d*x + c)) + 1)/(a*d*cos(d*x + c))
```

3.59.6 Sympy [F]

$$\int \frac{\tan^3(c+dx)}{a+a\sec(c+dx)}\,dx = \frac{\int \frac{\tan^3(c+dx)}{\sec(c+dx)+1}\,dx}{a}$$

input
```
integrate(tan(d*x+c)**3/(a+a*sec(d*x+c)),x)
```

output
```
Integral(tan(c + d*x)**3/(sec(c + d*x) + 1), x)/a
```

3.59.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{\tan^3(c+dx)}{a+a\sec(c+dx)}\,dx = \frac{\frac{\log(\cos(dx+c))}{a}+\frac{1}{a\cos(dx+c)}}{d}$$

input
```
integrate(tan(d*x+c)^3/(a+a*sec(d*x+c)),x, algorithm="maxima")
```

output
```
(log(cos(d*x + c))/a + 1/(a*cos(d*x + c)))/d
```

3.59.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 111 vs. $2(28) = 56$.

Time = 0.60 (sec) , antiderivative size = 111, normalized size of antiderivative = 3.96

$$\int \frac{\tan^3(c+dx)}{a+a\sec(c+dx)}\,dx = -\frac{\frac{\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right)}{a} - \frac{\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right)}{a} + \frac{\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1}{a\left(\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right)}}{d}$$

input
```
integrate(tan(d*x+c)^3/(a+a*sec(d*x+c)),x, algorithm="giac")
```

output
```
-(log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1))/a - log(abs(-(cos(d
*x + c) - 1)/(cos(d*x + c) + 1) - 1))/a + ((cos(d*x + c) - 1)/(cos(d*x + c
) + 1) - 1)/(a*((cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)))/d
```

3.59.9 Mupad [B] (verification not implemented)

Time = 13.94 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.57

$$\int \frac{\tan^3(c+dx)}{a+a\sec(c+dx)}\,dx = \frac{2}{d\left(a-a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2\right)} - \frac{2\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2\right)}{a\,d}$$

input
```
int(tan(c + d*x)^3/(a + a/cos(c + d*x)),x)
```

output
```
2/(d*(a - a*tan(c/2 + (d*x)/2)^2)) - (2*atanh(tan(c/2 + (d*x)/2)^2))/(a*d)
```

3.60 $\int \frac{\tan(c+dx)}{a+a\sec(c+dx)}\,dx$

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3.60.1 Optimal result

Integrand size = 19, antiderivative size = 17

$$\int \frac{\tan(c+dx)}{a+a\sec(c+dx)}\,dx = -\frac{\log(1+\cos(c+dx))}{ad}$$

output
```
-ln(1+cos(d*x+c))/a/d
```

3.60.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.12

$$\int \frac{\tan(c+dx)}{a+a\sec(c+dx)}\,dx = -\frac{2\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{ad}$$

input
```
Integrate[Tan[c + d*x]/(a + a*Sec[c + d*x]),x]
```

output
```
(-2*Log[Cos[(c + d*x)/2]])/(a*d)
```

3.60.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {3042, 25, 4367, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan(c+dx)}{a\sec(c+dx)+a}\,dx$$

$$\downarrow 3042$$

$$\int -\frac{\cot\left(c+dx+\frac{\pi}{2}\right)}{a\csc\left(c+dx+\frac{\pi}{2}\right)+a}dx$$

$$\downarrow 25$$

$$-\int \frac{\cot\left(\frac{1}{2}(2c+\pi)+dx\right)}{\csc\left(\frac{1}{2}(2c+\pi)+dx\right)a+a}dx$$

$$\downarrow 4367$$

$$-\frac{\int \frac{1}{\cos(c+dx)a+a}d\cos(c+dx)}{d}$$

$$\downarrow 16$$

$$-\frac{\log(\cos(c+dx)+1)}{ad}$$

input
```
Int[Tan[c + d*x]/(a + a*Sec[c + d*x]),x]
```

output
```
-(Log[1 + Cos[c + d*x]]/(a*d))
```

3.60.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4367
```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Simp[1/(a^(m - n - 1)*b^n*d)   Subst[Int[(a - b*x)^((m -
1)/2)*((a + b*x)^((m - 1)/2 + n)/x^(m + n)), x], x, Sin[c + d*x]], x] /; Fr
eeQ[{a, b, c, d}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && Integer
Q[n]
```

3.60.4 Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.59

method	result	size
derivativedivides	$\frac{-\ln(1+\sec(dx+c))+\ln(\sec(dx+c))}{da}$	27
default	$\frac{-\ln(1+\sec(dx+c))+\ln(\sec(dx+c))}{da}$	27
risch	$\frac{ix}{a}+\frac{2ic}{da}-\frac{2\ln\left(e^{i(dx+c)}+1\right)}{da}$	39

input
```
int(tan(d*x+c)/(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d/a*(-ln(1+sec(d*x+c))+ln(sec(d*x+c)))
```

3.60.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.12

$$\int \frac{\tan(c+dx)}{a+a\sec(c+dx)}\,dx = -\frac{\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)}{ad}$$

input
```
integrate(tan(d*x+c)/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

output
```
-log(1/2*cos(d*x + c) + 1/2)/(a*d)
```

3.60.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 41 vs. $2(14) = 28$.

Time = 2.02 (sec) , antiderivative size = 41, normalized size of antiderivative = 2.41

$$\int \frac{\tan(c+dx)}{a+a\sec(c+dx)}\,dx = \begin{cases} \frac{\log\left(\tan^2(c+dx)+1\right)}{2ad} - \frac{\log\left(\sec(c+dx)+1\right)}{ad} & \text{for } d \neq 0 \\ \frac{x\tan(c)}{a\sec(c)+a} & \text{otherwise} \end{cases}$$

input
```
integrate(tan(d*x+c)/(a+a*sec(d*x+c)),x)
```

output
```
Piecewise((log(tan(c + d*x)**2 + 1)/(2*a*d) - log(sec(c + d*x) + 1)/(a*d),
 Ne(d, 0)), (x*tan(c)/(a*sec(c) + a), True))
```

3.60.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \frac{\tan(c+dx)}{a+a\sec(c+dx)}\,dx = -\frac{\log\left(\cos(dx+c)+1\right)}{ad}$$

input
```
integrate(tan(d*x+c)/(a+a*sec(d*x+c)),x, algorithm="maxima")
```

output
```
-log(cos(d*x + c) + 1)/(a*d)
```

3.60.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.82

$$\int \frac{\tan(c+dx)}{a+a\sec(c+dx)}\,dx = \frac{\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right)}{ad}$$

input
```
integrate(tan(d*x+c)/(a+a*sec(d*x+c)),x, algorithm="giac")
```

output
```
log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1))/(a*d)
```

3.60.9 Mupad [B] (verification not implemented)

Time = 14.49 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.24

$$\int \frac{\tan(c+dx)}{a+a\sec(c+dx)}\,dx = \frac{\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+1\right)}{a\,d}$$

input
```
int(tan(c + d*x)/(a + a/cos(c + d*x)),x)
```

output
```
log(tan(c/2 + (d*x)/2)^2 + 1)/(a*d)
```

3.61 $\int \frac{\cot(c+dx)}{a+a\sec(c+dx)}\,dx$

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3.61.3 Rubi [A] (verified) 503
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3.61.6 Sympy [F] 505
3.61.7 Maxima [A] (verification not implemented) 505
3.61.8 Giac [A] (verification not implemented) 506
3.61.9 Mupad [B] (verification not implemented) 506

3.61.1 Optimal result

Integrand size = 19, antiderivative size = 61

$$\int \frac{\cot(c+dx)}{a+a\sec(c+dx)}\,dx = \frac{1}{2ad(1+\cos(c+dx))} + \frac{\log(1-\cos(c+dx))}{4ad} + \frac{3\log(1+\cos(c+dx))}{4ad}$$

output
```
1/2/a/d/(1+cos(d*x+c))+1/4*ln(1-cos(d*x+c))/a/d+3/4*ln(1+cos(d*x+c))/a/d
```

3.61.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.10

$$\int \frac{\cot(c+dx)}{a+a\sec(c+dx)}\,dx = \frac{\left(1+2\cos^2\left(\frac{1}{2}(c+dx)\right)\right)\left(3\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)+\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)\sec(c+dx)}{2ad(1+\sec(c+dx))}$$

input
```
Integrate[Cot[c + d*x]/(a + a*Sec[c + d*x]),x]
```

output
```
((1 + 2*Cos[(c + d*x)/2]^2*(3*Log[Cos[(c + d*x)/2]] + Log[Sin[(c + d*x)/2]
]))*Sec[c + d*x])/(2*a*d*(1 + Sec[c + d*x]))
```

3.61.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.84, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {3042, 25, 4367, 27, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot(c+dx)}{a\sec(c+dx)+a}\,dx$$

$$\downarrow 3042$$

$$\int -\frac{1}{\cot\left(c+dx+\frac{\pi}{2}\right)\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)}dx$$

$$\downarrow 25$$

$$-\int \frac{1}{\cot\left(\frac{1}{2}(2c+\pi)+dx\right)\left(\csc\left(\frac{1}{2}(2c+\pi)+dx\right)a+a\right)}dx$$

$$\downarrow 4367$$

$$-\frac{a^2\int \frac{\cos^2(c+dx)}{a^3(1-\cos(c+dx))(\cos(c+dx)+1)^2}\,d\cos(c+dx)}{d}$$

$$\downarrow 27$$

$$-\frac{\int \frac{\cos^2(c+dx)}{(1-\cos(c+dx))(\cos(c+dx)+1)^2}\,d\cos(c+dx)}{ad}$$

$$\downarrow 99$$

$$-\frac{\int \left(-\frac{3}{4(\cos(c+dx)+1)}+\frac{1}{2(\cos(c+dx)+1)^2}-\frac{1}{4(\cos(c+dx)-1)}\right)d\cos(c+dx)}{ad}$$

$$\downarrow 2009$$

$$-\frac{-\frac{1}{2(\cos(c+dx)+1)}-\frac{1}{4}\log(1-\cos(c+dx))-\frac{3}{4}\log(\cos(c+dx)+1)}{ad}$$

input
```
Int[Cot[c + d*x]/(a + a*Sec[c + d*x]),x]
```

output
```
-((-1/2*1/(1 + Cos[c + d*x]) - Log[1 - Cos[c + d*x]]/4 - (3*Log[1 + Cos[c
+ d*x]])/4)/(a*d))
```

3.61.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4367
```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Simp[1/(a^(m - n - 1)*b^n*d)   Subst[Int[(a - b*x)^((m -
1)/2)*((a + b*x)^((m - 1)/2 + n)/x^(m + n)), x], x, Sin[c + d*x]], x] /; Fr
eeQ[{a, b, c, d}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && Integer
Q[n]
```

3.61.4 Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.70

method	result	size
derivativedivides	$\frac{\frac{\ln(\cos(dx+c)-1)}{4}+\frac{1}{2\cos(dx+c)+2}+\frac{3\ln(\cos(dx+c)+1)}{4}}{da}$	43
default	$\frac{\frac{\ln(\cos(dx+c)-1)}{4}+\frac{1}{2\cos(dx+c)+2}+\frac{3\ln(\cos(dx+c)+1)}{4}}{da}$	43
risch	$-\frac{ix}{a}-\frac{2ic}{da}+\frac{e^{i(dx+c)}}{ad\left(e^{i(dx+c)}+1\right)^2}+\frac{3\ln\left(e^{i(dx+c)}+1\right)}{2da}+\frac{\ln\left(e^{i(dx+c)}-1\right)}{2da}$	88

input
```
int(cot(d*x+c)/(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d/a*(1/4*ln(cos(d*x+c)-1)+1/2/(cos(d*x+c)+1)+3/4*ln(cos(d*x+c)+1))
```

3.61.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.98

$$\int \frac{\cot(c+dx)}{a+a\sec(c+dx)}\,dx$$
$$= \frac{3\left(\cos\left(dx+c\right)+1\right)\log\left(\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)+\left(\cos\left(dx+c\right)+1\right)\log\left(-\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)+2}{4\left(ad\cos\left(dx+c\right)+ad\right)}$$

input
```
integrate(cot(d*x+c)/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

output
```
1/4*(3*(cos(d*x + c) + 1)*log(1/2*cos(d*x + c) + 1/2) + (cos(d*x + c) + 1)
*log(-1/2*cos(d*x + c) + 1/2) + 2)/(a*d*cos(d*x + c) + a*d)
```

3.61.6 Sympy [F]

$$\int \frac{\cot(c+dx)}{a+a\sec(c+dx)}\,dx = \frac{\int \frac{\cot(c+dx)}{\sec(c+dx)+1}\,dx}{a}$$

input
```
integrate(cot(d*x+c)/(a+a*sec(d*x+c)),x)
```

output
```
Integral(cot(c + d*x)/(sec(c + d*x) + 1), x)/a
```

3.61.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.77

$$\int \frac{\cot(c+dx)}{a+a\sec(c+dx)}\,dx = \frac{\frac{3\log(\cos(dx+c)+1)}{a}+\frac{\log(\cos(dx+c)-1)}{a}+\frac{2}{a\cos(dx+c)+a}}{4\,d}$$

input
```
integrate(cot(d*x+c)/(a+a*sec(d*x+c)),x, algorithm="maxima")
```

output
```
1/4*(3*log(cos(d*x + c) + 1)/a + log(cos(d*x + c) - 1)/a + 2/(a*cos(d*x +
c) + a))/d
```

3.61.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.41

$$\int \frac{\cot(c+dx)}{a+a\sec(c+dx)}\,dx = \frac{\frac{\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)}{a} - \frac{4\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right)}{a} - \frac{\cos(dx+c)-1}{a(\cos(dx+c)+1)}}{4\,d}$$

input
```
integrate(cot(d*x+c)/(a+a*sec(d*x+c)),x, algorithm="giac")
```

output
```
1/4*(log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1))/a - 4*log(abs(-(cos
(d*x + c) - 1)/(cos(d*x + c) + 1) + 1))/a - (cos(d*x + c) - 1)/(a*(cos(d*x
 + c) + 1)))/d
```

3.61.9 Mupad [B] (verification not implemented)

Time = 14.42 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.80

$$\int \frac{\cot(c+dx)}{a+a\sec(c+dx)}\,dx = \frac{\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{2} - \ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+1\right) + \frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2}{4}}{a\,d}$$

input
```
int(cot(c + d*x)/(a + a/cos(c + d*x)),x)
```

output
```
(log(tan(c/2 + (d*x)/2))/2 - log(tan(c/2 + (d*x)/2)^2 + 1) + tan(c/2 + (d*
x)/2)^2/4)/(a*d)
```

3.62 $\int \frac{\cot^3(c+dx)}{a+a\sec(c+dx)}\,dx$

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3.62.1 Optimal result

Integrand size = 21, antiderivative size = 103

$$\int \frac{\cot^3(c+dx)}{a+a\sec(c+dx)}\,dx = -\frac{1}{8ad(1-\cos(c+dx))} + \frac{1}{8ad(1+\cos(c+dx))^2} - \frac{3}{4ad(1+\cos(c+dx))} - \frac{5\log(1-\cos(c+dx))}{16ad} - \frac{11\log(1+\cos(c+dx))}{16ad}$$

output
```
-1/8/a/d/(1-cos(d*x+c))+1/8/a/d/(1+cos(d*x+c))^2-3/4/a/d/(1+cos(d*x+c))-5/
16*ln(1-cos(d*x+c))/a/d-11/16*ln(1+cos(d*x+c))/a/d
```

3.62.2 Mathematica [A] (verified)

Time = 0.42 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.94

$$\int \frac{\cot^3(c+dx)}{a+a\sec(c+dx)}\,dx = -\frac{\left(12+2\cot^2\left(\frac{1}{2}(c+dx)\right)+4\cos^2\left(\frac{1}{2}(c+dx)\right)\left(11\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)+5\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)-\sec\right.}{16ad(1+\sec(c+dx))}$$

input
```
Integrate[Cot[c + d*x]^3/(a + a*Sec[c + d*x]),x]
```

output
```
-1/16*((12 + 2*Cot[(c + d*x)/2]^2 + 4*Cos[(c + d*x)/2]^2*(11*Log[Cos[(c +
d*x)/2]] + 5*Log[Sin[(c + d*x)/2]]) - Sec[(c + d*x)/2]^2)*Sec[c + d*x])/(a
*d*(1 + Sec[c + d*x]))
```

3.62.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.79, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 25, 4367, 27, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^3(c+dx)}{a\sec(c+dx)+a}\,dx$$

$$\downarrow 3042$$

$$\int -\frac{1}{\cot\left(c+dx+\frac{\pi}{2}\right)^3\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)}dx$$

$$\downarrow 25$$

$$-\int \frac{1}{\cot\left(\frac{1}{2}(2c+\pi)+dx\right)^3\left(\csc\left(\frac{1}{2}(2c+\pi)+dx\right)a+a\right)}dx$$

$$\downarrow 4367$$

$$-\frac{a^4\int \frac{\cos^4(c+dx)}{a^5(1-\cos(c+dx))^2(\cos(c+dx)+1)^3}d\cos(c+dx)}{d}$$

$$\downarrow 27$$

$$-\frac{\int \frac{\cos^4(c+dx)}{(1-\cos(c+dx))^2(\cos(c+dx)+1)^3}d\cos(c+dx)}{ad}$$

$$\downarrow 99$$

$$-\frac{\int \left(\frac{11}{16(\cos(c+dx)+1)}-\frac{3}{4(\cos(c+dx)+1)^2}+\frac{1}{4(\cos(c+dx)+1)^3}+\frac{5}{16(\cos(c+dx)-1)}+\frac{1}{8(\cos(c+dx)-1)^2}\right)d\cos(c+dx)}{ad}$$

$$\downarrow 2009$$

$$-\frac{\frac{1}{8(1-\cos(c+dx))}+\frac{3}{4(\cos(c+dx)+1)}-\frac{1}{8(\cos(c+dx)+1)^2}+\frac{5}{16}\log(1-\cos(c+dx))+\frac{11}{16}\log(\cos(c+dx)+1)}{ad}$$

input
```
Int[Cot[c + d*x]^3/(a + a*Sec[c + d*x]),x]
```

output
```
-((1/(8*(1 - Cos[c + d*x])) - 1/(8*(1 + Cos[c + d*x])^2) + 3/(4*(1 + Cos[c
 + d*x])) + (5*Log[1 - Cos[c + d*x]])/16 + (11*Log[1 + Cos[c + d*x]])/16)/
(a*d))
```

3.62.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4367
```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Simp[1/(a^(m - n - 1)*b^n*d)   Subst[Int[(a - b*x)^((m -
1)/2)*((a + b*x)^((m - 1)/2 + n)/x^(m + n)), x], x, Sin[c + d*x]], x] /; Fr
eeQ[{a, b, c, d}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && Integer
Q[n]
```

3.62.4 Maple [A] (verified)

Time = 0.69 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.65

method	result
derivativedivides	$\frac{\frac{1}{8\cos(dx+c)-8}-\frac{5\ln(\cos(dx+c)-1)}{16}+\frac{1}{8(\cos(dx+c)+1)^2}-\frac{3}{4(\cos(dx+c)+1)}-\frac{11\ln(\cos(dx+c)+1)}{16}}{da}$
default	$\frac{\frac{1}{8\cos(dx+c)-8}-\frac{5\ln(\cos(dx+c)-1)}{16}+\frac{1}{8(\cos(dx+c)+1)^2}-\frac{3}{4(\cos(dx+c)+1)}-\frac{11\ln(\cos(dx+c)+1)}{16}}{da}$
risch	$\frac{ix}{a}+\frac{2ic}{da}-\frac{5e^{5i(dx+c)}-6e^{4i(dx+c)}-14e^{3i(dx+c)}-6e^{2i(dx+c)}+5e^{i(dx+c)}}{4da\left(e^{i(dx+c)}+1\right)^4\left(e^{i(dx+c)}-1\right)^2}-\frac{11\ln\left(e^{i(dx+c)}+1\right)}{8da}-\frac{5\ln\left(e^{i(dx+c)}-1\right.}{8da}$

input
```
int(cot(d*x+c)^3/(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d/a*(1/8/(cos(d*x+c)-1)-5/16*ln(cos(d*x+c)-1)+1/8/(cos(d*x+c)+1)^2-3/4/(
cos(d*x+c)+1)-11/16*ln(cos(d*x+c)+1))
```

3.62.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.35

$$\int \frac{\cot^3(c+dx)}{a+a\sec(c+dx)}\,dx =$$
$$-\frac{10\cos(dx+c)^2+11\left(\cos(dx+c)^3+\cos(dx+c)^2-\cos(dx+c)-1\right)\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)+5\left(c\right.}{16\left(ad\cos(dx+c)^3+ad\cos(dx+\right.}$$

input
```
integrate(cot(d*x+c)^3/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

output
```
-1/16*(10*cos(d*x + c)^2 + 11*(cos(d*x + c)^3 + cos(d*x + c)^2 - cos(d*x +
 c) - 1)*log(1/2*cos(d*x + c) + 1/2) + 5*(cos(d*x + c)^3 + cos(d*x + c)^2
- cos(d*x + c) - 1)*log(-1/2*cos(d*x + c) + 1/2) - 6*cos(d*x + c) - 12)/(a
*d*cos(d*x + c)^3 + a*d*cos(d*x + c)^2 - a*d*cos(d*x + c) - a*d)
```

3.62.6 Sympy [F]

$$\int \frac{\cot^3(c+dx)}{a+a\sec(c+dx)}\,dx = \frac{\int \frac{\cot^3(c+dx)}{\sec(c+dx)+1}\,dx}{a}$$

input
```
integrate(cot(d*x+c)**3/(a+a*sec(d*x+c)),x)
```

output
```
Integral(cot(c + d*x)**3/(sec(c + d*x) + 1), x)/a
```

3.62.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.88

$$\int \frac{\cot^3(c+dx)}{a+a\sec(c+dx)}\,dx = -\frac{\frac{2\left(5\cos(dx+c)^2-3\cos(dx+c)-6\right)}{a\cos(dx+c)^3+a\cos(dx+c)^2-a\cos(dx+c)-a}+\frac{11\log(\cos(dx+c)+1)}{a}+\frac{5\log(\cos(dx+c)-1)}{a}}{16\,d}$$

input
```
integrate(cot(d*x+c)^3/(a+a*sec(d*x+c)),x, algorithm="maxima")
```

output
```
-1/16*(2*(5*cos(d*x + c)^2 - 3*cos(d*x + c) - 6)/(a*cos(d*x + c)^3 + a*cos
(d*x + c)^2 - a*cos(d*x + c) - a) + 11*log(cos(d*x + c) + 1)/a + 5*log(cos
(d*x + c) - 1)/a)/d
```

3.62.8 Giac [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.52

$$\int \frac{\cot^3(c+dx)}{a+a\sec(c+dx)}\,dx = \frac{\frac{2\left(\frac{5(\cos(dx+c)-1)}{\cos(dx+c)+1}+1\right)(\cos(dx+c)+1)}{a(\cos(dx+c)-1)}-\frac{10\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)}{a}+\frac{32\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right)}{a}+\frac{\frac{10\,a(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{a(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}}{a^2}}{32\,d}$$

input
```
integrate(cot(d*x+c)^3/(a+a*sec(d*x+c)),x, algorithm="giac")
```

output
```
1/32*(2*(5*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)*(cos(d*x + c) + 1)/(
a*(cos(d*x + c) - 1)) - 10*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1
))/a + 32*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1))/a + (10*a*(
cos(d*x + c) - 1)/(cos(d*x + c) + 1) + a*(cos(d*x + c) - 1)^2/(cos(d*x + c
) + 1)^2)/a^2)/d
```

3.62.9 Mupad [B] (verification not implemented)

Time = 15.83 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.74

$$\int \frac{\cot^3(c+dx)}{a+a\sec(c+dx)}\,dx = -\frac{\frac{5\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{8} - \ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+1\right) + \frac{\cot\left(\frac{c}{2}+\frac{dx}{2}\right)^2}{16} + \frac{5\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2}{16} - \frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4}{32}}{a\,d}$$

input
```
int(cot(c + d*x)^3/(a + a/cos(c + d*x)),x)
```

output
```
-((5*log(tan(c/2 + (d*x)/2)))/8 - log(tan(c/2 + (d*x)/2)^2 + 1) + cot(c/2
+ (d*x)/2)^2/16 + (5*tan(c/2 + (d*x)/2)^2)/16 - tan(c/2 + (d*x)/2)^4/32)/(
a*d)
```

3.63 $\int \frac{\cot^5(c+dx)}{a+a\sec(c+dx)}\,dx$

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3.63.1 Optimal result

Integrand size = 21, antiderivative size = 145

$$\int \frac{\cot^5(c+dx)}{a+a\sec(c+dx)}\,dx = -\frac{1}{32ad(1-\cos(c+dx))^2} + \frac{1}{4ad(1-\cos(c+dx))} + \frac{1}{24ad(1+\cos(c+dx))^3} - \frac{9}{32ad(1+\cos(c+dx))^2} + \frac{15}{16ad(1+\cos(c+dx))} + \frac{11\log(1-\cos(c+dx))}{32ad} + \frac{21\log(1+\cos(c+dx))}{32ad}$$

output

```
-1/32/a/d/(1-cos(d*x+c))^2+1/4/a/d/(1-cos(d*x+c))+1/24/a/d/(1+cos(d*x+c))^
3-9/32/a/d/(1+cos(d*x+c))^2+15/16/a/d/(1+cos(d*x+c))+11/32*ln(1-cos(d*x+c)
)/a/d+21/32*ln(1+cos(d*x+c))/a/d
```

3.63.2 Mathematica [A] (verified)

Time = 0.38 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.93

$$\int \frac{\cot^5(c+dx)}{a+a\sec(c+dx)}\,dx = -\frac{\cos^2\left(\frac{1}{2}(c+dx)\right)\left(-48\csc^2\left(\frac{1}{2}(c+dx)\right)+3\csc^4\left(\frac{1}{2}(c+dx)\right)-504\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)-264\log\left(\sin\right.\right.}{192ad(1+\sec(c-}$$

input
```
Integrate[Cot[c + d*x]^5/(a + a*Sec[c + d*x]),x]
```

output
```
-1/192*(Cos[(c + d*x)/2]^2*(-48*Csc[(c + d*x)/2]^2 + 3*Csc[(c + d*x)/2]^4
- 504*Log[Cos[(c + d*x)/2]] - 264*Log[Sin[(c + d*x)/2]] - 180*Sec[(c + d*x
)/2]^2 + 27*Sec[(c + d*x)/2]^4 - 2*Sec[(c + d*x)/2]^6)*Sec[c + d*x])/(a*d*
(1 + Sec[c + d*x]))
```

3.63.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.77, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 25, 4367, 27, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^5(c+dx)}{a\sec(c+dx)+a}\,dx$$

$$\downarrow 3042$$

$$\int -\frac{1}{\cot\left(c+dx+\frac{\pi}{2}\right)^5\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)}dx$$

$$\downarrow 25$$

$$-\int \frac{1}{\cot\left(\frac{1}{2}(2c+\pi)+dx\right)^5\left(\csc\left(\frac{1}{2}(2c+\pi)+dx\right)a+a\right)}dx$$

$$\downarrow 4367$$

$$-\frac{a^6\int \frac{\cos^6(c+dx)}{a^7(1-\cos(c+dx))^3(\cos(c+dx)+1)^4}d\cos(c+dx)}{d}$$

$$\downarrow 27$$

$$-\frac{\int \frac{\cos^6(c+dx)}{(1-\cos(c+dx))^3(\cos(c+dx)+1)^4}d\cos(c+dx)}{ad}$$

$$\downarrow 99$$

$$-\frac{\int \left(-\frac{21}{32(\cos(c+dx)+1)}+\frac{15}{16(\cos(c+dx)+1)^2}-\frac{9}{16(\cos(c+dx)+1)^3}+\frac{1}{8(\cos(c+dx)+1)^4}-\frac{11}{32(\cos(c+dx)-1)}-\frac{1}{4(\cos(c+dx)-1)^2}-\frac{1}{16(}\right.}{ad}$$

$$\downarrow 2009$$

$$-\frac{-\frac{1}{4(1-\cos(c+dx))}-\frac{15}{16(\cos(c+dx)+1)}+\frac{1}{32(1-\cos(c+dx))^2}+\frac{9}{32(\cos(c+dx)+1)^2}-\frac{1}{24(\cos(c+dx)+1)^3}-\frac{11}{32}\log(1-\cos(c+dx))}{ad}$$

input
```
Int[Cot[c + d*x]^5/(a + a*Sec[c + d*x]),x]
```

output
```
-((1/(32*(1 - Cos[c + d*x])^2) - 1/(4*(1 - Cos[c + d*x])) - 1/(24*(1 + Cos
[c + d*x])^3) + 9/(32*(1 + Cos[c + d*x])^2) - 15/(16*(1 + Cos[c + d*x])) -
 (11*Log[1 - Cos[c + d*x]])/32 - (21*Log[1 + Cos[c + d*x]])/32)/(a*d))
```

3.63.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4367
```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Simp[1/(a^(m - n - 1)*b^n*d)   Subst[Int[(a - b*x)^((m -
1)/2)*((a + b*x)^((m - 1)/2 + n)/x^(m + n)), x], x, Sin[c + d*x]], x] /; Fr
eeQ[{a, b, c, d}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && Integer
Q[n]
```

3.63.4 Maple [A] (verified)

Time = 0.65 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.63

method	result
derivativedivides	$\frac{-\frac{1}{32(\cos(dx+c)-1)^2}-\frac{1}{4(\cos(dx+c)-1)}+\frac{11\ln(\cos(dx+c)-1)}{32}+\frac{1}{24(\cos(dx+c)+1)^3}-\frac{9}{32(\cos(dx+c)+1)^2}+\frac{15}{16(\cos(dx+c)+1)}+\frac{21\ln(c}{da}$
default	$\frac{-\frac{1}{32(\cos(dx+c)-1)^2}-\frac{1}{4(\cos(dx+c)-1)}+\frac{11\ln(\cos(dx+c)-1)}{32}+\frac{1}{24(\cos(dx+c)+1)^3}-\frac{9}{32(\cos(dx+c)+1)^2}+\frac{15}{16(\cos(dx+c)+1)}+\frac{21\ln(c}{da}$
risch	$-\frac{ix}{a}-\frac{2ic}{da}+\frac{33e^{9i(dx+c)}-78e^{8i(dx+c)}-184e^{7i(dx+c)}-2e^{6i(dx+c)}+270e^{5i(dx+c)}-2e^{4i(dx+c)}-184e^{3i(dx+c)}-78e}{24da\left(e^{i(dx+c)}+1\right)^6\left(e^{i(dx+c)}-1\right)^4}$

input
```
int(cot(d*x+c)^5/(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d/a*(-1/32/(cos(d*x+c)-1)^2-1/4/(cos(d*x+c)-1)+11/32*ln(cos(d*x+c)-1)+1/
24/(cos(d*x+c)+1)^3-9/32/(cos(d*x+c)+1)^2+15/16/(cos(d*x+c)+1)+21/32*ln(co
s(d*x+c)+1))
```

3.63.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.50

$$\int \frac{\cot^5(c+dx)}{a+a\sec(c+dx)}\,dx$$
$$= \frac{66\cos(dx+c)^4 - 78\cos(dx+c)^3 - 158\cos(dx+c)^2 + 63\left(\cos(dx+c)^5 + \cos(dx+c)^4 - 2\cos(dx+\right.}{9}$$

input
```
integrate(cot(d*x+c)^5/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

output
```
1/96*(66*cos(d*x + c)^4 - 78*cos(d*x + c)^3 - 158*cos(d*x + c)^2 + 63*(cos
(d*x + c)^5 + cos(d*x + c)^4 - 2*cos(d*x + c)^3 - 2*cos(d*x + c)^2 + cos(d
*x + c) + 1)*log(1/2*cos(d*x + c) + 1/2) + 33*(cos(d*x + c)^5 + cos(d*x +
c)^4 - 2*cos(d*x + c)^3 - 2*cos(d*x + c)^2 + cos(d*x + c) + 1)*log(-1/2*co
s(d*x + c) + 1/2) + 58*cos(d*x + c) + 88)/(a*d*cos(d*x + c)^5 + a*d*cos(d*
x + c)^4 - 2*a*d*cos(d*x + c)^3 - 2*a*d*cos(d*x + c)^2 + a*d*cos(d*x + c)
+ a*d)
```

3.63.6 Sympy [F]

$$\int \frac{\cot^5(c+dx)}{a+a\sec(c+dx)}\,dx = \frac{\int \frac{\cot^5(c+dx)}{\sec(c+dx)+1}\,dx}{a}$$

input
```
integrate(cot(d*x+c)**5/(a+a*sec(d*x+c)),x)
```

output
```
Integral(cot(c + d*x)**5/(sec(c + d*x) + 1), x)/a
```

3.63.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.90

$$\int \frac{\cot^5(c+dx)}{a+a\sec(c+dx)}\,dx = \frac{\frac{2\left(33\cos(dx+c)^4-39\cos(dx+c)^3-79\cos(dx+c)^2+29\cos(dx+c)+44\right)}{a\cos(dx+c)^5+a\cos(dx+c)^4-2a\cos(dx+c)^3-2a\cos(dx+c)^2+a\cos(dx+c)+a}+\frac{63\log(\cos(dx+c)+1)}{a}+\frac{33\log(\cos(dx+c)-1)}{a}}{96\,d}$$

input
```
integrate(cot(d*x+c)^5/(a+a*sec(d*x+c)),x, algorithm="maxima")
```

output
```
1/96*(2*(33*cos(d*x + c)^4 - 39*cos(d*x + c)^3 - 79*cos(d*x + c)^2 + 29*co
s(d*x + c) + 44)/(a*cos(d*x + c)^5 + a*cos(d*x + c)^4 - 2*a*cos(d*x + c)^3
 - 2*a*cos(d*x + c)^2 + a*cos(d*x + c) + a) + 63*log(cos(d*x + c) + 1)/a +
 33*log(cos(d*x + c) - 1)/a)/d
```

3.63.8 Giac [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.46

$$\int \frac{\cot^5(c+dx)}{a+a\sec(c+dx)}\,dx = -\frac{\frac{3\left(\frac{14(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{66(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}+1\right)(\cos(dx+c)+1)^2}{a(\cos(dx+c)-1)^2}-\frac{132\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)}{a}+\frac{384\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right)}{a}+\frac{132\,a^2(\cos(dx+c$$

input
```
integrate(cot(d*x+c)^5/(a+a*sec(d*x+c)),x, algorithm="giac")
```

output
```
-1/384*(3*(14*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 66*(cos(d*x + c) - 1
)^2/(cos(d*x + c) + 1)^2 + 1)*(cos(d*x + c) + 1)^2/(a*(cos(d*x + c) - 1)^2
) - 132*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1))/a + 384*log(abs(
-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1))/a + (132*a^2*(cos(d*x + c) -
1)/(cos(d*x + c) + 1) + 21*a^2*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 +
 2*a^2*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3)/a^3)/d
```

3.63.9 Mupad [B] (verification not implemented)

Time = 14.28 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.91

$$\int \frac{\cot^5(c+dx)}{a+a\sec(c+dx)}\,dx = \frac{11\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2}{32\,a\,d} - \frac{7\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4}{128\,a\,d} + \frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6}{192\,a\,d} + \frac{11\,\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{16\,a\,d} - \frac{\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+1\right)}{a\,d} + \frac{\cot\left(\frac{c}{2}+\frac{dx}{2}\right)^4\left(\frac{7\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2}{2}-\frac{1}{4}\right)}{32\,a\,d}$$

input
```
int(cot(c + d*x)^5/(a + a/cos(c + d*x)),x)
```

output
```
(11*tan(c/2 + (d*x)/2)^2)/(32*a*d) - (7*tan(c/2 + (d*x)/2)^4)/(128*a*d) +
tan(c/2 + (d*x)/2)^6/(192*a*d) + (11*log(tan(c/2 + (d*x)/2)))/(16*a*d) - l
og(tan(c/2 + (d*x)/2)^2 + 1)/(a*d) + (cot(c/2 + (d*x)/2)^4*((7*tan(c/2 + (
d*x)/2)^2)/2 - 1/4))/(32*a*d)
```

3.64 $\int \frac{\tan^8(c+dx)}{a+a\sec(c+dx)}\,dx$

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3.64.1 Optimal result

Integrand size = 21, antiderivative size = 105

$$\int \frac{\tan^8(c+dx)}{a+a\sec(c+dx)}\,dx = \frac{x}{a} - \frac{5\operatorname{arctanh}(\sin(c+dx))}{16ad} - \frac{(16-5\sec(c+dx))\tan(c+dx)}{16ad} + \frac{(8-5\sec(c+dx))\tan^3(c+dx)}{24ad} - \frac{(6-5\sec(c+dx))\tan^5(c+dx)}{30ad}$$

output

```
x/a-5/16*arctanh(sin(d*x+c))/a/d-1/16*(16-5*sec(d*x+c))*tan(d*x+c)/a/d+1/2
4*(8-5*sec(d*x+c))*tan(d*x+c)^3/a/d-1/30*(6-5*sec(d*x+c))*tan(d*x+c)^5/a/d
```

3.64.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 301 vs. $2(105) = 210$.

Time = 2.33 (sec) , antiderivative size = 301, normalized size of antiderivative = 2.87

$$\int \frac{\tan^8(c+dx)}{a+a\sec(c+dx)}\,dx = \frac{\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\left(2400\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\right.\right.\right.\right.$$

input

```
Integrate[Tan[c + d*x]^8/(a + a*Sec[c + d*x]),x]
```

output

```
(Cos[(c + d*x)/2]^2*Sec[c + d*x]*(2400*(Log[Cos[(c + d*x)/2] - Sin[(c + d*
x)/2]] - Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) + Sec[c]*Sec[c + d*x]^6
*(2400*d*x*Cos[c] + 1800*d*x*Cos[c + 2*d*x] + 1800*d*x*Cos[3*c + 2*d*x] +
720*d*x*Cos[3*c + 4*d*x] + 720*d*x*Cos[5*c + 4*d*x] + 120*d*x*Cos[5*c + 6*
d*x] + 120*d*x*Cos[7*c + 6*d*x] + 3680*Sin[c] + 450*Sin[d*x] + 450*Sin[2*c
 + d*x] - 3360*Sin[c + 2*d*x] + 2160*Sin[3*c + 2*d*x] - 25*Sin[2*c + 3*d*x
] - 25*Sin[4*c + 3*d*x] - 1488*Sin[3*c + 4*d*x] + 720*Sin[5*c + 4*d*x] + 1
65*Sin[4*c + 5*d*x] + 165*Sin[6*c + 5*d*x] - 368*Sin[5*c + 6*d*x])))/(3840
*a*d*(1 + Sec[c + d*x]))
```

3.64.3 Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.14, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.524$, Rules used = {3042, 4376, 25, 3042, 4369, 3042, 4369, 27, 3042, 4369, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^8(c+dx)}{a\sec(c+dx)+a}\,dx$$

$$\downarrow 3042$$

$$\int \frac{\cot\left(c+dx+\frac{\pi}{2}\right)^8}{a\csc\left(c+dx+\frac{\pi}{2}\right)+a}dx$$

$$\downarrow 4376$$

$$\frac{\int -\left((a-a\sec(c+dx))\tan^6(c+dx)\right)dx}{a^2}$$

$$\downarrow 25$$

$$-\frac{\int (a-a\sec(c+dx))\tan^6(c+dx)dx}{a^2}$$

$$\downarrow 3042$$

$$-\frac{\int \cot\left(c+dx+\frac{\pi}{2}\right)^6\left(a-a\csc\left(c+dx+\frac{\pi}{2}\right)\right)dx}{a^2}$$

$$\downarrow 4369$$

$$-\frac{\frac{\tan^5(c+dx)(6a-5a\sec(c+dx))}{30d}-\frac{1}{6}\int (6a-5a\sec(c+dx))\tan^4(c+dx)dx}{a^2}$$

$$\downarrow 3042$$

$$-\frac{\frac{\tan^5(c+dx)(6a-5a\sec(c+dx))}{30d}-\frac{1}{6}\int \cot\left(c+dx+\frac{\pi}{2}\right)^4\left(6a-5a\csc\left(c+dx+\frac{\pi}{2}\right)\right)dx}{a^2}$$

$$\downarrow 4369$$

$$-\frac{\frac{1}{6}\left(\frac{1}{4}\int 3(8a-5a\sec(c+dx))\tan^2(c+dx)dx-\frac{\tan^3(c+dx)(8a-5a\sec(c+dx))}{4d}\right)+\frac{\tan^5(c+dx)(6a-5a\sec(c+dx))}{30d}}{a^2}$$

$$\downarrow 27$$

$$-\frac{\frac{1}{6}\left(\frac{3}{4}\int(8a-5a\sec(c+dx))\tan^2(c+dx)dx-\frac{\tan^3(c+dx)(8a-5a\sec(c+dx))}{4d}\right)+\frac{\tan^5(c+dx)(6a-5a\sec(c+dx))}{30d}}{a^2}$$

$$\downarrow 3042$$

$$-\frac{\frac{1}{6}\left(\frac{3}{4}\int \cot\left(c+dx+\frac{\pi}{2}\right)^2\left(8a-5a\csc\left(c+dx+\frac{\pi}{2}\right)\right)dx-\frac{\tan^3(c+dx)(8a-5a\sec(c+dx))}{4d}\right)+\frac{\tan^5(c+dx)(6a-5a\sec(c+dx))}{30d}}{a^2}$$

$$\downarrow 4369$$

$$-\frac{\frac{1}{6}\left(\frac{3}{4}\left(\frac{\tan(c+dx)(16a-5a\sec(c+dx))}{2d}-\frac{1}{2}\int(16a-5a\sec(c+dx))dx\right)-\frac{\tan^3(c+dx)(8a-5a\sec(c+dx))}{4d}\right)+\frac{\tan^5(c+dx)(6a-5a\sec}{30d}}{a^2}$$

$$\downarrow 2009$$

$$-\frac{\frac{1}{6}\left(\frac{3}{4}\left(\frac{1}{2}\left(\frac{5a\operatorname{arctanh}(\sin(c+dx))}{d}-16ax\right)+\frac{\tan(c+dx)(16a-5a\sec(c+dx))}{2d}\right)-\frac{\tan^3(c+dx)(8a-5a\sec(c+dx))}{4d}\right)+\frac{\tan^5(c+dx)(6a-5}{30d}}{a^2}$$

input
```
Int[Tan[c + d*x]^8/(a + a*Sec[c + d*x]),x]
```

output
```
-((((6*a - 5*a*Sec[c + d*x])*Tan[c + d*x]^5)/(30*d) + (-1/4*((8*a - 5*a*Se
c[c + d*x])*Tan[c + d*x]^3)/d + (3*((-16*a*x + (5*a*ArcTanh[Sin[c + d*x]])
/d)/2 + ((16*a - 5*a*Sec[c + d*x])*Tan[c + d*x])/(2*d)))/4)/6)/a^2)
```

3.64.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4369
```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(-e)*(e*Cot[c + d*x])^(m - 1)*((a*m + b*(m - 1)*Csc
[c + d*x])/(d*m*(m - 1))), x] - Simp[e^2/m   Int[(e*Cot[c + d*x])^(m - 2)*(
a*m + b*(m - 1)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && GtQ[m
, 1]
```

rule 4376
```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Simp[a^(2*n)/e^(2*n)   Int[(e*Cot[c + d*x])^(m + 2*
n)/(-a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[a
^2 - b^2, 0] && ILtQ[n, 0]
```

3.64.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.19 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.85

method	result
risch	$\frac{x}{a}-\frac{i\left(165\,\mathrm{e}^{11i(dx+c)}+720\,\mathrm{e}^{10i(dx+c)}-25\,\mathrm{e}^{9i(dx+c)}+2160\,\mathrm{e}^{8i(dx+c)}+450\,\mathrm{e}^{7i(dx+c)}+3680\,\mathrm{e}^{6i(dx+c)}-450\,\mathrm{e}^{5i(dx+c)}+330\right.}{120da\left(\mathrm{e}^{2i(dx+c)}+1\right)^6}$
derivativedivides	$\frac{1}{6\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^6}+\frac{7}{10\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^5}+\frac{3}{4\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^4}-\frac{5}{12\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^3}-\frac{9}{16\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}+\frac{21}{16\left(\tan\left(\frac{dx}{2}\right.\right.}$
default	$\frac{1}{6\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^6}+\frac{7}{10\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^5}+\frac{3}{4\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^4}-\frac{5}{12\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^3}-\frac{9}{16\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}+\frac{21}{16\left(\tan\left(\frac{dx}{2}\right.\right.}$

input
```
int(tan(d*x+c)^8/(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
x/a-1/120*I*(165*exp(11*I*(d*x+c))+720*exp(10*I*(d*x+c))-25*exp(9*I*(d*x+c
))+2160*exp(8*I*(d*x+c))+450*exp(7*I*(d*x+c))+3680*exp(6*I*(d*x+c))-450*ex
p(5*I*(d*x+c))+3360*exp(4*I*(d*x+c))+25*exp(3*I*(d*x+c))+1488*exp(2*I*(d*x
+c))-165*exp(I*(d*x+c))+368)/d/a/(exp(2*I*(d*x+c))+1)^6+5/16/a/d*ln(exp(I*
(d*x+c))-I)-5/16/a/d*ln(exp(I*(d*x+c))+I)
```

3.64.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.21

$$\int \frac{\tan^8(c+dx)}{a+a\sec(c+dx)}\,dx$$
$$= \frac{480\,dx\cos(dx+c)^6 - 75\cos(dx+c)^6\log(\sin(dx+c)+1) + 75\cos(dx+c)^6\log(-\sin(dx+c)+1) -}{480}$$

input
```
integrate(tan(d*x+c)^8/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

output
```
1/480*(480*d*x*cos(d*x + c)^6 - 75*cos(d*x + c)^6*log(sin(d*x + c) + 1) +
75*cos(d*x + c)^6*log(-sin(d*x + c) + 1) - 2*(368*cos(d*x + c)^5 - 165*cos
(d*x + c)^4 - 176*cos(d*x + c)^3 + 130*cos(d*x + c)^2 + 48*cos(d*x + c) -
40)*sin(d*x + c))/(a*d*cos(d*x + c)^6)
```

3.64.6 Sympy [F]

$$\int \frac{\tan^8(c+dx)}{a+a\sec(c+dx)}\,dx = \frac{\int \frac{\tan^8(c+dx)}{\sec(c+dx)+1}\,dx}{a}$$

input
```
integrate(tan(d*x+c)**8/(a+a*sec(d*x+c)),x)
```

output
```
Integral(tan(c + d*x)**8/(sec(c + d*x) + 1), x)/a
```

3.64.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 329 vs. $2(97) = 194$.

Time = 0.31 (sec) , antiderivative size = 329, normalized size of antiderivative = 3.13

$$\int \frac{\tan^8(c+dx)}{a+a\sec(c+dx)}\,dx =$$

$$-\frac{\frac{2\left(\frac{165\sin(dx+c)}{\cos(dx+c)+1}-\frac{1095\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{3138\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{5118\sin(dx+c)^7}{(\cos(dx+c)+1)^7}+\frac{1945\sin(dx+c)^9}{(\cos(dx+c)+1)^9}-\frac{315\sin(dx+c)^{11}}{(\cos(dx+c)+1)^{11}}\right)}{a-\frac{6a\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{15a\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{20a\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{15a\sin(dx+c)^8}{(\cos(dx+c)+1)^8}-\frac{6a\sin(dx+c)^{10}}{(\cos(dx+c)+1)^{10}}+\frac{a\sin(dx+c)^{12}}{(\cos(dx+c)+1)^{12}}}-\frac{480\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}+$$

$$240\,d$$

input
```
integrate(tan(d*x+c)^8/(a+a*sec(d*x+c)),x, algorithm="maxima")
```

output
```
-1/240*(2*(165*sin(d*x + c)/(cos(d*x + c) + 1) - 1095*sin(d*x + c)^3/(cos(
d*x + c) + 1)^3 + 3138*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 5118*sin(d*x
 + c)^7/(cos(d*x + c) + 1)^7 + 1945*sin(d*x + c)^9/(cos(d*x + c) + 1)^9 - 3
15*sin(d*x + c)^11/(cos(d*x + c) + 1)^11)/(a - 6*a*sin(d*x + c)^2/(cos(d*x
 + c) + 1)^2 + 15*a*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 - 20*a*sin(d*x + c
)^6/(cos(d*x + c) + 1)^6 + 15*a*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 - 6*a*
sin(d*x + c)^10/(cos(d*x + c) + 1)^10 + a*sin(d*x + c)^12/(cos(d*x + c) +
1)^12) - 480*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a + 75*log(sin(d*x +
c)/(cos(d*x + c) + 1) + 1)/a - 75*log(sin(d*x + c)/(cos(d*x + c) + 1) - 1)
/a)/d
```

3.64.8 Giac [A] (verification not implemented)

Time = 4.80 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.42

$$\int \frac{\tan^8(c+dx)}{a+a\sec(c+dx)}\,dx$$

$$= \frac{\frac{240\,(dx+c)}{a}-\frac{75\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{a}+\frac{75\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{a}+\frac{2\left(315\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{11}-1945\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{9}+5118\tan\left(\frac{1}{2}\,d\right.\right.}{\left(\tan\right.}$$

$$240\,d$$

input
```
integrate(tan(d*x+c)^8/(a+a*sec(d*x+c)),x, algorithm="giac")
```

output
```
1/240*(240*(d*x + c)/a - 75*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a + 75*log(
abs(tan(1/2*d*x + 1/2*c) - 1))/a + 2*(315*tan(1/2*d*x + 1/2*c)^11 - 1945*t
an(1/2*d*x + 1/2*c)^9 + 5118*tan(1/2*d*x + 1/2*c)^7 - 3138*tan(1/2*d*x + 1
/2*c)^5 + 1095*tan(1/2*d*x + 1/2*c)^3 - 165*tan(1/2*d*x + 1/2*c))/((tan(1/
2*d*x + 1/2*c)^2 - 1)^6*a))/d
```

3.64.9 Mupad [B] (verification not implemented)

Time = 15.58 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.84

$$\int \frac{\tan^8(c+dx)}{a+a\sec(c+dx)}\,dx = \frac{x}{a} - \frac{5\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{8\,a\,d}$$
$$- \frac{-\frac{21\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{11}}{8} + \frac{389\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{9}}{24} - \frac{853\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{7}}{20} + \frac{523\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{5}}{20} - \frac{73\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{3}}{8} + \frac{11}{}}{d\left(a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{12} - 6\,a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{10} + 15\,a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{8} - 20\,a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{6} + 15\,a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right.}$$

input
```
int(tan(c + d*x)^8/(a + a/cos(c + d*x)),x)
```

output
```
x/a - (5*atanh(tan(c/2 + (d*x)/2)))/(8*a*d) - ((11*tan(c/2 + (d*x)/2))/8 -
 (73*tan(c/2 + (d*x)/2)^3)/8 + (523*tan(c/2 + (d*x)/2)^5)/20 - (853*tan(c/
2 + (d*x)/2)^7)/20 + (389*tan(c/2 + (d*x)/2)^9)/24 - (21*tan(c/2 + (d*x)/2
)^11)/8)/(d*(a - 6*a*tan(c/2 + (d*x)/2)^2 + 15*a*tan(c/2 + (d*x)/2)^4 - 20
*a*tan(c/2 + (d*x)/2)^6 + 15*a*tan(c/2 + (d*x)/2)^8 - 6*a*tan(c/2 + (d*x)/
2)^10 + a*tan(c/2 + (d*x)/2)^12))
```

3.65 $\int \frac{\tan^6(c+dx)}{a+a\sec(c+dx)}\,dx$

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3.65.1 Optimal result

Integrand size = 21, antiderivative size = 78

$$\int \frac{\tan^6(c+dx)}{a+a\sec(c+dx)}\,dx = -\frac{x}{a} + \frac{3\operatorname{arctanh}(\sin(c+dx))}{8ad} + \frac{(8-3\sec(c+dx))\tan(c+dx)}{8ad} - \frac{(4-3\sec(c+dx))\tan^3(c+dx)}{12ad}$$

output

```
-x/a+3/8*arctanh(sin(d*x+c))/a/d+1/8*(8-3*sec(d*x+c))*tan(d*x+c)/a/d-1/12*
(4-3*sec(d*x+c))*tan(d*x+c)^3/a/d
```

3.65.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 893 vs. $2(78) = 156$.

Time = 7.10 (sec) , antiderivative size = 893, normalized size of antiderivative = 11.45

$$
\begin{aligned}
&\int \frac{\tan^6(c+dx)}{a+a\sec(c+dx)}\,dx \\
&= -\frac{2x\cos^2\left(\frac{c}{2}+\frac{dx}{2}\right)\sec(c+dx)}{a+a\sec(c+dx)} \\
&\quad - \frac{3\cos^2\left(\frac{c}{2}+\frac{dx}{2}\right)\log\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)-\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\right)\sec(c+dx)}{4d(a+a\sec(c+dx))} \\
&\quad + \frac{3\cos^2\left(\frac{c}{2}+\frac{dx}{2}\right)\log\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)+\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\right)\sec(c+dx)}{4d(a+a\sec(c+dx))} \\
&\quad + \frac{\cos^2\left(\frac{c}{2}+\frac{dx}{2}\right)\sec(c+dx)}{8d(a+a\sec(c+dx))\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)-\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\right)^4} \\
&\quad - \frac{\cos^2\left(\frac{c}{2}+\frac{dx}{2}\right)\sec(c+dx)\sin\left(\frac{dx}{2}\right)}{3d(a+a\sec(c+dx))\left(\cos\left(\frac{c}{2}\right)-\sin\left(\frac{c}{2}\right)\right)\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)-\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\right)^3} \\
&\quad + \frac{\cos^2\left(\frac{c}{2}+\frac{dx}{2}\right)\sec(c+dx)\left(-19\cos\left(\frac{c}{2}\right)+11\sin\left(\frac{c}{2}\right)\right)}{24d(a+a\sec(c+dx))\left(\cos\left(\frac{c}{2}\right)-\sin\left(\frac{c}{2}\right)\right)\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)-\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\right)^2} \\
&\quad + \frac{8\cos^2\left(\frac{c}{2}+\frac{dx}{2}\right)\sec(c+dx)\sin\left(\frac{dx}{2}\right)}{3d(a+a\sec(c+dx))\left(\cos\left(\frac{c}{2}\right)-\sin\left(\frac{c}{2}\right)\right)\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)-\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\right)} \\
&\quad - \frac{\cos^2\left(\frac{c}{2}+\frac{dx}{2}\right)\sec(c+dx)}{8d(a+a\sec(c+dx))\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)+\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\right)^4} \\
&\quad - \frac{\cos^2\left(\frac{c}{2}+\frac{dx}{2}\right)\sec(c+dx)\sin\left(\frac{dx}{2}\right)}{3d(a+a\sec(c+dx))\left(\cos\left(\frac{c}{2}\right)+\sin\left(\frac{c}{2}\right)\right)\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)+\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\right)^3} \\
&\quad + \frac{\cos^2\left(\frac{c}{2}+\frac{dx}{2}\right)\sec(c+dx)\left(19\cos\left(\frac{c}{2}\right)+11\sin\left(\frac{c}{2}\right)\right)}{24d(a+a\sec(c+dx))\left(\cos\left(\frac{c}{2}\right)+\sin\left(\frac{c}{2}\right)\right)\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)+\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\right)^2} \\
&\quad + \frac{8\cos^2\left(\frac{c}{2}+\frac{dx}{2}\right)\sec(c+dx)\sin\left(\frac{dx}{2}\right)}{3d(a+a\sec(c+dx))\left(\cos\left(\frac{c}{2}\right)+\sin\left(\frac{c}{2}\right)\right)\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)+\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}
\end{aligned}
$$

input
```
Integrate[Tan[c + d*x]^6/(a + a*Sec[c + d*x]),x]
```

output

```
(-2*x*Cos[c/2 + (d*x)/2]^2*Sec[c + d*x])/(a + a*Sec[c + d*x]) - (3*Cos[c/2
 + (d*x)/2]^2*Log[Cos[c/2 + (d*x)/2] - Sin[c/2 + (d*x)/2]]*Sec[c + d*x])/(
4*d*(a + a*Sec[c + d*x])) + (3*Cos[c/2 + (d*x)/2]^2*Log[Cos[c/2 + (d*x)/2]
 + Sin[c/2 + (d*x)/2]]*Sec[c + d*x])/(4*d*(a + a*Sec[c + d*x])) + (Cos[c/2
 + (d*x)/2]^2*Sec[c + d*x])/(8*d*(a + a*Sec[c + d*x])*(Cos[c/2 + (d*x)/2]
- Sin[c/2 + (d*x)/2])^4) - (Cos[c/2 + (d*x)/2]^2*Sec[c + d*x]*Sin[(d*x)/2]
)/(3*d*(a + a*Sec[c + d*x])*(Cos[c/2] - Sin[c/2])*(Cos[c/2 + (d*x)/2] - Si
n[c/2 + (d*x)/2])^3) + (Cos[c/2 + (d*x)/2]^2*Sec[c + d*x]*(-19*Cos[c/2] +
11*Sin[c/2]))/(24*d*(a + a*Sec[c + d*x])*(Cos[c/2] - Sin[c/2])*(Cos[c/2 +
(d*x)/2] - Sin[c/2 + (d*x)/2])^2) + (8*Cos[c/2 + (d*x)/2]^2*Sec[c + d*x]*S
in[(d*x)/2])/(3*d*(a + a*Sec[c + d*x])*(Cos[c/2] - Sin[c/2])*(Cos[c/2 + (d
*x)/2] - Sin[c/2 + (d*x)/2])) - (Cos[c/2 + (d*x)/2]^2*Sec[c + d*x])/(8*d*(
a + a*Sec[c + d*x])*(Cos[c/2 + (d*x)/2] + Sin[c/2 + (d*x)/2])^4) - (Cos[c/
2 + (d*x)/2]^2*Sec[c + d*x]*Sin[(d*x)/2])/(3*d*(a + a*Sec[c + d*x])*(Cos[c
/2] + Sin[c/2])*(Cos[c/2 + (d*x)/2] + Sin[c/2 + (d*x)/2])^3) + (Cos[c/2 +
(d*x)/2]^2*Sec[c + d*x]*(19*Cos[c/2] + 11*Sin[c/2]))/(24*d*(a + a*Sec[c +
d*x])*(Cos[c/2] + Sin[c/2])*(Cos[c/2 + (d*x)/2] + Sin[c/2 + (d*x)/2])^2) +
 (8*Cos[c/2 + (d*x)/2]^2*Sec[c + d*x]*Sin[(d*x)/2])/(3*d*(a + a*Sec[c + d*
x])*(Cos[c/2] + Sin[c/2])*(Cos[c/2 + (d*x)/2] + Sin[c/2 + (d*x)/2]))
```

3.65.3 Rubi [A] (verified)

Time $= 0.43$ (sec) , antiderivative size $= 87$, normalized size of antiderivative $= 1.12$, number of steps used $= 8$, number of rules used $= 8$, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used $= \{3042, 4376, 25, 3042, 4369, 3042, 4369, 2009\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^6(c+dx)}{a\sec(c+dx)+a}\,dx$$

$$\downarrow 3042$$

$$\int \frac{\cot\left(c+dx+\frac{\pi}{2}\right)^6}{a\csc\left(c+dx+\frac{\pi}{2}\right)+a}dx$$

$$\downarrow 4376$$

$$\frac{\int -\left((a-a\sec(c+dx))\tan^4(c+dx)\right)dx}{a^2}$$

$$\downarrow 25$$

$$-\frac{\int (a-a\sec(c+dx))\tan^4(c+dx)dx}{a^2}$$

$$\downarrow 3042$$

$$-\frac{\int \cot\left(c+dx+\frac{\pi}{2}\right)^4\left(a-a\csc\left(c+dx+\frac{\pi}{2}\right)\right)dx}{a^2}$$

$$\downarrow 4369$$

$$-\frac{\frac{\tan^3(c+dx)(4a-3a\sec(c+dx))}{12d}-\frac{1}{4}\int (4a-3a\sec(c+dx))\tan^2(c+dx)dx}{a^2}$$

$$\downarrow 3042$$

$$-\frac{\frac{\tan^3(c+dx)(4a-3a\sec(c+dx))}{12d}-\frac{1}{4}\int \cot\left(c+dx+\frac{\pi}{2}\right)^2\left(4a-3a\csc\left(c+dx+\frac{\pi}{2}\right)\right)dx}{a^2}$$

$$\downarrow 4369$$

$$-\frac{\frac{1}{4}\left(\frac{1}{2}\int (8a-3a\sec(c+dx))dx-\frac{\tan(c+dx)(8a-3a\sec(c+dx))}{2d}\right)+\frac{\tan^3(c+dx)(4a-3a\sec(c+dx))}{12d}}{a^2}$$

$$\downarrow 2009$$

$$-\frac{\frac{1}{4}\left(\frac{1}{2}\left(8ax-\frac{3a\text{arctanh}(\sin(c+dx))}{d}\right)-\frac{\tan(c+dx)(8a-3a\sec(c+dx))}{2d}\right)+\frac{\tan^3(c+dx)(4a-3a\sec(c+dx))}{12d}}{a^2}$$

input
```
Int[Tan[c + d*x]^6/(a + a*Sec[c + d*x]),x]
```

output
```
-((((4*a - 3*a*Sec[c + d*x])*Tan[c + d*x]^3)/(12*d) + ((8*a*x - (3*a*ArcTa
nh[Sin[c + d*x]])/d)/2 - ((8*a - 3*a*Sec[c + d*x])*Tan[c + d*x])/(2*d))/4)
/a^2)
```

3.65.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4369
```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(-e)*(e*Cot[c + d*x])^(m - 1)*((a*m + b*(m - 1)*Csc
[c + d*x])/(d*m*(m - 1))), x] - Simp[e^2/m   Int[(e*Cot[c + d*x])^(m - 2)*(
a*m + b*(m - 1)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && GtQ[m
, 1]
```

rule 4376
```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Simp[a^(2*n)/e^(2*n)   Int[(e*Cot[c + d*x])^(m + 2*
n)/(-a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[a
^2 - b^2, 0] && ILtQ[n, 0]
```

3.65.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.95 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.94

method	result
risch	$-\frac{x}{a}+\frac{i\left(15\,\mathrm{e}^{7i(dx+c)}+48\,\mathrm{e}^{6i(dx+c)}-9\,\mathrm{e}^{5i(dx+c)}+96\,\mathrm{e}^{4i(dx+c)}+9\,\mathrm{e}^{3i(dx+c)}+80\,\mathrm{e}^{2i(dx+c)}-15\,\mathrm{e}^{i(dx+c)}+32\right)}{12da\left(\mathrm{e}^{2i(dx+c)}+1\right)^4}+\frac{3\ln\left(\mathrm{e}^{i}\right.}{8}$
derivativedivides	$\frac{\frac{1}{4\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^4}+\frac{5}{6\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^3}+\frac{3}{8\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}-\frac{11}{8\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}-\frac{3\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{8}-2\arctan\left(\tan\left(\right.\right.}{da}$
default	$\frac{\frac{1}{4\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^4}+\frac{5}{6\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^3}+\frac{3}{8\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}-\frac{11}{8\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}-\frac{3\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{8}-2\arctan\left(\tan\left(\right.\right.}{da}$

input
```
int(tan(d*x+c)^6/(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
-x/a+1/12*I*(15*exp(7*I*(d*x+c))+48*exp(6*I*(d*x+c))-9*exp(5*I*(d*x+c))+96
*exp(4*I*(d*x+c))+9*exp(3*I*(d*x+c))+80*exp(2*I*(d*x+c))-15*exp(I*(d*x+c))
+32)/d/a/(exp(2*I*(d*x+c))+1)^4+3/8/a/d*ln(exp(I*(d*x+c))+I)-3/8/a/d*ln(ex
p(I*(d*x+c))-I)
```

3.65.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.37

$$\int \frac{\tan^6(c+dx)}{a+a\sec(c+dx)}\,dx =$$
$$-\frac{48\,dx\cos(dx+c)^4 - 9\cos(dx+c)^4\log(\sin(dx+c)+1) + 9\cos(dx+c)^4\log(-\sin(dx+c)+1) - 2}{48\,ad\cos(dx+c)^4}$$

input
```
integrate(tan(d*x+c)^6/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

output
```
-1/48*(48*d*x*cos(d*x + c)^4 - 9*cos(d*x + c)^4*log(sin(d*x + c) + 1) + 9*
cos(d*x + c)^4*log(-sin(d*x + c) + 1) - 2*(32*cos(d*x + c)^3 - 15*cos(d*x
+ c)^2 - 8*cos(d*x + c) + 6)*sin(d*x + c))/(a*d*cos(d*x + c)^4)
```

3.65.6 Sympy [F]

$$\int \frac{\tan^6(c+dx)}{a+a\sec(c+dx)}\,dx = \frac{\int \frac{\tan^6(c+dx)}{\sec(c+dx)+1}\,dx}{a}$$

input
```
integrate(tan(d*x+c)**6/(a+a*sec(d*x+c)),x)
```

output
```
Integral(tan(c + d*x)**6/(sec(c + d*x) + 1), x)/a
```

3.65.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 247 vs. 2(72) = 144.

Time = 0.31 (sec) , antiderivative size = 247, normalized size of antiderivative = 3.17

$$\int \frac{\tan^6(c+dx)}{a+a\sec(c+dx)}\,dx$$
$$= \frac{\frac{2\left(\frac{15\sin(dx+c)}{\cos(dx+c)+1} - \frac{71\sin(dx+c)^3}{(\cos(dx+c)+1)^3} + \frac{137\sin(dx+c)^5}{(\cos(dx+c)+1)^5} - \frac{33\sin(dx+c)^7}{(\cos(dx+c)+1)^7}\right)}{a - \frac{4a\sin(dx+c)^2}{(\cos(dx+c)+1)^2} + \frac{6a\sin(dx+c)^4}{(\cos(dx+c)+1)^4} - \frac{4a\sin(dx+c)^6}{(\cos(dx+c)+1)^6} + \frac{a\sin(dx+c)^8}{(\cos(dx+c)+1)^8}} - \frac{48\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a} + \frac{9\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a} - \frac{9\log\left(\frac{\sin(dx}{a}\right.}$$
$$24\,d$$

input
```
integrate(tan(d*x+c)^6/(a+a*sec(d*x+c)),x, algorithm="maxima")
```

output
```
1/24*(2*(15*sin(d*x + c)/(cos(d*x + c) + 1) - 71*sin(d*x + c)^3/(cos(d*x +
 c) + 1)^3 + 137*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 33*sin(d*x + c)^7/(
cos(d*x + c) + 1)^7)/(a - 4*a*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 6*a*si
n(d*x + c)^4/(cos(d*x + c) + 1)^4 - 4*a*sin(d*x + c)^6/(cos(d*x + c) + 1)^
6 + a*sin(d*x + c)^8/(cos(d*x + c) + 1)^8) - 48*arctan(sin(d*x + c)/(cos(d
*x + c) + 1))/a + 9*log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/a - 9*log(sin
(d*x + c)/(cos(d*x + c) + 1) - 1)/a)/d
```

3.65.8 Giac [A] (verification not implemented)

Time = 2.13 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.58

$$\int \frac{\tan^6(c+dx)}{a+a\sec(c+dx)}\,dx = -\frac{\frac{24\,(dx+c)}{a} - \frac{9\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{a} + \frac{9\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{a} + \frac{2\left(33\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-137\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+71\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-15\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)^4 a}}{24\,d}$$

input
```
integrate(tan(d*x+c)^6/(a+a*sec(d*x+c)),x, algorithm="giac")
```

output
```
-1/24*(24*(d*x + c)/a - 9*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a + 9*log(abs
(tan(1/2*d*x + 1/2*c) - 1))/a + 2*(33*tan(1/2*d*x + 1/2*c)^7 - 137*tan(1/2
*d*x + 1/2*c)^5 + 71*tan(1/2*d*x + 1/2*c)^3 - 15*tan(1/2*d*x + 1/2*c))/((t
an(1/2*d*x + 1/2*c)^2 - 1)^4*a))/d
```

3.65.9 Mupad [B] (verification not implemented)

Time = 15.21 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.78

$$\int \frac{\tan^6(c+dx)}{a+a\sec(c+dx)}\,dx = \frac{3\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{4\,a\,d} - \frac{x}{a} + \frac{-\frac{11\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^7}{4} + \frac{137\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5}{12} - \frac{71\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3}{12} + \frac{5\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{4}}{d\left(a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^8 - 4\,a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6 + 6\,a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4 - 4\,a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2 + a\right)}$$

input
```
int(tan(c + d*x)^6/(a + a/cos(c + d*x)),x)
```

output
```
(3*atanh(tan(c/2 + (d*x)/2)))/(4*a*d) - x/a + ((5*tan(c/2 + (d*x)/2))/4 -
(71*tan(c/2 + (d*x)/2)^3)/12 + (137*tan(c/2 + (d*x)/2)^5)/12 - (11*tan(c/2
 + (d*x)/2)^7)/4)/(d*(a - 4*a*tan(c/2 + (d*x)/2)^2 + 6*a*tan(c/2 + (d*x)/2
)^4 - 4*a*tan(c/2 + (d*x)/2)^6 + a*tan(c/2 + (d*x)/2)^8))
```

3.66 $\int \frac{\tan^4(c+dx)}{a+a\sec(c+dx)}\,dx$

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3.66.1 Optimal result

Integrand size = 21, antiderivative size = 49

$$\int \frac{\tan^4(c+dx)}{a+a\sec(c+dx)}\,dx = \frac{x}{a} - \frac{\operatorname{arctanh}(\sin(c+dx))}{2ad} - \frac{(2-\sec(c+dx))\tan(c+dx)}{2ad}$$

output
```
x/a-1/2*arctanh(sin(d*x+c))/a/d-1/2*(2-sec(d*x+c))*tan(d*x+c)/a/d
```

3.66.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 241 vs. 2(49) = 98.

Time = 1.20 (sec) , antiderivative size = 241, normalized size of antiderivative = 4.92

$$\int \frac{\tan^4(c+dx)}{a+a\sec(c+dx)}\,dx$$
$$= \frac{\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\left(4x + \frac{2\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)}{d} - \frac{2\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)}{d} + \frac{}{d(\cos(\frac{1}{2}(c+}\right.}{2}$$

input
```
Integrate[Tan[c + d*x]^4/(a + a*Sec[c + d*x]),x]
```

output
```
(Cos[(c + d*x)/2]^2*Sec[c + d*x]*(4*x + (2*Log[Cos[(c + d*x)/2] - Sin[(c +
 d*x)/2]])/d - (2*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]])/d + 1/(d*(Cos[
(c + d*x)/2] - Sin[(c + d*x)/2])^2) - 1/(d*(Cos[(c + d*x)/2] + Sin[(c + d*
x)/2])^2) - (4*Sin[d*x])/(d*(Cos[c/2] - Sin[c/2])*(Cos[c/2] + Sin[c/2])*(C
os[(c + d*x)/2] - Sin[(c + d*x)/2])*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]))
))/(2*a*(1 + Sec[c + d*x]))
```

3.66.3 Rubi [A] (verified)

Time $= 0.33$ (sec) , antiderivative size $= 53$, normalized size of antiderivative $= 1.08$, number of steps used $= 6$, number of rules used $= 6$, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used $= \{3042, 4376, 25, 3042, 4369, 2009\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^4(c+dx)}{a\sec(c+dx)+a}\,dx$$
$$\downarrow 3042$$
$$\int \frac{\cot\left(c+dx+\frac{\pi}{2}\right)^4}{a\csc\left(c+dx+\frac{\pi}{2}\right)+a}dx$$
$$\downarrow 4376$$
$$\frac{\int -\left((a-a\sec(c+dx))\tan^2(c+dx)\right)dx}{a^2}$$
$$\downarrow 25$$
$$-\frac{\int (a-a\sec(c+dx))\tan^2(c+dx)dx}{a^2}$$
$$\downarrow 3042$$
$$-\frac{\int \cot\left(c+dx+\frac{\pi}{2}\right)^2\left(a-a\csc\left(c+dx+\frac{\pi}{2}\right)\right)dx}{a^2}$$
$$\downarrow 4369$$
$$-\frac{\frac{\tan(c+dx)(2a-a\sec(c+dx))}{2d}-\frac{1}{2}\int(2a-a\sec(c+dx))dx}{a^2}$$
$$\downarrow 2009$$
$$-\frac{\frac{1}{2}\left(\frac{a\operatorname{arctanh}(\sin(c+dx))}{d}-2ax\right)+\frac{\tan(c+dx)(2a-a\sec(c+dx))}{2d}}{a^2}$$

input
```
Int[Tan[c + d*x]^4/(a + a*Sec[c + d*x]),x]
```

output
```
-(((-2*a*x + (a*ArcTanh[Sin[c + d*x]])/d)/2 + ((2*a - a*Sec[c + d*x])*Tan[
c + d*x])/(2*d))/a^2)
```

3.66.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4369
```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(-e)*(e*Cot[c + d*x])^(m - 1)*((a*m + b*(m - 1)*Csc
[c + d*x])/(d*m*(m - 1))), x] - Simp[e^2/m   Int[(e*Cot[c + d*x])^(m - 2)*(
a*m + b*(m - 1)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && GtQ[m
, 1]
```

rule 4376
```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Simp[a^(2*n)/e^(2*n)   Int[(e*Cot[c + d*x])^(m + 2*
n)/(-a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[a
^2 - b^2, 0] && ILtQ[n, 0]
```

3.66.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.66 (sec) , antiderivative size = 104, normalized size of antiderivative = 2.12

method	result
risch	$\frac{x}{a} - \frac{i\left(e^{3i(dx+c)}+2\,e^{2i(dx+c)}-e^{i(dx+c)}+2\right)}{da\left(e^{2i(dx+c)}+1\right)^2} + \frac{\ln\left(e^{i(dx+c)}-i\right)}{2ad} - \frac{\ln\left(e^{i(dx+c)}+i\right)}{2ad}$
derivativedivides	$\frac{-\frac{1}{2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^2}+\frac{3}{2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}-\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{2}+\frac{1}{2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}+\frac{3}{2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-}{2}}{da}$
default	$\frac{-\frac{1}{2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^2}+\frac{3}{2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}-\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{2}+\frac{1}{2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}+\frac{3}{2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-}{2}}{da}$

input
```
int(tan(d*x+c)^4/(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
x/a-I*(exp(3*I*(d*x+c))+2*exp(2*I*(d*x+c))-exp(I*(d*x+c))+2)/d/a/(exp(2*I*
(d*x+c))+1)^2+1/2/a/d*ln(exp(I*(d*x+c))-I)-1/2/a/d*ln(exp(I*(d*x+c))+I)
```

3.66.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.76

$$\int \frac{\tan^4(c+dx)}{a+a\sec(c+dx)}\,dx$$
$$= \frac{4\,dx\cos(dx+c)^2 - \cos(dx+c)^2\log(\sin(dx+c)+1) + \cos(dx+c)^2\log(-\sin(dx+c)+1) - 2(2\cos$$
$$4\,ad\cos(dx+c)^2$$

input
```
integrate(tan(d*x+c)^4/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

output
```
1/4*(4*d*x*cos(d*x + c)^2 - cos(d*x + c)^2*log(sin(d*x + c) + 1) + cos(d*x
 + c)^2*log(-sin(d*x + c) + 1) - 2*(2*cos(d*x + c) - 1)*sin(d*x + c))/(a*d
*cos(d*x + c)^2)
```

3.66.6 Sympy [F]

$$\int \frac{\tan^4(c+dx)}{a+a\sec(c+dx)}\,dx = \frac{\int \frac{\tan^4(c+dx)}{\sec(c+dx)+1}\,dx}{a}$$

input
```
integrate(tan(d*x+c)**4/(a+a*sec(d*x+c)),x)
```

output
```
Integral(tan(c + d*x)**4/(sec(c + d*x) + 1), x)/a
```

3.66.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 163 vs. $2(43) = 86$.

Time = 0.30 (sec) , antiderivative size = 163, normalized size of antiderivative = 3.33

$$\int \frac{\tan^4(c+dx)}{a+a\sec(c+dx)}\,dx$$
$$= -\frac{\frac{2\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-\frac{3\sin(dx+c)^3}{(\cos(dx+c)+1)^3}\right)}{a-\frac{2a\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{a\sin(dx+c)^4}{(\cos(dx+c)+1)^4}} - \frac{4\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a} + \frac{\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a} - \frac{\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a}}{2d}$$

input
```
integrate(tan(d*x+c)^4/(a+a*sec(d*x+c)),x, algorithm="maxima")
```

output
```
-1/2*(2*(sin(d*x + c)/(cos(d*x + c) + 1) - 3*sin(d*x + c)^3/(cos(d*x + c)
+ 1)^3)/(a - 2*a*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + a*sin(d*x + c)^4/(c
os(d*x + c) + 1)^4) - 4*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a + log(si
n(d*x + c)/(cos(d*x + c) + 1) + 1)/a - log(sin(d*x + c)/(cos(d*x + c) + 1)
 - 1)/a)/d
```

3.66.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 96 vs. $2(43) = 86$.

Time = 0.89 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.96

$$\int \frac{\tan^4(c+dx)}{a+a\sec(c+dx)}\,dx$$
$$= \frac{\frac{2(dx+c)}{a} - \frac{\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)}{a} + \frac{\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)}{a} + \frac{2\left(3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)^2 a}}{2d}$$

input
```
integrate(tan(d*x+c)^4/(a+a*sec(d*x+c)),x, algorithm="giac")
```

output
```
1/2*(2*(d*x + c)/a - log(abs(tan(1/2*d*x + 1/2*c) + 1))/a + log(abs(tan(1/
2*d*x + 1/2*c) - 1))/a + 2*(3*tan(1/2*d*x + 1/2*c)^3 - tan(1/2*d*x + 1/2*c
))/((tan(1/2*d*x + 1/2*c)^2 - 1)^2*a))/d
```

3.66.9 Mupad [B] (verification not implemented)

Time = 14.40 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.69

$$\int \frac{\tan^4(c+dx)}{a+a\sec(c+dx)}\,dx = \frac{x}{a} - \frac{\operatorname{atanh}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{a\,d} - \frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right) - 3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3}{d\left(a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4 - 2\,a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2 + a\right)}$$

input
```
int(tan(c + d*x)^4/(a + a/cos(c + d*x)),x)
```

output
```
x/a - atanh(tan(c/2 + (d*x)/2))/(a*d) - (tan(c/2 + (d*x)/2) - 3*tan(c/2 +
(d*x)/2)^3)/(d*(a - 2*a*tan(c/2 + (d*x)/2)^2 + a*tan(c/2 + (d*x)/2)^4))
```

3.67 $\int \frac{\tan^2(c+dx)}{a+a\sec(c+dx)}\,dx$

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3.67.1 Optimal result

Integrand size = 21, antiderivative size = 21

$$\int \frac{\tan^2(c+dx)}{a+a\sec(c+dx)}\,dx = -\frac{x}{a}+\frac{\operatorname{arctanh}(\sin(c+dx))}{ad}$$

output
```
-x/a+arctanh(sin(d*x+c))/a/d
```

3.67.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 60 vs. 2(21) = 42.

Time = 0.37 (sec) , antiderivative size = 60, normalized size of antiderivative = 2.86

$$\int \frac{\tan^2(c+dx)}{a+a\sec(c+dx)}\,dx = -\frac{dx+\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)}{ad}$$

input
```
Integrate[Tan[c + d*x]^2/(a + a*Sec[c + d*x]),x]
```

output
```
-((d*x + Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[Cos[(c + d*x)/2] +
 Sin[(c + d*x)/2]])/(a*d))
```

3.67.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3042, 4376, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^2(c+dx)}{a\sec(c+dx)+a}\,dx$$

$$\downarrow 3042$$

$$\int \frac{\cot\left(c+dx+\frac{\pi}{2}\right)^2}{a\csc\left(c+dx+\frac{\pi}{2}\right)+a}dx$$

$$\downarrow 4376$$

$$\frac{\int(a\sec(c+dx)-a)dx}{a^2}$$

$$\downarrow 2009$$

$$\frac{\frac{a\text{arctanh}(\sin(c+dx))}{d}-ax}{a^2}$$

input
```
Int[Tan[c + d*x]^2/(a + a*Sec[c + d*x]),x]
```

output
```
(-(a*x) + (a*ArcTanh[Sin[c + d*x]])/d)/a^2
```

3.67.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4376
```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Simp[a^(2*n)/e^(2*n)   Int[(e*Cot[c + d*x])^(m + 2*
n)/(-a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[a
^2 - b^2, 0] && ILtQ[n, 0]
```

3.67.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.53 (sec) , antiderivative size = 49, normalized size of antiderivative = 2.33

method	result	size
risch	$-\frac{x}{a}-\frac{\ln\left(e^{i(dx+c)}-i\right)}{ad}+\frac{\ln\left(e^{i(dx+c)}+i\right)}{ad}$	49
derivativedivides	$\frac{-2\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{da}$	50
default	$\frac{-2\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{da}$	50

input
```
int(tan(d*x+c)^2/(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
-x/a-1/a/d*ln(exp(I*(d*x+c))-I)+1/a/d*ln(exp(I*(d*x+c))+I)
```

3.67.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.67

$$\int \frac{\tan^2(c+dx)}{a+a\sec(c+dx)}\,dx = -\frac{2\,dx-\log\left(\sin\left(dx+c\right)+1\right)+\log\left(-\sin\left(dx+c\right)+1\right)}{2\,ad}$$

input
```
integrate(tan(d*x+c)^2/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

output
```
-1/2*(2*d*x - log(sin(d*x + c) + 1) + log(-sin(d*x + c) + 1))/(a*d)
```

3.67.6 Sympy [F]

$$\int \frac{\tan^2(c+dx)}{a+a\sec(c+dx)}\,dx = \frac{\int \frac{\tan^2\left(c+dx\right)}{\sec\left(c+dx\right)+1}\,dx}{a}$$

input
```
integrate(tan(d*x+c)**2/(a+a*sec(d*x+c)),x)
```

output
```
Integral(tan(c + d*x)**2/(sec(c + d*x) + 1), x)/a
```

3.67.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 78 vs. $2(21) = 42$.

Time = 0.31 (sec) , antiderivative size = 78, normalized size of antiderivative = 3.71

$$\int \frac{\tan^2(c+dx)}{a+a\sec(c+dx)}\,dx = -\frac{\frac{2\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a} - \frac{\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a} + \frac{\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a}}{d}$$

input
```
integrate(tan(d*x+c)^2/(a+a*sec(d*x+c)),x, algorithm="maxima")
```

output
```
-(2*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a - log(sin(d*x + c)/(cos(d*x
+ c) + 1) + 1)/a + log(sin(d*x + c)/(cos(d*x + c) + 1) - 1)/a)/d
```

3.67.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 50 vs. $2(21) = 42$.

Time = 0.45 (sec) , antiderivative size = 50, normalized size of antiderivative = 2.38

$$\int \frac{\tan^2(c+dx)}{a+a\sec(c+dx)}\,dx = -\frac{\frac{dx+c}{a} - \frac{\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{a} + \frac{\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{a}}{d}$$

input
```
integrate(tan(d*x+c)^2/(a+a*sec(d*x+c)),x, algorithm="giac")
```

output
```
-((d*x + c)/a - log(abs(tan(1/2*d*x + 1/2*c) + 1))/a + log(abs(tan(1/2*d*x
 + 1/2*c) - 1))/a)/d
```

3.67.9 Mupad [B] (verification not implemented)

Time = 13.83 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.19

$$\int \frac{\tan^2(c+dx)}{a+a\sec(c+dx)}\,dx = \frac{2\,\operatorname{atanh}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{a\,d} - \frac{x}{a}$$

input
```
int(tan(c + d*x)^2/(a + a/cos(c + d*x)),x)
```

output
```
(2*atanh(tan(c/2 + (d*x)/2)))/(a*d) - x/a
```

3.68 $\int \frac{\cot^2(c+dx)}{a+a\sec(c+dx)}\,dx$

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3.68.1 Optimal result

Integrand size = 21, antiderivative size = 61

$$\int \frac{\cot^2(c+dx)}{a+a\sec(c+dx)}\,dx = -\frac{x}{a} - \frac{\cot(c+dx)(3-2\sec(c+dx))}{3ad} + \frac{\cot^3(c+dx)(1-\sec(c+dx))}{3ad}$$

output
```
-x/a-1/3*cot(d*x+c)*(3-2*sec(d*x+c))/a/d+1/3*cot(d*x+c)^3*(1-sec(d*x+c))/a
/d
```

3.68.2 Mathematica [A] (verified)

Time = 1.02 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.64

$$\int \frac{\cot^2(c+dx)}{a+a\sec(c+dx)}\,dx = \frac{\sec(c+dx)\left(-12dx\cos^2\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\left(3\cot\left(\frac{1}{2}(c+dx)\right)\csc\left(\frac{c}{2}\right)+13\sec\left(\frac{c}{2}\right)\right)\sin\left(\frac{dx}{2}\right)-\right.}{6ad(1+\sec(c+dx))}$$

input
```
Integrate[Cot[c + d*x]^2/(a + a*Sec[c + d*x]),x]
```

output
```
(Sec[c + d*x]*(-12*d*x*Cos[(c + d*x)/2]^2 + Cos[(c + d*x)/2]*(3*Cot[(c + d
*x)/2]*Csc[c/2] + 13*Sec[c/2])*Sin[(d*x)/2] - Tan[(c + d*x)/2]))/(6*a*d*(1
 + Sec[c + d*x]))
```

3.68.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.05, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3042, 4376, 25, 3042, 4370, 25, 3042, 4370, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^2(c+dx)}{a\sec(c+dx)+a}\,dx$$

$$\downarrow 3042$$

$$\int \frac{1}{\cot\left(c+dx+\frac{\pi}{2}\right)^2\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)}dx$$

$$\downarrow 4376$$

$$\frac{\int -\cot^4(c+dx)(a-a\sec(c+dx))dx}{a^2}$$

$$\downarrow 25$$

$$-\frac{\int \cot^4(c+dx)(a-a\sec(c+dx))dx}{a^2}$$

$$\downarrow 3042$$

$$-\frac{\int \frac{a-a\csc\left(c+dx+\frac{\pi}{2}\right)}{\cot\left(c+dx+\frac{\pi}{2}\right)^4}dx}{a^2}$$

$$\downarrow 4370$$

$$-\frac{\frac{1}{3}\int -\cot^2(c+dx)(3a-2a\sec(c+dx))dx-\frac{\cot^3(c+dx)(a-a\sec(c+dx))}{3d}}{a^2}$$

$$\downarrow 25$$

$$-\frac{-\frac{1}{3}\int \cot^2(c+dx)(3a-2a\sec(c+dx))dx-\frac{\cot^3(c+dx)(a-a\sec(c+dx))}{3d}}{a^2}$$

$$\downarrow 3042$$

$$-\frac{-\frac{1}{3}\int \frac{3a-2a\csc\left(c+dx+\frac{\pi}{2}\right)}{\cot\left(c+dx+\frac{\pi}{2}\right)^2}dx-\frac{\cot^3(c+dx)(a-a\sec(c+dx))}{3d}}{a^2}$$

$$\downarrow 4370$$

$$-\frac{\frac{1}{3}\left(\frac{\cot(c+dx)(3a-2a\sec(c+dx))}{d}-\int -3adx\right)-\frac{\cot^3(c+dx)(a-a\sec(c+dx))}{3d}}{a^2}$$

$$\downarrow 24$$

$$-\frac{\frac{1}{3}\left(\frac{\cot(c+dx)(3a-2a\sec(c+dx))}{d}+3ax\right)-\frac{\cot^3(c+dx)(a-a\sec(c+dx))}{3d}}{a^2}$$

input
```
Int[Cot[c + d*x]^2/(a + a*Sec[c + d*x]),x]
```

output
```
-((-1/3*(Cot[c + d*x]^3*(a - a*Sec[c + d*x]))/d + (3*a*x + (Cot[c + d*x]*(
3*a - 2*a*Sec[c + d*x]))/d)/3)/a^2)
```

3.68.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4370
```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(-(e*Cot[c + d*x])^(m + 1))*((a + b*Csc[c + d*x])/(
d*e*(m + 1))), x] - Simp[1/(e^2*(m + 1))   Int[(e*Cot[c + d*x])^(m + 2)*(a*
(m + 1) + b*(m + 2)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && L
tQ[m, -1]
```

rule 4376
```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Simp[a^(2*n)/e^(2*n)   Int[(e*Cot[c + d*x])^(m + 2*
n)/(-a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[a
^2 - b^2, 0] && ILtQ[n, 0]
```

3.68.4 Maple [A] (verified)

Time = 0.62 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.97

method	result	size
derivativedivides	$\frac{-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{3}+4\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-8\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\frac{1}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}}{4da}$	59
default	$\frac{-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{3}+4\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-8\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\frac{1}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}}{4da}$	59
risch	$-\frac{x}{a}+\frac{2i\left(3\,e^{3i(dx+c)}-5\,e^{i(dx+c)}-4\right)}{3da\left(e^{i(dx+c)}+1\right)^3\left(e^{i(dx+c)}-1\right)}$	67

input
```
int(cot(d*x+c)^2/(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/4/d/a*(-1/3*tan(1/2*d*x+1/2*c)^3+4*tan(1/2*d*x+1/2*c)-8*arctan(tan(1/2*d
*x+1/2*c))-1/tan(1/2*d*x+1/2*c))
```

3.68.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.05

$$\int \frac{\cot^2(c+dx)}{a+a\sec(c+dx)}\,dx = -\frac{4\cos(dx+c)^2+3\,(dx\cos(dx+c)+dx)\sin(dx+c)+\cos(dx+c)-2}{3\,(ad\cos(dx+c)+ad)\sin(dx+c)}$$

input
```
integrate(cot(d*x+c)^2/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

output
```
-1/3*(4*cos(d*x + c)^2 + 3*(d*x*cos(d*x + c) + d*x)*sin(d*x + c) + cos(d*x
 + c) - 2)/((a*d*cos(d*x + c) + a*d)*sin(d*x + c))
```

3.68.6 Sympy [F]

$$\int \frac{\cot^2(c+dx)}{a+a\sec(c+dx)}\,dx = \frac{\int \frac{\cot^2(c+dx)}{\sec(c+dx)+1}\,dx}{a}$$

input
```
integrate(cot(d*x+c)**2/(a+a*sec(d*x+c)),x)
```

output
```
Integral(cot(c + d*x)**2/(sec(c + d*x) + 1), x)/a
```

3.68.7 Maxima [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.52

$$\int \frac{\cot^2(c+dx)}{a+a\sec(c+dx)}\,dx = \frac{\frac{\frac{12\sin(dx+c)}{\cos(dx+c)+1} - \frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3}}{a} - \frac{24\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a} - \frac{3(\cos(dx+c)+1)}{a\sin(dx+c)}}{12\,d}$$

input
```
integrate(cot(d*x+c)^2/(a+a*sec(d*x+c)),x, algorithm="maxima")
```

output
```
1/12*((12*sin(d*x + c)/(cos(d*x + c) + 1) - sin(d*x + c)^3/(cos(d*x + c) +
 1)^3)/a - 24*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a - 3*(cos(d*x + c)
+ 1)/(a*sin(d*x + c)))/d
```

3.68.8 Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.08

$$\int \frac{\cot^2(c+dx)}{a+a\sec(c+dx)}\,dx = -\frac{\frac{12(dx+c)}{a} + \frac{a^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3 - 12a^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{a^3} + \frac{3}{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}}{12\,d}$$

input
```
integrate(cot(d*x+c)^2/(a+a*sec(d*x+c)),x, algorithm="giac")
```

output
```
-1/12*(12*(d*x + c)/a + (a^2*tan(1/2*d*x + 1/2*c)^3 - 12*a^2*tan(1/2*d*x +
 1/2*c))/a^3 + 3/(a*tan(1/2*d*x + 1/2*c)))/d
```

3.68.9 Mupad [B] (verification not implemented)

Time = 14.22 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.07

$$\int \frac{\cot^2(c+dx)}{a+a\sec(c+dx)}\,dx = -\frac{x}{a} - \frac{\frac{4\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^4}{3} - \frac{7\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^2}{6} + \frac{1}{12}}{a\,d\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^3\sin\left(\frac{c}{2}+\frac{dx}{2}\right)}$$

input
```
int(cot(c + d*x)^2/(a + a/cos(c + d*x)),x)
```

output
```
- x/a - ((4*cos(c/2 + (d*x)/2)^4)/3 - (7*cos(c/2 + (d*x)/2)^2)/6 + 1/12)/(
a*d*cos(c/2 + (d*x)/2)^3*sin(c/2 + (d*x)/2))
```

3.69 $\int \frac{\cot^4(c+dx)}{a+a\sec(c+dx)}\,dx$

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3.69.1 Optimal result

Integrand size = 21, antiderivative size = 88

$$\int \frac{\cot^4(c+dx)}{a+a\sec(c+dx)}\,dx = \frac{x}{a} + \frac{\cot(c+dx)(15-8\sec(c+dx))}{15ad} - \frac{\cot^3(c+dx)(5-4\sec(c+dx))}{15ad} + \frac{\cot^5(c+dx)(1-\sec(c+dx))}{5ad}$$

output
```
x/a+1/15*cot(d*x+c)*(15-8*sec(d*x+c))/a/d-1/15*cot(d*x+c)^3*(5-4*sec(d*x+c
))/a/d+1/5*cot(d*x+c)^5*(1-sec(d*x+c))/a/d
```

3.69.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 254 vs. 2(88) = 176.

Time = 1.07 (sec) , antiderivative size = 254, normalized size of antiderivative = 2.89

$$\int \frac{\cot^4(c+dx)}{a+a\sec(c+dx)}\,dx = \frac{\csc\left(\frac{c}{2}\right)\csc^3(c+dx)\sec\left(\frac{c}{2}\right)\sec(c+dx)(360dx\cos(dx) - 360dx\cos(2c+dx) + 120dx\cos(c+2dx) - 120d}{\ldots}$$

input
```
Integrate[Cot[c + d*x]^4/(a + a*Sec[c + d*x]),x]
```

output

```
(Csc[c/2]*Csc[c + d*x]^3*Sec[c/2]*Sec[c + d*x]*(360*d*x*Cos[d*x] - 360*d*x
*Cos[2*c + d*x] + 120*d*x*Cos[c + 2*d*x] - 120*d*x*Cos[3*c + 2*d*x] - 120*
d*x*Cos[2*c + 3*d*x] + 120*d*x*Cos[4*c + 3*d*x] - 60*d*x*Cos[3*c + 4*d*x]
+ 60*d*x*Cos[5*c + 4*d*x] - 200*Sin[c] - 584*Sin[d*x] + 534*Sin[c + d*x] +
 178*Sin[2*(c + d*x)] - 178*Sin[3*(c + d*x)] - 89*Sin[4*(c + d*x)] - 520*S
in[2*c + d*x] - 248*Sin[c + 2*d*x] - 120*Sin[3*c + 2*d*x] + 248*Sin[2*c +
3*d*x] + 120*Sin[4*c + 3*d*x] + 184*Sin[3*c + 4*d*x]))/(1920*a*d*(1 + Sec[
c + d*x]))
```

3.69.3 Rubi [A] (verified)

Time $= 0.53$ (sec) , antiderivative size $= 98$, normalized size of antiderivative $= 1.11$, number of steps used $= 12$, number of rules used $= 12$, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used $= \{3042, 4376, 25, 3042, 4370, 25, 3042, 4370, 25, 3042, 4370, 24\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^4(c+dx)}{a\sec(c+dx)+a}\,dx$$

$$\downarrow 3042$$

$$\int \frac{1}{\cot\left(c+dx+\frac{\pi}{2}\right)^4\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)}dx$$

$$\downarrow 4376$$

$$\frac{\int -\cot^6(c+dx)(a-a\sec(c+dx))dx}{a^2}$$

$$\downarrow 25$$

$$-\frac{\int \cot^6(c+dx)(a-a\sec(c+dx))dx}{a^2}$$

$$\downarrow 3042$$

$$-\frac{\int \frac{a-a\csc\left(c+dx+\frac{\pi}{2}\right)}{\cot\left(c+dx+\frac{\pi}{2}\right)^6}dx}{a^2}$$

$$\downarrow 4370$$

$$-\frac{\frac{1}{5}\int -\cot^4(c+dx)(5a-4a\sec(c+dx))dx-\frac{\cot^5(c+dx)(a-a\sec(c+dx))}{5d}}{a^2}$$

$$\downarrow 25$$

$$-\frac{-\frac{1}{5}\int \cot^4(c+dx)(5a-4a\sec(c+dx))dx - \frac{\cot^5(c+dx)(a-a\sec(c+dx))}{5d}}{a^2}$$

$$\downarrow 3042$$

$$-\frac{-\frac{1}{5}\int \frac{5a-4a\csc\left(c+dx+\frac{\pi}{2}\right)}{\cot\left(c+dx+\frac{\pi}{2}\right)^4}dx - \frac{\cot^5(c+dx)(a-a\sec(c+dx))}{5d}}{a^2}$$

$$\downarrow 4370$$

$$-\frac{\frac{1}{5}\left(\frac{\cot^3(c+dx)(5a-4a\sec(c+dx))}{3d} - \frac{1}{3}\int -\cot^2(c+dx)(15a-8a\sec(c+dx))dx\right) - \frac{\cot^5(c+dx)(a-a\sec(c+dx))}{5d}}{a^2}$$

$$\downarrow 25$$

$$-\frac{\frac{1}{5}\left(\frac{1}{3}\int \cot^2(c+dx)(15a-8a\sec(c+dx))dx + \frac{\cot^3(c+dx)(5a-4a\sec(c+dx))}{3d}\right) - \frac{\cot^5(c+dx)(a-a\sec(c+dx))}{5d}}{a^2}$$

$$\downarrow 3042$$

$$-\frac{\frac{1}{5}\left(\frac{1}{3}\int \frac{15a-8a\csc\left(c+dx+\frac{\pi}{2}\right)}{\cot\left(c+dx+\frac{\pi}{2}\right)^2}dx + \frac{\cot^3(c+dx)(5a-4a\sec(c+dx))}{3d}\right) - \frac{\cot^5(c+dx)(a-a\sec(c+dx))}{5d}}{a^2}$$

$$\downarrow 4370$$

$$-\frac{\frac{1}{5}\left(\frac{1}{3}\left(\int -15a dx - \frac{\cot(c+dx)(15a-8a\sec(c+dx))}{d}\right) + \frac{\cot^3(c+dx)(5a-4a\sec(c+dx))}{3d}\right) - \frac{\cot^5(c+dx)(a-a\sec(c+dx))}{5d}}{a^2}$$

$$\downarrow 24$$

$$-\frac{\frac{1}{5}\left(\frac{\cot^3(c+dx)(5a-4a\sec(c+dx))}{3d} + \frac{1}{3}\left(-\frac{\cot(c+dx)(15a-8a\sec(c+dx))}{d} - 15ax\right)\right) - \frac{\cot^5(c+dx)(a-a\sec(c+dx))}{5d}}{a^2}$$

input
```
Int[Cot[c + d*x]^4/(a + a*Sec[c + d*x]),x]
```

output
```
-((-1/5*(Cot[c + d*x]^5*(a - a*Sec[c + d*x]))/d + ((Cot[c + d*x]^3*(5*a -
4*a*Sec[c + d*x]))/(3*d) + (-15*a*x - (Cot[c + d*x]*(15*a - 8*a*Sec[c + d*
x]))/d)/3)/5)/a^2)
```

3.69.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4370
```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(-(e*Cot[c + d*x])^(m + 1))*((a + b*Csc[c + d*x])/(
d*e*(m + 1))), x] - Simp[1/(e^2*(m + 1))   Int[(e*Cot[c + d*x])^(m + 2)*(a*
(m + 1) + b*(m + 2)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && L
tQ[m, -1]
```

rule 4376
```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Simp[a^(2*n)/e^(2*n)   Int[(e*Cot[c + d*x])^(m + 2*
n)/(-a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[a
^2 - b^2, 0] && ILtQ[n, 0]
```

3.69.4 Maple [A] (verified)

Time = 0.66 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.97

method	result	size
derivativedivides	$\frac{-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{5}+2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3-16\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+32\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\frac{1}{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}+\frac{6}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}}{16da}$	85
default	$\frac{-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{5}+2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3-16\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+32\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\frac{1}{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}+\frac{6}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}}{16da}$	85
risch	$\frac{x}{a}-\frac{2i\left(15\,e^{7i(dx+c)}-15\,e^{6i(dx+c)}-65\,e^{5i(dx+c)}-25\,e^{4i(dx+c)}+73\,e^{3i(dx+c)}+31\,e^{2i(dx+c)}-31\,e^{i(dx+c)}-23\right)}{15da\left(e^{i(dx+c)}+1\right)^5\left(e^{i(dx+c)}-1\right)^3}$	121

input
```
int(cot(d*x+c)^4/(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/16/d/a*(-1/5*tan(1/2*d*x+1/2*c)^5+2*tan(1/2*d*x+1/2*c)^3-16*tan(1/2*d*x+
1/2*c)+32*arctan(tan(1/2*d*x+1/2*c))-1/3/tan(1/2*d*x+1/2*c)^3+6/tan(1/2*d*
x+1/2*c))
```

3.69.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.52

$$\int \frac{\cot^4(c+dx)}{a+a\sec(c+dx)}\,dx$$
$$= \frac{23\cos(dx+c)^4 + 8\cos(dx+c)^3 - 27\cos(dx+c)^2 + 15\left(dx\cos(dx+c)^3 + dx\cos(dx+c)^2 - dx\cos(d\right.}{15\left(ad\cos(dx+c)^3 + ad\cos(dx+c)^2 - ad\cos(dx+c) - ad\right)\sin(}$$

input
```
integrate(cot(d*x+c)^4/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

output
```
1/15*(23*cos(d*x + c)^4 + 8*cos(d*x + c)^3 - 27*cos(d*x + c)^2 + 15*(d*x*c
os(d*x + c)^3 + d*x*cos(d*x + c)^2 - d*x*cos(d*x + c) - d*x)*sin(d*x + c)
- 7*cos(d*x + c) + 8)/((a*d*cos(d*x + c)^3 + a*d*cos(d*x + c)^2 - a*d*cos(
d*x + c) - a*d)*sin(d*x + c))
```

3.69.6 Sympy [F]

$$\int \frac{\cot^4(c+dx)}{a+a\sec(c+dx)}\,dx = \frac{\int \frac{\cot^4(c+dx)}{\sec(c+dx)+1}\,dx}{a}$$

input
```
integrate(cot(d*x+c)**4/(a+a*sec(d*x+c)),x)
```

output
```
Integral(cot(c + d*x)**4/(sec(c + d*x) + 1), x)/a
```

3.69.7 Maxima [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.56

$$\int \frac{\cot^4(c+dx)}{a+a\sec(c+dx)}\,dx = -\frac{\frac{3\left(\frac{80\sin(dx+c)}{\cos(dx+c)+1}-\frac{10\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{\sin(dx+c)^5}{(\cos(dx+c)+1)^5}\right)}{a}-\frac{480\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}-\frac{5\left(\frac{18\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-1\right)(\cos(dx+c)+1)^3}{a\sin(dx+c)^3}}{240\,d}$$

input
```
integrate(cot(d*x+c)^4/(a+a*sec(d*x+c)),x, algorithm="maxima")
```

output
```
-1/240*(3*(80*sin(d*x + c)/(cos(d*x + c) + 1) - 10*sin(d*x + c)^3/(cos(d*x
 + c) + 1)^3 + sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/a - 480*arctan(sin(d*x
 + c)/(cos(d*x + c) + 1))/a - 5*(18*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 -
1)*(cos(d*x + c) + 1)^3/(a*sin(d*x + c)^3))/d
```

3.69.8 Giac [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.11

$$\int \frac{\cot^4(c+dx)}{a+a\sec(c+dx)}\,dx = \frac{\frac{240\,(dx+c)}{a}+\frac{5\left(18\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)}{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3}-\frac{3\left(a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-10\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+80\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{a^5}}{240\,d}$$

input
```
integrate(cot(d*x+c)^4/(a+a*sec(d*x+c)),x, algorithm="giac")
```

output
```
1/240*(240*(d*x + c)/a + 5*(18*tan(1/2*d*x + 1/2*c)^2 - 1)/(a*tan(1/2*d*x
+ 1/2*c)^3) - 3*(a^4*tan(1/2*d*x + 1/2*c)^5 - 10*a^4*tan(1/2*d*x + 1/2*c)^
3 + 80*a^4*tan(1/2*d*x + 1/2*c))/a^5)/d
```

3.69.9 Mupad [B] (verification not implemented)

Time = 14.41 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.80

$$\int \frac{\cot^4(c+dx)}{a+a\sec(c+dx)}\,dx =$$
$$-\frac{5\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^8+3\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^8-30\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^2\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^6+240\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^4\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^4-}{240\,a\,d\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^5\sin\left(\frac{c}{2}+\frac{d}{2}\right.}$$

input
```
int(cot(c + d*x)^4/(a + a/cos(c + d*x)),x)
```

output
```
-(5*cos(c/2 + (d*x)/2)^8 + 3*sin(c/2 + (d*x)/2)^8 - 30*cos(c/2 + (d*x)/2)^
2*sin(c/2 + (d*x)/2)^6 + 240*cos(c/2 + (d*x)/2)^4*sin(c/2 + (d*x)/2)^4 - 9
0*cos(c/2 + (d*x)/2)^6*sin(c/2 + (d*x)/2)^2 - 240*cos(c/2 + (d*x)/2)^5*sin
(c/2 + (d*x)/2)^3*(c + d*x))/(240*a*d*cos(c/2 + (d*x)/2)^5*sin(c/2 + (d*x)
/2)^3)
```

3.70 $\int \frac{\cot^6(c+dx)}{a+a\sec(c+dx)}\,dx$

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3.70.1 Optimal result

Integrand size = 21, antiderivative size = 117

$$\int \frac{\cot^6(c+dx)}{a+a\sec(c+dx)}\,dx = -\frac{x}{a} + \frac{\cot^3(c+dx)(35-24\sec(c+dx))}{105ad} - \frac{\cot(c+dx)(35-16\sec(c+dx))}{35ad} - \frac{\cot^5(c+dx)(7-6\sec(c+dx))}{35ad} + \frac{\cot^7(c+dx)(1-\sec(c+dx))}{7ad}$$

output
```
-x/a+1/105*cot(d*x+c)^3*(35-24*sec(d*x+c))/a/d-1/35*cot(d*x+c)*(35-16*sec(
d*x+c))/a/d-1/35*cot(d*x+c)^5*(7-6*sec(d*x+c))/a/d+1/7*cot(d*x+c)^7*(1-sec
(d*x+c))/a/d
```

3.70.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 359 vs. 2(117) = 234.

Time = 1.63 (sec) , antiderivative size = 359, normalized size of antiderivative = 3.07

$$\int \frac{\cot^6(c+dx)}{a+a\sec(c+dx)}\,dx = \frac{\csc\left(\frac{c}{2}\right)\csc^5(c+dx)\sec\left(\frac{c}{2}\right)\sec(c+dx)(-16800dx\cos(dx)+16800dx\cos(2c+dx)-4200dx\cos(c+2dx$$

input
```
Integrate[Cot[c + d*x]^6/(a + a*Sec[c + d*x]),x]
```

output
```
(Csc[c/2]*Csc[c + d*x]^5*Sec[c/2]*Sec[c + d*x]*(-16800*d*x*Cos[d*x] + 1680
0*d*x*Cos[2*c + d*x] - 4200*d*x*Cos[c + 2*d*x] + 4200*d*x*Cos[3*c + 2*d*x]
 + 8400*d*x*Cos[2*c + 3*d*x] - 8400*d*x*Cos[4*c + 3*d*x] + 3360*d*x*Cos[3*
c + 4*d*x] - 3360*d*x*Cos[5*c + 4*d*x] - 1680*d*x*Cos[4*c + 5*d*x] + 1680*
d*x*Cos[6*c + 5*d*x] - 840*d*x*Cos[5*c + 6*d*x] + 840*d*x*Cos[7*c + 6*d*x]
 + 3136*Sin[c] + 30112*Sin[d*x] - 22860*Sin[c + d*x] - 5715*Sin[2*(c + d*x
)] + 11430*Sin[3*(c + d*x)] + 4572*Sin[4*(c + d*x)] - 2286*Sin[5*(c + d*x)
] - 1143*Sin[6*(c + d*x)] + 26208*Sin[2*c + d*x] + 14080*Sin[c + 2*d*x] -
16400*Sin[2*c + 3*d*x] - 11760*Sin[4*c + 3*d*x] - 7904*Sin[3*c + 4*d*x] -
3360*Sin[5*c + 4*d*x] + 3952*Sin[4*c + 5*d*x] + 1680*Sin[6*c + 5*d*x] + 28
16*Sin[5*c + 6*d*x]))/(107520*a*d*(1 + Sec[c + d*x]))
```

3.70.3 Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.07, number of steps used = 15, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.714$, Rules used = {3042, 4376, 25, 3042, 4370, 25, 3042, 4370, 25, 3042, 4370, 27, 3042, 4370, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^6(c+dx)}{a\sec(c+dx)+a}\,dx$$

$$\downarrow 3042$$

$$\int \frac{1}{\cot\left(c+dx+\frac{\pi}{2}\right)^6\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)}dx$$

$$\downarrow 4376$$

$$\frac{\int -\cot^8(c+dx)(a-a\sec(c+dx))dx}{a^2}$$

$$\downarrow 25$$

$$-\frac{\int \cot^8(c+dx)(a-a\sec(c+dx))dx}{a^2}$$

$$\downarrow 3042$$

$$-\frac{\int \frac{a-a\csc\left(c+dx+\frac{\pi}{2}\right)}{\cot\left(c+dx+\frac{\pi}{2}\right)^8}dx}{a^2}$$

$$\downarrow 4370$$

$$-\frac{\frac{1}{7}\int -\cot^6(c+dx)(7a-6a\sec(c+dx))dx - \frac{\cot^7(c+dx)(a-a\sec(c+dx))}{7d}}{a^2}$$

$$\downarrow 25$$

$$-\frac{-\frac{1}{7}\int \cot^6(c+dx)(7a-6a\sec(c+dx))dx - \frac{\cot^7(c+dx)(a-a\sec(c+dx))}{7d}}{a^2}$$

$$\downarrow 3042$$

$$-\frac{-\frac{1}{7}\int \frac{7a-6a\csc\left(c+dx+\frac{\pi}{2}\right)}{\cot\left(c+dx+\frac{\pi}{2}\right)^6}dx - \frac{\cot^7(c+dx)(a-a\sec(c+dx))}{7d}}{a^2}$$

$$\downarrow 4370$$

$$-\frac{\frac{1}{7}\left(\frac{\cot^5(c+dx)(7a-6a\sec(c+dx))}{5d} - \frac{1}{5}\int -\cot^4(c+dx)(35a-24a\sec(c+dx))dx\right) - \frac{\cot^7(c+dx)(a-a\sec(c+dx))}{7d}}{a^2}$$

$$\downarrow 25$$

$$-\frac{\frac{1}{7}\left(\frac{1}{5}\int \cot^4(c+dx)(35a-24a\sec(c+dx))dx + \frac{\cot^5(c+dx)(7a-6a\sec(c+dx))}{5d}\right) - \frac{\cot^7(c+dx)(a-a\sec(c+dx))}{7d}}{a^2}$$

$$\downarrow 3042$$

$$-\frac{\frac{1}{7}\left(\frac{1}{5}\int \frac{35a-24a\csc\left(c+dx+\frac{\pi}{2}\right)}{\cot\left(c+dx+\frac{\pi}{2}\right)^4}dx + \frac{\cot^5(c+dx)(7a-6a\sec(c+dx))}{5d}\right) - \frac{\cot^7(c+dx)(a-a\sec(c+dx))}{7d}}{a^2}$$

$$\downarrow 4370$$

$$-\frac{\frac{1}{7}\left(\frac{1}{5}\left(\frac{1}{3}\int -3\cot^2(c+dx)(35a-16a\sec(c+dx))dx - \frac{\cot^3(c+dx)(35a-24a\sec(c+dx))}{3d}\right) + \frac{\cot^5(c+dx)(7a-6a\sec(c+dx))}{5d}\right) -}{a^2}$$

$$\downarrow 27$$

$$-\frac{\frac{1}{7}\left(\frac{1}{5}\left(-\int \cot^2(c+dx)(35a-16a\sec(c+dx))dx - \frac{\cot^3(c+dx)(35a-24a\sec(c+dx))}{3d}\right) + \frac{\cot^5(c+dx)(7a-6a\sec(c+dx))}{5d}\right) -}{a^2}$$

$$\downarrow 3042$$

$$-\frac{\frac{1}{7}\left(\frac{1}{5}\left(-\int \frac{35a-16a\csc\left(c+dx+\frac{\pi}{2}\right)}{\cot\left(c+dx+\frac{\pi}{2}\right)^2}dx - \frac{\cot^3(c+dx)(35a-24a\sec(c+dx))}{3d}\right) + \frac{\cot^5(c+dx)(7a-6a\sec(c+dx))}{5d}\right) - \frac{\cot^7(c+dx)(a-a\sec(}{7d}}{a^2}$$

$$\downarrow 4370$$

$$-\frac{\frac{1}{7}\left(\frac{1}{5}\left(-\int -35a dx-\frac{\cot^3(c+dx)(35a-24a\sec(c+dx))}{3d}+\frac{\cot(c+dx)(35a-16a\sec(c+dx))}{d}\right)+\frac{\cot^5(c+dx)(7a-6a\sec(c+dx))}{5d}\right)-\frac{\cot^7}{}}{a^2}$$

$$\downarrow 24$$

$$-\frac{\frac{1}{7}\left(\frac{\cot^5(c+dx)(7a-6a\sec(c+dx))}{5d}+\frac{1}{5}\left(-\frac{\cot^3(c+dx)(35a-24a\sec(c+dx))}{3d}+\frac{\cot(c+dx)(35a-16a\sec(c+dx))}{d}+35ax\right)\right)-\frac{\cot^7(c+dx)}{}}{a^2}$$

input
```
Int[Cot[c + d*x]^6/(a + a*Sec[c + d*x]),x]
```

output
```
-((-1/7*(Cot[c + d*x]^7*(a - a*Sec[c + d*x]))/d + ((Cot[c + d*x]^5*(7*a -
6*a*Sec[c + d*x]))/(5*d) + (35*a*x - (Cot[c + d*x]^3*(35*a - 24*a*Sec[c +
d*x]))/(3*d) + (Cot[c + d*x]*(35*a - 16*a*Sec[c + d*x]))/d)/5)/7)/a^2)
```

3.70.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4370
```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(-(e*Cot[c + d*x])^(m + 1))*((a + b*Csc[c + d*x])/(
d*e*(m + 1))), x] - Simp[1/(e^2*(m + 1))   Int[(e*Cot[c + d*x])^(m + 2)*(a*
(m + 1) + b*(m + 2)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && L
tQ[m, -1]
```

rule 4376

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Simp[a^(2*n)/e^(2*n)   Int[(e*Cot[c + d*x])^(m + 2*
n)/(-a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[a
^2 - b^2, 0] && ILtQ[n, 0]
```

3.70.4 Maple [A] (verified)

Time = 0.73 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.95

method	result
derivativedivides	$\frac{-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{7}+\frac{8\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{5}-\frac{29\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{3}+64\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{1}{5\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}+\frac{8}{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}-\frac{29}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}-128a}{64da}$
default	$\frac{-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{7}+\frac{8\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{5}-\frac{29\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{3}+64\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{1}{5\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}+\frac{8}{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}-\frac{29}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}-128a}{64da}$
risch	$-\frac{x}{a}+\frac{2i\left(105\,e^{11i(dx+c)}-210\,e^{10i(dx+c)}-735\,e^{9i(dx+c)}+1638\,e^{7i(dx+c)}+196\,e^{6i(dx+c)}-1882\,e^{5i(dx+c)}-880\,e^{4i(dx+c)}\right.}{105da\left(e^{i(dx+c)}+1\right)^7\left(e^{i(dx+c)}-1\right)^5}$

input

```
int(cot(d*x+c)^6/(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

output

```
1/64/d/a*(-1/7*tan(1/2*d*x+1/2*c)^7+8/5*tan(1/2*d*x+1/2*c)^5-29/3*tan(1/2*
d*x+1/2*c)^3+64*tan(1/2*d*x+1/2*c)-1/5/tan(1/2*d*x+1/2*c)^5+8/3/tan(1/2*d*
x+1/2*c)^3-29/tan(1/2*d*x+1/2*c)-128*arctan(tan(1/2*d*x+1/2*c)))
```

3.70.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.69

$$\int \frac{\cot^6(c+dx)}{a+a\sec(c+dx)}\,dx =$$
$$-\frac{176\cos(dx+c)^6+71\cos(dx+c)^5-335\cos(dx+c)^4-125\cos(dx+c)^3+225\cos(dx+c)^2+105}{105\left(ad\cos(dx+c)^5+ad\cos(dx+c)^4-\right.}$$

input

```
integrate(cot(d*x+c)^6/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

output
```
-1/105*(176*cos(d*x + c)^6 + 71*cos(d*x + c)^5 - 335*cos(d*x + c)^4 - 125*
cos(d*x + c)^3 + 225*cos(d*x + c)^2 + 105*(d*x*cos(d*x + c)^5 + d*x*cos(d*
x + c)^4 - 2*d*x*cos(d*x + c)^3 - 2*d*x*cos(d*x + c)^2 + d*x*cos(d*x + c)
+ d*x)*sin(d*x + c) + 57*cos(d*x + c) - 48)/((a*d*cos(d*x + c)^5 + a*d*cos
(d*x + c)^4 - 2*a*d*cos(d*x + c)^3 - 2*a*d*cos(d*x + c)^2 + a*d*cos(d*x +
c) + a*d)*sin(d*x + c))
```

3.70.6 Sympy [F]

$$\int \frac{\cot^6(c+dx)}{a+a\sec(c+dx)}\,dx = \frac{\int \frac{\cot^6(c+dx)}{\sec(c+dx)+1}\,dx}{a}$$

input
```
integrate(cot(d*x+c)**6/(a+a*sec(d*x+c)),x)
```

output
```
Integral(cot(c + d*x)**6/(sec(c + d*x) + 1), x)/a
```

3.70.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.51

$$\int \frac{\cot^6(c+dx)}{a+a\sec(c+dx)}\,dx$$
$$= \frac{\frac{\frac{6720\sin(dx+c)}{\cos(dx+c)+1} - \frac{1015\sin(dx+c)^3}{(\cos(dx+c)+1)^3} + \frac{168\sin(dx+c)^5}{(\cos(dx+c)+1)^5} - \frac{15\sin(dx+c)^7}{(\cos(dx+c)+1)^7}}{a} - \frac{13440\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a} + \frac{7\left(\frac{40\sin(dx+c)^2}{(\cos(dx+c)+1)^2} - \frac{435\sin(dx+c)^4}{(\cos(dx+c)+1)^4} - 3\right)(c}{a\sin(dx+c)^5}}{6720\,d}$$

input
```
integrate(cot(d*x+c)^6/(a+a*sec(d*x+c)),x, algorithm="maxima")
```

output
```
1/6720*((6720*sin(d*x + c)/(cos(d*x + c) + 1) - 1015*sin(d*x + c)^3/(cos(d
*x + c) + 1)^3 + 168*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 15*sin(d*x + c)
^7/(cos(d*x + c) + 1)^7)/a - 13440*arctan(sin(d*x + c)/(cos(d*x + c) + 1))
/a + 7*(40*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 435*sin(d*x + c)^4/(cos(d
*x + c) + 1)^4 - 3)*(cos(d*x + c) + 1)^5/(a*sin(d*x + c)^5))/d
```

3.70.8 Giac [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.09

$$\int \frac{\cot^6(c+dx)}{a+a\sec(c+dx)}\,dx =$$
$$-\frac{\frac{6720\,(dx+c)}{a}+\frac{7\left(435\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-40\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+3\right)}{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5}+\frac{15\,a^6\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-168\,a^6\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+1015\,a^6\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right.}{a^7}}{6720\,d}$$

input
```
integrate(cot(d*x+c)^6/(a+a*sec(d*x+c)),x, algorithm="giac")
```

output
```
-1/6720*(6720*(d*x + c)/a + 7*(435*tan(1/2*d*x + 1/2*c)^4 - 40*tan(1/2*d*x
 + 1/2*c)^2 + 3)/(a*tan(1/2*d*x + 1/2*c)^5) + (15*a^6*tan(1/2*d*x + 1/2*c)
^7 - 168*a^6*tan(1/2*d*x + 1/2*c)^5 + 1015*a^6*tan(1/2*d*x + 1/2*c)^3 - 67
20*a^6*tan(1/2*d*x + 1/2*c))/a^7)/d
```

3.70.9 Mupad [B] (verification not implemented)

Time = 14.90 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.76

$$\int \frac{\cot^6(c+dx)}{a+a\sec(c+dx)}\,dx =$$
$$-\frac{21\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{12}+15\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{12}-168\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^2\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}+1015\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^4\sin\left(\frac{c}{2}+\right.}{}$$

input
```
int(cot(c + d*x)^6/(a + a/cos(c + d*x)),x)
```

output
```
-(21*cos(c/2 + (d*x)/2)^12 + 15*sin(c/2 + (d*x)/2)^12 - 168*cos(c/2 + (d*x
)/2)^2*sin(c/2 + (d*x)/2)^10 + 1015*cos(c/2 + (d*x)/2)^4*sin(c/2 + (d*x)/2
)^8 - 6720*cos(c/2 + (d*x)/2)^6*sin(c/2 + (d*x)/2)^6 + 3045*cos(c/2 + (d*x
)/2)^8*sin(c/2 + (d*x)/2)^4 - 280*cos(c/2 + (d*x)/2)^10*sin(c/2 + (d*x)/2)
^2 + 6720*cos(c/2 + (d*x)/2)^7*sin(c/2 + (d*x)/2)^5*(c + d*x))/(6720*a*d*c
os(c/2 + (d*x)/2)^7*sin(c/2 + (d*x)/2)^5)
```

3.71 $\int \frac{\tan^9(c+dx)}{(a+a\sec(c+dx))^2}\,dx$

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3.71.1 Optimal result

Integrand size = 21, antiderivative size = 120

$$\int \frac{\tan^9(c+dx)}{(a+a\sec(c+dx))^2}\,dx = -\frac{\log(\cos(c+dx))}{a^2d} - \frac{2\sec(c+dx)}{a^2d} - \frac{\sec^2(c+dx)}{2a^2d} + \frac{4\sec^3(c+dx)}{3a^2d} - \frac{\sec^4(c+dx)}{4a^2d} - \frac{2\sec^5(c+dx)}{5a^2d} + \frac{\sec^6(c+dx)}{6a^2d}$$

output

```
-ln(cos(d*x+c))/a^2/d-2*sec(d*x+c)/a^2/d-1/2*sec(d*x+c)^2/a^2/d+4/3*sec(d*
x+c)^3/a^2/d-1/4*sec(d*x+c)^4/a^2/d-2/5*sec(d*x+c)^5/a^2/d+1/6*sec(d*x+c)^
6/a^2/d
```

3.71.2 Mathematica [A] (verified)

Time = 0.38 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.04

$$\int \frac{\tan^9(c+dx)}{(a+a\sec(c+dx))^2}\,dx = -\frac{(312\cos(c+dx) + 5(14 + 28\cos(3(c+dx)) + 6\cos(4(c+dx)) + 12\cos(5(c+dx)) + 30\log(\cos(c+dx$$

input

```
Integrate[Tan[c + d*x]^9/(a + a*Sec[c + d*x])^2,x]
```

output
```
-1/480*((312*Cos[c + d*x] + 5*(14 + 28*Cos[3*(c + d*x)] + 6*Cos[4*(c + d*x
)] + 12*Cos[5*(c + d*x)] + 30*Log[Cos[c + d*x]] + 18*Cos[4*(c + d*x)]*Log[
Cos[c + d*x]] + 3*Cos[6*(c + d*x)]*Log[Cos[c + d*x]] + 9*Cos[2*(c + d*x)]*
(4 + 5*Log[Cos[c + d*x]])))*Sec[c + d*x]^6)/(a^2*d)
```

3.71.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.70, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 25, 4367, 27, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^9(c+dx)}{(a\sec(c+dx)+a)^2}\,dx$$

$$\downarrow 3042$$

$$\int -\frac{\cot\left(c+dx+\frac{\pi}{2}\right)^9}{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^2}dx$$

$$\downarrow 25$$

$$-\int \frac{\cot\left(\frac{1}{2}(2c+\pi)+dx\right)^9}{\left(\csc\left(\frac{1}{2}(2c+\pi)+dx\right)a+a\right)^2}dx$$

$$\downarrow 4367$$

$$-\frac{\int a^6(1-\cos(c+dx))^4(\cos(c+dx)+1)^2\sec^7(c+dx)d\cos(c+dx)}{a^8 d}$$

$$\downarrow 27$$

$$-\frac{\int (1-\cos(c+dx))^4(\cos(c+dx)+1)^2\sec^7(c+dx)d\cos(c+dx)}{a^2 d}$$

$$\downarrow 99$$

$$-\frac{\int \left(\sec^7(c+dx)-2\sec^6(c+dx)-\sec^5(c+dx)+4\sec^4(c+dx)-\sec^3(c+dx)-2\sec^2(c+dx)+\sec(c+dx)\right)}{a^2 d}$$

$$\downarrow 2009$$

$$-\frac{-\frac{1}{6}\sec^6(c+dx)+\frac{2}{5}\sec^5(c+dx)+\frac{1}{4}\sec^4(c+dx)-\frac{4}{3}\sec^3(c+dx)+\frac{1}{2}\sec^2(c+dx)+2\sec(c+dx)+\log(\cos(c}{a^2 d}$$

input
```
Int[Tan[c + d*x]^9/(a + a*Sec[c + d*x])^2,x]
```

output
```
-((Log[Cos[c + d*x]] + 2*Sec[c + d*x] + Sec[c + d*x]^2/2 - (4*Sec[c + d*x]
^3)/3 + Sec[c + d*x]^4/4 + (2*Sec[c + d*x]^5)/5 - Sec[c + d*x]^6/6)/(a^2*d
))
```

3.71.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4367
```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Simp[1/(a^(m - n - 1)*b^n*d)   Subst[Int[(a - b*x)^((m -
1)/2)*((a + b*x)^((m - 1)/2 + n)/x^(m + n)), x], x, Sin[c + d*x]], x] /; Fr
eeQ[{a, b, c, d}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && Integer
Q[n]
```

3.71.4 Maple [A] (verified)

Time = 1.68 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.65

method	result
derivativedivides	$\frac{-\ln(\cos(dx+c))+\frac{4}{3\cos(dx+c)^3}-\frac{1}{2\cos(dx+c)^2}-\frac{2}{5\cos(dx+c)^5}-\frac{2}{\cos(dx+c)}+\frac{1}{6\cos(dx+c)^6}-\frac{1}{4\cos(dx+c)^4}}{d\,a^2}$
default	$\frac{-\ln(\cos(dx+c))+\frac{4}{3\cos(dx+c)^3}-\frac{1}{2\cos(dx+c)^2}-\frac{2}{5\cos(dx+c)^5}-\frac{2}{\cos(dx+c)}+\frac{1}{6\cos(dx+c)^6}-\frac{1}{4\cos(dx+c)^4}}{d\,a^2}$
risch	$\frac{ix}{a^2}+\frac{2ic}{a^2d}-\frac{2\left(30\,e^{11i(dx+c)}+15\,e^{10i(dx+c)}+70\,e^{9i(dx+c)}+90\,e^{8i(dx+c)}+156\,e^{7i(dx+c)}+70\,e^{6i(dx+c)}+156\,e^{5i(dx+c)}+\right.}{15d\,a^2\left(e^{2i(dx+c)}+1\right)^6}$

input
```
int(tan(d*x+c)^9/(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/d/a^2*(-ln(cos(d*x+c))+4/3/cos(d*x+c)^3-1/2/cos(d*x+c)^2-2/5/cos(d*x+c)^
5-2/cos(d*x+c)+1/6/cos(d*x+c)^6-1/4/cos(d*x+c)^4)
```

3.71.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.71

$$\int \frac{\tan^9(c+dx)}{(a+a\sec(c+dx))^2}\,dx =$$
$$-\frac{60\cos(dx+c)^6\log(-\cos(dx+c))+120\cos(dx+c)^5+30\cos(dx+c)^4-80\cos(dx+c)^3+15\cos}{60\,a^2d\cos(dx+c)^6}$$

input
```
integrate(tan(d*x+c)^9/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

output
```
-1/60*(60*cos(d*x + c)^6*log(-cos(d*x + c)) + 120*cos(d*x + c)^5 + 30*cos(
d*x + c)^4 - 80*cos(d*x + c)^3 + 15*cos(d*x + c)^2 + 24*cos(d*x + c) - 10)
/(a^2*d*cos(d*x + c)^6)
```

3.71.6 Sympy [F]

$$\int \frac{\tan^9(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \frac{\int \frac{\tan^9(c+dx)}{\sec^2(c+dx)+2\sec(c+dx)+1}\,dx}{a^2}$$

input
```
integrate(tan(d*x+c)**9/(a+a*sec(d*x+c))**2,x)
```

output
```
Integral(tan(c + d*x)**9/(sec(c + d*x)**2 + 2*sec(c + d*x) + 1), x)/a**2
```

3.71.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.67

$$\int \frac{\tan^9(c+dx)}{(a+a\sec(c+dx))^2}\,dx = -\frac{\frac{60\log(\cos(dx+c))}{a^2} + \frac{120\cos(dx+c)^5+30\cos(dx+c)^4-80\cos(dx+c)^3+15\cos(dx+c)^2+24\cos(dx+c)-10}{a^2\cos(dx+c)^6}}{60\,d}$$

input
```
integrate(tan(d*x+c)^9/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

output
```
-1/60*(60*log(cos(d*x + c))/a^2 + (120*cos(d*x + c)^5 + 30*cos(d*x + c)^4
- 80*cos(d*x + c)^3 + 15*cos(d*x + c)^2 + 24*cos(d*x + c) - 10)/(a^2*cos(d
*x + c)^6))/d
```

3.71.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 223 vs. 2(110) = 220.

Time = 5.81 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.86

$$\int \frac{\tan^9(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \frac{\frac{60\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right)}{a^2} - \frac{60\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right)}{a^2} + \frac{\frac{234(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{1005(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}+\frac{2220(\cos(dx+c)-1)^3}{(\cos(dx+c)+1)^3}+\frac{2925(\cos(dx+c)-1)}{(\cos(dx+c)+1)^4}}{a^2\left(\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right)^6}}{60\,d}$$

input
```
integrate(tan(d*x+c)^9/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

output
```
1/60*(60*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1))/a^2 - 60*log
(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1))/a^2 + (234*(cos(d*x + c)
 - 1)/(cos(d*x + c) + 1) + 1005*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2
+ 2220*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 + 2925*(cos(d*x + c) - 1)
^4/(cos(d*x + c) + 1)^4 + 1002*(cos(d*x + c) - 1)^5/(cos(d*x + c) + 1)^5 +
 147*(cos(d*x + c) - 1)^6/(cos(d*x + c) + 1)^6 + 19)/(a^2*((cos(d*x + c) -
 1)/(cos(d*x + c) + 1) + 1)^6))/d
```

3.71.9 Mupad [B] (verification not implemented)

Time = 17.77 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.61

$$\int \frac{\tan^9(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \frac{2\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2\right)}{a^2\,d}$$
$$+\frac{-2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}+12\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^8+12\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6-20\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4+\frac{54\tan}{}}{d\left(a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{12}-6\,a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}+15\,a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^8-20\,a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6+15\,a^2\tan\left(\frac{c}{2}-\right.\right.}$$

input
```
int(tan(c + d*x)^9/(a + a/cos(c + d*x))^2,x)
```

output
```
(2*atanh(tan(c/2 + (d*x)/2)^2))/(a^2*d) + ((54*tan(c/2 + (d*x)/2)^2)/5 - 2
0*tan(c/2 + (d*x)/2)^4 + 12*tan(c/2 + (d*x)/2)^6 + 12*tan(c/2 + (d*x)/2)^8
 - 2*tan(c/2 + (d*x)/2)^10 - 32/15)/(d*(15*a^2*tan(c/2 + (d*x)/2)^4 - 6*a^
2*tan(c/2 + (d*x)/2)^2 - 20*a^2*tan(c/2 + (d*x)/2)^6 + 15*a^2*tan(c/2 + (d
*x)/2)^8 - 6*a^2*tan(c/2 + (d*x)/2)^10 + a^2*tan(c/2 + (d*x)/2)^12 + a^2))
```

3.72 $\int \frac{\tan^7(c+dx)}{(a+a\sec(c+dx))^2}\,dx$

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3.72.1 Optimal result

Integrand size = 21, antiderivative size = 65

$$\int \frac{\tan^7(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \frac{\log(\cos(c+dx))}{a^2 d} + \frac{2\sec(c+dx)}{a^2 d} - \frac{2\sec^3(c+dx)}{3a^2 d} + \frac{\sec^4(c+dx)}{4a^2 d}$$

output
```
ln(cos(d*x+c))/a^2/d+2*sec(d*x+c)/a^2/d-2/3*sec(d*x+c)^3/a^2/d+1/4*sec(d*x
+c)^4/a^2/d
```

3.72.2 Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.28

$$\int \frac{\tan^7(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \frac{(20\cos(c+dx) + 3(2 + 4\cos(3(c+dx)) + 3\log(\cos(c+dx)) + 4\cos(2(c+dx))\log(\cos(c+dx)) + \cos(4$$

input
```
Integrate[Tan[c + d*x]^7/(a + a*Sec[c + d*x])^2,x]
```

output
```
((20*Cos[c + d*x] + 3*(2 + 4*Cos[3*(c + d*x)] + 3*Log[Cos[c + d*x]] + 4*Co
s[2*(c + d*x)]*Log[Cos[c + d*x]] + Cos[4*(c + d*x)]*Log[Cos[c + d*x]]))*Se
c[c + d*x]^4)/(24*a^2*d)
```

3.72.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.77, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 25, 4367, 27, 84, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^7(c+dx)}{(a\sec(c+dx)+a)^2}\,dx$$

$$\downarrow 3042$$

$$\int -\frac{\cot\left(c+dx+\frac{\pi}{2}\right)^7}{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^2}dx$$

$$\downarrow 25$$

$$-\int \frac{\cot\left(\frac{1}{2}(2c+\pi)+dx\right)^7}{\left(\csc\left(\frac{1}{2}(2c+\pi)+dx\right)a+a\right)^2}dx$$

$$\downarrow 4367$$

$$-\frac{\int a^4(1-\cos(c+dx))^3(\cos(c+dx)+1)\sec^5(c+dx)d\cos(c+dx)}{a^6 d}$$

$$\downarrow 27$$

$$-\frac{\int (1-\cos(c+dx))^3(\cos(c+dx)+1)\sec^5(c+dx)d\cos(c+dx)}{a^2 d}$$

$$\downarrow 84$$

$$-\frac{\int \left(\sec^5(c+dx)-2\sec^4(c+dx)+2\sec^2(c+dx)-\sec(c+dx)\right)d\cos(c+dx)}{a^2 d}$$

$$\downarrow 2009$$

$$-\frac{-\frac{1}{4}\sec^4(c+dx)+\frac{2}{3}\sec^3(c+dx)-2\sec(c+dx)-\log(\cos(c+dx))}{a^2 d}$$

input
```
Int[Tan[c + d*x]^7/(a + a*Sec[c + d*x])^2,x]
```

output
```
-((-Log[Cos[c + d*x]] - 2*Sec[c + d*x] + (2*Sec[c + d*x]^3)/3 - Sec[c + d*
x]^4/4)/(a^2*d))
```

3.72.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 84
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && EqQ[b*e + a*f, 0] &&  !(ILtQ[n + p + 2, 0
] && GtQ[n + 2*p, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4367
```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Simp[1/(a^(m - n - 1)*b^n*d)   Subst[Int[(a - b*x)^((m -
1)/2)*((a + b*x)^((m - 1)/2 + n)/x^(m + n)), x], x, Sin[c + d*x]], x] /; Fr
eeQ[{a, b, c, d}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && Integer
Q[n]
```

3.72.4 Maple [A] (verified)

Time = 1.33 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.71

method	result	size
derivativedivides	$\frac{\frac{2}{\cos(dx+c)}+\frac{1}{4\cos(dx+c)^4}-\frac{2}{3\cos(dx+c)^3}+\ln(\cos(dx+c))}{d\,a^2}$	46
default	$\frac{\frac{2}{\cos(dx+c)}+\frac{1}{4\cos(dx+c)^4}-\frac{2}{3\cos(dx+c)^3}+\ln(\cos(dx+c))}{d\,a^2}$	46
risch	$-\frac{ix}{a^2}-\frac{2ic}{a^2d}+\frac{4\,e^{7i(dx+c)}+\frac{20\,e^{5i(dx+c)}}{3}+4\,e^{4i(dx+c)}+\frac{20\,e^{3i(dx+c)}}{3}+4\,e^{i(dx+c)}}{d\,a^2\left(e^{2i(dx+c)}+1\right)^4}+\frac{\ln\left(e^{2i(dx+c)}+1\right)}{a^2d}$	115

input
```
int(tan(d*x+c)^7/(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/d/a^2*(2/cos(d*x+c)+1/4/cos(d*x+c)^4-2/3/cos(d*x+c)^3+ln(cos(d*x+c)))
```

3.72.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.85

$$\int \frac{\tan^7(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \frac{12\cos(dx+c)^4\log(-\cos(dx+c)) + 24\cos(dx+c)^3 - 8\cos(dx+c) + 3}{12\,a^2 d\cos(dx+c)^4}$$

input
```
integrate(tan(d*x+c)^7/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

output
```
1/12*(12*cos(d*x + c)^4*log(-cos(d*x + c)) + 24*cos(d*x + c)^3 - 8*cos(d*x
 + c) + 3)/(a^2*d*cos(d*x + c)^4)
```

3.72.6 Sympy [F]

$$\int \frac{\tan^7(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \frac{\int \frac{\tan^7(c+dx)}{\sec^2(c+dx)+2\sec(c+dx)+1}\,dx}{a^2}$$

input
```
integrate(tan(d*x+c)**7/(a+a*sec(d*x+c))**2,x)
```

output
```
Integral(tan(c + d*x)**7/(sec(c + d*x)**2 + 2*sec(c + d*x) + 1), x)/a**2
```

3.72.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.77

$$\int \frac{\tan^7(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \frac{\frac{12\log(\cos(dx+c))}{a^2} + \frac{24\cos(dx+c)^3 - 8\cos(dx+c) + 3}{a^2\cos(dx+c)^4}}{12\,d}$$

input
```
integrate(tan(d*x+c)^7/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

output
```
1/12*(12*log(cos(d*x + c))/a^2 + (24*cos(d*x + c)^3 - 8*cos(d*x + c) + 3)/
(a^2*cos(d*x + c)^4))/d
```

3.72.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 180 vs. $2(61) = 122$.

Time = 3.47 (sec) , antiderivative size = 180, normalized size of antiderivative = 2.77

$$\int \frac{\tan^7(c+dx)}{(a+a\sec(c+dx))^2}\,dx = -\frac{\frac{12\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right)}{a^2} - \frac{12\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right)}{a^2} - \frac{\frac{4(\cos(dx+c)-1)}{\cos(dx+c)+1} - \frac{54(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2} - \frac{124(\cos(dx+c)-1)^3}{(\cos(dx+c)+1)^3} - \frac{25(\cos(dx+c)-1)^4}{(\cos(dx+c)+1)^4} + }{a^2\left(\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right)^4}}{12\,d}$$

input
```
integrate(tan(d*x+c)^7/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

output
```
-1/12*(12*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1))/a^2 - 12*lo
g(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1))/a^2 - (4*(cos(d*x + c)
- 1)/(cos(d*x + c) + 1) - 54*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 1
24*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 - 25*(cos(d*x + c) - 1)^4/(co
s(d*x + c) + 1)^4 + 7)/(a^2*((cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)^4)
)/d
```

3.72.9 Mupad [B] (verification not implemented)

Time = 15.77 (sec) , antiderivative size = 135, normalized size of antiderivative = 2.08

$$\int \frac{\tan^7(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \frac{2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6 + 8\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4 - \frac{26\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2}{3} + \frac{8}{3}}{d\left(a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^8 - 4a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6 + 6a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4 - 4a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2 + a^2\right)} - \frac{2\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2\right)}{a^2\,d}$$

input
```
int(tan(c + d*x)^7/(a + a/cos(c + d*x))^2,x)
```

output
```
(8*tan(c/2 + (d*x)/2)^4 - (26*tan(c/2 + (d*x)/2)^2)/3 + 2*tan(c/2 + (d*x)/
2)^6 + 8/3)/(d*(6*a^2*tan(c/2 + (d*x)/2)^4 - 4*a^2*tan(c/2 + (d*x)/2)^2 -
4*a^2*tan(c/2 + (d*x)/2)^6 + a^2*tan(c/2 + (d*x)/2)^8 + a^2)) - (2*atanh(t
an(c/2 + (d*x)/2)^2))/(a^2*d)
```

3.73 $\int \frac{\tan^5(c+dx)}{(a+a\sec(c+dx))^2}\,dx$

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3.73.1 Optimal result

Integrand size = 21, antiderivative size = 48

$$\int \frac{\tan^5(c+dx)}{(a+a\sec(c+dx))^2}\,dx = -\frac{\log(\cos(c+dx))}{a^2 d} - \frac{2\sec(c+dx)}{a^2 d} + \frac{\sec^2(c+dx)}{2a^2 d}$$

output
```
-ln(cos(d*x+c))/a^2/d-2*sec(d*x+c)/a^2/d+1/2*sec(d*x+c)^2/a^2/d
```

3.73.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.06

$$\int \frac{\tan^5(c+dx)}{(a+a\sec(c+dx))^2}\,dx = -\frac{(-1+4\cos(c+dx)+\log(\cos(c+dx))+\cos(2(c+dx))\log(\cos(c+dx)))\sec^2(c+dx)}{2a^2 d}$$

input
```
Integrate[Tan[c + d*x]^5/(a + a*Sec[c + d*x])^2,x]
```

output
```
-1/2*((-1 + 4*Cos[c + d*x] + Log[Cos[c + d*x]] + Cos[2*(c + d*x)]*Log[Cos[
c + d*x]])*Sec[c + d*x]^2)/(a^2*d)
```

3.73.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.75, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 25, 4367, 27, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^5(c+dx)}{(a\sec(c+dx)+a)^2}\,dx$$

$$\downarrow 3042$$

$$\int -\frac{\cot\left(c+dx+\frac{\pi}{2}\right)^5}{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^2}dx$$

$$\downarrow 25$$

$$-\int \frac{\cot\left(\frac{1}{2}(2c+\pi)+dx\right)^5}{\left(\csc\left(\frac{1}{2}(2c+\pi)+dx\right)a+a\right)^2}dx$$

$$\downarrow 4367$$

$$-\frac{\int a^2(1-\cos(c+dx))^2\sec^3(c+dx)d\cos(c+dx)}{a^4 d}$$

$$\downarrow 27$$

$$-\frac{\int (1-\cos(c+dx))^2\sec^3(c+dx)d\cos(c+dx)}{a^2 d}$$

$$\downarrow 49$$

$$-\frac{\int \left(\sec^3(c+dx)-2\sec^2(c+dx)+\sec(c+dx)\right)d\cos(c+dx)}{a^2 d}$$

$$\downarrow 2009$$

$$-\frac{-\frac{1}{2}\sec^2(c+dx)+2\sec(c+dx)+\log(\cos(c+dx))}{a^2 d}$$

input
```
Int[Tan[c + d*x]^5/(a + a*Sec[c + d*x])^2,x]
```

output
```
-((Log[Cos[c + d*x]] + 2*Sec[c + d*x] - Sec[c + d*x]^2/2)/(a^2*d))
```

3.73.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4367
```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Simp[1/(a^(m - n - 1)*b^n*d)   Subst[Int[(a - b*x)^((m -
1)/2)*((a + b*x)^((m - 1)/2 + n)/x^(m + n)), x], x, Sin[c + d*x]], x] /; Fr
eeQ[{a, b, c, d}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && Integer
Q[n]
```

3.73.4 Maple [A] (verified)

Time = 0.85 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.79

method	result	size
derivativedivides	$\frac{\frac{1}{2\cos(dx+c)^2}-\ln(\cos(dx+c))-\frac{2}{\cos(dx+c)}}{d\,a^2}$	38
default	$\frac{\frac{1}{2\cos(dx+c)^2}-\ln(\cos(dx+c))-\frac{2}{\cos(dx+c)}}{d\,a^2}$	38
risch	$\frac{ix}{a^2}+\frac{2ic}{a^2d}-\frac{2\left(2\,e^{3i(dx+c)}-e^{2i(dx+c)}+2\,e^{i(dx+c)}\right)}{d\,a^2\left(e^{2i(dx+c)}+1\right)^2}-\frac{\ln\left(e^{2i(dx+c)}+1\right)}{a^2d}$	94

input
```
int(tan(d*x+c)^5/(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/d/a^2*(1/2/cos(d*x+c)^2-ln(cos(d*x+c))-2/cos(d*x+c))
```

3.73.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.94

$$\int \frac{\tan^5(c+dx)}{(a+a\sec(c+dx))^2}\,dx = -\frac{2\cos(dx+c)^2\log(-\cos(dx+c)) + 4\cos(dx+c) - 1}{2a^2d\cos(dx+c)^2}$$

input
```
integrate(tan(d*x+c)^5/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

output
```
-1/2*(2*cos(d*x + c)^2*log(-cos(d*x + c)) + 4*cos(d*x + c) - 1)/(a^2*d*cos
(d*x + c)^2)
```

3.73.6 Sympy [F]

$$\int \frac{\tan^5(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \frac{\int \frac{\tan^5(c+dx)}{\sec^2(c+dx)+2\sec(c+dx)+1}\,dx}{a^2}$$

input
```
integrate(tan(d*x+c)**5/(a+a*sec(d*x+c))**2,x)
```

output
```
Integral(tan(c + d*x)**5/(sec(c + d*x)**2 + 2*sec(c + d*x) + 1), x)/a**2
```

3.73.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.83

$$\int \frac{\tan^5(c+dx)}{(a+a\sec(c+dx))^2}\,dx = -\frac{\frac{2\log(\cos(dx+c))}{a^2} + \frac{4\cos(dx+c)-1}{a^2\cos(dx+c)^2}}{2d}$$

input
```
integrate(tan(d*x+c)^5/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

output
```
-1/2*(2*log(cos(d*x + c))/a^2 + (4*cos(d*x + c) - 1)/(a^2*cos(d*x + c)^2))
/d
```

3.73.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 136 vs. $2(46) = 92$.

Time = 1.58 (sec) , antiderivative size = 136, normalized size of antiderivative = 2.83

$$\int \frac{\tan^5(c+dx)}{(a+a\sec(c+dx))^2}\,dx$$
$$= \frac{\frac{2\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right)}{a^2} - \frac{2\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right)}{a^2} - \frac{\frac{6(\cos(dx+c)-1)}{\cos(dx+c)+1} - \frac{3(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}+5}{a^2\left(\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right)^2}}{2\,d}$$

input
```
integrate(tan(d*x+c)^5/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

output
```
1/2*(2*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1))/a^2 - 2*log(ab
s(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1))/a^2 - (6*(cos(d*x + c) - 1)
/(cos(d*x + c) + 1) - 3*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 5)/(a^
2*((cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)^2))/d
```

3.73.9 Mupad [B] (verification not implemented)

Time = 14.19 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.60

$$\int \frac{\tan^5(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \frac{6\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2-4}{d\left(a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4 - 2a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2 + a^2\right)} + \frac{2\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2\right)}{a^2\,d}$$

input
```
int(tan(c + d*x)^5/(a + a/cos(c + d*x))^2,x)
```

output
```
(6*tan(c/2 + (d*x)/2)^2 - 4)/(d*(a^2*tan(c/2 + (d*x)/2)^4 - 2*a^2*tan(c/2
+ (d*x)/2)^2 + a^2)) + (2*atanh(tan(c/2 + (d*x)/2)^2))/(a^2*d)
```

3.74 $\int \frac{\tan^3(c+dx)}{(a+a\sec(c+dx))^2}\,dx$

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3.74.1 Optimal result

Integrand size = 21, antiderivative size = 33

$$\int \frac{\tan^3(c+dx)}{(a+a\sec(c+dx))^2}\,dx = -\frac{\log(\cos(c+dx))}{a^2 d} + \frac{2\log(1+\cos(c+dx))}{a^2 d}$$

output
```
-ln(cos(d*x+c))/a^2/d+2*ln(1+cos(d*x+c))/a^2/d
```

3.74.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.91

$$\int \frac{\tan^3(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \frac{4\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right) - \log(\cos(c+dx))}{a^2 d}$$

input
```
Integrate[Tan[c + d*x]^3/(a + a*Sec[c + d*x])^2,x]
```

output
```
(4*Log[Cos[(c + d*x)/2]] - Log[Cos[c + d*x]])/(a^2*d)
```

3.74.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.82, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3042, 25, 4367, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^3(c+dx)}{(a\sec(c+dx)+a)^2}\,dx$$

$$\downarrow 3042$$

$$\int -\frac{\cot\left(c+dx+\frac{\pi}{2}\right)^3}{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^2}dx$$

$$\downarrow 25$$

$$-\int \frac{\cot\left(\frac{1}{2}(2c+\pi)+dx\right)^3}{\left(\csc\left(\frac{1}{2}(2c+\pi)+dx\right)a+a\right)^2}dx$$

$$\downarrow 4367$$

$$-\frac{\int \frac{(1-\cos(c+dx))\sec(c+dx)}{\cos(c+dx)+1}d\cos(c+dx)}{a^2 d}$$

$$\downarrow 86$$

$$-\frac{\int \left(\sec(c+dx)-\frac{2}{\cos(c+dx)+1}\right)d\cos(c+dx)}{a^2 d}$$

$$\downarrow 2009$$

$$-\frac{\log(\cos(c+dx))-2\log(\cos(c+dx)+1)}{a^2 d}$$

input
```
Int[Tan[c + d*x]^3/(a + a*Sec[c + d*x])^2,x]
```

output
```
-((Log[Cos[c + d*x]] - 2*Log[1 + Cos[c + d*x]])/(a^2*d))
```

3.74.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4367
```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Simp[1/(a^(m - n - 1)*b^n*d)   Subst[Int[(a - b*x)^((m -
1)/2)*((a + b*x)^((m - 1)/2 + n)/x^(m + n)), x], x, Sin[c + d*x]], x] /; Fr
eeQ[{a, b, c, d}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && Integer
Q[n]
```

3.74.4 Maple [A] (verified)

Time = 0.69 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.88

method	result	size
derivativedivides	$\frac{-\ln(\cos(dx+c))+2\ln(\cos(dx+c)+1)}{d\,a^2}$	29
default	$\frac{-\ln(\cos(dx+c))+2\ln(\cos(dx+c)+1)}{d\,a^2}$	29
risch	$-\frac{ix}{a^2}-\frac{2ic}{a^2d}+\frac{4\ln\left(e^{i(dx+c)}+1\right)}{a^2d}-\frac{\ln\left(e^{2i(dx+c)}+1\right)}{a^2d}$	59

input
```
int(tan(d*x+c)^3/(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/d/a^2*(-ln(cos(d*x+c))+2*ln(cos(d*x+c)+1))
```

3.74.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.94

$$\int \frac{\tan^3(c+dx)}{(a+a\sec(c+dx))^2}\,dx = -\frac{\log\left(-\cos\left(dx+c\right)\right) - 2\log\left(\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)}{a^2 d}$$

input
```
integrate(tan(d*x+c)^3/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

output
```
-(log(-cos(d*x + c)) - 2*log(1/2*cos(d*x + c) + 1/2))/(a^2*d)
```

3.74.6 Sympy [F]

$$\int \frac{\tan^3(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \frac{\int \frac{\tan^3(c+dx)}{\sec^2(c+dx)+2\sec(c+dx)+1}\,dx}{a^2}$$

input
```
integrate(tan(d*x+c)**3/(a+a*sec(d*x+c))**2,x)
```

output
```
Integral(tan(c + d*x)**3/(sec(c + d*x)**2 + 2*sec(c + d*x) + 1), x)/a**2
```

3.74.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.94

$$\int \frac{\tan^3(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \frac{\frac{2\log(\cos(dx+c)+1)}{a^2} - \frac{\log(\cos(dx+c))}{a^2}}{d}$$

input
```
integrate(tan(d*x+c)^3/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

output
```
(2*log(cos(d*x + c) + 1)/a^2 - log(cos(d*x + c))/a^2)/d
```

3.74.8 Giac [A] (verification not implemented)

Time = 0.65 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00

$$\int \frac{\tan^3(c+dx)}{(a+a\sec(c+dx))^2}\,dx = -\frac{\log\left(\left|\frac{(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}-1\right|\right)}{a^2 d}$$

input
```
integrate(tan(d*x+c)^3/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

output
```
-log(abs((cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 1))/(a^2*d)
```

3.74.9 Mupad [B] (verification not implemented)

Time = 14.35 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.67

$$\int \frac{\tan^3(c+dx)}{(a+a\sec(c+dx))^2}\,dx = -\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4-1\right)}{a^2\,d}$$

input
```
int(tan(c + d*x)^3/(a + a/cos(c + d*x))^2,x)
```

output
```
-log(tan(c/2 + (d*x)/2)^4 - 1)/(a^2*d)
```

3.75 $\int \frac{\tan(c+dx)}{(a+a\sec(c+dx))^2}\,dx$

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3.75.1 Optimal result

Integrand size = 19, antiderivative size = 36

$$\int \frac{\tan(c+dx)}{(a+a\sec(c+dx))^2}\,dx = -\frac{1}{a^2 d(1+\cos(c+dx))} - \frac{\log(1+\cos(c+dx))}{a^2 d}$$

output
```
-1/a^2/d/(1+cos(d*x+c))-ln(1+cos(d*x+c))/a^2/d
```

3.75.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.56

$$\int \frac{\tan(c+dx)}{(a+a\sec(c+dx))^2}\,dx = -\frac{\left(1+2\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)+2\cos(c+dx)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)\right)\sec^2\left(\frac{1}{2}(c+dx)\right)}{2a^2 d}$$

input
```
Integrate[Tan[c + d*x]/(a + a*Sec[c + d*x])^2,x]
```

output
```
-1/2*((1 + 2*Log[Cos[(c + d*x)/2]] + 2*Cos[c + d*x]*Log[Cos[(c + d*x)/2]]
*Sec[(c + d*x)/2]^2)/(a^2*d)
```

3.75.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.78, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {3042, 25, 4367, 27, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan(c+dx)}{(a\sec(c+dx)+a)^2}\,dx$$

$$\downarrow 3042$$

$$\int -\frac{\cot\left(c+dx+\frac{\pi}{2}\right)}{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^2}dx$$

$$\downarrow 25$$

$$-\int \frac{\cot\left(\frac{1}{2}(2c+\pi)+dx\right)}{\left(\csc\left(\frac{1}{2}(2c+\pi)+dx\right)a+a\right)^2}dx$$

$$\downarrow 4367$$

$$-\frac{\int \frac{\cos(c+dx)}{a^2(\cos(c+dx)+1)^2}d\cos(c+dx)}{d}$$

$$\downarrow 27$$

$$-\frac{\int \frac{\cos(c+dx)}{(\cos(c+dx)+1)^2}d\cos(c+dx)}{a^2d}$$

$$\downarrow 49$$

$$-\frac{\int \left(\frac{1}{\cos(c+dx)+1}-\frac{1}{(\cos(c+dx)+1)^2}\right)d\cos(c+dx)}{a^2d}$$

$$\downarrow 2009$$

$$-\frac{\frac{1}{\cos(c+dx)+1}+\log(\cos(c+dx)+1)}{a^2d}$$

input
```
Int[Tan[c + d*x]/(a + a*Sec[c + d*x])^2,x]
```

output
```
-(((1 + Cos[c + d*x])^(-1) + Log[1 + Cos[c + d*x]])/(a^2*d))
```

3.75.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4367
```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Simp[1/(a^(m - n - 1)*b^n*d)   Subst[Int[(a - b*x)^((m -
1)/2)*((a + b*x)^((m - 1)/2 + n)/x^(m + n)), x], x, Sin[c + d*x]], x] /; Fr
eeQ[{a, b, c, d}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && Integer
Q[n]
```

3.75.4 Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.03

method	result	size
derivativedivides	$\frac{\ln(\sec(dx+c))+\frac{1}{1+\sec(dx+c)}-\ln(1+\sec(dx+c))}{d\,a^2}$	37
default	$\frac{\ln(\sec(dx+c))+\frac{1}{1+\sec(dx+c)}-\ln(1+\sec(dx+c))}{d\,a^2}$	37
risch	$\frac{ix}{a^2}+\frac{2ic}{a^2d}-\frac{2\,e^{i(dx+c)}}{a^2d\left(e^{i(dx+c)}+1\right)^2}-\frac{2\ln\left(e^{i(dx+c)}+1\right)}{a^2d}$	69

input
```
int(tan(d*x+c)/(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/d/a^2*(ln(sec(d*x+c))+1/(1+sec(d*x+c))-ln(1+sec(d*x+c)))
```

3.75.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.19

$$\int \frac{\tan(c+dx)}{(a+a\sec(c+dx))^2}\,dx = -\frac{(\cos(dx+c)+1)\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)+1}{a^2d\cos(dx+c)+a^2d}$$

input
```
integrate(tan(d*x+c)/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

output
```
-((cos(d*x + c) + 1)*log(1/2*cos(d*x + c) + 1/2) + 1)/(a^2*d*cos(d*x + c)
+ a^2*d)
```

3.75.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 177 vs. $2(31) = 62$.

Time = 11.78 (sec) , antiderivative size = 177, normalized size of antiderivative = 4.92

$$\int \frac{\tan(c+dx)}{(a+a\sec(c+dx))^2}\,dx$$
$$= \begin{cases} \frac{\log(\tan^2(c+dx)+1)\sec(c+dx)}{2a^2d\sec(c+dx)+2a^2d} + \frac{\log(\tan^2(c+dx)+1)}{2a^2d\sec(c+dx)+2a^2d} - \frac{2\log(\sec(c+dx)+1)\sec(c+dx)}{2a^2d\sec(c+dx)+2a^2d} - \frac{2\log(\sec(c+dx)+1)}{2a^2d\sec(c+dx)+2a^2d} + \frac{2}{2a^2d\sec(c+dx)+2a^2d} \\ \frac{x\tan(c)}{(a\sec(c)+a)^2} \end{cases}$$

input
```
integrate(tan(d*x+c)/(a+a*sec(d*x+c))**2,x)
```

output
```
Piecewise((log(tan(c + d*x)**2 + 1)*sec(c + d*x)/(2*a**2*d*sec(c + d*x) +
2*a**2*d) + log(tan(c + d*x)**2 + 1)/(2*a**2*d*sec(c + d*x) + 2*a**2*d) -
2*log(sec(c + d*x) + 1)*sec(c + d*x)/(2*a**2*d*sec(c + d*x) + 2*a**2*d) -
2*log(sec(c + d*x) + 1)/(2*a**2*d*sec(c + d*x) + 2*a**2*d) + 2/(2*a**2*d*s
ec(c + d*x) + 2*a**2*d), Ne(d, 0)), (x*tan(c)/(a*sec(c) + a)**2, True))
```

3.75.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.97

$$\int \frac{\tan(c+dx)}{(a+a\sec(c+dx))^2}\,dx = -\frac{\frac{1}{a^2\cos(dx+c)+a^2}+\frac{\log(\cos(dx+c)+1)}{a^2}}{d}$$

input
```
integrate(tan(d*x+c)/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

output
```
-(1/(a^2*cos(d*x + c) + a^2) + log(cos(d*x + c) + 1)/a^2)/d
```

3.75.8 Giac [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.58

$$\int \frac{\tan(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \frac{\frac{2\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right)}{a^2}+\frac{\cos(dx+c)-1}{a^2(\cos(dx+c)+1)}}{2\,d}$$

input
```
integrate(tan(d*x+c)/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

output
```
1/2*(2*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1))/a^2 + (cos(d*x
 + c) - 1)/(a^2*(cos(d*x + c) + 1)))/d
```

3.75.9 Mupad [B] (verification not implemented)

Time = 14.19 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.97

$$\int \frac{\tan(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \frac{\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+1\right)-\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2}{2}}{a^2\,d}$$

input
```
int(tan(c + d*x)/(a + a/cos(c + d*x))^2,x)
```

output
```
(log(tan(c/2 + (d*x)/2)^2 + 1) - tan(c/2 + (d*x)/2)^2/2)/(a^2*d)
```

3.76 $\int \frac{\cot(c+dx)}{(a+a\sec(c+dx))^2}\,dx$

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3.76.9 Mupad [B] (verification not implemented) . . . 594

3.76.1 Optimal result

Integrand size = 19, antiderivative size = 81

$$\int \frac{\cot(c+dx)}{(a+a\sec(c+dx))^2}\,dx = -\frac{1}{4a^2d(1+\cos(c+dx))^2} + \frac{5}{4a^2d(1+\cos(c+dx))} + \frac{\log(1-\cos(c+dx))}{8a^2d} + \frac{7\log(1+\cos(c+dx))}{8a^2d}$$

output
```
-1/4/a^2/d/(1+cos(d*x+c))^2+5/4/a^2/d/(1+cos(d*x+c))+1/8*ln(1-cos(d*x+c))/
a^2/d+7/8*ln(1+cos(d*x+c))/a^2/d
```

3.76.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.02

$$\int \frac{\cot(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \frac{\left(-1+10\cos^2\left(\frac{1}{2}(c+dx)\right)+4\cos^4\left(\frac{1}{2}(c+dx)\right)\left(7\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)+\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)\sec^2(c+\right.}{4a^2d(1+\sec(c+dx))^2}$$

input
```
Integrate[Cot[c + d*x]/(a + a*Sec[c + d*x])^2,x]
```

output
```
((-1 + 10*Cos[(c + d*x)/2]^2 + 4*Cos[(c + d*x)/2]^4*(7*Log[Cos[(c + d*x)/2
]] + Log[Sin[(c + d*x)/2]]))*Sec[c + d*x]^2)/(4*a^2*d*(1 + Sec[c + d*x])^2
)
```

3.76.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.80, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {3042, 25, 4367, 27, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot(c+dx)}{(a\sec(c+dx)+a)^2}\,dx$$

$$\downarrow 3042$$

$$\int -\frac{1}{\cot\left(c+dx+\frac{\pi}{2}\right)\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^2}dx$$

$$\downarrow 25$$

$$-\int \frac{1}{\cot\left(\frac{1}{2}(2c+\pi)+dx\right)\left(\csc\left(\frac{1}{2}(2c+\pi)+dx\right)a+a\right)^2}dx$$

$$\downarrow 4367$$

$$-\frac{a^2\int \frac{\cos^3(c+dx)}{a^4(1-\cos(c+dx))(\cos(c+dx)+1)^3}d\cos(c+dx)}{d}$$

$$\downarrow 27$$

$$-\frac{\int \frac{\cos^3(c+dx)}{(1-\cos(c+dx))(\cos(c+dx)+1)^3}d\cos(c+dx)}{a^2d}$$

$$\downarrow 99$$

$$-\frac{\int \left(-\frac{7}{8(\cos(c+dx)+1)}+\frac{5}{4(\cos(c+dx)+1)^2}-\frac{1}{2(\cos(c+dx)+1)^3}-\frac{1}{8(\cos(c+dx)-1)}\right)d\cos(c+dx)}{a^2d}$$

$$\downarrow 2009$$

$$-\frac{-\frac{5}{4(\cos(c+dx)+1)}+\frac{1}{4(\cos(c+dx)+1)^2}-\frac{1}{8}\log(1-\cos(c+dx))-\frac{7}{8}\log(\cos(c+dx)+1)}{a^2d}$$

input
```
Int[Cot[c + d*x]/(a + a*Sec[c + d*x])^2,x]
```

output
```
-((1/(4*(1 + Cos[c + d*x])^2) - 5/(4*(1 + Cos[c + d*x])) - Log[1 - Cos[c +
 d*x]]/8 - (7*Log[1 + Cos[c + d*x]])/8)/(a^2*d))
```

3.76.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4367
```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Simp[1/(a^(m - n - 1)*b^n*d)   Subst[Int[(a - b*x)^((m -
1)/2)*((a + b*x)^((m - 1)/2 + n)/x^(m + n)), x], x, Sin[c + d*x]], x] /; Fr
eeQ[{a, b, c, d}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && Integer
Q[n]
```

3.76.4 Maple [A] (verified)

Time = 0.59 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.68

method	result	size
derivativedivides	$\frac{\frac{\ln(\cos(dx+c)-1)}{8}-\frac{1}{4(\cos(dx+c)+1)^2}+\frac{5}{4(\cos(dx+c)+1)}+\frac{7\ln(\cos(dx+c)+1)}{8}}{d\,a^2}$	55
default	$\frac{\frac{\ln(\cos(dx+c)-1)}{8}-\frac{1}{4(\cos(dx+c)+1)^2}+\frac{5}{4(\cos(dx+c)+1)}+\frac{7\ln(\cos(dx+c)+1)}{8}}{d\,a^2}$	55
risch	$-\frac{ix}{a^2}-\frac{2ic}{a^2d}+\frac{5\,e^{3i(dx+c)}+8\,e^{2i(dx+c)}+5\,e^{i(dx+c)}}{2d\,a^2\left(e^{i(dx+c)}+1\right)^4}+\frac{7\ln\left(e^{i(dx+c)}+1\right)}{4a^2d}+\frac{\ln\left(e^{i(dx+c)}-1\right)}{4a^2d}$	114

input
```
int(cot(d*x+c)/(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/d/a^2*(1/8*ln(cos(d*x+c)-1)-1/4/(cos(d*x+c)+1)^2+5/4/(cos(d*x+c)+1)+7/8*
ln(cos(d*x+c)+1))
```

3.76.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.31

$$\int \frac{\cot(c+dx)}{(a+a\sec(c+dx))^2}\,dx$$
$$= \frac{7\left(\cos\left(dx+c\right)^2+2\cos\left(dx+c\right)+1\right)\log\left(\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)+\left(\cos\left(dx+c\right)^2+2\cos\left(dx+c\right)+1\right)\log($$
$$8\left(a^2d\cos\left(dx+c\right)^2+2\,a^2d\cos\left(dx+c\right)+a^2d\right)$$

input
```
integrate(cot(d*x+c)/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

output
```
1/8*(7*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*log(1/2*cos(d*x + c) + 1/2) +
 (cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*log(-1/2*cos(d*x + c) + 1/2) + 10*c
os(d*x + c) + 8)/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)
```

3.76.6 Sympy [F]

$$\int \frac{\cot(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \frac{\int \frac{\cot(c+dx)}{\sec^2(c+dx)+2\sec(c+dx)+1}\,dx}{a^2}$$

input
```
integrate(cot(d*x+c)/(a+a*sec(d*x+c))**2,x)
```

output
```
Integral(cot(c + d*x)/(sec(c + d*x)**2 + 2*sec(c + d*x) + 1), x)/a**2
```

3.76.7 Maxima [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.91

$$\int \frac{\cot(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \frac{\frac{2\,(5\,\cos(dx+c)+4)}{a^2\cos(dx+c)^2+2\,a^2\cos(dx+c)+a^2}+\frac{7\,\log(\cos(dx+c)+1)}{a^2}+\frac{\log(\cos(dx+c)-1)}{a^2}}{8\,d}$$

input
```
integrate(cot(d*x+c)/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

output
```
1/8*(2*(5*cos(d*x + c) + 4)/(a^2*cos(d*x + c)^2 + 2*a^2*cos(d*x + c) + a^2
) + 7*log(cos(d*x + c) + 1)/a^2 + log(cos(d*x + c) - 1)/a^2)/d
```

3.76.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.44

$$\int \frac{\cot(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \frac{\frac{2\,\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)}{a^2}-\frac{16\,\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right)}{a^2}-\frac{\frac{8\,a^2(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{a^2(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}}{a^4}}{16\,d}$$

input
```
integrate(cot(d*x+c)/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

output
```
1/16*(2*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1))/a^2 - 16*log(abs
(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1))/a^2 - (8*a^2*(cos(d*x + c) -
 1)/(cos(d*x + c) + 1) + a^2*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2)/a^
4)/d
```

3.76.9 Mupad [B] (verification not implemented)

Time = 14.25 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.77

$$\int \frac{\cot(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \frac{\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{4}-\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+1\right)+\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2}{2}-\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4}{16}}{a^2\,d}$$

input
```
int(cot(c + d*x)/(a + a/cos(c + d*x))^2,x)
```

output
```
(log(tan(c/2 + (d*x)/2))/4 - log(tan(c/2 + (d*x)/2)^2 + 1) + tan(c/2 + (d*
x)/2)^2/2 - tan(c/2 + (d*x)/2)^4/16)/(a^2*d)
```

3.77 $\int \frac{\cot^3(c+dx)}{(a+a\sec(c+dx))^2}\,dx$

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3.77.1 Optimal result

Integrand size = 21, antiderivative size = 123

$$\int \frac{\cot^3(c+dx)}{(a+a\sec(c+dx))^2}\,dx = -\frac{1}{16a^2d(1-\cos(c+dx))} - \frac{1}{12a^2d(1+\cos(c+dx))^3} + \frac{1}{2a^2d(1+\cos(c+dx))^2} - \frac{23}{16a^2d(1+\cos(c+dx))} - \frac{3\log(1-\cos(c+dx))}{16a^2d} - \frac{13\log(1+\cos(c+dx))}{16a^2d}$$

output

```
-1/16/a^2/d/(1-cos(d*x+c))-1/12/a^2/d/(1+cos(d*x+c))^3+1/2/a^2/d/(1+cos(d*
x+c))^2-23/16/a^2/d/(1+cos(d*x+c))-3/16*ln(1-cos(d*x+c))/a^2/d-13/16*ln(1+
cos(d*x+c))/a^2/d
```

3.77.2 Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.98

$$\int \frac{\cot^3(c+dx)}{(a+a\sec(c+dx))^2}\,dx = -\frac{\cos^4\left(\frac{1}{2}(c+dx)\right)\left(3\csc^2\left(\frac{1}{2}(c+dx)\right)+156\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)+36\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)+69\sec^2\left(\frac{1}{2}\right.\right.}{24a^2d(1+\sec(c+dx))^2}$$

input

```
Integrate[Cot[c + d*x]^3/(a + a*Sec[c + d*x])^2,x]
```

output
```
-1/24*(Cos[(c + d*x)/2]^4*(3*Csc[(c + d*x)/2]^2 + 156*Log[Cos[(c + d*x)/2]
] + 36*Log[Sin[(c + d*x)/2]] + 69*Sec[(c + d*x)/2]^2 - 12*Sec[(c + d*x)/2]
^4 + Sec[(c + d*x)/2]^6)*Sec[c + d*x]^2)/(a^2*d*(1 + Sec[c + d*x])^2)
```

3.77.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.77, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 25, 4367, 27, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^3(c+dx)}{(a\sec(c+dx)+a)^2}\,dx$$

$$\downarrow 3042$$

$$\int -\frac{1}{\cot\left(c+dx+\frac{\pi}{2}\right)^3\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^2}dx$$

$$\downarrow 25$$

$$-\int \frac{1}{\cot\left(\frac{1}{2}(2c+\pi)+dx\right)^3\left(\csc\left(\frac{1}{2}(2c+\pi)+dx\right)a+a\right)^2}dx$$

$$\downarrow 4367$$

$$-\frac{a^4\int \frac{\cos^5(c+dx)}{a^6(1-\cos(c+dx))^2(\cos(c+dx)+1)^4}d\cos(c+dx)}{d}$$

$$\downarrow 27$$

$$-\frac{\int \frac{\cos^5(c+dx)}{(1-\cos(c+dx))^2(\cos(c+dx)+1)^4}d\cos(c+dx)}{a^2d}$$

$$\downarrow 99$$

$$-\frac{\int \left(\frac{13}{16(\cos(c+dx)+1)}-\frac{23}{16(\cos(c+dx)+1)^2}+\frac{1}{(\cos(c+dx)+1)^3}-\frac{1}{4(\cos(c+dx)+1)^4}+\frac{3}{16(\cos(c+dx)-1)}+\frac{1}{16(\cos(c+dx)-1)^2}\right)d\cos(c}{a^2d}$$

$$\downarrow 2009$$

$$-\frac{\frac{1}{16(1-\cos(c+dx))}+\frac{23}{16(\cos(c+dx)+1)}-\frac{1}{2(\cos(c+dx)+1)^2}+\frac{1}{12(\cos(c+dx)+1)^3}+\frac{3}{16}\log(1-\cos(c+dx))+\frac{13}{16}\log(\cos(c+dx}{a^2d}$$

input
```
Int[Cot[c + d*x]^3/(a + a*Sec[c + d*x])^2,x]
```

output
```
-((1/(16*(1 - Cos[c + d*x])) + 1/(12*(1 + Cos[c + d*x])^3) - 1/(2*(1 + Cos
[c + d*x])^2) + 23/(16*(1 + Cos[c + d*x])) + (3*Log[1 - Cos[c + d*x]])/16
+ (13*Log[1 + Cos[c + d*x]])/16)/(a^2*d))
```

3.77.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4367
```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Simp[1/(a^(m - n - 1)*b^n*d)   Subst[Int[(a - b*x)^((m -
1)/2)*((a + b*x)^((m - 1)/2 + n)/x^(m + n)), x], x, Sin[c + d*x]], x] /; Fr
eeQ[{a, b, c, d}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && Integer
Q[n]
```

3.77.4 Maple [A] (verified)

Time = 0.70 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.64

method	result
derivativedivides	$\frac{\frac{1}{16\cos(dx+c)-16}-\frac{3\ln(\cos(dx+c)-1)}{16}-\frac{1}{12(\cos(dx+c)+1)^3}+\frac{1}{2(\cos(dx+c)+1)^2}-\frac{23}{16(\cos(dx+c)+1)}-\frac{13\ln(\cos(dx+c)+1)}{16}}{d\,a^2}$
default	$\frac{\frac{1}{16\cos(dx+c)-16}-\frac{3\ln(\cos(dx+c)-1)}{16}-\frac{1}{12(\cos(dx+c)+1)^3}+\frac{1}{2(\cos(dx+c)+1)^2}-\frac{23}{16(\cos(dx+c)+1)}-\frac{13\ln(\cos(dx+c)+1)}{16}}{d\,a^2}$
risch	$\frac{ix}{a^2}+\frac{2ic}{a^2d}-\frac{33\,e^{7i(dx+c)}+36\,e^{6i(dx+c)}-49\,e^{5i(dx+c)}-136\,e^{4i(dx+c)}-49\,e^{3i(dx+c)}+36\,e^{2i(dx+c)}+33\,e^{i(dx+c)}}{12d\,a^2\left(e^{i(dx+c)}+1\right)^6\left(e^{i(dx+c)}-1\right)^2}-\frac{3\ln$

input
```
int(cot(d*x+c)^3/(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/d/a^2*(1/16/(cos(d*x+c)-1)-3/16*ln(cos(d*x+c)-1)-1/12/(cos(d*x+c)+1)^3+1
/2/(cos(d*x+c)+1)^2-23/16/(cos(d*x+c)+1)-13/16*ln(cos(d*x+c)+1))
```

3.77.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.32

$$\int \frac{\cot^3(c+dx)}{(a+a\sec(c+dx))^2}\,dx =$$
$$-\frac{66\cos(dx+c)^3+36\cos(dx+c)^2+39\left(\cos(dx+c)^4+2\cos(dx+c)^3-2\cos(dx+c)-1\right)\log\left(\frac{1}{2}\,c\right.}{48\left(a^2d\cos(dx+c)^4+2\right.}$$

input
```
integrate(cot(d*x+c)^3/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

output
```
-1/48*(66*cos(d*x + c)^3 + 36*cos(d*x + c)^2 + 39*(cos(d*x + c)^4 + 2*cos(
d*x + c)^3 - 2*cos(d*x + c) - 1)*log(1/2*cos(d*x + c) + 1/2) + 9*(cos(d*x
+ c)^4 + 2*cos(d*x + c)^3 - 2*cos(d*x + c) - 1)*log(-1/2*cos(d*x + c) + 1/
2) - 74*cos(d*x + c) - 52)/(a^2*d*cos(d*x + c)^4 + 2*a^2*d*cos(d*x + c)^3
- 2*a^2*d*cos(d*x + c) - a^2*d)
```

3.77.6 Sympy [F]

$$\int \frac{\cot^3(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \frac{\int \frac{\cot^3(c+dx)}{\sec^2(c+dx)+2\sec(c+dx)+1}\,dx}{a^2}$$

input
```
integrate(cot(d*x+c)**3/(a+a*sec(d*x+c))**2,x)
```

output
```
Integral(cot(c + d*x)**3/(sec(c + d*x)**2 + 2*sec(c + d*x) + 1), x)/a**2
```

3.77.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.89

$$\int \frac{\cot^3(c+dx)}{(a+a\sec(c+dx))^2}\,dx = -\frac{\frac{2\left(33\cos(dx+c)^3+18\cos(dx+c)^2-37\cos(dx+c)-26\right)}{a^2\cos(dx+c)^4+2a^2\cos(dx+c)^3-2a^2\cos(dx+c)-a^2}+\frac{39\log(\cos(dx+c)+1)}{a^2}+\frac{9\log(\cos(dx+c)-1)}{a^2}}{48\,d}$$

input
```
integrate(cot(d*x+c)^3/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

output
```
-1/48*(2*(33*cos(d*x + c)^3 + 18*cos(d*x + c)^2 - 37*cos(d*x + c) - 26)/(a
^2*cos(d*x + c)^4 + 2*a^2*cos(d*x + c)^3 - 2*a^2*cos(d*x + c) - a^2) + 39*
log(cos(d*x + c) + 1)/a^2 + 9*log(cos(d*x + c) - 1)/a^2)/d
```

3.77.8 Giac [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.51

$$\int \frac{\cot^3(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \frac{\frac{3\left(\frac{6(\cos(dx+c)-1)}{\cos(dx+c)+1}+1\right)(\cos(dx+c)+1)}{a^2(\cos(dx+c)-1)}-\frac{18\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)}{a^2}+\frac{96\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right)}{a^2}+\frac{\frac{48a^4(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{9a^4(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}+}{a^6}}{96\,d}$$

input
```
integrate(cot(d*x+c)^3/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

output
```
1/96*(3*(6*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)*(cos(d*x + c) + 1)/(
a^2*(cos(d*x + c) - 1)) - 18*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) +
 1))/a^2 + 96*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1))/a^2 + (
48*a^4*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 9*a^4*(cos(d*x + c) - 1)^2/
(cos(d*x + c) + 1)^2 + a^4*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3)/a^6)
/d
```

3.77.9 Mupad [B] (verification not implemented)

Time = 14.18 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.72

$$\int \frac{\cot^3(c+dx)}{(a+a\sec(c+dx))^2}\,dx = -\frac{\frac{3\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{8} - \ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+1\right) + \frac{\cot\left(\frac{c}{2}+\frac{dx}{2}\right)^2}{32} + \frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2}{2} - \frac{3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4}{32} + \frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6}{96}}{a^2\,d}$$

input
```
int(cot(c + d*x)^3/(a + a/cos(c + d*x))^2,x)
```

output
```
-((3*log(tan(c/2 + (d*x)/2)))/8 - log(tan(c/2 + (d*x)/2)^2 + 1) + cot(c/2
+ (d*x)/2)^2/32 + tan(c/2 + (d*x)/2)^2/2 - (3*tan(c/2 + (d*x)/2)^4)/32 + t
an(c/2 + (d*x)/2)^6/96)/(a^2*d)
```

3.78 $\int \frac{\cot^5(c+dx)}{(a+a\sec(c+dx))^2}\,dx$

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3.78.1 Optimal result

Integrand size = 21, antiderivative size = 165

$$\int \frac{\cot^5(c+dx)}{(a+a\sec(c+dx))^2}\,dx = -\frac{1}{64a^2d(1-\cos(c+dx))^2} + \frac{9}{64a^2d(1-\cos(c+dx))} - \frac{1}{32a^2d(1+\cos(c+dx))^4} + \frac{11}{48a^2d(1+\cos(c+dx))^3} - \frac{3}{4a^2d(1+\cos(c+dx))^2} + \frac{51}{32a^2d(1+\cos(c+dx))} + \frac{29\log(1-\cos(c+dx))}{128a^2d} + \frac{99\log(1+\cos(c+dx))}{128a^2d}$$

output
```
-1/64/a^2/d/(1-cos(d*x+c))^2+9/64/a^2/d/(1-cos(d*x+c))-1/32/a^2/d/(1+cos(d
*x+c))^4+11/48/a^2/d/(1+cos(d*x+c))^3-3/4/a^2/d/(1+cos(d*x+c))^2+51/32/a^2
/d/(1+cos(d*x+c))+29/128*ln(1-cos(d*x+c))/a^2/d+99/128*ln(1+cos(d*x+c))/a^
2/d
```

3.78.2 Mathematica [A] (verified)

Time = 0.61 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.93

$$\int \frac{\cot^5(c+dx)}{(a+a\sec(c+dx))^2}\,dx = -\frac{\cos^4\left(\frac{1}{2}(c+dx)\right)\left(-108\csc^2\left(\frac{1}{2}(c+dx)\right)+6\csc^4\left(\frac{1}{2}(c+dx)\right)-24\left(99\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)+29\log\left(\text{si}\right.\right.\right.}{384}$$

input
```
Integrate[Cot[c + d*x]^5/(a + a*Sec[c + d*x])^2,x]
```

output
```
-1/384*(Cos[(c + d*x)/2]^4*(-108*Csc[(c + d*x)/2]^2 + 6*Csc[(c + d*x)/2]^4
 - 24*(99*Log[Cos[(c + d*x)/2]] + 29*Log[Sin[(c + d*x)/2]]) - 1224*Sec[(c
+ d*x)/2]^2 + 288*Sec[(c + d*x)/2]^4 - 44*Sec[(c + d*x)/2]^6 + 3*Sec[(c +
d*x)/2]^8)*Sec[c + d*x]^2)/(a^2*d*(1 + Sec[c + d*x])^2)
```

3.78.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.76, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 25, 4367, 27, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^5(c+dx)}{(a\sec(c+dx)+a)^2}\,dx$$

$$\downarrow 3042$$

$$\int -\frac{1}{\cot\left(c+dx+\frac{\pi}{2}\right)^5\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^2}dx$$

$$\downarrow 25$$

$$-\int \frac{1}{\cot\left(\frac{1}{2}(2c+\pi)+dx\right)^5\left(\csc\left(\frac{1}{2}(2c+\pi)+dx\right)a+a\right)^2}dx$$

$$\downarrow 4367$$

$$-\frac{a^6\int \frac{\cos^7(c+dx)}{a^8(1-\cos(c+dx))^3(\cos(c+dx)+1)^5}d\cos(c+dx)}{d}$$

$$\downarrow 27$$

$$-\frac{\int \frac{\cos^7(c+dx)}{(1-\cos(c+dx))^3(\cos(c+dx)+1)^5}d\cos(c+dx)}{a^2d}$$

$$\downarrow 99$$

$$-\frac{\int\left(-\frac{99}{128(\cos(c+dx)+1)}+\frac{51}{32(\cos(c+dx)+1)^2}-\frac{3}{2(\cos(c+dx)+1)^3}+\frac{11}{16(\cos(c+dx)+1)^4}-\frac{1}{8(\cos(c+dx)+1)^5}-\frac{29}{128(\cos(c+dx)-1)}-\right.}{a^2d}$$

$$\downarrow 2009$$

$$-\frac{-\frac{9}{64(1-\cos(c+dx))}-\frac{51}{32(\cos(c+dx)+1)}+\frac{1}{64(1-\cos(c+dx))^2}+\frac{3}{4(\cos(c+dx)+1)^2}-\frac{11}{48(\cos(c+dx)+1)^3}+\frac{1}{32(\cos(c+dx)+1)^4}-\frac{29}{128}\ln$$
$$a^2d$$

input
```
Int[Cot[c + d*x]^5/(a + a*Sec[c + d*x])^2,x]
```

output
```
-((1/(64*(1 - Cos[c + d*x])^2) - 9/(64*(1 - Cos[c + d*x])) + 1/(32*(1 + Co
s[c + d*x])^4) - 11/(48*(1 + Cos[c + d*x])^3) + 3/(4*(1 + Cos[c + d*x])^2)
 - 51/(32*(1 + Cos[c + d*x])) - (29*Log[1 - Cos[c + d*x]])/128 - (99*Log[1
 + Cos[c + d*x]])/128)/(a^2*d))
```

3.78.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4367
```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Simp[1/(a^(m - n - 1)*b^n*d)   Subst[Int[(a - b*x)^((m -
1)/2)*((a + b*x)^((m - 1)/2 + n)/x^(m + n)), x], x, Sin[c + d*x]], x] /; Fr
eeQ[{a, b, c, d}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && Integer
Q[n]
```

3.78.4 Maple [A] (verified)

Time = 0.72 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.62

method	result
derivativedivides	$\frac{-\frac{1}{64(\cos(dx+c)-1)^2}-\frac{9}{64(\cos(dx+c)-1)}+\frac{29\ln(\cos(dx+c)-1)}{128}-\frac{1}{32(\cos(dx+c)+1)^4}+\frac{11}{48(\cos(dx+c)+1)^3}-\frac{3}{4(\cos(dx+c)+1)^2}+\frac{32(\cos(dx+c)+1)^4}{d\,a^2}}{d\,a^2}$
default	$\frac{-\frac{1}{64(\cos(dx+c)-1)^2}-\frac{9}{64(\cos(dx+c)-1)}+\frac{29\ln(\cos(dx+c)-1)}{128}-\frac{1}{32(\cos(dx+c)+1)^4}+\frac{11}{48(\cos(dx+c)+1)^3}-\frac{3}{4(\cos(dx+c)+1)^2}+\frac{32(\cos(dx+c)+1)^4}{d\,a^2}}{d\,a^2}$
risch	$-\frac{ix}{a^2}-\frac{2ic}{a^2d}+\frac{279\,e^{11i(dx+c)}+156\,e^{10i(dx+c)}-1141\,e^{9i(dx+c)}-2080\,e^{8i(dx+c)}+670\,e^{7i(dx+c)}+2696\,e^{6i(dx+c)}+670\,e^{5}}{96d\,a^2\left(e^{i(dx+c)}+1\right)^8\left(e^{i(dx+c)}-1\right)}$

input
```
int(cot(d*x+c)^5/(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/d/a^2*(-1/64/(cos(d*x+c)-1)^2-9/64/(cos(d*x+c)-1)+29/128*ln(cos(d*x+c)-1
)-1/32/(cos(d*x+c)+1)^4+11/48/(cos(d*x+c)+1)^3-3/4/(cos(d*x+c)+1)^2+51/32/
(cos(d*x+c)+1)+99/128*ln(cos(d*x+c)+1))
```

3.78.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 283, normalized size of antiderivative = 1.72

$$\int \frac{\cot^5(c+dx)}{(a+a\sec(c+dx))^2}\,dx$$
$$= \frac{558\cos(dx+c)^5+156\cos(dx+c)^4-1268\cos(dx+c)^3-676\cos(dx+c)^2+297\left(\cos(dx+c)^6+2\,c\right.}{}$$

input
```
integrate(cot(d*x+c)^5/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

output
```
1/384*(558*cos(d*x + c)^5 + 156*cos(d*x + c)^4 - 1268*cos(d*x + c)^3 - 676
*cos(d*x + c)^2 + 297*(cos(d*x + c)^6 + 2*cos(d*x + c)^5 - cos(d*x + c)^4
- 4*cos(d*x + c)^3 - cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*log(1/2*cos(d*x
+ c) + 1/2) + 87*(cos(d*x + c)^6 + 2*cos(d*x + c)^5 - cos(d*x + c)^4 - 4*c
os(d*x + c)^3 - cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*log(-1/2*cos(d*x + c)
 + 1/2) + 686*cos(d*x + c) + 448)/(a^2*d*cos(d*x + c)^6 + 2*a^2*d*cos(d*x
+ c)^5 - a^2*d*cos(d*x + c)^4 - 4*a^2*d*cos(d*x + c)^3 - a^2*d*cos(d*x + c
)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)
```

3.78.6 Sympy [F]

$$\int \frac{\cot^5(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \frac{\int \frac{\cot^5(c+dx)}{\sec^2(c+dx)+2\sec(c+dx)+1}\,dx}{a^2}$$

input
```
integrate(cot(d*x+c)**5/(a+a*sec(d*x+c))**2,x)
```

output
```
Integral(cot(c + d*x)**5/(sec(c + d*x)**2 + 2*sec(c + d*x) + 1), x)/a**2
```

3.78.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.01

$$\int \frac{\cot^5(c+dx)}{(a+a\sec(c+dx))^2}\,dx$$
$$= \frac{\frac{2\left(279\cos(dx+c)^5+78\cos(dx+c)^4-634\cos(dx+c)^3-338\cos(dx+c)^2+343\cos(dx+c)+224\right)}{a^2\cos(dx+c)^6+2a^2\cos(dx+c)^5-a^2\cos(dx+c)^4-4a^2\cos(dx+c)^3-a^2\cos(dx+c)^2+2a^2\cos(dx+c)+a^2} + \frac{297\log(\cos(dx+c)+1)}{a^2} + \frac{87\log(\cos}{}}{384\,d}$$

input
```
integrate(cot(d*x+c)^5/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

output
```
1/384*(2*(279*cos(d*x + c)^5 + 78*cos(d*x + c)^4 - 634*cos(d*x + c)^3 - 33
8*cos(d*x + c)^2 + 343*cos(d*x + c) + 224)/(a^2*cos(d*x + c)^6 + 2*a^2*cos
(d*x + c)^5 - a^2*cos(d*x + c)^4 - 4*a^2*cos(d*x + c)^3 - a^2*cos(d*x + c)
^2 + 2*a^2*cos(d*x + c) + a^2) + 297*log(cos(d*x + c) + 1)/a^2 + 87*log(co
s(d*x + c) - 1)/a^2)/d
```

3.78.8 Giac [A] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 236, normalized size of antiderivative = 1.43

$$\int \frac{\cot^5(c+dx)}{(a+a\sec(c+dx))^2}\,dx =$$
$$-\frac{\frac{6\left(\frac{16(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{87(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}+1\right)(\cos(dx+c)+1)^2}{a^2(\cos(dx+c)-1)^2} - \frac{348\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)}{a^2} + \frac{1536\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right)}{a^2} + \frac{768\,a^6(\cos(d}{\cos(dx+}}{1536\,d}$$

input
```
integrate(cot(d*x+c)^5/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

output
```
-1/1536*(6*(16*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 87*(cos(d*x + c) -
1)^2/(cos(d*x + c) + 1)^2 + 1)*(cos(d*x + c) + 1)^2/(a^2*(cos(d*x + c) - 1
)^2) - 348*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1))/a^2 + 1536*lo
g(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1))/a^2 + (768*a^6*(cos(d*x
 + c) - 1)/(cos(d*x + c) + 1) + 174*a^6*(cos(d*x + c) - 1)^2/(cos(d*x + c)
 + 1)^2 + 32*a^6*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 + 3*a^6*(cos(d*
x + c) - 1)^4/(cos(d*x + c) + 1)^4)/a^8)/d
```

3.78.9 Mupad [B] (verification not implemented)

Time = 14.32 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.92

$$\int \frac{\cot^5(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2}{2\,a^2\,d} - \frac{29\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4}{256\,a^2\,d} + \frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6}{48\,a^2\,d} - \frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^8}{512\,a^2\,d} + \frac{29\,\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{64\,a^2\,d} - \frac{\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+1\right)}{a^2\,d} + \frac{\cot\left(\frac{c}{2}+\frac{dx}{2}\right)^4\left(4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2-\frac{1}{4}\right)}{64\,a^2\,d}$$

input
```
int(cot(c + d*x)^5/(a + a/cos(c + d*x))^2,x)
```

output
```
tan(c/2 + (d*x)/2)^2/(2*a^2*d) - (29*tan(c/2 + (d*x)/2)^4)/(256*a^2*d) + t
an(c/2 + (d*x)/2)^6/(48*a^2*d) - tan(c/2 + (d*x)/2)^8/(512*a^2*d) + (29*lo
g(tan(c/2 + (d*x)/2)))/(64*a^2*d) - log(tan(c/2 + (d*x)/2)^2 + 1)/(a^2*d)
+ (cot(c/2 + (d*x)/2)^4*(4*tan(c/2 + (d*x)/2)^2 - 1/4))/(64*a^2*d)
```

3.79 $\int \frac{\tan^8(c+dx)}{(a+a\sec(c+dx))^2}\,dx$

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3.79.1 Optimal result

Integrand size = 21, antiderivative size = 119

$$\int \frac{\tan^8(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \frac{x}{a^2} - \frac{3\operatorname{arctanh}(\sin(c+dx))}{4a^2d} - \frac{\tan(c+dx)}{a^2d} + \frac{3\sec(c+dx)\tan(c+dx)}{4a^2d} + \frac{\tan^3(c+dx)}{3a^2d} - \frac{\sec(c+dx)\tan^3(c+dx)}{2a^2d} + \frac{\tan^5(c+dx)}{5a^2d}$$

output
```
x/a^2-3/4*arctanh(sin(d*x+c))/a^2/d-tan(d*x+c)/a^2/d+3/4*sec(d*x+c)*tan(d*
x+c)/a^2/d+1/3*tan(d*x+c)^3/a^2/d-1/2*sec(d*x+c)*tan(d*x+c)^3/a^2/d+1/5*ta
n(d*x+c)^5/a^2/d
```

3.79.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 495 vs. 2(119) = 238.

Time = 5.79 (sec) , antiderivative size = 495, normalized size of antiderivative = 4.16

$$\int \frac{\tan^8(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \frac{\cos^4\left(\frac{1}{2}(c+dx)\right)\sec^2(c+dx)\left(240x + \frac{180\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)}{d} - \frac{180\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)}{d} - \frac{(29}{}\right.$$

input
```
Integrate[Tan[c + d*x]^8/(a + a*Sec[c + d*x])^2,x]
```

output
```
(Cos[(c + d*x)/2]^4*Sec[c + d*x]^2*(240*x + (180*Log[Cos[(c + d*x)/2] - Si
n[(c + d*x)/2]])/d - (180*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]])/d - ((
293*Cos[(d*x)/2] + 333*Cos[2*c + (3*d*x)/2] + 287*Cos[2*c + (5*d*x)/2] + 6
7*Cos[4*c + (7*d*x)/2] + 68*Cos[4*c + (9*d*x)/2])*Sec[c]*Sec[c + d*x]^5*Si
n[(d*x)/2])/(2*d) + (36*Sin[c/2])/(d*(Cos[c/2] - Sin[c/2])*(Cos[(c + d*x)/
2] - Sin[(c + d*x)/2])^4) - (151*Sin[c/2])/(d*(Cos[c/2] - Sin[c/2])*(Cos[(
c + d*x)/2] - Sin[(c + d*x)/2])^2) + (36*Sin[c/2])/(d*(Cos[c/2] + Sin[c/2]
)*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^4) - (151*Sin[c/2])/(d*(Cos[c/2] +
 Sin[c/2])*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2) + (Cos[c/2]*Sec[c]*Sec
[c + d*x]^4*(308*Sin[c/2] - 43*Sin[c/2 + d*x] - 43*Sin[(3*c)/2 + d*x] - 34
6*Sin[(3*c)/2 + 2*d*x] + 346*Sin[(5*c)/2 + 2*d*x] + 149*Sin[(5*c)/2 + 3*d*
x] + 149*Sin[(7*c)/2 + 3*d*x]))/(4*d)))/(60*a^2*(1 + Sec[c + d*x])^2)
```

3.79.3 Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.03, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3042, 4376, 3042, 4374, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^8(c+dx)}{(a\sec(c+dx)+a)^2}\,dx$$
$$\downarrow \ 3042$$
$$\int \frac{\cot\left(c+dx+\frac{\pi}{2}\right)^8}{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^2}dx$$
$$\downarrow \ 4376$$
$$\frac{\int (a-a\sec(c+dx))^2\tan^4(c+dx)dx}{a^4}$$
$$\downarrow \ 3042$$
$$\frac{\int \cot\left(c+dx+\frac{\pi}{2}\right)^4\left(a-a\csc\left(c+dx+\frac{\pi}{2}\right)\right)^2 dx}{a^4}$$
$$\downarrow \ 4374$$

$$\frac{\int \left(a^2 \tan^4(c+dx) + a^2 \sec^2(c+dx) \tan^4(c+dx) - 2a^2 \sec(c+dx) \tan^4(c+dx)\right) dx}{a^4}$$

$$\downarrow \text{2009}$$

$$\frac{-\frac{3a^2 \text{arctanh}(\sin(c+dx))}{4d} + \frac{a^2 \tan^5(c+dx)}{5d} + \frac{a^2 \tan^3(c+dx)}{3d} - \frac{a^2 \tan(c+dx)}{d} - \frac{a^2 \tan^3(c+dx) \sec(c+dx)}{2d} + \frac{3a^2 \tan(c+dx) \sec(c+dx)}{4d} +}{a^4}$$

input
```
Int[Tan[c + d*x]^8/(a + a*Sec[c + d*x])^2,x]
```

output
```
(a^2*x - (3*a^2*ArcTanh[Sin[c + d*x]])/(4*d) - (a^2*Tan[c + d*x])/d + (3*a
^2*Sec[c + d*x]*Tan[c + d*x])/(4*d) + (a^2*Tan[c + d*x]^3)/(3*d) - (a^2*Se
c[c + d*x]*Tan[c + d*x]^3)/(2*d) + (a^2*Tan[c + d*x]^5)/(5*d))/a^4
```

3.79.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4374
```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

rule 4376
```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Simp[a^(2*n)/e^(2*n)   Int[(e*Cot[c + d*x])^(m + 2*
n)/(-a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[a
^2 - b^2, 0] && ILtQ[n, 0]
```

3.79.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.42 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.35

method	result
risch	$\frac{x}{a^2}-\frac{i\left(75\,e^{9i(dx+c)}+60\,e^{8i(dx+c)}+30\,e^{7i(dx+c)}+360\,e^{6i(dx+c)}+320\,e^{4i(dx+c)}-30\,e^{3i(dx+c)}+280\,e^{2i(dx+c)}-75\,e^{i(dx+c}\right.}{30d\,a^2\left(e^{2i(dx+c)}+1\right)^5}$
derivativedivides	$-\frac{1}{5\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^5}-\frac{1}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^4}-\frac{19}{12\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^3}-\frac{1}{8\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}+\frac{7}{4\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{3\ln\left(\tan\left(\frac{dx}{2}+\right.\right.}{4}$
default	$-\frac{1}{5\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^5}-\frac{1}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^4}-\frac{19}{12\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^3}-\frac{1}{8\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}+\frac{7}{4\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{3\ln\left(\tan\left(\frac{dx}{2}+\right.\right.}{4}$

input
```
int(tan(d*x+c)^8/(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
x/a^2-1/30*I*(75*exp(9*I*(d*x+c))+60*exp(8*I*(d*x+c))+30*exp(7*I*(d*x+c))+
360*exp(6*I*(d*x+c))+320*exp(4*I*(d*x+c))-30*exp(3*I*(d*x+c))+280*exp(2*I*
(d*x+c))-75*exp(I*(d*x+c))+68)/d/a^2/(exp(2*I*(d*x+c))+1)^5+3/4/a^2/d*ln(e
xp(I*(d*x+c))-I)-3/4/a^2/d*ln(exp(I*(d*x+c))+I)
```

3.79.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.98

$$\int \frac{\tan^8(c+dx)}{(a+a\sec(c+dx))^2}\,dx$$
$$=\frac{120\,dx\cos(dx+c)^5-45\,\cos(dx+c)^5\log(\sin(dx+c)+1)+45\,\cos(dx+c)^5\log(-\sin(dx+c)+1)-}{120\,a^2d\cos(dx+}$$

input
```
integrate(tan(d*x+c)^8/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

output
```
1/120*(120*d*x*cos(d*x + c)^5 - 45*cos(d*x + c)^5*log(sin(d*x + c) + 1) +
45*cos(d*x + c)^5*log(-sin(d*x + c) + 1) - 2*(68*cos(d*x + c)^4 - 75*cos(d
*x + c)^3 + 4*cos(d*x + c)^2 + 30*cos(d*x + c) - 12)*sin(d*x + c))/(a^2*d*
cos(d*x + c)^5)
```

3.79.6 Sympy [F]

$$\int \frac{\tan^8(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \frac{\int \frac{\tan^8(c+dx)}{\sec^2(c+dx)+2\sec(c+dx)+1}\,dx}{a^2}$$

input
```
integrate(tan(d*x+c)**8/(a+a*sec(d*x+c))**2,x)
```

output
```
Integral(tan(c + d*x)**8/(sec(c + d*x)**2 + 2*sec(c + d*x) + 1), x)/a**2
```

3.79.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 301 vs. $2(109) = 218$.

Time $= 0.30$ (sec) , antiderivative size $= 301$, normalized size of antiderivative $= 2.53$

$$\int \frac{\tan^8(c+dx)}{(a+a\sec(c+dx))^2}\,dx = -\frac{\frac{2\left(\frac{15\sin(dx+c)}{\cos(dx+c)+1}-\frac{110\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{328\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{530\sin(dx+c)^7}{(\cos(dx+c)+1)^7}+\frac{105\sin(dx+c)^9}{(\cos(dx+c)+1)^9}\right)}{a^2-\frac{5a^2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{10a^2\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{10a^2\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{5a^2\sin(dx+c)^8}{(\cos(dx+c)+1)^8}-\frac{a^2\sin(dx+c)^{10}}{(\cos(dx+c)+1)^{10}}} - \frac{120\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^2} + \frac{45\log\left(\frac{\sin(dx+c}{\cos(dx+c)}\right.}{a^2}}{60\,d}$$

input
```
integrate(tan(d*x+c)^8/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

output
```
-1/60*(2*(15*sin(d*x + c)/(cos(d*x + c) + 1) - 110*sin(d*x + c)^3/(cos(d*x
 + c) + 1)^3 + 328*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 530*sin(d*x + c)^
7/(cos(d*x + c) + 1)^7 + 105*sin(d*x + c)^9/(cos(d*x + c) + 1)^9)/(a^2 - 5
*a^2*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 10*a^2*sin(d*x + c)^4/(cos(d*x
+ c) + 1)^4 - 10*a^2*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + 5*a^2*sin(d*x +
 c)^8/(cos(d*x + c) + 1)^8 - a^2*sin(d*x + c)^10/(cos(d*x + c) + 1)^10) -
120*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^2 + 45*log(sin(d*x + c)/(cos
(d*x + c) + 1) + 1)/a^2 - 45*log(sin(d*x + c)/(cos(d*x + c) + 1) - 1)/a^2)
/d
```

3.79.8 Giac [A] (verification not implemented)

Time = 4.86 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.14

$$\int \frac{\tan^8(c+dx)}{(a+a\sec(c+dx))^2}\,dx$$

$$= \frac{\frac{60\,(dx+c)}{a^2} - \frac{45\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{a^2} + \frac{45\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{a^2} + \frac{2\left(105\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9-530\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+328\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\right.\right.}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right.}}{60\,d}$$

input
```
integrate(tan(d*x+c)^8/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

output
```
1/60*(60*(d*x + c)/a^2 - 45*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^2 + 45*lo
g(abs(tan(1/2*d*x + 1/2*c) - 1))/a^2 + 2*(105*tan(1/2*d*x + 1/2*c)^9 - 530
*tan(1/2*d*x + 1/2*c)^7 + 328*tan(1/2*d*x + 1/2*c)^5 - 110*tan(1/2*d*x + 1
/2*c)^3 + 15*tan(1/2*d*x + 1/2*c))/((tan(1/2*d*x + 1/2*c)^2 - 1)^5*a^2))/d
```

3.79.9 Mupad [B] (verification not implemented)

Time = 15.05 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.50

$$\int \frac{\tan^8(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \frac{x}{a^2} - \frac{3\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{2\,a^2\,d}$$

$$+ \frac{\frac{7\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^9}{2} - \frac{53\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^7}{3} + \frac{164\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5}{15} - \frac{11\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3}{3} + \frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{2}}{d\left(a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{10} - 5\,a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^8 + 10\,a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6 - 10\,a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4 + 5\,a^2\tan\left(\frac{c}{2}+\right.\right.}$$

input
```
int(tan(c + d*x)^8/(a + a/cos(c + d*x))^2,x)
```

output
```
x/a^2 - (3*atanh(tan(c/2 + (d*x)/2)))/(2*a^2*d) + (tan(c/2 + (d*x)/2)/2 -
(11*tan(c/2 + (d*x)/2)^3)/3 + (164*tan(c/2 + (d*x)/2)^5)/15 - (53*tan(c/2
+ (d*x)/2)^7)/3 + (7*tan(c/2 + (d*x)/2)^9)/2)/(d*(5*a^2*tan(c/2 + (d*x)/2)
^2 - 10*a^2*tan(c/2 + (d*x)/2)^4 + 10*a^2*tan(c/2 + (d*x)/2)^6 - 5*a^2*tan
(c/2 + (d*x)/2)^8 + a^2*tan(c/2 + (d*x)/2)^10 - a^2))
```

3.80 $\int \frac{\tan^6(c+dx)}{(a+a\sec(c+dx))^2}\,dx$

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3.80.1 Optimal result

Integrand size = 21, antiderivative size = 72

$$\int \frac{\tan^6(c+dx)}{(a+a\sec(c+dx))^2}\,dx = -\frac{x}{a^2} + \frac{\operatorname{arctanh}(\sin(c+dx))}{a^2 d} + \frac{\tan(c+dx)}{a^2 d} - \frac{\sec(c+dx)\tan(c+dx)}{a^2 d} + \frac{\tan^3(c+dx)}{3a^2 d}$$

output
```
-x/a^2+arctanh(sin(d*x+c))/a^2/d+tan(d*x+c)/a^2/d-sec(d*x+c)*tan(d*x+c)/a^
2/d+1/3*tan(d*x+c)^3/a^2/d
```

3.80.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 767 vs. $2(72) = 144$.

Time = 7.08 (sec) , antiderivative size = 767, normalized size of antiderivative = 10.65

$$
\begin{aligned}
&\int \frac{\tan^6(c+dx)}{(a+a\sec(c+dx))^2}\,dx \\
&= -\frac{4x\cos^4\left(\frac{c}{2}+\frac{dx}{2}\right)\sec^2(c+dx)}{(a+a\sec(c+dx))^2} \\
&\quad - \frac{4\cos^4\left(\frac{c}{2}+\frac{dx}{2}\right)\log\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)-\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\right)\sec^2(c+dx)}{d(a+a\sec(c+dx))^2} \\
&\quad + \frac{4\cos^4\left(\frac{c}{2}+\frac{dx}{2}\right)\log\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)+\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\right)\sec^2(c+dx)}{d(a+a\sec(c+dx))^2} \\
&\quad + \frac{2\cos^4\left(\frac{c}{2}+\frac{dx}{2}\right)\sec^2(c+dx)\sin\left(\frac{dx}{2}\right)}{3d(a+a\sec(c+dx))^2\left(\cos\left(\frac{c}{2}\right)-\sin\left(\frac{c}{2}\right)\right)\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)-\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\right)^3} \\
&\quad + \frac{\cos^4\left(\frac{c}{2}+\frac{dx}{2}\right)\sec^2(c+dx)\left(-5\cos\left(\frac{c}{2}\right)+7\sin\left(\frac{c}{2}\right)\right)}{3d(a+a\sec(c+dx))^2\left(\cos\left(\frac{c}{2}\right)-\sin\left(\frac{c}{2}\right)\right)\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)-\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\right)^2} \\
&\quad + \frac{8\cos^4\left(\frac{c}{2}+\frac{dx}{2}\right)\sec^2(c+dx)\sin\left(\frac{dx}{2}\right)}{3d(a+a\sec(c+dx))^2\left(\cos\left(\frac{c}{2}\right)-\sin\left(\frac{c}{2}\right)\right)\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)-\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\right)} \\
&\quad + \frac{2\cos^4\left(\frac{c}{2}+\frac{dx}{2}\right)\sec^2(c+dx)\sin\left(\frac{dx}{2}\right)}{3d(a+a\sec(c+dx))^2\left(\cos\left(\frac{c}{2}\right)+\sin\left(\frac{c}{2}\right)\right)\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)+\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\right)^3} \\
&\quad + \frac{\cos^4\left(\frac{c}{2}+\frac{dx}{2}\right)\sec^2(c+dx)\left(5\cos\left(\frac{c}{2}\right)+7\sin\left(\frac{c}{2}\right)\right)}{3d(a+a\sec(c+dx))^2\left(\cos\left(\frac{c}{2}\right)+\sin\left(\frac{c}{2}\right)\right)\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)+\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\right)^2} \\
&\quad + \frac{8\cos^4\left(\frac{c}{2}+\frac{dx}{2}\right)\sec^2(c+dx)\sin\left(\frac{dx}{2}\right)}{3d(a+a\sec(c+dx))^2\left(\cos\left(\frac{c}{2}\right)+\sin\left(\frac{c}{2}\right)\right)\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)+\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}
\end{aligned}
$$

input
```
Integrate[Tan[c + d*x]^6/(a + a*Sec[c + d*x])^2,x]
```

output
```
(-4*x*Cos[c/2 + (d*x)/2]^4*Sec[c + d*x]^2)/(a + a*Sec[c + d*x])^2 - (4*Cos
[c/2 + (d*x)/2]^4*Log[Cos[c/2 + (d*x)/2] - Sin[c/2 + (d*x)/2]]*Sec[c + d*x
]^2)/(d*(a + a*Sec[c + d*x])^2) + (4*Cos[c/2 + (d*x)/2]^4*Log[Cos[c/2 + (d
*x)/2] + Sin[c/2 + (d*x)/2]]*Sec[c + d*x]^2)/(d*(a + a*Sec[c + d*x])^2) +
(2*Cos[c/2 + (d*x)/2]^4*Sec[c + d*x]^2*Sin[(d*x)/2])/(3*d*(a + a*Sec[c + d
*x])^2*(Cos[c/2] - Sin[c/2])*(Cos[c/2 + (d*x)/2] - Sin[c/2 + (d*x)/2])^3)
+ (Cos[c/2 + (d*x)/2]^4*Sec[c + d*x]^2*(-5*Cos[c/2] + 7*Sin[c/2]))/(3*d*(a
 + a*Sec[c + d*x])^2*(Cos[c/2] - Sin[c/2])*(Cos[c/2 + (d*x)/2] - Sin[c/2 +
 (d*x)/2])^2) + (8*Cos[c/2 + (d*x)/2]^4*Sec[c + d*x]^2*Sin[(d*x)/2])/(3*d*
(a + a*Sec[c + d*x])^2*(Cos[c/2] - Sin[c/2])*(Cos[c/2 + (d*x)/2] - Sin[c/2
 + (d*x)/2])) + (2*Cos[c/2 + (d*x)/2]^4*Sec[c + d*x]^2*Sin[(d*x)/2])/(3*d*
(a + a*Sec[c + d*x])^2*(Cos[c/2] + Sin[c/2])*(Cos[c/2 + (d*x)/2] + Sin[c/2
 + (d*x)/2])^3) + (Cos[c/2 + (d*x)/2]^4*Sec[c + d*x]^2*(5*Cos[c/2] + 7*Sin
[c/2]))/(3*d*(a + a*Sec[c + d*x])^2*(Cos[c/2] + Sin[c/2])*(Cos[c/2 + (d*x)
/2] + Sin[c/2 + (d*x)/2])^2) + (8*Cos[c/2 + (d*x)/2]^4*Sec[c + d*x]^2*Sin[
(d*x)/2])/(3*d*(a + a*Sec[c + d*x])^2*(Cos[c/2] + Sin[c/2])*(Cos[c/2 + (d*
x)/2] + Sin[c/2 + (d*x)/2]))
```

3.80.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.06, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3042, 4376, 3042, 4374, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^6(c+dx)}{(a\sec(c+dx)+a)^2}\,dx$$

$$\downarrow 3042$$

$$\int \frac{\cot\left(c+dx+\frac{\pi}{2}\right)^6}{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^2}dx$$

$$\downarrow 4376$$

$$\frac{\int (a-a\sec(c+dx))^2\tan^2(c+dx)dx}{a^4}$$

$$\downarrow 3042$$

$$\frac{\int \cot\left(c+dx+\frac{\pi}{2}\right)^2\left(a-a\csc\left(c+dx+\frac{\pi}{2}\right)\right)^2 dx}{a^4}$$

$$\downarrow 4374$$

$$\frac{\int \left(a^2 \tan^2(c+dx) + a^2 \sec^2(c+dx)\tan^2(c+dx) - 2a^2 \sec(c+dx)\tan^2(c+dx)\right) dx}{a^4}$$

$$\downarrow 2009$$

$$\frac{\frac{a^2 \text{arctanh}(\sin(c+dx))}{d} + \frac{a^2 \tan^3(c+dx)}{3d} + \frac{a^2 \tan(c+dx)}{d} - \frac{a^2 \tan(c+dx)\sec(c+dx)}{d} - a^2 x}{a^4}$$

input
```
Int[Tan[c + d*x]^6/(a + a*Sec[c + d*x])^2,x]
```

output
```
(-(a^2*x) + (a^2*ArcTanh[Sin[c + d*x]])/d + (a^2*Tan[c + d*x])/d - (a^2*Se
c[c + d*x]*Tan[c + d*x])/d + (a^2*Tan[c + d*x]^3)/(3*d))/a^4
```

3.80.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4374
```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

rule 4376
```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Simp[a^(2*n)/e^(2*n)   Int[(e*Cot[c + d*x])^(m + 2*
n)/(-a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[a
^2 - b^2, 0] && ILtQ[n, 0]
```

3.80.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.06 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.47

method	result
risch	$-\frac{x}{a^2}+\frac{2i\left(3\,e^{5i(dx+c)}+6\,e^{2i(dx+c)}-3\,e^{i(dx+c)}+2\right)}{3d\,a^2\left(e^{2i(dx+c)}+1\right)^3}-\frac{\ln\left(e^{i(dx+c)}-i\right)}{a^2d}+\frac{\ln\left(e^{i(dx+c)}+i\right)}{a^2d}$
derivativedivides	$\frac{-\frac{1}{3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^3}-\frac{3}{2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}-\frac{2}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1}-\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)-2\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\frac{1}{3\left(\tan\left(\frac{dx}{2}+\right.\right.}}{a^2d}$
default	$\frac{-\frac{1}{3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^3}-\frac{3}{2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}-\frac{2}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1}-\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)-2\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\frac{1}{3\left(\tan\left(\frac{dx}{2}+\right.\right.}}{a^2d}$

input
```
int(tan(d*x+c)^6/(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
-x/a^2+2/3*I*(3*exp(5*I*(d*x+c))+6*exp(2*I*(d*x+c))-3*exp(I*(d*x+c))+2)/d/
a^2/(exp(2*I*(d*x+c))+1)^3-1/a^2/d*ln(exp(I*(d*x+c))-I)+1/a^2/d*ln(exp(I*(
d*x+c))+I)
```

3.80.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.35

$$\int \frac{\tan^6(c+dx)}{(a+a\sec(c+dx))^2}\,dx = -\frac{6\,dx\cos(dx+c)^3-3\cos(dx+c)^3\log(\sin(dx+c)+1)+3\cos(dx+c)^3\log(-\sin(dx+c)+1)-2}{6\,a^2d\cos(dx+c)^3}$$

input
```
integrate(tan(d*x+c)^6/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

output
```
-1/6*(6*d*x*cos(d*x + c)^3 - 3*cos(d*x + c)^3*log(sin(d*x + c) + 1) + 3*co
s(d*x + c)^3*log(-sin(d*x + c) + 1) - 2*(2*cos(d*x + c)^2 - 3*cos(d*x + c)
 + 1)*sin(d*x + c))/(a^2*d*cos(d*x + c)^3)
```

3.80.6 Sympy [F]

$$\int \frac{\tan^6(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \frac{\int \frac{\tan^6(c+dx)}{\sec^2(c+dx)+2\sec(c+dx)+1}\,dx}{a^2}$$

input
```
integrate(tan(d*x+c)**6/(a+a*sec(d*x+c))**2,x)
```

output
```
Integral(tan(c + d*x)**6/(sec(c + d*x)**2 + 2*sec(c + d*x) + 1), x)/a**2
```

3.80.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 196 vs. 2(70) = 140.

Time = 0.31 (sec) , antiderivative size = 196, normalized size of antiderivative = 2.72

$$\int \frac{\tan^6(c+dx)}{(a+a\sec(c+dx))^2}\,dx = -\frac{\frac{4\left(\frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{3\sin(dx+c)^5}{(\cos(dx+c)+1)^5}\right)}{a^2-\frac{3a^2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{3a^2\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{a^2\sin(dx+c)^6}{(\cos(dx+c)+1)^6}}+\frac{6\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^2}-\frac{3\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a^2}+\frac{3\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a^2}}{3\,d}$$

input
```
integrate(tan(d*x+c)^6/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

output
```
-1/3*(4*(sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 3*sin(d*x + c)^5/(cos(d*x +
 c) + 1)^5)/(a^2 - 3*a^2*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 3*a^2*sin(d
*x + c)^4/(cos(d*x + c) + 1)^4 - a^2*sin(d*x + c)^6/(cos(d*x + c) + 1)^6)
+ 6*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^2 - 3*log(sin(d*x + c)/(cos(
d*x + c) + 1) + 1)/a^2 + 3*log(sin(d*x + c)/(cos(d*x + c) + 1) - 1)/a^2)/d
```

3.80.8 Giac [A] (verification not implemented)

Time = 2.22 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.38

$$\int \frac{\tan^6(c+dx)}{(a+a\sec(c+dx))^2}\,dx = -\frac{\frac{3(dx+c)}{a^2}-\frac{3\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)}{a^2}+\frac{3\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)}{a^2}+\frac{4\left(3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5-\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3\right)}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)^3a^2}}{3\,d}$$

input
```
integrate(tan(d*x+c)^6/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

output
```
-1/3*(3*(d*x + c)/a^2 - 3*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^2 + 3*log(a
bs(tan(1/2*d*x + 1/2*c) - 1))/a^2 + 4*(3*tan(1/2*d*x + 1/2*c)^5 - tan(1/2*
d*x + 1/2*c)^3)/((tan(1/2*d*x + 1/2*c)^2 - 1)^3*a^2))/d
```

3.80.9 Mupad [B] (verification not implemented)

Time = 14.48 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.54

$$\int \frac{\tan^6(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \frac{2\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{a^2\,d} - \frac{x}{a^2} + \frac{\frac{4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3}{3} - 4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5}{d\left(a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6 - 3\,a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4 + 3\,a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2 - a^2\right)}$$

input
```
int(tan(c + d*x)^6/(a + a/cos(c + d*x))^2,x)
```

output
```
(2*atanh(tan(c/2 + (d*x)/2)))/(a^2*d) - x/a^2 + ((4*tan(c/2 + (d*x)/2)^3)/
3 - 4*tan(c/2 + (d*x)/2)^5)/(d*(3*a^2*tan(c/2 + (d*x)/2)^2 - 3*a^2*tan(c/2
 + (d*x)/2)^4 + a^2*tan(c/2 + (d*x)/2)^6 - a^2))
```

3.81 $\int \frac{\tan^4(c+dx)}{(a+a\sec(c+dx))^2}\,dx$

3.81.1 Optimal result . . . 621
3.81.2 Mathematica [B] (verified) . . . 621
3.81.3 Rubi [A] (verified) . . . 622
3.81.4 Maple [C] (verified) . . . 624
3.81.5 Fricas [A] (verification not implemented) . . . 624
3.81.6 Sympy [F] . . . 625
3.81.7 Maxima [B] (verification not implemented) . . . 625
3.81.8 Giac [B] (verification not implemented) . . . 625
3.81.9 Mupad [B] (verification not implemented) . . . 626

3.81.1 Optimal result

Integrand size = 21, antiderivative size = 34

$$\int \frac{\tan^4(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \frac{x}{a^2} - \frac{2\operatorname{arctanh}(\sin(c+dx))}{a^2 d} + \frac{\tan(c+dx)}{a^2 d}$$

output
```
x/a^2-2*arctanh(sin(d*x+c))/a^2/d+tan(d*x+c)/a^2/d
```

3.81.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 177 vs. 2(34) = 68.

Time = 1.05 (sec) , antiderivative size = 177, normalized size of antiderivative = 5.21

$$\int \frac{\tan^4(c+dx)}{(a+a\sec(c+dx))^2}\,dx$$
$$= \frac{4\cos^4\left(\frac{1}{2}(c+dx)\right)\sec^2(c+dx)\left(dx + 2\log\left(\cos\left(\frac{1}{2}(c+dx)\right) - \sin\left(\frac{1}{2}(c+dx)\right)\right) - 2\log\left(\cos\left(\frac{1}{2}(c+dx)\right) +\right.}{a^2 d(1+\sec(c+$$

input
```
Integrate[Tan[c + d*x]^4/(a + a*Sec[c + d*x])^2,x]
```

output
```
(4*Cos[(c + d*x)/2]^4*Sec[c + d*x]^2*(d*x + 2*Log[Cos[(c + d*x)/2] - Sin[(
c + d*x)/2]] - 2*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + Sin[d*x]/((Cos
[c/2] - Sin[c/2])*(Cos[c/2] + Sin[c/2])*(Cos[(c + d*x)/2] - Sin[(c + d*x)/
2])*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]))))/(a^2*d*(1 + Sec[c + d*x])^2)
```

3.81.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.12, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {3042, 4376, 3042, 4260, 3042, 4254, 24, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^4(c+dx)}{(a\sec(c+dx)+a)^2}\,dx$$

$$\downarrow 3042$$

$$\int \frac{\cot\left(c+dx+\frac{\pi}{2}\right)^4}{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^2}dx$$

$$\downarrow 4376$$

$$\frac{\int (a\sec(c+dx)-a)^2 dx}{a^4}$$

$$\downarrow 3042$$

$$\frac{\int \left(a\csc\left(c+dx+\frac{\pi}{2}\right)-a\right)^2 dx}{a^4}$$

$$\downarrow 4260$$

$$\frac{a^2\int \sec^2(c+dx)dx - 2a^2\int \sec(c+dx)dx + a^2x}{a^4}$$

$$\downarrow 3042$$

$$\frac{-2a^2\int \csc\left(c+dx+\frac{\pi}{2}\right)dx + a^2\int \csc\left(c+dx+\frac{\pi}{2}\right)^2 dx + a^2x}{a^4}$$

$$\downarrow 4254$$

$$\frac{-\frac{a^2\int 1 d(-\tan(c+dx))}{d} - 2a^2\int \csc\left(c+dx+\frac{\pi}{2}\right)dx + a^2x}{a^4}$$

$$\downarrow 24$$

$$\frac{-2a^2\int \csc\left(c+dx+\frac{\pi}{2}\right)dx + \frac{a^2\tan(c+dx)}{d} + a^2x}{a^4}$$

$$\downarrow 4257$$

$$\frac{-\frac{2a^2\text{arctanh}(\sin(c+dx))}{d} + \frac{a^2\tan(c+dx)}{d} + a^2x}{a^4}$$

input
```
Int[Tan[c + d*x]^4/(a + a*Sec[c + d*x])^2,x]
```

output
```
(a^2*x - (2*a^2*ArcTanh[Sin[c + d*x]])/d + (a^2*Tan[c + d*x])/d)/a^4
```

3.81.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 4260
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^2, x_Symbol] :> Simp[a^2*x, x] +
 (Simp[2*a*b   Int[Csc[c + d*x], x], x] + Simp[b^2   Int[Csc[c + d*x]^2, x]
, x]) /; FreeQ[{a, b, c, d}, x]
```

rule 4376
```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Simp[a^(2*n)/e^(2*n)   Int[(e*Cot[c + d*x])^(m + 2*
n)/(-a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[a
^2 - b^2, 0] && ILtQ[n, 0]
```

3.81.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.72 (sec) , antiderivative size = 71, normalized size of antiderivative = 2.09

method	result	size
risch	$\frac{x}{a^2}+\frac{2i}{d\,a^2\left(e^{2i(dx+c)}+1\right)}+\frac{2\ln\left(e^{i(dx+c)}-i\right)}{a^2d}-\frac{2\ln\left(e^{i(dx+c)}+i\right)}{a^2d}$	71
derivativedivides	$\frac{-\frac{1}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1}-2\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)+2\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\frac{1}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1}+2\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{a^2d}$	80
default	$\frac{-\frac{1}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1}-2\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)+2\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\frac{1}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1}+2\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{a^2d}$	80

input
```
int(tan(d*x+c)^4/(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
x/a^2+2*I/d/a^2/(exp(2*I*(d*x+c))+1)+2/a^2/d*ln(exp(I*(d*x+c))-I)-2/a^2/d*
ln(exp(I*(d*x+c))+I)
```

3.81.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.94

$$\int \frac{\tan^4(c+dx)}{(a+a\sec(c+dx))^2}\,dx$$
$$= \frac{dx\cos(dx+c)-\cos(dx+c)\log(\sin(dx+c)+1)+\cos(dx+c)\log(-\sin(dx+c)+1)+\sin(dx+c)}{a^2d\cos(dx+c)}$$

input
```
integrate(tan(d*x+c)^4/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

output
```
(d*x*cos(d*x + c) - cos(d*x + c)*log(sin(d*x + c) + 1) + cos(d*x + c)*log(
-sin(d*x + c) + 1) + sin(d*x + c))/(a^2*d*cos(d*x + c))
```

3.81.6 Sympy [F]

$$\int \frac{\tan^4(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \frac{\int \frac{\tan^4(c+dx)}{\sec^2(c+dx)+2\sec(c+dx)+1}\,dx}{a^2}$$

input
```
integrate(tan(d*x+c)**4/(a+a*sec(d*x+c))**2,x)
```

output
```
Integral(tan(c + d*x)**4/(sec(c + d*x)**2 + 2*sec(c + d*x) + 1), x)/a**2
```

3.81.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 123 vs. $2(34) = 68$.

Time = 0.30 (sec) , antiderivative size = 123, normalized size of antiderivative = 3.62

$$\int \frac{\tan^4(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \frac{2\left(\frac{\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^2} - \frac{\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a^2} + \frac{\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a^2} + \frac{\sin(dx+c)}{\left(a^2-\frac{a^2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right)(\cos(dx+c)+1)}\right)}{d}$$

input
```
integrate(tan(d*x+c)^4/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

output
```
2*(arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^2 - log(sin(d*x + c)/(cos(d*x
 + c) + 1) + 1)/a^2 + log(sin(d*x + c)/(cos(d*x + c) + 1) - 1)/a^2 + sin(d
*x + c)/((a^2 - a^2*sin(d*x + c)^2/(cos(d*x + c) + 1)^2)*(cos(d*x + c) + 1
)))/d
```

3.81.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 79 vs. $2(34) = 68$.

Time = 0.95 (sec) , antiderivative size = 79, normalized size of antiderivative = 2.32

$$\int \frac{\tan^4(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \frac{\frac{dx+c}{a^2} - \frac{2\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)}{a^2} + \frac{2\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)}{a^2} - \frac{2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)a^2}}{d}$$

input
```
integrate(tan(d*x+c)^4/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

output
```
((d*x + c)/a^2 - 2*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^2 + 2*log(abs(tan(
1/2*d*x + 1/2*c) - 1))/a^2 - 2*tan(1/2*d*x + 1/2*c)/((tan(1/2*d*x + 1/2*c)
^2 - 1)*a^2))/d
```

3.81.9 Mupad [B] (verification not implemented)

Time = 14.21 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.79

$$\int \frac{\tan^4(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \frac{x}{a^2} - \frac{4\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{a^2\,d} - \frac{2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{d\left(a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2 - a^2\right)}$$

input
```
int(tan(c + d*x)^4/(a + a/cos(c + d*x))^2,x)
```

output
```
x/a^2 - (4*atanh(tan(c/2 + (d*x)/2)))/(a^2*d) - (2*tan(c/2 + (d*x)/2))/(d*
(a^2*tan(c/2 + (d*x)/2)^2 - a^2))
```

3.82 $\int \frac{\tan^2(c+dx)}{(a+a\sec(c+dx))^2}\,dx$

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3.82.2 Mathematica [A] (verified) . . . 627
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3.82.5 Fricas [A] (verification not implemented) . . . 630
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3.82.8 Giac [A] (verification not implemented) . . . 631
3.82.9 Mupad [B] (verification not implemented) . . . 631

3.82.1 Optimal result

Integrand size = 21, antiderivative size = 33

$$\int \frac{\tan^2(c+dx)}{(a+a\sec(c+dx))^2}\,dx = -\frac{x}{a^2} + \frac{2\tan(c+dx)}{ad(a+a\sec(c+dx))}$$

output
```
-x/a^2+2*tan(d*x+c)/a/d/(a+a*sec(d*x+c))
```

3.82.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.27

$$\int \frac{\tan^2(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \frac{-\frac{2\arctan\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{d} + \frac{2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{d}}{a^2}$$

input
```
Integrate[Tan[c + d*x]^2/(a + a*Sec[c + d*x])^2,x]
```

output
```
((-2*ArcTan[Tan[c/2 + (d*x)/2]])/d + (2*Tan[c/2 + (d*x)/2])/d)/a^2
```

3.82.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.18, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3042, 4376, 3042, 4374, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^2(c+dx)}{(a\sec(c+dx)+a)^2}\,dx$$

$$\downarrow 3042$$

$$\int \frac{\cot\left(c+dx+\frac{\pi}{2}\right)^2}{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^2}dx$$

$$\downarrow 4376$$

$$\frac{\int \cot^2(c+dx)(a-a\sec(c+dx))^2dx}{a^4}$$

$$\downarrow 3042$$

$$\frac{\int \frac{(a-a\csc(c+dx+\frac{\pi}{2}))^2}{\cot(c+dx+\frac{\pi}{2})^2}dx}{a^4}$$

$$\downarrow 4374$$

$$\frac{\int \left(\cot^2(c+dx)a^2+\csc^2(c+dx)a^2-2\cot(c+dx)\csc(c+dx)a^2\right)dx}{a^4}$$

$$\downarrow 2009$$

$$\frac{-\frac{2a^2\cot(c+dx)}{d}+\frac{2a^2\csc(c+dx)}{d}+a^2(-x)}{a^4}$$

input
```
Int[Tan[c + d*x]^2/(a + a*Sec[c + d*x])^2,x]
```

output
```
(-(a^2*x) - (2*a^2*Cot[c + d*x])/d + (2*a^2*Csc[c + d*x])/d)/a^4
```

3.82.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4374
```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

rule 4376
```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Simp[a^(2*n)/e^(2*n)   Int[(e*Cot[c + d*x])^(m + 2*
n)/(-a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[a
^2 - b^2, 0] && ILtQ[n, 0]
```

3.82.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.62 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.91

method	result	size
risch	$-\frac{x}{a^2}+\frac{4i}{a^2 d\left(e^{i(dx+c)}+1\right)}$	30
derivativedivides	$\frac{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a^2 d}$	31
default	$\frac{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a^2 d}$	31

input
```
int(tan(d*x+c)^2/(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
-x/a^2+4*I/a^2/d/(exp(I*(d*x+c))+1)
```

3.82.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.27

$$\int \frac{\tan^2(c+dx)}{(a+a\sec(c+dx))^2}\,dx = -\frac{dx\cos(dx+c)+dx-2\sin(dx+c)}{a^2d\cos(dx+c)+a^2d}$$

input
```
integrate(tan(d*x+c)^2/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

output
```
-(d*x*cos(d*x + c) + d*x - 2*sin(d*x + c))/(a^2*d*cos(d*x + c) + a^2*d)
```

3.82.6 Sympy [F]

$$\int \frac{\tan^2(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \frac{\int \frac{\tan^2(c+dx)}{\sec^2(c+dx)+2\sec(c+dx)+1}\,dx}{a^2}$$

input
```
integrate(tan(d*x+c)**2/(a+a*sec(d*x+c))**2,x)
```

output
```
Integral(tan(c + d*x)**2/(sec(c + d*x)**2 + 2*sec(c + d*x) + 1), x)/a**2
```

3.82.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.48

$$\int \frac{\tan^2(c+dx)}{(a+a\sec(c+dx))^2}\,dx = -\frac{2\left(\frac{\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^2}-\frac{\sin(dx+c)}{a^2(\cos(dx+c)+1)}\right)}{d}$$

input
```
integrate(tan(d*x+c)^2/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

output
```
-2*(arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^2 - sin(d*x + c)/(a^2*(cos(d
*x + c) + 1)))/d
```

3.82.8 Giac [A] (verification not implemented)

Time = 0.51 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.88

$$\int \frac{\tan^2(c+dx)}{(a+a\sec(c+dx))^2}\,dx = -\frac{\frac{dx+c}{a^2} - \frac{2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^2}}{d}$$

input
```
integrate(tan(d*x+c)^2/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

output
```
-((d*x + c)/a^2 - 2*tan(1/2*d*x + 1/2*c)/a^2)/d
```

3.82.9 Mupad [B] (verification not implemented)

Time = 13.99 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.67

$$\int \frac{\tan^2(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \frac{2\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)-\frac{dx}{2}\right)}{a^2\,d}$$

input
```
int(tan(c + d*x)^2/(a + a/cos(c + d*x))^2,x)
```

output
```
(2*(tan(c/2 + (d*x)/2) - (d*x)/2))/(a^2*d)
```

3.83 $\int \frac{\cot^2(c+dx)}{(a+a\sec(c+dx))^2}\,dx$

3.83.1 Optimal result 632
3.83.2 Mathematica [A] (verified) 632
3.83.3 Rubi [A] (verified) 633
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3.83.7 Maxima [A] (verification not implemented) 636
3.83.8 Giac [A] (verification not implemented) 636
3.83.9 Mupad [B] (verification not implemented) 637

3.83.1 Optimal result

Integrand size = 21, antiderivative size = 107

$$\int \frac{\cot^2(c+dx)}{(a+a\sec(c+dx))^2}\,dx = -\frac{x}{a^2} - \frac{\cot(c+dx)}{a^2 d} + \frac{\cot^3(c+dx)}{3a^2 d} - \frac{2\cot^5(c+dx)}{5a^2 d} + \frac{2\csc(c+dx)}{a^2 d} - \frac{4\csc^3(c+dx)}{3a^2 d} + \frac{2\csc^5(c+dx)}{5a^2 d}$$

output
```
-x/a^2-cot(d*x+c)/a^2/d+1/3*cot(d*x+c)^3/a^2/d-2/5*cot(d*x+c)^5/a^2/d+2*cs
c(d*x+c)/a^2/d-4/3*csc(d*x+c)^3/a^2/d+2/5*csc(d*x+c)^5/a^2/d
```

3.83.2 Mathematica [A] (verified)

Time = 1.48 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.39

$$\int \frac{\cot^2(c+dx)}{(a+a\sec(c+dx))^2}\,dx$$
$$= \frac{\sec^2(c+dx)\left(-120dx\cos^4\left(\frac{1}{2}(c+dx)\right) - 31\cos\left(\frac{1}{2}(c+dx)\right)\sec\left(\frac{c}{2}\right)\sin\left(\frac{dx}{2}\right) + \cos^3\left(\frac{1}{2}(c+dx)\right)\left(15\cot\left(\frac{1}{2}\right.\right.\right.}{30a^2 d(1+\sec(c+$$

input
```
Integrate[Cot[c + d*x]^2/(a + a*Sec[c + d*x])^2,x]
```

output
```
(Sec[c + d*x]^2*(-120*d*x*Cos[(c + d*x)/2]^4 - 31*Cos[(c + d*x)/2]*Sec[c/2
]*Sin[(d*x)/2] + Cos[(c + d*x)/2]^3*(15*Cot[(c + d*x)/2]*Csc[c/2] + 193*Se
c[c/2])*Sin[(d*x)/2] - 31*Cos[(c + d*x)/2]^2*Tan[c/2] + 3*Tan[(c + d*x)/2]
))/(30*a^2*d*(1 + Sec[c + d*x])^2)
```

3.83.3 Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.04, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3042, 4376, 3042, 4374, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^2(c+dx)}{(a\sec(c+dx)+a)^2}\,dx$$

$$\downarrow 3042$$

$$\int \frac{1}{\cot\left(c+dx+\frac{\pi}{2}\right)^2\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^2}dx$$

$$\downarrow 4376$$

$$\frac{\int \cot^6(c+dx)(a-a\sec(c+dx))^2dx}{a^4}$$

$$\downarrow 3042$$

$$\frac{\int \frac{(a-a\csc(c+dx+\frac{\pi}{2}))^2}{\cot(c+dx+\frac{\pi}{2})^6}dx}{a^4}$$

$$\downarrow 4374$$

$$\frac{\int \left(a^2\cot^6(c+dx)-2a^2\csc(c+dx)\cot^5(c+dx)+a^2\csc^2(c+dx)\cot^4(c+dx)\right)dx}{a^4}$$

$$\downarrow 2009$$

$$\frac{-\frac{2a^2\cot^5(c+dx)}{5d}+\frac{a^2\cot^3(c+dx)}{3d}-\frac{a^2\cot(c+dx)}{d}+\frac{2a^2\csc^5(c+dx)}{5d}-\frac{4a^2\csc^3(c+dx)}{3d}+\frac{2a^2\csc(c+dx)}{d}-a^2x}{a^4}$$

input
```
Int[Cot[c + d*x]^2/(a + a*Sec[c + d*x])^2,x]
```

output
```
(-(a^2*x) - (a^2*Cot[c + d*x])/d + (a^2*Cot[c + d*x]^3)/(3*d) - (2*a^2*Cot
[c + d*x]^5)/(5*d) + (2*a^2*Csc[c + d*x])/d - (4*a^2*Csc[c + d*x]^3)/(3*d)
 + (2*a^2*Csc[c + d*x]^5)/(5*d))/a^4
```

3.83.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4374
```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

rule 4376
```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Simp[a^(2*n)/e^(2*n)   Int[(e*Cot[c + d*x])^(m + 2*
n)/(-a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[a
^2 - b^2, 0] && ILtQ[n, 0]
```

3.83.4 Maple [A] (verified)

Time = 0.66 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.67

method	result	size
derivativedivides	$\frac{\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{5}-\frac{5\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{3}+11\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{1}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}-16\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8da^2}$	72
default	$\frac{\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{5}-\frac{5\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{3}+11\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{1}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}-16\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8da^2}$	72
risch	$-\frac{x}{a^2}+\frac{4i\left(15\,e^{5i(dx+c)}+30\,e^{4i(dx+c)}+10\,e^{3i(dx+c)}-35\,e^{2i(dx+c)}-37\,e^{i(dx+c)}-13\right)}{15da^2\left(e^{i(dx+c)}+1\right)^5\left(e^{i(dx+c)}-1\right)}$	100

input
```
int(cot(d*x+c)^2/(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/8/d/a^2*(1/5*tan(1/2*d*x+1/2*c)^5-5/3*tan(1/2*d*x+1/2*c)^3+11*tan(1/2*d*
x+1/2*c)-1/tan(1/2*d*x+1/2*c)-16*arctan(tan(1/2*d*x+1/2*c)))
```

3.83.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.99

$$\int \frac{\cot^2(c+dx)}{(a+a\sec(c+dx))^2}\,dx = -\frac{26\cos(dx+c)^3+22\cos(dx+c)^2+15\left(dx\cos(dx+c)^2+2\,dx\cos(dx+c)+dx\right)\sin(dx+c)-17\,c}{15\left(a^2d\cos(dx+c)^2+2\,a^2d\cos(dx+c)+a^2d\right)\sin(dx+c)}$$

input
```
integrate(cot(d*x+c)^2/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

output
```
-1/15*(26*cos(d*x + c)^3 + 22*cos(d*x + c)^2 + 15*(d*x*cos(d*x + c)^2 + 2*
d*x*cos(d*x + c) + d*x)*sin(d*x + c) - 17*cos(d*x + c) - 16)/((a^2*d*cos(d
*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)*sin(d*x + c))
```

3.83.6 Sympy [F]

$$\int \frac{\cot^2(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \frac{\int \frac{\cot^2(c+dx)}{\sec^2(c+dx)+2\sec(c+dx)+1}\,dx}{a^2}$$

input
```
integrate(cot(d*x+c)**2/(a+a*sec(d*x+c))**2,x)
```

output
```
Integral(cot(c + d*x)**2/(sec(c + d*x)**2 + 2*sec(c + d*x) + 1), x)/a**2
```

3.83.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.06

$$\int \frac{\cot^2(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \frac{\frac{\frac{165\sin(dx+c)}{\cos(dx+c)+1}-\frac{25\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{3\sin(dx+c)^5}{(\cos(dx+c)+1)^5}}{a^2}-\frac{240\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^2}-\frac{15(\cos(dx+c)+1)}{a^2\sin(dx+c)}}{120\,d}$$

input
```
integrate(cot(d*x+c)^2/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

output
```
1/120*((165*sin(d*x + c)/(cos(d*x + c) + 1) - 25*sin(d*x + c)^3/(cos(d*x +
 c) + 1)^3 + 3*sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/a^2 - 240*arctan(sin(d
*x + c)/(cos(d*x + c) + 1))/a^2 - 15*(cos(d*x + c) + 1)/(a^2*sin(d*x + c))
)/d
```

3.83.8 Giac [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.79

$$\int \frac{\cot^2(c+dx)}{(a+a\sec(c+dx))^2}\,dx = -\frac{\frac{120\,(dx+c)}{a^2}+\frac{15}{a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}-\frac{3\,a^8\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-25\,a^8\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+165\,a^8\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^{10}}}{120\,d}$$

input
```
integrate(cot(d*x+c)^2/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

output
```
-1/120*(120*(d*x + c)/a^2 + 15/(a^2*tan(1/2*d*x + 1/2*c)) - (3*a^8*tan(1/2
*d*x + 1/2*c)^5 - 25*a^8*tan(1/2*d*x + 1/2*c)^3 + 165*a^8*tan(1/2*d*x + 1/
2*c))/a^10)/d
```

3.83.9 Mupad [B] (verification not implemented)

Time = 14.34 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.73

$$\int \frac{\cot^2(c+dx)}{(a+a\sec(c+dx))^2}\,dx = -\frac{x}{a^2} - \frac{\frac{26\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^6}{15} - \frac{28\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^4}{15} + \frac{17\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^2}{60} - \frac{1}{40}}{a^2\,d\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^5\sin\left(\frac{c}{2}+\frac{dx}{2}\right)}$$

input
```
int(cot(c + d*x)^2/(a + a/cos(c + d*x))^2,x)
```

output
```
- x/a^2 - ((17*cos(c/2 + (d*x)/2)^2)/60 - (28*cos(c/2 + (d*x)/2)^4)/15 + (
26*cos(c/2 + (d*x)/2)^6)/15 - 1/40)/(a^2*d*cos(c/2 + (d*x)/2)^5*sin(c/2 +
(d*x)/2))
```

3.84 $\int \frac{\cot^4(c+dx)}{(a+a\sec(c+dx))^2}\,dx$

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3.84.1 Optimal result

Integrand size = 21, antiderivative size = 139

$$\int \frac{\cot^4(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \frac{x}{a^2} + \frac{\cot(c+dx)}{a^2 d} - \frac{\cot^3(c+dx)}{3a^2 d} + \frac{\cot^5(c+dx)}{5a^2 d} - \frac{2\cot^7(c+dx)}{7a^2 d} - \frac{2\csc(c+dx)}{a^2 d} + \frac{2\csc^3(c+dx)}{a^2 d} - \frac{6\csc^5(c+dx)}{5a^2 d} + \frac{2\csc^7(c+dx)}{7a^2 d}$$

output

```
x/a^2+cot(d*x+c)/a^2/d-1/3*cot(d*x+c)^3/a^2/d+1/5*cot(d*x+c)^5/a^2/d-2/7*c
ot(d*x+c)^7/a^2/d-2*csc(d*x+c)/a^2/d+2*csc(d*x+c)^3/a^2/d-6/5*csc(d*x+c)^5
/a^2/d+2/7*csc(d*x+c)^7/a^2/d
```

3.84.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 314 vs. 2(139) = 278.

Time = 1.36 (sec) , antiderivative size = 314, normalized size of antiderivative = 2.26

$$\int \frac{\cot^4(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \frac{\csc\left(\frac{c}{2}\right)\csc^3(c+dx)\sec\left(\frac{c}{2}\right)\sec^2(c+dx)(5880dx\cos(dx) - 5880dx\cos(2c+dx) + 3360dx\cos(c+2dx) -$$

input
```
Integrate[Cot[c + d*x]^4/(a + a*Sec[c + d*x])^2,x]
```

output
```
(Csc[c/2]*Csc[c + d*x]^3*Sec[c/2]*Sec[c + d*x]^2*(5880*d*x*Cos[d*x] - 5880
*d*x*Cos[2*c + d*x] + 3360*d*x*Cos[c + 2*d*x] - 3360*d*x*Cos[3*c + 2*d*x]
- 1260*d*x*Cos[2*c + 3*d*x] + 1260*d*x*Cos[4*c + 3*d*x] - 1680*d*x*Cos[3*c
 + 4*d*x] + 1680*d*x*Cos[5*c + 4*d*x] - 420*d*x*Cos[4*c + 5*d*x] + 420*d*x
*Cos[6*c + 5*d*x] - 4032*Sin[c] - 9632*Sin[d*x] + 16002*Sin[c + d*x] + 914
4*Sin[2*(c + d*x)] - 3429*Sin[3*(c + d*x)] - 4572*Sin[4*(c + d*x)] - 1143*
Sin[5*(c + d*x)] - 11760*Sin[2*c + d*x] - 8864*Sin[c + 2*d*x] - 3360*Sin[3
*c + 2*d*x] + 2064*Sin[2*c + 3*d*x] + 2520*Sin[4*c + 3*d*x] + 4432*Sin[3*c
 + 4*d*x] + 1680*Sin[5*c + 4*d*x] + 1528*Sin[4*c + 5*d*x]))/(26880*a^2*d*(
1 + Sec[c + d*x])^2)
```

3.84.3 Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.03, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3042, 4376, 3042, 4374, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^4(c+dx)}{(a\sec(c+dx)+a)^2}\,dx$$

$$\downarrow 3042$$

$$\int \frac{1}{\cot\left(c+dx+\frac{\pi}{2}\right)^4\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^2}dx$$

$$\downarrow 4376$$

$$\frac{\int \cot^8(c+dx)(a-a\sec(c+dx))^2dx}{a^4}$$

$$\downarrow 3042$$

$$\frac{\int \frac{\left(a-a\csc\left(c+dx+\frac{\pi}{2}\right)\right)^2}{\cot\left(c+dx+\frac{\pi}{2}\right)^8}dx}{a^4}$$

$$\downarrow 4374$$

$$\frac{\int \left(a^2\cot^8(c+dx)-2a^2\csc(c+dx)\cot^7(c+dx)+a^2\csc^2(c+dx)\cot^6(c+dx)\right)dx}{a^4}$$

$$\downarrow 2009$$

$$\frac{-\frac{2a^2\cot^7(c+dx)}{7d}+\frac{a^2\cot^5(c+dx)}{5d}-\frac{a^2\cot^3(c+dx)}{3d}+\frac{a^2\cot(c+dx)}{d}+\frac{2a^2\csc^7(c+dx)}{7d}-\frac{6a^2\csc^5(c+dx)}{5d}+\frac{2a^2\csc^3(c+dx)}{d}-\frac{2a^2\csc(c-}{d}}{a^4}$$

input
```
Int[Cot[c + d*x]^4/(a + a*Sec[c + d*x])^2,x]
```

output
```
(a^2*x + (a^2*Cot[c + d*x])/d - (a^2*Cot[c + d*x]^3)/(3*d) + (a^2*Cot[c +
d*x]^5)/(5*d) - (2*a^2*Cot[c + d*x]^7)/(7*d) - (2*a^2*Csc[c + d*x])/d + (2
*a^2*Csc[c + d*x]^3)/d - (6*a^2*Csc[c + d*x]^5)/(5*d) + (2*a^2*Csc[c + d*x
]^7)/(7*d))/a^4
```

3.84.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4374
```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

rule 4376
```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Simp[a^(2*n)/e^(2*n)   Int[(e*Cot[c + d*x])^(m + 2*
n)/(-a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[a
^2 - b^2, 0] && ILtQ[n, 0]
```

3.84.4 Maple [A] (verified)

Time = 0.62 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.71

method	result
derivativedivides	$\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{7}-\frac{7\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{5}+\frac{22\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{3}-42\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+64\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\frac{1}{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}+\frac{7}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}$ / $32da^2$
default	$\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{7}-\frac{7\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{5}+\frac{22\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{3}-42\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+64\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\frac{1}{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}+\frac{7}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}$ / $32da^2$
risch	$\frac{x}{a^2}-\frac{2i\left(210\,e^{9i(dx+c)}+315\,e^{8i(dx+c)}-420\,e^{7i(dx+c)}-1470\,e^{6i(dx+c)}-504\,e^{5i(dx+c)}+1204\,e^{4i(dx+c)}+1108\,e^{3i(dx+c)}-\right.}{105da^2\left(e^{i(dx+c)}+1\right)^7\left(e^{i(dx+c)}-1\right)^3}$

input
```
int(cot(d*x+c)^4/(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/32/d/a^2*(1/7*tan(1/2*d*x+1/2*c)^7-7/5*tan(1/2*d*x+1/2*c)^5+22/3*tan(1/2
*d*x+1/2*c)^3-42*tan(1/2*d*x+1/2*c)+64*arctan(tan(1/2*d*x+1/2*c))-1/3/tan(
1/2*d*x+1/2*c)^3+7/tan(1/2*d*x+1/2*c))
```

3.84.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.11

$$\int \frac{\cot^4(c+dx)}{(a+a\sec(c+dx))^2}\,dx$$
$$= \frac{191\cos(dx+c)^5+172\cos(dx+c)^4-253\cos(dx+c)^3-258\cos(dx+c)^2+105\left(dx\cos(dx+c)^4+2\right.}{105\left(a^2d\cos(dx+c)^4+2a^2d\cos(dx+c)^3-2a^2d\cos\right.}$$

input
```
integrate(cot(d*x+c)^4/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

output
```
1/105*(191*cos(d*x + c)^5 + 172*cos(d*x + c)^4 - 253*cos(d*x + c)^3 - 258*
cos(d*x + c)^2 + 105*(d*x*cos(d*x + c)^4 + 2*d*x*cos(d*x + c)^3 - 2*d*x*co
s(d*x + c) - d*x)*sin(d*x + c) + 87*cos(d*x + c) + 96)/((a^2*d*cos(d*x + c
)^4 + 2*a^2*d*cos(d*x + c)^3 - 2*a^2*d*cos(d*x + c) - a^2*d)*sin(d*x + c))
```

3.84.6 Sympy [F]

$$\int \frac{\cot^4(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \frac{\int \frac{\cot^4(c+dx)}{\sec^2(c+dx)+2\sec(c+dx)+1}\,dx}{a^2}$$

input
```
integrate(cot(d*x+c)**4/(a+a*sec(d*x+c))**2,x)
```

output
```
Integral(cot(c + d*x)**4/(sec(c + d*x)**2 + 2*sec(c + d*x) + 1), x)/a**2
```

3.84.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.13

$$\int \frac{\cot^4(c+dx)}{(a+a\sec(c+dx))^2}\,dx = -\frac{\frac{4410\sin(dx+c)}{\cos(dx+c)+1} - \frac{770\sin(dx+c)^3}{(\cos(dx+c)+1)^3} + \frac{147\sin(dx+c)^5}{(\cos(dx+c)+1)^5} - \frac{15\sin(dx+c)^7}{(\cos(dx+c)+1)^7}}{a^2} - \frac{6720\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^2} - \frac{35\left(\frac{21\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-1\right)(\cos(dx+c)+1)}{a^2\sin(dx+c)^3}}{3360\,d}$$

input
```
integrate(cot(d*x+c)^4/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

output
```
-1/3360*((4410*sin(d*x + c)/(cos(d*x + c) + 1) - 770*sin(d*x + c)^3/(cos(d
*x + c) + 1)^3 + 147*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 15*sin(d*x + c)
^7/(cos(d*x + c) + 1)^7)/a^2 - 6720*arctan(sin(d*x + c)/(cos(d*x + c) + 1)
)/a^2 - 35*(21*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 1)*(cos(d*x + c) + 1)
^3/(a^2*sin(d*x + c)^3))/d
```

3.84.8 Giac [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.82

$$\int \frac{\cot^4(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \frac{\frac{3360\,(dx+c)}{a^2} + \frac{35\left(21\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)}{a^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3} + \frac{15a^{12}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^7 - 147a^{12}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5 + 770a^{12}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3 - 4410a^{12}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{a^{14}}}{3360\,d}$$

input
```
integrate(cot(d*x+c)^4/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

output
```
1/3360*(3360*(d*x + c)/a^2 + 35*(21*tan(1/2*d*x + 1/2*c)^2 - 1)/(a^2*tan(1
/2*d*x + 1/2*c)^3) + (15*a^12*tan(1/2*d*x + 1/2*c)^7 - 147*a^12*tan(1/2*d*
x + 1/2*c)^5 + 770*a^12*tan(1/2*d*x + 1/2*c)^3 - 4410*a^12*tan(1/2*d*x + 1
/2*c))/a^14)/d
```

3.84.9 Mupad [B] (verification not implemented)

Time = 14.93 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.31

$$\int \frac{\cot^4(c+dx)}{(a+a\sec(c+dx))^2}\,dx$$
$$= \frac{15\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{10} - 35\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{10} - 147\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^2\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^8 + 770\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^4\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^6}{3360\,a^2\,d\cos($$

input
```
int(cot(c + d*x)^4/(a + a/cos(c + d*x))^2,x)
```

output
```
(15*sin(c/2 + (d*x)/2)^10 - 35*cos(c/2 + (d*x)/2)^10 - 147*cos(c/2 + (d*x)
/2)^2*sin(c/2 + (d*x)/2)^8 + 770*cos(c/2 + (d*x)/2)^4*sin(c/2 + (d*x)/2)^6
 - 4410*cos(c/2 + (d*x)/2)^6*sin(c/2 + (d*x)/2)^4 + 735*cos(c/2 + (d*x)/2)
^8*sin(c/2 + (d*x)/2)^2 + 3360*cos(c/2 + (d*x)/2)^7*sin(c/2 + (d*x)/2)^3*(
c + d*x))/(3360*a^2*d*cos(c/2 + (d*x)/2)^7*sin(c/2 + (d*x)/2)^3)
```

3.85 $\int \frac{\cot^6(c+dx)}{(a+a\sec(c+dx))^2}\,dx$

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3.85.1 Optimal result

Integrand size = 21, antiderivative size = 179

$$\int \frac{\cot^6(c+dx)}{(a+a\sec(c+dx))^2}\,dx = -\frac{x}{a^2} - \frac{\cot(c+dx)}{a^2 d} + \frac{\cot^3(c+dx)}{3a^2 d} - \frac{\cot^5(c+dx)}{5a^2 d} + \frac{\cot^7(c+dx)}{7a^2 d} - \frac{2\cot^9(c+dx)}{9a^2 d} + \frac{2\csc(c+dx)}{a^2 d} - \frac{8\csc^3(c+dx)}{3a^2 d} + \frac{12\csc^5(c+dx)}{5a^2 d} - \frac{8\csc^7(c+dx)}{7a^2 d} + \frac{2\csc^9(c+dx)}{9a^2 d}$$

output

```
-x/a^2-cot(d*x+c)/a^2/d+1/3*cot(d*x+c)^3/a^2/d-1/5*cot(d*x+c)^5/a^2/d+1/7*
cot(d*x+c)^7/a^2/d-2/9*cot(d*x+c)^9/a^2/d+2*csc(d*x+c)/a^2/d-8/3*csc(d*x+c
)^3/a^2/d+12/5*csc(d*x+c)^5/a^2/d-8/7*csc(d*x+c)^7/a^2/d+2/9*csc(d*x+c)^9/
a^2/d
```

3.85.2 Mathematica [A] (verified)

Time = 6.50 (sec) , antiderivative size = 276, normalized size of antiderivative = 1.54

$$\int \frac{\cot^6(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \frac{\cos^4\left(\frac{1}{2}(c+dx)\right)\sec^2(c+dx)\left(-63\cot^2\left(\frac{c}{2}\right)\csc^2\left(\frac{1}{2}(c+dx)\right)\left(-17+\csc^2\left(\frac{1}{2}(c+dx)\right)\right) - \frac{63}{8}\cot\left(\frac{c}{2}\right)(5120$$

input
```
Integrate[Cot[c + d*x]^6/(a + a*Sec[c + d*x])^2,x]
```

output
```
(Cos[(c + d*x)/2]^4*Sec[c + d*x]^2*(-63*Cot[c/2]^2*Csc[(c + d*x)/2]^2*(-17
 + Csc[(c + d*x)/2]^2) - (63*Cot[c/2]*(5120*d*x + (-543 + 736*Cos[c + d*x]
 - 201*Cos[2*(c + d*x)])*Csc[c/2]*Csc[(c + d*x)/2]^5*Sin[(d*x)/2]))/8 + (C
sc[c/2]*Sec[(c + d*x)/2]^9*(4360986*Sin[(d*x)/2] - 3688020*Sin[c + (d*x)/2
] + 3365964*Sin[c + (3*d*x)/2] - 2000040*Sin[2*c + (3*d*x)/2] + 1660896*Si
n[2*c + (5*d*x)/2] - 638820*Sin[3*c + (5*d*x)/2] + 479484*Sin[3*c + (7*d*x
)/2] - 95445*Sin[4*c + (7*d*x)/2] + 63881*Sin[4*c + (9*d*x)/2]))/256)*Tan[
c/2])/(10080*a^2*d*(1 + Sec[c + d*x])^2)
```

3.85.3 Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.02, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3042, 4376, 3042, 4374, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^6(c+dx)}{(a\sec(c+dx)+a)^2}\,dx$$

$$\downarrow 3042$$

$$\int \frac{1}{\cot\left(c+dx+\frac{\pi}{2}\right)^6\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^2}dx$$

$$\downarrow 4376$$

$$\frac{\int \cot^{10}(c+dx)(a-a\sec(c+dx))^2dx}{a^4}$$

$$\downarrow 3042$$

$$\frac{\int \frac{(a-a\csc(c+dx+\frac{\pi}{2}))^2}{\cot(c+dx+\frac{\pi}{2})^{10}}dx}{a^4}$$

$$\downarrow 4374$$

$$\frac{\int \left(a^2\cot^{10}(c+dx)-2a^2\csc(c+dx)\cot^9(c+dx)+a^2\csc^2(c+dx)\cot^8(c+dx)\right)dx}{a^4}$$

$$\downarrow 2009$$

$$\frac{-\frac{2a^2\cot^9(c+dx)}{9d}+\frac{a^2\cot^7(c+dx)}{7d}-\frac{a^2\cot^5(c+dx)}{5d}+\frac{a^2\cot^3(c+dx)}{3d}-\frac{a^2\cot(c+dx)}{d}+\frac{2a^2\csc^9(c+dx)}{9d}-\frac{8a^2\csc^7(c+dx)}{7d}+\frac{12a^2\csc^5(c}{5d}}{a^4}$$

input
```
Int[Cot[c + d*x]^6/(a + a*Sec[c + d*x])^2,x]
```

output
```
(-(a^2*x) - (a^2*Cot[c + d*x])/d + (a^2*Cot[c + d*x]^3)/(3*d) - (a^2*Cot[c
 + d*x]^5)/(5*d) + (a^2*Cot[c + d*x]^7)/(7*d) - (2*a^2*Cot[c + d*x]^9)/(9*
d) + (2*a^2*Csc[c + d*x])/d - (8*a^2*Csc[c + d*x]^3)/(3*d) + (12*a^2*Csc[c
 + d*x]^5)/(5*d) - (8*a^2*Csc[c + d*x]^7)/(7*d) + (2*a^2*Csc[c + d*x]^9)/(
9*d))/a^4
```

3.85.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4374
```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

rule 4376
```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Simp[a^(2*n)/e^(2*n)   Int[(e*Cot[c + d*x])^(m + 2*
n)/(-a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[a
^2 - b^2, 0] && ILtQ[n, 0]
```

3.85.4 Maple [A] (verified)

Time = 0.67 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.69

method	result
derivativedivides	$\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^9}{9}-\frac{9\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{7}+\frac{37\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{5}-31\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3+163\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-256\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\frac{}{5\tan($ $128da^2$
default	$\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^9}{9}-\frac{9\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{7}+\frac{37\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{5}-31\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3+163\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-256\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\frac{}{5\tan($ $128da^2$
risch	$-\frac{x}{a^2}+\frac{4i\left(315\,e^{13i(dx+c)}+315\,e^{12i(dx+c)}-1470\,e^{11i(dx+c)}-3360\,e^{10i(dx+c)}+1113\,e^{9i(dx+c)}+6447\,e^{8i(dx+c)}+2028\,e^{7}\right.}{315da^2\left(e^{i(dx+c)}+1\right.}$

input
```
int(cot(d*x+c)^6/(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/128/d/a^2*(1/9*tan(1/2*d*x+1/2*c)^9-9/7*tan(1/2*d*x+1/2*c)^7+37/5*tan(1/
2*d*x+1/2*c)^5-31*tan(1/2*d*x+1/2*c)^3+163*tan(1/2*d*x+1/2*c)-256*arctan(t
an(1/2*d*x+1/2*c))-1/5/tan(1/2*d*x+1/2*c)^5+3/tan(1/2*d*x+1/2*c)^3-37/tan(
1/2*d*x+1/2*c))
```

3.85.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 250, normalized size of antiderivative = 1.40

$$\int \frac{\cot^6(c+dx)}{(a+a\sec(c+dx))^2}\,dx =$$
$$-\frac{598\cos(dx+c)^7+566\cos(dx+c)^6-1212\cos(dx+c)^5-1310\cos(dx+c)^4+860\cos(dx+c)^3+}{315\left(a^2d\cos(dx+c)^6+2a^2d\right.}$$

input
```
integrate(cot(d*x+c)^6/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

output
```
-1/315*(598*cos(d*x + c)^7 + 566*cos(d*x + c)^6 - 1212*cos(d*x + c)^5 - 13
10*cos(d*x + c)^4 + 860*cos(d*x + c)^3 + 1014*cos(d*x + c)^2 + 315*(d*x*co
s(d*x + c)^6 + 2*d*x*cos(d*x + c)^5 - d*x*cos(d*x + c)^4 - 4*d*x*cos(d*x +
 c)^3 - d*x*cos(d*x + c)^2 + 2*d*x*cos(d*x + c) + d*x)*sin(d*x + c) - 197*
cos(d*x + c) - 256)/((a^2*d*cos(d*x + c)^6 + 2*a^2*d*cos(d*x + c)^5 - a^2*
d*cos(d*x + c)^4 - 4*a^2*d*cos(d*x + c)^3 - a^2*d*cos(d*x + c)^2 + 2*a^2*d
*cos(d*x + c) + a^2*d)*sin(d*x + c))
```

3.85.6 Sympy [F]

$$\int \frac{\cot^6(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \frac{\int \frac{\cot^6(c+dx)}{\sec^2(c+dx)+2\sec(c+dx)+1}\,dx}{a^2}$$

input
```
integrate(cot(d*x+c)**6/(a+a*sec(d*x+c))**2,x)
```

output
```
Integral(cot(c + d*x)**6/(sec(c + d*x)**2 + 2*sec(c + d*x) + 1), x)/a**2
```

3.85.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.10

$$\int \frac{\cot^6(c+dx)}{(a+a\sec(c+dx))^2}\,dx$$
$$= \frac{\frac{51345\sin(dx+c)}{\cos(dx+c)+1} - \frac{9765\sin(dx+c)^3}{(\cos(dx+c)+1)^3} + \frac{2331\sin(dx+c)^5}{(\cos(dx+c)+1)^5} - \frac{405\sin(dx+c)^7}{(\cos(dx+c)+1)^7} + \frac{35\sin(dx+c)^9}{(\cos(dx+c)+1)^9}}{a^2} - \frac{80640\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^2} + \frac{63\left(\frac{15\sin(dx+c)^2}{(\cos(dx+c)+1)^2} - \frac{1}{(c}\right.}{a^{}}$$
$$\frac{}{40320\,d}$$

input
```
integrate(cot(d*x+c)^6/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

output
```
1/40320*((51345*sin(d*x + c)/(cos(d*x + c) + 1) - 9765*sin(d*x + c)^3/(cos
(d*x + c) + 1)^3 + 2331*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 405*sin(d*x
+ c)^7/(cos(d*x + c) + 1)^7 + 35*sin(d*x + c)^9/(cos(d*x + c) + 1)^9)/a^2
- 80640*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^2 + 63*(15*sin(d*x + c)^
2/(cos(d*x + c) + 1)^2 - 185*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 - 1)*(cos
(d*x + c) + 1)^5/(a^2*sin(d*x + c)^5))/d
```

3.85.8 Giac [A] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.80

$$\int \frac{\cot^6(c+dx)}{(a+a\sec(c+dx))^2}\,dx =$$
$$-\frac{\frac{40320\,(dx+c)}{a^2} + \frac{63\left(185\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4 - 15\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+1\right)}{a^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5} - \frac{35\,a^{16}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^9 - 405\,a^{16}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^7+2331\,a^{16}\tan\left(\frac{1}{2}d}{a^{18}}}{40320\,d}$$

input
```
integrate(cot(d*x+c)^6/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

output
```
-1/40320*(40320*(d*x + c)/a^2 + 63*(185*tan(1/2*d*x + 1/2*c)^4 - 15*tan(1/
2*d*x + 1/2*c)^2 + 1)/(a^2*tan(1/2*d*x + 1/2*c)^5) - (35*a^16*tan(1/2*d*x
+ 1/2*c)^9 - 405*a^16*tan(1/2*d*x + 1/2*c)^7 + 2331*a^16*tan(1/2*d*x + 1/2
*c)^5 - 9765*a^16*tan(1/2*d*x + 1/2*c)^3 + 51345*a^16*tan(1/2*d*x + 1/2*c)
)/a^18)/d
```

3.85.9 Mupad [B] (verification not implemented)

Time = 15.78 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.28

$$\int \frac{\cot^6(c+dx)}{(a+a\sec(c+dx))^2}\,dx =$$
$$-\frac{63\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{14}-35\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{14}+405\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{2}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{12}-2331\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{4}\sin\left(\frac{c}{2}+$$

input
```
int(cot(c + d*x)^6/(a + a/cos(c + d*x))^2,x)
```

output
```
-(63*cos(c/2 + (d*x)/2)^14 - 35*sin(c/2 + (d*x)/2)^14 + 405*cos(c/2 + (d*x
)/2)^2*sin(c/2 + (d*x)/2)^12 - 2331*cos(c/2 + (d*x)/2)^4*sin(c/2 + (d*x)/2
)^10 + 9765*cos(c/2 + (d*x)/2)^6*sin(c/2 + (d*x)/2)^8 - 51345*cos(c/2 + (d
*x)/2)^8*sin(c/2 + (d*x)/2)^6 + 11655*cos(c/2 + (d*x)/2)^10*sin(c/2 + (d*x
)/2)^4 - 945*cos(c/2 + (d*x)/2)^12*sin(c/2 + (d*x)/2)^2 + 40320*cos(c/2 +
(d*x)/2)^9*sin(c/2 + (d*x)/2)^5*(c + d*x))/(40320*a^2*d*cos(c/2 + (d*x)/2)
^9*sin(c/2 + (d*x)/2)^5)
```

3.86 $\int \frac{\tan^{11}(c+dx)}{(a+a\sec(c+dx))^3}\,dx$

3.86.1 Optimal result 650
3.86.2 Mathematica [A] (verified) 650
3.86.3 Rubi [A] (verified) 651
3.86.4 Maple [A] (verified) 653
3.86.5 Fricas [A] (verification not implemented) 653
3.86.6 Sympy [F] 654
3.86.7 Maxima [A] (verification not implemented) 654
3.86.8 Giac [A] (verification not implemented) 654
3.86.9 Mupad [B] (verification not implemented) 655

3.86.1 Optimal result

Integrand size = 21, antiderivative size = 137

$$\int \frac{\tan^{11}(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{\log(\cos(c+dx))}{a^3 d} + \frac{3\sec(c+dx)}{a^3 d} - \frac{\sec^2(c+dx)}{2a^3 d} - \frac{5\sec^3(c+dx)}{3a^3 d} + \frac{5\sec^4(c+dx)}{4a^3 d} + \frac{\sec^5(c+dx)}{5a^3 d} - \frac{\sec^6(c+dx)}{2a^3 d} + \frac{\sec^7(c+dx)}{7a^3 d}$$

output
```
ln(cos(d*x+c))/a^3/d+3*sec(d*x+c)/a^3/d-1/2*sec(d*x+c)^2/a^3/d-5/3*sec(d*x
+c)^3/a^3/d+5/4*sec(d*x+c)^4/a^3/d+1/5*sec(d*x+c)^5/a^3/d-1/2*sec(d*x+c)^6
/a^3/d+1/7*sec(d*x+c)^7/a^3/d
```

3.86.2 Mathematica [A] (verified)

Time = 0.45 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.02

$$\int \frac{\tan^{11}(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{(3732 + 4522\cos(2(c+dx)) + 1050\cos(3(c+dx)) + 2380\cos(4(c+dx)) - 210\cos(5(c+dx)) + 630\cos($$

input
```
Integrate[Tan[c + d*x]^11/(a + a*Sec[c + d*x])^3,x]
```

output
```
((3732 + 4522*Cos[2*(c + d*x)] + 1050*Cos[3*(c + d*x)] + 2380*Cos[4*(c + d
*x)] - 210*Cos[5*(c + d*x)] + 630*Cos[6*(c + d*x)] + 2205*Cos[3*(c + d*x)]
*Log[Cos[c + d*x]] + 735*Cos[5*(c + d*x)]*Log[Cos[c + d*x]] + 105*Cos[7*(c
 + d*x)]*Log[Cos[c + d*x]] + 105*Cos[c + d*x]*(8 + 35*Log[Cos[c + d*x]]))*
Sec[c + d*x]^7)/(6720*a^3*d)
```

3.86.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.72, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 25, 4367, 27, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^{11}(c+dx)}{(a\sec(c+dx)+a)^3}\,dx$$

$$\downarrow 3042$$

$$\int -\frac{\cot\left(c+dx+\frac{\pi}{2}\right)^{11}}{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^3}dx$$

$$\downarrow 25$$

$$-\int \frac{\cot\left(\frac{1}{2}(2c+\pi)+dx\right)^{11}}{\left(\csc\left(\frac{1}{2}(2c+\pi)+dx\right)a+a\right)^3}dx$$

$$\downarrow 4367$$

$$-\frac{\int a^7(1-\cos(c+dx))^5(\cos(c+dx)+1)^2\sec^8(c+dx)d\cos(c+dx)}{a^{10}d}$$

$$\downarrow 27$$

$$-\frac{\int(1-\cos(c+dx))^5(\cos(c+dx)+1)^2\sec^8(c+dx)d\cos(c+dx)}{a^3d}$$

$$\downarrow 99$$

$$-\frac{\int\left(\sec^8(c+dx)-3\sec^7(c+dx)+\sec^6(c+dx)+5\sec^5(c+dx)-5\sec^4(c+dx)-\sec^3(c+dx)+3\sec^2(c+dx)\right.}{a^3d}$$

$$\downarrow 2009$$

$$-\frac{-\frac{1}{7}\sec^7(c+dx)+\frac{1}{2}\sec^6(c+dx)-\frac{1}{5}\sec^5(c+dx)-\frac{5}{4}\sec^4(c+dx)+\frac{5}{3}\sec^3(c+dx)+\frac{1}{2}\sec^2(c+dx)-3\sec(c+$$
$$a^3d$$

input
```
Int[Tan[c + d*x]^11/(a + a*Sec[c + d*x])^3,x]
```

output
```
-((-Log[Cos[c + d*x]] - 3*Sec[c + d*x] + Sec[c + d*x]^2/2 + (5*Sec[c + d*x
]^3)/3 - (5*Sec[c + d*x]^4)/4 - Sec[c + d*x]^5/5 + Sec[c + d*x]^6/2 - Sec[
c + d*x]^7/7)/(a^3*d))
```

3.86.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4367
```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Simp[1/(a^(m - n - 1)*b^n*d)   Subst[Int[(a - b*x)^((m -
1)/2)*((a + b*x)^((m - 1)/2 + n)/x^(m + n)), x], x, Sin[c + d*x]], x] /; Fr
eeQ[{a, b, c, d}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && Integer
Q[n]
```

3.86.4 Maple [A] (verified)

Time = 4.21 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.63

method	result
derivativedivides	$\frac{\ln(\cos(dx+c))+\frac{3}{\cos(dx+c)}+\frac{1}{5\cos(dx+c)^5}-\frac{1}{2\cos(dx+c)^2}+\frac{5}{4\cos(dx+c)^4}-\frac{1}{2\cos(dx+c)^6}+\frac{1}{7\cos(dx+c)^7}-\frac{5}{3\cos(dx+c)^3}}{d\,a^3}$
default	$\frac{\ln(\cos(dx+c))+\frac{3}{\cos(dx+c)}+\frac{1}{5\cos(dx+c)^5}-\frac{1}{2\cos(dx+c)^2}+\frac{5}{4\cos(dx+c)^4}-\frac{1}{2\cos(dx+c)^6}+\frac{1}{7\cos(dx+c)^7}-\frac{5}{3\cos(dx+c)^3}}{d\,a^3}$
risch	$-\frac{ix}{a^3}-\frac{2ic}{a^3d}+\frac{6\,e^{13i(dx+c)}-2\,e^{12i(dx+c)}+\frac{68\,e^{11i(dx+c)}}{3}+10\,e^{10i(dx+c)}+\frac{646\,e^{9i(dx+c)}}{15}+8\,e^{8i(dx+c)}+\frac{2488\,e^{7i(dx+c)}}{35}+}{d\,a^3\left(e^{2i(dx+c)}+1\right)^7}$

input
```
int(tan(d*x+c)^11/(a+a*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/d/a^3*(ln(cos(d*x+c))+3/cos(d*x+c)+1/5/cos(d*x+c)^5-1/2/cos(d*x+c)^2+5/4
/cos(d*x+c)^4-1/2/cos(d*x+c)^6+1/7/cos(d*x+c)^7-5/3/cos(d*x+c)^3)
```

3.86.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.69

$$\int \frac{\tan^{11}(c+dx)}{(a+a\sec(c+dx))^3}\,dx$$
$$= \frac{420\,\cos(dx+c)^7\log(-\cos(dx+c)) + 1260\,\cos(dx+c)^6 - 210\,\cos(dx+c)^5 - 700\,\cos(dx+c)^4 + 525}{420\,a^3 d\cos(dx+c)^7}$$

input
```
integrate(tan(d*x+c)^11/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

output
```
1/420*(420*cos(d*x + c)^7*log(-cos(d*x + c)) + 1260*cos(d*x + c)^6 - 210*c
os(d*x + c)^5 - 700*cos(d*x + c)^4 + 525*cos(d*x + c)^3 + 84*cos(d*x + c)^
2 - 210*cos(d*x + c) + 60)/(a^3*d*cos(d*x + c)^7)
```

3.86.6 Sympy [F]

$$\int \frac{\tan^{11}(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{\int \frac{\tan^{11}(c+dx)}{\sec^3(c+dx)+3\sec^2(c+dx)+3\sec(c+dx)+1}\,dx}{a^3}$$

input
```
integrate(tan(d*x+c)**11/(a+a*sec(d*x+c))**3,x)
```

output
```
Integral(tan(c + d*x)**11/(sec(c + d*x)**3 + 3*sec(c + d*x)**2 + 3*sec(c +
 d*x) + 1), x)/a**3
```

3.86.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.66

$$\int \frac{\tan^{11}(c+dx)}{(a+a\sec(c+dx))^3}\,dx$$
$$= \frac{\frac{420\log(\cos(dx+c))}{a^3} + \frac{1260\cos(dx+c)^6 - 210\cos(dx+c)^5 - 700\cos(dx+c)^4 + 525\cos(dx+c)^3 + 84\cos(dx+c)^2 - 210\cos(dx+c) + 60}{a^3\cos(dx+c)^7}}{420\,d}$$

input
```
integrate(tan(d*x+c)^11/(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

output
```
1/420*(420*log(cos(d*x + c))/a^3 + (1260*cos(d*x + c)^6 - 210*cos(d*x + c)
^5 - 700*cos(d*x + c)^4 + 525*cos(d*x + c)^3 + 84*cos(d*x + c)^2 - 210*cos
(d*x + c) + 60)/(a^3*cos(d*x + c)^7))/d
```

3.86.8 Giac [A] (verification not implemented)

Time = 20.53 (sec) , antiderivative size = 246, normalized size of antiderivative = 1.80

$$\int \frac{\tan^{11}(c+dx)}{(a+a\sec(c+dx))^3}\,dx =$$
$$\frac{420\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right)}{a^3} - \frac{420\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right)}{a^3} - \frac{\frac{1393(\cos(dx+c)-1)}{\cos(dx+c)+1} + \frac{819(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2} - \frac{6755(\cos(dx+c)-1)^3}{(\cos(dx+c)+1)^3} - \frac{20195(\cos(dx+c}{}}{a^3\left(\frac{\cos(}{\cos(}\right.}$$
$$- \frac{}{420\,d}$$

input
```
integrate(tan(d*x+c)^11/(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

output
```
-1/420*(420*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1))/a^3 - 420
*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1))/a^3 - (1393*(cos(d*x
 + c) - 1)/(cos(d*x + c) + 1) + 819*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1
)^2 - 6755*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 - 20195*(cos(d*x + c)
 - 1)^4/(cos(d*x + c) + 1)^4 - 28749*(cos(d*x + c) - 1)^5/(cos(d*x + c) +
1)^5 - 8463*(cos(d*x + c) - 1)^6/(cos(d*x + c) + 1)^6 - 1089*(cos(d*x + c)
 - 1)^7/(cos(d*x + c) + 1)^7 + 319)/(a^3*((cos(d*x + c) - 1)/(cos(d*x + c)
 + 1) + 1)^7))/d
```

3.86.9 Mupad [B] (verification not implemented)

Time = 18.40 (sec) , antiderivative size = 225, normalized size of antiderivative = 1.64

$$\int \frac{\tan^{11}(c+dx)}{(a+a\sec(c+dx))^3}\,dx = -\frac{2\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2\right)}{a^3\,d}$$
$$-\frac{-2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{12}+14\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}+\frac{128\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^8}{3}-\frac{224\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6}{3}+\frac{282\,\mathrm{ta}}{}}{d\left(a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{14}-7\,a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{12}+21\,a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}-35\,a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^8+35\,a^3\tan\left(\frac{c}{2}\right.\right.}$$

input
```
int(tan(c + d*x)^11/(a + a/cos(c + d*x))^3,x)
```

output
```
- (2*atanh(tan(c/2 + (d*x)/2)^2))/(a^3*d) - ((282*tan(c/2 + (d*x)/2)^4)/5
- (322*tan(c/2 + (d*x)/2)^2)/15 - (224*tan(c/2 + (d*x)/2)^6)/3 + (128*tan(
c/2 + (d*x)/2)^8)/3 + 14*tan(c/2 + (d*x)/2)^10 - 2*tan(c/2 + (d*x)/2)^12 +
 352/105)/(d*(7*a^3*tan(c/2 + (d*x)/2)^2 - 21*a^3*tan(c/2 + (d*x)/2)^4 + 3
5*a^3*tan(c/2 + (d*x)/2)^6 - 35*a^3*tan(c/2 + (d*x)/2)^8 + 21*a^3*tan(c/2
+ (d*x)/2)^10 - 7*a^3*tan(c/2 + (d*x)/2)^12 + a^3*tan(c/2 + (d*x)/2)^14 -
a^3))
```

3.87 $\int \frac{\tan^9(c+dx)}{(a+a\sec(c+dx))^3}\,dx$

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3.87.1 Optimal result

Integrand size = 21, antiderivative size = 99

$$\int \frac{\tan^9(c+dx)}{(a+a\sec(c+dx))^3}\,dx = -\frac{\log(\cos(c+dx))}{a^3d} - \frac{3\sec(c+dx)}{a^3d} + \frac{\sec^2(c+dx)}{a^3d} + \frac{2\sec^3(c+dx)}{3a^3d} - \frac{3\sec^4(c+dx)}{4a^3d} + \frac{\sec^5(c+dx)}{5a^3d}$$

output
```
-ln(cos(d*x+c))/a^3/d-3*sec(d*x+c)/a^3/d+sec(d*x+c)^2/a^3/d+2/3*sec(d*x+c)
^3/a^3/d-3/4*sec(d*x+c)^4/a^3/d+1/5*sec(d*x+c)^5/a^3/d
```

3.87.2 Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.94

$$\int \frac{\tan^9(c+dx)}{(a+a\sec(c+dx))^3}\,dx = -\frac{(142+280\cos(2(c+dx))+90\cos(4(c+dx))+150\cos(c+dx)\log(\cos(c+dx))+15\cos(5(c+dx))\,l}{240a^3d}$$

input
```
Integrate[Tan[c + d*x]^9/(a + a*Sec[c + d*x])^3,x]
```

output
```
-1/240*((142 + 280*Cos[2*(c + d*x)] + 90*Cos[4*(c + d*x)] + 150*Cos[c + d*
x]*Log[Cos[c + d*x]] + 15*Cos[5*(c + d*x)]*Log[Cos[c + d*x]] + 15*Cos[3*(c
 + d*x)]*(-4 + 5*Log[Cos[c + d*x]]))*Sec[c + d*x]^5)/(a^3*d)
```

3.87.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.71, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 25, 4367, 27, 84, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^9(c+dx)}{(a\sec(c+dx)+a)^3}\,dx$$

$$\downarrow 3042$$

$$\int -\frac{\cot\left(c+dx+\frac{\pi}{2}\right)^9}{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^3}dx$$

$$\downarrow 25$$

$$-\int \frac{\cot\left(\frac{1}{2}(2c+\pi)+dx\right)^9}{\left(\csc\left(\frac{1}{2}(2c+\pi)+dx\right)a+a\right)^3}dx$$

$$\downarrow 4367$$

$$-\frac{\int a^5(1-\cos(c+dx))^4(\cos(c+dx)+1)\sec^6(c+dx)d\cos(c+dx)}{a^8 d}$$

$$\downarrow 27$$

$$-\frac{\int (1-\cos(c+dx))^4(\cos(c+dx)+1)\sec^6(c+dx)d\cos(c+dx)}{a^3 d}$$

$$\downarrow 84$$

$$-\frac{\int \left(\sec^6(c+dx)-3\sec^5(c+dx)+2\sec^4(c+dx)+2\sec^3(c+dx)-3\sec^2(c+dx)+\sec(c+dx)\right)d\cos(c+dx)}{a^3 d}$$

$$\downarrow 2009$$

$$-\frac{-\frac{1}{5}\sec^5(c+dx)+\frac{3}{4}\sec^4(c+dx)-\frac{2}{3}\sec^3(c+dx)-\sec^2(c+dx)+3\sec(c+dx)+\log(\cos(c+dx))}{a^3 d}$$

input
```
Int[Tan[c + d*x]^9/(a + a*Sec[c + d*x])^3,x]
```

output
```
-((Log[Cos[c + d*x]] + 3*Sec[c + d*x] - Sec[c + d*x]^2 - (2*Sec[c + d*x]^3
)/3 + (3*Sec[c + d*x]^4)/4 - Sec[c + d*x]^5/5)/(a^3*d))
```

3.87.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 84
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && EqQ[b*e + a*f, 0] &&  !(ILtQ[n + p + 2, 0
] && GtQ[n + 2*p, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4367
```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Simp[1/(a^(m - n - 1)*b^n*d)   Subst[Int[(a - b*x)^((m -
1)/2)*((a + b*x)^((m - 1)/2 + n)/x^(m + n)), x], x, Sin[c + d*x]], x] /; Fr
eeQ[{a, b, c, d}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && Integer
Q[n]
```

3.87.4 Maple [A] (verified)

Time = 2.39 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.67

method	result
derivativedivides	$\frac{\frac{1}{5\cos(dx+c)^5}-\ln(\cos(dx+c))+\frac{2}{3\cos(dx+c)^3}-\frac{3}{4\cos(dx+c)^4}-\frac{3}{\cos(dx+c)}+\frac{1}{\cos(dx+c)^2}}{d\,a^3}$
default	$\frac{\frac{1}{5\cos(dx+c)^5}-\ln(\cos(dx+c))+\frac{2}{3\cos(dx+c)^3}-\frac{3}{4\cos(dx+c)^4}-\frac{3}{\cos(dx+c)}+\frac{1}{\cos(dx+c)^2}}{d\,a^3}$
risch	$\frac{ix}{a^3}+\frac{2ic}{a^3d}-\frac{2\left(45\,e^{9i(dx+c)}-30\,e^{8i(dx+c)}+140\,e^{7i(dx+c)}+142\,e^{5i(dx+c)}+140\,e^{3i(dx+c)}-30\,e^{2i(dx+c)}+45\,e^{i(dx+c)}\right)}{15d\,a^3\left(e^{2i(dx+c)}+1\right)^5}$

input
```
int(tan(d*x+c)^9/(a+a*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/d/a^3*(1/5/cos(d*x+c)^5-ln(cos(d*x+c))+2/3/cos(d*x+c)^3-3/4/cos(d*x+c)^4
-3/cos(d*x+c)+1/cos(d*x+c)^2)
```

3.87.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.76

$$\int \frac{\tan^9(c+dx)}{(a+a\sec(c+dx))^3}\,dx =$$
$$-\frac{60\cos(dx+c)^5\log(-\cos(dx+c)) + 180\cos(dx+c)^4 - 60\cos(dx+c)^3 - 40\cos(dx+c)^2 + 45\cos}{60\,a^3 d\cos(dx+c)^5}$$

input
```
integrate(tan(d*x+c)^9/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

output
```
-1/60*(60*cos(d*x + c)^5*log(-cos(d*x + c)) + 180*cos(d*x + c)^4 - 60*cos(
d*x + c)^3 - 40*cos(d*x + c)^2 + 45*cos(d*x + c) - 12)/(a^3*d*cos(d*x + c)
^5)
```

3.87.6 Sympy [F]

$$\int \frac{\tan^9(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{\int \frac{\tan^9(c+dx)}{\sec^3(c+dx)+3\sec^2(c+dx)+3\sec(c+dx)+1}\,dx}{a^3}$$

input
```
integrate(tan(d*x+c)**9/(a+a*sec(d*x+c))**3,x)
```

output
```
Integral(tan(c + d*x)**9/(sec(c + d*x)**3 + 3*sec(c + d*x)**2 + 3*sec(c +
d*x) + 1), x)/a**3
```

3.87.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.71

$$\int \frac{\tan^9(c+dx)}{(a+a\sec(c+dx))^3}\,dx = -\frac{\frac{60\log(\cos(dx+c))}{a^3} + \frac{180\cos(dx+c)^4 - 60\cos(dx+c)^3 - 40\cos(dx+c)^2 + 45\cos(dx+c) - 12}{a^3\cos(dx+c)^5}}{60\,d}$$

input
```
integrate(tan(d*x+c)^9/(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

output
```
-1/60*(60*log(cos(d*x + c))/a^3 + (180*cos(d*x + c)^4 - 60*cos(d*x + c)^3
- 40*cos(d*x + c)^2 + 45*cos(d*x + c) - 12)/(a^3*cos(d*x + c)^5))/d
```

3.87.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 202 vs. 2(93) = 186.

Time = 6.25 (sec) , antiderivative size = 202, normalized size of antiderivative = 2.04

$$\int \frac{\tan^9(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{\frac{60\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right)}{a^3} - \frac{60\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right)}{a^3} - \frac{\frac{475\,(\cos(dx+c)-1)}{\cos(dx+c)+1} + \frac{590\,(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2} - \frac{50\,(\cos(dx+c)-1)^3}{(\cos(dx+c)+1)^3} - \frac{805\,(\cos(dx+c)-1)^4}{(\cos(dx+c)+1)^4} - \cdots}{a^3\left(\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right)^5}}{60\,d}$$

input
```
integrate(tan(d*x+c)^9/(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

output
```
1/60*(60*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1))/a^3 - 60*log
(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1))/a^3 - (475*(cos(d*x + c)
 - 1)/(cos(d*x + c) + 1) + 590*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 -
 50*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 - 805*(cos(d*x + c) - 1)^4/(
cos(d*x + c) + 1)^4 - 137*(cos(d*x + c) - 1)^5/(cos(d*x + c) + 1)^5 + 119)
/(a^3*((cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)^5))/d
```

3.87.9 Mupad [B] (verification not implemented)

Time = 18.70 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.69

$$\int \frac{\tan^9(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{2\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2\right)}{a^3\,d}$$
$$-\frac{2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^8+22\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6-\frac{98\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4}{3}+\frac{58\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2}{3}-\frac{64}{15}}{d\left(a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}-5\,a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^8+10\,a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6-10\,a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4+5\,a^3\tan\left(\frac{c}{2}+\right.\right.}$$

input
```
int(tan(c + d*x)^9/(a + a/cos(c + d*x))^3,x)
```

output
```
(2*atanh(tan(c/2 + (d*x)/2)^2))/(a^3*d) - ((58*tan(c/2 + (d*x)/2)^2)/3 - (
98*tan(c/2 + (d*x)/2)^4)/3 + 22*tan(c/2 + (d*x)/2)^6 + 2*tan(c/2 + (d*x)/2
)^8 - 64/15)/(d*(5*a^3*tan(c/2 + (d*x)/2)^2 - 10*a^3*tan(c/2 + (d*x)/2)^4
+ 10*a^3*tan(c/2 + (d*x)/2)^6 - 5*a^3*tan(c/2 + (d*x)/2)^8 + a^3*tan(c/2 +
 (d*x)/2)^10 - a^3))
```

3.88 $\int \frac{\tan^7(c+dx)}{(a+a\sec(c+dx))^3}\,dx$

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3.88.7 Maxima [A] (verification not implemented) . . . 665
3.88.8 Giac [B] (verification not implemented) . . . 666
3.88.9 Mupad [B] (verification not implemented) . . . 666

3.88.1 Optimal result

Integrand size = 21, antiderivative size = 65

$$\int \frac{\tan^7(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{\log(\cos(c+dx))}{a^3 d} + \frac{3\sec(c+dx)}{a^3 d} - \frac{3\sec^2(c+dx)}{2a^3 d} + \frac{\sec^3(c+dx)}{3a^3 d}$$

output

```
ln(cos(d*x+c))/a^3/d+3*sec(d*x+c)/a^3/d-3/2*sec(d*x+c)^2/a^3/d+1/3*sec(d*x
+c)^3/a^3/d
```

3.88.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.98

$$\int \frac{\tan^7(c+dx)}{(a+a\sec(c+dx))^3}\,dx$$
$$= \frac{(22+18\cos(2(c+dx))+9\cos(c+dx)(-2+\log(\cos(c+dx)))+3\cos(3(c+dx))\log(\cos(c+dx)))\sec^3(}{12a^3 d}$$

input

```
Integrate[Tan[c + d*x]^7/(a + a*Sec[c + d*x])^3,x]
```

output

```
((22 + 18*Cos[2*(c + d*x)] + 9*Cos[c + d*x]*(-2 + Log[Cos[c + d*x]]) + 3*C
os[3*(c + d*x)]*Log[Cos[c + d*x]])*Sec[c + d*x]^3)/(12*a^3*d)
```

3.88.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.77, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 25, 4367, 27, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^7(c+dx)}{(a\sec(c+dx)+a)^3}\,dx$$

$$\downarrow 3042$$

$$\int -\frac{\cot\left(c+dx+\frac{\pi}{2}\right)^7}{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^3}dx$$

$$\downarrow 25$$

$$-\int \frac{\cot\left(\frac{1}{2}(2c+\pi)+dx\right)^7}{\left(\csc\left(\frac{1}{2}(2c+\pi)+dx\right)a+a\right)^3}dx$$

$$\downarrow 4367$$

$$-\frac{\int a^3(1-\cos(c+dx))^3\sec^4(c+dx)d\cos(c+dx)}{a^6 d}$$

$$\downarrow 27$$

$$-\frac{\int(1-\cos(c+dx))^3\sec^4(c+dx)d\cos(c+dx)}{a^3 d}$$

$$\downarrow 49$$

$$-\frac{\int\left(\sec^4(c+dx)-3\sec^3(c+dx)+3\sec^2(c+dx)-\sec(c+dx)\right)d\cos(c+dx)}{a^3 d}$$

$$\downarrow 2009$$

$$-\frac{-\frac{1}{3}\sec^3(c+dx)+\frac{3}{2}\sec^2(c+dx)-3\sec(c+dx)-\log(\cos(c+dx))}{a^3 d}$$

input
```
Int[Tan[c + d*x]^7/(a + a*Sec[c + d*x])^3,x]
```

output
```
-((-Log[Cos[c + d*x]] - 3*Sec[c + d*x] + (3*Sec[c + d*x]^2)/2 - Sec[c + d*
x]^3/3)/(a^3*d))
```

3.88.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4367
```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Simp[1/(a^(m - n - 1)*b^n*d)   Subst[Int[(a - b*x)^((m -
1)/2)*((a + b*x)^((m - 1)/2 + n)/x^(m + n)), x], x, Sin[c + d*x]], x] /; Fr
eeQ[{a, b, c, d}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && Integer
Q[n]
```

3.88.4 Maple [A] (verified)

Time = 1.62 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.71

method	result	size
derivativedivides	$\frac{\frac{1}{3\cos(dx+c)^3}+\ln(\cos(dx+c))-\frac{3}{2\cos(dx+c)^2}+\frac{3}{\cos(dx+c)}}{d\,a^3}$	46
default	$\frac{\frac{1}{3\cos(dx+c)^3}+\ln(\cos(dx+c))-\frac{3}{2\cos(dx+c)^2}+\frac{3}{\cos(dx+c)}}{d\,a^3}$	46
risch	$-\frac{ix}{a^3}-\frac{2ic}{a^3d}+\frac{6\,e^{5i(dx+c)}-6\,e^{4i(dx+c)}+\frac{44\,e^{3i(dx+c)}}{3}-6\,e^{2i(dx+c)}+6\,e^{i(dx+c)}}{d\,a^3\left(e^{2i(dx+c)}+1\right)^3}+\frac{\ln\left(e^{2i(dx+c)}+1\right)}{a^3d}$	115

input
```
int(tan(d*x+c)^7/(a+a*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/d/a^3*(1/3/cos(d*x+c)^3+ln(cos(d*x+c))-3/2/cos(d*x+c)^2+3/cos(d*x+c))
```

3.88.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.85

$$\int \frac{\tan^7(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{6\cos(dx+c)^3\log(-\cos(dx+c)) + 18\cos(dx+c)^2 - 9\cos(dx+c) + 2}{6\,a^3 d\cos(dx+c)^3}$$

input
```
integrate(tan(d*x+c)^7/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

output
```
1/6*(6*cos(d*x + c)^3*log(-cos(d*x + c)) + 18*cos(d*x + c)^2 - 9*cos(d*x +
 c) + 2)/(a^3*d*cos(d*x + c)^3)
```

3.88.6 Sympy [F]

$$\int \frac{\tan^7(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{\int \frac{\tan^7(c+dx)}{\sec^3(c+dx)+3\sec^2(c+dx)+3\sec(c+dx)+1}\,dx}{a^3}$$

input
```
integrate(tan(d*x+c)**7/(a+a*sec(d*x+c))**3,x)
```

output
```
Integral(tan(c + d*x)**7/(sec(c + d*x)**3 + 3*sec(c + d*x)**2 + 3*sec(c +
d*x) + 1), x)/a**3
```

3.88.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.77

$$\int \frac{\tan^7(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{\frac{6\log(\cos(dx+c))}{a^3} + \frac{18\cos(dx+c)^2 - 9\cos(dx+c) + 2}{a^3\cos(dx+c)^3}}{6\,d}$$

input
```
integrate(tan(d*x+c)^7/(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

output
```
1/6*(6*log(cos(d*x + c))/a^3 + (18*cos(d*x + c)^2 - 9*cos(d*x + c) + 2)/(a
^3*cos(d*x + c)^3))/d
```

3.88.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 158 vs. $2(61) = 122$.

Time = 3.60 (sec) , antiderivative size = 158, normalized size of antiderivative = 2.43

$$\int \frac{\tan^7(c+dx)}{(a+a\sec(c+dx))^3}\,dx = -\frac{\frac{6\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right)}{a^3} - \frac{6\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right)}{a^3} - \frac{\frac{75(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{51(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}-\frac{11(\cos(dx+c)-1)^3}{(\cos(dx+c)+1)^3}+29}{a^3\left(\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right)^3}}{6\,d}$$

input
```
integrate(tan(d*x+c)^7/(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

output
```
-1/6*(6*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1))/a^3 - 6*log(a
bs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1))/a^3 - (75*(cos(d*x + c) -
1)/(cos(d*x + c) + 1) + 51*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 11*
(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 + 29)/(a^3*((cos(d*x + c) - 1)/(
cos(d*x + c) + 1) + 1)^3))/d
```

3.88.9 Mupad [B] (verification not implemented)

Time = 15.50 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.68

$$\int \frac{\tan^7(c+dx)}{(a+a\sec(c+dx))^3}\,dx = -\frac{14\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4 - 18\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2 + \frac{20}{3}}{d\left(a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6 - 3a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4 + 3a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2 - a^3\right)} - \frac{2\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2\right)}{a^3 d}$$

input
```
int(tan(c + d*x)^7/(a + a/cos(c + d*x))^3,x)
```

output
```
- (14*tan(c/2 + (d*x)/2)^4 - 18*tan(c/2 + (d*x)/2)^2 + 20/3)/(d*(3*a^3*tan
(c/2 + (d*x)/2)^2 - 3*a^3*tan(c/2 + (d*x)/2)^4 + a^3*tan(c/2 + (d*x)/2)^6
- a^3)) - (2*atanh(tan(c/2 + (d*x)/2)^2))/(a^3*d)
```

3.89 $\int \frac{\tan^5(c+dx)}{(a+a\sec(c+dx))^3}\,dx$

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3.89.1 Optimal result

Integrand size = 21, antiderivative size = 46

$$\int \frac{\tan^5(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{3\log(\cos(c+dx))}{a^3d} - \frac{4\log(1+\cos(c+dx))}{a^3d} + \frac{\sec(c+dx)}{a^3d}$$

output
```
3*ln(cos(d*x+c))/a^3/d-4*ln(1+cos(d*x+c))/a^3/d+sec(d*x+c)/a^3/d
```

3.89.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.78

$$\int \frac{\tan^5(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{-8\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right) + 3\log(\cos(c+dx)) + \sec(c+dx)}{a^3d}$$

input
```
Integrate[Tan[c + d*x]^5/(a + a*Sec[c + d*x])^3,x]
```

output
```
(-8*Log[Cos[(c + d*x)/2]] + 3*Log[Cos[c + d*x]] + Sec[c + d*x])/(a^3*d)
```

3.89.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.80, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 25, 4367, 27, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^5(c+dx)}{(a\sec(c+dx)+a)^3}\,dx$$

$$\downarrow 3042$$

$$\int -\frac{\cot\left(c+dx+\frac{\pi}{2}\right)^5}{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^3}dx$$

$$\downarrow 25$$

$$-\int \frac{\cot\left(\frac{1}{2}(2c+\pi)+dx\right)^5}{\left(\csc\left(\frac{1}{2}(2c+\pi)+dx\right)a+a\right)^3}dx$$

$$\downarrow 4367$$

$$-\frac{\int \frac{a(1-\cos(c+dx))^2\sec^2(c+dx)}{\cos(c+dx)+1}d\cos(c+dx)}{a^4d}$$

$$\downarrow 27$$

$$-\frac{\int \frac{(1-\cos(c+dx))^2\sec^2(c+dx)}{\cos(c+dx)+1}d\cos(c+dx)}{a^3d}$$

$$\downarrow 99$$

$$-\frac{\int \left(\sec^2(c+dx)-3\sec(c+dx)+\frac{4}{\cos(c+dx)+1}\right)d\cos(c+dx)}{a^3d}$$

$$\downarrow 2009$$

$$-\frac{-\sec(c+dx)-3\log(\cos(c+dx))+4\log(\cos(c+dx)+1)}{a^3d}$$

input
```
Int[Tan[c + d*x]^5/(a + a*Sec[c + d*x])^3,x]
```

output
```
-((-3*Log[Cos[c + d*x]] + 4*Log[1 + Cos[c + d*x]] - Sec[c + d*x])/(a^3*d))
```

3.89.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4367
```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Simp[1/(a^(m - n - 1)*b^n*d)   Subst[Int[(a - b*x)^((m -
1)/2)*((a + b*x)^((m - 1)/2 + n)/x^(m + n)), x], x, Sin[c + d*x]], x] /; Fr
eeQ[{a, b, c, d}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && Integer
Q[n]
```

3.89.4 Maple [A] (verified)

Time = 1.04 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.80

method	result	size
derivativedivides	$\frac{-4\ln(\cos(dx+c)+1)+\frac{1}{\cos(dx+c)}+3\ln(\cos(dx+c))}{d\,a^3}$	37
default	$\frac{-4\ln(\cos(dx+c)+1)+\frac{1}{\cos(dx+c)}+3\ln(\cos(dx+c))}{d\,a^3}$	37
risch	$\frac{ix}{a^3}+\frac{2ic}{a^3d}+\frac{2\,\mathrm{e}^{i(dx+c)}}{d\,a^3\left(\mathrm{e}^{2i(dx+c)}+1\right)}-\frac{8\ln\left(\mathrm{e}^{i(dx+c)}+1\right)}{a^3d}+\frac{3\ln\left(\mathrm{e}^{2i(dx+c)}+1\right)}{a^3d}$	89

input
```
int(tan(d*x+c)^5/(a+a*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/d/a^3*(-4*ln(cos(d*x+c)+1)+1/cos(d*x+c)+3*ln(cos(d*x+c)))
```

3.89.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.15

$$\int \frac{\tan^5(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{3\cos(dx+c)\log(-\cos(dx+c)) - 4\cos(dx+c)\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)+1}{a^3 d\cos(dx+c)}$$

input
```
integrate(tan(d*x+c)^5/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

output
```
(3*cos(d*x + c)*log(-cos(d*x + c)) - 4*cos(d*x + c)*log(1/2*cos(d*x + c) +
 1/2) + 1)/(a^3*d*cos(d*x + c))
```

3.89.6 Sympy [F]

$$\int \frac{\tan^5(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{\int \frac{\tan^5(c+dx)}{\sec^3(c+dx)+3\sec^2(c+dx)+3\sec(c+dx)+1}\,dx}{a^3}$$

input
```
integrate(tan(d*x+c)**5/(a+a*sec(d*x+c))**3,x)
```

output
```
Integral(tan(c + d*x)**5/(sec(c + d*x)**3 + 3*sec(c + d*x)**2 + 3*sec(c +
d*x) + 1), x)/a**3
```

3.89.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.98

$$\int \frac{\tan^5(c+dx)}{(a+a\sec(c+dx))^3}\,dx = -\frac{\frac{4\log(\cos(dx+c)+1)}{a^3} - \frac{3\log(\cos(dx+c))}{a^3} - \frac{1}{a^3\cos(dx+c)}}{d}$$

input
```
integrate(tan(d*x+c)^5/(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

output
```
-(4*log(cos(d*x + c) + 1)/a^3 - 3*log(cos(d*x + c))/a^3 - 1/(a^3*cos(d*x +
 c)))/d
```

3.89.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 112 vs. $2(46) = 92$.

Time = 1.67 (sec) , antiderivative size = 112, normalized size of antiderivative = 2.43

$$\int \frac{\tan^5(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{\frac{\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right)}{a^3} + \frac{3\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right)}{a^3} - \frac{\frac{3(\cos(dx+c)-1)}{\cos(dx+c)+1}+1}{a^3\left(\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right)}}{d}$$

input
```
integrate(tan(d*x+c)^5/(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

output
```
(log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1))/a^3 + 3*log(abs(-(co
s(d*x + c) - 1)/(cos(d*x + c) + 1) - 1))/a^3 - (3*(cos(d*x + c) - 1)/(cos(
d*x + c) + 1) + 1)/(a^3*((cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)))/d
```

3.89.9 Mupad [B] (verification not implemented)

Time = 14.65 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.57

$$\int \frac{\tan^5(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{3\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2-1\right)}{a^3\,d} - \frac{2}{d\left(a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2-a^3\right)} + \frac{\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+1\right)}{a^3\,d}$$

input
```
int(tan(c + d*x)^5/(a + a/cos(c + d*x))^3,x)
```

output
```
(3*log(tan(c/2 + (d*x)/2)^2 - 1))/(a^3*d) - 2/(d*(a^3*tan(c/2 + (d*x)/2)^2
 - a^3)) + log(tan(c/2 + (d*x)/2)^2 + 1)/(a^3*d)
```

3.90 $\int \frac{\tan^3(c+dx)}{(a+a\sec(c+dx))^3}\,dx$

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3.90.1 Optimal result

Integrand size = 21, antiderivative size = 35

$$\int \frac{\tan^3(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{2}{a^3 d(1+\cos(c+dx))} + \frac{\log(1+\cos(c+dx))}{a^3 d}$$

output
```
2/a^3/d/(1+cos(d*x+c))+ln(1+cos(d*x+c))/a^3/d
```

3.90.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.94

$$\int \frac{\tan^3(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{2\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right) + \tan^2\left(\frac{1}{2}(c+dx)\right)}{a^3 d}$$

input
```
Integrate[Tan[c + d*x]^3/(a + a*Sec[c + d*x])^3,x]
```

output
```
(2*Log[Cos[(c + d*x)/2]] + Tan[(c + d*x)/2]^2)/(a^3*d)
```

3.90.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.91, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 25, 4367, 27, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^3(c+dx)}{(a\sec(c+dx)+a)^3}\,dx$$

$$\downarrow 3042$$

$$\int -\frac{\cot\left(c+dx+\frac{\pi}{2}\right)^3}{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^3}dx$$

$$\downarrow 25$$

$$-\int \frac{\cot\left(\frac{1}{2}(2c+\pi)+dx\right)^3}{\left(\csc\left(\frac{1}{2}(2c+\pi)+dx\right)a+a\right)^3}dx$$

$$\downarrow 4367$$

$$-\frac{\int \frac{1-\cos(c+dx)}{a(\cos(c+dx)+1)^2}d\cos(c+dx)}{a^2 d}$$

$$\downarrow 27$$

$$-\frac{\int \frac{1-\cos(c+dx)}{(\cos(c+dx)+1)^2}d\cos(c+dx)}{a^3 d}$$

$$\downarrow 49$$

$$-\frac{\int \left(\frac{2}{(\cos(c+dx)+1)^2}+\frac{1}{-\cos(c+dx)-1}\right)d\cos(c+dx)}{a^3 d}$$

$$\downarrow 2009$$

$$-\frac{-\frac{2}{\cos(c+dx)+1}-\log(\cos(c+dx)+1)}{a^3 d}$$

input
```
Int[Tan[c + d*x]^3/(a + a*Sec[c + d*x])^3,x]
```

output
```
-((-2/(1 + Cos[c + d*x]) - Log[1 + Cos[c + d*x]])/(a^3*d))
```

3.90.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4367
```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Simp[1/(a^(m - n - 1)*b^n*d)   Subst[Int[(a - b*x)^((m -
1)/2)*((a + b*x)^((m - 1)/2 + n)/x^(m + n)), x], x, Sin[c + d*x]], x] /; Fr
eeQ[{a, b, c, d}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && Integer
Q[n]
```

3.90.4 Maple [A] (verified)

Time = 0.81 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.86

method	result	size
derivativedivides	$\frac{\ln(\cos(dx+c)+1)+\frac{2}{\cos(dx+c)+1}}{d\,a^3}$	30
default	$\frac{\ln(\cos(dx+c)+1)+\frac{2}{\cos(dx+c)+1}}{d\,a^3}$	30
risch	$-\frac{ix}{a^3}-\frac{2ic}{a^3d}+\frac{4\,e^{i(dx+c)}}{a^3d\left(e^{i(dx+c)}+1\right)^2}+\frac{2\ln\left(e^{i(dx+c)}+1\right)}{a^3d}$	69

input
```
int(tan(d*x+c)^3/(a+a*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/d/a^3*(ln(cos(d*x+c)+1)+2/(cos(d*x+c)+1))
```

3.90.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.20

$$\int \frac{\tan^3(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{(\cos(dx+c)+1)\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)+2}{a^3 d\cos(dx+c)+a^3 d}$$

input
```
integrate(tan(d*x+c)^3/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

output
```
((cos(d*x + c) + 1)*log(1/2*cos(d*x + c) + 1/2) + 2)/(a^3*d*cos(d*x + c) +
 a^3*d)
```

3.90.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 457 vs. $2(29) = 58$.

Time = 18.80 (sec) , antiderivative size = 457, normalized size of antiderivative = 13.06

$$\int \frac{\tan^3(c+dx)}{(a+a\sec(c+dx))^3}\,dx$$
$$= \begin{cases} -\frac{\log\left(\tan^2(c+dx)+1\right)\sec^2(c+dx)}{2a^3 d\sec^2(c+dx)+4a^3 d\sec(c+dx)+2a^3 d} - \frac{2\log\left(\tan^2(c+dx)+1\right)\sec(c+dx)}{2a^3 d\sec^2(c+dx)+4a^3 d\sec(c+dx)+2a^3 d} - \frac{\log\left(\tan^2(c+dx)+1\right)}{2a^3 d\sec^2(c+dx)+4a^3 d\sec(c+dx)+2a^3 d} + \frac{}{2} \\ \frac{x\tan^3(c)}{(a\sec(c)+a)^3} \end{cases}$$

input
```
integrate(tan(d*x+c)**3/(a+a*sec(d*x+c))**3,x)
```

output
```
Piecewise((-log(tan(c + d*x)**2 + 1)*sec(c + d*x)**2/(2*a**3*d*sec(c + d*x
)**2 + 4*a**3*d*sec(c + d*x) + 2*a**3*d) - 2*log(tan(c + d*x)**2 + 1)*sec(
c + d*x)/(2*a**3*d*sec(c + d*x)**2 + 4*a**3*d*sec(c + d*x) + 2*a**3*d) - l
og(tan(c + d*x)**2 + 1)/(2*a**3*d*sec(c + d*x)**2 + 4*a**3*d*sec(c + d*x)
 + 2*a**3*d) + 2*log(sec(c + d*x) + 1)*sec(c + d*x)**2/(2*a**3*d*sec(c + d*
x)**2 + 4*a**3*d*sec(c + d*x) + 2*a**3*d) + 4*log(sec(c + d*x) + 1)*sec(c
 + d*x)/(2*a**3*d*sec(c + d*x)**2 + 4*a**3*d*sec(c + d*x) + 2*a**3*d) + 2*l
og(sec(c + d*x) + 1)/(2*a**3*d*sec(c + d*x)**2 + 4*a**3*d*sec(c + d*x) + 2
*a**3*d) + tan(c + d*x)**2/(2*a**3*d*sec(c + d*x)**2 + 4*a**3*d*sec(c + d*
x) + 2*a**3*d) - 2*sec(c + d*x)/(2*a**3*d*sec(c + d*x)**2 + 4*a**3*d*sec(c
 + d*x) + 2*a**3*d) - 2/(2*a**3*d*sec(c + d*x)**2 + 4*a**3*d*sec(c + d*x)
 + 2*a**3*d), Ne(d, 0)), (x*tan(c)**3/(a*sec(c) + a)**3, True))
```

3.90.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.03

$$\int \frac{\tan^3(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{\frac{2}{a^3\cos(dx+c)+a^3} + \frac{\log(\cos(dx+c)+1)}{a^3}}{d}$$

input
```
integrate(tan(d*x+c)^3/(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

output
```
(2/(a^3*cos(d*x + c) + a^3) + log(cos(d*x + c) + 1)/a^3)/d
```

3.90.8 Giac [A] (verification not implemented)

Time = 0.74 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.60

$$\int \frac{\tan^3(c+dx)}{(a+a\sec(c+dx))^3}\,dx = -\frac{\frac{\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right)}{a^3} + \frac{\cos(dx+c)-1}{a^3(\cos(dx+c)+1)}}{d}$$

input
```
integrate(tan(d*x+c)^3/(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

output
```
-(log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1))/a^3 + (cos(d*x + c)
 - 1)/(a^3*(cos(d*x + c) + 1)))/d
```

3.90.9 Mupad [B] (verification not implemented)

Time = 14.51 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.03

$$\int \frac{\tan^3(c+dx)}{(a+a\sec(c+dx))^3}\,dx = -\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+1\right)-\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2}{a^3\,d}$$

input
```
int(tan(c + d*x)^3/(a + a/cos(c + d*x))^3,x)
```

output
```
-(log(tan(c/2 + (d*x)/2)^2 + 1) - tan(c/2 + (d*x)/2)^2)/(a^3*d)
```

3.91 $\int \frac{\tan(c+dx)}{(a+a\sec(c+dx))^3}\,dx$

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3.91.1 Optimal result

Integrand size = 19, antiderivative size = 56

$$\int \frac{\tan(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{1}{2a^3d(1+\cos(c+dx))^2} - \frac{2}{a^3d(1+\cos(c+dx))} - \frac{\log(1+\cos(c+dx))}{a^3d}$$

output
```
1/2/a^3/d/(1+cos(d*x+c))^2-2/a^3/d/(1+cos(d*x+c))-ln(1+cos(d*x+c))/a^3/d
```

3.91.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.41

$$\int \frac{\tan(c+dx)}{(a+a\sec(c+dx))^3}\,dx = -\frac{\cos^2\left(\frac{1}{2}(c+dx)\right)\left(-1+8\cos^2\left(\frac{1}{2}(c+dx)\right)+16\cos^4\left(\frac{1}{2}(c+dx)\right)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)\right)\sec^3(c+dx)}{a^3d(1+\sec(c+dx))^3}$$

input
```
Integrate[Tan[c + d*x]/(a + a*Sec[c + d*x])^3,x]
```

output
```
-((Cos[(c + d*x)/2]^2*(-1 + 8*Cos[(c + d*x)/2]^2 + 16*Cos[(c + d*x)/2]^4*L
og[Cos[(c + d*x)/2]])*Sec[c + d*x]^3)/(a^3*d*(1 + Sec[c + d*x])^3))
```

3.91.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.79, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {3042, 25, 4367, 27, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan(c+dx)}{(a\sec(c+dx)+a)^3}\,dx$$

$$\downarrow 3042$$

$$\int -\frac{\cot\left(c+dx+\frac{\pi}{2}\right)}{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^3}dx$$

$$\downarrow 25$$

$$-\int \frac{\cot\left(\frac{1}{2}(2c+\pi)+dx\right)}{\left(\csc\left(\frac{1}{2}(2c+\pi)+dx\right)a+a\right)^3}dx$$

$$\downarrow 4367$$

$$-\frac{\int \frac{\cos^2(c+dx)}{a^3(\cos(c+dx)+1)^3}d\cos(c+dx)}{d}$$

$$\downarrow 27$$

$$-\frac{\int \frac{\cos^2(c+dx)}{(\cos(c+dx)+1)^3}d\cos(c+dx)}{a^3d}$$

$$\downarrow 49$$

$$-\frac{\int \left(\frac{1}{\cos(c+dx)+1}-\frac{2}{(\cos(c+dx)+1)^2}+\frac{1}{(\cos(c+dx)+1)^3}\right)d\cos(c+dx)}{a^3d}$$

$$\downarrow 2009$$

$$-\frac{\frac{2}{\cos(c+dx)+1}-\frac{1}{2(\cos(c+dx)+1)^2}+\log(\cos(c+dx)+1)}{a^3d}$$

input
```
Int[Tan[c + d*x]/(a + a*Sec[c + d*x])^3,x]
```

output
```
-((-1/2*1/(1 + Cos[c + d*x])^2 + 2/(1 + Cos[c + d*x]) + Log[1 + Cos[c + d*
x]])/(a^3*d))
```

3.91.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4367
```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Simp[1/(a^(m - n - 1)*b^n*d)   Subst[Int[(a - b*x)^((m -
1)/2)*((a + b*x)^((m - 1)/2 + n)/x^(m + n)), x], x, Sin[c + d*x]], x] /; Fr
eeQ[{a, b, c, d}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && Integer
Q[n]
```

3.91.4 Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.88

method	result	size
derivativedivides	$\frac{\ln(\sec(dx+c))+\frac{1}{2(1+\sec(dx+c))^2}+\frac{1}{1+\sec(dx+c)}-\ln(1+\sec(dx+c))}{d\,a^3}$	49
default	$\frac{\ln(\sec(dx+c))+\frac{1}{2(1+\sec(dx+c))^2}+\frac{1}{1+\sec(dx+c)}-\ln(1+\sec(dx+c))}{d\,a^3}$	49
risch	$\frac{ix}{a^3}+\frac{2ic}{a^3d}-\frac{2\left(2\,e^{3i(dx+c)}+3\,e^{2i(dx+c)}+2\,e^{i(dx+c)}\right)}{d\,a^3\left(e^{i(dx+c)}+1\right)^4}-\frac{2\ln\left(e^{i(dx+c)}+1\right)}{a^3d}$	94

input
```
int(tan(d*x+c)/(a+a*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/d/a^3*(ln(sec(d*x+c))+1/2/(1+sec(d*x+c))^2+1/(1+sec(d*x+c))-ln(1+sec(d*x
+c)))
```

3.91.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.36

$$\int \frac{\tan(c+dx)}{(a+a\sec(c+dx))^3}\,dx = -\frac{2\left(\cos(dx+c)^2+2\cos(dx+c)+1\right)\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)+4\cos(dx+c)+3}{2\left(a^3d\cos(dx+c)^2+2a^3d\cos(dx+c)+a^3d\right)}$$

input
```
integrate(tan(d*x+c)/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

output
```
-1/2*(2*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*log(1/2*cos(d*x + c) + 1/2)
+ 4*cos(d*x + c) + 3)/(a^3*d*cos(d*x + c)^2 + 2*a^3*d*cos(d*x + c) + a^3*d
)
```

3.91.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 411 vs. $2(48) = 96$.

Time = 19.01 (sec) , antiderivative size = 411, normalized size of antiderivative = 7.34

$$\int \frac{\tan(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \begin{cases} \frac{\log\left(\tan^2(c+dx)+1\right)\sec^2(c+dx)}{2a^3d\sec^2(c+dx)+4a^3d\sec(c+dx)+2a^3d} + \frac{2\log\left(\tan^2(c+dx)+1\right)\sec(c+dx)}{2a^3d\sec^2(c+dx)+4a^3d\sec(c+dx)+2a^3d} + \frac{\log\left(\tan^2(c+dx)+1\right)}{2a^3d\sec^2(c+dx)+4a^3d\sec(c+dx)+2a^3d} - \frac{}{2a^3} \\ \frac{x\tan(c)}{(a\sec(c)+a)^3} \end{cases}$$

input
```
integrate(tan(d*x+c)/(a+a*sec(d*x+c))**3,x)
```

output
```
Piecewise((log(tan(c + d*x)**2 + 1)*sec(c + d*x)**2/(2*a**3*d*sec(c + d*x)
**2 + 4*a**3*d*sec(c + d*x) + 2*a**3*d) + 2*log(tan(c + d*x)**2 + 1)*sec(c
 + d*x)/(2*a**3*d*sec(c + d*x)**2 + 4*a**3*d*sec(c + d*x) + 2*a**3*d) + lo
g(tan(c + d*x)**2 + 1)/(2*a**3*d*sec(c + d*x)**2 + 4*a**3*d*sec(c + d*x) +
 2*a**3*d) - 2*log(sec(c + d*x) + 1)*sec(c + d*x)**2/(2*a**3*d*sec(c + d*x
)**2 + 4*a**3*d*sec(c + d*x) + 2*a**3*d) - 4*log(sec(c + d*x) + 1)*sec(c +
 d*x)/(2*a**3*d*sec(c + d*x)**2 + 4*a**3*d*sec(c + d*x) + 2*a**3*d) - 2*lo
g(sec(c + d*x) + 1)/(2*a**3*d*sec(c + d*x)**2 + 4*a**3*d*sec(c + d*x) + 2*
a**3*d) + 2*sec(c + d*x)/(2*a**3*d*sec(c + d*x)**2 + 4*a**3*d*sec(c + d*x)
 + 2*a**3*d) + 3/(2*a**3*d*sec(c + d*x)**2 + 4*a**3*d*sec(c + d*x) + 2*a**
3*d), Ne(d, 0)), (x*tan(c)/(a*sec(c) + a)**3, True))
```

3.91.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.07

$$\int \frac{\tan(c+dx)}{(a+a\sec(c+dx))^3}\,dx = -\frac{\frac{4\cos(dx+c)+3}{a^3\cos(dx+c)^2+2a^3\cos(dx+c)+a^3}+\frac{2\log(\cos(dx+c)+1)}{a^3}}{2d}$$

input
```
integrate(tan(d*x+c)/(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

output
```
-1/2*((4*cos(d*x + c) + 3)/(a^3*cos(d*x + c)^2 + 2*a^3*cos(d*x + c) + a^3)
 + 2*log(cos(d*x + c) + 1)/a^3)/d
```

3.91.8 Giac [A] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.55

$$\int \frac{\tan(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{\frac{8\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right)}{a^3}+\frac{\frac{6a^3(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{a^3(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}}{a^6}}{8d}$$

input
```
integrate(tan(d*x+c)/(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

output
```
1/8*(8*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1))/a^3 + (6*a^3*(
cos(d*x + c) - 1)/(cos(d*x + c) + 1) + a^3*(cos(d*x + c) - 1)^2/(cos(d*x +
 c) + 1)^2)/a^6)/d
```

3.91.9 Mupad [B] (verification not implemented)

Time = 14.63 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.86

$$\int \frac{\tan(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+1\right) - \frac{3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2}{4} + \frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4}{8}}{a^3\,d}$$

input
```
int(tan(c + d*x)/(a + a/cos(c + d*x))^3,x)
```

output
```
(log(tan(c/2 + (d*x)/2)^2 + 1) - (3*tan(c/2 + (d*x)/2)^2)/4 + tan(c/2 + (d
*x)/2)^4/8)/(a^3*d)
```

3.92 $\int \frac{\cot(c+dx)}{(a+a\sec(c+dx))^3}\,dx$

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3.92.9 Mupad [B] (verification not implemented) 690

3.92.1 Optimal result

Integrand size = 19, antiderivative size = 101

$$\int \frac{\cot(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{1}{6a^3d(1+\cos(c+dx))^3} - \frac{7}{8a^3d(1+\cos(c+dx))^2} + \frac{17}{8a^3d(1+\cos(c+dx))} + \frac{\log(1-\cos(c+dx))}{16a^3d} + \frac{15\log(1+\cos(c+dx))}{16a^3d}$$

output

```
1/6/a^3/d/(1+cos(d*x+c))^3-7/8/a^3/d/(1+cos(d*x+c))^2+17/8/a^3/d/(1+cos(d*
x+c))+1/16*ln(1-cos(d*x+c))/a^3/d+15/16*ln(1+cos(d*x+c))/a^3/d
```

3.92.2 Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.96

$$\int \frac{\cot(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{\left(2-21\cos^2\left(\frac{1}{2}(c+dx)\right)+102\cos^4\left(\frac{1}{2}(c+dx)\right)+12\cos^6\left(\frac{1}{2}(c+dx)\right)\left(15\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)+\log\left(\sin\right.\right.\right.}{12a^3d(1+\sec(c+dx))^3}$$

input

```
Integrate[Cot[c + d*x]/(a + a*Sec[c + d*x])^3,x]
```

output
```
((2 - 21*Cos[(c + d*x)/2]^2 + 102*Cos[(c + d*x)/2]^4 + 12*Cos[(c + d*x)/2]
^6*(15*Log[Cos[(c + d*x)/2]] + Log[Sin[(c + d*x)/2]]))*Sec[c + d*x]^3)/(12
*a^3*d*(1 + Sec[c + d*x])^3)
```

3.92.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.78, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {3042, 25, 4367, 27, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot(c+dx)}{(a\sec(c+dx)+a)^3}\,dx$$

$$\downarrow 3042$$

$$\int -\frac{1}{\cot\left(c+dx+\frac{\pi}{2}\right)\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^3}dx$$

$$\downarrow 25$$

$$-\int \frac{1}{\cot\left(\frac{1}{2}(2c+\pi)+dx\right)\left(\csc\left(\frac{1}{2}(2c+\pi)+dx\right)a+a\right)^3}dx$$

$$\downarrow 4367$$

$$-\frac{a^2\int \frac{\cos^4(c+dx)}{a^5(1-\cos(c+dx))(\cos(c+dx)+1)^4}\,d\cos(c+dx)}{d}$$

$$\downarrow 27$$

$$-\frac{\int \frac{\cos^4(c+dx)}{(1-\cos(c+dx))(\cos(c+dx)+1)^4}\,d\cos(c+dx)}{a^3d}$$

$$\downarrow 99$$

$$-\frac{\int \left(-\frac{15}{16(\cos(c+dx)+1)}+\frac{17}{8(\cos(c+dx)+1)^2}-\frac{7}{4(\cos(c+dx)+1)^3}+\frac{1}{2(\cos(c+dx)+1)^4}-\frac{1}{16(\cos(c+dx)-1)}\right)d\cos(c+dx)}{a^3d}$$

$$\downarrow 2009$$

$$-\frac{-\frac{17}{8(\cos(c+dx)+1)}+\frac{7}{8(\cos(c+dx)+1)^2}-\frac{1}{6(\cos(c+dx)+1)^3}-\frac{1}{16}\log(1-\cos(c+dx))-\frac{15}{16}\log(\cos(c+dx)+1)}{a^3d}$$

input
```
Int[Cot[c + d*x]/(a + a*Sec[c + d*x])^3,x]
```

output
```
-((-1/6*1/(1 + Cos[c + d*x])^3 + 7/(8*(1 + Cos[c + d*x])^2) - 17/(8*(1 + C
os[c + d*x])) - Log[1 - Cos[c + d*x]]/16 - (15*Log[1 + Cos[c + d*x]])/16)/
(a^3*d))
```

3.92.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4367
```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Simp[1/(a^(m - n - 1)*b^n*d)   Subst[Int[(a - b*x)^((m -
1)/2)*((a + b*x)^((m - 1)/2 + n)/x^(m + n)), x], x, Sin[c + d*x]], x] /; Fr
eeQ[{a, b, c, d}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && Integer
Q[n]
```

3.92.4 Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.66

method	result
derivativedivides	$\frac{\frac{\ln(\cos(dx+c)-1)}{16}+\frac{1}{6(\cos(dx+c)+1)^3}-\frac{7}{8(\cos(dx+c)+1)^2}+\frac{17}{8(\cos(dx+c)+1)}+\frac{15\ln(\cos(dx+c)+1)}{16}}{d\,a^3}$
default	$\frac{\frac{\ln(\cos(dx+c)-1)}{16}+\frac{1}{6(\cos(dx+c)+1)^3}-\frac{7}{8(\cos(dx+c)+1)^2}+\frac{17}{8(\cos(dx+c)+1)}+\frac{15\ln(\cos(dx+c)+1)}{16}}{d\,a^3}$
risch	$-\frac{ix}{a^3}-\frac{2ic}{a^3d}+\frac{51\,e^{5i(dx+c)}+162\,e^{4i(dx+c)}+238\,e^{3i(dx+c)}+162\,e^{2i(dx+c)}+51\,e^{i(dx+c)}}{12d\,a^3\left(e^{i(dx+c)}+1\right)^6}+\frac{\ln\left(e^{i(dx+c)}-1\right)}{8a^3d}+\frac{15\ln(e}{}$

input
```
int(cot(d*x+c)/(a+a*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/d/a^3*(1/16*ln(cos(d*x+c)-1)+1/6/(cos(d*x+c)+1)^3-7/8/(cos(d*x+c)+1)^2+1
7/8/(cos(d*x+c)+1)+15/16*ln(cos(d*x+c)+1))
```

3.92.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.50

$$\int \frac{\cot(c+dx)}{(a+a\sec(c+dx))^3}\,dx$$
$$= \frac{102\cos(dx+c)^2+45\left(\cos(dx+c)^3+3\cos(dx+c)^2+3\cos(dx+c)+1\right)\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)+3}{48\left(a^3d\cos(dx+c)^3+3\,a^3d\cos(d}$$

input
```
integrate(cot(d*x+c)/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

output
```
1/48*(102*cos(d*x + c)^2 + 45*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + 3*cos(d
*x + c) + 1)*log(1/2*cos(d*x + c) + 1/2) + 3*(cos(d*x + c)^3 + 3*cos(d*x +
 c)^2 + 3*cos(d*x + c) + 1)*log(-1/2*cos(d*x + c) + 1/2) + 162*cos(d*x + c
) + 68)/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x +
 c) + a^3*d)
```

3.92.6 Sympy [F]

$$\int \frac{\cot(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{\int \frac{\cot(c+dx)}{\sec^3(c+dx)+3\sec^2(c+dx)+3\sec(c+dx)+1}\,dx}{a^3}$$

input
```
integrate(cot(d*x+c)/(a+a*sec(d*x+c))**3,x)
```

output
```
Integral(cot(c + d*x)/(sec(c + d*x)**3 + 3*sec(c + d*x)**2 + 3*sec(c + d*x
) + 1), x)/a**3
```

3.92.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.97

$$\int \frac{\cot(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{\frac{2\left(51\cos(dx+c)^2+81\cos(dx+c)+34\right)}{a^3\cos(dx+c)^3+3a^3\cos(dx+c)^2+3a^3\cos(dx+c)+a^3} + \frac{45\log(\cos(dx+c)+1)}{a^3} + \frac{3\log(\cos(dx+c)-1)}{a^3}}{48\,d}$$

input
```
integrate(cot(d*x+c)/(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

output
```
1/48*(2*(51*cos(d*x + c)^2 + 81*cos(d*x + c) + 34)/(a^3*cos(d*x + c)^3 + 3
*a^3*cos(d*x + c)^2 + 3*a^3*cos(d*x + c) + a^3) + 45*log(cos(d*x + c) + 1)
/a^3 + 3*log(cos(d*x + c) - 1)/a^3)/d
```

3.92.8 Giac [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.42

$$\int \frac{\cot(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{\frac{6\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)}{a^3} - \frac{96\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right)}{a^3} - \frac{\frac{66\,a^6(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{15\,a^6(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}+\frac{2\,a^6(\cos(dx+c)-1)^3}{(\cos(dx+c)+1)^3}}{a^9}}{96\,d}$$

input
```
integrate(cot(d*x+c)/(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

output
```
1/96*(6*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1))/a^3 - 96*log(abs
(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1))/a^3 - (66*a^6*(cos(d*x + c)
- 1)/(cos(d*x + c) + 1) + 15*a^6*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2
 + 2*a^6*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3)/a^9)/d
```

3.92.9 Mupad [B] (verification not implemented)

Time = 14.26 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.74

$$\int \frac{\cot(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{8} - \ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+1\right) + \frac{11\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2}{16} - \frac{5\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4}{32} + \frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6}{48}}{a^3\,d}$$

input
```
int(cot(c + d*x)/(a + a/cos(c + d*x))^3,x)
```

output
```
(log(tan(c/2 + (d*x)/2))/8 - log(tan(c/2 + (d*x)/2)^2 + 1) + (11*tan(c/2 +
 (d*x)/2)^2)/16 - (5*tan(c/2 + (d*x)/2)^4)/32 + tan(c/2 + (d*x)/2)^6/48)/(
a^3*d)
```

3.93 $\int \frac{\cot^3(c+dx)}{(a+a\sec(c+dx))^3}\,dx$

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3.93.1 Optimal result

Integrand size = 21, antiderivative size = 143

$$\int \frac{\cot^3(c+dx)}{(a+a\sec(c+dx))^3}\,dx = -\frac{1}{32a^3d(1-\cos(c+dx))} + \frac{1}{16a^3d(1+\cos(c+dx))^4} - \frac{5}{12a^3d(1+\cos(c+dx))^3} + \frac{39}{32a^3d(1+\cos(c+dx))^2} - \frac{9}{4a^3d(1+\cos(c+dx))} - \frac{7\log(1-\cos(c+dx))}{64a^3d} - \frac{57\log(1+\cos(c+dx))}{64a^3d}$$

output
```
-1/32/a^3/d/(1-cos(d*x+c))+1/16/a^3/d/(1+cos(d*x+c))^4-5/12/a^3/d/(1+cos(d
*x+c))^3+39/32/a^3/d/(1+cos(d*x+c))^2-9/4/a^3/d/(1+cos(d*x+c))-7/64*ln(1-c
os(d*x+c))/a^3/d-57/64*ln(1+cos(d*x+c))/a^3/d
```

3.93.2 Mathematica [A] (verified)

Time = 0.49 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.97

$$\int \frac{\cot^3(c+dx)}{(a+a\sec(c+dx))^3}\,dx = -\frac{\cos^4\left(\frac{1}{2}(c+dx)\right)\left(864+12\cot^2\left(\frac{1}{2}(c+dx)\right)+24\cos^2\left(\frac{1}{2}(c+dx)\right)\left(57\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)+7\log\left(\sin\right.\right.\right.}{96a^3d(1+\sec(c$$

input
```
Integrate[Cot[c + d*x]^3/(a + a*Sec[c + d*x])^3,x]
```

output
```
-1/96*(Cos[(c + d*x)/2]^4*(864 + 12*Cot[(c + d*x)/2]^2 + 24*Cos[(c + d*x)/
2]^2*(57*Log[Cos[(c + d*x)/2]] + 7*Log[Sin[(c + d*x)/2]]) - 234*Sec[(c + d
*x)/2]^2 + 40*Sec[(c + d*x)/2]^4 - 3*Sec[(c + d*x)/2]^6)*Sec[c + d*x]^3)/(
a^3*d*(1 + Sec[c + d*x])^3)
```

3.93.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.76, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 25, 4367, 27, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^3(c+dx)}{(a\sec(c+dx)+a)^3}\,dx$$

$$\downarrow 3042$$

$$\int -\frac{1}{\cot\left(c+dx+\frac{\pi}{2}\right)^3\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^3}dx$$

$$\downarrow 25$$

$$-\int \frac{1}{\cot\left(\frac{1}{2}(2c+\pi)+dx\right)^3\left(\csc\left(\frac{1}{2}(2c+\pi)+dx\right)a+a\right)^3}dx$$

$$\downarrow 4367$$

$$-\frac{a^4\int \frac{\cos^6(c+dx)}{a^7(1-\cos(c+dx))^2(\cos(c+dx)+1)^5}d\cos(c+dx)}{d}$$

$$\downarrow 27$$

$$-\frac{\int \frac{\cos^6(c+dx)}{(1-\cos(c+dx))^2(\cos(c+dx)+1)^5}d\cos(c+dx)}{a^3d}$$

$$\downarrow 99$$

$$-\frac{\int\left(\frac{57}{64(\cos(c+dx)+1)}-\frac{9}{4(\cos(c+dx)+1)^2}+\frac{39}{16(\cos(c+dx)+1)^3}-\frac{5}{4(\cos(c+dx)+1)^4}+\frac{1}{4(\cos(c+dx)+1)^5}+\frac{7}{64(\cos(c+dx)-1)}+\frac{}{32(\cos}\right.}{a^3d}$$

$$\downarrow 2009$$

$$-\frac{\frac{1}{32(1-\cos(c+dx))}+\frac{9}{4(\cos(c+dx)+1)}-\frac{39}{32(\cos(c+dx)+1)^2}+\frac{5}{12(\cos(c+dx)+1)^3}-\frac{1}{16(\cos(c+dx)+1)^4}+\frac{7}{64}\log(1-\cos(c+dx))+}{a^3d}$$

input
```
Int[Cot[c + d*x]^3/(a + a*Sec[c + d*x])^3,x]
```

output
```
-((1/(32*(1 - Cos[c + d*x])) - 1/(16*(1 + Cos[c + d*x])^4) + 5/(12*(1 + Co
s[c + d*x])^3) - 39/(32*(1 + Cos[c + d*x])^2) + 9/(4*(1 + Cos[c + d*x])) +
 (7*Log[1 - Cos[c + d*x]])/64 + (57*Log[1 + Cos[c + d*x]])/64)/(a^3*d))
```

3.93.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4367
```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Simp[1/(a^(m - n - 1)*b^n*d)   Subst[Int[(a - b*x)^((m -
1)/2)*((a + b*x)^((m - 1)/2 + n)/x^(m + n)), x], x, Sin[c + d*x]], x] /; Fr
eeQ[{a, b, c, d}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && Integer
Q[n]
```

3.93.4 Maple [A] (verified)

Time = 0.75 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.64

method	result
derivativedivides	$\frac{\frac{1}{16(\cos(dx+c)+1)^4}-\frac{5}{12(\cos(dx+c)+1)^3}+\frac{39}{32(\cos(dx+c)+1)^2}-\frac{9}{4(\cos(dx+c)+1)}-\frac{57\ln(\cos(dx+c)+1)}{64}+\frac{1}{32\cos(dx+c)-32}-\frac{7\ln(\cos($
default	$\frac{\frac{1}{16(\cos(dx+c)+1)^4}-\frac{5}{12(\cos(dx+c)+1)^3}+\frac{39}{32(\cos(dx+c)+1)^2}-\frac{9}{4(\cos(dx+c)+1)}-\frac{57\ln(\cos(dx+c)+1)}{64}+\frac{1}{32\cos(dx+c)-32}-\frac{7\ln(\cos($
risch	$\frac{ix}{a^3}+\frac{2ic}{a^3d}-\frac{213\,e^{9i(dx+c)}+606\,e^{8i(dx+c)}+472\,e^{7i(dx+c)}-846\,e^{6i(dx+c)}-1658\,e^{5i(dx+c)}-846\,e^{4i(dx+c)}+472\,e^{3i(dx+c}}{48da^3\left(e^{i(dx+c)}+1\right)^8\left(e^{i(dx+c)}-1\right)^2}$

input
```
int(cot(d*x+c)^3/(a+a*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/d/a^3*(1/16/(cos(d*x+c)+1)^4-5/12/(cos(d*x+c)+1)^3+39/32/(cos(d*x+c)+1)^
2-9/4/(cos(d*x+c)+1)-57/64*ln(cos(d*x+c)+1)+1/32/(cos(d*x+c)-1)-7/64*ln(co
s(d*x+c)-1))
```

3.93.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.68

$$\int \frac{\cot^3(c+dx)}{(a+a\sec(c+dx))^3}\,dx =$$
$$-\frac{426\cos(dx+c)^4+606\cos(dx+c)^3-190\cos(dx+c)^2+171\left(\cos(dx+c)^5+3\cos(dx+c)^4+2\,co}$$

input
```
integrate(cot(d*x+c)^3/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

output
```
-1/192*(426*cos(d*x + c)^4 + 606*cos(d*x + c)^3 - 190*cos(d*x + c)^2 + 171
*(cos(d*x + c)^5 + 3*cos(d*x + c)^4 + 2*cos(d*x + c)^3 - 2*cos(d*x + c)^2
 - 3*cos(d*x + c) - 1)*log(1/2*cos(d*x + c) + 1/2) + 21*(cos(d*x + c)^5 + 3
*cos(d*x + c)^4 + 2*cos(d*x + c)^3 - 2*cos(d*x + c)^2 - 3*cos(d*x + c) - 1
)*log(-1/2*cos(d*x + c) + 1/2) - 666*cos(d*x + c) - 272)/(a^3*d*cos(d*x +
c)^5 + 3*a^3*d*cos(d*x + c)^4 + 2*a^3*d*cos(d*x + c)^3 - 2*a^3*d*cos(d*x +
 c)^2 - 3*a^3*d*cos(d*x + c) - a^3*d)
```

3.93.6 Sympy [F]

$$\int \frac{\cot^3(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{\int \frac{\cot^3(c+dx)}{\sec^3(c+dx)+3\sec^2(c+dx)+3\sec(c+dx)+1}\,dx}{a^3}$$

input
```
integrate(cot(d*x+c)**3/(a+a*sec(d*x+c))**3,x)
```

output
```
Integral(cot(c + d*x)**3/(sec(c + d*x)**3 + 3*sec(c + d*x)**2 + 3*sec(c +
d*x) + 1), x)/a**3
```

3.93.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.02

$$\int \frac{\cot^3(c+dx)}{(a+a\sec(c+dx))^3}\,dx = -\frac{\frac{2\left(213\cos(dx+c)^4+303\cos(dx+c)^3-95\cos(dx+c)^2-333\cos(dx+c)-136\right)}{a^3\cos(dx+c)^5+3a^3\cos(dx+c)^4+2a^3\cos(dx+c)^3-2a^3\cos(dx+c)^2-3a^3\cos(dx+c)-a^3}+\frac{171\log(\cos(dx+c)+1)}{a^3}+\frac{21\log(\cos(dx+c)-1)}{a^3}}{192\,d}$$

input
```
integrate(cot(d*x+c)^3/(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

output
```
-1/192*(2*(213*cos(d*x + c)^4 + 303*cos(d*x + c)^3 - 95*cos(d*x + c)^2 - 3
33*cos(d*x + c) - 136)/(a^3*cos(d*x + c)^5 + 3*a^3*cos(d*x + c)^4 + 2*a^3*
cos(d*x + c)^3 - 2*a^3*cos(d*x + c)^2 - 3*a^3*cos(d*x + c) - a^3) + 171*lo
g(cos(d*x + c) + 1)/a^3 + 21*log(cos(d*x + c) - 1)/a^3)/d
```

3.93.8 Giac [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.48

$$\int \frac{\cot^3(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{\frac{12\left(\frac{7(\cos(dx+c)-1)}{\cos(dx+c)+1}+1\right)(\cos(dx+c)+1)}{a^3(\cos(dx+c)-1)}-\frac{84\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)}{a^3}+\frac{768\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right)}{a^3}+\frac{\frac{504\,a^9(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{132\,a^9(\cos(dx+c)+1)}{(\cos(dx+c)+1)}}{}}{768\,d}$$

input
```
integrate(cot(d*x+c)^3/(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

output
```
1/768*(12*(7*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)*(cos(d*x + c) + 1)
/(a^3*(cos(d*x + c) - 1)) - 84*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c)
 + 1))/a^3 + 768*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1))/a^3
+ (504*a^9*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 132*a^9*(cos(d*x + c) -
 1)^2/(cos(d*x + c) + 1)^2 + 28*a^9*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1
)^3 + 3*a^9*(cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4)/a^12)/d
```

3.93.9 Mupad [B] (verification not implemented)

Time = 14.66 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.71

$$\int \frac{\cot^3(c+dx)}{(a+a\sec(c+dx))^3}\,dx = -\frac{\frac{7\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{32} - \ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+1\right) + \frac{\cot\left(\frac{c}{2}+\frac{dx}{2}\right)^2}{64} + \frac{21\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2}{32} - \frac{11\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4}{64} + \frac{7\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6}{192} -}{a^3\,d}$$

input
```
int(cot(c + d*x)^3/(a + a/cos(c + d*x))^3,x)
```

output
```
-((7*log(tan(c/2 + (d*x)/2)))/32 - log(tan(c/2 + (d*x)/2)^2 + 1) + cot(c/2
 + (d*x)/2)^2/64 + (21*tan(c/2 + (d*x)/2)^2)/32 - (11*tan(c/2 + (d*x)/2)^4
)/64 + (7*tan(c/2 + (d*x)/2)^6)/192 - tan(c/2 + (d*x)/2)^8/256)/(a^3*d)
```

3.94 $\int \frac{\cot^5(c+dx)}{(a+a\sec(c+dx))^3}\,dx$

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3.94.1 Optimal result

Integrand size = 21, antiderivative size = 185

$$\begin{aligned}\int \frac{\cot^5(c+dx)}{(a+a\sec(c+dx))^3}\,dx = &-\frac{1}{128a^3d(1-\cos(c+dx))^2}\\ &+\frac{5}{64a^3d(1-\cos(c+dx))}+\frac{1}{40a^3d(1+\cos(c+dx))^5}\\ &-\frac{13}{64a^3d(1+\cos(c+dx))^4}+\frac{35}{48a^3d(1+\cos(c+dx))^3}\\ &-\frac{99}{64a^3d(1+\cos(c+dx))^2}+\frac{303}{128a^3d(1+\cos(c+dx))}\\ &+\frac{37\log(1-\cos(c+dx))}{256a^3d}+\frac{219\log(1+\cos(c+dx))}{256a^3d}\end{aligned}$$

output

```
-1/128/a^3/d/(1-cos(d*x+c))^2+5/64/a^3/d/(1-cos(d*x+c))+1/40/a^3/d/(1+cos(
d*x+c))^5-13/64/a^3/d/(1+cos(d*x+c))^4+35/48/a^3/d/(1+cos(d*x+c))^3-99/64/
a^3/d/(1+cos(d*x+c))^2+303/128/a^3/d/(1+cos(d*x+c))+37/256*ln(1-cos(d*x+c)
)/a^3/d+219/256*ln(1+cos(d*x+c))/a^3/d
```

3.94.2 Mathematica [A] (verified)

Time = 0.89 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.91

$$\int \frac{\cot^5(c+dx)}{(a+a\sec(c+dx))^3}\,dx$$
$$= \frac{\left(12-195\cos^2\left(\frac{1}{2}(c+dx)\right)+1400\cos^4\left(\frac{1}{2}(c+dx)\right)+60\cos^8\left(\frac{1}{2}(c+dx)\right)\left(303+10\cot^2\left(\frac{1}{2}(c+dx)\right)\right)-3\right.}{}$$

input
```
Integrate[Cot[c + d*x]^5/(a + a*Sec[c + d*x])^3,x]
```

output
```
((12 - 195*Cos[(c + d*x)/2]^2 + 1400*Cos[(c + d*x)/2]^4 + 60*Cos[(c + d*x)
/2]^8*(303 + 10*Cot[(c + d*x)/2]^2) - 30*Cos[(c + d*x)/2]^6*(198 + Cot[(c
+ d*x)/2]^4) + 120*Cos[(c + d*x)/2]^10*(219*Log[Cos[(c + d*x)/2]] + 37*Log
[Sin[(c + d*x)/2]]))*Sec[(c + d*x)/2]^4*Sec[c + d*x]^3)/(1920*a^3*d*(1 + S
ec[c + d*x])^3)
```

3.94.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.75, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 25, 4367, 27, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^5(c+dx)}{(a\sec(c+dx)+a)^3}\,dx$$
$$\downarrow 3042$$
$$\int -\frac{1}{\cot\left(c+dx+\frac{\pi}{2}\right)^5\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^3}dx$$
$$\downarrow 25$$
$$-\int \frac{1}{\cot\left(\frac{1}{2}(2c+\pi)+dx\right)^5\left(\csc\left(\frac{1}{2}(2c+\pi)+dx\right)a+a\right)^3}dx$$
$$\downarrow 4367$$
$$-\frac{a^6\int \frac{\cos^8(c+dx)}{a^9(1-\cos(c+dx))^3(\cos(c+dx)+1)^6}\,d\cos(c+dx)}{d}$$

$$\downarrow 27$$

$$-\frac{\int \frac{\cos^8(c+dx)}{(1-\cos(c+dx))^3(\cos(c+dx)+1)^6}\,d\cos(c+dx)}{a^3 d}$$

$$\downarrow 99$$

$$-\frac{\int \left(-\frac{219}{256(\cos(c+dx)+1)}+\frac{303}{128(\cos(c+dx)+1)^2}-\frac{99}{32(\cos(c+dx)+1)^3}+\frac{35}{16(\cos(c+dx)+1)^4}-\frac{13}{16(\cos(c+dx)+1)^5}+\frac{1}{8(\cos(c+dx)+1)^6}-\right.}{a^3 d}$$

$$\downarrow 2009$$

$$-\frac{-\frac{5}{64(1-\cos(c+dx))}-\frac{303}{128(\cos(c+dx)+1)}+\frac{1}{128(1-\cos(c+dx))^2}+\frac{99}{64(\cos(c+dx)+1)^2}-\frac{35}{48(\cos(c+dx)+1)^3}+\frac{13}{64(\cos(c+dx)+1)^4}-\frac{40}{}}{a^3 d}$$

input
```
Int[Cot[c + d*x]^5/(a + a*Sec[c + d*x])^3,x]
```

output
```
-((1/(128*(1 - Cos[c + d*x])^2) - 5/(64*(1 - Cos[c + d*x])) - 1/(40*(1 + C
os[c + d*x])^5) + 13/(64*(1 + Cos[c + d*x])^4) - 35/(48*(1 + Cos[c + d*x])
^3) + 99/(64*(1 + Cos[c + d*x])^2) - 303/(128*(1 + Cos[c + d*x])) - (37*Lo
g[1 - Cos[c + d*x]])/256 - (219*Log[1 + Cos[c + d*x]])/256)/(a^3*d))
```

3.94.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4367
```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Simp[1/(a^(m - n - 1)*b^n*d)   Subst[Int[(a - b*x)^((m -
1)/2)*((a + b*x)^((m - 1)/2 + n)/x^(m + n)), x], x, Sin[c + d*x]], x] /; Fr
eeQ[{a, b, c, d}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && Integer
Q[n]
```

3.94.4 Maple [A] (verified)

Time = 0.74 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.62

method	result
derivativedivides	$\frac{-\frac{1}{128(\cos(dx+c)-1)^2}-\frac{5}{64(\cos(dx+c)-1)}+\frac{37\ln(\cos(dx+c)-1)}{256}+\frac{1}{40(\cos(dx+c)+1)^5}-\frac{13}{64(\cos(dx+c)+1)^4}+\frac{35}{48(\cos(dx+c)+1)^3}-\frac{64(}{d\,a^3}$
default	$\frac{-\frac{1}{128(\cos(dx+c)-1)^2}-\frac{5}{64(\cos(dx+c)-1)}+\frac{37\ln(\cos(dx+c)-1)}{256}+\frac{1}{40(\cos(dx+c)+1)^5}-\frac{13}{64(\cos(dx+c)+1)^4}+\frac{35}{48(\cos(dx+c)+1)^3}-\frac{64(}{d\,a^3}$
risch	$-\frac{ix}{a^3}-\frac{2ic}{a^3d}+\frac{4395\,e^{13i(dx+c)}+11010\,e^{12i(dx+c)}-1390\,e^{11i(dx+c)}-47190\,e^{10i(dx+c)}-50987\,e^{9i(dx+c)}+25428\,e^{8i(dx}}{960d\,a^3($

input
```
int(cot(d*x+c)^5/(a+a*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/d/a^3*(-1/128/(cos(d*x+c)-1)^2-5/64/(cos(d*x+c)-1)+37/256*ln(cos(d*x+c)-
1)+1/40/(cos(d*x+c)+1)^5-13/64/(cos(d*x+c)+1)^4+35/48/(cos(d*x+c)+1)^3-99/
64/(cos(d*x+c)+1)^2+303/128/(cos(d*x+c)+1)+219/256*ln(cos(d*x+c)+1))
```

3.94.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 317, normalized size of antiderivative = 1.71

$$\int \frac{\cot^5(c+dx)}{(a+a\sec(c+dx))^3}\,dx$$
$$= \frac{8790\cos(dx+c)^6+11010\cos(dx+c)^5-13880\cos(dx+c)^4-25560\cos(dx+c)^3-734\cos(dx+c)}{}$$

input
```
integrate(cot(d*x+c)^5/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

output
```
1/3840*(8790*cos(d*x + c)^6 + 11010*cos(d*x + c)^5 - 13880*cos(d*x + c)^4
- 25560*cos(d*x + c)^3 - 734*cos(d*x + c)^2 + 3285*(cos(d*x + c)^7 + 3*cos
(d*x + c)^6 + cos(d*x + c)^5 - 5*cos(d*x + c)^4 - 5*cos(d*x + c)^3 + cos(d
*x + c)^2 + 3*cos(d*x + c) + 1)*log(1/2*cos(d*x + c) + 1/2) + 555*(cos(d*x
 + c)^7 + 3*cos(d*x + c)^6 + cos(d*x + c)^5 - 5*cos(d*x + c)^4 - 5*cos(d*x
 + c)^3 + cos(d*x + c)^2 + 3*cos(d*x + c) + 1)*log(-1/2*cos(d*x + c) + 1/2
) + 13878*cos(d*x + c) + 5536)/(a^3*d*cos(d*x + c)^7 + 3*a^3*d*cos(d*x + c
)^6 + a^3*d*cos(d*x + c)^5 - 5*a^3*d*cos(d*x + c)^4 - 5*a^3*d*cos(d*x + c)
^3 + a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)
```

3.94.6 Sympy [F]

$$\int \frac{\cot^5(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{\int \frac{\cot^5(c+dx)}{\sec^3(c+dx)+3\sec^2(c+dx)+3\sec(c+dx)+1}\,dx}{a^3}$$

input
```
integrate(cot(d*x+c)**5/(a+a*sec(d*x+c))**3,x)
```

output
```
Integral(cot(c + d*x)**5/(sec(c + d*x)**3 + 3*sec(c + d*x)**2 + 3*sec(c +
d*x) + 1), x)/a**3
```

3.94.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.02

$$\int \frac{\cot^5(c+dx)}{(a+a\sec(c+dx))^3}\,dx$$
$$= \frac{2\left(4395\cos(dx+c)^6+5505\cos(dx+c)^5-6940\cos(dx+c)^4-12780\cos(dx+c)^3-367\cos(dx+c)^2+6939\cos(dx+c)+2768\right)}{a^3\cos(dx+c)^7+3a^3\cos(dx+c)^6+a^3\cos(dx+c)^5-5a^3\cos(dx+c)^4-5a^3\cos(dx+c)^3+a^3\cos(dx+c)^2+3a^3\cos(dx+c)+a^3} + \frac{3285\log(\cos(dx+c)}{3840\,d}$$

input
```
integrate(cot(d*x+c)^5/(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

output
```
1/3840*(2*(4395*cos(d*x + c)^6 + 5505*cos(d*x + c)^5 - 6940*cos(d*x + c)^4
 - 12780*cos(d*x + c)^3 - 367*cos(d*x + c)^2 + 6939*cos(d*x + c) + 2768)/(
a^3*cos(d*x + c)^7 + 3*a^3*cos(d*x + c)^6 + a^3*cos(d*x + c)^5 - 5*a^3*cos
(d*x + c)^4 - 5*a^3*cos(d*x + c)^3 + a^3*cos(d*x + c)^2 + 3*a^3*cos(d*x +
c) + a^3) + 3285*log(cos(d*x + c) + 1)/a^3 + 555*log(cos(d*x + c) - 1)/a^3
)/d
```

3.94.8 Giac [A] (verification not implemented)

Time = 0.49 (sec) , antiderivative size = 261, normalized size of antiderivative = 1.41

$$\int \frac{\cot^5(c+dx)}{(a+a\sec(c+dx))^3}\,dx = -\frac{\frac{30\left(\frac{18(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{111(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}+1\right)(\cos(dx+c)+1)^2}{a^3(\cos(dx+c)-1)^2} - \frac{2220\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)}{a^3} + \frac{15360\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right)}{a^3} + \frac{9780\,a^{1}}{c}}{15360\,d}$$

input
```
integrate(cot(d*x+c)^5/(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

output
```
-1/15360*(30*(18*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 111*(cos(d*x + c)
 - 1)^2/(cos(d*x + c) + 1)^2 + 1)*(cos(d*x + c) + 1)^2/(a^3*(cos(d*x + c)
- 1)^2) - 2220*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1))/a^3 + 153
60*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1))/a^3 + (9780*a^12*(
cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 2790*a^12*(cos(d*x + c) - 1)^2/(cos
(d*x + c) + 1)^2 + 740*a^12*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 + 13
5*a^12*(cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4 + 12*a^12*(cos(d*x + c) -
 1)^5/(cos(d*x + c) + 1)^5)/a^15)/d
```

3.94.9 Mupad [B] (verification not implemented)

Time = 14.54 (sec) , antiderivative size = 170, normalized size of antiderivative = 0.92

$$\int \frac{\cot^5(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{163\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2}{256\,a^3\,d} - \frac{93\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4}{512\,a^3\,d} + \frac{37\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6}{768\,a^3\,d} - \frac{9\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^8}{1024\,a^3\,d} + \frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}}{1280\,a^3\,d} + \frac{37\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{128\,a^3\,d} - \frac{\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+1\right)}{a^3\,d} + \frac{\cot\left(\frac{c}{2}+\frac{dx}{2}\right)^4\left(\frac{9\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2}{2}-\frac{1}{4}\right)}{128\,a^3\,d}$$

input
```
int(cot(c + d*x)^5/(a + a/cos(c + d*x))^3,x)
```

output
```
(163*tan(c/2 + (d*x)/2)^2)/(256*a^3*d) - (93*tan(c/2 + (d*x)/2)^4)/(512*a^
3*d) + (37*tan(c/2 + (d*x)/2)^6)/(768*a^3*d) - (9*tan(c/2 + (d*x)/2)^8)/(1
024*a^3*d) + tan(c/2 + (d*x)/2)^10/(1280*a^3*d) + (37*log(tan(c/2 + (d*x)/
2)))/(128*a^3*d) - log(tan(c/2 + (d*x)/2)^2 + 1)/(a^3*d) + (cot(c/2 + (d*x
)/2)^4*((9*tan(c/2 + (d*x)/2)^2)/2 - 1/4))/(128*a^3*d)
```

3.95 $\int \frac{\tan^{12}(c+dx)}{(a+a\sec(c+dx))^3}\,dx$

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3.95.8 Giac [A] (verification not implemented) . . . 708
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3.95.1 Optimal result

Integrand size = 21, antiderivative size = 237

$$\begin{aligned}\int \frac{\tan^{12}(c+dx)}{(a+a\sec(c+dx))^3}\,dx = {}& \frac{x}{a^3} - \frac{125\operatorname{arctanh}(\sin(c+dx))}{128a^3d} \\ & - \frac{\tan(c+dx)}{a^3d} + \frac{115\sec(c+dx)\tan(c+dx)}{128a^3d} \\ & + \frac{5\sec^3(c+dx)\tan(c+dx)}{64a^3d} + \frac{\tan^3(c+dx)}{3a^3d} \\ & - \frac{5\sec(c+dx)\tan^3(c+dx)}{8a^3d} - \frac{5\sec^3(c+dx)\tan^3(c+dx)}{48a^3d} \\ & - \frac{\tan^5(c+dx)}{5a^3d} + \frac{\sec(c+dx)\tan^5(c+dx)}{2a^3d} \\ & + \frac{\sec^3(c+dx)\tan^5(c+dx)}{8a^3d} - \frac{3\tan^7(c+dx)}{7a^3d}\end{aligned}$$

output

```
x/a^3-125/128*arctanh(sin(d*x+c))/a^3/d-tan(d*x+c)/a^3/d+115/128*sec(d*x+c
)*tan(d*x+c)/a^3/d+5/64*sec(d*x+c)^3*tan(d*x+c)/a^3/d+1/3*tan(d*x+c)^3/a^3
/d-5/8*sec(d*x+c)*tan(d*x+c)^3/a^3/d-5/48*sec(d*x+c)^3*tan(d*x+c)^3/a^3/d-
1/5*tan(d*x+c)^5/a^3/d+1/2*sec(d*x+c)*tan(d*x+c)^5/a^3/d+1/8*sec(d*x+c)^3*
tan(d*x+c)^5/a^3/d-3/7*tan(d*x+c)^7/a^3/d
```

3.95.2 Mathematica [A] (verified)

Time = 3.59 (sec) , antiderivative size = 362, normalized size of antiderivative = 1.53

$$\int \frac{\tan^{12}(c+dx)}{(a+a\sec(c+dx))^3}\,dx$$
$$= \frac{\cos^6\left(\frac{1}{2}(c+dx)\right)\sec^3(c+dx)\left(1680000\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\right.\right.}{}$$

input
```
Integrate[Tan[c + d*x]^12/(a + a*Sec[c + d*x])^3,x]
```

output
```
(Cos[(c + d*x)/2]^6*Sec[c + d*x]^3*(1680000*(Log[Cos[(c + d*x)/2] - Sin[(c
 + d*x)/2]] - Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) + Sec[c]*Sec[c + d
*x]^8*(470400*d*x*Cos[c] + 376320*d*x*Cos[c + 2*d*x] + 376320*d*x*Cos[3*c
 + 2*d*x] + 188160*d*x*Cos[3*c + 4*d*x] + 188160*d*x*Cos[5*c + 4*d*x] + 537
60*d*x*Cos[5*c + 6*d*x] + 53760*d*x*Cos[7*c + 6*d*x] + 6720*d*x*Cos[7*c +
8*d*x] + 6720*d*x*Cos[9*c + 8*d*x] + 519680*Sin[c] + 133175*Sin[d*x] + 133
175*Sin[2*c + d*x] - 544768*Sin[c + 2*d*x] + 286720*Sin[3*c + 2*d*x] + 635
95*Sin[2*c + 3*d*x] + 63595*Sin[4*c + 3*d*x] - 254464*Sin[3*c + 4*d*x] + 1
61280*Sin[5*c + 4*d*x] + 65135*Sin[4*c + 5*d*x] + 65135*Sin[6*c + 5*d*x] -
 118784*Sin[5*c + 6*d*x] + 27195*Sin[6*c + 7*d*x] + 27195*Sin[8*c + 7*d*x]
 - 14848*Sin[7*c + 8*d*x])))/(215040*a^3*d*(1 + Sec[c + d*x])^3)
```

3.95.3 Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 242, normalized size of antiderivative = 1.02, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 4376, 25, 3042, 4374, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^{12}(c+dx)}{(a\sec(c+dx)+a)^3}\,dx$$
$$\downarrow 3042$$
$$\int \frac{\cot\left(c+dx+\frac{\pi}{2}\right)^{12}}{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^3}dx$$
$$\downarrow 4376$$

$$\frac{\int -(a-a\sec(c+dx))^3\tan^6(c+dx)dx}{a^6}$$

$$\downarrow 25$$

$$-\frac{\int (a-a\sec(c+dx))^3\tan^6(c+dx)dx}{a^6}$$

$$\downarrow 3042$$

$$-\frac{\int \cot\left(c+dx+\frac{\pi}{2}\right)^6\left(a-a\csc\left(c+dx+\frac{\pi}{2}\right)\right)^3 dx}{a^6}$$

$$\downarrow 4374$$

$$-\frac{\int \left(a^3\tan^6(c+dx)-a^3\sec^3(c+dx)\tan^6(c+dx)+3a^3\sec^2(c+dx)\tan^6(c+dx)-3a^3\sec(c+dx)\tan^6(c+dx\right.}{a^6}$$

$$\downarrow 2009$$

$$-\frac{125a^3\text{arctanh}(\sin(c+dx))}{128d}+\frac{3a^3\tan^7(c+dx)}{7d}+\frac{a^3\tan^5(c+dx)}{5d}-\frac{a^3\tan^3(c+dx)}{3d}+\frac{a^3\tan(c+dx)}{d}-\frac{a^3\tan^5(c+dx)\sec^3(c+dx)}{8d}+\frac{5a^3t}{}$$

input
```
Int[Tan[c + d*x]^12/(a + a*Sec[c + d*x])^3,x]
```

output
```
-((-(a^3*x) + (125*a^3*ArcTanh[Sin[c + d*x]])/(128*d) + (a^3*Tan[c + d*x])
/d - (115*a^3*Sec[c + d*x]*Tan[c + d*x])/(128*d) - (5*a^3*Sec[c + d*x]^3*T
an[c + d*x])/(64*d) - (a^3*Tan[c + d*x]^3)/(3*d) + (5*a^3*Sec[c + d*x]*Tan
[c + d*x]^3)/(8*d) + (5*a^3*Sec[c + d*x]^3*Tan[c + d*x]^3)/(48*d) + (a^3*T
an[c + d*x]^5)/(5*d) - (a^3*Sec[c + d*x]*Tan[c + d*x]^5)/(2*d) - (a^3*Sec[
c + d*x]^3*Tan[c + d*x]^5)/(8*d) + (3*a^3*Tan[c + d*x]^7)/(7*d))/a^6)
```

3.95.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4374
```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

rule 4376
```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Simp[a^(2*n)/e^(2*n)   Int[(e*Cot[c + d*x])^(m + 2*
n)/(-a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[a
^2 - b^2, 0] && ILtQ[n, 0]
```

3.95.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 5.02 (sec) , antiderivative size = 227, normalized size of antiderivative = 0.96

method	result
risch	$\frac{x}{a^3} - \frac{i\left(27195\,e^{15i(dx+c)}+65135\,e^{13i(dx+c)}+161280\,e^{12i(dx+c)}+63595\,e^{11i(dx+c)}+286720\,e^{10i(dx+c)}+133175\,e^{9i(dx+c}\right.}{}$
derivativedivides	$\frac{1}{8\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^8}+\frac{13}{14\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^7}+\frac{65}{24\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^6}+\frac{143}{40\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^5}+\frac{79}{64\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^4}-\frac{4}{32\left(\tan\left(\frac{dx}{2}\right.\right.}$
default	$\frac{1}{8\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^8}+\frac{13}{14\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^7}+\frac{65}{24\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^6}+\frac{143}{40\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^5}+\frac{79}{64\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^4}-\frac{4}{32\left(\tan\left(\frac{dx}{2}\right.\right.}$

input
```
int(tan(d*x+c)^12/(a+a*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
x/a^3-1/6720*I*(27195*exp(15*I*(d*x+c))+65135*exp(13*I*(d*x+c))+161280*exp
(12*I*(d*x+c))+63595*exp(11*I*(d*x+c))+286720*exp(10*I*(d*x+c))+133175*exp
(9*I*(d*x+c))+519680*exp(8*I*(d*x+c))-133175*exp(7*I*(d*x+c))+544768*exp(6
*I*(d*x+c))-63595*exp(5*I*(d*x+c))+254464*exp(4*I*(d*x+c))-65135*exp(3*I*(
d*x+c))+118784*exp(2*I*(d*x+c))-27195*exp(I*(d*x+c))+14848)/d/a^3/(exp(2*I
*(d*x+c))+1)^8-125/128/a^3/d*ln(exp(I*(d*x+c))+I)+125/128/a^3/d*ln(exp(I*(
d*x+c))-I)
```

3.95.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.62

$$\int \frac{\tan^{12}(c+dx)}{(a+a\sec(c+dx))^3}\,dx$$
$$= \frac{26880\,dx\cos(dx+c)^8 - 13125\cos(dx+c)^8\log(\sin(dx+c)+1) + 13125\cos(dx+c)^8\log(-\sin(dx+$$

input
```
integrate(tan(d*x+c)^12/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

output
```
1/26880*(26880*d*x*cos(d*x + c)^8 - 13125*cos(d*x + c)^8*log(sin(d*x + c)
+ 1) + 13125*cos(d*x + c)^8*log(-sin(d*x + c) + 1) - 2*(14848*cos(d*x + c)
^7 - 27195*cos(d*x + c)^6 + 7424*cos(d*x + c)^5 + 17710*cos(d*x + c)^4 - 1
4592*cos(d*x + c)^3 - 1960*cos(d*x + c)^2 + 5760*cos(d*x + c) - 1680)*sin(
d*x + c))/(a^3*d*cos(d*x + c)^8)
```

3.95.6 Sympy [F]

$$\int \frac{\tan^{12}(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{\int \frac{\tan^{12}(c+dx)}{\sec^3(c+dx)+3\sec^2(c+dx)+3\sec(c+dx)+1}\,dx}{a^3}$$

input
```
integrate(tan(d*x+c)**12/(a+a*sec(d*x+c))**3,x)
```

output
```
Integral(tan(c + d*x)**12/(sec(c + d*x)**3 + 3*sec(c + d*x)**2 + 3*sec(c +
 d*x) + 1), x)/a**3
```

3.95.7 Maxima [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 429, normalized size of antiderivative = 1.81

$$\int \frac{\tan^{12}(c+dx)}{(a+a\sec(c+dx))^3}\,dx =$$
$$-\frac{2\left(\frac{315\sin(dx+c)}{\cos(dx+c)+1} - \frac{11375\sin(dx+c)^3}{(\cos(dx+c)+1)^3} + \frac{79723\sin(dx+c)^5}{(\cos(dx+c)+1)^5} - \frac{269879\sin(dx+c)^7}{(\cos(dx+c)+1)^7} + \frac{550089\sin(dx+c)^9}{(\cos(dx+c)+1)^9} - \frac{749973\sin(dx+c)^{11}}{(\cos(dx+c)+1)^{11}} + \frac{212625\sin(dx+c)^{13}}{(\cos(dx+c)+1)^{13}} - \frac{26565\,s}{(\cos(dx}\right.}{a^3 - \frac{8a^3\sin(dx+c)^2}{(\cos(dx+c)+1)^2} + \frac{28a^3\sin(dx+c)^4}{(\cos(dx+c)+1)^4} - \frac{56a^3\sin(dx+c)^6}{(\cos(dx+c)+1)^6} + \frac{70a^3\sin(dx+c)^8}{(\cos(dx+c)+1)^8} - \frac{56a^3\sin(dx+c)^{10}}{(\cos(dx+c)+1)^{10}} + \frac{28a^3\sin(dx+c)^{12}}{(\cos(dx+c)+1)^{12}} - \frac{8a^3\sin(dx+c)^{14}}{(\cos(dx+c)+1)^{14}} + \frac{a^3\sin(d}{(\cos(dx+}}$$
$$13440\,d$$

input
```
integrate(tan(d*x+c)^12/(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

output
```
-1/13440*(2*(315*sin(d*x + c)/(cos(d*x + c) + 1) - 11375*sin(d*x + c)^3/(c
os(d*x + c) + 1)^3 + 79723*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 269879*si
n(d*x + c)^7/(cos(d*x + c) + 1)^7 + 550089*sin(d*x + c)^9/(cos(d*x + c) +
1)^9 - 749973*sin(d*x + c)^11/(cos(d*x + c) + 1)^11 + 212625*sin(d*x + c)^
13/(cos(d*x + c) + 1)^13 - 26565*sin(d*x + c)^15/(cos(d*x + c) + 1)^15)/(a
^3 - 8*a^3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 28*a^3*sin(d*x + c)^4/(co
s(d*x + c) + 1)^4 - 56*a^3*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + 70*a^3*si
n(d*x + c)^8/(cos(d*x + c) + 1)^8 - 56*a^3*sin(d*x + c)^10/(cos(d*x + c) +
 1)^10 + 28*a^3*sin(d*x + c)^12/(cos(d*x + c) + 1)^12 - 8*a^3*sin(d*x + c)
^14/(cos(d*x + c) + 1)^14 + a^3*sin(d*x + c)^16/(cos(d*x + c) + 1)^16) - 2
6880*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^3 + 13125*log(sin(d*x + c)/
(cos(d*x + c) + 1) + 1)/a^3 - 13125*log(sin(d*x + c)/(cos(d*x + c) + 1) -
1)/a^3)/d
```

3.95.8 Giac [A] (verification not implemented)

Time = 25.91 (sec) , antiderivative size = 175, normalized size of antiderivative = 0.74

$$\int \frac{\tan^{12}(c+dx)}{(a+a\sec(c+dx))^3}\,dx$$

$$= \frac{\frac{13440\,(dx+c)}{a^3} - \frac{13125\,\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{a^3} + \frac{13125\,\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{a^3} + \frac{2\left(26565\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{15} - 212625\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{13}\right.}{1}$$

input
```
integrate(tan(d*x+c)^12/(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

output
```
1/13440*(13440*(d*x + c)/a^3 - 13125*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^
3 + 13125*log(abs(tan(1/2*d*x + 1/2*c) - 1))/a^3 + 2*(26565*tan(1/2*d*x +
1/2*c)^15 - 212625*tan(1/2*d*x + 1/2*c)^13 + 749973*tan(1/2*d*x + 1/2*c)^1
1 - 550089*tan(1/2*d*x + 1/2*c)^9 + 269879*tan(1/2*d*x + 1/2*c)^7 - 79723*
tan(1/2*d*x + 1/2*c)^5 + 11375*tan(1/2*d*x + 1/2*c)^3 - 315*tan(1/2*d*x +
1/2*c))/((tan(1/2*d*x + 1/2*c)^2 - 1)^8*a^3))/d
```

3.95.9 Mupad [B] (verification not implemented)

Time = 15.70 (sec) , antiderivative size = 265, normalized size of antiderivative = 1.12

$$\int \frac{\tan^{12}(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{x}{a^3} - \frac{125\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{64\,a^3\,d}$$
$$- \frac{-\frac{253\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{15}}{64} + \frac{2025\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{13}}{64} - \frac{35713\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{11}}{320} + \frac{183363\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{9}}{2240} - \frac{269879\,t}{}}{d\left(a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{16} - 8\,a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{14} + 28\,a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{12} - 56\,a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{10} + 70\,a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{8} - 56\,a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{6} + 28\,a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{4} - 8\,a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{2} + a^3\right)}$$

input
```
int(tan(c + d*x)^12/(a + a/cos(c + d*x))^3,x)
```

output
```
x/a^3 - (125*atanh(tan(c/2 + (d*x)/2)))/(64*a^3*d) - ((3*tan(c/2 + (d*x)/2
))/64 - (325*tan(c/2 + (d*x)/2)^3)/192 + (11389*tan(c/2 + (d*x)/2)^5)/960
- (269879*tan(c/2 + (d*x)/2)^7)/6720 + (183363*tan(c/2 + (d*x)/2)^9)/2240
- (35713*tan(c/2 + (d*x)/2)^11)/320 + (2025*tan(c/2 + (d*x)/2)^13)/64 - (2
53*tan(c/2 + (d*x)/2)^15)/64)/(d*(28*a^3*tan(c/2 + (d*x)/2)^4 - 8*a^3*tan(
c/2 + (d*x)/2)^2 - 56*a^3*tan(c/2 + (d*x)/2)^6 + 70*a^3*tan(c/2 + (d*x)/2)
^8 - 56*a^3*tan(c/2 + (d*x)/2)^10 + 28*a^3*tan(c/2 + (d*x)/2)^12 - 8*a^3*t
an(c/2 + (d*x)/2)^14 + a^3*tan(c/2 + (d*x)/2)^16 + a^3))
```

3.96 $\int \frac{\tan^{10}(c+dx)}{(a+a\sec(c+dx))^3}\,dx$

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3.96.1 Optimal result

Integrand size = 21, antiderivative size = 169

$$\int \frac{\tan^{10}(c+dx)}{(a+a\sec(c+dx))^3}\,dx = -\frac{x}{a^3} + \frac{19\operatorname{arctanh}(\sin(c+dx))}{16a^3d} + \frac{\tan(c+dx)}{a^3d} - \frac{17\sec(c+dx)\tan(c+dx)}{16a^3d} - \frac{\sec^3(c+dx)\tan(c+dx)}{8a^3d} - \frac{\tan^3(c+dx)}{3a^3d} + \frac{3\sec(c+dx)\tan^3(c+dx)}{4a^3d} + \frac{\sec^3(c+dx)\tan^3(c+dx)}{6a^3d} - \frac{3\tan^5(c+dx)}{5a^3d}$$

output

```
-x/a^3+19/16*arctanh(sin(d*x+c))/a^3/d+tan(d*x+c)/a^3/d-17/16*sec(d*x+c)*t
an(d*x+c)/a^3/d-1/8*sec(d*x+c)^3*tan(d*x+c)/a^3/d-1/3*tan(d*x+c)^3/a^3/d+3
/4*sec(d*x+c)*tan(d*x+c)^3/a^3/d+1/6*sec(d*x+c)^3*tan(d*x+c)^3/a^3/d-3/5*t
an(d*x+c)^5/a^3/d
```

3.96.2 Mathematica [A] (verified)

Time = 3.63 (sec) , antiderivative size = 303, normalized size of antiderivative = 1.79

$$\int \frac{\tan^{10}(c+dx)}{(a+a\sec(c+dx))^3}\,dx = -\frac{\cos^6\left(\frac{1}{2}(c+dx)\right)\sec^3(c+dx)\left(9120\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)\right.$$

input
```
Integrate[Tan[c + d*x]^10/(a + a*Sec[c + d*x])^3,x]
```

output
```
-1/960*(Cos[(c + d*x)/2]^6*Sec[c + d*x]^3*(9120*(Log[Cos[(c + d*x)/2] - Si
n[(c + d*x)/2]] - Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) + Sec[c]*Sec[c
 + d*x]^6*(2400*d*x*Cos[c] + 1800*d*x*Cos[c + 2*d*x] + 1800*d*x*Cos[3*c +
2*d*x] + 720*d*x*Cos[3*c + 4*d*x] + 720*d*x*Cos[5*c + 4*d*x] + 120*d*x*Cos
[5*c + 6*d*x] + 120*d*x*Cos[7*c + 6*d*x] + 1760*Sin[c] - 210*Sin[d*x] - 21
0*Sin[2*c + d*x] - 1440*Sin[c + 2*d*x] + 1200*Sin[3*c + 2*d*x] + 865*Sin[2
*c + 3*d*x] + 865*Sin[4*c + 3*d*x] - 1296*Sin[3*c + 4*d*x] - 240*Sin[5*c +
 4*d*x] + 435*Sin[4*c + 5*d*x] + 435*Sin[6*c + 5*d*x] - 176*Sin[5*c + 6*d*
x])))/(a^3*d*(1 + Sec[c + d*x])^3)
```

3.96.3 Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.03, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 4376, 25, 3042, 4374, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^{10}(c+dx)}{(a\sec(c+dx)+a)^3}\,dx$$
$$\downarrow 3042$$
$$\int \frac{\cot\left(c+dx+\frac{\pi}{2}\right)^{10}}{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^3}dx$$
$$\downarrow 4376$$
$$\frac{\int -(a-a\sec(c+dx))^3\tan^4(c+dx)dx}{a^6}$$
$$\downarrow 25$$
$$-\frac{\int (a-a\sec(c+dx))^3\tan^4(c+dx)dx}{a^6}$$
$$\downarrow 3042$$
$$-\frac{\int \cot\left(c+dx+\frac{\pi}{2}\right)^4\left(a-a\csc\left(c+dx+\frac{\pi}{2}\right)\right)^3 dx}{a^6}$$
$$\downarrow 4374$$

$$-\frac{\int \left(a^3\tan^4(c+dx)-a^3\sec^3(c+dx)\tan^4(c+dx)+3a^3\sec^2(c+dx)\tan^4(c+dx)-3a^3\sec(c+dx)\tan^4(c+dx\right.}{a^6}$$

$$\downarrow \text{2009}$$

$$-\frac{-\frac{19a^3\text{arctanh}(\sin(c+dx))}{16d}+\frac{3a^3\tan^5(c+dx)}{5d}+\frac{a^3\tan^3(c+dx)}{3d}-\frac{a^3\tan(c+dx)}{d}-\frac{a^3\tan^3(c+dx)\sec^3(c+dx)}{6d}+\frac{a^3\tan(c+dx)\sec^3(c+d}{8d}}{a^6}$$

input
```
Int[Tan[c + d*x]^10/(a + a*Sec[c + d*x])^3,x]
```

output
```
-((a^3*x - (19*a^3*ArcTanh[Sin[c + d*x]])/(16*d) - (a^3*Tan[c + d*x])/d +
(17*a^3*Sec[c + d*x]*Tan[c + d*x])/(16*d) + (a^3*Sec[c + d*x]^3*Tan[c + d*
x])/(8*d) + (a^3*Tan[c + d*x]^3)/(3*d) - (3*a^3*Sec[c + d*x]*Tan[c + d*x]^
3)/(4*d) - (a^3*Sec[c + d*x]^3*Tan[c + d*x]^3)/(6*d) + (3*a^3*Tan[c + d*x]
^5)/(5*d))/a^6)
```

3.96.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4374
```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

rule 4376
```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Simp[a^(2*n)/e^(2*n)   Int[(e*Cot[c + d*x])^(m + 2*
n)/(-a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[a
^2 - b^2, 0] && ILtQ[n, 0]
```

3.96.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 3.21 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.15

method	result
risch	$-\frac{x}{a^3}+\frac{i\left(435\,\mathrm{e}^{11i(dx+c)}-240\,\mathrm{e}^{10i(dx+c)}+865\,\mathrm{e}^{9i(dx+c)}+1200\,\mathrm{e}^{8i(dx+c)}-210\,\mathrm{e}^{7i(dx+c)}+1760\,\mathrm{e}^{6i(dx+c)}+210\,\mathrm{e}^{5i(dx+c)}\right.}{120d\,a^3\left(\mathrm{e}^{2i(dx+c)}+1\right)^6}$
derivativedivides	$\frac{-2\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\frac{1}{6\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^6}+\frac{11}{10\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^5}+\frac{11}{4\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^4}+\frac{11}{4\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^3}-\frac{}{16\left(\tan\right.}}{}$
default	$\frac{-2\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\frac{1}{6\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^6}+\frac{11}{10\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^5}+\frac{11}{4\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^4}+\frac{11}{4\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^3}-\frac{}{16\left(\tan\right.}}{}$

input
```
int(tan(d*x+c)^10/(a+a*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
-x/a^3+1/120*I*(435*exp(11*I*(d*x+c))-240*exp(10*I*(d*x+c))+865*exp(9*I*(d
*x+c))+1200*exp(8*I*(d*x+c))-210*exp(7*I*(d*x+c))+1760*exp(6*I*(d*x+c))+21
0*exp(5*I*(d*x+c))+1440*exp(4*I*(d*x+c))-865*exp(3*I*(d*x+c))+1296*exp(2*I
*(d*x+c))-435*exp(I*(d*x+c))+176)/d/a^3/(exp(2*I*(d*x+c))+1)^6+19/16/a^3/d
*ln(exp(I*(d*x+c))+I)-19/16/a^3/d*ln(exp(I*(d*x+c))-I)
```

3.96.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.75

$$\int \frac{\tan^{10}(c+dx)}{(a+a\sec(c+dx))^3}\,dx =$$
$$-\frac{480\,dx\cos(dx+c)^6-285\,\cos(dx+c)^6\log(\sin(dx+c)+1)+285\,\cos(dx+c)^6\log(-\sin(dx+c)+}{}$$

input
```
integrate(tan(d*x+c)^10/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

output
```
-1/480*(480*d*x*cos(d*x + c)^6 - 285*cos(d*x + c)^6*log(sin(d*x + c) + 1)
+ 285*cos(d*x + c)^6*log(-sin(d*x + c) + 1) - 2*(176*cos(d*x + c)^5 - 435*
cos(d*x + c)^4 + 208*cos(d*x + c)^3 + 110*cos(d*x + c)^2 - 144*cos(d*x + c
) + 40)*sin(d*x + c))/(a^3*d*cos(d*x + c)^6)
```

3.96.6 Sympy [F]

$$\int \frac{\tan^{10}(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{\int \frac{\tan^{10}(c+dx)}{\sec^3(c+dx)+3\sec^2(c+dx)+3\sec(c+dx)+1}\,dx}{a^3}$$

input
```
integrate(tan(d*x+c)**10/(a+a*sec(d*x+c))**3,x)
```

output
```
Integral(tan(c + d*x)**10/(sec(c + d*x)**3 + 3*sec(c + d*x)**2 + 3*sec(c +
 d*x) + 1), x)/a**3
```

3.96.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 343 vs. 2(155) = 310.

Time = 0.30 (sec) , antiderivative size = 343, normalized size of antiderivative = 2.03

$$\int \frac{\tan^{10}(c+dx)}{(a+a\sec(c+dx))^3}\,dx = -\frac{\frac{2\left(\frac{45\sin(dx+c)}{\cos(dx+c)+1}-\frac{95\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{366\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{1746\sin(dx+c)^7}{(\cos(dx+c)+1)^7}-\frac{3135\sin(dx+c)^9}{(\cos(dx+c)+1)^9}+\frac{525\sin(dx+c)^{11}}{(\cos(dx+c)+1)^{11}}\right)}{a^3-\frac{6a^3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{15a^3\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{20a^3\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{15a^3\sin(dx+c)^8}{(\cos(dx+c)+1)^8}-\frac{6a^3\sin(dx+c)^{10}}{(\cos(dx+c)+1)^{10}}+\frac{a^3\sin(dx+c)^{12}}{(\cos(dx+c)+1)^{12}}}+\frac{480\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^3}-}{240\,d}$$

input
```
integrate(tan(d*x+c)^10/(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

output
```
-1/240*(2*(45*sin(d*x + c)/(cos(d*x + c) + 1) - 95*sin(d*x + c)^3/(cos(d*x
 + c) + 1)^3 - 366*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 1746*sin(d*x + c)
^7/(cos(d*x + c) + 1)^7 - 3135*sin(d*x + c)^9/(cos(d*x + c) + 1)^9 + 525*s
in(d*x + c)^11/(cos(d*x + c) + 1)^11)/(a^3 - 6*a^3*sin(d*x + c)^2/(cos(d*x
 + c) + 1)^2 + 15*a^3*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 - 20*a^3*sin(d*x
 + c)^6/(cos(d*x + c) + 1)^6 + 15*a^3*sin(d*x + c)^8/(cos(d*x + c) + 1)^8
- 6*a^3*sin(d*x + c)^10/(cos(d*x + c) + 1)^10 + a^3*sin(d*x + c)^12/(cos(d
*x + c) + 1)^12) + 480*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^3 - 285*l
og(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/a^3 + 285*log(sin(d*x + c)/(cos(d*
x + c) + 1) - 1)/a^3)/d
```

3.96.8 Giac [A] (verification not implemented)

Time = 12.48 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.88

$$\int \frac{\tan^{10}(c+dx)}{(a+a\sec(c+dx))^3}\,dx = -\frac{\frac{240\,(dx+c)}{a^3} - \frac{285\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{a^3} + \frac{285\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{a^3} + \frac{2\left(525\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{11}-3135\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{9}+1746\,ta\right.}{\left(t\right.}}{240\,d}$$

input
```
integrate(tan(d*x+c)^10/(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

output
```
-1/240*(240*(d*x + c)/a^3 - 285*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^3 + 2
85*log(abs(tan(1/2*d*x + 1/2*c) - 1))/a^3 + 2*(525*tan(1/2*d*x + 1/2*c)^11
 - 3135*tan(1/2*d*x + 1/2*c)^9 + 1746*tan(1/2*d*x + 1/2*c)^7 - 366*tan(1/2
*d*x + 1/2*c)^5 - 95*tan(1/2*d*x + 1/2*c)^3 + 45*tan(1/2*d*x + 1/2*c))/((t
an(1/2*d*x + 1/2*c)^2 - 1)^6*a^3))/d
```

3.96.9 Mupad [B] (verification not implemented)

Time = 15.04 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.23

$$\int \frac{\tan^{10}(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{19\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{8\,a^3\,d} - \frac{x}{a^3} - \frac{\frac{35\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{11}}{8} - \frac{209\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{9}}{8} + \frac{291\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{7}}{20} - \frac{61\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{5}}{20} - \frac{19\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{3}}{24} +}{d\left(a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{12} - 6\,a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{10} + 15\,a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{8} - 20\,a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{6} + 15\,a^3\tan\left(\frac{c}{2}-\right.\right.}$$

input
```
int(tan(c + d*x)^10/(a + a/cos(c + d*x))^3,x)
```

output
```
(19*atanh(tan(c/2 + (d*x)/2)))/(8*a^3*d) - x/a^3 - ((3*tan(c/2 + (d*x)/2))
/8 - (19*tan(c/2 + (d*x)/2)^3)/24 - (61*tan(c/2 + (d*x)/2)^5)/20 + (291*ta
n(c/2 + (d*x)/2)^7)/20 - (209*tan(c/2 + (d*x)/2)^9)/8 + (35*tan(c/2 + (d*x
)/2)^11)/8)/(d*(15*a^3*tan(c/2 + (d*x)/2)^4 - 6*a^3*tan(c/2 + (d*x)/2)^2 -
 20*a^3*tan(c/2 + (d*x)/2)^6 + 15*a^3*tan(c/2 + (d*x)/2)^8 - 6*a^3*tan(c/2
 + (d*x)/2)^10 + a^3*tan(c/2 + (d*x)/2)^12 + a^3))
```

3.97 $\int \frac{\tan^8(c+dx)}{(a+a\sec(c+dx))^3}\,dx$

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3.97.1 Optimal result

Integrand size = 21, antiderivative size = 99

$$\int \frac{\tan^8(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{x}{a^3} - \frac{13\operatorname{arctanh}(\sin(c+dx))}{8a^3d} - \frac{\tan(c+dx)}{a^3d} + \frac{11\sec(c+dx)\tan(c+dx)}{8a^3d} + \frac{\sec^3(c+dx)\tan(c+dx)}{4a^3d} - \frac{\tan^3(c+dx)}{a^3d}$$

output
```
x/a^3-13/8*arctanh(sin(d*x+c))/a^3/d-tan(d*x+c)/a^3/d+11/8*sec(d*x+c)*tan(
d*x+c)/a^3/d+1/4*sec(d*x+c)^3*tan(d*x+c)/a^3/d-tan(d*x+c)^3/a^3/d
```

3.97.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 230 vs. 2(99) = 198.

Time = 2.07 (sec) , antiderivative size = 230, normalized size of antiderivative = 2.32

$$\int \frac{\tan^8(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{\sec^4(c+dx)\left(24dx + 39\log\left(\cos\left(\frac{1}{2}(c+dx)\right) - \sin\left(\frac{1}{2}(c+dx)\right)\right) + 4\cos(2(c+dx))\left(8dx + 13\log\left(\cos\left(\frac{1}{2}(\right.\right.\right.\right.$$

input
```
Integrate[Tan[c + d*x]^8/(a + a*Sec[c + d*x])^3,x]
```

output
```
(Sec[c + d*x]^4*(24*d*x + 39*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 4*
Cos[2*(c + d*x)]*(8*d*x + 13*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - 13
*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) + Cos[4*(c + d*x)]*(8*d*x + 13*
Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - 13*Log[Cos[(c + d*x)/2] + Sin[(
c + d*x)/2]]) - 39*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + 38*Sin[c + d
*x] - 32*Sin[2*(c + d*x)] + 22*Sin[3*(c + d*x)]))/(64*a^3*d)
```

3.97.3 Rubi [A] (verified)

Time $= 0.45$ (sec) , antiderivative size $= 103$, normalized size of antiderivative $= 1.04$, number of steps used $= 6$, number of rules used $= 6$, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used $= \{3042, 4376, 25, 3042, 4374, 2009\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^8(c+dx)}{(a\sec(c+dx)+a)^3}\,dx$$

$$\downarrow 3042$$

$$\int \frac{\cot\left(c+dx+\frac{\pi}{2}\right)^8}{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^3}dx$$

$$\downarrow 4376$$

$$\frac{\int -(a-a\sec(c+dx))^3\tan^2(c+dx)dx}{a^6}$$

$$\downarrow 25$$

$$-\frac{\int (a-a\sec(c+dx))^3\tan^2(c+dx)dx}{a^6}$$

$$\downarrow 3042$$

$$-\frac{\int \cot\left(c+dx+\frac{\pi}{2}\right)^2\left(a-a\csc\left(c+dx+\frac{\pi}{2}\right)\right)^3 dx}{a^6}$$

$$\downarrow 4374$$

$$-\frac{\int \left(-\sec^3(c+dx)\tan^2(c+dx)a^3+3\sec^2(c+dx)\tan^2(c+dx)a^3-3\sec(c+dx)\tan^2(c+dx)a^3+\tan^2(c+dx)\right.}{a^6}$$

$$\downarrow 2009$$

$$-\frac{\frac{13a^3\text{arctanh}(\sin(c+dx))}{8d}+\frac{a^3\tan^3(c+dx)}{d}+\frac{a^3\tan(c+dx)}{d}-\frac{a^3\tan(c+dx)\sec^3(c+dx)}{4d}-\frac{11a^3\tan(c+dx)\sec(c+dx)}{8d}-a^3x}{a^6}$$

input
```
Int[Tan[c + d*x]^8/(a + a*Sec[c + d*x])^3,x]
```

output
```
-((-(a^3*x) + (13*a^3*ArcTanh[Sin[c + d*x]])/(8*d) + (a^3*Tan[c + d*x])/d
- (11*a^3*Sec[c + d*x]*Tan[c + d*x])/(8*d) - (a^3*Sec[c + d*x]^3*Tan[c + d
*x])/(4*d) + (a^3*Tan[c + d*x]^3)/d)/a^6)
```

3.97.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4374
```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

rule 4376
```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Simp[a^(2*n)/e^(2*n)   Int[(e*Cot[c + d*x])^(m + 2*
n)/(-a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[a
^2 - b^2, 0] && ILtQ[n, 0]
```

3.97.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.01 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.39

method	result
risch	$\frac{x}{a^3}-\frac{i\left(11\,\mathrm{e}^{7i(dx+c)}-16\,\mathrm{e}^{6i(dx+c)}+19\,\mathrm{e}^{5i(dx+c)}-19\,\mathrm{e}^{3i(dx+c)}+16\,\mathrm{e}^{2i(dx+c)}-11\,\mathrm{e}^{i(dx+c)}\right)}{4da^3\left(\mathrm{e}^{2i(dx+c)}+1\right)^4}-\frac{13\ln\left(\mathrm{e}^{i(dx+c)}+i\right)}{8a^3d}+\frac{13}{}$
derivativedivides	$\frac{-\frac{1}{4\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^4}+\frac{3}{2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^3}-\frac{27}{8\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^2}+\frac{21}{8\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}-\frac{13\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{8}+\frac{1}{4\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}}{a^3}$
default	$\frac{-\frac{1}{4\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^4}+\frac{3}{2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^3}-\frac{27}{8\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^2}+\frac{21}{8\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}-\frac{13\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{8}+\frac{1}{4\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}}{a^3}$

input
```
int(tan(d*x+c)^8/(a+a*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
x/a^3-1/4*I/d/a^3/(exp(2*I*(d*x+c))+1)^4*(11*exp(7*I*(d*x+c))-16*exp(6*I*(
d*x+c))+19*exp(5*I*(d*x+c))-19*exp(3*I*(d*x+c))+16*exp(2*I*(d*x+c))-11*exp
(I*(d*x+c)))-13/8/a^3/d*ln(exp(I*(d*x+c))+I)+13/8/a^3/d*ln(exp(I*(d*x+c))-
I)
```

3.97.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.98

$$\int \frac{\tan^8(c+dx)}{(a+a\sec(c+dx))^3}\,dx$$
$$=\frac{16\,dx\cos\left(dx+c\right)^4-13\,\cos\left(dx+c\right)^4\log\left(\sin\left(dx+c\right)+1\right)+13\,\cos\left(dx+c\right)^4\log\left(-\sin\left(dx+c\right)+1\right)+2}{16\,a^3d\cos\left(dx+c\right)^4}$$

input
```
integrate(tan(d*x+c)^8/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

output
```
1/16*(16*d*x*cos(d*x + c)^4 - 13*cos(d*x + c)^4*log(sin(d*x + c) + 1) + 13
*cos(d*x + c)^4*log(-sin(d*x + c) + 1) + 2*(11*cos(d*x + c)^2 - 8*cos(d*x
+ c) + 2)*sin(d*x + c))/(a^3*d*cos(d*x + c)^4)
```

3.97.6 Sympy [F]

$$\int \frac{\tan^8(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{\int \frac{\tan^8(c+dx)}{\sec^3(c+dx)+3\sec^2(c+dx)+3\sec(c+dx)+1}\,dx}{a^3}$$

input
```
integrate(tan(d*x+c)**8/(a+a*sec(d*x+c))**3,x)
```

output
```
Integral(tan(c + d*x)**8/(sec(c + d*x)**3 + 3*sec(c + d*x)**2 + 3*sec(c +
d*x) + 1), x)/a**3
```

3.97.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 257 vs. 2(93) = 186.

Time = 0.30 (sec) , antiderivative size = 257, normalized size of antiderivative = 2.60

$$\int \frac{\tan^8(c+dx)}{(a+a\sec(c+dx))^3}\,dx$$
$$= \frac{\frac{2\left(\frac{5\sin(dx+c)}{\cos(dx+c)+1}-\frac{13\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{3\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{21\sin(dx+c)^7}{(\cos(dx+c)+1)^7}\right)}{a^3-\frac{4a^3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{6a^3\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{4a^3\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{a^3\sin(dx+c)^8}{(\cos(dx+c)+1)^8}}+\frac{16\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^3}-\frac{13\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a^3}+\frac{13\log\left(\frac{\sin}{\cos(}\right.}{8\,d}$$

input
```
integrate(tan(d*x+c)^8/(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

output
```
1/8*(2*(5*sin(d*x + c)/(cos(d*x + c) + 1) - 13*sin(d*x + c)^3/(cos(d*x + c
) + 1)^3 + 3*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 21*sin(d*x + c)^7/(cos(
d*x + c) + 1)^7)/(a^3 - 4*a^3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 6*a^3*
sin(d*x + c)^4/(cos(d*x + c) + 1)^4 - 4*a^3*sin(d*x + c)^6/(cos(d*x + c) +
 1)^6 + a^3*sin(d*x + c)^8/(cos(d*x + c) + 1)^8) + 16*arctan(sin(d*x + c)/
(cos(d*x + c) + 1))/a^3 - 13*log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/a^3
+ 13*log(sin(d*x + c)/(cos(d*x + c) + 1) - 1)/a^3)/d
```

3.97.8 Giac [A] (verification not implemented)

Time = 5.00 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.24

$$\int \frac{\tan^8(c+dx)}{(a+a\sec(c+dx))^3}\,dx$$

$$= \frac{\frac{8\,(dx+c)}{a^3} - \frac{13\,\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{a^3} + \frac{13\,\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{a^3} + \frac{2\left(21\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+3\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-13\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+\right.}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)^4 a^3}}{8\,d}$$

input
```
integrate(tan(d*x+c)^8/(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

output
```
1/8*(8*(d*x + c)/a^3 - 13*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^3 + 13*log(
abs(tan(1/2*d*x + 1/2*c) - 1))/a^3 + 2*(21*tan(1/2*d*x + 1/2*c)^7 + 3*tan(
1/2*d*x + 1/2*c)^5 - 13*tan(1/2*d*x + 1/2*c)^3 + 5*tan(1/2*d*x + 1/2*c))/(
(tan(1/2*d*x + 1/2*c)^2 - 1)^4*a^3))/d
```

3.97.9 Mupad [B] (verification not implemented)

Time = 14.98 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.49

$$\int \frac{\tan^8(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{x}{a^3} - \frac{13\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{4\,a^3\,d}$$
$$+ \frac{\frac{21\,\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^7}{4} + \frac{3\,\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5}{4} - \frac{13\,\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3}{4} + \frac{5\,\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{4}}{d\left(a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^8 - 4\,a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6 + 6\,a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4 - 4\,a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2 + a^3\right)}$$

input
```
int(tan(c + d*x)^8/(a + a/cos(c + d*x))^3,x)
```

output
```
x/a^3 - (13*atanh(tan(c/2 + (d*x)/2)))/(4*a^3*d) + ((5*tan(c/2 + (d*x)/2))
/4 - (13*tan(c/2 + (d*x)/2)^3)/4 + (3*tan(c/2 + (d*x)/2)^5)/4 + (21*tan(c/
2 + (d*x)/2)^7)/4)/(d*(6*a^3*tan(c/2 + (d*x)/2)^4 - 4*a^3*tan(c/2 + (d*x)/
2)^2 - 4*a^3*tan(c/2 + (d*x)/2)^6 + a^3*tan(c/2 + (d*x)/2)^8 + a^3))
```

3.98 $\int \frac{\tan^6(c+dx)}{(a+a\sec(c+dx))^3}\,dx$

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3.98.1 Optimal result

Integrand size = 21, antiderivative size = 66

$$\int \frac{\tan^6(c+dx)}{(a+a\sec(c+dx))^3}\,dx = -\frac{x}{a^3} + \frac{7\operatorname{arctanh}(\sin(c+dx))}{2a^3d} - \frac{5\tan(c+dx)}{2a^3d} - \frac{(1-\sec(c+dx))\tan(c+dx)}{2a^3d}$$

output

```
-x/a^3+7/2*arctanh(sin(d*x+c))/a^3/d-5/2*tan(d*x+c)/a^3/d-1/2*(1-sec(d*x+c
))*tan(d*x+c)/a^3/d
```

3.98.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 241 vs. 2(66) = 132.

Time = 1.73 (sec) , antiderivative size = 241, normalized size of antiderivative = 3.65

$$\int \frac{\tan^6(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{2\cos^6\left(\frac{1}{2}(c+dx)\right)\sec^3(c+dx)\left(-4x - \frac{14\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)}{d} + \frac{14\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)}{d} + \frac{}{d(c}\right.}{}$$

input

```
Integrate[Tan[c + d*x]^6/(a + a*Sec[c + d*x])^3,x]
```

output
```
(2*Cos[(c + d*x)/2]^6*Sec[c + d*x]^3*(-4*x - (14*Log[Cos[(c + d*x)/2] - Si
n[(c + d*x)/2]])/d + (14*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]])/d + 1/(
d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^2) - 1/(d*(Cos[(c + d*x)/2] + Sin[
(c + d*x)/2])^2) - (12*Sin[d*x])/(d*(Cos[c/2] - Sin[c/2])*(Cos[c/2] + Sin[
c/2])*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])*(Cos[(c + d*x)/2] + Sin[(c + d
*x)/2])))))/(a^3*(1 + Sec[c + d*x])^3)
```

3.98.3 Rubi [A] (verified)

Time $= 0.51$ (sec) , antiderivative size $= 74$, normalized size of antiderivative $= 1.12$, number of steps used $= 11$, number of rules used $= 10$, $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$, Rules used $= \{3042, 4376, 3042, 4262, 3042, 4402, 3042, 4254, 24, 4257\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^6(c+dx)}{(a\sec(c+dx)+a)^3}\,dx$$
$$\downarrow 3042$$
$$\int \frac{\cot\left(c+dx+\frac{\pi}{2}\right)^6}{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^3}dx$$
$$\downarrow 4376$$
$$\frac{\int (a\sec(c+dx)-a)^3 dx}{a^6}$$
$$\downarrow 3042$$
$$\frac{\int \left(a\csc\left(c+dx+\frac{\pi}{2}\right)-a\right)^3 dx}{a^6}$$
$$\downarrow 4262$$
$$\frac{-\frac{1}{2}a\int (2a-5a\sec(c+dx))(a-a\sec(c+dx))dx - \frac{\tan(c+dx)(a^3-a^3\sec(c+dx))}{2d}}{a^6}$$
$$\downarrow 3042$$
$$\frac{-\frac{1}{2}a\int \left(2a-5a\csc\left(c+dx+\frac{\pi}{2}\right)\right)\left(a-a\csc\left(c+dx+\frac{\pi}{2}\right)\right)dx - \frac{\tan(c+dx)(a^3-a^3\sec(c+dx))}{2d}}{a^6}$$
$$\downarrow 4402$$
$$\frac{-\frac{1}{2}a\left(5a^2\int \sec^2(c+dx)dx - 7a^2\int \sec(c+dx)dx + 2a^2x\right) - \frac{\tan(c+dx)(a^3-a^3\sec(c+dx))}{2d}}{a^6}$$

$$\downarrow 3042$$

$$\frac{-\frac{1}{2}a\left(-7a^2\int \csc\left(c+dx+\frac{\pi}{2}\right)dx+5a^2\int \csc\left(c+dx+\frac{\pi}{2}\right)^2dx+2a^2x\right)-\frac{\tan(c+dx)(a^3-a^3\sec(c+dx))}{2d}}{a^6}$$

$$\downarrow 4254$$

$$\frac{-\frac{1}{2}a\left(-\frac{5a^2\int 1d(-\tan(c+dx))}{d}-7a^2\int \csc\left(c+dx+\frac{\pi}{2}\right)dx+2a^2x\right)-\frac{\tan(c+dx)(a^3-a^3\sec(c+dx))}{2d}}{a^6}$$

$$\downarrow 24$$

$$\frac{-\frac{1}{2}a\left(-7a^2\int \csc\left(c+dx+\frac{\pi}{2}\right)dx+\frac{5a^2\tan(c+dx)}{d}+2a^2x\right)-\frac{\tan(c+dx)(a^3-a^3\sec(c+dx))}{2d}}{a^6}$$

$$\downarrow 4257$$

$$\frac{-\frac{\tan(c+dx)(a^3-a^3\sec(c+dx))}{2d}-\frac{1}{2}a\left(-\frac{7a^2\text{arctanh}(\sin(c+dx))}{d}+\frac{5a^2\tan(c+dx)}{d}+2a^2x\right)}{a^6}$$

input
```
Int[Tan[c + d*x]^6/(a + a*Sec[c + d*x])^3,x]
```

output
```
(-1/2*((a^3 - a^3*Sec[c + d*x])*Tan[c + d*x])/d - (a*(2*a^2*x - (7*a^2*Arc
Tanh[Sin[c + d*x]])/d + (5*a^2*Tan[c + d*x])/d))/2)/a^6
```

3.98.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 4262
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Simp[(-b^2)*C
ot[c + d*x]*((a + b*Csc[c + d*x])^(n - 2)/(d*(n - 1))), x] + Simp[a/(n - 1)
   Int[(a + b*Csc[c + d*x])^(n - 2)*(a*(n - 1) + b*(3*n - 4)*Csc[c + d*x]),
 x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && GtQ[n, 1] && Inte
gerQ[2*n]
```

rule 4376
```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Simp[a^(2*n)/e^(2*n)   Int[(e*Cot[c + d*x])^(m + 2*
n)/(-a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[a
^2 - b^2, 0] && ILtQ[n, 0]
```

rule 4402
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))*(csc[(e_.) + (f_.)*(x_)]*(d_.) +
 (c_)), x_Symbol] :> Simp[a*c*x, x] + (Simp[b*d   Int[Csc[e + f*x]^2, x], x
] + Simp[(b*c + a*d)   Int[Csc[e + f*x], x], x]) /; FreeQ[{a, b, c, d, e, f
}, x] && NeQ[b*c - a*d, 0] && NeQ[b*c + a*d, 0]
```

3.98.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.28 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.59

method	result
risch	$-\frac{x}{a^3}-\frac{i\left(e^{3i(dx+c)}+6e^{2i(dx+c)}-e^{i(dx+c)}+6\right)}{da^3\left(e^{2i(dx+c)}+1\right)^2}-\frac{7\ln\left(e^{i(dx+c)}-i\right)}{2a^3d}+\frac{7\ln\left(e^{i(dx+c)}+i\right)}{2a^3d}$
derivativedivides	$\frac{\frac{1}{2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}+\frac{7}{2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}-\frac{7\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{2}-\frac{1}{2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^2}+\frac{7}{2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}+\frac{7\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right.}{2}}{a^3d}$
default	$\frac{\frac{1}{2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}+\frac{7}{2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}-\frac{7\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{2}-\frac{1}{2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^2}+\frac{7}{2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}+\frac{7\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right.}{2}}{a^3d}$

input
```
int(tan(d*x+c)^6/(a+a*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
-x/a^3-I*(exp(3*I*(d*x+c))+6*exp(2*I*(d*x+c))-exp(I*(d*x+c))+6)/d/a^3/(exp
(2*I*(d*x+c))+1)^2-7/2/a^3/d*ln(exp(I*(d*x+c))-I)+7/2/a^3/d*ln(exp(I*(d*x+
c))+I)
```

3.98.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.32

$$\int \frac{\tan^6(c+dx)}{(a+a\sec(c+dx))^3}\,dx = -\frac{4\,dx\cos(dx+c)^2 - 7\cos(dx+c)^2\log(\sin(dx+c)+1) + 7\cos(dx+c)^2\log(-\sin(dx+c)+1) + 2}{4\,a^3 d\cos(dx+c)^2}$$

input
```
integrate(tan(d*x+c)^6/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

output
```
-1/4*(4*d*x*cos(d*x + c)^2 - 7*cos(d*x + c)^2*log(sin(d*x + c) + 1) + 7*co
s(d*x + c)^2*log(-sin(d*x + c) + 1) + 2*(6*cos(d*x + c) - 1)*sin(d*x + c))
/(a^3*d*cos(d*x + c)^2)
```

3.98.6 Sympy [F]

$$\int \frac{\tan^6(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{\int \frac{\tan^6(c+dx)}{\sec^3(c+dx)+3\sec^2(c+dx)+3\sec(c+dx)+1}\,dx}{a^3}$$

input
```
integrate(tan(d*x+c)**6/(a+a*sec(d*x+c))**3,x)
```

output
```
Integral(tan(c + d*x)**6/(sec(c + d*x)**3 + 3*sec(c + d*x)**2 + 3*sec(c +
d*x) + 1), x)/a**3
```

3.98.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 171 vs. 2(58) = 116.

Time = 0.31 (sec) , antiderivative size = 171, normalized size of antiderivative = 2.59

$$\int \frac{\tan^6(c+dx)}{(a+a\sec(c+dx))^3}\,dx = -\frac{\frac{2\left(\frac{5\sin(dx+c)}{\cos(dx+c)+1} - \frac{7\sin(dx+c)^3}{(\cos(dx+c)+1)^3}\right)}{a^3 - \frac{2a^3\sin(dx+c)^2}{(\cos(dx+c)+1)^2} + \frac{a^3\sin(dx+c)^4}{(\cos(dx+c)+1)^4}} + \frac{4\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^3} - \frac{7\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a^3} + \frac{7\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a^3}}{2\,d}$$

input
```
integrate(tan(d*x+c)^6/(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

output
```
-1/2*(2*(5*sin(d*x + c)/(cos(d*x + c) + 1) - 7*sin(d*x + c)^3/(cos(d*x + c
) + 1)^3)/(a^3 - 2*a^3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + a^3*sin(d*x +
 c)^4/(cos(d*x + c) + 1)^4) + 4*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^
3 - 7*log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/a^3 + 7*log(sin(d*x + c)/(c
os(d*x + c) + 1) - 1)/a^3)/d
```

3.98.8 Giac [A] (verification not implemented)

Time = 2.25 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.47

$$\int \frac{\tan^6(c+dx)}{(a+a\sec(c+dx))^3}\,dx = -\frac{\frac{2\,(dx+c)}{a^3} - \frac{7\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{a^3} + \frac{7\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{a^3} - \frac{2\left(7\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-5\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)^2 a^3}}{2\,d}$$

input
```
integrate(tan(d*x+c)^6/(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

output
```
-1/2*(2*(d*x + c)/a^3 - 7*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^3 + 7*log(a
bs(tan(1/2*d*x + 1/2*c) - 1))/a^3 - 2*(7*tan(1/2*d*x + 1/2*c)^3 - 5*tan(1/
2*d*x + 1/2*c))/((tan(1/2*d*x + 1/2*c)^2 - 1)^2*a^3))/d
```

3.98.9 Mupad [B] (verification not implemented)

Time = 14.73 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.39

$$\int \frac{\tan^6(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{7\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{a^3\,d} - \frac{x}{a^3} - \frac{5\tan\left(\frac{c}{2}+\frac{dx}{2}\right) - 7\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3}{d\left(a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4 - 2\,a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2 + a^3\right)}$$

input
```
int(tan(c + d*x)^6/(a + a/cos(c + d*x))^3,x)
```

output
```
(7*atanh(tan(c/2 + (d*x)/2)))/(a^3*d) - x/a^3 - (5*tan(c/2 + (d*x)/2) - 7*
tan(c/2 + (d*x)/2)^3)/(d*(a^3*tan(c/2 + (d*x)/2)^4 - 2*a^3*tan(c/2 + (d*x)
/2)^2 + a^3))
```

3.99 $\int \frac{\tan^4(c+dx)}{(a+a\sec(c+dx))^3}\,dx$

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3.99.2 Mathematica [B] (verified) 729
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3.99.8 Giac [A] (verification not implemented) 733
3.99.9 Mupad [B] (verification not implemented) 733

3.99.1 Optimal result

Integrand size = 21, antiderivative size = 46

$$\int \frac{\tan^4(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{x}{a^3} + \frac{\operatorname{arctanh}(\sin(c+dx))}{a^3 d} - \frac{4\tan(c+dx)}{a^2 d(a+a\sec(c+dx))}$$

output
```
x/a^3+arctanh(sin(d*x+c))/a^3/d-4*tan(d*x+c)/a^2/d/(a+a*sec(d*x+c))
```

3.99.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 117 vs. 2(46) = 92.

Time = 0.85 (sec) , antiderivative size = 117, normalized size of antiderivative = 2.54

$$\int \frac{\tan^4(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{8\cos^5\left(\frac{1}{2}(c+dx)\right)\sec^3(c+dx)\left(\cos\left(\frac{1}{2}(c+dx)\right)\left(dx-\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)+\log\left(\cos\right.\right.\right.}{a^3 d(1+\sec(c+dx))^3}$$

input
```
Integrate[Tan[c + d*x]^4/(a + a*Sec[c + d*x])^3,x]
```

output
```
(8*Cos[(c + d*x)/2]^5*Sec[c + d*x]^3*(Cos[(c + d*x)/2]*(d*x - Log[Cos[(c +
 d*x)/2] - Sin[(c + d*x)/2]] + Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) -
 4*Sec[c/2]*Sin[(d*x)/2]))/(a^3*d*(1 + Sec[c + d*x])^3)
```

3.99.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.20, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 4376, 25, 3042, 4374, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^4(c+dx)}{(a\sec(c+dx)+a)^3}\,dx$$

$$\downarrow 3042$$

$$\int \frac{\cot\left(c+dx+\frac{\pi}{2}\right)^4}{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^3}dx$$

$$\downarrow 4376$$

$$\frac{\int -\cot^2(c+dx)(a-a\sec(c+dx))^3dx}{a^6}$$

$$\downarrow 25$$

$$-\frac{\int \cot^2(c+dx)(a-a\sec(c+dx))^3dx}{a^6}$$

$$\downarrow 3042$$

$$-\frac{\int \frac{(a-a\csc(c+dx+\frac{\pi}{2}))^3}{\cot\left(c+dx+\frac{\pi}{2}\right)^2}dx}{a^6}$$

$$\downarrow 4374$$

$$-\frac{\int \left(\cot^2(c+dx)a^3+3\csc^2(c+dx)a^3-3\cot(c+dx)\csc(c+dx)a^3-\csc^2(c+dx)\sec(c+dx)a^3\right)dx}{a^6}$$

$$\downarrow 2009$$

$$-\frac{-\frac{a^3\text{arctanh}(\sin(c+dx))}{d}-\frac{4a^3\cot(c+dx)}{d}+\frac{4a^3\csc(c+dx)}{d}+a^3(-x)}{a^6}$$

input
```
Int[Tan[c + d*x]^4/(a + a*Sec[c + d*x])^3,x]
```

output
```
-((-(a^3*x) - (a^3*ArcTanh[Sin[c + d*x]])/d - (4*a^3*Cot[c + d*x])/d + (4*
a^3*Csc[c + d*x])/d)/a^6)
```

3.99.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4374
```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

rule 4376
```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Simp[a^(2*n)/e^(2*n)   Int[(e*Cot[c + d*x])^(m + 2*
n)/(-a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[a
^2 - b^2, 0] && ILtQ[n, 0]
```

3.99.4 Maple [A] (verified)

Time = 0.77 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.33

method	result	size
derivativedivides	$\frac{-4\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)-\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+2\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a^3 d}$	61
default	$\frac{-4\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)-\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+2\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a^3 d}$	61
risch	$\frac{x}{a^3}-\frac{8i}{a^3 d\left(e^{i(dx+c)}+1\right)}+\frac{\ln\left(e^{i(dx+c)}+i\right)}{a^3 d}-\frac{\ln\left(e^{i(dx+c)}-i\right)}{a^3 d}$	70

input
```
int(tan(d*x+c)^4/(a+a*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
4/d/a^3*(-tan(1/2*d*x+1/2*c)+1/4*ln(tan(1/2*d*x+1/2*c)+1)-1/4*ln(tan(1/2*d
*x+1/2*c)-1)+1/2*arctan(tan(1/2*d*x+1/2*c)))
```

3.99.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.80

$$\int \frac{\tan^4(c+dx)}{(a+a\sec(c+dx))^3}\,dx$$
$$= \frac{2\,dx\cos(dx+c)+2\,dx+(\cos(dx+c)+1)\log(\sin(dx+c)+1)-(\cos(dx+c)+1)\log(-\sin(dx+c)}{2\,(a^3d\cos(dx+c)+a^3d)}$$

input
```
integrate(tan(d*x+c)^4/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

output
```
1/2*(2*d*x*cos(d*x + c) + 2*d*x + (cos(d*x + c) + 1)*log(sin(d*x + c) + 1)
 - (cos(d*x + c) + 1)*log(-sin(d*x + c) + 1) - 8*sin(d*x + c))/(a^3*d*cos(
d*x + c) + a^3*d)
```

3.99.6 Sympy [F]

$$\int \frac{\tan^4(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{\int \frac{\tan^4(c+dx)}{\sec^3(c+dx)+3\sec^2(c+dx)+3\sec(c+dx)+1}\,dx}{a^3}$$

input
```
integrate(tan(d*x+c)**4/(a+a*sec(d*x+c))**3,x)
```

output
```
Integral(tan(c + d*x)**4/(sec(c + d*x)**3 + 3*sec(c + d*x)**2 + 3*sec(c +
d*x) + 1), x)/a**3
```

3.99.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 98 vs. 2(46) = 92.

Time = 0.30 (sec) , antiderivative size = 98, normalized size of antiderivative = 2.13

$$\int \frac{\tan^4(c+dx)}{(a+a\sec(c+dx))^3}\,dx$$
$$= \frac{\frac{2\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^3}+\frac{\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a^3}-\frac{\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a^3}-\frac{4\sin(dx+c)}{a^3(\cos(dx+c)+1)}}{d}$$

input
```
integrate(tan(d*x+c)^4/(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

output
```
(2*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^3 + log(sin(d*x + c)/(cos(d*x
 + c) + 1) + 1)/a^3 - log(sin(d*x + c)/(cos(d*x + c) + 1) - 1)/a^3 - 4*sin
(d*x + c)/(a^3*(cos(d*x + c) + 1)))/d
```

3.99.8 Giac [A] (verification not implemented)

Time = 1.07 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.37

$$\int \frac{\tan^4(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{\frac{dx+c}{a^3} + \frac{\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{a^3} - \frac{\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{a^3} - \frac{4\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^3}}{d}$$

input
```
integrate(tan(d*x+c)^4/(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

output
```
((d*x + c)/a^3 + log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^3 - log(abs(tan(1/2*
d*x + 1/2*c) - 1))/a^3 - 4*tan(1/2*d*x + 1/2*c)/a^3)/d
```

3.99.9 Mupad [B] (verification not implemented)

Time = 14.48 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.80

$$\int \frac{\tan^4(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{x}{a^3} + \frac{2\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right) - 4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{a^3\,d}$$

input
```
int(tan(c + d*x)^4/(a + a/cos(c + d*x))^3,x)
```

output
```
x/a^3 + (2*atanh(tan(c/2 + (d*x)/2)) - 4*tan(c/2 + (d*x)/2))/(a^3*d)
```

3.100 $\int \frac{\tan^2(c+dx)}{(a+a\sec(c+dx))^3}\,dx$

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3.100.1 Optimal result

Integrand size = 21, antiderivative size = 60

$$\int \frac{\tan^2(c+dx)}{(a+a\sec(c+dx))^3}\,dx = -\frac{x}{a^3} + \frac{2\tan(c+dx)}{a^2 d(a+a\sec(c+dx))} - \frac{\tan^3(c+dx)}{3d(a+a\sec(c+dx))^3}$$

output
```
-x/a^3+2*tan(d*x+c)/a^2/d/(a+a*sec(d*x+c))-1/3*tan(d*x+c)^3/d/(a+a*sec(d*x
+c))^3
```

3.100.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 125 vs. $2(60) = 120$.

Time = 0.32 (sec) , antiderivative size = 125, normalized size of antiderivative = 2.08

$$\int \frac{\tan^2(c+dx)}{(a+a\sec(c+dx))^3}\,dx = -\frac{\sec\left(\frac{c}{2}\right)\sec^3\left(\frac{1}{2}(c+dx)\right)\left(180dx\cos\left(\frac{dx}{2}\right) + 180dx\cos\left(c+\frac{dx}{2}\right) + 60dx\cos\left(c+\frac{3dx}{2}\right) + 60dx\cos\left(2c+\frac{3d}{2}\right)\right.}{480a^3d}$$

input
```
Integrate[Tan[c + d*x]^2/(a + a*Sec[c + d*x])^3,x]
```

output
```
-1/480*(Sec[c/2]*Sec[(c + d*x)/2]^3*(180*d*x*Cos[(d*x)/2] + 180*d*x*Cos[c
+ (d*x)/2] + 60*d*x*Cos[c + (3*d*x)/2] + 60*d*x*Cos[2*c + (3*d*x)/2] - 471
*Sin[(d*x)/2] + 351*Sin[c + (d*x)/2] - 277*Sin[c + (3*d*x)/2] - 3*Sin[2*c
+ (3*d*x)/2]))/(a^3*d)
```

3.100.3 Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.23, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 4376, 25, 3042, 4374, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^2(c+dx)}{(a\sec(c+dx)+a)^3}\,dx$$

$$\downarrow 3042$$

$$\int \frac{\cot\left(c+dx+\frac{\pi}{2}\right)^2}{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^3}dx$$

$$\downarrow 4376$$

$$\frac{\int -\cot^4(c+dx)(a-a\sec(c+dx))^3dx}{a^6}$$

$$\downarrow 25$$

$$-\frac{\int \cot^4(c+dx)(a-a\sec(c+dx))^3dx}{a^6}$$

$$\downarrow 3042$$

$$-\frac{\int \frac{(a-a\csc(c+dx+\frac{\pi}{2}))^3}{\cot\left(c+dx+\frac{\pi}{2}\right)^4}dx}{a^6}$$

$$\downarrow 4374$$

$$-\frac{\int \left(a^3\cot^4(c+dx)-3a^3\csc(c+dx)\cot^3(c+dx)+3a^3\csc^2(c+dx)\cot^2(c+dx)-a^3\csc^3(c+dx)\cot(c+dx)\right)}{a^6}$$

$$\downarrow 2009$$

$$-\frac{-\frac{4a^3\cot^3(c+dx)}{3d}+\frac{a^3\cot(c+dx)}{d}+\frac{4a^3\csc^3(c+dx)}{3d}-\frac{3a^3\csc(c+dx)}{d}+a^3x}{a^6}$$

input
```
Int[Tan[c + d*x]^2/(a + a*Sec[c + d*x])^3,x]
```

output
```
-((a^3*x + (a^3*Cot[c + d*x])/d - (4*a^3*Cot[c + d*x]^3)/(3*d) - (3*a^3*Cs
c[c + d*x])/d + (4*a^3*Csc[c + d*x]^3)/(3*d))/a^6)
```

3.100.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4374
```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

rule 4376
```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Simp[a^(2*n)/e^(2*n)   Int[(e*Cot[c + d*x])^(m + 2*
n)/(-a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[a
^2 - b^2, 0] && ILtQ[n, 0]
```

3.100.4 Maple [A] (verified)

Time = 0.68 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.75

method	result	size
derivativedivides	$\frac{-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{3}+2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d\,a^3}$	45
default	$\frac{-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{3}+2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d\,a^3}$	45
risch	$-\frac{x}{a^3}+\frac{2i\left(9\,e^{2i(dx+c)}+12\,e^{i(dx+c)}+7\right)}{3d\,a^3\left(e^{i(dx+c)}+1\right)^3}$	54

input
```
int(tan(d*x+c)^2/(a+a*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/d/a^3*(-1/3*tan(1/2*d*x+1/2*c)^3+2*tan(1/2*d*x+1/2*c)-2*arctan(tan(1/2*d
*x+1/2*c)))
```

3.100.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.33

$$\int \frac{\tan^2(c+dx)}{(a+a\sec(c+dx))^3}\,dx = -\frac{3\,dx\cos(dx+c)^2+6\,dx\cos(dx+c)+3\,dx-(7\cos(dx+c)+5)\sin(dx+c)}{3\left(a^3d\cos(dx+c)^2+2\,a^3d\cos(dx+c)+a^3d\right)}$$

input
```
integrate(tan(d*x+c)^2/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

output
```
-1/3*(3*d*x*cos(d*x + c)^2 + 6*d*x*cos(d*x + c) + 3*d*x - (7*cos(d*x + c)
+ 5)*sin(d*x + c))/(a^3*d*cos(d*x + c)^2 + 2*a^3*d*cos(d*x + c) + a^3*d)
```

3.100.6 Sympy [F]

$$\int \frac{\tan^2(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{\int \frac{\tan^2(c+dx)}{\sec^3(c+dx)+3\sec^2(c+dx)+3\sec(c+dx)+1}\,dx}{a^3}$$

input
```
integrate(tan(d*x+c)**2/(a+a*sec(d*x+c))**3,x)
```

output
```
Integral(tan(c + d*x)**2/(sec(c + d*x)**3 + 3*sec(c + d*x)**2 + 3*sec(c +
d*x) + 1), x)/a**3
```

3.100.7 Maxima [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.20

$$\int \frac{\tan^2(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{\frac{6\sin(dx+c)}{\cos(dx+c)+1}-\frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3}}{a^3} - \frac{6\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^3}}{3\,d}$$

input
```
integrate(tan(d*x+c)^2/(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

output
```
1/3*((6*sin(d*x + c)/(cos(d*x + c) + 1) - sin(d*x + c)^3/(cos(d*x + c) + 1
)^3)/a^3 - 6*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^3)/d
```

3.100.8 Giac [A] (verification not implemented)

Time = 0.61 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.83

$$\int \frac{\tan^2(c+dx)}{(a+a\sec(c+dx))^3}\,dx = -\frac{\frac{3\,(dx+c)}{a^3} + \frac{a^6\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3 - 6\,a^6\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^9}}{3\,d}$$

input
```
integrate(tan(d*x+c)^2/(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

output
```
-1/3*(3*(d*x + c)/a^3 + (a^6*tan(1/2*d*x + 1/2*c)^3 - 6*a^6*tan(1/2*d*x +
1/2*c))/a^9)/d
```

3.100.9 Mupad [B] (verification not implemented)

Time = 13.81 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.58

$$\int \frac{\tan^2(c+dx)}{(a+a\sec(c+dx))^3}\,dx = -\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3 - 6\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right) + 3\,d\,x}{3\,a^3\,d}$$

input
```
int(tan(c + d*x)^2/(a + a/cos(c + d*x))^3,x)
```

output
```
-(tan(c/2 + (d*x)/2)^3 - 6*tan(c/2 + (d*x)/2) + 3*d*x)/(3*a^3*d)
```

3.101 $\int \frac{\cot^2(c+dx)}{(a+a\sec(c+dx))^3}\,dx$

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3.101.1 Optimal result

Integrand size = 21, antiderivative size = 143

$$\int \frac{\cot^2(c+dx)}{(a+a\sec(c+dx))^3}\,dx = -\frac{x}{a^3} - \frac{\cot(c+dx)}{a^3 d} + \frac{\cot^3(c+dx)}{3a^3 d} - \frac{\cot^5(c+dx)}{5a^3 d} + \frac{4\cot^7(c+dx)}{7a^3 d} + \frac{3\csc(c+dx)}{a^3 d} - \frac{10\csc^3(c+dx)}{3a^3 d} + \frac{11\csc^5(c+dx)}{5a^3 d} - \frac{4\csc^7(c+dx)}{7a^3 d}$$

output
```
-x/a^3-cot(d*x+c)/a^3/d+1/3*cot(d*x+c)^3/a^3/d-1/5*cot(d*x+c)^5/a^3/d+4/7*
cot(d*x+c)^7/a^3/d+3*csc(d*x+c)/a^3/d-10/3*csc(d*x+c)^3/a^3/d+11/5*csc(d*x
+c)^5/a^3/d-4/7*csc(d*x+c)^7/a^3/d
```

3.101.2 Mathematica [A] (verified)

Time = 1.64 (sec) , antiderivative size = 252, normalized size of antiderivative = 1.76

$$\int \frac{\cot^2(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{\csc\left(\frac{c}{2}\right)\csc\left(\frac{1}{2}(c+dx)\right)\sec\left(\frac{c}{2}\right)\sec^7\left(\frac{1}{2}(c+dx)\right)\left(-5880dx\cos(dx) + 5880dx\cos(2c+dx) - 5880dx\cos(c+\right.$$

input
```
Integrate[Cot[c + d*x]^2/(a + a*Sec[c + d*x])^3,x]
```

output

```
(Csc[c/2]*Csc[(c + d*x)/2]*Sec[c/2]*Sec[(c + d*x)/2]^7*(-5880*d*x*Cos[d*x]
 + 5880*d*x*Cos[2*c + d*x] - 5880*d*x*Cos[c + 2*d*x] + 5880*d*x*Cos[3*c +
2*d*x] - 2520*d*x*Cos[2*c + 3*d*x] + 2520*d*x*Cos[4*c + 3*d*x] - 420*d*x*C
os[3*c + 4*d*x] + 420*d*x*Cos[5*c + 4*d*x] + 4200*Sin[c] + 11032*Sin[d*x]
- 23282*Sin[c + d*x] - 23282*Sin[2*(c + d*x)] - 9978*Sin[3*(c + d*x)] - 16
63*Sin[4*(c + d*x)] + 13720*Sin[2*c + d*x] + 15512*Sin[c + 2*d*x] + 9240*S
in[3*c + 2*d*x] + 8088*Sin[2*c + 3*d*x] + 2520*Sin[4*c + 3*d*x] + 1768*Sin
[3*c + 4*d*x]))/(215040*a^3*d)
```

3.101.3 Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.02, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 4376, 25, 3042, 4374, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^2(c+dx)}{(a\sec(c+dx)+a)^3}\,dx$$

$$\downarrow 3042$$

$$\int \frac{1}{\cot\left(c+dx+\frac{\pi}{2}\right)^2\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^3}dx$$

$$\downarrow 4376$$

$$\frac{\int -\cot^8(c+dx)(a-a\sec(c+dx))^3dx}{a^6}$$

$$\downarrow 25$$

$$-\frac{\int \cot^8(c+dx)(a-a\sec(c+dx))^3dx}{a^6}$$

$$\downarrow 3042$$

$$-\frac{\int \frac{\left(a-a\csc\left(c+dx+\frac{\pi}{2}\right)\right)^3}{\cot\left(c+dx+\frac{\pi}{2}\right)^8}dx}{a^6}$$

$$\downarrow 4374$$

$$-\frac{\int \left(a^3\cot^8(c+dx)-3a^3\csc(c+dx)\cot^7(c+dx)+3a^3\csc^2(c+dx)\cot^6(c+dx)-a^3\csc^3(c+dx)\cot^5(c+dx)\right.}{a^6}$$

$$\downarrow 2009$$

$$-\frac{-\frac{4a^3\cot^7(c+dx)}{7d}+\frac{a^3\cot^5(c+dx)}{5d}-\frac{a^3\cot^3(c+dx)}{3d}+\frac{a^3\cot(c+dx)}{d}+\frac{4a^3\csc^7(c+dx)}{7d}-\frac{11a^3\csc^5(c+dx)}{5d}+\frac{10a^3\csc^3(c+dx)}{3d}-\frac{3a^3\,c}{}}{a^6}$$

input
```
Int[Cot[c + d*x]^2/(a + a*Sec[c + d*x])^3,x]
```

output
```
-((a^3*x + (a^3*Cot[c + d*x])/d - (a^3*Cot[c + d*x]^3)/(3*d) + (a^3*Cot[c
+ d*x]^5)/(5*d) - (4*a^3*Cot[c + d*x]^7)/(7*d) - (3*a^3*Csc[c + d*x])/d +
(10*a^3*Csc[c + d*x]^3)/(3*d) - (11*a^3*Csc[c + d*x]^5)/(5*d) + (4*a^3*Csc
[c + d*x]^7)/(7*d))/a^6)
```

3.101.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4374
```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

rule 4376
```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Simp[a^(2*n)/e^(2*n)   Int[(e*Cot[c + d*x])^(m + 2*
n)/(-a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[a
^2 - b^2, 0] && ILtQ[n, 0]
```

3.101.4 Maple [A] (verified)

Time = 0.64 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.59

method	result
derivativedivides	$\frac{-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{7}+\frac{6\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{5}-\frac{16\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{3}+26\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{1}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}-32\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{16da^3}$
default	$\frac{-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{7}+\frac{6\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{5}-\frac{16\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{3}+26\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{1}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}-32\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{16da^3}$
risch	$-\frac{x}{a^3}+\frac{2i\left(315\,e^{7i(dx+c)}+1155\,e^{6i(dx+c)}+1715\,e^{5i(dx+c)}+525\,e^{4i(dx+c)}-1379\,e^{3i(dx+c)}-1939\,e^{2i(dx+c)}-1011\,e^{i(dx+}\right.}{105da^3\left(e^{i(dx+c)}+1\right)^7\left(e^{i(dx+c)}-1\right)}$

input
```
int(cot(d*x+c)^2/(a+a*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/16/d/a^3*(-1/7*tan(1/2*d*x+1/2*c)^7+6/5*tan(1/2*d*x+1/2*c)^5-16/3*tan(1/
2*d*x+1/2*c)^3+26*tan(1/2*d*x+1/2*c)-1/tan(1/2*d*x+1/2*c)-32*arctan(tan(1/
2*d*x+1/2*c)))
```

3.101.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.99

$$\int \frac{\cot^2(c+dx)}{(a+a\sec(c+dx))^3}\,dx =$$
$$-\frac{221\cos(dx+c)^4+348\cos(dx+c)^3-25\cos(dx+c)^2+105\left(dx\cos(dx+c)^3+3\,dx\cos(dx+c)^2+\right.}{105\left(a^3d\cos(dx+c)^3+3\,a^3d\cos(dx+c)^2+3\,a^3d\cos(dx+c\right.}$$

input
```
integrate(cot(d*x+c)^2/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

output
```
-1/105*(221*cos(d*x + c)^4 + 348*cos(d*x + c)^3 - 25*cos(d*x + c)^2 + 105*
(d*x*cos(d*x + c)^3 + 3*d*x*cos(d*x + c)^2 + 3*d*x*cos(d*x + c) + d*x)*sin
(d*x + c) - 303*cos(d*x + c) - 136)/((a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d
*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)*sin(d*x + c))
```

3.101.6 Sympy [F]

$$\int \frac{\cot^2(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{\int \frac{\cot^2(c+dx)}{\sec^3(c+dx)+3\sec^2(c+dx)+3\sec(c+dx)+1}\,dx}{a^3}$$

input
```
integrate(cot(d*x+c)**2/(a+a*sec(d*x+c))**3,x)
```

output
```
Integral(cot(c + d*x)**2/(sec(c + d*x)**3 + 3*sec(c + d*x)**2 + 3*sec(c +
d*x) + 1), x)/a**3
```

3.101.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.93

$$\int \frac{\cot^2(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{\frac{2730\sin(dx+c)}{\cos(dx+c)+1} - \frac{560\sin(dx+c)^3}{(\cos(dx+c)+1)^3} + \frac{126\sin(dx+c)^5}{(\cos(dx+c)+1)^5} - \frac{15\sin(dx+c)^7}{(\cos(dx+c)+1)^7}}{a^3} - \frac{3360\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^3} - \frac{105(\cos(dx+c)+1)}{a^3\sin(dx+c)}}{1680\,d}$$

input
```
integrate(cot(d*x+c)^2/(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

output
```
1/1680*((2730*sin(d*x + c)/(cos(d*x + c) + 1) - 560*sin(d*x + c)^3/(cos(d*
x + c) + 1)^3 + 126*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 15*sin(d*x + c)^
7/(cos(d*x + c) + 1)^7)/a^3 - 3360*arctan(sin(d*x + c)/(cos(d*x + c) + 1))
/a^3 - 105*(cos(d*x + c) + 1)/(a^3*sin(d*x + c)))/d
```

3.101.8 Giac [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.69

$$\int \frac{\cot^2(c+dx)}{(a+a\sec(c+dx))^3}\,dx = -\frac{\frac{1680\,(dx+c)}{a^3} + \frac{105}{a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)} + \frac{15\,a^{18}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7 - 126\,a^{18}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5 + 560\,a^{18}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3 - 2730\,a^{18}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^{21}}}{1680\,d}$$

input
```
integrate(cot(d*x+c)^2/(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

output
```
-1/1680*(1680*(d*x + c)/a^3 + 105/(a^3*tan(1/2*d*x + 1/2*c)) + (15*a^18*ta
n(1/2*d*x + 1/2*c)^7 - 126*a^18*tan(1/2*d*x + 1/2*c)^5 + 560*a^18*tan(1/2*
d*x + 1/2*c)^3 - 2730*a^18*tan(1/2*d*x + 1/2*c))/a^21)/d
```

3.101.9 Mupad [B] (verification not implemented)

Time = 14.09 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.64

$$\int \frac{\cot^2(c+dx)}{(a+a\sec(c+dx))^3}\,dx$$
$$= -\frac{x}{a^3} - \frac{\frac{221\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^8}{105} - \frac{268\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^6}{105} + \frac{257\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^4}{420} - \frac{31\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^2}{280} + \frac{1}{112}}{a^3\,d\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^7\sin\left(\frac{c}{2}+\frac{dx}{2}\right)}$$

input
```
int(cot(c + d*x)^2/(a + a/cos(c + d*x))^3,x)
```

output
```
- x/a^3 - ((257*cos(c/2 + (d*x)/2)^4)/420 - (31*cos(c/2 + (d*x)/2)^2)/280
- (268*cos(c/2 + (d*x)/2)^6)/105 + (221*cos(c/2 + (d*x)/2)^8)/105 + 1/112)
/(a^3*d*cos(c/2 + (d*x)/2)^7*sin(c/2 + (d*x)/2))
```

3.102 $\int \frac{\cot^4(c+dx)}{(a+a\sec(c+dx))^3}\,dx$

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3.102.1 Optimal result

Integrand size = 21, antiderivative size = 177

$$\int \frac{\cot^4(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{x}{a^3} + \frac{\cot(c+dx)}{a^3 d} - \frac{\cot^3(c+dx)}{3a^3 d} + \frac{\cot^5(c+dx)}{5a^3 d} - \frac{\cot^7(c+dx)}{7a^3 d} + \frac{4\cot^9(c+dx)}{9a^3 d} - \frac{3\csc(c+dx)}{a^3 d} + \frac{13\csc^3(c+dx)}{3a^3 d} - \frac{21\csc^5(c+dx)}{5a^3 d} + \frac{15\csc^7(c+dx)}{7a^3 d} - \frac{4\csc^9(c+dx)}{9a^3 d}$$

output

```
x/a^3+cot(d*x+c)/a^3/d-1/3*cot(d*x+c)^3/a^3/d+1/5*cot(d*x+c)^5/a^3/d-1/7*c
ot(d*x+c)^7/a^3/d+4/9*cot(d*x+c)^9/a^3/d-3*csc(d*x+c)/a^3/d+13/3*csc(d*x+c
)^3/a^3/d-21/5*csc(d*x+c)^5/a^3/d+15/7*csc(d*x+c)^7/a^3/d-4/9*csc(d*x+c)^9
/a^3/d
```

3.102.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 366 vs. $2(177) = 354$.

Time = 1.89 (sec) , antiderivative size = 366, normalized size of antiderivative = 2.07

$$\int \frac{\cot^4(c+dx)}{(a+a\sec(c+dx))^3}\,dx$$
$$= \frac{\csc\left(\frac{c}{2}\right)\csc^3(2(c+dx))\sec\left(\frac{c}{2}\right)(181440dx\cos(dx) - 181440dx\cos(2c+dx) + 136080dx\cos(c+2dx) - 13$$

input
```
Integrate[Cot[c + d*x]^4/(a + a*Sec[c + d*x])^3,x]
```

output
```
(Csc[c/2]*Csc[2*(c + d*x)]^3*Sec[c/2]*(181440*d*x*Cos[d*x] - 181440*d*x*Co
s[2*c + d*x] + 136080*d*x*Cos[c + 2*d*x] - 136080*d*x*Cos[3*c + 2*d*x] - 1
0080*d*x*Cos[2*c + 3*d*x] + 10080*d*x*Cos[4*c + 3*d*x] - 60480*d*x*Cos[3*c
 + 4*d*x] + 60480*d*x*Cos[5*c + 4*d*x] - 30240*d*x*Cos[4*c + 5*d*x] + 3024
0*d*x*Cos[6*c + 5*d*x] - 5040*d*x*Cos[5*c + 6*d*x] + 5040*d*x*Cos[7*c + 6*
d*x] - 169344*Sin[c] - 338112*Sin[d*x] + 675036*Sin[c + d*x] + 506277*Sin[
2*(c + d*x)] - 37502*Sin[3*(c + d*x)] - 225012*Sin[4*(c + d*x)] - 112506*S
in[5*(c + d*x)] - 18751*Sin[6*(c + d*x)] - 431424*Sin[2*c + d*x] - 375552*
Sin[c + 2*d*x] - 201600*Sin[3*c + 2*d*x] - 41248*Sin[2*c + 3*d*x] + 84000*
Sin[4*c + 3*d*x] + 155712*Sin[3*c + 4*d*x] + 100800*Sin[5*c + 4*d*x] + 980
16*Sin[4*c + 5*d*x] + 30240*Sin[6*c + 5*d*x] + 21376*Sin[5*c + 6*d*x]))/(8
0640*a^3*d*(1 + Sec[c + d*x])^3)
```

3.102.3 Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.04, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 4376, 25, 3042, 4374, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^4(c+dx)}{(a\sec(c+dx)+a)^3}\,dx$$

$$\downarrow 3042$$

$$\int \frac{1}{\cot\left(c+dx+\frac{\pi}{2}\right)^4\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^3}dx$$

$$\downarrow 4376$$

$$\frac{\int -\cot^{10}(c+dx)(a-a\sec(c+dx))^3dx}{a^6}$$

$$\downarrow 25$$

$$-\frac{\int \cot^{10}(c+dx)(a-a\sec(c+dx))^3dx}{a^6}$$

$$\downarrow 3042$$

$$-\frac{\int \frac{(a-a\csc(c+dx+\frac{\pi}{2}))^3}{\cot(c+dx+\frac{\pi}{2})^{10}}dx}{a^6}$$

$$\downarrow 4374$$

$$-\frac{\int \left(a^3\cot^{10}(c+dx)-3a^3\csc(c+dx)\cot^9(c+dx)+3a^3\csc^2(c+dx)\cot^8(c+dx)-a^3\csc^3(c+dx)\cot^7(c+dx\right.}{a^6}$$

$$\downarrow 2009$$

$$-\frac{-\frac{4a^3\cot^9(c+dx)}{9d}+\frac{a^3\cot^7(c+dx)}{7d}-\frac{a^3\cot^5(c+dx)}{5d}+\frac{a^3\cot^3(c+dx)}{3d}-\frac{a^3\cot(c+dx)}{d}+\frac{4a^3\csc^9(c+dx)}{9d}-\frac{15a^3\csc^7(c+dx)}{7d}+\frac{21a^3\csc}{}}{a^6}$$

input
```
Int[Cot[c + d*x]^4/(a + a*Sec[c + d*x])^3,x]
```

output
```
-((-(a^3*x) - (a^3*Cot[c + d*x])/d + (a^3*Cot[c + d*x]^3)/(3*d) - (a^3*Cot
[c + d*x]^5)/(5*d) + (a^3*Cot[c + d*x]^7)/(7*d) - (4*a^3*Cot[c + d*x]^9)/(
9*d) + (3*a^3*Csc[c + d*x])/d - (13*a^3*Csc[c + d*x]^3)/(3*d) + (21*a^3*Cs
c[c + d*x]^5)/(5*d) - (15*a^3*Csc[c + d*x]^7)/(7*d) + (4*a^3*Csc[c + d*x]^
9)/(9*d))/a^6)
```

3.102.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4374
```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

rule 4376
```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Simp[a^(2*n)/e^(2*n)   Int[(e*Cot[c + d*x])^(m + 2*
n)/(-a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[a
^2 - b^2, 0] && ILtQ[n, 0]
```

3.102.4 Maple [A] (verified)

Time = 0.64 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.63

method	result
derivativedivides	$\frac{-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^9}{9}+\frac{8\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{7}-\frac{29\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{5}+\frac{64\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{3}-99\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+128\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\frac{}{3\tan(}}{64da^3}$
default	$\frac{-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^9}{9}+\frac{8\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{7}-\frac{29\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{5}+\frac{64\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{3}-99\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+128\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\frac{}{3\tan(}}{64da^3}$
risch	$\frac{x}{a^3}-\frac{2i\left(945\,e^{11i(dx+c)}+3150\,e^{10i(dx+c)}+2625\,e^{9i(dx+c)}-6300\,e^{8i(dx+c)}-13482\,e^{7i(dx+c)}-5292\,e^{6i(dx+c)}+10566\,e^{5i(}{315da^3\left(e^{i(dx+c)}+1\right)^9\left(e^{i(dx+c)}-1\right)}$

input
```
int(cot(d*x+c)^4/(a+a*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/64/d/a^3*(-1/9*tan(1/2*d*x+1/2*c)^9+8/7*tan(1/2*d*x+1/2*c)^7-29/5*tan(1/
2*d*x+1/2*c)^5+64/3*tan(1/2*d*x+1/2*c)^3-99*tan(1/2*d*x+1/2*c)+128*arctan(
tan(1/2*d*x+1/2*c))-1/3/tan(1/2*d*x+1/2*c)^3+8/tan(1/2*d*x+1/2*c))
```

3.102.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.22

$$\int \frac{\cot^4(c+dx)}{(a+a\sec(c+dx))^3}\,dx$$
$$= \frac{668\cos(dx+c)^6+1059\cos(dx+c)^5-573\cos(dx+c)^4-1813\cos(dx+c)^3-393\cos(dx+c)^2+31}{315\left(a^3d\cos(dx+c)^5+3\,a^3d\cos(dx+c)^4\right.}$$

input
```
integrate(cot(d*x+c)^4/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

output
```
1/315*(668*cos(d*x + c)^6 + 1059*cos(d*x + c)^5 - 573*cos(d*x + c)^4 - 181
3*cos(d*x + c)^3 - 393*cos(d*x + c)^2 + 315*(d*x*cos(d*x + c)^5 + 3*d*x*co
s(d*x + c)^4 + 2*d*x*cos(d*x + c)^3 - 2*d*x*cos(d*x + c)^2 - 3*d*x*cos(d*x
 + c) - d*x)*sin(d*x + c) + 789*cos(d*x + c) + 368)/((a^3*d*cos(d*x + c)^5
 + 3*a^3*d*cos(d*x + c)^4 + 2*a^3*d*cos(d*x + c)^3 - 2*a^3*d*cos(d*x + c)^
2 - 3*a^3*d*cos(d*x + c) - a^3*d)*sin(d*x + c))
```

3.102.6 Sympy [F]

$$\int \frac{\cot^4(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{\int \frac{\cot^4(c+dx)}{\sec^3(c+dx)+3\sec^2(c+dx)+3\sec(c+dx)+1}\,dx}{a^3}$$

input
```
integrate(cot(d*x+c)**4/(a+a*sec(d*x+c))**3,x)
```

output
```
Integral(cot(c + d*x)**4/(sec(c + d*x)**3 + 3*sec(c + d*x)**2 + 3*sec(c +
d*x) + 1), x)/a**3
```

3.102.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.00

$$\int \frac{\cot^4(c+dx)}{(a+a\sec(c+dx))^3}\,dx = -\frac{\frac{31185\sin(dx+c)}{\cos(dx+c)+1} - \frac{6720\sin(dx+c)^3}{(\cos(dx+c)+1)^3} + \frac{1827\sin(dx+c)^5}{(\cos(dx+c)+1)^5} - \frac{360\sin(dx+c)^7}{(\cos(dx+c)+1)^7} + \frac{35\sin(dx+c)^9}{(\cos(dx+c)+1)^9}}{a^3} - \frac{40320\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^3} - \frac{105\left(\frac{24\sin(dx+c)^2}{(\cos(dx+c)+1)^2}}{a^3\sin}$$

$$\frac{}{20160\,d}$$

input
```
integrate(cot(d*x+c)^4/(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

output
```
-1/20160*((31185*sin(d*x + c)/(cos(d*x + c) + 1) - 6720*sin(d*x + c)^3/(co
s(d*x + c) + 1)^3 + 1827*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 360*sin(d*x
 + c)^7/(cos(d*x + c) + 1)^7 + 35*sin(d*x + c)^9/(cos(d*x + c) + 1)^9)/a^3
 - 40320*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^3 - 105*(24*sin(d*x + c
)^2/(cos(d*x + c) + 1)^2 - 1)*(cos(d*x + c) + 1)^3/(a^3*sin(d*x + c)^3))/d
```

3.102.8 Giac [A] (verification not implemented)

Time = 0.47 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.74

$$\int \frac{\cot^4(c+dx)}{(a+a\sec(c+dx))^3}\,dx$$
$$= \frac{\frac{20160\,(dx+c)}{a^3} + \frac{105\left(24\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)}{a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3} - \frac{35\,a^{24}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9-360\,a^{24}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+1827\,a^{24}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-6720\,a^{24}\tan\left(\frac{1}{2}\right.}{a^{27}}}{20160\,d}$$

input
```
integrate(cot(d*x+c)^4/(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

output
```
1/20160*(20160*(d*x + c)/a^3 + 105*(24*tan(1/2*d*x + 1/2*c)^2 - 1)/(a^3*ta
n(1/2*d*x + 1/2*c)^3) - (35*a^24*tan(1/2*d*x + 1/2*c)^9 - 360*a^24*tan(1/2
*d*x + 1/2*c)^7 + 1827*a^24*tan(1/2*d*x + 1/2*c)^5 - 6720*a^24*tan(1/2*d*x
 + 1/2*c)^3 + 31185*a^24*tan(1/2*d*x + 1/2*c))/a^27)/d
```

3.102.9 Mupad [B] (verification not implemented)

Time = 14.28 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.16

$$\int \frac{\cot^4(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^9(c+dx) - \cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{11}(c+dx)}{a^3\,d\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^9 - \cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{11}\right)}$$
$$- \frac{\frac{668\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{12}}{315} - \frac{983\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}}{210} + \frac{346\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^8}{105} - \frac{2291\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^6}{2520} + \frac{173\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^4}{840} - \frac{19\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^2}{672} + \frac{1}{576}}{a^3\,d\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^9\sin\left(\frac{c}{2}+\frac{dx}{2}\right) - \cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{11}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}$$

input
```
int(cot(c + d*x)^4/(a + a/cos(c + d*x))^3,x)
```

output
```
(cos(c/2 + (d*x)/2)^9*(c + d*x) - cos(c/2 + (d*x)/2)^11*(c + d*x))/(a^3*d*
(cos(c/2 + (d*x)/2)^9 - cos(c/2 + (d*x)/2)^11)) - ((173*cos(c/2 + (d*x)/2)
^4)/840 - (19*cos(c/2 + (d*x)/2)^2)/672 - (2291*cos(c/2 + (d*x)/2)^6)/2520
 + (346*cos(c/2 + (d*x)/2)^8)/105 - (983*cos(c/2 + (d*x)/2)^10)/210 + (668
*cos(c/2 + (d*x)/2)^12)/315 + 1/576)/(a^3*d*(cos(c/2 + (d*x)/2)^9*sin(c/2
+ (d*x)/2) - cos(c/2 + (d*x)/2)^11*sin(c/2 + (d*x)/2)))
```

3.103 $\int \frac{\cot^6(c+dx)}{(a+a\sec(c+dx))^3}\,dx$

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3.103.1 Optimal result

Integrand size = 21, antiderivative size = 215

$$\begin{aligned}\int \frac{\cot^6(c+dx)}{(a+a\sec(c+dx))^3}\,dx = &-\frac{x}{a^3}-\frac{\cot(c+dx)}{a^3d}+\frac{\cot^3(c+dx)}{3a^3d}-\frac{\cot^5(c+dx)}{5a^3d}\\&+\frac{\cot^7(c+dx)}{7a^3d}-\frac{\cot^9(c+dx)}{9a^3d}+\frac{4\cot^{11}(c+dx)}{11a^3d}\\&+\frac{3\csc(c+dx)}{a^3d}-\frac{16\csc^3(c+dx)}{3a^3d}+\frac{34\csc^5(c+dx)}{5a^3d}\\&-\frac{36\csc^7(c+dx)}{7a^3d}+\frac{19\csc^9(c+dx)}{9a^3d}-\frac{4\csc^{11}(c+dx)}{11a^3d}\end{aligned}$$

output

```
-x/a^3-cot(d*x+c)/a^3/d+1/3*cot(d*x+c)^3/a^3/d-1/5*cot(d*x+c)^5/a^3/d+1/7*
cot(d*x+c)^7/a^3/d-1/9*cot(d*x+c)^9/a^3/d+4/11*cot(d*x+c)^11/a^3/d+3*csc(d
*x+c)/a^3/d-16/3*csc(d*x+c)^3/a^3/d+34/5*csc(d*x+c)^5/a^3/d-36/7*csc(d*x+c
)^7/a^3/d+19/9*csc(d*x+c)^9/a^3/d-4/11*csc(d*x+c)^11/a^3/d
```

3.103.2 Mathematica [A] (verified)

Time = 4.22 (sec) , antiderivative size = 394, normalized size of antiderivative = 1.83

$$\int \frac{\cot^6(c+dx)}{(a+a\sec(c+dx))^3}\,dx =$$
$$-\frac{\cos^6\left(\frac{1}{2}(c+dx)\right)\sec^3(c+dx)\left(231(-25+28\cos(c+dx))\cot^2\left(\frac{c}{2}\right)\csc^4\left(\frac{1}{2}(c+dx)\right)+561145\sec^2\left(\frac{1}{2}(c+\right.\right.}{}$$

input
```
Integrate[Cot[c + d*x]^6/(a + a*Sec[c + d*x])^3,x]
```

output
```
-1/110880*(Cos[(c + d*x)/2]^6*Sec[c + d*x]^3*(231*(-25 + 28*Cos[c + d*x])*
Cot[c/2]^2*Csc[(c + d*x)/2]^4 + 561145*Sec[(c + d*x)/2]^2 - 184650*Sec[(c
+ d*x)/2]^4 + 41320*Sec[(c + d*x)/2]^6 - 5425*Sec[(c + d*x)/2]^8 + 315*Sec
[(c + d*x)/2]^10 - 1736335*Csc[c/2]*Sec[(c + d*x)/2]*Sin[(d*x)/2] + 561145
*Csc[c/2]*Sec[(c + d*x)/2]^3*Sin[(d*x)/2] - 184650*Csc[c/2]*Sec[(c + d*x)/
2]^5*Sin[(d*x)/2] + 41320*Csc[c/2]*Sec[(c + d*x)/2]^7*Sin[(d*x)/2] - 5425*
Csc[c/2]*Sec[(c + d*x)/2]^9*Sin[(d*x)/2] + 315*Csc[c/2]*Sec[(c + d*x)/2]^1
1*Sin[(d*x)/2] + 6468*Csc[c/2]^3*Csc[(c + d*x)/2]^3*Sin[c]*Sin[(d*x)/2] +
231*Cot[c/2]*(3840*d*x - Csc[c/2]*Csc[(c + d*x)/2]*(743 + 3*Csc[(c + d*x)/
2]^4)*Sin[(d*x)/2]))*Tan[c/2])/(a^3*d*(1 + Sec[c + d*x])^3)
```

3.103.3 Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.01, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 4376, 25, 3042, 4374, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^6(c+dx)}{(a\sec(c+dx)+a)^3}\,dx$$

$$\downarrow 3042$$

$$\int \frac{1}{\cot\left(c+dx+\frac{\pi}{2}\right)^6\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^3}dx$$

$$\downarrow 4376$$

$$\frac{\int -\cot^{12}(c+dx)(a-a\sec(c+dx))^3dx}{a^6}$$

$$\downarrow 25$$

$$-\frac{\int \cot^{12}(c+dx)(a-a\sec(c+dx))^3dx}{a^6}$$

$$\downarrow 3042$$

$$-\frac{\int \frac{\left(a-a\csc\left(c+dx+\frac{\pi}{2}\right)\right)^3}{\cot\left(c+dx+\frac{\pi}{2}\right)^{12}}dx}{a^6}$$

$$\downarrow 4374$$

$$-\frac{\int \left(a^3\cot^{12}(c+dx)-3a^3\csc(c+dx)\cot^{11}(c+dx)+3a^3\csc^2(c+dx)\cot^{10}(c+dx)-a^3\csc^3(c+dx)\cot^9(c+d\right.}{a^6}$$

$$\downarrow 2009$$

$$-\frac{-\frac{4a^3\cot^{11}(c+dx)}{11d}+\frac{a^3\cot^9(c+dx)}{9d}-\frac{a^3\cot^7(c+dx)}{7d}+\frac{a^3\cot^5(c+dx)}{5d}-\frac{a^3\cot^3(c+dx)}{3d}+\frac{a^3\cot(c+dx)}{d}+\frac{4a^3\csc^{11}(c+dx)}{11d}-\frac{19a^3\cs}{}}{a^6}$$

input
```
Int[Cot[c + d*x]^6/(a + a*Sec[c + d*x])^3,x]
```

output
```
-((a^3*x + (a^3*Cot[c + d*x])/d - (a^3*Cot[c + d*x]^3)/(3*d) + (a^3*Cot[c
+ d*x]^5)/(5*d) - (a^3*Cot[c + d*x]^7)/(7*d) + (a^3*Cot[c + d*x]^9)/(9*d)
- (4*a^3*Cot[c + d*x]^11)/(11*d) - (3*a^3*Csc[c + d*x])/d + (16*a^3*Csc[c
+ d*x]^3)/(3*d) - (34*a^3*Csc[c + d*x]^5)/(5*d) + (36*a^3*Csc[c + d*x]^7)/
(7*d) - (19*a^3*Csc[c + d*x]^9)/(9*d) + (4*a^3*Csc[c + d*x]^11)/(11*d))/a^
6)
```

3.103.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4374
```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

rule 4376
```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Simp[a^(2*n)/e^(2*n)   Int[(e*Cot[c + d*x])^(m + 2*
n)/(-a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[a
^2 - b^2, 0] && ILtQ[n, 0]
```

3.103.4 Maple [A] (verified)

Time = 0.90 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.64

method	result
derivativedivides	$\frac{-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{11}}{11}+\frac{10\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{9}}{9}-\frac{46\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{7}}{7}+26\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}-\frac{256\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{3}+382\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-512\arctan($
default	$\frac{-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{11}}{11}+\frac{10\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{9}}{9}-\frac{46\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{7}}{7}+26\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}-\frac{256\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{3}+382\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-512\arctan($
risch	$-\frac{x}{a^3}+\frac{2i(10395\,e^{15i(dx+c)}+31185\,e^{14i(dx+c)}+1155\,e^{13i(dx+c)}-148995\,e^{12i(dx+c)}-190113\,e^{11i(dx+c)}+117117\,e^{10i(d}$

input
```
int(cot(d*x+c)^6/(a+a*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/256/d/a^3*(-1/11*tan(1/2*d*x+1/2*c)^11+10/9*tan(1/2*d*x+1/2*c)^9-46/7*ta
n(1/2*d*x+1/2*c)^7+26*tan(1/2*d*x+1/2*c)^5-256/3*tan(1/2*d*x+1/2*c)^3+382*
tan(1/2*d*x+1/2*c)-512*arctan(tan(1/2*d*x+1/2*c))-1/5/tan(1/2*d*x+1/2*c)^5
+10/3/tan(1/2*d*x+1/2*c)^3-46/tan(1/2*d*x+1/2*c))
```

3.103.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 282, normalized size of antiderivative = 1.31

$$\int \frac{\cot^6(c+dx)}{(a+a\sec(c+dx))^3}\,dx =$$
$$-\frac{7453\cos(dx+c)^8+11964\cos(dx+c)^7-11866\cos(dx+c)^6-30542\cos(dx+c)^5+90\cos(dx+c$$
$$3465\left(a^3d\cos(dx\right.$$

input
```
integrate(cot(d*x+c)^6/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

output
```
-1/3465*(7453*cos(d*x + c)^8 + 11964*cos(d*x + c)^7 - 11866*cos(d*x + c)^6
 - 30542*cos(d*x + c)^5 + 90*cos(d*x + c)^4 + 26438*cos(d*x + c)^3 + 8539*
cos(d*x + c)^2 + 3465*(d*x*cos(d*x + c)^7 + 3*d*x*cos(d*x + c)^6 + d*x*cos
(d*x + c)^5 - 5*d*x*cos(d*x + c)^4 - 5*d*x*cos(d*x + c)^3 + d*x*cos(d*x +
c)^2 + 3*d*x*cos(d*x + c) + d*x)*sin(d*x + c) - 7671*cos(d*x + c) - 3712)/
((a^3*d*cos(d*x + c)^7 + 3*a^3*d*cos(d*x + c)^6 + a^3*d*cos(d*x + c)^5 - 5
*a^3*d*cos(d*x + c)^4 - 5*a^3*d*cos(d*x + c)^3 + a^3*d*cos(d*x + c)^2 + 3*
a^3*d*cos(d*x + c) + a^3*d)*sin(d*x + c))
```

3.103.6 Sympy [F]

$$\int \frac{\cot^6(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{\int \frac{\cot^6(c+dx)}{\sec^3(c+dx)+3\sec^2(c+dx)+3\sec(c+dx)+1}\,dx}{a^3}$$

input
```
integrate(cot(d*x+c)**6/(a+a*sec(d*x+c))**3,x)
```

output
```
Integral(cot(c + d*x)**6/(sec(c + d*x)**3 + 3*sec(c + d*x)**2 + 3*sec(c +
d*x) + 1), x)/a**3
```

3.103.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.01

$$\int \frac{\cot^6(c+dx)}{(a+a\sec(c+dx))^3}\,dx$$
$$= \frac{5\left(\frac{264726\sin(dx+c)}{\cos(dx+c)+1} - \frac{59136\sin(dx+c)^3}{(\cos(dx+c)+1)^3} + \frac{18018\sin(dx+c)^5}{(\cos(dx+c)+1)^5} - \frac{4554\sin(dx+c)^7}{(\cos(dx+c)+1)^7} + \frac{770\sin(dx+c)^9}{(\cos(dx+c)+1)^9} - \frac{63\sin(dx+c)^{11}}{(\cos(dx+c)+1)^{11}}\right)}{a^3} - \frac{1774080\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^3}$$
$$\frac{}{887040\,d}$$

input
```
integrate(cot(d*x+c)^6/(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

output
```
1/887040*(5*(264726*sin(d*x + c)/(cos(d*x + c) + 1) - 59136*sin(d*x + c)^3
/(cos(d*x + c) + 1)^3 + 18018*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 4554*s
in(d*x + c)^7/(cos(d*x + c) + 1)^7 + 770*sin(d*x + c)^9/(cos(d*x + c) + 1)
^9 - 63*sin(d*x + c)^11/(cos(d*x + c) + 1)^11)/a^3 - 1774080*arctan(sin(d*
x + c)/(cos(d*x + c) + 1))/a^3 + 231*(50*sin(d*x + c)^2/(cos(d*x + c) + 1)
^2 - 690*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 - 3)*(cos(d*x + c) + 1)^5/(a^
3*sin(d*x + c)^5))/d
```

3.103.8 Giac [A] (verification not implemented)

Time = 0.53 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.74

$$\int \frac{\cot^6(c+dx)}{(a+a\sec(c+dx))^3}\,dx =$$

$$-\frac{\frac{887040\,(dx+c)}{a^3}+\frac{231\left(690\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-50\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+3\right)}{a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5}+\frac{5\left(63\,a^{30}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{11}-770\,a^{30}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9+4554\,a^{30}\,\mathrm{ta}}{}}{887040\,d}$$

input
```
integrate(cot(d*x+c)^6/(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

output
```
-1/887040*(887040*(d*x + c)/a^3 + 231*(690*tan(1/2*d*x + 1/2*c)^4 - 50*tan
(1/2*d*x + 1/2*c)^2 + 3)/(a^3*tan(1/2*d*x + 1/2*c)^5) + 5*(63*a^30*tan(1/2
*d*x + 1/2*c)^11 - 770*a^30*tan(1/2*d*x + 1/2*c)^9 + 4554*a^30*tan(1/2*d*x
 + 1/2*c)^7 - 18018*a^30*tan(1/2*d*x + 1/2*c)^5 + 59136*a^30*tan(1/2*d*x +
 1/2*c)^3 - 264726*a^30*tan(1/2*d*x + 1/2*c))/a^33)/d
```

3.103.9 Mupad [B] (verification not implemented)

Time = 14.92 (sec) , antiderivative size = 254, normalized size of antiderivative = 1.18

$$\int \frac{\cot^6(c+dx)}{(a+a\sec(c+dx))^3}\,dx =$$

$$-\frac{693\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{16}+315\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{16}-3850\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^2\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{14}+22770\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^4\sin(}{}$$

input
```
int(cot(c + d*x)^6/(a + a/cos(c + d*x))^3,x)
```

output
```
-(693*cos(c/2 + (d*x)/2)^16 + 315*sin(c/2 + (d*x)/2)^16 - 3850*cos(c/2 + (
d*x)/2)^2*sin(c/2 + (d*x)/2)^14 + 22770*cos(c/2 + (d*x)/2)^4*sin(c/2 + (d*
x)/2)^12 - 90090*cos(c/2 + (d*x)/2)^6*sin(c/2 + (d*x)/2)^10 + 295680*cos(c
/2 + (d*x)/2)^8*sin(c/2 + (d*x)/2)^8 - 1323630*cos(c/2 + (d*x)/2)^10*sin(c
/2 + (d*x)/2)^6 + 159390*cos(c/2 + (d*x)/2)^12*sin(c/2 + (d*x)/2)^4 - 1155
0*cos(c/2 + (d*x)/2)^14*sin(c/2 + (d*x)/2)^2 + 887040*cos(c/2 + (d*x)/2)^1
1*sin(c/2 + (d*x)/2)^5*(c + d*x))/(887040*a^3*d*cos(c/2 + (d*x)/2)^11*sin(
c/2 + (d*x)/2)^5)
```

3.104 $\int (a + a\sec(c+dx))(e\tan(c+dx))^{5/2}\,dx$

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3.104.1 Optimal result

Integrand size = 23, antiderivative size = 310

$$\int (a + a\sec(c+dx))(e\tan(c+dx))^{5/2}\,dx = \frac{ae^{5/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}d} - \frac{ae^{5/2}\arctan\left(1+\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}d} - \frac{ae^{5/2}\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}d} + \frac{ae^{5/2}\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)+\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}d} + \frac{6ae^2\cos(c+dx)E\left(c-\frac{\pi}{4}+dx\middle|2\right)\sqrt{e\tan(c+dx)}}{5d\sqrt{\sin(2c+2dx)}} - \frac{6ae\cos(c+dx)(e\tan(c+dx))^{3/2}}{5d} + \frac{2e(5a+3a\sec(c+dx))(e\tan(c+dx))^{3/2}}{15d}$$

output

```
1/2*a*e^(5/2)*arctan(1-2^(1/2)*(e*tan(d*x+c))^(1/2)/e^(1/2))/d*2^(1/2)-1/2
*a*e^(5/2)*arctan(1+2^(1/2)*(e*tan(d*x+c))^(1/2)/e^(1/2))/d*2^(1/2)-1/4*a*
e^(5/2)*ln(e^(1/2)-2^(1/2)*(e*tan(d*x+c))^(1/2)+e^(1/2)*tan(d*x+c))/d*2^(1
/2)+1/4*a*e^(5/2)*ln(e^(1/2)+2^(1/2)*(e*tan(d*x+c))^(1/2)+e^(1/2)*tan(d*x+
c))/d*2^(1/2)-6/5*a*e^2*cos(d*x+c)*(sin(c+1/4*Pi+d*x)^2)^(1/2)/sin(c+1/4*P
i+d*x)*EllipticE(cos(c+1/4*Pi+d*x),2^(1/2))*(e*tan(d*x+c))^(1/2)/d/sin(2*d
*x+2*c)^(1/2)-6/5*a*e*cos(d*x+c)*(e*tan(d*x+c))^(3/2)/d+2/15*e*(5*a+3*a*se
c(d*x+c))*(e*tan(d*x+c))^(3/2)/d
```

3.104.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 2.58 (sec) , antiderivative size = 186, normalized size of antiderivative = 0.60

$$\int (a + a\sec(c+dx))(e\tan(c+dx))^{5/2}\,dx = \frac{a(1+\cos(c+dx))\csc(c+dx)\sec^2\left(\frac{1}{2}(c+dx)\right)\left(12+20\cos(c+dx)-36\cos^2(c+dx)+\frac{24\,\mathrm{H}}{}\right.$$

input
```
Integrate[(a + a*Sec[c + d*x])*(e*Tan[c + d*x])^(5/2),x]
```

output
```
(a*(1 + Cos[c + d*x])*Csc[c + d*x]*Sec[(c + d*x)/2]^2*(12 + 20*Cos[c + d*x
] - 36*Cos[c + d*x]^2 + (24*Hypergeometric2F1[3/4, 3/2, 7/4, -Tan[c + d*x]
^2])/Sqrt[Sec[c + d*x]^2] + 15*ArcSin[Cos[c + d*x] - Sin[c + d*x]]*Cot[c +
 d*x]^2*Sqrt[Sin[2*(c + d*x)]] + 15*Cot[c + d*x]^2*Log[Cos[c + d*x] + Sin[
c + d*x] + Sqrt[Sin[2*(c + d*x)]]]*Sqrt[Sin[2*(c + d*x)]])*(e*Tan[c + d*x]
)^(5/2))/(60*d)
```

3.104.3 Rubi [A] (warning: unable to verify)

Time = 1.09 (sec) , antiderivative size = 295, normalized size of antiderivative = 0.95, number of steps used = 24, number of rules used = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 1.000$, Rules used = {3042, 4369, 27, 3042, 4372, 3042, 3093, 3042, 3095, 3042, 3052, 3042, 3119, 3957, 266, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sec(c+dx)+a)(e\tan(c+dx))^{5/2}\,dx$$

$$\downarrow 3042$$

$$\int \left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)\left(-e\cot\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}dx$$

$$\downarrow 4369$$

$$\frac{2e(3a\sec(c+dx)+5a)(e\tan(c+dx))^{3/2}}{15d} - \frac{2}{5}e^2\int \frac{1}{2}(3\sec(c+dx)a+5a)\sqrt{e\tan(c+dx)}dx$$

$$\downarrow 27$$

$$\frac{2e(3a\sec(c+dx)+5a)(e\tan(c+dx))^{3/2}}{15d}-\frac{1}{5}e^2\int(3\sec(c+dx)a+5a)\sqrt{e\tan(c+dx)}dx$$

$$\downarrow 3042$$

$$\frac{2e(3a\sec(c+dx)+5a)(e\tan(c+dx))^{3/2}}{15d}-\frac{1}{5}e^2\int\sqrt{-e\cot\left(c+dx+\frac{\pi}{2}\right)}\left(3\csc\left(c+dx+\frac{\pi}{2}\right)a+5a\right)dx$$

$$\downarrow 4372$$

$$\frac{2e(3a\sec(c+dx)+5a)(e\tan(c+dx))^{3/2}}{15d}-\frac{1}{5}e^2\left(5a\int\sqrt{e\tan(c+dx)}dx+3a\int\sec(c+dx)\sqrt{e\tan(c+dx)}dx\right)$$

$$\downarrow 3042$$

$$\frac{2e(3a\sec(c+dx)+5a)(e\tan(c+dx))^{3/2}}{15d}-\frac{1}{5}e^2\left(5a\int\sqrt{e\tan(c+dx)}dx+3a\int\sec(c+dx)\sqrt{e\tan(c+dx)}dx\right)$$

$$\downarrow 3093$$

$$\frac{2e(3a\sec(c+dx)+5a)(e\tan(c+dx))^{3/2}}{15d}-\frac{1}{5}e^2\left(5a\int\sqrt{e\tan(c+dx)}dx+3a\left(\frac{2\cos(c+dx)(e\tan(c+dx))^{3/2}}{de}-2\int\cos(c+dx)\sqrt{e\tan(c+dx)}dx\right)\right)$$

$$\downarrow 3042$$

$$\frac{2e(3a\sec(c+dx)+5a)(e\tan(c+dx))^{3/2}}{15d}-\frac{1}{5}e^2\left(5a\int\sqrt{e\tan(c+dx)}dx+3a\left(\frac{2\cos(c+dx)(e\tan(c+dx))^{3/2}}{de}-2\int\frac{\sqrt{e\tan(c+dx)}}{\sec(c+dx)}dx\right)\right)$$

$$\downarrow 3095$$

$$\frac{2e(3a\sec(c+dx)+5a)(e\tan(c+dx))^{3/2}}{15d}-\frac{1}{5}e^2\left(5a\int\sqrt{e\tan(c+dx)}dx+3a\left(\frac{2\cos(c+dx)(e\tan(c+dx))^{3/2}}{de}-\frac{2\sqrt{\cos(c+dx)}\sqrt{e\tan(c+dx)}\int\sqrt{\cos(c+}{\sqrt{\sin(c+dx)}}\right.\right.$$

$$\downarrow 3042$$

$$\frac{2e(3a\sec(c+dx)+5a)(e\tan(c+dx))^{3/2}}{15d}-\frac{1}{5}e^2\left(5a\int\sqrt{e\tan(c+dx)}dx+3a\left(\frac{2\cos(c+dx)(e\tan(c+dx))^{3/2}}{de}-\frac{2\sqrt{\cos(c+dx)}\sqrt{e\tan(c+dx)}\int\sqrt{\cos(c+}{\sqrt{\sin(c+dx)}}\right.\right.$$

$$\downarrow 3052$$

$$\frac{2e(3a\sec(c+dx)+5a)(e\tan(c+dx))^{3/2}}{15d} -$$

$$\frac{1}{5}e^2\left(5a\int\sqrt{e\tan(c+dx)}dx+3a\left(\frac{2\cos(c+dx)(e\tan(c+dx))^{3/2}}{de}-\frac{2\cos(c+dx)\sqrt{e\tan(c+dx)}\int\sqrt{\sin(2c+2}\right.\right.$$

$$\downarrow 3042$$

$$\frac{2e(3a\sec(c+dx)+5a)(e\tan(c+dx))^{3/2}}{15d} -$$

$$\frac{1}{5}e^2\left(5a\int\sqrt{e\tan(c+dx)}dx+3a\left(\frac{2\cos(c+dx)(e\tan(c+dx))^{3/2}}{de}-\frac{2\cos(c+dx)\sqrt{e\tan(c+dx)}\int\sqrt{\sin(2c+2}\right.\right.$$

$$\downarrow 3119$$

$$\frac{2e(3a\sec(c+dx)+5a)(e\tan(c+dx))^{3/2}}{15d} -$$

$$\frac{1}{5}e^2\left(5a\int\sqrt{e\tan(c+dx)}dx+3a\left(\frac{2\cos(c+dx)(e\tan(c+dx))^{3/2}}{de}-\frac{2\cos(c+dx)E\left(c+dx-\frac{\pi}{4}\middle|2\right)\sqrt{e\tan(c+}\right.\right.$$

$$\downarrow 3957$$

$$\frac{2e(3a\sec(c+dx)+5a)(e\tan(c+dx))^{3/2}}{15d} -$$

$$\frac{1}{5}e^2\left(\frac{5ae\int\frac{\sqrt{e\tan(c+dx)}}{\tan^2(c+dx)e^2+e^2}d(e\tan(c+dx))}{d}+3a\left(\frac{2\cos(c+dx)(e\tan(c+dx))^{3/2}}{de}-\frac{2\cos(c+dx)E\left(c+dx-\frac{\pi}{4}\middle|2\right.}{d\sqrt{\sin(2c+2}}\right.\right.$$

$$\downarrow 266$$

$$\frac{2e(3a\sec(c+dx)+5a)(e\tan(c+dx))^{3/2}}{15d} -$$

$$\frac{1}{5}e^2\left(\frac{10ae\int\frac{e^2\tan^2(c+dx)}{e^4\tan^4(c+dx)+e^2}d\sqrt{e\tan(c+dx)}}{d}+3a\left(\frac{2\cos(c+dx)(e\tan(c+dx))^{3/2}}{de}-\frac{2\cos(c+dx)E\left(c+dx-\frac{\pi}{4}\right.}{d\sqrt{\sin(2c+}}\right.\right.$$

$$\downarrow 826$$

$$\frac{2e(3a\sec(c+dx)+5a)(e\tan(c+dx))^{3/2}}{15d} -$$

$$\frac{1}{5}e^2\left(\frac{10ae\left(\frac{1}{2}\int\frac{e^2\tan^2(c+dx)+e}{e^4\tan^4(c+dx)+e^2}d\sqrt{e\tan(c+dx)}-\frac{1}{2}\int\frac{e-e^2\tan^2(c+dx)}{e^4\tan^4(c+dx)+e^2}d\sqrt{e\tan(c+dx)}\right)}{d}+3a\left(\frac{2\cos(c+dx)(e\tan}{de}\right.\right.$$

$$\downarrow 1476$$

$$\frac{2e(3a\sec(c+dx)+5a)(e\tan(c+dx))^{3/2}}{15d} -$$

$$\frac{1}{5}e^2\left(\frac{10ae\left(\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{e^2\tan^2(c+dx)-\sqrt{2}e^{3/2}\tan(c+dx)+e}d\sqrt{e\tan(c+dx)}+\frac{1}{2}\int \frac{1}{e^2\tan^2(c+dx)+\sqrt{2}e^{3/2}\tan(c+dx)+e}d\sqrt{e\tan(c+\right.\right.\right.}{d}\right.$$

$$\downarrow 1082$$

$$\frac{2e(3a\sec(c+dx)+5a)(e\tan(c+dx))^{3/2}}{15d} -$$

$$\frac{1}{5}e^2\left(\frac{10ae\left(\frac{1}{2}\left(\frac{\int \frac{1}{-e^2\tan^2(c+dx)-1}d\left(1-\sqrt{2}\sqrt{e}\tan(c+dx)\right)}{\sqrt{2}\sqrt{e}}-\frac{\int \frac{1}{-e^2\tan^2(c+dx)-1}d\left(\sqrt{2}\sqrt{e}\tan(c+dx)+1\right)}{\sqrt{2}\sqrt{e}}\right)\right)-\frac{1}{2}\int \frac{e-e^2\tan^2(c+dx)}{e^4\tan^4(c+dx)+e^2}d\sqrt{}\right.$$

$$\downarrow 217$$

$$\frac{2e(3a\sec(c+dx)+5a)(e\tan(c+dx))^{3/2}}{15d} -$$

$$\frac{1}{5}e^2\left(\frac{10ae\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{e}\tan(c+dx)+1\right)}{\sqrt{2}\sqrt{e}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{e}\tan(c+dx)\right)}{\sqrt{2}\sqrt{e}}\right)-\frac{1}{2}\int \frac{e-e^2\tan^2(c+dx)}{e^4\tan^4(c+dx)+e^2}d\sqrt{e\tan(c+dx)}\right)}{d}+3a\right.$$

$$\downarrow 1479$$

$$\frac{2e(3a\sec(c+dx)+5a)(e\tan(c+dx))^{3/2}}{15d} -$$

$$\frac{1}{5}e^2\left(\frac{10ae\left(\frac{1}{2}\left(\frac{\int -\frac{\sqrt{2}\sqrt{e}-2\sqrt{e\tan(c+dx)}}{e^2\tan^2(c+dx)-\sqrt{2}e^{3/2}\tan(c+dx)+e}d\sqrt{e\tan(c+dx)}}{2\sqrt{2}\sqrt{e}}+\frac{\int -\frac{\sqrt{2}\left(\sqrt{e}+\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{e^2\tan^2(c+dx)+\sqrt{2}e^{3/2}\tan(c+dx)+e}d\sqrt{e\tan(c+dx)}}{2\sqrt{2}\sqrt{e}}\right)+\frac{1}{2}\left(\right.\right.\right.}{d}\right.$$

$$\downarrow 25$$

$$\frac{2e(3a\sec(c+dx)+5a)(e\tan(c+dx))^{3/2}}{15d} -$$

$$\frac{1}{5}e^2\left(\frac{10ae\left(\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}\sqrt{e}-2\sqrt{e\tan(c+dx)}}{e^2\tan^2(c+dx)-\sqrt{2}e^{3/2}\tan(c+dx)+e}d\sqrt{e\tan(c+dx)}}{2\sqrt{2}\sqrt{e}}-\frac{\int \frac{\sqrt{2}\left(\sqrt{e}+\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{e^2\tan^2(c+dx)+\sqrt{2}e^{3/2}\tan(c+dx)+e}d\sqrt{e\tan(c+dx)}}{2\sqrt{2}\sqrt{e}}\right)+\frac{1}{2}\left(\right.\right.\right.}{d}\right.$$

$$\downarrow 27$$

$$\frac{1}{5}e^2\left(\frac{2e(3a\sec(c+dx)+5a)(e\tan(c+dx))^{3/2}}{15d} - \frac{10ae\left(\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}\sqrt{e}-2\sqrt{e\tan(c+dx)}}{e^2\tan^2(c+dx)-\sqrt{2}e^{3/2}\tan(c+dx)+e}d\sqrt{e\tan(c+dx)}}{2\sqrt{2}\sqrt{e}} - \frac{\int \frac{\sqrt{e}+\sqrt{2}\sqrt{e\tan(c+dx)}}{e^2\tan^2(c+dx)+\sqrt{2}e^{3/2}\tan(c+dx)+e}d\sqrt{e\tan(c+dx)}}{2\sqrt{e}}\right)+\frac{1}{2}\left(\text{arct}\right.\right.}{d}\right.$$

$$\downarrow 1103$$

$$\frac{1}{5}e^2\left(\frac{2e(3a\sec(c+dx)+5a)(e\tan(c+dx))^{3/2}}{15d} - \frac{10ae\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{e}\tan(c+dx)+1\right)}{\sqrt{2}\sqrt{e}} - \frac{\arctan\left(1-\sqrt{2}\sqrt{e}\tan(c+dx)\right)}{\sqrt{2}\sqrt{e}}\right)+\frac{1}{2}\left(\frac{\log\left(-\sqrt{2}e^{3/2}\tan(c+dx)+e^2\tan^2(c+dx)+e\right)}{2\sqrt{2}\sqrt{e}} - \text{lo}\right.\right.}{d}\right.$$

input
```
Int[(a + a*Sec[c + d*x])*(e*Tan[c + d*x])^(5/2),x]
```

output
```
(2*e*(5*a + 3*a*Sec[c + d*x])*(e*Tan[c + d*x])^(3/2))/(15*d) - (e^2*((10*a
*e*((-(ArcTan[1 - Sqrt[2]*Sqrt[e]*Tan[c + d*x]]/(Sqrt[2]*Sqrt[e])) + ArcTa
n[1 + Sqrt[2]*Sqrt[e]*Tan[c + d*x]]/(Sqrt[2]*Sqrt[e]))/2 + (Log[e - Sqrt[2
]*e^(3/2)*Tan[c + d*x] + e^2*Tan[c + d*x]^2]/(2*Sqrt[2]*Sqrt[e]) - Log[e +
 Sqrt[2]*e^(3/2)*Tan[c + d*x] + e^2*Tan[c + d*x]^2]/(2*Sqrt[2]*Sqrt[e]))/2
))/d + 3*a*((-2*Cos[c + d*x]*EllipticE[c - Pi/4 + d*x, 2]*Sqrt[e*Tan[c + d
*x]])/(d*Sqrt[Sin[2*c + 2*d*x]]) + (2*Cos[c + d*x]*(e*Tan[c + d*x])^(3/2))
/(d*e))))/5
```

3.104.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3052
```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Simp[Sqrt[a*Sin[e + f*x]]*(Sqrt[b*Cos[e + f*x]]/Sqrt[Sin[2*e
 + 2*f*x]])   Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

rule 3093
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[a^2*(a*Sec[e + f*x])^(m - 2)*((b*Tan[e + f*x])^(n +
1)/(b*f*(m + n - 1))), x] + Simp[a^2*((m - 2)/(m + n - 1))   Int[(a*Sec[e +
 f*x])^(m - 2)*(b*Tan[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (
GtQ[m, 1] || (EqQ[m, 1] && EqQ[n, 1/2])) && NeQ[m + n - 1, 0] && IntegersQ[
2*m, 2*n]
```

rule 3095
```
Int[Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]]/sec[(e_.) + (f_.)*(x_)], x_Symbol]
:> Simp[Sqrt[Cos[e + f*x]]*(Sqrt[b*Tan[e + f*x]]/Sqrt[Sin[e + f*x]])   Int[
Sqrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]], x], x] /; FreeQ[{b, e, f}, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

rule 4369
```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(-e)*(e*Cot[c + d*x])^(m - 1)*((a*m + b*(m - 1)*Csc
[c + d*x])/(d*m*(m - 1))), x] - Simp[e^2/m   Int[(e*Cot[c + d*x])^(m - 2)*(
a*m + b*(m - 1)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && GtQ[m
, 1]
```

rule 4372
```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(e*Cot[c + d*x])^m, x], x] + Simp[b   Int[
(e*Cot[c + d*x])^m*Csc[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, m}, x]
```

3.104.4 Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 579 vs. $2(274) = 548$.

Time = 6.00 (sec) , antiderivative size = 580, normalized size of antiderivative = 1.87

method	result
parts	$\frac{2ae\left(\frac{(e\tan(dx+c))^{\frac{3}{2}}}{3}-\frac{e^2\sqrt{2}\left(\ln\left(\frac{e\tan(dx+c)-\left(e^2\right)^{\frac{1}{4}}\sqrt{e\tan(dx+c)}\sqrt{2}+\sqrt{e^2}}{e\tan(dx+c)+\left(e^2\right)^{\frac{1}{4}}\sqrt{e\tan(dx+c)}\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{e\tan(dx+c)}}{\left(e^2\right)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\frac{\sqrt{2}\sqrt{e\tan(dx+c)}}{\left(e^2\right)^{\frac{1}{4}}}\right.\right.}{8\left(e^2\right)^{\frac{1}{4}}}\right.}{d}$
default	Expression too large to display

input
```
int((a+a*sec(d*x+c))*(e*tan(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2*a/d*e*(1/3*(e*tan(d*x+c))^(3/2)-1/8*e^2/(e^2)^(1/4)*2^(1/2)*(ln((e*tan(d
*x+c)-(e^2)^(1/4)*(e*tan(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2))/(e*tan(d*x+c)+
(e^2)^(1/4)*(e*tan(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2)))+2*arctan(2^(1/2)/(e
^2)^(1/4)*(e*tan(d*x+c))^(1/2)+1)-2*arctan(-2^(1/2)/(e^2)^(1/4)*(e*tan(d*x
+c))^(1/2)+1)))-1/5*a/d*2^(1/2)*(-6*(csc(d*x+c)-cot(d*x+c)+1)^(1/2)*(cot(d
*x+c)-csc(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)*EllipticE((csc(d*x
+c)-cot(d*x+c)+1)^(1/2),1/2*2^(1/2))*cos(d*x+c)^3+3*(csc(d*x+c)-cot(d*x+c)
+1)^(1/2)*(cot(d*x+c)-csc(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)*El
lipticF((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2*2^(1/2))*cos(d*x+c)^3-6*(csc(d
*x+c)-cot(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(
d*x+c))^(1/2)*EllipticE((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2*2^(1/2))*cos(d
*x+c)^2+3*(csc(d*x+c)-cot(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c)+1)^(1/2)*
(cot(d*x+c)-csc(d*x+c))^(1/2)*EllipticF((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/
2*2^(1/2))*cos(d*x+c)^2+3*cos(d*x+c)^3*2^(1/2)-4*2^(1/2)*cos(d*x+c)^2+2^(1
/2))*e^2*(e*tan(d*x+c))^(1/2)/(cos(d*x+c)^2-1)*sec(d*x+c)*tan(d*x+c)
```

3.104.5 Fricas [F(-1)]

Timed out.

$$\int (a + a\sec(c + dx))(e\tan(c + dx))^{5/2}\,dx = \text{Timed out}$$

input
```
integrate((a+a*sec(d*x+c))*(e*tan(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.104.6 Sympy [F]

$$\int (a + a\sec(c + dx))(e\tan(c + dx))^{5/2}\,dx = a\left(\int (e\tan(c + dx))^{\frac{5}{2}}\,dx + \int (e\tan(c + dx))^{\frac{5}{2}}\sec(c + dx)\,dx\right)$$

input
```
integrate((a+a*sec(d*x+c))*(e*tan(d*x+c))**(5/2),x)
```

output
```
a*(Integral((e*tan(c + d*x))**(5/2), x) + Integral((e*tan(c + d*x))**(5/2)
*sec(c + d*x), x))
```

3.104.7 Maxima [F(-2)]

Exception generated.

$$\int (a + a\sec(c + dx))(e\tan(c + dx))^{5/2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+a*sec(d*x+c))*(e*tan(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.104.8 Giac [F]

$$\int (a + a\sec(c + dx))(e\tan(c + dx))^{5/2}\,dx = \int (a\sec(dx + c) + a)(e\tan(dx + c))^{\frac{5}{2}}\,dx$$

input
```
integrate((a+a*sec(d*x+c))*(e*tan(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
integrate((a*sec(d*x + c) + a)*(e*tan(d*x + c))^(5/2), x)
```

3.104.9 Mupad [F(-1)]

Timed out.

$$\int (a + a\sec(c + dx))(e\tan(c + dx))^{5/2}\,dx = \int (e\tan(c + d\,x))^{5/2}\left(a + \frac{a}{\cos(c + d\,x)}\right)\,dx$$

input
```
int((e*tan(c + d*x))^(5/2)*(a + a/cos(c + d*x)),x)
```

output
```
int((e*tan(c + d*x))^(5/2)*(a + a/cos(c + d*x)), x)
```

3.105 $\int (a + a\sec(c+dx))(e\tan(c+dx))^{3/2}\,dx$

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3.105.6 Sympy [F] . . . 777
3.105.7 Maxima [F(-2)] . . . 777
3.105.8 Giac [F] . . . 777
3.105.9 Mupad [F(-1)] . . . 778

3.105.1 Optimal result

Integrand size = 23, antiderivative size = 282

$$\begin{aligned}\int (a + a\sec(c+dx))(e\tan(c+dx))^{3/2}\,dx &= \frac{ae^{3/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}d}\\ &- \frac{ae^{3/2}\arctan\left(1+\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}d}\\ &+ \frac{ae^{3/2}\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}d}\\ &- \frac{ae^{3/2}\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)+\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}d}\\ &- \frac{ae^2\,\text{EllipticF}\left(c-\frac{\pi}{4}+dx,2\right)\sec(c+dx)\sqrt{\sin(2c+2dx)}}{3d\sqrt{e\tan(c+dx)}}\\ &+ \frac{2e(3a+a\sec(c+dx))\sqrt{e\tan(c+dx)}}{3d}\end{aligned}$$

output

```
1/2*a*e^(3/2)*arctan(1-2^(1/2)*(e*tan(d*x+c))^(1/2)/e^(1/2))/d*2^(1/2)-1/2
*a*e^(3/2)*arctan(1+2^(1/2)*(e*tan(d*x+c))^(1/2)/e^(1/2))/d*2^(1/2)+1/4*a*
e^(3/2)*ln(e^(1/2)-2^(1/2)*(e*tan(d*x+c))^(1/2)+e^(1/2)*tan(d*x+c))/d*2^(1
/2)-1/4*a*e^(3/2)*ln(e^(1/2)+2^(1/2)*(e*tan(d*x+c))^(1/2)+e^(1/2)*tan(d*x+
c))/d*2^(1/2)+1/3*a*e^2*(sin(c+1/4*Pi+d*x)^2)^(1/2)/sin(c+1/4*Pi+d*x)*Elli
pticF(cos(c+1/4*Pi+d*x),2^(1/2))*sec(d*x+c)*sin(2*d*x+2*c)^(1/2)/d/(e*tan(
d*x+c))^(1/2)+2/3*e*(3*a+a*sec(d*x+c))*(e*tan(d*x+c))^(1/2)/d
```

3.105.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.15 (sec) , antiderivative size = 214, normalized size of antiderivative = 0.76

$$\int (a + a\sec(c+dx))(e\tan(c+dx))^{3/2}\,dx =$$

$$-\frac{ae\cos(2(c+dx))\csc\left(\frac{1}{2}(c+dx)\right)\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{\sec^2(c+dx)}\sqrt{e\tan(c+dx)}\left(4\sqrt[4]{-1}\,\text{EllipticF}\left(i\text{arcsin}\right.\right.}{}$$

input
```
Integrate[(a + a*Sec[c + d*x])*(e*Tan[c + d*x])^(3/2),x]
```

output
```
-1/12*(a*e*Cos[2*(c + d*x)]*Csc[(c + d*x)/2]*Sec[(c + d*x)/2]*Sqrt[Sec[c +
 d*x]^2]*Sqrt[e*Tan[c + d*x]]*(4*(-1)^(1/4)*EllipticF[I*ArcSinh[(-1)^(1/4)
*Sqrt[Tan[c + d*x]]], -1]*Sqrt[Tan[c + d*x]] + Sqrt[Sec[c + d*x]^2]*(12*Si
n[c + d*x] + 3*ArcSin[Cos[c + d*x] - Sin[c + d*x]]*Sqrt[Sin[2*(c + d*x)]]
 - 3*Log[Cos[c + d*x] + Sin[c + d*x] + Sqrt[Sin[2*(c + d*x)]]]*Sqrt[Sin[2*(
c + d*x)]] + 4*Tan[c + d*x])))/(d*(-1 + Tan[c + d*x]^2))
```

3.105.3 Rubi [A] (warning: unable to verify)

Time = 0.94 (sec) , antiderivative size = 270, normalized size of antiderivative = 0.96, number of steps used = 22, number of rules used = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.913$, Rules used = {3042, 4369, 27, 3042, 4372, 3042, 3094, 3042, 3053, 3042, 3120, 3957, 266, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sec(c+dx)+a)(e\tan(c+dx))^{3/2}\,dx$$

$$\downarrow 3042$$

$$\int \left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)\left(-e\cot\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}dx$$

$$\downarrow 4369$$

$$\frac{2e(a\sec(c+dx)+3a)\sqrt{e\tan(c+dx)}}{3d}-\frac{2}{3}e^2\int\frac{\sec(c+dx)a+3a}{2\sqrt{e\tan(c+dx)}}dx$$

$$\downarrow 27$$

$$\frac{2e(a\sec(c+dx)+3a)\sqrt{e\tan(c+dx)}}{3d}-\frac{1}{3}e^2\int\frac{\sec(c+dx)a+3a}{\sqrt{e\tan(c+dx)}}dx$$

$$\downarrow 3042$$

$$\frac{2e(a\sec(c+dx)+3a)\sqrt{e\tan(c+dx)}}{3d}-\frac{1}{3}e^2\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)a+3a}{\sqrt{-e\cot\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\downarrow 4372$$

$$\frac{2e(a\sec(c+dx)+3a)\sqrt{e\tan(c+dx)}}{3d}-\frac{1}{3}e^2\left(3a\int\frac{1}{\sqrt{e\tan(c+dx)}}dx+a\int\frac{\sec(c+dx)}{\sqrt{e\tan(c+dx)}}dx\right)$$

$$\downarrow 3042$$

$$\frac{2e(a\sec(c+dx)+3a)\sqrt{e\tan(c+dx)}}{3d}-\frac{1}{3}e^2\left(3a\int\frac{1}{\sqrt{e\tan(c+dx)}}dx+a\int\frac{\sec(c+dx)}{\sqrt{e\tan(c+dx)}}dx\right)$$

$$\downarrow 3094$$

$$\frac{2e(a\sec(c+dx)+3a)\sqrt{e\tan(c+dx)}}{3d}-\frac{1}{3}e^2\left(3a\int\frac{1}{\sqrt{e\tan(c+dx)}}dx+\frac{a\sqrt{\sin(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}\sqrt{\sin(c+dx)}}dx}{\sqrt{\cos(c+dx)}\sqrt{e\tan(c+dx)}}\right)$$

$$\downarrow 3042$$

$$\frac{2e(a\sec(c+dx)+3a)\sqrt{e\tan(c+dx)}}{3d}-\frac{1}{3}e^2\left(3a\int\frac{1}{\sqrt{e\tan(c+dx)}}dx+\frac{a\sqrt{\sin(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}\sqrt{\sin(c+dx)}}dx}{\sqrt{\cos(c+dx)}\sqrt{e\tan(c+dx)}}\right)$$

$$\downarrow 3053$$

$$\frac{2e(a\sec(c+dx)+3a)\sqrt{e\tan(c+dx)}}{3d}-\frac{1}{3}e^2\left(3a\int\frac{1}{\sqrt{e\tan(c+dx)}}dx+\frac{a\sqrt{\sin(2c+2dx)}\sec(c+dx)\int\frac{1}{\sqrt{\sin(2c+2dx)}}dx}{\sqrt{e\tan(c+dx)}}\right)$$

$$\downarrow 3042$$

$$\frac{2e(a\sec(c+dx)+3a)\sqrt{e\tan(c+dx)}}{3d} -$$

$$\frac{1}{3}e^2\left(3a\int\frac{1}{\sqrt{e\tan(c+dx)}}dx+\frac{a\sqrt{\sin(2c+2dx)}\sec(c+dx)\int\frac{1}{\sqrt{\sin(2c+2dx)}}dx}{\sqrt{e\tan(c+dx)}}\right)$$

$$\downarrow 3120$$

$$\frac{2e(a\sec(c+dx)+3a)\sqrt{e\tan(c+dx)}}{3d} -$$

$$\frac{1}{3}e^2\left(3a\int\frac{1}{\sqrt{e\tan(c+dx)}}dx+\frac{a\sqrt{\sin(2c+2dx)}\sec(c+dx)\,\text{EllipticF}\left(c+dx-\frac{\pi}{4},2\right)}{d\sqrt{e\tan(c+dx)}}\right)$$

$$\downarrow 3957$$

$$\frac{2e(a\sec(c+dx)+3a)\sqrt{e\tan(c+dx)}}{3d} -$$

$$\frac{1}{3}e^2\left(\frac{3ae\int\frac{1}{\sqrt{e\tan(c+dx)}(\tan^2(c+dx)e^2+e^2)}d(e\tan(c+dx))}{d}+\frac{a\sqrt{\sin(2c+2dx)}\sec(c+dx)\,\text{EllipticF}\left(c+dx-\frac{\pi}{4},2\right)}{d\sqrt{e\tan(c+dx)}}\right)$$

$$\downarrow 266$$

$$\frac{2e(a\sec(c+dx)+3a)\sqrt{e\tan(c+dx)}}{3d} -$$

$$\frac{1}{3}e^2\left(\frac{6ae\int\frac{1}{e^4\tan^4(c+dx)+e^2}d\sqrt{e\tan(c+dx)}}{d}+\frac{a\sqrt{\sin(2c+2dx)}\sec(c+dx)\,\text{EllipticF}\left(c+dx-\frac{\pi}{4},2\right)}{d\sqrt{e\tan(c+dx)}}\right)$$

$$\downarrow 755$$

$$\frac{2e(a\sec(c+dx)+3a)\sqrt{e\tan(c+dx)}}{3d} -$$

$$\frac{1}{3}e^2\left(\frac{6ae\left(\frac{\int\frac{e-e^2\tan^2(c+dx)}{e^4\tan^4(c+dx)+e^2}d\sqrt{e\tan(c+dx)}}{2e}+\frac{\int\frac{e^2\tan^2(c+dx)+e}{e^4\tan^4(c+dx)+e^2}d\sqrt{e\tan(c+dx)}}{2e}\right)}{d}+\frac{a\sqrt{\sin(2c+2dx)}\sec(c+dx)\,\text{EllipticF}}{d\sqrt{e\tan(c+dx)}}\right)$$

$$\downarrow 1476$$

$$\frac{2e(a\sec(c+dx)+3a)\sqrt{e\tan(c+dx)}}{3d} -$$

$$\frac{1}{3}e^2\left(\frac{6ae\left(\frac{\frac{1}{2}\int\frac{1}{e^2\tan^2(c+dx)-\sqrt{2}e^{3/2}\tan(c+dx)+e}d\sqrt{e\tan(c+dx)}+\frac{1}{2}\int\frac{1}{e^2\tan^2(c+dx)+\sqrt{2}e^{3/2}\tan(c+dx)+e}d\sqrt{e\tan(c+dx)}}{2e}+\frac{\int\frac{e-e^2\tan^2(c+dx)}{e^4\tan^4(c+dx)+e}}{2}\right)}{d}\right)$$

$$\downarrow 1082$$

$$\frac{2e(a\sec(c+dx)+3a)\sqrt{e\tan(c+dx)}}{3d} -$$

$$\frac{1}{3}e^2\left(\frac{6ae\left(\frac{\int \frac{1}{-e^2\tan^2(c+dx)-1}d\left(1-\sqrt{2}\sqrt{e}\tan(c+dx)\right)}{\sqrt{2}\sqrt{e}} - \frac{\int \frac{1}{-e^2\tan^2(c+dx)-1}d\left(\sqrt{2}\sqrt{e}\tan(c+dx)+1\right)}{\sqrt{2}\sqrt{e}}}{2e} + \frac{\int \frac{e-e^2\tan^2(c+dx)}{e^4\tan^4(c+dx)+e^2}d\sqrt{e\tan(c+dx)}}{2e}\right)}{d} + \frac{a}{}\right.$$

$$\downarrow 217$$

$$\frac{2e(a\sec(c+dx)+3a)\sqrt{e\tan(c+dx)}}{3d} -$$

$$\frac{1}{3}e^2\left(\frac{6ae\left(\frac{\int \frac{e-e^2\tan^2(c+dx)}{e^4\tan^4(c+dx)+e^2}d\sqrt{e\tan(c+dx)}}{2e} + \frac{\frac{\arctan\left(\sqrt{2}\sqrt{e}\tan(c+dx)+1\right)}{\sqrt{2}\sqrt{e}} - \frac{\arctan\left(1-\sqrt{2}\sqrt{e}\tan(c+dx)\right)}{\sqrt{2}\sqrt{e}}}{2e}\right)}{d} + \frac{a\sqrt{\sin(2c+2dx)}\sec(c}{d\sqrt{e}}\right.$$

$$\downarrow 1479$$

$$\frac{2e(a\sec(c+dx)+3a)\sqrt{e\tan(c+dx)}}{3d} -$$

$$\frac{1}{3}e^2\left(\frac{6ae\left(\frac{-\frac{\int -\frac{\sqrt{2}\sqrt{e}-2\sqrt{e\tan(c+dx)}}{e^2\tan^2(c+dx)-\sqrt{2}e^{3/2}\tan(c+dx)+e}d\sqrt{e\tan(c+dx)}}{2\sqrt{2}\sqrt{e}} - \frac{\int -\frac{\sqrt{2}\left(\sqrt{e}+\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{e^2\tan^2(c+dx)+\sqrt{2}e^{3/2}\tan(c+dx)+e}d\sqrt{e\tan(c+dx)}}{2\sqrt{2}\sqrt{e}}}{2e} + \frac{\arctan\left(\sqrt{2}\sqrt{e}\tan(c+d\right.}{\sqrt{2}\sqrt{e}}\right)}{d}\right.$$

$$\downarrow 25$$

$$\frac{2e(a\sec(c+dx)+3a)\sqrt{e\tan(c+dx)}}{3d} -$$

$$\frac{1}{3}e^2\left(\frac{6ae\left(\frac{\int \frac{\sqrt{2}\sqrt{e}-2\sqrt{e\tan(c+dx)}}{e^2\tan^2(c+dx)-\sqrt{2}e^{3/2}\tan(c+dx)+e}d\sqrt{e\tan(c+dx)}}{2\sqrt{2}\sqrt{e}} + \frac{\int \frac{\sqrt{2}\left(\sqrt{e}+\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{e^2\tan^2(c+dx)+\sqrt{2}e^{3/2}\tan(c+dx)+e}d\sqrt{e\tan(c+dx)}}{2\sqrt{2}\sqrt{e}}}{2e} + \frac{\arctan\left(\sqrt{2}\sqrt{e}\tan(c+dx)+1\right.}{\sqrt{2}\sqrt{e}}\right)}{d}\right.$$

$$\downarrow 27$$

$$\frac{2e(a\sec(c+dx)+3a)\sqrt{e\tan(c+dx)}}{3d} -$$

$$\frac{1}{3}e^2\left(\frac{6ae\left(\frac{\int \frac{\sqrt{2}\sqrt{e}-2\sqrt{e\tan(c+dx)}}{e^2\tan^2(c+dx)-\sqrt{2}e^{3/2}\tan(c+dx)+e}d\sqrt{e\tan(c+dx)}}{2\sqrt{2}\sqrt{e}}+\frac{\int \frac{\sqrt{e}+\sqrt{2}\sqrt{e\tan(c+dx)}}{e^2\tan^2(c+dx)+\sqrt{2}e^{3/2}\tan(c+dx)+e}d\sqrt{e\tan(c+dx)}}{2\sqrt{e}}}{2e}+\frac{\arctan\left(\sqrt{2}\sqrt{e}\tan(c+dx)+1\right.}{\sqrt{2}\sqrt{e}}\right.}{d}$$

$$\downarrow 1103$$

$$\frac{2e(a\sec(c+dx)+3a)\sqrt{e\tan(c+dx)}}{3d} -$$

$$\frac{1}{3}e^2\left(\frac{6ae\left(\frac{\arctan\left(\sqrt{2}\sqrt{e}\tan(c+dx)+1\right)}{\sqrt{2}\sqrt{e}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{e}\tan(c+dx)\right)}{\sqrt{2}\sqrt{e}}}{2e}+\frac{\frac{\log\left(\sqrt{2}e^{3/2}\tan(c+dx)+e^2\tan^2(c+dx)+e\right)}{2\sqrt{2}\sqrt{e}}-\frac{\log\left(-\sqrt{2}e^{3/2}\tan(c+dx)+e^2\tan^2(c\right.}{2\sqrt{2}\sqrt{e}}}{2e}\right.}{d}$$

input
```
Int[(a + a*Sec[c + d*x])*(e*Tan[c + d*x])^(3/2),x]
```

output
```
(2*e*(3*a + a*Sec[c + d*x])*Sqrt[e*Tan[c + d*x]])/(3*d) - (e^2*((6*a*e*((-
(ArcTan[1 - Sqrt[2]*Sqrt[e]*Tan[c + d*x]]/(Sqrt[2]*Sqrt[e])) + ArcTan[1 +
Sqrt[2]*Sqrt[e]*Tan[c + d*x]]/(Sqrt[2]*Sqrt[e]))/(2*e) + (-1/2*Log[e - Sqr
t[2]*e^(3/2)*Tan[c + d*x] + e^2*Tan[c + d*x]^2]/(Sqrt[2]*Sqrt[e]) + Log[e
+ Sqrt[2]*e^(3/2)*Tan[c + d*x] + e^2*Tan[c + d*x]^2]/(2*Sqrt[2]*Sqrt[e]))/
(2*e)))/d + (a*EllipticF[c - Pi/4 + d*x, 2]*Sec[c + d*x]*Sqrt[Sin[2*c + 2*
d*x]])/(d*Sqrt[e*Tan[c + d*x]])))/3
```

3.105.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3053
```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Simp[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]])   Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f
}, x]
```

rule 3094
```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]], x_Symbol]
:> Simp[Sqrt[Sin[e + f*x]]/(Sqrt[Cos[e + f*x]]*Sqrt[b*Tan[e + f*x]])   Int[
1/(Sqrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]]), x], x] /; FreeQ[{b, e, f}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

rule 4369
```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(-e)*(e*Cot[c + d*x])^(m - 1)*((a*m + b*(m - 1)*Csc
[c + d*x])/(d*m*(m - 1))), x] - Simp[e^2/m   Int[(e*Cot[c + d*x])^(m - 2)*(
a*m + b*(m - 1)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && GtQ[m
, 1]
```

rule 4372
```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(e*Cot[c + d*x])^m, x], x] + Simp[b   Int[
(e*Cot[c + d*x])^m*Csc[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, m}, x]
```

3.105.4 Maple [A] (verified)

Time = 5.59 (sec) , antiderivative size = 372, normalized size of antiderivative = 1.32

method	result
parts	$\frac{2ae\left(\sqrt{e\tan(dx+c)}-\frac{\left(e^2\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\tan(dx+c)+\left(e^2\right)^{\frac{1}{4}}\sqrt{e\tan(dx+c)}\sqrt{2}+\sqrt{e^2}}{e\tan(dx+c)-\left(e^2\right)^{\frac{1}{4}}\sqrt{e\tan(dx+c)}\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{e\tan(dx+c)}}{\left(e^2\right)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\frac{\sqrt{2}\sqrt{e\tan(}\right.\right.\right.}{8}\right.}{d}$
default	Expression too large to display

input
```
int((a+a*sec(d*x+c))*(e*tan(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2*a/d*e*((e*tan(d*x+c))^(1/2)-1/8*(e^2)^(1/4)*2^(1/2)*(ln((e*tan(d*x+c)+(e
^2)^(1/4)*(e*tan(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2))/(e*tan(d*x+c)-(e^2)^(1
/4)*(e*tan(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2)))+2*arctan(2^(1/2)/(e^2)^(1/4
)*(e*tan(d*x+c))^(1/2)+1)-2*arctan(-2^(1/2)/(e^2)^(1/4)*(e*tan(d*x+c))^(1/
2)+1)))-1/3*a/d*2^(1/2)*(-(csc(d*x+c)-cot(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(
d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)*EllipticF((csc(d*x+c)-cot(d*
x+c)+1)^(1/2),1/2*2^(1/2))*cos(d*x+c)^2-(csc(d*x+c)-cot(d*x+c)+1)^(1/2)*(c
ot(d*x+c)-csc(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)*EllipticF((csc
(d*x+c)-cot(d*x+c)+1)^(1/2),1/2*2^(1/2))*cos(d*x+c)+2^(1/2)*sin(d*x+c))*e*
tan(d*x+c)*(e*tan(d*x+c))^(1/2)/(cos(d*x+c)^2-1)
```

3.105.5 Fricas [F(-1)]

Timed out.

$$\int (a + a\sec(c + dx))(e\tan(c + dx))^{3/2}\,dx = \text{Timed out}$$

input
```
integrate((a+a*sec(d*x+c))*(e*tan(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.105.6 Sympy [F]

$$\int (a + a\sec(c+dx))(e\tan(c+dx))^{3/2}\,dx = a\left(\int (e\tan(c+dx))^{\frac{3}{2}}\,dx + \int (e\tan(c+dx))^{\frac{3}{2}}\sec(c+dx)\,dx\right)$$

input
```
integrate((a+a*sec(d*x+c))*(e*tan(d*x+c))**(3/2),x)
```

output
```
a*(Integral((e*tan(c + d*x))**(3/2), x) + Integral((e*tan(c + d*x))**(3/2)
*sec(c + d*x), x))
```

3.105.7 Maxima [F(-2)]

Exception generated.

$$\int (a + a\sec(c+dx))(e\tan(c+dx))^{3/2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+a*sec(d*x+c))*(e*tan(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.105.8 Giac [F]

$$\int (a + a\sec(c+dx))(e\tan(c+dx))^{3/2}\,dx = \int (a\sec(dx+c)+a)(e\tan(dx+c))^{\frac{3}{2}}\,dx$$

input
```
integrate((a+a*sec(d*x+c))*(e*tan(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
integrate((a*sec(d*x + c) + a)*(e*tan(d*x + c))^(3/2), x)
```

3.105.9 Mupad [F(-1)]

Timed out.

$$\int (a + a\sec(c + dx))(e\tan(c + dx))^{3/2}\,dx = \int (e\tan(c + d\,x))^{3/2}\left(a + \frac{a}{\cos(c + d\,x)}\right)\,dx$$

input
```
int((e*tan(c + d*x))^(3/2)*(a + a/cos(c + d*x)),x)
```

output
```
int((e*tan(c + d*x))^(3/2)*(a + a/cos(c + d*x)), x)
```

3.106 $\int (a + a\sec(c+dx))\sqrt{e\tan(c+dx)}\,dx$

3.106.1 Optimal result . . . 779
3.106.2 Mathematica [C] (verified) . . . 780
3.106.3 Rubi [A] (warning: unable to verify) . . . 780
3.106.4 Maple [B] (verified) . . . 786
3.106.5 Fricas [F(-1)] . . . 787
3.106.6 Sympy [F] . . . 787
3.106.7 Maxima [F(-2)] . . . 787
3.106.8 Giac [F] . . . 788
3.106.9 Mupad [F(-1)] . . . 788

3.106.1 Optimal result

Integrand size = 23, antiderivative size = 272

$$\begin{aligned}
&\int (a + a\sec(c+dx))\sqrt{e\tan(c+dx)}\,dx \\
&= -\frac{a\sqrt{e}\arctan\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}d} + \frac{a\sqrt{e}\arctan\left(1+\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}d} \\
&\quad + \frac{a\sqrt{e}\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}d} \\
&\quad - \frac{a\sqrt{e}\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)+\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}d} \\
&\quad - \frac{2a\cos(c+dx)E\left(c-\frac{\pi}{4}+dx\middle|2\right)\sqrt{e\tan(c+dx)}}{d\sqrt{\sin(2c+2dx)}} + \frac{2a\cos(c+dx)(e\tan(c+dx))^{3/2}}{de}
\end{aligned}$$

output

```
-1/2*a*arctan(1-2^(1/2)*(e*tan(d*x+c))^(1/2)/e^(1/2))*e^(1/2)/d*2^(1/2)+1/
2*a*arctan(1+2^(1/2)*(e*tan(d*x+c))^(1/2)/e^(1/2))*e^(1/2)/d*2^(1/2)+1/4*a
*ln(e^(1/2)-2^(1/2)*(e*tan(d*x+c))^(1/2)+e^(1/2)*tan(d*x+c))*e^(1/2)/d*2^(
1/2)-1/4*a*ln(e^(1/2)+2^(1/2)*(e*tan(d*x+c))^(1/2)+e^(1/2)*tan(d*x+c))*e^(
1/2)/d*2^(1/2)+2*a*cos(d*x+c)*(sin(c+1/4*Pi+d*x)^2)^(1/2)/sin(c+1/4*Pi+d*x
)*EllipticE(cos(c+1/4*Pi+d*x),2^(1/2))*(e*tan(d*x+c))^(1/2)/d/sin(2*d*x+2*
c)^(1/2)+2*a*cos(d*x+c)*(e*tan(d*x+c))^(3/2)/d/e
```

3.106.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 1.38 (sec) , antiderivative size = 182, normalized size of antiderivative = 0.67

$$\int (a + a\sec(c+dx))\sqrt{e\tan(c+dx)}\,dx =$$

$$-\frac{a(1+\cos(c+dx))\csc(c+dx)\sec^2\left(\frac{1}{2}(c+dx)\right)\sqrt{e\tan(c+dx)}\left(3\sqrt{\sec^2(c+dx)}\left(-4\sin^2(c+dx)+\text{ar}\right.\right.}{}$$

input
```
Integrate[(a + a*Sec[c + d*x])*Sqrt[e*Tan[c + d*x]],x]
```

output
```
-1/12*(a*(1 + Cos[c + d*x])*Csc[c + d*x]*Sec[(c + d*x)/2]^2*Sqrt[e*Tan[c +
 d*x]]*(3*Sqrt[Sec[c + d*x]^2]*(-4*Sin[c + d*x]^2 + ArcSin[Cos[c + d*x] -
Sin[c + d*x]]*Sqrt[Sin[2*(c + d*x)]] + Log[Cos[c + d*x] + Sin[c + d*x] + S
qrt[Sin[2*(c + d*x)]]]*Sqrt[Sin[2*(c + d*x)]]) + 8*Hypergeometric2F1[3/4,
3/2, 7/4, -Tan[c + d*x]^2]*Tan[c + d*x]^2))/(d*Sqrt[Sec[c + d*x]^2])
```

3.106.3 Rubi [A] (warning: unable to verify)

Time = 0.88 (sec) , antiderivative size = 253, normalized size of antiderivative = 0.93, number of steps used = 21, number of rules used = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.870$, Rules used = {3042, 4372, 3042, 3093, 3042, 3095, 3042, 3052, 3042, 3119, 3957, 266, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sec(c+dx)+a)\sqrt{e\tan(c+dx)}\,dx$$

$$\downarrow 3042$$

$$\int \left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)\sqrt{-e\cot\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\downarrow 4372$$

$$a\int \sqrt{e\tan(c+dx)}dx + a\int \sec(c+dx)\sqrt{e\tan(c+dx)}dx$$

$$\downarrow 3042$$

$$a\int \sqrt{e\tan(c+dx)}dx + a\int \sec(c+dx)\sqrt{e\tan(c+dx)}dx$$

$$\downarrow 3093$$

$$a\int \sqrt{e\tan(c+dx)}dx + a\left(\frac{2\cos(c+dx)(e\tan(c+dx))^{3/2}}{de} - 2\int \cos(c+dx)\sqrt{e\tan(c+dx)}dx\right)$$

$$\downarrow 3042$$

$$a\int \sqrt{e\tan(c+dx)}dx + a\left(\frac{2\cos(c+dx)(e\tan(c+dx))^{3/2}}{de} - 2\int \frac{\sqrt{e\tan(c+dx)}}{\sec(c+dx)}dx\right)$$

$$\downarrow 3095$$

$$a\int \sqrt{e\tan(c+dx)}dx +$$
$$a\left(\frac{2\cos(c+dx)(e\tan(c+dx))^{3/2}}{de} - \frac{2\sqrt{\cos(c+dx)}\sqrt{e\tan(c+dx)}\int \sqrt{\cos(c+dx)}\sqrt{\sin(c+dx)}dx}{\sqrt{\sin(c+dx)}}\right)$$

$$\downarrow 3042$$

$$a\int \sqrt{e\tan(c+dx)}dx +$$
$$a\left(\frac{2\cos(c+dx)(e\tan(c+dx))^{3/2}}{de} - \frac{2\sqrt{\cos(c+dx)}\sqrt{e\tan(c+dx)}\int \sqrt{\cos(c+dx)}\sqrt{\sin(c+dx)}dx}{\sqrt{\sin(c+dx)}}\right)$$

$$\downarrow 3052$$

$$a\int \sqrt{e\tan(c+dx)}dx +$$
$$a\left(\frac{2\cos(c+dx)(e\tan(c+dx))^{3/2}}{de} - \frac{2\cos(c+dx)\sqrt{e\tan(c+dx)}\int \sqrt{\sin(2c+2dx)}dx}{\sqrt{\sin(2c+2dx)}}\right)$$

$$\downarrow 3042$$

$$a\int \sqrt{e\tan(c+dx)}dx +$$
$$a\left(\frac{2\cos(c+dx)(e\tan(c+dx))^{3/2}}{de} - \frac{2\cos(c+dx)\sqrt{e\tan(c+dx)}\int \sqrt{\sin(2c+2dx)}dx}{\sqrt{\sin(2c+2dx)}}\right)$$

$$\downarrow 3119$$

$$a\int \sqrt{e\tan(c+dx)}dx +$$
$$a\left(\frac{2\cos(c+dx)(e\tan(c+dx))^{3/2}}{de} - \frac{2\cos(c+dx)E\left(c+dx-\frac{\pi}{4}\middle|2\right)\sqrt{e\tan(c+dx)}}{d\sqrt{\sin(2c+2dx)}}\right)$$

$$\downarrow 3957$$

$$\frac{ae\int\frac{\sqrt{e\tan(c+dx)}}{\tan^2(c+dx)e^2+e^2}d(e\tan(c+dx))}{d}+$$
$$a\left(\frac{2\cos(c+dx)(e\tan(c+dx))^{3/2}}{de}-\frac{2\cos(c+dx)E\left(c+dx-\frac{\pi}{4}\middle|2\right)\sqrt{e\tan(c+dx)}}{d\sqrt{\sin(2c+2dx)}}\right)$$

$$\downarrow 266$$

$$\frac{2ae\int\frac{e^2\tan^2(c+dx)}{e^4\tan^4(c+dx)+e^2}d\sqrt{e\tan(c+dx)}}{d}+$$
$$a\left(\frac{2\cos(c+dx)(e\tan(c+dx))^{3/2}}{de}-\frac{2\cos(c+dx)E\left(c+dx-\frac{\pi}{4}\middle|2\right)\sqrt{e\tan(c+dx)}}{d\sqrt{\sin(2c+2dx)}}\right)$$

$$\downarrow 826$$

$$\frac{2ae\left(\frac{1}{2}\int\frac{e^2\tan^2(c+dx)+e}{e^4\tan^4(c+dx)+e^2}d\sqrt{e\tan(c+dx)}-\frac{1}{2}\int\frac{e-e^2\tan^2(c+dx)}{e^4\tan^4(c+dx)+e^2}d\sqrt{e\tan(c+dx)}\right)}{d}+$$
$$a\left(\frac{2\cos(c+dx)(e\tan(c+dx))^{3/2}}{de}-\frac{2\cos(c+dx)E\left(c+dx-\frac{\pi}{4}\middle|2\right)\sqrt{e\tan(c+dx)}}{d\sqrt{\sin(2c+2dx)}}\right)$$

$$\downarrow 1476$$

$$\frac{2ae\left(\frac{1}{2}\left(\frac{1}{2}\int\frac{1}{e^2\tan^2(c+dx)-\sqrt{2}e^{3/2}\tan(c+dx)+e}d\sqrt{e\tan(c+dx)}+\frac{1}{2}\int\frac{1}{e^2\tan^2(c+dx)+\sqrt{2}e^{3/2}\tan(c+dx)+e}d\sqrt{e\tan(c+dx)}\right)\right.}{d}$$
$$a\left(\frac{2\cos(c+dx)(e\tan(c+dx))^{3/2}}{de}-\frac{2\cos(c+dx)E\left(c+dx-\frac{\pi}{4}\middle|2\right)\sqrt{e\tan(c+dx)}}{d\sqrt{\sin(2c+2dx)}}\right)$$

$$\downarrow 1082$$

$$\frac{2ae\left(\frac{1}{2}\left(\frac{\int\frac{1}{-e^2\tan^2(c+dx)-1}d\left(1-\sqrt{2}\sqrt{e}\tan(c+dx)\right)}{\sqrt{2}\sqrt{e}}-\frac{\int\frac{1}{-e^2\tan^2(c+dx)-1}d\left(\sqrt{2}\sqrt{e}\tan(c+dx)+1\right)}{\sqrt{2}\sqrt{e}}\right)-\frac{1}{2}\int\frac{e-e^2\tan^2(c+dx)}{e^4\tan^4(c+dx)+e^2}d\sqrt{e\tan(\right.}{d}$$
$$a\left(\frac{2\cos(c+dx)(e\tan(c+dx))^{3/2}}{de}-\frac{2\cos(c+dx)E\left(c+dx-\frac{\pi}{4}\middle|2\right)\sqrt{e\tan(c+dx)}}{d\sqrt{\sin(2c+2dx)}}\right)$$

$$\downarrow 217$$

$$\frac{2ae\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{e}\tan(c+dx)+1\right)}{\sqrt{2}\sqrt{e}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{e}\tan(c+dx)\right)}{\sqrt{2}\sqrt{e}}\right)-\frac{1}{2}\int\frac{e-e^2\tan^2(c+dx)}{e^4\tan^4(c+dx)+e^2}d\sqrt{e\tan(c+dx)}\right)}{d}+$$
$$a\left(\frac{2\cos(c+dx)(e\tan(c+dx))^{3/2}}{de}-\frac{2\cos(c+dx)E\left(c+dx-\frac{\pi}{4}\middle|2\right)\sqrt{e\tan(c+dx)}}{d\sqrt{\sin(2c+2dx)}}\right)$$

$$\downarrow 1479$$

$$\frac{2ae\left(\frac{1}{2}\left(\frac{\int -\frac{\sqrt{2}\sqrt{e}-2\sqrt{e\tan(c+dx)}}{e^2\tan^2(c+dx)-\sqrt{2}e^{3/2}\tan(c+dx)+e}d\sqrt{e\tan(c+dx)}}{2\sqrt{2}\sqrt{e}}+\frac{\int -\frac{\sqrt{2}\left(\sqrt{e}+\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{e^2\tan^2(c+dx)+\sqrt{2}e^{3/2}\tan(c+dx)+e}d\sqrt{e\tan(c+dx)}}{2\sqrt{2}\sqrt{e}}\right)+\frac{1}{2}\left(\arctan\left(\sqrt{2}\right.\right.\right.}{a\left(\frac{2\cos(c+dx)(e\tan(c+dx))^{3/2}}{de}-\frac{2\cos(c+dx)E\left(c+dx-\frac{\pi}{4}\middle|2\right)\sqrt{e\tan(c+dx)}}{d\sqrt{\sin(2c+2dx)}}\right)}$$

$$\downarrow 25$$

$$\frac{2ae\left(\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}\sqrt{e}-2\sqrt{e\tan(c+dx)}}{e^2\tan^2(c+dx)-\sqrt{2}e^{3/2}\tan(c+dx)+e}d\sqrt{e\tan(c+dx)}}{2\sqrt{2}\sqrt{e}}-\frac{\int \frac{\sqrt{2}\left(\sqrt{e}+\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{e^2\tan^2(c+dx)+\sqrt{2}e^{3/2}\tan(c+dx)+e}d\sqrt{e\tan(c+dx)}}{2\sqrt{2}\sqrt{e}}\right)+\frac{1}{2}\left(\arctan\left(\sqrt{2}\sqrt{\right.\right.\right.}{a\left(\frac{2\cos(c+dx)(e\tan(c+dx))^{3/2}}{de}-\frac{2\cos(c+dx)E\left(c+dx-\frac{\pi}{4}\middle|2\right)\sqrt{e\tan(c+dx)}}{d\sqrt{\sin(2c+2dx)}}\right)}$$

$$\downarrow 27$$

$$\frac{2ae\left(\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}\sqrt{e}-2\sqrt{e\tan(c+dx)}}{e^2\tan^2(c+dx)-\sqrt{2}e^{3/2}\tan(c+dx)+e}d\sqrt{e\tan(c+dx)}}{2\sqrt{2}\sqrt{e}}-\frac{\int \frac{\sqrt{e}+\sqrt{2}\sqrt{e\tan(c+dx)}}{e^2\tan^2(c+dx)+\sqrt{2}e^{3/2}\tan(c+dx)+e}d\sqrt{e\tan(c+dx)}}{2\sqrt{e}}\right)+\frac{1}{2}\left(\arctan\left(\sqrt{2}\sqrt{\right.\right.\right.}{a\left(\frac{2\cos(c+dx)(e\tan(c+dx))^{3/2}}{de}-\frac{2\cos(c+dx)E\left(c+dx-\frac{\pi}{4}\middle|2\right)\sqrt{e\tan(c+dx)}}{d\sqrt{\sin(2c+2dx)}}\right)}$$

$$\downarrow 1103$$

$$\frac{2ae\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{e}\tan(c+dx)+1\right)}{\sqrt{2}\sqrt{e}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{e}\tan(c+dx)\right)}{\sqrt{2}\sqrt{e}}\right)+\frac{1}{2}\left(\frac{\log\left(-\sqrt{2}e^{3/2}\tan(c+dx)+e^2\tan^2(c+dx)+e\right)}{2\sqrt{2}\sqrt{e}}-\frac{\log\left(\sqrt{2}e^{3/}\right.\right.\right.}{a\left(\frac{2\cos(c+dx)(e\tan(c+dx))^{3/2}}{de}-\frac{2\cos(c+dx)E\left(c+dx-\frac{\pi}{4}\middle|2\right)\sqrt{e\tan(c+dx)}}{d\sqrt{\sin(2c+2dx)}}\right)}$$

input
```
Int[(a + a*Sec[c + d*x])*Sqrt[e*Tan[c + d*x]],x]
```

output
```
(2*a*e*((-(ArcTan[1 - Sqrt[2]*Sqrt[e]*Tan[c + d*x]]/(Sqrt[2]*Sqrt[e])) + A
rcTan[1 + Sqrt[2]*Sqrt[e]*Tan[c + d*x]]/(Sqrt[2]*Sqrt[e]))/2 + (Log[e - Sq
rt[2]*e^(3/2)*Tan[c + d*x] + e^2*Tan[c + d*x]^2]/(2*Sqrt[2]*Sqrt[e]) - Log
[e + Sqrt[2]*e^(3/2)*Tan[c + d*x] + e^2*Tan[c + d*x]^2]/(2*Sqrt[2]*Sqrt[e]
))/2))/d + a*((-2*Cos[c + d*x]*EllipticE[c - Pi/4 + d*x, 2]*Sqrt[e*Tan[c +
 d*x]])/(d*Sqrt[Sin[2*c + 2*d*x]]) + (2*Cos[c + d*x]*(e*Tan[c + d*x])^(3/2
))/(d*e))
```

3.106.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3052
```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Simp[Sqrt[a*Sin[e + f*x]]*(Sqrt[b*Cos[e + f*x]]/Sqrt[Sin[2*e
 + 2*f*x]])   Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

rule 3093
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[a^2*(a*Sec[e + f*x])^(m - 2)*((b*Tan[e + f*x])^(n +
1)/(b*f*(m + n - 1))), x] + Simp[a^2*((m - 2)/(m + n - 1))   Int[(a*Sec[e +
 f*x])^(m - 2)*(b*Tan[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (
GtQ[m, 1] || (EqQ[m, 1] && EqQ[n, 1/2])) && NeQ[m + n - 1, 0] && IntegersQ[
2*m, 2*n]
```

rule 3095
```
Int[Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]]/sec[(e_.) + (f_.)*(x_)], x_Symbol]
:> Simp[Sqrt[Cos[e + f*x]]*(Sqrt[b*Tan[e + f*x]]/Sqrt[Sin[e + f*x]])   Int[
Sqrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]], x], x] /; FreeQ[{b, e, f}, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

rule 4372
```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(e*Cot[c + d*x])^m, x], x] + Simp[b   Int[
(e*Cot[c + d*x])^m*Csc[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, m}, x]
```

3.106.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 504 vs. $2(244) = 488$.

Time = 5.04 (sec) , antiderivative size = 505, normalized size of antiderivative = 1.86

method	result
parts	$\frac{ae\sqrt{2}\left(\ln\left(\frac{e\tan(dx+c)-\left(e^2\right)^{\frac{1}{4}}\sqrt{e\tan(dx+c)}\sqrt{2}+\sqrt{e^2}}{e\tan(dx+c)+\left(e^2\right)^{\frac{1}{4}}\sqrt{e\tan(dx+c)}\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{e\tan(dx+c)}}{\left(e^2\right)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\frac{\sqrt{2}\sqrt{e\tan(dx+c)}}{\left(e^2\right)^{\frac{1}{4}}}+1\right)\right)}{4d\left(e^2\right)^{\frac{1}{4}}} - \frac{a\sqrt{}}{}$
default	Expression too large to display

input
```
int((a+a*sec(d*x+c))*(e*tan(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/4*a/d*e/(e^2)^(1/4)*2^(1/2)*(ln((e*tan(d*x+c)-(e^2)^(1/4)*(e*tan(d*x+c))
^(1/2)*2^(1/2)+(e^2)^(1/2))/(e*tan(d*x+c)+(e^2)^(1/4)*(e*tan(d*x+c))^(1/2)
*2^(1/2)+(e^2)^(1/2)))+2*arctan(2^(1/2)/(e^2)^(1/4)*(e*tan(d*x+c))^(1/2)+1
)-2*arctan(-2^(1/2)/(e^2)^(1/4)*(e*tan(d*x+c))^(1/2)+1))-a/d*2^(1/2)*(-2*(
csc(d*x+c)-cot(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c)+1)^(1/2)*(cot(d*x+c)
-csc(d*x+c))^(1/2)*EllipticE((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2*2^(1/2))*
cos(d*x+c)+(csc(d*x+c)-cot(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c)+1)^(1/2)
*(cot(d*x+c)-csc(d*x+c))^(1/2)*EllipticF((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1
/2*2^(1/2))*cos(d*x+c)-2*(csc(d*x+c)-cot(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d
*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)*EllipticE((csc(d*x+c)-cot(d*x
+c)+1)^(1/2),1/2*2^(1/2))+(csc(d*x+c)-cot(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(
d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)*EllipticF((csc(d*x+c)-cot(d*
x+c)+1)^(1/2),1/2*2^(1/2))+2^(1/2)*cos(d*x+c)-2^(1/2))*(e*tan(d*x+c))^(1/2
)*csc(d*x+c)
```

3.106.5 Fricas [F(-1)]

Timed out.

$$\int (a + a\sec(c + dx))\sqrt{e\tan(c + dx)}\,dx = \text{Timed out}$$

input
```
integrate((a+a*sec(d*x+c))*(e*tan(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.106.6 Sympy [F]

$$\int (a + a\sec(c + dx))\sqrt{e\tan(c + dx)}\,dx = a\left(\int \sqrt{e\tan(c + dx)}\,dx + \int \sqrt{e\tan(c + dx)}\sec(c + dx)\,dx\right)$$

input
```
integrate((a+a*sec(d*x+c))*(e*tan(d*x+c))**(1/2),x)
```

output
```
a*(Integral(sqrt(e*tan(c + d*x)), x) + Integral(sqrt(e*tan(c + d*x))*sec(c
 + d*x), x))
```

3.106.7 Maxima [F(-2)]

Exception generated.

$$\int (a + a\sec(c + dx))\sqrt{e\tan(c + dx)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+a*sec(d*x+c))*(e*tan(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.106.8 Giac [F]

$$\int (a + a\sec(c + dx))\sqrt{e\tan(c + dx)}\,dx = \int (a\sec(dx + c) + a)\sqrt{e\tan(dx + c)}\,dx$$

input
```
integrate((a+a*sec(d*x+c))*(e*tan(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate((a*sec(d*x + c) + a)*sqrt(e*tan(d*x + c)), x)
```

3.106.9 Mupad [F(-1)]

Timed out.

$$\int (a + a\sec(c + dx))\sqrt{e\tan(c + dx)}\,dx = \int \sqrt{e\tan(c + d\,x)}\left(a + \frac{a}{\cos(c + d\,x)}\right)dx$$

input
```
int((e*tan(c + d*x))^(1/2)*(a + a/cos(c + d*x)),x)
```

output
```
int((e*tan(c + d*x))^(1/2)*(a + a/cos(c + d*x)), x)
```

3.107 $\int \frac{a+a\sec(c+dx)}{\sqrt{e\tan(c+dx)}}\,dx$

3.107.1 Optimal result . . . 789
3.107.2 Mathematica [C] (warning: unable to verify) . . . 790
3.107.3 Rubi [A] (warning: unable to verify) . . . 790
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3.107.5 Fricas [F(-1)] . . . 796
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3.107.8 Giac [F] . . . 797
3.107.9 Mupad [F(-1)] . . . 797

3.107.1 Optimal result

Integrand size = 23, antiderivative size = 244

$$\begin{aligned}\int \frac{a+a\sec(c+dx)}{\sqrt{e\tan(c+dx)}}\,dx = &-\frac{a\arctan\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}d\sqrt{e}}+\frac{a\arctan\left(1+\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}d\sqrt{e}}\\ &-\frac{a\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}d\sqrt{e}}\\ &+\frac{a\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)+\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}d\sqrt{e}}\\ &+\frac{a\,\text{EllipticF}\left(c-\frac{\pi}{4}+dx,2\right)\sec(c+dx)\sqrt{\sin(2c+2dx)}}{d\sqrt{e\tan(c+dx)}}\end{aligned}$$

output

```
-1/2*a*arctan(1-2^(1/2)*(e*tan(d*x+c))^(1/2)/e^(1/2))/d*2^(1/2)/e^(1/2)+1/
2*a*arctan(1+2^(1/2)*(e*tan(d*x+c))^(1/2)/e^(1/2))/d*2^(1/2)/e^(1/2)-1/4*a
*ln(e^(1/2)-2^(1/2)*(e*tan(d*x+c))^(1/2)+e^(1/2)*tan(d*x+c))/d*2^(1/2)/e^(
1/2)+1/4*a*ln(e^(1/2)+2^(1/2)*(e*tan(d*x+c))^(1/2)+e^(1/2)*tan(d*x+c))/d*2
^(1/2)/e^(1/2)-a*(sin(c+1/4*Pi+d*x)^2)^(1/2)/sin(c+1/4*Pi+d*x)*EllipticF(c
os(c+1/4*Pi+d*x),2^(1/2))*sec(d*x+c)*sin(2*d*x+2*c)^(1/2)/d/(e*tan(d*x+c))
^(1/2)
```

3.107.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 0.87 (sec) , antiderivative size = 220, normalized size of antiderivative = 0.90

$$\int \frac{a + a\sec(c+dx)}{\sqrt{e\tan(c+dx)}}\,dx$$

$$= \frac{20a\,\text{AppellF1}\left(\frac{1}{4},\frac{1}{2},1,\frac{5}{4},\tan^2\left(\frac{1}{2}(c+\right.\right.}{d\left(2\left(2\,\text{AppellF1}\left(\frac{5}{4},\frac{1}{2},2,\frac{9}{4},\tan^2\left(\frac{1}{2}(c+dx)\right),-\tan^2\left(\frac{1}{2}(c+dx)\right)\right)\right)-\text{AppellF1}\left(\frac{5}{4},\frac{3}{2},1,\frac{9}{4},\tan^2\left(\frac{1}{2}(c+dx)\right)\right.\right.}$$

input
```
Integrate[(a + a*Sec[c + d*x])/Sqrt[e*Tan[c + d*x]],x]
```

output
```
(20*a*AppellF1[1/4, 1/2, 1, 5/4, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*
Cos[(c + d*x)/2]^2*(1 + Sec[c + d*x])*Sin[c + d*x])/(d*(2*(2*AppellF1[5/4,
 1/2, 2, 9/4, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] - AppellF1[5/4, 3/2
, 1, 9/4, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*(-1 + Cos[c + d*x]) +
5*AppellF1[1/4, 1/2, 1, 5/4, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*(1 +
 Cos[c + d*x]))*Sqrt[e*Tan[c + d*x]])
```

3.107.3 Rubi [A] (warning: unable to verify)

Time = 0.76 (sec) , antiderivative size = 230, normalized size of antiderivative = 0.94, number of steps used = 19, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.783$, Rules used = {3042, 4372, 3042, 3094, 3042, 3053, 3042, 3120, 3957, 266, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a\sec(c+dx)+a}{\sqrt{e\tan(c+dx)}}\,dx$$

$$\downarrow 3042$$

$$\int \frac{a\csc\left(c+dx+\frac{\pi}{2}\right)+a}{\sqrt{-e\cot\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\downarrow 4372$$

$$a\int \frac{1}{\sqrt{e\tan(c+dx)}}dx + a\int \frac{\sec(c+dx)}{\sqrt{e\tan(c+dx)}}dx$$

$$\downarrow 3042$$

$$a\int \frac{1}{\sqrt{e\tan(c+dx)}}dx + a\int \frac{\sec(c+dx)}{\sqrt{e\tan(c+dx)}}dx$$

$$\downarrow 3094$$

$$a\int \frac{1}{\sqrt{e\tan(c+dx)}}dx + \frac{a\sqrt{\sin(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{\sin(c+dx)}}dx}{\sqrt{\cos(c+dx)}\sqrt{e\tan(c+dx)}}$$

$$\downarrow 3042$$

$$a\int \frac{1}{\sqrt{e\tan(c+dx)}}dx + \frac{a\sqrt{\sin(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{\sin(c+dx)}}dx}{\sqrt{\cos(c+dx)}\sqrt{e\tan(c+dx)}}$$

$$\downarrow 3053$$

$$a\int \frac{1}{\sqrt{e\tan(c+dx)}}dx + \frac{a\sqrt{\sin(2c+2dx)}\sec(c+dx)\int \frac{1}{\sqrt{\sin(2c+2dx)}}dx}{\sqrt{e\tan(c+dx)}}$$

$$\downarrow 3042$$

$$a\int \frac{1}{\sqrt{e\tan(c+dx)}}dx + \frac{a\sqrt{\sin(2c+2dx)}\sec(c+dx)\int \frac{1}{\sqrt{\sin(2c+2dx)}}dx}{\sqrt{e\tan(c+dx)}}$$

$$\downarrow 3120$$

$$a\int \frac{1}{\sqrt{e\tan(c+dx)}}dx + \frac{a\sqrt{\sin(2c+2dx)}\sec(c+dx)\,\text{EllipticF}\left(c+dx-\frac{\pi}{4},2\right)}{d\sqrt{e\tan(c+dx)}}$$

$$\downarrow 3957$$

$$\frac{ae\int \frac{1}{\sqrt{e\tan(c+dx)}(\tan^2(c+dx)e^2+e^2)}d(e\tan(c+dx))}{d} + \frac{a\sqrt{\sin(2c+2dx)}\sec(c+dx)\,\text{EllipticF}\left(c+dx-\frac{\pi}{4},2\right)}{d\sqrt{e\tan(c+dx)}}$$

$$\downarrow 266$$

$$\frac{2ae\int \frac{1}{e^4\tan^4(c+dx)+e^2}d\sqrt{e\tan(c+dx)}}{d} + \frac{a\sqrt{\sin(2c+2dx)}\sec(c+dx)\,\text{EllipticF}\left(c+dx-\frac{\pi}{4},2\right)}{d\sqrt{e\tan(c+dx)}}$$

$$\downarrow 755$$

$$\frac{2ae\left(\frac{\int \frac{e-e^2\tan^2(c+dx)}{e^4\tan^4(c+dx)+e^2}d\sqrt{e\tan(c+dx)}}{2e}+\frac{\int \frac{e^2\tan^2(c+dx)+e}{e^4\tan^4(c+dx)+e^2}d\sqrt{e\tan(c+dx)}}{2e}\right)}{\frac{a\sqrt{\sin(2c+2dx)}\sec(c+dx)\,\text{EllipticF}\left(c+dx-\frac{\pi}{4},2\right)}{d\sqrt{e\tan(c+dx)}}}+$$

$$\downarrow 1476$$

$$\frac{2ae\left(\frac{\frac{1}{2}\int \frac{1}{e^2\tan^2(c+dx)-\sqrt{2}e^{3/2}\tan(c+dx)+e}d\sqrt{e\tan(c+dx)}+\frac{1}{2}\int \frac{1}{e^2\tan^2(c+dx)+\sqrt{2}e^{3/2}\tan(c+dx)+e}d\sqrt{e\tan(c+dx)}}{2e}+\frac{\int \frac{e-e^2\tan^2(c+dx)}{e^4\tan^4(c+dx)+e^2}d\sqrt{e\t}}{2e}\right)}{\frac{a\sqrt{\sin(2c+2dx)}\sec(c+dx)\,\text{EllipticF}\left(c+dx-\frac{\pi}{4},2\right)}{d\sqrt{e\tan(c+dx)}}}$$

$$\downarrow 1082$$

$$\frac{2ae\left(\frac{\frac{\int \frac{1}{-e^2\tan^2(c+dx)-1}d\left(1-\sqrt{2}\sqrt{e}\tan(c+dx)\right)}{\sqrt{2}\sqrt{e}}-\frac{\int \frac{1}{-e^2\tan^2(c+dx)-1}d\left(\sqrt{2}\sqrt{e}\tan(c+dx)+1\right)}{\sqrt{2}\sqrt{e}}}{2e}+\frac{\int \frac{e-e^2\tan^2(c+dx)}{e^4\tan^4(c+dx)+e^2}d\sqrt{e\tan(c+dx)}}{2e}\right)}{\frac{a\sqrt{\sin(2c+2dx)}\sec(c+dx)\,\text{EllipticF}\left(c+dx-\frac{\pi}{4},2\right)}{d\sqrt{e\tan(c+dx)}}}+$$

$$\downarrow 217$$

$$\frac{2ae\left(\frac{\int \frac{e-e^2\tan^2(c+dx)}{e^4\tan^4(c+dx)+e^2}d\sqrt{e\tan(c+dx)}}{2e}+\frac{\frac{\arctan\left(\sqrt{2}\sqrt{e}\tan(c+dx)+1\right)}{\sqrt{2}\sqrt{e}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{e}\tan(c+dx)\right)}{\sqrt{2}\sqrt{e}}}{2e}\right)}{\frac{a\sqrt{\sin(2c+2dx)}\sec(c+dx)\,\text{EllipticF}\left(c+dx-\frac{\pi}{4},2\right)}{d\sqrt{e\tan(c+dx)}}}+$$

$$\downarrow 1479$$

$$\frac{2ae\left(\frac{-\frac{\int -\frac{\sqrt{2}\sqrt{e}-2\sqrt{e\tan(c+dx)}}{e^2\tan^2(c+dx)-\sqrt{2}e^{3/2}\tan(c+dx)+e}d\sqrt{e\tan(c+dx)}}{2\sqrt{2}\sqrt{e}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{e}+\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{e^2\tan^2(c+dx)+\sqrt{2}e^{3/2}\tan(c+dx)+e}d\sqrt{e\tan(c+dx)}}{2\sqrt{2}\sqrt{e}}}{2e}+\frac{\arctan\left(\sqrt{2}\sqrt{e}\tan(c+dx)+1\right)}{\sqrt{2}\sqrt{e}}-}{2}\right)}{\frac{a\sqrt{\sin(2c+2dx)}\sec(c+dx)\,\text{EllipticF}\left(c+dx-\frac{\pi}{4},2\right)}{d\sqrt{e\tan(c+dx)}}}$$

$$\downarrow 25$$

3.107. $\int \frac{a+a\sec(c+dx)}{\sqrt{e\tan(c+dx)}}\,dx$

$$\frac{2ae\left(\frac{\int \frac{\sqrt{2}\sqrt{e}-2\sqrt{e\tan(c+dx)}}{e^2\tan^2(c+dx)-\sqrt{2}e^{3/2}\tan(c+dx)+e}d\sqrt{e\tan(c+dx)}}{2\sqrt{2}\sqrt{e}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{e}+\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{e^2\tan^2(c+dx)+\sqrt{2}e^{3/2}\tan(c+dx)+e}d\sqrt{e\tan(c+dx)}}{2\sqrt{2}\sqrt{e}}}{2e}+\frac{\frac{\arctan\left(\sqrt{2}\sqrt{e}\tan(c+dx)+1\right)}{\sqrt{2}\sqrt{e}}-\frac{\arcta}{}}{2e}\right)}{\frac{a\sqrt{\sin(2c+2dx)}\sec(c+dx)\,\text{EllipticF}\left(c+dx-\frac{\pi}{4},2\right)}{d\sqrt{e\tan(c+dx)}}}$$

$$\downarrow 27$$

$$\frac{2ae\left(\frac{\int \frac{\sqrt{2}\sqrt{e}-2\sqrt{e\tan(c+dx)}}{e^2\tan^2(c+dx)-\sqrt{2}e^{3/2}\tan(c+dx)+e}d\sqrt{e\tan(c+dx)}}{2\sqrt{2}\sqrt{e}}+\frac{\int \frac{\sqrt{e}+\sqrt{2}\sqrt{e\tan(c+dx)}}{e^2\tan^2(c+dx)+\sqrt{2}e^{3/2}\tan(c+dx)+e}d\sqrt{e\tan(c+dx)}}{2\sqrt{e}}}{2e}+\frac{\frac{\arctan\left(\sqrt{2}\sqrt{e}\tan(c+dx)+1\right)}{\sqrt{2}\sqrt{e}}-\frac{\arcta}{}}{2e}\right)}{\frac{a\sqrt{\sin(2c+2dx)}\sec(c+dx)\,\text{EllipticF}\left(c+dx-\frac{\pi}{4},2\right)}{d\sqrt{e\tan(c+dx)}}}$$

$$\downarrow 1103$$

$$\frac{2ae\left(\frac{\arctan\left(\sqrt{2}\sqrt{e}\tan(c+dx)+1\right)}{\sqrt{2}\sqrt{e}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{e}\tan(c+dx)\right)}{\sqrt{2}\sqrt{e}}}{2e}+\frac{\frac{\log\left(\sqrt{2}e^{3/2}\tan(c+dx)+e^2\tan^2(c+dx)+e\right)}{2\sqrt{2}\sqrt{e}}-\frac{\log\left(-\sqrt{2}e^{3/2}\tan(c+dx)+e^2\tan^2(c+dx)+e\right.}{2\sqrt{2}\sqrt{e}}}{2e}\right)}{\frac{a\sqrt{\sin(2c+2dx)}\sec(c+dx)\,\text{EllipticF}\left(c+dx-\frac{\pi}{4},2\right)}{d\sqrt{e\tan(c+dx)}}}$$

input
```
Int[(a + a*Sec[c + d*x])/Sqrt[e*Tan[c + d*x]],x]
```

output
```
(2*a*e*((-(ArcTan[1 - Sqrt[2]*Sqrt[e]*Tan[c + d*x]]/(Sqrt[2]*Sqrt[e])) + A
rcTan[1 + Sqrt[2]*Sqrt[e]*Tan[c + d*x]]/(Sqrt[2]*Sqrt[e]))/(2*e) + (-1/2*L
og[e - Sqrt[2]*e^(3/2)*Tan[c + d*x] + e^2*Tan[c + d*x]^2]/(Sqrt[2]*Sqrt[e]
) + Log[e + Sqrt[2]*e^(3/2)*Tan[c + d*x] + e^2*Tan[c + d*x]^2]/(2*Sqrt[2]*
Sqrt[e]))/(2*e)))/d + (a*EllipticF[c - Pi/4 + d*x, 2]*Sec[c + d*x]*Sqrt[Si
n[2*c + 2*d*x]])/(d*Sqrt[e*Tan[c + d*x]])
```

3.107.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3053
```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Simp[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]])   Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f
}, x]
```

rule 3094
```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]], x_Symbol]
:> Simp[Sqrt[Sin[e + f*x]]/(Sqrt[Cos[e + f*x]]*Sqrt[b*Tan[e + f*x]])   Int[
1/(Sqrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]]), x], x] /; FreeQ[{b, e, f}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

rule 4372
```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(e*Cot[c + d*x])^m, x], x] + Simp[b   Int[
(e*Cot[c + d*x])^m*Csc[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, m}, x]
```

3.107.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 4.42 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.58

method	result
default	$\frac{\left(\frac{1}{2}-\frac{i}{2}\right)a\sqrt{2}\left(i\,\text{EllipticPi}\left(\sqrt{\csc(dx+c)-\cot(dx+c)+1},\frac{1}{2}-\frac{i}{2},\frac{\sqrt{2}}{2}\right)+\text{EllipticPi}\left(\sqrt{\csc(dx+c)-\cot(dx+c)+1},\frac{1}{2}+\frac{i}{2},\frac{\sqrt{2}}{2}\right)\right)\sqrt{\cot(dx+c)-}}{d\sqrt{e\tan(dx+c)}}$
parts	$\frac{a(e^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\tan(dx+c)+(e^2)^{\frac{1}{4}}\sqrt{e\tan(dx+c)}\sqrt{2}+\sqrt{e^2}}{e\tan(dx+c)-(e^2)^{\frac{1}{4}}\sqrt{e\tan(dx+c)}\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{e\tan(dx+c)}}{(e^2)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\frac{\sqrt{2}\sqrt{e\tan(dx+c)}}{(e^2)^{\frac{1}{4}}}+1\right)\right)}{4de}$

input
```
int((a+a*sec(d*x+c))/(e*tan(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(1/2-1/2*I)*a/d*2^(1/2)*(I*EllipticPi((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2-
1/2*I,1/2*2^(1/2))+EllipticPi((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2+1/2*I,1/
2*2^(1/2)))*(cot(d*x+c)-csc(d*x+c))^(1/2)*(cot(d*x+c)-csc(d*x+c)+1)^(1/2)*
(csc(d*x+c)-cot(d*x+c)+1)^(1/2)/(e*tan(d*x+c))^(1/2)*(1+sec(d*x+c))
```

3.107.5 Fricas [F(-1)]

Timed out.

$$\int \frac{a+a\sec(c+dx)}{\sqrt{e\tan(c+dx)}}\,dx = \text{Timed out}$$

input
```
integrate((a+a*sec(d*x+c))/(e*tan(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.107.6 Sympy [F]

$$\int \frac{a+a\sec(c+dx)}{\sqrt{e\tan(c+dx)}}\,dx = a\left(\int \frac{1}{\sqrt{e\tan(c+dx)}}\,dx + \int \frac{\sec(c+dx)}{\sqrt{e\tan(c+dx)}}\,dx\right)$$

input
```
integrate((a+a*sec(d*x+c))/(e*tan(d*x+c))**(1/2),x)
```

output
```
a*(Integral(1/sqrt(e*tan(c + d*x)), x) + Integral(sec(c + d*x)/sqrt(e*tan(
c + d*x)), x))
```

3.107.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{a + a\sec(c+dx)}{\sqrt{e\tan(c+dx)}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+a*sec(d*x+c))/(e*tan(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.107.8 Giac [F]

$$\int \frac{a + a\sec(c+dx)}{\sqrt{e\tan(c+dx)}}\,dx = \int \frac{a\sec(dx+c) + a}{\sqrt{e\tan(dx+c)}}\,dx$$

input
```
integrate((a+a*sec(d*x+c))/(e*tan(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate((a*sec(d*x + c) + a)/sqrt(e*tan(d*x + c)), x)
```

3.107.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a + a\sec(c+dx)}{\sqrt{e\tan(c+dx)}}\,dx = \int \frac{a + \frac{a}{\cos(c+dx)}}{\sqrt{e\tan(c+dx)}}\,dx$$

input
```
int((a + a/cos(c + d*x))/(e*tan(c + d*x))^(1/2),x)
```

output
```
int((a + a/cos(c + d*x))/(e*tan(c + d*x))^(1/2), x)
```

3.108 $\int \frac{a+a\sec(c+dx)}{(e\tan(c+dx))^{3/2}}\,dx$

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3.108.8 Giac [F] . . . 807
3.108.9 Mupad [F(-1)] . . . 808

3.108.1 Optimal result

Integrand size = 23, antiderivative size = 305

$$\begin{aligned}\int \frac{a+a\sec(c+dx)}{(e\tan(c+dx))^{3/2}}\,dx &= \frac{a\arctan\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}de^{3/2}}\\ &-\frac{a\arctan\left(1+\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}de^{3/2}}\\ &-\frac{a\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}de^{3/2}}\\ &+\frac{a\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)+\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}de^{3/2}}\\ &-\frac{2(a+a\sec(c+dx))}{de\sqrt{e\tan(c+dx)}}-\frac{2a\cos(c+dx)E\left(c-\frac{\pi}{4}+dx\middle|2\right)\sqrt{e\tan(c+dx)}}{de^2\sqrt{\sin(2c+2dx)}}\\ &+\frac{2a\cos(c+dx)(e\tan(c+dx))^{3/2}}{de^3}\end{aligned}$$

output

```
1/2*a*arctan(1-2^(1/2)*(e*tan(d*x+c))^(1/2)/e^(1/2))/d/e^(3/2)*2^(1/2)-1/2
*a*arctan(1+2^(1/2)*(e*tan(d*x+c))^(1/2)/e^(1/2))/d/e^(3/2)*2^(1/2)-1/4*a*
ln(e^(1/2)-2^(1/2)*(e*tan(d*x+c))^(1/2)+e^(1/2)*tan(d*x+c))/d/e^(3/2)*2^(1
/2)+1/4*a*ln(e^(1/2)+2^(1/2)*(e*tan(d*x+c))^(1/2)+e^(1/2)*tan(d*x+c))/d/e^
(3/2)*2^(1/2)-2*(a+a*sec(d*x+c))/d/e/(e*tan(d*x+c))^(1/2)+2*a*cos(d*x+c)*(
sin(c+1/4*Pi+d*x)^2)^(1/2)/sin(c+1/4*Pi+d*x)*EllipticE(cos(c+1/4*Pi+d*x),2
^(1/2))*(e*tan(d*x+c))^(1/2)/d/e^2/sin(2*d*x+2*c)^(1/2)+2*a*cos(d*x+c)*(e*
tan(d*x+c))^(3/2)/d/e^3
```

3.108.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 2.63 (sec) , antiderivative size = 196, normalized size of antiderivative = 0.64

$$\int \frac{a + a\sec(c+dx)}{(e\tan(c+dx))^{3/2}}\,dx =$$

$$-\frac{a(1+\cos(c+dx))\csc(c+dx)\sec^2\left(\frac{1}{2}(c+dx)\right)\sqrt{e\tan(c+dx)}\left(3\sqrt{\sec^2(c+dx)}\left(2+4\cos(c+dx)+2\cos\right.\right.}{}$$

input
```
Integrate[(a + a*Sec[c + d*x])/(e*Tan[c + d*x])^(3/2),x]
```

output
```
-1/12*(a*(1 + Cos[c + d*x])*Csc[c + d*x]*Sec[(c + d*x)/2]^2*Sqrt[e*Tan[c +
 d*x]]*(3*Sqrt[Sec[c + d*x]^2]*(2 + 4*Cos[c + d*x] + 2*Cos[2*(c + d*x)] -
ArcSin[Cos[c + d*x] - Sin[c + d*x]]*Sqrt[Sin[2*(c + d*x)]] - Log[Cos[c + d
*x] + Sin[c + d*x] + Sqrt[Sin[2*(c + d*x)]]]*Sqrt[Sin[2*(c + d*x)]]) + 8*H
ypergeometric2F1[3/4, 3/2, 7/4, -Tan[c + d*x]^2]*Tan[c + d*x]^2))/(d*e^2*S
qrt[Sec[c + d*x]^2])
```

3.108.3 Rubi [A] (warning: unable to verify)

Time = 1.12 (sec) , antiderivative size = 290, normalized size of antiderivative = 0.95, number of steps used = 24, number of rules used = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 1.000$, Rules used = {3042, 4370, 27, 3042, 4372, 3042, 3093, 3042, 3095, 3042, 3052, 3042, 3119, 3957, 266, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a\sec(c+dx)+a}{(e\tan(c+dx))^{3/2}}\,dx$$

$$\downarrow 3042$$

$$\int \frac{a\csc\left(c+dx+\frac{\pi}{2}\right)+a}{\left(-e\cot\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx$$

$$\downarrow 4370$$

$$\frac{2\int -\frac{1}{2}(a-a\sec(c+dx))\sqrt{e\tan(c+dx)}dx}{e^2} - \frac{2(a\sec(c+dx)+a)}{de\sqrt{e\tan(c+dx)}}$$

$$\downarrow 27$$

$$-\frac{\int (a-a\sec(c+dx))\sqrt{e\tan(c+dx)}dx}{e^2}-\frac{2(a\sec(c+dx)+a)}{de\sqrt{e\tan(c+dx)}}$$

$$\downarrow 3042$$

$$-\frac{\int \sqrt{-e\cot\left(c+dx+\frac{\pi}{2}\right)}\left(a-a\csc\left(c+dx+\frac{\pi}{2}\right)\right)dx}{e^2}-\frac{2(a\sec(c+dx)+a)}{de\sqrt{e\tan(c+dx)}}$$

$$\downarrow 4372$$

$$-\frac{a\int \sqrt{e\tan(c+dx)}dx-a\int \sec(c+dx)\sqrt{e\tan(c+dx)}dx}{e^2}-\frac{2(a\sec(c+dx)+a)}{de\sqrt{e\tan(c+dx)}}$$

$$\downarrow 3042$$

$$-\frac{a\int \sqrt{e\tan(c+dx)}dx-a\int \sec(c+dx)\sqrt{e\tan(c+dx)}dx}{e^2}-\frac{2(a\sec(c+dx)+a)}{de\sqrt{e\tan(c+dx)}}$$

$$\downarrow 3093$$

$$-\frac{a\int \sqrt{e\tan(c+dx)}dx-a\left(\frac{2\cos(c+dx)(e\tan(c+dx))^{3/2}}{de}-2\int \cos(c+dx)\sqrt{e\tan(c+dx)}dx\right)}{e^2}-\frac{2(a\sec(c+dx)+a)}{de\sqrt{e\tan(c+dx)}}$$

$$\downarrow 3042$$

$$-\frac{a\int \sqrt{e\tan(c+dx)}dx-a\left(\frac{2\cos(c+dx)(e\tan(c+dx))^{3/2}}{de}-2\int \frac{\sqrt{e\tan(c+dx)}}{\sec(c+dx)}dx\right)}{e^2}-\frac{2(a\sec(c+dx)+a)}{de\sqrt{e\tan(c+dx)}}$$

$$\downarrow 3095$$

$$-\frac{a\int \sqrt{e\tan(c+dx)}dx-a\left(\frac{2\cos(c+dx)(e\tan(c+dx))^{3/2}}{de}-\frac{2\sqrt{\cos(c+dx)}\sqrt{e\tan(c+dx)}\int \sqrt{\cos(c+dx)}\sqrt{\sin(c+dx)}dx}{\sqrt{\sin(c+dx)}}\right)}{e^2}-\frac{2(a\sec(c+dx)+a)}{de\sqrt{e\tan(c+dx)}}$$

$$\downarrow 3042$$

$$-\frac{a\int \sqrt{e\tan(c+dx)}dx-a\left(\frac{2\cos(c+dx)(e\tan(c+dx))^{3/2}}{de}-\frac{2\sqrt{\cos(c+dx)}\sqrt{e\tan(c+dx)}\int \sqrt{\cos(c+dx)}\sqrt{\sin(c+dx)}dx}{\sqrt{\sin(c+dx)}}\right)}{e^2}-\frac{2(a\sec(c+dx)+a)}{de\sqrt{e\tan(c+dx)}}$$

$$\downarrow 3052$$

$$-\frac{a\int\sqrt{e\tan(c+dx)}dx-a\left(\frac{2\cos(c+dx)(e\tan(c+dx))^{3/2}}{de}-\frac{2\cos(c+dx)\sqrt{e\tan(c+dx)}\int\sqrt{\sin(2c+2dx)}dx}{\sqrt{\sin(2c+2dx)}}\right)}{\frac{e^2}{2(a\sec(c+dx)+a)}{de\sqrt{e\tan(c+dx)}}}-$$

$$\downarrow 3042$$

$$-\frac{a\int\sqrt{e\tan(c+dx)}dx-a\left(\frac{2\cos(c+dx)(e\tan(c+dx))^{3/2}}{de}-\frac{2\cos(c+dx)\sqrt{e\tan(c+dx)}\int\sqrt{\sin(2c+2dx)}dx}{\sqrt{\sin(2c+2dx)}}\right)}{\frac{e^2}{2(a\sec(c+dx)+a)}{de\sqrt{e\tan(c+dx)}}}-$$

$$\downarrow 3119$$

$$-\frac{a\int\sqrt{e\tan(c+dx)}dx-a\left(\frac{2\cos(c+dx)(e\tan(c+dx))^{3/2}}{de}-\frac{2\cos(c+dx)E\left(c+dx-\frac{\pi}{4}|2\right)\sqrt{e\tan(c+dx)}}{d\sqrt{\sin(2c+2dx)}}\right)}{\frac{e^2}{2(a\sec(c+dx)+a)}{de\sqrt{e\tan(c+dx)}}}-$$

$$\downarrow 3957$$

$$-\frac{\frac{ae\int\frac{\sqrt{e\tan(c+dx)}}{\tan^2(c+dx)e^2+e^2}d(e\tan(c+dx))}{d}-a\left(\frac{2\cos(c+dx)(e\tan(c+dx))^{3/2}}{de}-\frac{2\cos(c+dx)E\left(c+dx-\frac{\pi}{4}|2\right)\sqrt{e\tan(c+dx)}}{d\sqrt{\sin(2c+2dx)}}\right)}{\frac{e^2}{2(a\sec(c+dx)+a)}{de\sqrt{e\tan(c+dx)}}}-$$

$$\downarrow 266$$

$$-\frac{\frac{2ae\int\frac{e^2\tan^2(c+dx)}{e^4\tan^4(c+dx)+e^2}d\sqrt{e\tan(c+dx)}}{d}-a\left(\frac{2\cos(c+dx)(e\tan(c+dx))^{3/2}}{de}-\frac{2\cos(c+dx)E\left(c+dx-\frac{\pi}{4}|2\right)\sqrt{e\tan(c+dx)}}{d\sqrt{\sin(2c+2dx)}}\right)}{\frac{e^2}{2(a\sec(c+dx)+a)}{de\sqrt{e\tan(c+dx)}}}-$$

$$\downarrow 826$$

$$-\frac{\frac{2ae\left(\frac{1}{2}\int\frac{e^2\tan^2(c+dx)+e}{e^4\tan^4(c+dx)+e^2}d\sqrt{e\tan(c+dx)}-\frac{1}{2}\int\frac{e-e^2\tan^2(c+dx)}{e^4\tan^4(c+dx)+e^2}d\sqrt{e\tan(c+dx)}\right)}{d}-a\left(\frac{2\cos(c+dx)(e\tan(c+dx))^{3/2}}{de}-\frac{2\cos(c+dx)E(c+d}{d\sqrt{\sin}}\right.}{\frac{e^2}{2(a\sec(c+dx)+a)}{de\sqrt{e\tan(c+dx)}}}$$

$$\downarrow 1476$$

$$-\frac{\frac{2ae\left(\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{e^2\tan^2(c+dx)-\sqrt{2}e^{3/2}\tan(c+dx)+e}d\sqrt{e\tan(c+dx)}+\frac{1}{2}\int \frac{1}{e^2\tan^2(c+dx)+\sqrt{2}e^{3/2}\tan(c+dx)+e}d\sqrt{e\tan(c+dx)}\right)-\frac{1}{2}\int \frac{e-e^2\tan^2(c+dx)}{e^4\tan^4(c+dx)+e^2}d\sqrt{e\tan(c+dx)}\right)}{d}}{e^2}}{\frac{2(a\sec(c+dx)+a)}{de\sqrt{e\tan(c+dx)}}}$$

$$\downarrow 1082$$

$$-\frac{\frac{2ae\left(\frac{1}{2}\left(\frac{\int \frac{1}{-e^2\tan^2(c+dx)-1}d\left(1-\sqrt{2}\sqrt{e}\tan(c+dx)\right)}{\sqrt{2}\sqrt{e}}-\frac{\int \frac{1}{-e^2\tan^2(c+dx)-1}d\left(\sqrt{2}\sqrt{e}\tan(c+dx)+1\right)}{\sqrt{2}\sqrt{e}}\right)-\frac{1}{2}\int \frac{e-e^2\tan^2(c+dx)}{e^4\tan^4(c+dx)+e^2}d\sqrt{e\tan(c+dx)}\right)}{d}-a\left(}{e^2}}{\frac{2(a\sec(c+dx)+a)}{de\sqrt{e\tan(c+dx)}}}$$

$$\downarrow 217$$

$$-\frac{\frac{2ae\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{e}\tan(c+dx)+1\right)}{\sqrt{2}\sqrt{e}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{e}\tan(c+dx)\right)}{\sqrt{2}\sqrt{e}}\right)-\frac{1}{2}\int \frac{e-e^2\tan^2(c+dx)}{e^4\tan^4(c+dx)+e^2}d\sqrt{e\tan(c+dx)}\right)}{d}-a\left(\frac{2\cos(c+dx)(e\tan(c+dx))^{3/2}}{de}\right.}{e^2}}{\frac{2(a\sec(c+dx)+a)}{de\sqrt{e\tan(c+dx)}}}$$

$$\downarrow 1479$$

$$-\frac{\frac{2ae\left(\frac{1}{2}\left(\int -\frac{\sqrt{2}\sqrt{e}-2\sqrt{e\tan(c+dx)}}{e^2\tan^2(c+dx)-\sqrt{2}e^{3/2}\tan(c+dx)+e}d\sqrt{e\tan(c+dx)}+\int -\frac{\sqrt{2}\left(\sqrt{e}+\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{e^2\tan^2(c+dx)+\sqrt{2}e^{3/2}\tan(c+dx)+e}d\sqrt{e\tan(c+dx)}\right)\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{e}\tan(c+dx)\right.}{\sqrt{2}\sqrt{e}}\right.}{d}}{e^2}}{\frac{2(a\sec(c+dx)+a)}{de\sqrt{e\tan(c+dx)}}}$$

$$\downarrow 25$$

$$-\frac{\frac{2ae\left(\frac{1}{2}\left(-\int \frac{\sqrt{2}\sqrt{e}-2\sqrt{e\tan(c+dx)}}{e^2\tan^2(c+dx)-\sqrt{2}e^{3/2}\tan(c+dx)+e}d\sqrt{e\tan(c+dx)}-\int \frac{\sqrt{2}\left(\sqrt{e}+\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{e^2\tan^2(c+dx)+\sqrt{2}e^{3/2}\tan(c+dx)+e}d\sqrt{e\tan(c+dx)}\right)\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{e}\tan(c+dx)+\right.\right.}{\sqrt{2}\sqrt{e}}\right.}{d}}{e^2}}{\frac{2(a\sec(c+dx)+a)}{de\sqrt{e\tan(c+dx)}}}$$

$$\downarrow 27$$

$$-\frac{2ae\left(\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}\sqrt{e}-2\sqrt{e\tan(c+dx)}}{e^2\tan^2(c+dx)-\sqrt{2}e^{3/2}\tan(c+dx)+e}d\sqrt{e\tan(c+dx)}}{2\sqrt{2}\sqrt{e}}-\frac{\int \frac{\sqrt{e}+\sqrt{2}\sqrt{e\tan(c+dx)}}{e^2\tan^2(c+dx)+\sqrt{2}e^{3/2}\tan(c+dx)+e}d\sqrt{e\tan(c+dx)}}{2\sqrt{e}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{e}\tan(c+dx)+\right.\right.}{\sqrt{2}\sqrt{e}}}{d}}{e^2}$$

$$\frac{2(a\sec(c+dx)+a)}{de\sqrt{e\tan(c+dx)}}$$

$$\downarrow 1103$$

$$-\frac{2ae\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{e}\tan(c+dx)+1\right)}{\sqrt{2}\sqrt{e}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{e}\tan(c+dx)\right)}{\sqrt{2}\sqrt{e}}\right)+\frac{1}{2}\left(\frac{\log\left(-\sqrt{2}e^{3/2}\tan(c+dx)+e^2\tan^2(c+dx)+e\right)}{2\sqrt{2}\sqrt{e}}-\frac{\log\left(\sqrt{2}e^{3/2}\tan(c+dx)+e^2\tan^2(c+\right.}{2\sqrt{2}\sqrt{e}}}{d}}{e^2}$$

$$\frac{2(a\sec(c+dx)+a)}{de\sqrt{e\tan(c+dx)}}$$

input
```
Int[(a + a*Sec[c + d*x])/(e*Tan[c + d*x])^(3/2),x]
```

output
```
(-2*(a + a*Sec[c + d*x]))/(d*e*Sqrt[e*Tan[c + d*x]]) - ((2*a*e*((-(ArcTan[
1 - Sqrt[2]*Sqrt[e]*Tan[c + d*x]]/(Sqrt[2]*Sqrt[e])) + ArcTan[1 + Sqrt[2]*
Sqrt[e]*Tan[c + d*x]]/(Sqrt[2]*Sqrt[e]))/2 + (Log[e - Sqrt[2]*e^(3/2)*Tan[
c + d*x] + e^2*Tan[c + d*x]^2]/(2*Sqrt[2]*Sqrt[e]) - Log[e + Sqrt[2]*e^(3/
2)*Tan[c + d*x] + e^2*Tan[c + d*x]^2]/(2*Sqrt[2]*Sqrt[e]))/2))/d - a*((-2*
Cos[c + d*x]*EllipticE[c - Pi/4 + d*x, 2]*Sqrt[e*Tan[c + d*x]])/(d*Sqrt[Si
n[2*c + 2*d*x]]) + (2*Cos[c + d*x]*(e*Tan[c + d*x])^(3/2))/(d*e)))/e^2
```

3.108.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3052
```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Simp[Sqrt[a*Sin[e + f*x]]*(Sqrt[b*Cos[e + f*x]]/Sqrt[Sin[2*e
 + 2*f*x]])   Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

rule 3093
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[a^2*(a*Sec[e + f*x])^(m - 2)*((b*Tan[e + f*x])^(n +
1)/(b*f*(m + n - 1))), x] + Simp[a^2*((m - 2)/(m + n - 1))   Int[(a*Sec[e +
 f*x])^(m - 2)*(b*Tan[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (
GtQ[m, 1] || (EqQ[m, 1] && EqQ[n, 1/2])) && NeQ[m + n - 1, 0] && IntegersQ[
2*m, 2*n]
```

rule 3095
```
Int[Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]]/sec[(e_.) + (f_.)*(x_)], x_Symbol]
:> Simp[Sqrt[Cos[e + f*x]]*(Sqrt[b*Tan[e + f*x]]/Sqrt[Sin[e + f*x]])   Int[
Sqrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]], x], x] /; FreeQ[{b, e, f}, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

rule 4370
```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(-(e*Cot[c + d*x])^(m + 1))*((a + b*Csc[c + d*x])/(
d*e*(m + 1))), x] - Simp[1/(e^2*(m + 1))   Int[(e*Cot[c + d*x])^(m + 2)*(a*
(m + 1) + b*(m + 2)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && L
tQ[m, -1]
```

rule 4372
```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(e*Cot[c + d*x])^m, x], x] + Simp[b   Int[
(e*Cot[c + d*x])^m*Csc[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, m}, x]
```

3.108.4 Maple [A] (verified)

Time = 4.70 (sec) , antiderivative size = 511, normalized size of antiderivative = 1.68

method	result
parts	$\frac{2ae\left(-\frac{1}{e^2\sqrt{e\tan(dx+c)}}-\frac{\sqrt{2}\left(\ln\left(\frac{e\tan(dx+c)-\left(e^2\right)^{\frac{1}{4}}\sqrt{e\tan(dx+c)}\sqrt{2}+\sqrt{e^2}}{e\tan(dx+c)+\left(e^2\right)^{\frac{1}{4}}\sqrt{e\tan(dx+c)}\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{e\tan(dx+c)}}{\left(e^2\right)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\frac{\sqrt{2}\sqrt{e\tan(dx+c)}}{\left(e^2\right)^{\frac{1}{4}}}\right.\right.}{8e^2\left(e^2\right)^{\frac{1}{4}}}\right.}{d}$
default	Expression too large to display

input
```
int((a+a*sec(d*x+c))/(e*tan(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2*a/d*e*(-1/e^2/(e*tan(d*x+c))^(1/2)-1/8/e^2/(e^2)^(1/4)*2^(1/2)*(ln((e*ta
n(d*x+c)-(e^2)^(1/4)*(e*tan(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2))/(e*tan(d*x+
c)+(e^2)^(1/4)*(e*tan(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2)))+2*arctan(2^(1/2)
/(e^2)^(1/4)*(e*tan(d*x+c))^(1/2)+1)-2*arctan(-2^(1/2)/(e^2)^(1/4)*(e*tan(
d*x+c))^(1/2)+1)))-a/d*2^(1/2)/e/(e*tan(d*x+c))^(1/2)*(-2*(csc(d*x+c)-cot(
d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1
/2)*EllipticE((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2*2^(1/2))+(csc(d*x+c)-cot
(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c))^(
1/2)*EllipticF((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2*2^(1/2))-2*(csc(d*x+c)-
cot(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c)
)^(1/2)*EllipticE((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2*2^(1/2))*sec(d*x+c)+
(csc(d*x+c)-cot(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c)+1)^(1/2)*(cot(d*x+c
)-csc(d*x+c))^(1/2)*EllipticF((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2*2^(1/2))
*sec(d*x+c)+2^(1/2))
```

3.108.5 Fricas [F(-1)]

Timed out.

$$\int \frac{a + a\sec(c + dx)}{(e\tan(c + dx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((a+a*sec(d*x+c))/(e*tan(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.108.6 Sympy [F]

$$\int \frac{a + a\sec(c+dx)}{(e\tan(c+dx))^{3/2}}\,dx = a\left(\int \frac{1}{(e\tan(c+dx))^{\frac{3}{2}}}\,dx + \int \frac{\sec(c+dx)}{(e\tan(c+dx))^{\frac{3}{2}}}\,dx\right)$$

input
```
integrate((a+a*sec(d*x+c))/(e*tan(d*x+c))**(3/2),x)
```

output
```
a*(Integral((e*tan(c + d*x))**(-3/2), x) + Integral(sec(c + d*x)/(e*tan(c
+ d*x))**(3/2), x))
```

3.108.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{a + a\sec(c+dx)}{(e\tan(c+dx))^{3/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+a*sec(d*x+c))/(e*tan(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.108.8 Giac [F]

$$\int \frac{a + a\sec(c+dx)}{(e\tan(c+dx))^{3/2}}\,dx = \int \frac{a\sec(dx+c)+a}{(e\tan(dx+c))^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+a*sec(d*x+c))/(e*tan(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
integrate((a*sec(d*x + c) + a)/(e*tan(d*x + c))^(3/2), x)
```

3.108.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a + a\sec(c + dx)}{(e\tan(c + dx))^{3/2}}\,dx = \int \frac{a + \frac{a}{\cos(c+d\,x)}}{(e\tan(c + d\,x))^{3/2}}\,dx$$

input
```
int((a + a/cos(c + d*x))/(e*tan(c + d*x))^(3/2),x)
```

output
```
int((a + a/cos(c + d*x))/(e*tan(c + d*x))^(3/2), x)
```

3.109 $\int \frac{a+a\sec(c+dx)}{(e\tan(c+dx))^{5/2}}\,dx$

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3.109.1 Optimal result

Integrand size = 23, antiderivative size = 282

$$\begin{aligned}\int \frac{a+a\sec(c+dx)}{(e\tan(c+dx))^{5/2}}\,dx &= \frac{a\arctan\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}de^{5/2}} \\ &- \frac{a\arctan\left(1+\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}de^{5/2}} \\ &+ \frac{a\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}de^{5/2}} \\ &- \frac{a\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)+\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}de^{5/2}} - \frac{2(a+a\sec(c+dx))}{3de(e\tan(c+dx))^{3/2}} \\ &- \frac{a\,\text{EllipticF}\left(c-\frac{\pi}{4}+dx,2\right)\sec(c+dx)\sqrt{\sin(2c+2dx)}}{3de^2\sqrt{e\tan(c+dx)}}\end{aligned}$$

output

```
1/2*a*arctan(1-2^(1/2)*(e*tan(d*x+c))^(1/2)/e^(1/2))/d/e^(5/2)*2^(1/2)-1/2
*a*arctan(1+2^(1/2)*(e*tan(d*x+c))^(1/2)/e^(1/2))/d/e^(5/2)*2^(1/2)+1/4*a*
ln(e^(1/2)-2^(1/2)*(e*tan(d*x+c))^(1/2)+e^(1/2)*tan(d*x+c))/d/e^(5/2)*2^(1
/2)-1/4*a*ln(e^(1/2)+2^(1/2)*(e*tan(d*x+c))^(1/2)+e^(1/2)*tan(d*x+c))/d/e^
(5/2)*2^(1/2)+1/3*a*(sin(c+1/4*Pi+d*x)^2)^(1/2)/sin(c+1/4*Pi+d*x)*Elliptic
F(cos(c+1/4*Pi+d*x),2^(1/2))*sec(d*x+c)*sin(2*d*x+2*c)^(1/2)/d/e^2/(e*tan(
d*x+c))^(1/2)-2/3*(a+a*sec(d*x+c))/d/e/(e*tan(d*x+c))^(3/2)
```

3.109.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.64 (sec) , antiderivative size = 200, normalized size of antiderivative = 0.71

$$\int \frac{a + a\sec(c+dx)}{(e\tan(c+dx))^{5/2}}\,dx =$$
$$-\frac{a\csc(c+dx)\left(\sqrt{\sec^2(c+dx)}\left(2\cot\left(\frac{1}{2}(c+dx)\right)+2\cos\left(\frac{3}{2}(c+dx)\right)\csc\left(\frac{1}{2}(c+dx)\right)-3\arcsin(\cos(c+dx\right.\right.}$$

input
```
Integrate[(a + a*Sec[c + d*x])/(e*Tan[c + d*x])^(5/2),x]
```

output
```
-1/6*(a*Csc[c + d*x]*(Sqrt[Sec[c + d*x]^2]*(2*Cot[(c + d*x)/2] + 2*Cos[(3*
(c + d*x))/2]*Csc[(c + d*x)/2] - 3*ArcSin[Cos[c + d*x] - Sin[c + d*x]]*Sqr
t[Sin[2*(c + d*x)]] + 3*Log[Cos[c + d*x] + Sin[c + d*x] + Sqrt[Sin[2*(c +
d*x)]]]*Sqrt[Sin[2*(c + d*x)]]) - 4*(-1)^(1/4)*EllipticF[I*ArcSinh[(-1)^(1
/4)*Sqrt[Tan[c + d*x]]], -1]*Sqrt[Tan[c + d*x]])*Sqrt[e*Tan[c + d*x]])/(d*
e^3*Sqrt[Sec[c + d*x]^2])
```

3.109.3 Rubi [A] (warning: unable to verify)

Time = 0.93 (sec) , antiderivative size = 270, normalized size of antiderivative = 0.96, number of steps used = 22, number of rules used = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.913$, Rules used = {3042, 4370, 27, 3042, 4372, 3042, 3094, 3042, 3053, 3042, 3120, 3957, 266, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a\sec(c+dx)+a}{(e\tan(c+dx))^{5/2}}\,dx$$
$$\downarrow 3042$$
$$\int \frac{a\csc\left(c+dx+\frac{\pi}{2}\right)+a}{\left(-e\cot\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}dx$$
$$\downarrow 4370$$
$$\frac{2\int -\frac{\sec(c+dx)a+3a}{2\sqrt{e\tan(c+dx)}}dx}{3e^2} - \frac{2(a\sec(c+dx)+a)}{3de(e\tan(c+dx))^{3/2}}$$

$$\downarrow 27$$

$$-\frac{\int \frac{\sec(c+dx)a+3a}{\sqrt{e\tan(c+dx)}}dx}{3e^2}-\frac{2(a\sec(c+dx)+a)}{3de(e\tan(c+dx))^{3/2}}$$

$$\downarrow 3042$$

$$-\frac{\int \frac{\csc(c+dx+\frac{\pi}{2})a+3a}{\sqrt{-e\cot(c+dx+\frac{\pi}{2})}}dx}{3e^2}-\frac{2(a\sec(c+dx)+a)}{3de(e\tan(c+dx))^{3/2}}$$

$$\downarrow 4372$$

$$-\frac{3a\int \frac{1}{\sqrt{e\tan(c+dx)}}dx+a\int \frac{\sec(c+dx)}{\sqrt{e\tan(c+dx)}}dx}{3e^2}-\frac{2(a\sec(c+dx)+a)}{3de(e\tan(c+dx))^{3/2}}$$

$$\downarrow 3042$$

$$-\frac{3a\int \frac{1}{\sqrt{e\tan(c+dx)}}dx+a\int \frac{\sec(c+dx)}{\sqrt{e\tan(c+dx)}}dx}{3e^2}-\frac{2(a\sec(c+dx)+a)}{3de(e\tan(c+dx))^{3/2}}$$

$$\downarrow 3094$$

$$-\frac{3a\int \frac{1}{\sqrt{e\tan(c+dx)}}dx+\frac{a\sqrt{\sin(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{\sin(c+dx)}}dx}{\sqrt{\cos(c+dx)}\sqrt{e\tan(c+dx)}}}{3e^2}-\frac{2(a\sec(c+dx)+a)}{3de(e\tan(c+dx))^{3/2}}$$

$$\downarrow 3042$$

$$-\frac{3a\int \frac{1}{\sqrt{e\tan(c+dx)}}dx+\frac{a\sqrt{\sin(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{\sin(c+dx)}}dx}{\sqrt{\cos(c+dx)}\sqrt{e\tan(c+dx)}}}{3e^2}-\frac{2(a\sec(c+dx)+a)}{3de(e\tan(c+dx))^{3/2}}$$

$$\downarrow 3053$$

$$-\frac{3a\int \frac{1}{\sqrt{e\tan(c+dx)}}dx+\frac{a\sqrt{\sin(2c+2dx)}\sec(c+dx)\int \frac{1}{\sqrt{\sin(2c+2dx)}}dx}{\sqrt{e\tan(c+dx)}}}{3e^2}-\frac{2(a\sec(c+dx)+a)}{3de(e\tan(c+dx))^{3/2}}$$

$$\downarrow 3042$$

$$-\frac{3a\int \frac{1}{\sqrt{e\tan(c+dx)}}dx+\frac{a\sqrt{\sin(2c+2dx)}\sec(c+dx)\int \frac{1}{\sqrt{\sin(2c+2dx)}}dx}{\sqrt{e\tan(c+dx)}}}{3e^2}-\frac{2(a\sec(c+dx)+a)}{3de(e\tan(c+dx))^{3/2}}$$

$$\downarrow 3120$$

$$-\frac{3a\int \frac{1}{\sqrt{e\tan(c+dx)}}dx+\frac{a\sqrt{\sin(2c+2dx)}\sec(c+dx)\,\text{EllipticF}\left(c+dx-\frac{\pi}{4},2\right)}{d\sqrt{e\tan(c+dx)}}}{3e^2}-\frac{2(a\sec(c+dx)+a)}{3de(e\tan(c+dx))^{3/2}}$$

$$\downarrow 3957$$

$$-\frac{\frac{3ae\int \frac{1}{\sqrt{e\tan(c+dx)}\left(\tan^2(c+dx)e^2+e^2\right)}d(e\tan(c+dx))}{d}+\frac{a\sqrt{\sin(2c+2dx)}\sec(c+dx)\,\text{EllipticF}\left(c+dx-\frac{\pi}{4},2\right)}{d\sqrt{e\tan(c+dx)}}}{3e^2}-\frac{2(a\sec(c+dx)+a)}{3de(e\tan(c+dx))^{3/2}}$$

$$\downarrow 266$$

$$-\frac{\frac{6ae\int \frac{1}{e^4\tan^4(c+dx)+e^2}d\sqrt{e\tan(c+dx)}}{d}+\frac{a\sqrt{\sin(2c+2dx)}\sec(c+dx)\,\text{EllipticF}\left(c+dx-\frac{\pi}{4},2\right)}{d\sqrt{e\tan(c+dx)}}}{3e^2}-\frac{2(a\sec(c+dx)+a)}{3de(e\tan(c+dx))^{3/2}}$$

$$\downarrow 755$$

$$-\frac{\frac{6ae\left(\frac{\int \frac{e-e^2\tan^2(c+dx)}{e^4\tan^4(c+dx)+e^2}d\sqrt{e\tan(c+dx)}}{2e}+\frac{\int \frac{e^2\tan^2(c+dx)+e}{e^4\tan^4(c+dx)+e^2}d\sqrt{e\tan(c+dx)}}{2e}\right)}{d}+\frac{a\sqrt{\sin(2c+2dx)}\sec(c+dx)\,\text{EllipticF}\left(c+dx-\frac{\pi}{4},2\right)}{d\sqrt{e\tan(c+dx)}}}{3e^2}-\frac{2(a\sec(c+dx)+a)}{3de(e\tan(c+dx))^{3/2}}$$

$$\downarrow 1476$$

$$-\frac{\frac{6ae\left(\frac{\frac{1}{2}\int \frac{1}{e^2\tan^2(c+dx)-\sqrt{2}e^{3/2}\tan(c+dx)+e}d\sqrt{e\tan(c+dx)}+\frac{1}{2}\int \frac{1}{e^2\tan^2(c+dx)+\sqrt{2}e^{3/2}\tan(c+dx)+e}d\sqrt{e\tan(c+dx)}}{2e}+\frac{\int \frac{e-e^2\tan^2(c+dx)}{e^4\tan^4(c+dx)+e^2}d\sqrt{e\tan(c+d}}{2e}\right)}{d}}{3e^2}$$

$$-\frac{2(a\sec(c+dx)+a)}{3de(e\tan(c+dx))^{3/2}}$$

$$\downarrow 1082$$

$$-\frac{\frac{6ae\left(\frac{\frac{\int \frac{1}{-e^2\tan^2(c+dx)-1}d\left(1-\sqrt{2}\sqrt{e}\tan(c+dx)\right)}{\sqrt{2}\sqrt{e}}-\frac{\int \frac{1}{-e^2\tan^2(c+dx)-1}d\left(\sqrt{2}\sqrt{e}\tan(c+dx)+1\right)}{\sqrt{2}\sqrt{e}}}{2e}+\frac{\int \frac{e-e^2\tan^2(c+dx)}{e^4\tan^4(c+dx)+e^2}d\sqrt{e\tan(c+dx)}}{2e}\right)}{d}+\frac{a\sqrt{\sin(2c+2}}{d}}{3e^2}$$

$$-\frac{2(a\sec(c+dx)+a)}{3de(e\tan(c+dx))^{3/2}}$$

$$\downarrow 217$$

$$-\frac{6ae\left(\frac{\int \frac{e-e^2\tan^2(c+dx)}{e^4\tan^4(c+dx)+e^2}d\sqrt{e\tan(c+dx)}}{2e}+\frac{\frac{\arctan\left(\sqrt{2}\sqrt{e}\tan(c+dx)+1\right)}{\sqrt{2}\sqrt{e}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{e}\tan(c+dx)\right)}{\sqrt{2}\sqrt{e}}}{2e}\right)}{d}+\frac{a\sqrt{\sin(2c+2dx)}\sec(c+dx)\,\text{EllipticF}\left(c+dx\right.}{d\sqrt{e\tan(c+dx)}}}{\frac{2(a\sec(c+dx)+a)}{3de(e\tan(c+dx))^{3/2}}}\Bigg/3e^2$$

$$\downarrow 1479$$

$$-\frac{6ae\left(\frac{-\frac{\int -\frac{\sqrt{2}\sqrt{e}-2\sqrt{e\tan(c+dx)}}{e^2\tan^2(c+dx)-\sqrt{2}e^{3/2}\tan(c+dx)+e}d\sqrt{e\tan(c+dx)}}{2\sqrt{2}\sqrt{e}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{e}+\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{e^2\tan^2(c+dx)+\sqrt{2}e^{3/2}\tan(c+dx)+e}d\sqrt{e\tan(c+dx)}}{2\sqrt{2}\sqrt{e}}}{2e}+\frac{\frac{\arctan\left(\sqrt{2}\sqrt{e}\tan(c+dx)+1\right)}{\sqrt{2}\sqrt{e}}-\frac{a}{2e}}{d}}{\frac{2(a\sec(c+dx)+a)}{3de(e\tan(c+dx))^{3/2}}}\Bigg/3e^2$$

$$\downarrow 25$$

$$-\frac{6ae\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt{e}-2\sqrt{e\tan(c+dx)}}{e^2\tan^2(c+dx)-\sqrt{2}e^{3/2}\tan(c+dx)+e}d\sqrt{e\tan(c+dx)}}{2\sqrt{2}\sqrt{e}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{e}+\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{e^2\tan^2(c+dx)+\sqrt{2}e^{3/2}\tan(c+dx)+e}d\sqrt{e\tan(c+dx)}}{2\sqrt{2}\sqrt{e}}}{2e}+\frac{\frac{\arctan\left(\sqrt{2}\sqrt{e}\tan(c+dx)+1\right)}{\sqrt{2}\sqrt{e}}-\frac{\arctan}{2e}}{d}}{\frac{2(a\sec(c+dx)+a)}{3de(e\tan(c+dx))^{3/2}}}\Bigg/3e^2$$

$$\downarrow 27$$

$$-\frac{6ae\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt{e}-2\sqrt{e\tan(c+dx)}}{e^2\tan^2(c+dx)-\sqrt{2}e^{3/2}\tan(c+dx)+e}d\sqrt{e\tan(c+dx)}}{2\sqrt{2}\sqrt{e}}+\frac{\int \frac{\sqrt{e}+\sqrt{2}\sqrt{e\tan(c+dx)}}{e^2\tan^2(c+dx)+\sqrt{2}e^{3/2}\tan(c+dx)+e}d\sqrt{e\tan(c+dx)}}{2\sqrt{e}}}{2e}+\frac{\frac{\arctan\left(\sqrt{2}\sqrt{e}\tan(c+dx)+1\right)}{\sqrt{2}\sqrt{e}}-\frac{\arctan}{2e}}{d}}{\frac{2(a\sec(c+dx)+a)}{3de(e\tan(c+dx))^{3/2}}}\Bigg/3e^2$$

$$\downarrow 1103$$

$$-\frac{\frac{2(a\sec(c+dx)+a)}{3de(e\tan(c+dx))^{3/2}}-\frac{6ae\left(\frac{\frac{\arctan\left(\sqrt{2}\sqrt{e}\tan(c+dx)+1\right)}{\sqrt{2}\sqrt{e}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{e}\tan(c+dx)\right)}{\sqrt{2}\sqrt{e}}}{2e}+\frac{\frac{\log\left(\sqrt{2}e^{3/2}\tan(c+dx)+e^2\tan^2(c+dx)+e\right)}{2\sqrt{2}\sqrt{e}}-\frac{\log\left(-\sqrt{2}e^{3/2}\tan(c+dx)+e^2\tan^2(c+dx)+e\right)}{2\sqrt{2}\sqrt{e}}}{2e}\right)}{d}}{3e^2}$$

input
```
Int[(a + a*Sec[c + d*x])/(e*Tan[c + d*x])^(5/2),x]
```

output
```
(-2*(a + a*Sec[c + d*x]))/(3*d*e*(e*Tan[c + d*x])^(3/2)) - ((6*a*e*((-(Arc
Tan[1 - Sqrt[2]*Sqrt[e]*Tan[c + d*x]]/(Sqrt[2]*Sqrt[e])) + ArcTan[1 + Sqrt
[2]*Sqrt[e]*Tan[c + d*x]]/(Sqrt[2]*Sqrt[e]))/(2*e) + (-1/2*Log[e - Sqrt[2]
*e^(3/2)*Tan[c + d*x] + e^2*Tan[c + d*x]^2]/(Sqrt[2]*Sqrt[e]) + Log[e + Sq
rt[2]*e^(3/2)*Tan[c + d*x] + e^2*Tan[c + d*x]^2]/(2*Sqrt[2]*Sqrt[e]))/(2*e
)))/d + (a*EllipticF[c - Pi/4 + d*x, 2]*Sec[c + d*x]*Sqrt[Sin[2*c + 2*d*x]
])/(d*Sqrt[e*Tan[c + d*x]]))/(3*e^2)
```

3.109.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3053
```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Simp[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]])   Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f
}, x]
```

rule 3094
```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]], x_Symbol]
:> Simp[Sqrt[Sin[e + f*x]]/(Sqrt[Cos[e + f*x]]*Sqrt[b*Tan[e + f*x]])   Int[
1/(Sqrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]]), x], x] /; FreeQ[{b, e, f}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

rule 4370
```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(-(e*Cot[c + d*x])^(m + 1))*((a + b*Csc[c + d*x])/(
d*e*(m + 1))), x] - Simp[1/(e^2*(m + 1))   Int[(e*Cot[c + d*x])^(m + 2)*(a*
(m + 1) + b*(m + 2)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && L
tQ[m, -1]
```

rule 4372
```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(e*Cot[c + d*x])^m, x], x] + Simp[b   Int[
(e*Cot[c + d*x])^m*Csc[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, m}, x]
```

3.109.4 Maple [A] (verified)

Time = 4.78 (sec) , antiderivative size = 458, normalized size of antiderivative = 1.62

method	result
parts	$\frac{2ae\left(-\frac{1}{3e^2(e\tan(dx+c))^{\frac{3}{2}}}-\frac{\left(e^2\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\tan(dx+c)+\left(e^2\right)^{\frac{1}{4}}\sqrt{e\tan(dx+c)}\sqrt{2}+\sqrt{e^2}}{e\tan(dx+c)-\left(e^2\right)^{\frac{1}{4}}\sqrt{e\tan(dx+c)}\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{e\tan(dx+c)}}{\left(e^2\right)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\frac{\sqrt{2}\sqrt{e}}{}\right.\right.}{8e^4}\right.}{d}$
default	$\frac{a\sqrt{2}\,(\cos(dx+c)+1)\left(3i\sin(dx+c)\sqrt{\csc(dx+c)-\cot(dx+c)+1}\,\sqrt{\cot(dx+c)-\csc(dx+c)+1}\,\sqrt{\cot(dx+c)-\csc(dx+c)}\,\text{EllipticPi}\left(\sqrt{\csc}\right.\right.$

input
```
int((a+a*sec(d*x+c))/(e*tan(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2*a/d*e*(-1/3/e^2/(e*tan(d*x+c))^(3/2)-1/8/e^4*(e^2)^(1/4)*2^(1/2)*(ln((e*
tan(d*x+c)+(e^2)^(1/4)*(e*tan(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2))/(e*tan(d*
x+c)-(e^2)^(1/4)*(e*tan(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2)))+2*arctan(2^(1/
2)/(e^2)^(1/4)*(e*tan(d*x+c))^(1/2)+1)-2*arctan(-2^(1/2)/(e^2)^(1/4)*(e*ta
n(d*x+c))^(1/2)+1)))+1/6*a/d*2^(1/2)*(1-cos(d*x+c))^2*(2*(csc(d*x+c)-cot(d
*x+c)+1)^(1/2)*(2-2*csc(d*x+c)+2*cot(d*x+c))^(1/2)*(cot(d*x+c)-csc(d*x+c))
^(1/2)*EllipticF((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2*2^(1/2))*(-cot(d*x+c)
+csc(d*x+c))-(1-cos(d*x+c))^4*csc(d*x+c)^4+1)/((1-cos(d*x+c))^3*csc(d*x+c)
^3+cot(d*x+c)-csc(d*x+c))^(1/2)/((1-cos(d*x+c))*((1-cos(d*x+c))^2*csc(d*x+
c)^2-1)*csc(d*x+c))^(1/2)/((1-cos(d*x+c))^2*csc(d*x+c)^2-1)^2/(-e/((1-cos(
d*x+c))^2*csc(d*x+c)^2-1)*(-cot(d*x+c)+csc(d*x+c)))^(5/2)*csc(d*x+c)^2
```

3.109.5 Fricas [F(-1)]

Timed out.

$$\int \frac{a + a\sec(c + dx)}{(e\tan(c + dx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((a+a*sec(d*x+c))/(e*tan(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.109.6 Sympy [F]

$$\int \frac{a + a\sec(c + dx)}{(e\tan(c + dx))^{5/2}}\,dx = a\left(\int \frac{1}{(e\tan(c + dx))^{\frac{5}{2}}}\,dx + \int \frac{\sec(c + dx)}{(e\tan(c + dx))^{\frac{5}{2}}}\,dx\right)$$

input
```
integrate((a+a*sec(d*x+c))/(e*tan(d*x+c))**(5/2),x)
```

output
```
a*(Integral((e*tan(c + d*x))**(-5/2), x) + Integral(sec(c + d*x)/(e*tan(c
+ d*x))**(5/2), x))
```

3.109.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{a + a\sec(c+dx)}{(e\tan(c+dx))^{5/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+a*sec(d*x+c))/(e*tan(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.109.8 Giac [F]

$$\int \frac{a + a\sec(c+dx)}{(e\tan(c+dx))^{5/2}}\,dx = \int \frac{a\sec(dx+c)+a}{(e\tan(dx+c))^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+a*sec(d*x+c))/(e*tan(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
integrate((a*sec(d*x + c) + a)/(e*tan(d*x + c))^(5/2), x)
```

3.109.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a + a\sec(c+dx)}{(e\tan(c+dx))^{5/2}}\,dx = \int \frac{a+\frac{a}{\cos(c+dx)}}{(e\tan(c+dx))^{5/2}}\,dx$$

input
```
int((a + a/cos(c + d*x))/(e*tan(c + d*x))^(5/2),x)
```

output
```
int((a + a/cos(c + d*x))/(e*tan(c + d*x))^(5/2), x)
```

3.110 $\int \frac{a+a\sec(c+dx)}{(e\tan(c+dx))^{7/2}}\,dx$

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3.110.1 Optimal result

Integrand size = 23, antiderivative size = 346

$$\int \frac{a+a\sec(c+dx)}{(e\tan(c+dx))^{7/2}}\,dx = -\frac{a\arctan\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}de^{7/2}} + \frac{a\arctan\left(1+\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}de^{7/2}} + \frac{a\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}de^{7/2}} - \frac{a\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)+\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}de^{7/2}} - \frac{2(a+a\sec(c+dx))}{5de(e\tan(c+dx))^{5/2}} + \frac{2(5a+3a\sec(c+dx))}{5de^3\sqrt{e\tan(c+dx)}} + \frac{6a\cos(c+dx)E\left(c-\frac{\pi}{4}+dx\middle|2\right)\sqrt{e\tan(c+dx)}}{5de^4\sqrt{\sin(2c+2dx)}} - \frac{6a\cos(c+dx)(e\tan(c+dx))^{3/2}}{5de^5}$$

output

```
-1/2*a*arctan(1-2^(1/2)*(e*tan(d*x+c))^(1/2)/e^(1/2))/d/e^(7/2)*2^(1/2)+1/
2*a*arctan(1+2^(1/2)*(e*tan(d*x+c))^(1/2)/e^(1/2))/d/e^(7/2)*2^(1/2)+1/4*a
*ln(e^(1/2)-2^(1/2)*(e*tan(d*x+c))^(1/2)+e^(1/2)*tan(d*x+c))/d/e^(7/2)*2^(
1/2)-1/4*a*ln(e^(1/2)+2^(1/2)*(e*tan(d*x+c))^(1/2)+e^(1/2)*tan(d*x+c))/d/e
^(7/2)*2^(1/2)+2/5*(5*a+3*a*sec(d*x+c))/d/e^3/(e*tan(d*x+c))^(1/2)-6/5*a*c
os(d*x+c)*(sin(c+1/4*Pi+d*x)^2)^(1/2)/sin(c+1/4*Pi+d*x)*EllipticE(cos(c+1/
4*Pi+d*x),2^(1/2))*(e*tan(d*x+c))^(1/2)/d/e^4/sin(2*d*x+2*c)^(1/2)-2/5*(a+
a*sec(d*x+c))/d/e/(e*tan(d*x+c))^(5/2)-6/5*a*cos(d*x+c)*(e*tan(d*x+c))^(3/
2)/d/e^5
```

3.110.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 2.75 (sec) , antiderivative size = 254, normalized size of antiderivative = 0.73

$$\int \frac{a + a\sec(c+dx)}{(e\tan(c+dx))^{7/2}}\,dx =$$

$$-\frac{a\csc\left(\frac{1}{2}(c+dx)\right)\sec\left(\frac{1}{2}(c+dx)\right)(1+\sec(c+dx))\left(2\cot\left(\frac{1}{2}(c+dx)\right) - 19\sin(c+dx) + 12\sin^2(c+dx)\,\mathrm{t}\right.}{}$$

input
```
Integrate[(a + a*Sec[c + d*x])/(e*Tan[c + d*x])^(7/2),x]
```

output
```
-1/20*(a*Csc[(c + d*x)/2]*Sec[(c + d*x)/2]*(1 + Sec[c + d*x])*(2*Cot[(c +
d*x)/2] - 19*Sin[c + d*x] + 12*Sin[c + d*x]^2*Tan[(c + d*x)/2] - 8*Hyperge
ometric2F1[3/4, 3/2, 7/4, -Tan[c + d*x]^2]*Sqrt[Sec[c + d*x]^2]*Sin[c + d*
x]^2*Tan[(c + d*x)/2] + 5*ArcSin[Cos[c + d*x] - Sin[c + d*x]]*Sqrt[Sin[2*(
c + d*x)]]*Tan[(c + d*x)/2] + 5*Log[Cos[c + d*x] + Sin[c + d*x] + Sqrt[Sin
[2*(c + d*x)]]]*Sqrt[Sin[2*(c + d*x)]]*Tan[(c + d*x)/2] + 5*Sin[c + d*x]*T
an[(c + d*x)/2]^2))/(d*e^3*Sqrt[e*Tan[c + d*x]])
```

3.110.3 Rubi [A] (warning: unable to verify)

Time = 1.30 (sec) , antiderivative size = 333, normalized size of antiderivative = 0.96, number of steps used = 27, number of rules used = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 1.130$, Rules used = {3042, 4370, 27, 3042, 4370, 27, 3042, 4372, 3042, 3093, 3042, 3095, 3042, 3052, 3042, 3119, 3957, 266, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a\sec(c+dx)+a}{(e\tan(c+dx))^{7/2}}\,dx$$

$$\downarrow 3042$$

$$\int \frac{a\csc\left(c+dx+\frac{\pi}{2}\right)+a}{\left(-e\cot\left(c+dx+\frac{\pi}{2}\right)\right)^{7/2}}dx$$

$$\downarrow 4370$$

$$\frac{2\int -\frac{3\sec(c+dx)a+5a}{2(e\tan(c+dx))^{3/2}}dx}{5e^2}-\frac{2(a\sec(c+dx)+a)}{5de(e\tan(c+dx))^{5/2}}$$

$$\downarrow 27$$

$$-\frac{\int \frac{3\sec(c+dx)a+5a}{(e\tan(c+dx))^{3/2}}dx}{5e^2}-\frac{2(a\sec(c+dx)+a)}{5de(e\tan(c+dx))^{5/2}}$$

$$\downarrow 3042$$

$$-\frac{\int \frac{3\csc\left(c+dx+\frac{\pi}{2}\right)a+5a}{\left(-e\cot\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{5e^2}-\frac{2(a\sec(c+dx)+a)}{5de(e\tan(c+dx))^{5/2}}$$

$$\downarrow 4370$$

$$-\frac{\frac{2\int -\frac{1}{2}(5a-3a\sec(c+dx))\sqrt{e\tan(c+dx)}dx}{e^2}-\frac{2(3a\sec(c+dx)+5a)}{de\sqrt{e\tan(c+dx)}}}{5e^2}-\frac{2(a\sec(c+dx)+a)}{5de(e\tan(c+dx))^{5/2}}$$

$$\downarrow 27$$

$$-\frac{-\frac{\int (5a-3a\sec(c+dx))\sqrt{e\tan(c+dx)}dx}{e^2}-\frac{2(3a\sec(c+dx)+5a)}{de\sqrt{e\tan(c+dx)}}}{5e^2}-\frac{2(a\sec(c+dx)+a)}{5de(e\tan(c+dx))^{5/2}}$$

$$\downarrow 3042$$

$$-\frac{-\frac{\int \sqrt{-e\cot\left(c+dx+\frac{\pi}{2}\right)}\left(5a-3a\csc\left(c+dx+\frac{\pi}{2}\right)\right)dx}{e^2}-\frac{2(3a\sec(c+dx)+5a)}{de\sqrt{e\tan(c+dx)}}}{5e^2}-\frac{2(a\sec(c+dx)+a)}{5de(e\tan(c+dx))^{5/2}}$$

$$\downarrow 4372$$

$$-\frac{-\frac{5a\int \sqrt{e\tan(c+dx)}dx-3a\int \sec(c+dx)\sqrt{e\tan(c+dx)}dx}{e^2}-\frac{2(3a\sec(c+dx)+5a)}{de\sqrt{e\tan(c+dx)}}}{5e^2}-\frac{2(a\sec(c+dx)+a)}{5de(e\tan(c+dx))^{5/2}}$$

$$\downarrow 3042$$

$$-\frac{-\frac{5a\int \sqrt{e\tan(c+dx)}dx-3a\int \sec(c+dx)\sqrt{e\tan(c+dx)}dx}{e^2}-\frac{2(3a\sec(c+dx)+5a)}{de\sqrt{e\tan(c+dx)}}}{5e^2}-\frac{2(a\sec(c+dx)+a)}{5de(e\tan(c+dx))^{5/2}}$$

$$\downarrow 3093$$

$$-\frac{-\frac{5a\int \sqrt{e\tan(c+dx)}dx-3a\left(\frac{2\cos(c+dx)(e\tan(c+dx))^{3/2}}{de}-2\int \cos(c+dx)\sqrt{e\tan(c+dx)}dx\right)}{e^2}-\frac{2(3a\sec(c+dx)+5a)}{de\sqrt{e\tan(c+dx)}}}{5e^2}-\frac{2(a\sec(c+dx)+a)}{5de(e\tan(c+dx))^{5/2}}$$

$$\downarrow 3042$$

$$-\frac{-\frac{5a\int\sqrt{e\tan(c+dx)}dx-3a\left(\frac{2\cos(c+dx)(e\tan(c+dx))^{3/2}}{de}-2\int\frac{\sqrt{e\tan(c+dx)}}{\sec(c+dx)}dx\right)}{e^2}-\frac{2(3a\sec(c+dx)+5a)}{de\sqrt{e\tan(c+dx)}}}{\frac{2(a\sec(c+dx)+a)}{5de(e\tan(c+dx))^{5/2}}}-\frac{5e^2}{}$$

$$\downarrow 3095$$

$$-\frac{-\frac{5a\int\sqrt{e\tan(c+dx)}dx-3a\left(\frac{2\cos(c+dx)(e\tan(c+dx))^{3/2}}{de}-\frac{2\sqrt{\cos(c+dx)}\sqrt{e\tan(c+dx)}\int\sqrt{\cos(c+dx)}\sqrt{\sin(c+dx)}dx}{\sqrt{\sin(c+dx)}}\right)}{e^2}-\frac{2(3a\sec(c+dx)+5a)}{de\sqrt{e\tan(c+dx)}}}{\frac{2(a\sec(c+dx)+a)}{5de(e\tan(c+dx))^{5/2}}}-\frac{5e^2}{}$$

$$\downarrow 3042$$

$$-\frac{-\frac{5a\int\sqrt{e\tan(c+dx)}dx-3a\left(\frac{2\cos(c+dx)(e\tan(c+dx))^{3/2}}{de}-\frac{2\sqrt{\cos(c+dx)}\sqrt{e\tan(c+dx)}\int\sqrt{\cos(c+dx)}\sqrt{\sin(c+dx)}dx}{\sqrt{\sin(c+dx)}}\right)}{e^2}-\frac{2(3a\sec(c+dx)+5a)}{de\sqrt{e\tan(c+dx)}}}{\frac{2(a\sec(c+dx)+a)}{5de(e\tan(c+dx))^{5/2}}}-\frac{5e^2}{}$$

$$\downarrow 3052$$

$$-\frac{-\frac{5a\int\sqrt{e\tan(c+dx)}dx-3a\left(\frac{2\cos(c+dx)(e\tan(c+dx))^{3/2}}{de}-\frac{2\cos(c+dx)\sqrt{e\tan(c+dx)}\int\sqrt{\sin(2c+2dx)}dx}{\sqrt{\sin(2c+2dx)}}\right)}{e^2}-\frac{2(3a\sec(c+dx)+5a)}{de\sqrt{e\tan(c+dx)}}}{\frac{2(a\sec(c+dx)+a)}{5de(e\tan(c+dx))^{5/2}}}-\frac{5e^2}{}$$

$$\downarrow 3042$$

$$-\frac{-\frac{5a\int\sqrt{e\tan(c+dx)}dx-3a\left(\frac{2\cos(c+dx)(e\tan(c+dx))^{3/2}}{de}-\frac{2\cos(c+dx)\sqrt{e\tan(c+dx)}\int\sqrt{\sin(2c+2dx)}dx}{\sqrt{\sin(2c+2dx)}}\right)}{e^2}-\frac{2(3a\sec(c+dx)+5a)}{de\sqrt{e\tan(c+dx)}}}{\frac{2(a\sec(c+dx)+a)}{5de(e\tan(c+dx))^{5/2}}}-\frac{5e^2}{}$$

$$\downarrow 3119$$

$$-\frac{-\frac{5a\int\sqrt{e\tan(c+dx)}dx-3a\left(\frac{2\cos(c+dx)(e\tan(c+dx))^{3/2}}{de}-\frac{2\cos(c+dx)E\left(c+dx-\frac{\pi}{4}\middle|2\right)\sqrt{e\tan(c+dx)}}{d\sqrt{\sin(2c+2dx)}}\right)}{e^2}-\frac{2(3a\sec(c+dx)+5a)}{de\sqrt{e\tan(c+dx)}}}{\frac{2(a\sec(c+dx)+a)}{5de(e\tan(c+dx))^{5/2}}}-\frac{5e^2}{}$$

$$\downarrow 3957$$

$$-\frac{-\frac{5ae\int \frac{\sqrt{e\tan(c+dx)}}{\tan^2(c+dx)e^2+e^2}\,d(e\tan(c+dx))}{d}-3a\left(\frac{2\cos(c+dx)(e\tan(c+dx))^{3/2}}{de}-\frac{2\cos(c+dx)E\left(c+dx-\frac{\pi}{4}\middle|2\right)\sqrt{e\tan(c+dx)}}{d\sqrt{\sin(2c+2dx)}}\right)}{e^2}-\frac{2(3a\sec(c+dx)+5a)}{de\sqrt{e\tan(c+dx)}}}{5e^2}$$
$$\frac{2(a\sec(c+dx)+a)}{5de(e\tan(c+dx))^{5/2}}$$

$$\downarrow 266$$

$$-\frac{-\frac{10ae\int \frac{e^2\tan^2(c+dx)}{e^4\tan^4(c+dx)+e^2}\,d\sqrt{e\tan(c+dx)}}{d}-3a\left(\frac{2\cos(c+dx)(e\tan(c+dx))^{3/2}}{de}-\frac{2\cos(c+dx)E\left(c+dx-\frac{\pi}{4}\middle|2\right)\sqrt{e\tan(c+dx)}}{d\sqrt{\sin(2c+2dx)}}\right)}{e^2}-\frac{2(3a\sec(c+dx)+5a)}{de\sqrt{e\tan(c+dx)}}}{5e^2}$$
$$\frac{2(a\sec(c+dx)+a)}{5de(e\tan(c+dx))^{5/2}}$$

$$\downarrow 826$$

$$-\frac{-\frac{10ae\left(\frac{1}{2}\int \frac{e^2\tan^2(c+dx)+e}{e^4\tan^4(c+dx)+e^2}\,d\sqrt{e\tan(c+dx)}-\frac{1}{2}\int \frac{e-e^2\tan^2(c+dx)}{e^4\tan^4(c+dx)+e^2}\,d\sqrt{e\tan(c+dx)}\right)}{d}-3a\left(\frac{2\cos(c+dx)(e\tan(c+dx))^{3/2}}{de}-\frac{2\cos(c+dx)E\left(c+dx-\frac{\pi}{4}\middle|2\right)\sqrt{e}}{d\sqrt{\sin(2c+2dx)}}\right)}{e^2}}{5e^2}$$
$$\frac{2(a\sec(c+dx)+a)}{5de(e\tan(c+dx))^{5/2}}$$

$$\downarrow 1476$$

$$-\frac{-\frac{10ae\left(\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{e^2\tan^2(c+dx)-\sqrt{2}e^{3/2}\tan(c+dx)+e}\,d\sqrt{e\tan(c+dx)}+\frac{1}{2}\int \frac{1}{e^2\tan^2(c+dx)+\sqrt{2}e^{3/2}\tan(c+dx)+e}\,d\sqrt{e\tan(c+dx)}\right)-\frac{1}{2}\int \frac{e-e^2\tan^2(c+dx)}{e^4\tan^4(c+dx)+e^2}\,d\sqrt{e}\right)}{d}}{e^2}}{5e^2}$$
$$\frac{2(a\sec(c+dx)+a)}{5de(e\tan(c+dx))^{5/2}}$$

$$\downarrow 1082$$

$$-\frac{-\frac{10ae\left(\frac{1}{2}\left(\frac{\int \frac{1}{-e^2\tan^2(c+dx)-1}\,d\left(1-\sqrt{2}\sqrt{e}\tan(c+dx)\right)}{\sqrt{2}\sqrt{e}}-\frac{\int \frac{1}{-e^2\tan^2(c+dx)-1}\,d\left(\sqrt{2}\sqrt{e}\tan(c+dx)+1\right)}{\sqrt{2}\sqrt{e}}\right)-\frac{1}{2}\int \frac{e-e^2\tan^2(c+dx)}{e^4\tan^4(c+dx)+e^2}\,d\sqrt{e\tan(c+dx)}\right)}{d}-3a\left(\frac{2c}{}\right.}{e^2}}{5e^2}$$
$$\frac{2(a\sec(c+dx)+a)}{5de(e\tan(c+dx))^{5/2}}$$

$$\downarrow 217$$

$$-\frac{10ae\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{e}\tan(c+dx)+1\right)}{\sqrt{2}\sqrt{e}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{e}\tan(c+dx)\right)}{\sqrt{2}\sqrt{e}}\right)-\frac{1}{2}\int\frac{e-e^2\tan^2(c+dx)}{e^4\tan^4(c+dx)+e^2}d\sqrt{e\tan(c+dx)}\right)}{d}-3a\left(\frac{2\cos(c+dx)(e\tan(c+dx))^{3/2}}{de}-\frac{2\cos}{}\right)}{e^2}}{5e^2}$$

$$\frac{2(a\sec(c+dx)+a)}{5de(e\tan(c+dx))^{5/2}}$$

$$\downarrow 1479$$

$$-\frac{10ae\left(\frac{1}{2}\left(\frac{\int-\frac{\sqrt{2}\sqrt{e}-2\sqrt{e\tan(c+dx)}}{e^2\tan^2(c+dx)-\sqrt{2}e^{3/2}\tan(c+dx)+e}d\sqrt{e\tan(c+dx)}}{2\sqrt{2}\sqrt{e}}+\frac{\int-\frac{\sqrt{2}\left(\sqrt{e}+\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{e^2\tan^2(c+dx)+\sqrt{2}e^{3/2}\tan(c+dx)+e}d\sqrt{e\tan(c+dx)}}{2\sqrt{2}\sqrt{e}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{e}\tan(c+d\right.}{\sqrt{2}\sqrt{e}}\right.}{d}-3a\left(\frac{2\cos(c+dx)(e\tan(c+dx))^{3/2}}{de}-\frac{2\cos}{}\right)}{e^2}}{5e^2}$$

$$\frac{2(a\sec(c+dx)+a)}{5de(e\tan(c+dx))^{5/2}}$$

$$\downarrow 25$$

$$-\frac{10ae\left(\frac{1}{2}\left(-\frac{\int\frac{\sqrt{2}\sqrt{e}-2\sqrt{e\tan(c+dx)}}{e^2\tan^2(c+dx)-\sqrt{2}e^{3/2}\tan(c+dx)+e}d\sqrt{e\tan(c+dx)}}{2\sqrt{2}\sqrt{e}}-\frac{\int\frac{\sqrt{2}\left(\sqrt{e}+\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{e^2\tan^2(c+dx)+\sqrt{2}e^{3/2}\tan(c+dx)+e}d\sqrt{e\tan(c+dx)}}{2\sqrt{2}\sqrt{e}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{e}\tan(c+dx)\right.}{\sqrt{2}\sqrt{e}}\right.}{d}-3a\left(\frac{2\cos(c+dx)(e\tan(c+dx))^{3/2}}{de}-\frac{2\cos}{}\right)}{e^2}}{5e^2}$$

$$\frac{2(a\sec(c+dx)+a)}{5de(e\tan(c+dx))^{5/2}}$$

$$\downarrow 27$$

$$-\frac{10ae\left(\frac{1}{2}\left(-\frac{\int\frac{\sqrt{2}\sqrt{e}-2\sqrt{e\tan(c+dx)}}{e^2\tan^2(c+dx)-\sqrt{2}e^{3/2}\tan(c+dx)+e}d\sqrt{e\tan(c+dx)}}{2\sqrt{2}\sqrt{e}}-\frac{\int\frac{\sqrt{e}+\sqrt{2}\sqrt{e\tan(c+dx)}}{e^2\tan^2(c+dx)+\sqrt{2}e^{3/2}\tan(c+dx)+e}d\sqrt{e\tan(c+dx)}}{2\sqrt{e}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{e}\tan(c+dx)\right.}{\sqrt{2}\sqrt{e}}\right.}{d}-3a\left(\frac{2\cos(c+dx)(e\tan(c+dx))^{3/2}}{de}-\frac{2\cos}{}\right)}{e^2}}{5e^2}$$

$$\frac{2(a\sec(c+dx)+a)}{5de(e\tan(c+dx))^{5/2}}$$

$$\downarrow 1103$$

$$\frac{10ae\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{e}\tan(c+dx)+1\right)}{\sqrt{2}\sqrt{e}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{e}\tan(c+dx)\right)}{\sqrt{2}\sqrt{e}}\right)+\frac{1}{2}\left(\frac{\log\left(-\sqrt{2}e^{3/2}\tan(c+dx)+e^2\tan^2(c+dx)+e\right)}{2\sqrt{2}\sqrt{e}}-\frac{\log\left(\sqrt{2}e^{3/2}\tan(c+dx)+e^2\tan^2\right.\right.\right.}{2\sqrt{2}\sqrt{e}}}{d}$$

$$-\frac{}{e^2}$$

$$-\frac{}{5e^2}$$

$$\frac{2(a\sec(c+dx)+a)}{5de(e\tan(c+dx))^{5/2}}$$

input
```
Int[(a + a*Sec[c + d*x])/(e*Tan[c + d*x])^(7/2),x]
```

output
```
(-2*(a + a*Sec[c + d*x]))/(5*d*e*(e*Tan[c + d*x])^(5/2)) - ((-2*(5*a + 3*a
*Sec[c + d*x]))/(d*e*Sqrt[e*Tan[c + d*x]]) - ((10*a*e*((-(ArcTan[1 - Sqrt[
2]*Sqrt[e]*Tan[c + d*x]]/(Sqrt[2]*Sqrt[e])) + ArcTan[1 + Sqrt[2]*Sqrt[e]*T
an[c + d*x]]/(Sqrt[2]*Sqrt[e]))/2 + (Log[e - Sqrt[2]*e^(3/2)*Tan[c + d*x]
+ e^2*Tan[c + d*x]^2]/(2*Sqrt[2]*Sqrt[e]) - Log[e + Sqrt[2]*e^(3/2)*Tan[c
+ d*x] + e^2*Tan[c + d*x]^2]/(2*Sqrt[2]*Sqrt[e]))/2))/d - 3*a*((-2*Cos[c +
 d*x]*EllipticE[c - Pi/4 + d*x, 2]*Sqrt[e*Tan[c + d*x]])/(d*Sqrt[Sin[2*c +
 2*d*x]]) + (2*Cos[c + d*x]*(e*Tan[c + d*x])^(3/2))/(d*e)))/e^2)/(5*e^2)
```

3.110.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3052
```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Simp[Sqrt[a*Sin[e + f*x]]*(Sqrt[b*Cos[e + f*x]]/Sqrt[Sin[2*e
 + 2*f*x]])   Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

rule 3093
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[a^2*(a*Sec[e + f*x])^(m - 2)*((b*Tan[e + f*x])^(n +
1)/(b*f*(m + n - 1))), x] + Simp[a^2*((m - 2)/(m + n - 1))   Int[(a*Sec[e +
 f*x])^(m - 2)*(b*Tan[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (
GtQ[m, 1] || (EqQ[m, 1] && EqQ[n, 1/2])) && NeQ[m + n - 1, 0] && IntegersQ[
2*m, 2*n]
```

rule 3095
```
Int[Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]]/sec[(e_.) + (f_.)*(x_)], x_Symbol]
:> Simp[Sqrt[Cos[e + f*x]]*(Sqrt[b*Tan[e + f*x]]/Sqrt[Sin[e + f*x]])   Int[
Sqrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]], x], x] /; FreeQ[{b, e, f}, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

rule 4370
```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(-(e*Cot[c + d*x])^(m + 1))*((a + b*Csc[c + d*x])/(
d*e*(m + 1))), x] - Simp[1/(e^2*(m + 1))   Int[(e*Cot[c + d*x])^(m + 2)*(a*
(m + 1) + b*(m + 2)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && L
tQ[m, -1]
```

rule 4372
```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(e*Cot[c + d*x])^m, x], x] + Simp[b   Int[
(e*Cot[c + d*x])^m*Csc[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, m}, x]
```

3.110.4 Maple [A] (verified)

Time = 4.59 (sec) , antiderivative size = 546, normalized size of antiderivative = 1.58

method	result
parts	$\frac{2ae\left(-\frac{1}{5e^2(e\tan(dx+c))^{\frac{5}{2}}}+\frac{1}{e^4\sqrt{e\tan(dx+c)}}+\frac{\sqrt{2}\left(\ln\left(\frac{e\tan(dx+c)-\left(e^2\right)^{\frac{1}{4}}\sqrt{e\tan(dx+c)}\sqrt{2}+\sqrt{e^2}}{e\tan(dx+c)+\left(e^2\right)^{\frac{1}{4}}\sqrt{e\tan(dx+c)}\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{e\tan(dx+c)}}{\left(e^2\right)^{\frac{1}{4}}}+1\right)-2\arcta}{8e^4\left(e^2\right)^{\frac{1}{4}}}\right)}{d}$
default	$-\frac{a\sqrt{2}\left(5i\sin(dx+c)^2\sqrt{\cot(dx+c)-\csc(dx+c)+1}\sqrt{\cot(dx+c)-\csc(dx+c)}\,\text{EllipticPi}\left(\sqrt{\csc(dx+c)-\cot(dx+c)+1},\frac{1}{2}-\frac{i}{2},\frac{\sqrt{2}}{2}\right)\sqrt{\csc}\right.$

input
```
int((a+a*sec(d*x+c))/(e*tan(d*x+c))^(7/2),x,method=_RETURNVERBOSE)
```

output
```
2*a/d*e*(-1/5/e^2/(e*tan(d*x+c))^(5/2)+1/e^4/(e*tan(d*x+c))^(1/2)+1/8/e^4/
(e^2)^(1/4)*2^(1/2)*(ln((e*tan(d*x+c)-(e^2)^(1/4)*(e*tan(d*x+c))^(1/2)*2^(
1/2)+(e^2)^(1/2))/(e*tan(d*x+c)+(e^2)^(1/4)*(e*tan(d*x+c))^(1/2)*2^(1/2)+(
e^2)^(1/2)))+2*arctan(2^(1/2)/(e^2)^(1/4)*(e*tan(d*x+c))^(1/2)+1)-2*arctan
(-2^(1/2)/(e^2)^(1/4)*(e*tan(d*x+c))^(1/2)+1)))+1/5*a/d*2^(1/2)/(e*tan(d*x
+c))^(1/2)/e^3*(-6*(csc(d*x+c)-cot(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c)+
1)^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)*EllipticE((csc(d*x+c)-cot(d*x+c)+1)
^(1/2),1/2*2^(1/2))+3*(csc(d*x+c)-cot(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+
c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)*EllipticF((csc(d*x+c)-cot(d*x+c)
+1)^(1/2),1/2*2^(1/2))-6*(csc(d*x+c)-cot(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d
*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)*EllipticE((csc(d*x+c)-cot(d*x
+c)+1)^(1/2),1/2*2^(1/2))*sec(d*x+c)+3*(csc(d*x+c)-cot(d*x+c)+1)^(1/2)*(co
t(d*x+c)-csc(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)*EllipticF((csc(
d*x+c)-cot(d*x+c)+1)^(1/2),1/2*2^(1/2))*sec(d*x+c)+3*2^(1/2)-2^(1/2)*cot(d
*x+c)*csc(d*x+c))
```

3.110.5 Fricas [F(-1)]

Timed out.

$$\int \frac{a+a\sec(c+dx)}{(e\tan(c+dx))^{7/2}}\,dx = \text{Timed out}$$

input
```
integrate((a+a*sec(d*x+c))/(e*tan(d*x+c))^(7/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.110.6 Sympy [F]

$$\int \frac{a + a\sec(c+dx)}{(e\tan(c+dx))^{7/2}}\,dx = a\left(\int \frac{1}{(e\tan(c+dx))^{\frac{7}{2}}}\,dx + \int \frac{\sec(c+dx)}{(e\tan(c+dx))^{\frac{7}{2}}}\,dx\right)$$

input
```
integrate((a+a*sec(d*x+c))/(e*tan(d*x+c))**(7/2),x)
```

output
```
a*(Integral((e*tan(c + d*x))**(-7/2), x) + Integral(sec(c + d*x)/(e*tan(c
+ d*x))**(7/2), x))
```

3.110.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{a + a\sec(c+dx)}{(e\tan(c+dx))^{7/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+a*sec(d*x+c))/(e*tan(d*x+c))^(7/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.110.8 Giac [F]

$$\int \frac{a + a\sec(c+dx)}{(e\tan(c+dx))^{7/2}}\,dx = \int \frac{a\sec(dx+c)+a}{(e\tan(dx+c))^{\frac{7}{2}}}\,dx$$

input
```
integrate((a+a*sec(d*x+c))/(e*tan(d*x+c))^(7/2),x, algorithm="giac")
```

output
```
integrate((a*sec(d*x + c) + a)/(e*tan(d*x + c))^(7/2), x)
```

3.110.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a + a\sec(c + dx)}{(e\tan(c + dx))^{7/2}}\,dx = \int \frac{a + \frac{a}{\cos(c+d\,x)}}{(e\tan(c + d\,x))^{7/2}}\,dx$$

input
```
int((a + a/cos(c + d*x))/(e*tan(c + d*x))^(7/2),x)
```

output
```
int((a + a/cos(c + d*x))/(e*tan(c + d*x))^(7/2), x)
```

3.111 $\int (a + a\sec(c+dx))^2 (e\tan(c+dx))^{5/2}\,dx$

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3.111.1 Optimal result

Integrand size = 25, antiderivative size = 366

$$\int (a + a\sec(c+dx))^2 (e\tan(c+dx))^{5/2}\,dx = \frac{a^2 e^{5/2}\arctan\left(1 - \frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}d}$$
$$- \frac{a^2 e^{5/2}\arctan\left(1 + \frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}d}$$
$$- \frac{a^2 e^{5/2}\log\left(\sqrt{e} + \sqrt{e}\tan(c+dx) - \sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}d}$$
$$+ \frac{a^2 e^{5/2}\log\left(\sqrt{e} + \sqrt{e}\tan(c+dx) + \sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}d}$$
$$+ \frac{12a^2e^2\cos(c+dx)E\left(c - \frac{\pi}{4} + dx\middle|2\right)\sqrt{e\tan(c+dx)}}{5d\sqrt{\sin(2c+2dx)}}$$
$$+ \frac{2a^2 e(e\tan(c+dx))^{3/2}}{3d} - \frac{12a^2 e\cos(c+dx)(e\tan(c+dx))^{3/2}}{5d}$$
$$+ \frac{4a^2 e\sec(c+dx)(e\tan(c+dx))^{3/2}}{5d} + \frac{2a^2(e\tan(c+dx))^{7/2}}{7de}$$

output
```
1/2*a^2*e^(5/2)*arctan(1-2^(1/2)*(e*tan(d*x+c))^(1/2)/e^(1/2))/d*2^(1/2)-1
/2*a^2*e^(5/2)*arctan(1+2^(1/2)*(e*tan(d*x+c))^(1/2)/e^(1/2))/d*2^(1/2)-1/
4*a^2*e^(5/2)*ln(e^(1/2)-2^(1/2)*(e*tan(d*x+c))^(1/2)+e^(1/2)*tan(d*x+c))/
d*2^(1/2)+1/4*a^2*e^(5/2)*ln(e^(1/2)+2^(1/2)*(e*tan(d*x+c))^(1/2)+e^(1/2)*
tan(d*x+c))/d*2^(1/2)-12/5*a^2*e^2*cos(d*x+c)*(sin(c+1/4*Pi+d*x)^2)^(1/2)/
sin(c+1/4*Pi+d*x)*EllipticE(cos(c+1/4*Pi+d*x),2^(1/2))*(e*tan(d*x+c))^(1/2
)/d/sin(2*d*x+2*c)^(1/2)+2/3*a^2*e*(e*tan(d*x+c))^(3/2)/d-12/5*a^2*e*cos(d
*x+c)*(e*tan(d*x+c))^(3/2)/d+4/5*a^2*e*sec(d*x+c)*(e*tan(d*x+c))^(3/2)/d+2
/7*a^2*(e*tan(d*x+c))^(7/2)/d/e
```

3.111.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 12.89 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.32

$$\int (a + a\sec(c + dx))^2 (e\tan(c + dx))^{5/2}\,dx = \frac{2a^2 e \cos^4\left(\frac{1}{2}(c+dx)\right)\sec^4\left(\frac{1}{2}\arctan(\tan(c+dx))\right)(e\tan(c+dx))^{3/2}\left(35 - 42\,\text{Hypergeometri}\right.$$

input
```
Integrate[(a + a*Sec[c + d*x])^2*(e*Tan[c + d*x])^(5/2),x]
```

output
```
(2*a^2*e*Cos[(c + d*x)/2]^4*Sec[ArcTan[Tan[c + d*x]]/2]^4*(e*Tan[c + d*x])
^(3/2)*(35 - 42*Hypergeometric2F1[1/2, 3/4, 7/4, -Tan[c + d*x]^2] - 35*Hyp
ergeometric2F1[3/4, 1, 7/4, -Tan[c + d*x]^2] + 42*Sqrt[Sec[c + d*x]^2] + 1
5*Tan[c + d*x]^2))/(105*d)
```

3.111.3 Rubi [A] (verified)

Time = 0.81 (sec) , antiderivative size = 366, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {3042, 4374, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sec(c + dx) + a)^2 (e\tan(c + dx))^{5/2}\,dx$$

$$\downarrow 3042$$

$$\int \left(a \csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^2 \left(-e \cot\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2} dx$$

$$\downarrow 4374$$

$$\int \left(a^2(e\tan(c+dx))^{5/2}+a^2\sec^2(c+dx)(e\tan(c+dx))^{5/2}+2a^2\sec(c+dx)(e\tan(c+dx))^{5/2}\right) dx$$

$$\downarrow 2009$$

$$\frac{a^2e^{5/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}d}-\frac{a^2e^{5/2}\arctan\left(\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}d}-$$
$$\frac{a^2e^{5/2}\log\left(\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}d}+$$
$$\frac{a^2e^{5/2}\log\left(\sqrt{e}\tan(c+dx)+\sqrt{2}\sqrt{e\tan(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}d}+$$
$$\frac{12a^2e^2\cos(c+dx)E\left(c+dx-\frac{\pi}{4}\middle|2\right)\sqrt{e\tan(c+dx)}}{5d\sqrt{\sin(2c+2dx)}}+\frac{2a^2(e\tan(c+dx))^{7/2}}{7de}+$$
$$\frac{2a^2e(e\tan(c+dx))^{3/2}}{3d}-\frac{12a^2e\cos(c+dx)(e\tan(c+dx))^{3/2}}{5d}+\frac{4a^2e\sec(c+dx)(e\tan(c+dx))^{3/2}}{5d}$$

input

```
Int[(a + a*Sec[c + d*x])^2*(e*Tan[c + d*x])^(5/2),x]
```

output

```
(a^2*e^(5/2)*ArcTan[1 - (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[2]*
d) - (a^2*e^(5/2)*ArcTan[1 + (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sqr
t[2]*d) - (a^2*e^(5/2)*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] - Sqrt[2]*Sqrt[e
*Tan[c + d*x]]])/(2*Sqrt[2]*d) + (a^2*e^(5/2)*Log[Sqrt[e] + Sqrt[e]*Tan[c
+ d*x] + Sqrt[2]*Sqrt[e*Tan[c + d*x]]])/(2*Sqrt[2]*d) + (12*a^2*e^2*Cos[c
+ d*x]*EllipticE[c - Pi/4 + d*x, 2]*Sqrt[e*Tan[c + d*x]])/(5*d*Sqrt[Sin[2*
c + 2*d*x]]) + (2*a^2*e*(e*Tan[c + d*x])^(3/2))/(3*d) - (12*a^2*e*Cos[c +
d*x]*(e*Tan[c + d*x])^(3/2))/(5*d) + (4*a^2*e*Sec[c + d*x]*(e*Tan[c + d*x]
)^(3/2))/(5*d) + (2*a^2*(e*Tan[c + d*x])^(7/2))/(7*d*e)
```

3.111.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4374
```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

3.111.4 Maple [A] (warning: unable to verify)

Time = 6.60 (sec) , antiderivative size = 605, normalized size of antiderivative = 1.65

method	result
parts	$2a^2e\left(\frac{(e\tan(dx+c))^{\frac{3}{2}}}{3}-\frac{e^2\sqrt{2}\left(\ln\left(\frac{e\tan(dx+c)-\left(e^2\right)^{\frac{1}{4}}\sqrt{e\tan(dx+c)}\sqrt{2}+\sqrt{e^2}}{e\tan(dx+c)+\left(e^2\right)^{\frac{1}{4}}\sqrt{e\tan(dx+c)}\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{e\tan(dx+c)}}{\left(e^2\right)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\frac{\sqrt{2}\sqrt{e\tan(dx+}}{\left(e^2\right)^{\frac{1}{4}}}\right.\right.}{8\left(e^2\right)^{\frac{1}{4}}}\right.$
	$\left.\left.\vphantom{\frac{1}{1}}\right)\right)$
	d
default	Expression too large to display

input
```
int((a+a*sec(d*x+c))^2*(e*tan(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2*a^2/d*e*(1/3*(e*tan(d*x+c))^(3/2)-1/8*e^2/(e^2)^(1/4)*2^(1/2)*(ln((e*tan
(d*x+c)-(e^2)^(1/4)*(e*tan(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2))/(e*tan(d*x+c
)+(e^2)^(1/4)*(e*tan(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2)))+2*arctan(2^(1/2)/
(e^2)^(1/4)*(e*tan(d*x+c))^(1/2)+1)-2*arctan(-2^(1/2)/(e^2)^(1/4)*(e*tan(d
*x+c))^(1/2)+1)))+2/7*a^2*(e*tan(d*x+c))^(7/2)/d/e-2/5*a^2/d*2^(1/2)*(-6*(
csc(d*x+c)-cot(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c)+1)^(1/2)*(cot(d*x+c)
-csc(d*x+c))^(1/2)*EllipticE((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2*2^(1/2))*
cos(d*x+c)^3+3*(csc(d*x+c)-cot(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c)+1)^(
1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)*EllipticF((csc(d*x+c)-cot(d*x+c)+1)^(1/
2),1/2*2^(1/2))*cos(d*x+c)^3-6*(csc(d*x+c)-cot(d*x+c)+1)^(1/2)*(cot(d*x+c)
-csc(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)*EllipticE((csc(d*x+c)-c
ot(d*x+c)+1)^(1/2),1/2*2^(1/2))*cos(d*x+c)^2+3*(csc(d*x+c)-cot(d*x+c)+1)^(
1/2)*(cot(d*x+c)-csc(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)*Ellipti
cF((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2*2^(1/2))*cos(d*x+c)^2+3*cos(d*x+c)^
3*2^(1/2)-4*2^(1/2)*cos(d*x+c)^2+2^(1/2))*e^2*(e*tan(d*x+c))^(1/2)/(cos(d*
x+c)^2-1)*sec(d*x+c)*tan(d*x+c)
```

3.111.5 Fricas [F(-1)]

Timed out.

$$\int (a + a\sec(c + dx))^2 (e\tan(c + dx))^{5/2}\, dx = \text{Timed out}$$

input
```
integrate((a+a*sec(d*x+c))^2*(e*tan(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.111.6 Sympy [F(-1)]

Timed out.

$$\int (a + a\sec(c + dx))^2 (e\tan(c + dx))^{5/2}\, dx = \text{Timed out}$$

input
```
integrate((a+a*sec(d*x+c))**2*(e*tan(d*x+c))**(5/2),x)
```

output
```
Timed out
```

3.111.7 Maxima [F(-2)]

Exception generated.

$$\int (a + a\sec(c + dx))^2 (e\tan(c + dx))^{5/2}\, dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+a*sec(d*x+c))^2*(e*tan(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.111.8 Giac [F]

$$\int (a + a\sec(c + dx))^2 (e\tan(c + dx))^{5/2}\, dx = \int (a\sec(dx + c) + a)^2 (e\tan(dx + c))^{\frac{5}{2}}\, dx$$

input
```
integrate((a+a*sec(d*x+c))^2*(e*tan(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
integrate((a*sec(d*x + c) + a)^2*(e*tan(d*x + c))^(5/2), x)
```

3.111.9 Mupad [F(-1)]

Timed out.

$$\int (a + a\sec(c + dx))^2 (e\tan(c + dx))^{5/2}\, dx = \int (e\tan(c + dx))^{5/2} \left(a + \frac{a}{\cos(c + dx)}\right)^2 dx$$

input
```
int((e*tan(c + d*x))^(5/2)*(a + a/cos(c + d*x))^2,x)
```

output
```
int((e*tan(c + d*x))^(5/2)*(a + a/cos(c + d*x))^2, x)
```

3.112 $\int (a + a\sec(c+dx))^2 (e\tan(c+dx))^{3/2}\,dx$

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3.112.1 Optimal result

Integrand size = 25, antiderivative size = 335

$$\int (a + a\sec(c+dx))^2 (e\tan(c+dx))^{3/2}\,dx = \frac{a^2 e^{3/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}d} - \frac{a^2 e^{3/2}\arctan\left(1+\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}d} + \frac{a^2 e^{3/2}\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}d} - \frac{a^2 e^{3/2}\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)+\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}d} - \frac{2a^2e^2\,\text{EllipticF}\left(c-\frac{\pi}{4}+dx,2\right)\sec(c+dx)\sqrt{\sin(2c+2dx)}}{3d\sqrt{e\tan(c+dx)}} + \frac{2a^2 e\sqrt{e\tan(c+dx)}}{d} + \frac{4a^2 e\sec(c+dx)\sqrt{e\tan(c+dx)}}{3d} + \frac{2a^2(e\tan(c+dx))^{5/2}}{5de}$$

output

```
1/2*a^2*e^(3/2)*arctan(1-2^(1/2)*(e*tan(d*x+c))^(1/2)/e^(1/2))/d*2^(1/2)-1
/2*a^2*e^(3/2)*arctan(1+2^(1/2)*(e*tan(d*x+c))^(1/2)/e^(1/2))/d*2^(1/2)+1/
4*a^2*e^(3/2)*ln(e^(1/2)-2^(1/2)*(e*tan(d*x+c))^(1/2)+e^(1/2)*tan(d*x+c))/
d*2^(1/2)-1/4*a^2*e^(3/2)*ln(e^(1/2)+2^(1/2)*(e*tan(d*x+c))^(1/2)+e^(1/2)*
tan(d*x+c))/d*2^(1/2)+2/3*a^2*e^2*(sin(c+1/4*Pi+d*x)^2)^(1/2)/sin(c+1/4*Pi
+d*x)*EllipticF(cos(c+1/4*Pi+d*x),2^(1/2))*sec(d*x+c)*sin(2*d*x+2*c)^(1/2)
/d/(e*tan(d*x+c))^(1/2)+2*a^2*e*(e*tan(d*x+c))^(1/2)/d+4/3*a^2*e*sec(d*x+c
)*(e*tan(d*x+c))^(1/2)/d+2/5*a^2*(e*tan(d*x+c))^(5/2)/d/e
```

3.112.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 26.79 (sec) , antiderivative size = 257, normalized size of antiderivative = 0.77

$$\int (a + a\sec(c+dx))^2 (e\tan(c+dx))^{3/2}\,dx = \frac{a^2\cos^4\left(\frac{1}{2}(c+dx)\right)\sec^4\left(\frac{1}{2}\arctan(\tan(c+dx))\right)(e\tan(c+dx))^{3/2}\left(30\sqrt{2}\arctan\left(1-\sqrt{2}\sqrt{t}\right.\right.}{}$$

input
```
Integrate[(a + a*Sec[c + d*x])^2*(e*Tan[c + d*x])^(3/2),x]
```

output
```
(a^2*Cos[(c + d*x)/2]^4*Sec[ArcTan[Tan[c + d*x]]/2]^4*(e*Tan[c + d*x])^(3/
2)*(30*Sqrt[2]*ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]] - 30*Sqrt[2]*ArcTan[
1 + Sqrt[2]*Sqrt[Tan[c + d*x]]] + 15*Sqrt[2]*Log[1 - Sqrt[2]*Sqrt[Tan[c +
d*x]] + Tan[c + d*x]] - 15*Sqrt[2]*Log[1 + Sqrt[2]*Sqrt[Tan[c + d*x]] + Ta
n[c + d*x]] + 120*Sqrt[Tan[c + d*x]] - 80*Hypergeometric2F1[1/4, 1/2, 5/4,
 -Tan[c + d*x]^2]*Sqrt[Tan[c + d*x]] + 80*Sqrt[Sec[c + d*x]^2]*Sqrt[Tan[c
+ d*x]] + 24*Tan[c + d*x]^(5/2)))/(60*d*Tan[c + d*x]^(3/2))
```

3.112.3 Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 335, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {3042, 4374, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sec(c+dx)+a)^2 (e\tan(c+dx))^{3/2}\,dx$$

$$\downarrow 3042$$

$$\int \left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^2 \left(-e\cot\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2} dx$$

$$\downarrow 4374$$

$$\int \left(a^2(e\tan(c+dx))^{3/2} + a^2\sec^2(c+dx)(e\tan(c+dx))^{3/2} + 2a^2\sec(c+dx)(e\tan(c+dx))^{3/2}\right) dx$$

$$\downarrow \text{2009}$$

$$\frac{a^2e^{3/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}d}-\frac{a^2e^{3/2}\arctan\left(\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}d}+$$
$$\frac{a^2e^{3/2}\log\left(\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}d}-$$
$$\frac{a^2e^{3/2}\log\left(\sqrt{e}\tan(c+dx)+\sqrt{2}\sqrt{e\tan(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}d}-$$
$$\frac{2a^2e^2\sqrt{\sin(2c+2dx)}\sec(c+dx)\,\text{EllipticF}\left(c+dx-\frac{\pi}{4},2\right)}{3d\sqrt{e\tan(c+dx)}}+\frac{2a^2(e\tan(c+dx))^{5/2}}{5de}+$$
$$\frac{2a^2e\sqrt{e\tan(c+dx)}}{d}+\frac{4a^2e\sec(c+dx)\sqrt{e\tan(c+dx)}}{3d}$$

input
```
Int[(a + a*Sec[c + d*x])^2*(e*Tan[c + d*x])^(3/2),x]
```

output
```
(a^2*e^(3/2)*ArcTan[1 - (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[2]*
d) - (a^2*e^(3/2)*ArcTan[1 + (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sqr
t[2]*d) + (a^2*e^(3/2)*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] - Sqrt[2]*Sqrt[e
*Tan[c + d*x]]])/(2*Sqrt[2]*d) - (a^2*e^(3/2)*Log[Sqrt[e] + Sqrt[e]*Tan[c
+ d*x] + Sqrt[2]*Sqrt[e*Tan[c + d*x]]])/(2*Sqrt[2]*d) - (2*a^2*e^2*Ellipti
cF[c - Pi/4 + d*x, 2]*Sec[c + d*x]*Sqrt[Sin[2*c + 2*d*x]])/(3*d*Sqrt[e*Tan
[c + d*x]]) + (2*a^2*e*Sqrt[e*Tan[c + d*x]])/d + (4*a^2*e*Sec[c + d*x]*Sqr
t[e*Tan[c + d*x]])/(3*d) + (2*a^2*(e*Tan[c + d*x])^(5/2))/(5*d*e)
```

3.112.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4374
```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

3.112.4 Maple [A] (verified)

Time = 6.59 (sec) , antiderivative size = 397, normalized size of antiderivative = 1.19

method	result
parts	$\frac{2a^2e\left(\sqrt{e\tan(dx+c)}-\frac{\left(e^2\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\tan(dx+c)+\left(e^2\right)^{\frac{1}{4}}\sqrt{e\tan(dx+c)}\sqrt{2}+\sqrt{e^2}}{e\tan(dx+c)-\left(e^2\right)^{\frac{1}{4}}\sqrt{e\tan(dx+c)}\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{e\tan(dx+c)}}{\left(e^2\right)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\frac{\sqrt{2}\sqrt{e\tan(dx+c)}}{\left(e^2\right)^{\frac{1}{4}}}\right.\right.}{8}}{d}$
default	Expression too large to display

input
```
int((a+a*sec(d*x+c))^2*(e*tan(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2*a^2/d*e*((e*tan(d*x+c))^(1/2)-1/8*(e^2)^(1/4)*2^(1/2)*(ln((e*tan(d*x+c)+
(e^2)^(1/4)*(e*tan(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2))/(e*tan(d*x+c)-(e^2)^
(1/4)*(e*tan(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2)))+2*arctan(2^(1/2)/(e^2)^(1
/4)*(e*tan(d*x+c))^(1/2)+1)-2*arctan(-2^(1/2)/(e^2)^(1/4)*(e*tan(d*x+c))^(
1/2)+1)))+2/5*a^2*(e*tan(d*x+c))^(5/2)/d/e-2/3*a^2/d*2^(1/2)*(-(csc(d*x+c)
-cot(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c
))^(1/2)*EllipticF((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2*2^(1/2))*cos(d*x+c)
^2-(csc(d*x+c)-cot(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c)+1)^(1/2)*(cot(d*
x+c)-csc(d*x+c))^(1/2)*EllipticF((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2*2^(1/
2))*cos(d*x+c)+2^(1/2)*sin(d*x+c))*e*tan(d*x+c)*(e*tan(d*x+c))^(1/2)/(cos(
d*x+c)^2-1)
```

3.112.5 Fricas [F(-1)]

Timed out.

$$\int (a + a\sec(c + dx))^2 (e\tan(c + dx))^{3/2}\, dx = \text{Timed out}$$

input
```
integrate((a+a*sec(d*x+c))^2*(e*tan(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.112.6 Sympy [F]

$$\int (a + a\sec(c+dx))^2 (e\tan(c+dx))^{3/2}\,dx = a^2\left(\int (e\tan(c+dx))^{\frac{3}{2}}\,dx + \int 2(e\tan(c+dx))^{\frac{3}{2}}\sec(c+dx)\,dx + \int (e\tan(c+dx))^{\frac{3}{2}}\sec^2(c+dx)\,dx\right)$$

input
```
integrate((a+a*sec(d*x+c))**2*(e*tan(d*x+c))**(3/2),x)
```

output
```
a**2*(Integral((e*tan(c + d*x))**(3/2), x) + Integral(2*(e*tan(c + d*x))**
(3/2)*sec(c + d*x), x) + Integral((e*tan(c + d*x))**(3/2)*sec(c + d*x)**2,
 x))
```

3.112.7 Maxima [F(-2)]

Exception generated.

$$\int (a + a\sec(c+dx))^2 (e\tan(c+dx))^{3/2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+a*sec(d*x+c))^2*(e*tan(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.112.8 Giac [F]

$$\int (a + a\sec(c+dx))^2 (e\tan(c+dx))^{3/2}\,dx = \int (a\sec(dx+c)+a)^2 (e\tan(dx+c))^{\frac{3}{2}}\,dx$$

input
```
integrate((a+a*sec(d*x+c))^2*(e*tan(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
integrate((a*sec(d*x + c) + a)^2*(e*tan(d*x + c))^(3/2), x)
```

3.112.9 Mupad [F(-1)]

Timed out.

$$\int (a + a\sec(c + dx))^2 (e\tan(c + dx))^{3/2}\, dx = \int (e\tan(c + dx))^{3/2} \left(a + \frac{a}{\cos(c + dx)}\right)^2 dx$$

input
```
int((e*tan(c + d*x))^(3/2)*(a + a/cos(c + d*x))^2,x)
```

output
```
int((e*tan(c + d*x))^(3/2)*(a + a/cos(c + d*x))^2, x)
```

3.113 $\int (a + a\sec(c+dx))^2 \sqrt{e\tan(c+dx)}\,dx$

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3.113.1 Optimal result

Integrand size = 25, antiderivative size = 309

$$\begin{aligned}
&\int (a + a\sec(c+dx))^2 \sqrt{e\tan(c+dx)}\,dx \\
&= -\frac{a^2\sqrt{e}\arctan\left(1 - \frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}d} + \frac{a^2\sqrt{e}\arctan\left(1 + \frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}d} \\
&\quad + \frac{a^2\sqrt{e}\log\left(\sqrt{e} + \sqrt{e}\tan(c+dx) - \sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}d} \\
&\quad - \frac{a^2\sqrt{e}\log\left(\sqrt{e} + \sqrt{e}\tan(c+dx) + \sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}d} \\
&\quad - \frac{4a^2\cos(c+dx)E\left(c - \frac{\pi}{4} + dx\middle|2\right)\sqrt{e\tan(c+dx)}}{d\sqrt{\sin(2c+2dx)}} \\
&\quad + \frac{2a^2(e\tan(c+dx))^{3/2}}{3de} + \frac{4a^2\cos(c+dx)(e\tan(c+dx))^{3/2}}{de}
\end{aligned}$$

output

```
-1/2*a^2*arctan(1-2^(1/2)*(e*tan(d*x+c))^(1/2)/e^(1/2))*e^(1/2)/d*2^(1/2)+
1/2*a^2*arctan(1+2^(1/2)*(e*tan(d*x+c))^(1/2)/e^(1/2))*e^(1/2)/d*2^(1/2)+1
/4*a^2*ln(e^(1/2)-2^(1/2)*(e*tan(d*x+c))^(1/2)+e^(1/2)*tan(d*x+c))*e^(1/2)
/d*2^(1/2)-1/4*a^2*ln(e^(1/2)+2^(1/2)*(e*tan(d*x+c))^(1/2)+e^(1/2)*tan(d*x
+c))*e^(1/2)/d*2^(1/2)+4*a^2*cos(d*x+c)*(sin(c+1/4*Pi+d*x)^2)^(1/2)/sin(c+
1/4*Pi+d*x)*EllipticE(cos(c+1/4*Pi+d*x),2^(1/2))*(e*tan(d*x+c))^(1/2)/d/si
n(2*d*x+2*c)^(1/2)+2/3*a^2*(e*tan(d*x+c))^(3/2)/d/e+4*a^2*cos(d*x+c)*(e*ta
n(d*x+c))^(3/2)/d/e
```

3.113.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 11.68 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.34

$$\int (a + a\sec(c+dx))^2 \sqrt{e\tan(c+dx)}\,dx$$
$$= \frac{4a^2\cos^5\left(\frac{1}{2}(c+dx)\right)\left(1 + 2\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},-\tan^2(c+dx)\right) + \text{Hypergeometric2F1}\left(\frac{3}{4},1,\frac{7}{4},-\right.\right.}{3d}$$

input
```
Integrate[(a + a*Sec[c + d*x])^2*Sqrt[e*Tan[c + d*x]],x]
```

output
```
(4*a^2*Cos[(c + d*x)/2]^5*(1 + 2*Hypergeometric2F1[1/2, 3/4, 7/4, -Tan[c +
 d*x]^2] + Hypergeometric2F1[3/4, 1, 7/4, -Tan[c + d*x]^2])*Sec[c + d*x]*S
ec[ArcTan[Tan[c + d*x]]/2]^4*Sin[(c + d*x)/2]*Sqrt[e*Tan[c + d*x]])/(3*d)
```

3.113.3 Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 309, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {3042, 4374, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sec(c+dx)+a)^2\sqrt{e\tan(c+dx)}\,dx$$
$$\downarrow 3042$$
$$\int \left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^2\sqrt{-e\cot\left(c+dx+\frac{\pi}{2}\right)}dx$$
$$\downarrow 4374$$
$$\int \left(a^2\sqrt{e\tan(c+dx)} + a^2\sec^2(c+dx)\sqrt{e\tan(c+dx)} + 2a^2\sec(c+dx)\sqrt{e\tan(c+dx)}\right)dx$$
$$\downarrow 2009$$

$$-\frac{a^2\sqrt{e}\arctan\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}d}+\frac{a^2\sqrt{e}\arctan\left(\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}d}+\frac{2a^2(e\tan(c+dx))^{3/2}}{3de}+$$
$$\frac{a^2\sqrt{e}\log\left(\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}d}-$$
$$\frac{a^2\sqrt{e}\log\left(\sqrt{e}\tan(c+dx)+\sqrt{2}\sqrt{e\tan(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}d}+\frac{4a^2\cos(c+dx)(e\tan(c+dx))^{3/2}}{de}-$$
$$\frac{4a^2\cos(c+dx)E\left(c+dx-\frac{\pi}{4}\middle|2\right)\sqrt{e\tan(c+dx)}}{d\sqrt{\sin(2c+2dx)}}$$

input
```
Int[(a + a*Sec[c + d*x])^2*Sqrt[e*Tan[c + d*x]],x]
```

output
```
-((a^2*Sqrt[e]*ArcTan[1 - (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[2
]*d)) + (a^2*Sqrt[e]*ArcTan[1 + (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(
Sqrt[2]*d) + (a^2*Sqrt[e]*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] - Sqrt[2]*Sqr
t[e*Tan[c + d*x]]])/(2*Sqrt[2]*d) - (a^2*Sqrt[e]*Log[Sqrt[e] + Sqrt[e]*Tan
[c + d*x] + Sqrt[2]*Sqrt[e*Tan[c + d*x]]])/(2*Sqrt[2]*d) - (4*a^2*Cos[c +
d*x]*EllipticE[c - Pi/4 + d*x, 2]*Sqrt[e*Tan[c + d*x]])/(d*Sqrt[Sin[2*c +
2*d*x]]) + (2*a^2*(e*Tan[c + d*x])^(3/2))/(3*d*e) + (4*a^2*Cos[c + d*x]*(e
*Tan[c + d*x])^(3/2))/(d*e)
```

3.113.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4374
```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

3.113.4 Maple [A] (verified)

Time = 5.43 (sec) , antiderivative size = 530, normalized size of antiderivative = 1.72

method	result
parts	$\frac{a^2 e\sqrt{2}\left(\ln\left(\frac{e\tan(dx+c)-\left(e^2\right)^{\frac{1}{4}}\sqrt{e\tan(dx+c)}\sqrt{2}+\sqrt{e^2}}{e\tan(dx+c)+\left(e^2\right)^{\frac{1}{4}}\sqrt{e\tan(dx+c)}\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{e\tan(dx+c)}}{\left(e^2\right)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\frac{\sqrt{2}\sqrt{e\tan(dx+c)}}{\left(e^2\right)^{\frac{1}{4}}}+1\right)\right)}{4d\left(e^2\right)^{\frac{1}{4}}}+\frac{2}{}$
default	Expression too large to display

input
```
int((a+a*sec(d*x+c))^2*(e*tan(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/4*a^2/d*e/(e^2)^(1/4)*2^(1/2)*(ln((e*tan(d*x+c)-(e^2)^(1/4)*(e*tan(d*x+c
))^(1/2)*2^(1/2)+(e^2)^(1/2))/(e*tan(d*x+c)+(e^2)^(1/4)*(e*tan(d*x+c))^(1/
2)*2^(1/2)+(e^2)^(1/2)))+2*arctan(2^(1/2)/(e^2)^(1/4)*(e*tan(d*x+c))^(1/2)
+1)-2*arctan(-2^(1/2)/(e^2)^(1/4)*(e*tan(d*x+c))^(1/2)+1))+2/3*a^2*(e*tan(
d*x+c))^(3/2)/d/e-2*a^2/d*2^(1/2)*(-2*(csc(d*x+c)-cot(d*x+c)+1)^(1/2)*(cot
(d*x+c)-csc(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)*EllipticE((csc(d
*x+c)-cot(d*x+c)+1)^(1/2),1/2*2^(1/2))*cos(d*x+c)+(csc(d*x+c)-cot(d*x+c)+1
)^(1/2)*(cot(d*x+c)-csc(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)*Elli
pticF((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2*2^(1/2))*cos(d*x+c)-2*(csc(d*x+c
)-cot(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+
c))^(1/2)*EllipticE((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2*2^(1/2))+(csc(d*x+
c)-cot(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x
+c))^(1/2)*EllipticF((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2*2^(1/2))+2^(1/2)*
cos(d*x+c)-2^(1/2))*(e*tan(d*x+c))^(1/2)*csc(d*x+c)
```

3.113.5 Fricas [F(-1)]

Timed out.

$$\int (a + a\sec(c + dx))^2 \sqrt{e\tan(c + dx)}\, dx = \text{Timed out}$$

input
```
integrate((a+a*sec(d*x+c))^2*(e*tan(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.113.6 Sympy [F]

$$\int (a + a\sec(c+dx))^2 \sqrt{e\tan(c+dx)}\,dx = a^2\left(\int \sqrt{e\tan(c+dx)}\,dx + \int 2\sqrt{e\tan(c+dx)}\sec(c+dx)\,dx + \int \sqrt{e\tan(c+dx)}\sec^2(c+dx)\,dx\right)$$

input
```
integrate((a+a*sec(d*x+c))**2*(e*tan(d*x+c))**(1/2),x)
```

output
```
a**2*(Integral(sqrt(e*tan(c + d*x)), x) + Integral(2*sqrt(e*tan(c + d*x))*
sec(c + d*x), x) + Integral(sqrt(e*tan(c + d*x))*sec(c + d*x)**2, x))
```

3.113.7 Maxima [F(-2)]

Exception generated.

$$\int (a + a\sec(c+dx))^2 \sqrt{e\tan(c+dx)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+a*sec(d*x+c))^2*(e*tan(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.113.8 Giac [F]

$$\int (a + a\sec(c+dx))^2 \sqrt{e\tan(c+dx)}\,dx = \int (a\sec(dx+c)+a)^2 \sqrt{e\tan(dx+c)}\,dx$$

input
```
integrate((a+a*sec(d*x+c))^2*(e*tan(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate((a*sec(d*x + c) + a)^2*sqrt(e*tan(d*x + c)), x)
```

3.113.9 Mupad [F(-1)]

Timed out.

$$\int (a + a\sec(c + dx))^2 \sqrt{e\tan(c + dx)}\, dx = \int \sqrt{e\tan(c + dx)} \left(a + \frac{a}{\cos(c + dx)}\right)^2 dx$$

input
```
int((e*tan(c + d*x))^(1/2)*(a + a/cos(c + d*x))^2,x)
```

output
```
int((e*tan(c + d*x))^(1/2)*(a + a/cos(c + d*x))^2, x)
```

3.114 $\int \frac{(a+a\sec(c+dx))^2}{\sqrt{e\tan(c+dx)}}\,dx$

3.114.1 Optimal result . . . 849
3.114.2 Mathematica [C] (warning: unable to verify) . . . 850
3.114.3 Rubi [A] (verified) . . . 850
3.114.4 Maple [A] (verified) . . . 852
3.114.5 Fricas [F(-1)] . . . 852
3.114.6 Sympy [F] . . . 853
3.114.7 Maxima [F(-2)] . . . 853
3.114.8 Giac [F] . . . 853
3.114.9 Mupad [F(-1)] . . . 854

3.114.1 Optimal result

Integrand size = 25, antiderivative size = 278

$$\begin{aligned}\int \frac{(a+a\sec(c+dx))^2}{\sqrt{e\tan(c+dx)}}\,dx &= -\frac{a^2\arctan\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}d\sqrt{e}} \\ &+ \frac{a^2\arctan\left(1+\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}d\sqrt{e}} \\ &- \frac{a^2\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}d\sqrt{e}} \\ &+ \frac{a^2\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)+\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}d\sqrt{e}} \\ &+ \frac{2a^2\,\text{EllipticF}\left(c-\frac{\pi}{4}+dx,2\right)\sec(c+dx)\sqrt{\sin(2c+2dx)}}{d\sqrt{e\tan(c+dx)}} \\ &+ \frac{2a^2\sqrt{e\tan(c+dx)}}{de}\end{aligned}$$

output

```
-1/2*a^2*arctan(1-2^(1/2)*(e*tan(d*x+c))^(1/2)/e^(1/2))/d*2^(1/2)/e^(1/2)+
1/2*a^2*arctan(1+2^(1/2)*(e*tan(d*x+c))^(1/2)/e^(1/2))/d*2^(1/2)/e^(1/2)-1
/4*a^2*ln(e^(1/2)-2^(1/2)*(e*tan(d*x+c))^(1/2)+e^(1/2)*tan(d*x+c))/d*2^(1/
2)/e^(1/2)+1/4*a^2*ln(e^(1/2)+2^(1/2)*(e*tan(d*x+c))^(1/2)+e^(1/2)*tan(d*x
+c))/d*2^(1/2)/e^(1/2)-2*a^2*(sin(c+1/4*Pi+d*x)^2)^(1/2)/sin(c+1/4*Pi+d*x)
*EllipticF(cos(c+1/4*Pi+d*x),2^(1/2))*sec(d*x+c)*sin(2*d*x+2*c)^(1/2)/d/(e
*tan(d*x+c))^(1/2)+2*a^2*(e*tan(d*x+c))^(1/2)/d/e
```

3.114.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 13.01 (sec) , antiderivative size = 220, normalized size of antiderivative = 0.79

$$\int \frac{(a+a\sec(c+dx))^2}{\sqrt{e\tan(c+dx)}}\,dx$$
$$= \frac{a^2\cos^4\left(\frac{1}{2}(c+dx)\right)\sec^4\left(\frac{1}{2}\arctan(\tan(c+dx))\right)\left(-2\sqrt{2}\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)+2\sqrt{2}\arctan\left(1\right.\right.}{}$$

input
```
Integrate[(a + a*Sec[c + d*x])^2/Sqrt[e*Tan[c + d*x]],x]
```

output
```
(a^2*Cos[(c + d*x)/2]^4*Sec[ArcTan[Tan[c + d*x]]/2]^4*(-2*Sqrt[2]*ArcTan[1
 - Sqrt[2]*Sqrt[Tan[c + d*x]]] + 2*Sqrt[2]*ArcTan[1 + Sqrt[2]*Sqrt[Tan[c +
 d*x]]] - Sqrt[2]*Log[1 - Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]] + Sqr
t[2]*Log[1 + Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]] + 8*Sqrt[Tan[c + d
*x]] + 16*Hypergeometric2F1[1/4, 1/2, 5/4, -Tan[c + d*x]^2]*Sqrt[Tan[c + d
*x]])*Sqrt[Tan[c + d*x]])/(4*d*Sqrt[e*Tan[c + d*x]])
```

3.114.3 Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 278, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {3042, 4374, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sec(c+dx)+a)^2}{\sqrt{e\tan(c+dx)}}\,dx$$
$$\downarrow 3042$$
$$\int \frac{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^2}{\sqrt{-e\cot\left(c+dx+\frac{\pi}{2}\right)}}dx$$
$$\downarrow 4374$$
$$\int \left(\frac{a^2}{\sqrt{e\tan(c+dx)}}+\frac{a^2\sec^2(c+dx)}{\sqrt{e\tan(c+dx)}}+\frac{2a^2\sec(c+dx)}{\sqrt{e\tan(c+dx)}}\right)dx$$

$$\downarrow 2009$$

$$-\frac{a^2 \arctan\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}d\sqrt{e}}+\frac{a^2 \arctan\left(\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}d\sqrt{e}}+\frac{2a^2\sqrt{e\tan(c+dx)}}{de}-$$
$$\frac{a^2 \log\left(\sqrt{e\tan(c+dx)}-\sqrt{2}\sqrt{e\tan(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}d\sqrt{e}}+$$
$$\frac{a^2 \log\left(\sqrt{e\tan(c+dx)}+\sqrt{2}\sqrt{e\tan(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}d\sqrt{e}}+$$
$$\frac{2a^2\sqrt{\sin(2c+2dx)}\sec(c+dx)\,\text{EllipticF}\left(c+dx-\frac{\pi}{4},2\right)}{d\sqrt{e\tan(c+dx)}}$$

input
```
Int[(a + a*Sec[c + d*x])^2/Sqrt[e*Tan[c + d*x]],x]
```

output
```
-((a^2*ArcTan[1 - (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[2]*d*Sqrt
[e])) + (a^2*ArcTan[1 + (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[2]*
d*Sqrt[e]) - (a^2*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] - Sqrt[2]*Sqrt[e*Tan[
c + d*x]]])/(2*Sqrt[2]*d*Sqrt[e]) + (a^2*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x
] + Sqrt[2]*Sqrt[e*Tan[c + d*x]]])/(2*Sqrt[2]*d*Sqrt[e]) + (2*a^2*Elliptic
F[c - Pi/4 + d*x, 2]*Sec[c + d*x]*Sqrt[Sin[2*c + 2*d*x]])/(d*Sqrt[e*Tan[c
+ d*x]]) + (2*a^2*Sqrt[e*Tan[c + d*x]])/(d*e)
```

3.114.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4374
```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

3.114.4 Maple [A] (verified)

Time = 5.46 (sec) , antiderivative size = 269, normalized size of antiderivative = 0.97

method	result
parts	$\frac{a^2 (e^2)^{\frac{1}{4}} \sqrt{2} \left(\ln\left(\frac{e\tan(dx+c)+(e^2)^{\frac{1}{4}} \sqrt{e\tan(dx+c)}\sqrt{2}+\sqrt{e^2}}{e\tan(dx+c)-(e^2)^{\frac{1}{4}} \sqrt{e\tan(dx+c)}\sqrt{2}+\sqrt{e^2}} \right) + 2\arctan\left(\frac{\sqrt{2}\sqrt{e\tan(dx+c)}}{(e^2)^{\frac{1}{4}}}+1 \right) - 2\arctan\left(-\frac{\sqrt{2}\sqrt{e\tan(dx+c)}}{(e^2)^{\frac{1}{4}}}+1 \right) \right)}{4de}$
default	Expression too large to display

input
```
int((a+a*sec(d*x+c))^2/(e*tan(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/4*a^2/d/e*(e^2)^(1/4)*2^(1/2)*(ln((e*tan(d*x+c)+(e^2)^(1/4)*(e*tan(d*x+c
))^(1/2)*2^(1/2)+(e^2)^(1/2))/(e*tan(d*x+c)-(e^2)^(1/4)*(e*tan(d*x+c))^(1/
2)*2^(1/2)+(e^2)^(1/2)))+2*arctan(2^(1/2)/(e^2)^(1/4)*(e*tan(d*x+c))^(1/2)
+1)-2*arctan(-2^(1/2)/(e^2)^(1/4)*(e*tan(d*x+c))^(1/2)+1))+2*a^2*(e*tan(d*
x+c))^(1/2)/d/e+2*a^2/d*2^(1/2)*EllipticF((csc(d*x+c)-cot(d*x+c)+1)^(1/2),
1/2*2^(1/2))*(cot(d*x+c)-csc(d*x+c))^(1/2)*(cot(d*x+c)-csc(d*x+c)+1)^(1/2)
*(csc(d*x+c)-cot(d*x+c)+1)^(1/2)/(e*tan(d*x+c))^(1/2)*(1+sec(d*x+c))
```

3.114.5 Fricas [F(-1)]

Timed out.

$$\int \frac{(a + a\sec(c + dx))^2}{\sqrt{e\tan(c + dx)}}\,dx = \text{Timed out}$$

input
```
integrate((a+a*sec(d*x+c))^2/(e*tan(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.114.6 Sympy [F]

$$\int \frac{(a+a\sec(c+dx))^2}{\sqrt{e\tan(c+dx)}}\,dx = a^2\left(\int \frac{1}{\sqrt{e\tan(c+dx)}}\,dx + \int \frac{2\sec(c+dx)}{\sqrt{e\tan(c+dx)}}\,dx + \int \frac{\sec^2(c+dx)}{\sqrt{e\tan(c+dx)}}\,dx\right)$$

input
```
integrate((a+a*sec(d*x+c))**2/(e*tan(d*x+c))**(1/2),x)
```

output
```
a**2*(Integral(1/sqrt(e*tan(c + d*x)), x) + Integral(2*sec(c + d*x)/sqrt(e
*tan(c + d*x)), x) + Integral(sec(c + d*x)**2/sqrt(e*tan(c + d*x)), x))
```

3.114.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+a\sec(c+dx))^2}{\sqrt{e\tan(c+dx)}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+a*sec(d*x+c))^2/(e*tan(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.114.8 Giac [F]

$$\int \frac{(a+a\sec(c+dx))^2}{\sqrt{e\tan(c+dx)}}\,dx = \int \frac{(a\sec(dx+c)+a)^2}{\sqrt{e\tan(dx+c)}}\,dx$$

input
```
integrate((a+a*sec(d*x+c))^2/(e*tan(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate((a*sec(d*x + c) + a)^2/sqrt(e*tan(d*x + c)), x)
```

3.114.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a + a\sec(c + dx))^2}{\sqrt{e\tan(c + dx)}}\,dx = \int \frac{\left(a + \frac{a}{\cos(c+dx)}\right)^2}{\sqrt{e\tan(c + dx)}}\,dx$$

input
```
int((a + a/cos(c + d*x))^2/(e*tan(c + d*x))^(1/2),x)
```

output
```
int((a + a/cos(c + d*x))^2/(e*tan(c + d*x))^(1/2), x)
```

3.115 $\int \frac{(a+a\sec(c+dx))^2}{(e\tan(c+dx))^{3/2}}\,dx$

3.115.1 Optimal result . . . 855
3.115.2 Mathematica [C] (warning: unable to verify) . . . 856
3.115.3 Rubi [A] (verified) . . . 856
3.115.4 Maple [A] (verified) . . . 858
3.115.5 Fricas [F(-1)] . . . 858
3.115.6 Sympy [F] . . . 859
3.115.7 Maxima [F(-2)] . . . 859
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3.115.1 Optimal result

Integrand size = 25, antiderivative size = 310

$$\begin{aligned}\int \frac{(a+a\sec(c+dx))^2}{(e\tan(c+dx))^{3/2}}\,dx &= \frac{a^2\arctan\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}de^{3/2}}\\ &- \frac{a^2\arctan\left(1+\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}de^{3/2}}\\ &- \frac{a^2\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}de^{3/2}}\\ &+ \frac{a^2\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)+\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}de^{3/2}} - \frac{4a^2}{de\sqrt{e\tan(c+dx)}}\\ &- \frac{4a^2\cos(c+dx)}{de\sqrt{e\tan(c+dx)}} - \frac{4a^2\cos(c+dx)E\left(c-\frac{\pi}{4}+dx\middle|2\right)\sqrt{e\tan(c+dx)}}{de^2\sqrt{\sin(2c+2dx)}}\end{aligned}$$

output

```
1/2*a^2*arctan(1-2^(1/2)*(e*tan(d*x+c))^(1/2)/e^(1/2))/d/e^(3/2)*2^(1/2)-1
/2*a^2*arctan(1+2^(1/2)*(e*tan(d*x+c))^(1/2)/e^(1/2))/d/e^(3/2)*2^(1/2)-1/
4*a^2*ln(e^(1/2)-2^(1/2)*(e*tan(d*x+c))^(1/2)+e^(1/2)*tan(d*x+c))/d/e^(3/2
)*2^(1/2)+1/4*a^2*ln(e^(1/2)+2^(1/2)*(e*tan(d*x+c))^(1/2)+e^(1/2)*tan(d*x+
c))/d/e^(3/2)*2^(1/2)-4*a^2/d/e/(e*tan(d*x+c))^(1/2)-4*a^2*cos(d*x+c)/d/e/
(e*tan(d*x+c))^(1/2)+4*a^2*cos(d*x+c)*(sin(c+1/4*Pi+d*x)^2)^(1/2)/sin(c+1/
4*Pi+d*x)*EllipticE(cos(c+1/4*Pi+d*x),2^(1/2))*(e*tan(d*x+c))^(1/2)/d/e^2/
sin(2*d*x+2*c)^(1/2)
```

3.115.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 7.76 (sec) , antiderivative size = 238, normalized size of antiderivative = 0.77

$$\int \frac{(a+a\sec(c+dx))^2}{(e\tan(c+dx))^{3/2}}\,dx = \frac{a^2\left(-24\left(1+e^{i(c+dx)}+e^{2i(c+dx)}+e^{3i(c+dx)}\right)-3\sqrt{-1+e^{4i(c+dx)}}\arctan\left(\sqrt{-1+\right.\right.}$$

input
```
Integrate[(a + a*Sec[c + d*x])^2/(e*Tan[c + d*x])^(3/2),x]
```

output
```
(a^2*(-24*(1 + E^(I*(c + d*x)) + E^((2*I)*(c + d*x)) + E^((3*I)*(c + d*x))
) - 3*Sqrt[-1 + E^((4*I)*(c + d*x))]*ArcTan[Sqrt[-1 + E^((4*I)*(c + d*x))]
] + 6*Sqrt[-1 + E^((2*I)*(c + d*x))]*Sqrt[1 + E^((2*I)*(c + d*x))]*ArcTanh
[Sqrt[(-1 + E^((2*I)*(c + d*x)))/(1 + E^((2*I)*(c + d*x)))]] + 8*E^((3*I)*
(c + d*x))*Sqrt[1 - E^((4*I)*(c + d*x))]*Hypergeometric2F1[1/2, 3/4, 7/4,
E^((4*I)*(c + d*x))]))/(6*d*e*(1 + E^((2*I)*(c + d*x)))*Sqrt[e*Tan[c + d*x
]])
```

3.115.3 Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 310, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {3042, 4374, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sec(c+dx)+a)^2}{(e\tan(c+dx))^{3/2}}\,dx$$

$$\downarrow 3042$$

$$\int \frac{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^2}{\left(-e\cot\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx$$

$$\downarrow 4374$$

$$\int \left(\frac{a^2}{(e\tan(c+dx))^{3/2}}+\frac{a^2\sec^2(c+dx)}{(e\tan(c+dx))^{3/2}}+\frac{2a^2\sec(c+dx)}{(e\tan(c+dx))^{3/2}}\right)dx$$

$$\downarrow 2009$$

$$\frac{a^2 \arctan\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}de^{3/2}} - \frac{a^2 \arctan\left(\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}de^{3/2}} - \frac{a^2 \log\left(\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}de^{3/2}} + \frac{a^2 \log\left(\sqrt{e}\tan(c+dx)+\sqrt{2}\sqrt{e\tan(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}de^{3/2}} - \frac{4a^2\cos(c+dx)E\left(c+dx-\frac{\pi}{4}\middle|2\right)\sqrt{e\tan(c+dx)}}{de^2\sqrt{\sin(2c+2dx)}} - \frac{4a^2}{de\sqrt{e\tan(c+dx)}} - \frac{4a^2\cos(c+dx)}{de\sqrt{e\tan(c+dx)}}$$

input
```
Int[(a + a*Sec[c + d*x])^2/(e*Tan[c + d*x])^(3/2),x]
```

output
```
(a^2*ArcTan[1 - (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[2]*d*e^(3/2
)) - (a^2*ArcTan[1 + (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[2]*d*e
^(3/2)) - (a^2*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] - Sqrt[2]*Sqrt[e*Tan[c +
 d*x]]])/(2*Sqrt[2]*d*e^(3/2)) + (a^2*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] +
 Sqrt[2]*Sqrt[e*Tan[c + d*x]]])/(2*Sqrt[2]*d*e^(3/2)) - (4*a^2)/(d*e*Sqrt[
e*Tan[c + d*x]]) - (4*a^2*Cos[c + d*x])/(d*e*Sqrt[e*Tan[c + d*x]]) - (4*a^
2*Cos[c + d*x]*EllipticE[c - Pi/4 + d*x, 2]*Sqrt[e*Tan[c + d*x]])/(d*e^2*S
qrt[Sin[2*c + 2*d*x]])
```

3.115.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4374
```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

3.115.4 Maple [A] (verified)

Time = 4.95 (sec) , antiderivative size = 536, normalized size of antiderivative = 1.73

method	result
parts	$\frac{2a^2e\left(-\frac{1}{e^2\sqrt{e\tan(dx+c)}}-\frac{\sqrt{2}\left(\ln\left(\frac{e\tan(dx+c)-\left(e^2\right)^{\frac{1}{4}}\sqrt{e\tan(dx+c)}\sqrt{2}+\sqrt{e^2}}{e\tan(dx+c)+\left(e^2\right)^{\frac{1}{4}}\sqrt{e\tan(dx+c)}\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{e\tan(dx+c)}}{\left(e^2\right)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\frac{\sqrt{2}\sqrt{e\tan(dx+}\right.\right.\right.}{\left(e^2\right)^{\frac{1}{4}}}}{8e^2\left(e^2\right)^{\frac{1}{4}}}}{d}$
default	Expression too large to display

input
```
int((a+a*sec(d*x+c))^2/(e*tan(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2*a^2/d*e*(-1/e^2/(e*tan(d*x+c))^(1/2)-1/8/e^2/(e^2)^(1/4)*2^(1/2)*(ln((e*
tan(d*x+c)-(e^2)^(1/4)*(e*tan(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2))/(e*tan(d*
x+c)+(e^2)^(1/4)*(e*tan(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2)))+2*arctan(2^(1/
2)/(e^2)^(1/4)*(e*tan(d*x+c))^(1/2)+1)-2*arctan(-2^(1/2)/(e^2)^(1/4)*(e*ta
n(d*x+c))^(1/2)+1)))-2*a^2/d/e/(e*tan(d*x+c))^(1/2)-2*a^2/d*2^(1/2)/e/(e*t
an(d*x+c))^(1/2)*(-2*(csc(d*x+c)-cot(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c
)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)*EllipticE((csc(d*x+c)-cot(d*x+c)+
1)^(1/2),1/2*2^(1/2))+(csc(d*x+c)-cot(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+
c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)*EllipticF((csc(d*x+c)-cot(d*x+c)
+1)^(1/2),1/2*2^(1/2))-2*(csc(d*x+c)-cot(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d
*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)*EllipticE((csc(d*x+c)-cot(d*x
+c)+1)^(1/2),1/2*2^(1/2))*sec(d*x+c)+(csc(d*x+c)-cot(d*x+c)+1)^(1/2)*(cot(
d*x+c)-csc(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)*EllipticF((csc(d*
x+c)-cot(d*x+c)+1)^(1/2),1/2*2^(1/2))*sec(d*x+c)+2^(1/2))
```

3.115.5 Fricas [F(-1)]

Timed out.

$$\int \frac{(a+a\sec(c+dx))^2}{(e\tan(c+dx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((a+a*sec(d*x+c))^2/(e*tan(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.115.6 Sympy [F]

$$\int \frac{(a+a\sec(c+dx))^2}{(e\tan(c+dx))^{3/2}}\,dx = a^2\left(\int \frac{1}{(e\tan(c+dx))^{\frac{3}{2}}}\,dx + \int \frac{2\sec(c+dx)}{(e\tan(c+dx))^{\frac{3}{2}}}\,dx + \int \frac{\sec^2(c+dx)}{(e\tan(c+dx))^{\frac{3}{2}}}\,dx\right)$$

input
```
integrate((a+a*sec(d*x+c))**2/(e*tan(d*x+c))**(3/2),x)
```

output
```
a**2*(Integral((e*tan(c + d*x))**(-3/2), x) + Integral(2*sec(c + d*x)/(e*t
an(c + d*x))**(3/2), x) + Integral(sec(c + d*x)**2/(e*tan(c + d*x))**(3/2)
, x))
```

3.115.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+a\sec(c+dx))^2}{(e\tan(c+dx))^{3/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+a*sec(d*x+c))^2/(e*tan(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.115.8 Giac [F]

$$\int \frac{(a+a\sec(c+dx))^2}{(e\tan(c+dx))^{3/2}}\,dx = \int \frac{(a\sec(dx+c)+a)^2}{(e\tan(dx+c))^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+a*sec(d*x+c))^2/(e*tan(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
integrate((a*sec(d*x + c) + a)^2/(e*tan(d*x + c))^(3/2), x)
```

3.115.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a + a\sec(c + dx))^2}{(e\tan(c + dx))^{3/2}}\,dx = \int \frac{\left(a + \frac{a}{\cos(c+dx)}\right)^2}{(e\tan(c + dx))^{3/2}}\,dx$$

input
```
int((a + a/cos(c + d*x))^2/(e*tan(c + d*x))^(3/2),x)
```

output
```
int((a + a/cos(c + d*x))^2/(e*tan(c + d*x))^(3/2), x)
```

3.116 $\int \frac{(a+a\sec(c+dx))^2}{(e\tan(c+dx))^{5/2}}\,dx$

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3.116.1 Optimal result

Integrand size = 25, antiderivative size = 316

$$\int \frac{(a+a\sec(c+dx))^2}{(e\tan(c+dx))^{5/2}}\,dx = \frac{a^2\arctan\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}de^{5/2}} - \frac{a^2\arctan\left(1+\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}de^{5/2}} + \frac{a^2\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}de^{5/2}} - \frac{a^2\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)+\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}de^{5/2}} - \frac{4a^2}{3de(e\tan(c+dx))^{3/2}} - \frac{4a^2\sec(c+dx)}{3de(e\tan(c+dx))^{3/2}} - \frac{2a^2\,\text{EllipticF}\left(c-\frac{\pi}{4}+dx,2\right)\sec(c+dx)\sqrt{\sin(2c+2dx)}}{3de^2\sqrt{e\tan(c+dx)}}$$

output

```
1/2*a^2*arctan(1-2^(1/2)*(e*tan(d*x+c))^(1/2)/e^(1/2))/d/e^(5/2)*2^(1/2)-1
/2*a^2*arctan(1+2^(1/2)*(e*tan(d*x+c))^(1/2)/e^(1/2))/d/e^(5/2)*2^(1/2)+1/
4*a^2*ln(e^(1/2)-2^(1/2)*(e*tan(d*x+c))^(1/2)+e^(1/2)*tan(d*x+c))/d/e^(5/2
)*2^(1/2)-1/4*a^2*ln(e^(1/2)+2^(1/2)*(e*tan(d*x+c))^(1/2)+e^(1/2)*tan(d*x+
c))/d/e^(5/2)*2^(1/2)+2/3*a^2*(sin(c+1/4*Pi+d*x)^2)^(1/2)/sin(c+1/4*Pi+d*x
)*EllipticF(cos(c+1/4*Pi+d*x),2^(1/2))*sec(d*x+c)*sin(2*d*x+2*c)^(1/2)/d/e
^2/(e*tan(d*x+c))^(1/2)-4/3*a^2/d/e/(e*tan(d*x+c))^(3/2)-4/3*a^2*sec(d*x+c
)/d/e/(e*tan(d*x+c))^(3/2)
```

3.116.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 15.82 (sec) , antiderivative size = 224, normalized size of antiderivative = 0.71

$$\int \frac{(a+a\sec(c+dx))^2}{(e\tan(c+dx))^{5/2}}\,dx =$$

$$-\frac{a^2\cos^2\left(\frac{1}{2}(c+dx)\right)\cos(c+dx)\cot\left(\frac{1}{2}(c+dx)\right)\sec^4\left(\frac{1}{2}\arctan(\tan(c+dx))\right)\left(16\,\text{Hypergeometric2F1}\left(-\frac{3}{4}\right.\right.}{}$$

input
```
Integrate[(a + a*Sec[c + d*x])^2/(e*Tan[c + d*x])^(5/2),x]
```

output
```
-1/24*(a^2*Cos[(c + d*x)/2]^2*Cos[c + d*x]*Cot[(c + d*x)/2]*Sec[ArcTan[Tan
[c + d*x]]/2]^4*(16*Hypergeometric2F1[-3/4, 1/2, 1/4, -Tan[c + d*x]^2] + 1
6*Hypergeometric2F1[-3/4, 1, 1/4, -Tan[c + d*x]^2] + 3*Sqrt[2]*(2*ArcTan[1
 - Sqrt[2]*Sqrt[Tan[c + d*x]]] - 2*ArcTan[1 + Sqrt[2]*Sqrt[Tan[c + d*x]]]
+ Log[1 - Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]] - Log[1 + Sqrt[2]*Sqr
t[Tan[c + d*x]] + Tan[c + d*x]])*Tan[c + d*x]^(3/2)))/(d*e^2*Sqrt[e*Tan[c
+ d*x]])
```

3.116.3 Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 316, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {3042, 4374, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sec(c+dx)+a)^2}{(e\tan(c+dx))^{5/2}}\,dx$$

$$\downarrow 3042$$

$$\int \frac{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^2}{\left(-e\cot\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}dx$$

$$\downarrow 4374$$

$$\int \left(\frac{a^2}{(e\tan(c+dx))^{5/2}}+\frac{a^2\sec^2(c+dx)}{(e\tan(c+dx))^{5/2}}+\frac{2a^2\sec(c+dx)}{(e\tan(c+dx))^{5/2}}\right)dx$$

$$\downarrow 2009$$

$$\frac{a^2 \arctan\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}de^{5/2}} - \frac{a^2 \arctan\left(\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}de^{5/2}} + \frac{a^2 \log\left(\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}de^{5/2}} - \frac{a^2 \log\left(\sqrt{e}\tan(c+dx)+\sqrt{2}\sqrt{e\tan(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}de^{5/2}} - \frac{2a^2\sqrt{\sin(2c+2dx)}\sec(c+dx)\,\text{EllipticF}\left(c+dx-\frac{\pi}{4},2\right)}{3de^2\sqrt{e\tan(c+dx)}} - \frac{4a^2}{3de(e\tan(c+dx))^{3/2}} - \frac{4a^2\sec(c+dx)}{3de(e\tan(c+dx))^{3/2}}$$

input
```
Int[(a + a*Sec[c + d*x])^2/(e*Tan[c + d*x])^(5/2),x]
```

output
```
(a^2*ArcTan[1 - (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[2]*d*e^(5/2
)) - (a^2*ArcTan[1 + (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[2]*d*e
^(5/2)) + (a^2*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] - Sqrt[2]*Sqrt[e*Tan[c +
 d*x]]])/(2*Sqrt[2]*d*e^(5/2)) - (a^2*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] +
 Sqrt[2]*Sqrt[e*Tan[c + d*x]]])/(2*Sqrt[2]*d*e^(5/2)) - (4*a^2)/(3*d*e*(e*
Tan[c + d*x])^(3/2)) - (4*a^2*Sec[c + d*x])/(3*d*e*(e*Tan[c + d*x])^(3/2))
 - (2*a^2*EllipticF[c - Pi/4 + d*x, 2]*Sec[c + d*x]*Sqrt[Sin[2*c + 2*d*x]]
)/(3*d*e^2*Sqrt[e*Tan[c + d*x]])
```

3.116.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4374
```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

3.116.4 Maple [A] (verified)

Time = 5.55 (sec) , antiderivative size = 483, normalized size of antiderivative = 1.53

method	result
parts	$\frac{2a^2e\left(-\frac{1}{3e^2(e\tan(dx+c))^{\frac{3}{2}}}-\frac{(e^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\tan(dx+c)+(e^2)^{\frac{1}{4}}\sqrt{e\tan(dx+c)}\sqrt{2}+\sqrt{e^2}}{e\tan(dx+c)-(e^2)^{\frac{1}{4}}\sqrt{e\tan(dx+c)}\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{e\tan(dx+c)}}{(e^2)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\frac{\sqrt{2}\sqrt{}\right.\right.\right.}{8e^4}}{d}$
default	$\frac{a^2\sqrt{2}\,(\cos(dx+c)+1)\left(3i\sin(dx+c)\sqrt{\csc(dx+c)-\cot(dx+c)+1}\,\sqrt{\cot(dx+c)-\csc(dx+c)+1}\,\sqrt{\cot(dx+c)-\csc(dx+c)}\,\text{EllipticPi}\left(\sqrt{c}\right.\right.}{}$

input
```
int((a+a*sec(d*x+c))^2/(e*tan(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2*a^2/d*e*(-1/3/e^2/(e*tan(d*x+c))^(3/2)-1/8/e^4*(e^2)^(1/4)*2^(1/2)*(ln((
e*tan(d*x+c)+(e^2)^(1/4)*(e*tan(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2))/(e*tan(
d*x+c)-(e^2)^(1/4)*(e*tan(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2)))+2*arctan(2^(
1/2)/(e^2)^(1/4)*(e*tan(d*x+c))^(1/2)+1)-2*arctan(-2^(1/2)/(e^2)^(1/4)*(e*
tan(d*x+c))^(1/2)+1)))-2/3*a^2/d/e/(e*tan(d*x+c))^(3/2)+1/3*a^2/d*2^(1/2)*
(1-cos(d*x+c))^2*(2*(csc(d*x+c)-cot(d*x+c)+1)^(1/2)*(2-2*csc(d*x+c)+2*cot(
d*x+c))^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)*EllipticF((csc(d*x+c)-cot(d*x+
c)+1)^(1/2),1/2*2^(1/2))*(-cot(d*x+c)+csc(d*x+c))-(1-cos(d*x+c))^4*csc(d*x
+c)^4+1)/((1-cos(d*x+c))^3*csc(d*x+c)^3+cot(d*x+c)-csc(d*x+c))^(1/2)/((1-c
os(d*x+c))*((1-cos(d*x+c))^2*csc(d*x+c)^2-1)*csc(d*x+c))^(1/2)/((1-cos(d*x
+c))^2*csc(d*x+c)^2-1)^2/(-e/((1-cos(d*x+c))^2*csc(d*x+c)^2-1)*(-cot(d*x+c
)+csc(d*x+c)))^(5/2)*csc(d*x+c)^2
```

3.116.5 Fricas [F(-1)]

Timed out.

$$\int \frac{(a+a\sec(c+dx))^2}{(e\tan(c+dx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((a+a*sec(d*x+c))^2/(e*tan(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.116.6 Sympy [F]

$$\int \frac{(a+a\sec(c+dx))^2}{(e\tan(c+dx))^{5/2}}\,dx = a^2\left(\int \frac{1}{(e\tan(c+dx))^{\frac{5}{2}}}\,dx + \int \frac{2\sec(c+dx)}{(e\tan(c+dx))^{\frac{5}{2}}}\,dx + \int \frac{\sec^2(c+dx)}{(e\tan(c+dx))^{\frac{5}{2}}}\,dx\right)$$

input
```
integrate((a+a*sec(d*x+c))**2/(e*tan(d*x+c))**(5/2),x)
```

output
```
a**2*(Integral((e*tan(c + d*x))**(-5/2), x) + Integral(2*sec(c + d*x)/(e*t
an(c + d*x))**(5/2), x) + Integral(sec(c + d*x)**2/(e*tan(c + d*x))**(5/2)
, x))
```

3.116.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+a\sec(c+dx))^2}{(e\tan(c+dx))^{5/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+a*sec(d*x+c))^2/(e*tan(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.116.8 Giac [F]

$$\int \frac{(a+a\sec(c+dx))^2}{(e\tan(c+dx))^{5/2}}\,dx = \int \frac{(a\sec(dx+c)+a)^2}{(e\tan(dx+c))^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+a*sec(d*x+c))^2/(e*tan(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
integrate((a*sec(d*x + c) + a)^2/(e*tan(d*x + c))^(5/2), x)
```

3.116.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a + a\sec(c + dx))^2}{(e\tan(c + dx))^{5/2}}\,dx = \int \frac{\left(a + \frac{a}{\cos(c+dx)}\right)^2}{(e\tan(c + dx))^{5/2}}\,dx$$

input
```
int((a + a/cos(c + d*x))^2/(e*tan(c + d*x))^(5/2),x)
```

output
```
int((a + a/cos(c + d*x))^2/(e*tan(c + d*x))^(5/2), x)
```

3.117 $\int \frac{(a+a\sec(c+dx))^2}{(e\tan(c+dx))^{7/2}}\,dx$

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3.117.1 Optimal result

Integrand size = 25, antiderivative size = 370

$$\begin{aligned}\int \frac{(a+a\sec(c+dx))^2}{(e\tan(c+dx))^{7/2}}\,dx &= -\frac{a^2\arctan\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}de^{7/2}} \\ &+ \frac{a^2\arctan\left(1+\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}de^{7/2}} \\ &+ \frac{a^2\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}de^{7/2}} \\ &- \frac{a^2\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)+\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}de^{7/2}} \\ &- \frac{4a^2}{5de(e\tan(c+dx))^{5/2}} - \frac{4a^2\sec(c+dx)}{5de(e\tan(c+dx))^{5/2}} + \frac{2a^2}{de^3\sqrt{e\tan(c+dx)}} \\ &+ \frac{12a^2\cos(c+dx)}{5de^3\sqrt{e\tan(c+dx)}} + \frac{12a^2\cos(c+dx)E\left(c-\frac{\pi}{4}+dx\middle|2\right)\sqrt{e\tan(c+dx)}}{5de^4\sqrt{\sin(2c+2dx)}}\end{aligned}$$

output
```
-1/2*a^2*arctan(1-2^(1/2)*(e*tan(d*x+c))^(1/2)/e^(1/2))/d/e^(7/2)*2^(1/2)+
1/2*a^2*arctan(1+2^(1/2)*(e*tan(d*x+c))^(1/2)/e^(1/2))/d/e^(7/2)*2^(1/2)+1
/4*a^2*ln(e^(1/2)-2^(1/2)*(e*tan(d*x+c))^(1/2)+e^(1/2)*tan(d*x+c))/d/e^(7/
2)*2^(1/2)-1/4*a^2*ln(e^(1/2)+2^(1/2)*(e*tan(d*x+c))^(1/2)+e^(1/2)*tan(d*x
+c))/d/e^(7/2)*2^(1/2)+2*a^2/d/e^3/(e*tan(d*x+c))^(1/2)+12/5*a^2*cos(d*x+c
)/d/e^3/(e*tan(d*x+c))^(1/2)-12/5*a^2*cos(d*x+c)*(sin(c+1/4*Pi+d*x)^2)^(1/
2)/sin(c+1/4*Pi+d*x)*EllipticE(cos(c+1/4*Pi+d*x),2^(1/2))*(e*tan(d*x+c))^(
1/2)/d/e^4/sin(2*d*x+2*c)^(1/2)-4/5*a^2/d/e/(e*tan(d*x+c))^(5/2)-4/5*a^2*s
ec(d*x+c)/d/e/(e*tan(d*x+c))^(5/2)
```

3.117.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 14.67 (sec) , antiderivative size = 2820, normalized size of antiderivative = 7.62

$$\int \frac{(a+a\sec(c+dx))^2}{(e\tan(c+dx))^{7/2}}\,dx = \text{Result too large to show}$$

input
```
Integrate[(a + a*Sec[c + d*x])^2/(e*Tan[c + d*x])^(7/2),x]
```

output
```
(Sec[c/2 + (d*x)/2]^4*(a + a*Sec[c + d*x])^2*((7*Cot[c/2])/(10*d) - (Cot[c
/2]*Csc[c/2 + (d*x)/2]^2)/(20*d) - (3*(4*Cos[c/2] - Cos[(3*c)/2] + Cos[(5*
c)/2])*Cos[d*x]*Sec[2*c]*Sin[c/2])/(10*d*(-1 + 2*Cos[c])) - (7*Csc[c/2]*Cs
c[c/2 + (d*x)/2]*Sin[(d*x)/2])/(10*d) + (Csc[c/2]*Csc[c/2 + (d*x)/2]^3*Sin
[(d*x)/2])/(20*d) - (3*(2 - 5*Cos[c] + 6*Cos[2*c] + Cos[3*c])*Sec[2*c]*Sin
[d*x])/(20*d*(-1 + 2*Cos[c])))*Sin[c + d*x]^2*Tan[c + d*x]^2)/(e*Tan[c + d
*x])^(7/2) + ((E^((2*I)*c)*Sqrt[-1 + E^((4*I)*(c + d*x))]*ArcTan[Sqrt[-1 +
 E^((4*I)*(c + d*x))]] - 2*Sqrt[-1 + E^((2*I)*(c + d*x))]*Sqrt[1 + E^((2*I
)*(c + d*x))]*ArcTanh[Sqrt[(-1 + E^((2*I)*(c + d*x)))/(1 + E^((2*I)*(c + d
*x)))]])*Cos[c + d*x]^2*Sec[2*c]*Sec[c/2 + (d*x)/2]^4*(a + a*Sec[c + d*x])
^2*Tan[c + d*x]^(7/2))/(16*d*E^(I*c)*Sqrt[((-I)*(-1 + E^((2*I)*(c + d*x)))
)/(1 + E^((2*I)*(c + d*x)))]*(1 + E^((2*I)*(c + d*x)))*(-1 + 2*Cos[c])*(e*
Tan[c + d*x])^(7/2)) + ((-(E^((4*I)*c)*Sqrt[-1 + E^((4*I)*(c + d*x))]*ArcT
an[Sqrt[-1 + E^((4*I)*(c + d*x))]]) + 2*Sqrt[-1 + E^((2*I)*(c + d*x))]*Sqr
t[1 + E^((2*I)*(c + d*x))]*ArcTanh[Sqrt[(-1 + E^((2*I)*(c + d*x)))/(1 + E^
((2*I)*(c + d*x)))]])*Cos[c + d*x]^2*Sec[2*c]*Sec[c/2 + (d*x)/2]^4*(a + a*
Sec[c + d*x])^2*Tan[c + d*x]^(7/2))/(16*d*E^((2*I)*c)*Sqrt[((-I)*(-1 + E^(
(2*I)*(c + d*x))))/(1 + E^((2*I)*(c + d*x)))]*(1 + E^((2*I)*(c + d*x)))*(-
1 + 2*Cos[c])*(e*Tan[c + d*x])^(7/2)) - ((-(E^((6*I)*c)*Sqrt[-1 + E^((4*I)
*(c + d*x))]*ArcTan[Sqrt[-1 + E^((4*I)*(c + d*x))]]) + 2*Sqrt[-1 + E^((...
```

3.117.3 Rubi [A] (verified)

Time = 0.77 (sec) , antiderivative size = 370, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {3042, 4374, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sec(c+dx)+a)^2}{(e\tan(c+dx))^{7/2}}\,dx$$

$$\downarrow 3042$$

$$\int \frac{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^2}{\left(-e\cot\left(c+dx+\frac{\pi}{2}\right)\right)^{7/2}}dx$$

$$\downarrow 4374$$

$$\int \left(\frac{a^2}{(e\tan(c+dx))^{7/2}}+\frac{a^2\sec^2(c+dx)}{(e\tan(c+dx))^{7/2}}+\frac{2a^2\sec(c+dx)}{(e\tan(c+dx))^{7/2}}\right)dx$$

$$\downarrow 2009$$

$$-\frac{a^2\arctan\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}de^{7/2}}+\frac{a^2\arctan\left(\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}de^{7/2}}+$$
$$\frac{a^2\log\left(\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}de^{7/2}}-$$
$$\frac{a^2\log\left(\sqrt{e}\tan(c+dx)+\sqrt{2}\sqrt{e\tan(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}de^{7/2}}+$$
$$\frac{12a^2\cos(c+dx)E\left(c+dx-\frac{\pi}{4}\middle|2\right)\sqrt{e\tan(c+dx)}}{5de^4\sqrt{\sin(2c+2dx)}}+\frac{2a^2}{de^3\sqrt{e\tan(c+dx)}}+\frac{12a^2\cos(c+dx)}{5de^3\sqrt{e\tan(c+dx)}}-$$
$$\frac{4a^2}{5de(e\tan(c+dx))^{5/2}}-\frac{4a^2\sec(c+dx)}{5de(e\tan(c+dx))^{5/2}}$$

input
```
Int[(a + a*Sec[c + d*x])^2/(e*Tan[c + d*x])^(7/2),x]
```

output
```
-((a^2*ArcTan[1 - (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[2]*d*e^(7
/2))) + (a^2*ArcTan[1 + (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[2]*
d*e^(7/2)) + (a^2*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] - Sqrt[2]*Sqrt[e*Tan[
c + d*x]]])/(2*Sqrt[2]*d*e^(7/2)) - (a^2*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x
] + Sqrt[2]*Sqrt[e*Tan[c + d*x]]])/(2*Sqrt[2]*d*e^(7/2)) - (4*a^2)/(5*d*e*
(e*Tan[c + d*x])^(5/2)) - (4*a^2*Sec[c + d*x])/(5*d*e*(e*Tan[c + d*x])^(5/
2)) + (2*a^2)/(d*e^3*Sqrt[e*Tan[c + d*x]]) + (12*a^2*Cos[c + d*x])/(5*d*e^
3*Sqrt[e*Tan[c + d*x]]) + (12*a^2*Cos[c + d*x]*EllipticE[c - Pi/4 + d*x, 2
]*Sqrt[e*Tan[c + d*x]])/(5*d*e^4*Sqrt[Sin[2*c + 2*d*x]])
```

3.117.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4374
```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

3.117.4 Maple [A] (verified)

Time = 4.91 (sec) , antiderivative size = 571, normalized size of antiderivative = 1.54

method	result
parts	$\frac{2a^2e\left(-\frac{1}{5e^2(e\tan(dx+c))^{\frac{5}{2}}}+\frac{1}{e^4\sqrt{e\tan(dx+c)}}+\frac{\sqrt{2}\left(\ln\left(\frac{e\tan(dx+c)-\left(e^2\right)^{\frac{1}{4}}\sqrt{e\tan(dx+c)}\sqrt{2}+\sqrt{e^2}}{e\tan(dx+c)+\left(e^2\right)^{\frac{1}{4}}\sqrt{e\tan(dx+c)}\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{e\tan(dx+c)}}{\left(e^2\right)^{\frac{1}{4}}}+1\right)-2\arc}{8e^4\left(e^2\right)^{\frac{1}{4}}}\right.}{d}$
default	$\frac{a^2\sqrt{2}\left(5i\sin(dx+c)^2\sqrt{\cot(dx+c)-\csc(dx+c)+1}\sqrt{\cot(dx+c)-\csc(dx+c)}\sqrt{\csc(dx+c)-\cot(dx+c)+1}\,\text{EllipticPi}\left(\sqrt{\csc(dx+c)-\cot\right.}$

input
```
int((a+a*sec(d*x+c))^2/(e*tan(d*x+c))^(7/2),x,method=_RETURNVERBOSE)
```

output
```
2*a^2/d*e*(-1/5/e^2/(e*tan(d*x+c))^(5/2)+1/e^4/(e*tan(d*x+c))^(1/2)+1/8/e^
4/(e^2)^(1/4)*2^(1/2)*(ln((e*tan(d*x+c)-(e^2)^(1/4)*(e*tan(d*x+c))^(1/2)*2
^(1/2)+(e^2)^(1/2))/(e*tan(d*x+c)+(e^2)^(1/4)*(e*tan(d*x+c))^(1/2)*2^(1/2)
+(e^2)^(1/2)))+2*arctan(2^(1/2)/(e^2)^(1/4)*(e*tan(d*x+c))^(1/2)+1)-2*arct
an(-2^(1/2)/(e^2)^(1/4)*(e*tan(d*x+c))^(1/2)+1)))-2/5*a^2/d/e/(e*tan(d*x+c
))^(5/2)+2/5*a^2/d*2^(1/2)/(e*tan(d*x+c))^(1/2)/e^3*(-6*(csc(d*x+c)-cot(d*
x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2
)*EllipticE((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2*2^(1/2))+3*(csc(d*x+c)-cot
(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c))^(
1/2)*EllipticF((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2*2^(1/2))-6*(csc(d*x+c)-
cot(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c)
)^(1/2)*EllipticE((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2*2^(1/2))*sec(d*x+c)+
3*(csc(d*x+c)-cot(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c)+1)^(1/2)*(cot(d*x
+c)-csc(d*x+c))^(1/2)*EllipticF((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2*2^(1/2
))*sec(d*x+c)+3*2^(1/2)-2^(1/2)*cot(d*x+c)*csc(d*x+c))
```

3.117.5 Fricas [F(-1)]

Timed out.

$$\int \frac{(a+a\sec(c+dx))^2}{(e\tan(c+dx))^{7/2}}\,dx = \text{Timed out}$$

input
```
integrate((a+a*sec(d*x+c))^2/(e*tan(d*x+c))^(7/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.117.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+a\sec(c+dx))^2}{(e\tan(c+dx))^{7/2}}\,dx = \text{Timed out}$$

input
```
integrate((a+a*sec(d*x+c))**2/(e*tan(d*x+c))**(7/2),x)
```

output
```
Timed out
```

3.117.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+a\sec(c+dx))^2}{(e\tan(c+dx))^{7/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+a*sec(d*x+c))^2/(e*tan(d*x+c))^(7/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.117.8 Giac [F]

$$\int \frac{(a+a\sec(c+dx))^2}{(e\tan(c+dx))^{7/2}}\,dx = \int \frac{(a\sec(dx+c)+a)^2}{(e\tan(dx+c))^{\frac{7}{2}}}\,dx$$

input
```
integrate((a+a*sec(d*x+c))^2/(e*tan(d*x+c))^(7/2),x, algorithm="giac")
```

output
```
integrate((a*sec(d*x + c) + a)^2/(e*tan(d*x + c))^(7/2), x)
```

3.117.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+a\sec(c+dx))^2}{(e\tan(c+dx))^{7/2}}\,dx = \int \frac{\left(a+\frac{a}{\cos(c+dx)}\right)^2}{(e\tan(c+dx))^{7/2}}\,dx$$

input
```
int((a + a/cos(c + d*x))^2/(e*tan(c + d*x))^(7/2),x)
```

output
```
int((a + a/cos(c + d*x))^2/(e*tan(c + d*x))^(7/2), x)
```

3.118 $\int \frac{(e\tan(c+dx))^{11/2}}{a+a\sec(c+dx)}\,dx$

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3.118.1 Optimal result

Integrand size = 25, antiderivative size = 330

$$\int \frac{(e\tan(c+dx))^{11/2}}{a+a\sec(c+dx)}\,dx = \frac{e^{11/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}ad} - \frac{e^{11/2}\arctan\left(1+\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}ad} + \frac{e^{11/2}\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}ad} - \frac{e^{11/2}\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)+\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}ad} + \frac{5e^6\,\text{EllipticF}\left(c-\frac{\pi}{4}+dx,2\right)\sec(c+dx)\sqrt{\sin(2c+2dx)}}{21ad\sqrt{e\tan(c+dx)}} + \frac{2e^5(21-5\sec(c+dx))\sqrt{e\tan(c+dx)}}{21ad} - \frac{2e^3(7-5\sec(c+dx))(e\tan(c+dx))^{5/2}}{35ad}$$

output
```
1/2*e^(11/2)*arctan(1-2^(1/2)*(e*tan(d*x+c))^(1/2)/e^(1/2))/a/d*2^(1/2)-1/
2*e^(11/2)*arctan(1+2^(1/2)*(e*tan(d*x+c))^(1/2)/e^(1/2))/a/d*2^(1/2)+1/4*
e^(11/2)*ln(e^(1/2)-2^(1/2)*(e*tan(d*x+c))^(1/2)+e^(1/2)*tan(d*x+c))/a/d*2
^(1/2)-1/4*e^(11/2)*ln(e^(1/2)+2^(1/2)*(e*tan(d*x+c))^(1/2)+e^(1/2)*tan(d*
x+c))/a/d*2^(1/2)-5/21*e^6*(sin(c+1/4*Pi+d*x)^2)^(1/2)/sin(c+1/4*Pi+d*x)*E
llipticF(cos(c+1/4*Pi+d*x),2^(1/2))*sec(d*x+c)*sin(2*d*x+2*c)^(1/2)/a/d/(e
*tan(d*x+c))^(1/2)+2/21*e^5*(21-5*sec(d*x+c))*(e*tan(d*x+c))^(1/2)/a/d-2/3
5*e^3*(7-5*sec(d*x+c))*(e*tan(d*x+c))^(5/2)/a/d
```

3.118.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 16.03 (sec) , antiderivative size = 332, normalized size of antiderivative = 1.01

$$\int \frac{(e\tan(c+dx))^{11/2}}{a+a\sec(c+dx)}\,dx = \frac{e^5\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\left(1+\sqrt{\sec^2(c+dx)}\right)\sqrt{e\tan(c+dx)}\left(70\sqrt{2}\arct$$

input
```
Integrate[(e*Tan[c + d*x])^(11/2)/(a + a*Sec[c + d*x]),x]
```

output
```
(e^5*Cos[(c + d*x)/2]^2*Sec[c + d*x]*(1 + Sqrt[Sec[c + d*x]^2])*Sqrt[e*Tan
[c + d*x]]*(70*Sqrt[2]*ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]] - 70*Sqrt[2]
*ArcTan[1 + Sqrt[2]*Sqrt[Tan[c + d*x]]] + 35*Sqrt[2]*Log[1 - Sqrt[2]*Sqrt[
Tan[c + d*x]] + Tan[c + d*x]] - 35*Sqrt[2]*Log[1 + Sqrt[2]*Sqrt[Tan[c + d*
x]] + Tan[c + d*x]] + 280*Sqrt[Tan[c + d*x]] - 320*Hypergeometric2F1[-1/2,
 1/4, 5/4, -Tan[c + d*x]^2]*Sqrt[Tan[c + d*x]] + 280*Hypergeometric2F1[1/4
, 1/2, 5/4, -Tan[c + d*x]^2]*Sqrt[Tan[c + d*x]] + 40*Sqrt[Sec[c + d*x]^2]*
Sqrt[Tan[c + d*x]] - 56*Tan[c + d*x]^(5/2) + 40*Sqrt[Sec[c + d*x]^2]*Tan[c
 + d*x]^(5/2)))/(70*a*d*(1 + Sec[c + d*x])^2*Sqrt[Tan[c + d*x]])
```

3.118.3 Rubi [A] (warning: unable to verify)

Time = 1.30 (sec) , antiderivative size = 321, normalized size of antiderivative = 0.97, number of steps used = 28, number of rules used = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 1.080$, Rules used = {3042, 4376, 25, 3042, 4369, 27, 3042, 4369, 27, 3042, 4372, 3042, 3094, 3042, 3053, 3042, 3120, 3957, 266, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e\tan(c+dx))^{11/2}}{a\sec(c+dx)+a}\,dx$$

$$\downarrow 3042$$

$$\int \frac{\left(-e\cot\left(c+dx+\frac{\pi}{2}\right)\right)^{11/2}}{a\csc\left(c+dx+\frac{\pi}{2}\right)+a}dx$$

$$\downarrow 4376$$

$$\frac{e^2\int -\left((a-a\sec(c+dx))(e\tan(c+dx))^{7/2}\right)dx}{a^2}$$

$$\downarrow 25$$

$$-\frac{e^2\int (a-a\sec(c+dx))(e\tan(c+dx))^{7/2}dx}{a^2}$$

$$\downarrow 3042$$

$$-\frac{e^2\int \left(-e\cot\left(c+dx+\frac{\pi}{2}\right)\right)^{7/2}\left(a-a\csc\left(c+dx+\frac{\pi}{2}\right)\right)dx}{a^2}$$

$$\downarrow 4369$$

$$-\frac{e^2\left(\frac{2e(7a-5a\sec(c+dx))(e\tan(c+dx))^{5/2}}{35d}-\frac{2}{7}e^2\int \frac{1}{2}(7a-5a\sec(c+dx))(e\tan(c+dx))^{3/2}dx\right)}{a^2}$$

$$\downarrow 27$$

$$-\frac{e^2\left(\frac{2e(7a-5a\sec(c+dx))(e\tan(c+dx))^{5/2}}{35d}-\frac{1}{7}e^2\int (7a-5a\sec(c+dx))(e\tan(c+dx))^{3/2}dx\right)}{a^2}$$

$$\downarrow 3042$$

$$-\frac{e^2\left(\frac{2e(7a-5a\sec(c+dx))(e\tan(c+dx))^{5/2}}{35d}-\frac{1}{7}e^2\int \left(-e\cot\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}\left(7a-5a\csc\left(c+dx+\frac{\pi}{2}\right)\right)dx\right)}{a^2}$$

$$\downarrow 4369$$

$$-\frac{e^2\left(\frac{2e(7a-5a\sec(c+dx))(e\tan(c+dx))^{5/2}}{35d}-\frac{1}{7}e^2\left(\frac{2e(21a-5a\sec(c+dx))\sqrt{e\tan(c+dx)}}{3d}-\frac{2}{3}e^2\int \frac{21a-5a\sec(c+dx)}{2\sqrt{e\tan(c+dx)}}dx\right)\right)}{a^2}$$

$$\downarrow 27$$

$$-\frac{e^2\left(\frac{2e(7a-5a\sec(c+dx))(e\tan(c+dx))^{5/2}}{35d}-\frac{1}{7}e^2\left(\frac{2e(21a-5a\sec(c+dx))\sqrt{e\tan(c+dx)}}{3d}-\frac{1}{3}e^2\int \frac{21a-5a\sec(c+dx)}{\sqrt{e\tan(c+dx)}}dx\right)\right)}{a^2}$$

$$\downarrow 3042$$

$$-\frac{e^2\left(\frac{2e(7a-5a\sec(c+dx))(e\tan(c+dx))^{5/2}}{35d}-\frac{1}{7}e^2\left(\frac{2e(21a-5a\sec(c+dx))\sqrt{e\tan(c+dx)}}{3d}-\frac{1}{3}e^2\int\frac{21a-5a\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{-e\cot\left(c+dx+\frac{\pi}{2}\right)}}dx\right)\right)}{a^2}$$

$$\downarrow 4372$$

$$-\frac{e^2\left(\frac{2e(7a-5a\sec(c+dx))(e\tan(c+dx))^{5/2}}{35d}-\frac{1}{7}e^2\left(\frac{2e(21a-5a\sec(c+dx))\sqrt{e\tan(c+dx)}}{3d}-\frac{1}{3}e^2\left(21a\int\frac{1}{\sqrt{e\tan(c+dx)}}dx-5a\int\frac{\sec(}{\sqrt{e\ta}}\right.\right.\right.}{a^2}$$

$$\downarrow 3042$$

$$-\frac{e^2\left(\frac{2e(7a-5a\sec(c+dx))(e\tan(c+dx))^{5/2}}{35d}-\frac{1}{7}e^2\left(\frac{2e(21a-5a\sec(c+dx))\sqrt{e\tan(c+dx)}}{3d}-\frac{1}{3}e^2\left(21a\int\frac{1}{\sqrt{e\tan(c+dx)}}dx-5a\int\frac{\sec(}{\sqrt{e\ta}}\right.\right.\right.}{a^2}$$

$$\downarrow 3094$$

$$-\frac{e^2\left(\frac{2e(7a-5a\sec(c+dx))(e\tan(c+dx))^{5/2}}{35d}-\frac{1}{7}e^2\left(\frac{2e(21a-5a\sec(c+dx))\sqrt{e\tan(c+dx)}}{3d}-\frac{1}{3}e^2\left(21a\int\frac{1}{\sqrt{e\tan(c+dx)}}dx-\frac{5a\sqrt{\sin(c+}}{\sqrt{c}}\right.\right.\right.}{a^2}$$

$$\downarrow 3042$$

$$-\frac{e^2\left(\frac{2e(7a-5a\sec(c+dx))(e\tan(c+dx))^{5/2}}{35d}-\frac{1}{7}e^2\left(\frac{2e(21a-5a\sec(c+dx))\sqrt{e\tan(c+dx)}}{3d}-\frac{1}{3}e^2\left(21a\int\frac{1}{\sqrt{e\tan(c+dx)}}dx-\frac{5a\sqrt{\sin(c+}}{\sqrt{c}}\right.\right.\right.}{a^2}$$

$$\downarrow 3053$$

$$-\frac{e^2\left(\frac{2e(7a-5a\sec(c+dx))(e\tan(c+dx))^{5/2}}{35d}-\frac{1}{7}e^2\left(\frac{2e(21a-5a\sec(c+dx))\sqrt{e\tan(c+dx)}}{3d}-\frac{1}{3}e^2\left(21a\int\frac{1}{\sqrt{e\tan(c+dx)}}dx-\frac{5a\sqrt{\sin(2c}}{}\right.\right.\right.}{a^2}$$

$$\downarrow 3042$$

$$-\frac{e^2\left(\frac{2e(7a-5a\sec(c+dx))(e\tan(c+dx))^{5/2}}{35d}-\frac{1}{7}e^2\left(\frac{2e(21a-5a\sec(c+dx))\sqrt{e\tan(c+dx)}}{3d}-\frac{1}{3}e^2\left(21a\int\frac{1}{\sqrt{e\tan(c+dx)}}dx-\frac{5a\sqrt{\sin(2c}}{}\right.\right.\right.}{a^2}$$

$$\downarrow 3120$$

$$-\frac{e^2\left(\frac{2e(7a-5a\sec(c+dx))(e\tan(c+dx))^{5/2}}{35d}-\frac{1}{7}e^2\left(\frac{2e(21a-5a\sec(c+dx))\sqrt{e\tan(c+dx)}}{3d}-\frac{1}{3}e^2\left(21a\int\frac{1}{\sqrt{e\tan(c+dx)}}dx-\frac{5a\sqrt{\sin(2c}}{a^2}\right.\right.\right.$$

$$\downarrow 3957$$

$$-\frac{e^2\left(\frac{2e(7a-5a\sec(c+dx))(e\tan(c+dx))^{5/2}}{35d}-\frac{1}{7}e^2\left(\frac{2e(21a-5a\sec(c+dx))\sqrt{e\tan(c+dx)}}{3d}-\frac{1}{3}e^2\left(\frac{21ae\int\frac{1}{\sqrt{e\tan(c+dx)}\left(\tan^2(c+dx)e^2+e^2\right)}d}{d}\right.\right.\right.}{a^2}$$

$$\downarrow 266$$

$$-\frac{e^2\left(\frac{2e(7a-5a\sec(c+dx))(e\tan(c+dx))^{5/2}}{35d}-\frac{1}{7}e^2\left(\frac{2e(21a-5a\sec(c+dx))\sqrt{e\tan(c+dx)}}{3d}-\frac{1}{3}e^2\left(\frac{42ae\int\frac{1}{e^4\tan^4(c+dx)+e^2}d\sqrt{e\tan(c+dx)}}{d}\right.\right.\right.}{a^2}$$

$$\downarrow 755$$

$$-\frac{e^2\left(\frac{2e(7a-5a\sec(c+dx))(e\tan(c+dx))^{5/2}}{35d}-\frac{1}{7}e^2\left(\frac{2e(21a-5a\sec(c+dx))\sqrt{e\tan(c+dx)}}{3d}-\frac{1}{3}e^2\left(\frac{42ae\left(\frac{\int\frac{e-e^2\tan^2(c+dx)}{e^4\tan^4(c+dx)+e^2}d\sqrt{e\tan(c+dx)}}{2e}\right.\right.\right.\right.}{d}}{a^2}$$

$$\downarrow 1476$$

$$-\frac{e^2\left(\frac{2e(7a-5a\sec(c+dx))(e\tan(c+dx))^{5/2}}{35d}-\frac{1}{7}e^2\left(\frac{2e(21a-5a\sec(c+dx))\sqrt{e\tan(c+dx)}}{3d}-\frac{1}{3}e^2\left(\frac{42ae\left(\frac{1}{2}\int\frac{1}{e^2\tan^2(c+dx)-\sqrt{2}e^{3/2}\tan(c+\right.\right.\right.\right.}{}$$

$$\downarrow 1082$$

$$-\frac{e^2\left(\frac{2e(7a-5a\sec(c+dx))(e\tan(c+dx))^{5/2}}{35d}-\frac{1}{7}e^2\left(\frac{2e(21a-5a\sec(c+dx))\sqrt{e\tan(c+dx)}}{3d}-\frac{1}{3}e^2\left(\frac{42ae\left(\frac{\int\frac{1}{-e^2\tan^2(c+dx)-1}d\left(1-\sqrt{2}\sqrt{e}\tan\right.\right.\right.\right.}{\sqrt{2}\sqrt{e}}\right.\right.\right.\right.}{}$$

$$\downarrow 217$$

$$-\frac{e^2\left(\frac{2e(7a-5a\sec(c+dx))(e\tan(c+dx))^{5/2}}{35d}-\frac{1}{7}e^2\left(\frac{2e(21a-5a\sec(c+dx))\sqrt{e\tan(c+dx)}}{3d}-\frac{1}{3}e^2\left(42ae\left(\frac{\int\frac{e-e^2\tan^2(c+dx)}{e^4\tan^4(c+dx)+e^2}d\sqrt{e\tan(c+dx)}}{2e}\right.\right.\right.\right.}{a^2}$$

$$\downarrow 1479$$

$$-\frac{e^2\left(\frac{2e(7a-5a\sec(c+dx))(e\tan(c+dx))^{5/2}}{35d}-\frac{1}{7}e^2\left(\frac{2e(21a-5a\sec(c+dx))\sqrt{e\tan(c+dx)}}{3d}-\frac{1}{3}e^2\left(42ae\left(-\frac{\int-\frac{\sqrt{2}\sqrt{e}-2\sqrt{e\tan(c+dx)}}{e^2\tan^2(c+dx)-\sqrt{2}e^{3/2}\tan(c+dx)}}{2\sqrt{2}\sqrt{e}}\right.\right.\right.\right.}{a^2}$$

$$\downarrow 25$$

$$-\frac{e^2\left(\frac{2e(7a-5a\sec(c+dx))(e\tan(c+dx))^{5/2}}{35d}-\frac{1}{7}e^2\left(\frac{2e(21a-5a\sec(c+dx))\sqrt{e\tan(c+dx)}}{3d}-\frac{1}{3}e^2\left(42ae\left(\frac{\int\frac{\sqrt{2}\sqrt{e}-2\sqrt{e\tan(c+dx)}}{e^2\tan^2(c+dx)-\sqrt{2}e^{3/2}\tan(c+dx)}}{2\sqrt{2}\sqrt{e}}\right.\right.\right.\right.}{a^2}$$

$$\downarrow 27$$

$$-\frac{e^2\left(\frac{2e(7a-5a\sec(c+dx))(e\tan(c+dx))^{5/2}}{35d}-\frac{1}{7}e^2\left(\frac{2e(21a-5a\sec(c+dx))\sqrt{e\tan(c+dx)}}{3d}-\frac{1}{3}e^2\left(42ae\left(\frac{\int\frac{\sqrt{2}\sqrt{e}-2\sqrt{e\tan(c+dx)}}{e^2\tan^2(c+dx)-\sqrt{2}e^{3/2}\tan(c+dx)}}{2\sqrt{2}\sqrt{e}}\right.\right.\right.\right.}{a^2}$$

$$\downarrow 1103$$

$$-\frac{e^2\left(\frac{2e(7a-5a\sec(c+dx))(e\tan(c+dx))^{5/2}}{35d}-\frac{1}{7}e^2\left(\frac{2e(21a-5a\sec(c+dx))\sqrt{e\tan(c+dx)}}{3d}-\frac{1}{3}e^2\left(\frac{42ae\left(\frac{\arctan\left(\sqrt{2}\sqrt{e}\tan(c+dx)+1\right)}{\sqrt{2}\sqrt{e}}-\frac{\arcta}{2e}\right.\right.\right.\right.}$$

input
```
Int[(e*Tan[c + d*x])^(11/2)/(a + a*Sec[c + d*x]),x]
```

output
```
-((e^2*((2*e*(7*a - 5*a*Sec[c + d*x])*(e*Tan[c + d*x])^(5/2))/(35*d) - (e^
2*((2*e*(21*a - 5*a*Sec[c + d*x])*Sqrt[e*Tan[c + d*x]])/(3*d) - (e^2*((42*
a*e*((-(ArcTan[1 - Sqrt[2]*Sqrt[e]*Tan[c + d*x]]/(Sqrt[2]*Sqrt[e])) + ArcT
an[1 + Sqrt[2]*Sqrt[e]*Tan[c + d*x]]/(Sqrt[2]*Sqrt[e]))/(2*e) + (-1/2*Log[
e - Sqrt[2]*e^(3/2)*Tan[c + d*x] + e^2*Tan[c + d*x]^2]/(Sqrt[2]*Sqrt[e]) +
 Log[e + Sqrt[2]*e^(3/2)*Tan[c + d*x] + e^2*Tan[c + d*x]^2]/(2*Sqrt[2]*Sqr
t[e]))/(2*e)))/d - (5*a*EllipticF[c - Pi/4 + d*x, 2]*Sec[c + d*x]*Sqrt[Sin
[2*c + 2*d*x]])/(d*Sqrt[e*Tan[c + d*x]])))/3))/7))/a^2)
```

3.118.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3053
```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Simp[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]])   Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f
}, x]
```

rule 3094
```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]], x_Symbol]
:> Simp[Sqrt[Sin[e + f*x]]/(Sqrt[Cos[e + f*x]]*Sqrt[b*Tan[e + f*x]])   Int[
1/(Sqrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]]), x], x] /; FreeQ[{b, e, f}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

rule 4369
```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(-e)*(e*Cot[c + d*x])^(m - 1)*((a*m + b*(m - 1)*Csc
[c + d*x])/(d*m*(m - 1))), x] - Simp[e^2/m   Int[(e*Cot[c + d*x])^(m - 2)*(
a*m + b*(m - 1)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && GtQ[m
, 1]
```

rule 4372
```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(e*Cot[c + d*x])^m, x], x] + Simp[b   Int[
(e*Cot[c + d*x])^m*Csc[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, m}, x]
```

rule 4376
```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Simp[a^(2*n)/e^(2*n)   Int[(e*Cot[c + d*x])^(m + 2*
n)/(-a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[a
^2 - b^2, 0] && ILtQ[n, 0]
```

3.118.4 Maple [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 4.64 (sec) , antiderivative size = 1026, normalized size of antiderivative = 3.11

method	result	size
default	Expression too large to display	1026

input
```
int((e*tan(d*x+c))^(11/2)/(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/210/a/d*2^(1/2)*(105*I*(csc(d*x+c)-cot(d*x+c)+1)^(1/2)*EllipticPi((csc(d
*x+c)-cot(d*x+c)+1)^(1/2),1/2-1/2*I,1/2*2^(1/2))*(cot(d*x+c)-csc(d*x+c)+1)
^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)*cos(d*x+c)^4-105*I*(csc(d*x+c)-cot(d*
x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2
)*EllipticPi((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2+1/2*I,1/2*2^(1/2))*cos(d*
x+c)^4+105*I*(csc(d*x+c)-cot(d*x+c)+1)^(1/2)*EllipticPi((csc(d*x+c)-cot(d*
x+c)+1)^(1/2),1/2-1/2*I,1/2*2^(1/2))*(cot(d*x+c)-csc(d*x+c)+1)^(1/2)*(cot(
d*x+c)-csc(d*x+c))^(1/2)*cos(d*x+c)^3-105*I*(csc(d*x+c)-cot(d*x+c)+1)^(1/2
)*(cot(d*x+c)-csc(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)*EllipticPi
((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2+1/2*I,1/2*2^(1/2))*cos(d*x+c)^3+105*(
cot(d*x+c)-csc(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)*(csc(d*x+c)-c
ot(d*x+c)+1)^(1/2)*EllipticPi((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2-1/2*I,1/
2*2^(1/2))*cos(d*x+c)^4-260*(cot(d*x+c)-csc(d*x+c)+1)^(1/2)*(cot(d*x+c)-cs
c(d*x+c))^(1/2)*(csc(d*x+c)-cot(d*x+c)+1)^(1/2)*EllipticF((csc(d*x+c)-cot(
d*x+c)+1)^(1/2),1/2*2^(1/2))*cos(d*x+c)^4+105*(cot(d*x+c)-csc(d*x+c)+1)^(1
/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)*EllipticPi((csc(d*x+c)-cot(d*x+c)+1)^(1/
2),1/2+1/2*I,1/2*2^(1/2))*(csc(d*x+c)-cot(d*x+c)+1)^(1/2)*cos(d*x+c)^4+105
*(cot(d*x+c)-csc(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)*(csc(d*x+c)
-cot(d*x+c)+1)^(1/2)*EllipticPi((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2-1/2*I,
1/2*2^(1/2))*cos(d*x+c)^3-260*(csc(d*x+c)-cot(d*x+c)+1)^(1/2)*(cot(d*x+...
```

3.118.5 Fricas [F(-1)]

Timed out.

$$\int \frac{(e\tan(c+dx))^{11/2}}{a+a\sec(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((e*tan(d*x+c))^(11/2)/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

output
```
Timed out
```

3.118.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(e\tan(c+dx))^{11/2}}{a+a\sec(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((e*tan(d*x+c))**(11/2)/(a+a*sec(d*x+c)),x)
```

output
```
Timed out
```

3.118.7 Maxima [F]

$$\int \frac{(e\tan(c+dx))^{11/2}}{a+a\sec(c+dx)}\,dx = \int \frac{(e\tan(dx+c))^{\frac{11}{2}}}{a\sec(dx+c)+a}\,dx$$

input
```
integrate((e*tan(d*x+c))^(11/2)/(a+a*sec(d*x+c)),x, algorithm="maxima")
```

output
```
integrate((e*tan(d*x + c))^(11/2)/(a*sec(d*x + c) + a), x)
```

3.118.8 Giac [F]

$$\int \frac{(e\tan(c+dx))^{11/2}}{a+a\sec(c+dx)}\,dx = \int \frac{(e\tan(dx+c))^{\frac{11}{2}}}{a\sec(dx+c)+a}\,dx$$

input
```
integrate((e*tan(d*x+c))^(11/2)/(a+a*sec(d*x+c)),x, algorithm="giac")
```

output
```
integrate((e*tan(d*x + c))^(11/2)/(a*sec(d*x + c) + a), x)
```

3.118.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(e\tan(c+dx))^{11/2}}{a+a\sec(c+dx)}\,dx = \int \frac{\cos(c+d\,x)\,(e\tan(c+d\,x))^{11/2}}{a\,(\cos(c+d\,x)+1)}\,dx$$

input
```
int((e*tan(c + d*x))^(11/2)/(a + a/cos(c + d*x)),x)
```

output
```
int((cos(c + d*x)*(e*tan(c + d*x))^(11/2))/(a*(cos(c + d*x) + 1)), x)
```

3.119 $\int \frac{(e\tan(c+dx))^{9/2}}{a+a\sec(c+dx)}\,dx$

3.119.1 Optimal result . . . 885
3.119.2 Mathematica [C] (warning: unable to verify) . . . 886
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3.119.9 Mupad [F(-1)] . . . 895

3.119.1 Optimal result

Integrand size = 25, antiderivative size = 326

$$\int \frac{(e\tan(c+dx))^{9/2}}{a+a\sec(c+dx)}\,dx = -\frac{e^{9/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}ad} + \frac{e^{9/2}\arctan\left(1+\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}ad} + \frac{e^{9/2}\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}ad} - \frac{e^{9/2}\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)+\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}ad} + \frac{6e^4\cos(c+dx)E\left(c-\frac{\pi}{4}+dx\middle|2\right)\sqrt{e\tan(c+dx)}}{5ad\sqrt{\sin(2c+2dx)}} - \frac{6e^3\cos(c+dx)(e\tan(c+dx))^{3/2}}{5ad} - \frac{2e^3(5-3\sec(c+dx))(e\tan(c+dx))^{3/2}}{15ad}$$

output

```
-1/2*e^(9/2)*arctan(1-2^(1/2)*(e*tan(d*x+c))^(1/2)/e^(1/2))/a/d*2^(1/2)+1/
2*e^(9/2)*arctan(1+2^(1/2)*(e*tan(d*x+c))^(1/2)/e^(1/2))/a/d*2^(1/2)+1/4*e
^(9/2)*ln(e^(1/2)-2^(1/2)*(e*tan(d*x+c))^(1/2)+e^(1/2)*tan(d*x+c))/a/d*2^(
1/2)-1/4*e^(9/2)*ln(e^(1/2)+2^(1/2)*(e*tan(d*x+c))^(1/2)+e^(1/2)*tan(d*x+c
))/a/d*2^(1/2)-6/5*e^4*cos(d*x+c)*(sin(c+1/4*Pi+d*x)^2)^(1/2)/sin(c+1/4*Pi
+d*x)*EllipticE(cos(c+1/4*Pi+d*x),2^(1/2))*(e*tan(d*x+c))^(1/2)/a/d/sin(2*
d*x+2*c)^(1/2)-6/5*e^3*cos(d*x+c)*(e*tan(d*x+c))^(3/2)/a/d-2/15*e^3*(5-3*s
ec(d*x+c))*(e*tan(d*x+c))^(3/2)/a/d
```

3.119.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 17.80 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.40

$$\int \frac{(e\tan(c+dx))^{9/2}}{a+a\sec(c+dx)}\,dx = \frac{4e^3\cos^2\left(\frac{1}{2}(c+dx)\right)\left(-1+\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{3}{4},\frac{7}{4},-\tan^2(c+dx)\right)-\text{Hy}\right.}{}$$

input
```
Integrate[(e*Tan[c + d*x])^(9/2)/(a + a*Sec[c + d*x]),x]
```

output
```
(4*e^3*Cos[(c + d*x)/2]^2*(-1 + Hypergeometric2F1[-1/2, 3/4, 7/4, -Tan[c +
 d*x]^2] - Hypergeometric2F1[1/2, 3/4, 7/4, -Tan[c + d*x]^2] + Hypergeomet
ric2F1[3/4, 1, 7/4, -Tan[c + d*x]^2])*Sec[c + d*x]*(1 + Sqrt[Sec[c + d*x]^
2])*(e*Tan[c + d*x])^(3/2))/(3*a*d*(1 + Sec[c + d*x])^2)
```

3.119.3 Rubi [A] (warning: unable to verify)

Time = 1.25 (sec) , antiderivative size = 303, normalized size of antiderivative = 0.93, number of steps used = 27, number of rules used = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 1.040$, Rules used = {3042, 4376, 25, 3042, 4369, 27, 3042, 4372, 3042, 3093, 3042, 3095, 3042, 3052, 3042, 3119, 3957, 266, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e\tan(c+dx))^{9/2}}{a\sec(c+dx)+a}\,dx$$
$$\downarrow 3042$$
$$\int \frac{\left(-e\cot\left(c+dx+\frac{\pi}{2}\right)\right)^{9/2}}{a\csc\left(c+dx+\frac{\pi}{2}\right)+a}dx$$
$$\downarrow 4376$$
$$\frac{e^2\int -((a-a\sec(c+dx))(e\tan(c+dx))^{5/2})\,dx}{a^2}$$
$$\downarrow 25$$
$$-\frac{e^2\int (a-a\sec(c+dx))(e\tan(c+dx))^{5/2}dx}{a^2}$$

$$\downarrow 3042$$

$$-\frac{e^2\int\left(-e\cot\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}\left(a-a\csc\left(c+dx+\frac{\pi}{2}\right)\right)dx}{a^2}$$

$$\downarrow 4369$$

$$-\frac{e^2\left(\frac{2e(5a-3a\sec(c+dx))(e\tan(c+dx))^{3/2}}{15d}-\frac{2}{5}e^2\int\frac{1}{2}(5a-3a\sec(c+dx))\sqrt{e\tan(c+dx)}dx\right)}{a^2}$$

$$\downarrow 27$$

$$-\frac{e^2\left(\frac{2e(5a-3a\sec(c+dx))(e\tan(c+dx))^{3/2}}{15d}-\frac{1}{5}e^2\int(5a-3a\sec(c+dx))\sqrt{e\tan(c+dx)}dx\right)}{a^2}$$

$$\downarrow 3042$$

$$-\frac{e^2\left(\frac{2e(5a-3a\sec(c+dx))(e\tan(c+dx))^{3/2}}{15d}-\frac{1}{5}e^2\int\sqrt{-e\cot\left(c+dx+\frac{\pi}{2}\right)}\left(5a-3a\csc\left(c+dx+\frac{\pi}{2}\right)\right)dx\right)}{a^2}$$

$$\downarrow 4372$$

$$-\frac{e^2\left(\frac{2e(5a-3a\sec(c+dx))(e\tan(c+dx))^{3/2}}{15d}-\frac{1}{5}e^2\left(5a\int\sqrt{e\tan(c+dx)}dx-3a\int\sec(c+dx)\sqrt{e\tan(c+dx)}dx\right)\right)}{a^2}$$

$$\downarrow 3042$$

$$-\frac{e^2\left(\frac{2e(5a-3a\sec(c+dx))(e\tan(c+dx))^{3/2}}{15d}-\frac{1}{5}e^2\left(5a\int\sqrt{e\tan(c+dx)}dx-3a\int\sec(c+dx)\sqrt{e\tan(c+dx)}dx\right)\right)}{a^2}$$

$$\downarrow 3093$$

$$-\frac{e^2\left(\frac{2e(5a-3a\sec(c+dx))(e\tan(c+dx))^{3/2}}{15d}-\frac{1}{5}e^2\left(5a\int\sqrt{e\tan(c+dx)}dx-3a\left(\frac{2\cos(c+dx)(e\tan(c+dx))^{3/2}}{de}-2\int\cos(c+dx\right.\right.\right.}{a^2}$$

$$\downarrow 3042$$

$$-\frac{e^2\left(\frac{2e(5a-3a\sec(c+dx))(e\tan(c+dx))^{3/2}}{15d}-\frac{1}{5}e^2\left(5a\int\sqrt{e\tan(c+dx)}dx-3a\left(\frac{2\cos(c+dx)(e\tan(c+dx))^{3/2}}{de}-2\int\frac{\sqrt{e\tan(c+dx}}{\sec(c+dx)}\right.\right.\right.}{a^2}$$

$$\downarrow 3095$$

$$-\frac{e^2\left(\frac{2e(5a-3a\sec(c+dx))(e\tan(c+dx))^{3/2}}{15d}-\frac{1}{5}e^2\left(5a\int\sqrt{e\tan(c+dx)}dx-3a\left(\frac{2\cos(c+dx)(e\tan(c+dx))^{3/2}}{de}-\frac{2\sqrt{\cos(c+dx)}\sqrt{e}}{\right.\right.\right.}{a^2}$$

$$\downarrow 3042$$

$$-\frac{e^2\left(\frac{2e(5a-3a\sec(c+dx))(e\tan(c+dx))^{3/2}}{15d}-\frac{1}{5}e^2\left(5a\int\sqrt{e\tan(c+dx)}dx-3a\left(\frac{2\cos(c+dx)(e\tan(c+dx))^{3/2}}{de}-\frac{2\sqrt{\cos(c+dx)}\sqrt{e}}{}\right.\right.\right.}{a^2}$$

$$\downarrow 3052$$

$$-\frac{e^2\left(\frac{2e(5a-3a\sec(c+dx))(e\tan(c+dx))^{3/2}}{15d}-\frac{1}{5}e^2\left(5a\int\sqrt{e\tan(c+dx)}dx-3a\left(\frac{2\cos(c+dx)(e\tan(c+dx))^{3/2}}{de}-\frac{2\cos(c+dx)\sqrt{e\tan}}{\sqrt{}}\right.\right.\right.}{a^2}$$

$$\downarrow 3042$$

$$-\frac{e^2\left(\frac{2e(5a-3a\sec(c+dx))(e\tan(c+dx))^{3/2}}{15d}-\frac{1}{5}e^2\left(5a\int\sqrt{e\tan(c+dx)}dx-3a\left(\frac{2\cos(c+dx)(e\tan(c+dx))^{3/2}}{de}-\frac{2\cos(c+dx)\sqrt{e\tan}}{\sqrt{}}\right.\right.\right.}{a^2}$$

$$\downarrow 3119$$

$$-\frac{e^2\left(\frac{2e(5a-3a\sec(c+dx))(e\tan(c+dx))^{3/2}}{15d}-\frac{1}{5}e^2\left(5a\int\sqrt{e\tan(c+dx)}dx-3a\left(\frac{2\cos(c+dx)(e\tan(c+dx))^{3/2}}{de}-\frac{2\cos(c+dx)E(c-}{d\sqrt{}}\right.\right.\right.}{a^2}$$

$$\downarrow 3957$$

$$-\frac{e^2\left(\frac{2e(5a-3a\sec(c+dx))(e\tan(c+dx))^{3/2}}{15d}-\frac{1}{5}e^2\left(\frac{5ae\int\frac{\sqrt{e\tan(c+dx)}}{\tan^2(c+dx)e^2+e^2}d(e\tan(c+dx))}{d}-3a\left(\frac{2\cos(c+dx)(e\tan(c+dx))^{3/2}}{de}-\frac{2\cos(c-}{}\right.\right.\right.}{a^2}$$

$$\downarrow 266$$

$$-\frac{e^2\left(\frac{2e(5a-3a\sec(c+dx))(e\tan(c+dx))^{3/2}}{15d}-\frac{1}{5}e^2\left(\frac{10ae\int\frac{e^2\tan^2(c+dx)}{e^4\tan^4(c+dx)+e^2}d\sqrt{e\tan(c+dx)}}{d}-3a\left(\frac{2\cos(c+dx)(e\tan(c+dx))^{3/2}}{de}-\frac{2\cos(}{}\right.\right.\right.}{a^2}$$

$$\downarrow 826$$

$$-\frac{e^2\left(\frac{2e(5a-3a\sec(c+dx))(e\tan(c+dx))^{3/2}}{15d}-\frac{1}{5}e^2\left(\frac{10ae\left(\frac{1}{2}\int\frac{e^2\tan^2(c+dx)+e}{e^4\tan^4(c+dx)+e^2}d\sqrt{e\tan(c+dx)}-\frac{1}{2}\int\frac{e-e^2\tan^2(c+dx)}{e^4\tan^4(c+dx)+e^2}d\sqrt{e\tan(c+dx)}\right)}{d}-\right.\right.\right.}{a^2}$$

$$\downarrow 1476$$

3.119. $\int\frac{(e\tan(c+dx))^{9/2}}{a+a\sec(c+dx)}dx$

$$-\frac{e^2\left(\frac{2e(5a-3a\sec(c+dx))(e\tan(c+dx))^{3/2}}{15d}-\frac{1}{5}e^2\left(\frac{10ae\left(\frac{1}{2}\left(\frac{1}{2}\int\frac{1}{e^2\tan^2(c+dx)-\sqrt{2}e^{3/2}\tan(c+dx)+e}d\sqrt{e\tan(c+dx)}+\frac{1}{2}\int\frac{1}{e^2\tan^2(c+dx)+\sqrt{2}e}\right.\right.\right.}{d}\right.\right.}{a^2}$$

$$\downarrow 1082$$

$$-\frac{e^2\left(\frac{2e(5a-3a\sec(c+dx))(e\tan(c+dx))^{3/2}}{15d}-\frac{1}{5}e^2\left(\frac{10ae\left(\frac{1}{2}\left(\frac{\int\frac{1}{-e^2\tan^2(c+dx)-1}d\left(1-\sqrt{2}\sqrt{e}\tan(c+dx)\right)}{\sqrt{2}\sqrt{e}}-\frac{\int\frac{1}{-e^2\tan^2(c+dx)-1}d\left(\sqrt{2}\sqrt{e}\tan(c+\right.\right.\right.}{d}\right.\right.}{a^2}$$

$$\downarrow 217$$

$$-\frac{e^2\left(\frac{2e(5a-3a\sec(c+dx))(e\tan(c+dx))^{3/2}}{15d}-\frac{1}{5}e^2\left(\frac{10ae\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{e}\tan(c+dx)+1\right)}{\sqrt{2}\sqrt{e}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{e}\tan(c+dx)\right)}{\sqrt{2}\sqrt{e}}\right)-\frac{1}{2}\int\frac{e-e^2\tan^2(c+dx)}{e^4\tan^4(c+dx)+e^}\right.}{d}\right.}{a^2}$$

$$\downarrow 1479$$

$$-\frac{e^2\left(\frac{2e(5a-3a\sec(c+dx))(e\tan(c+dx))^{3/2}}{15d}-\frac{1}{5}e^2\left(\frac{10ae\left(\frac{1}{2}\left(\frac{\int-\frac{\sqrt{2}\sqrt{e}-2\sqrt{e\tan(c+dx)}}{e^2\tan^2(c+dx)-\sqrt{2}e^{3/2}\tan(c+dx)+e}d\sqrt{e\tan(c+dx)}}{2\sqrt{2}\sqrt{e}}+\frac{\int-\frac{\sqrt{2}\left(\sqrt{e}+\sqrt{2}\sqrt{e\tan}\right.}{e^2\tan^2(c+dx)+\sqrt{2}e^{3/}}\right.\right.\right.}{d}\right.\right.}{a^2}$$

$$\downarrow 25$$

$$-\frac{e^2\left(\frac{2e(5a-3a\sec(c+dx))(e\tan(c+dx))^{3/2}}{15d}-\frac{1}{5}e^2\left(\frac{10ae\left(\frac{1}{2}\left(-\frac{\int\frac{\sqrt{2}\sqrt{e}-2\sqrt{e\tan(c+dx)}}{e^2\tan^2(c+dx)-\sqrt{2}e^{3/2}\tan(c+dx)+e}d\sqrt{e\tan(c+dx)}}{2\sqrt{2}\sqrt{e}}-\frac{\int\frac{\sqrt{2}\left(\sqrt{e}+\sqrt{2}\sqrt{e\tan}\right.}{e^2\tan^2(c+dx)+\sqrt{2}e^{3/2}}\right.\right.\right.}{d}\right.\right.}{a^2}$$

$$\downarrow 27$$

$$-\frac{e^2\left(\frac{2e(5a-3a\sec(c+dx))(e\tan(c+dx))^{3/2}}{15d}-\frac{1}{5}e^2\left(\frac{10ae\left(\frac{1}{2}\left(-\frac{\int\frac{\sqrt{2}\sqrt{e}-2\sqrt{e\tan(c+dx)}}{e^2\tan^2(c+dx)-\sqrt{2}e^{3/2}\tan(c+dx)+e}d\sqrt{e\tan(c+dx)}}{2\sqrt{2}\sqrt{e}}-\int\frac{\sqrt{e}+\sqrt{2}\sqrt{e\tan(c}}{e^2\tan^2(c+dx)+\sqrt{2}e^{3/2}}\right.\right.\right.}{2}}$$

$$\downarrow 1103$$

$$-\frac{e^2\left(\frac{2e(5a-3a\sec(c+dx))(e\tan(c+dx))^{3/2}}{15d}-\frac{1}{5}e^2\left(\frac{10ae\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{e}\tan(c+dx)+1\right)}{\sqrt{2}\sqrt{e}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{e}\tan(c+dx)\right)}{\sqrt{2}\sqrt{e}}\right)+\frac{1}{2}\left(\frac{\log\left(-\sqrt{2}e^{3/2}\tan(\right.}{d}\right.\right.\right.}$$

input
```
Int[(e*Tan[c + d*x])^(9/2)/(a + a*Sec[c + d*x]),x]
```

output
```
-((e^2*((2*e*(5*a - 3*a*Sec[c + d*x])*(e*Tan[c + d*x])^(3/2))/(15*d) - (e^
2*((10*a*e*((-(ArcTan[1 - Sqrt[2]*Sqrt[e]*Tan[c + d*x]]/(Sqrt[2]*Sqrt[e]))
 + ArcTan[1 + Sqrt[2]*Sqrt[e]*Tan[c + d*x]]/(Sqrt[2]*Sqrt[e]))/2 + (Log[e
 - Sqrt[2]*e^(3/2)*Tan[c + d*x] + e^2*Tan[c + d*x]^2]/(2*Sqrt[2]*Sqrt[e]) -
 Log[e + Sqrt[2]*e^(3/2)*Tan[c + d*x] + e^2*Tan[c + d*x]^2]/(2*Sqrt[2]*Sqr
t[e]))/2))/d - 3*a*((-2*Cos[c + d*x]*EllipticE[c - Pi/4 + d*x, 2]*Sqrt[e*T
an[c + d*x]])/(d*Sqrt[Sin[2*c + 2*d*x]]) + (2*Cos[c + d*x]*(e*Tan[c + d*x]
)^(3/2))/(d*e))))/5))/a^2)
```

3.119.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3052
```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Simp[Sqrt[a*Sin[e + f*x]]*(Sqrt[b*Cos[e + f*x]]/Sqrt[Sin[2*e
 + 2*f*x]])   Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

rule 3093
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[a^2*(a*Sec[e + f*x])^(m - 2)*((b*Tan[e + f*x])^(n +
1)/(b*f*(m + n - 1))), x] + Simp[a^2*((m - 2)/(m + n - 1))   Int[(a*Sec[e +
 f*x])^(m - 2)*(b*Tan[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (
GtQ[m, 1] || (EqQ[m, 1] && EqQ[n, 1/2])) && NeQ[m + n - 1, 0] && IntegersQ[
2*m, 2*n]
```

rule 3095
```
Int[Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]]/sec[(e_.) + (f_.)*(x_)], x_Symbol]
:> Simp[Sqrt[Cos[e + f*x]]*(Sqrt[b*Tan[e + f*x]]/Sqrt[Sin[e + f*x]])   Int[
Sqrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]], x], x] /; FreeQ[{b, e, f}, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

rule 4369
```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(-e)*(e*Cot[c + d*x])^(m - 1)*((a*m + b*(m - 1)*Csc
[c + d*x])/(d*m*(m - 1))), x] - Simp[e^2/m   Int[(e*Cot[c + d*x])^(m - 2)*(
a*m + b*(m - 1)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && GtQ[m
, 1]
```

rule 4372
```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(e*Cot[c + d*x])^m, x], x] + Simp[b   Int[
(e*Cot[c + d*x])^m*Csc[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, m}, x]
```

rule 4376
```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Simp[a^(2*n)/e^(2*n)   Int[(e*Cot[c + d*x])^(m + 2*
n)/(-a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[a
^2 - b^2, 0] && ILtQ[n, 0]
```

3.119.4 Maple [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 3.86 (sec) , antiderivative size = 1166, normalized size of antiderivative = 3.58

method	result	size
default	Expression too large to display	1166

input
```
int((e*tan(d*x+c))^(9/2)/(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/30/a/d*2^(1/2)*e^4*(-15*I*(cot(d*x+c)-csc(d*x+c)+1)^(1/2)*(cot(d*x+c)-cs
c(d*x+c))^(1/2)*(csc(d*x+c)-cot(d*x+c)+1)^(1/2)*EllipticPi((csc(d*x+c)-cot
(d*x+c)+1)^(1/2),1/2+1/2*I,1/2*2^(1/2))*cos(d*x+c)^2-15*I*(cot(d*x+c)-csc(
d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)*(csc(d*x+c)-cot(d*x+c)+1)^(1
/2)*EllipticPi((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2+1/2*I,1/2*2^(1/2))*cos(
d*x+c)^3+15*I*(cot(d*x+c)-csc(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2
)*(csc(d*x+c)-cot(d*x+c)+1)^(1/2)*EllipticPi((csc(d*x+c)-cot(d*x+c)+1)^(1/
2),1/2-1/2*I,1/2*2^(1/2))*cos(d*x+c)^3+15*I*(cot(d*x+c)-csc(d*x+c)+1)^(1/2
)*(cot(d*x+c)-csc(d*x+c))^(1/2)*(csc(d*x+c)-cot(d*x+c)+1)^(1/2)*EllipticPi
((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2-1/2*I,1/2*2^(1/2))*cos(d*x+c)^2+18*(c
sc(d*x+c)-cot(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c)+1)^(1/2)*(cot(d*x+c)-
csc(d*x+c))^(1/2)*EllipticF((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2*2^(1/2))*c
os(d*x+c)^3-15*(cot(d*x+c)-csc(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1/
2)*(csc(d*x+c)-cot(d*x+c)+1)^(1/2)*EllipticPi((csc(d*x+c)-cot(d*x+c)+1)^(1
/2),1/2-1/2*I,1/2*2^(1/2))*cos(d*x+c)^3-15*(cot(d*x+c)-csc(d*x+c)+1)^(1/2)
*(cot(d*x+c)-csc(d*x+c))^(1/2)*EllipticPi((csc(d*x+c)-cot(d*x+c)+1)^(1/2),
1/2+1/2*I,1/2*2^(1/2))*(csc(d*x+c)-cot(d*x+c)+1)^(1/2)*cos(d*x+c)^3-36*(cs
c(d*x+c)-cot(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c)+1)^(1/2)*(cot(d*x+c)-c
sc(d*x+c))^(1/2)*EllipticE((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2*2^(1/2))*co
s(d*x+c)^3+18*(csc(d*x+c)-cot(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c)+1)...
```

3.119.5 Fricas [F(-1)]

Timed out.

$$\int \frac{(e\tan(c+dx))^{9/2}}{a+a\sec(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((e*tan(d*x+c))^(9/2)/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

output
```
Timed out
```

3.119.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(e\tan(c+dx))^{9/2}}{a+a\sec(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((e*tan(d*x+c))**(9/2)/(a+a*sec(d*x+c)),x)
```

output
```
Timed out
```

3.119.7 Maxima [F]

$$\int \frac{(e\tan(c+dx))^{9/2}}{a+a\sec(c+dx)}\,dx = \int \frac{(e\tan(dx+c))^{\frac{9}{2}}}{a\sec(dx+c)+a}\,dx$$

input
```
integrate((e*tan(d*x+c))^(9/2)/(a+a*sec(d*x+c)),x, algorithm="maxima")
```

output
```
integrate((e*tan(d*x + c))^(9/2)/(a*sec(d*x + c) + a), x)
```

3.119.8 Giac [F]

$$\int \frac{(e\tan(c+dx))^{9/2}}{a+a\sec(c+dx)}\,dx = \int \frac{(e\tan(dx+c))^{\frac{9}{2}}}{a\sec(dx+c)+a}\,dx$$

input
```
integrate((e*tan(d*x+c))^(9/2)/(a+a*sec(d*x+c)),x, algorithm="giac")
```

output
```
integrate((e*tan(d*x + c))^(9/2)/(a*sec(d*x + c) + a), x)
```

3.119.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(e\tan(c+dx))^{9/2}}{a+a\sec(c+dx)}\,dx = \int \frac{\cos(c+dx)\,(e\tan(c+dx))^{9/2}}{a\,(\cos(c+dx)+1)}\,dx$$

input
```
int((e*tan(c + d*x))^(9/2)/(a + a/cos(c + d*x)),x)
```

output
```
int((cos(c + d*x)*(e*tan(c + d*x))^(9/2))/(a*(cos(c + d*x) + 1)), x)
```

3.120 $\int \frac{(e\tan(c+dx))^{7/2}}{a+a\sec(c+dx)}\,dx$

3.120.1 Optimal result . . . 896
3.120.2 Mathematica [C] (warning: unable to verify) . . . 897
3.120.3 Rubi [A] (warning: unable to verify) . . . 897
3.120.4 Maple [C] (warning: unable to verify) . . . 904
3.120.5 Fricas [F(-1)] . . . 904
3.120.6 Sympy [F(-1)] . . . 905
3.120.7 Maxima [F] . . . 905
3.120.8 Giac [F] . . . 905
3.120.9 Mupad [F(-1)] . . . 906

3.120.1 Optimal result

Integrand size = 25, antiderivative size = 295

$$\begin{aligned}\int \frac{(e\tan(c+dx))^{7/2}}{a+a\sec(c+dx)}\,dx &= -\frac{e^{7/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}ad}\\ &+\frac{e^{7/2}\arctan\left(1+\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}ad}\\ &-\frac{e^{7/2}\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}ad}\\ &+\frac{e^{7/2}\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)+\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}ad}\\ &-\frac{e^4\,\text{EllipticF}\left(c-\frac{\pi}{4}+dx,2\right)\sec(c+dx)\sqrt{\sin(2c+2dx)}}{3ad\sqrt{e\tan(c+dx)}}\\ &-\frac{2e^3(3-\sec(c+dx))\sqrt{e\tan(c+dx)}}{3ad}\end{aligned}$$

output

```
-1/2*e^(7/2)*arctan(1-2^(1/2)*(e*tan(d*x+c))^(1/2)/e^(1/2))/a/d*2^(1/2)+1/
2*e^(7/2)*arctan(1+2^(1/2)*(e*tan(d*x+c))^(1/2)/e^(1/2))/a/d*2^(1/2)-1/4*e
^(7/2)*ln(e^(1/2)-2^(1/2)*(e*tan(d*x+c))^(1/2)+e^(1/2)*tan(d*x+c))/a/d*2^(
1/2)+1/4*e^(7/2)*ln(e^(1/2)+2^(1/2)*(e*tan(d*x+c))^(1/2)+e^(1/2)*tan(d*x+c
))/a/d*2^(1/2)+1/3*e^4*(sin(c+1/4*Pi+d*x)^2)^(1/2)/sin(c+1/4*Pi+d*x)*Ellip
ticF(cos(c+1/4*Pi+d*x),2^(1/2))*sec(d*x+c)*sin(2*d*x+2*c)^(1/2)/a/d/(e*tan
(d*x+c))^(1/2)-2/3*e^3*(3-sec(d*x+c))*(e*tan(d*x+c))^(1/2)/a/d
```

3.120.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 22.33 (sec) , antiderivative size = 271, normalized size of antiderivative = 0.92

$$\int \frac{(e\tan(c+dx))^{7/2}}{a+a\sec(c+dx)}\,dx = \frac{e^3\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\left(1+\sqrt{\sec^2(c+dx)}\right)\left(-2\sqrt{2}\arctan\left(1-\sqrt{2}\sqrt{\text{ta}}\right.\right.}{}$$

input
```
Integrate[(e*Tan[c + d*x])^(7/2)/(a + a*Sec[c + d*x]),x]
```

output
```
(e^3*Cos[(c + d*x)/2]^2*Sec[c + d*x]*(1 + Sqrt[Sec[c + d*x]^2])*(-2*Sqrt[2
]*ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]] + 2*Sqrt[2]*ArcTan[1 + Sqrt[2]*Sq
rt[Tan[c + d*x]]] - Sqrt[2]*Log[1 - Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d
*x]] + Sqrt[2]*Log[1 + Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]] - 8*Sqrt
[Tan[c + d*x]] + 8*Hypergeometric2F1[-1/2, 1/4, 5/4, -Tan[c + d*x]^2]*Sqrt
[Tan[c + d*x]] - 8*Hypergeometric2F1[1/4, 1/2, 5/4, -Tan[c + d*x]^2]*Sqrt[
Tan[c + d*x]])*Sqrt[e*Tan[c + d*x]])/(2*a*d*(1 + Sec[c + d*x])^2*Sqrt[Tan[
c + d*x]])
```

3.120.3 Rubi [A] (warning: unable to verify)

Time = 1.09 (sec) , antiderivative size = 280, normalized size of antiderivative = 0.95, number of steps used = 25, number of rules used = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.960$, Rules used = {3042, 4376, 25, 3042, 4369, 27, 3042, 4372, 3042, 3094, 3042, 3053, 3042, 3120, 3957, 266, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e\tan(c+dx))^{7/2}}{a\sec(c+dx)+a}\,dx$$

$$\downarrow 3042$$

$$\int \frac{(-e\cot\left(c+dx+\frac{\pi}{2}\right))^{7/2}}{a\csc\left(c+dx+\frac{\pi}{2}\right)+a}dx$$

$$\downarrow 4376$$

$$\frac{e^2\int -((a-a\sec(c+dx))(e\tan(c+dx))^{3/2})\,dx}{a^2}$$

$$\downarrow 25$$

$$-\frac{e^2\int (a-a\sec(c+dx))(e\tan(c+dx))^{3/2}dx}{a^2}$$

$$\downarrow 3042$$

$$-\frac{e^2\int \left(-e\cot\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}\left(a-a\csc\left(c+dx+\frac{\pi}{2}\right)\right)dx}{a^2}$$

$$\downarrow 4369$$

$$-\frac{e^2\left(\frac{2e(3a-a\sec(c+dx))\sqrt{e\tan(c+dx)}}{3d}-\frac{2}{3}e^2\int\frac{3a-a\sec(c+dx)}{2\sqrt{e\tan(c+dx)}}dx\right)}{a^2}$$

$$\downarrow 27$$

$$-\frac{e^2\left(\frac{2e(3a-a\sec(c+dx))\sqrt{e\tan(c+dx)}}{3d}-\frac{1}{3}e^2\int\frac{3a-a\sec(c+dx)}{\sqrt{e\tan(c+dx)}}dx\right)}{a^2}$$

$$\downarrow 3042$$

$$-\frac{e^2\left(\frac{2e(3a-a\sec(c+dx))\sqrt{e\tan(c+dx)}}{3d}-\frac{1}{3}e^2\int\frac{3a-a\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{-e\cot\left(c+dx+\frac{\pi}{2}\right)}}dx\right)}{a^2}$$

$$\downarrow 4372$$

$$-\frac{e^2\left(\frac{2e(3a-a\sec(c+dx))\sqrt{e\tan(c+dx)}}{3d}-\frac{1}{3}e^2\left(3a\int\frac{1}{\sqrt{e\tan(c+dx)}}dx-a\int\frac{\sec(c+dx)}{\sqrt{e\tan(c+dx)}}dx\right)\right)}{a^2}$$

$$\downarrow 3042$$

$$-\frac{e^2\left(\frac{2e(3a-a\sec(c+dx))\sqrt{e\tan(c+dx)}}{3d}-\frac{1}{3}e^2\left(3a\int\frac{1}{\sqrt{e\tan(c+dx)}}dx-a\int\frac{\sec(c+dx)}{\sqrt{e\tan(c+dx)}}dx\right)\right)}{a^2}$$

$$\downarrow 3094$$

$$-\frac{e^2\left(\frac{2e(3a-a\sec(c+dx))\sqrt{e\tan(c+dx)}}{3d}-\frac{1}{3}e^2\left(3a\int\frac{1}{\sqrt{e\tan(c+dx)}}dx-\frac{a\sqrt{\sin(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}\sqrt{\sin(c+dx)}}dx}{\sqrt{\cos(c+dx)}\sqrt{e\tan(c+dx)}}\right)\right)}{a^2}$$

$$\downarrow 3042$$

$$-\frac{e^2\left(\frac{2e(3a-a\sec(c+dx))\sqrt{e\tan(c+dx)}}{3d}-\frac{1}{3}e^2\left(3a\int\frac{1}{\sqrt{e\tan(c+dx)}}dx-\frac{a\sqrt{\sin(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}\sqrt{\sin(c+dx)}}dx}{\sqrt{\cos(c+dx)}\sqrt{e\tan(c+dx)}}\right)\right)}{a^2}$$

$$\downarrow 3053$$

$$-\frac{e^2\left(\frac{2e(3a-a\sec(c+dx))\sqrt{e\tan(c+dx)}}{3d}-\frac{1}{3}e^2\left(3a\int\frac{1}{\sqrt{e\tan(c+dx)}}dx-\frac{a\sqrt{\sin(2c+2dx)}\sec(c+dx)\int\frac{1}{\sqrt{\sin(2c+2dx)}}dx}{\sqrt{e\tan(c+dx)}}\right)\right)}{a^2}$$

$$\downarrow 3042$$

$$-\frac{e^2\left(\frac{2e(3a-a\sec(c+dx))\sqrt{e\tan(c+dx)}}{3d}-\frac{1}{3}e^2\left(3a\int\frac{1}{\sqrt{e\tan(c+dx)}}dx-\frac{a\sqrt{\sin(2c+2dx)}\sec(c+dx)\int\frac{1}{\sqrt{\sin(2c+2dx)}}dx}{\sqrt{e\tan(c+dx)}}\right)\right)}{a^2}$$

$$\downarrow 3120$$

$$-\frac{e^2\left(\frac{2e(3a-a\sec(c+dx))\sqrt{e\tan(c+dx)}}{3d}-\frac{1}{3}e^2\left(3a\int\frac{1}{\sqrt{e\tan(c+dx)}}dx-\frac{a\sqrt{\sin(2c+2dx)}\sec(c+dx)\,\text{EllipticF}\left(c+dx-\frac{\pi}{4},2\right)}{d\sqrt{e\tan(c+dx)}}\right)\right)}{a^2}$$

$$\downarrow 3957$$

$$-\frac{e^2\left(\frac{2e(3a-a\sec(c+dx))\sqrt{e\tan(c+dx)}}{3d}-\frac{1}{3}e^2\left(\frac{3ae\int\frac{1}{\sqrt{e\tan(c+dx)}\left(\tan^2(c+dx)e^2+e^2\right)}d(e\tan(c+dx))}{d}-\frac{a\sqrt{\sin(2c+2dx)}\sec(c+dx)\,\text{Elliptic}}{d\sqrt{e\tan(c+dx)}}\right.\right.}{a^2}$$

$$\downarrow 266$$

$$-\frac{e^2\left(\frac{2e(3a-a\sec(c+dx))\sqrt{e\tan(c+dx)}}{3d}-\frac{1}{3}e^2\left(\frac{6ae\int\frac{1}{e^4\tan^4(c+dx)+e^2}d\sqrt{e\tan(c+dx)}}{d}-\frac{a\sqrt{\sin(2c+2dx)}\sec(c+dx)\,\text{EllipticF}\left(c+dx-\frac{\pi}{4},2\right)}{d\sqrt{e\tan(c+dx)}}\right.\right.}{a^2}$$

$$\downarrow 755$$

$$-\frac{e^2\left(\frac{2e(3a-a\sec(c+dx))\sqrt{e\tan(c+dx)}}{3d}-\frac{1}{3}e^2\left(\frac{6ae\left(\frac{\int\frac{e-e^2\tan^2(c+dx)}{e^4\tan^4(c+dx)+e^2}d\sqrt{e\tan(c+dx)}}{2e}+\frac{\int\frac{e^2\tan^2(c+dx)+e}{e^4\tan^4(c+dx)+e^2}d\sqrt{e\tan(c+dx)}}{2e}\right)}{d}-\frac{a\sqrt{\sin(2c+\right.\right.}{a^2}$$

$$\downarrow 1476$$

$$-\frac{e^2\left(\frac{2e(3a-a\sec(c+dx))\sqrt{e\tan(c+dx)}}{3d}-\frac{1}{3}e^2\left(\frac{6ae\left(\frac{\frac{1}{2}\int\frac{1}{e^2\tan^2(c+dx)-\sqrt{2}e^{3/2}\tan(c+dx)+e}d\sqrt{e\tan(c+dx)}+\frac{1}{2}\int\frac{1}{e^2\tan^2(c+dx)+\sqrt{2}e^{3/2}\tan(c+d}}{2e}\right)}{d}\right.\right.}{a^2}$$

3.120. $\int\frac{(e\tan(c+dx))^{7/2}}{a+a\sec(c+dx)}dx$

$$\downarrow 1082$$

$$-\frac{e^2\left(\frac{2e(3a-a\sec(c+dx))\sqrt{e\tan(c+dx)}}{3d}-\frac{1}{3}e^2\left(\frac{6ae\left(\frac{\int\frac{1}{-e^2\tan^2(c+dx)-1}d\left(1-\sqrt{2}\sqrt{e}\tan(c+dx)\right)}{\sqrt{2}\sqrt{e}}-\frac{\int\frac{1}{-e^2\tan^2(c+dx)-1}d\left(\sqrt{2}\sqrt{e}\tan(c+dx)+1\right)}{\sqrt{2}\sqrt{e}}}{2e}+\right.\right.\right.}{d}}{a^2}$$

$$\downarrow 217$$

$$-\frac{e^2\left(\frac{2e(3a-a\sec(c+dx))\sqrt{e\tan(c+dx)}}{3d}-\frac{1}{3}e^2\left(\frac{6ae\left(\frac{\int\frac{e-e^2\tan^2(c+dx)}{e^4\tan^4(c+dx)+e^2}d\sqrt{e\tan(c+dx)}}{2e}+\frac{\arctan\left(\sqrt{2}\sqrt{e}\tan(c+dx)+1\right)}{\sqrt{2}\sqrt{e}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{e}\tan(c+dx\right.\right.\right.\right.}{\sqrt{2}\sqrt{e}}}{d}}{a^2}$$

$$\downarrow 1479$$

$$-\frac{e^2\left(\frac{2e(3a-a\sec(c+dx))\sqrt{e\tan(c+dx)}}{3d}-\frac{1}{3}e^2\left(\frac{6ae\left(\frac{-\frac{\int-\frac{\sqrt{2}\sqrt{e}-2\sqrt{e\tan(c+dx)}}{e^2\tan^2(c+dx)-\sqrt{2}e^{3/2}\tan(c+dx)+e}d\sqrt{e\tan(c+dx)}}{2\sqrt{2}\sqrt{e}}-\frac{\int-\frac{\sqrt{2}\left(\sqrt{e}+\sqrt{2}\sqrt{e\tan(c+dx)}\right.}{e^2\tan^2(c+dx)+\sqrt{2}e^{3/2}\tan(c+dx)}}{2\sqrt{2}\sqrt{e}}}{2e}\right.\right.\right.}{d}}{a^2}$$

$$\downarrow 25$$

$$-\frac{e^2\left(\frac{2e(3a-a\sec(c+dx))\sqrt{e\tan(c+dx)}}{3d}-\frac{1}{3}e^2\left(\frac{6ae\left(\frac{\int\frac{\sqrt{2}\sqrt{e}-2\sqrt{e\tan(c+dx)}}{e^2\tan^2(c+dx)-\sqrt{2}e^{3/2}\tan(c+dx)+e}d\sqrt{e\tan(c+dx)}}{2\sqrt{2}\sqrt{e}}+\frac{\int\frac{\sqrt{2}\left(\sqrt{e}+\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{e^2\tan^2(c+dx)+\sqrt{2}e^{3/2}\tan(c+dx)+}}{2\sqrt{2}\sqrt{e}}}{2e}\right.\right.\right.}{d}}{a^2}$$

$$\downarrow 27$$

$$-\frac{e^2\left(\frac{2e(3a-a\sec(c+dx))\sqrt{e\tan(c+dx)}}{3d}-\frac{1}{3}e^2\left(\frac{6ae\left(\frac{\int\frac{\sqrt{2}\sqrt{e}-2\sqrt{e\tan(c+dx)}}{e^2\tan^2(c+dx)-\sqrt{2}e^{3/2}\tan(c+dx)+e}d\sqrt{e\tan(c+dx)}}{2\sqrt{2}\sqrt{e}}+\frac{\int\frac{\sqrt{e}+\sqrt{2}\sqrt{e\tan(c+dx)}}{e^2\tan^2(c+dx)+\sqrt{2}e^{3/2}\tan(c+dx)+}}{2\sqrt{e}}\right)}{2e}\right)}{d}\right)}{a^2}$$

$$\downarrow 1103$$

$$-\frac{e^2\left(\frac{2e(3a-a\sec(c+dx))\sqrt{e\tan(c+dx)}}{3d}-\frac{1}{3}e^2\left(\frac{6ae\left(\frac{\arctan\left(\sqrt{2}\sqrt{e}\tan(c+dx)+1\right)}{\sqrt{2}\sqrt{e}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{e}\tan(c+dx)\right)}{\sqrt{2}\sqrt{e}}+\frac{\log\left(\sqrt{2}e^{3/2}\tan(c+dx)+e^2\tan^2(\right.}{2\sqrt{2}\sqrt{e}}\right)}{2e}\right)}{d}\right)}{a^2}$$

input
```
Int[(e*Tan[c + d*x])^(7/2)/(a + a*Sec[c + d*x]),x]
```

output
```
-((e^2*((2*e*(3*a - a*Sec[c + d*x])*Sqrt[e*Tan[c + d*x]])/(3*d) - (e^2*((6
*a*e*((-(ArcTan[1 - Sqrt[2]*Sqrt[e]*Tan[c + d*x]]/(Sqrt[2]*Sqrt[e])) + Arc
Tan[1 + Sqrt[2]*Sqrt[e]*Tan[c + d*x]]/(Sqrt[2]*Sqrt[e]))/(2*e) + (-1/2*Log
[e - Sqrt[2]*e^(3/2)*Tan[c + d*x] + e^2*Tan[c + d*x]^2]/(Sqrt[2]*Sqrt[e])
+ Log[e + Sqrt[2]*e^(3/2)*Tan[c + d*x] + e^2*Tan[c + d*x]^2]/(2*Sqrt[2]*Sq
rt[e]))/(2*e)))/d - (a*EllipticF[c - Pi/4 + d*x, 2]*Sec[c + d*x]*Sqrt[Sin[
2*c + 2*d*x]])/(d*Sqrt[e*Tan[c + d*x]])))/3))/a^2)
```

3.120.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3053
```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Simp[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]])   Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f
}, x]
```

rule 3094
```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]], x_Symbol]
:> Simp[Sqrt[Sin[e + f*x]]/(Sqrt[Cos[e + f*x]]*Sqrt[b*Tan[e + f*x]])   Int[
1/(Sqrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]]), x], x] /; FreeQ[{b, e, f}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

rule 4369
```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(-e)*(e*Cot[c + d*x])^(m - 1)*((a*m + b*(m - 1)*Csc
[c + d*x])/(d*m*(m - 1))), x] - Simp[e^2/m   Int[(e*Cot[c + d*x])^(m - 2)*(
a*m + b*(m - 1)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && GtQ[m
, 1]
```

rule 4372
```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(e*Cot[c + d*x])^m, x], x] + Simp[b   Int[
(e*Cot[c + d*x])^m*Csc[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, m}, x]
```

rule 4376
```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Simp[a^(2*n)/e^(2*n)   Int[(e*Cot[c + d*x])^(m + 2*
n)/(-a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[a
^2 - b^2, 0] && ILtQ[n, 0]
```

3.120.4 Maple [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 4.03 (sec) , antiderivative size = 976, normalized size of antiderivative = 3.31

method	result	size
default	Expression too large to display	976

input
```
int((e*tan(d*x+c))^(7/2)/(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
-1/6/a/d*2^(1/2)*(3*I*(cot(d*x+c)-csc(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+
c))^(1/2)*(csc(d*x+c)-cot(d*x+c)+1)^(1/2)*EllipticPi((csc(d*x+c)-cot(d*x+c
)+1)^(1/2),1/2+1/2*I,1/2*2^(1/2))*cos(d*x+c)^2-3*I*(cot(d*x+c)-csc(d*x+c)+
1)^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)*(csc(d*x+c)-cot(d*x+c)+1)^(1/2)*Ell
ipticPi((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2-1/2*I,1/2*2^(1/2))*cos(d*x+c)^
2+3*I*(cot(d*x+c)-csc(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)*(csc(d
*x+c)-cot(d*x+c)+1)^(1/2)*EllipticPi((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2+1
/2*I,1/2*2^(1/2))*cos(d*x+c)-3*I*(cot(d*x+c)-csc(d*x+c)+1)^(1/2)*(cot(d*x+
c)-csc(d*x+c))^(1/2)*(csc(d*x+c)-cot(d*x+c)+1)^(1/2)*EllipticPi((csc(d*x+c
)-cot(d*x+c)+1)^(1/2),1/2-1/2*I,1/2*2^(1/2))*cos(d*x+c)+8*(csc(d*x+c)-cot(
d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1
/2)*EllipticF((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2*2^(1/2))*cos(d*x+c)^2-3*
(cot(d*x+c)-csc(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)*(csc(d*x+c)-
cot(d*x+c)+1)^(1/2)*EllipticPi((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2+1/2*I,1
/2*2^(1/2))*cos(d*x+c)^2-3*(cot(d*x+c)-csc(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc
(d*x+c))^(1/2)*(csc(d*x+c)-cot(d*x+c)+1)^(1/2)*EllipticPi((csc(d*x+c)-cot(
d*x+c)+1)^(1/2),1/2-1/2*I,1/2*2^(1/2))*cos(d*x+c)^2+8*(csc(d*x+c)-cot(d*x+
c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)*
EllipticF((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2*2^(1/2))*cos(d*x+c)-3*(cot(d
*x+c)-csc(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)*EllipticPi((csc...
```

3.120.5 Fricas [F(-1)]

Timed out.

$$\int \frac{(e\tan(c+dx))^{7/2}}{a+a\sec(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((e*tan(d*x+c))^(7/2)/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

output
```
Timed out
```

3.120.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(e\tan(c+dx))^{7/2}}{a+a\sec(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((e*tan(d*x+c))**(7/2)/(a+a*sec(d*x+c)),x)
```

output
```
Timed out
```

3.120.7 Maxima [F]

$$\int \frac{(e\tan(c+dx))^{7/2}}{a+a\sec(c+dx)}\,dx = \int \frac{(e\tan(dx+c))^{\frac{7}{2}}}{a\sec(dx+c)+a}\,dx$$

input
```
integrate((e*tan(d*x+c))^(7/2)/(a+a*sec(d*x+c)),x, algorithm="maxima")
```

output
```
integrate((e*tan(d*x + c))^(7/2)/(a*sec(d*x + c) + a), x)
```

3.120.8 Giac [F]

$$\int \frac{(e\tan(c+dx))^{7/2}}{a+a\sec(c+dx)}\,dx = \int \frac{(e\tan(dx+c))^{\frac{7}{2}}}{a\sec(dx+c)+a}\,dx$$

input
```
integrate((e*tan(d*x+c))^(7/2)/(a+a*sec(d*x+c)),x, algorithm="giac")
```

output
```
integrate((e*tan(d*x + c))^(7/2)/(a*sec(d*x + c) + a), x)
```

3.120.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(e\tan(c+dx))^{7/2}}{a+a\sec(c+dx)}\,dx = \int \frac{\cos(c+dx)\,(e\tan(c+dx))^{7/2}}{a\,(\cos(c+dx)+1)}\,dx$$

input
```
int((e*tan(c + d*x))^(7/2)/(a + a/cos(c + d*x)),x)
```

output
```
int((cos(c + d*x)*(e*tan(c + d*x))^(7/2))/(a*(cos(c + d*x) + 1)), x)
```

3.121 $\int \frac{(e\tan(c+dx))^{5/2}}{a+a\sec(c+dx)}\,dx$

3.121.1 Optimal result 907
3.121.2 Mathematica [C] (warning: unable to verify) 908
3.121.3 Rubi [A] (warning: unable to verify) 908
3.121.4 Maple [C] (warning: unable to verify) 914
3.121.5 Fricas [F(-1)] 915
3.121.6 Sympy [F] 915
3.121.7 Maxima [F] 915
3.121.8 Giac [F] 916
3.121.9 Mupad [F(-1)] 916

3.121.1 Optimal result

Integrand size = 25, antiderivative size = 285

$$\begin{aligned}\int \frac{(e\tan(c+dx))^{5/2}}{a+a\sec(c+dx)}\,dx &= \frac{e^{5/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}ad}\\ &- \frac{e^{5/2}\arctan\left(1+\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}ad}\\ &- \frac{e^{5/2}\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}ad}\\ &+ \frac{e^{5/2}\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)+\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}ad}\\ &- \frac{2e^2\cos(c+dx)E\left(c-\frac{\pi}{4}+dx\middle|2\right)\sqrt{e\tan(c+dx)}}{ad\sqrt{\sin(2c+2dx)}}\\ &+ \frac{2e\cos(c+dx)(e\tan(c+dx))^{3/2}}{ad}\end{aligned}$$

output

```
1/2*e^(5/2)*arctan(1-2^(1/2)*(e*tan(d*x+c))^(1/2)/e^(1/2))/a/d*2^(1/2)-1/2
*e^(5/2)*arctan(1+2^(1/2)*(e*tan(d*x+c))^(1/2)/e^(1/2))/a/d*2^(1/2)-1/4*e^
(5/2)*ln(e^(1/2)-2^(1/2)*(e*tan(d*x+c))^(1/2)+e^(1/2)*tan(d*x+c))/a/d*2^(1
/2)+1/4*e^(5/2)*ln(e^(1/2)+2^(1/2)*(e*tan(d*x+c))^(1/2)+e^(1/2)*tan(d*x+c)
)/a/d*2^(1/2)+2*e^2*cos(d*x+c)*(sin(c+1/4*Pi+d*x)^2)^(1/2)/sin(c+1/4*Pi+d*
x)*EllipticE(cos(c+1/4*Pi+d*x),2^(1/2))*(e*tan(d*x+c))^(1/2)/a/d/sin(2*d*x
+2*c)^(1/2)+2*e*cos(d*x+c)*(e*tan(d*x+c))^(3/2)/a/d
```

3.121.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 16.52 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.37

$$\int \frac{(e\tan(c+dx))^{5/2}}{a+a\sec(c+dx)}\,dx = \frac{4\cos^2\left(\frac{1}{2}(c+dx)\right)\csc(c+dx)\left(\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},-\tan^2(c+dx)\right)-\text{H}}{3ad(1+s}$$

input
```
Integrate[(e*Tan[c + d*x])^(5/2)/(a + a*Sec[c + d*x]),x]
```

output
```
(4*Cos[(c + d*x)/2]^2*Csc[c + d*x]*(Hypergeometric2F1[1/2, 3/4, 7/4, -Tan[
c + d*x]^2] - Hypergeometric2F1[3/4, 1, 7/4, -Tan[c + d*x]^2])*(1 + Sqrt[S
ec[c + d*x]^2])*(e*Tan[c + d*x])^(5/2))/(3*a*d*(1 + Sec[c + d*x])^2)
```

3.121.3 Rubi [A] (warning: unable to verify)

Time = 1.03 (sec) , antiderivative size = 262, normalized size of antiderivative = 0.92, number of steps used = 24, number of rules used = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.920$, Rules used = {3042, 4376, 25, 3042, 4372, 3042, 3093, 3042, 3095, 3042, 3052, 3042, 3119, 3957, 266, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e\tan(c+dx))^{5/2}}{a\sec(c+dx)+a}\,dx$$
$$\downarrow 3042$$
$$\int \frac{\left(-e\cot\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}{a\csc\left(c+dx+\frac{\pi}{2}\right)+a}dx$$
$$\downarrow 4376$$
$$\frac{e^2\int -\left((a-a\sec(c+dx))\sqrt{e\tan(c+dx)}\right)dx}{a^2}$$
$$\downarrow 25$$
$$-\frac{e^2\int (a-a\sec(c+dx))\sqrt{e\tan(c+dx)}dx}{a^2}$$
$$\downarrow 3042$$

$$-\frac{e^2\int\sqrt{-e\cot\left(c+dx+\frac{\pi}{2}\right)}\left(a-a\csc\left(c+dx+\frac{\pi}{2}\right)\right)dx}{a^2}$$

$$\downarrow 4372$$

$$-\frac{e^2\left(a\int\sqrt{e\tan(c+dx)}dx-a\int\sec(c+dx)\sqrt{e\tan(c+dx)}dx\right)}{a^2}$$

$$\downarrow 3042$$

$$-\frac{e^2\left(a\int\sqrt{e\tan(c+dx)}dx-a\int\sec(c+dx)\sqrt{e\tan(c+dx)}dx\right)}{a^2}$$

$$\downarrow 3093$$

$$-\frac{e^2\left(a\int\sqrt{e\tan(c+dx)}dx-a\left(\frac{2\cos(c+dx)(e\tan(c+dx))^{3/2}}{de}-2\int\cos(c+dx)\sqrt{e\tan(c+dx)}dx\right)\right)}{a^2}$$

$$\downarrow 3042$$

$$-\frac{e^2\left(a\int\sqrt{e\tan(c+dx)}dx-a\left(\frac{2\cos(c+dx)(e\tan(c+dx))^{3/2}}{de}-2\int\frac{\sqrt{e\tan(c+dx)}}{\sec(c+dx)}dx\right)\right)}{a^2}$$

$$\downarrow 3095$$

$$-\frac{e^2\left(a\int\sqrt{e\tan(c+dx)}dx-a\left(\frac{2\cos(c+dx)(e\tan(c+dx))^{3/2}}{de}-\frac{2\sqrt{\cos(c+dx)}\sqrt{e\tan(c+dx)}\int\sqrt{\cos(c+dx)}\sqrt{\sin(c+dx)}dx}{\sqrt{\sin(c+dx)}}\right)\right)}{a^2}$$

$$\downarrow 3042$$

$$-\frac{e^2\left(a\int\sqrt{e\tan(c+dx)}dx-a\left(\frac{2\cos(c+dx)(e\tan(c+dx))^{3/2}}{de}-\frac{2\sqrt{\cos(c+dx)}\sqrt{e\tan(c+dx)}\int\sqrt{\cos(c+dx)}\sqrt{\sin(c+dx)}dx}{\sqrt{\sin(c+dx)}}\right)\right)}{a^2}$$

$$\downarrow 3052$$

$$-\frac{e^2\left(a\int\sqrt{e\tan(c+dx)}dx-a\left(\frac{2\cos(c+dx)(e\tan(c+dx))^{3/2}}{de}-\frac{2\cos(c+dx)\sqrt{e\tan(c+dx)}\int\sqrt{\sin(2c+2dx)}dx}{\sqrt{\sin(2c+2dx)}}\right)\right)}{a^2}$$

$$\downarrow 3042$$

$$-\frac{e^2\left(a\int\sqrt{e\tan(c+dx)}dx-a\left(\frac{2\cos(c+dx)(e\tan(c+dx))^{3/2}}{de}-\frac{2\cos(c+dx)\sqrt{e\tan(c+dx)}\int\sqrt{\sin(2c+2dx)}dx}{\sqrt{\sin(2c+2dx)}}\right)\right)}{a^2}$$

$$\downarrow 3119$$

$$-\frac{e^2\left(a\int\sqrt{e\tan(c+dx)}dx-a\left(\frac{2\cos(c+dx)(e\tan(c+dx))^{3/2}}{de}-\frac{2\cos(c+dx)E\left(c+dx-\frac{\pi}{4}\middle|2\right)\sqrt{e\tan(c+dx)}}{d\sqrt{\sin(2c+2dx)}}\right)\right)}{a^2}$$

$$\downarrow 3957$$

$$-\frac{e^2\left(\frac{ae\int \frac{\sqrt{e\tan(c+dx)}}{\tan^2(c+dx)e^2+e^2}d(e\tan(c+dx))}{d}-a\left(\frac{2\cos(c+dx)(e\tan(c+dx))^{3/2}}{de}-\frac{2\cos(c+dx)E\left(c+dx-\frac{\pi}{4}|2\right)\sqrt{e\tan(c+dx)}}{d\sqrt{\sin(2c+2dx)}}\right)\right)}{a^2}$$

$$\downarrow 266$$

$$-\frac{e^2\left(\frac{2ae\int \frac{e^2\tan^2(c+dx)}{e^4\tan^4(c+dx)+e^2}d\sqrt{e\tan(c+dx)}}{d}-a\left(\frac{2\cos(c+dx)(e\tan(c+dx))^{3/2}}{de}-\frac{2\cos(c+dx)E\left(c+dx-\frac{\pi}{4}|2\right)\sqrt{e\tan(c+dx)}}{d\sqrt{\sin(2c+2dx)}}\right)\right)}{a^2}$$

$$\downarrow 826$$

$$-\frac{e^2\left(\frac{2ae\left(\frac{1}{2}\int \frac{e^2\tan^2(c+dx)+e}{e^4\tan^4(c+dx)+e^2}d\sqrt{e\tan(c+dx)}-\frac{1}{2}\int \frac{e-e^2\tan^2(c+dx)}{e^4\tan^4(c+dx)+e^2}d\sqrt{e\tan(c+dx)}\right)}{d}-a\left(\frac{2\cos(c+dx)(e\tan(c+dx))^{3/2}}{de}-\frac{2\cos(c+dx)E}{d}\right)\right)}{a^2}$$

$$\downarrow 1476$$

$$-\frac{e^2\left(\frac{2ae\left(\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{e^2\tan^2(c+dx)-\sqrt{2}e^{3/2}\tan(c+dx)+e}d\sqrt{e\tan(c+dx)}+\frac{1}{2}\int \frac{1}{e^2\tan^2(c+dx)+\sqrt{2}e^{3/2}\tan(c+dx)+e}d\sqrt{e\tan(c+dx)}\right)-\frac{1}{2}\int \frac{e-e^2\tan^2(c-}{e^4\tan^4(c+dx}\right)}{d}\right)}{a^2}$$

$$\downarrow 1082$$

$$-\frac{e^2\left(\frac{2ae\left(\frac{1}{2}\left(\frac{\int \frac{1}{-e^2\tan^2(c+dx)-1}d\left(1-\sqrt{2}\sqrt{e}\tan(c+dx)\right)}{\sqrt{2}\sqrt{e}}-\frac{\int \frac{1}{-e^2\tan^2(c+dx)-1}d\left(\sqrt{2}\sqrt{e}\tan(c+dx)+1\right)}{\sqrt{2}\sqrt{e}}\right)-\frac{1}{2}\int \frac{e-e^2\tan^2(c+dx)}{e^4\tan^4(c+dx)+e^2}d\sqrt{e\tan(c+dx)}\right)}{d}-\right)}{a^2}$$

$$\downarrow 217$$

$$-\frac{e^2\left(\frac{2ae\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{e}\tan(c+dx)+1\right)}{\sqrt{2}\sqrt{e}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{e}\tan(c+dx)\right)}{\sqrt{2}\sqrt{e}}\right)-\frac{1}{2}\int \frac{e-e^2\tan^2(c+dx)}{e^4\tan^4(c+dx)+e^2}d\sqrt{e\tan(c+dx)}\right)}{d}-a\left(\frac{2\cos(c+dx)(e\tan(c+dx}{de}\right.\right.}{a^2}$$

$$\downarrow 1479$$

$$-\frac{e^2\left(\frac{2ae\left(\frac{1}{2}\left(\frac{\int -\frac{\sqrt{2}\sqrt{e}-2\sqrt{e\tan(c+dx)}}{e^2\tan^2(c+dx)-\sqrt{2}e^{3/2}\tan(c+dx)+e}d\sqrt{e\tan(c+dx)}}{2\sqrt{2}\sqrt{e}}+\frac{\int -\frac{\sqrt{2}\left(\sqrt{e}+\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{e^2\tan^2(c+dx)+\sqrt{2}e^{3/2}\tan(c+dx)+e}d\sqrt{e\tan(c+dx)}}{2\sqrt{2}\sqrt{e}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{e}\tan(}{\sqrt{2}\sqrt{e}}\right.\right.}{d}\right.}{a^2}$$

$$\downarrow 25$$

$$-\frac{e^2\left(\frac{2ae\left(\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}\sqrt{e}-2\sqrt{e\tan(c+dx)}}{e^2\tan^2(c+dx)-\sqrt{2}e^{3/2}\tan(c+dx)+e}d\sqrt{e\tan(c+dx)}}{2\sqrt{2}\sqrt{e}}-\frac{\int \frac{\sqrt{2}\left(\sqrt{e}+\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{e^2\tan^2(c+dx)+\sqrt{2}e^{3/2}\tan(c+dx)+e}d\sqrt{e\tan(c+dx)}}{2\sqrt{2}\sqrt{e}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{e}\tan(c\right.\right.}{\sqrt{2}\sqrt{e}}\right.}{d}\right.}{a^2}$$

$$\downarrow 27$$

$$-\frac{e^2\left(\frac{2ae\left(\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}\sqrt{e}-2\sqrt{e\tan(c+dx)}}{e^2\tan^2(c+dx)-\sqrt{2}e^{3/2}\tan(c+dx)+e}d\sqrt{e\tan(c+dx)}}{2\sqrt{2}\sqrt{e}}-\frac{\int \frac{\sqrt{e}+\sqrt{2}\sqrt{e\tan(c+dx)}}{e^2\tan^2(c+dx)+\sqrt{2}e^{3/2}\tan(c+dx)+e}d\sqrt{e\tan(c+dx)}}{2\sqrt{e}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{e}\tan(c\right.\right.}{\sqrt{2}\sqrt{e}}\right.}{d}\right.}{a^2}$$

$$\downarrow 1103$$

$$-\frac{e^2\left(\frac{2ae\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{e}\tan(c+dx)+1\right)}{\sqrt{2}\sqrt{e}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{e}\tan(c+dx)\right)}{\sqrt{2}\sqrt{e}}\right)+\frac{1}{2}\left(\frac{\log\left(-\sqrt{2}e^{3/2}\tan(c+dx)+e^2\tan^2(c+dx)+e\right)}{2\sqrt{2}\sqrt{e}}-\frac{\log\left(\sqrt{2}e^{3/2}\tan(c+dx)+e^2t\right.}{2\sqrt{2}\sqrt{e}}\right.}{d}\right.}{a^2}$$

input
```
Int[(e*Tan[c + d*x])^(5/2)/(a + a*Sec[c + d*x]),x]
```

output
```
-((e^2*((2*a*e*((-(ArcTan[1 - Sqrt[2]*Sqrt[e]*Tan[c + d*x]]/(Sqrt[2]*Sqrt[
e])) + ArcTan[1 + Sqrt[2]*Sqrt[e]*Tan[c + d*x]]/(Sqrt[2]*Sqrt[e]))/2 + (Lo
g[e - Sqrt[2]*e^(3/2)*Tan[c + d*x] + e^2*Tan[c + d*x]^2]/(2*Sqrt[2]*Sqrt[e
]) - Log[e + Sqrt[2]*e^(3/2)*Tan[c + d*x] + e^2*Tan[c + d*x]^2]/(2*Sqrt[2]
*Sqrt[e]))/2))/d - a*((-2*Cos[c + d*x]*EllipticE[c - Pi/4 + d*x, 2]*Sqrt[e
*Tan[c + d*x]])/(d*Sqrt[Sin[2*c + 2*d*x]]) + (2*Cos[c + d*x]*(e*Tan[c + d*
x])^(3/2))/(d*e))))/a^2)
```

3.121. $\int \frac{(e\tan(c+dx))^{5/2}}{a+a\sec(c+dx)}\,dx$

3.121.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3052
```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Simp[Sqrt[a*Sin[e + f*x]]*(Sqrt[b*Cos[e + f*x]]/Sqrt[Sin[2*e
 + 2*f*x]])   Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

rule 3093
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[a^2*(a*Sec[e + f*x])^(m - 2)*((b*Tan[e + f*x])^(n +
1)/(b*f*(m + n - 1))), x] + Simp[a^2*((m - 2)/(m + n - 1))   Int[(a*Sec[e +
 f*x])^(m - 2)*(b*Tan[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (
GtQ[m, 1] || (EqQ[m, 1] && EqQ[n, 1/2])) && NeQ[m + n - 1, 0] && IntegersQ[
2*m, 2*n]
```

rule 3095
```
Int[Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]]/sec[(e_.) + (f_.)*(x_)], x_Symbol]
:> Simp[Sqrt[Cos[e + f*x]]*(Sqrt[b*Tan[e + f*x]]/Sqrt[Sin[e + f*x]])   Int[
Sqrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]], x], x] /; FreeQ[{b, e, f}, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

rule 4372
```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(e*Cot[c + d*x])^m, x], x] + Simp[b   Int[
(e*Cot[c + d*x])^m*Csc[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, m}, x]
```

rule 4376
```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Simp[a^(2*n)/e^(2*n)   Int[(e*Cot[c + d*x])^(m + 2*
n)/(-a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[a
^2 - b^2, 0] && ILtQ[n, 0]
```

3.121.4 Maple [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 3.64 (sec) , antiderivative size = 1100, normalized size of antiderivative = 3.86

method	result	size
default	Expression too large to display	1100

input
```
int((e*tan(d*x+c))^(5/2)/(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
-1/2/a/d*2^(1/2)*(-e/((1-cos(d*x+c))^2*csc(d*x+c)^2-1)*(-cot(d*x+c)+csc(d*
x+c)))^(5/2)*((1-cos(d*x+c))^2*csc(d*x+c)^2-1)^3*(I*(csc(d*x+c)-cot(d*x+c)
+1)^(1/2)*(2-2*csc(d*x+c)+2*cot(d*x+c))^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2
)*EllipticPi((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2-1/2*I,1/2*2^(1/2))*((1-co
s(d*x+c))*(-cot(d*x+c)+csc(d*x+c)-1)*(csc(d*x+c)-cot(d*x+c)+1)*csc(d*x+c))
^(1/2)-I*(csc(d*x+c)-cot(d*x+c)+1)^(1/2)*(2-2*csc(d*x+c)+2*cot(d*x+c))^(1/
2)*(cot(d*x+c)-csc(d*x+c))^(1/2)*EllipticPi((csc(d*x+c)-cot(d*x+c)+1)^(1/2
),1/2+1/2*I,1/2*2^(1/2))*((1-cos(d*x+c))*(-cot(d*x+c)+csc(d*x+c)-1)*(csc(d
*x+c)-cot(d*x+c)+1)*csc(d*x+c))^(1/2)-4*(csc(d*x+c)-cot(d*x+c)+1)^(1/2)*(2
-2*csc(d*x+c)+2*cot(d*x+c))^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)*EllipticE(
(csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2*2^(1/2))*((1-cos(d*x+c))*(-cot(d*x+c)+
csc(d*x+c)-1)*(csc(d*x+c)-cot(d*x+c)+1)*csc(d*x+c))^(1/2)+2*(csc(d*x+c)-co
t(d*x+c)+1)^(1/2)*(2-2*csc(d*x+c)+2*cot(d*x+c))^(1/2)*(cot(d*x+c)-csc(d*x+
c))^(1/2)*EllipticF((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2*2^(1/2))*((1-cos(d
*x+c))*(-cot(d*x+c)+csc(d*x+c)-1)*(csc(d*x+c)-cot(d*x+c)+1)*csc(d*x+c))^(1
/2)-(csc(d*x+c)-cot(d*x+c)+1)^(1/2)*(2-2*csc(d*x+c)+2*cot(d*x+c))^(1/2)*(c
ot(d*x+c)-csc(d*x+c))^(1/2)*EllipticPi((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2
-1/2*I,1/2*2^(1/2))*((1-cos(d*x+c))*(-cot(d*x+c)+csc(d*x+c)-1)*(csc(d*x+c)
-cot(d*x+c)+1)*csc(d*x+c))^(1/2)-(csc(d*x+c)-cot(d*x+c)+1)^(1/2)*(2-2*csc(
d*x+c)+2*cot(d*x+c))^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)*EllipticPi((cs...
```

3.121.5 Fricas [F(-1)]

Timed out.

$$\int \frac{(e\tan(c+dx))^{5/2}}{a+a\sec(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((e*tan(d*x+c))^(5/2)/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

output
```
Timed out
```

3.121.6 Sympy [F]

$$\int \frac{(e\tan(c+dx))^{5/2}}{a+a\sec(c+dx)}\,dx = \frac{\int \frac{(e\tan(c+dx))^{\frac{5}{2}}}{\sec(c+dx)+1}\,dx}{a}$$

input
```
integrate((e*tan(d*x+c))**(5/2)/(a+a*sec(d*x+c)),x)
```

output
```
Integral((e*tan(c + d*x))**(5/2)/(sec(c + d*x) + 1), x)/a
```

3.121.7 Maxima [F]

$$\int \frac{(e\tan(c+dx))^{5/2}}{a+a\sec(c+dx)}\,dx = \int \frac{(e\tan(dx+c))^{\frac{5}{2}}}{a\sec(dx+c)+a}\,dx$$

input
```
integrate((e*tan(d*x+c))^(5/2)/(a+a*sec(d*x+c)),x, algorithm="maxima")
```

output
```
integrate((e*tan(d*x + c))^(5/2)/(a*sec(d*x + c) + a), x)
```

3.121.8 Giac [F]

$$\int \frac{(e\tan(c+dx))^{5/2}}{a+a\sec(c+dx)}\,dx = \int \frac{(e\tan(dx+c))^{\frac{5}{2}}}{a\sec(dx+c)+a}\,dx$$

input
```
integrate((e*tan(d*x+c))^(5/2)/(a+a*sec(d*x+c)),x, algorithm="giac")
```

output
```
integrate((e*tan(d*x + c))^(5/2)/(a*sec(d*x + c) + a), x)
```

3.121.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(e\tan(c+dx))^{5/2}}{a+a\sec(c+dx)}\,dx = \int \frac{\cos(c+dx)\,(e\tan(c+dx))^{5/2}}{a\,(\cos(c+dx)+1)}\,dx$$

input
```
int((e*tan(c + d*x))^(5/2)/(a + a/cos(c + d*x)),x)
```

output
```
int((cos(c + d*x)*(e*tan(c + d*x))^(5/2))/(a*(cos(c + d*x) + 1)), x)
```

3.122 $\int \frac{(e\tan(c+dx))^{3/2}}{a+a\sec(c+dx)}\,dx$

3.122.1 Optimal result 917
3.122.2 Mathematica [C] (warning: unable to verify) 918
3.122.3 Rubi [A] (warning: unable to verify) 919
3.122.4 Maple [C] (warning: unable to verify) 925
3.122.5 Fricas [F(-1)] 925
3.122.6 Sympy [F] 926
3.122.7 Maxima [F] 926
3.122.8 Giac [F] 926
3.122.9 Mupad [F(-1)] 927

3.122.1 Optimal result

Integrand size = 25, antiderivative size = 257

$$\begin{aligned}\int \frac{(e\tan(c+dx))^{3/2}}{a+a\sec(c+dx)}\,dx &= \frac{e^{3/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}ad}\\ &- \frac{e^{3/2}\arctan\left(1+\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}ad}\\ &+ \frac{e^{3/2}\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}ad}\\ &- \frac{e^{3/2}\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)+\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}ad}\\ &+ \frac{e^2\,\text{EllipticF}\left(c-\frac{\pi}{4}+dx,2\right)\sec(c+dx)\sqrt{\sin(2c+2dx)}}{ad\sqrt{e\tan(c+dx)}}\end{aligned}$$

output

```
1/2*e^(3/2)*arctan(1-2^(1/2)*(e*tan(d*x+c))^(1/2)/e^(1/2))/a/d*2^(1/2)-1/2
*e^(3/2)*arctan(1+2^(1/2)*(e*tan(d*x+c))^(1/2)/e^(1/2))/a/d*2^(1/2)+1/4*e^
(3/2)*ln(e^(1/2)-2^(1/2)*(e*tan(d*x+c))^(1/2)+e^(1/2)*tan(d*x+c))/a/d*2^(1
/2)-1/4*e^(3/2)*ln(e^(1/2)+2^(1/2)*(e*tan(d*x+c))^(1/2)+e^(1/2)*tan(d*x+c)
)/a/d*2^(1/2)-e^2*(sin(c+1/4*Pi+d*x)^2)^(1/2)/sin(c+1/4*Pi+d*x)*EllipticF(
cos(c+1/4*Pi+d*x),2^(1/2))*sec(d*x+c)*sin(2*d*x+2*c)^(1/2)/a/d/(e*tan(d*x+
c))^(1/2)
```

3.122.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 15.61 (sec) , antiderivative size = 1211, normalized size of antiderivative = 4.71

$$\int \frac{(e\tan(c+dx))^{3/2}}{a+a\sec(c+dx)}\,dx = \frac{\cos^2\left(\frac{c}{2}+\frac{dx}{2}\right)\csc(c+dx)\left(\frac{8\cos(c)\cos(dx)\sec(2c)\sin^2\left(\frac{c}{2}\right)}{d} - \frac{16\cos\left(\frac{c}{2}\right)\sec(2c)\sin^3\left(\frac{c}{2}\right)\sin(dx)}{d}\right)(e$$
$$- \frac{2e^{-i(c+dx)}\sqrt{-\frac{i\left(-1+e^{2i(c+dx)}\right)}{1+e^{2i(c+dx)}}}\left(1+e^{2i(c+dx)}\right)\cos^2\left(\frac{c}{2}+\frac{dx}{2}\right)\sec(2c)\sec(c+dx)(e\tan(c+dx))^{3/2}}{d(a+a\sec(c+dx))\tan^{\frac{3}{2}}(c+dx)}$$
$$- \frac{e^{-2ic}\sqrt{-\frac{i\left(-1+e^{2i(c+dx)}\right)}{1+e^{2i(c+dx)}}}\left(e^{4ic}\sqrt{-1+e^{4i(c+dx)}}\arctan\left(\sqrt{-1+e^{4i(c+dx)}}\right)+2\sqrt{-1+e^{2i(c+dx)}}\sqrt{1+e^{2i(c+dx)}}\text{arc}\right.}{2d\left(-1+e^{2i(c+dx)}\right)(a+a\sec(c+dx))\tan$$
$$- \frac{e^{-2ic}\sqrt{-\frac{i\left(-1+e^{2i(c+dx)}\right)}{1+e^{2i(c+dx)}}}\left(\sqrt{-1+e^{4i(c+dx)}}\arctan\left(\sqrt{-1+e^{4i(c+dx)}}\right)+2e^{4ic}\sqrt{-1+e^{2i(c+dx)}}\sqrt{1+e^{2i(c+dx)}}\text{arc}\right.}{2d\left(-1+e^{2i(c+dx)}\right)(a+a\sec(c+dx))\tan$$
$$+ \frac{e^{-i(2c+dx)}\sqrt{-\frac{i\left(-1+e^{2i(c+dx)}\right)}{1+e^{2i(c+dx)}}}\cos^2\left(\frac{c}{2}+\frac{dx}{2}\right)\left(3\left(-1+e^{4i(c+dx)}\right)+e^{4i(c+dx)}\left(-1+e^{2ic}\right)\sqrt{1-e^{4i(c+dx)}}\,\text{Hypergeo}\right.}{3d\left(-1+e^{2i(c+dx)}\right)(a+a\sec(c+dx))\tan^{\frac{3}{2}}(c$$
$$- \frac{e^{-idx}\sqrt{-\frac{i\left(-1+e^{2i(c+dx)}\right)}{1+e^{2i(c+dx)}}}\cos^2\left(\frac{c}{2}+\frac{dx}{2}\right)\left(3-3e^{4i(c+dx)}+e^{2i(c+2dx)}\left(-1+e^{2ic}\right)\sqrt{1-e^{4i(c+dx)}}\,\text{Hypergeometric2}\right.}{3d\left(-1+e^{2i(c+dx)}\right)(a+a\sec(c+dx))\tan^{\frac{3}{2}}(c+d$$
$$- \frac{4\sqrt[4]{-1}\cos^2\left(\frac{c}{2}+\frac{dx}{2}\right)\text{EllipticF}\left(i\,\text{arcsinh}\left(\sqrt[4]{-1}\sqrt{\tan(c+dx)}\right),-1\right)\sec^4(c+dx)(e\tan(c+dx))^{3/2}}{d(a+a\sec(c+dx))\tan^{\frac{3}{2}}(c+dx)\left(1+\tan^2(c+dx)\right)^{3/2}}$$

input
```
Integrate[(e*Tan[c + d*x])^(3/2)/(a + a*Sec[c + d*x]),x]
```

output

```
(Cos[c/2 + (d*x)/2]^2*Csc[c + d*x]*((8*Cos[c]*Cos[d*x]*Sec[2*c]*Sin[c/2]^2
)/d - (16*Cos[c/2]*Sec[2*c]*Sin[c/2]^3*Sin[d*x])/d)*(e*Tan[c + d*x])^(3/2)
)/(a + a*Sec[c + d*x]) - (2*Sqrt[((-I)*(-1 + E^((2*I)*(c + d*x))))/(1 + E^
((2*I)*(c + d*x)))]*(1 + E^((2*I)*(c + d*x)))*Cos[c/2 + (d*x)/2]^2*Sec[2*c
]*Sec[c + d*x]*(e*Tan[c + d*x])^(3/2))/(d*E^(I*(c + d*x))*(a + a*Sec[c + d
*x])*Tan[c + d*x]^(3/2)) - (Sqrt[((-I)*(-1 + E^((2*I)*(c + d*x))))/(1 + E^
((2*I)*(c + d*x)))]*(E^((4*I)*c)*Sqrt[-1 + E^((4*I)*(c + d*x))]*ArcTan[Sqr
t[-1 + E^((4*I)*(c + d*x))]] + 2*Sqrt[-1 + E^((2*I)*(c + d*x))]*Sqrt[1 + E
^((2*I)*(c + d*x))]*ArcTanh[Sqrt[(-1 + E^((2*I)*(c + d*x)))/(1 + E^((2*I)*
(c + d*x)))]])*Cos[c/2 + (d*x)/2]^2*Sec[2*c]*Sec[c + d*x]*(e*Tan[c + d*x])
^(3/2))/(2*d*E^((2*I)*c)*(-1 + E^((2*I)*(c + d*x)))*(a + a*Sec[c + d*x])*T
an[c + d*x]^(3/2)) - (Sqrt[((-I)*(-1 + E^((2*I)*(c + d*x))))/(1 + E^((2*I)
*(c + d*x)))]*(Sqrt[-1 + E^((4*I)*(c + d*x))]*ArcTan[Sqrt[-1 + E^((4*I)*(c
 + d*x))]] + 2*E^((4*I)*c)*Sqrt[-1 + E^((2*I)*(c + d*x))]*Sqrt[1 + E^((2*I
)*(c + d*x))]*ArcTanh[Sqrt[(-1 + E^((2*I)*(c + d*x)))/(1 + E^((2*I)*(c + d
*x)))]])*Cos[c/2 + (d*x)/2]^2*Sec[2*c]*Sec[c + d*x]*(e*Tan[c + d*x])^(3/2)
)/(2*d*E^((2*I)*c)*(-1 + E^((2*I)*(c + d*x)))*(a + a*Sec[c + d*x])*Tan[c +
 d*x]^(3/2)) + (Sqrt[((-I)*(-1 + E^((2*I)*(c + d*x))))/(1 + E^((2*I)*(c +
d*x)))]*Cos[c/2 + (d*x)/2]^2*(3*(-1 + E^((4*I)*(c + d*x))) + E^((4*I)*(c +
 d*x))*(-1 + E^((2*I)*c))*Sqrt[1 - E^((4*I)*(c + d*x))]*Hypergeometric2...
```

3.122.3 Rubi [A] (warning: unable to verify)

Time = 0.87 (sec) , antiderivative size = 239, normalized size of antiderivative = 0.93, number of steps used = 22, number of rules used = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.840$, Rules used = {3042, 4376, 25, 3042, 4372, 3042, 3094, 3042, 3053, 3042, 3120, 3957, 266, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e\tan(c+dx))^{3/2}}{a\sec(c+dx)+a}\,dx$$

$$\downarrow 3042$$

$$\int \frac{\left(-e\cot\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}{a\csc\left(c+dx+\frac{\pi}{2}\right)+a}dx$$

$$\downarrow 4376$$

$$\frac{e^2\int -\frac{a-a\sec(c+dx)}{\sqrt{e\tan(c+dx)}}dx}{a^2}$$

$$\downarrow 25$$

$$-\frac{e^2\int\frac{a-a\sec(c+dx)}{\sqrt{e\tan(c+dx)}}dx}{a^2}$$

$$\downarrow 3042$$

$$-\frac{e^2\int\frac{a-a\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{-e\cot\left(c+dx+\frac{\pi}{2}\right)}}dx}{a^2}$$

$$\downarrow 4372$$

$$-\frac{e^2\left(a\int\frac{1}{\sqrt{e\tan(c+dx)}}dx-a\int\frac{\sec(c+dx)}{\sqrt{e\tan(c+dx)}}dx\right)}{a^2}$$

$$\downarrow 3042$$

$$-\frac{e^2\left(a\int\frac{1}{\sqrt{e\tan(c+dx)}}dx-a\int\frac{\sec(c+dx)}{\sqrt{e\tan(c+dx)}}dx\right)}{a^2}$$

$$\downarrow 3094$$

$$-\frac{e^2\left(a\int\frac{1}{\sqrt{e\tan(c+dx)}}dx-\frac{a\sqrt{\sin(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}\sqrt{\sin(c+dx)}}dx}{\sqrt{\cos(c+dx)}\sqrt{e\tan(c+dx)}}\right)}{a^2}$$

$$\downarrow 3042$$

$$-\frac{e^2\left(a\int\frac{1}{\sqrt{e\tan(c+dx)}}dx-\frac{a\sqrt{\sin(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}\sqrt{\sin(c+dx)}}dx}{\sqrt{\cos(c+dx)}\sqrt{e\tan(c+dx)}}\right)}{a^2}$$

$$\downarrow 3053$$

$$-\frac{e^2\left(a\int\frac{1}{\sqrt{e\tan(c+dx)}}dx-\frac{a\sqrt{\sin(2c+2dx)}\sec(c+dx)\int\frac{1}{\sqrt{\sin(2c+2dx)}}dx}{\sqrt{e\tan(c+dx)}}\right)}{a^2}$$

$$\downarrow 3042$$

$$-\frac{e^2\left(a\int\frac{1}{\sqrt{e\tan(c+dx)}}dx-\frac{a\sqrt{\sin(2c+2dx)}\sec(c+dx)\int\frac{1}{\sqrt{\sin(2c+2dx)}}dx}{\sqrt{e\tan(c+dx)}}\right)}{a^2}$$

$$\downarrow 3120$$

$$-\frac{e^2\left(a\int\frac{1}{\sqrt{e\tan(c+dx)}}dx-\frac{a\sqrt{\sin(2c+2dx)}\sec(c+dx)\,\text{EllipticF}\left(c+dx-\frac{\pi}{4},2\right)}{d\sqrt{e\tan(c+dx)}}\right)}{a^2}$$

$$\downarrow 3957$$

$$-\frac{e^2\left(\frac{ae\int \frac{1}{\sqrt{e\tan(c+dx)}\left(\tan^2(c+dx)e^2+e^2\right)}d(e\tan(c+dx))}{d}-\frac{a\sqrt{\sin(2c+2dx)}\sec(c+dx)\,\text{EllipticF}\left(c+dx-\frac{\pi}{4},2\right)}{d\sqrt{e\tan(c+dx)}}\right)}{a^2}$$

$$\downarrow 266$$

$$-\frac{e^2\left(\frac{2ae\int \frac{1}{e^4\tan^4(c+dx)+e^2}d\sqrt{e\tan(c+dx)}}{d}-\frac{a\sqrt{\sin(2c+2dx)}\sec(c+dx)\,\text{EllipticF}\left(c+dx-\frac{\pi}{4},2\right)}{d\sqrt{e\tan(c+dx)}}\right)}{a^2}$$

$$\downarrow 755$$

$$-\frac{e^2\left(\frac{2ae\left(\frac{\int \frac{e-e^2\tan^2(c+dx)}{e^4\tan^4(c+dx)+e^2}d\sqrt{e\tan(c+dx)}}{2e}+\frac{\int \frac{e^2\tan^2(c+dx)+e}{e^4\tan^4(c+dx)+e^2}d\sqrt{e\tan(c+dx)}}{2e}\right)}{d}-\frac{a\sqrt{\sin(2c+2dx)}\sec(c+dx)\,\text{EllipticF}\left(c+dx-\frac{\pi}{4},2\right)}{d\sqrt{e\tan(c+dx)}}\right)}{a^2}$$

$$\downarrow 1476$$

$$-\frac{e^2\left(\frac{2ae\left(\frac{\frac{1}{2}\int \frac{1}{e^2\tan^2(c+dx)-\sqrt{2}e^{3/2}\tan(c+dx)+e}d\sqrt{e\tan(c+dx)}+\frac{1}{2}\int \frac{1}{e^2\tan^2(c+dx)+\sqrt{2}e^{3/2}\tan(c+dx)+e}d\sqrt{e\tan(c+dx)}}{2e}+\frac{\int \frac{e-e^2\tan^2(c+dx)}{e^4\tan^4(c+dx)+e^2}d\sqrt{e\tan}}{2e}\right)}{d}\right)}{a^2}$$

$$\downarrow 1082$$

$$-\frac{e^2\left(\frac{2ae\left(\frac{\int \frac{1}{-e^2\tan^2(c+dx)-1}d\left(1-\sqrt{2}\sqrt{e}\tan(c+dx)\right)}{\sqrt{2}\sqrt{e}}-\frac{\int \frac{1}{-e^2\tan^2(c+dx)-1}d\left(\sqrt{2}\sqrt{e}\tan(c+dx)+1\right)}{\sqrt{2}\sqrt{e}}}{2e}+\frac{\int \frac{e-e^2\tan^2(c+dx)}{e^4\tan^4(c+dx)+e^2}d\sqrt{e\tan(c+dx)}}{2e}\right)}{d}-\frac{a\sqrt{\sin(}}{}\right)}{a^2}$$

$$\downarrow 217$$

$$-\frac{e^2\left(\frac{2ae\left(\frac{\int \frac{e-e^2\tan^2(c+dx)}{e^4\tan^4(c+dx)+e^2}d\sqrt{e\tan(c+dx)}}{2e}+\frac{\frac{\arctan\left(\sqrt{2}\sqrt{e}\tan(c+dx)+1\right)}{\sqrt{2}\sqrt{e}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{e}\tan(c+dx)\right)}{\sqrt{2}\sqrt{e}}}{2e}\right)}{d}-\frac{a\sqrt{\sin(2c+2dx)}\sec(c+dx)\,\text{EllipticF}(}{d\sqrt{e\tan(c+dx)}}\right)}{a^2}$$

3.122. $\int \frac{(e\tan(c+dx))^{3/2}}{a+a\sec(c+dx)}\,dx$

$$\downarrow 1479$$

$$-\frac{e^2\left(2ae\left(\frac{-\frac{\int -\frac{\sqrt{2}\sqrt{e}-2\sqrt{e\tan(c+dx)}}{e^2\tan^2(c+dx)-\sqrt{2}e^{3/2}\tan(c+dx)+e}d\sqrt{e\tan(c+dx)}}{2\sqrt{2}\sqrt{e}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{e}+\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{e^2\tan^2(c+dx)+\sqrt{2}e^{3/2}\tan(c+dx)+e}d\sqrt{e\tan(c+dx)}}{2\sqrt{2}\sqrt{e}}}{2e}+\frac{\arctan\left(\sqrt{2}\sqrt{e}\tan(c+dx)+\right.}{\sqrt{2}\sqrt{e}}\right.}{d}}{a^2}$$

$$\downarrow 25$$

$$-\frac{e^2\left(2ae\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt{e}-2\sqrt{e\tan(c+dx)}}{e^2\tan^2(c+dx)-\sqrt{2}e^{3/2}\tan(c+dx)+e}d\sqrt{e\tan(c+dx)}}{2\sqrt{2}\sqrt{e}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{e}+\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{e^2\tan^2(c+dx)+\sqrt{2}e^{3/2}\tan(c+dx)+e}d\sqrt{e\tan(c+dx)}}{2\sqrt{2}\sqrt{e}}}{2e}+\frac{\arctan\left(\sqrt{2}\sqrt{e}\tan(c+dx)+1\right)}{\sqrt{2}\sqrt{e}}\right.}{d}}{a^2}$$

$$\downarrow 27$$

$$-\frac{e^2\left(2ae\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt{e}-2\sqrt{e\tan(c+dx)}}{e^2\tan^2(c+dx)-\sqrt{2}e^{3/2}\tan(c+dx)+e}d\sqrt{e\tan(c+dx)}}{2\sqrt{2}\sqrt{e}}+\frac{\int \frac{\sqrt{e}+\sqrt{2}\sqrt{e\tan(c+dx)}}{e^2\tan^2(c+dx)+\sqrt{2}e^{3/2}\tan(c+dx)+e}d\sqrt{e\tan(c+dx)}}{2\sqrt{e}}}{2e}+\frac{\arctan\left(\sqrt{2}\sqrt{e}\tan(c+dx)+1\right)}{\sqrt{2}\sqrt{e}}\right.}{d}}{a^2}$$

$$\downarrow 1103$$

$$-\frac{e^2\left(2ae\left(\frac{\arctan\left(\sqrt{2}\sqrt{e}\tan(c+dx)+1\right)}{\sqrt{2}\sqrt{e}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{e}\tan(c+dx)\right)}{\sqrt{2}\sqrt{e}}}{2e}+\frac{\log\left(\sqrt{2}e^{3/2}\tan(c+dx)+e^2\tan^2(c+dx)+e\right)}{2\sqrt{2}\sqrt{e}}-\frac{\log\left(-\sqrt{2}e^{3/2}\tan(c+dx)+e^2\tan^2(c+dx)\right.}{2\sqrt{2}\sqrt{e}}\right.}{d}}{a^2}$$

input
```
Int[(e*Tan[c + d*x])^(3/2)/(a + a*Sec[c + d*x]),x]
```

output
```
-((e^2*((2*a*e*((-(ArcTan[1 - Sqrt[2]*Sqrt[e]*Tan[c + d*x]]/(Sqrt[2]*Sqrt[
e])) + ArcTan[1 + Sqrt[2]*Sqrt[e]*Tan[c + d*x]]/(Sqrt[2]*Sqrt[e]))/(2*e) +
 (-1/2*Log[e - Sqrt[2]*e^(3/2)*Tan[c + d*x] + e^2*Tan[c + d*x]^2]/(Sqrt[2]
*Sqrt[e]) + Log[e + Sqrt[2]*e^(3/2)*Tan[c + d*x] + e^2*Tan[c + d*x]^2]/(2*
Sqrt[2]*Sqrt[e]))/(2*e)))/d - (a*EllipticF[c - Pi/4 + d*x, 2]*Sec[c + d*x]
*Sqrt[Sin[2*c + 2*d*x]])/(d*Sqrt[e*Tan[c + d*x]])))/a^2)
```

3.122.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3053
```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Simp[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]])   Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f
}, x]
```

rule 3094
```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]], x_Symbol]
:> Simp[Sqrt[Sin[e + f*x]]/(Sqrt[Cos[e + f*x]]*Sqrt[b*Tan[e + f*x]])   Int[
1/(Sqrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]]), x], x] /; FreeQ[{b, e, f}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

rule 4372
```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(e*Cot[c + d*x])^m, x], x] + Simp[b   Int[
(e*Cot[c + d*x])^m*Csc[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, m}, x]
```

rule 4376
```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Simp[a^(2*n)/e^(2*n)   Int[(e*Cot[c + d*x])^(m + 2*
n)/(-a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[a
^2 - b^2, 0] && ILtQ[n, 0]
```

3.122.4 Maple [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 3.75 (sec) , antiderivative size = 205, normalized size of antiderivative = 0.80

method	result
default	$\frac{(\frac{1}{2}-\frac{i}{2})\sqrt{2}\left(i\,\mathrm{EllipticPi}\left(\sqrt{\csc(dx+c)-\cot(dx+c)+1},\frac{1}{2}-\frac{i}{2},\frac{\sqrt{2}}{2}\right)+\mathrm{EllipticPi}\left(\sqrt{\csc(dx+c)-\cot(dx+c)+1},\frac{1}{2}+\frac{i}{2},\frac{\sqrt{2}}{2}\right)-2\,\mathrm{EllipticF}\left(\sqrt{}\right.\right.$

input
```
int((e*tan(d*x+c))^(3/2)/(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
(1/2-1/2*I)/a/d*2^(1/2)*(I*EllipticPi((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2-
1/2*I,1/2*2^(1/2))+EllipticPi((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2+1/2*I,1/
2*2^(1/2))-2*EllipticF((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2*2^(1/2))-2*I*El
lipticF((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2*2^(1/2)))*(e*tan(d*x+c))^(1/2)
*(csc(d*x+c)-cot(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c)+1)^(1/2)*(cot(d*x+
c)-csc(d*x+c))^(1/2)*e/(cos(d*x+c)-1)*sin(d*x+c)
```

3.122.5 Fricas [F(-1)]

Timed out.

$$\int \frac{(e\tan(c+dx))^{3/2}}{a+a\sec(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((e*tan(d*x+c))^(3/2)/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

output
```
Timed out
```

3.122.6 Sympy [F]

$$\int \frac{(e\tan(c+dx))^{3/2}}{a+a\sec(c+dx)}\,dx = \frac{\int \frac{(e\tan(c+dx))^{\frac{3}{2}}}{\sec(c+dx)+1}\,dx}{a}$$

input
```
integrate((e*tan(d*x+c))**(3/2)/(a+a*sec(d*x+c)),x)
```

output
```
Integral((e*tan(c + d*x))**(3/2)/(sec(c + d*x) + 1), x)/a
```

3.122.7 Maxima [F]

$$\int \frac{(e\tan(c+dx))^{3/2}}{a+a\sec(c+dx)}\,dx = \int \frac{(e\tan(dx+c))^{\frac{3}{2}}}{a\sec(dx+c)+a}\,dx$$

input
```
integrate((e*tan(d*x+c))^(3/2)/(a+a*sec(d*x+c)),x, algorithm="maxima")
```

output
```
integrate((e*tan(d*x + c))^(3/2)/(a*sec(d*x + c) + a), x)
```

3.122.8 Giac [F]

$$\int \frac{(e\tan(c+dx))^{3/2}}{a+a\sec(c+dx)}\,dx = \int \frac{(e\tan(dx+c))^{\frac{3}{2}}}{a\sec(dx+c)+a}\,dx$$

input
```
integrate((e*tan(d*x+c))^(3/2)/(a+a*sec(d*x+c)),x, algorithm="giac")
```

output
```
integrate((e*tan(d*x + c))^(3/2)/(a*sec(d*x + c) + a), x)
```

3.122.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(e\tan(c+dx))^{3/2}}{a+a\sec(c+dx)}\,dx = \int \frac{\cos(c+dx)\,(e\tan(c+dx))^{3/2}}{a\,(\cos(c+dx)+1)}\,dx$$

input
```
int((e*tan(c + d*x))^(3/2)/(a + a/cos(c + d*x)),x)
```

output
```
int((cos(c + d*x)*(e*tan(c + d*x))^(3/2))/(a*(cos(c + d*x) + 1)), x)
```

3.123 $\int \frac{\sqrt{e\tan(c+dx)}}{a+a\sec(c+dx)}\,dx$

3.123.1 Optimal result . . . 928
3.123.2 Mathematica [C] (warning: unable to verify) . . . 929
3.123.3 Rubi [A] (warning: unable to verify) . . . 929
3.123.4 Maple [C] (verified) . . . 937
3.123.5 Fricas [F(-1)] . . . 937
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3.123.7 Maxima [F] . . . 938
3.123.8 Giac [F] . . . 938
3.123.9 Mupad [F(-1)] . . . 939

3.123.1 Optimal result

Integrand size = 25, antiderivative size = 315

$$\begin{aligned}\int \frac{\sqrt{e\tan(c+dx)}}{a+a\sec(c+dx)}\,dx &= -\frac{\sqrt{e}\arctan\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}ad}+\frac{\sqrt{e}\arctan\left(1+\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}ad}\\ &+\frac{\sqrt{e}\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}ad}\\ &-\frac{\sqrt{e}\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)+\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}ad}\\ &+\frac{2e(1-\sec(c+dx))}{ad\sqrt{e\tan(c+dx)}}\\ &-\frac{2\cos(c+dx)E\left(c-\frac{\pi}{4}+dx\middle|2\right)\sqrt{e\tan(c+dx)}}{ad\sqrt{\sin(2c+2dx)}}\\ &+\frac{2\cos(c+dx)(e\tan(c+dx))^{3/2}}{ade}\end{aligned}$$

output

```
-1/2*arctan(1-2^(1/2)*(e*tan(d*x+c))^(1/2)/e^(1/2))*e^(1/2)/a/d*2^(1/2)+1/
2*arctan(1+2^(1/2)*(e*tan(d*x+c))^(1/2)/e^(1/2))*e^(1/2)/a/d*2^(1/2)+1/4*l
n(e^(1/2)-2^(1/2)*(e*tan(d*x+c))^(1/2)+e^(1/2)*tan(d*x+c))*e^(1/2)/a/d*2^(
1/2)-1/4*ln(e^(1/2)+2^(1/2)*(e*tan(d*x+c))^(1/2)+e^(1/2)*tan(d*x+c))*e^(1/
2)/a/d*2^(1/2)+2*e*(1-sec(d*x+c))/a/d/(e*tan(d*x+c))^(1/2)+2*cos(d*x+c)*(s
in(c+1/4*Pi+d*x)^2)^(1/2)/sin(c+1/4*Pi+d*x)*EllipticE(cos(c+1/4*Pi+d*x),2^
(1/2))*(e*tan(d*x+c))^(1/2)/a/d/sin(2*d*x+2*c)^(1/2)+2*cos(d*x+c)*(e*tan(d
*x+c))^(3/2)/a/d/e
```

3.123.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.58 (sec) , antiderivative size = 2715, normalized size of antiderivative = 8.62

$$\int \frac{\sqrt{e\tan(c+dx)}}{a+a\sec(c+dx)}\,dx = \text{Result too large to show}$$

input
```
Integrate[Sqrt[e*Tan[c + d*x]]/(a + a*Sec[c + d*x]),x]
```

output
```
(Cos[c/2 + (d*x)/2]^2*Sec[c + d*x]*((-2*Cos[c/2]*Cos[d*x]*Sec[2*c]*(4*Sin[
c/2] + Sin[(3*c)/2] + Sin[(5*c)/2]))/(d*(1 + 2*Cos[c])) - (4*Sec[c/2]*Sec[
c/2 + (d*x)/2]*Sin[(d*x)/2])/d - ((-2 - 5*Cos[c] - 6*Cos[2*c] + Cos[3*c])*
Sec[2*c]*Sin[d*x])/(d*(1 + 2*Cos[c])) - (4*Tan[c/2])/d)*Sqrt[e*Tan[c + d*x
]])/(a + a*Sec[c + d*x]) + ((E^((2*I)*c)*Sqrt[-1 + E^((4*I)*(c + d*x))]*Ar
cTan[Sqrt[-1 + E^((4*I)*(c + d*x))]] - 2*Sqrt[-1 + E^((2*I)*(c + d*x))]*Sq
rt[1 + E^((2*I)*(c + d*x))]*ArcTanh[Sqrt[(-1 + E^((2*I)*(c + d*x)))/(1 + E
^((2*I)*(c + d*x)))]])*Cos[c/2 + (d*x)/2]^2*Sec[2*c]*Sec[c + d*x]*Sqrt[e*T
an[c + d*x]])/(2*d*E^(I*c)*Sqrt[((-I)*(-1 + E^((2*I)*(c + d*x))))/(1 + E^(
(2*I)*(c + d*x)))]*(1 + E^((2*I)*(c + d*x)))*(1 + 2*Cos[c])*(a + a*Sec[c +
 d*x])*Sqrt[Tan[c + d*x]]) - ((-(E^((4*I)*c)*Sqrt[-1 + E^((4*I)*(c + d*x))
]*ArcTan[Sqrt[-1 + E^((4*I)*(c + d*x))]]) + 2*Sqrt[-1 + E^((2*I)*(c + d*x)
)]*Sqrt[1 + E^((2*I)*(c + d*x))]*ArcTanh[Sqrt[(-1 + E^((2*I)*(c + d*x)))/(
1 + E^((2*I)*(c + d*x)))]])*Cos[c/2 + (d*x)/2]^2*Sec[2*c]*Sec[c + d*x]*Sqr
t[e*Tan[c + d*x]])/(2*d*E^((2*I)*c)*Sqrt[((-I)*(-1 + E^((2*I)*(c + d*x))))
/(1 + E^((2*I)*(c + d*x)))]*(1 + E^((2*I)*(c + d*x)))*(1 + 2*Cos[c])*(a +
a*Sec[c + d*x])*Sqrt[Tan[c + d*x]]) - ((-(E^((6*I)*c)*Sqrt[-1 + E^((4*I)*(
c + d*x))]*ArcTan[Sqrt[-1 + E^((4*I)*(c + d*x))]]) + 2*Sqrt[-1 + E^((2*I)*
(c + d*x))]*Sqrt[1 + E^((2*I)*(c + d*x))]*ArcTanh[Sqrt[(-1 + E^((2*I)*(c +
 d*x)))/(1 + E^((2*I)*(c + d*x)))]])*Cos[c/2 + (d*x)/2]^2*Sec[2*c]*Sec[...
```

3.123.3 Rubi [A] (warning: unable to verify)

Time = 1.29 (sec) , antiderivative size = 298, normalized size of antiderivative = 0.95, number of steps used = 27, number of rules used = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 1.040$, Rules used = {3042, 4376, 25, 3042, 4370, 27, 3042, 4372, 3042, 3093, 3042, 3095, 3042, 3052, 3042, 3119, 3957, 266, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{e\tan(c+dx)}}{a\sec(c+dx)+a}\,dx$$

$$\downarrow 3042$$

$$\int \frac{\sqrt{-e\cot\left(c+dx+\frac{\pi}{2}\right)}}{a\csc\left(c+dx+\frac{\pi}{2}\right)+a}dx$$

$$\downarrow 4376$$

$$\frac{e^2\int -\frac{a-a\sec(c+dx)}{(e\tan(c+dx))^{3/2}}dx}{a^2}$$

$$\downarrow 25$$

$$-\frac{e^2\int \frac{a-a\sec(c+dx)}{(e\tan(c+dx))^{3/2}}dx}{a^2}$$

$$\downarrow 3042$$

$$-\frac{e^2\int \frac{a-a\csc\left(c+dx+\frac{\pi}{2}\right)}{\left(-e\cot\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{a^2}$$

$$\downarrow 4370$$

$$-\frac{e^2\left(\frac{2\int -\frac{1}{2}(\sec(c+dx)a+a)\sqrt{e\tan(c+dx)}dx}{e^2}-\frac{2(a-a\sec(c+dx))}{de\sqrt{e\tan(c+dx)}}\right)}{a^2}$$

$$\downarrow 27$$

$$-\frac{e^2\left(-\frac{\int(\sec(c+dx)a+a)\sqrt{e\tan(c+dx)}dx}{e^2}-\frac{2(a-a\sec(c+dx))}{de\sqrt{e\tan(c+dx)}}\right)}{a^2}$$

$$\downarrow 3042$$

$$-\frac{e^2\left(-\frac{\int\sqrt{-e\cot\left(c+dx+\frac{\pi}{2}\right)}\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)dx}{e^2}-\frac{2(a-a\sec(c+dx))}{de\sqrt{e\tan(c+dx)}}\right)}{a^2}$$

$$\downarrow 4372$$

$$-\frac{e^2\left(-\frac{a\int\sqrt{e\tan(c+dx)}dx+a\int\sec(c+dx)\sqrt{e\tan(c+dx)}dx}{e^2}-\frac{2(a-a\sec(c+dx))}{de\sqrt{e\tan(c+dx)}}\right)}{a^2}$$

$$\downarrow 3042$$

$$-\frac{e^2\left(-\frac{a\int\sqrt{e\tan(c+dx)}dx+a\int\sec(c+dx)\sqrt{e\tan(c+dx)}dx}{e^2}-\frac{2(a-a\sec(c+dx))}{de\sqrt{e\tan(c+dx)}}\right)}{a^2}$$

$$\downarrow 3093$$

$$-\frac{e^2\left(-\frac{a\int\sqrt{e\tan(c+dx)}dx+a\left(\frac{2\cos(c+dx)(e\tan(c+dx))^{3/2}}{de}-2\int\cos(c+dx)\sqrt{e\tan(c+dx)}dx\right)}{e^2}-\frac{2(a-a\sec(c+dx))}{de\sqrt{e\tan(c+dx)}}\right)}{a^2}$$

$$\downarrow 3042$$

$$-\frac{e^2\left(-\frac{a\int\sqrt{e\tan(c+dx)}dx+a\left(\frac{2\cos(c+dx)(e\tan(c+dx))^{3/2}}{de}-2\int\frac{\sqrt{e\tan(c+dx)}}{\sec(c+dx)}dx\right)}{e^2}-\frac{2(a-a\sec(c+dx))}{de\sqrt{e\tan(c+dx)}}\right)}{a^2}$$

$$\downarrow 3095$$

$$-\frac{e^2\left(-\frac{a\int\sqrt{e\tan(c+dx)}dx+a\left(\frac{2\cos(c+dx)(e\tan(c+dx))^{3/2}}{de}-\frac{2\sqrt{\cos(c+dx)}\sqrt{e\tan(c+dx)}\int\sqrt{\cos(c+dx)}\sqrt{\sin(c+dx)}dx}{\sqrt{\sin(c+dx)}}\right)}{e^2}-\frac{2(a-a\sec(c+dx))}{de\sqrt{e\tan(c+dx)}}\right)}{a^2}$$

$$\downarrow 3042$$

$$-\frac{e^2\left(-\frac{a\int\sqrt{e\tan(c+dx)}dx+a\left(\frac{2\cos(c+dx)(e\tan(c+dx))^{3/2}}{de}-\frac{2\sqrt{\cos(c+dx)}\sqrt{e\tan(c+dx)}\int\sqrt{\cos(c+dx)}\sqrt{\sin(c+dx)}dx}{\sqrt{\sin(c+dx)}}\right)}{e^2}-\frac{2(a-a\sec(c+dx))}{de\sqrt{e\tan(c+dx)}}\right)}{a^2}$$

$$\downarrow 3052$$

$$-\frac{e^2\left(-\frac{a\int\sqrt{e\tan(c+dx)}dx+a\left(\frac{2\cos(c+dx)(e\tan(c+dx))^{3/2}}{de}-\frac{2\cos(c+dx)\sqrt{e\tan(c+dx)}\int\sqrt{\sin(2c+2dx)}dx}{\sqrt{\sin(2c+2dx)}}\right)}{e^2}-\frac{2(a-a\sec(c+dx))}{de\sqrt{e\tan(c+dx)}}\right)}{a^2}$$

$$\downarrow 3042$$

$$-\frac{e^2\left(-\frac{a\int\sqrt{e\tan(c+dx)}dx+a\left(\frac{2\cos(c+dx)(e\tan(c+dx))^{3/2}}{de}-\frac{2\cos(c+dx)\sqrt{e\tan(c+dx)}\int\sqrt{\sin(2c+2dx)}dx}{\sqrt{\sin(2c+2dx)}}\right)}{e^2}-\frac{2(a-a\sec(c+dx))}{de\sqrt{e\tan(c+dx)}}\right)}{a^2}$$

$$\downarrow 3119$$

$$-\frac{e^2\left(-\frac{a\int\sqrt{e\tan(c+dx)}dx+a\left(\frac{2\cos(c+dx)(e\tan(c+dx))^{3/2}}{de}-\frac{2\cos(c+dx)E\left(c+dx-\frac{\pi}{4}\middle|2\right)\sqrt{e\tan(c+dx)}}{d\sqrt{\sin(2c+2dx)}}\right)}{e^2}-\frac{2(a-a\sec(c+dx))}{de\sqrt{e\tan(c+dx)}}\right)}{a^2}$$

$$\downarrow 3957$$

$$-\frac{e^2\left(-\frac{\frac{ae\int\frac{\sqrt{e\tan(c+dx)}}{\tan^2(c+dx)e^2+e^2}d(e\tan(c+dx))}{d}+a\left(\frac{2\cos(c+dx)(e\tan(c+dx))^{3/2}}{de}-\frac{2\cos(c+dx)E\left(c+dx-\frac{\pi}{4}\middle|2\right)\sqrt{e\tan(c+dx)}}{d\sqrt{\sin(2c+2dx)}}\right)}{e^2}-\frac{2(a-a\sec(c+dx))}{de\sqrt{e\tan(c+dx)}}\right)}{a^2}$$

$$\downarrow 266$$

$$-\frac{e^2\left(-\frac{\frac{2ae\int\frac{e^2\tan^2(c+dx)}{e^4\tan^4(c+dx)+e^2}d\sqrt{e\tan(c+dx)}}{d}+a\left(\frac{2\cos(c+dx)(e\tan(c+dx))^{3/2}}{de}-\frac{2\cos(c+dx)E\left(c+dx-\frac{\pi}{4}\middle|2\right)\sqrt{e\tan(c+dx)}}{d\sqrt{\sin(2c+2dx)}}\right)}{e^2}-\frac{2(a-a\sec(c+dx))}{de\sqrt{e\tan(c+dx)}}\right)}{a^2}$$

$$\downarrow 826$$

$$-\frac{e^2\left(-\frac{\frac{2ae\left(\frac{1}{2}\int\frac{e^2\tan^2(c+dx)+e}{e^4\tan^4(c+dx)+e^2}d\sqrt{e\tan(c+dx)}-\frac{1}{2}\int\frac{e-e^2\tan^2(c+dx)}{e^4\tan^4(c+dx)+e^2}d\sqrt{e\tan(c+dx)}\right)}{d}+a\left(\frac{2\cos(c+dx)(e\tan(c+dx))^{3/2}}{de}-\frac{2\cos(c+dx)E\left(c+dx-\frac{\pi}{4}\middle|2\right)}{d\sqrt{\sin(2c+2dx}}\right.\right.}{e^2}\right.}{a^2}$$

$$\downarrow 1476$$

$$-\frac{e^2\left(-\frac{\frac{2ae\left(\frac{1}{2}\left(\frac{1}{2}\int\frac{1}{e^2\tan^2(c+dx)-\sqrt{2}e^{3/2}\tan(c+dx)+e}d\sqrt{e\tan(c+dx)}+\frac{1}{2}\int\frac{1}{e^2\tan^2(c+dx)+\sqrt{2}e^{3/2}\tan(c+dx)+e}d\sqrt{e\tan(c+dx)}\right)-\frac{1}{2}\int\frac{e-e^2\tan^2(c+dx)}{e^4\tan^4(c+dx)+e^2}\right.}{d}}{e^2}\right.}{a^2}$$

$$\downarrow 1082$$

$$-\frac{e^2\left(-\frac{\frac{2ae\left(\frac{1}{2}\left(\frac{\int\frac{1}{-e^2\tan^2(c+dx)-1}d\left(1-\sqrt{2}\sqrt{e}\tan(c+dx)\right)}{\sqrt{2}\sqrt{e}}-\frac{\int\frac{1}{-e^2\tan^2(c+dx)-1}d\left(\sqrt{2}\sqrt{e}\tan(c+dx)+1\right)}{\sqrt{2}\sqrt{e}}\right)-\frac{1}{2}\int\frac{e-e^2\tan^2(c+dx)}{e^4\tan^4(c+dx)+e^2}d\sqrt{e\tan(c+dx)}\right)}{d}}{e^2}\right.}{a^2}$$

$$\downarrow 217$$

$$e^2\left(-\frac{\frac{2ae\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{e}\tan(c+dx)+1\right)}{\sqrt{2}\sqrt{e}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{e}\tan(c+dx)\right)}{\sqrt{2}\sqrt{e}}\right)-\frac{1}{2}\int\frac{e-e^2\tan^2(c+dx)}{e^4\tan^4(c+dx)+e^2}d\sqrt{e\tan(c+dx)}\right)}{d}}{e^2}+a\left(\frac{2\cos(c+dx)(e\tan(c+dx))^{3/2}}{de}-\right.\right.$$

$$-\frac{}{a^2}$$

$$\downarrow 1479$$

$$e^2\left(-\frac{\frac{2ae\left(\frac{1}{2}\left(\frac{\int-\frac{\sqrt{2}\sqrt{e}-2\sqrt{e\tan(c+dx)}}{e^2\tan^2(c+dx)-\sqrt{2}e^{3/2}\tan(c+dx)+e}d\sqrt{e\tan(c+dx)}}{2\sqrt{2}\sqrt{e}}+\frac{\int-\frac{\sqrt{2}\left(\sqrt{e}+\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{e^2\tan^2(c+dx)+\sqrt{2}e^{3/2}\tan(c+dx)+e}d\sqrt{e\tan(c+dx)}}{2\sqrt{2}\sqrt{e}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{e}\tan(\right.\right.\right.}{d}}{e^2}$$

$$-\frac{}{a^2}$$

$$\downarrow 25$$

$$e^2\left(-\frac{\frac{2ae\left(\frac{1}{2}\left(-\frac{\int\frac{\sqrt{2}\sqrt{e}-2\sqrt{e\tan(c+dx)}}{e^2\tan^2(c+dx)-\sqrt{2}e^{3/2}\tan(c+dx)+e}d\sqrt{e\tan(c+dx)}}{2\sqrt{2}\sqrt{e}}-\frac{\int\frac{\sqrt{2}\left(\sqrt{e}+\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{e^2\tan^2(c+dx)+\sqrt{2}e^{3/2}\tan(c+dx)+e}d\sqrt{e\tan(c+dx)}}{2\sqrt{2}\sqrt{e}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{e}\tan(c\right.\right.\right.}{d}}{e^2}$$

$$-\frac{}{a^2}$$

$$\downarrow 27$$

$$e^2\left(-\frac{\frac{2ae\left(\frac{1}{2}\left(-\frac{\int\frac{\sqrt{2}\sqrt{e}-2\sqrt{e\tan(c+dx)}}{e^2\tan^2(c+dx)-\sqrt{2}e^{3/2}\tan(c+dx)+e}d\sqrt{e\tan(c+dx)}}{2\sqrt{2}\sqrt{e}}-\frac{\int\frac{\sqrt{e}+\sqrt{2}\sqrt{e\tan(c+dx)}}{e^2\tan^2(c+dx)+\sqrt{2}e^{3/2}\tan(c+dx)+e}d\sqrt{e\tan(c+dx)}}{2\sqrt{e}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{e}\tan(c\right.\right.\right.}{d}}{e^2}$$

$$-\frac{}{a^2}$$

$$\downarrow 1103$$

$$e^2\left(-\frac{\frac{2ae\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{e}\tan(c+dx)+1\right)}{\sqrt{2}\sqrt{e}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{e}\tan(c+dx)\right)}{\sqrt{2}\sqrt{e}}\right)+\frac{1}{2}\left(\frac{\log\left(-\sqrt{2}e^{3/2}\tan(c+dx)+e^2\tan^2(c+dx)+e\right)}{2\sqrt{2}\sqrt{e}}-\frac{\log\left(\sqrt{2}e^{3/2}\tan(c+dx)+e^2\right.}{d}\right.}{e^2}\right.$$
$$\left.-\frac{}{a^2}\right)$$

input
```
Int[Sqrt[e*Tan[c + d*x]]/(a + a*Sec[c + d*x]),x]
```

output
```
-((e^2*((-2*(a - a*Sec[c + d*x]))/(d*e*Sqrt[e*Tan[c + d*x]]) - ((2*a*e*((-
(ArcTan[1 - Sqrt[2]*Sqrt[e]*Tan[c + d*x]]/(Sqrt[2]*Sqrt[e])) + ArcTan[1 +
Sqrt[2]*Sqrt[e]*Tan[c + d*x]]/(Sqrt[2]*Sqrt[e]))/2 + (Log[e - Sqrt[2]*e^(3
/2)*Tan[c + d*x] + e^2*Tan[c + d*x]^2]/(2*Sqrt[2]*Sqrt[e]) - Log[e + Sqrt[
2]*e^(3/2)*Tan[c + d*x] + e^2*Tan[c + d*x]^2]/(2*Sqrt[2]*Sqrt[e]))/2))/d +
 a*((-2*Cos[c + d*x]*EllipticE[c - Pi/4 + d*x, 2]*Sqrt[e*Tan[c + d*x]])/(d
*Sqrt[Sin[2*c + 2*d*x]]) + (2*Cos[c + d*x]*(e*Tan[c + d*x])^(3/2))/(d*e)))
/e^2))/a^2)
```

3.123.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3052
```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Simp[Sqrt[a*Sin[e + f*x]]*(Sqrt[b*Cos[e + f*x]]/Sqrt[Sin[2*e
 + 2*f*x]])   Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

rule 3093
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[a^2*(a*Sec[e + f*x])^(m - 2)*((b*Tan[e + f*x])^(n +
1)/(b*f*(m + n - 1))), x] + Simp[a^2*((m - 2)/(m + n - 1))   Int[(a*Sec[e +
 f*x])^(m - 2)*(b*Tan[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (
GtQ[m, 1] || (EqQ[m, 1] && EqQ[n, 1/2])) && NeQ[m + n - 1, 0] && IntegersQ[
2*m, 2*n]
```

rule 3095
```
Int[Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]]/sec[(e_.) + (f_.)*(x_)], x_Symbol]
:> Simp[Sqrt[Cos[e + f*x]]*(Sqrt[b*Tan[e + f*x]]/Sqrt[Sin[e + f*x]])   Int[
Sqrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]], x], x] /; FreeQ[{b, e, f}, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

rule 4370
```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(-(e*Cot[c + d*x])^(m + 1))*((a + b*Csc[c + d*x])/(
d*e*(m + 1))), x] - Simp[1/(e^2*(m + 1))   Int[(e*Cot[c + d*x])^(m + 2)*(a*
(m + 1) + b*(m + 2)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && L
tQ[m, -1]
```

rule 4372
```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(e*Cot[c + d*x])^m, x], x] + Simp[b   Int[
(e*Cot[c + d*x])^m*Csc[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, m}, x]
```

rule 4376
```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Simp[a^(2*n)/e^(2*n)   Int[(e*Cot[c + d*x])^(m + 2*
n)/(-a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[a
^2 - b^2, 0] && ILtQ[n, 0]
```

3.123.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.36 (sec) , antiderivative size = 256, normalized size of antiderivative = 0.81

method	result
default	$\frac{(\frac{1}{2}-\frac{i}{2})\sqrt{2}\left(2i\,\text{EllipticE}\left(\sqrt{\csc(dx+c)-\cot(dx+c)+1},\frac{\sqrt{2}}{2}\right)-i\,\text{EllipticF}\left(\sqrt{\csc(dx+c)-\cot(dx+c)+1},\frac{\sqrt{2}}{2}\right)-i\,\text{EllipticPi}\left(\sqrt{\csc(dx+c)}\right.\right.$

input
```
int((e*tan(d*x+c))^(1/2)/(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
(1/2-1/2*I)/a/d*2^(1/2)*(2*I*EllipticE((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2
*2^(1/2))-I*EllipticF((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2*2^(1/2))-I*Ellip
ticPi((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2+1/2*I,1/2*2^(1/2))-EllipticPi((c
sc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2-1/2*I,1/2*2^(1/2))+2*EllipticE((csc(d*x+
c)-cot(d*x+c)+1)^(1/2),1/2*2^(1/2))-EllipticF((csc(d*x+c)-cot(d*x+c)+1)^(1
/2),1/2*2^(1/2)))*(e*tan(d*x+c))^(1/2)*(csc(d*x+c)-cot(d*x+c)+1)^(1/2)*(co
t(d*x+c)-csc(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)*(cot(d*x+c)+csc
(d*x+c))
```

3.123.5 Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{e\tan(c+dx)}}{a+a\sec(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((e*tan(d*x+c))^(1/2)/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

output
```
Timed out
```

3.123.6 Sympy [F]

$$\int \frac{\sqrt{e\tan(c+dx)}}{a+a\sec(c+dx)}\,dx = \frac{\int \frac{\sqrt{e\tan(c+dx)}}{\sec(c+dx)+1}\,dx}{a}$$

input
```
integrate((e*tan(d*x+c))**(1/2)/(a+a*sec(d*x+c)),x)
```

output
```
Integral(sqrt(e*tan(c + d*x))/(sec(c + d*x) + 1), x)/a
```

3.123.7 Maxima [F]

$$\int \frac{\sqrt{e\tan(c+dx)}}{a+a\sec(c+dx)}\,dx = \int \frac{\sqrt{e\tan(dx+c)}}{a\sec(dx+c)+a}\,dx$$

input
```
integrate((e*tan(d*x+c))^(1/2)/(a+a*sec(d*x+c)),x, algorithm="maxima")
```

output
```
integrate(sqrt(e*tan(d*x + c))/(a*sec(d*x + c) + a), x)
```

3.123.8 Giac [F]

$$\int \frac{\sqrt{e\tan(c+dx)}}{a+a\sec(c+dx)}\,dx = \int \frac{\sqrt{e\tan(dx+c)}}{a\sec(dx+c)+a}\,dx$$

input
```
integrate((e*tan(d*x+c))^(1/2)/(a+a*sec(d*x+c)),x, algorithm="giac")
```

output
```
integrate(sqrt(e*tan(d*x + c))/(a*sec(d*x + c) + a), x)
```

3.123.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{e\tan(c+dx)}}{a+a\sec(c+dx)}\,dx = \int \frac{\cos(c+dx)\,\sqrt{e\tan(c+dx)}}{a\,(\cos(c+dx)+1)}\,dx$$

input
```
int((e*tan(c + d*x))^(1/2)/(a + a/cos(c + d*x)),x)
```

output
```
int((cos(c + d*x)*(e*tan(c + d*x))^(1/2))/(a*(cos(c + d*x) + 1)), x)
```

3.124 $\int \frac{1}{(a+a\sec(c+dx))\sqrt{e\tan(c+dx)}}\,dx$

3.124.1 Optimal result . . . 940
3.124.2 Mathematica [C] (warning: unable to verify) . . . 941
3.124.3 Rubi [A] (warning: unable to verify) . . . 942
3.124.4 Maple [C] (verified) . . . 949
3.124.5 Fricas [F(-1)] . . . 950
3.124.6 Sympy [F] . . . 950
3.124.7 Maxima [F] . . . 951
3.124.8 Giac [F] . . . 951
3.124.9 Mupad [F(-1)] . . . 951

3.124.1 Optimal result

Integrand size = 25, antiderivative size = 290

$$\begin{aligned}
&\int \frac{1}{(a+a\sec(c+dx))\sqrt{e\tan(c+dx)}}\,dx \\
&= -\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}ad\sqrt{e}} + \frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}ad\sqrt{e}} \\
&\quad - \frac{\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}ad\sqrt{e}} \\
&\quad + \frac{\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)+\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}ad\sqrt{e}} + \frac{2e(1-\sec(c+dx))}{3ad(e\tan(c+dx))^{3/2}} \\
&\quad - \frac{\text{EllipticF}\left(c-\frac{\pi}{4}+dx,2\right)\sec(c+dx)\sqrt{\sin(2c+2dx)}}{3ad\sqrt{e\tan(c+dx)}}
\end{aligned}$$

output

```
-1/2*arctan(1-2^(1/2)*(e*tan(d*x+c))^(1/2)/e^(1/2))/a/d*2^(1/2)/e^(1/2)+1/
2*arctan(1+2^(1/2)*(e*tan(d*x+c))^(1/2)/e^(1/2))/a/d*2^(1/2)/e^(1/2)-1/4*l
n(e^(1/2)-2^(1/2)*(e*tan(d*x+c))^(1/2)+e^(1/2)*tan(d*x+c))/a/d*2^(1/2)/e^(
1/2)+1/4*ln(e^(1/2)+2^(1/2)*(e*tan(d*x+c))^(1/2)+e^(1/2)*tan(d*x+c))/a/d*2
^(1/2)/e^(1/2)+1/3*(sin(c+1/4*Pi+d*x)^2)^(1/2)/sin(c+1/4*Pi+d*x)*EllipticF
(cos(c+1/4*Pi+d*x),2^(1/2))*sec(d*x+c)*sin(2*d*x+2*c)^(1/2)/a/d/(e*tan(d*x
+c))^(1/2)+2/3*e*(1-sec(d*x+c))/a/d/(e*tan(d*x+c))^(3/2)
```

3.124.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.24 (sec) , antiderivative size = 1253, normalized size of antiderivative = 4.32

$$\int \frac{1}{(a+a\sec(c+dx))\sqrt{e\tan(c+dx)}}\,dx$$

$$= \frac{2e^{-i(c+dx)}\sqrt{-\frac{i\left(-1+e^{2i(c+dx)}\right)}{1+e^{2i(c+dx)}}}\left(1+e^{2i(c+dx)}\right)\cos^2\left(\frac{c}{2}+\frac{dx}{2}\right)\sec(2c)\sec(c+dx)\sqrt{\tan(c+dx)}}{3d(a+a\sec(c+dx))\sqrt{e\tan(c+dx)}}$$

$$+\frac{e^{-2ic}\sqrt{-\frac{i\left(-1+e^{2i(c+dx)}\right)}{1+e^{2i(c+dx)}}}\left(e^{4ic}\sqrt{-1+e^{4i(c+dx)}}\arctan\left(\sqrt{-1+e^{4i(c+dx)}}\right)+2\sqrt{-1+e^{2i(c+dx)}}\sqrt{1+e^{2i(c+dx)}}\right.}{2d\left(-1+e^{2i(c+dx)}\right)(a+a\sec(c+dx))\sqrt{e}}$$

$$+\frac{e^{-2ic}\sqrt{-\frac{i\left(-1+e^{2i(c+dx)}\right)}{1+e^{2i(c+dx)}}}\left(\sqrt{-1+e^{4i(c+dx)}}\arctan\left(\sqrt{-1+e^{4i(c+dx)}}\right)+2e^{4ic}\sqrt{-1+e^{2i(c+dx)}}\sqrt{1+e^{2i(c+dx)}}\right.}{2d\left(-1+e^{2i(c+dx)}\right)(a+a\sec(c+dx))\sqrt{e}}$$

$$-\frac{e^{-i(2c+dx)}\sqrt{-\frac{i\left(-1+e^{2i(c+dx)}\right)}{1+e^{2i(c+dx)}}}\cos^2\left(\frac{c}{2}+\frac{dx}{2}\right)\left(3\left(-1+e^{4i(c+dx)}\right)+e^{4i(c+dx)}(-1+e^{2ic})\sqrt{1-e^{4i(c+dx)}}\,\text{Hyperg}\right.}{3d\left(-1+e^{2i(c+dx)}\right)(a+a\sec(c+dx))\sqrt{e\tan}}$$

$$+\frac{e^{-idx}\sqrt{-\frac{i\left(-1+e^{2i(c+dx)}\right)}{1+e^{2i(c+dx)}}}\cos^2\left(\frac{c}{2}+\frac{dx}{2}\right)\left(3-3e^{4i(c+dx)}+e^{2i(c+2dx)}(-1+e^{2ic})\sqrt{1-e^{4i(c+dx)}}\,\text{Hypergeometr}\right.}{3d\left(-1+e^{2i(c+dx)}\right)(a+a\sec(c+dx))\sqrt{e\tan(c+}$$

$$+\frac{\cos^2\left(\frac{c}{2}+\frac{dx}{2}\right)\sec(c+dx)\left(-\frac{4}{3d}+\frac{2(3-2\cos(c)+3\cos(2c))\cos(dx)\sec(2c)}{3d}+\frac{2\sec^2\left(\frac{c}{2}+\frac{dx}{2}\right)}{3d}-\frac{2\sec(2c)(-2\sin(c)+3\sin(2c))}{3d}\right.}{(a+a\sec(c+dx))\sqrt{e\tan(c+dx)}}$$

$$+\frac{4\sqrt[4]{-1}\cos^2\left(\frac{c}{2}+\frac{dx}{2}\right)\text{EllipticF}\left(i\text{arcsinh}\left(\sqrt[4]{-1}\sqrt{\tan(c+dx)}\right),-1\right)\sec^4(c+dx)\sqrt{\tan(c+dx)}}{3d(a+a\sec(c+dx))\sqrt{e\tan(c+dx)}\left(1+\tan^2(c+dx)\right)^{3/2}}$$

input
```
Integrate[1/((a + a*Sec[c + d*x])*Sqrt[e*Tan[c + d*x]]),x]
```

output

```
(2*Sqrt[((-I)*(-1 + E^((2*I)*(c + d*x))))/(1 + E^((2*I)*(c + d*x)))]*(1 +
E^((2*I)*(c + d*x)))*Cos[c/2 + (d*x)/2]^2*Sec[2*c]*Sec[c + d*x]*Sqrt[Tan[c
 + d*x]])/(3*d*E^(I*(c + d*x))*(a + a*Sec[c + d*x])*Sqrt[e*Tan[c + d*x]])
+ (Sqrt[((-I)*(-1 + E^((2*I)*(c + d*x))))/(1 + E^((2*I)*(c + d*x)))]*(E^((
4*I)*c)*Sqrt[-1 + E^((4*I)*(c + d*x))]*ArcTan[Sqrt[-1 + E^((4*I)*(c + d*x)
)]] + 2*Sqrt[-1 + E^((2*I)*(c + d*x))]*Sqrt[1 + E^((2*I)*(c + d*x))]*ArcTa
nh[Sqrt[(-1 + E^((2*I)*(c + d*x)))/(1 + E^((2*I)*(c + d*x)))]])*Cos[c/2 +
(d*x)/2]^2*Sec[2*c]*Sec[c + d*x]*Sqrt[Tan[c + d*x]])/(2*d*E^((2*I)*c)*(-1
+ E^((2*I)*(c + d*x)))*(a + a*Sec[c + d*x])*Sqrt[e*Tan[c + d*x]]) + (Sqrt[
((-I)*(-1 + E^((2*I)*(c + d*x))))/(1 + E^((2*I)*(c + d*x)))]*(Sqrt[-1 + E^
((4*I)*(c + d*x))]*ArcTan[Sqrt[-1 + E^((4*I)*(c + d*x))]] + 2*E^((4*I)*c)*
Sqrt[-1 + E^((2*I)*(c + d*x))]*Sqrt[1 + E^((2*I)*(c + d*x))]*ArcTanh[Sqrt[
(-1 + E^((2*I)*(c + d*x)))/(1 + E^((2*I)*(c + d*x)))]])*Cos[c/2 + (d*x)/2]
^2*Sec[2*c]*Sec[c + d*x]*Sqrt[Tan[c + d*x]])/(2*d*E^((2*I)*c)*(-1 + E^((2*
I)*(c + d*x)))*(a + a*Sec[c + d*x])*Sqrt[e*Tan[c + d*x]]) - (Sqrt[((-I)*(-
1 + E^((2*I)*(c + d*x))))/(1 + E^((2*I)*(c + d*x)))]*Cos[c/2 + (d*x)/2]^2*
(3*(-1 + E^((4*I)*(c + d*x))) + E^((4*I)*(c + d*x))*(-1 + E^((2*I)*c))*Sqr
t[1 - E^((4*I)*(c + d*x))]*Hypergeometric2F1[1/2, 3/4, 7/4, E^((4*I)*(c +
d*x))])*Sec[2*c]*Sec[c + d*x]*Sqrt[Tan[c + d*x]])/(3*d*E^(I*(2*c + d*x))*(
-1 + E^((2*I)*(c + d*x)))*(a + a*Sec[c + d*x])*Sqrt[e*Tan[c + d*x]]) + ...
```

3.124.3 Rubi [A] (warning: unable to verify)

Time = 1.10 (sec) , antiderivative size = 280, normalized size of antiderivative = 0.97, number of steps used = 25, number of rules used = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.960$, Rules used = {3042, 4376, 25, 3042, 4370, 27, 3042, 4372, 3042, 3094, 3042, 3053, 3042, 3120, 3957, 266, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a\sec(c+dx)+a)\sqrt{e\tan(c+dx)}}\,dx$$

$$\downarrow 3042$$

$$\int \frac{1}{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)\sqrt{-e\cot\left(c+dx+\frac{\pi}{2}\right)}}\,dx$$

$$\downarrow 4376$$

$$\frac{e^2\int -\frac{a-a\sec(c+dx)}{(e\tan(c+dx))^{5/2}}\,dx}{a^2}$$

$$\downarrow 25$$

$$-\frac{e^2\int \frac{a-a\sec(c+dx)}{(e\tan(c+dx))^{5/2}}dx}{a^2}$$

$$\downarrow 3042$$

$$-\frac{e^2\int \frac{a-a\csc\left(c+dx+\frac{\pi}{2}\right)}{\left(-e\cot\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}dx}{a^2}$$

$$\downarrow 4370$$

$$-\frac{e^2\left(\frac{2\int -\frac{3a-a\sec(c+dx)}{2\sqrt{e\tan(c+dx)}}dx}{3e^2}-\frac{2(a-a\sec(c+dx))}{3de(e\tan(c+dx))^{3/2}}\right)}{a^2}$$

$$\downarrow 27$$

$$-\frac{e^2\left(-\frac{\int \frac{3a-a\sec(c+dx)}{\sqrt{e\tan(c+dx)}}dx}{3e^2}-\frac{2(a-a\sec(c+dx))}{3de(e\tan(c+dx))^{3/2}}\right)}{a^2}$$

$$\downarrow 3042$$

$$-\frac{e^2\left(-\frac{\int \frac{3a-a\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{-e\cot\left(c+dx+\frac{\pi}{2}\right)}}dx}{3e^2}-\frac{2(a-a\sec(c+dx))}{3de(e\tan(c+dx))^{3/2}}\right)}{a^2}$$

$$\downarrow 4372$$

$$-\frac{e^2\left(-\frac{3a\int \frac{1}{\sqrt{e\tan(c+dx)}}dx-a\int \frac{\sec(c+dx)}{\sqrt{e\tan(c+dx)}}dx}{3e^2}-\frac{2(a-a\sec(c+dx))}{3de(e\tan(c+dx))^{3/2}}\right)}{a^2}$$

$$\downarrow 3042$$

$$-\frac{e^2\left(-\frac{3a\int \frac{1}{\sqrt{e\tan(c+dx)}}dx-a\int \frac{\sec(c+dx)}{\sqrt{e\tan(c+dx)}}dx}{3e^2}-\frac{2(a-a\sec(c+dx))}{3de(e\tan(c+dx))^{3/2}}\right)}{a^2}$$

$$\downarrow 3094$$

$$-\frac{e^2\left(-\frac{3a\int \frac{1}{\sqrt{e\tan(c+dx)}}dx-\frac{a\sqrt{\sin(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{\sin(c+dx)}}dx}{\sqrt{\cos(c+dx)}\sqrt{e\tan(c+dx)}}}{3e^2}-\frac{2(a-a\sec(c+dx))}{3de(e\tan(c+dx))^{3/2}}\right)}{a^2}$$

$$\downarrow 3042$$

$$-\frac{e^2\left(-\frac{3a\int\frac{1}{\sqrt{e\tan(c+dx)}}dx-\frac{a\sqrt{\sin(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}\sqrt{\sin(c+dx)}}dx}{\sqrt{\cos(c+dx)}\sqrt{e\tan(c+dx)}}}{3e^2}-\frac{2(a-a\sec(c+dx))}{3de(e\tan(c+dx))^{3/2}}\right)}{a^2}$$

$$\downarrow 3053$$

$$-\frac{e^2\left(-\frac{3a\int\frac{1}{\sqrt{e\tan(c+dx)}}dx-\frac{a\sqrt{\sin(2c+2dx)}\sec(c+dx)\int\frac{1}{\sqrt{\sin(2c+2dx)}}dx}{\sqrt{e\tan(c+dx)}}}{3e^2}-\frac{2(a-a\sec(c+dx))}{3de(e\tan(c+dx))^{3/2}}\right)}{a^2}$$

$$\downarrow 3042$$

$$-\frac{e^2\left(-\frac{3a\int\frac{1}{\sqrt{e\tan(c+dx)}}dx-\frac{a\sqrt{\sin(2c+2dx)}\sec(c+dx)\int\frac{1}{\sqrt{\sin(2c+2dx)}}dx}{\sqrt{e\tan(c+dx)}}}{3e^2}-\frac{2(a-a\sec(c+dx))}{3de(e\tan(c+dx))^{3/2}}\right)}{a^2}$$

$$\downarrow 3120$$

$$-\frac{e^2\left(-\frac{3a\int\frac{1}{\sqrt{e\tan(c+dx)}}dx-\frac{a\sqrt{\sin(2c+2dx)}\sec(c+dx)\,\text{EllipticF}\left(c+dx-\frac{\pi}{4},2\right)}{d\sqrt{e\tan(c+dx)}}}{3e^2}-\frac{2(a-a\sec(c+dx))}{3de(e\tan(c+dx))^{3/2}}\right)}{a^2}$$

$$\downarrow 3957$$

$$-\frac{e^2\left(-\frac{\frac{3ae\int\frac{1}{\sqrt{e\tan(c+dx)}\left(\tan^2(c+dx)e^2+e^2\right)}d(e\tan(c+dx))}{d}-\frac{a\sqrt{\sin(2c+2dx)}\sec(c+dx)\,\text{EllipticF}\left(c+dx-\frac{\pi}{4},2\right)}{d\sqrt{e\tan(c+dx)}}}{3e^2}-\frac{2(a-a\sec(c+dx))}{3de(e\tan(c+dx))^{3/2}}\right)}{a^2}$$

$$\downarrow 266$$

$$-\frac{e^2\left(-\frac{\frac{6ae\int\frac{1}{e^4\tan^4(c+dx)+e^2}d\sqrt{e\tan(c+dx)}}{d}-\frac{a\sqrt{\sin(2c+2dx)}\sec(c+dx)\,\text{EllipticF}\left(c+dx-\frac{\pi}{4},2\right)}{d\sqrt{e\tan(c+dx)}}}{3e^2}-\frac{2(a-a\sec(c+dx))}{3de(e\tan(c+dx))^{3/2}}\right)}{a^2}$$

$$\downarrow 755$$

$$-\frac{e^2\left(-\frac{6ae\left(\frac{\int\frac{e-e^2\tan^2(c+dx)}{e^4\tan^4(c+dx)+e^2}d\sqrt{e\tan(c+dx)}}{2e}+\frac{\int\frac{e^2\tan^2(c+dx)+e}{e^4\tan^4(c+dx)+e^2}d\sqrt{e\tan(c+dx)}}{2e}\right)}{d}-\frac{a\sqrt{\sin(2c+2dx)}\sec(c+dx)\,\text{EllipticF}\left(c+dx-\frac{\pi}{4},2\right)}{d\sqrt{e\tan(c+dx)}}}{3e^2}-\frac{2(a-a\sec(c+dx))}{3de(e\tan(c+dx))^{3/2}}\right)}{a^2}$$

3.124. $\int\frac{1}{(a+a\sec(c+dx))\sqrt{e\tan(c+dx)}}dx$

$$\downarrow 1476$$

$$e^2\left(-\frac{6ae\left(\frac{\frac{1}{2}\int\frac{1}{e^2\tan^2(c+dx)-\sqrt{2}e^{3/2}\tan(c+dx)+e}d\sqrt{e\tan(c+dx)}+\frac{1}{2}\int\frac{1}{e^2\tan^2(c+dx)+\sqrt{2}e^{3/2}\tan(c+dx)+e}d\sqrt{e\tan(c+dx)}}{2e}+\frac{\int\frac{e-e^2\tan^2(c+dx)}{e^4\tan^4(c+dx)+e^2}d\sqrt{e\t}}{2e}\right)}{d}}{3e^2}\right)-\frac{}{a^2}$$

$$\downarrow 1082$$

$$e^2\left(-\frac{6ae\left(\frac{\frac{\int\frac{1}{-e^2\tan^2(c+dx)-1}d\left(1-\sqrt{2}\sqrt{e}\tan(c+dx)\right)}{\sqrt{2}\sqrt{e}}-\frac{\int\frac{1}{-e^2\tan^2(c+dx)-1}d\left(\sqrt{2}\sqrt{e}\tan(c+dx)+1\right)}{\sqrt{2}\sqrt{e}}}{2e}+\frac{\int\frac{e-e^2\tan^2(c+dx)}{e^4\tan^4(c+dx)+e^2}d\sqrt{e\tan(c+dx)}}{2e}\right)}{d}-\frac{a\sqrt{\sin(2c}}{3e^2}\right)-\frac{}{a^2}$$

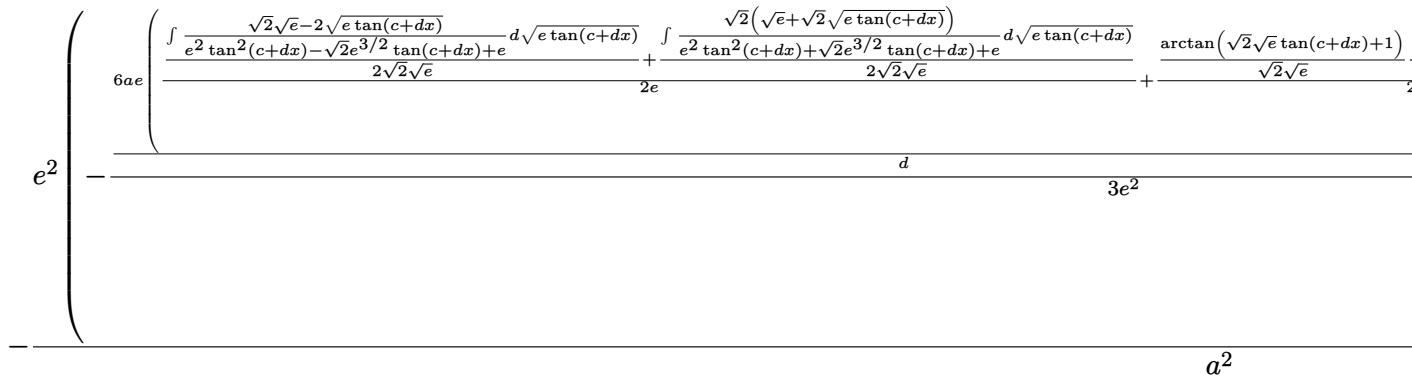
$$\downarrow 217$$

$$e^2\left(-\frac{6ae\left(\frac{\int\frac{e-e^2\tan^2(c+dx)}{e^4\tan^4(c+dx)+e^2}d\sqrt{e\tan(c+dx)}}{2e}+\frac{\frac{\arctan\left(\sqrt{2}\sqrt{e}\tan(c+dx)+1\right)}{\sqrt{2}\sqrt{e}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{e}\tan(c+dx)\right)}{\sqrt{2}\sqrt{e}}}{2e}\right)}{d}-\frac{a\sqrt{\sin(2c+2dx)}\sec(c+dx)\,\text{EllipticF}\left(c+dx\right.}{d\sqrt{e\tan(c+dx)}}}{3e^2}\right)-\frac{}{a^2}$$

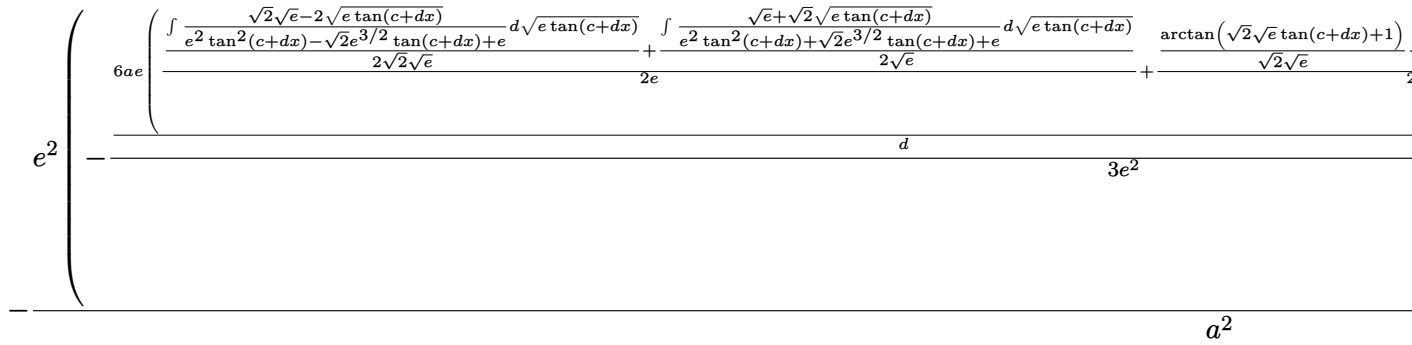
$$\downarrow 1479$$

$$e^2\left(-\frac{6ae\left(\frac{-\frac{\int-\frac{\sqrt{2}\sqrt{e}-2\sqrt{e\tan(c+dx)}}{e^2\tan^2(c+dx)-\sqrt{2}e^{3/2}\tan(c+dx)+e}d\sqrt{e\tan(c+dx)}}{2\sqrt{2}\sqrt{e}}-\frac{\int-\frac{\sqrt{2}\left(\sqrt{e}+\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{e^2\tan^2(c+dx)+\sqrt{2}e^{3/2}\tan(c+dx)+e}d\sqrt{e\tan(c+dx)}}{2\sqrt{2}\sqrt{e}}}{2e}+\frac{\arctan\left(\sqrt{2}\sqrt{e}\tan(c+dx)\right.}{\sqrt{2}\sqrt{e}}}{d}}{3e^2}\right)-\frac{}{a^2}$$

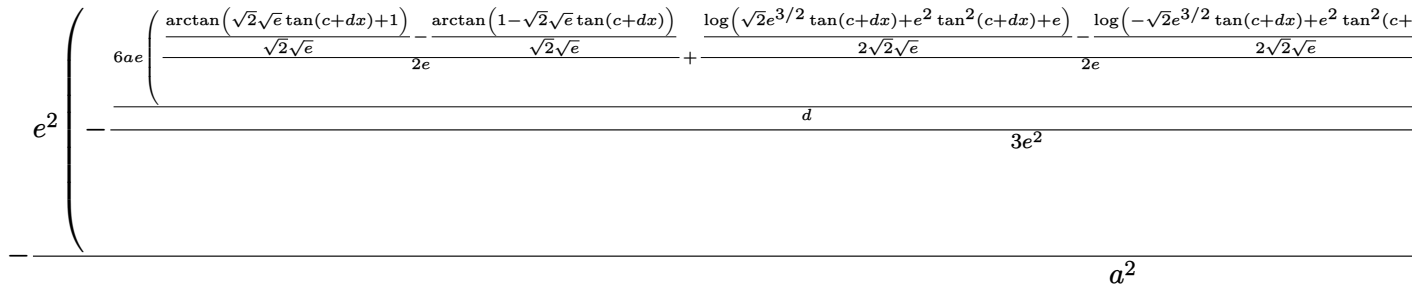
$$\downarrow 25$$



$\downarrow$ 27



$\downarrow$ 1103



input
```
Int[1/((a + a*Sec[c + d*x])*Sqrt[e*Tan[c + d*x]]),x]
```

output
```
-((e^2*((-2*(a - a*Sec[c + d*x]))/(3*d*e*(e*Tan[c + d*x])^(3/2)) - ((6*a*e
*((-(ArcTan[1 - Sqrt[2]*Sqrt[e]*Tan[c + d*x]]/(Sqrt[2]*Sqrt[e])) + ArcTan[
1 + Sqrt[2]*Sqrt[e]*Tan[c + d*x]]/(Sqrt[2]*Sqrt[e]))/(2*e) + (-1/2*Log[e -
 Sqrt[2]*e^(3/2)*Tan[c + d*x] + e^2*Tan[c + d*x]^2]/(Sqrt[2]*Sqrt[e]) + Lo
g[e + Sqrt[2]*e^(3/2)*Tan[c + d*x] + e^2*Tan[c + d*x]^2]/(2*Sqrt[2]*Sqrt[e
]))/(2*e)))/d - (a*EllipticF[c - Pi/4 + d*x, 2]*Sec[c + d*x]*Sqrt[Sin[2*c
+ 2*d*x]])/(d*Sqrt[e*Tan[c + d*x]]))/(3*e^2)))/a^2)
```

3.124.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3053
```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Simp[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]])   Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f
}, x]
```

rule 3094
```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]], x_Symbol]
:> Simp[Sqrt[Sin[e + f*x]]/(Sqrt[Cos[e + f*x]]*Sqrt[b*Tan[e + f*x]])   Int[
1/(Sqrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]]), x], x] /; FreeQ[{b, e, f}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

rule 4370
```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(-(e*Cot[c + d*x])^(m + 1))*((a + b*Csc[c + d*x])/(
d*e*(m + 1))), x] - Simp[1/(e^2*(m + 1))   Int[(e*Cot[c + d*x])^(m + 2)*(a*
(m + 1) + b*(m + 2)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && L
tQ[m, -1]
```

rule 4372
```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(e*Cot[c + d*x])^m, x], x] + Simp[b   Int[
(e*Cot[c + d*x])^m*Csc[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, m}, x]
```

rule 4376
```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Simp[a^(2*n)/e^(2*n)   Int[(e*Cot[c + d*x])^(m + 2*
n)/(-a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[a
^2 - b^2, 0] && ILtQ[n, 0]
```

3.124.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 3.75 (sec) , antiderivative size = 611, normalized size of antiderivative = 2.11

method	result
default	$-\frac{\sqrt{2}\left(3i\sqrt{\csc(dx+c)-\cot(dx+c)+1}\,\sqrt{2-2\csc(dx+c)+2\cot(dx+c)}\,\sqrt{\cot(dx+c)-\csc(dx+c)}\,\text{EllipticPi}\left(\sqrt{\csc(dx+c)-\cot(dx+c)+\right.\right.$

input
```
int(1/(a+a*sec(d*x+c))/(e*tan(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/6/a/d*2^(1/2)*(3*I*(csc(d*x+c)-cot(d*x+c)+1)^(1/2)*(2-2*csc(d*x+c)+2*co
t(d*x+c))^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)*EllipticPi((csc(d*x+c)-cot(d
*x+c)+1)^(1/2),1/2-1/2*I,1/2*2^(1/2))-3*I*(csc(d*x+c)-cot(d*x+c)+1)^(1/2)*
(2-2*csc(d*x+c)+2*cot(d*x+c))^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)*Elliptic
Pi((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2+1/2*I,1/2*2^(1/2))-8*(csc(d*x+c)-co
t(d*x+c)+1)^(1/2)*(2-2*csc(d*x+c)+2*cot(d*x+c))^(1/2)*(cot(d*x+c)-csc(d*x+
c))^(1/2)*EllipticF((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2*2^(1/2))+3*(csc(d*
x+c)-cot(d*x+c)+1)^(1/2)*(2-2*csc(d*x+c)+2*cot(d*x+c))^(1/2)*(cot(d*x+c)-c
sc(d*x+c))^(1/2)*EllipticPi((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2-1/2*I,1/2*
2^(1/2))+3*(csc(d*x+c)-cot(d*x+c)+1)^(1/2)*(2-2*csc(d*x+c)+2*cot(d*x+c))^(
1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)*EllipticPi((csc(d*x+c)-cot(d*x+c)+1)^(1
/2),1/2+1/2*I,1/2*2^(1/2))+2*(1-cos(d*x+c))^3*csc(d*x+c)^3-2*csc(d*x+c)+2*
cot(d*x+c))*(1-cos(d*x+c))/((1-cos(d*x+c))^3*csc(d*x+c)^3+cot(d*x+c)-csc(d
*x+c))^(1/2)/((1-cos(d*x+c))*((1-cos(d*x+c))^2*csc(d*x+c)^2-1)*csc(d*x+c))
^(1/2)/(-e/((1-cos(d*x+c))^2*csc(d*x+c)^2-1)*(-cot(d*x+c)+csc(d*x+c)))^(1/
2)*csc(d*x+c)
```

3.124.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a+a\sec(c+dx))\sqrt{e\tan(c+dx)}}\,dx = \text{Timed out}$$

input
```
integrate(1/(a+a*sec(d*x+c))/(e*tan(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.124.6 Sympy [F]

$$\int \frac{1}{(a+a\sec(c+dx))\sqrt{e\tan(c+dx)}}\,dx = \frac{\int \frac{1}{\sqrt{e\tan(c+dx)}\sec(c+dx)+\sqrt{e\tan(c+dx)}}\,dx}{a}$$

input
```
integrate(1/(a+a*sec(d*x+c))/(e*tan(d*x+c))**(1/2),x)
```

output
```
Integral(1/(sqrt(e*tan(c + d*x))*sec(c + d*x) + sqrt(e*tan(c + d*x))), x)/
a
```

3.124.7 Maxima [F]

$$\int \frac{1}{(a+a\sec(c+dx))\sqrt{e\tan(c+dx)}}\,dx = \int \frac{1}{(a\sec(dx+c)+a)\sqrt{e\tan(dx+c)}}\,dx$$

input
```
integrate(1/(a+a*sec(d*x+c))/(e*tan(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/((a*sec(d*x + c) + a)*sqrt(e*tan(d*x + c))), x)
```

3.124.8 Giac [F]

$$\int \frac{1}{(a+a\sec(c+dx))\sqrt{e\tan(c+dx)}}\,dx = \int \frac{1}{(a\sec(dx+c)+a)\sqrt{e\tan(dx+c)}}\,dx$$

input
```
integrate(1/(a+a*sec(d*x+c))/(e*tan(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(1/((a*sec(d*x + c) + a)*sqrt(e*tan(d*x + c))), x)
```

3.124.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+a\sec(c+dx))\sqrt{e\tan(c+dx)}}\,dx = \int \frac{\cos(c+dx)}{a\sqrt{e\tan(c+dx)}\,(\cos(c+dx)+1)}\,dx$$

input
```
int(1/((e*tan(c + d*x))^(1/2)*(a + a/cos(c + d*x))),x)
```

output
```
int(cos(c + d*x)/(a*(e*tan(c + d*x))^(1/2)*(cos(c + d*x) + 1)), x)
```

3.125 $\int \frac{1}{(a+a\sec(c+dx))(e\tan(c+dx))^{3/2}}\,dx$

3.125.1 Optimal result . . . 952
3.125.2 Mathematica [C] (warning: unable to verify) . . . 953
3.125.3 Rubi [A] (warning: unable to verify) . . . 953
3.125.4 Maple [C] (warning: unable to verify) . . . 961
3.125.5 Fricas [F(-1)] . . . 962
3.125.6 Sympy [F] . . . 963
3.125.7 Maxima [F] . . . 963
3.125.8 Giac [F] . . . 963
3.125.9 Mupad [F(-1)] . . . 964

3.125.1 Optimal result

Integrand size = 25, antiderivative size = 359

$$\int \frac{1}{(a+a\sec(c+dx))(e\tan(c+dx))^{3/2}}\,dx = \frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}ade^{3/2}} - \frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}ade^{3/2}} - \frac{\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}ade^{3/2}} + \frac{\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)+\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}ade^{3/2}} + \frac{2e(1-\sec(c+dx))}{5ad(e\tan(c+dx))^{5/2}} - \frac{2(5-3\sec(c+dx))}{5ade\sqrt{e\tan(c+dx)}} + \frac{6\cos(c+dx)E\left(c-\frac{\pi}{4}+dx\middle|2\right)\sqrt{e\tan(c+dx)}}{5ade^2\sqrt{\sin(2c+2dx)}} - \frac{6\cos(c+dx)(e\tan(c+dx))^{3/2}}{5ade^3}$$

output

```
1/2*arctan(1-2^(1/2)*(e*tan(d*x+c))^(1/2)/e^(1/2))/a/d/e^(3/2)*2^(1/2)-1/2
*arctan(1+2^(1/2)*(e*tan(d*x+c))^(1/2)/e^(1/2))/a/d/e^(3/2)*2^(1/2)-1/4*ln
(e^(1/2)-2^(1/2)*(e*tan(d*x+c))^(1/2)+e^(1/2)*tan(d*x+c))/a/d/e^(3/2)*2^(1
/2)+1/4*ln(e^(1/2)+2^(1/2)*(e*tan(d*x+c))^(1/2)+e^(1/2)*tan(d*x+c))/a/d/e^
(3/2)*2^(1/2)-2/5*(5-3*sec(d*x+c))/a/d/e/(e*tan(d*x+c))^(1/2)-6/5*cos(d*x+
c)*(sin(c+1/4*Pi+d*x)^2)^(1/2)/sin(c+1/4*Pi+d*x)*EllipticE(cos(c+1/4*Pi+d*
x),2^(1/2))*(e*tan(d*x+c))^(1/2)/a/d/e^2/sin(2*d*x+2*c)^(1/2)+2/5*e*(1-sec
(d*x+c))/a/d/(e*tan(d*x+c))^(5/2)-6/5*cos(d*x+c)*(e*tan(d*x+c))^(3/2)/a/d/
e^3
```

3.125.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 13.99 (sec) , antiderivative size = 180, normalized size of antiderivative = 0.50

$$\int \frac{1}{(a+a\sec(c+dx))(e\tan(c+dx))^{3/2}}\,dx =$$

$$-\frac{4\csc(c+dx)\left(15\cot^2(c+dx)-3\cot^4(c+dx)+3\cot^4(c+dx)\,\text{Hypergeometric2F1}\left(-\frac{5}{4},-\frac{1}{2},-\frac{1}{4},-\tan^2\right.\right.}{}$$

input
```
Integrate[1/((a + a*Sec[c + d*x])*(e*Tan[c + d*x])^(3/2)),x]
```

output
```
(-4*Csc[c + d*x]*(15*Cot[c + d*x]^2 - 3*Cot[c + d*x]^4 + 3*Cot[c + d*x]^4*
Hypergeometric2F1[-5/4, -1/2, -1/4, -Tan[c + d*x]^2] - 15*Cot[c + d*x]^2*H
ypergeometric2F1[-1/2, -1/4, 3/4, -Tan[c + d*x]^2] - 5*Hypergeometric2F1[1
/2, 3/4, 7/4, -Tan[c + d*x]^2] + 5*Hypergeometric2F1[3/4, 1, 7/4, -Tan[c +
 d*x]^2])*(1 + Sqrt[Sec[c + d*x]^2])*Sin[(c + d*x)/2]^2*Sqrt[e*Tan[c + d*x
]])/(15*a*d*e^2)
```

3.125.3 Rubi [A] (warning: unable to verify)

Time = 1.59 (sec) , antiderivative size = 342, normalized size of antiderivative = 0.95, number of steps used = 30, number of rules used = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 1.160$, Rules used = {3042, 4376, 25, 3042, 4370, 27, 3042, 4370, 27, 3042, 4372, 3042, 3093, 3042, 3095, 3042, 3052, 3042, 3119, 3957, 266, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a\sec(c+dx)+a)(e\tan(c+dx))^{3/2}}\,dx$$

$$\downarrow 3042$$

$$\int \frac{1}{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)\left(-e\cot\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx$$

$$\downarrow 4376$$

$$\frac{e^2\int -\frac{a-a\sec(c+dx)}{(e\tan(c+dx))^{7/2}}dx}{a^2}$$

$$\downarrow 25$$

$$-\frac{e^2\int \frac{a-a\sec(c+dx)}{(e\tan(c+dx))^{7/2}}dx}{a^2}$$

$$\downarrow 3042$$

$$-\frac{e^2\int \frac{a-a\csc\left(c+dx+\frac{\pi}{2}\right)}{\left(-e\cot\left(c+dx+\frac{\pi}{2}\right)\right)^{7/2}}dx}{a^2}$$

$$\downarrow 4370$$

$$-\frac{e^2\left(\frac{2\int -\frac{5a-3a\sec(c+dx)}{2(e\tan(c+dx))^{3/2}}dx}{5e^2}-\frac{2(a-a\sec(c+dx))}{5de(e\tan(c+dx))^{5/2}}\right)}{a^2}$$

$$\downarrow 27$$

$$-\frac{e^2\left(-\frac{\int \frac{5a-3a\sec(c+dx)}{(e\tan(c+dx))^{3/2}}dx}{5e^2}-\frac{2(a-a\sec(c+dx))}{5de(e\tan(c+dx))^{5/2}}\right)}{a^2}$$

$$\downarrow 3042$$

$$-\frac{e^2\left(-\frac{\int \frac{5a-3a\csc\left(c+dx+\frac{\pi}{2}\right)}{\left(-e\cot\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{5e^2}-\frac{2(a-a\sec(c+dx))}{5de(e\tan(c+dx))^{5/2}}\right)}{a^2}$$

$$\downarrow 4370$$

$$-\frac{e^2\left(-\frac{2\int -\frac{1}{2}(3\sec(c+dx)a+5a)\sqrt{e\tan(c+dx)}dx}{e^2}-\frac{2(5a-3a\sec(c+dx))}{de\sqrt{e\tan(c+dx)}}}{5e^2}-\frac{2(a-a\sec(c+dx))}{5de(e\tan(c+dx))^{5/2}}\right)}{a^2}$$

$$\downarrow 27$$

$$-\frac{e^2\left(-\frac{-\int(3\sec(c+dx)a+5a)\sqrt{e\tan(c+dx)}dx}{e^2}-\frac{2(5a-3a\sec(c+dx))}{de\sqrt{e\tan(c+dx)}}}{5e^2}-\frac{2(a-a\sec(c+dx))}{5de(e\tan(c+dx))^{5/2}}\right)}{a^2}$$

$$\downarrow 3042$$

$$-\frac{e^2\left(-\frac{-\int\sqrt{-e\cot\left(c+dx+\frac{\pi}{2}\right)}\left(3\csc\left(c+dx+\frac{\pi}{2}\right)a+5a\right)dx}{e^2}-\frac{2(5a-3a\sec(c+dx))}{de\sqrt{e\tan(c+dx)}}}{5e^2}-\frac{2(a-a\sec(c+dx))}{5de(e\tan(c+dx))^{5/2}}\right)}{a^2}$$

$$\downarrow 4372$$

$$-\frac{e^2\left(-\frac{-\frac{5a\int\sqrt{e\tan(c+dx)}dx+3a\int\sec(c+dx)\sqrt{e\tan(c+dx)}dx}{e^2}-\frac{2(5a-3a\sec(c+dx))}{de\sqrt{e\tan(c+dx)}}}{5e^2}-\frac{2(a-a\sec(c+dx))}{5de(e\tan(c+dx))^{5/2}}\right)}{a^2}$$

$$\downarrow 3042$$

$$-\frac{e^2\left(-\frac{-\frac{5a\int\sqrt{e\tan(c+dx)}dx+3a\int\sec(c+dx)\sqrt{e\tan(c+dx)}dx}{e^2}-\frac{2(5a-3a\sec(c+dx))}{de\sqrt{e\tan(c+dx)}}}{5e^2}-\frac{2(a-a\sec(c+dx))}{5de(e\tan(c+dx))^{5/2}}\right)}{a^2}$$

$$\downarrow 3093$$

$$-\frac{e^2\left(-\frac{-\frac{5a\int\sqrt{e\tan(c+dx)}dx+3a\left(\frac{2\cos(c+dx)(e\tan(c+dx))^{3/2}}{de}-2\int\cos(c+dx)\sqrt{e\tan(c+dx)}dx\right)}{e^2}-\frac{2(5a-3a\sec(c+dx))}{de\sqrt{e\tan(c+dx)}}}{5e^2}-\frac{2(a-a\sec(c+dx))}{5de(e\tan(c+dx))^{5/2}}\right)}{a^2}$$

$$\downarrow 3042$$

$$-\frac{e^2\left(-\frac{-\frac{5a\int\sqrt{e\tan(c+dx)}dx+3a\left(\frac{2\cos(c+dx)(e\tan(c+dx))^{3/2}}{de}-2\int\frac{\sqrt{e\tan(c+dx)}}{\sec(c+dx)}dx\right)}{e^2}-\frac{2(5a-3a\sec(c+dx))}{de\sqrt{e\tan(c+dx)}}}{5e^2}-\frac{2(a-a\sec(c+dx))}{5de(e\tan(c+dx))^{5/2}}\right)}{a^2}$$

$$\downarrow 3095$$

$$-\frac{e^2\left(-\frac{-\frac{5a\int\sqrt{e\tan(c+dx)}dx+3a\left(\frac{2\cos(c+dx)(e\tan(c+dx))^{3/2}}{de}-\frac{2\sqrt{\cos(c+dx)}\sqrt{e\tan(c+dx)}\int\sqrt{\cos(c+dx)}\sqrt{\sin(c+dx)}dx}{\sqrt{\sin(c+dx)}}\right)}{e^2}-\frac{2(5a-3a\sec(c+dx))}{de\sqrt{e\tan(c+dx)}}}{5e^2}-\frac{2(a-a\sec(c+dx))}{5de(e\tan(c+dx))^{5/2}}\right)}{a^2}$$

$$\downarrow 3042$$

$$-\frac{e^2\left(-\frac{-\frac{5a\int\sqrt{e\tan(c+dx)}dx+3a\left(\frac{2\cos(c+dx)(e\tan(c+dx))^{3/2}}{de}-\frac{2\sqrt{\cos(c+dx)}\sqrt{e\tan(c+dx)}\int\sqrt{\cos(c+dx)}\sqrt{\sin(c+dx)}dx}{\sqrt{\sin(c+dx)}}\right)}{e^2}-\frac{2(5a-3a\sec(c+dx))}{de\sqrt{e\tan(c+dx)}}}{5e^2}-\frac{2(a-a\sec(c+dx))}{5de(e\tan(c+dx))^{5/2}}\right)}{a^2}$$

$$\downarrow 3052$$

$$e^2\left(-\frac{-\frac{5a\int\sqrt{e\tan(c+dx)}dx+3a\left(\frac{2\cos(c+dx)(e\tan(c+dx))^{3/2}}{de}-\frac{2\cos(c+dx)\sqrt{e\tan(c+dx)}\int\sqrt{\sin(2c+2dx)}dx}{\sqrt{\sin(2c+2dx)}}\right)}{e^2}-\frac{2(5a-3a\sec(c+dx))}{de\sqrt{e\tan(c+dx)}}}{5e^2}-\frac{2(a-a\sec(c+\right.$$
$$\left.-\frac{}{a^2}\right.$$

$$\downarrow 3042$$

$$e^2\left(-\frac{-\frac{5a\int\sqrt{e\tan(c+dx)}dx+3a\left(\frac{2\cos(c+dx)(e\tan(c+dx))^{3/2}}{de}-\frac{2\cos(c+dx)\sqrt{e\tan(c+dx)}\int\sqrt{\sin(2c+2dx)}dx}{\sqrt{\sin(2c+2dx)}}\right)}{e^2}-\frac{2(5a-3a\sec(c+dx))}{de\sqrt{e\tan(c+dx)}}}{5e^2}-\frac{2(a-a\sec(c+\right.$$
$$\left.-\frac{}{a^2}\right.$$

$$\downarrow 3119$$

$$e^2\left(-\frac{-\frac{5a\int\sqrt{e\tan(c+dx)}dx+3a\left(\frac{2\cos(c+dx)(e\tan(c+dx))^{3/2}}{de}-\frac{2\cos(c+dx)E\left(c+dx-\frac{\pi}{4}\middle|2\right)\sqrt{e\tan(c+dx)}}{d\sqrt{\sin(2c+2dx)}}\right)}{e^2}-\frac{2(5a-3a\sec(c+dx))}{de\sqrt{e\tan(c+dx)}}}{5e^2}-\frac{2(a-a\sec(c+dx)}{5de(e\tan(c+dx))}\right.$$
$$\left.-\frac{}{a^2}\right.$$

$$\downarrow 3957$$

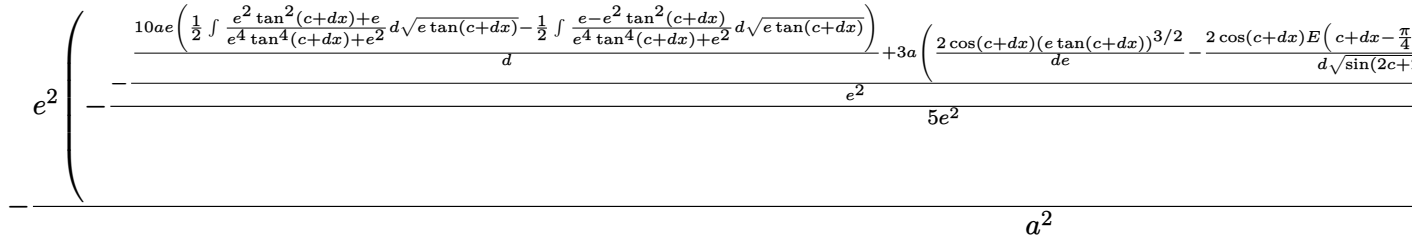
$$e^2\left(-\frac{-\frac{5ae\int\frac{\sqrt{e\tan(c+dx)}}{\tan^2(c+dx)e^2+e^2}d(e\tan(c+dx))}{d}+3a\left(\frac{2\cos(c+dx)(e\tan(c+dx))^{3/2}}{de}-\frac{2\cos(c+dx)E\left(c+dx-\frac{\pi}{4}\middle|2\right)\sqrt{e\tan(c+dx)}}{d\sqrt{\sin(2c+2dx)}}\right)}{e^2}-\frac{2(5a-3a\sec(c+dx))}{de\sqrt{e\tan(c+dx)}}-\right.$$
$$\left.-\frac{}{a^2}\right.$$

$$\downarrow 266$$

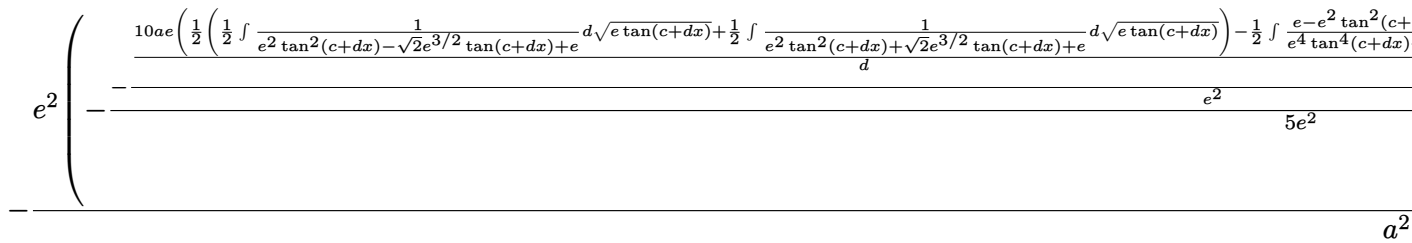
$$e^2\left(-\frac{-\frac{10ae\int\frac{e^2\tan^2(c+dx)}{e^4\tan^4(c+dx)+e^2}d\sqrt{e\tan(c+dx)}}{d}+3a\left(\frac{2\cos(c+dx)(e\tan(c+dx))^{3/2}}{de}-\frac{2\cos(c+dx)E\left(c+dx-\frac{\pi}{4}\middle|2\right)\sqrt{e\tan(c+dx)}}{d\sqrt{\sin(2c+2dx)}}\right)}{e^2}-\frac{2(5a-3a\sec(c+dx))}{de\sqrt{e\tan(c+dx)}}-\right.$$
$$\left.-\frac{}{a^2}\right.$$

$$\downarrow 826$$

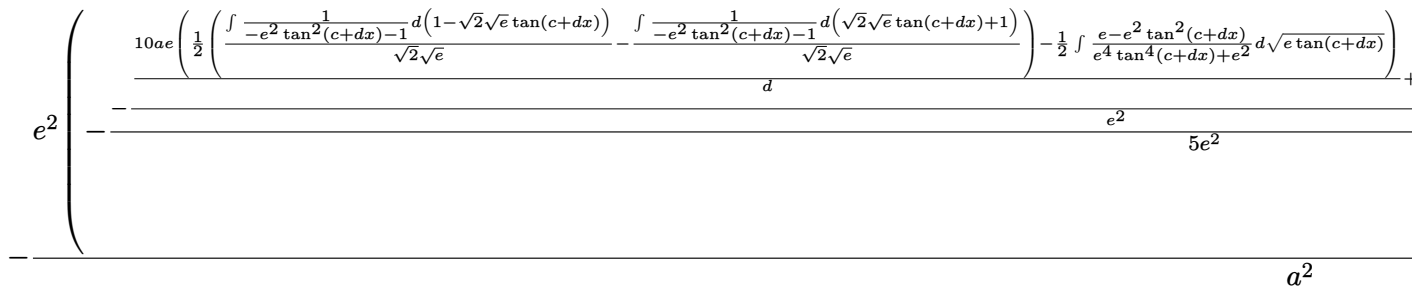
3.125. $\int \frac{1}{(a+a\sec(c+dx))(e\tan(c+dx))^{3/2}}\,dx$



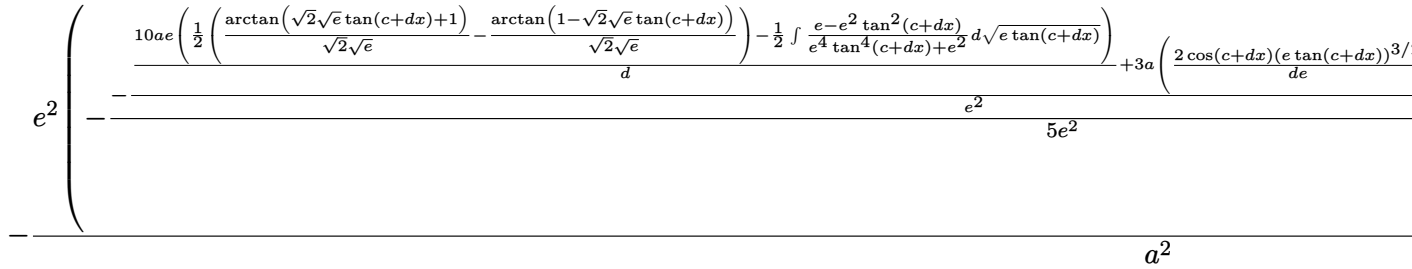
$$\downarrow 1476$$



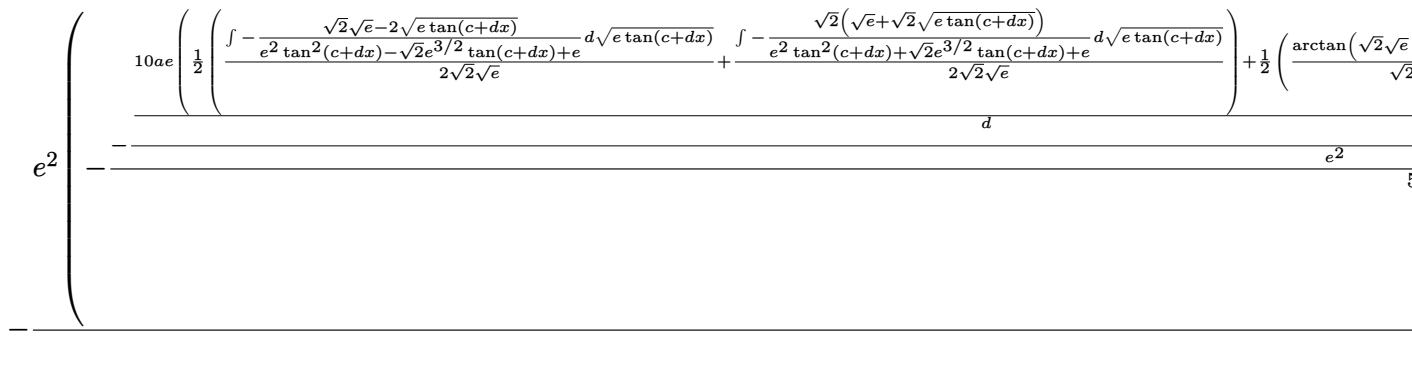
$$\downarrow 1082$$



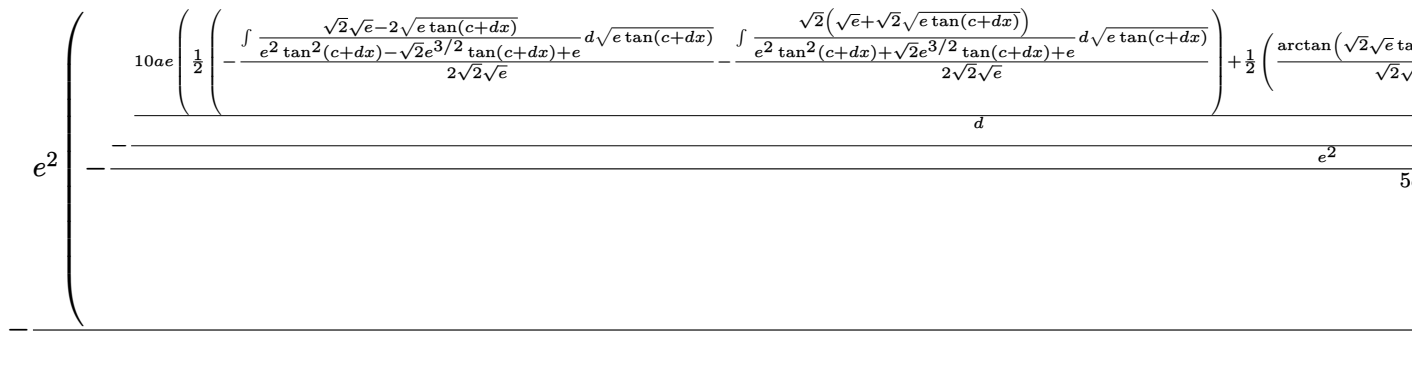
$$\downarrow 217$$



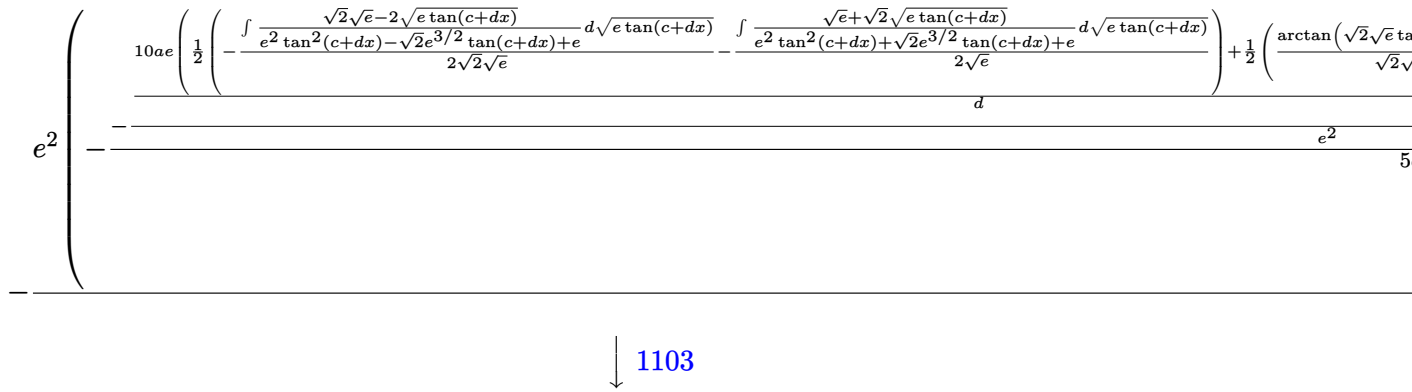
$$\downarrow 1479$$



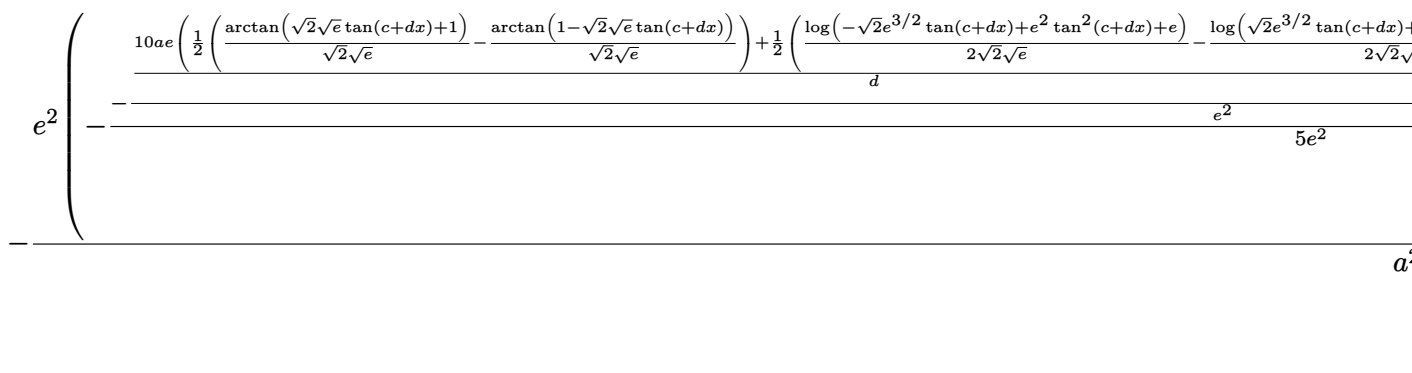
$\downarrow$ 25



$\downarrow$ 27



$\downarrow$ 1103



input
```
Int[1/((a + a*Sec[c + d*x])*(e*Tan[c + d*x])^(3/2)),x]
```

output
```
-((e^2*((-2*(a - a*Sec[c + d*x]))/(5*d*e*(e*Tan[c + d*x])^(5/2)) - ((-2*(5
*a - 3*a*Sec[c + d*x]))/(d*e*Sqrt[e*Tan[c + d*x]]) - ((10*a*e*((-(ArcTan[1
 - Sqrt[2]*Sqrt[e]*Tan[c + d*x]]/(Sqrt[2]*Sqrt[e])) + ArcTan[1 + Sqrt[2]*S
qrt[e]*Tan[c + d*x]]/(Sqrt[2]*Sqrt[e]))/2 + (Log[e - Sqrt[2]*e^(3/2)*Tan[c
 + d*x] + e^2*Tan[c + d*x]^2]/(2*Sqrt[2]*Sqrt[e]) - Log[e + Sqrt[2]*e^(3/2
)*Tan[c + d*x] + e^2*Tan[c + d*x]^2]/(2*Sqrt[2]*Sqrt[e]))/2))/d + 3*a*((-2
*Cos[c + d*x]*EllipticE[c - Pi/4 + d*x, 2]*Sqrt[e*Tan[c + d*x]])/(d*Sqrt[S
in[2*c + 2*d*x]]) + (2*Cos[c + d*x]*(e*Tan[c + d*x])^(3/2))/(d*e)))/e^2)/(
5*e^2)))/a^2)
```

3.125.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3052
```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Simp[Sqrt[a*Sin[e + f*x]]*(Sqrt[b*Cos[e + f*x]]/Sqrt[Sin[2*e
 + 2*f*x]])   Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

rule 3093
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[a^2*(a*Sec[e + f*x])^(m - 2)*((b*Tan[e + f*x])^(n +
1)/(b*f*(m + n - 1))), x] + Simp[a^2*((m - 2)/(m + n - 1))   Int[(a*Sec[e +
 f*x])^(m - 2)*(b*Tan[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (
GtQ[m, 1] || (EqQ[m, 1] && EqQ[n, 1/2])) && NeQ[m + n - 1, 0] && IntegersQ[
2*m, 2*n]
```

rule 3095
```
Int[Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]]/sec[(e_.) + (f_.)*(x_)], x_Symbol]
:> Simp[Sqrt[Cos[e + f*x]]*(Sqrt[b*Tan[e + f*x]]/Sqrt[Sin[e + f*x]])   Int[
Sqrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]], x], x] /; FreeQ[{b, e, f}, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

rule 4370
```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(-(e*Cot[c + d*x])^(m + 1))*((a + b*Csc[c + d*x])/(
d*e*(m + 1))), x] - Simp[1/(e^2*(m + 1))   Int[(e*Cot[c + d*x])^(m + 2)*(a*
(m + 1) + b*(m + 2)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && L
tQ[m, -1]
```

rule 4372
```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(e*Cot[c + d*x])^m, x], x] + Simp[b   Int[
(e*Cot[c + d*x])^m*Csc[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, m}, x]
```

rule 4376
```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Simp[a^(2*n)/e^(2*n)   Int[(e*Cot[c + d*x])^(m + 2*
n)/(-a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[a
^2 - b^2, 0] && ILtQ[n, 0]
```

3.125.4 Maple [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 2.99 (sec) , antiderivative size = 1116, normalized size of antiderivative = 3.11

method	result	size
default	Expression too large to display	1116

input
```
int(1/(a+a*sec(d*x+c))/(e*tan(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output

```
-1/10/a/d*2^(1/2)*(5*I*(csc(d*x+c)-cot(d*x+c)+1)^(1/2)*(2-2*csc(d*x+c)+2*c
ot(d*x+c))^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)*EllipticPi((csc(d*x+c)-cot(
d*x+c)+1)^(1/2),1/2-1/2*I,1/2*2^(1/2))*((1-cos(d*x+c))*((1-cos(d*x+c))^2*c
sc(d*x+c)^2-1)*csc(d*x+c))^(1/2)-5*I*(csc(d*x+c)-cot(d*x+c)+1)^(1/2)*(2-2*
csc(d*x+c)+2*cot(d*x+c))^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)*EllipticPi((c
sc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2+1/2*I,1/2*2^(1/2))*((1-cos(d*x+c))*((1-c
os(d*x+c))^2*csc(d*x+c)^2-1)*csc(d*x+c))^(1/2)+12*(csc(d*x+c)-cot(d*x+c)+1
)^(1/2)*(2-2*csc(d*x+c)+2*cot(d*x+c))^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)*
EllipticE((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2*2^(1/2))*((1-cos(d*x+c))*((1
-cos(d*x+c))^2*csc(d*x+c)^2-1)*csc(d*x+c))^(1/2)-6*(csc(d*x+c)-cot(d*x+c)+
1)^(1/2)*(2-2*csc(d*x+c)+2*cot(d*x+c))^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)
*EllipticF((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2*2^(1/2))*((1-cos(d*x+c))*((
1-cos(d*x+c))^2*csc(d*x+c)^2-1)*csc(d*x+c))^(1/2)-5*(csc(d*x+c)-cot(d*x+c)
+1)^(1/2)*(2-2*csc(d*x+c)+2*cot(d*x+c))^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2
)*EllipticPi((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2-1/2*I,1/2*2^(1/2))*((1-co
s(d*x+c))*((1-cos(d*x+c))^2*csc(d*x+c)^2-1)*csc(d*x+c))^(1/2)-5*(csc(d*x+c
)-cot(d*x+c)+1)^(1/2)*(2-2*csc(d*x+c)+2*cot(d*x+c))^(1/2)*(cot(d*x+c)-csc(
d*x+c))^(1/2)*EllipticPi((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2+1/2*I,1/2*2^(
1/2))*((1-cos(d*x+c))*((1-cos(d*x+c))^2*csc(d*x+c)^2-1)*csc(d*x+c))^(1/2)+
((1-cos(d*x+c))*((1-cos(d*x+c))^2*csc(d*x+c)^2-1)*csc(d*x+c))^(1/2)*(1-...
```

3.125.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a+a\sec(c+dx))(e\tan(c+dx))^{3/2}}\,dx = \text{Timed out}$$

input

```
integrate(1/(a+a*sec(d*x+c))/(e*tan(d*x+c))^(3/2),x, algorithm="fricas")
```

output

```
Timed out
```

3.125.6 Sympy [F]

$$\int \frac{1}{(a+a\sec(c+dx))(e\tan(c+dx))^{3/2}}\,dx = \frac{\int \frac{1}{(e\tan(c+dx))^{\frac{3}{2}}\sec(c+dx)+(e\tan(c+dx))^{\frac{3}{2}}}\,dx}{a}$$

input
```
integrate(1/(a+a*sec(d*x+c))/(e*tan(d*x+c))**(3/2),x)
```

output
```
Integral(1/((e*tan(c + d*x))**(3/2)*sec(c + d*x) + (e*tan(c + d*x))**(3/2)
), x)/a
```

3.125.7 Maxima [F]

$$\int \frac{1}{(a+a\sec(c+dx))(e\tan(c+dx))^{3/2}}\,dx = \int \frac{1}{(a\sec(dx+c)+a)\,(e\tan(dx+c))^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a+a*sec(d*x+c))/(e*tan(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((a*sec(d*x + c) + a)*(e*tan(d*x + c))^(3/2)), x)
```

3.125.8 Giac [F]

$$\int \frac{1}{(a+a\sec(c+dx))(e\tan(c+dx))^{3/2}}\,dx = \int \frac{1}{(a\sec(dx+c)+a)\,(e\tan(dx+c))^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a+a*sec(d*x+c))/(e*tan(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
integrate(1/((a*sec(d*x + c) + a)*(e*tan(d*x + c))^(3/2)), x)
```

3.125.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+a\sec(c+dx))(e\tan(c+dx))^{3/2}}\,dx = \int \frac{\cos(c+dx)}{a\,(e\tan(c+dx))^{3/2}\,(\cos(c+dx)+1)}\,dx$$

input
```
int(1/((e*tan(c + d*x))^(3/2)*(a + a/cos(c + d*x))),x)
```

output
```
int(cos(c + d*x)/(a*(e*tan(c + d*x))^(3/2)*(cos(c + d*x) + 1)), x)
```

3.126 $\int \frac{1}{(a+a\sec(c+dx))(e\tan(c+dx))^{5/2}}\,dx$

3.126.1 Optimal result . . . 965
3.126.2 Mathematica [C] (warning: unable to verify) . . . 966
3.126.3 Rubi [A] (warning: unable to verify) . . . 967
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3.126.8 Giac [F] . . . 977
3.126.9 Mupad [F(-1)] . . . 978

3.126.1 Optimal result

Integrand size = 25, antiderivative size = 328

$$\int \frac{1}{(a+a\sec(c+dx))(e\tan(c+dx))^{5/2}}\,dx = \frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}ade^{5/2}} - \frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}ade^{5/2}} + \frac{\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}ade^{5/2}} - \frac{\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)+\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}ade^{5/2}} + \frac{2e(1-\sec(c+dx))}{7ad(e\tan(c+dx))^{7/2}} - \frac{2(7-5\sec(c+dx))}{21ade(e\tan(c+dx))^{3/2}} + \frac{5\,\text{EllipticF}\left(c-\frac{\pi}{4}+dx,2\right)\sec(c+dx)\sqrt{\sin(2c+2dx)}}{21ade^2\sqrt{e\tan(c+dx)}}$$

output

```
1/2*arctan(1-2^(1/2)*(e*tan(d*x+c))^(1/2)/e^(1/2))/a/d/e^(5/2)*2^(1/2)-1/2
*arctan(1+2^(1/2)*(e*tan(d*x+c))^(1/2)/e^(1/2))/a/d/e^(5/2)*2^(1/2)+1/4*ln
(e^(1/2)-2^(1/2)*(e*tan(d*x+c))^(1/2)+e^(1/2)*tan(d*x+c))/a/d/e^(5/2)*2^(1
/2)-1/4*ln(e^(1/2)+2^(1/2)*(e*tan(d*x+c))^(1/2)+e^(1/2)*tan(d*x+c))/a/d/e^
(5/2)*2^(1/2)-5/21*(sin(c+1/4*Pi+d*x)^2)^(1/2)/sin(c+1/4*Pi+d*x)*EllipticF
(cos(c+1/4*Pi+d*x),2^(1/2))*sec(d*x+c)*sin(2*d*x+2*c)^(1/2)/a/d/e^2/(e*tan
(d*x+c))^(1/2)+2/7*e*(1-sec(d*x+c))/a/d/(e*tan(d*x+c))^(7/2)-2/21*(7-5*sec
(d*x+c))/a/d/e/(e*tan(d*x+c))^(3/2)
```

3.126.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.62 (sec) , antiderivative size = 1299, normalized size of antiderivative = 3.96

$$\int \frac{1}{(a+a\sec(c+dx))(e\tan(c+dx))^{5/2}}\,dx =$$

$$-\frac{10e^{-i(c+dx)}\sqrt{-\frac{i\left(-1+e^{2i(c+dx)}\right)}{1+e^{2i(c+dx)}}}\left(1+e^{2i(c+dx)}\right)\cos^2\left(\frac{c}{2}+\frac{dx}{2}\right)\sec(2c)\sec(c+dx)\tan^{\frac{5}{2}}(c+dx)}{21d(a+a\sec(c+dx))(e\tan(c+dx))^{5/2}}$$

$$-\frac{e^{-2ic}\sqrt{-\frac{i\left(-1+e^{2i(c+dx)}\right)}{1+e^{2i(c+dx)}}}\left(e^{4ic}\sqrt{-1+e^{4i(c+dx)}}\arctan\left(\sqrt{-1+e^{4i(c+dx)}}\right)+2\sqrt{-1+e^{2i(c+dx)}}\sqrt{1+e^{2i(c+dx)}}\text{arc}\right.}{2d\left(-1+e^{2i(c+dx)}\right)(a+a\sec(c+dx))(e\tan(c}$$

$$-\frac{e^{-2ic}\sqrt{-\frac{i\left(-1+e^{2i(c+dx)}\right)}{1+e^{2i(c+dx)}}}\left(\sqrt{-1+e^{4i(c+dx)}}\arctan\left(\sqrt{-1+e^{4i(c+dx)}}\right)+2e^{4ic}\sqrt{-1+e^{2i(c+dx)}}\sqrt{1+e^{2i(c+dx)}}\text{arc}\right.}{2d\left(-1+e^{2i(c+dx)}\right)(a+a\sec(c+dx))(e\tan(c}$$

$$+\frac{e^{-i(2c+dx)}\sqrt{-\frac{i\left(-1+e^{2i(c+dx)}\right)}{1+e^{2i(c+dx)}}}\cos^2\left(\frac{c}{2}+\frac{dx}{2}\right)\left(3\left(-1+e^{4i(c+dx)}\right)+e^{4i(c+dx)}\left(-1+e^{2ic}\right)\sqrt{1-e^{4i(c+dx)}}\,\text{Hypergeo}\right.}{3d\left(-1+e^{2i(c+dx)}\right)(a+a\sec(c+dx))(e\tan(c+d}$$

$$-\frac{e^{-idx}\sqrt{-\frac{i\left(-1+e^{2i(c+dx)}\right)}{1+e^{2i(c+dx)}}}\cos^2\left(\frac{c}{2}+\frac{dx}{2}\right)\left(3-3e^{4i(c+dx)}+e^{2i(c+2dx)}\left(-1+e^{2ic}\right)\sqrt{1-e^{4i(c+dx)}}\,\text{Hypergeometric2}\right.}{3d\left(-1+e^{2i(c+dx)}\right)(a+a\sec(c+dx))(e\tan(c+dx))}$$

$$+\frac{\cos^2\left(\frac{c}{2}+\frac{dx}{2}\right)\sec(c+dx)\left(\frac{40}{21d}-\frac{\csc^2\left(\frac{c}{2}+\frac{dx}{2}\right)}{6d}-\frac{2(21-10\cos(c)+21\cos(2c))\cos(dx)\sec(2c)}{21d}-\frac{13\sec^2\left(\frac{c}{2}+\frac{dx}{2}\right)}{14d}+\frac{\sec^4\left(\frac{c}{2}+\frac{dx}{2}\right)}{14d}\right.}{(a+a\sec(c+dx))(e\tan(c+dx))^{5/2}}$$

$$-\frac{20\sqrt[4]{-1}\cos^2\left(\frac{c}{2}+\frac{dx}{2}\right)\text{EllipticF}\left(i\text{arcsinh}\left(\sqrt[4]{-1}\sqrt{\tan(c+dx)}\right),-1\right)\sec^4(c+dx)\tan^{\frac{5}{2}}(c+dx)}{21d(a+a\sec(c+dx))(e\tan(c+dx))^{5/2}\left(1+\tan^2(c+dx)\right)^{3/2}}$$

input
```
Integrate[1/((a + a*Sec[c + d*x])*(e*Tan[c + d*x])^(5/2)),x]
```

output

```
(-10*Sqrt[((-I)*(-1 + E^((2*I)*(c + d*x))))/(1 + E^((2*I)*(c + d*x)))]*(1
+ E^((2*I)*(c + d*x)))*Cos[c/2 + (d*x)/2]^2*Sec[2*c]*Sec[c + d*x]*Tan[c +
d*x]^(5/2))/(21*d*E^(I*(c + d*x))*(a + a*Sec[c + d*x])*(e*Tan[c + d*x])^(5
/2)) - (Sqrt[((-I)*(-1 + E^((2*I)*(c + d*x))))/(1 + E^((2*I)*(c + d*x)))]*
(E^((4*I)*c)*Sqrt[-1 + E^((4*I)*(c + d*x))]*ArcTan[Sqrt[-1 + E^((4*I)*(c +
 d*x))]] + 2*Sqrt[-1 + E^((2*I)*(c + d*x))]*Sqrt[1 + E^((2*I)*(c + d*x))]*
ArcTanh[Sqrt[(-1 + E^((2*I)*(c + d*x)))/(1 + E^((2*I)*(c + d*x)))]])*Cos[c
/2 + (d*x)/2]^2*Sec[2*c]*Sec[c + d*x]*Tan[c + d*x]^(5/2))/(2*d*E^((2*I)*c)
*(-1 + E^((2*I)*(c + d*x)))*(a + a*Sec[c + d*x])*(e*Tan[c + d*x])^(5/2)) -
 (Sqrt[((-I)*(-1 + E^((2*I)*(c + d*x))))/(1 + E^((2*I)*(c + d*x)))]*(Sqrt[
-1 + E^((4*I)*(c + d*x))]*ArcTan[Sqrt[-1 + E^((4*I)*(c + d*x))]] + 2*E^((4
*I)*c)*Sqrt[-1 + E^((2*I)*(c + d*x))]*Sqrt[1 + E^((2*I)*(c + d*x))]*ArcTan
h[Sqrt[(-1 + E^((2*I)*(c + d*x)))/(1 + E^((2*I)*(c + d*x)))]])*Cos[c/2 + (
d*x)/2]^2*Sec[2*c]*Sec[c + d*x]*Tan[c + d*x]^(5/2))/(2*d*E^((2*I)*c)*(-1 +
 E^((2*I)*(c + d*x)))*(a + a*Sec[c + d*x])*(e*Tan[c + d*x])^(5/2)) + (Sqrt
[((-I)*(-1 + E^((2*I)*(c + d*x))))/(1 + E^((2*I)*(c + d*x)))]*Cos[c/2 + (d
*x)/2]^2*(3*(-1 + E^((4*I)*(c + d*x))) + E^((4*I)*(c + d*x))*(-1 + E^((2*I
)*c))*Sqrt[1 - E^((4*I)*(c + d*x))]*Hypergeometric2F1[1/2, 3/4, 7/4, E^((4
*I)*(c + d*x))])*Sec[2*c]*Sec[c + d*x]*Tan[c + d*x]^(5/2))/(3*d*E^(I*(2*c
+ d*x))*(-1 + E^((2*I)*(c + d*x)))*(a + a*Sec[c + d*x])*(e*Tan[c + d*x]...
```

3.126.3 Rubi [A] (warning: unable to verify)

Time = 1.33 (sec) , antiderivative size = 323, normalized size of antiderivative = 0.98, number of steps used = 28, number of rules used = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 1.080$, Rules used = {3042, 4376, 25, 3042, 4370, 27, 3042, 4370, 27, 3042, 4372, 3042, 3094, 3042, 3053, 3042, 3120, 3957, 266, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a\sec(c+dx)+a)(e\tan(c+dx))^{5/2}}\,dx$$

$$\downarrow 3042$$

$$\int \frac{1}{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)\left(-e\cot\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}dx$$

$$\downarrow 4376$$

$$\frac{e^2\int -\frac{a-a\sec(c+dx)}{(e\tan(c+dx))^{9/2}}dx}{a^2}$$

$$\downarrow 25$$

$$-\frac{e^2\int \frac{a-a\sec(c+dx)}{(e\tan(c+dx))^{9/2}}dx}{a^2}$$

$$\downarrow 3042$$

$$-\frac{e^2\int \frac{a-a\csc\left(c+dx+\frac{\pi}{2}\right)}{\left(-e\cot\left(c+dx+\frac{\pi}{2}\right)\right)^{9/2}}dx}{a^2}$$

$$\downarrow 4370$$

$$-\frac{e^2\left(\frac{2\int -\frac{7a-5a\sec(c+dx)}{2(e\tan(c+dx))^{5/2}}dx}{7e^2}-\frac{2(a-a\sec(c+dx))}{7de(e\tan(c+dx))^{7/2}}\right)}{a^2}$$

$$\downarrow 27$$

$$-\frac{e^2\left(-\frac{\int \frac{7a-5a\sec(c+dx)}{(e\tan(c+dx))^{5/2}}dx}{7e^2}-\frac{2(a-a\sec(c+dx))}{7de(e\tan(c+dx))^{7/2}}\right)}{a^2}$$

$$\downarrow 3042$$

$$-\frac{e^2\left(-\frac{\int \frac{7a-5a\csc\left(c+dx+\frac{\pi}{2}\right)}{\left(-e\cot\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}dx}{7e^2}-\frac{2(a-a\sec(c+dx))}{7de(e\tan(c+dx))^{7/2}}\right)}{a^2}$$

$$\downarrow 4370$$

$$-\frac{e^2\left(-\frac{\frac{2\int -\frac{21a-5a\sec(c+dx)}{2\sqrt{e\tan(c+dx)}}dx}{3e^2}-\frac{2(7a-5a\sec(c+dx))}{3de(e\tan(c+dx))^{3/2}}}{7e^2}-\frac{2(a-a\sec(c+dx))}{7de(e\tan(c+dx))^{7/2}}\right)}{a^2}$$

$$\downarrow 27$$

$$-\frac{e^2\left(-\frac{-\frac{\int \frac{21a-5a\sec(c+dx)}{\sqrt{e\tan(c+dx)}}dx}{3e^2}-\frac{2(7a-5a\sec(c+dx))}{3de(e\tan(c+dx))^{3/2}}}{7e^2}-\frac{2(a-a\sec(c+dx))}{7de(e\tan(c+dx))^{7/2}}\right)}{a^2}$$

$$\downarrow 3042$$

$$-\frac{e^2\left(-\frac{\int \frac{21a-5a\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{-e\cot\left(c+dx+\frac{\pi}{2}\right)}}dx}{3e^2}-\frac{2(7a-5a\sec(c+dx))}{3de(e\tan(c+dx))^{3/2}}-\frac{2(a-a\sec(c+dx))}{7de(e\tan(c+dx))^{7/2}}\right)}{7e^2}}{a^2}$$

$$\downarrow 4372$$

$$-\frac{e^2\left(-\frac{-\frac{21a\int\frac{1}{\sqrt{e\tan(c+dx)}}dx-5a\int\frac{\sec(c+dx)}{\sqrt{e\tan(c+dx)}}dx}{3e^2}-\frac{2(7a-5a\sec(c+dx))}{3de(e\tan(c+dx))^{3/2}}}{7e^2}-\frac{2(a-a\sec(c+dx))}{7de(e\tan(c+dx))^{7/2}}\right)}{a^2}$$

$$\downarrow 3042$$

$$-\frac{e^2\left(-\frac{-\frac{21a\int\frac{1}{\sqrt{e\tan(c+dx)}}dx-5a\int\frac{\sec(c+dx)}{\sqrt{e\tan(c+dx)}}dx}{3e^2}-\frac{2(7a-5a\sec(c+dx))}{3de(e\tan(c+dx))^{3/2}}}{7e^2}-\frac{2(a-a\sec(c+dx))}{7de(e\tan(c+dx))^{7/2}}\right)}{a^2}$$

$$\downarrow 3094$$

$$-\frac{e^2\left(-\frac{-\frac{21a\int\frac{1}{\sqrt{e\tan(c+dx)}}dx-\frac{5a\sqrt{\sin(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}\sqrt{\sin(c+dx)}}dx}{\sqrt{\cos(c+dx)}\sqrt{e\tan(c+dx)}}}{3e^2}-\frac{2(7a-5a\sec(c+dx))}{3de(e\tan(c+dx))^{3/2}}}{7e^2}-\frac{2(a-a\sec(c+dx))}{7de(e\tan(c+dx))^{7/2}}\right)}{a^2}$$

$$\downarrow 3042$$

$$-\frac{e^2\left(-\frac{-\frac{21a\int\frac{1}{\sqrt{e\tan(c+dx)}}dx-\frac{5a\sqrt{\sin(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}\sqrt{\sin(c+dx)}}dx}{\sqrt{\cos(c+dx)}\sqrt{e\tan(c+dx)}}}{3e^2}-\frac{2(7a-5a\sec(c+dx))}{3de(e\tan(c+dx))^{3/2}}}{7e^2}-\frac{2(a-a\sec(c+dx))}{7de(e\tan(c+dx))^{7/2}}\right)}{a^2}$$

$$\downarrow 3053$$

$$-\frac{e^2\left(-\frac{-\frac{21a\int\frac{1}{\sqrt{e\tan(c+dx)}}dx-\frac{5a\sqrt{\sin(2c+2dx)}\sec(c+dx)\int\frac{1}{\sqrt{\sin(2c+2dx)}}dx}{\sqrt{e\tan(c+dx)}}}{3e^2}-\frac{2(7a-5a\sec(c+dx))}{3de(e\tan(c+dx))^{3/2}}}{7e^2}-\frac{2(a-a\sec(c+dx))}{7de(e\tan(c+dx))^{7/2}}\right)}{a^2}$$

$$\downarrow 3042$$

$$-\frac{e^2\left(-\frac{-\frac{21a\int\frac{1}{\sqrt{e\tan(c+dx)}}dx-\frac{5a\sqrt{\sin(2c+2dx)}\sec(c+dx)\int\frac{1}{\sqrt{\sin(2c+2dx)}}dx}{\sqrt{e\tan(c+dx)}}}{3e^2}-\frac{2(7a-5a\sec(c+dx))}{3de(e\tan(c+dx))^{3/2}}}{7e^2}-\frac{2(a-a\sec(c+dx))}{7de(e\tan(c+dx))^{7/2}}\right)}{a^2}$$

$$\downarrow 3120$$

$$-\frac{e^2\left(-\frac{-\frac{21a\int\frac{1}{\sqrt{e\tan(c+dx)}}dx-\frac{5a\sqrt{\sin(2c+2dx)}\sec(c+dx)\,\text{EllipticF}\left(c+dx-\frac{\pi}{4},2\right)}{d\sqrt{e\tan(c+dx)}}}{3e^2}-\frac{2(7a-5a\sec(c+dx))}{3de(e\tan(c+dx))^{3/2}}}{7e^2}-\frac{2(a-a\sec(c+dx))}{7de(e\tan(c+dx))^{7/2}}\right)}{a^2}$$

$$\downarrow 3957$$

$$-\frac{e^2\left(-\frac{-\frac{21ae\int\frac{1}{\sqrt{e\tan(c+dx)}\left(\tan^2(c+dx)e^2+e^2\right)}d(e\tan(c+dx))}{d}-\frac{5a\sqrt{\sin(2c+2dx)}\sec(c+dx)\,\text{EllipticF}\left(c+dx-\frac{\pi}{4},2\right)}{d\sqrt{e\tan(c+dx)}}}{3e^2}-\frac{2(7a-5a\sec(c+dx))}{3de(e\tan(c+dx))^{3/2}}}{7e^2}-\frac{2(a-a\sec(c+dx))}{7de(e\tan(c+dx))^{7/2}}\right)}{a^2}$$

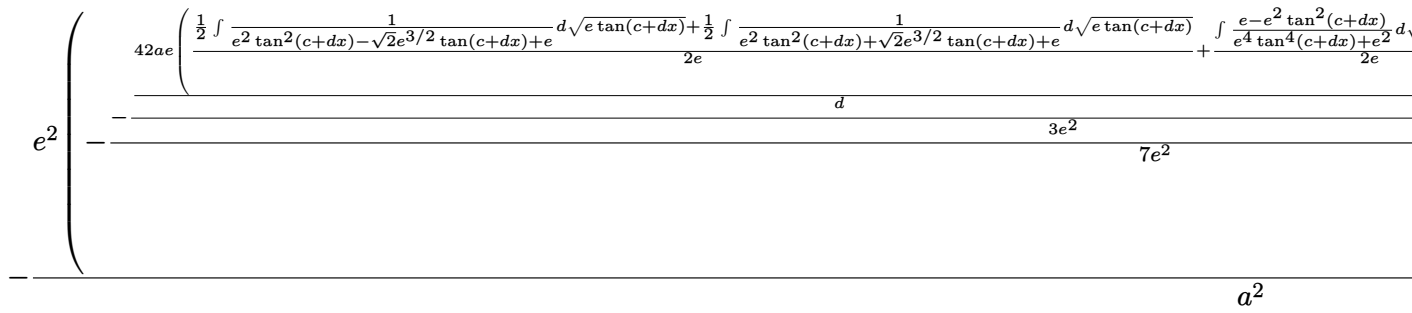
$$\downarrow 266$$

$$-\frac{e^2\left(-\frac{-\frac{42ae\int\frac{1}{e^4\tan^4(c+dx)+e^2}d\sqrt{e\tan(c+dx)}}{d}-\frac{5a\sqrt{\sin(2c+2dx)}\sec(c+dx)\,\text{EllipticF}\left(c+dx-\frac{\pi}{4},2\right)}{d\sqrt{e\tan(c+dx)}}}{3e^2}-\frac{2(7a-5a\sec(c+dx))}{3de(e\tan(c+dx))^{3/2}}}{7e^2}-\frac{2(a-a\sec(c+dx))}{7de(e\tan(c+dx))^{7/2}}\right)}{a^2}$$

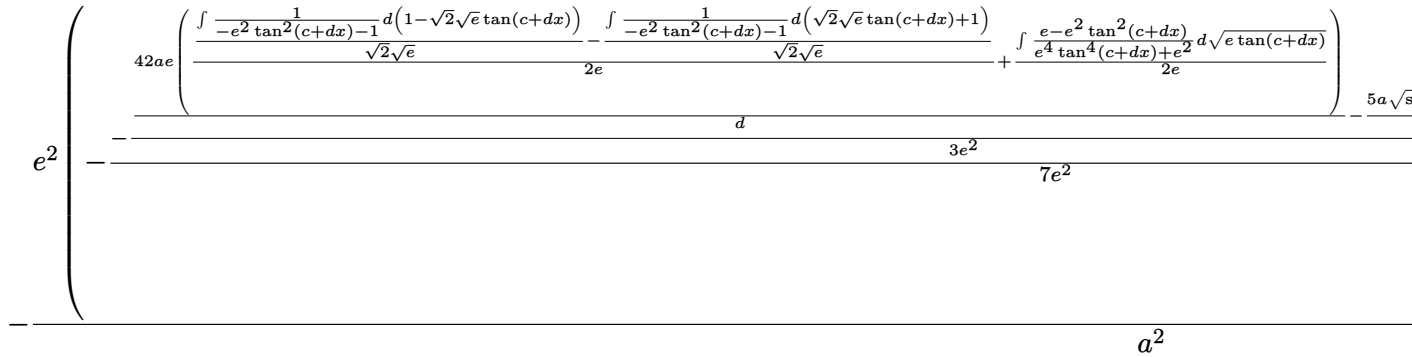
$$\downarrow 755$$

$$-\frac{e^2\left(-\frac{-\frac{42ae\left(\frac{\int\frac{e-e^2\tan^2(c+dx)}{e^4\tan^4(c+dx)+e^2}d\sqrt{e\tan(c+dx)}}{2e}+\frac{\int\frac{e^2\tan^2(c+dx)+e}{e^4\tan^4(c+dx)+e^2}d\sqrt{e\tan(c+dx)}}{2e}\right)}{d}-\frac{5a\sqrt{\sin(2c+2dx)}\sec(c+dx)\,\text{EllipticF}\left(c+dx-\frac{\pi}{4},2\right)}{d\sqrt{e\tan(c+dx)}}}{3e^2}-\frac{2(7a-5a}{3de(e\tan}}{7e^2}\right)}{a^2}$$

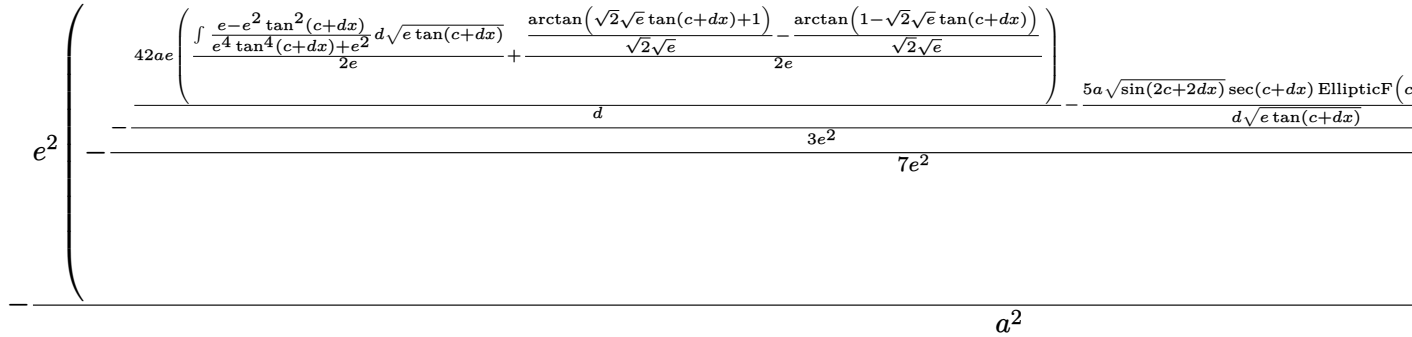
$$\downarrow 1476$$



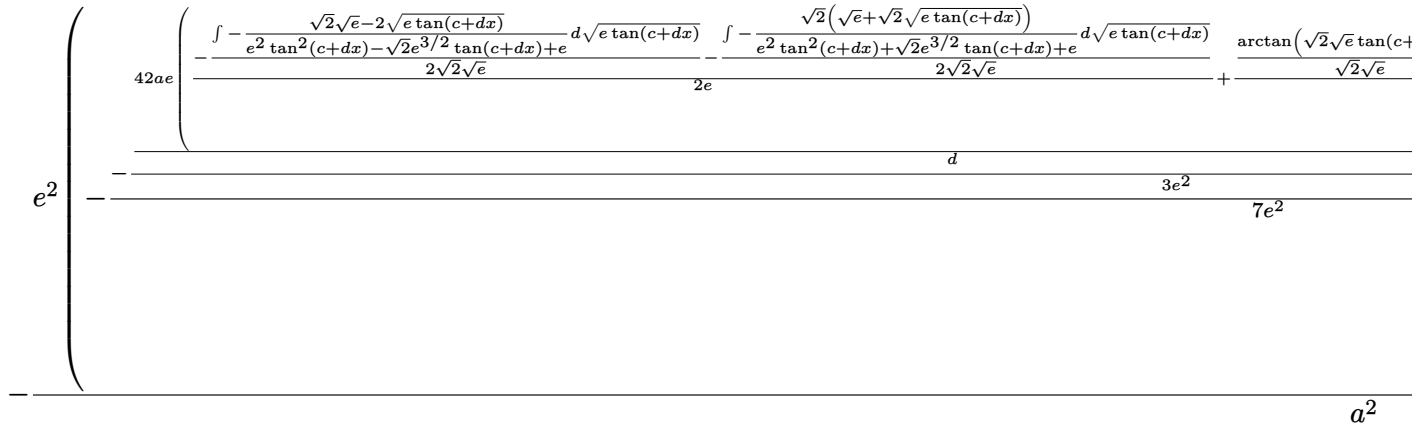
↓ 1082



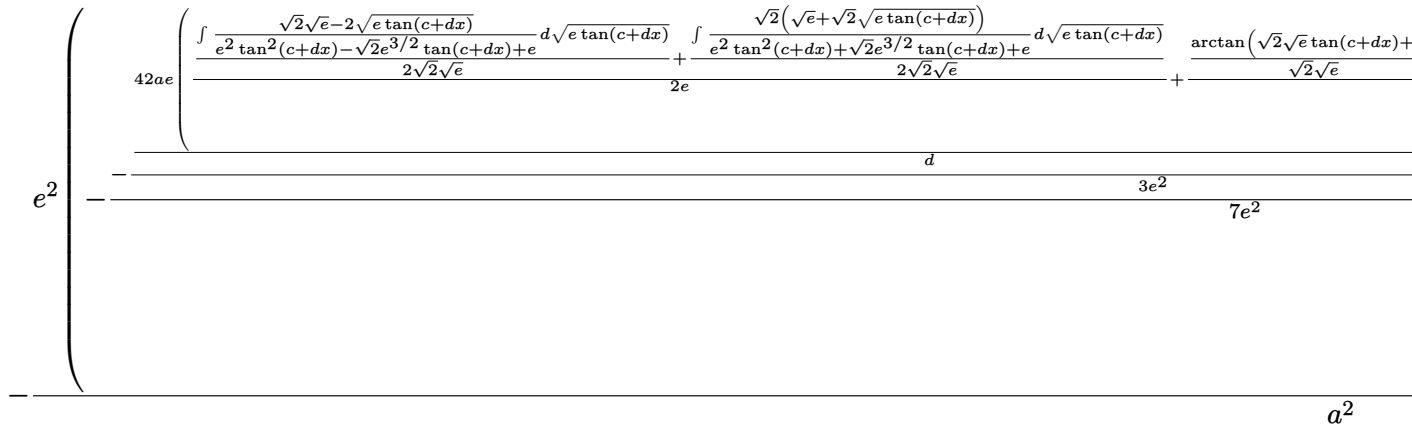
↓ 217



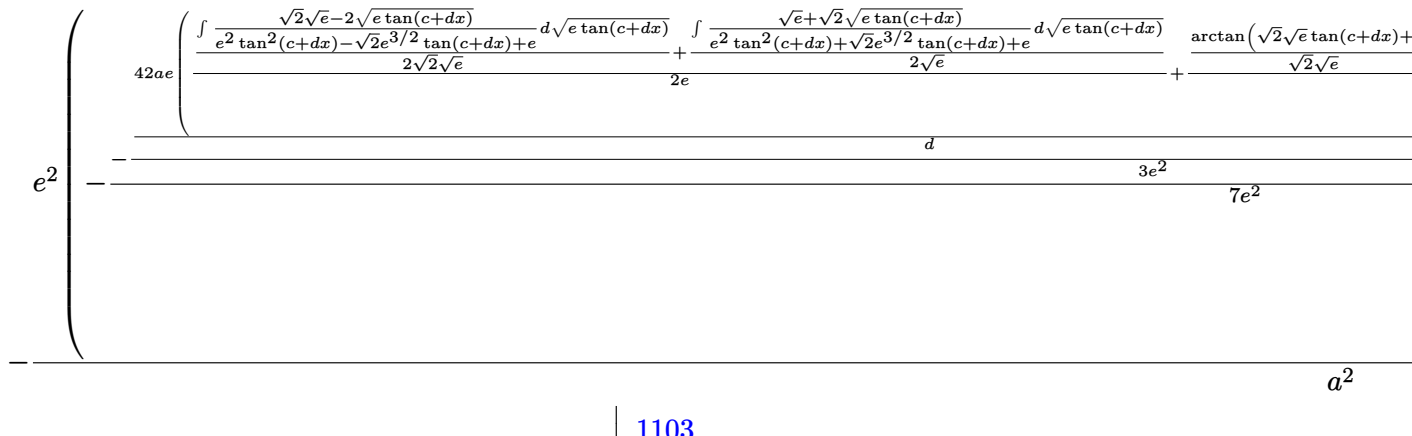
↓ 1479



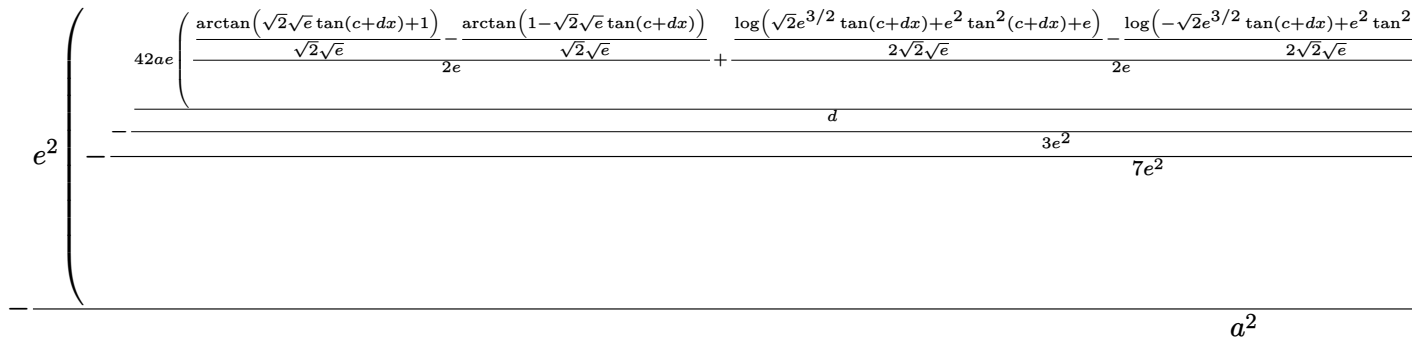
↓ 25



↓ 27



↓ 1103



input
```
Int[1/((a + a*Sec[c + d*x])*(e*Tan[c + d*x])^(5/2)),x]
```

output
```
-((e^2*((-2*(a - a*Sec[c + d*x]))/(7*d*e*(e*Tan[c + d*x])^(7/2)) - ((-2*(7
*a - 5*a*Sec[c + d*x]))/(3*d*e*(e*Tan[c + d*x])^(3/2)) - ((42*a*e*((-(ArcT
an[1 - Sqrt[2]*Sqrt[e]*Tan[c + d*x]]/(Sqrt[2]*Sqrt[e])) + ArcTan[1 + Sqrt[
2]*Sqrt[e]*Tan[c + d*x]]/(Sqrt[2]*Sqrt[e]))/(2*e) + (-1/2*Log[e - Sqrt[2]*
e^(3/2)*Tan[c + d*x] + e^2*Tan[c + d*x]^2]/(Sqrt[2]*Sqrt[e]) + Log[e + Sqr
t[2]*e^(3/2)*Tan[c + d*x] + e^2*Tan[c + d*x]^2]/(2*Sqrt[2]*Sqrt[e]))/(2*e)
))/d - (5*a*EllipticF[c - Pi/4 + d*x, 2]*Sec[c + d*x]*Sqrt[Sin[2*c + 2*d*x
]])/(d*Sqrt[e*Tan[c + d*x]]))/(3*e^2))/(7*e^2)))/a^2)
```

3.126.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3053
```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Simp[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]])   Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f
}, x]
```

rule 3094
```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]], x_Symbol]
:> Simp[Sqrt[Sin[e + f*x]]/(Sqrt[Cos[e + f*x]]*Sqrt[b*Tan[e + f*x]])   Int[
1/(Sqrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]]), x], x] /; FreeQ[{b, e, f}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

rule 4370
```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(-(e*Cot[c + d*x])^(m + 1))*((a + b*Csc[c + d*x])/(
d*e*(m + 1))), x] - Simp[1/(e^2*(m + 1))   Int[(e*Cot[c + d*x])^(m + 2)*(a*
(m + 1) + b*(m + 2)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && L
tQ[m, -1]
```

rule 4372
```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(e*Cot[c + d*x])^m, x], x] + Simp[b   Int[
(e*Cot[c + d*x])^m*Csc[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, m}, x]
```

rule 4376
```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Simp[a^(2*n)/e^(2*n)   Int[(e*Cot[c + d*x])^(m + 2*
n)/(-a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[a
^2 - b^2, 0] && ILtQ[n, 0]
```

3.126.4 Maple [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 3.87 (sec) , antiderivative size = 744, normalized size of antiderivative = 2.27

method	result
default	$\frac{\sqrt{2}\,(1-\cos(dx+c))^2\left(42i\sqrt{\csc(dx+c)-\cot(dx+c)+1}\,\sqrt{2-2\csc(dx+c)+2\cot(dx+c)}\,\sqrt{\cot(dx+c)-\csc(dx+c)}\,\text{EllipticPi}\left(\sqrt{\csc(dx+c}\right.\right.$

input
```
int(1/(a+a*sec(d*x+c))/(e*tan(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/84/a/d*2^(1/2)*(1-cos(d*x+c))^2*(42*I*(csc(d*x+c)-cot(d*x+c)+1)^(1/2)*(2
-2*csc(d*x+c)+2*cot(d*x+c))^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)*EllipticPi
((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2-1/2*I,1/2*2^(1/2))*(-cot(d*x+c)+csc(d
*x+c))-42*I*(csc(d*x+c)-cot(d*x+c)+1)^(1/2)*(2-2*csc(d*x+c)+2*cot(d*x+c))^
(1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)*EllipticPi((csc(d*x+c)-cot(d*x+c)+1)^(
1/2),1/2+1/2*I,1/2*2^(1/2))*(-cot(d*x+c)+csc(d*x+c))-3*(1-cos(d*x+c))^6*cs
c(d*x+c)^6-104*(csc(d*x+c)-cot(d*x+c)+1)^(1/2)*(2-2*csc(d*x+c)+2*cot(d*x+c
))^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)*EllipticF((csc(d*x+c)-cot(d*x+c)+1)
^(1/2),1/2*2^(1/2))*(-cot(d*x+c)+csc(d*x+c))+42*(csc(d*x+c)-cot(d*x+c)+1)^
(1/2)*(2-2*csc(d*x+c)+2*cot(d*x+c))^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)*El
lipticPi((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2-1/2*I,1/2*2^(1/2))*(-cot(d*x+
c)+csc(d*x+c))+42*(csc(d*x+c)-cot(d*x+c)+1)^(1/2)*(2-2*csc(d*x+c)+2*cot(d*
x+c))^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)*EllipticPi((csc(d*x+c)-cot(d*x+c
)+1)^(1/2),1/2+1/2*I,1/2*2^(1/2))*(-cot(d*x+c)+csc(d*x+c))+33*(1-cos(d*x+c
))^4*csc(d*x+c)^4-37*(1-cos(d*x+c))^2*csc(d*x+c)^2+7)/((1-cos(d*x+c))^3*cs
c(d*x+c)^3+cot(d*x+c)-csc(d*x+c))^(1/2)/((1-cos(d*x+c))*((1-cos(d*x+c))^2*
csc(d*x+c)^2-1)*csc(d*x+c))^(1/2)/((1-cos(d*x+c))^2*csc(d*x+c)^2-1)^2/(-e/
((1-cos(d*x+c))^2*csc(d*x+c)^2-1)*(-cot(d*x+c)+csc(d*x+c)))^(5/2)*csc(d*x+
c)^2
```

3.126.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a+a\sec(c+dx))(e\tan(c+dx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/(a+a*sec(d*x+c))/(e*tan(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.126.6 Sympy [F]

$$\int \frac{1}{(a+a\sec(c+dx))(e\tan(c+dx))^{5/2}}\,dx = \frac{\int \frac{1}{(e\tan(c+dx))^{\frac{5}{2}}\sec(c+dx)+(e\tan(c+dx))^{\frac{5}{2}}}\,dx}{a}$$

input
```
integrate(1/(a+a*sec(d*x+c))/(e*tan(d*x+c))**(5/2),x)
```

output
```
Integral(1/((e*tan(c + d*x))**(5/2)*sec(c + d*x) + (e*tan(c + d*x))**(5/2)
), x)/a
```

3.126.7 Maxima [F]

$$\int \frac{1}{(a+a\sec(c+dx))(e\tan(c+dx))^{5/2}}\,dx = \int \frac{1}{(a\sec(dx+c)+a)(e\tan(dx+c))^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(a+a*sec(d*x+c))/(e*tan(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
integrate(1/((a*sec(d*x + c) + a)*(e*tan(d*x + c))^(5/2)), x)
```

3.126.8 Giac [F]

$$\int \frac{1}{(a+a\sec(c+dx))(e\tan(c+dx))^{5/2}}\,dx = \int \frac{1}{(a\sec(dx+c)+a)(e\tan(dx+c))^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(a+a*sec(d*x+c))/(e*tan(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
integrate(1/((a*sec(d*x + c) + a)*(e*tan(d*x + c))^(5/2)), x)
```

3.126.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+a\sec(c+dx))(e\tan(c+dx))^{5/2}}\,dx = \int \frac{\cos(c+dx)}{a\,(e\tan(c+dx))^{5/2}\,(\cos(c+dx)+1)}\,dx$$

input
```
int(1/((e*tan(c + d*x))^(5/2)*(a + a/cos(c + d*x))),x)
```

output
```
int(cos(c + d*x)/(a*(e*tan(c + d*x))^(5/2)*(cos(c + d*x) + 1)), x)
```

3.127 $\int \frac{(e\tan(c+dx))^{13/2}}{(a+a\sec(c+dx))^2}\,dx$

3.127.1 Optimal result . . . 979
3.127.2 Mathematica [F] . . . 980
3.127.3 Rubi [A] (verified) . . . 980
3.127.4 Maple [C] (warning: unable to verify) . . . 982
3.127.5 Fricas [F(-1)] . . . 983
3.127.6 Sympy [F(-1)] . . . 983
3.127.7 Maxima [F(-1)] . . . 983
3.127.8 Giac [F] . . . 984
3.127.9 Mupad [F(-1)] . . . 984

3.127.1 Optimal result

Integrand size = 25, antiderivative size = 372

$$\begin{aligned}\int \frac{(e\tan(c+dx))^{13/2}}{(a+a\sec(c+dx))^2}\,dx &= \frac{e^{13/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}a^2d}\\ &- \frac{e^{13/2}\arctan\left(1+\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}a^2d}\\ &- \frac{e^{13/2}\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}a^2d}\\ &+ \frac{e^{13/2}\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)+\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}a^2d}\\ &- \frac{12e^6\cos(c+dx)E\left(c-\frac{\pi}{4}+dx\middle|2\right)\sqrt{e\tan(c+dx)}}{5a^2d\sqrt{\sin(2c+2dx)}}\\ &+ \frac{2e^5(e\tan(c+dx))^{3/2}}{3a^2d}+\frac{12e^5\cos(c+dx)(e\tan(c+dx))^{3/2}}{5a^2d}\\ &- \frac{4e^5\sec(c+dx)(e\tan(c+dx))^{3/2}}{5a^2d}+\frac{2e^3(e\tan(c+dx))^{7/2}}{7a^2d}\end{aligned}$$

output
```
1/2*e^(13/2)*arctan(1-2^(1/2)*(e*tan(d*x+c))^(1/2)/e^(1/2))/a^2/d*2^(1/2)-
1/2*e^(13/2)*arctan(1+2^(1/2)*(e*tan(d*x+c))^(1/2)/e^(1/2))/a^2/d*2^(1/2)-
1/4*e^(13/2)*ln(e^(1/2)-2^(1/2)*(e*tan(d*x+c))^(1/2)+e^(1/2)*tan(d*x+c))/a
^2/d*2^(1/2)+1/4*e^(13/2)*ln(e^(1/2)+2^(1/2)*(e*tan(d*x+c))^(1/2)+e^(1/2)*
tan(d*x+c))/a^2/d*2^(1/2)+12/5*e^6*cos(d*x+c)*(sin(c+1/4*Pi+d*x)^2)^(1/2)/
sin(c+1/4*Pi+d*x)*EllipticE(cos(c+1/4*Pi+d*x),2^(1/2))*(e*tan(d*x+c))^(1/2
)/a^2/d/sin(2*d*x+2*c)^(1/2)+2/3*e^5*(e*tan(d*x+c))^(3/2)/a^2/d+12/5*e^5*c
os(d*x+c)*(e*tan(d*x+c))^(3/2)/a^2/d-4/5*e^5*sec(d*x+c)*(e*tan(d*x+c))^(3/
2)/a^2/d+2/7*e^3*(e*tan(d*x+c))^(7/2)/a^2/d
```

3.127.2 Mathematica [F]

$$\int \frac{(e\tan(c+dx))^{13/2}}{(a+a\sec(c+dx))^2}\,dx = \int \frac{(e\tan(c+dx))^{13/2}}{(a+a\sec(c+dx))^2}\,dx$$

input
```
Integrate[(e*Tan[c + d*x])^(13/2)/(a + a*Sec[c + d*x])^2,x]
```

output
```
Integrate[(e*Tan[c + d*x])^(13/2)/(a + a*Sec[c + d*x])^2, x]
```

3.127.3 Rubi [A] (verified)

Time = 0.84 (sec) , antiderivative size = 373, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3042, 4376, 3042, 4374, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e\tan(c+dx))^{13/2}}{(a\sec(c+dx)+a)^2}\,dx$$
$$\downarrow \text{3042}$$
$$\int \frac{\left(-e\cot\left(c+dx+\frac{\pi}{2}\right)\right)^{13/2}}{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^2}dx$$
$$\downarrow \text{4376}$$
$$\frac{e^4\int(a-a\sec(c+dx))^2(e\tan(c+dx))^{5/2}dx}{a^4}$$

$$\downarrow 3042$$

$$\frac{e^4 \int \left(-e \cot\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2} \left(a-a \csc\left(c+dx+\frac{\pi}{2}\right)\right)^2 dx}{a^4}$$

$$\downarrow 4374$$

$$\frac{e^4 \int \left(a^2 (e \tan(c+dx))^{5/2} + a^2 \sec^2(c+dx)(e \tan(c+dx))^{5/2} - 2a^2 \sec(c+dx)(e \tan(c+dx))^{5/2}\right) dx}{a^4}$$

$$\downarrow 2009$$

$$e^4 \left(\frac{a^2 e^{5/2} \arctan\left(1-\frac{\sqrt{2}\sqrt{e \tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}d} - \frac{a^2 e^{5/2} \arctan\left(\frac{\sqrt{2}\sqrt{e \tan(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}d} - \frac{a^2 e^{5/2} \log\left(\sqrt{e} \tan(c+dx)-\sqrt{2}\sqrt{e \tan(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}d} + \frac{a^2 e^{5}}{}\right.$$

input
```
Int[(e*Tan[c + d*x])^(13/2)/(a + a*Sec[c + d*x])^2,x]
```

output
```
(e^4*((a^2*e^(5/2)*ArcTan[1 - (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sq
rt[2]*d) - (a^2*e^(5/2)*ArcTan[1 + (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]]
)/(Sqrt[2]*d) - (a^2*e^(5/2)*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] - Sqrt[2]*
Sqrt[e*Tan[c + d*x]]])/(2*Sqrt[2]*d) + (a^2*e^(5/2)*Log[Sqrt[e] + Sqrt[e]*
Tan[c + d*x] + Sqrt[2]*Sqrt[e*Tan[c + d*x]]])/(2*Sqrt[2]*d) - (12*a^2*e^2*
Cos[c + d*x]*EllipticE[c - Pi/4 + d*x, 2]*Sqrt[e*Tan[c + d*x]])/(5*d*Sqrt[
Sin[2*c + 2*d*x]]) + (2*a^2*e*(e*Tan[c + d*x])^(3/2))/(3*d) + (12*a^2*e*Co
s[c + d*x]*(e*Tan[c + d*x])^(3/2))/(5*d) - (4*a^2*e*Sec[c + d*x]*(e*Tan[c
+ d*x])^(3/2))/(5*d) + (2*a^2*(e*Tan[c + d*x])^(7/2))/(7*d*e)))/a^4
```

3.127.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4374
```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

rule 4376
```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Simp[a^(2*n)/e^(2*n)   Int[(e*Cot[c + d*x])^(m + 2*
n)/(-a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[a
^2 - b^2, 0] && ILtQ[n, 0]
```

3.127.4 Maple [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 5.19 (sec) , antiderivative size = 1181, normalized size of antiderivative = 3.17

method	result	size
default	Expression too large to display	1181

input
```
int((e*tan(d*x+c))^(13/2)/(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
-1/210/a^2/d*2^(1/2)*e^6*(105*I*(cot(d*x+c)-csc(d*x+c)+1)^(1/2)*(cot(d*x+c
)-csc(d*x+c))^(1/2)*EllipticPi((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2+1/2*I,1
/2*2^(1/2))*(csc(d*x+c)-cot(d*x+c)+1)^(1/2)*cos(d*x+c)^3-105*I*(cot(d*x+c)
-csc(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)*(csc(d*x+c)-cot(d*x+c)+
1)^(1/2)*EllipticPi((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2-1/2*I,1/2*2^(1/2))
*cos(d*x+c)^3+105*I*(cot(d*x+c)-csc(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c)
)^(1/2)*EllipticPi((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2+1/2*I,1/2*2^(1/2))*
(csc(d*x+c)-cot(d*x+c)+1)^(1/2)*cos(d*x+c)^4-105*I*(cot(d*x+c)-csc(d*x+c)+
1)^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)*(csc(d*x+c)-cot(d*x+c)+1)^(1/2)*Ell
ipticPi((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2-1/2*I,1/2*2^(1/2))*cos(d*x+c)^
4+105*(cot(d*x+c)-csc(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)*Elliptic
icPi((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2+1/2*I,1/2*2^(1/2))*(csc(d*x+c)-co
t(d*x+c)+1)^(1/2)*cos(d*x+c)^4+504*(cot(d*x+c)-csc(d*x+c)+1)^(1/2)*(cot(d*
x+c)-csc(d*x+c))^(1/2)*(csc(d*x+c)-cot(d*x+c)+1)^(1/2)*EllipticE((csc(d*x+
c)-cot(d*x+c)+1)^(1/2),1/2*2^(1/2))*cos(d*x+c)^4-252*(cot(d*x+c)-csc(d*x+c
)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)*(csc(d*x+c)-cot(d*x+c)+1)^(1/2)*E
llipticF((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2*2^(1/2))*cos(d*x+c)^4+105*(co
t(d*x+c)-csc(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)*(csc(d*x+c)-cot
(d*x+c)+1)^(1/2)*EllipticPi((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2-1/2*I,1/2*
2^(1/2))*cos(d*x+c)^4+105*(cot(d*x+c)-csc(d*x+c)+1)^(1/2)*(cot(d*x+c)-c...
```

3.127.5 Fricas [F(-1)]

Timed out.

$$\int \frac{(e\tan(c+dx))^{13/2}}{(a+a\sec(c+dx))^2}\,dx = \text{Timed out}$$

input
```
integrate((e*tan(d*x+c))^(13/2)/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

output
```
Timed out
```

3.127.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(e\tan(c+dx))^{13/2}}{(a+a\sec(c+dx))^2}\,dx = \text{Timed out}$$

input
```
integrate((e*tan(d*x+c))**(13/2)/(a+a*sec(d*x+c))**2,x)
```

output
```
Timed out
```

3.127.7 Maxima [F(-1)]

Timed out.

$$\int \frac{(e\tan(c+dx))^{13/2}}{(a+a\sec(c+dx))^2}\,dx = \text{Timed out}$$

input
```
integrate((e*tan(d*x+c))^(13/2)/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

output
```
Timed out
```

3.127.8 Giac [F]

$$\int \frac{(e\tan(c+dx))^{13/2}}{(a+a\sec(c+dx))^2}\,dx = \int \frac{(e\tan(dx+c))^{\frac{13}{2}}}{(a\sec(dx+c)+a)^2}\,dx$$

input
```
integrate((e*tan(d*x+c))^(13/2)/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

output
```
integrate((e*tan(d*x + c))^(13/2)/(a*sec(d*x + c) + a)^2, x)
```

3.127.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(e\tan(c+dx))^{13/2}}{(a+a\sec(c+dx))^2}\,dx = \int \frac{\cos(c+dx)^2\,(e\tan(c+dx))^{13/2}}{a^2\,(\cos(c+dx)+1)^2}\,dx$$

input
```
int((e*tan(c + d*x))^(13/2)/(a + a/cos(c + d*x))^2,x)
```

output
```
int((cos(c + d*x)^2*(e*tan(c + d*x))^(13/2))/(a^2*(cos(c + d*x) + 1)^2), x
)
```

3.128 $\int \frac{(e\tan(c+dx))^{11/2}}{(a+a\sec(c+dx))^2}\,dx$

3.128.1 Optimal result . . . 985
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3.128.3 Rubi [A] (verified) . . . 986
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3.128.8 Giac [F] . . . 990
3.128.9 Mupad [F(-1)] . . . 990

3.128.1 Optimal result

Integrand size = 25, antiderivative size = 339

$$\begin{aligned}\int \frac{(e\tan(c+dx))^{11/2}}{(a+a\sec(c+dx))^2}\,dx &= \frac{e^{11/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}a^2d}\\ &- \frac{e^{11/2}\arctan\left(1+\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}a^2d}\\ &+ \frac{e^{11/2}\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}a^2d}\\ &- \frac{e^{11/2}\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)+\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}a^2d}\\ &+ \frac{2e^6\,\text{EllipticF}\left(c-\frac{\pi}{4}+dx,2\right)\sec(c+dx)\sqrt{\sin(2c+2dx)}}{3a^2d\sqrt{e\tan(c+dx)}}\\ &+ \frac{2e^5\sqrt{e\tan(c+dx)}}{a^2d} - \frac{4e^5\sec(c+dx)\sqrt{e\tan(c+dx)}}{3a^2d} + \frac{2e^3(e\tan(c+dx))^{5/2}}{5a^2d}\end{aligned}$$

output
```
1/2*e^(11/2)*arctan(1-2^(1/2)*(e*tan(d*x+c))^(1/2)/e^(1/2))/a^2/d*2^(1/2)-
1/2*e^(11/2)*arctan(1+2^(1/2)*(e*tan(d*x+c))^(1/2)/e^(1/2))/a^2/d*2^(1/2)+
1/4*e^(11/2)*ln(e^(1/2)-2^(1/2)*(e*tan(d*x+c))^(1/2)+e^(1/2)*tan(d*x+c))/a
^2/d*2^(1/2)-1/4*e^(11/2)*ln(e^(1/2)+2^(1/2)*(e*tan(d*x+c))^(1/2)+e^(1/2)*
tan(d*x+c))/a^2/d*2^(1/2)-2/3*e^6*(sin(c+1/4*Pi+d*x)^2)^(1/2)/sin(c+1/4*Pi
+d*x)*EllipticF(cos(c+1/4*Pi+d*x),2^(1/2))*sec(d*x+c)*sin(2*d*x+2*c)^(1/2)
/a^2/d/(e*tan(d*x+c))^(1/2)+2*e^5*(e*tan(d*x+c))^(1/2)/a^2/d-4/3*e^5*sec(d
*x+c)*(e*tan(d*x+c))^(1/2)/a^2/d+2/5*e^3*(e*tan(d*x+c))^(5/2)/a^2/d
```

3.128.2 Mathematica [F]

$$\int \frac{(e\tan(c+dx))^{11/2}}{(a+a\sec(c+dx))^2}\,dx = \int \frac{(e\tan(c+dx))^{11/2}}{(a+a\sec(c+dx))^2}\,dx$$

input
```
Integrate[(e*Tan[c + d*x])^(11/2)/(a + a*Sec[c + d*x])^2,x]
```

output
```
Integrate[(e*Tan[c + d*x])^(11/2)/(a + a*Sec[c + d*x])^2, x]
```

3.128.3 Rubi [A] (verified)

Time = 0.77 (sec) , antiderivative size = 342, normalized size of antiderivative = 1.01, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3042, 4376, 3042, 4374, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e\tan(c+dx))^{11/2}}{(a\sec(c+dx)+a)^2}\,dx$$
$$\downarrow \text{3042}$$
$$\int \frac{\left(-e\cot\left(c+dx+\frac{\pi}{2}\right)\right)^{11/2}}{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^2}dx$$
$$\downarrow \text{4376}$$
$$\frac{e^4\int(a-a\sec(c+dx))^2(e\tan(c+dx))^{3/2}dx}{a^4}$$

$$\downarrow 3042$$

$$\frac{e^4 \int \left(-e \cot\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2} \left(a-a \csc\left(c+dx+\frac{\pi}{2}\right)\right)^2 dx}{a^4}$$

$$\downarrow 4374$$

$$\frac{e^4 \int \left(\sec^2(c+dx)(e\tan(c+dx))^{3/2}a^2 - 2\sec(c+dx)(e\tan(c+dx))^{3/2}a^2 + (e\tan(c+dx))^{3/2}a^2\right) dx}{a^4}$$

$$\downarrow 2009$$

$$e^4\left(\frac{a^2e^{3/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}d} - \frac{a^2e^{3/2}\arctan\left(\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}d} + \frac{a^2e^{3/2}\log\left(\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}d} - \frac{a^2e^{3}}{}\right.$$

input
```
Int[(e*Tan[c + d*x])^(11/2)/(a + a*Sec[c + d*x])^2,x]
```

output
```
(e^4*((a^2*e^(3/2)*ArcTan[1 - (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sq
rt[2]*d) - (a^2*e^(3/2)*ArcTan[1 + (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]]
)/(Sqrt[2]*d) + (a^2*e^(3/2)*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] - Sqrt[2]*
Sqrt[e*Tan[c + d*x]]])/(2*Sqrt[2]*d) - (a^2*e^(3/2)*Log[Sqrt[e] + Sqrt[e]*
Tan[c + d*x] + Sqrt[2]*Sqrt[e*Tan[c + d*x]]])/(2*Sqrt[2]*d) + (2*a^2*e^2*E
llipticF[c - Pi/4 + d*x, 2]*Sec[c + d*x]*Sqrt[Sin[2*c + 2*d*x]])/(3*d*Sqrt
[e*Tan[c + d*x]]) + (2*a^2*e*Sqrt[e*Tan[c + d*x]])/d - (4*a^2*e*Sec[c + d*
x]*Sqrt[e*Tan[c + d*x]])/(3*d) + (2*a^2*(e*Tan[c + d*x])^(5/2))/(5*d*e)))/
a^4
```

3.128.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4374
```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

rule 4376
```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Simp[a^(2*n)/e^(2*n)   Int[(e*Cot[c + d*x])^(m + 2*
n)/(-a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[a
^2 - b^2, 0] && ILtQ[n, 0]
```

3.128.4 Maple [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 4.51 (sec) , antiderivative size = 1007, normalized size of antiderivative = 2.97

method	result	size
default	Expression too large to display	1007

input
```
int((e*tan(d*x+c))^(11/2)/(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
-1/30/a^2/d*2^(1/2)*(15*I*(cot(d*x+c)-csc(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(
d*x+c))^(1/2)*EllipticPi((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2+1/2*I,1/2*2^(
1/2))*(csc(d*x+c)-cot(d*x+c)+1)^(1/2)*cos(d*x+c)^3-15*I*(cot(d*x+c)-csc(d*
x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)*(csc(d*x+c)-cot(d*x+c)+1)^(1/2
)*EllipticPi((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2-1/2*I,1/2*2^(1/2))*cos(d*
x+c)^3+15*I*(cot(d*x+c)-csc(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)*
EllipticPi((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2+1/2*I,1/2*2^(1/2))*(csc(d*x
+c)-cot(d*x+c)+1)^(1/2)*cos(d*x+c)^2-15*I*(cot(d*x+c)-csc(d*x+c)+1)^(1/2)*
(cot(d*x+c)-csc(d*x+c))^(1/2)*(csc(d*x+c)-cot(d*x+c)+1)^(1/2)*EllipticPi((
csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2-1/2*I,1/2*2^(1/2))*cos(d*x+c)^2-15*(cot
(d*x+c)-csc(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)*EllipticPi((csc(
d*x+c)-cot(d*x+c)+1)^(1/2),1/2+1/2*I,1/2*2^(1/2))*(csc(d*x+c)-cot(d*x+c)+1
)^(1/2)*cos(d*x+c)^3-15*(cot(d*x+c)-csc(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*
x+c))^(1/2)*(csc(d*x+c)-cot(d*x+c)+1)^(1/2)*EllipticPi((csc(d*x+c)-cot(d*x
+c)+1)^(1/2),1/2-1/2*I,1/2*2^(1/2))*cos(d*x+c)^3+50*(csc(d*x+c)-cot(d*x+c)
+1)^(1/2)*(cot(d*x+c)-csc(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)*El
lipticF((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2*2^(1/2))*cos(d*x+c)^3-15*(cot(
d*x+c)-csc(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)*(csc(d*x+c)-cot(d
*x+c)+1)^(1/2)*EllipticPi((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2+1/2*I,1/2*2^
(1/2))*cos(d*x+c)^2-15*(cot(d*x+c)-csc(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(...
```

3.128.5 Fricas [F(-1)]

Timed out.

$$\int \frac{(e\tan(c+dx))^{11/2}}{(a+a\sec(c+dx))^2}\,dx = \text{Timed out}$$

input
```
integrate((e*tan(d*x+c))^(11/2)/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

output
```
Timed out
```

3.128.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(e\tan(c+dx))^{11/2}}{(a+a\sec(c+dx))^2}\,dx = \text{Timed out}$$

input
```
integrate((e*tan(d*x+c))**(11/2)/(a+a*sec(d*x+c))**2,x)
```

output
```
Timed out
```

3.128.7 Maxima [F(-1)]

Timed out.

$$\int \frac{(e\tan(c+dx))^{11/2}}{(a+a\sec(c+dx))^2}\,dx = \text{Timed out}$$

input
```
integrate((e*tan(d*x+c))^(11/2)/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

output
```
Timed out
```

3.128.8 Giac [F]

$$\int \frac{(e\tan(c+dx))^{11/2}}{(a+a\sec(c+dx))^2}\,dx = \int \frac{(e\tan(dx+c))^{\frac{11}{2}}}{(a\sec(dx+c)+a)^2}\,dx$$

input
```
integrate((e*tan(d*x+c))^(11/2)/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

output
```
integrate((e*tan(d*x + c))^(11/2)/(a*sec(d*x + c) + a)^2, x)
```

3.128.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(e\tan(c+dx))^{11/2}}{(a+a\sec(c+dx))^2}\,dx = \int \frac{\cos(c+dx)^2\,(e\tan(c+dx))^{11/2}}{a^2\,(\cos(c+dx)+1)^2}\,dx$$

input
```
int((e*tan(c + d*x))^(11/2)/(a + a/cos(c + d*x))^2,x)
```

output
```
int((cos(c + d*x)^2*(e*tan(c + d*x))^(11/2))/(a^2*(cos(c + d*x) + 1)^2), x
)
```

3.129 $\int \frac{(e\tan(c+dx))^{9/2}}{(a+a\sec(c+dx))^2}\,dx$

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3.129.1 Optimal result

Integrand size = 25, antiderivative size = 312

$$\begin{aligned}\int \frac{(e\tan(c+dx))^{9/2}}{(a+a\sec(c+dx))^2}\,dx &= -\frac{e^{9/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}a^2d}\\ &+\frac{e^{9/2}\arctan\left(1+\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}a^2d}\\ &+\frac{e^{9/2}\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}a^2d}\\ &-\frac{e^{9/2}\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)+\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}a^2d}\\ &+\frac{4e^4\cos(c+dx)E\left(c-\frac{\pi}{4}+dx\middle|2\right)\sqrt{e\tan(c+dx)}}{a^2d\sqrt{\sin(2c+2dx)}}\\ &+\frac{2e^3(e\tan(c+dx))^{3/2}}{3a^2d}-\frac{4e^3\cos(c+dx)(e\tan(c+dx))^{3/2}}{a^2d}\end{aligned}$$

output

```
-1/2*e^(9/2)*arctan(1-2^(1/2)*(e*tan(d*x+c))^(1/2)/e^(1/2))/a^2/d*2^(1/2)+
1/2*e^(9/2)*arctan(1+2^(1/2)*(e*tan(d*x+c))^(1/2)/e^(1/2))/a^2/d*2^(1/2)+1
/4*e^(9/2)*ln(e^(1/2)-2^(1/2)*(e*tan(d*x+c))^(1/2)+e^(1/2)*tan(d*x+c))/a^2
/d*2^(1/2)-1/4*e^(9/2)*ln(e^(1/2)+2^(1/2)*(e*tan(d*x+c))^(1/2)+e^(1/2)*tan
(d*x+c))/a^2/d*2^(1/2)-4*e^4*cos(d*x+c)*(sin(c+1/4*Pi+d*x)^2)^(1/2)/sin(c+
1/4*Pi+d*x)*EllipticE(cos(c+1/4*Pi+d*x),2^(1/2))*(e*tan(d*x+c))^(1/2)/a^2/
d/sin(2*d*x+2*c)^(1/2)+2/3*e^3*(e*tan(d*x+c))^(3/2)/a^2/d-4*e^3*cos(d*x+c)
*(e*tan(d*x+c))^(3/2)/a^2/d
```

3.129.2 Mathematica [F]

$$\int \frac{(e\tan(c+dx))^{9/2}}{(a+a\sec(c+dx))^2}\,dx = \int \frac{(e\tan(c+dx))^{9/2}}{(a+a\sec(c+dx))^2}\,dx$$

input
```
Integrate[(e*Tan[c + d*x])^(9/2)/(a + a*Sec[c + d*x])^2,x]
```

output
```
Integrate[(e*Tan[c + d*x])^(9/2)/(a + a*Sec[c + d*x])^2, x]
```

3.129.3 Rubi [A] (verified)

Time = 0.74 (sec) , antiderivative size = 316, normalized size of antiderivative = 1.01, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3042, 4376, 3042, 4374, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e\tan(c+dx))^{9/2}}{(a\sec(c+dx)+a)^2}\,dx$$
$$\downarrow \text{3042}$$
$$\int \frac{\left(-e\cot\left(c+dx+\frac{\pi}{2}\right)\right)^{9/2}}{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^2}dx$$
$$\downarrow \text{4376}$$
$$\frac{e^4\int (a-a\sec(c+dx))^2\sqrt{e\tan(c+dx)}dx}{a^4}$$
$$\downarrow \text{3042}$$
$$\frac{e^4\int \sqrt{-e\cot\left(c+dx+\frac{\pi}{2}\right)}\left(a-a\csc\left(c+dx+\frac{\pi}{2}\right)\right)^2 dx}{a^4}$$
$$\downarrow \text{4374}$$
$$\frac{e^4\int \left(\sec^2(c+dx)\sqrt{e\tan(c+dx)}a^2-2\sec(c+dx)\sqrt{e\tan(c+dx)}a^2+\sqrt{e\tan(c+dx)}a^2\right)dx}{a^4}$$
$$\downarrow \text{2009}$$

$$e^4\left(-\frac{a^2\sqrt{e}\arctan\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}d}+\frac{a^2\sqrt{e}\arctan\left(\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}d}+\frac{2a^2(e\tan(c+dx))^{3/2}}{3de}+\frac{a^2\sqrt{e}\log\left(\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e}\ta\right.}{2\sqrt{2}d}$$

input
```
Int[(e*Tan[c + d*x])^(9/2)/(a + a*Sec[c + d*x])^2,x]
```

output
```
(e^4*(-((a^2*Sqrt[e]*ArcTan[1 - (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(
Sqrt[2]*d)) + (a^2*Sqrt[e]*ArcTan[1 + (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[
e]])/(Sqrt[2]*d) + (a^2*Sqrt[e]*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] - Sqrt[
2]*Sqrt[e*Tan[c + d*x]]])/(2*Sqrt[2]*d) - (a^2*Sqrt[e]*Log[Sqrt[e] + Sqrt[
e]*Tan[c + d*x] + Sqrt[2]*Sqrt[e*Tan[c + d*x]]])/(2*Sqrt[2]*d) + (4*a^2*Co
s[c + d*x]*EllipticE[c - Pi/4 + d*x, 2]*Sqrt[e*Tan[c + d*x]])/(d*Sqrt[Sin[
2*c + 2*d*x]]) + (2*a^2*(e*Tan[c + d*x])^(3/2))/(3*d*e) - (4*a^2*Cos[c + d
*x]*(e*Tan[c + d*x])^(3/2))/(d*e)))/a^4
```

3.129.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4374
```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

rule 4376
```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Simp[a^(2*n)/e^(2*n)   Int[(e*Cot[c + d*x])^(m + 2*
n)/(-a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[a
^2 - b^2, 0] && ILtQ[n, 0]
```

3.129.4 Maple [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 4.92 (sec) , antiderivative size = 1141, normalized size of antiderivative = 3.66

method	result	size
default	Expression too large to display	1141

input
```
int((e*tan(d*x+c))^(9/2)/(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
-1/6/a^2/d*2^(1/2)*e^4*(3*I*(cot(d*x+c)-csc(d*x+c)+1)^(1/2)*(cot(d*x+c)-cs
c(d*x+c))^(1/2)*(csc(d*x+c)-cot(d*x+c)+1)^(1/2)*EllipticPi((csc(d*x+c)-cot
(d*x+c)+1)^(1/2),1/2+1/2*I,1/2*2^(1/2))*cos(d*x+c)^2-3*I*(cot(d*x+c)-csc(d
*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)*(csc(d*x+c)-cot(d*x+c)+1)^(1/
2)*EllipticPi((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2-1/2*I,1/2*2^(1/2))*cos(d
*x+c)^2+3*I*(cot(d*x+c)-csc(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)*
(csc(d*x+c)-cot(d*x+c)+1)^(1/2)*EllipticPi((csc(d*x+c)-cot(d*x+c)+1)^(1/2)
,1/2+1/2*I,1/2*2^(1/2))*cos(d*x+c)-3*I*(cot(d*x+c)-csc(d*x+c)+1)^(1/2)*(co
t(d*x+c)-csc(d*x+c))^(1/2)*(csc(d*x+c)-cot(d*x+c)+1)^(1/2)*EllipticPi((csc
(d*x+c)-cot(d*x+c)+1)^(1/2),1/2-1/2*I,1/2*2^(1/2))*cos(d*x+c)+24*(csc(d*x+
c)-cot(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x
+c))^(1/2)*EllipticE((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2*2^(1/2))*cos(d*x+
c)^2-12*(csc(d*x+c)-cot(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c)+1)^(1/2)*(c
ot(d*x+c)-csc(d*x+c))^(1/2)*EllipticF((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2*
2^(1/2))*cos(d*x+c)^2+3*(cot(d*x+c)-csc(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*
x+c))^(1/2)*(csc(d*x+c)-cot(d*x+c)+1)^(1/2)*EllipticPi((csc(d*x+c)-cot(d*x
+c)+1)^(1/2),1/2+1/2*I,1/2*2^(1/2))*cos(d*x+c)^2+3*(cot(d*x+c)-csc(d*x+c)+
1)^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)*(csc(d*x+c)-cot(d*x+c)+1)^(1/2)*Ell
ipticPi((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2-1/2*I,1/2*2^(1/2))*cos(d*x+c)^
2+24*(csc(d*x+c)-cot(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c)+1)^(1/2)*(c...
```

3.129.5 Fricas [F(-1)]

Timed out.

$$\int \frac{(e\tan(c+dx))^{9/2}}{(a+a\sec(c+dx))^2}\,dx = \text{Timed out}$$

input
```
integrate((e*tan(d*x+c))^(9/2)/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

output
```
Timed out
```

3.129.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(e\tan(c+dx))^{9/2}}{(a+a\sec(c+dx))^2}\,dx = \text{Timed out}$$

input
```
integrate((e*tan(d*x+c))**(9/2)/(a+a*sec(d*x+c))**2,x)
```

output
```
Timed out
```

3.129.7 Maxima [F]

$$\int \frac{(e\tan(c+dx))^{9/2}}{(a+a\sec(c+dx))^2}\,dx = \int \frac{(e\tan(dx+c))^{\frac{9}{2}}}{(a\sec(dx+c)+a)^2}\,dx$$

input
```
integrate((e*tan(d*x+c))^(9/2)/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

output
```
integrate((e*tan(d*x + c))^(9/2)/(a*sec(d*x + c) + a)^2, x)
```

3.129.8 Giac [F]

$$\int \frac{(e\tan(c+dx))^{9/2}}{(a+a\sec(c+dx))^2}\,dx = \int \frac{(e\tan(dx+c))^{\frac{9}{2}}}{(a\sec(dx+c)+a)^2}\,dx$$

input
```
integrate((e*tan(d*x+c))^(9/2)/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

output
```
integrate((e*tan(d*x + c))^(9/2)/(a*sec(d*x + c) + a)^2, x)
```

3.129.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(e\tan(c+dx))^{9/2}}{(a+a\sec(c+dx))^2}\,dx = \int \frac{\cos(c+dx)^2\,(e\tan(c+dx))^{9/2}}{a^2\,(\cos(c+dx)+1)^2}\,dx$$

input
```
int((e*tan(c + d*x))^(9/2)/(a + a/cos(c + d*x))^2,x)
```

output
```
int((cos(c + d*x)^2*(e*tan(c + d*x))^(9/2))/(a^2*(cos(c + d*x) + 1)^2), x)
```

3.130 $\int \frac{(e\tan(c+dx))^{7/2}}{(a+a\sec(c+dx))^2}\,dx$

3.130.1 Optimal result . . . 997
3.130.2 Mathematica [F] . . . 998
3.130.3 Rubi [A] (verified) . . . 998
3.130.4 Maple [C] (warning: unable to verify) . . . 1000
3.130.5 Fricas [F(-1)] . . . 1000
3.130.6 Sympy [F(-1)] . . . 1001
3.130.7 Maxima [F(-1)] . . . 1001
3.130.8 Giac [F] . . . 1001
3.130.9 Mupad [F(-1)] . . . 1002

3.130.1 Optimal result

Integrand size = 25, antiderivative size = 281

$$\begin{aligned}\int \frac{(e\tan(c+dx))^{7/2}}{(a+a\sec(c+dx))^2}\,dx &= -\frac{e^{7/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}a^2d}\\ &+\frac{e^{7/2}\arctan\left(1+\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}a^2d}\\ &-\frac{e^{7/2}\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}a^2d}\\ &+\frac{e^{7/2}\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)+\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}a^2d}\\ &-\frac{2e^4\,\text{EllipticF}\left(c-\frac{\pi}{4}+dx,2\right)\sec(c+dx)\sqrt{\sin(2c+2dx)}}{a^2d\sqrt{e\tan(c+dx)}}+\frac{2e^3\sqrt{e\tan(c+dx)}}{a^2d}\end{aligned}$$

output

```
-1/2*e^(7/2)*arctan(1-2^(1/2)*(e*tan(d*x+c))^(1/2)/e^(1/2))/a^2/d*2^(1/2)+
1/2*e^(7/2)*arctan(1+2^(1/2)*(e*tan(d*x+c))^(1/2)/e^(1/2))/a^2/d*2^(1/2)-1
/4*e^(7/2)*ln(e^(1/2)-2^(1/2)*(e*tan(d*x+c))^(1/2)+e^(1/2)*tan(d*x+c))/a^2
/d*2^(1/2)+1/4*e^(7/2)*ln(e^(1/2)+2^(1/2)*(e*tan(d*x+c))^(1/2)+e^(1/2)*tan
(d*x+c))/a^2/d*2^(1/2)+2*e^4*(sin(c+1/4*Pi+d*x)^2)^(1/2)/sin(c+1/4*Pi+d*x)
*EllipticF(cos(c+1/4*Pi+d*x),2^(1/2))*sec(d*x+c)*sin(2*d*x+2*c)^(1/2)/a^2/
d/(e*tan(d*x+c))^(1/2)+2*e^3*(e*tan(d*x+c))^(1/2)/a^2/d
```

3.130.2 Mathematica [F]

$$\int \frac{(e\tan(c+dx))^{7/2}}{(a+a\sec(c+dx))^2}\,dx = \int \frac{(e\tan(c+dx))^{7/2}}{(a+a\sec(c+dx))^2}\,dx$$

input
```
Integrate[(e*Tan[c + d*x])^(7/2)/(a + a*Sec[c + d*x])^2,x]
```

output
```
Integrate[(e*Tan[c + d*x])^(7/2)/(a + a*Sec[c + d*x])^2, x]
```

3.130.3 Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 285, normalized size of antiderivative = 1.01, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3042, 4376, 3042, 4374, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e\tan(c+dx))^{7/2}}{(a\sec(c+dx)+a)^2}\,dx$$
$$\downarrow 3042$$
$$\int \frac{\left(-e\cot\left(c+dx+\frac{\pi}{2}\right)\right)^{7/2}}{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^2}dx$$
$$\downarrow 4376$$
$$\frac{e^4\int \frac{(a-a\sec(c+dx))^2}{\sqrt{e\tan(c+dx)}}dx}{a^4}$$
$$\downarrow 3042$$
$$\frac{e^4\int \frac{\left(a-a\csc\left(c+dx+\frac{\pi}{2}\right)\right)^2}{\sqrt{-e\cot\left(c+dx+\frac{\pi}{2}\right)}}dx}{a^4}$$
$$\downarrow 4374$$
$$\frac{e^4\int \left(\frac{\sec^2(c+dx)a^2}{\sqrt{e\tan(c+dx)}}-\frac{2\sec(c+dx)a^2}{\sqrt{e\tan(c+dx)}}+\frac{a^2}{\sqrt{e\tan(c+dx)}}\right)dx}{a^4}$$
$$\downarrow 2009$$

$$\frac{e^4\left(-\frac{a^2\arctan\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}d\sqrt{e}}+\frac{a^2\arctan\left(\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}d\sqrt{e}}+\frac{2a^2\sqrt{e\tan(c+dx)}}{de}-\frac{a^2\log\left(\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}+\sqrt{}\right.}{2\sqrt{2}d\sqrt{e}}\right.}{a^4}$$

input
```
Int[(e*Tan[c + d*x])^(7/2)/(a + a*Sec[c + d*x])^2,x]
```

output
```
(e^4*(-((a^2*ArcTan[1 - (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[2]*
d*Sqrt[e])) + (a^2*ArcTan[1 + (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sq
rt[2]*d*Sqrt[e]) - (a^2*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] - Sqrt[2]*Sqrt[
e*Tan[c + d*x]]])/(2*Sqrt[2]*d*Sqrt[e]) + (a^2*Log[Sqrt[e] + Sqrt[e]*Tan[c
 + d*x] + Sqrt[2]*Sqrt[e*Tan[c + d*x]]])/(2*Sqrt[2]*d*Sqrt[e]) - (2*a^2*El
lipticF[c - Pi/4 + d*x, 2]*Sec[c + d*x]*Sqrt[Sin[2*c + 2*d*x]])/(d*Sqrt[e*
Tan[c + d*x]]) + (2*a^2*Sqrt[e*Tan[c + d*x]])/(d*e)))/a^4
```

3.130.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4374
```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

rule 4376
```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Simp[a^(2*n)/e^(2*n)   Int[(e*Cot[c + d*x])^(m + 2*
n)/(-a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[a
^2 - b^2, 0] && ILtQ[n, 0]
```

3.130.4 Maple [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 6.34 (sec) , antiderivative size = 961, normalized size of antiderivative = 3.42

method	result	size
default	Expression too large to display	961

input
```
int((e*tan(d*x+c))^(7/2)/(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
-1/2/a^2/d*2^(1/2)*(-e/((1-cos(d*x+c))^2*csc(d*x+c)^2-1)*(-cot(d*x+c)+csc(
d*x+c)))^(7/2)*((1-cos(d*x+c))^2*csc(d*x+c)^2-1)^4*(I*(csc(d*x+c)-cot(d*x+
c)+1)^(1/2)*(2-2*csc(d*x+c)+2*cot(d*x+c))^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1
/2)*EllipticPi((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2-1/2*I,1/2*2^(1/2))*((1-
cos(d*x+c))*(-cot(d*x+c)+csc(d*x+c)-1)*(csc(d*x+c)-cot(d*x+c)+1)*csc(d*x+c
))^(1/2)-I*(csc(d*x+c)-cot(d*x+c)+1)^(1/2)*(2-2*csc(d*x+c)+2*cot(d*x+c))^(
1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)*EllipticPi((csc(d*x+c)-cot(d*x+c)+1)^(1
/2),1/2+1/2*I,1/2*2^(1/2))*((1-cos(d*x+c))*(-cot(d*x+c)+csc(d*x+c)-1)*(csc
(d*x+c)-cot(d*x+c)+1)*csc(d*x+c))^(1/2)-6*(csc(d*x+c)-cot(d*x+c)+1)^(1/2)*
(2-2*csc(d*x+c)+2*cot(d*x+c))^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)*Elliptic
F((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2*2^(1/2))*((1-cos(d*x+c))*(-cot(d*x+c
)+csc(d*x+c)-1)*(csc(d*x+c)-cot(d*x+c)+1)*csc(d*x+c))^(1/2)+(csc(d*x+c)-co
t(d*x+c)+1)^(1/2)*(2-2*csc(d*x+c)+2*cot(d*x+c))^(1/2)*(cot(d*x+c)-csc(d*x+
c))^(1/2)*EllipticPi((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2-1/2*I,1/2*2^(1/2)
)*((1-cos(d*x+c))*(-cot(d*x+c)+csc(d*x+c)-1)*(csc(d*x+c)-cot(d*x+c)+1)*csc
(d*x+c))^(1/2)+(csc(d*x+c)-cot(d*x+c)+1)^(1/2)*(2-2*csc(d*x+c)+2*cot(d*x+c
))^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)*EllipticPi((csc(d*x+c)-cot(d*x+c)+1
)^(1/2),1/2+1/2*I,1/2*2^(1/2))*((1-cos(d*x+c))*(-cot(d*x+c)+csc(d*x+c)-1)*
(csc(d*x+c)-cot(d*x+c)+1)*csc(d*x+c))^(1/2)+4*((1-cos(d*x+c))^3*csc(d*x+c)
^3+cot(d*x+c)-csc(d*x+c))^(1/2)*(-cot(d*x+c)+csc(d*x+c)))/(1-cos(d*x+c)...
```

3.130.5 Fricas [F(-1)]

Timed out.

$$\int \frac{(e\tan(c+dx))^{7/2}}{(a+a\sec(c+dx))^2}\,dx = \text{Timed out}$$

input
```
integrate((e*tan(d*x+c))^(7/2)/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

output
```
Timed out
```

3.130.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(e\tan(c+dx))^{7/2}}{(a+a\sec(c+dx))^2}\,dx = \text{Timed out}$$

input
```
integrate((e*tan(d*x+c))**(7/2)/(a+a*sec(d*x+c))**2,x)
```

output
```
Timed out
```

3.130.7 Maxima [F(-1)]

Timed out.

$$\int \frac{(e\tan(c+dx))^{7/2}}{(a+a\sec(c+dx))^2}\,dx = \text{Timed out}$$

input
```
integrate((e*tan(d*x+c))^(7/2)/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

output
```
Timed out
```

3.130.8 Giac [F]

$$\int \frac{(e\tan(c+dx))^{7/2}}{(a+a\sec(c+dx))^2}\,dx = \int \frac{(e\tan(dx+c))^{\frac{7}{2}}}{(a\sec(dx+c)+a)^2}\,dx$$

input
```
integrate((e*tan(d*x+c))^(7/2)/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

output
```
integrate((e*tan(d*x + c))^(7/2)/(a*sec(d*x + c) + a)^2, x)
```

3.130.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(e\tan(c+dx))^{7/2}}{(a+a\sec(c+dx))^2}\,dx = \int \frac{\cos(c+dx)^2\,(e\tan(c+dx))^{7/2}}{a^2\,(\cos(c+dx)+1)^2}\,dx$$

input
```
int((e*tan(c + d*x))^(7/2)/(a + a/cos(c + d*x))^2,x)
```

output
```
int((cos(c + d*x)^2*(e*tan(c + d*x))^(7/2))/(a^2*(cos(c + d*x) + 1)^2), x)
```

3.131 $\int \frac{(e\tan(c+dx))^{5/2}}{(a+a\sec(c+dx))^2}\,dx$

3.131.1 Optimal result . . . 1003
3.131.2 Mathematica [C] (warning: unable to verify) . . . 1004
3.131.3 Rubi [A] (verified) . . . 1005
3.131.4 Maple [C] (warning: unable to verify) . . . 1006
3.131.5 Fricas [F(-1)] . . . 1007
3.131.6 Sympy [F] . . . 1007
3.131.7 Maxima [F] . . . 1008
3.131.8 Giac [F] . . . 1008
3.131.9 Mupad [F(-1)] . . . 1008

3.131.1 Optimal result

Integrand size = 25, antiderivative size = 310

$$\int \frac{(e\tan(c+dx))^{5/2}}{(a+a\sec(c+dx))^2}\,dx = \frac{e^{5/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}a^2d} - \frac{e^{5/2}\arctan\left(1+\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}a^2d} - \frac{e^{5/2}\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}a^2d} + \frac{e^{5/2}\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)+\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}a^2d} - \frac{4e^3}{a^2d\sqrt{e\tan(c+dx)}} + \frac{4e^3\cos(c+dx)}{a^2d\sqrt{e\tan(c+dx)}} + \frac{4e^2\cos(c+dx)E\left(c-\frac{\pi}{4}+dx\middle|2\right)\sqrt{e\tan(c+dx)}}{a^2d\sqrt{\sin(2c+2dx)}}$$

output

```
1/2*e^(5/2)*arctan(1-2^(1/2)*(e*tan(d*x+c))^(1/2)/e^(1/2))/a^2/d*2^(1/2)-1
/2*e^(5/2)*arctan(1+2^(1/2)*(e*tan(d*x+c))^(1/2)/e^(1/2))/a^2/d*2^(1/2)-1/
4*e^(5/2)*ln(e^(1/2)-2^(1/2)*(e*tan(d*x+c))^(1/2)+e^(1/2)*tan(d*x+c))/a^2/
d*2^(1/2)+1/4*e^(5/2)*ln(e^(1/2)+2^(1/2)*(e*tan(d*x+c))^(1/2)+e^(1/2)*tan(
d*x+c))/a^2/d*2^(1/2)-4*e^3/a^2/d/(e*tan(d*x+c))^(1/2)+4*e^3*cos(d*x+c)/a^
2/d/(e*tan(d*x+c))^(1/2)-4*e^2*cos(d*x+c)*(sin(c+1/4*Pi+d*x)^2)^(1/2)/sin(
c+1/4*Pi+d*x)*EllipticE(cos(c+1/4*Pi+d*x),2^(1/2))*(e*tan(d*x+c))^(1/2)/a^
2/d/sin(2*d*x+2*c)^(1/2)
```

3.131.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 8.36 (sec) , antiderivative size = 812, normalized size of antiderivative = 2.62

$$\int \frac{(e\tan(c+dx))^{5/2}}{(a+a\sec(c+dx))^2}\,dx = \frac{\cos^4\left(\frac{c}{2}+\frac{dx}{2}\right)\csc^2(c+dx)\left(\frac{32\cos\left(\frac{c}{2}\right)\cos(dx)\sec(2c)\sin\left(\frac{c}{2}\right)}{d}+\frac{16\sec\left(\frac{c}{2}\right)\sec\left(\frac{c}{2}+\frac{dx}{2}\right)\sin\left(\frac{dx}{2}\right)}{d}\right.}{(a+a\sec(c+dx))^2}$$

$$+\frac{e^{-2ic}\left(-e^{4ic}\sqrt{-1+e^{4i(c+dx)}}\arctan\left(\sqrt{-1+e^{4i(c+dx)}}\right)+2\sqrt{-1+e^{2i(c+dx)}}\sqrt{1+e^{2i(c+dx)}}\operatorname{arctanh}\left(\sqrt{\frac{-1+e^{2i(c}}{1+e^{2i(c-}}\right.\right.}{d\sqrt{-\frac{i\left(-1+e^{2i(c+dx)}\right)}{1+e^{2i(c+dx)}}}\left(1+e^{2i(c+dx)}\right)(a+a\sec(c+dx))^2\tan$$

$$-\frac{e^{-2ic}\left(\sqrt{-1+e^{4i(c+dx)}}\arctan\left(\sqrt{-1+e^{4i(c+dx)}}\right)-2e^{4ic}\sqrt{-1+e^{2i(c+dx)}}\sqrt{1+e^{2i(c+dx)}}\operatorname{arctanh}\left(\sqrt{\frac{-1+e^{2i(c+}}{1+e^{2i(c+d}}\right.\right.}{d\sqrt{-\frac{i\left(-1+e^{2i(c+dx)}\right)}{1+e^{2i(c+dx)}}}\left(1+e^{2i(c+dx)}\right)(a+a\sec(c+dx))^2\tan^{\frac{5}{}}$$

$$-\frac{8e^{i(c-dx)}\cos^4\left(\frac{c}{2}+\frac{dx}{2}\right)\left(3-3e^{4i(c+dx)}+e^{4idx}(1+e^{4ic})\sqrt{1-e^{4i(c+dx)}}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},e^{4i(c+dx)}\right.\right.}{3d\sqrt{-\frac{i\left(-1+e^{2i(c+dx)}\right)}{1+e^{2i(c+dx)}}}\left(1+e^{2i(c+dx)}\right)(a+a\sec(c+dx))^2\tan^{\frac{5}{2}}(c+dx$$

input
```
Integrate[(e*Tan[c + d*x])^(5/2)/(a + a*Sec[c + d*x])^2,x]
```

output
```
(Cos[c/2 + (d*x)/2]^4*Csc[c + d*x]^2*((32*Cos[c/2]*Cos[d*x]*Sec[2*c]*Sin[c
/2])/d + (16*Sec[c/2]*Sec[c/2 + (d*x)/2]*Sin[(d*x)/2])/d - (16*Cos[c]*Sec[
2*c]*Sin[d*x])/d + (16*Tan[c/2])/d)*(e*Tan[c + d*x])^(5/2))/(a + a*Sec[c +
 d*x])^2 + ((-(E^((4*I)*c)*Sqrt[-1 + E^((4*I)*(c + d*x))]*ArcTan[Sqrt[-1 +
 E^((4*I)*(c + d*x))]]) + 2*Sqrt[-1 + E^((2*I)*(c + d*x))]*Sqrt[1 + E^((2*
I)*(c + d*x))]*ArcTanh[Sqrt[(-1 + E^((2*I)*(c + d*x)))/(1 + E^((2*I)*(c +
d*x)))]])*Cos[c/2 + (d*x)/2]^4*Sec[2*c]*Sec[c + d*x]^2*(e*Tan[c + d*x])^(5
/2))/(d*E^((2*I)*c)*Sqrt[((-I)*(-1 + E^((2*I)*(c + d*x))))/(1 + E^((2*I)*(
c + d*x)))]*(1 + E^((2*I)*(c + d*x)))*(a + a*Sec[c + d*x])^2*Tan[c + d*x]^
(5/2)) - ((Sqrt[-1 + E^((4*I)*(c + d*x))]*ArcTan[Sqrt[-1 + E^((4*I)*(c + d
*x))]] - 2*E^((4*I)*c)*Sqrt[-1 + E^((2*I)*(c + d*x))]*Sqrt[1 + E^((2*I)*(c
 + d*x))]*ArcTanh[Sqrt[(-1 + E^((2*I)*(c + d*x)))/(1 + E^((2*I)*(c + d*x))
)]])*Cos[c/2 + (d*x)/2]^4*Sec[2*c]*Sec[c + d*x]^2*(e*Tan[c + d*x])^(5/2))/
(d*E^((2*I)*c)*Sqrt[((-I)*(-1 + E^((2*I)*(c + d*x))))/(1 + E^((2*I)*(c + d
*x)))]*(1 + E^((2*I)*(c + d*x)))*(a + a*Sec[c + d*x])^2*Tan[c + d*x]^(5/2)
) - (8*E^(I*(c - d*x))*Cos[c/2 + (d*x)/2]^4*(3 - 3*E^((4*I)*(c + d*x)) + E
^((4*I)*d*x)*(1 + E^((4*I)*c))*Sqrt[1 - E^((4*I)*(c + d*x))]*Hypergeometri
c2F1[1/2, 3/4, 7/4, E^((4*I)*(c + d*x))])*Sec[2*c]*Sec[c + d*x]^2*(e*Tan[c
 + d*x])^(5/2))/(3*d*Sqrt[((-I)*(-1 + E^((2*I)*(c + d*x))))/(1 + E^((2*I)*
(c + d*x)))]*(1 + E^((2*I)*(c + d*x)))*(a + a*Sec[c + d*x])^2*Tan[c + d...
```

3.131.3 Rubi [A] (verified)

Time = 0.76 (sec) , antiderivative size = 317, normalized size of antiderivative = 1.02, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3042, 4376, 3042, 4374, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e\tan(c+dx))^{5/2}}{(a\sec(c+dx)+a)^2}\,dx$$

$$\downarrow 3042$$

$$\int \frac{\left(-e\cot\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^2}dx$$

$$\downarrow 4376$$

$$\frac{e^4\int \frac{(a-a\sec(c+dx))^2}{(e\tan(c+dx))^{3/2}}dx}{a^4}$$

$$\downarrow 3042$$

$$\frac{e^4\int \frac{\left(a-a\csc\left(c+dx+\frac{\pi}{2}\right)\right)^2}{\left(-e\cot\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{a^4}$$

$$\downarrow 4374$$

$$\frac{e^4\int \left(\frac{\sec^2(c+dx)a^2}{(e\tan(c+dx))^{3/2}}-\frac{2\sec(c+dx)a^2}{(e\tan(c+dx))^{3/2}}+\frac{a^2}{(e\tan(c+dx))^{3/2}}\right)dx}{a^4}$$

$$\downarrow 2009$$

$$e^4\left(\frac{a^2\arctan\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}de^{3/2}}-\frac{a^2\arctan\left(\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}de^{3/2}}-\frac{a^2\log\left(\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}de^{3/2}}+\frac{a^2\log\left(\sqrt{e}\tan(c+\right.}{a^4}$$

input
```
Int[(e*Tan[c + d*x])^(5/2)/(a + a*Sec[c + d*x])^2,x]
```

output
```
(e^4*((a^2*ArcTan[1 - (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[2]*d*
e^(3/2)) - (a^2*ArcTan[1 + (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[
2]*d*e^(3/2)) - (a^2*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] - Sqrt[2]*Sqrt[e*T
an[c + d*x]]])/(2*Sqrt[2]*d*e^(3/2)) + (a^2*Log[Sqrt[e] + Sqrt[e]*Tan[c +
d*x] + Sqrt[2]*Sqrt[e*Tan[c + d*x]]])/(2*Sqrt[2]*d*e^(3/2)) - (4*a^2)/(d*e
*Sqrt[e*Tan[c + d*x]]) + (4*a^2*Cos[c + d*x])/(d*e*Sqrt[e*Tan[c + d*x]]) +
 (4*a^2*Cos[c + d*x]*EllipticE[c - Pi/4 + d*x, 2]*Sqrt[e*Tan[c + d*x]])/(d
*e^2*Sqrt[Sin[2*c + 2*d*x]]))/a^4
```

3.131.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4374
```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

rule 4376
```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Simp[a^(2*n)/e^(2*n)   Int[(e*Cot[c + d*x])^(m + 2*
n)/(-a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[a
^2 - b^2, 0] && ILtQ[n, 0]
```

3.131.4 Maple [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 4.49 (sec) , antiderivative size = 261, normalized size of antiderivative = 0.84

method	result
default	$-\frac{\sqrt{2}\,e^2\left(i\,\text{EllipticPi}\left(\sqrt{\csc(dx+c)-\cot(dx+c)+1},\frac{1}{2}+\frac{i}{2},\frac{\sqrt{2}}{2}\right)-i\,\text{EllipticPi}\left(\sqrt{\csc(dx+c)-\cot(dx+c)+1},\frac{1}{2}-\frac{i}{2},\frac{\sqrt{2}}{2}\right)+\text{EllipticPi}\left(\sqrt{\csc}\right.\right.$

input
```
int((e*tan(d*x+c))^(5/2)/(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
-1/2/a^2/d*2^(1/2)*e^2*(I*EllipticPi((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2+1
/2*I,1/2*2^(1/2))-I*EllipticPi((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2-1/2*I,1
/2*2^(1/2))+EllipticPi((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2+1/2*I,1/2*2^(1/
2))+EllipticPi((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2-1/2*I,1/2*2^(1/2))-8*El
lipticE((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2*2^(1/2))+4*EllipticF((csc(d*x+
c)-cot(d*x+c)+1)^(1/2),1/2*2^(1/2)))*(e*tan(d*x+c))^(1/2)*(csc(d*x+c)-cot(
d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1
/2)*sin(d*x+c)/(cos(d*x+c)-1)
```

3.131.5 Fricas [F(-1)]

Timed out.

$$\int \frac{(e\tan(c+dx))^{5/2}}{(a+a\sec(c+dx))^2}\,dx = \text{Timed out}$$

input
```
integrate((e*tan(d*x+c))^(5/2)/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

output
```
Timed out
```

3.131.6 Sympy [F]

$$\int \frac{(e\tan(c+dx))^{5/2}}{(a+a\sec(c+dx))^2}\,dx = \frac{\int \frac{(e\tan(c+dx))^{\frac{5}{2}}}{\sec^2(c+dx)+2\sec(c+dx)+1}\,dx}{a^2}$$

input
```
integrate((e*tan(d*x+c))**(5/2)/(a+a*sec(d*x+c))**2,x)
```

output
```
Integral((e*tan(c + d*x))**(5/2)/(sec(c + d*x)**2 + 2*sec(c + d*x) + 1), x
)/a**2
```

3.131.7 Maxima [F]

$$\int \frac{(e\tan(c+dx))^{5/2}}{(a+a\sec(c+dx))^2}\,dx = \int \frac{(e\tan(dx+c))^{\frac{5}{2}}}{(a\sec(dx+c)+a)^2}\,dx$$

input
```
integrate((e*tan(d*x+c))^(5/2)/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

output
```
integrate((e*tan(d*x + c))^(5/2)/(a*sec(d*x + c) + a)^2, x)
```

3.131.8 Giac [F]

$$\int \frac{(e\tan(c+dx))^{5/2}}{(a+a\sec(c+dx))^2}\,dx = \int \frac{(e\tan(dx+c))^{\frac{5}{2}}}{(a\sec(dx+c)+a)^2}\,dx$$

input
```
integrate((e*tan(d*x+c))^(5/2)/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

output
```
integrate((e*tan(d*x + c))^(5/2)/(a*sec(d*x + c) + a)^2, x)
```

3.131.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(e\tan(c+dx))^{5/2}}{(a+a\sec(c+dx))^2}\,dx = \int \frac{\cos(c+dx)^2\,(e\tan(c+dx))^{5/2}}{a^2\,(\cos(c+dx)+1)^2}\,dx$$

input
```
int((e*tan(c + d*x))^(5/2)/(a + a/cos(c + d*x))^2,x)
```

output
```
int((cos(c + d*x)^2*(e*tan(c + d*x))^(5/2))/(a^2*(cos(c + d*x) + 1)^2), x)
```

3.132 $\int \frac{(e\tan(c+dx))^{3/2}}{(a+a\sec(c+dx))^2}\,dx$

3.132.1 Optimal result . . . 1009
3.132.2 Mathematica [F] . . . 1010
3.132.3 Rubi [A] (verified) . . . 1010
3.132.4 Maple [C] (warning: unable to verify) . . . 1012
3.132.5 Fricas [F(-1)] . . . 1012
3.132.6 Sympy [F] . . . 1013
3.132.7 Maxima [F(-1)] . . . 1013
3.132.8 Giac [F] . . . 1013
3.132.9 Mupad [F(-1)] . . . 1014

3.132.1 Optimal result

Integrand size = 25, antiderivative size = 316

$$\int \frac{(e\tan(c+dx))^{3/2}}{(a+a\sec(c+dx))^2}\,dx = \frac{e^{3/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}a^2d} - \frac{e^{3/2}\arctan\left(1+\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}a^2d} + \frac{e^{3/2}\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}a^2d} - \frac{e^{3/2}\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)+\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}a^2d} - \frac{4e^3}{3a^2d(e\tan(c+dx))^{3/2}} + \frac{4e^3\sec(c+dx)}{3a^2d(e\tan(c+dx))^{3/2}} + \frac{2e^2\,\text{EllipticF}\left(c-\frac{\pi}{4}+dx,2\right)\sec(c+dx)\sqrt{\sin(2c+2dx)}}{3a^2d\sqrt{e\tan(c+dx)}}$$

output

```
1/2*e^(3/2)*arctan(1-2^(1/2)*(e*tan(d*x+c))^(1/2)/e^(1/2))/a^2/d*2^(1/2)-1
/2*e^(3/2)*arctan(1+2^(1/2)*(e*tan(d*x+c))^(1/2)/e^(1/2))/a^2/d*2^(1/2)+1/
4*e^(3/2)*ln(e^(1/2)-2^(1/2)*(e*tan(d*x+c))^(1/2)+e^(1/2)*tan(d*x+c))/a^2/
d*2^(1/2)-1/4*e^(3/2)*ln(e^(1/2)+2^(1/2)*(e*tan(d*x+c))^(1/2)+e^(1/2)*tan(
d*x+c))/a^2/d*2^(1/2)-2/3*e^2*(sin(c+1/4*Pi+d*x)^2)^(1/2)/sin(c+1/4*Pi+d*x
)*EllipticF(cos(c+1/4*Pi+d*x),2^(1/2))*sec(d*x+c)*sin(2*d*x+2*c)^(1/2)/a^2
/d/(e*tan(d*x+c))^(1/2)-4/3*e^3/a^2/d/(e*tan(d*x+c))^(3/2)+4/3*e^3*sec(d*x
+c)/a^2/d/(e*tan(d*x+c))^(3/2)
```

3.132.2 Mathematica [F]

$$\int \frac{(e\tan(c+dx))^{3/2}}{(a+a\sec(c+dx))^2}\,dx = \int \frac{(e\tan(c+dx))^{3/2}}{(a+a\sec(c+dx))^2}\,dx$$

input
```
Integrate[(e*Tan[c + d*x])^(3/2)/(a + a*Sec[c + d*x])^2,x]
```

output
```
Integrate[(e*Tan[c + d*x])^(3/2)/(a + a*Sec[c + d*x])^2, x]
```

3.132.3 Rubi [A] (verified)

Time = 0.78 (sec) , antiderivative size = 323, normalized size of antiderivative = 1.02, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3042, 4376, 3042, 4374, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e\tan(c+dx))^{3/2}}{(a\sec(c+dx)+a)^2}\,dx$$
$$\downarrow 3042$$
$$\int \frac{\left(-e\cot\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^2}dx$$
$$\downarrow 4376$$
$$\frac{e^4\int \frac{(a-a\sec(c+dx))^2}{(e\tan(c+dx))^{5/2}}dx}{a^4}$$
$$\downarrow 3042$$
$$\frac{e^4\int \frac{\left(a-a\csc\left(c+dx+\frac{\pi}{2}\right)\right)^2}{\left(-e\cot\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}dx}{a^4}$$
$$\downarrow 4374$$
$$\frac{e^4\int \left(\frac{\sec^2(c+dx)a^2}{(e\tan(c+dx))^{5/2}}-\frac{2\sec(c+dx)a^2}{(e\tan(c+dx))^{5/2}}+\frac{a^2}{(e\tan(c+dx))^{5/2}}\right)dx}{a^4}$$
$$\downarrow 2009$$

$$e^4\left(\frac{a^2\arctan\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}de^{5/2}}-\frac{a^2\arctan\left(\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}de^{5/2}}+\frac{a^2\log\left(\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}de^{5/2}}-\frac{a^2\log\left(\sqrt{e}\tan(c+\right.}{a^4}$$

input
```
Int[(e*Tan[c + d*x])^(3/2)/(a + a*Sec[c + d*x])^2,x]
```

output
```
(e^4*((a^2*ArcTan[1 - (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[2]*d*
e^(5/2)) - (a^2*ArcTan[1 + (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[
2]*d*e^(5/2)) + (a^2*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] - Sqrt[2]*Sqrt[e*T
an[c + d*x]]])/(2*Sqrt[2]*d*e^(5/2)) - (a^2*Log[Sqrt[e] + Sqrt[e]*Tan[c +
d*x] + Sqrt[2]*Sqrt[e*Tan[c + d*x]]])/(2*Sqrt[2]*d*e^(5/2)) - (4*a^2)/(3*d
*e*(e*Tan[c + d*x])^(3/2)) + (4*a^2*Sec[c + d*x])/(3*d*e*(e*Tan[c + d*x])^
(3/2)) + (2*a^2*EllipticF[c - Pi/4 + d*x, 2]*Sec[c + d*x]*Sqrt[Sin[2*c + 2
*d*x]])/(3*d*e^2*Sqrt[e*Tan[c + d*x]])))/a^4
```

3.132.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4374
```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

rule 4376
```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Simp[a^(2*n)/e^(2*n)   Int[(e*Cot[c + d*x])^(m + 2*
n)/(-a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[a
^2 - b^2, 0] && ILtQ[n, 0]
```

3.132.4 Maple [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 5.27 (sec) , antiderivative size = 638, normalized size of antiderivative = 2.02

method	result
default	$-\frac{\sqrt{2}\left(-\frac{e(-\cot(dx+c)+\csc(dx+c))}{(1-\cos(dx+c))^2\csc(dx+c)^2-1}\right)^{\frac{3}{2}}\left((1-\cos(dx+c))^2\csc(dx+c)^2-1\right)^2\left(-3i\sqrt{\csc(dx+c)-\cot(dx+c)+1}\sqrt{2-2\csc(dx+c)+2\cot(dx+c)}\right.$

input
```
int((e*tan(d*x+c))^(3/2)/(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
-1/6/a^2/d*2^(1/2)*(-e/((1-cos(d*x+c))^2*csc(d*x+c)^2-1)*(-cot(d*x+c)+csc(
d*x+c)))^(3/2)*((1-cos(d*x+c))^2*csc(d*x+c)^2-1)^2*(-3*I*(csc(d*x+c)-cot(d
*x+c)+1)^(1/2)*(2-2*csc(d*x+c)+2*cot(d*x+c))^(1/2)*(cot(d*x+c)-csc(d*x+c))
^(1/2)*EllipticPi((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2-1/2*I,1/2*2^(1/2))+3
*I*(csc(d*x+c)-cot(d*x+c)+1)^(1/2)*(2-2*csc(d*x+c)+2*cot(d*x+c))^(1/2)*(co
t(d*x+c)-csc(d*x+c))^(1/2)*EllipticPi((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2+
1/2*I,1/2*2^(1/2))+10*(csc(d*x+c)-cot(d*x+c)+1)^(1/2)*(2-2*csc(d*x+c)+2*co
t(d*x+c))^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)*EllipticF((csc(d*x+c)-cot(d*
x+c)+1)^(1/2),1/2*2^(1/2))-3*(csc(d*x+c)-cot(d*x+c)+1)^(1/2)*(2-2*csc(d*x+
c)+2*cot(d*x+c))^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)*EllipticPi((csc(d*x+c
)-cot(d*x+c)+1)^(1/2),1/2-1/2*I,1/2*2^(1/2))-3*(csc(d*x+c)-cot(d*x+c)+1)^(
1/2)*(2-2*csc(d*x+c)+2*cot(d*x+c))^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)*Ell
ipticPi((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2+1/2*I,1/2*2^(1/2))-4*(1-cos(d*
x+c))^3*csc(d*x+c)^3+4*csc(d*x+c)-4*cot(d*x+c))/(1-cos(d*x+c))*sin(d*x+c)/
((1-cos(d*x+c))*((1-cos(d*x+c))^2*csc(d*x+c)^2-1)*csc(d*x+c))^(1/2)/((1-co
s(d*x+c))^3*csc(d*x+c)^3+cot(d*x+c)-csc(d*x+c))^(1/2)
```

3.132.5 Fricas [F(-1)]

Timed out.

$$\int \frac{(e\tan(c+dx))^{3/2}}{(a+a\sec(c+dx))^2}\,dx = \text{Timed out}$$

input
```
integrate((e*tan(d*x+c))^(3/2)/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

output
```
Timed out
```

3.132.6 Sympy [F]

$$\int \frac{(e\tan(c+dx))^{3/2}}{(a+a\sec(c+dx))^2}\,dx = \frac{\int \frac{(e\tan(c+dx))^{\frac{3}{2}}}{\sec^2(c+dx)+2\sec(c+dx)+1}\,dx}{a^2}$$

input
```
integrate((e*tan(d*x+c))**(3/2)/(a+a*sec(d*x+c))**2,x)
```

output
```
Integral((e*tan(c + d*x))**(3/2)/(sec(c + d*x)**2 + 2*sec(c + d*x) + 1), x
)/a**2
```

3.132.7 Maxima [F(-1)]

Timed out.

$$\int \frac{(e\tan(c+dx))^{3/2}}{(a+a\sec(c+dx))^2}\,dx = \text{Timed out}$$

input
```
integrate((e*tan(d*x+c))^(3/2)/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

output
```
Timed out
```

3.132.8 Giac [F]

$$\int \frac{(e\tan(c+dx))^{3/2}}{(a+a\sec(c+dx))^2}\,dx = \int \frac{(e\tan(dx+c))^{\frac{3}{2}}}{(a\sec(dx+c)+a)^2}\,dx$$

input
```
integrate((e*tan(d*x+c))^(3/2)/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

output
```
integrate((e*tan(d*x + c))^(3/2)/(a*sec(d*x + c) + a)^2, x)
```

3.132.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(e\tan(c+dx))^{3/2}}{(a+a\sec(c+dx))^2}\,dx = \int \frac{\cos(c+dx)^2\,(e\tan(c+dx))^{3/2}}{a^2\,(\cos(c+dx)+1)^2}\,dx$$

input
```
int((e*tan(c + d*x))^(3/2)/(a + a/cos(c + d*x))^2,x)
```

output
```
int((cos(c + d*x)^2*(e*tan(c + d*x))^(3/2))/(a^2*(cos(c + d*x) + 1)^2), x)
```

3.133 $\int \frac{\sqrt{e\tan(c+dx)}}{(a+a\sec(c+dx))^2}\,dx$

3.133.1 Optimal result 1015
3.133.2 Mathematica [C] (warning: unable to verify) 1016
3.133.3 Rubi [A] (verified) 1017
3.133.4 Maple [C] (warning: unable to verify) 1018
3.133.5 Fricas [F(-1)] 1019
3.133.6 Sympy [F] 1020
3.133.7 Maxima [F] 1020
3.133.8 Giac [F] 1020
3.133.9 Mupad [F(-1)] 1021

3.133.1 Optimal result

Integrand size = 25, antiderivative size = 363

$$\begin{aligned}\int \frac{\sqrt{e\tan(c+dx)}}{(a+a\sec(c+dx))^2}\,dx &= -\frac{\sqrt{e}\arctan\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}a^2d}\\ &+\frac{\sqrt{e}\arctan\left(1+\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}a^2d}\\ &+\frac{\sqrt{e}\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}a^2d}\\ &-\frac{\sqrt{e}\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)+\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}a^2d}\\ &-\frac{4e^3}{5a^2d(e\tan(c+dx))^{5/2}}+\frac{4e^3\sec(c+dx)}{5a^2d(e\tan(c+dx))^{5/2}}\\ &+\frac{2e}{a^2d\sqrt{e\tan(c+dx)}}-\frac{12e\cos(c+dx)}{5a^2d\sqrt{e\tan(c+dx)}}\\ &-\frac{12\cos(c+dx)E\left(c-\frac{\pi}{4}+dx\middle|2\right)\sqrt{e\tan(c+dx)}}{5a^2d\sqrt{\sin(2c+2dx)}}\end{aligned}$$

output
```
-1/2*arctan(1-2^(1/2)*(e*tan(d*x+c))^(1/2)/e^(1/2))*e^(1/2)/a^2/d*2^(1/2)+
1/2*arctan(1+2^(1/2)*(e*tan(d*x+c))^(1/2)/e^(1/2))*e^(1/2)/a^2/d*2^(1/2)+1
/4*ln(e^(1/2)-2^(1/2)*(e*tan(d*x+c))^(1/2)+e^(1/2)*tan(d*x+c))*e^(1/2)/a^2
/d*2^(1/2)-1/4*ln(e^(1/2)+2^(1/2)*(e*tan(d*x+c))^(1/2)+e^(1/2)*tan(d*x+c))
*e^(1/2)/a^2/d*2^(1/2)+2*e/a^2/d/(e*tan(d*x+c))^(1/2)-12/5*e*cos(d*x+c)/a^
2/d/(e*tan(d*x+c))^(1/2)+12/5*cos(d*x+c)*(sin(c+1/4*Pi+d*x)^2)^(1/2)/sin(c
+1/4*Pi+d*x)*EllipticE(cos(c+1/4*Pi+d*x),2^(1/2))*(e*tan(d*x+c))^(1/2)/a^2
/d/sin(2*d*x+2*c)^(1/2)-4/5*e^3/a^2/d/(e*tan(d*x+c))^(5/2)+4/5*e^3*sec(d*x
+c)/a^2/d/(e*tan(d*x+c))^(5/2)
```

3.133.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.04 (sec) , antiderivative size = 2792, normalized size of antiderivative = 7.69

$$\int \frac{\sqrt{e\tan(c+dx)}}{(a+a\sec(c+dx))^2}\,dx = \text{Result too large to show}$$

input
```
Integrate[Sqrt[e*Tan[c + d*x]]/(a + a*Sec[c + d*x])^2,x]
```

output
```
(Cos[c/2 + (d*x)/2]^4*Sec[c + d*x]^2*((-24*Cos[c/2]*Cos[d*x]*Sec[2*c]*(4*S
in[c/2] + Sin[(3*c)/2] + Sin[(5*c)/2]))/(5*d*(1 + 2*Cos[c])) - (56*Sec[c/2
]*Sec[c/2 + (d*x)/2]*Sin[(d*x)/2])/(5*d) + (4*Sec[c/2]*Sec[c/2 + (d*x)/2]^
3*Sin[(d*x)/2])/(5*d) - (12*(-2 - 5*Cos[c] - 6*Cos[2*c] + Cos[3*c])*Sec[2*
c]*Sin[d*x])/(5*d*(1 + 2*Cos[c])) - (56*Tan[c/2])/(5*d) + (4*Sec[c/2 + (d*
x)/2]^2*Tan[c/2])/(5*d))*Sqrt[e*Tan[c + d*x]])/(a + a*Sec[c + d*x])^2 + ((
E^((2*I)*c)*Sqrt[-1 + E^((4*I)*(c + d*x))]*ArcTan[Sqrt[-1 + E^((4*I)*(c +
d*x))]] - 2*Sqrt[-1 + E^((2*I)*(c + d*x))]*Sqrt[1 + E^((2*I)*(c + d*x))]*A
rcTanh[Sqrt[(-1 + E^((2*I)*(c + d*x)))/(1 + E^((2*I)*(c + d*x)))]])*Cos[c/
2 + (d*x)/2]^4*Sec[2*c]*Sec[c + d*x]^2*Sqrt[e*Tan[c + d*x]])/(d*E^(I*c)*Sq
rt[((-I)*(-1 + E^((2*I)*(c + d*x))))/(1 + E^((2*I)*(c + d*x)))]*(1 + E^((2
*I)*(c + d*x)))*(1 + 2*Cos[c])*(a + a*Sec[c + d*x])^2*Sqrt[Tan[c + d*x]])
- ((-(E^((4*I)*c)*Sqrt[-1 + E^((4*I)*(c + d*x))]*ArcTan[Sqrt[-1 + E^((4*I)
*(c + d*x))]]) + 2*Sqrt[-1 + E^((2*I)*(c + d*x))]*Sqrt[1 + E^((2*I)*(c + d
*x))]*ArcTanh[Sqrt[(-1 + E^((2*I)*(c + d*x)))/(1 + E^((2*I)*(c + d*x)))]])
*Cos[c/2 + (d*x)/2]^4*Sec[2*c]*Sec[c + d*x]^2*Sqrt[e*Tan[c + d*x]])/(d*E^(
(2*I)*c)*Sqrt[((-I)*(-1 + E^((2*I)*(c + d*x))))/(1 + E^((2*I)*(c + d*x)))]
*(1 + E^((2*I)*(c + d*x)))*(1 + 2*Cos[c])*(a + a*Sec[c + d*x])^2*Sqrt[Tan[
c + d*x]]) - ((-(E^((6*I)*c)*Sqrt[-1 + E^((4*I)*(c + d*x))]*ArcTan[Sqrt[-1
 + E^((4*I)*(c + d*x))]]) + 2*Sqrt[-1 + E^((2*I)*(c + d*x))]*Sqrt[1 + E...
```

3.133.3 Rubi [A] (verified)

Time = 0.84 (sec) , antiderivative size = 377, normalized size of antiderivative = 1.04, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3042, 4376, 3042, 4374, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{e\tan(c+dx)}}{(a\sec(c+dx)+a)^2}\,dx$$

$$\downarrow 3042$$

$$\int \frac{\sqrt{-e\cot\left(c+dx+\frac{\pi}{2}\right)}}{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^2}dx$$

$$\downarrow 4376$$

$$\frac{e^4\int \frac{(a-a\sec(c+dx))^2}{(e\tan(c+dx))^{7/2}}dx}{a^4}$$

$$\downarrow 3042$$

$$\frac{e^4\int \frac{\left(a-a\csc\left(c+dx+\frac{\pi}{2}\right)\right)^2}{\left(-e\cot\left(c+dx+\frac{\pi}{2}\right)\right)^{7/2}}dx}{a^4}$$

$$\downarrow 4374$$

$$\frac{e^4\int \left(\frac{\sec^2(c+dx)a^2}{(e\tan(c+dx))^{7/2}}-\frac{2\sec(c+dx)a^2}{(e\tan(c+dx))^{7/2}}+\frac{a^2}{(e\tan(c+dx))^{7/2}}\right)dx}{a^4}$$

$$\downarrow 2009$$

$$e^4\left(-\frac{a^2\arctan\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}de^{7/2}}+\frac{a^2\arctan\left(\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}de^{7/2}}+\frac{a^2\log\left(\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}de^{7/2}}-\frac{a^2\log\left(\sqrt{e}\tan(\right.}{}\right.$$

input
```
Int[Sqrt[e*Tan[c + d*x]]/(a + a*Sec[c + d*x])^2,x]
```

output
```
(e^4*(-((a^2*ArcTan[1 - (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[2]*
d*e^(7/2))) + (a^2*ArcTan[1 + (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sq
rt[2]*d*e^(7/2)) + (a^2*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] - Sqrt[2]*Sqrt[
e*Tan[c + d*x]]])/(2*Sqrt[2]*d*e^(7/2)) - (a^2*Log[Sqrt[e] + Sqrt[e]*Tan[c
 + d*x] + Sqrt[2]*Sqrt[e*Tan[c + d*x]]])/(2*Sqrt[2]*d*e^(7/2)) - (4*a^2)/(
5*d*e*(e*Tan[c + d*x])^(5/2)) + (4*a^2*Sec[c + d*x])/(5*d*e*(e*Tan[c + d*x
])^(5/2)) + (2*a^2)/(d*e^3*Sqrt[e*Tan[c + d*x]]) - (12*a^2*Cos[c + d*x])/(
5*d*e^3*Sqrt[e*Tan[c + d*x]]) - (12*a^2*Cos[c + d*x]*EllipticE[c - Pi/4 +
d*x, 2]*Sqrt[e*Tan[c + d*x]])/(5*d*e^4*Sqrt[Sin[2*c + 2*d*x]]))/a^4
```

3.133.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4374
```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

rule 4376
```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Simp[a^(2*n)/e^(2*n)   Int[(e*Cot[c + d*x])^(m + 2*
n)/(-a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[a
^2 - b^2, 0] && ILtQ[n, 0]
```

3.133.4 Maple [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 4.23 (sec) , antiderivative size = 705, normalized size of antiderivative = 1.94

method	result
default	$\frac{\sqrt{2}\sqrt{-\frac{e(-\cot(dx+c)+\csc(dx+c))}{(1-\cos(dx+c))^2\csc(dx+c)^2-1}}\left((1-\cos(dx+c))^2\csc(dx+c)^2-1\right)\left(5i\sqrt{\csc(dx+c)-\cot(dx+c)+1}\sqrt{2-2\csc(dx+c)+2\cot(dx+c}\right.$

input
```
int((e*tan(d*x+c))^(1/2)/(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/10/a^2/d*2^(1/2)*(-e/((1-cos(d*x+c))^2*csc(d*x+c)^2-1)*(-cot(d*x+c)+csc(
d*x+c)))^(1/2)*((1-cos(d*x+c))^2*csc(d*x+c)^2-1)*(5*I*(csc(d*x+c)-cot(d*x+
c)+1)^(1/2)*(2-2*csc(d*x+c)+2*cot(d*x+c))^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1
/2)*EllipticPi((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2-1/2*I,1/2*2^(1/2))-5*I*
(csc(d*x+c)-cot(d*x+c)+1)^(1/2)*(2-2*csc(d*x+c)+2*cot(d*x+c))^(1/2)*(cot(d
*x+c)-csc(d*x+c))^(1/2)*EllipticPi((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2+1/2
*I,1/2*2^(1/2))+24*(csc(d*x+c)-cot(d*x+c)+1)^(1/2)*(2-2*csc(d*x+c)+2*cot(d
*x+c))^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)*EllipticE((csc(d*x+c)-cot(d*x+c
)+1)^(1/2),1/2*2^(1/2))-12*(csc(d*x+c)-cot(d*x+c)+1)^(1/2)*(2-2*csc(d*x+c)
+2*cot(d*x+c))^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)*EllipticF((csc(d*x+c)-c
ot(d*x+c)+1)^(1/2),1/2*2^(1/2))-5*(csc(d*x+c)-cot(d*x+c)+1)^(1/2)*(2-2*csc
(d*x+c)+2*cot(d*x+c))^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)*EllipticPi((csc(
d*x+c)-cot(d*x+c)+1)^(1/2),1/2-1/2*I,1/2*2^(1/2))-5*(csc(d*x+c)-cot(d*x+c)
+1)^(1/2)*(2-2*csc(d*x+c)+2*cot(d*x+c))^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2
)*EllipticPi((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2+1/2*I,1/2*2^(1/2))+2*(1-c
os(d*x+c))^4*csc(d*x+c)^4-2*(1-cos(d*x+c))^2*csc(d*x+c)^2)/((1-cos(d*x+c))
*((1-cos(d*x+c))^2*csc(d*x+c)^2-1)*csc(d*x+c))^(1/2)/((1-cos(d*x+c))^3*csc
(d*x+c)^3+cot(d*x+c)-csc(d*x+c))^(1/2)
```

3.133.5 Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{e\tan(c+dx)}}{(a+a\sec(c+dx))^2}\,dx = \text{Timed out}$$

input
```
integrate((e*tan(d*x+c))^(1/2)/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

output
```
Timed out
```

3.133.6 Sympy [F]

$$\int \frac{\sqrt{e\tan(c+dx)}}{(a+a\sec(c+dx))^2}\,dx = \frac{\int \frac{\sqrt{e\tan(c+dx)}}{\sec^2(c+dx)+2\sec(c+dx)+1}\,dx}{a^2}$$

input
```
integrate((e*tan(d*x+c))**(1/2)/(a+a*sec(d*x+c))**2,x)
```

output
```
Integral(sqrt(e*tan(c + d*x))/(sec(c + d*x)**2 + 2*sec(c + d*x) + 1), x)/a
**2
```

3.133.7 Maxima [F]

$$\int \frac{\sqrt{e\tan(c+dx)}}{(a+a\sec(c+dx))^2}\,dx = \int \frac{\sqrt{e\tan(dx+c)}}{(a\sec(dx+c)+a)^2}\,dx$$

input
```
integrate((e*tan(d*x+c))^(1/2)/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

output
```
integrate(sqrt(e*tan(d*x + c))/(a*sec(d*x + c) + a)^2, x)
```

3.133.8 Giac [F]

$$\int \frac{\sqrt{e\tan(c+dx)}}{(a+a\sec(c+dx))^2}\,dx = \int \frac{\sqrt{e\tan(dx+c)}}{(a\sec(dx+c)+a)^2}\,dx$$

input
```
integrate((e*tan(d*x+c))^(1/2)/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

output
```
integrate(sqrt(e*tan(d*x + c))/(a*sec(d*x + c) + a)^2, x)
```

3.133.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{e\tan(c+dx)}}{(a+a\sec(c+dx))^2}\,dx = \int \frac{\cos(c+dx)^2\sqrt{e\tan(c+dx)}}{a^2(\cos(c+dx)+1)^2}\,dx$$

input
```
int((e*tan(c + d*x))^(1/2)/(a + a/cos(c + d*x))^2,x)
```

output
```
int((cos(c + d*x)^2*(e*tan(c + d*x))^(1/2))/(a^2*(cos(c + d*x) + 1)^2), x)
```

3.134 $\int \frac{1}{(a+a\sec(c+dx))^2\sqrt{e\tan(c+dx)}}\,dx$

3.134.1 Optimal result 1022
3.134.2 Mathematica [C] (warning: unable to verify) 1023
3.134.3 Rubi [A] (verified) 1024
3.134.4 Maple [C] (warning: unable to verify) 1026
3.134.5 Fricas [F(-1)] 1027
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3.134.7 Maxima [F] 1027
3.134.8 Giac [F] 1028
3.134.9 Mupad [F(-1)] 1028

3.134.1 Optimal result

Integrand size = 25, antiderivative size = 365

$$\begin{aligned}
&\int \frac{1}{(a+a\sec(c+dx))^2\sqrt{e\tan(c+dx)}}\,dx \\
&= -\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}a^2d\sqrt{e}} + \frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}a^2d\sqrt{e}} \\
&\quad - \frac{\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}a^2d\sqrt{e}} \\
&\quad + \frac{\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)+\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}a^2d\sqrt{e}} - \frac{4e^3}{7a^2d(e\tan(c+dx))^{7/2}} \\
&\quad + \frac{4e^3\sec(c+dx)}{7a^2d(e\tan(c+dx))^{7/2}} + \frac{2e}{3a^2d(e\tan(c+dx))^{3/2}} - \frac{20e\sec(c+dx)}{21a^2d(e\tan(c+dx))^{3/2}} \\
&\quad - \frac{10\,\text{EllipticF}\left(c-\frac{\pi}{4}+dx,2\right)\sec(c+dx)\sqrt{\sin(2c+2dx)}}{21a^2d\sqrt{e\tan(c+dx)}}
\end{aligned}$$

output
```
-1/2*arctan(1-2^(1/2)*(e*tan(d*x+c))^(1/2)/e^(1/2))/a^2/d*2^(1/2)/e^(1/2)+
1/2*arctan(1+2^(1/2)*(e*tan(d*x+c))^(1/2)/e^(1/2))/a^2/d*2^(1/2)/e^(1/2)-1
/4*ln(e^(1/2)-2^(1/2)*(e*tan(d*x+c))^(1/2)+e^(1/2)*tan(d*x+c))/a^2/d*2^(1/
2)/e^(1/2)+1/4*ln(e^(1/2)+2^(1/2)*(e*tan(d*x+c))^(1/2)+e^(1/2)*tan(d*x+c))
/a^2/d*2^(1/2)/e^(1/2)+10/21*(sin(c+1/4*Pi+d*x)^2)^(1/2)/sin(c+1/4*Pi+d*x)
*EllipticF(cos(c+1/4*Pi+d*x),2^(1/2))*sec(d*x+c)*sin(2*d*x+2*c)^(1/2)/a^2/
d/(e*tan(d*x+c))^(1/2)-4/7*e^3/a^2/d/(e*tan(d*x+c))^(7/2)+4/7*e^3*sec(d*x+
c)/a^2/d/(e*tan(d*x+c))^(7/2)+2/3*e/a^2/d/(e*tan(d*x+c))^(3/2)-20/21*e*sec
(d*x+c)/a^2/d/(e*tan(d*x+c))^(3/2)
```

3.134.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 12.51 (sec) , antiderivative size = 1281, normalized size of antiderivative = 3.51

$$\int \frac{1}{(a+a\sec(c+dx))^2\sqrt{e\tan(c+dx)}}\,dx$$

$$= \frac{40e^{-i(c+dx)}\sqrt{-\frac{i\left(-1+e^{2i(c+dx)}\right)}{1+e^{2i(c+dx)}}}\left(1+e^{2i(c+dx)}\right)\cos^4\left(\frac{c}{2}+\frac{dx}{2}\right)\sec(2c)\sec^2(c+dx)\sqrt{\tan(c+dx)}}{21d(a+a\sec(c+dx))^2\sqrt{e\tan(c+dx)}}$$

$$+\frac{e^{-2ic}\sqrt{-\frac{i\left(-1+e^{2i(c+dx)}\right)}{1+e^{2i(c+dx)}}}\left(e^{4ic}\sqrt{-1+e^{4i(c+dx)}}\arctan\left(\sqrt{-1+e^{4i(c+dx)}}\right)+2\sqrt{-1+e^{2i(c+dx)}}\sqrt{1+e^{2i(c+dx)}}\right.}{d\left(-1+e^{2i(c+dx)}\right)(a+a\sec(c+dx))^2\sqrt{e}}$$

$$+\frac{e^{-2ic}\sqrt{-\frac{i\left(-1+e^{2i(c+dx)}\right)}{1+e^{2i(c+dx)}}}\left(\sqrt{-1+e^{4i(c+dx)}}\arctan\left(\sqrt{-1+e^{4i(c+dx)}}\right)+2e^{4ic}\sqrt{-1+e^{2i(c+dx)}}\sqrt{1+e^{2i(c+dx)}}\right.}{d\left(-1+e^{2i(c+dx)}\right)(a+a\sec(c+dx))^2\sqrt{e}}$$

$$-\frac{2e^{-i(2c+dx)}\sqrt{-\frac{i\left(-1+e^{2i(c+dx)}\right)}{1+e^{2i(c+dx)}}}\cos^4\left(\frac{c}{2}+\frac{dx}{2}\right)\left(3\left(-1+e^{4i(c+dx)}\right)+e^{4i(c+dx)}(-1+e^{2ic})\sqrt{1-e^{4i(c+dx)}}\,\text{Hype}\right.}{3d\left(-1+e^{2i(c+dx)}\right)(a+a\sec(c+dx))^2\sqrt{e}\,\text{ta}}$$

$$+\frac{2e^{-idx}\sqrt{-\frac{i\left(-1+e^{2i(c+dx)}\right)}{1+e^{2i(c+dx)}}}\cos^4\left(\frac{c}{2}+\frac{dx}{2}\right)\left(3-3e^{4i(c+dx)}+e^{2i(c+2dx)}(-1+e^{2ic})\sqrt{1-e^{4i(c+dx)}}\,\text{Hypergeomet}\right.}{3d\left(-1+e^{2i(c+dx)}\right)(a+a\sec(c+dx))^2\sqrt{e\tan(c}}$$

$$+\frac{\cos^4\left(\frac{c}{2}+\frac{dx}{2}\right)\sec^2(c+dx)\left(-\frac{104}{21d}+\frac{4(21-20\cos(c)+21\cos(2c))\cos(dx)\sec(2c)}{21d}+\frac{64\sec^2\left(\frac{c}{2}+\frac{dx}{2}\right)}{21d}-\frac{2\sec^4\left(\frac{c}{2}+\frac{dx}{2}\right)}{7d}-\frac{4\,\text{se}}{}\right.}{(a+a\sec(c+dx))^2\sqrt{e\tan(c+dx)}}$$

$$+\frac{80\sqrt[4]{-1}\cos^4\left(\frac{c}{2}+\frac{dx}{2}\right)\text{EllipticF}\left(i\text{arcsinh}\left(\sqrt[4]{-1}\sqrt{\tan(c+dx)}\right),-1\right)\sec^5(c+dx)\sqrt{\tan(c+dx)}}{21d(a+a\sec(c+dx))^2\sqrt{e\tan(c+dx)}\left(1+\tan^2(c+dx)\right)^{3/2}}$$

input
```
Integrate[1/((a + a*Sec[c + d*x])^2*Sqrt[e*Tan[c + d*x]]),x]
```

output

```
(40*Sqrt[((-I)*(-1 + E^((2*I)*(c + d*x))))/(1 + E^((2*I)*(c + d*x)))]*(1 +
 E^((2*I)*(c + d*x)))*Cos[c/2 + (d*x)/2]^4*Sec[2*c]*Sec[c + d*x]^2*Sqrt[Ta
n[c + d*x]])/(21*d*E^(I*(c + d*x))*(a + a*Sec[c + d*x])^2*Sqrt[e*Tan[c + d
*x]]) + (Sqrt[((-I)*(-1 + E^((2*I)*(c + d*x))))/(1 + E^((2*I)*(c + d*x)))]
*(E^((4*I)*c)*Sqrt[-1 + E^((4*I)*(c + d*x))]*ArcTan[Sqrt[-1 + E^((4*I)*(c
+ d*x))]] + 2*Sqrt[-1 + E^((2*I)*(c + d*x))]*Sqrt[1 + E^((2*I)*(c + d*x))]
*ArcTanh[Sqrt[(-1 + E^((2*I)*(c + d*x)))/(1 + E^((2*I)*(c + d*x)))]])*Cos[
c/2 + (d*x)/2]^4*Sec[2*c]*Sec[c + d*x]^2*Sqrt[Tan[c + d*x]])/(d*E^((2*I)*c
)*(-1 + E^((2*I)*(c + d*x)))*(a + a*Sec[c + d*x])^2*Sqrt[e*Tan[c + d*x]])
+ (Sqrt[((-I)*(-1 + E^((2*I)*(c + d*x))))/(1 + E^((2*I)*(c + d*x)))]*(Sqrt
[-1 + E^((4*I)*(c + d*x))]*ArcTan[Sqrt[-1 + E^((4*I)*(c + d*x))]] + 2*E^((
4*I)*c)*Sqrt[-1 + E^((2*I)*(c + d*x))]*Sqrt[1 + E^((2*I)*(c + d*x))]*ArcTa
nh[Sqrt[(-1 + E^((2*I)*(c + d*x)))/(1 + E^((2*I)*(c + d*x)))]])*Cos[c/2 +
(d*x)/2]^4*Sec[2*c]*Sec[c + d*x]^2*Sqrt[Tan[c + d*x]])/(d*E^((2*I)*c)*(-1
+ E^((2*I)*(c + d*x)))*(a + a*Sec[c + d*x])^2*Sqrt[e*Tan[c + d*x]]) - (2*S
qrt[((-I)*(-1 + E^((2*I)*(c + d*x))))/(1 + E^((2*I)*(c + d*x)))]*Cos[c/2 +
 (d*x)/2]^4*(3*(-1 + E^((4*I)*(c + d*x))) + E^((4*I)*(c + d*x))*(-1 + E^((
2*I)*c))*Sqrt[1 - E^((4*I)*(c + d*x))]*Hypergeometric2F1[1/2, 3/4, 7/4, E^
((4*I)*(c + d*x))])*Sec[2*c]*Sec[c + d*x]^2*Sqrt[Tan[c + d*x]])/(3*d*E^(I*
(2*c + d*x))*(-1 + E^((2*I)*(c + d*x)))*(a + a*Sec[c + d*x])^2*Sqrt[e*T...
```

3.134.3 Rubi [A] (verified)

Time = 0.85 (sec) , antiderivative size = 379, normalized size of antiderivative = 1.04, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3042, 4376, 3042, 4374, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a\sec(c+dx)+a)^2\sqrt{e\tan(c+dx)}}\,dx$$

$$\downarrow 3042$$

$$\int \frac{1}{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^2\sqrt{-e\cot\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\downarrow 4376$$

$$\frac{e^4\int \frac{(a-a\sec(c+dx))^2}{(e\tan(c+dx))^{9/2}}dx}{a^4}$$

$$\downarrow 3042$$

$$\frac{e^4 \int \frac{\left(a-a\csc\left(c+dx+\frac{\pi}{2}\right)\right)^2}{\left(-e\cot\left(c+dx+\frac{\pi}{2}\right)\right)^{9/2}}dx}{a^4}$$

$$\downarrow 4374$$

$$\frac{e^4 \int \left(\frac{\sec^2(c+dx)a^2}{(e\tan(c+dx))^{9/2}} - \frac{2\sec(c+dx)a^2}{(e\tan(c+dx))^{9/2}} + \frac{a^2}{(e\tan(c+dx))^{9/2}}\right)dx}{a^4}$$

$$\downarrow 2009$$

$$e^4\left(-\frac{a^2\arctan\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}de^{9/2}} + \frac{a^2\arctan\left(\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}de^{9/2}} - \frac{a^2\log\left(\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}de^{9/2}} + \frac{a^2\log\left(\sqrt{e}\tan(\right.}{}\right.$$

input

```
Int[1/((a + a*Sec[c + d*x])^2*Sqrt[e*Tan[c + d*x]]),x]
```

output

```
(e^4*(-((a^2*ArcTan[1 - (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[2]*
d*e^(9/2))) + (a^2*ArcTan[1 + (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sq
rt[2]*d*e^(9/2)) - (a^2*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] - Sqrt[2]*Sqrt[
e*Tan[c + d*x]]])/(2*Sqrt[2]*d*e^(9/2)) + (a^2*Log[Sqrt[e] + Sqrt[e]*Tan[c
 + d*x] + Sqrt[2]*Sqrt[e*Tan[c + d*x]]])/(2*Sqrt[2]*d*e^(9/2)) - (4*a^2)/(
7*d*e*(e*Tan[c + d*x])^(7/2)) + (4*a^2*Sec[c + d*x])/(7*d*e*(e*Tan[c + d*x
])^(7/2)) + (2*a^2)/(3*d*e^3*(e*Tan[c + d*x])^(3/2)) - (20*a^2*Sec[c + d*x
])/(21*d*e^3*(e*Tan[c + d*x])^(3/2)) - (10*a^2*EllipticF[c - Pi/4 + d*x, 2
]*Sec[c + d*x]*Sqrt[Sin[2*c + 2*d*x]])/(21*d*e^4*Sqrt[e*Tan[c + d*x]])))/a
^4
```

3.134.3.1 Defintions of rubi rules used

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4374
```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

rule 4376
```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Simp[a^(2*n)/e^(2*n)   Int[(e*Cot[c + d*x])^(m + 2*
n)/(-a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[a
^2 - b^2, 0] && ILtQ[n, 0]
```

3.134.4 Maple [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 5.60 (sec) , antiderivative size = 633, normalized size of antiderivative = 1.73

method	result
default	$-\frac{\sqrt{2}\left(21i\sqrt{\csc(dx+c)-\cot(dx+c)+1}\,\sqrt{2-2\csc(dx+c)+2\cot(dx+c)}\,\sqrt{\cot(dx+c)-\csc(dx+c)}\,\text{EllipticPi}\left(\sqrt{\csc(dx+c)-\cot(dx+c)}\right.\right.$

input
```
int(1/(a+a*sec(d*x+c))^2/(e*tan(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/42/a^2/d*2^(1/2)*(21*I*(csc(d*x+c)-cot(d*x+c)+1)^(1/2)*(2-2*csc(d*x+c)+
2*cot(d*x+c))^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)*EllipticPi((csc(d*x+c)-c
ot(d*x+c)+1)^(1/2),1/2-1/2*I,1/2*2^(1/2))-21*I*(csc(d*x+c)-cot(d*x+c)+1)^(
1/2)*(2-2*csc(d*x+c)+2*cot(d*x+c))^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)*Ell
ipticPi((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2+1/2*I,1/2*2^(1/2))-3*(1-cos(d*
x+c))^5*csc(d*x+c)^5-62*(csc(d*x+c)-cot(d*x+c)+1)^(1/2)*(2-2*csc(d*x+c)+2*
cot(d*x+c))^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)*EllipticF((csc(d*x+c)-cot(
d*x+c)+1)^(1/2),1/2*2^(1/2))+21*(csc(d*x+c)-cot(d*x+c)+1)^(1/2)*(2-2*csc(d
*x+c)+2*cot(d*x+c))^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)*EllipticPi((csc(d*
x+c)-cot(d*x+c)+1)^(1/2),1/2-1/2*I,1/2*2^(1/2))+21*(csc(d*x+c)-cot(d*x+c)+
1)^(1/2)*(2-2*csc(d*x+c)+2*cot(d*x+c))^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)
*EllipticPi((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2+1/2*I,1/2*2^(1/2))+26*(1-c
os(d*x+c))^3*csc(d*x+c)^3-23*csc(d*x+c)+23*cot(d*x+c))*(1-cos(d*x+c))/((1-
cos(d*x+c))^3*csc(d*x+c)^3+cot(d*x+c)-csc(d*x+c))^(1/2)/((1-cos(d*x+c))*((
1-cos(d*x+c))^2*csc(d*x+c)^2-1)*csc(d*x+c))^(1/2)/(-e/((1-cos(d*x+c))^2*cs
c(d*x+c)^2-1)*(-cot(d*x+c)+csc(d*x+c)))^(1/2)*csc(d*x+c)
```

3.134.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a+a\sec(c+dx))^2\sqrt{e\tan(c+dx)}}\,dx = \text{Timed out}$$

input
```
integrate(1/(a+a*sec(d*x+c))^2/(e*tan(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.134.6 Sympy [F]

$$\int \frac{1}{(a+a\sec(c+dx))^2\sqrt{e\tan(c+dx)}}\,dx = \frac{\int \frac{1}{\sqrt{e\tan(c+dx)}\sec^2(c+dx)+2\sqrt{e\tan(c+dx)}\sec(c+dx)+\sqrt{e\tan(c+dx)}}\,dx}{a^2}$$

input
```
integrate(1/(a+a*sec(d*x+c))**2/(e*tan(d*x+c))**(1/2),x)
```

output
```
Integral(1/(sqrt(e*tan(c + d*x))*sec(c + d*x)**2 + 2*sqrt(e*tan(c + d*x))*
sec(c + d*x) + sqrt(e*tan(c + d*x))), x)/a**2
```

3.134.7 Maxima [F]

$$\int \frac{1}{(a+a\sec(c+dx))^2\sqrt{e\tan(c+dx)}}\,dx = \int \frac{1}{(a\sec(dx+c)+a)^2\sqrt{e\tan(dx+c)}}\,dx$$

input
```
integrate(1/(a+a*sec(d*x+c))^2/(e*tan(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/((a*sec(d*x + c) + a)^2*sqrt(e*tan(d*x + c))), x)
```

3.134.8 Giac [F]

$$\int \frac{1}{(a+a\sec(c+dx))^2\sqrt{e\tan(c+dx)}}\,dx = \int \frac{1}{(a\sec(dx+c)+a)^2\sqrt{e\tan(dx+c)}}\,dx$$

input
```
integrate(1/(a+a*sec(d*x+c))^2/(e*tan(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(1/((a*sec(d*x + c) + a)^2*sqrt(e*tan(d*x + c))), x)
```

3.134.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+a\sec(c+dx))^2\sqrt{e\tan(c+dx)}}\,dx = \int \frac{\cos(c+dx)^2}{a^2\sqrt{e\tan(c+dx)}\,(\cos(c+dx)+1)^2}\,dx$$

input
```
int(1/((e*tan(c + d*x))^(1/2)*(a + a/cos(c + d*x))^2),x)
```

output
```
int(cos(c + d*x)^2/(a^2*(e*tan(c + d*x))^(1/2)*(cos(c + d*x) + 1)^2), x)
```

3.135 $\int \sqrt{a + a\sec(c+dx)}\tan^5(c+dx)\,dx$

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3.135.2 Mathematica [A] (verified) . . . 1029
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3.135.9 Mupad [F(-1)] . . . 1034

3.135.1 Optimal result

Integrand size = 23, antiderivative size = 147

$$\int \sqrt{a + a\sec(c+dx)}\tan^5(c+dx)\,dx = -\frac{2\sqrt{a}\operatorname{arctanh}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{a}}\right)}{d} + \frac{2\sqrt{a+a\sec(c+dx)}}{d} + \frac{2(a+a\sec(c+dx))^{3/2}}{3ad} + \frac{2(a+a\sec(c+dx))^{5/2}}{5a^2d} - \frac{6(a+a\sec(c+dx))^{7/2}}{7a^3d} + \frac{2(a+a\sec(c+dx))^{9/2}}{9a^4d}$$

output

```
2/3*(a+a*sec(d*x+c))^(3/2)/a/d+2/5*(a+a*sec(d*x+c))^(5/2)/a^2/d-6/7*(a+a*s
ec(d*x+c))^(7/2)/a^3/d+2/9*(a+a*sec(d*x+c))^(9/2)/a^4/d-2*arctanh((a+a*sec
(d*x+c))^(1/2)/a^(1/2))*a^(1/2)/d+2*(a+a*sec(d*x+c))^(1/2)/d
```

3.135.2 Mathematica [A] (verified)

Time = 0.62 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.69

$$\int \sqrt{a + a\sec(c+dx)}\tan^5(c+dx)\,dx = \frac{2\sqrt{a(1+\sec(c+dx))}\left(-315\operatorname{arctanh}\left(\sqrt{1+\sec(c+dx)}\right) + \sqrt{1+\sec(c+dx)}(383 - 34\sec(c+dx) - 13\right.}{315d\sqrt{1+\sec(c+dx)}}$$

input
```
Integrate[Sqrt[a + a*Sec[c + d*x]]*Tan[c + d*x]^5,x]
```

output
```
(2*Sqrt[a*(1 + Sec[c + d*x])]*(-315*ArcTanh[Sqrt[1 + Sec[c + d*x]]] + Sqrt
[1 + Sec[c + d*x]]*(383 - 34*Sec[c + d*x] - 132*Sec[c + d*x]^2 + 5*Sec[c +
 d*x]^3 + 35*Sec[c + d*x]^4)))/(315*d*Sqrt[1 + Sec[c + d*x]])
```

3.135.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.91, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3042, 25, 4368, 27, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^5(c+dx)\sqrt{a\sec(c+dx)+a}\,dx$$

$$\downarrow 3042$$

$$\int -\cot\left(c+dx+\frac{\pi}{2}\right)^5\sqrt{a\csc\left(c+dx+\frac{\pi}{2}\right)+a}dx$$

$$\downarrow 25$$

$$-\int \cot\left(\frac{1}{2}(2c+\pi)+dx\right)^5\sqrt{\csc\left(\frac{1}{2}(2c+\pi)+dx\right)a+a}dx$$

$$\downarrow 4368$$

$$\frac{\int a^2\cos(c+dx)(1-\sec(c+dx))^2(\sec(c+dx)a+a)^{5/2}d\sec(c+dx)}{a^4d}$$

$$\downarrow 27$$

$$\frac{\int \cos(c+dx)(1-\sec(c+dx))^2(\sec(c+dx)a+a)^{5/2}d\sec(c+dx)}{a^2d}$$

$$\downarrow 99$$

$$\frac{\int\left(\frac{(\sec(c+dx)a+a)^{7/2}}{a}+\cos(c+dx)(\sec(c+dx)a+a)^{5/2}-3(\sec(c+dx)a+a)^{5/2}\right)d\sec(c+dx)}{a^2d}$$

$$\downarrow 2009$$

$$\frac{-2a^{5/2}\operatorname{arctanh}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{a}}\right)+\frac{2(a\sec(c+dx)+a)^{9/2}}{9a^2}+2a^2\sqrt{a\sec(c+dx)+a}-\frac{6(a\sec(c+dx)+a)^{7/2}}{7a}+\frac{2}{5}(a\sec(c+d}{a^2d}$$

input
```
Int[Sqrt[a + a*Sec[c + d*x]]*Tan[c + d*x]^5,x]
```

output
```
(-2*a^(5/2)*ArcTanh[Sqrt[a + a*Sec[c + d*x]]/Sqrt[a]] + 2*a^2*Sqrt[a + a*S
ec[c + d*x]] + (2*a*(a + a*Sec[c + d*x])^(3/2))/3 + (2*(a + a*Sec[c + d*x]
)^(5/2))/5 - (6*(a + a*Sec[c + d*x])^(7/2))/(7*a) + (2*(a + a*Sec[c + d*x]
)^(9/2))/(9*a^2))/(a^2*d)
```

3.135.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4368
```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Simp[-(d*b^(m - 1))^(-1)   Subst[Int[(-a + b*x)^((m - 1)/2
)*((a + b*x)^((m - 1)/2 + n)/x), x], x, Csc[c + d*x]], x] /; FreeQ[{a, b, c
, d, n}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[n]
```

3.135.4 Maple [A] (verified)

Time = 5.03 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.69

method	result
default	$\frac{2\sqrt{a(1+\sec(dx+c))}\left(315\arctan\left(\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}\right)\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}+383-34\sec(dx+c)-132\sec(dx+c)^2+5\sec(dx+c)^3+35\sec(dx+c)\right.}{315d}$

input
```
int((a+a*sec(d*x+c))^(1/2)*tan(d*x+c)^5,x,method=_RETURNVERBOSE)
```

output
```
2/315/d*(a*(1+sec(d*x+c)))^(1/2)*(315*arctan((-cos(d*x+c)/(cos(d*x+c)+1))^
(1/2))*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+383-34*sec(d*x+c)-132*sec(d*x+c)
^2+5*sec(d*x+c)^3+35*sec(d*x+c)^4)
```

3.135.5 Fricas [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 299, normalized size of antiderivative = 2.03

$$\int \sqrt{a+a\sec(c+dx)}\tan^5(c+dx)\,dx$$

$$= \left[\frac{315\sqrt{a}\cos(dx+c)^4\log\left(-8\,a\cos(dx+c)^2+4\left(2\cos(dx+c)^2+\cos(dx+c)\right)\sqrt{a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}-8\,a\right.}{630}\right.$$

input
```
integrate((a+a*sec(d*x+c))^(1/2)*tan(d*x+c)^5,x, algorithm="fricas")
```

output
```
[1/630*(315*sqrt(a)*cos(d*x + c)^4*log(-8*a*cos(d*x + c)^2 + 4*(2*cos(d*x
+ c)^2 + cos(d*x + c))*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)) - 8
*a*cos(d*x + c) - a) + 4*(383*cos(d*x + c)^4 - 34*cos(d*x + c)^3 - 132*cos
(d*x + c)^2 + 5*cos(d*x + c) + 35)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))
)/(d*cos(d*x + c)^4), 1/315*(315*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*
x + c) + a)/cos(d*x + c))*cos(d*x + c)/(2*a*cos(d*x + c) + a))*cos(d*x + c
)^4 + 2*(383*cos(d*x + c)^4 - 34*cos(d*x + c)^3 - 132*cos(d*x + c)^2 + 5*c
os(d*x + c) + 35)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)))/(d*cos(d*x + c)
^4)]
```

3.135.6 Sympy [F]

$$\int \sqrt{a + a\sec(c+dx)}\tan^5(c+dx)\,dx = \int \sqrt{a\left(\sec\left(c+dx\right)+1\right)}\tan^5\left(c+dx\right)dx$$

input
```
integrate((a+a*sec(d*x+c))**(1/2)*tan(d*x+c)**5,x)
```

output
```
Integral(sqrt(a*(sec(c + d*x) + 1))*tan(c + d*x)**5, x)
```

3.135.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.99

$$\int \sqrt{a + a\sec(c+dx)}\tan^5(c+dx)\,dx$$
$$= \frac{315\sqrt{a}\log\left(\frac{\sqrt{a+\frac{a}{\cos(dx+c)}}-\sqrt{a}}{\sqrt{a+\frac{a}{\cos(dx+c)}}+\sqrt{a}}\right) + 630\sqrt{a+\frac{a}{\cos(dx+c)}} + \frac{70\left(a+\frac{a}{\cos(dx+c)}\right)^{\frac{9}{2}}}{a^4} - \frac{270\left(a+\frac{a}{\cos(dx+c)}\right)^{\frac{7}{2}}}{a^3} + \frac{126\left(a+\frac{a}{\cos(dx+c)}\right)^{\frac{5}{2}}}{a^2}}{315\,d}$$

input
```
integrate((a+a*sec(d*x+c))^(1/2)*tan(d*x+c)^5,x, algorithm="maxima")
```

output
```
1/315*(315*sqrt(a)*log((sqrt(a + a/cos(d*x + c)) - sqrt(a))/(sqrt(a + a/co
s(d*x + c)) + sqrt(a))) + 630*sqrt(a + a/cos(d*x + c)) + 70*(a + a/cos(d*x
 + c))^(9/2)/a^4 - 270*(a + a/cos(d*x + c))^(7/2)/a^3 + 126*(a + a/cos(d*x
 + c))^(5/2)/a^2 + 210*(a + a/cos(d*x + c))^(3/2)/a)/d
```

3.135.8 Giac [F]

$$\int \sqrt{a + a\sec(c+dx)}\tan^5(c+dx)\,dx = \int \sqrt{a\sec\left(dx+c\right)+a}\tan\left(dx+c\right)^5\,dx$$

input
```
integrate((a+a*sec(d*x+c))^(1/2)*tan(d*x+c)^5,x, algorithm="giac")
```

output
```
sage0*x
```

3.135.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{a + a\sec(c + dx)}\tan^5(c + dx)\,dx = \int \tan(c + d\,x)^5 \sqrt{a + \frac{a}{\cos(c + d\,x)}}\,dx$$

input
```
int(tan(c + d*x)^5*(a + a/cos(c + d*x))^(1/2),x)
```

output
```
int(tan(c + d*x)^5*(a + a/cos(c + d*x))^(1/2), x)
```

3.136 $\int \sqrt{a+a\sec(c+dx)}\tan^3(c+dx)\,dx$

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3.136.2 Mathematica [A] (verified) 1035
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3.136.1 Optimal result

Integrand size = 23, antiderivative size = 99

$$\int \sqrt{a+a\sec(c+dx)}\tan^3(c+dx)\,dx = \frac{2\sqrt{a}\operatorname{arctanh}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{a}}\right)}{d} - \frac{2\sqrt{a+a\sec(c+dx)}}{d} - \frac{2(a+a\sec(c+dx))^{3/2}}{3ad} + \frac{2(a+a\sec(c+dx))^{5/2}}{5a^2d}$$

output
```
-2/3*(a+a*sec(d*x+c))^(3/2)/a/d+2/5*(a+a*sec(d*x+c))^(5/2)/a^2/d+2*arctanh
((a+a*sec(d*x+c))^(1/2)/a^(1/2))*a^(1/2)/d-2*(a+a*sec(d*x+c))^(1/2)/d
```

3.136.2 Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.81

$$\int \sqrt{a+a\sec(c+dx)}\tan^3(c+dx)\,dx = \frac{2\sqrt{a(1+\sec(c+dx))}\left(15\operatorname{arctanh}\left(\sqrt{1+\sec(c+dx)}\right)+\sqrt{1+\sec(c+dx)}(-17+\sec(c+dx)+3\sec^2(c\right.}{15d\sqrt{1+\sec(c+dx)}}$$

input
```
Integrate[Sqrt[a + a*Sec[c + d*x]]*Tan[c + d*x]^3,x]
```

output
```
(2*Sqrt[a*(1 + Sec[c + d*x])]*(15*ArcTanh[Sqrt[1 + Sec[c + d*x]]] + Sqrt[1
 + Sec[c + d*x]]*(-17 + Sec[c + d*x] + 3*Sec[c + d*x]^2)))/(15*d*Sqrt[1 +
Sec[c + d*x]])
```

3.136.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.96, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.435$, Rules used = {3042, 25, 4368, 25, 27, 90, 60, 60, 73, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^3(c+dx)\sqrt{a\sec(c+dx)+a}\,dx$$

$$\downarrow 3042$$

$$\int -\cot\left(c+dx+\frac{\pi}{2}\right)^3\sqrt{a\csc\left(c+dx+\frac{\pi}{2}\right)+a}dx$$

$$\downarrow 25$$

$$-\int \cot\left(\frac{1}{2}(2c+\pi)+dx\right)^3\sqrt{\csc\left(\frac{1}{2}(2c+\pi)+dx\right)a+a}dx$$

$$\downarrow 4368$$

$$\frac{\int -a\cos(c+dx)(1-\sec(c+dx))(\sec(c+dx)a+a)^{3/2}d\sec(c+dx)}{a^2d}$$

$$\downarrow 25$$

$$-\frac{\int a\cos(c+dx)(1-\sec(c+dx))(\sec(c+dx)a+a)^{3/2}d\sec(c+dx)}{a^2d}$$

$$\downarrow 27$$

$$-\frac{\int \cos(c+dx)(1-\sec(c+dx))(\sec(c+dx)a+a)^{3/2}d\sec(c+dx)}{ad}$$

$$\downarrow 90$$

$$-\frac{\int \cos(c+dx)(\sec(c+dx)a+a)^{3/2}d\sec(c+dx)-\frac{2(a\sec(c+dx)+a)^{5/2}}{5a}}{ad}$$

$$\downarrow 60$$

$$-\frac{a\int \cos(c+dx)\sqrt{\sec(c+dx)a+a}d\sec(c+dx)-\frac{2(a\sec(c+dx)+a)^{5/2}}{5a}+\frac{2}{3}(a\sec(c+dx)+a)^{3/2}}{ad}$$

$$\downarrow 60$$

$$-\frac{a\left(a\int \frac{\cos(c+dx)}{\sqrt{\sec(c+dx)a+a}}d\sec(c+dx)+2\sqrt{a\sec(c+dx)+a}\right)-\frac{2(a\sec(c+dx)+a)^{5/2}}{5a}+\frac{2}{3}(a\sec(c+dx)+a)^{3/2}}{ad}$$

$$\downarrow 73$$

$$-\frac{a\left(2\int \frac{1}{\frac{\sec(c+dx)a+a}{a}-1}d\sqrt{\sec(c+dx)a+a}+2\sqrt{a\sec(c+dx)+a}\right)-\frac{2(a\sec(c+dx)+a)^{5/2}}{5a}+\frac{2}{3}(a\sec(c+dx)+a)^{3/2}}{ad}$$

$$\downarrow 220$$

$$-\frac{a\left(2\sqrt{a\sec(c+dx)+a}-2\sqrt{a}\operatorname{arctanh}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{a}}\right)\right)-\frac{2(a\sec(c+dx)+a)^{5/2}}{5a}+\frac{2}{3}(a\sec(c+dx)+a)^{3/2}}{ad}$$

input
```
Int[Sqrt[a + a*Sec[c + d*x]]*Tan[c + d*x]^3,x]
```

output
```
-(((2*(a + a*Sec[c + d*x])^(3/2))/3 - (2*(a + a*Sec[c + d*x])^(5/2))/(5*a)
 + a*(-2*Sqrt[a]*ArcTanh[Sqrt[a + a*Sec[c + d*x]]/Sqrt[a]] + 2*Sqrt[a + a*
Sec[c + d*x]]))/(a*d))
```

3.136.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4368
```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Simp[-(d*b^(m - 1))^(-1)   Subst[Int[(-a + b*x)^((m - 1)/2
)*((a + b*x)^((m - 1)/2 + n)/x), x], x, Csc[c + d*x]], x] /; FreeQ[{a, b, c
, d, n}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[n]
```

3.136.4 Maple [A] (verified)

Time = 4.74 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.82

method	result	size
default	$-\frac{2\sqrt{a(1+\sec(dx+c))}\left(15\arctan\left(\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}\right)\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}+17-\sec(dx+c)-3\sec(dx+c)^2\right)}{15d}$	81

input
```
int((a+a*sec(d*x+c))^(1/2)*tan(d*x+c)^3,x,method=_RETURNVERBOSE)
```

output
```
-2/15/d*(a*(1+sec(d*x+c)))^(1/2)*(15*arctan((-cos(d*x+c)/(cos(d*x+c)+1))^(
1/2))*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+17-sec(d*x+c)-3*sec(d*x+c)^2)
```

3.136.5 Fricas [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 259, normalized size of antiderivative = 2.62

$$\int \sqrt{a+a\sec(c+dx)}\tan^3(c+dx)\,dx$$

$$= \left[\frac{15\sqrt{a}\cos(dx+c)^2\log\left(-8\,a\cos(dx+c)^2-4\left(2\cos(dx+c)^2+\cos(dx+c)\right)\sqrt{a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}-8\,a\,c\right.\right.}{30\,d\cos(dx+c)^2}$$

$$\left.\left.-\frac{15\sqrt{-a}\arctan\left(\frac{2\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)}{2\,a\cos(dx+c)+a}\right)\cos(dx+c)^2+2\left(17\cos(dx+c)^2-\cos(dx+c)-3\right)\sqrt{\frac{a}{}}}{15\,d\cos(dx+c)^2}\right.\right.$$

input
```
integrate((a+a*sec(d*x+c))^(1/2)*tan(d*x+c)^3,x, algorithm="fricas")
```

output
```
[1/30*(15*sqrt(a)*cos(d*x + c)^2*log(-8*a*cos(d*x + c)^2 - 4*(2*cos(d*x +
c)^2 + cos(d*x + c))*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)) - 8*a
*cos(d*x + c) - a) - 4*(17*cos(d*x + c)^2 - cos(d*x + c) - 3)*sqrt((a*cos(
d*x + c) + a)/cos(d*x + c)))/(d*cos(d*x + c)^2), -1/15*(15*sqrt(-a)*arctan
(2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(2*a*cos(
d*x + c) + a))*cos(d*x + c)^2 + 2*(17*cos(d*x + c)^2 - cos(d*x + c) - 3)*s
qrt((a*cos(d*x + c) + a)/cos(d*x + c)))/(d*cos(d*x + c)^2)]
```

3.136.6 Sympy [F]

$$\int \sqrt{a+a\sec(c+dx)}\tan^3(c+dx)\,dx = \int \sqrt{a\left(\sec(c+dx)+1\right)}\tan^3(c+dx)\,dx$$

input
```
integrate((a+a*sec(d*x+c))**(1/2)*tan(d*x+c)**3,x)
```

output
```
Integral(sqrt(a*(sec(c + d*x) + 1))*tan(c + d*x)**3, x)
```

3.136.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.08

$$\int \sqrt{a+a\sec(c+dx)}\tan^3(c+dx)\,dx$$
$$= -\frac{15\sqrt{a}\log\left(\frac{\sqrt{a+\frac{a}{\cos(dx+c)}}-\sqrt{a}}{\sqrt{a+\frac{a}{\cos(dx+c)}}+\sqrt{a}}\right)+30\sqrt{a+\frac{a}{\cos(dx+c)}}-\frac{6\left(a+\frac{a}{\cos(dx+c)}\right)^{\frac{5}{2}}}{a^2}+\frac{10\left(a+\frac{a}{\cos(dx+c)}\right)^{\frac{3}{2}}}{a}}{15\,d}$$

input
```
integrate((a+a*sec(d*x+c))^(1/2)*tan(d*x+c)^3,x, algorithm="maxima")
```

output
```
-1/15*(15*sqrt(a)*log((sqrt(a + a/cos(d*x + c)) - sqrt(a))/(sqrt(a + a/cos
(d*x + c)) + sqrt(a))) + 30*sqrt(a + a/cos(d*x + c)) - 6*(a + a/cos(d*x +
c))^(5/2)/a^2 + 10*(a + a/cos(d*x + c))^(3/2)/a)/d
```

3.136.8 Giac [F]

$$\int \sqrt{a+a\sec(c+dx)}\tan^3(c+dx)\,dx = \int \sqrt{a\sec(dx+c)+a}\tan(dx+c)^3\,dx$$

input
```
integrate((a+a*sec(d*x+c))^(1/2)*tan(d*x+c)^3,x, algorithm="giac")
```

output
```
sage0*x
```

3.136.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+a\sec(c+dx)}\tan^3(c+dx)\,dx = \int \tan(c+d\,x)^3\sqrt{a+\frac{a}{\cos(c+d\,x)}}\,dx$$

input
```
int(tan(c + d*x)^3*(a + a/cos(c + d*x))^(1/2),x)
```

output
```
int(tan(c + d*x)^3*(a + a/cos(c + d*x))^(1/2), x)
```

3.137 $\int \sqrt{a + a\sec(c+dx)}\tan(c+dx)\,dx$

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3.137.8 Giac [F] 1045
3.137.9 Mupad [B] (verification not implemented) 1046

3.137.1 Optimal result

Integrand size = 21, antiderivative size = 51

$$\int \sqrt{a + a\sec(c+dx)}\tan(c+dx)\,dx = -\frac{2\sqrt{a}\operatorname{arctanh}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{a}}\right)}{d} + \frac{2\sqrt{a+a\sec(c+dx)}}{d}$$

output
```
-2*arctanh((a+a*sec(d*x+c))^(1/2)/a^(1/2))*a^(1/2)/d+2*(a+a*sec(d*x+c))^(1
/2)/d
```

3.137.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.18

$$\int \sqrt{a + a\sec(c+dx)}\tan(c+dx)\,dx = \frac{\sqrt{a(1+\sec(c+dx))}\left(-2\operatorname{arctanh}\left(\sqrt{1+\sec(c+dx)}\right) + 2\sqrt{1+\sec(c+dx)}\right)}{d\sqrt{1+\sec(c+dx)}}$$

input
```
Integrate[Sqrt[a + a*Sec[c + d*x]]*Tan[c + d*x],x]
```

output
```
(Sqrt[a*(1 + Sec[c + d*x])]*(-2*ArcTanh[Sqrt[1 + Sec[c + d*x]]] + 2*Sqrt[1
 + Sec[c + d*x]]))/(d*Sqrt[1 + Sec[c + d*x]])
```

3.137.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.96, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 25, 4368, 60, 73, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan(c+dx)\sqrt{a\sec(c+dx)+a}\,dx$$

$$\downarrow 3042$$

$$\int -\cot\left(c+dx+\frac{\pi}{2}\right)\sqrt{a\csc\left(c+dx+\frac{\pi}{2}\right)+a}dx$$

$$\downarrow 25$$

$$-\int \cot\left(\frac{1}{2}(2c+\pi)+dx\right)\sqrt{\csc\left(\frac{1}{2}(2c+\pi)+dx\right)a+a}dx$$

$$\downarrow 4368$$

$$\frac{\int \cos(c+dx)\sqrt{\sec(c+dx)a+a}d\sec(c+dx)}{d}$$

$$\downarrow 60$$

$$\frac{a\int \frac{\cos(c+dx)}{\sqrt{\sec(c+dx)a+a}}d\sec(c+dx)+2\sqrt{a\sec(c+dx)+a}}{d}$$

$$\downarrow 73$$

$$\frac{2\int \frac{1}{\frac{\sec(c+dx)a+a}{a}-1}d\sqrt{\sec(c+dx)a+a}+2\sqrt{a\sec(c+dx)+a}}{d}$$

$$\downarrow 220$$

$$\frac{2\sqrt{a\sec(c+dx)+a}-2\sqrt{a}\operatorname{arctanh}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{a}}\right)}{d}$$

input
```
Int[Sqrt[a + a*Sec[c + d*x]]*Tan[c + d*x],x]
```

output
```
(-2*Sqrt[a]*ArcTanh[Sqrt[a + a*Sec[c + d*x]]/Sqrt[a]] + 2*Sqrt[a + a*Sec[c
 + d*x]])/d
```

3.137.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4368
```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Simp[-(d*b^(m - 1))^(-1)   Subst[Int[(-a + b*x)^((m - 1)/2
)*((a + b*x)^((m - 1)/2 + n)/x), x], x, Csc[c + d*x]], x] /; FreeQ[{a, b, c
, d, n}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[n]
```

3.137.4 Maple [A] (verified)

Time = 0.93 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.82

method	result	size
derivativedivides	$\frac{2\sqrt{a+a\sec(dx+c)}-2\sqrt{a}\,\operatorname{arctanh}\left(\frac{\sqrt{a+a\sec(dx+c)}}{\sqrt{a}}\right)}{d}$	42
default	$\frac{2\sqrt{a+a\sec(dx+c)}-2\sqrt{a}\,\operatorname{arctanh}\left(\frac{\sqrt{a+a\sec(dx+c)}}{\sqrt{a}}\right)}{d}$	42

input
```
int((a+a*sec(d*x+c))^(1/2)*tan(d*x+c),x,method=_RETURNVERBOSE)
```

output
```
1/d*(2*(a+a*sec(d*x+c))^(1/2)-2*a^(1/2)*arctanh((a+a*sec(d*x+c))^(1/2)/a^(
1/2)))
```

3.137.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 184, normalized size of antiderivative = 3.61

$$\int \sqrt{a+a\sec(c+dx)}\tan(c+dx)\,dx$$

$$= \left[\frac{\sqrt{a}\log\left(-8\,a\cos(dx+c)^2+4\left(2\cos(dx+c)^2+\cos(dx+c)\right)\sqrt{a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}-8\,a\cos(dx+c)-a\right)}{2\,d}\right.$$

input
```
integrate((a+a*sec(d*x+c))^(1/2)*tan(d*x+c),x, algorithm="fricas")
```

output
```
[1/2*(sqrt(a)*log(-8*a*cos(d*x + c)^2 + 4*(2*cos(d*x + c)^2 + cos(d*x + c)
)*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)) - 8*a*cos(d*x + c) - a)
+ 4*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)))/d, (sqrt(-a)*arctan(2*sqrt(-a
)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(2*a*cos(d*x + c) +
 a)) + 2*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)))/d]
```

3.137.6 Sympy [F]

$$\int \sqrt{a + a\sec(c+dx)}\tan(c+dx)\,dx = \int \sqrt{a\left(\sec\left(c+dx\right)+1\right)}\tan\left(c+dx\right)dx$$

input
```
integrate((a+a*sec(d*x+c))**(1/2)*tan(d*x+c),x)
```

output
```
Integral(sqrt(a*(sec(c + d*x) + 1))*tan(c + d*x), x)
```

3.137.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.31

$$\int \sqrt{a + a\sec(c+dx)}\tan(c+dx)\,dx = \frac{\sqrt{a}\log\left(\frac{\sqrt{a+\frac{a}{\cos(dx+c)}}-\sqrt{a}}{\sqrt{a+\frac{a}{\cos(dx+c)}}+\sqrt{a}}\right) + 2\sqrt{a+\frac{a}{\cos(dx+c)}}}{d}$$

input
```
integrate((a+a*sec(d*x+c))^(1/2)*tan(d*x+c),x, algorithm="maxima")
```

output
```
(sqrt(a)*log((sqrt(a + a/cos(d*x + c)) - sqrt(a))/(sqrt(a + a/cos(d*x + c)
) + sqrt(a))) + 2*sqrt(a + a/cos(d*x + c)))/d
```

3.137.8 Giac [F]

$$\int \sqrt{a + a\sec(c+dx)}\tan(c+dx)\,dx = \int \sqrt{a\sec\left(dx+c\right)+a}\tan\left(dx+c\right)\,dx$$

input
```
integrate((a+a*sec(d*x+c))^(1/2)*tan(d*x+c),x, algorithm="giac")
```

output
```
sage0*x
```

3.137.9 Mupad [B] (verification not implemented)

Time = 14.58 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.92

$$\int \sqrt{a + a\sec(c+dx)}\tan(c+dx)\,dx = \frac{2\sqrt{a+\frac{a}{\cos(c+d\,x)}}}{d} - \frac{2\sqrt{a}\,\operatorname{atanh}\left(\frac{\sqrt{a+\frac{a}{\cos(c+d\,x)}}}{\sqrt{a}}\right)}{d}$$

input
```
int(tan(c + d*x)*(a + a/cos(c + d*x))^(1/2),x)
```

output
```
(2*(a + a/cos(c + d*x))^(1/2))/d - (2*a^(1/2)*atanh((a + a/cos(c + d*x))^(
1/2)/a^(1/2)))/d
```

3.138 $\int \cot(c+dx)\sqrt{a+a\sec(c+dx)}\,dx$

3.138.1 Optimal result 1047
3.138.2 Mathematica [A] (verified) 1047
3.138.3 Rubi [A] (verified) 1048
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3.138.6 Sympy [F] 1051
3.138.7 Maxima [F] 1051
3.138.8 Giac [F] 1052
3.138.9 Mupad [F(-1)] 1052

3.138.1 Optimal result

Integrand size = 21, antiderivative size = 73

$$\int \cot(c+dx)\sqrt{a+a\sec(c+dx)}\,dx = \frac{2\sqrt{a}\operatorname{arctanh}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{a}}\right)}{d} - \frac{\sqrt{2}\sqrt{a}\operatorname{arctanh}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{d}$$

output
```
2*arctanh((a+a*sec(d*x+c))^(1/2)/a^(1/2))*a^(1/2)/d-arctanh(1/2*(a+a*sec(d
*x+c))^(1/2)*2^(1/2)/a^(1/2))*2^(1/2)*a^(1/2)/d
```

3.138.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.99

$$\int \cot(c+dx)\sqrt{a+a\sec(c+dx)}\,dx = \frac{\left(2\operatorname{arctanh}\left(\sqrt{1+\sec(c+dx)}\right) - \sqrt{2}\operatorname{arctanh}\left(\frac{\sqrt{1+\sec(c+dx)}}{\sqrt{2}}\right)\right)\sqrt{a(1+\sec(c+dx))}}{d\sqrt{1+\sec(c+dx)}}$$

input
```
Integrate[Cot[c + d*x]*Sqrt[a + a*Sec[c + d*x]],x]
```

output
```
((2*ArcTanh[Sqrt[1 + Sec[c + d*x]]] - Sqrt[2]*ArcTanh[Sqrt[1 + Sec[c + d*x
]]/Sqrt[2]])*Sqrt[a*(1 + Sec[c + d*x])])/(d*Sqrt[1 + Sec[c + d*x]])
```

3.138.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.99, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3042, 25, 4368, 25, 27, 97, 73, 219, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot(c+dx)\sqrt{a\sec(c+dx)+a}\,dx$$

$$\downarrow 3042$$

$$\int -\frac{\sqrt{a\csc\left(c+dx+\frac{\pi}{2}\right)+a}}{\cot\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\downarrow 25$$

$$-\int \frac{\sqrt{\csc\left(\frac{1}{2}(2c+\pi)+dx\right)a+a}}{\cot\left(\frac{1}{2}(2c+\pi)+dx\right)}dx$$

$$\downarrow 4368$$

$$\frac{a^2\int -\frac{\cos(c+dx)}{a(1-\sec(c+dx))\sqrt{\sec(c+dx)a+a}}d\sec(c+dx)}{d}$$

$$\downarrow 25$$

$$-\frac{a^2\int \frac{\cos(c+dx)}{a(1-\sec(c+dx))\sqrt{\sec(c+dx)a+a}}d\sec(c+dx)}{d}$$

$$\downarrow 27$$

$$-\frac{a\int \frac{\cos(c+dx)}{(1-\sec(c+dx))\sqrt{\sec(c+dx)a+a}}d\sec(c+dx)}{d}$$

$$\downarrow 97$$

$$-\frac{a\left(\int \frac{1}{(1-\sec(c+dx))\sqrt{\sec(c+dx)a+a}}d\sec(c+dx)+\int \frac{\cos(c+dx)}{\sqrt{\sec(c+dx)a+a}}d\sec(c+dx)\right)}{d}$$

$$\downarrow 73$$

$$-\frac{a\left(\frac{2\int \frac{1}{2-\frac{\sec(c+dx)a+a}{a}}d\sqrt{\sec(c+dx)a+a}}{a}+\frac{2\int \frac{1}{\frac{\sec(c+dx)a+a}{a}-1}d\sqrt{\sec(c+dx)a+a}}{a}\right)}{d}$$

$$\downarrow 219$$

$$-\frac{a\left(\frac{2\int \frac{1}{\frac{\sec(c+dx)a+a}{a}-1}d\sqrt{\sec(c+dx)a+a}}{a}+\frac{\sqrt{2}\text{arctanh}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{2}\sqrt{a}}\right)}{\sqrt{a}}\right)}{d}$$

$$\downarrow 220$$

$$-\frac{a\left(\frac{\sqrt{2}\text{arctanh}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{2}\sqrt{a}}\right)}{\sqrt{a}}-\frac{2\text{arctanh}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{a}}\right)}{\sqrt{a}}\right)}{d}$$

input
```
Int[Cot[c + d*x]*Sqrt[a + a*Sec[c + d*x]],x]
```

output
```
-((a*((-2*ArcTanh[Sqrt[a + a*Sec[c + d*x]]/Sqrt[a]])/Sqrt[a] + (Sqrt[2]*Ar
cTanh[Sqrt[a + a*Sec[c + d*x]]/(Sqrt[2]*Sqrt[a])])/Sqrt[a]))/d)
```

3.138.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 97
```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_] :> Simp[b/(b*c - a*d)   Int[(e + f*x)^p/(a + b*x), x], x] - Simp[d/(b*c
 - a*d)   Int[(e + f*x)^p/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, p},
 x] &&  !IntegerQ[p]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4368
```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Simp[-(d*b^(m - 1))^(-1)   Subst[Int[(-a + b*x)^((m - 1)/2
)*((a + b*x)^((m - 1)/2 + n)/x), x], x, Csc[c + d*x]], x] /; FreeQ[{a, b, c
, d, n}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[n]
```

3.138.4 Maple [A] (verified)

Time = 2.37 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.26

method	result	size
default	$-\frac{\sqrt{a(1+\sec(dx+c))}\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}\left(2\arctan\left(\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}\right)+\sqrt{2}\arctan\left(\frac{\sqrt{2}}{2\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}}\right)\right)}{d}$	92

input
```
int(cot(d*x+c)*(a+a*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/d*(a*(1+sec(d*x+c)))^(1/2)*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(2*arctan
((-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))+2^(1/2)*arctan(1/2*2^(1/2)/(-cos(d*x+
c)/(cos(d*x+c)+1))^(1/2)))
```

3.138.5 Fricas [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 242, normalized size of antiderivative = 3.32

$$\int \cot(c+dx)\sqrt{a+a\sec(c+dx)}\,dx$$

$$= \left[\frac{\sqrt{2}\sqrt{a}\log\left(-\frac{2\sqrt{2}\sqrt{a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)-3\,a\cos(dx+c)-a}{\cos(dx+c)-1}\right)+2\sqrt{a}\log\left(-2\,a\cos(dx+c)-2\sqrt{a}\sqrt{\frac{a\cos(dx+c)}{\cos(dx+c)}}\right.\right.}{2\,d}$$

input
```
integrate(cot(d*x+c)*(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
[1/2*(sqrt(2)*sqrt(a)*log(-(2*sqrt(2)*sqrt(a)*sqrt((a*cos(d*x + c) + a)/co
s(d*x + c))*cos(d*x + c) - 3*a*cos(d*x + c) - a)/(cos(d*x + c) - 1)) + 2*s
qrt(a)*log(-2*a*cos(d*x + c) - 2*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x
 + c))*cos(d*x + c) - a))/d, (sqrt(2)*sqrt(-a)*arctan(sqrt(2)*sqrt(-a)*sqr
t((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(a*cos(d*x + c) + a)) -
2*sqrt(-a)*arctan(sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x
 + c)/(a*cos(d*x + c) + a)))/d]
```

3.138.6 Sympy [F]

$$\int \cot(c+dx)\sqrt{a+a\sec(c+dx)}\,dx = \int \sqrt{a\left(\sec\left(c+dx\right)+1\right)}\cot\left(c+dx\right)dx$$

input
```
integrate(cot(d*x+c)*(a+a*sec(d*x+c))**(1/2),x)
```

output
```
Integral(sqrt(a*(sec(c + d*x) + 1))*cot(c + d*x), x)
```

3.138.7 Maxima [F]

$$\int \cot(c+dx)\sqrt{a+a\sec(c+dx)}\,dx = \int \sqrt{a\sec\left(dx+c\right)+a}\cot\left(dx+c\right)\,dx$$

input
```
integrate(cot(d*x+c)*(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(a*sec(d*x + c) + a)*cot(d*x + c), x)
```

3.138.8 Giac [F]

$$\int \cot(c+dx)\sqrt{a+a\sec(c+dx)}\,dx = \int \sqrt{a\sec(dx+c)+a}\cot(dx+c)\,dx$$

input
```
integrate(cot(d*x+c)*(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.138.9 Mupad [F(-1)]

Timed out.

$$\int \cot(c+dx)\sqrt{a+a\sec(c+dx)}\,dx = \int \cot(c+d\,x)\,\sqrt{a+\frac{a}{\cos(c+d\,x)}}\,dx$$

input
```
int(cot(c + d*x)*(a + a/cos(c + d*x))^(1/2),x)
```

output
```
int(cot(c + d*x)*(a + a/cos(c + d*x))^(1/2), x)
```

3.139 $\int \cot^3(c+dx)\sqrt{a+a\sec(c+dx)}\,dx$

3.139.1 Optimal result 1053
3.139.2 Mathematica [C] (verified) 1053
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3.139.1 Optimal result

Integrand size = 23, antiderivative size = 131

$$\int \cot^3(c+dx)\sqrt{a+a\sec(c+dx)}\,dx = -\frac{2\sqrt{a}\operatorname{arctanh}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{a}}\right)}{d} + \frac{7\sqrt{a}\operatorname{arctanh}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{4\sqrt{2}d} + \frac{a}{4d\sqrt{a+a\sec(c+dx)}} + \frac{a}{2d(1-\sec(c+dx))\sqrt{a+a\sec(c+dx)}}$$

output

```
-2*arctanh((a+a*sec(d*x+c))^(1/2)/a^(1/2))*a^(1/2)/d+7/8*arctanh(1/2*(a+a*
sec(d*x+c))^(1/2)*2^(1/2)/a^(1/2))*2^(1/2)*a^(1/2)/d+1/4*a/d/(a+a*sec(d*x+
c))^(1/2)+1/2*a/d/(1-sec(d*x+c))/(a+a*sec(d*x+c))^(1/2)
```

3.139.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.55 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.66

$$\int \cot^3(c+dx)\sqrt{a+a\sec(c+dx)}\,dx$$
$$= \frac{\cot^2(c+dx)\left(-2-7\,\text{Hypergeometric2F1}\left(-\tfrac{1}{2},1,\tfrac{1}{2},\tfrac{1}{2}(1+\sec(c+dx))\right)(-1+\sec(c+dx))+8\,\text{Hyperge}}{4d}$$

input
```
Integrate[Cot[c + d*x]^3*Sqrt[a + a*Sec[c + d*x]],x]
```

output
```
(Cot[c + d*x]^2*(-2 - 7*Hypergeometric2F1[-1/2, 1, 1/2, (1 + Sec[c + d*x])
/2]*(-1 + Sec[c + d*x]) + 8*Hypergeometric2F1[-1/2, 1, 1/2, 1 + Sec[c + d*
x]]*(-1 + Sec[c + d*x]))*Sqrt[a*(1 + Sec[c + d*x])])/(4*d)
```

3.139.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.05, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.522$, Rules used = {3042, 25, 4368, 27, 114, 27, 169, 27, 174, 73, 219, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^3(c+dx)\sqrt{a\sec(c+dx)+a}\,dx$$

$$\downarrow 3042$$

$$\int -\frac{\sqrt{a\csc\left(c+dx+\frac{\pi}{2}\right)+a}}{\cot\left(c+dx+\frac{\pi}{2}\right)^3}dx$$

$$\downarrow 25$$

$$-\int \frac{\sqrt{\csc\left(\frac{1}{2}(2c+\pi)+dx\right)a+a}}{\cot\left(\frac{1}{2}(2c+\pi)+dx\right)^3}dx$$

$$\downarrow 4368$$

$$\frac{a^4\int \frac{\cos(c+dx)}{a^2(1-\sec(c+dx))^2(\sec(c+dx)a+a)^{3/2}}d\sec(c+dx)}{d}$$

$$\downarrow 27$$

$$\frac{a^2\int \frac{\cos(c+dx)}{(1-\sec(c+dx))^2(\sec(c+dx)a+a)^{3/2}}d\sec(c+dx)}{d}$$

$$\downarrow 114$$

$$\frac{a^2\left(\frac{1}{2a(1-\sec(c+dx))\sqrt{a\sec(c+dx)+a}}-\frac{\int -\frac{a\cos(c+dx)(3\sec(c+dx)+4)}{2(1-\sec(c+dx))(\sec(c+dx)a+a)^{3/2}}d\sec(c+dx)}{2a}\right)}{d}$$

$$\downarrow 27$$

$$\frac{a^2\left(\frac{1}{4}\int \frac{\cos(c+dx)(3\sec(c+dx)+4)}{(1-\sec(c+dx))(\sec(c+dx)a+a)^{3/2}}d\sec(c+dx)+\frac{1}{2a(1-\sec(c+dx))\sqrt{a\sec(c+dx)+a}}\right)}{d}$$

$$\downarrow 169$$

$$\frac{a^2\left(\frac{1}{4}\left(\frac{\int \frac{a\cos(c+dx)(8-\sec(c+dx))}{2(1-\sec(c+dx))\sqrt{\sec(c+dx)a+a}}d\sec(c+dx)}{a^2}+\frac{1}{a\sqrt{a\sec(c+dx)+a}}\right)+\frac{1}{2a(1-\sec(c+dx))\sqrt{a\sec(c+dx)+a}}\right)}{d}$$

$$\downarrow 27$$

$$\frac{a^2\left(\frac{1}{4}\left(\frac{\int \frac{\cos(c+dx)(8-\sec(c+dx))}{(1-\sec(c+dx))\sqrt{\sec(c+dx)a+a}}d\sec(c+dx)}{2a}+\frac{1}{a\sqrt{a\sec(c+dx)+a}}\right)+\frac{1}{2a(1-\sec(c+dx))\sqrt{a\sec(c+dx)+a}}\right)}{d}$$

$$\downarrow 174$$

$$\frac{a^2\left(\frac{1}{4}\left(\frac{7\int \frac{1}{(1-\sec(c+dx))\sqrt{\sec(c+dx)a+a}}d\sec(c+dx)+8\int \frac{\cos(c+dx)}{\sqrt{\sec(c+dx)a+a}}d\sec(c+dx)}{2a}+\frac{1}{a\sqrt{a\sec(c+dx)+a}}\right)+\frac{1}{2a(1-\sec(c+dx))\sqrt{a\sec(c+dx)+a}}\right)}{d}$$

$$\downarrow 73$$

$$\frac{a^2\left(\frac{1}{4}\left(\frac{14\int \frac{1}{2-\frac{\sec(c+dx)a+a}{a}}d\sqrt{\sec(c+dx)a+a}}{a}+\frac{16\int \frac{1}{\frac{\sec(c+dx)a+a}{a}-1}d\sqrt{\sec(c+dx)a+a}}{a}}{2a}+\frac{1}{a\sqrt{a\sec(c+dx)+a}}\right)+\frac{1}{2a(1-\sec(c+dx))\sqrt{a\sec(c+dx)+a}}\right)}{d}$$

$$\downarrow 219$$

$$\frac{a^2\left(\frac{1}{4}\left(\frac{16\int \frac{1}{\frac{\sec(c+dx)a+a}{a}-1}d\sqrt{\sec(c+dx)a+a}}{a}+\frac{7\sqrt{2}\operatorname{arctanh}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{2}\sqrt{a}}\right)}{\sqrt{a}}}{2a}+\frac{1}{a\sqrt{a\sec(c+dx)+a}}\right)+\frac{1}{2a(1-\sec(c+dx))\sqrt{a\sec(c+dx)+a}}\right)}{d}$$

$$\downarrow 220$$

$$\frac{a^2\left(\frac{1}{4}\left(\frac{\frac{7\sqrt{2}\operatorname{arctanh}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{2}\sqrt{a}}\right)}{\sqrt{a}}-\frac{16\operatorname{arctanh}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{a}}\right)}{\sqrt{a}}}{2a}+\frac{1}{a\sqrt{a\sec(c+dx)+a}}\right)+\frac{1}{2a(1-\sec(c+dx))\sqrt{a\sec(c+dx)+a}}\right)}{d}$$

input
```
Int[Cot[c + d*x]^3*Sqrt[a + a*Sec[c + d*x]],x]
```

3.139. $\int \cot^3(c+dx)\sqrt{a+a\sec(c+dx)}\,dx$

output
```
(a^2*(1/(2*a*(1 - Sec[c + d*x])*Sqrt[a + a*Sec[c + d*x]]) + (((-16*ArcTanh
[Sqrt[a + a*Sec[c + d*x]]/Sqrt[a]])/Sqrt[a] + (7*Sqrt[2]*ArcTanh[Sqrt[a +
a*Sec[c + d*x]]/(Sqrt[2]*Sqrt[a])])/Sqrt[a])/(2*a) + 1/(a*Sqrt[a + a*Sec[c
 + d*x]]))/4))/d
```

3.139.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 114
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[1/((m + 1)*(b*c - a*d)*(b*e
 - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1)
 - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (IntegerQ[n] ||
 IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

rule 169
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && IntegersQ[
2*m, 2*n, 2*p]
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4368
```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Simp[-(d*b^(m - 1))^(-1)   Subst[Int[(-a + b*x)^((m - 1)/2
)*((a + b*x)^((m - 1)/2 + n)/x), x], x, Csc[c + d*x]], x] /; FreeQ[{a, b, c
, d, n}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[n]
```

3.139.4 Maple [A] (verified)

Time = 2.62 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.05

method	result
default	$-\frac{\sqrt{a(1+\sec(dx+c))}\left(-7\sqrt{2}\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}\arctan\left(\frac{\sqrt{2}}{2\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}}\right)-16\arctan\left(\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}\right)\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}+6\cot(dx\right.}{8d}$

input
```
int(cot(d*x+c)^3*(a+a*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/8/d*(a*(1+sec(d*x+c)))^(1/2)*(-7*2^(1/2)*(-cos(d*x+c)/(cos(d*x+c)+1))^(
1/2)*arctan(1/2*2^(1/2)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))-16*arctan((-co
s(d*x+c)/(cos(d*x+c)+1))^(1/2))*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+6*cot(d
*x+c)^2-2*cot(d*x+c)*csc(d*x+c))
```

3.139.5 Fricas [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 426, normalized size of antiderivative = 3.25

$$\int \cot^3(c+dx)\sqrt{a+a\sec(c+dx)}\,dx$$

$$= \left[\frac{8\left(\cos\left(dx+c\right)^2-1\right)\sqrt{a}\log\left(-8\,a\cos\left(dx+c\right)^2+4\left(2\,\cos\left(dx+c\right)^2+\cos\left(dx+c\right)\right)\sqrt{a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}-\right.}{}\right.$$

$$\left.-\frac{7\left(\sqrt{2}\cos\left(dx+c\right)^2-\sqrt{2}\right)\sqrt{-a}\arctan\left(\frac{\sqrt{2}\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)}{a\cos(dx+c)+a}\right)-8\left(\cos\left(dx+c\right)^2-1\right)\sqrt{-a}\arctan}{8\left(d\cos\left(dx+c\right)^2-d\right)}\right]$$

input
```
integrate(cot(d*x+c)^3*(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
[1/16*(8*(cos(d*x + c)^2 - 1)*sqrt(a)*log(-8*a*cos(d*x + c)^2 + 4*(2*cos(d
*x + c)^2 + cos(d*x + c))*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))
- 8*a*cos(d*x + c) - a) + 7*(sqrt(2)*cos(d*x + c)^2 - sqrt(2))*sqrt(a)*log
((2*sqrt(2)*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c) +
 3*a*cos(d*x + c) + a)/(cos(d*x + c) - 1)) + 4*(3*cos(d*x + c)^2 - cos(d*x
 + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)))/(d*cos(d*x + c)^2 - d), -1
/8*(7*(sqrt(2)*cos(d*x + c)^2 - sqrt(2))*sqrt(-a)*arctan(sqrt(2)*sqrt(-a)*
sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(a*cos(d*x + c) + a))
 - 8*(cos(d*x + c)^2 - 1)*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c)
+ a)/cos(d*x + c))*cos(d*x + c)/(2*a*cos(d*x + c) + a)) - 2*(3*cos(d*x + c
)^2 - cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)))/(d*cos(d*x +
c)^2 - d)]
```

3.139.6 Sympy [F]

$$\int \cot^3(c+dx)\sqrt{a+a\sec(c+dx)}\,dx = \int \sqrt{a\left(\sec\left(c+dx\right)+1\right)}\cot^3\left(c+dx\right)dx$$

input
```
integrate(cot(d*x+c)**3*(a+a*sec(d*x+c))**(1/2),x)
```

output
```
Integral(sqrt(a*(sec(c + d*x) + 1))*cot(c + d*x)**3, x)
```

3.139.7 Maxima [F]

$$\int \cot^3(c+dx)\sqrt{a+a\sec(c+dx)}\,dx = \int \sqrt{a\sec(dx+c)+a}\cot(dx+c)^3\,dx$$

input
```
integrate(cot(d*x+c)^3*(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(a*sec(d*x + c) + a)*cot(d*x + c)^3, x)
```

3.139.8 Giac [F]

$$\int \cot^3(c+dx)\sqrt{a+a\sec(c+dx)}\,dx = \int \sqrt{a\sec(dx+c)+a}\cot(dx+c)^3\,dx$$

input
```
integrate(cot(d*x+c)^3*(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.139.9 Mupad [F(-1)]

Timed out.

$$\int \cot^3(c+dx)\sqrt{a+a\sec(c+dx)}\,dx = \int \cot(c+d\,x)^3\sqrt{a+\frac{a}{\cos(c+d\,x)}}\,dx$$

input
```
int(cot(c + d*x)^3*(a + a/cos(c + d*x))^(1/2),x)
```

output
```
int(cot(c + d*x)^3*(a + a/cos(c + d*x))^(1/2), x)
```

3.140 $\int \cot^5(c+dx)\sqrt{a+a\sec(c+dx)}\,dx$

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3.140.1 Optimal result

Integrand size = 23, antiderivative size = 193

$$\begin{aligned}\int \cot^5(c+dx)\sqrt{a+a\sec(c+dx)}\,dx = {} & \frac{2\sqrt{a}\operatorname{arctanh}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{a}}\right)}{d} \\ & - \frac{107\sqrt{a}\operatorname{arctanh}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{64\sqrt{2}d} \\ & + \frac{43a^2}{96d(a+a\sec(c+dx))^{3/2}} \\ & - \frac{a^2}{4d(1-\sec(c+dx))^2(a+a\sec(c+dx))^{3/2}} \\ & - \frac{15a^2}{16d(1-\sec(c+dx))(a+a\sec(c+dx))^{3/2}} \\ & - \frac{21a}{64d\sqrt{a+a\sec(c+dx)}}\end{aligned}$$

output

```
43/96*a^2/d/(a+a*sec(d*x+c))^(3/2)-1/4*a^2/d/(1-sec(d*x+c))^2/(a+a*sec(d*x
+c))^(3/2)-15/16*a^2/d/(1-sec(d*x+c))/(a+a*sec(d*x+c))^(3/2)+2*arctanh((a+
a*sec(d*x+c))^(1/2)/a^(1/2))*a^(1/2)/d-107/128*arctanh(1/2*(a+a*sec(d*x+c)
)^(1/2)*2^(1/2)/a^(1/2))*2^(1/2)*a^(1/2)/d-21/64*a/d/(a+a*sec(d*x+c))^(1/2
)
```

3.140.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.59 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.53

$$\int \cot^5(c+dx)\sqrt{a+a\sec(c+dx)}\,dx$$
$$= \frac{\cot^4(c+dx)\left(-2\left(57+32\,\text{Hypergeometric2F1}\left(-\tfrac{3}{2},1,-\tfrac{1}{2},1+\sec(c+dx)\right)\right)(-1+\sec(c+dx))^2-45\sec(\right.}{}$$

input
```
Integrate[Cot[c + d*x]^5*Sqrt[a + a*Sec[c + d*x]],x]
```

output
```
(Cot[c + d*x]^4*(-2*(57 + 32*Hypergeometric2F1[-3/2, 1, -1/2, 1 + Sec[c +
d*x]]*(-1 + Sec[c + d*x])^2 - 45*Sec[c + d*x]) + 107*Hypergeometric2F1[-3/
2, 1, -1/2, (1 + Sec[c + d*x])/2]*(-1 + Sec[c + d*x])^2)*Sqrt[a*(1 + Sec[c
 + d*x])])/(96*d)
```

3.140.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.07, number of steps used = 18, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.739$, Rules used = {3042, 25, 4368, 25, 27, 114, 27, 168, 27, 169, 27, 169, 27, 174, 73, 219, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^5(c+dx)\sqrt{a\sec(c+dx)+a}\,dx$$
$$\downarrow 3042$$
$$\int -\frac{\sqrt{a\csc\left(c+dx+\frac{\pi}{2}\right)+a}}{\cot\left(c+dx+\frac{\pi}{2}\right)^5}dx$$
$$\downarrow 25$$
$$-\int \frac{\sqrt{\csc\left(\frac{1}{2}(2c+\pi)+dx\right)a+a}}{\cot\left(\frac{1}{2}(2c+\pi)+dx\right)^5}dx$$
$$\downarrow 4368$$

$$\frac{a^6 \int -\frac{\cos(c+dx)}{a^3(1-\sec(c+dx))^3(\sec(c+dx)a+a)^{5/2}}\,d\sec(c+dx)}{d}$$

$$\downarrow 25$$

$$-\frac{a^6 \int \frac{\cos(c+dx)}{a^3(1-\sec(c+dx))^3(\sec(c+dx)a+a)^{5/2}}\,d\sec(c+dx)}{d}$$

$$\downarrow 27$$

$$-\frac{a^3 \int \frac{\cos(c+dx)}{(1-\sec(c+dx))^3(\sec(c+dx)a+a)^{5/2}}\,d\sec(c+dx)}{d}$$

$$\downarrow 114$$

$$-\frac{a^3\left(\frac{1}{4a(1-\sec(c+dx))^2(a\sec(c+dx)+a)^{3/2}} - \frac{\int -\frac{a\cos(c+dx)(7\sec(c+dx)+8)}{2(1-\sec(c+dx))^2(\sec(c+dx)a+a)^{5/2}}\,d\sec(c+dx)}{4a}\right)}{d}$$

$$\downarrow 27$$

$$-\frac{a^3\left(\frac{1}{8}\int \frac{\cos(c+dx)(7\sec(c+dx)+8)}{(1-\sec(c+dx))^2(\sec(c+dx)a+a)^{5/2}}\,d\sec(c+dx) + \frac{1}{4a(1-\sec(c+dx))^2(a\sec(c+dx)+a)^{3/2}}\right)}{d}$$

$$\downarrow 168$$

$$-\frac{a^3\left(\frac{1}{8}\left(\frac{15}{2a(1-\sec(c+dx))(a\sec(c+dx)+a)^{3/2}} - \frac{\int -\frac{a\cos(c+dx)(75\sec(c+dx)+32)}{2(1-\sec(c+dx))(\sec(c+dx)a+a)^{5/2}}\,d\sec(c+dx)}{2a}\right) + \frac{1}{4a(1-\sec(c+dx))^2(a\sec(c+dx)+a)^{3/}}\right.}{d}$$

$$\downarrow 27$$

$$-\frac{a^3\left(\frac{1}{8}\left(\frac{1}{4}\int \frac{\cos(c+dx)(75\sec(c+dx)+32)}{(1-\sec(c+dx))(\sec(c+dx)a+a)^{5/2}}\,d\sec(c+dx) + \frac{15}{2a(1-\sec(c+dx))(a\sec(c+dx)+a)^{3/2}}\right) + \frac{1}{4a(1-\sec(c+dx))^2(a\sec(c+dx)}\right.}{d}$$

$$\downarrow 169$$

$$-\frac{a^3\left(\frac{1}{8}\left(\frac{1}{4}\left(\frac{\int \frac{3a\cos(c+dx)(43\sec(c+dx)+64)}{2(1-\sec(c+dx))(\sec(c+dx)a+a)^{3/2}}\,d\sec(c+dx)}{3a^2} - \frac{43}{3a(a\sec(c+dx)+a)^{3/2}}\right) + \frac{15}{2a(1-\sec(c+dx))(a\sec(c+dx)+a)^{3/2}}\right) + \frac{}{4a(1-}\right.}{d}$$

$$\downarrow 27$$

$$-\frac{a^3\left(\frac{1}{8}\left(\frac{1}{4}\left(\frac{\int \frac{\cos(c+dx)(43\sec(c+dx)+64)}{(1-\sec(c+dx))(\sec(c+dx)a+a)^{3/2}}\,d\sec(c+dx)}{2a} - \frac{43}{3a(a\sec(c+dx)+a)^{3/2}}\right) + \frac{15}{2a(1-\sec(c+dx))(a\sec(c+dx)+a)^{3/2}}\right) + \frac{}{4a(1-}\right.}{d}$$

3.140. $\int \cot^5(c+dx)\sqrt{a+a\sec(c+dx)}\,dx$

$$\downarrow 169$$

$$-\frac{a^3\left(\frac{1}{8}\left(\frac{1}{4}\left(\frac{\int \frac{a\cos(c+dx)(128-21\sec(c+dx))}{2(1-\sec(c+dx))\sqrt{\sec(c+dx)a+a}}d\sec(c+dx)}{a^2}+\frac{21}{a\sqrt{a\sec(c+dx)+a}}\right)}{2a}-\frac{43}{3a(a\sec(c+dx)+a)^{3/2}}\right)+\frac{15}{2a(1-\sec(c+dx))(a\sec(c+dx)+a)}\right.}{d}$$

$$\downarrow 27$$

$$-\frac{a^3\left(\frac{1}{8}\left(\frac{1}{4}\left(\frac{\int \frac{\cos(c+dx)(128-21\sec(c+dx))}{(1-\sec(c+dx))\sqrt{\sec(c+dx)a+a}}d\sec(c+dx)}{2a}+\frac{21}{a\sqrt{a\sec(c+dx)+a}}\right)}{2a}-\frac{43}{3a(a\sec(c+dx)+a)^{3/2}}\right)+\frac{15}{2a(1-\sec(c+dx))(a\sec(c+dx)+a)^3}\right.}{d}$$

$$\downarrow 174$$

$$-\frac{a^3\left(\frac{1}{8}\left(\frac{1}{4}\left(\frac{107\int \frac{1}{(1-\sec(c+dx))\sqrt{\sec(c+dx)a+a}}d\sec(c+dx)+128\int \frac{\cos(c+dx)}{\sqrt{\sec(c+dx)a+a}}d\sec(c+dx)}{2a}+\frac{21}{a\sqrt{a\sec(c+dx)+a}}\right)}{2a}-\frac{43}{3a(a\sec(c+dx)+a)^{3/2}}\right)+\right.}{d}$$

$$\downarrow 73$$

$$-\frac{a^3\left(\frac{1}{8}\left(\frac{1}{4}\left(\frac{214\int \frac{1}{2-\frac{\sec(c+dx)a+a}{a}}d\sqrt{\sec(c+dx)a+a}}{a}+\frac{256\int \frac{1}{\frac{\sec(c+dx)a+a}{a}-1}d\sqrt{\sec(c+dx)a+a}}{a}}{2a}+\frac{21}{a\sqrt{a\sec(c+dx)+a}}\right)}{2a}-\frac{43}{3a(a\sec(c+dx)+a)^{3/2}}\right)+\right.}{d}$$

$$\downarrow 219$$

$$-\frac{a^3\left(\frac{1}{8}\left(\frac{1}{4}\left(\frac{256\int \frac{1}{\frac{\sec(c+dx)a+a}{a}-1}d\sqrt{\sec(c+dx)a+a}}{a}+\frac{107\sqrt{2}\operatorname{arctanh}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{2}\sqrt{a}}\right)}{\sqrt{a}}}{2a}+\frac{21}{a\sqrt{a\sec(c+dx)+a}}\right)}{2a}-\frac{43}{3a(a\sec(c+dx)+a)^{3/2}}\right)+\frac{2}{}\right.}{d}$$

$$\downarrow 220$$

$$-\frac{a^3\left(\frac{1}{8}\left(\frac{1}{4}\left(\frac{\frac{107\sqrt{2}\operatorname{arctanh}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{2}\sqrt{a}}\right)}{\sqrt{a}}-\frac{256\operatorname{arctanh}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{a}}\right)}{\sqrt{a}}}{2a}+\frac{21}{a\sqrt{a\sec(c+dx)+a}}\right)-\frac{43}{3a(a\sec(c+dx)+a)^{3/2}}\right)+\frac{}{2a(1-s}\right.}{d}$$

input
```
Int[Cot[c + d*x]^5*Sqrt[a + a*Sec[c + d*x]],x]
```

output
```
-((a^3*(1/(4*a*(1 - Sec[c + d*x])^2*(a + a*Sec[c + d*x])^(3/2)) + (15/(2*a
*(1 - Sec[c + d*x])*(a + a*Sec[c + d*x])^(3/2)) + (-43/(3*a*(a + a*Sec[c +
 d*x])^(3/2)) + (((-256*ArcTanh[Sqrt[a + a*Sec[c + d*x]]/Sqrt[a]])/Sqrt[a]
 + (107*Sqrt[2]*ArcTanh[Sqrt[a + a*Sec[c + d*x]]/(Sqrt[2]*Sqrt[a])])/Sqrt[
a])/(2*a) + 21/(a*Sqrt[a + a*Sec[c + d*x]]))/(2*a))/4)/8))/d)
```

3.140.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 114
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[1/((m + 1)*(b*c - a*d)*(b*e
 - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1)
 - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (IntegerQ[n] ||
 IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

rule 168
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

rule 169
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && IntegersQ[
2*m, 2*n, 2*p]
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4368
```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Simp[-(d*b^(m - 1))^(-1)   Subst[Int[(-a + b*x)^((m - 1)/2
)*((a + b*x)^((m - 1)/2 + n)/x), x], x, Csc[c + d*x]], x] /; FreeQ[{a, b, c
, d, n}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[n]
```

3.140.4 Maple [A] (verified)

Time = 3.04 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.90

method	result
default	$-\frac{\sqrt{a(1+\sec(dx+c))}\left(321\sqrt{2}\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}\arctan\left(\frac{\sqrt{2}}{2\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}}\right)+768\arctan\left(\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}\right)\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}+410\cos\right.}{384d}$

input
```
int(cot(d*x+c)^5*(a+a*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/384/d*(a*(1+sec(d*x+c)))^(1/2)*(321*2^(1/2)*(-cos(d*x+c)/(cos(d*x+c)+1)
)^(1/2)*arctan(1/2*2^(1/2)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))+768*arctan(
(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+410
*cot(d*x+c)^4-142*cot(d*x+c)^3*csc(d*x+c)-298*cot(d*x+c)^2*csc(d*x+c)^2+12
6*csc(d*x+c)^3*cot(d*x+c))
```

3.140.5 Fricas [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 529, normalized size of antiderivative = 2.74

$$\int \cot^5(c+dx)\sqrt{a+a\sec(c+dx)}\,dx$$

$$= \left[\frac{384\left(\cos\left(dx+c\right)^4 - 2\,\cos\left(dx+c\right)^2 + 1\right)\sqrt{a}\log\left(-8\,a\cos\left(dx+c\right)^2 - 4\left(2\,\cos\left(dx+c\right)^2 + \cos\left(dx+c\right)\right)\right.}{}\right.$$

input
```
integrate(cot(d*x+c)^5*(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
[1/768*(384*(cos(d*x + c)^4 - 2*cos(d*x + c)^2 + 1)*sqrt(a)*log(-8*a*cos(d
*x + c)^2 - 4*(2*cos(d*x + c)^2 + cos(d*x + c))*sqrt(a)*sqrt((a*cos(d*x +
c) + a)/cos(d*x + c)) - 8*a*cos(d*x + c) - a) + 321*(sqrt(2)*cos(d*x + c)^
4 - 2*sqrt(2)*cos(d*x + c)^2 + sqrt(2))*sqrt(a)*log(-(2*sqrt(2)*sqrt(a)*sq
rt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c) - 3*a*cos(d*x + c) - a)
/(cos(d*x + c) - 1)) - 4*(205*cos(d*x + c)^4 - 71*cos(d*x + c)^3 - 149*cos
(d*x + c)^2 + 63*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)))/(d
*cos(d*x + c)^4 - 2*d*cos(d*x + c)^2 + d), 1/384*(321*(sqrt(2)*cos(d*x + c
)^4 - 2*sqrt(2)*cos(d*x + c)^2 + sqrt(2))*sqrt(-a)*arctan(sqrt(2)*sqrt(-a)
*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(a*cos(d*x + c) + a)
) - 384*(cos(d*x + c)^4 - 2*cos(d*x + c)^2 + 1)*sqrt(-a)*arctan(2*sqrt(-a)
*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(2*a*cos(d*x + c) +
a)) - 2*(205*cos(d*x + c)^4 - 71*cos(d*x + c)^3 - 149*cos(d*x + c)^2 + 63*
cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)))/(d*cos(d*x + c)^4 -
 2*d*cos(d*x + c)^2 + d)]
```

3.140.6 Sympy [F]

$$\int \cot^5(c+dx)\sqrt{a+a\sec(c+dx)}\,dx = \int \sqrt{a\left(\sec\left(c+dx\right)+1\right)}\cot^5\left(c+dx\right)dx$$

input
```
integrate(cot(d*x+c)**5*(a+a*sec(d*x+c))**(1/2),x)
```

output
```
Integral(sqrt(a*(sec(c + d*x) + 1))*cot(c + d*x)**5, x)
```

3.140.7 Maxima [F(-1)]

Timed out.

$$\int \cot^5(c+dx)\sqrt{a+a\sec(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate(cot(d*x+c)^5*(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
Timed out
```

3.140.8 Giac [F]

$$\int \cot^5(c+dx)\sqrt{a+a\sec(c+dx)}\,dx = \int \sqrt{a\sec(dx+c)+a}\cot(dx+c)^5\,dx$$

input
```
integrate(cot(d*x+c)^5*(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.140.9 Mupad [F(-1)]

Timed out.

$$\int \cot^5(c+dx)\sqrt{a+a\sec(c+dx)}\,dx = \int \cot(c+d\,x)^5\sqrt{a+\frac{a}{\cos(c+d\,x)}}\,dx$$

input
```
int(cot(c + d*x)^5*(a + a/cos(c + d*x))^(1/2),x)
```

output
```
int(cot(c + d*x)^5*(a + a/cos(c + d*x))^(1/2), x)
```

3.141 $\int \sqrt{a + a\sec(c+dx)}\tan^6(c+dx)\,dx$

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3.141.1 Optimal result

Integrand size = 23, antiderivative size = 222

$$\begin{aligned}\int \sqrt{a + a\sec(c+dx)}\tan^6(c+dx)\,dx = &-\frac{2\sqrt{a}\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d} \\ &+ \frac{2a\tan(c+dx)}{d\sqrt{a+a\sec(c+dx)}} - \frac{2a^2\tan^3(c+dx)}{3d(a+a\sec(c+dx))^{3/2}} \\ &+ \frac{2a^3\tan^5(c+dx)}{5d(a+a\sec(c+dx))^{5/2}} \\ &+ \frac{2a^4\tan^7(c+dx)}{d(a+a\sec(c+dx))^{7/2}} \\ &+ \frac{10a^5\tan^9(c+dx)}{9d(a+a\sec(c+dx))^{9/2}} \\ &+ \frac{2a^6\tan^{11}(c+dx)}{11d(a+a\sec(c+dx))^{11/2}}\end{aligned}$$

output

```
-2*arctan(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))*a^(1/2)/d+2*a*tan(d*x
+c)/d/(a+a*sec(d*x+c))^(1/2)-2/3*a^2*tan(d*x+c)^3/d/(a+a*sec(d*x+c))^(3/2)
+2/5*a^3*tan(d*x+c)^5/d/(a+a*sec(d*x+c))^(5/2)+2*a^4*tan(d*x+c)^7/d/(a+a*s
ec(d*x+c))^(7/2)+10/9*a^5*tan(d*x+c)^9/d/(a+a*sec(d*x+c))^(9/2)+2/11*a^6*t
an(d*x+c)^11/d/(a+a*sec(d*x+c))^(11/2)
```

3.141.2 Mathematica [A] (verified)

Time = 7.81 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.60

$$\int \sqrt{a+a\sec(c+dx)}\tan^6(c+dx)\,dx =$$

$$-\frac{\sec\left(\frac{1}{2}(c+dx)\right)\sec^5(c+dx)\sqrt{a(1+\sec(c+dx))}\left(3960\sqrt{2}\arcsin\left(\sqrt{2}\sin\left(\frac{1}{2}(c+dx)\right)\right)\cos^{\frac{11}{2}}(c+dx)+\right.}{}$$

input
```
Integrate[Sqrt[a + a*Sec[c + d*x]]*Tan[c + d*x]^6,x]
```

output
```
-1/3960*(Sec[(c + d*x)/2]*Sec[c + d*x]^5*Sqrt[a*(1 + Sec[c + d*x])]*(3960*
Sqrt[2]*ArcSin[Sqrt[2]*Sin[(c + d*x)/2]]*Cos[c + d*x]^(11/2) + 792*Sin[(c
+ d*x)/2] - 1386*Sin[(3*(c + d*x))/2] + 495*Sin[(5*(c + d*x))/2] - 616*Sin
[(7*(c + d*x))/2] - 247*Sin[(11*(c + d*x))/2]))/d
```

3.141.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 205, normalized size of antiderivative = 0.92, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3042, 4375, 364, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^6(c+dx)\sqrt{a\sec(c+dx)+a}\,dx$$

$$\downarrow 3042$$

$$\int \cot\left(c+dx+\frac{\pi}{2}\right)^6\sqrt{a\csc\left(c+dx+\frac{\pi}{2}\right)+a}dx$$

$$\downarrow 4375$$

$$-\frac{2a^4\int \frac{\tan^6(c+dx)\left(\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+2\right)^3}{(\sec(c+dx)a+a)^3\left(\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+1\right)}d\left(-\frac{\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)}{d}$$

$$\downarrow 364$$

$$-\frac{2a^4\int\left(\frac{a^2\tan^{10}(c+dx)}{(\sec(c+dx)a+a)^5}+\frac{5a\tan^8(c+dx)}{(\sec(c+dx)a+a)^4}+\frac{7\tan^6(c+dx)}{(\sec(c+dx)a+a)^3}+\frac{\tan^4(c+dx)}{a(\sec(c+dx)a+a)^2}-\frac{\tan^2(c+dx)}{a^2(\sec(c+dx)a+a)}-\frac{1}{a^3\left(\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+1\right)}\right.}{d}$$

$$\downarrow 2009$$

$$-\frac{2a^4\left(\frac{\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{a^{7/2}}-\frac{\tan(c+dx)}{a^3\sqrt{a\sec(c+dx)+a}}-\frac{a^2\tan^{11}(c+dx)}{11(a\sec(c+dx)+a)^{11/2}}+\frac{\tan^3(c+dx)}{3a^2(a\sec(c+dx)+a)^{3/2}}-\frac{5a\tan^9(c+dx)}{9(a\sec(c+dx)+a)^{9/2}}-\right.}{d}$$

input
```
Int[Sqrt[a + a*Sec[c + d*x]]*Tan[c + d*x]^6,x]
```

output
```
(-2*a^4*(ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]]/a^(7/2) -
 Tan[c + d*x]/(a^3*Sqrt[a + a*Sec[c + d*x]]) + Tan[c + d*x]^3/(3*a^2*(a +
a*Sec[c + d*x])^(3/2)) - Tan[c + d*x]^5/(5*a*(a + a*Sec[c + d*x])^(5/2)) -
 Tan[c + d*x]^7/(a + a*Sec[c + d*x])^(7/2) - (5*a*Tan[c + d*x]^9)/(9*(a +
a*Sec[c + d*x])^(9/2)) - (a^2*Tan[c + d*x]^11)/(11*(a + a*Sec[c + d*x])^(1
1/2))))/d
```

3.141.3.1 Defintions of rubi rules used

rule 364
```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_))/((c_) + (d_.)*(x_)^2),
x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*((a + b*x^2)^p/(c + d*x^2)), x], x
] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && (In
tegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4375
```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Simp[-2*(a^(m/2 + n + 1/2)/d)   Subst[Int[x^m*((2 + a*x^2
)^(m/2 + n - 1/2)/(1 + a*x^2)), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]
]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && I
ntegerQ[n - 1/2]
```

3.141.4 Maple [A] (verified)

Time = 5.35 (sec) , antiderivative size = 233, normalized size of antiderivative = 1.05

method	result
default	$\frac{2\sqrt{a(1+\sec(dx+c))}\left(-495\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,\operatorname{arctanh}\left(\frac{\sin(dx+c)}{(\cos(dx+c)+1)\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}}\right)\cos(dx+c)-495\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,\operatorname{arctanh}\left(\frac{}{(c}\right.\right.}{}$

input
```
int((a+a*sec(d*x+c))^(1/2)*tan(d*x+c)^6,x,method=_RETURNVERBOSE)
```

output
```
2/495/d*(a*(1+sec(d*x+c)))^(1/2)/(cos(d*x+c)+1)*(-495*(-cos(d*x+c)/(cos(d*
x+c)+1))^(1/2)*arctanh(sin(d*x+c)/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+
1))^(1/2))*cos(d*x+c)-495*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctanh(sin(d
*x+c)/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))+494*sin(d*x+c)+24
7*tan(d*x+c)-186*sec(d*x+c)*tan(d*x+c)-155*tan(d*x+c)*sec(d*x+c)^2+50*tan(
d*x+c)*sec(d*x+c)^3+45*sec(d*x+c)^4*tan(d*x+c))
```

3.141.5 Fricas [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 371, normalized size of antiderivative = 1.67

$$\int \sqrt{a+a\sec(c+dx)}\tan^6(c+dx)\,dx$$
$$= \left[\frac{495\left(\cos\left(dx+c\right)^6+\cos\left(dx+c\right)^5\right)\sqrt{-a}\log\left(\frac{2\,a\cos(dx+c)^2+2\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)\sin(dx+c)+a\cos(dx+c)-a}{\cos(dx+c)+1}\right)}{495}\right.$$

input
```
integrate((a+a*sec(d*x+c))^(1/2)*tan(d*x+c)^6,x, algorithm="fricas")
```

output
```
[1/495*(495*(cos(d*x + c)^6 + cos(d*x + c)^5)*sqrt(-a)*log((2*a*cos(d*x +
c)^2 + 2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin
(d*x + c) + a*cos(d*x + c) - a)/(cos(d*x + c) + 1)) + 2*(494*cos(d*x + c)^
5 + 247*cos(d*x + c)^4 - 186*cos(d*x + c)^3 - 155*cos(d*x + c)^2 + 50*cos(
d*x + c) + 45)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(d*co
s(d*x + c)^6 + d*cos(d*x + c)^5), 2/495*(495*(cos(d*x + c)^6 + cos(d*x + c
)^5)*sqrt(a)*arctan(sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(
sqrt(a)*sin(d*x + c))) + (494*cos(d*x + c)^5 + 247*cos(d*x + c)^4 - 186*co
s(d*x + c)^3 - 155*cos(d*x + c)^2 + 50*cos(d*x + c) + 45)*sqrt((a*cos(d*x
+ c) + a)/cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^6 + d*cos(d*x + c)^5
)]
```

3.141.6 Sympy [F]

$$\int \sqrt{a + a\sec(c + dx)}\tan^6(c + dx)\,dx = \int \sqrt{a\left(\sec\left(c + dx\right) + 1\right)}\tan^6\left(c + dx\right)\,dx$$

input
```
integrate((a+a*sec(d*x+c))**(1/2)*tan(d*x+c)**6,x)
```

output
```
Integral(sqrt(a*(sec(c + d*x) + 1))*tan(c + d*x)**6, x)
```

3.141.7 Maxima [F]

$$\int \sqrt{a + a\sec(c + dx)}\tan^6(c + dx)\,dx = \int \sqrt{a\sec\left(dx + c\right) + a}\tan\left(dx + c\right)^6\,dx$$

input
```
integrate((a+a*sec(d*x+c))^(1/2)*tan(d*x+c)^6,x, algorithm="maxima")
```

output
```
1/990*(495*((cos(2*d*x + 2*c)^4 + sin(2*d*x + 2*c)^4 + 4*cos(2*d*x + 2*c)^
3 + 2*(cos(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sin(2*d*x + 2*c)^2 + 6
*cos(2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c) + 1)*arctan2((cos(2*d*x + 2*c)^2
+ sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2
*d*x + 2*c), cos(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c
)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(
2*d*x + 2*c) + 1)) + 1) - (cos(2*d*x + 2*c)^4 + sin(2*d*x + 2*c)^4 + 4*cos
(2*d*x + 2*c)^3 + 2*(cos(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sin(2*d*
x + 2*c)^2 + 6*cos(2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c) + 1)*arctan2((cos(2
*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2
*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + s
in(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*
x + 2*c), cos(2*d*x + 2*c) + 1)) - 1) - 2*(d*cos(2*d*x + 2*c)^4 + d*sin(2*
d*x + 2*c)^4 + 4*d*cos(2*d*x + 2*c)^3 + 6*d*cos(2*d*x + 2*c)^2 + 2*(d*cos(
2*d*x + 2*c)^2 + 2*d*cos(2*d*x + 2*c) + d)*sin(2*d*x + 2*c)^2 + 4*d*cos(2*
d*x + 2*c) + d)*integrate((((cos(14*d*x + 14*c)*cos(2*d*x + 2*c) + 6*cos(1
2*d*x + 12*c)*cos(2*d*x + 2*c) + 15*cos(10*d*x + 10*c)*cos(2*d*x + 2*c) +
20*cos(8*d*x + 8*c)*cos(2*d*x + 2*c) + 15*cos(6*d*x + 6*c)*cos(2*d*x + 2*c
) + 6*cos(4*d*x + 4*c)*cos(2*d*x + 2*c) + cos(2*d*x + 2*c)^2 + sin(14*d*x
+ 14*c)*sin(2*d*x + 2*c) + 6*sin(12*d*x + 12*c)*sin(2*d*x + 2*c) + 15*s...
```

3.141.8 Giac [F]

$$\int \sqrt{a + a\sec(c + dx)}\tan^6(c + dx)\,dx = \int \sqrt{a\sec(dx + c) + a}\tan(dx + c)^6\,dx$$

input
```
integrate((a+a*sec(d*x+c))^(1/2)*tan(d*x+c)^6,x, algorithm="giac")
```

output
```
sage0*x
```

3.141.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{a + a\sec(c + dx)}\tan^6(c + dx)\,dx = \int \tan(c + d\,x)^6 \sqrt{a + \frac{a}{\cos(c + d\,x)}}\,dx$$

input
```
int(tan(c + d*x)^6*(a + a/cos(c + d*x))^(1/2),x)
```

output
```
int(tan(c + d*x)^6*(a + a/cos(c + d*x))^(1/2), x)
```

3.142 $\int \sqrt{a + a\sec(c+dx)}\tan^4(c+dx)\,dx$

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3.142.2 Mathematica [A] (verified) 1077
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3.142.8 Giac [F] 1081
3.142.9 Mupad [F(-1)] 1082

3.142.1 Optimal result

Integrand size = 23, antiderivative size = 160

$$\begin{aligned}\int \sqrt{a + a\sec(c+dx)}\tan^4(c+dx)\,dx = {} & \frac{2\sqrt{a}\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d} \\ & - \frac{2a\tan(c+dx)}{d\sqrt{a+a\sec(c+dx)}} + \frac{2a^2\tan^3(c+dx)}{3d(a+a\sec(c+dx))^{3/2}} \\ & + \frac{6a^3\tan^5(c+dx)}{5d(a+a\sec(c+dx))^{5/2}} \\ & + \frac{2a^4\tan^7(c+dx)}{7d(a+a\sec(c+dx))^{7/2}}\end{aligned}$$

output

```
2*arctan(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))*a^(1/2)/d-2*a*tan(d*x+
c)/d/(a+a*sec(d*x+c))^(1/2)+2/3*a^2*tan(d*x+c)^3/d/(a+a*sec(d*x+c))^(3/2)+
6/5*a^3*tan(d*x+c)^5/d/(a+a*sec(d*x+c))^(5/2)+2/7*a^4*tan(d*x+c)^7/d/(a+a*
sec(d*x+c))^(7/2)
```

3.142.2 Mathematica [A] (verified)

Time = 6.66 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.69

$$\int \sqrt{a+a\sec(c+dx)}\tan^4(c+dx)\,dx$$
$$= \frac{\sec\left(\frac{1}{2}(c+dx)\right)\sec^3(c+dx)\sqrt{a(1+\sec(c+dx))}\left(105\sqrt{2}\arcsin\left(\sqrt{2}\sin\left(\frac{1}{2}(c+dx)\right)\right)\cos^{\frac{7}{2}}(c+dx)+35\,\mathrm{si}\right.}{105d}$$

input
```
Integrate[Sqrt[a + a*Sec[c + d*x]]*Tan[c + d*x]^4,x]
```

output
```
(Sec[(c + d*x)/2]*Sec[c + d*x]^3*Sqrt[a*(1 + Sec[c + d*x])]*(105*Sqrt[2]*A
rcSin[Sqrt[2]*Sin[(c + d*x)/2]]*Cos[c + d*x]^(7/2) + 35*Sin[(c + d*x)/2] -
 28*Sin[(3*(c + d*x))/2] - 23*Sin[(7*(c + d*x))/2]))/(105*d)
```

3.142.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.93, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3042, 4375, 364, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^4(c+dx)\sqrt{a\sec(c+dx)+a}\,dx$$
$$\downarrow 3042$$
$$\int \cot\left(c+dx+\frac{\pi}{2}\right)^4\sqrt{a\csc\left(c+dx+\frac{\pi}{2}\right)+a}dx$$
$$\downarrow 4375$$
$$-\frac{2a^3\int \frac{\tan^4(c+dx)\left(\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+2\right)^2}{(\sec(c+dx)a+a)^2\left(\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+1\right)}d\left(-\frac{\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)}{d}$$
$$\downarrow 364$$
$$-\frac{2a^3\int\left(\frac{a\tan^6(c+dx)}{(\sec(c+dx)a+a)^3}+\frac{3\tan^4(c+dx)}{(\sec(c+dx)a+a)^2}+\frac{\tan^2(c+dx)}{a(\sec(c+dx)a+a)}+\frac{1}{a^2\left(\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+1\right)}-\frac{1}{a^2}\right)d\left(-\frac{\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)}{d}$$

$$\downarrow 2009$$

$$-\frac{2a^3\left(-\frac{\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{a^{5/2}}+\frac{\tan(c+dx)}{a^2\sqrt{a\sec(c+dx)+a}}-\frac{a\tan^7(c+dx)}{7(a\sec(c+dx)+a)^{7/2}}-\frac{3\tan^5(c+dx)}{5(a\sec(c+dx)+a)^{5/2}}-\frac{\tan^3(c+dx)}{3a(a\sec(c+dx)+a)^{3/2}}\right)}{d}$$

input
```
Int[Sqrt[a + a*Sec[c + d*x]]*Tan[c + d*x]^4,x]
```

output
```
(-2*a^3*(-(ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]]/a^(5/2)
) + Tan[c + d*x]/(a^2*Sqrt[a + a*Sec[c + d*x]]) - Tan[c + d*x]^3/(3*a*(a +
 a*Sec[c + d*x])^(3/2)) - (3*Tan[c + d*x]^5)/(5*(a + a*Sec[c + d*x])^(5/2)
) - (a*Tan[c + d*x]^7)/(7*(a + a*Sec[c + d*x])^(7/2))))/d
```

3.142.3.1 Defintions of rubi rules used

rule 364
```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_))/((c_) + (d_.)*(x_)^2),
x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*((a + b*x^2)^p/(c + d*x^2)), x], x
] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && (In
tegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4375
```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Simp[-2*(a^(m/2 + n + 1/2)/d)   Subst[Int[x^m*((2 + a*x^2
)^(m/2 + n - 1/2)/(1 + a*x^2)), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]
]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && I
ntegerQ[n - 1/2]
```

3.142.4 Maple [A] (verified)

Time = 5.18 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.26

method	result
default	$\frac{2\sqrt{a(1+\sec(dx+c))}\left(105\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,\operatorname{arctanh}\left(\frac{\sin(dx+c)}{(\cos(dx+c)+1)\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}}\right)\cos(dx+c)+105\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,\operatorname{arctanh}\left(\frac{}{(\cos}\right.}{105d(\cos(dx+c)+1)}$

input
```
int((a+a*sec(d*x+c))^(1/2)*tan(d*x+c)^4,x,method=_RETURNVERBOSE)
```

output
```
2/105/d*(a*(1+sec(d*x+c)))^(1/2)/(cos(d*x+c)+1)*(105*(-cos(d*x+c)/(cos(d*x
+c)+1))^(1/2)*arctanh(sin(d*x+c)/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1
))^(1/2))*cos(d*x+c)+105*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctanh(sin(d*
x+c)/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))-92*sin(d*x+c)-46*t
an(d*x+c)+18*sec(d*x+c)*tan(d*x+c)+15*tan(d*x+c)*sec(d*x+c)^2)
```

3.142.5 Fricas [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 331, normalized size of antiderivative = 2.07

$$\int \sqrt{a+a\sec(c+dx)}\tan^4(c+dx)\,dx$$

$$= \left[\frac{105\left(\cos(dx+c)^4+\cos(dx+c)^3\right)\sqrt{-a}\log\left(\frac{2\,a\cos(dx+c)^2-2\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)\sin(dx+c)+a\cos(dx+c)-a}{\cos(dx+c)+1}\right)}{105\left(d\cos(dx+c)^4+d\,c\right.}\right.$$

$$\left.-\frac{2\left(105\left(\cos(dx+c)^4+\cos(dx+c)^3\right)\sqrt{a}\arctan\left(\frac{\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)}{\sqrt{a}\sin(dx+c)}\right)+\left(92\cos(dx+c)^3+46\cos(\right.\right.}{105\left(d\cos(dx+c)^4+d\cos(dx+c)^3\right)}$$

input
```
integrate((a+a*sec(d*x+c))^(1/2)*tan(d*x+c)^4,x, algorithm="fricas")
```

output
```
[1/105*(105*(cos(d*x + c)^4 + cos(d*x + c)^3)*sqrt(-a)*log((2*a*cos(d*x +
c)^2 - 2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin
(d*x + c) + a*cos(d*x + c) - a)/(cos(d*x + c) + 1)) - 2*(92*cos(d*x + c)^3
 + 46*cos(d*x + c)^2 - 18*cos(d*x + c) - 15)*sqrt((a*cos(d*x + c) + a)/cos
(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^4 + d*cos(d*x + c)^3), -2/105*(10
5*(cos(d*x + c)^4 + cos(d*x + c)^3)*sqrt(a)*arctan(sqrt((a*cos(d*x + c) +
a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c))) + (92*cos(d*x + c)^3
 + 46*cos(d*x + c)^2 - 18*cos(d*x + c) - 15)*sqrt((a*cos(d*x + c) + a)/cos
(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^4 + d*cos(d*x + c)^3)]
```

3.142.6 Sympy [F]

$$\int \sqrt{a + a\sec(c + dx)}\tan^4(c + dx)\,dx = \int \sqrt{a\left(\sec\left(c + dx\right) + 1\right)}\tan^4\left(c + dx\right)\,dx$$

input
```
integrate((a+a*sec(d*x+c))**(1/2)*tan(d*x+c)**4,x)
```

output
```
Integral(sqrt(a*(sec(c + d*x) + 1))*tan(c + d*x)**4, x)
```

3.142.7 Maxima [F]

$$\int \sqrt{a + a\sec(c + dx)}\tan^4(c + dx)\,dx = \int \sqrt{a\sec\left(dx + c\right) + a}\tan\left(dx + c\right)^4\,dx$$

input
```
integrate((a+a*sec(d*x+c))^(1/2)*tan(d*x+c)^4,x, algorithm="maxima")
```

output
```
-1/210*(105*((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c)
 + 1)*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c
) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)), (co
s(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(
1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + 1) - (cos(2*d*x + 2
*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*arctan2((cos(2*d*x +
2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan
2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*
x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c
), cos(2*d*x + 2*c) + 1)) - 1) - 2*(d*cos(2*d*x + 2*c)^2 + d*sin(2*d*x + 2
*c)^2 + 2*d*cos(2*d*x + 2*c) + d)*integrate((((cos(10*d*x + 10*c)*cos(2*d*
x + 2*c) + 4*cos(8*d*x + 8*c)*cos(2*d*x + 2*c) + 6*cos(6*d*x + 6*c)*cos(2*
d*x + 2*c) + 4*cos(4*d*x + 4*c)*cos(2*d*x + 2*c) + cos(2*d*x + 2*c)^2 + si
n(10*d*x + 10*c)*sin(2*d*x + 2*c) + 4*sin(8*d*x + 8*c)*sin(2*d*x + 2*c) +
6*sin(6*d*x + 6*c)*sin(2*d*x + 2*c) + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c)
+ sin(2*d*x + 2*c)^2)*cos(9/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))
 + (cos(2*d*x + 2*c)*sin(10*d*x + 10*c) + 4*cos(2*d*x + 2*c)*sin(8*d*x + 8
*c) + 6*cos(2*d*x + 2*c)*sin(6*d*x + 6*c) + 4*cos(2*d*x + 2*c)*sin(4*d*x +
 4*c) - cos(10*d*x + 10*c)*sin(2*d*x + 2*c) - 4*cos(8*d*x + 8*c)*sin(2*d*x
 + 2*c) - 6*cos(6*d*x + 6*c)*sin(2*d*x + 2*c) - 4*cos(4*d*x + 4*c)*sin(...
```

3.142.8 Giac [F]

$$\int \sqrt{a + a\sec(c + dx)}\tan^4(c + dx)\,dx = \int \sqrt{a\sec(dx + c) + a}\tan(dx + c)^4\,dx$$

input
```
integrate((a+a*sec(d*x+c))^(1/2)*tan(d*x+c)^4,x, algorithm="giac")
```

output
```
sage0*x
```

3.142.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{a + a\sec(c + dx)}\tan^4(c + dx)\,dx = \int \tan(c + d\,x)^4 \sqrt{a + \frac{a}{\cos(c + d\,x)}}\,dx$$

input
```
int(tan(c + d*x)^4*(a + a/cos(c + d*x))^(1/2),x)
```

output
```
int(tan(c + d*x)^4*(a + a/cos(c + d*x))^(1/2), x)
```

3.143 $\int \sqrt{a + a\sec(c+dx)}\tan^2(c+dx)\,dx$

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3.143.1 Optimal result

Integrand size = 23, antiderivative size = 96

$$\int \sqrt{a + a\sec(c+dx)}\tan^2(c+dx)\,dx = -\frac{2\sqrt{a}\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d} + \frac{2a\tan(c+dx)}{d\sqrt{a+a\sec(c+dx)}} + \frac{2a^2\tan^3(c+dx)}{3d(a+a\sec(c+dx))^{3/2}}$$

output
```
-2*arctan(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))*a^(1/2)/d+2*a*tan(d*x
+c)/d/(a+a*sec(d*x+c))^(1/2)+2/3*a^2*tan(d*x+c)^3/d/(a+a*sec(d*x+c))^(3/2)
```

3.143.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 3.48 (sec) , antiderivative size = 226, normalized size of antiderivative = 2.35

$$\int \sqrt{a + a\sec(c+dx)}\tan^2(c+dx)\,dx = \frac{8\sqrt{2}\left(\frac{1}{1+\sec(c+dx)}\right)^{7/2}\sqrt{a(1+\sec(c+dx))}\left(-\frac{\cos(c+dx)(7+3\cos(c+dx))\csc^4\left(\frac{1}{2}(c+dx)\right)\sec^2\left(\frac{1}{2}(c+dx)\right)\left(-3\operatorname{arctanh}\left(\sqrt{1-se}\right.\right.\right.}{24\sqrt{1-\sec(c+dx)}}$$

input
```
Integrate[Sqrt[a + a*Sec[c + d*x]]*Tan[c + d*x]^2,x]
```

output
```
(8*Sqrt[2]*((1 + Sec[c + d*x])^(-1))^(7/2)*Sqrt[a*(1 + Sec[c + d*x])]*(-1/
24*(Cos[c + d*x]*(7 + 3*Cos[c + d*x])*Csc[(c + d*x)/2]^4*Sec[(c + d*x)/2]^
2*(-3*ArcTanh[Sqrt[1 - Sec[c + d*x]]]*Cos[c + d*x] + (-1 + 4*Cos[c + d*x])
*Sqrt[1 - Sec[c + d*x]]))/Sqrt[1 - Sec[c + d*x]] - (4*Hypergeometric2F1[2,
 7/2, 9/2, -2*Sec[c + d*x]*Sin[(c + d*x)/2]^2]*Sec[c + d*x]*Tan[(c + d*x)/
2]^2)/7)*Tan[c + d*x]^3)/(3*d*(1 - Tan[(c + d*x)/2]^2)^(5/2))
```

3.143.3 Rubi [A] (verified)

Time $= 0.25$ (sec) , antiderivative size $= 93$, normalized size of antiderivative $= 0.97$, number of steps used $= 6$, number of rules used $= 5$, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used $= \{3042, 4375, 363, 262, 216\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^2(c+dx)\sqrt{a\sec(c+dx)+a}\,dx$$

$$\downarrow 3042$$

$$\int \cot\left(c+dx+\frac{\pi}{2}\right)^2\sqrt{a\csc\left(c+dx+\frac{\pi}{2}\right)+a}dx$$

$$\downarrow 4375$$

$$-\frac{2a^2\int \frac{\tan^2(c+dx)\left(\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+2\right)}{(\sec(c+dx)a+a)\left(\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+1\right)}d\left(-\frac{\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)}{d}$$

$$\downarrow 363$$

$$-\frac{2a^2\left(\int \frac{\tan^2(c+dx)}{(\sec(c+dx)a+a)\left(\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+1\right)}d\left(-\frac{\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)-\frac{\tan^3(c+dx)}{3(a\sec(c+dx)+a)^{3/2}}\right)}{d}$$

$$\downarrow 262$$

$$-\frac{2a^2\left(-\frac{\int \frac{1}{\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+1}d\left(-\frac{\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)}{a}-\frac{\tan^3(c+dx)}{3(a\sec(c+dx)+a)^{3/2}}-\frac{\tan(c+dx)}{a\sqrt{a\sec(c+dx)+a}}\right)}{d}$$

$$\downarrow 216$$

$$-\frac{2a^2\left(\frac{\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{a^{3/2}}-\frac{\tan^3(c+dx)}{3(a\sec(c+dx)+a)^{3/2}}-\frac{\tan(c+dx)}{a\sqrt{a\sec(c+dx)+a}}\right)}{d}$$

input
```
Int[Sqrt[a + a*Sec[c + d*x]]*Tan[c + d*x]^2,x]
```

output
```
(-2*a^2*(ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]]/a^(3/2) -
 Tan[c + d*x]/(a*Sqrt[a + a*Sec[c + d*x]]) - Tan[c + d*x]^3/(3*(a + a*Sec[
c + d*x])^(3/2))))/d
```

3.143.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 363
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(b*e*(m + 2*p + 3))),
 x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(b*(m + 2*p + 3))   Int[(e*x)^
m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a*d
, 0] && NeQ[m + 2*p + 3, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4375
```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Simp[-2*(a^(m/2 + n + 1/2)/d)   Subst[Int[x^m*((2 + a*x^2
)^(m/2 + n - 1/2)/(1 + a*x^2)), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]
]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && I
ntegerQ[n - 1/2]
```

3.143.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 185 vs. $2(84) = 168$.

Time = 4.73 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.94

method	result
default	$-\frac{\left(3\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{2}\,(-\cot(dx+c)+\csc(dx+c))}{\sqrt{(1-\cos(dx+c))^2\csc(dx+c)^2-1}}\right)\left((1-\cos(dx+c))^2\csc(dx+c)^2-1\right)^{\frac{3}{2}}-2(1-\cos(dx+c))^3\csc(dx+c)^3+6\csc(dx+c)\right.$ $3d(-\cot(dx+c)+\csc(dx+c)-1)(\csc(dx+c)-\cot(dx+c)+1)$

input
```
int((a+a*sec(d*x+c))^(1/2)*tan(d*x+c)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/3/d*(3*2^(1/2)*arctanh(2^(1/2)/((1-cos(d*x+c))^2*csc(d*x+c)^2-1)^(1/2)*
(-cot(d*x+c)+csc(d*x+c)))*((1-cos(d*x+c))^2*csc(d*x+c)^2-1)^(3/2)-2*(1-cos
(d*x+c))^3*csc(d*x+c)^3+6*csc(d*x+c)-6*cot(d*x+c))*(-2*a/((1-cos(d*x+c))^2
*csc(d*x+c)^2-1))^(1/2)/(-cot(d*x+c)+csc(d*x+c)-1)/(csc(d*x+c)-cot(d*x+c)+
1)
```

3.143.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 283, normalized size of antiderivative = 2.95

$$\int \sqrt{a + a\sec(c+dx)}\tan^2(c+dx)\,dx$$
$$= \left[\frac{3\left(\cos\left(dx+c\right)^2+\cos\left(dx+c\right)\right)\sqrt{-a}\log\left(\frac{2\,a\cos(dx+c)^2+2\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)\sin(dx+c)+a\cos(dx+c)-a}{\cos(dx+c)+1}\right)+}{3\left(d\cos\left(dx+c\right)^2+d\cos\left(dx+c\right)\right)}\right.$$

input
```
integrate((a+a*sec(d*x+c))^(1/2)*tan(d*x+c)^2,x, algorithm="fricas")
```

output
```
[1/3*(3*(cos(d*x + c)^2 + cos(d*x + c))*sqrt(-a)*log((2*a*cos(d*x + c)^2 +
 2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x +
 c) + a*cos(d*x + c) - a)/(cos(d*x + c) + 1)) + 2*sqrt((a*cos(d*x + c) + a
)/cos(d*x + c))*(2*cos(d*x + c) + 1)*sin(d*x + c))/(d*cos(d*x + c)^2 + d*c
os(d*x + c)), 2/3*(3*(cos(d*x + c)^2 + cos(d*x + c))*sqrt(a)*arctan(sqrt((
a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c))) + s
qrt((a*cos(d*x + c) + a)/cos(d*x + c))*(2*cos(d*x + c) + 1)*sin(d*x + c))/
(d*cos(d*x + c)^2 + d*cos(d*x + c))]
```

3.143.6 Sympy [F]

$$\int \sqrt{a + a\sec(c + dx)}\tan^2(c + dx)\,dx = \int \sqrt{a\left(\sec\left(c + dx\right) + 1\right)}\tan^2\left(c + dx\right)\,dx$$

input
```
integrate((a+a*sec(d*x+c))**(1/2)*tan(d*x+c)**2,x)
```

output
```
Integral(sqrt(a*(sec(c + d*x) + 1))*tan(c + d*x)**2, x)
```

3.143.7 Maxima [F]

$$\int \sqrt{a + a\sec(c + dx)}\tan^2(c + dx)\,dx = \int \sqrt{a\sec\left(dx + c\right) + a}\tan\left(dx + c\right)^2\,dx$$

input
```
integrate((a+a*sec(d*x+c))^(1/2)*tan(d*x+c)^2,x, algorithm="maxima")
```

output
```
-1/6*(3*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)
^(3/4)*(2*d*integrate((((cos(6*d*x + 6*c)*cos(2*d*x + 2*c) + 2*cos(4*d*x +
 4*c)*cos(2*d*x + 2*c) + cos(2*d*x + 2*c)^2 + sin(6*d*x + 6*c)*sin(2*d*x +
 2*c) + 2*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + sin(2*d*x + 2*c)^2)*cos(5/2*
arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + (cos(2*d*x + 2*c)*sin(6*d*x
 + 6*c) + 2*cos(2*d*x + 2*c)*sin(4*d*x + 4*c) - cos(6*d*x + 6*c)*sin(2*d*x
 + 2*c) - 2*cos(4*d*x + 4*c)*sin(2*d*x + 2*c))*sin(5/2*arctan2(sin(2*d*x +
 2*c), cos(2*d*x + 2*c))))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2
*c) + 1)) - ((cos(2*d*x + 2*c)*sin(6*d*x + 6*c) + 2*cos(2*d*x + 2*c)*sin(4
*d*x + 4*c) - cos(6*d*x + 6*c)*sin(2*d*x + 2*c) - 2*cos(4*d*x + 4*c)*sin(2
*d*x + 2*c))*cos(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - (cos(6
*d*x + 6*c)*cos(2*d*x + 2*c) + 2*cos(4*d*x + 4*c)*cos(2*d*x + 2*c) + cos(2
*d*x + 2*c)^2 + sin(6*d*x + 6*c)*sin(2*d*x + 2*c) + 2*sin(4*d*x + 4*c)*sin
(2*d*x + 2*c) + sin(2*d*x + 2*c)^2)*sin(5/2*arctan2(sin(2*d*x + 2*c), cos(
2*d*x + 2*c))))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)))/
(((2*(2*cos(4*d*x + 4*c) + cos(2*d*x + 2*c))*cos(6*d*x + 6*c) + cos(6*d*x
 + 6*c)^2 + 4*cos(4*d*x + 4*c)^2 + 4*cos(4*d*x + 4*c)*cos(2*d*x + 2*c) + co
s(2*d*x + 2*c)^2 + 2*(2*sin(4*d*x + 4*c) + sin(2*d*x + 2*c))*sin(6*d*x + 6
*c) + sin(6*d*x + 6*c)^2 + 4*sin(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2
*d*x + 2*c) + sin(2*d*x + 2*c)^2)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos...
```

3.143.8 Giac [F]

$$\int \sqrt{a + a\sec(c + dx)}\tan^2(c + dx)\,dx = \int \sqrt{a\sec\left(dx + c\right) + a}\tan\left(dx + c\right)^2\,dx$$

input
```
integrate((a+a*sec(d*x+c))^(1/2)*tan(d*x+c)^2,x, algorithm="giac")
```

output
```
sage0*x
```

3.143.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{a + a\sec(c + dx)}\tan^2(c + dx)\,dx = \int \tan(c + d\,x)^2\sqrt{a + \frac{a}{\cos(c + d\,x)}}\,dx$$

input
```
int(tan(c + d*x)^2*(a + a/cos(c + d*x))^(1/2),x)
```

output
```
int(tan(c + d*x)^2*(a + a/cos(c + d*x))^(1/2), x)
```

3.144 $\int \cot^2(c+dx)\sqrt{a+a\sec(c+dx)}\,dx$

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3.144.1 Optimal result

Integrand size = 23, antiderivative size = 109

$$\int \cot^2(c+dx)\sqrt{a+a\sec(c+dx)}\,dx = -\frac{2\sqrt{a}\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d} + \frac{\sqrt{a}\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{\sqrt{2}d} - \frac{\cot(c+dx)\sqrt{a+a\sec(c+dx)}}{d}$$

output
```
-2*arctan(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))*a^(1/2)/d+1/2*arctan(
1/2*a^(1/2)*tan(d*x+c)*2^(1/2)/(a+a*sec(d*x+c))^(1/2))*a^(1/2)/d*2^(1/2)-c
ot(d*x+c)*(a+a*sec(d*x+c))^(1/2)/d
```

3.144.2 Mathematica [A] (verified)

Time = 3.34 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.14

$$\int \cot^2(c+dx)\sqrt{a+a\sec(c+dx)}\,dx = \frac{\sqrt{a(1+\sec(c+dx))}\left(-\frac{4\arctan\left(\frac{\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{\frac{1}{1+\sec(c+dx)}}}\right)\sqrt{\frac{\cos(c+dx)}{(1+\cos(c+dx))^2}}}{\sqrt{\frac{1}{1+\cos(c+dx)}}} - 2\cot(c+dx) + \sqrt{2}\arcsin\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)\right.}{2d}$$

input
```
Integrate[Cot[c + d*x]^2*Sqrt[a + a*Sec[c + d*x]],x]
```

output
```
(Sqrt[a*(1 + Sec[c + d*x])]*((-4*ArcTan[Tan[(c + d*x)/2]/Sqrt[(1 + Sec[c +
 d*x])^(-1)]]*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])^2])/Sqrt[(1 + Cos[c + d
*x])^(-1)] - 2*Cot[c + d*x] + Sqrt[2]*ArcSin[Tan[(c + d*x)/2]]*Sqrt[(1 + S
ec[c + d*x])^(-1)]))/(2*d)
```

3.144.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.04, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {3042, 4375, 382, 25, 27, 397, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^2(c+dx)\sqrt{a\sec(c+dx)+a}\,dx$$

$$\downarrow 3042$$

$$\int \frac{\sqrt{a\csc\left(c+dx+\frac{\pi}{2}\right)+a}}{\cot\left(c+dx+\frac{\pi}{2}\right)^2}dx$$

$$\downarrow 4375$$

$$-\frac{2\int \frac{\cot^2(c+dx)(\sec(c+dx)a+a)}{\left(\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+1\right)\left(\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+2\right)}d\left(-\frac{\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)}{d}$$

$$\downarrow 382$$

$$-\frac{2\left(\frac{1}{2}\int -\frac{a\left(\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+3\right)}{\left(\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+1\right)\left(\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+2\right)}d\left(-\frac{\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)+\frac{1}{2}\cot(c+dx)\sqrt{a\sec(c+dx)+a}\right)}{d}$$

$$\downarrow 25$$

$$-\frac{2\left(\frac{1}{2}\cot(c+dx)\sqrt{a\sec(c+dx)+a}-\frac{1}{2}\int \frac{a\left(\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+3\right)}{\left(\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+1\right)\left(\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+2\right)}d\left(-\frac{\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)\right)}{d}$$

$$\downarrow 27$$

$$-\frac{2\left(\frac{1}{2}\cot(c+dx)\sqrt{a\sec(c+dx)+a}-\frac{1}{2}a\int\frac{\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+3}{\left(\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+1\right)\left(\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+2\right)}d\left(-\frac{\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)\right)}{d}$$

$$\downarrow 397$$

$$-\frac{2\left(\frac{1}{2}\cot(c+dx)\sqrt{a\sec(c+dx)+a}-\frac{1}{2}a\left(2\int\frac{1}{\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+1}d\left(-\frac{\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)-\int\frac{1}{\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+2}d\left(-\frac{\tan(c+dx)}{\sqrt{\sec(c+dx)}}\right.\right.\right.}{d}$$

$$\downarrow 216$$

$$-\frac{2\left(\frac{1}{2}\cot(c+dx)\sqrt{a\sec(c+dx)+a}-\frac{1}{2}a\left(\frac{\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{2}\sqrt{a}}-\frac{2\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}}\right)\right)}{d}$$

input
```
Int[Cot[c + d*x]^2*Sqrt[a + a*Sec[c + d*x]],x]
```

output
```
(-2*(-1/2*(a*((-2*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])
/Sqrt[a] + ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]]
)]/(Sqrt[2]*Sqrt[a]))) + (Cot[c + d*x]*Sqrt[a + a*Sec[c + d*x]])/2))/d
```

3.144.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 382
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)
, x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/
(a*c*e*(m + 1))), x] - Simp[1/(a*c*e^2*(m + 1))   Int[(e*x)^(m + 2)*(a + b*
x^2)^p*(c + d*x^2)^q*Simp[(b*c + a*d)*(m + 3) + 2*(b*c*p + a*d*q) + b*d*(m
+ 2*p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[
b*c - a*d, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, d, e, m, 2, p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4375
```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Simp[-2*(a^(m/2 + n + 1/2)/d)   Subst[Int[x^m*((2 + a*x^2
)^(m/2 + n - 1/2)/(1 + a*x^2)), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]
]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && I
ntegerQ[n - 1/2]
```

3.144.4 Maple [A] (verified)

Time = 2.81 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.48

method	result
default	$\frac{\sqrt{a(1+\sec(dx+c))}\left(\ln\left(\csc(dx+c)-\cot(dx+c)+\sqrt{\cot(dx+c)^2-2\cot(dx+c)\csc(dx+c)+\csc(dx+c)^2-1}\right)\sqrt{2}\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}-4\sqrt{-}}{2d}$

input
```
int(cot(d*x+c)^2*(a+a*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2/d*(a*(1+sec(d*x+c)))^(1/2)*(ln(csc(d*x+c)-cot(d*x+c)+(cot(d*x+c)^2-2*c
ot(d*x+c)*csc(d*x+c)+csc(d*x+c)^2-1)^(1/2))*2^(1/2)*(-cos(d*x+c)/(cos(d*x+
c)+1))^(1/2)-4*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctanh(sin(d*x+c)/(cos(
d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))-2*cot(d*x+c))
```

3.144.5 Fricas [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 422, normalized size of antiderivative = 3.87

$$\int \cot^2(c+dx)\sqrt{a+a\sec(c+dx)}\,dx$$

$$= \left[\frac{\sqrt{2}\sqrt{-a}\log\left(-\frac{2\sqrt{2}\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)\sin(dx+c)-3\,a\cos(dx+c)^2-2\,a\cos(dx+c)+a}{\cos(dx+c)^2+2\cos(dx+c)+1}\right)\sin(dx+c)+2\sqrt{-a}\log\right.$$

$$\left.-\frac{\sqrt{2}\sqrt{a}\arctan\left(\frac{\sqrt{2}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)}{\sqrt{a}\sin(dx+c)}\right)\sin(dx+c)+2\sqrt{a}\arctan\left(\frac{2\sqrt{a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)\sin(dx+c)}{2\,a\cos(dx+c)^2+a\cos(dx+c)-a}\right)}{2\,d\sin(dx+c)}\right.$$

input
```
integrate(cot(d*x+c)^2*(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
[1/4*(sqrt(2)*sqrt(-a)*log(-(2*sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/
cos(d*x + c))*cos(d*x + c)*sin(d*x + c) - 3*a*cos(d*x + c)^2 - 2*a*cos(d*x
 + c) + a)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1))*sin(d*x + c) + 2*sqrt(-a
)*log(-(8*a*cos(d*x + c)^3 + 4*(2*cos(d*x + c)^2 - cos(d*x + c))*sqrt(-a)*
sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c) - 7*a*cos(d*x + c) +
a)/(cos(d*x + c) + 1))*sin(d*x + c) - 4*sqrt((a*cos(d*x + c) + a)/cos(d*x
+ c))*cos(d*x + c))/(d*sin(d*x + c)), -1/2*(sqrt(2)*sqrt(a)*arctan(sqrt(2)
*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c
)))*sin(d*x + c) + 2*sqrt(a)*arctan(2*sqrt(a)*sqrt((a*cos(d*x + c) + a)/co
s(d*x + c))*cos(d*x + c)*sin(d*x + c)/(2*a*cos(d*x + c)^2 + a*cos(d*x + c)
 - a))*sin(d*x + c) + 2*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x +
c))/(d*sin(d*x + c))]
```

3.144.6 Sympy [F]

$$\int \cot^2(c+dx)\sqrt{a+a\sec(c+dx)}\,dx = \int \sqrt{a\left(\sec\left(c+dx\right)+1\right)}\cot^2\left(c+dx\right)dx$$

input
```
integrate(cot(d*x+c)**2*(a+a*sec(d*x+c))**(1/2),x)
```

output
```
Integral(sqrt(a*(sec(c + d*x) + 1))*cot(c + d*x)**2, x)
```

3.144.7 Maxima [F]

$$\int \cot^2(c+dx)\sqrt{a+a\sec(c+dx)}\,dx = \int \sqrt{a\sec(dx+c)+a}\cot(dx+c)^2\,dx$$

input
```
integrate(cot(d*x+c)^2*(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(a*sec(d*x + c) + a)*cot(d*x + c)^2, x)
```

3.144.8 Giac [F]

$$\int \cot^2(c+dx)\sqrt{a+a\sec(c+dx)}\,dx = \int \sqrt{a\sec(dx+c)+a}\cot(dx+c)^2\,dx$$

input
```
integrate(cot(d*x+c)^2*(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.144.9 Mupad [F(-1)]

Timed out.

$$\int \cot^2(c+dx)\sqrt{a+a\sec(c+dx)}\,dx = \int \cot(c+d\,x)^2\sqrt{a+\frac{a}{\cos(c+d\,x)}}\,dx$$

input
```
int(cot(c + d*x)^2*(a + a/cos(c + d*x))^(1/2),x)
```

output
```
int(cot(c + d*x)^2*(a + a/cos(c + d*x))^(1/2), x)
```

3.145 $\int \cot^4(c+dx)\sqrt{a+a\sec(c+dx)}\,dx$

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3.145.1 Optimal result

Integrand size = 23, antiderivative size = 196

$$\begin{aligned}
&\int \cot^4(c+dx)\sqrt{a+a\sec(c+dx)}\,dx \\
&= \frac{2\sqrt{a}\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d} - \frac{9\sqrt{a}\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{8\sqrt{2}d} \\
&\quad + \frac{7\cot(c+dx)\sqrt{a+a\sec(c+dx)}}{8d} + \frac{\cot^3(c+dx)(a+a\sec(c+dx))^{3/2}}{12ad} \\
&\quad - \frac{\cos(c+dx)\cot^3(c+dx)\sec^2\left(\frac{1}{2}(c+dx)\right)(a+a\sec(c+dx))^{3/2}}{4ad}
\end{aligned}$$

output

```
1/12*cot(d*x+c)^3*(a+a*sec(d*x+c))^(3/2)/a/d-1/4*cos(d*x+c)*cot(d*x+c)^3*s
ec(1/2*d*x+1/2*c)^2*(a+a*sec(d*x+c))^(3/2)/a/d+2*arctan(a^(1/2)*tan(d*x+c)
/(a+a*sec(d*x+c))^(1/2))*a^(1/2)/d-9/16*arctan(1/2*a^(1/2)*tan(d*x+c)*2^(1
/2)/(a+a*sec(d*x+c))^(1/2))*a^(1/2)/d*2^(1/2)+7/8*cot(d*x+c)*(a+a*sec(d*x+
c))^(1/2)/d
```

3.145.2 Mathematica [A] (warning: unable to verify)

Time = 7.21 (sec) , antiderivative size = 232, normalized size of antiderivative = 1.18

$$\int \cot^4(c+dx)\sqrt{a+a\sec(c+dx)}\,dx$$

$$= \frac{\left(-9\arcsin\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)+16\sqrt{2}\arctan\left(\frac{\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{\frac{\cos(c+dx)}{1+\cos(c+dx)}}}\right)\right)\sqrt{\frac{\cos(c+dx)}{1+\cos(c+dx)}}\sqrt{\sec^2\left(\frac{1}{2}(c+dx)\right)}\sqrt{1+\sec(c}}{16d\sqrt{\sec(c+dx)}}$$

$$+\frac{\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{a(1+\sec(c+dx))}\left(\frac{2}{3}\csc\left(\frac{1}{2}(c+dx)\right)-\frac{1}{24}\csc^3\left(\frac{1}{2}(c+dx)\right)-\frac{31}{24}\sin\left(\frac{1}{2}(c+dx)\right)+\frac{1}{16}\right.}{d}$$

input
```
Integrate[Cot[c + d*x]^4*Sqrt[a + a*Sec[c + d*x]],x]
```

output
```
((-9*ArcSin[Tan[(c + d*x)/2]] + 16*Sqrt[2]*ArcTan[Tan[(c + d*x)/2]/Sqrt[Co
s[c + d*x]/(1 + Cos[c + d*x])]])*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqr
t[Sec[(c + d*x)/2]^2]*Sqrt[1 + Sec[c + d*x]]*Sqrt[a*(1 + Sec[c + d*x])])/(
16*d*Sqrt[Sec[c + d*x]]) + (Sec[(c + d*x)/2]*Sqrt[a*(1 + Sec[c + d*x])]*((
2*Csc[(c + d*x)/2])/3 - Csc[(c + d*x)/2]^3/24 - (31*Sin[(c + d*x)/2])/24 +
 (Sec[(c + d*x)/2]*Tan[(c + d*x)/2])/16))/d
```

3.145.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.05, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.478$, Rules used = {3042, 4375, 374, 25, 27, 445, 27, 445, 27, 397, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^4(c+dx)\sqrt{a\sec(c+dx)+a}\,dx$$

$$\downarrow 3042$$

$$\int \frac{\sqrt{a\csc\left(c+dx+\frac{\pi}{2}\right)+a}}{\cot\left(c+dx+\frac{\pi}{2}\right)^4}dx$$

$$\downarrow 4375$$

$$-\frac{2\int \frac{\cot^4(c+dx)(\sec(c+dx)a+a)^2}{\left(\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+1\right)\left(\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+2\right)^2} d\left(-\frac{\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)}{ad}$$

$$\downarrow 374$$

$$-\frac{2\left(\frac{\int -\frac{a\cot^4(c+dx)(\sec(c+dx)a+a)^2\left(\frac{5a\tan^2(c+dx)}{\sec(c+dx)a+a}+1\right)}{\left(\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+1\right)\left(\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+2\right)} d\left(-\frac{\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)}{4a} + \frac{\cot^3(c+dx)(a\sec(c+dx)+a)^{3/2}}{4\left(\frac{a\tan^2(c+dx)}{a\sec(c+dx)+a}+2\right)}\right)}{ad}$$

$$\downarrow 25$$

$$-\frac{2\left(\frac{\cot^3(c+dx)(a\sec(c+dx)+a)^{3/2}}{4\left(\frac{a\tan^2(c+dx)}{a\sec(c+dx)+a}+2\right)} - \frac{\int \frac{a\cot^4(c+dx)(\sec(c+dx)a+a)^2\left(\frac{5a\tan^2(c+dx)}{\sec(c+dx)a+a}+1\right)}{\left(\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+1\right)\left(\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+2\right)} d\left(-\frac{\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)}{4a}\right)}{ad}$$

$$\downarrow 27$$

$$-\frac{2\left(\frac{\cot^3(c+dx)(a\sec(c+dx)+a)^{3/2}}{4\left(\frac{a\tan^2(c+dx)}{a\sec(c+dx)+a}+2\right)} - \frac{1}{4}\int \frac{\cot^4(c+dx)(\sec(c+dx)a+a)^2\left(\frac{5a\tan^2(c+dx)}{\sec(c+dx)a+a}+1\right)}{\left(\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+1\right)\left(\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+2\right)} d\left(-\frac{\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)\right)}{ad}$$

$$\downarrow 445$$

$$-\frac{2\left(\frac{1}{4}\left(\frac{1}{6}\int -\frac{3a\cot^2(c+dx)(\sec(c+dx)a+a)\left(7-\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}\right)}{\left(\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+1\right)\left(\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+2\right)} d\left(-\frac{\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right) - \frac{1}{6}\cot^3(c+dx)(a\sec(c+dx)+a)^{3/2}\right)\right)}{ad}$$

$$\downarrow 27$$

$$-\frac{2\left(\frac{1}{4}\left(-\frac{1}{2}a\int \frac{\cot^2(c+dx)(\sec(c+dx)a+a)\left(7-\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}\right)}{\left(\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+1\right)\left(\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+2\right)} d\left(-\frac{\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right) - \frac{1}{6}\cot^3(c+dx)(a\sec(c+dx)+a)^{3/2}\right)\right)}{ad}$$

$$\downarrow 445$$

$$-\frac{2\left(\frac{1}{4}\left(-\frac{1}{2}a\left(\frac{7}{2}\cot(c+dx)\sqrt{a\sec(c+dx)+a} - \frac{1}{2}\int \frac{a\left(\frac{7a\tan^2(c+dx)}{\sec(c+dx)a+a}+23\right)}{\left(\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+1\right)\left(\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+2\right)} d\left(-\frac{\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)\right) - \frac{1}{6}\text{co}\right)\right)}{ad}$$

$$\downarrow 27$$

$$-\frac{2\left(\frac{1}{4}\left(-\frac{1}{2}a\left(\frac{7}{2}\cot(c+dx)\sqrt{a\sec(c+dx)+a}-\frac{1}{2}a\int\frac{\frac{7a\tan^2(c+dx)}{\sec(c+dx)a+a}+23}{\left(\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+1\right)\left(\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+2\right)}d\left(-\frac{\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)\right)-\frac{1}{6}\text{co}\right.\right.}{ad}$$

$$\downarrow 397$$

$$-\frac{2\left(\frac{1}{4}\left(-\frac{1}{2}a\left(\frac{7}{2}\cot(c+dx)\sqrt{a\sec(c+dx)+a}-\frac{1}{2}a\left(16\int\frac{1}{\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+1}d\left(-\frac{\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)-9\int\frac{1}{\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+2}d\right.\right.\right.\right.}{ad}$$

$$\downarrow 216$$

$$-\frac{2\left(\frac{1}{4}\left(-\frac{1}{2}a\left(\frac{7}{2}\cot(c+dx)\sqrt{a\sec(c+dx)+a}-\frac{1}{2}a\left(\frac{9\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{2}\sqrt{a}}-\frac{16\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}}\right)\right)-\frac{1}{6}\right.\right.}{ad}$$

input
```
Int[Cot[c + d*x]^4*Sqrt[a + a*Sec[c + d*x]],x]
```

output
```
(-2*((-1/6*(Cot[c + d*x]^3*(a + a*Sec[c + d*x])^(3/2)) - (a*(-1/2*(a*((-16
*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/Sqrt[a] + (9*Arc
Tan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])])/(Sqrt[2]*S
qrt[a]))) + (7*Cot[c + d*x]*Sqrt[a + a*Sec[c + d*x]])/2))/2)/4 + (Cot[c +
d*x]^3*(a + a*Sec[c + d*x])^(3/2))/(4*(2 + (a*Tan[c + d*x]^2)/(a + a*Sec[c
 + d*x])))))/(a*d)
```

3.145.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 374
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q
 + 1)/(a*e*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(e*x)^m*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[b*c*(m + 1) + 2*(b*c -
a*d)*(p + 1) + d*b*(m + 2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b,
c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IntBinomialQ[a, b,
 c, d, e, m, 2, p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 445
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^2*(m + 1))
 Int[(g*x)^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*f*c*(m + 1) - e*(b*c
+ a*d)*(m + 2 + 1) - e*2*(b*c*p + a*d*q) - b*e*d*(m + 2*(p + q + 2) + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && LtQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4375
```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Simp[-2*(a^(m/2 + n + 1/2)/d)   Subst[Int[x^m*((2 + a*x^2
)^(m/2 + n - 1/2)/(1 + a*x^2)), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]
]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && I
ntegerQ[n - 1/2]
```

3.145.4 Maple [A] (verified)

Time = 2.78 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.00

method	result
default	$\frac{\sqrt{a(1+\sec(dx+c))}\left(-27\ln\left(\csc(dx+c)-\cot(dx+c)+\sqrt{\cot(dx+c)^2-2\cot(dx+c)\csc(dx+c)+\csc(dx+c)^2-1}\right)\sqrt{2}\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}+\cdots}{48}$

input
```
int(cot(d*x+c)^4*(a+a*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/48/d*(a*(1+sec(d*x+c)))^(1/2)*(-27*ln(csc(d*x+c)-cot(d*x+c)+(cot(d*x+c)^
2-2*cot(d*x+c)*csc(d*x+c)+csc(d*x+c)^2-1)^(1/2))*2^(1/2)*(-cos(d*x+c)/(cos
(d*x+c)+1))^(1/2)+96*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctanh(sin(d*x+c)
/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))-62*cot(d*x+c)^3+4*cot(
d*x+c)^2*csc(d*x+c)+42*cot(d*x+c)*csc(d*x+c)^2)
```

3.145.5 Fricas [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 547, normalized size of antiderivative = 2.79

$$\int \cot^4(c+dx)\sqrt{a+a\sec(c+dx)}\,dx$$
$$= \left[\frac{27\left(\sqrt{2}\cos\left(dx+c\right)^2-\sqrt{2}\right)\sqrt{-a}\log\left(\frac{2\sqrt{2}\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)\sin(dx+c)+3\,a\cos(dx+c)^2+2\,a\cos(dx+c)-a}{\cos(dx+c)^2+2\,\cos(dx+c)+1}\right)\sin\right.$$

input
```
integrate(cot(d*x+c)^4*(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
[1/96*(27*(sqrt(2)*cos(d*x + c)^2 - sqrt(2))*sqrt(-a)*log((2*sqrt(2)*sqrt(
-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x + c) + 3*
a*cos(d*x + c)^2 + 2*a*cos(d*x + c) - a)/(cos(d*x + c)^2 + 2*cos(d*x + c)
+ 1))*sin(d*x + c) + 48*(cos(d*x + c)^2 - 1)*sqrt(-a)*log(-(8*a*cos(d*x +
c)^3 - 4*(2*cos(d*x + c)^2 - cos(d*x + c))*sqrt(-a)*sqrt((a*cos(d*x + c) +
 a)/cos(d*x + c))*sin(d*x + c) - 7*a*cos(d*x + c) + a)/(cos(d*x + c) + 1))
*sin(d*x + c) + 4*(31*cos(d*x + c)^3 - 2*cos(d*x + c)^2 - 21*cos(d*x + c))
*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)))/((d*cos(d*x + c)^2 - d)*sin(d*x
+ c)), 1/48*(48*(cos(d*x + c)^2 - 1)*sqrt(a)*arctan(2*sqrt(a)*sqrt((a*cos(
d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x + c)/(2*a*cos(d*x + c)^2
+ a*cos(d*x + c) - a))*sin(d*x + c) + 27*(sqrt(2)*cos(d*x + c)^2 - sqrt(2)
)*sqrt(a)*arctan(sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x +
 c)/(sqrt(a)*sin(d*x + c)))*sin(d*x + c) + 2*(31*cos(d*x + c)^3 - 2*cos(d*
x + c)^2 - 21*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)))/((d*c
os(d*x + c)^2 - d)*sin(d*x + c))]
```

3.145.6 Sympy [F]

$$\int \cot^4(c+dx)\sqrt{a+a\sec(c+dx)}\,dx = \int \sqrt{a\left(\sec\left(c+dx\right)+1\right)}\cot^4\left(c+dx\right)dx$$

input
```
integrate(cot(d*x+c)**4*(a+a*sec(d*x+c))**(1/2),x)
```

output
```
Integral(sqrt(a*(sec(c + d*x) + 1))*cot(c + d*x)**4, x)
```

3.145.7 Maxima [F]

$$\int \cot^4(c+dx)\sqrt{a+a\sec(c+dx)}\,dx = \int \sqrt{a\sec\left(dx+c\right)+a}\cot\left(dx+c\right)^4\,dx$$

input
```
integrate(cot(d*x+c)^4*(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(a*sec(d*x + c) + a)*cot(d*x + c)^4, x)
```

3.145.8 Giac [F]

$$\int \cot^4(c+dx)\sqrt{a+a\sec(c+dx)}\,dx = \int \sqrt{a\sec\left(dx+c\right)+a}\cot\left(dx+c\right)^4\,dx$$

input
```
integrate(cot(d*x+c)^4*(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.145.9 Mupad [F(-1)]

Timed out.

$$\int \cot^4(c+dx)\sqrt{a+a\sec(c+dx)}\,dx = \int \cot(c+d\,x)^4\sqrt{a+\frac{a}{\cos(c+d\,x)}}\,dx$$

input
```
int(cot(c + d*x)^4*(a + a/cos(c + d*x))^(1/2),x)
```

output
```
int(cot(c + d*x)^4*(a + a/cos(c + d*x))^(1/2), x)
```

3.146 $\int \cot^6(c+dx)\sqrt{a+a\sec(c+dx)}\,dx$

3.146.1 Optimal result 1104
3.146.2 Mathematica [A] (verified) 1105
3.146.3 Rubi [A] (verified) 1105
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3.146.1 Optimal result

Integrand size = 23, antiderivative size = 280

$$\begin{aligned}
&\int \cot^6(c+dx)\sqrt{a+a\sec(c+dx)}\,dx \\
&= -\frac{2\sqrt{a}\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d} + \frac{151\sqrt{a}\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{128\sqrt{2}d} \\
&\quad - \frac{105\cot(c+dx)\sqrt{a+a\sec(c+dx)}}{128d} - \frac{23\cot^3(c+dx)(a+a\sec(c+dx))^{3/2}}{192ad} \\
&\quad + \frac{87\cot^5(c+dx)(a+a\sec(c+dx))^{5/2}}{160a^2d} \\
&\quad - \frac{17\cos(c+dx)\cot^5(c+dx)\sec^2\left(\frac{1}{2}(c+dx)\right)(a+a\sec(c+dx))^{5/2}}{32a^2d} \\
&\quad - \frac{\cos^2(c+dx)\cot^5(c+dx)\sec^4\left(\frac{1}{2}(c+dx)\right)(a+a\sec(c+dx))^{5/2}}{16a^2d}
\end{aligned}$$

output

```
-23/192*cot(d*x+c)^3*(a+a*sec(d*x+c))^(3/2)/a/d+87/160*cot(d*x+c)^5*(a+a*s
ec(d*x+c))^(5/2)/a^2/d-17/32*cos(d*x+c)*cot(d*x+c)^5*sec(1/2*d*x+1/2*c)^2*
(a+a*sec(d*x+c))^(5/2)/a^2/d-1/16*cos(d*x+c)^2*cot(d*x+c)^5*sec(1/2*d*x+1/
2*c)^4*(a+a*sec(d*x+c))^(5/2)/a^2/d-2*arctan(a^(1/2)*tan(d*x+c)/(a+a*sec(d
*x+c))^(1/2))*a^(1/2)/d+151/256*arctan(1/2*a^(1/2)*tan(d*x+c)*2^(1/2)/(a+a
*sec(d*x+c))^(1/2))*a^(1/2)/d*2^(1/2)-105/128*cot(d*x+c)*(a+a*sec(d*x+c))^
(1/2)/d
```

3.146.2 Mathematica [A] (verified)

Time = 4.66 (sec) , antiderivative size = 210, normalized size of antiderivative = 0.75

$$\int \cot^6(c+dx)\sqrt{a+a\sec(c+dx)}\,dx$$

$$= \frac{\sqrt{a(1+\sec(c+dx))}\left(-\frac{(5207+172\cos(c+dx)-4572\cos(2(c+dx))-556\cos(3(c+dx))+2821\cos(4(c+dx)))\csc^5(c+dx)}{4\sqrt{\sec(c+dx)}} - 7680\arctan\right.}{}$$

input
```
Integrate[Cot[c + d*x]^6*Sqrt[a + a*Sec[c + d*x]],x]
```

output
```
(Sqrt[a*(1 + Sec[c + d*x])]*(-1/4*((5207 + 172*Cos[c + d*x] - 4572*Cos[2*(
c + d*x)] - 556*Cos[3*(c + d*x)] + 2821*Cos[4*(c + d*x)])*Csc[c + d*x]^5)/
Sqrt[Sec[c + d*x]] - 7680*ArcTan[Tan[(c + d*x)/2]/Sqrt[(1 + Sec[c + d*x])^
(-1)]]*Sqrt[Sec[c + d*x]/(1 + Sec[c + d*x])^2]*Sqrt[1 + Sec[c + d*x]] + 22
65*ArcSin[Tan[(c + d*x)/2]]*Sqrt[Sec[(c + d*x)/2]^2]*Sqrt[(1 + Sec[c + d*x
])^(-1)]*Sqrt[1 + Sec[c + d*x]]))/(3840*d*Sqrt[Sec[c + d*x]])
```

3.146.3 Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 295, normalized size of antiderivative = 1.05, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.652$, Rules used = {3042, 4375, 374, 25, 27, 441, 27, 445, 27, 445, 27, 445, 27, 397, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^6(c+dx)\sqrt{a\sec(c+dx)+a}\,dx$$

$$\downarrow 3042$$

$$\int \frac{\sqrt{a\csc\left(c+dx+\frac{\pi}{2}\right)+a}}{\cot\left(c+dx+\frac{\pi}{2}\right)^6}dx$$

$$\downarrow 4375$$

$$-\frac{2\int \frac{\cot^6(c+dx)(\sec(c+dx)a+a)^3}{\left(\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+1\right)\left(\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+2\right)^3}d\left(-\frac{\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)}{a^2d}$$

$$\downarrow 374$$

$$-\frac{2\left(\frac{\int -\frac{a\cot^6(c+dx)(\sec(c+dx)a+a)^3\left(\frac{9a\tan^2(c+dx)}{\sec(c+dx)a+a}+1\right)}{\left(\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+1\right)\left(\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+2\right)^2}d\left(-\frac{\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)}{8a}+\frac{\cot^5(c+dx)(a\sec(c+dx)+a)^{5/2}}{8\left(\frac{a\tan^2(c+dx)}{a\sec(c+dx)+a}+2\right)^2}\right)}{a^2d}$$

$$\downarrow 25$$

$$-\frac{2\left(\frac{\cot^5(c+dx)(a\sec(c+dx)+a)^{5/2}}{8\left(\frac{a\tan^2(c+dx)}{a\sec(c+dx)+a}+2\right)^2}-\frac{\int\frac{a\cot^6(c+dx)(\sec(c+dx)a+a)^3\left(\frac{9a\tan^2(c+dx)}{\sec(c+dx)a+a}+1\right)}{\left(\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+1\right)\left(\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+2\right)^2}d\left(-\frac{\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)}{8a}\right)}{a^2d}$$

$$\downarrow 27$$

$$-\frac{2\left(\frac{\cot^5(c+dx)(a\sec(c+dx)+a)^{5/2}}{8\left(\frac{a\tan^2(c+dx)}{a\sec(c+dx)+a}+2\right)^2}-\frac{1}{8}\int\frac{\cot^6(c+dx)(\sec(c+dx)a+a)^3\left(\frac{9a\tan^2(c+dx)}{\sec(c+dx)a+a}+1\right)}{\left(\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+1\right)\left(\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+2\right)^2}d\left(-\frac{\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)\right)}{a^2d}$$

$$\downarrow 441$$

$$-\frac{2\left(\frac{1}{8}\left(\frac{17\cot^5(c+dx)(a\sec(c+dx)+a)^{5/2}}{4\left(\frac{a\tan^2(c+dx)}{a\sec(c+dx)+a}+2\right)}-\frac{\int\frac{a\cot^6(c+dx)(\sec(c+dx)a+a)^3\left(\frac{119a\tan^2(c+dx)}{\sec(c+dx)a+a}+87\right)}{\left(\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+1\right)\left(\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+2\right)}d\left(-\frac{\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)}{4a}\right)+\frac{\cot^5(c+dx)(a\sec(c+dx)+a)^{5/2}}{8\left(\frac{a\tan^2(c+dx)}{a\sec(c+dx)+a}+2\right)^2}\right)}{a^2d}$$

$$\downarrow 27$$

$$-\frac{2\left(\frac{1}{8}\left(\frac{17\cot^5(c+dx)(a\sec(c+dx)+a)^{5/2}}{4\left(\frac{a\tan^2(c+dx)}{a\sec(c+dx)+a}+2\right)}-\frac{1}{4}\int\frac{\cot^6(c+dx)(\sec(c+dx)a+a)^3\left(\frac{119a\tan^2(c+dx)}{\sec(c+dx)a+a}+87\right)}{\left(\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+1\right)\left(\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+2\right)}d\left(-\frac{\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)\right)+\frac{\cot^5(c+dx)(a\sec(c+dx)+a)^{5/2}}{8\left(\frac{a\tan^2(c+dx)}{a\sec(c+dx)+a}+2\right)^2}\right)}{a^2d}$$

$$\downarrow 445$$

$$-\frac{2\left(\frac{1}{8}\left(\frac{1}{4}\left(\frac{1}{10}\int\frac{5a\cot^4(c+dx)(\sec(c+dx)a+a)^2\left(\frac{87a\tan^2(c+dx)}{\sec(c+dx)a+a}+23\right)}{\left(\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+1\right)\left(\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+2\right)}d\left(-\frac{\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)-\frac{87}{10}\cot^5(c+dx)(a\sec(c+dx)+\right.\right.\right.\right.}{a^2d}$$

$$\downarrow 27$$

$$-\frac{2\left(\frac{1}{8}\left(\frac{1}{4}\left(\frac{1}{2}a\int\frac{\cot^4(c+dx)(\sec(c+dx)a+a)^2\left(\frac{87a\tan^2(c+dx)}{\sec(c+dx)a+a}+23\right)}{\left(\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+1\right)\left(\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+2\right)}d\left(-\frac{\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)-\frac{87}{10}\cot^5(c+dx)(a\sec(c+dx)+a)\right.\right.\right.}{a^2d}$$

$$\downarrow 445$$

$$-\frac{2\left(\frac{1}{8}\left(\frac{1}{4}\left(\frac{1}{2}a\left(\frac{23}{6}\cot^3(c+dx)(a\sec(c+dx)+a)^{3/2}-\frac{1}{6}\int-\frac{3a\cot^2(c+dx)(\sec(c+dx)a+a)\left(105-\frac{23a\tan^2(c+dx)}{\sec(c+dx)a+a}\right)}{\left(\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+1\right)\left(\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+2\right)}d\left(-\frac{\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)\right.\right.\right.}{a^2d}$$

$$\downarrow 27$$

$$-\frac{2\left(\frac{1}{8}\left(\frac{1}{4}\left(\frac{1}{2}a\left(\frac{1}{2}a\int\frac{\cot^2(c+dx)(\sec(c+dx)a+a)\left(105-\frac{23a\tan^2(c+dx)}{\sec(c+dx)a+a}\right)}{\left(\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+1\right)\left(\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+2\right)}d\left(-\frac{\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)+\frac{23}{6}\cot^3(c+dx)(a\sec(c+dx)\right.\right.\right.\right.}{a^2d}$$

$$\downarrow 445$$

$$-\frac{2\left(\frac{1}{8}\left(\frac{1}{4}\left(\frac{1}{2}a\left(\frac{1}{2}a\left(\frac{105}{2}\cot(c+dx)\sqrt{a\sec(c+dx)+a}-\frac{1}{2}\int\frac{a\left(\frac{105a\tan^2(c+dx)}{\sec(c+dx)a+a}+361\right)}{\left(\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+1\right)\left(\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+2\right)}d\left(-\frac{\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)\right.\right.\right.\right.}{a^2d}$$

$$\downarrow 27$$

$$-\frac{2\left(\frac{1}{8}\left(\frac{1}{4}\left(\frac{1}{2}a\left(\frac{1}{2}a\left(\frac{105}{2}\cot(c+dx)\sqrt{a\sec(c+dx)+a}-\frac{1}{2}a\int\frac{\frac{105a\tan^2(c+dx)}{\sec(c+dx)a+a}+361}{\left(\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+1\right)\left(\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+2\right)}d\left(-\frac{\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)\right.\right.\right.\right.}{a^2d}$$

$$\downarrow 397$$

$$-\frac{2\left(\frac{1}{8}\left(\frac{1}{4}\left(\frac{1}{2}a\left(\frac{1}{2}a\left(\frac{105}{2}\cot(c+dx)\sqrt{a\sec(c+dx)+a}-\frac{1}{2}a\left(256\int\frac{1}{\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+1}d\left(-\frac{\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)-151\int\right.\right.\right.\right.\right.}{a^2d}$$

$$\downarrow 216$$

$$-\frac{2\left(\frac{1}{8}\left(\frac{1}{4}\left(\frac{1}{2}a\left(\frac{1}{2}a\left(\frac{105}{2}\cot(c+dx)\sqrt{a\sec(c+dx)+a}-\frac{1}{2}a\left(\frac{151\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{2}\sqrt{a}}-\frac{256\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right.\right.\right.\right.\right.\right.}{a^2d}$$

input
```
Int[Cot[c + d*x]^6*Sqrt[a + a*Sec[c + d*x]],x]
```

output
```
(-2*((Cot[c + d*x]^5*(a + a*Sec[c + d*x])^(5/2))/(8*(2 + (a*Tan[c + d*x]^2
)/(a + a*Sec[c + d*x]))^2) + (((-87*Cot[c + d*x]^5*(a + a*Sec[c + d*x])^(5
/2))/10 + (a*((23*Cot[c + d*x]^3*(a + a*Sec[c + d*x])^(3/2))/6 + (a*(-1/2*
(a*((-256*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/Sqrt[a]
 + (151*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])])
/(Sqrt[2]*Sqrt[a]))) + (105*Cot[c + d*x]*Sqrt[a + a*Sec[c + d*x]])/2))/2))
/2)/4 + (17*Cot[c + d*x]^5*(a + a*Sec[c + d*x])^(5/2))/(4*(2 + (a*Tan[c +
d*x]^2)/(a + a*Sec[c + d*x])))))/8))/(a^2*d)
```

3.146.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 374
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q
 + 1)/(a*e*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(e*x)^m*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[b*c*(m + 1) + 2*(b*c -
a*d)*(p + 1) + d*b*(m + 2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b,
c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IntBinomialQ[a, b,
 c, d, e, m, 2, p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 441
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*(g*x)^(m + 1)*(a
+ b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(a*g*2*(b*c - a*d)*(p + 1))), x] + Si
mp[1/(a*2*(b*c - a*d)*(p + 1))   Int[(g*x)^m*(a + b*x^2)^(p + 1)*(c + d*x^2
)^q*Simp[c*(b*e - a*f)*(m + 1) + e*2*(b*c - a*d)*(p + 1) + d*(b*e - a*f)*(m
 + 2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m, q},
 x] && LtQ[p, -1]
```

rule 445
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^2*(m + 1))
 Int[(g*x)^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*f*c*(m + 1) - e*(b*c
+ a*d)*(m + 2 + 1) - e*2*(b*c*p + a*d*q) - b*e*d*(m + 2*(p + q + 2) + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && LtQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4375
```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Simp[-2*(a^(m/2 + n + 1/2)/d)   Subst[Int[x^m*((2 + a*x^2
)^(m/2 + n - 1/2)/(1 + a*x^2)), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]
]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && I
ntegerQ[n - 1/2]
```

3.146.4 Maple [A] (verified)

Time = 2.65 (sec) , antiderivative size = 226, normalized size of antiderivative = 0.81

method	result
default	$\frac{\left(2265\sin(dx+c)^5\sqrt{2}\ln\left(\csc(dx+c)-\cot(dx+c)+\sqrt{\cot(dx+c)^2-2\cot(dx+c)\csc(dx+c)+\csc(dx+c)^2-1}\right)\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}-7680\sin$

input
```
int(cot(d*x+c)^6*(a+a*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3840/d*(2265*sin(d*x+c)^5*2^(1/2)*ln(csc(d*x+c)-cot(d*x+c)+(cot(d*x+c)^2
-2*cot(d*x+c)*csc(d*x+c)+csc(d*x+c)^2-1)^(1/2))*(-cos(d*x+c)/(cos(d*x+c)+1
))^(1/2)-7680*sin(d*x+c)^5*arctanh(sin(d*x+c)/(cos(d*x+c)+1)/(-cos(d*x+c)/
(cos(d*x+c)+1))^(1/2))*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)-5642*cos(d*x+c)^
5+556*cos(d*x+c)^4+7928*cos(d*x+c)^3-460*cos(d*x+c)^2-3150*cos(d*x+c))*(a*
(1+sec(d*x+c)))^(1/2)*csc(d*x+c)^5
```

3.146.5 Fricas [A] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 646, normalized size of antiderivative = 2.31

$$\int \cot^6(c+dx)\sqrt{a+a\sec(c+dx)}\,dx$$

$$= \left[\frac{2265\left(\sqrt{2}\cos(dx+c)^4 - 2\sqrt{2}\cos(dx+c)^2 + \sqrt{2}\right)\sqrt{-a}\log\left(-\frac{2\sqrt{2}\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)\sin(dx+c) - 3a\cos}{\cos(dx+c)^2+2\cos(dx+c)+1}\right.\right.$$

$$\left. \left. -\frac{3840\left(\cos(dx+c)^4 - 2\cos(dx+c)^2 + 1\right)\sqrt{a}\arctan\left(\frac{2\sqrt{a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)\sin(dx+c)}{2a\cos(dx+c)^2+a\cos(dx+c)-a}\right)\sin(dx+c) + \right. \right.$$

input
```
integrate(cot(d*x+c)^6*(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
[1/7680*(2265*(sqrt(2)*cos(d*x + c)^4 - 2*sqrt(2)*cos(d*x + c)^2 + sqrt(2)
)*sqrt(-a)*log(-(2*sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)
)*cos(d*x + c)*sin(d*x + c) - 3*a*cos(d*x + c)^2 - 2*a*cos(d*x + c) + a)/(
cos(d*x + c)^2 + 2*cos(d*x + c) + 1))*sin(d*x + c) + 3840*(cos(d*x + c)^4
- 2*cos(d*x + c)^2 + 1)*sqrt(-a)*log(-(8*a*cos(d*x + c)^3 + 4*(2*cos(d*x +
 c)^2 - cos(d*x + c))*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin
(d*x + c) - 7*a*cos(d*x + c) + a)/(cos(d*x + c) + 1))*sin(d*x + c) - 4*(28
21*cos(d*x + c)^5 - 278*cos(d*x + c)^4 - 3964*cos(d*x + c)^3 + 230*cos(d*x
 + c)^2 + 1575*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)))/((d*
cos(d*x + c)^4 - 2*d*cos(d*x + c)^2 + d)*sin(d*x + c)), -1/3840*(3840*(cos
(d*x + c)^4 - 2*cos(d*x + c)^2 + 1)*sqrt(a)*arctan(2*sqrt(a)*sqrt((a*cos(d
*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x + c)/(2*a*cos(d*x + c)^2 +
 a*cos(d*x + c) - a))*sin(d*x + c) + 2265*(sqrt(2)*cos(d*x + c)^4 - 2*sqrt
(2)*cos(d*x + c)^2 + sqrt(2))*sqrt(a)*arctan(sqrt(2)*sqrt((a*cos(d*x + c)
+ a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c)))*sin(d*x + c) + 2*(
2821*cos(d*x + c)^5 - 278*cos(d*x + c)^4 - 3964*cos(d*x + c)^3 + 230*cos(d
*x + c)^2 + 1575*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)))/((
d*cos(d*x + c)^4 - 2*d*cos(d*x + c)^2 + d)*sin(d*x + c))]
```

3.146.6 Sympy [F]

$$\int \cot^6(c+dx)\sqrt{a+a\sec(c+dx)}\,dx = \int \sqrt{a\left(\sec\left(c+dx\right)+1\right)}\cot^6\left(c+dx\right)dx$$

input
```
integrate(cot(d*x+c)**6*(a+a*sec(d*x+c))**(1/2),x)
```

output
```
Integral(sqrt(a*(sec(c + d*x) + 1))*cot(c + d*x)**6, x)
```

3.146.7 Maxima [F(-1)]

Timed out.

$$\int \cot^6(c+dx)\sqrt{a+a\sec(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate(cot(d*x+c)^6*(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
Timed out
```

3.146.8 Giac [F]

$$\int \cot^6(c+dx)\sqrt{a+a\sec(c+dx)}\,dx = \int \sqrt{a\sec(dx+c)+a}\cot(dx+c)^6\,dx$$

input
```
integrate(cot(d*x+c)^6*(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.146.9 Mupad [F(-1)]

Timed out.

$$\int \cot^6(c+dx)\sqrt{a+a\sec(c+dx)}\,dx = \int \cot(c+d\,x)^6\sqrt{a+\frac{a}{\cos(c+d\,x)}}\,dx$$

input
```
int(cot(c + d*x)^6*(a + a/cos(c + d*x))^(1/2),x)
```

output
```
int(cot(c + d*x)^6*(a + a/cos(c + d*x))^(1/2), x)
```

3.147 $\int (a + a\sec(c+dx))^{3/2}\tan^5(c+dx)\,dx$

3.147.1 Optimal result . . . 1113
3.147.2 Mathematica [A] (verified) . . . 1113
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3.147.9 Mupad [F(-1)] . . . 1118

3.147.1 Optimal result

Integrand size = 23, antiderivative size = 169

$$\int (a + a\sec(c+dx))^{3/2}\tan^5(c+dx)\,dx = -\frac{2a^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{a}}\right)}{d} + \frac{2a\sqrt{a+a\sec(c+dx)}}{d} + \frac{2(a+a\sec(c+dx))^{3/2}}{3d} + \frac{2(a+a\sec(c+dx))^{5/2}}{5ad} + \frac{2(a+a\sec(c+dx))^{7/2}}{7a^2d} - \frac{2(a+a\sec(c+dx))^{9/2}}{3a^3d} + \frac{2(a+a\sec(c+dx))^{11/2}}{11a^4d}$$

output

```
-2*a^(3/2)*arctanh((a+a*sec(d*x+c))^(1/2)/a^(1/2))/d+2/3*(a+a*sec(d*x+c))^
(3/2)/d+2/5*(a+a*sec(d*x+c))^(5/2)/a/d+2/7*(a+a*sec(d*x+c))^(7/2)/a^2/d-2/
3*(a+a*sec(d*x+c))^(9/2)/a^3/d+2/11*(a+a*sec(d*x+c))^(11/2)/a^4/d+2*a*(a+a
*sec(d*x+c))^(1/2)/d
```

3.147.2 Mathematica [A] (verified)

Time = 0.43 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.66

$$\int (a + a\sec(c+dx))^{3/2}\tan^5(c+dx)\,dx = \frac{2(a(1+\sec(c+dx)))^{3/2}\left(-1155\operatorname{arctanh}\left(\sqrt{1+\sec(c+dx)}\right) + \sqrt{1+\sec(c+dx)}(1656+327\sec(c+dx)\right.}{1155d(1+\sec(c+dx))}$$

input
```
Integrate[(a + a*Sec[c + d*x])^(3/2)*Tan[c + d*x]^5,x]
```

output
```
(2*(a*(1 + Sec[c + d*x]))^(3/2)*(-1155*ArcTanh[Sqrt[1 + Sec[c + d*x]]] + S
qrt[1 + Sec[c + d*x]]*(1656 + 327*Sec[c + d*x] - 534*Sec[c + d*x]^2 - 325*
Sec[c + d*x]^3 + 140*Sec[c + d*x]^4 + 105*Sec[c + d*x]^5)))/(1155*d*(1 + S
ec[c + d*x])^(3/2))
```

3.147.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.92, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3042, 25, 4368, 27, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^5(c+dx)(a\sec(c+dx)+a)^{3/2}\,dx$$

$$\downarrow 3042$$

$$\int -\cot\left(c+dx+\frac{\pi}{2}\right)^5\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^{3/2}dx$$

$$\downarrow 25$$

$$-\int \cot\left(\frac{1}{2}(2c+\pi)+dx\right)^5\left(\csc\left(\frac{1}{2}(2c+\pi)+dx\right)a+a\right)^{3/2}dx$$

$$\downarrow 4368$$

$$\frac{\int a^2\cos(c+dx)(1-\sec(c+dx))^2(\sec(c+dx)a+a)^{7/2}d\sec(c+dx)}{a^4d}$$

$$\downarrow 27$$

$$\frac{\int \cos(c+dx)(1-\sec(c+dx))^2(\sec(c+dx)a+a)^{7/2}d\sec(c+dx)}{a^2d}$$

$$\downarrow 99$$

$$\frac{\int\left(\frac{(\sec(c+dx)a+a)^{9/2}}{a}+\cos(c+dx)(\sec(c+dx)a+a)^{7/2}-3(\sec(c+dx)a+a)^{7/2}\right)d\sec(c+dx)}{a^2d}$$

$$\downarrow 2009$$

$$\frac{-2a^{7/2}\operatorname{arctanh}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{a}}\right)+2a^3\sqrt{a\sec(c+dx)+a}+\frac{2(a\sec(c+dx)+a)^{11/2}}{11a^2}+\frac{2}{3}a^2(a\sec(c+dx)+a)^{3/2}-\frac{2(a\sec}{a^2d}$$

input
```
Int[(a + a*Sec[c + d*x])^(3/2)*Tan[c + d*x]^5,x]
```

output
```
(-2*a^(7/2)*ArcTanh[Sqrt[a + a*Sec[c + d*x]]/Sqrt[a]] + 2*a^3*Sqrt[a + a*S
ec[c + d*x]] + (2*a^2*(a + a*Sec[c + d*x])^(3/2))/3 + (2*a*(a + a*Sec[c +
d*x])^(5/2))/5 + (2*(a + a*Sec[c + d*x])^(7/2))/7 - (2*(a + a*Sec[c + d*x]
)^(9/2))/(3*a) + (2*(a + a*Sec[c + d*x])^(11/2))/(11*a^2))/(a^2*d)
```

3.147.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4368
```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Simp[-(d*b^(m - 1))^(-1)   Subst[Int[(-a + b*x)^((m - 1)/2
)*((a + b*x)^((m - 1)/2 + n)/x), x], x, Csc[c + d*x]], x] /; FreeQ[{a, b, c
, d, n}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[n]
```

3.147.4 Maple [A] (verified)

Time = 4.93 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.66

method	result
default	$\frac{2a\sqrt{a(1+\sec(dx+c))}\left(1155\arctan\left(\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}\right)\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}+1656+327\sec(dx+c)-534\sec(dx+c)^2-325\sec(dx+c)^3+140\right.}{1155d}$

input
```
int((a+a*sec(d*x+c))^(3/2)*tan(d*x+c)^5,x,method=_RETURNVERBOSE)
```

output
```
2/1155/d*a*(a*(1+sec(d*x+c)))^(1/2)*(1155*arctan((-cos(d*x+c)/(cos(d*x+c)+
1))^(1/2))*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+1656+327*sec(d*x+c)-534*sec(
d*x+c)^2-325*sec(d*x+c)^3+140*sec(d*x+c)^4+105*sec(d*x+c)^5)
```

3.147.5 Fricas [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 334, normalized size of antiderivative = 1.98

$$\int (a + a\sec(c + dx))^{3/2}\tan^5(c + dx)\,dx = \left[\frac{1155\,a^{\frac{3}{2}}\cos(dx+c)^5\log\left(-8\,a\cos(dx+c)^2+4\left(2\cos(dx+c)^2+\cos(dx+c)\right)\sqrt{a}\sqrt{\frac{a\cos(dx+c)}{\cos(dx+c}\right.\right.$$

input
```
integrate((a+a*sec(d*x+c))^(3/2)*tan(d*x+c)^5,x, algorithm="fricas")
```

output
```
[1/2310*(1155*a^(3/2)*cos(d*x + c)^5*log(-8*a*cos(d*x + c)^2 + 4*(2*cos(d*
x + c)^2 + cos(d*x + c))*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)) -
 8*a*cos(d*x + c) - a) + 4*(1656*a*cos(d*x + c)^5 + 327*a*cos(d*x + c)^4 -
 534*a*cos(d*x + c)^3 - 325*a*cos(d*x + c)^2 + 140*a*cos(d*x + c) + 105*a)
*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)))/(d*cos(d*x + c)^5), 1/1155*(1155
*sqrt(-a)*a*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(
d*x + c)/(2*a*cos(d*x + c) + a))*cos(d*x + c)^5 + 2*(1656*a*cos(d*x + c)^5
 + 327*a*cos(d*x + c)^4 - 534*a*cos(d*x + c)^3 - 325*a*cos(d*x + c)^2 + 14
0*a*cos(d*x + c) + 105*a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)))/(d*cos(
d*x + c)^5)]
```

3.147.6 Sympy [F]

$$\int (a + a\sec(c+dx))^{3/2}\tan^5(c+dx)\,dx = \int (a(\sec(c+dx)+1))^{\frac{3}{2}}\tan^5(c+dx)\,dx$$

input
```
integrate((a+a*sec(d*x+c))**(3/2)*tan(d*x+c)**5,x)
```

output
```
Integral((a*(sec(c + d*x) + 1))**(3/2)*tan(c + d*x)**5, x)
```

3.147.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 162, normalized size of antiderivative = 0.96

$$\int (a + a\sec(c+dx))^{3/2}\tan^5(c+dx)\,dx = \frac{1155\,a^{\frac{3}{2}}\log\left(\frac{\sqrt{a+\frac{a}{\cos(dx+c)}}-\sqrt{a}}{\sqrt{a+\frac{a}{\cos(dx+c)}}+\sqrt{a}}\right) + 770\left(a+\frac{a}{\cos(dx+c)}\right)^{\frac{3}{2}} + \frac{210\left(a+\frac{a}{\cos(dx+c)}\right)^{\frac{11}{2}}}{a^4} - \frac{770\left(a+\frac{a}{\cos(dx+c)}\right)^{\frac{9}{2}}}{a^3} + \frac{33}{1155\,d}$$

input
```
integrate((a+a*sec(d*x+c))^(3/2)*tan(d*x+c)^5,x, algorithm="maxima")
```

output
```
1/1155*(1155*a^(3/2)*log((sqrt(a + a/cos(d*x + c)) - sqrt(a))/(sqrt(a + a/
cos(d*x + c)) + sqrt(a))) + 770*(a + a/cos(d*x + c))^(3/2) + 210*(a + a/co
s(d*x + c))^(11/2)/a^4 - 770*(a + a/cos(d*x + c))^(9/2)/a^3 + 330*(a + a/c
os(d*x + c))^(7/2)/a^2 + 462*(a + a/cos(d*x + c))^(5/2)/a + 2310*sqrt(a +
a/cos(d*x + c))*a)/d
```

3.147.8 Giac [F]

$$\int (a + a\sec(c+dx))^{3/2}\tan^5(c+dx)\,dx = \int (a\sec(dx+c)+a)^{\frac{3}{2}}\tan(dx+c)^5\,dx$$

input
```
integrate((a+a*sec(d*x+c))^(3/2)*tan(d*x+c)^5,x, algorithm="giac")
```

output
```
sage0*x
```

3.147.9 Mupad [F(-1)]

Timed out.

$$\int (a + a\sec(c + dx))^{3/2} \tan^5(c + dx)\, dx = \int \tan(c + d\,x)^5 \left(a + \frac{a}{\cos(c + d\,x)}\right)^{3/2} dx$$

input
```
int(tan(c + d*x)^5*(a + a/cos(c + d*x))^(3/2),x)
```

output
```
int(tan(c + d*x)^5*(a + a/cos(c + d*x))^(3/2), x)
```

3.148 $\int (a + a\sec(c+dx))^{3/2}\tan^3(c+dx)\,dx$

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3.148.1 Optimal result

Integrand size = 23, antiderivative size = 121

$$\int (a + a\sec(c+dx))^{3/2}\tan^3(c+dx)\,dx = \frac{2a^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{a}}\right)}{d} - \frac{2a\sqrt{a+a\sec(c+dx)}}{d} - \frac{2(a+a\sec(c+dx))^{3/2}}{3d} - \frac{2(a+a\sec(c+dx))^{5/2}}{5ad} + \frac{2(a+a\sec(c+dx))^{7/2}}{7a^2d}$$

output
```
2*a^(3/2)*arctanh((a+a*sec(d*x+c))^(1/2)/a^(1/2))/d-2/3*(a+a*sec(d*x+c))^(
3/2)/d-2/5*(a+a*sec(d*x+c))^(5/2)/a/d+2/7*(a+a*sec(d*x+c))^(7/2)/a^2/d-2*a
*(a+a*sec(d*x+c))^(1/2)/d
```

3.148.2 Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.76

$$\int (a + a\sec(c+dx))^{3/2}\tan^3(c+dx)\,dx = \frac{2(a(1+\sec(c+dx)))^{3/2}\left(105\operatorname{arctanh}\left(\sqrt{1+\sec(c+dx)}\right) + \sqrt{1+\sec(c+dx)}(-146 - 32\sec(c+dx)\right.}{105d(1+\sec(c+dx))^{3/2}}$$

input
```
Integrate[(a + a*Sec[c + d*x])^(3/2)*Tan[c + d*x]^3,x]
```

output
```
(2*(a*(1 + Sec[c + d*x]))^(3/2)*(105*ArcTanh[Sqrt[1 + Sec[c + d*x]]] + Sqr
t[1 + Sec[c + d*x]]*(-146 - 32*Sec[c + d*x] + 24*Sec[c + d*x]^2 + 15*Sec[c
 + d*x]^3)))/(105*d*(1 + Sec[c + d*x])^(3/2))
```

3.148.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.96, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.478$, Rules used = {3042, 25, 4368, 25, 27, 90, 60, 60, 60, 73, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^3(c+dx)(a\sec(c+dx)+a)^{3/2}\,dx$$

$$\downarrow 3042$$

$$\int -\cot\left(c+dx+\frac{\pi}{2}\right)^3\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^{3/2}dx$$

$$\downarrow 25$$

$$-\int \cot\left(\frac{1}{2}(2c+\pi)+dx\right)^3\left(\csc\left(\frac{1}{2}(2c+\pi)+dx\right)a+a\right)^{3/2}dx$$

$$\downarrow 4368$$

$$\frac{\int -a\cos(c+dx)(1-\sec(c+dx))(\sec(c+dx)a+a)^{5/2}d\sec(c+dx)}{a^2d}$$

$$\downarrow 25$$

$$-\frac{\int a\cos(c+dx)(1-\sec(c+dx))(\sec(c+dx)a+a)^{5/2}d\sec(c+dx)}{a^2d}$$

$$\downarrow 27$$

$$-\frac{\int \cos(c+dx)(1-\sec(c+dx))(\sec(c+dx)a+a)^{5/2}d\sec(c+dx)}{ad}$$

$$\downarrow 90$$

$$-\frac{\int \cos(c+dx)(\sec(c+dx)a+a)^{5/2}d\sec(c+dx)-\frac{2(a\sec(c+dx)+a)^{7/2}}{7a}}{ad}$$

$$\downarrow 60$$

$$-\frac{a\int \cos(c+dx)(\sec(c+dx)a+a)^{3/2}d\sec(c+dx)-\frac{2(a\sec(c+dx)+a)^{7/2}}{7a}+\frac{2}{5}(a\sec(c+dx)+a)^{5/2}}{ad}$$

$$\downarrow 60$$

$$-\frac{a\left(a\int \cos(c+dx)\sqrt{\sec(c+dx)a+a}d\sec(c+dx)+\frac{2}{3}(a\sec(c+dx)+a)^{3/2}\right)-\frac{2(a\sec(c+dx)+a)^{7/2}}{7a}+\frac{2}{5}(a\sec(c+$$
$$ad$$

$$\downarrow 60$$

$$-\frac{a\left(a\left(a\int \frac{\cos(c+dx)}{\sqrt{\sec(c+dx)a+a}}d\sec(c+dx)+2\sqrt{a\sec(c+dx)+a}\right)+\frac{2}{3}(a\sec(c+dx)+a)^{3/2}\right)-\frac{2(a\sec(c+dx)+a)^{7/2}}{7a}+\frac{2}{5}$$
$$ad$$

$$\downarrow 73$$

$$-\frac{a\left(a\left(2\int \frac{1}{\frac{\sec(c+dx)a+a}{a}-1}d\sqrt{\sec(c+dx)a+a}+2\sqrt{a\sec(c+dx)+a}\right)+\frac{2}{3}(a\sec(c+dx)+a)^{3/2}\right)-\frac{2(a\sec(c+dx)+}{7a}$$
$$ad$$

$$\downarrow 220$$

$$-\frac{a\left(a\left(2\sqrt{a\sec(c+dx)+a}-2\sqrt{a}\operatorname{arctanh}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{a}}\right)\right)+\frac{2}{3}(a\sec(c+dx)+a)^{3/2}\right)-\frac{2(a\sec(c+dx)+a)^{7/2}}{7a}+\frac{2}{5}($$
$$ad$$

input
```
Int[(a + a*Sec[c + d*x])^(3/2)*Tan[c + d*x]^3,x]
```

output
```
-(((2*(a + a*Sec[c + d*x])^(5/2))/5 - (2*(a + a*Sec[c + d*x])^(7/2))/(7*a)
 + a*((2*(a + a*Sec[c + d*x])^(3/2))/3 + a*(-2*Sqrt[a]*ArcTanh[Sqrt[a + a*
Sec[c + d*x]]/Sqrt[a]] + 2*Sqrt[a + a*Sec[c + d*x]])))/(a*d))
```

3.148.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4368
```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Simp[-(d*b^(m - 1))^(-1)   Subst[Int[(-a + b*x)^((m - 1)/2
)*((a + b*x)^((m - 1)/2 + n)/x), x], x, Csc[c + d*x]], x] /; FreeQ[{a, b, c
, d, n}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[n]
```

3.148.4 Maple [A] (verified)

Time = 4.26 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.76

method	result	siz
default	$-\frac{2a\sqrt{a(1+\sec(dx+c))}\left(105\arctan\left(\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}\right)\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}+146+32\sec(dx+c)-24\sec(dx+c)^2-15\sec(dx+c)^3\right)}{105d}$	92

input
```
int((a+a*sec(d*x+c))^(3/2)*tan(d*x+c)^3,x,method=_RETURNVERBOSE)
```

output
```
-2/105/d*a*(a*(1+sec(d*x+c)))^(1/2)*(105*arctan((-cos(d*x+c)/(cos(d*x+c)+1
))^(1/2))*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+146+32*sec(d*x+c)-24*sec(d*x+
c)^2-15*sec(d*x+c)^3)
```

3.148.5 Fricas [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 290, normalized size of antiderivative = 2.40

$$\int (a + a\sec(c + dx))^{3/2}\tan^3(c + dx)\,dx = \left[\frac{105\,a^{\frac{3}{2}}\cos(dx+c)^3\log\left(-8\,a\cos(dx+c)^2 - 4\left(2\cos(dx+c)^2+\cos(dx+c)\right)\sqrt{a}\sqrt{\frac{a\cos(dx+c)+}{\cos(dx+c)}}\right.\right.$$

$$\left.\left.-\frac{105\sqrt{-a}a\arctan\left(\frac{2\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)}{2\,a\cos(dx+c)+a}\right)\cos(dx+c)^3 + 2\left(146\,a\cos(dx+c)^3 + 32\,a\cos(dx+c)^2 -\right.}{105\,d\cos(dx+c)^3}\right.$$

input
```
integrate((a+a*sec(d*x+c))^(3/2)*tan(d*x+c)^3,x, algorithm="fricas")
```

output
```
[1/210*(105*a^(3/2)*cos(d*x + c)^3*log(-8*a*cos(d*x + c)^2 - 4*(2*cos(d*x
+ c)^2 + cos(d*x + c))*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)) - 8
*a*cos(d*x + c) - a) - 4*(146*a*cos(d*x + c)^3 + 32*a*cos(d*x + c)^2 - 24*
a*cos(d*x + c) - 15*a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)))/(d*cos(d*x
 + c)^3), -1/105*(105*sqrt(-a)*a*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) +
a)/cos(d*x + c))*cos(d*x + c)/(2*a*cos(d*x + c) + a))*cos(d*x + c)^3 + 2*(
146*a*cos(d*x + c)^3 + 32*a*cos(d*x + c)^2 - 24*a*cos(d*x + c) - 15*a)*sqr
t((a*cos(d*x + c) + a)/cos(d*x + c)))/(d*cos(d*x + c)^3)]
```

3.148.6 Sympy [F]

$$\int (a + a\sec(c+dx))^{3/2}\tan^3(c+dx)\,dx = \int (a(\sec(c+dx)+1))^{\frac{3}{2}}\tan^3(c+dx)\,dx$$

input
```
integrate((a+a*sec(d*x+c))**(3/2)*tan(d*x+c)**3,x)
```

output
```
Integral((a*(sec(c + d*x) + 1))**(3/2)*tan(c + d*x)**3, x)
```

3.148.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.02

$$\int (a + a\sec(c+dx))^{3/2}\tan^3(c+dx)\,dx =$$
$$-\frac{105\,a^{\frac{3}{2}}\log\left(\frac{\sqrt{a+\frac{a}{\cos(dx+c)}}-\sqrt{a}}{\sqrt{a+\frac{a}{\cos(dx+c)}}+\sqrt{a}}\right)+70\left(a+\frac{a}{\cos(dx+c)}\right)^{\frac{3}{2}}-\frac{30\left(a+\frac{a}{\cos(dx+c)}\right)^{\frac{7}{2}}}{a^2}+\frac{42\left(a+\frac{a}{\cos(dx+c)}\right)^{\frac{5}{2}}}{a}+210\sqrt{a+\frac{a}{\cos(dx}}}{105\,d}$$

input
```
integrate((a+a*sec(d*x+c))^(3/2)*tan(d*x+c)^3,x, algorithm="maxima")
```

output
```
-1/105*(105*a^(3/2)*log((sqrt(a + a/cos(d*x + c)) - sqrt(a))/(sqrt(a + a/c
os(d*x + c)) + sqrt(a))) + 70*(a + a/cos(d*x + c))^(3/2) - 30*(a + a/cos(d
*x + c))^(7/2)/a^2 + 42*(a + a/cos(d*x + c))^(5/2)/a + 210*sqrt(a + a/cos(
d*x + c))*a)/d
```

3.148.8 Giac [F]

$$\int (a + a\sec(c+dx))^{3/2}\tan^3(c+dx)\,dx = \int (a\sec(dx+c)+a)^{\frac{3}{2}}\tan(dx+c)^3\,dx$$

input
```
integrate((a+a*sec(d*x+c))^(3/2)*tan(d*x+c)^3,x, algorithm="giac")
```

output
```
sage0*x
```

3.148.9 Mupad [F(-1)]

Timed out.

$$\int (a + a\sec(c + dx))^{3/2} \tan^3(c + dx)\, dx = \int \tan(c + d\,x)^3 \left(a + \frac{a}{\cos(c + d\,x)}\right)^{3/2} dx$$

input
```
int(tan(c + d*x)^3*(a + a/cos(c + d*x))^(3/2),x)
```

output
```
int(tan(c + d*x)^3*(a + a/cos(c + d*x))^(3/2), x)
```

3.149 $\int (a + a\sec(c+dx))^{3/2}\tan(c+dx)\,dx$

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3.149.1 Optimal result

Integrand size = 21, antiderivative size = 73

$$\int (a + a\sec(c+dx))^{3/2}\tan(c+dx)\,dx = -\frac{2a^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{a}}\right)}{d} + \frac{2a\sqrt{a+a\sec(c+dx)}}{d} + \frac{2(a+a\sec(c+dx))^{3/2}}{3d}$$

output
```
-2*a^(3/2)*arctanh((a+a*sec(d*x+c))^(1/2)/a^(1/2))/d+2/3*(a+a*sec(d*x+c))^
(3/2)/d+2*a*(a+a*sec(d*x+c))^(1/2)/d
```

3.149.2 Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.96

$$\int (a + a\sec(c+dx))^{3/2}\tan(c+dx)\,dx = \frac{2(a(1+\sec(c+dx)))^{3/2}\left(-3\operatorname{arctanh}\left(\sqrt{1+\sec(c+dx)}\right) + \sqrt{1+\sec(c+dx)}(4+\sec(c+dx))\right)}{3d(1+\sec(c+dx))^{3/2}}$$

input
```
Integrate[(a + a*Sec[c + d*x])^(3/2)*Tan[c + d*x],x]
```

output
```
(2*(a*(1 + Sec[c + d*x]))^(3/2)*(-3*ArcTanh[Sqrt[1 + Sec[c + d*x]]] + Sqrt
[1 + Sec[c + d*x]]*(4 + Sec[c + d*x])))/(3*d*(1 + Sec[c + d*x])^(3/2))
```

3.149.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.96, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 25, 4368, 60, 60, 73, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan(c+dx)(a\sec(c+dx)+a)^{3/2}\,dx$$

$$\downarrow 3042$$

$$\int -\cot\left(c+dx+\frac{\pi}{2}\right)\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^{3/2}dx$$

$$\downarrow 25$$

$$-\int \cot\left(\frac{1}{2}(2c+\pi)+dx\right)\left(\csc\left(\frac{1}{2}(2c+\pi)+dx\right)a+a\right)^{3/2}dx$$

$$\downarrow 4368$$

$$\frac{\int \cos(c+dx)(\sec(c+dx)a+a)^{3/2}d\sec(c+dx)}{d}$$

$$\downarrow 60$$

$$\frac{a\int \cos(c+dx)\sqrt{\sec(c+dx)a+a}d\sec(c+dx)+\frac{2}{3}(a\sec(c+dx)+a)^{3/2}}{d}$$

$$\downarrow 60$$

$$\frac{a\left(a\int \frac{\cos(c+dx)}{\sqrt{\sec(c+dx)a+a}}d\sec(c+dx)+2\sqrt{a\sec(c+dx)+a}\right)+\frac{2}{3}(a\sec(c+dx)+a)^{3/2}}{d}$$

$$\downarrow 73$$

$$\frac{a\left(2\int \frac{1}{\frac{\sec(c+dx)a+a}{a}-1}d\sqrt{\sec(c+dx)a+a}+2\sqrt{a\sec(c+dx)+a}\right)+\frac{2}{3}(a\sec(c+dx)+a)^{3/2}}{d}$$

$$\downarrow 220$$

$$\frac{a\left(2\sqrt{a\sec(c+dx)+a}-2\sqrt{a}\operatorname{arctanh}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{a}}\right)\right)+\frac{2}{3}(a\sec(c+dx)+a)^{3/2}}{d}$$

input
```
Int[(a + a*Sec[c + d*x])^(3/2)*Tan[c + d*x],x]
```

output
```
((2*(a + a*Sec[c + d*x])^(3/2))/3 + a*(-2*Sqrt[a]*ArcTanh[Sqrt[a + a*Sec[c
 + d*x]]/Sqrt[a]] + 2*Sqrt[a + a*Sec[c + d*x]]))/d
```

3.149.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4368
```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Simp[-(d*b^(m - 1))^(-1)   Subst[Int[(-a + b*x)^((m - 1)/2
)*((a + b*x)^((m - 1)/2 + n)/x), x], x, Csc[c + d*x]], x] /; FreeQ[{a, b, c
, d, n}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[n]
```

3.149.4 Maple [A] (verified)

Time = 0.75 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.78

method	result	size
derivativedivides	$\frac{\frac{2(a+a\sec(dx+c))^{\frac{3}{2}}}{3}+2\sqrt{a+a\sec(dx+c)}\,a-2a^{\frac{3}{2}}\operatorname{arctanh}\left(\frac{\sqrt{a+a\sec(dx+c)}}{\sqrt{a}}\right)}{d}$	57
default	$\frac{\frac{2(a+a\sec(dx+c))^{\frac{3}{2}}}{3}+2\sqrt{a+a\sec(dx+c)}\,a-2a^{\frac{3}{2}}\operatorname{arctanh}\left(\frac{\sqrt{a+a\sec(dx+c)}}{\sqrt{a}}\right)}{d}$	57

input
```
int((a+a*sec(d*x+c))^(3/2)*tan(d*x+c),x,method=_RETURNVERBOSE)
```

output
```
1/d*(2/3*(a+a*sec(d*x+c))^(3/2)+2*(a+a*sec(d*x+c))^(1/2)*a-2*a^(3/2)*arcta
nh((a+a*sec(d*x+c))^(1/2)/a^(1/2)))
```

3.149.5 Fricas [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 238, normalized size of antiderivative = 3.26

$$\int (a + a\sec(c + dx))^{3/2}\tan(c + dx)\,dx = \left[\frac{3\,a^{\frac{3}{2}}\cos(dx+c)\log\left(-8\,a\cos(dx+c)^2+4\left(2\cos(dx+c)^2+\cos(dx+c)\right)\sqrt{a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}-\right.}{6\,d\cos(dx+c)}\right.$$

input
```
integrate((a+a*sec(d*x+c))^(3/2)*tan(d*x+c),x, algorithm="fricas")
```

output
```
[1/6*(3*a^(3/2)*cos(d*x + c)*log(-8*a*cos(d*x + c)^2 + 4*(2*cos(d*x + c)^2
 + cos(d*x + c))*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)) - 8*a*cos
(d*x + c) - a) + 4*(4*a*cos(d*x + c) + a)*sqrt((a*cos(d*x + c) + a)/cos(d*
x + c)))/(d*cos(d*x + c)), 1/3*(3*sqrt(-a)*a*arctan(2*sqrt(-a)*sqrt((a*cos
(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(2*a*cos(d*x + c) + a))*cos(d*x
+ c) + 2*(4*a*cos(d*x + c) + a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)))/(
d*cos(d*x + c))]
```

3.149.6 Sympy [F]

$$\int (a + a\sec(c+dx))^{3/2}\tan(c+dx)\,dx = \int (a(\sec(c+dx)+1))^{\frac{3}{2}}\tan(c+dx)\,dx$$

input
```
integrate((a+a*sec(d*x+c))**(3/2)*tan(d*x+c),x)
```

output
```
Integral((a*(sec(c + d*x) + 1))**(3/2)*tan(c + d*x), x)
```

3.149.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.18

$$\int (a + a\sec(c+dx))^{3/2}\tan(c + dx)\,dx = \frac{3\,a^{\frac{3}{2}}\log\left(\frac{\sqrt{a+\frac{a}{\cos(dx+c)}}-\sqrt{a}}{\sqrt{a+\frac{a}{\cos(dx+c)}}+\sqrt{a}}\right) + 2\left(a+\frac{a}{\cos(dx+c)}\right)^{\frac{3}{2}} + 6\sqrt{a+\frac{a}{\cos(dx+c)}}\,a}{3\,d}$$

input
```
integrate((a+a*sec(d*x+c))^(3/2)*tan(d*x+c),x, algorithm="maxima")
```

output
```
1/3*(3*a^(3/2)*log((sqrt(a + a/cos(d*x + c)) - sqrt(a))/(sqrt(a + a/cos(d*
x + c)) + sqrt(a))) + 2*(a + a/cos(d*x + c))^(3/2) + 6*sqrt(a + a/cos(d*x
+ c))*a)/d
```

3.149.8 Giac [F]

$$\int (a + a\sec(c+dx))^{3/2}\tan(c+dx)\,dx = \int (a\sec(dx+c)+a)^{\frac{3}{2}}\tan(dx+c)\,dx$$

input
```
integrate((a+a*sec(d*x+c))^(3/2)*tan(d*x+c),x, algorithm="giac")
```

output
```
sage0*x
```

3.149.9 Mupad [B] (verification not implemented)

Time = 14.33 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.92

$$\int (a + a\sec(c+dx))^{3/2}\tan(c+dx)\,dx = \frac{2\left(a+\frac{a}{\cos(c+dx)}\right)^{3/2}}{3\,d} - \frac{2\,a^{3/2}\,\mathrm{atanh}\left(\frac{\sqrt{a+\frac{a}{\cos(c+dx)}}}{\sqrt{a}}\right)}{d} + \frac{2\,a\sqrt{a+\frac{a}{\cos(c+dx)}}}{d}$$

input
```
int(tan(c + d*x)*(a + a/cos(c + d*x))^(3/2),x)
```

output
```
(2*(a + a/cos(c + d*x))^(3/2))/(3*d) - (2*a^(3/2)*atanh((a + a/cos(c + d*x
))^(1/2)/a^(1/2)))/d + (2*a*(a + a/cos(c + d*x))^(1/2))/d
```

3.150 $\int \cot(c+dx)(a+a\sec(c+dx))^{3/2}\,dx$

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3.150.2 Mathematica [A] (verified) . . . 1132
3.150.3 Rubi [A] (verified) . . . 1133
3.150.4 Maple [A] (verified) . . . 1135
3.150.5 Fricas [B] (verification not implemented) . . . 1135
3.150.6 Sympy [F] . . . 1136
3.150.7 Maxima [F] . . . 1136
3.150.8 Giac [F] . . . 1137
3.150.9 Mupad [F(-1)] . . . 1137

3.150.1 Optimal result

Integrand size = 21, antiderivative size = 73

$$\int \cot(c+dx)(a+a\sec(c+dx))^{3/2}\,dx = \frac{2a^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{a}}\right)}{d} - \frac{2\sqrt{2}a^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{d}$$

output
```
2*a^(3/2)*arctanh((a+a*sec(d*x+c))^(1/2)/a^(1/2))/d-2*a^(3/2)*arctanh(1/2*
(a+a*sec(d*x+c))^(1/2)*2^(1/2)/a^(1/2))*2^(1/2)/d
```

3.150.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.99

$$\int \cot(c+dx)(a+a\sec(c+dx))^{3/2}\,dx = \frac{\left(2\operatorname{arctanh}\left(\sqrt{1+\sec(c+dx)}\right) - 2\sqrt{2}\operatorname{arctanh}\left(\frac{\sqrt{1+\sec(c+dx)}}{\sqrt{2}}\right)\right)(a(1+\sec(c+dx)))^{3/2}}{d(1+\sec(c+dx))^{3/2}}$$

input
```
Integrate[Cot[c + d*x]*(a + a*Sec[c + d*x])^(3/2),x]
```

output
```
((2*ArcTanh[Sqrt[1 + Sec[c + d*x]]] - 2*Sqrt[2]*ArcTanh[Sqrt[1 + Sec[c + d
*x]]/Sqrt[2]])*(a*(1 + Sec[c + d*x]))^(3/2))/(d*(1 + Sec[c + d*x])^(3/2))
```

3.150.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3042, 25, 4368, 25, 27, 94, 73, 219, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot(c+dx)(a\sec(c+dx)+a)^{3/2}\,dx$$

$$\downarrow 3042$$

$$\int -\frac{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^{3/2}}{\cot\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\downarrow 25$$

$$-\int \frac{\left(\csc\left(\frac{1}{2}(2c+\pi)+dx\right)a+a\right)^{3/2}}{\cot\left(\frac{1}{2}(2c+\pi)+dx\right)}dx$$

$$\downarrow 4368$$

$$\frac{a^2\int -\frac{\cos(c+dx)\sqrt{\sec(c+dx)a+a}}{a(1-\sec(c+dx))}d\sec(c+dx)}{d}$$

$$\downarrow 25$$

$$-\frac{a^2\int \frac{\cos(c+dx)\sqrt{\sec(c+dx)a+a}}{a(1-\sec(c+dx))}d\sec(c+dx)}{d}$$

$$\downarrow 27$$

$$-\frac{a\int \frac{\cos(c+dx)\sqrt{\sec(c+dx)a+a}}{1-\sec(c+dx)}d\sec(c+dx)}{d}$$

$$\downarrow 94$$

$$-\frac{a\left(2a\int \frac{1}{(1-\sec(c+dx))\sqrt{\sec(c+dx)a+a}}d\sec(c+dx)+a\int \frac{\cos(c+dx)}{\sqrt{\sec(c+dx)a+a}}d\sec(c+dx)\right)}{d}$$

$$\downarrow 73$$

$$-\frac{a\left(4\int \frac{1}{2-\frac{\sec(c+dx)a+a}{a}}d\sqrt{\sec(c+dx)a+a}+2\int \frac{1}{\frac{\sec(c+dx)a+a}{a}-1}d\sqrt{\sec(c+dx)a+a}\right)}{d}$$

$$\downarrow 219$$

$$-\frac{a\left(2\int \frac{1}{\frac{\sec(c+dx)a+a}{a}-1}d\sqrt{\sec(c+dx)a+a}+2\sqrt{2}\sqrt{a}\operatorname{arctanh}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{2}\sqrt{a}}\right)\right)}{d}$$

$$\downarrow 220$$

$$-\frac{a\left(2\sqrt{2}\sqrt{a}\operatorname{arctanh}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{2}\sqrt{a}}\right)-2\sqrt{a}\operatorname{arctanh}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{a}}\right)\right)}{d}$$

input
```
Int[Cot[c + d*x]*(a + a*Sec[c + d*x])^(3/2),x]
```

output
```
-((a*(-2*Sqrt[a]*ArcTanh[Sqrt[a + a*Sec[c + d*x]]/Sqrt[a]] + 2*Sqrt[2]*Sqr
t[a]*ArcTanh[Sqrt[a + a*Sec[c + d*x]]/(Sqrt[2]*Sqrt[a])]))/d)
```

3.150.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 94
```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[(e + f*x)^(p - 1)/(a + b*x), x],
x] - Simp[(d*e - c*f)/(b*c - a*d)   Int[(e + f*x)^(p - 1)/(c + d*x), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && LtQ[0, p, 1]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4368
```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Simp[-(d*b^(m - 1))^(-1)   Subst[Int[(-a + b*x)^((m - 1)/2
)*((a + b*x)^((m - 1)/2 + n)/x), x], x, Csc[c + d*x]], x] /; FreeQ[{a, b, c
, d, n}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[n]
```

3.150.4 Maple [A] (verified)

Time = 1.53 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.25

method	result	size
default	$-\frac{2a\sqrt{a(1+\sec(dx+c))}\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}\left(\arctan\left(\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}\right)+\sqrt{2}\arctan\left(\frac{\sqrt{2}}{2\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}}\right)\right)}{d}$	91

input
```
int(cot(d*x+c)*(a+a*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/d*a*(a*(1+sec(d*x+c)))^(1/2)*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(arctan
((-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))+2^(1/2)*arctan(1/2*2^(1/2)/(-cos(d*x+
c)/(cos(d*x+c)+1))^(1/2)))
```

3.150.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 119 vs. $2(58) = 116$.

Time = 0.33 (sec) , antiderivative size = 243, normalized size of antiderivative = 3.33

$$\int \cot(c+dx)(a+a\sec(c+dx))^{3/2}\,dx = \left[\frac{\sqrt{2}a^{\frac{3}{2}}\log\left(-\frac{2\sqrt{2}\sqrt{a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)-3a\cos(dx+c)-a}{\cos(dx+c)-1}\right)+a^{\frac{3}{2}}\log\left(-2a\cos(dx+c)-2\sqrt{a}\sqrt{}\right.}{d}\right.$$

input
```
integrate(cot(d*x+c)*(a+a*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
[(sqrt(2)*a^(3/2)*log(-(2*sqrt(2)*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*
x + c))*cos(d*x + c) - 3*a*cos(d*x + c) - a)/(cos(d*x + c) - 1)) + a^(3/2)
*log(-2*a*cos(d*x + c) - 2*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))
*cos(d*x + c) - a))/d, 2*(sqrt(2)*sqrt(-a)*a*arctan(sqrt(2)*sqrt(-a)*sqrt(
(a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(a*cos(d*x + c) + a)) - sq
rt(-a)*a*arctan(sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x +
 c)/(a*cos(d*x + c) + a)))/d]
```

3.150.6 Sympy [F]

$$\int \cot(c+dx)(a+a\sec(c+dx))^{3/2}\,dx = \int (a(\sec(c+dx)+1))^{\frac{3}{2}}\cot(c+dx)\,dx$$

input
```
integrate(cot(d*x+c)*(a+a*sec(d*x+c))**(3/2),x)
```

output
```
Integral((a*(sec(c + d*x) + 1))**(3/2)*cot(c + d*x), x)
```

3.150.7 Maxima [F]

$$\int \cot(c+dx)(a+a\sec(c+dx))^{3/2}\,dx = \int (a\sec(dx+c)+a)^{\frac{3}{2}}\cot(dx+c)\,dx$$

input
```
integrate(cot(d*x+c)*(a+a*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate((a*sec(d*x + c) + a)^(3/2)*cot(d*x + c), x)
```

3.150.8 Giac [F]

$$\int \cot(c+dx)(a+a\sec(c+dx))^{3/2}\,dx = \int (a\sec(dx+c)+a)^{\frac{3}{2}}\cot(dx+c)\,dx$$

input
```
integrate(cot(d*x+c)*(a+a*sec(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.150.9 Mupad [F(-1)]

Timed out.

$$\int \cot(c+dx)(a+a\sec(c+dx))^{3/2}\,dx = \int \cot(c+d\,x)\left(a+\frac{a}{\cos(c+d\,x)}\right)^{3/2}dx$$

input
```
int(cot(c + d*x)*(a + a/cos(c + d*x))^(3/2),x)
```

output
```
int(cot(c + d*x)*(a + a/cos(c + d*x))^(3/2), x)
```

3.151 $\int \cot^3(c+dx)(a+a\sec(c+dx))^{3/2}\,dx$

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3.151.1 Optimal result

Integrand size = 23, antiderivative size = 109

$$\int \cot^3(c+dx)(a+a\sec(c+dx))^{3/2}\,dx = -\frac{2a^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{a}}\right)}{d} + \frac{5a^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{2\sqrt{2}d} + \frac{a\sqrt{a+a\sec(c+dx)}}{2d(1-\sec(c+dx))}$$

output
```
-2*a^(3/2)*arctanh((a+a*sec(d*x+c))^(1/2)/a^(1/2))/d+5/4*a^(3/2)*arctanh(1
/2*(a+a*sec(d*x+c))^(1/2)*2^(1/2)/a^(1/2))*2^(1/2)/d+1/2*a*(a+a*sec(d*x+c)
)^(1/2)/d/(1-sec(d*x+c))
```

3.151.2 Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.92

$$\int \cot^3(c+dx)(a+a\sec(c+dx))^{3/2}\,dx = \frac{(a(1+\sec(c+dx)))^{3/2}\left(-2\operatorname{arctanh}\left(\sqrt{1+\sec(c+dx)}\right) + \frac{5\operatorname{arctanh}\left(\frac{\sqrt{1+\sec(c+dx)}}{\sqrt{2}}\right)}{2\sqrt{2}} - \frac{\sqrt{1+\sec(c}}{2(-1+\sec(}\right.}{d(1+\sec(c+dx))^{3/2}}$$

input
```
Integrate[Cot[c + d*x]^3*(a + a*Sec[c + d*x])^(3/2),x]
```

output
```
((a*(1 + Sec[c + d*x]))^(3/2)*(-2*ArcTanh[Sqrt[1 + Sec[c + d*x]]] + (5*Arc
Tanh[Sqrt[1 + Sec[c + d*x]]/Sqrt[2]])/(2*Sqrt[2]) - Sqrt[1 + Sec[c + d*x]]
/(2*(-1 + Sec[c + d*x]))))/(d*(1 + Sec[c + d*x])^(3/2))
```

3.151.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.03, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.435$, Rules used = {3042, 25, 4368, 27, 114, 27, 174, 73, 219, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^3(c+dx)(a\sec(c+dx)+a)^{3/2}\,dx$$

$$\downarrow 3042$$

$$\int -\frac{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^{3/2}}{\cot\left(c+dx+\frac{\pi}{2}\right)^3}dx$$

$$\downarrow 25$$

$$-\int \frac{\left(\csc\left(\frac{1}{2}(2c+\pi)+dx\right)a+a\right)^{3/2}}{\cot\left(\frac{1}{2}(2c+\pi)+dx\right)^3}dx$$

$$\downarrow 4368$$

$$\frac{a^4\int \frac{\cos(c+dx)}{a^2(1-\sec(c+dx))^2\sqrt{\sec(c+dx)a+a}}d\sec(c+dx)}{d}$$

$$\downarrow 27$$

$$\frac{a^2\int \frac{\cos(c+dx)}{(1-\sec(c+dx))^2\sqrt{\sec(c+dx)a+a}}d\sec(c+dx)}{d}$$

$$\downarrow 114$$

$$\frac{a^2\left(\frac{\sqrt{a\sec(c+dx)+a}}{2a(1-\sec(c+dx))}-\frac{\int -\frac{a\cos(c+dx)(\sec(c+dx)+4)}{2(1-\sec(c+dx))\sqrt{\sec(c+dx)a+a}}d\sec(c+dx)}{2a}\right)}{d}$$

$$\downarrow 27$$

$$\frac{a^2\left(\frac{1}{4}\int \frac{\cos(c+dx)(\sec(c+dx)+4)}{(1-\sec(c+dx))\sqrt{\sec(c+dx)a+a}}d\sec(c+dx)+\frac{\sqrt{a\sec(c+dx)+a}}{2a(1-\sec(c+dx))}\right)}{d}$$

$$\downarrow 174$$

$$\frac{a^2\left(\frac{1}{4}\left(5\int\frac{1}{(1-\sec(c+dx))\sqrt{\sec(c+dx)a+a}}d\sec(c+dx)+4\int\frac{\cos(c+dx)}{\sqrt{\sec(c+dx)a+a}}d\sec(c+dx)\right)+\frac{\sqrt{a\sec(c+dx)+a}}{2a(1-\sec(c+dx))}\right)}{d}$$

$$\downarrow 73$$

$$\frac{a^2\left(\frac{1}{4}\left(\frac{10\int\frac{1}{2-\frac{\sec(c+dx)a+a}{a}}d\sqrt{\sec(c+dx)a+a}}{a}+\frac{8\int\frac{1}{\frac{\sec(c+dx)a+a}{a}-1}d\sqrt{\sec(c+dx)a+a}}{a}\right)+\frac{\sqrt{a\sec(c+dx)+a}}{2a(1-\sec(c+dx))}\right)}{d}$$

$$\downarrow 219$$

$$\frac{a^2\left(\frac{1}{4}\left(\frac{8\int\frac{1}{\frac{\sec(c+dx)a+a}{a}-1}d\sqrt{\sec(c+dx)a+a}}{a}+\frac{5\sqrt{2}\operatorname{arctanh}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{2}\sqrt{a}}\right)}{\sqrt{a}}\right)+\frac{\sqrt{a\sec(c+dx)+a}}{2a(1-\sec(c+dx))}\right)}{d}$$

$$\downarrow 220$$

$$\frac{a^2\left(\frac{1}{4}\left(\frac{5\sqrt{2}\operatorname{arctanh}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{2}\sqrt{a}}\right)}{\sqrt{a}}-\frac{8\operatorname{arctanh}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{a}}\right)}{\sqrt{a}}\right)+\frac{\sqrt{a\sec(c+dx)+a}}{2a(1-\sec(c+dx))}\right)}{d}$$

input
```
Int[Cot[c + d*x]^3*(a + a*Sec[c + d*x])^(3/2),x]
```

output
```
(a^2*(((-8*ArcTanh[Sqrt[a + a*Sec[c + d*x]]/Sqrt[a]])/Sqrt[a] + (5*Sqrt[2]
*ArcTanh[Sqrt[a + a*Sec[c + d*x]]/(Sqrt[2]*Sqrt[a])])/Sqrt[a])/4 + Sqrt[a
+ a*Sec[c + d*x]]/(2*a*(1 - Sec[c + d*x]))))/d
```

3.151.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 114
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[1/((m + 1)*(b*c - a*d)*(b*e
 - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1)
 - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (IntegerQ[n] ||
 IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4368
```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Simp[-(d*b^(m - 1))^(-1)   Subst[Int[(-a + b*x)^((m - 1)/2
)*((a + b*x)^((m - 1)/2 + n)/x), x], x, Csc[c + d*x]], x] /; FreeQ[{a, b, c
, d, n}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[n]
```

3.151.4 Maple [A] (verified)

Time = 1.94 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.27

method	result
default	$\frac{a\sqrt{a(1+\sec(dx+c))}\left(5\sqrt{2}\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}\arctan\left(\frac{\sqrt{2}}{2\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}}\right)+8\arctan\left(\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}\right)\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}-2\cot(dx+c)\right.}{4d}$

input
```
int(cot(d*x+c)^3*(a+a*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/4/d*a*(a*(1+sec(d*x+c)))^(1/2)*(5*2^(1/2)*(-cos(d*x+c)/(cos(d*x+c)+1))^(
1/2)*arctan(1/2*2^(1/2)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))+8*arctan((-cos
(d*x+c)/(cos(d*x+c)+1))^(1/2))*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)-2*cot(d*
x+c)^2-2*cot(d*x+c)*csc(d*x+c))
```

3.151.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 186 vs. 2(86) = 172.

Time = 0.33 (sec) , antiderivative size = 378, normalized size of antiderivative = 3.47

$$\int \cot^3(c+dx)(a+a\sec(c+dx))^{3/2}\,dx = \left[\frac{4\,a\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)+8\,(a\cos(dx+c)-a)\sqrt{a}\log\left(-2\,a\cos(dx+c)+2\sqrt{a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sin(dx+c)+a\right)-5\,(\sqrt{2}a\cos(dx+c)-\sqrt{2}a)\sqrt{-a}\arctan\left(\frac{\sqrt{2}\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)}{a\cos(dx+c)+a}\right)-8\,(a\cos(dx+c)-a)\sqrt{-a}\arctan\left(\frac{\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sin(dx+c)}{a\cos(dx+c)+a}\right)}{4\,(d\cos(dx+c)-d)}\right]$$

input
```
integrate(cot(d*x+c)^3*(a+a*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
[1/8*(4*a*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c) + 8*(a*cos(
d*x + c) - a)*sqrt(a)*log(-2*a*cos(d*x + c) + 2*sqrt(a)*sqrt((a*cos(d*x +
c) + a)/cos(d*x + c))*cos(d*x + c) - a) + 5*(sqrt(2)*a*cos(d*x + c) - sqrt
(2)*a)*sqrt(a)*log((2*sqrt(2)*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x +
c))*cos(d*x + c) + 3*a*cos(d*x + c) + a)/(cos(d*x + c) - 1)))/(d*cos(d*x +
 c) - d), -1/4*(5*(sqrt(2)*a*cos(d*x + c) - sqrt(2)*a)*sqrt(-a)*arctan(sqr
t(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(a*cos(
d*x + c) + a)) - 8*(a*cos(d*x + c) - a)*sqrt(-a)*arctan(sqrt(-a)*sqrt((a*c
os(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(a*cos(d*x + c) + a)) - 2*a*sq
rt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c))/(d*cos(d*x + c) - d)]
```

3.151.6 Sympy [F]

$$\int \cot^3(c+dx)(a+a\sec(c+dx))^{3/2}\,dx = \int (a(\sec(c+dx)+1))^{\frac{3}{2}}\cot^3(c+dx)\,dx$$

input
```
integrate(cot(d*x+c)**3*(a+a*sec(d*x+c))**(3/2),x)
```

output
```
Integral((a*(sec(c + d*x) + 1))**(3/2)*cot(c + d*x)**3, x)
```

3.151.7 Maxima [F]

$$\int \cot^3(c+dx)(a+a\sec(c+dx))^{3/2}\,dx = \int (a\sec(dx+c)+a)^{\frac{3}{2}}\cot(dx+c)^3\,dx$$

input
```
integrate(cot(d*x+c)^3*(a+a*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate((a*sec(d*x + c) + a)^(3/2)*cot(d*x + c)^3, x)
```

3.151.8 Giac [F]

$$\int \cot^3(c+dx)(a+a\sec(c+dx))^{3/2}\,dx = \int (a\sec(dx+c)+a)^{\frac{3}{2}}\cot(dx+c)^3\,dx$$

input
```
integrate(cot(d*x+c)^3*(a+a*sec(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.151.9 Mupad [F(-1)]

Timed out.

$$\int \cot^3(c+dx)(a+a\sec(c+dx))^{3/2}\,dx = \int \cot(c+d\,x)^3\left(a+\frac{a}{\cos(c+d\,x)}\right)^{3/2}dx$$

input
```
int(cot(c + d*x)^3*(a + a/cos(c + d*x))^(3/2),x)
```

output
```
int(cot(c + d*x)^3*(a + a/cos(c + d*x))^(3/2), x)
```

3.152 $\int \cot^5(c+dx)(a+a\sec(c+dx))^{3/2}\,dx$

3.152.1 Optimal result 1145
3.152.2 Mathematica [C] (verified) 1145
3.152.3 Rubi [A] (verified) 1146
3.152.4 Maple [A] (verified) 1150
3.152.5 Fricas [B] (verification not implemented) 1150
3.152.6 Sympy [F(-1)] 1151
3.152.7 Maxima [F(-1)] 1151
3.152.8 Giac [F] 1152
3.152.9 Mupad [F(-1)] 1152

3.152.1 Optimal result

Integrand size = 23, antiderivative size = 171

$$\int \cot^5(c+dx)(a+a\sec(c+dx))^{3/2}\,dx = \frac{2a^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{a}}\right)}{d} - \frac{71a^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{32\sqrt{2}d} + \frac{7a^2}{32d\sqrt{a+a\sec(c+dx)}} - \frac{a^2}{4d(1-\sec(c+dx))^2\sqrt{a+a\sec(c+dx)}} - \frac{13a^2}{16d(1-\sec(c+dx))\sqrt{a+a\sec(c+dx)}}$$

output

```
2*a^(3/2)*arctanh((a+a*sec(d*x+c))^(1/2)/a^(1/2))/d-71/64*a^(3/2)*arctanh(
1/2*(a+a*sec(d*x+c))^(1/2)*2^(1/2)/a^(1/2))*2^(1/2)/d+7/32*a^2/d/(a+a*sec(
d*x+c))^(1/2)-1/4*a^2/d/(1-sec(d*x+c))^2/(a+a*sec(d*x+c))^(1/2)-13/16*a^2/
d/(1-sec(d*x+c))/(a+a*sec(d*x+c))^(1/2)
```

3.152.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.72 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.61

$$\int \cot^5(c+dx)(a+a\sec(c+dx))^{3/2}\,dx = \frac{a^2\left(-34+71\,\text{Hypergeometric2F1}\left(-\tfrac{1}{2},1,\tfrac{1}{2},\tfrac{1}{2}(1+\sec(c+dx))\right)(-1+\sec(c+dx))^2-64\,\text{Hy}}{32d(-1+\sec(c+dx))^2\sqrt{a}}$$

input
```
Integrate[Cot[c + d*x]^5*(a + a*Sec[c + d*x])^(3/2),x]
```

output
```
(a^2*(-34 + 71*Hypergeometric2F1[-1/2, 1, 1/2, (1 + Sec[c + d*x])/2]*(-1 +
 Sec[c + d*x])^2 - 64*Hypergeometric2F1[-1/2, 1, 1/2, 1 + Sec[c + d*x]]*(-
1 + Sec[c + d*x])^2 + 26*Sec[c + d*x]))/(32*d*(-1 + Sec[c + d*x])^2*Sqrt[a
*(1 + Sec[c + d*x])])
```

3.152.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.04, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.652$, Rules used = {3042, 25, 4368, 25, 27, 114, 27, 168, 27, 169, 27, 174, 73, 219, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^5(c+dx)(a\sec(c+dx)+a)^{3/2}\,dx$$

$$\downarrow 3042$$

$$\int -\frac{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^{3/2}}{\cot\left(c+dx+\frac{\pi}{2}\right)^5}dx$$

$$\downarrow 25$$

$$-\int \frac{\left(\csc\left(\frac{1}{2}(2c+\pi)+dx\right)a+a\right)^{3/2}}{\cot\left(\frac{1}{2}(2c+\pi)+dx\right)^5}dx$$

$$\downarrow 4368$$

$$\frac{a^6\int -\frac{\cos(c+dx)}{a^3(1-\sec(c+dx))^3(\sec(c+dx)a+a)^{3/2}}d\sec(c+dx)}{d}$$

$$\downarrow 25$$

$$-\frac{a^6\int \frac{\cos(c+dx)}{a^3(1-\sec(c+dx))^3(\sec(c+dx)a+a)^{3/2}}d\sec(c+dx)}{d}$$

$$\downarrow 27$$

$$-\frac{a^3\int \frac{\cos(c+dx)}{(1-\sec(c+dx))^3(\sec(c+dx)a+a)^{3/2}}d\sec(c+dx)}{d}$$

$$\downarrow 114$$

$$-\frac{a^3\left(\frac{1}{4a(1-\sec(c+dx))^2\sqrt{a\sec(c+dx)+a}}-\frac{\int-\frac{a\cos(c+dx)(5\sec(c+dx)+8)}{2(1-\sec(c+dx))^2(\sec(c+dx)a+a)^{3/2}}d\sec(c+dx)}{4a}\right)}{d}$$

$$\downarrow 27$$

$$-\frac{a^3\left(\frac{1}{8}\int\frac{\cos(c+dx)(5\sec(c+dx)+8)}{(1-\sec(c+dx))^2(\sec(c+dx)a+a)^{3/2}}d\sec(c+dx)+\frac{1}{4a(1-\sec(c+dx))^2\sqrt{a\sec(c+dx)+a}}\right)}{d}$$

$$\downarrow 168$$

$$-\frac{a^3\left(\frac{1}{8}\left(\frac{13}{2a(1-\sec(c+dx))\sqrt{a\sec(c+dx)+a}}-\frac{\int-\frac{a\cos(c+dx)(39\sec(c+dx)+32)}{2(1-\sec(c+dx))(\sec(c+dx)a+a)^{3/2}}d\sec(c+dx)}{2a}\right)+\frac{1}{4a(1-\sec(c+dx))^2\sqrt{a\sec(c+dx)+a}}\right)}{d}$$

$$\downarrow 27$$

$$-\frac{a^3\left(\frac{1}{8}\left(\frac{1}{4}\int\frac{\cos(c+dx)(39\sec(c+dx)+32)}{(1-\sec(c+dx))(\sec(c+dx)a+a)^{3/2}}d\sec(c+dx)+\frac{13}{2a(1-\sec(c+dx))\sqrt{a\sec(c+dx)+a}}\right)+\frac{1}{4a(1-\sec(c+dx))^2\sqrt{a\sec(c+dx)+a}}\right)}{d}$$

$$\downarrow 169$$

$$-\frac{a^3\left(\frac{1}{8}\left(\frac{1}{4}\left(\frac{\int\frac{a\cos(c+dx)(7\sec(c+dx)+64)}{2(1-\sec(c+dx))\sqrt{\sec(c+dx)a+a}}d\sec(c+dx)}{a^2}-\frac{7}{a\sqrt{a\sec(c+dx)+a}}\right)+\frac{13}{2a(1-\sec(c+dx))\sqrt{a\sec(c+dx)+a}}\right)+\frac{}{4a(1-\sec(c+dx)}\right.}{d}$$

$$\downarrow 27$$

$$-\frac{a^3\left(\frac{1}{8}\left(\frac{1}{4}\left(\frac{\int\frac{\cos(c+dx)(7\sec(c+dx)+64)}{(1-\sec(c+dx))\sqrt{\sec(c+dx)a+a}}d\sec(c+dx)}{2a}-\frac{7}{a\sqrt{a\sec(c+dx)+a}}\right)+\frac{13}{2a(1-\sec(c+dx))\sqrt{a\sec(c+dx)+a}}\right)+\frac{}{4a(1-\sec(c+dx)}\right.}{d}$$

$$\downarrow 174$$

$$-\frac{a^3\left(\frac{1}{8}\left(\frac{1}{4}\left(\frac{71\int\frac{1}{(1-\sec(c+dx))\sqrt{\sec(c+dx)a+a}}d\sec(c+dx)+64\int\frac{\cos(c+dx)}{\sqrt{\sec(c+dx)a+a}}d\sec(c+dx)}{2a}-\frac{7}{a\sqrt{a\sec(c+dx)+a}}\right)+\frac{13}{2a(1-\sec(c+dx))\sqrt{a}}\right.\right.}{d}$$

$$\downarrow 73$$

$$-\frac{a^3\left(\frac{1}{8}\left(\frac{1}{4}\left(\frac{\frac{142\int\frac{1}{2-\frac{\sec(c+dx)a+a}{a}}d\sqrt{\sec(c+dx)a+a}}{a}+\frac{128\int\frac{1}{\frac{\sec(c+dx)a+a}{a}-1}d\sqrt{\sec(c+dx)a+a}}{a}}{2a}-\frac{7}{a\sqrt{a\sec(c+dx)+a}}\right)+\frac{13}{2a(1-\sec(c+dx))\sqrt{a}}\right.\right.}{d}$$

3.152. $\int \cot^5(c+dx)(a+a\sec(c+dx))^{3/2}\,dx$

$$\downarrow 219$$

$$-\frac{a^3\left(\frac{1}{8}\left(\frac{1}{4}\left(\frac{\frac{128\int \frac{1}{\frac{\sec(c+dx)a+a}{a}-1}d\sqrt{\sec(c+dx)a+a}}{a}+\frac{71\sqrt{2}\operatorname{arctanh}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{2}\sqrt{a}}\right)}{\sqrt{a}}}{2a}-\frac{7}{a\sqrt{a\sec(c+dx)+a}}\right)+\frac{13}{2a(1-\sec(c+dx))\sqrt{a\sec(}}\right.\right.}{d}$$

$$\downarrow 220$$

$$-\frac{a^3\left(\frac{1}{8}\left(\frac{1}{4}\left(\frac{\frac{71\sqrt{2}\operatorname{arctanh}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{2}\sqrt{a}}\right)}{\sqrt{a}}-\frac{128\operatorname{arctanh}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{a}}\right)}{\sqrt{a}}}{2a}-\frac{7}{a\sqrt{a\sec(c+dx)+a}}\right)+\frac{13}{2a(1-\sec(c+dx))\sqrt{a\sec(c+dx}}\right.\right.}{d}$$

input
```
Int[Cot[c + d*x]^5*(a + a*Sec[c + d*x])^(3/2),x]
```

output
```
-((a^3*(1/(4*a*(1 - Sec[c + d*x])^2*Sqrt[a + a*Sec[c + d*x]]) + (13/(2*a*(
1 - Sec[c + d*x])*Sqrt[a + a*Sec[c + d*x]]) + (((-128*ArcTanh[Sqrt[a + a*S
ec[c + d*x]]/Sqrt[a]])/Sqrt[a] + (71*Sqrt[2]*ArcTanh[Sqrt[a + a*Sec[c + d*
x]]/(Sqrt[2]*Sqrt[a])])/Sqrt[a])/(2*a) - 7/(a*Sqrt[a + a*Sec[c + d*x]]))/4
)/8))/d)
```

3.152.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 114
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[1/((m + 1)*(b*c - a*d)*(b*e
 - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1)
 - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (IntegerQ[n] ||
 IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

rule 168
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

rule 169
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && IntegersQ[
2*m, 2*n, 2*p]
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4368
```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Simp[-(d*b^(m - 1))^(-1)  Subst[Int[(-a + b*x)^((m - 1)/2
)*((a + b*x)^((m - 1)/2 + n)/x), x], x, Csc[c + d*x]], x] /; FreeQ[{a, b, c
, d, n}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[n]
```

3.152.4 Maple [A] (verified)

Time = 1.96 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.02

method	result
default	$-\frac{a\sqrt{a(1+\sec(dx+c))}\left(71\sqrt{2}\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}\arctan\left(\frac{\sqrt{2}}{2\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}}\right)+128\arctan\left(\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}\right)\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}+54\cot$
	$64d$

input
```
int(cot(d*x+c)^5*(a+a*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/64/d*a*(a*(1+sec(d*x+c)))^(1/2)*(71*2^(1/2)*(-cos(d*x+c)/(cos(d*x+c)+1)
)^(1/2)*arctan(1/2*2^(1/2)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))+128*arctan(
(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+54*
cot(d*x+c)^4+30*cot(d*x+c)^3*csc(d*x+c)-38*cot(d*x+c)^2*csc(d*x+c)^2-14*cs
c(d*x+c)^3*cot(d*x+c))
```

3.152.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 280 vs. $2(138) = 276$.

Time = 0.42 (sec) , antiderivative size = 589, normalized size of antiderivative = 3.44

$$\int \cot^5(c+dx)(a+a\sec(c$$

$$+dx))^{3/2}\,dx = \left[\frac{64\left(a\cos(dx+c)^3 - a\cos(dx+c)^2 - a\cos(dx+c) + a\right)\sqrt{a}\log\left(-8\,a\cos(dx+c)^2 - 4\,(2\right.}{\right.}$$

input
```
integrate(cot(d*x+c)^5*(a+a*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
[1/128*(64*(a*cos(d*x + c)^3 - a*cos(d*x + c)^2 - a*cos(d*x + c) + a)*sqrt
(a)*log(-8*a*cos(d*x + c)^2 - 4*(2*cos(d*x + c)^2 + cos(d*x + c))*sqrt(a)*
sqrt((a*cos(d*x + c) + a)/cos(d*x + c)) - 8*a*cos(d*x + c) - a) + 71*(sqrt
(2)*a*cos(d*x + c)^3 - sqrt(2)*a*cos(d*x + c)^2 - sqrt(2)*a*cos(d*x + c) +
 sqrt(2)*a)*sqrt(a)*log(-(2*sqrt(2)*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(
d*x + c))*cos(d*x + c) - 3*a*cos(d*x + c) - a)/(cos(d*x + c) - 1)) - 4*(27
*a*cos(d*x + c)^3 - 12*a*cos(d*x + c)^2 - 7*a*cos(d*x + c))*sqrt((a*cos(d*
x + c) + a)/cos(d*x + c)))/(d*cos(d*x + c)^3 - d*cos(d*x + c)^2 - d*cos(d*
x + c) + d), 1/64*(71*(sqrt(2)*a*cos(d*x + c)^3 - sqrt(2)*a*cos(d*x + c)^2
 - sqrt(2)*a*cos(d*x + c) + sqrt(2)*a)*sqrt(-a)*arctan(sqrt(2)*sqrt(-a)*sq
rt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(a*cos(d*x + c) + a)) -
 64*(a*cos(d*x + c)^3 - a*cos(d*x + c)^2 - a*cos(d*x + c) + a)*sqrt(-a)*ar
ctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(2*a*
cos(d*x + c) + a)) - 2*(27*a*cos(d*x + c)^3 - 12*a*cos(d*x + c)^2 - 7*a*co
s(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)))/(d*cos(d*x + c)^3 - d
*cos(d*x + c)^2 - d*cos(d*x + c) + d)]
```

3.152.6 Sympy [F(-1)]

Timed out.

$$\int \cot^5(c+dx)(a+a\sec(c+dx))^{3/2}\,dx = \text{Timed out}$$

input
```
integrate(cot(d*x+c)**5*(a+a*sec(d*x+c))**(3/2),x)
```

output
```
Timed out
```

3.152.7 Maxima [F(-1)]

Timed out.

$$\int \cot^5(c+dx)(a+a\sec(c+dx))^{3/2}\,dx = \text{Timed out}$$

input
```
integrate(cot(d*x+c)^5*(a+a*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
Timed out
```

3.152.8 Giac [F]

$$\int \cot^5(c+dx)(a+a\sec(c+dx))^{3/2}\,dx = \int (a\sec(dx+c)+a)^{\frac{3}{2}}\cot(dx+c)^5\,dx$$

input
```
integrate(cot(d*x+c)^5*(a+a*sec(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.152.9 Mupad [F(-1)]

Timed out.

$$\int \cot^5(c+dx)(a+a\sec(c+dx))^{3/2}\,dx = \int \cot(c+d\,x)^5\left(a+\frac{a}{\cos(c+d\,x)}\right)^{3/2}dx$$

input
```
int(cot(c + d*x)^5*(a + a/cos(c + d*x))^(3/2),x)
```

output
```
int(cot(c + d*x)^5*(a + a/cos(c + d*x))^(3/2), x)
```

3.153 $\int (a + a\sec(c+dx))^{3/2}\tan^6(c+dx)\,dx$

3.153.1 Optimal result 1153
3.153.2 Mathematica [A] (verified) 1154
3.153.3 Rubi [A] (verified) 1154
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3.153.9 Mupad [F(-1)] 1158

3.153.1 Optimal result

Integrand size = 23, antiderivative size = 258

$$\int (a + a\sec(c+dx))^{3/2}\tan^6(c+dx)\,dx = -\frac{2a^{3/2}\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d} + \frac{2a^2\tan(c+dx)}{d\sqrt{a+a\sec(c+dx)}} - \frac{2a^3\tan^3(c+dx)}{3d(a+a\sec(c+dx))^{3/2}} + \frac{2a^4\tan^5(c+dx)}{5d(a+a\sec(c+dx))^{5/2}} + \frac{30a^5\tan^7(c+dx)}{7d(a+a\sec(c+dx))^{7/2}} + \frac{34a^6\tan^9(c+dx)}{9d(a+a\sec(c+dx))^{9/2}} + \frac{14a^7\tan^{11}(c+dx)}{11d(a+a\sec(c+dx))^{11/2}} + \frac{2a^8\tan^{13}(c+dx)}{13d(a+a\sec(c+dx))^{13/2}}$$

output

```
-2*a^(3/2)*arctan(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))/d+2*a^2*tan(d
*x+c)/d/(a+a*sec(d*x+c))^(1/2)-2/3*a^3*tan(d*x+c)^3/d/(a+a*sec(d*x+c))^(3/
2)+2/5*a^4*tan(d*x+c)^5/d/(a+a*sec(d*x+c))^(5/2)+30/7*a^5*tan(d*x+c)^7/d/(
a+a*sec(d*x+c))^(7/2)+34/9*a^6*tan(d*x+c)^9/d/(a+a*sec(d*x+c))^(9/2)+14/11
*a^7*tan(d*x+c)^11/d/(a+a*sec(d*x+c))^(11/2)+2/13*a^8*tan(d*x+c)^13/d/(a+a
*sec(d*x+c))^(13/2)
```

3.153.2 Mathematica [A] (verified)

Time = 9.38 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.57

$$\int (a + a\sec(c+dx))^{3/2}\tan^6(c+dx)\,dx = \frac{a\sec\left(\frac{1}{2}(c+dx)\right)\sec^6(c+dx)\sqrt{a(1+\sec(c+dx))}\Big(-1441440\sqrt{2}\arcsin\left(\sqrt{2}\sin\left(\frac{1}{2}(c+dx)\right)\right)\cos$$

input
```
Integrate[(a + a*Sec[c + d*x])^(3/2)*Tan[c + d*x]^6,x]
```

output
```
(a*Sec[(c + d*x)/2]*Sec[c + d*x]^6*Sqrt[a*(1 + Sec[c + d*x])]*(-1441440*Sq
rt[2]*ArcSin[Sqrt[2]*Sin[(c + d*x)/2]]*Cos[c + d*x]^(13/2) + 164736*Sin[(c
 + d*x)/2] + 81081*Sin[(3*(c + d*x))/2] + 134849*Sin[(5*(c + d*x))/2] + 98
176*Sin[(9*(c + d*x))/2] + 45045*Sin[(11*(c + d*x))/2] + 32429*Sin[(13*(c
+ d*x))/2]))/(1441440*d)
```

3.153.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 236, normalized size of antiderivative = 0.91, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3042, 4375, 364, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^6(c+dx)(a\sec(c+dx)+a)^{3/2}\,dx$$

$$\downarrow 3042$$

$$\int \cot\left(c+dx+\frac{\pi}{2}\right)^6\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^{3/2}dx$$

$$\downarrow 4375$$

$$-\frac{2a^5\int \frac{\tan^6(c+dx)\left(\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+2\right)^4}{(\sec(c+dx)a+a)^3\left(\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+1\right)}d\left(-\frac{\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)}{d}$$

$$\downarrow 364$$

$$-\frac{2a^5\int\left(\frac{a^3\tan^{12}(c+dx)}{(\sec(c+dx)a+a)^6}+\frac{7a^2\tan^{10}(c+dx)}{(\sec(c+dx)a+a)^5}+\frac{17a\tan^8(c+dx)}{(\sec(c+dx)a+a)^4}+\frac{15\tan^6(c+dx)}{(\sec(c+dx)a+a)^3}+\frac{\tan^4(c+dx)}{a(\sec(c+dx)a+a)^2}-\frac{\tan^2(c+dx)}{a^2(\sec(c+dx)a+a)}-\right.}{d}$$

$$\downarrow 2009$$

$$-\frac{2a^5\left(\frac{\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{a^{7/2}}-\frac{a^3\tan^{13}(c+dx)}{13(a\sec(c+dx)+a)^{13/2}}-\frac{\tan(c+dx)}{a^3\sqrt{a\sec(c+dx)+a}}-\frac{7a^2\tan^{11}(c+dx)}{11(a\sec(c+dx)+a)^{11/2}}+\frac{\tan^3(c+dx)}{3a^2(a\sec(c+dx)+a)^{3/2}}-\right.}{d}$$

input
```
Int[(a + a*Sec[c + d*x])^(3/2)*Tan[c + d*x]^6,x]
```

output
```
(-2*a^5*(ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]]/a^(7/2) -
 Tan[c + d*x]/(a^3*Sqrt[a + a*Sec[c + d*x]]) + Tan[c + d*x]^3/(3*a^2*(a +
a*Sec[c + d*x])^(3/2)) - Tan[c + d*x]^5/(5*a*(a + a*Sec[c + d*x])^(5/2)) -
 (15*Tan[c + d*x]^7)/(7*(a + a*Sec[c + d*x])^(7/2)) - (17*a*Tan[c + d*x]^9
)/(9*(a + a*Sec[c + d*x])^(9/2)) - (7*a^2*Tan[c + d*x]^11)/(11*(a + a*Sec[
c + d*x])^(11/2)) - (a^3*Tan[c + d*x]^13)/(13*(a + a*Sec[c + d*x])^(13/2))
))/d
```

3.153.3.1 Defintions of rubi rules used

rule 364
```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_))/((c_) + (d_.)*(x_)^2),
x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*((a + b*x^2)^p/(c + d*x^2)), x], x
] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && (In
tegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4375
```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Simp[-2*(a^(m/2 + n + 1/2)/d)   Subst[Int[x^m*((2 + a*x^2
)^(m/2 + n - 1/2)/(1 + a*x^2)), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]
]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && I
ntegerQ[n - 1/2]
```

3.153.4 Maple [A] (verified)

Time = 4.03 (sec) , antiderivative size = 250, normalized size of antiderivative = 0.97

method	result
default	$-\frac{2a\sqrt{a(1+\sec(dx+c))}\left(45045\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,\operatorname{arctanh}\left(\frac{\sin(dx+c)}{(\cos(dx+c)+1)\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}}\right)\cos(dx+c)+45045\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,\text{arcta}$

input
```
int((a+a*sec(d*x+c))^(3/2)*tan(d*x+c)^6,x,method=_RETURNVERBOSE)
```

output
```
-2/45045/d*a*(a*(1+sec(d*x+c)))^(1/2)/(cos(d*x+c)+1)*(45045*(-cos(d*x+c)/(
cos(d*x+c)+1))^(1/2)*arctanh(sin(d*x+c)/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d
*x+c)+1))^(1/2))*cos(d*x+c)+45045*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arcta
nh(sin(d*x+c)/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))-32429*sin
(d*x+c)-38737*tan(d*x+c)+4731*sec(d*x+c)*tan(d*x+c)+26465*tan(d*x+c)*sec(d
*x+c)^2+6265*tan(d*x+c)*sec(d*x+c)^3-7875*sec(d*x+c)^4*tan(d*x+c)-3465*tan
(d*x+c)*sec(d*x+c)^5)
```

3.153.5 Fricas [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 415, normalized size of antiderivative = 1.61

$$\int (a + a\sec(c + dx))^{3/2}\tan^6(c + dx)\,dx = \left[\frac{45045\left(a\cos(dx+c)^7 + a\cos(dx+c)^6\right)\sqrt{-a}\log\left(\frac{2\,a\cos(dx+c)^2+2\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)\sin(dx+c)}{\cos(dx+c)+1}\right.\right.$$

input
```
integrate((a+a*sec(d*x+c))^(3/2)*tan(d*x+c)^6,x, algorithm="fricas")
```

output
```
[1/45045*(45045*(a*cos(d*x + c)^7 + a*cos(d*x + c)^6)*sqrt(-a)*log((2*a*co
s(d*x + c)^2 + 2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x
+ c)*sin(d*x + c) + a*cos(d*x + c) - a)/(cos(d*x + c) + 1)) + 2*(32429*a*c
os(d*x + c)^6 + 38737*a*cos(d*x + c)^5 - 4731*a*cos(d*x + c)^4 - 26465*a*c
os(d*x + c)^3 - 6265*a*cos(d*x + c)^2 + 7875*a*cos(d*x + c) + 3465*a)*sqrt
((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^7 + d*co
s(d*x + c)^6), 2/45045*(45045*(a*cos(d*x + c)^7 + a*cos(d*x + c)^6)*sqrt(a
)*arctan(sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin
(d*x + c))) + (32429*a*cos(d*x + c)^6 + 38737*a*cos(d*x + c)^5 - 4731*a*co
s(d*x + c)^4 - 26465*a*cos(d*x + c)^3 - 6265*a*cos(d*x + c)^2 + 7875*a*cos
(d*x + c) + 3465*a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/
(d*cos(d*x + c)^7 + d*cos(d*x + c)^6)]
```

3.153.6 Sympy [F]

$$\int (a + a\sec(c + dx))^{3/2} \tan^6(c + dx)\, dx = \int (a(\sec(c + dx) + 1))^{\frac{3}{2}} \tan^6(c + dx)\, dx$$

input
```
integrate((a+a*sec(d*x+c))**(3/2)*tan(d*x+c)**6,x)
```

output
```
Integral((a*(sec(c + d*x) + 1))**(3/2)*tan(c + d*x)**6, x)
```

3.153.7 Maxima [F(-1)]

Timed out.

$$\int (a + a\sec(c + dx))^{3/2} \tan^6(c + dx)\, dx = \text{Timed out}$$

input
```
integrate((a+a*sec(d*x+c))^(3/2)*tan(d*x+c)^6,x, algorithm="maxima")
```

output
```
Timed out
```

3.153.8 Giac [F]

$$\int (a + a\sec(c + dx))^{3/2} \tan^6(c + dx)\, dx = \int (a\sec(dx + c) + a)^{\frac{3}{2}} \tan(dx + c)^6\, dx$$

input
```
integrate((a+a*sec(d*x+c))^(3/2)*tan(d*x+c)^6,x, algorithm="giac")
```

output
```
sage0*x
```

3.153.9 Mupad [F(-1)]

Timed out.

$$\int (a + a\sec(c + dx))^{3/2} \tan^6(c + dx)\, dx = \int \tan(c + d\,x)^6 \left(a + \frac{a}{\cos(c + d\,x)}\right)^{3/2} dx$$

input
```
int(tan(c + d*x)^6*(a + a/cos(c + d*x))^(3/2),x)
```

output
```
int(tan(c + d*x)^6*(a + a/cos(c + d*x))^(3/2), x)
```

3.154 $\int (a + a\sec(c+dx))^{3/2}\tan^4(c+dx)\,dx$

3.154.1 Optimal result 1159
3.154.2 Mathematica [A] (verified) 1159
3.154.3 Rubi [A] (verified) 1160
3.154.4 Maple [A] (verified) 1161
3.154.5 Fricas [A] (verification not implemented) 1162
3.154.6 Sympy [F] 1163
3.154.7 Maxima [F(-1)] 1163
3.154.8 Giac [F] 1163
3.154.9 Mupad [F(-1)] 1164

3.154.1 Optimal result

Integrand size = 23, antiderivative size = 194

$$\int (a + a\sec(c+dx))^{3/2}\tan^4(c+dx)\,dx = \frac{2a^{3/2}\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d} - \frac{2a^2\tan(c+dx)}{d\sqrt{a+a\sec(c+dx)}} + \frac{2a^3\tan^3(c+dx)}{3d(a+a\sec(c+dx))^{3/2}} + \frac{14a^4\tan^5(c+dx)}{5d(a+a\sec(c+dx))^{5/2}} + \frac{10a^5\tan^7(c+dx)}{7d(a+a\sec(c+dx))^{7/2}} + \frac{2a^6\tan^9(c+dx)}{9d(a+a\sec(c+dx))^{9/2}}$$

output

```
2*a^(3/2)*arctan(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))/d-2*a^2*tan(d*
x+c)/d/(a+a*sec(d*x+c))^(1/2)+2/3*a^3*tan(d*x+c)^3/d/(a+a*sec(d*x+c))^(3/2
)+14/5*a^4*tan(d*x+c)^5/d/(a+a*sec(d*x+c))^(5/2)+10/7*a^5*tan(d*x+c)^7/d/(
a+a*sec(d*x+c))^(7/2)+2/9*a^6*tan(d*x+c)^9/d/(a+a*sec(d*x+c))^(9/2)
```

3.154.2 Mathematica [A] (verified)

Time = 7.15 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.63

$$\int (a + a\sec(c+dx))^{3/2}\tan^4(c+dx)\,dx = \frac{a\sec\left(\frac{1}{2}(c+dx)\right)\sec^4(c+dx)\sqrt{a(1+\sec(c+dx))}\left(2520\sqrt{2}\arcsin\left(\sqrt{2}\sin\left(\frac{1}{2}(c+dx)\right)\right)\cos^{\frac{9}{2}}(c+\right.}{2520}$$

input
```
Integrate[(a + a*Sec[c + d*x])^(3/2)*Tan[c + d*x]^4,x]
```

output
```
(a*Sec[(c + d*x)/2]*Sec[c + d*x]^4*Sqrt[a*(1 + Sec[c + d*x])]*(2520*Sqrt[2
]*ArcSin[Sqrt[2]*Sin[(c + d*x)/2]]*Cos[c + d*x]^(9/2) + 126*Sin[(c + d*x)/
2] - 288*Sin[(5*(c + d*x))/2] - 315*Sin[(7*(c + d*x))/2] - 169*Sin[(9*(c +
 d*x))/2]))/(2520*d)
```

3.154.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 178, normalized size of antiderivative = 0.92, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3042, 4375, 364, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^4(c+dx)(a\sec(c+dx)+a)^{3/2}\,dx$$

$$\downarrow 3042$$

$$\int \cot\left(c+dx+\frac{\pi}{2}\right)^4\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^{3/2}dx$$

$$\downarrow 4375$$

$$-\frac{2a^4\int \frac{\tan^4(c+dx)\left(\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+2\right)^3}{(\sec(c+dx)a+a)^2\left(\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+1\right)}d\left(-\frac{\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)}{d}$$

$$\downarrow 364$$

$$-\frac{2a^4\int\left(\frac{a^2\tan^8(c+dx)}{(\sec(c+dx)a+a)^4}+\frac{5a\tan^6(c+dx)}{(\sec(c+dx)a+a)^3}+\frac{7\tan^4(c+dx)}{(\sec(c+dx)a+a)^2}+\frac{\tan^2(c+dx)}{a(\sec(c+dx)a+a)}+\frac{1}{a^2\left(\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+1\right)}-\frac{1}{a^2}\right)d\left(-\frac{\tan(c}{\sqrt{\sec(c-}\right.}{d}$$

$$\downarrow 2009$$

$$-\frac{2a^4\left(-\frac{\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{a^{5/2}}-\frac{a^2\tan^9(c+dx)}{9(a\sec(c+dx)+a)^{9/2}}+\frac{\tan(c+dx)}{a^2\sqrt{a\sec(c+dx)+a}}-\frac{5a\tan^7(c+dx)}{7(a\sec(c+dx)+a)^{7/2}}-\frac{7\tan^5(c+dx)}{5(a\sec(c+dx)+a)^{5/2}}-\frac{}{3a(}\right.}{d}$$

input
```
Int[(a + a*Sec[c + d*x])^(3/2)*Tan[c + d*x]^4,x]
```

output
```
(-2*a^4*(-(ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]]/a^(5/2)
) + Tan[c + d*x]/(a^2*Sqrt[a + a*Sec[c + d*x]]) - Tan[c + d*x]^3/(3*a*(a +
 a*Sec[c + d*x])^(3/2)) - (7*Tan[c + d*x]^5)/(5*(a + a*Sec[c + d*x])^(5/2)
) - (5*a*Tan[c + d*x]^7)/(7*(a + a*Sec[c + d*x])^(7/2)) - (a^2*Tan[c + d*x
]^9)/(9*(a + a*Sec[c + d*x])^(9/2))))/d
```

3.154.3.1 Defintions of rubi rules used

rule 364
```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_))/((c_) + (d_.)*(x_)^2),
x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*((a + b*x^2)^p/(c + d*x^2)), x], x
] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && (In
tegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4375
```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Simp[-2*(a^(m/2 + n + 1/2)/d)   Subst[Int[x^m*((2 + a*x^2
)^(m/2 + n - 1/2)/(1 + a*x^2)), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]
]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && I
ntegerQ[n - 1/2]
```

3.154.4 Maple [A] (verified)

Time = 3.72 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.12

method	result
default	$\frac{2a\sqrt{a(1+\sec(dx+c))}\left(315\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,\operatorname{arctanh}\left(\frac{\sin(dx+c)}{(\cos(dx+c)+1)\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}}\right)\cos(dx+c)+315\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,\operatorname{arctanh}\left(\frac{}{(co}\right.}{315d}$

input
```
int((a+a*sec(d*x+c))^(3/2)*tan(d*x+c)^4,x,method=_RETURNVERBOSE)
```

output
```
2/315/d*a*(a*(1+sec(d*x+c)))^(1/2)/(cos(d*x+c)+1)*(315*(-cos(d*x+c)/(cos(d
*x+c)+1))^(1/2)*arctanh(sin(d*x+c)/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)
+1))^(1/2))*cos(d*x+c)+315*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctanh(sin(
d*x+c)/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))-169*sin(d*x+c)-2
42*tan(d*x+c)-24*sec(d*x+c)*tan(d*x+c)+85*tan(d*x+c)*sec(d*x+c)^2+35*tan(d
*x+c)*sec(d*x+c)^3)
```

3.154.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 371, normalized size of antiderivative = 1.91

$$\int (a + a\sec(c+dx))^{3/2}\tan^4(c$$

$$+dx)\,dx = \left[\frac{315\left(a\cos(dx+c)^5 + a\cos(dx+c)^4\right)\sqrt{-a}\log\left(\frac{2\,a\cos(dx+c)^2 - 2\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)\sin(dx+c) + a}{\cos(dx+c)+1}\right.}{3}\right.$$

$$\left.-\frac{2\left(315\left(a\cos(dx+c)^5 + a\cos(dx+c)^4\right)\sqrt{a}\arctan\left(\frac{\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)}{\sqrt{a}\sin(dx+c)}\right) + \left(169\,a\cos(dx+c)^4 + 242\right.\right.}{315\left(d\cos(dx+c)^5 + d\cos\right.}\right]$$

input
```
integrate((a+a*sec(d*x+c))^(3/2)*tan(d*x+c)^4,x, algorithm="fricas")
```

output
```
[1/315*(315*(a*cos(d*x + c)^5 + a*cos(d*x + c)^4)*sqrt(-a)*log((2*a*cos(d*
x + c)^2 - 2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)
*sin(d*x + c) + a*cos(d*x + c) - a)/(cos(d*x + c) + 1)) - 2*(169*a*cos(d*x
 + c)^4 + 242*a*cos(d*x + c)^3 + 24*a*cos(d*x + c)^2 - 85*a*cos(d*x + c) -
 35*a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(d*cos(d*x +
c)^5 + d*cos(d*x + c)^4), -2/315*(315*(a*cos(d*x + c)^5 + a*cos(d*x + c)^4
)*sqrt(a)*arctan(sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqr
t(a)*sin(d*x + c))) + (169*a*cos(d*x + c)^4 + 242*a*cos(d*x + c)^3 + 24*a*
cos(d*x + c)^2 - 85*a*cos(d*x + c) - 35*a)*sqrt((a*cos(d*x + c) + a)/cos(d
*x + c))*sin(d*x + c))/(d*cos(d*x + c)^5 + d*cos(d*x + c)^4)]
```

3.154.6 Sympy [F]

$$\int (a + a\sec(c+dx))^{3/2}\tan^4(c+dx)\,dx = \int (a(\sec(c+dx)+1))^{\frac{3}{2}}\tan^4(c+dx)\,dx$$

input
```
integrate((a+a*sec(d*x+c))**(3/2)*tan(d*x+c)**4,x)
```

output
```
Integral((a*(sec(c + d*x) + 1))**(3/2)*tan(c + d*x)**4, x)
```

3.154.7 Maxima [F(-1)]

Timed out.

$$\int (a + a\sec(c+dx))^{3/2}\tan^4(c+dx)\,dx = \text{Timed out}$$

input
```
integrate((a+a*sec(d*x+c))^(3/2)*tan(d*x+c)^4,x, algorithm="maxima")
```

output
```
Timed out
```

3.154.8 Giac [F]

$$\int (a + a\sec(c+dx))^{3/2}\tan^4(c+dx)\,dx = \int (a\sec(dx+c)+a)^{\frac{3}{2}}\tan(dx+c)^4\,dx$$

input
```
integrate((a+a*sec(d*x+c))^(3/2)*tan(d*x+c)^4,x, algorithm="giac")
```

output
```
sage0*x
```

3.154.9 Mupad [F(-1)]

Timed out.

$$\int (a + a\sec(c + dx))^{3/2} \tan^4(c + dx)\, dx = \int \tan(c + d\,x)^4 \left(a + \frac{a}{\cos(c + d\,x)}\right)^{3/2} dx$$

input
```
int(tan(c + d*x)^4*(a + a/cos(c + d*x))^(3/2),x)
```

output
```
int(tan(c + d*x)^4*(a + a/cos(c + d*x))^(3/2), x)
```

3.155 $\int (a + a\sec(c+dx))^{3/2}\tan^2(c+dx)\,dx$

3.155.1 Optimal result 1165
3.155.2 Mathematica [A] (verified) 1165
3.155.3 Rubi [A] (verified) 1166
3.155.4 Maple [A] (warning: unable to verify) 1167
3.155.5 Fricas [A] (verification not implemented) 1168
3.155.6 Sympy [F] 1168
3.155.7 Maxima [F] 1169
3.155.8 Giac [F] 1169
3.155.9 Mupad [F(-1)] 1170

3.155.1 Optimal result

Integrand size = 23, antiderivative size = 128

$$\int (a + a\sec(c+dx))^{3/2}\tan^2(c+dx)\,dx = -\frac{2a^{3/2}\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d} + \frac{2a^2\tan(c+dx)}{d\sqrt{a+a\sec(c+dx)}} + \frac{2a^3\tan^3(c+dx)}{d(a+a\sec(c+dx))^{3/2}} + \frac{2a^4\tan^5(c+dx)}{5d(a+a\sec(c+dx))^{5/2}}$$

output
```
-2*a^(3/2)*arctan(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))/d+2*a^2*tan(d
*x+c)/d/(a+a*sec(d*x+c))^(1/2)+2*a^3*tan(d*x+c)^3/d/(a+a*sec(d*x+c))^(3/2)
+2/5*a^4*tan(d*x+c)^5/d/(a+a*sec(d*x+c))^(5/2)
```

3.155.2 Mathematica [A] (verified)

Time = 6.79 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.76

$$\int (a + a\sec(c+dx))^{3/2}\tan^2(c + dx)\,dx = \frac{a\sec\left(\frac{1}{2}(c+dx)\right)\sec^2(c+dx)\sqrt{a(1+\sec(c+dx))}\left(-10\sqrt{2}\arcsin\left(\sqrt{2}\sin\left(\frac{1}{2}(c+dx)\right)\right)\cos^{\frac{5}{2}}(c+\right.}{10d}$$

input
```
Integrate[(a + a*Sec[c + d*x])^(3/2)*Tan[c + d*x]^2,x]
```

output
```
(a*Sec[(c + d*x)/2]*Sec[c + d*x]^2*Sqrt[a*(1 + Sec[c + d*x])]*(-10*Sqrt[2]
*ArcSin[Sqrt[2]*Sin[(c + d*x)/2]]*Cos[c + d*x]^(5/2) + 5*Sin[(3*(c + d*x))
/2] + Sin[(5*(c + d*x))/2]))/(10*d)
```

3.155.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.92, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3042, 4375, 364, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^2(c+dx)(a\sec(c+dx)+a)^{3/2}\,dx$$

$$\downarrow 3042$$

$$\int \cot\left(c+dx+\frac{\pi}{2}\right)^2\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^{3/2}dx$$

$$\downarrow 4375$$

$$-\frac{2a^3\int \frac{\tan^2(c+dx)\left(\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+2\right)^2}{(\sec(c+dx)a+a)\left(\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+1\right)}d\left(-\frac{\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)}{d}$$

$$\downarrow 364$$

$$-\frac{2a^3\int\left(\frac{a\tan^4(c+dx)}{(\sec(c+dx)a+a)^2}+\frac{3\tan^2(c+dx)}{\sec(c+dx)a+a}+\frac{1}{a}-\frac{1}{a\left(\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+1\right)}\right)d\left(-\frac{\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)}{d}$$

$$\downarrow 2009$$

$$-\frac{2a^3\left(\frac{\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{a^{3/2}}-\frac{a\tan^5(c+dx)}{5(a\sec(c+dx)+a)^{5/2}}-\frac{\tan^3(c+dx)}{(a\sec(c+dx)+a)^{3/2}}-\frac{\tan(c+dx)}{a\sqrt{a\sec(c+dx)+a}}\right)}{d}$$

input
```
Int[(a + a*Sec[c + d*x])^(3/2)*Tan[c + d*x]^2,x]
```

output
```
(-2*a^3*(ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]]/a^(3/2) -
 Tan[c + d*x]/(a*Sqrt[a + a*Sec[c + d*x]]) - Tan[c + d*x]^3/(a + a*Sec[c +
 d*x])^(3/2) - (a*Tan[c + d*x]^5)/(5*(a + a*Sec[c + d*x])^(5/2))))/d
```

3.155.3.1 Defintions of rubi rules used

rule 364
```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_))/((c_) + (d_.)*(x_)^2),
x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*((a + b*x^2)^p/(c + d*x^2)), x], x
] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && (In
tegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4375
```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Simp[-2*(a^(m/2 + n + 1/2)/d)   Subst[Int[x^m*((2 + a*x^2
)^(m/2 + n - 1/2)/(1 + a*x^2)), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]
]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && I
ntegerQ[n - 1/2]
```

3.155.4 Maple [A] (warning: unable to verify)

Time = 3.48 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.46

method	result
default	$-\frac{a\left(5\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{2}\,(-\cot(dx+c)+\csc(dx+c))}{\sqrt{(1-\cos(dx+c))^2\csc(dx+c)^2-1}}\right)\left((1-\cos(dx+c))^2\csc(dx+c)^2-1\right)^{\frac{5}{2}}+2(1-\cos(dx+c))^5\csc(dx+c)^5-10\csc(dx+$ $5d(\csc(dx+c)-\cot(dx+c)+1)^2(-\cot(dx+c)+\csc(dx+c)-1)^2$

input
```
int((a+a*sec(d*x+c))^(3/2)*tan(d*x+c)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/5/d*a*(5*2^(1/2)*arctanh(2^(1/2)/((1-cos(d*x+c))^2*csc(d*x+c)^2-1)^(1/2
)*(-cot(d*x+c)+csc(d*x+c)))*((1-cos(d*x+c))^2*csc(d*x+c)^2-1)^(5/2)+2*(1-c
os(d*x+c))^5*csc(d*x+c)^5-10*csc(d*x+c)+10*cot(d*x+c))*(-2*a/((1-cos(d*x+c
))^2*csc(d*x+c)^2-1))^(1/2)/(csc(d*x+c)-cot(d*x+c)+1)^2/(-cot(d*x+c)+csc(d
*x+c)-1)^2
```

3.155.5 Fricas [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 321, normalized size of antiderivative = 2.51

$$\int (a + a\sec(c + dx))^{3/2} \tan^2(c$$

$$+dx)\,dx = \left[\frac{5\left(a\cos\left(dx+c\right)^3 + a\cos\left(dx+c\right)^2\right)\sqrt{-a}\log\left(\frac{2\,a\cos(dx+c)^2+2\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)\sin(dx+c)+a\cos}{\cos(dx+c)+1}\right.}{5\left(d\cos\left(dx+c\right)^3 + d\cos(\right.} \right.$$

input
```
integrate((a+a*sec(d*x+c))^(3/2)*tan(d*x+c)^2,x, algorithm="fricas")
```

output
```
[1/5*(5*(a*cos(d*x + c)^3 + a*cos(d*x + c)^2)*sqrt(-a)*log((2*a*cos(d*x +
c)^2 + 2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin
(d*x + c) + a*cos(d*x + c) - a)/(cos(d*x + c) + 1)) + 2*(a*cos(d*x + c)^2
+ 3*a*cos(d*x + c) + a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x +
c))/(d*cos(d*x + c)^3 + d*cos(d*x + c)^2), 2/5*(5*(a*cos(d*x + c)^3 + a*co
s(d*x + c)^2)*sqrt(a)*arctan(sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d
*x + c)/(sqrt(a)*sin(d*x + c))) + (a*cos(d*x + c)^2 + 3*a*cos(d*x + c) + a
)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^3
+ d*cos(d*x + c)^2)]
```

3.155.6 Sympy [F]

$$\int (a + a\sec(c + dx))^{3/2} \tan^2(c + dx)\,dx = \int (a(\sec(c + dx) + 1))^{\frac{3}{2}} \tan^2(c + dx)\,dx$$

input
```
integrate((a+a*sec(d*x+c))**(3/2)*tan(d*x+c)**2,x)
```

output
```
Integral((a*(sec(c + d*x) + 1))**(3/2)*tan(c + d*x)**2, x)
```

3.155.7 Maxima [F]

$$\int (a + a\sec(c+dx))^{3/2}\tan^2(c+dx)\,dx = \int (a\sec(dx+c)+a)^{\frac{3}{2}}\tan(dx+c)^2\,dx$$

input
```
integrate((a+a*sec(d*x+c))^(3/2)*tan(d*x+c)^2,x, algorithm="maxima")
```

output
```
1/10*(5*((a*cos(2*d*x + 2*c)^2 + a*sin(2*d*x + 2*c)^2 + 2*a*cos(2*d*x + 2*
c) + a)*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2
*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)), (
cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*co
s(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + 1) - (a*cos(2*d*x
 + 2*c)^2 + a*sin(2*d*x + 2*c)^2 + 2*a*cos(2*d*x + 2*c) + a)*arctan2((cos(
2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/
2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 +
sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d
*x + 2*c), cos(2*d*x + 2*c) + 1)) - 1) - 2*(a*d*cos(2*d*x + 2*c)^2 + a*d*s
in(2*d*x + 2*c)^2 + 2*a*d*cos(2*d*x + 2*c) + a*d)*integrate((cos(2*d*x + 2
*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(((cos(6*d*x +
6*c)*cos(2*d*x + 2*c) + 2*cos(4*d*x + 4*c)*cos(2*d*x + 2*c) + cos(2*d*x +
2*c)^2 + sin(6*d*x + 6*c)*sin(2*d*x + 2*c) + 2*sin(4*d*x + 4*c)*sin(2*d*x
+ 2*c) + sin(2*d*x + 2*c)^2)*cos(7/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
 2*c))) + (cos(2*d*x + 2*c)*sin(6*d*x + 6*c) + 2*cos(2*d*x + 2*c)*sin(4*d*
x + 4*c) - cos(6*d*x + 6*c)*sin(2*d*x + 2*c) - 2*cos(4*d*x + 4*c)*sin(2*d*
x + 2*c))*sin(7/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*cos(3/2*ar
ctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - ((cos(2*d*x + 2*c)*sin(6*
d*x + 6*c) + 2*cos(2*d*x + 2*c)*sin(4*d*x + 4*c) - cos(6*d*x + 6*c)*sin...
```

3.155.8 Giac [F]

$$\int (a + a\sec(c+dx))^{3/2}\tan^2(c+dx)\,dx = \int (a\sec(dx+c)+a)^{\frac{3}{2}}\tan(dx+c)^2\,dx$$

input
```
integrate((a+a*sec(d*x+c))^(3/2)*tan(d*x+c)^2,x, algorithm="giac")
```

output
```
sage0*x
```

3.155.9 Mupad [F(-1)]

Timed out.

$$\int (a + a\sec(c + dx))^{3/2} \tan^2(c + dx)\, dx = \int \tan(c + d\,x)^2 \left(a + \frac{a}{\cos(c + d\,x)}\right)^{3/2} dx$$

input
```
int(tan(c + d*x)^2*(a + a/cos(c + d*x))^(3/2),x)
```

output
```
int(tan(c + d*x)^2*(a + a/cos(c + d*x))^(3/2), x)
```

3.156 $\int \cot^2(c+dx)(a+a\sec(c+dx))^{3/2}\,dx$

3.156.1 Optimal result 1171
3.156.2 Mathematica [A] (warning: unable to verify) 1171
3.156.3 Rubi [A] (verified) 1172
3.156.4 Maple [A] (verified) 1173
3.156.5 Fricas [B] (verification not implemented) 1174
3.156.6 Sympy [F] 1174
3.156.7 Maxima [F] 1175
3.156.8 Giac [F] 1175
3.156.9 Mupad [F(-1)] 1175

3.156.1 Optimal result

Integrand size = 23, antiderivative size = 64

$$\int \cot^2(c+dx)(a+a\sec(c+dx))^{3/2}\,dx = -\frac{2a^{3/2}\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d} - \frac{2a\cot(c+dx)\sqrt{a+a\sec(c+dx)}}{d}$$

output
```
-2*a^(3/2)*arctan(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))/d-2*a*cot(d*x
+c)*(a+a*sec(d*x+c))^(1/2)/d
```

3.156.2 Mathematica [A] (warning: unable to verify)

Time = 0.38 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.59

$$\int \cot^2(c+dx)(a+a\sec(c+dx))^{3/2}\,dx = -\frac{2\cot(c+dx)\sqrt{\frac{1}{1+\sec(c+dx)}}(a(1+\sec(c+dx)))^{3/2}\left(\sqrt{\cos(c+dx)}\sqrt{\frac{1}{1+\cos(c+dx)}} + \arcsin\left(\frac{\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{\frac{1}{1+\cos(c+dx)}}}\right)\tan\right.}{d}$$

input
```
Integrate[Cot[c + d*x]^2*(a + a*Sec[c + d*x])^(3/2),x]
```

output
```
(-2*Cot[c + d*x]*Sqrt[(1 + Sec[c + d*x])^(-1)]*(a*(1 + Sec[c + d*x]))^(3/2
)*(Sqrt[Cos[c + d*x]]*Sqrt[(1 + Cos[c + d*x])^(-1)] + ArcSin[Tan[(c + d*x)
/2]/Sqrt[(1 + Cos[c + d*x])^(-1)]]*Tan[(c + d*x)/2]))/d
```

3.156.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.95, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3042, 4375, 264, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^2(c+dx)(a\sec(c+dx)+a)^{3/2}\,dx$$

$$\downarrow 3042$$

$$\int \frac{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^{3/2}}{\cot\left(c+dx+\frac{\pi}{2}\right)^2}dx$$

$$\downarrow 4375$$

$$-\frac{2a\int \frac{\cot^2(c+dx)(\sec(c+dx)a+a)}{\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+1}d\left(-\frac{\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)}{d}$$

$$\downarrow 264$$

$$-\frac{2a\left(\cot(c+dx)\sqrt{a\sec(c+dx)+a}-a\int \frac{1}{\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+1}d\left(-\frac{\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)\right)}{d}$$

$$\downarrow 216$$

$$-\frac{2a\left(\sqrt{a}\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)+\cot(c+dx)\sqrt{a\sec(c+dx)+a}\right)}{d}$$

input
```
Int[Cot[c + d*x]^2*(a + a*Sec[c + d*x])^(3/2),x]
```

output
```
(-2*a*(Sqrt[a]*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]] + C
ot[c + d*x]*Sqrt[a + a*Sec[c + d*x]]))/d
```

3.156.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4375
```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Simp[-2*(a^(m/2 + n + 1/2)/d)   Subst[Int[x^m*((2 + a*x^2
)^(m/2 + n - 1/2)/(1 + a*x^2)), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]
]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && I
ntegerQ[n - 1/2]
```

3.156.4 Maple [A] (verified)

Time = 1.94 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.33

method	result	size
default	$-\frac{2a\sqrt{a(1+\sec(dx+c))}\left(\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,\operatorname{arctanh}\left(\frac{\sin(dx+c)}{(\cos(dx+c)+1)\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}}\right)+\cot(dx+c)\right)}{d}$	85

input
```
int(cot(d*x+c)^2*(a+a*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/d*a*(a*(1+sec(d*x+c)))^(1/2)*((-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctan
h(sin(d*x+c)/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))+cot(d*x+c)
)
```

3.156.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 118 vs. $2(56) = 112$.

Time $= 0.34$ (sec) , antiderivative size $= 264$, normalized size of antiderivative $= 4.12$

$$\int \cot^2(c+dx)(a+a\sec(c+dx))^{3/2}\,dx = \left[\frac{\sqrt{-a}a\log\left(-\frac{8\,a\cos(dx+c)^3+4\left(2\cos(dx+c)^2-\cos(dx+c)\right)\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sin(dx+c)-7\,a\cos(dx+c)+a}{\cos(dx+c)+1}\right)\sin(dx+c)}{2\,d\sin(dx+c)}\right.$$
$$\left.-\frac{a^{\frac{3}{2}}\arctan\left(\frac{2\sqrt{a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)\sin(dx+c)}{2\,a\cos(dx+c)^2+a\cos(dx+c)-a}\right)\sin(dx+c)+2\,a\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)}{d\sin(dx+c)}\right]$$

input
```
integrate(cot(d*x+c)^2*(a+a*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
[1/2*(sqrt(-a)*a*log(-(8*a*cos(d*x + c)^3 + 4*(2*cos(d*x + c)^2 - cos(d*x
+ c))*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c) - 7*a*
cos(d*x + c) + a)/(cos(d*x + c) + 1))*sin(d*x + c) - 4*a*sqrt((a*cos(d*x +
 c) + a)/cos(d*x + c))*cos(d*x + c))/(d*sin(d*x + c)), -(a^(3/2)*arctan(2*
sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x + c)/
(2*a*cos(d*x + c)^2 + a*cos(d*x + c) - a))*sin(d*x + c) + 2*a*sqrt((a*cos(
d*x + c) + a)/cos(d*x + c))*cos(d*x + c))/(d*sin(d*x + c))]
```

3.156.6 Sympy [F]

$$\int \cot^2(c+dx)(a+a\sec(c+dx))^{3/2}\,dx = \int (a(\sec(c+dx)+1))^{\frac{3}{2}}\cot^2(c+dx)\,dx$$

input
```
integrate(cot(d*x+c)**2*(a+a*sec(d*x+c))**(3/2),x)
```

output
```
Integral((a*(sec(c + d*x) + 1))**(3/2)*cot(c + d*x)**2, x)
```

3.156.7 Maxima [F]

$$\int \cot^2(c+dx)(a+a\sec(c+dx))^{3/2}\,dx = \int (a\sec(dx+c)+a)^{\frac{3}{2}}\cot(dx+c)^2\,dx$$

input
```
integrate(cot(d*x+c)^2*(a+a*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate((a*sec(d*x + c) + a)^(3/2)*cot(d*x + c)^2, x)
```

3.156.8 Giac [F]

$$\int \cot^2(c+dx)(a+a\sec(c+dx))^{3/2}\,dx = \int (a\sec(dx+c)+a)^{\frac{3}{2}}\cot(dx+c)^2\,dx$$

input
```
integrate(cot(d*x+c)^2*(a+a*sec(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.156.9 Mupad [F(-1)]

Timed out.

$$\int \cot^2(c+dx)(a+a\sec(c+dx))^{3/2}\,dx = \int \cot(c+d\,x)^2\left(a+\frac{a}{\cos(c+d\,x)}\right)^{3/2}dx$$

input
```
int(cot(c + d*x)^2*(a + a/cos(c + d*x))^(3/2),x)
```

output
```
int(cot(c + d*x)^2*(a + a/cos(c + d*x))^(3/2), x)
```

3.157 $\int \cot^4(c+dx)(a+a\sec(c+dx))^{3/2}\,dx$

3.157.1 Optimal result 1176
3.157.2 Mathematica [A] (warning: unable to verify) 1176
3.157.3 Rubi [A] (verified) 1177
3.157.4 Maple [A] (verified) 1180
3.157.5 Fricas [A] (verification not implemented) 1180
3.157.6 Sympy [F(-1)] 1181
3.157.7 Maxima [F(-1)] 1181
3.157.8 Giac [F] 1182
3.157.9 Mupad [F(-1)] 1182

3.157.1 Optimal result

Integrand size = 23, antiderivative size = 144

$$\int \cot^4(c+dx)(a+a\sec(c+dx))^{3/2}\,dx = \frac{2a^{3/2}\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d} - \frac{a^{3/2}\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{2\sqrt{2}d} + \frac{3a\cot(c+dx)\sqrt{a+a\sec(c+dx)}}{2d} - \frac{\cot^3(c+dx)(a+a\sec(c+dx))^{3/2}}{3d}$$

output
```
2*a^(3/2)*arctan(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))/d-1/3*cot(d*x+
c)^3*(a+a*sec(d*x+c))^(3/2)/d-1/4*a^(3/2)*arctan(1/2*a^(1/2)*tan(d*x+c)*2^
(1/2)/(a+a*sec(d*x+c))^(1/2))/d*2^(1/2)+3/2*a*cot(d*x+c)*(a+a*sec(d*x+c))^
(1/2)/d
```

3.157.2 Mathematica [A] (warning: unable to verify)

Time = 6.69 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.57

$$\int \cot^4(c+dx)(a+a\sec(c+dx))^{3/2}\,dx = -\frac{\left(\arcsin\left(\tan\left(\frac{1}{2}(c+dx)\right)\right) - 4\sqrt{2}\arctan\left(\frac{\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{\frac{\cos(c+dx)}{1+\cos(c+dx)}}}\right)\right)\sqrt{\frac{\cos(c+dx)}{1+\cos(c+dx)}}\sec^4\left(\frac{1}{2}(c+dx)\right)\sqrt{1+\sec(c+dx)}}{8d\sqrt{\sec^2\left(\frac{1}{2}(c+dx)\right)}\sec^{\frac{3}{2}}(c+dx)} + \frac{\cos(c+dx)\sec^3\left(\frac{1}{2}(c+dx)\right)(a(1+\sec(c+dx)))^{3/2}\left(\frac{13}{24}\csc\left(\frac{1}{2}(c+dx)\right) - \frac{1}{24}\csc^3\left(\frac{1}{2}(c+dx)\right) - \frac{11}{12}\sin\left(\frac{1}{2}(c\right.\right.}{d}$$

input
```
Integrate[Cot[c + d*x]^4*(a + a*Sec[c + d*x])^(3/2),x]
```

output
```
-1/8*((ArcSin[Tan[(c + d*x)/2]] - 4*Sqrt[2]*ArcTan[Tan[(c + d*x)/2]/Sqrt[C
os[c + d*x]/(1 + Cos[c + d*x])]])*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Se
c[(c + d*x)/2]^4*Sqrt[1 + Sec[c + d*x]]*(a*(1 + Sec[c + d*x]))^(3/2))/(d*S
qrt[Sec[(c + d*x)/2]^2]*Sec[c + d*x]^(3/2)) + (Cos[c + d*x]*Sec[(c + d*x)/
2]^3*(a*(1 + Sec[c + d*x]))^(3/2)*((13*Csc[(c + d*x)/2])/24 - Csc[(c + d*x
)/2]^3/24 - (11*Sin[(c + d*x)/2])/12))/d
```

3.157.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.01, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {3042, 4375, 382, 27, 445, 27, 397, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^4(c+dx)(a\sec(c+dx)+a)^{3/2}\,dx$$

$$\downarrow 3042$$

$$\int \frac{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^{3/2}}{\cot\left(c+dx+\frac{\pi}{2}\right)^4}dx$$

$$\downarrow 4375$$

$$-\frac{2\int \frac{\cot^4(c+dx)(\sec(c+dx)a+a)^2}{\left(\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+1\right)\left(\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+2\right)}d\left(-\frac{\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)}{d}$$

$$\downarrow 382$$

$$-\frac{2\left(\frac{1}{6}\int -\frac{3a\cot^2(c+dx)(\sec(c+dx)a+a)\left(\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+3\right)}{\left(\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+1\right)\left(\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+2\right)}d\left(-\frac{\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)+\frac{1}{6}\cot^3(c+dx)(a\sec(c+dx)+a)^{3/2}\right)}{d}$$

$$\downarrow 27$$

$$-\frac{2\left(\frac{1}{6}\cot^3(c+dx)(a\sec(c+dx)+a)^{3/2}-\frac{1}{2}a\int \frac{\cot^2(c+dx)(\sec(c+dx)a+a)\left(\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+3\right)}{\left(\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+1\right)\left(\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+2\right)}d\left(-\frac{\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)\right)}{d}$$

$$\downarrow 445$$

$$-\frac{2\left(\frac{1}{6}\cot^3(c+dx)(a\sec(c+dx)+a)^{3/2}-\frac{1}{2}a\left(\frac{3}{2}\cot(c+dx)\sqrt{a\sec(c+dx)+a}-\frac{1}{2}\int\frac{a\left(\frac{3a\tan^2(c+dx)}{\sec(c+dx)a+a}+7\right)}{\left(\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+1\right)\left(\frac{a\tan^2(c+dx)}{\sec(c+dx)a+}\right.}\right.}{d}$$

$$\downarrow 27$$

$$-\frac{2\left(\frac{1}{6}\cot^3(c+dx)(a\sec(c+dx)+a)^{3/2}-\frac{1}{2}a\left(\frac{3}{2}\cot(c+dx)\sqrt{a\sec(c+dx)+a}-\frac{1}{2}a\int\frac{\frac{3a\tan^2(c+dx)}{\sec(c+dx)a+a}+7}{\left(\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+1\right)\left(\frac{a\tan^2(c+d}{\sec(c+dx)a}\right.}\right.}{d}$$

$$\downarrow 397$$

$$-\frac{2\left(\frac{1}{6}\cot^3(c+dx)(a\sec(c+dx)+a)^{3/2}-\frac{1}{2}a\left(\frac{3}{2}\cot(c+dx)\sqrt{a\sec(c+dx)+a}-\frac{1}{2}a\left(4\int\frac{1}{\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+1}d\left(-\frac{t}{\sqrt{se}}\right.\right.\right.\right.}{d}$$

$$\downarrow 216$$

$$-\frac{2\left(\frac{1}{6}\cot^3(c+dx)(a\sec(c+dx)+a)^{3/2}-\frac{1}{2}a\left(\frac{3}{2}\cot(c+dx)\sqrt{a\sec(c+dx)+a}-\frac{1}{2}a\left(\frac{\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{2}\sqrt{a}}-\right.\right.\right.}{d}$$

input
```
Int[Cot[c + d*x]^4*(a + a*Sec[c + d*x])^(3/2),x]
```

output
```
(-2*((Cot[c + d*x]^3*(a + a*Sec[c + d*x])^(3/2))/6 - (a*(-1/2*(a*((-4*ArcT
an[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/Sqrt[a] + ArcTan[(Sqr
t[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])]/(Sqrt[2]*Sqrt[a])))
 + (3*Cot[c + d*x]*Sqrt[a + a*Sec[c + d*x]])/2))/2))/d
```

3.157.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 382
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)
, x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/
(a*c*e*(m + 1))), x] - Simp[1/(a*c*e^2*(m + 1))   Int[(e*x)^(m + 2)*(a + b*
x^2)^p*(c + d*x^2)^q*Simp[(b*c + a*d)*(m + 3) + 2*(b*c*p + a*d*q) + b*d*(m
+ 2*p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[
b*c - a*d, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, d, e, m, 2, p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 445
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^2*(m + 1))
 Int[(g*x)^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*f*c*(m + 1) - e*(b*c
+ a*d)*(m + 2 + 1) - e*2*(b*c*p + a*d*q) - b*e*d*(m + 2*(p + q + 2) + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && LtQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4375
```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Simp[-2*(a^(m/2 + n + 1/2)/d)   Subst[Int[x^m*((2 + a*x^2
)^(m/2 + n - 1/2)/(1 + a*x^2)), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]
]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && I
ntegerQ[n - 1/2]
```

3.157.4 Maple [A] (verified)

Time = 1.74 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.37

method	result
default	$-\frac{a\sqrt{a(1+\sec(dx+c))}\left(3\ln\left(\csc(dx+c)-\cot(dx+c)+\sqrt{\cot(dx+c)^2-2\cot(dx+c)\csc(dx+c)+\csc(dx+c)^2-1}\right)\sqrt{2}\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}-$

input
```
int(cot(d*x+c)^4*(a+a*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/12/d*a*(a*(1+sec(d*x+c)))^(1/2)*(3*ln(csc(d*x+c)-cot(d*x+c)+(cot(d*x+c)
^2-2*cot(d*x+c)*csc(d*x+c)+csc(d*x+c)^2-1)^(1/2))*2^(1/2)*(-cos(d*x+c)/(co
s(d*x+c)+1))^(1/2)-24*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctanh(sin(d*x+c
)/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))+22*cot(d*x+c)^3+4*cot
(d*x+c)^2*csc(d*x+c)-18*cot(d*x+c)*csc(d*x+c)^2)
```

3.157.5 Fricas [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 531, normalized size of antiderivative = 3.69

$$\int \cot^4(c+dx)(a+a\sec(c+dx))^{3/2}\,dx = \left[\frac{3\left(\sqrt{2}a\cos\left(dx+c\right)-\sqrt{2}a\right)\sqrt{-a}\log\left(\frac{2\sqrt{2}\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)\sin(dx+c)+3\,a\cos(dx+c)^2+2\,a\cos(\right.}{\cos(dx+c)^2+2\,\cos(dx+c)+1}\right.$$

input
```
integrate(cot(d*x+c)^4*(a+a*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
[1/24*(3*(sqrt(2)*a*cos(d*x + c) - sqrt(2)*a)*sqrt(-a)*log((2*sqrt(2)*sqrt
(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x + c) + 3
*a*cos(d*x + c)^2 + 2*a*cos(d*x + c) - a)/(cos(d*x + c)^2 + 2*cos(d*x + c)
 + 1))*sin(d*x + c) + 12*(a*cos(d*x + c) - a)*sqrt(-a)*log(-(8*a*cos(d*x +
 c)^3 - 4*(2*cos(d*x + c)^2 - cos(d*x + c))*sqrt(-a)*sqrt((a*cos(d*x + c)
+ a)/cos(d*x + c))*sin(d*x + c) - 7*a*cos(d*x + c) + a)/(cos(d*x + c) + 1)
)*sin(d*x + c) + 4*(11*a*cos(d*x + c)^2 - 9*a*cos(d*x + c))*sqrt((a*cos(d*
x + c) + a)/cos(d*x + c)))/((d*cos(d*x + c) - d)*sin(d*x + c)), 1/12*(12*(
a*cos(d*x + c) - a)*sqrt(a)*arctan(2*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos
(d*x + c))*cos(d*x + c)*sin(d*x + c)/(2*a*cos(d*x + c)^2 + a*cos(d*x + c)
- a))*sin(d*x + c) + 3*(sqrt(2)*a*cos(d*x + c) - sqrt(2)*a)*sqrt(a)*arctan
(sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin
(d*x + c)))*sin(d*x + c) + 2*(11*a*cos(d*x + c)^2 - 9*a*cos(d*x + c))*sqrt
((a*cos(d*x + c) + a)/cos(d*x + c)))/((d*cos(d*x + c) - d)*sin(d*x + c))]
```

3.157.6 Sympy [F(-1)]

Timed out.

$$\int \cot^4(c+dx)(a+a\sec(c+dx))^{3/2}\,dx = \text{Timed out}$$

input
```
integrate(cot(d*x+c)**4*(a+a*sec(d*x+c))**(3/2),x)
```

output
```
Timed out
```

3.157.7 Maxima [F(-1)]

Timed out.

$$\int \cot^4(c+dx)(a+a\sec(c+dx))^{3/2}\,dx = \text{Timed out}$$

input
```
integrate(cot(d*x+c)^4*(a+a*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
Timed out
```

3.157.8 Giac [F]

$$\int \cot^4(c+dx)(a+a\sec(c+dx))^{3/2}\,dx = \int (a\sec(dx+c)+a)^{\frac{3}{2}}\cot(dx+c)^4\,dx$$

input
```
integrate(cot(d*x+c)^4*(a+a*sec(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.157.9 Mupad [F(-1)]

Timed out.

$$\int \cot^4(c+dx)(a+a\sec(c+dx))^{3/2}\,dx = \int \cot(c+d\,x)^4\left(a+\frac{a}{\cos(c+d\,x)}\right)^{3/2}dx$$

input
```
int(cot(c + d*x)^4*(a + a/cos(c + d*x))^(3/2),x)
```

output
```
int(cot(c + d*x)^4*(a + a/cos(c + d*x))^(3/2), x)
```

3.158 $\int \cot^6(c+dx)(a+a\sec(c+dx))^{3/2}\,dx$

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3.158.1 Optimal result

Integrand size = 23, antiderivative size = 226

$$\int \cot^6(c+dx)(a+a\sec(c+dx))^{3/2}\,dx = -\frac{2a^{3/2}\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d} + \frac{11a^{3/2}\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{16\sqrt{2}d} - \frac{21a\cot(c+dx)\sqrt{a+a\sec(c+dx)}}{16d} + \frac{5\cot^3(c+dx)(a+a\sec(c+dx))^{3/2}}{24d} + \frac{3\cot^5(c+dx)(a+a\sec(c+dx))^{5/2}}{20ad} - \frac{\cos(c+dx)\cot^5(c+dx)\sec^2\left(\frac{1}{2}(c+dx)\right)(a+a\sec(c+dx))^{5/2}}{4ad}$$

output

```
-2*a^(3/2)*arctan(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))/d+5/24*cot(d*
x+c)^3*(a+a*sec(d*x+c))^(3/2)/d+3/20*cot(d*x+c)^5*(a+a*sec(d*x+c))^(5/2)/a
/d-1/4*cos(d*x+c)*cot(d*x+c)^5*sec(1/2*d*x+1/2*c)^2*(a+a*sec(d*x+c))^(5/2)
/a/d+11/32*a^(3/2)*arctan(1/2*a^(1/2)*tan(d*x+c)*2^(1/2)/(a+a*sec(d*x+c))^
(1/2))/d*2^(1/2)-21/16*a*cot(d*x+c)*(a+a*sec(d*x+c))^(1/2)/d
```

3.158.2 Mathematica [A] (verified)

Time = 7.46 (sec) , antiderivative size = 235, normalized size of antiderivative = 1.04

$$\int \cot^6(c+dx)(a+a\sec(c+dx))^{3/2}\,dx = \frac{(a(1+\sec(c+dx)))^{3/2}\left(165\arcsin\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)\sec^4\left(\frac{1}{2}(c+dx)\right)\sqrt{\frac{1}{1+\sec(c+dx)}}\sqrt{1+\sec(}\right.}{}$$

input
```
Integrate[Cot[c + d*x]^6*(a + a*Sec[c + d*x])^(3/2),x]
```

output
```
((a*(1 + Sec[c + d*x]))^(3/2)*(165*ArcSin[Tan[(c + d*x)/2]]*Sec[(c + d*x)/
2]^4*Sqrt[(1 + Sec[c + d*x])^(-1)]*Sqrt[1 + Sec[c + d*x]] + 2*Sqrt[2]*(-24
0*ArcTan[Tan[(c + d*x)/2]/Sqrt[(1 + Sec[c + d*x])^(-1)]]*Sec[(c + d*x)/2]^
4*Sqrt[(1 + Sec[c + d*x])^(-1)]*Sqrt[1 + Sec[c + d*x]] + (Sqrt[(1 + Cos[c
+ d*x])^(-1)]*Csc[c + d*x]^5*(281 - 279*Sec[c + d*x] + Cos[2*(c + d*x)]*(-
449 + 351*Sec[c + d*x]))/Sec[c + d*x]^(3/2))))/(960*d*Sqrt[Sec[(c + d*x)/
2]^2]*Sec[c + d*x]^(3/2))
```

3.158.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 238, normalized size of antiderivative = 1.05, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.565$, Rules used = {3042, 4375, 374, 25, 27, 445, 27, 445, 27, 445, 27, 397, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^6(c+dx)(a\sec(c+dx)+a)^{3/2}\,dx$$

$$\downarrow 3042$$

$$\int \frac{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^{3/2}}{\cot\left(c+dx+\frac{\pi}{2}\right)^6}dx$$

$$\downarrow 4375$$

$$-\frac{2\int \frac{\cot^6(c+dx)(\sec(c+dx)a+a)^3}{\left(\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+1\right)\left(\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+2\right)^2}d\left(-\frac{\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)}{ad}$$

$$\downarrow 374$$

$$-\frac{2\left(\frac{\int -\frac{a\cot^6(c+dx)(\sec(c+dx)a+a)^3\left(\frac{7a\tan^2(c+dx)}{\sec(c+dx)a+a}+3\right)}{\left(\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+1\right)\left(\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+2\right)}d\left(-\frac{\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)}{4a}+\frac{\cot^5(c+dx)(a\sec(c+dx)+a)^{5/2}}{4\left(\frac{a\tan^2(c+dx)}{a\sec(c+dx)+a}+2\right)}\right)}{ad}$$

$$\downarrow 25$$

$$-\frac{2\left(\frac{\cot^5(c+dx)(a\sec(c+dx)+a)^{5/2}}{4\left(\frac{a\tan^2(c+dx)}{a\sec(c+dx)+a}+2\right)}-\frac{\int \frac{a\cot^6(c+dx)(\sec(c+dx)a+a)^3\left(\frac{7a\tan^2(c+dx)}{\sec(c+dx)a+a}+3\right)}{\left(\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+1\right)\left(\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+2\right)}d\left(-\frac{\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)}{4a}\right)}{ad}$$

$$\downarrow 27$$

$$-\frac{2\left(\frac{\cot^5(c+dx)(a\sec(c+dx)+a)^{5/2}}{4\left(\frac{a\tan^2(c+dx)}{a\sec(c+dx)+a}+2\right)}-\frac{1}{4}\int \frac{\cot^6(c+dx)(\sec(c+dx)a+a)^3\left(\frac{7a\tan^2(c+dx)}{\sec(c+dx)a+a}+3\right)}{\left(\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+1\right)\left(\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+2\right)}d\left(-\frac{\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)\right)}{ad}$$

$$\downarrow 445$$

$$-\frac{2\left(\frac{1}{4}\left(\frac{1}{10}\int -\frac{5a\cot^4(c+dx)(\sec(c+dx)a+a)^2\left(5-\frac{3a\tan^2(c+dx)}{\sec(c+dx)a+a}\right)}{\left(\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+1\right)\left(\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+2\right)}d\left(-\frac{\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)-\frac{3}{10}\cot^5(c+dx)(a\sec(c+dx)+a)^5\right.\right.}{ad}$$

$$\downarrow 27$$

$$-\frac{2\left(\frac{1}{4}\left(-\frac{1}{2}a\int \frac{\cot^4(c+dx)(\sec(c+dx)a+a)^2\left(5-\frac{3a\tan^2(c+dx)}{\sec(c+dx)a+a}\right)}{\left(\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+1\right)\left(\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+2\right)}d\left(-\frac{\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)-\frac{3}{10}\cot^5(c+dx)(a\sec(c+dx)+a)^{5/2}\right.\right.}{ad}$$

$$\downarrow 445$$

$$-\frac{2\left(\frac{1}{4}\left(-\frac{1}{2}a\left(\frac{5}{6}\cot^3(c+dx)(a\sec(c+dx)+a)^{3/2}-\frac{1}{6}\int \frac{3a\cot^2(c+dx)(\sec(c+dx)a+a)\left(\frac{5a\tan^2(c+dx)}{\sec(c+dx)a+a}+21\right)}{\left(\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+1\right)\left(\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+2\right)}d\left(-\frac{\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right.\right.\right.\right.}{ad}$$

$$\downarrow 27$$

$$-\frac{2\left(\frac{1}{4}\left(-\frac{1}{2}a\left(\frac{5}{6}\cot^3(c+dx)(a\sec(c+dx)+a)^{3/2}-\frac{1}{2}a\int\frac{\cot^2(c+dx)(\sec(c+dx)a+a)\left(\frac{5a\tan^2(c+dx)}{\sec(c+dx)a+a}+21\right)}{\left(\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+1\right)\left(\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+2\right)}d\left(-\frac{\tan(c+dx)}{\sqrt{\sec(c+dx)a}}\right.\right.\right.}{ad}$$

$$\downarrow 445$$

$$-\frac{2\left(\frac{1}{4}\left(-\frac{1}{2}a\left(\frac{5}{6}\cot^3(c+dx)(a\sec(c+dx)+a)^{3/2}-\frac{1}{2}a\left(\frac{21}{2}\cot(c+dx)\sqrt{a\sec(c+dx)+a}-\frac{1}{2}\int\frac{a\left(\frac{21a\tan^2}{\sec(c+d}\right.\right.\right.\right.}{\left(\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+1\right.}$$

$$\downarrow 27$$

$$-\frac{2\left(\frac{1}{4}\left(-\frac{1}{2}a\left(\frac{5}{6}\cot^3(c+dx)(a\sec(c+dx)+a)^{3/2}-\frac{1}{2}a\left(\frac{21}{2}\cot(c+dx)\sqrt{a\sec(c+dx)+a}-\frac{1}{2}a\int\frac{\frac{21a\tan^2}{\sec(c+d}\right.\right.\right.\right.}{\left(\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+1\right.}$$

$$\downarrow 397$$

$$-\frac{2\left(\frac{1}{4}\left(-\frac{1}{2}a\left(\frac{5}{6}\cot^3(c+dx)(a\sec(c+dx)+a)^{3/2}-\frac{1}{2}a\left(\frac{21}{2}\cot(c+dx)\sqrt{a\sec(c+dx)+a}-\frac{1}{2}a\left(32\int\frac{1}{\frac{a\tan^2(c+dx)}{\sec(c+dx)a+}}\right.\right.\right.\right.\right.}{}$$

$$\downarrow 216$$

$$-\frac{2\left(\frac{1}{4}\left(-\frac{1}{2}a\left(\frac{5}{6}\cot^3(c+dx)(a\sec(c+dx)+a)^{3/2}-\frac{1}{2}a\left(\frac{21}{2}\cot(c+dx)\sqrt{a\sec(c+dx)+a}-\frac{1}{2}a\left(\frac{11\arctan\left(\frac{\sqrt{a}}{\sqrt{2}\sqrt{}}\right.\right.\right.\right.\right.\right.}{\sqrt{2}\sqrt{}}$$

input
```
Int[Cot[c + d*x]^6*(a + a*Sec[c + d*x])^(3/2),x]
```

output
```
(-2*(((-3*Cot[c + d*x]^5*(a + a*Sec[c + d*x])^(5/2))/10 - (a*((5*Cot[c + d
*x]^3*(a + a*Sec[c + d*x])^(3/2))/6 - (a*(-1/2*(a*((-32*ArcTan[(Sqrt[a]*Ta
n[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/Sqrt[a] + (11*ArcTan[(Sqrt[a]*Tan[c
 + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])])/(Sqrt[2]*Sqrt[a])) + (21*Co
t[c + d*x]*Sqrt[a + a*Sec[c + d*x]])/2))/2))/2)/4 + (Cot[c + d*x]^5*(a + a
*Sec[c + d*x])^(5/2))/(4*(2 + (a*Tan[c + d*x]^2)/(a + a*Sec[c + d*x])))))/
(a*d)
```

3.158.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 374
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q
 + 1)/(a*e*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(e*x)^m*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[b*c*(m + 1) + 2*(b*c -
a*d)*(p + 1) + d*b*(m + 2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b,
c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IntBinomialQ[a, b,
 c, d, e, m, 2, p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 445
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^2*(m + 1))
 Int[(g*x)^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*f*c*(m + 1) - e*(b*c
+ a*d)*(m + 2 + 1) - e*2*(b*c*p + a*d*q) - b*e*d*(m + 2*(p + q + 2) + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && LtQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4375

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Simp[-2*(a^(m/2 + n + 1/2)/d)   Subst[Int[x^m*((2 + a*x^2
)^(m/2 + n - 1/2)/(1 + a*x^2)), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]
]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && I
ntegerQ[n - 1/2]
```

3.158.4 Maple [A] (verified)

Time = 1.85 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.00

method	result
default	$-\frac{a\left(-165\sin(dx+c)^5\sqrt{2}\,\ln\left(\csc(dx+c)-\cot(dx+c)+\sqrt{\cot(dx+c)^2-2\cot(dx+c)\csc(dx+c)+\csc(dx+c)^2-1}\right)\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}+96\right.}{}$

input

```
int(cot(d*x+c)^6*(a+a*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output

```
-1/480/d*a*(-165*sin(d*x+c)^5*2^(1/2)*ln(csc(d*x+c)-cot(d*x+c)+(cot(d*x+c)
^2-2*cot(d*x+c)*csc(d*x+c)+csc(d*x+c)^2-1)^(1/2))*(-cos(d*x+c)/(cos(d*x+c)
+1))^(1/2)+960*sin(d*x+c)^5*arctanh(sin(d*x+c)/(cos(d*x+c)+1)/(-cos(d*x+c)
/(cos(d*x+c)+1))^(1/2))*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+898*cos(d*x+c)^
5+196*cos(d*x+c)^4-1432*cos(d*x+c)^3-100*cos(d*x+c)^2+630*cos(d*x+c))*(a*(
1+sec(d*x+c)))^(1/2)*csc(d*x+c)^5
```

3.158.5 Fricas [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 708, normalized size of antiderivative = 3.13

$$\int \cot^6(c+dx)(a+a\sec(c+dx))^{3/2}\,dx = \left[\frac{165\left(\sqrt{2}a\cos(dx+c)^3-\sqrt{2}a\cos(dx+c)^2-\sqrt{2}a\cos(dx+c)+\sqrt{2}a\right)\sqrt{-a}\log\left(-\frac{2\sqrt{2}\sqrt{-a}}{}\right.\right.$$

$$-\frac{480\left(a\cos(dx+c)^3-a\cos(dx+c)^2-a\cos(dx+c)+a\right)\sqrt{a}\arctan\left(\frac{2\sqrt{a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)\sin(dx+c)}{2a\cos(dx+c)^2+a\cos(dx+c)-a}\right)}{}$$

input
```
integrate(cot(d*x+c)^6*(a+a*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
[1/960*(165*(sqrt(2)*a*cos(d*x + c)^3 - sqrt(2)*a*cos(d*x + c)^2 - sqrt(2)
*a*cos(d*x + c) + sqrt(2)*a)*sqrt(-a)*log(-(2*sqrt(2)*sqrt(-a)*sqrt((a*cos
(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x + c) - 3*a*cos(d*x + c)^
2 - 2*a*cos(d*x + c) + a)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1))*sin(d*x +
 c) + 480*(a*cos(d*x + c)^3 - a*cos(d*x + c)^2 - a*cos(d*x + c) + a)*sqrt(
-a)*log(-(8*a*cos(d*x + c)^3 + 4*(2*cos(d*x + c)^2 - cos(d*x + c))*sqrt(-a
)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c) - 7*a*cos(d*x + c)
+ a)/(cos(d*x + c) + 1))*sin(d*x + c) - 4*(449*a*cos(d*x + c)^4 - 351*a*co
s(d*x + c)^3 - 365*a*cos(d*x + c)^2 + 315*a*cos(d*x + c))*sqrt((a*cos(d*x
+ c) + a)/cos(d*x + c)))/((d*cos(d*x + c)^3 - d*cos(d*x + c)^2 - d*cos(d*x
 + c) + d)*sin(d*x + c)), -1/480*(480*(a*cos(d*x + c)^3 - a*cos(d*x + c)^2
 - a*cos(d*x + c) + a)*sqrt(a)*arctan(2*sqrt(a)*sqrt((a*cos(d*x + c) + a)/
cos(d*x + c))*cos(d*x + c)*sin(d*x + c)/(2*a*cos(d*x + c)^2 + a*cos(d*x +
c) - a))*sin(d*x + c) + 165*(sqrt(2)*a*cos(d*x + c)^3 - sqrt(2)*a*cos(d*x
+ c)^2 - sqrt(2)*a*cos(d*x + c) + sqrt(2)*a)*sqrt(a)*arctan(sqrt(2)*sqrt((
a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c)))*sin
(d*x + c) + 2*(449*a*cos(d*x + c)^4 - 351*a*cos(d*x + c)^3 - 365*a*cos(d*x
 + c)^2 + 315*a*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)))/((d
*cos(d*x + c)^3 - d*cos(d*x + c)^2 - d*cos(d*x + c) + d)*sin(d*x + c))]
```

3.158.6 Sympy [F(-1)]

Timed out.

$$\int \cot^6(c+dx)(a+a\sec(c+dx))^{3/2}\,dx = \text{Timed out}$$

input
```
integrate(cot(d*x+c)**6*(a+a*sec(d*x+c))**(3/2),x)
```

output
```
Timed out
```

3.158.7 Maxima [F(-1)]

Timed out.

$$\int \cot^6(c+dx)(a+a\sec(c+dx))^{3/2}\,dx = \text{Timed out}$$

input
```
integrate(cot(d*x+c)^6*(a+a*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
Timed out
```

3.158.8 Giac [F]

$$\int \cot^6(c+dx)(a+a\sec(c+dx))^{3/2}\,dx = \int (a\sec(dx+c)+a)^{\frac{3}{2}}\cot(dx+c)^6\,dx$$

input
```
integrate(cot(d*x+c)^6*(a+a*sec(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.158.9 Mupad [F(-1)]

Timed out.

$$\int \cot^6(c+dx)(a+a\sec(c+dx))^{3/2}\,dx = \int \cot(c+d\,x)^6\left(a+\frac{a}{\cos(c+d\,x)}\right)^{3/2}dx$$

input
```
int(cot(c + d*x)^6*(a + a/cos(c + d*x))^(3/2),x)
```

output
```
int(cot(c + d*x)^6*(a + a/cos(c + d*x))^(3/2), x)
```

3.159 $\int (a + a\sec(c+dx))^{5/2}\tan^5(c+dx)\,dx$

3.159.1 Optimal result 1191
3.159.2 Mathematica [A] (verified) 1192
3.159.3 Rubi [A] (verified) 1192
3.159.4 Maple [A] (verified) 1194
3.159.5 Fricas [A] (verification not implemented) 1194
3.159.6 Sympy [F(-1)] 1195
3.159.7 Maxima [A] (verification not implemented) 1195
3.159.8 Giac [F] 1196
3.159.9 Mupad [F(-1)] 1196

3.159.1 Optimal result

Integrand size = 23, antiderivative size = 193

$$\int (a + a\sec(c+dx))^{5/2}\tan^5(c+dx)\,dx = -\frac{2a^{5/2}\operatorname{arctanh}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{a}}\right)}{d} + \frac{2a^2\sqrt{a+a\sec(c+dx)}}{d} + \frac{2a(a+a\sec(c+dx))^{3/2}}{3d} + \frac{2(a+a\sec(c+dx))^{5/2}}{5d} + \frac{2(a+a\sec(c+dx))^{7/2}}{7ad} + \frac{2(a+a\sec(c+dx))^{9/2}}{9a^2d} - \frac{6(a+a\sec(c+dx))^{11/2}}{11a^3d} + \frac{2(a+a\sec(c+dx))^{13/2}}{13a^4d}$$

output

```
-2*a^(5/2)*arctanh((a+a*sec(d*x+c))^(1/2)/a^(1/2))/d+2/3*a*(a+a*sec(d*x+c)
)^(3/2)/d+2/5*(a+a*sec(d*x+c))^(5/2)/d+2/7*(a+a*sec(d*x+c))^(7/2)/a/d+2/9*
(a+a*sec(d*x+c))^(9/2)/a^2/d-6/11*(a+a*sec(d*x+c))^(11/2)/a^3/d+2/13*(a+a*
sec(d*x+c))^(13/2)/a^4/d+2*a^2*(a+a*sec(d*x+c))^(1/2)/d
```

3.159.2 Mathematica [A] (verified)

Time = 0.55 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.81

$$\int (a + a\sec(c + dx))^{5/2} \tan^5(c + dx)\, dx = \frac{(a(1 + \sec(c + dx)))^{5/2} \left(-2\text{arctanh}\left(\sqrt{1 + \sec(c + dx)}\right) + 2\sqrt{1 + \sec(c + dx)} + \frac{2}{3}(1 + \sec(c + d$$

input
```
Integrate[(a + a*Sec[c + d*x])^(5/2)*Tan[c + d*x]^5,x]
```

output
```
((a*(1 + Sec[c + d*x]))^(5/2)*(-2*ArcTanh[Sqrt[1 + Sec[c + d*x]]] + 2*Sqrt
[1 + Sec[c + d*x]] + (2*(1 + Sec[c + d*x])^(3/2))/3 + (2*(1 + Sec[c + d*x]
)^(5/2))/5 + (2*(1 + Sec[c + d*x])^(7/2))/7 + (2*(1 + Sec[c + d*x])^(9/2))
/9 - (6*(1 + Sec[c + d*x])^(11/2))/11 + (2*(1 + Sec[c + d*x])^(13/2))/13))
/(d*(1 + Sec[c + d*x])^(5/2))
```

3.159.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 176, normalized size of antiderivative = 0.91, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3042, 25, 4368, 27, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^5(c + dx)(a\sec(c + dx) + a)^{5/2}\, dx$$

$$\downarrow 3042$$

$$\int -\cot\left(c + dx + \frac{\pi}{2}\right)^5 \left(a\csc\left(c + dx + \frac{\pi}{2}\right) + a\right)^{5/2} dx$$

$$\downarrow 25$$

$$-\int \cot\left(\frac{1}{2}(2c + \pi) + dx\right)^5 \left(\csc\left(\frac{1}{2}(2c + \pi) + dx\right)a + a\right)^{5/2} dx$$

$$\downarrow 4368$$

$$\frac{\int a^2 \cos(c + dx)(1 - \sec(c + dx))^2(\sec(c + dx)a + a)^{9/2} d\sec(c + dx)}{a^4 d}$$

$$\downarrow 27$$

$$\frac{\int \cos(c+dx)(1-\sec(c+dx))^2(\sec(c+dx)a+a)^{9/2}d\sec(c+dx)}{a^2d}$$

$$\downarrow 99$$

$$\frac{\int \left(\frac{(\sec(c+dx)a+a)^{11/2}}{a}+\cos(c+dx)(\sec(c+dx)a+a)^{9/2}-3(\sec(c+dx)a+a)^{9/2}\right)d\sec(c+dx)}{a^2d}$$

$$\downarrow 2009$$

$$\frac{-2a^{9/2}\text{arctanh}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{a}}\right)+2a^4\sqrt{a\sec(c+dx)+a}+\frac{2}{3}a^3(a\sec(c+dx)+a)^{3/2}+\frac{2(a\sec(c+dx)+a)^{13/2}}{13a^2}+\frac{2}{5}a^2(}{a^2d}$$

input
```
Int[(a + a*Sec[c + d*x])^(5/2)*Tan[c + d*x]^5,x]
```

output
```
(-2*a^(9/2)*ArcTanh[Sqrt[a + a*Sec[c + d*x]]/Sqrt[a]] + 2*a^4*Sqrt[a + a*S
ec[c + d*x]] + (2*a^3*(a + a*Sec[c + d*x])^(3/2))/3 + (2*a^2*(a + a*Sec[c
+ d*x])^(5/2))/5 + (2*a*(a + a*Sec[c + d*x])^(7/2))/7 + (2*(a + a*Sec[c +
d*x])^(9/2))/9 - (6*(a + a*Sec[c + d*x])^(11/2))/(11*a) + (2*(a + a*Sec[c
+ d*x])^(13/2))/(13*a^2))/(a^2*d)
```

3.159.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4368
```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Simp[-(d*b^(m - 1))^(-1)   Subst[Int[(-a + b*x)^((m - 1)/2
)*((a + b*x)^((m - 1)/2 + n)/x), x], x, Csc[c + d*x]], x] /; FreeQ[{a, b, c
, d, n}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[n]
```

3.159.4 Maple [A] (verified)

Time = 262.51 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.64

method	result
default	$\frac{2a^2\sqrt{a(1+\sec(dx+c))}\left(45045\arctan\left(\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}\right)\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}+71689+31723\sec(dx+c)-12531\sec(dx+c)^2-27095\sec(d$ $45045d$

input
```
int((a+a*sec(d*x+c))^(5/2)*tan(d*x+c)^5,x,method=_RETURNVERBOSE)
```

output
```
2/45045/d*a^2*(a*(1+sec(d*x+c)))^(1/2)*(45045*arctan((-cos(d*x+c)/(cos(d*x
+c)+1))^(1/2))*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+71689+31723*sec(d*x+c)-1
2531*sec(d*x+c)^2-27095*sec(d*x+c)^3-4445*sec(d*x+c)^4+8505*sec(d*x+c)^5+3
465*sec(d*x+c)^6)
```

3.159.5 Fricas [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 386, normalized size of antiderivative = 2.00

$$\int (a + a\sec(c + dx))^{5/2}\tan^5(c + dx)\,dx = \left[\frac{45045\,a^{\frac{5}{2}}\cos(dx+c)^6\log\left(-8\,a\cos(dx+c)^2+4\left(2\cos(dx+c)^2+\cos(dx+c)\right)\sqrt{a}\sqrt{\frac{a\cos(dx+c}{\cos(dx+}\right.\right.$$

input
```
integrate((a+a*sec(d*x+c))^(5/2)*tan(d*x+c)^5,x, algorithm="fricas")
```

output
```
[1/90090*(45045*a^(5/2)*cos(d*x + c)^6*log(-8*a*cos(d*x + c)^2 + 4*(2*cos(
d*x + c)^2 + cos(d*x + c))*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))
 - 8*a*cos(d*x + c) - a) + 4*(71689*a^2*cos(d*x + c)^6 + 31723*a^2*cos(d*x
 + c)^5 - 12531*a^2*cos(d*x + c)^4 - 27095*a^2*cos(d*x + c)^3 - 4445*a^2*c
os(d*x + c)^2 + 8505*a^2*cos(d*x + c) + 3465*a^2)*sqrt((a*cos(d*x + c) + a
)/cos(d*x + c)))/(d*cos(d*x + c)^6), 1/45045*(45045*sqrt(-a)*a^2*arctan(2*
sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(2*a*cos(d*x
 + c) + a))*cos(d*x + c)^6 + 2*(71689*a^2*cos(d*x + c)^6 + 31723*a^2*cos(d
*x + c)^5 - 12531*a^2*cos(d*x + c)^4 - 27095*a^2*cos(d*x + c)^3 - 4445*a^2
*cos(d*x + c)^2 + 8505*a^2*cos(d*x + c) + 3465*a^2)*sqrt((a*cos(d*x + c) +
 a)/cos(d*x + c)))/(d*cos(d*x + c)^6)]
```

3.159.6 Sympy [F(-1)]

Timed out.

$$\int (a + a\sec(c + dx))^{5/2} \tan^5(c + dx)\, dx = \text{Timed out}$$

input
```
integrate((a+a*sec(d*x+c))**(5/2)*tan(d*x+c)**5,x)
```

output
```
Timed out
```

3.159.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 181, normalized size of antiderivative = 0.94

$$\int (a + a\sec(c + dx))^{5/2} \tan^5(c + dx)\, dx = \frac{45045\, a^{\frac{5}{2}} \log\left(\frac{\sqrt{a+\frac{a}{\cos(dx+c)}}-\sqrt{a}}{\sqrt{a+\frac{a}{\cos(dx+c)}}+\sqrt{a}}\right) + 18018\left(a+\frac{a}{\cos(dx+c)}\right)^{\frac{5}{2}} + \frac{6930\left(a+\frac{a}{\cos(dx+c)}\right)^{\frac{13}{2}}}{a^4} - \frac{24570\left(a+\frac{a}{\cos(dx+c)}\right)}{a^3}}{450}$$

input
```
integrate((a+a*sec(d*x+c))^(5/2)*tan(d*x+c)^5,x, algorithm="maxima")
```

output
```
1/45045*(45045*a^(5/2)*log((sqrt(a + a/cos(d*x + c)) - sqrt(a))/(sqrt(a +
a/cos(d*x + c)) + sqrt(a))) + 18018*(a + a/cos(d*x + c))^(5/2) + 6930*(a +
 a/cos(d*x + c))^(13/2)/a^4 - 24570*(a + a/cos(d*x + c))^(11/2)/a^3 + 1001
0*(a + a/cos(d*x + c))^(9/2)/a^2 + 12870*(a + a/cos(d*x + c))^(7/2)/a + 30
030*(a + a/cos(d*x + c))^(3/2)*a + 90090*sqrt(a + a/cos(d*x + c))*a^2)/d
```

3.159.8 Giac [F]

$$\int (a + a\sec(c + dx))^{5/2} \tan^5(c + dx)\, dx = \int (a\sec(dx + c) + a)^{\frac{5}{2}} \tan(dx + c)^5\, dx$$

input
```
integrate((a+a*sec(d*x+c))^(5/2)*tan(d*x+c)^5,x, algorithm="giac")
```

output
```
sage0*x
```

3.159.9 Mupad [F(-1)]

Timed out.

$$\int (a + a\sec(c + dx))^{5/2} \tan^5(c + dx)\, dx = \int \tan(c + d\,x)^5 \left(a + \frac{a}{\cos(c + d\,x)}\right)^{5/2} dx$$

input
```
int(tan(c + d*x)^5*(a + a/cos(c + d*x))^(5/2),x)
```

output
```
int(tan(c + d*x)^5*(a + a/cos(c + d*x))^(5/2), x)
```

3.160 $\int (a + a\sec(c+dx))^{5/2}\tan^3(c+dx)\,dx$

3.160.1 Optimal result 1197
3.160.2 Mathematica [A] (verified) 1197
3.160.3 Rubi [A] (verified) 1198
3.160.4 Maple [A] (verified) 1201
3.160.5 Fricas [A] (verification not implemented) 1201
3.160.6 Sympy [F] 1202
3.160.7 Maxima [A] (verification not implemented) 1202
3.160.8 Giac [F] 1203
3.160.9 Mupad [F(-1)] 1203

3.160.1 Optimal result

Integrand size = 23, antiderivative size = 145

$$\int (a + a\sec(c+dx))^{5/2}\tan^3(c+dx)\,dx = \frac{2a^{5/2}\operatorname{arctanh}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{a}}\right)}{d} - \frac{2a^2\sqrt{a+a\sec(c+dx)}}{d} - \frac{2a(a+a\sec(c+dx))^{3/2}}{3d} - \frac{2(a+a\sec(c+dx))^{5/2}}{5d} - \frac{2(a+a\sec(c+dx))^{7/2}}{7ad} + \frac{2(a+a\sec(c+dx))^{9/2}}{9a^2d}$$

output
```
2*a^(5/2)*arctanh((a+a*sec(d*x+c))^(1/2)/a^(1/2))/d-2/3*a*(a+a*sec(d*x+c))
^(3/2)/d-2/5*(a+a*sec(d*x+c))^(5/2)/d-2/7*(a+a*sec(d*x+c))^(7/2)/a/d+2/9*(
a+a*sec(d*x+c))^(9/2)/a^2/d-2*a^2*(a+a*sec(d*x+c))^(1/2)/d
```

3.160.2 Mathematica [A] (verified)

Time = 0.44 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.70

$$\int (a + a\sec(c+dx))^{5/2}\tan^3(c+dx)\,dx = \frac{2(a(1+\sec(c+dx)))^{5/2}\left(315\operatorname{arctanh}\left(\sqrt{1+\sec(c+dx)}\right) + \sqrt{1+\sec(c+dx)}(-493-226\sec(c+dx)\right.}{315d(1+\sec(c+dx))^{5/2}}$$

input
```
Integrate[(a + a*Sec[c + d*x])^(5/2)*Tan[c + d*x]^3,x]
```

output
```
(2*(a*(1 + Sec[c + d*x]))^(5/2)*(315*ArcTanh[Sqrt[1 + Sec[c + d*x]]] + Sqr
t[1 + Sec[c + d*x]]*(-493 - 226*Sec[c + d*x] + 12*Sec[c + d*x]^2 + 95*Sec[
c + d*x]^3 + 35*Sec[c + d*x]^4)))/(315*d*(1 + Sec[c + d*x])^(5/2))
```

3.160.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.94, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.522$, Rules used = {3042, 25, 4368, 25, 27, 90, 60, 60, 60, 60, 73, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^3(c+dx)(a\sec(c+dx)+a)^{5/2}\,dx$$

$$\downarrow 3042$$

$$\int -\cot\left(c+dx+\frac{\pi}{2}\right)^3\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^{5/2}dx$$

$$\downarrow 25$$

$$-\int \cot\left(\frac{1}{2}(2c+\pi)+dx\right)^3\left(\csc\left(\frac{1}{2}(2c+\pi)+dx\right)a+a\right)^{5/2}dx$$

$$\downarrow 4368$$

$$\frac{\int -a\cos(c+dx)(1-\sec(c+dx))(\sec(c+dx)a+a)^{7/2}d\sec(c+dx)}{a^2d}$$

$$\downarrow 25$$

$$-\frac{\int a\cos(c+dx)(1-\sec(c+dx))(\sec(c+dx)a+a)^{7/2}d\sec(c+dx)}{a^2d}$$

$$\downarrow 27$$

$$-\frac{\int \cos(c+dx)(1-\sec(c+dx))(\sec(c+dx)a+a)^{7/2}d\sec(c+dx)}{ad}$$

$$\downarrow 90$$

$$-\frac{\int \cos(c+dx)(\sec(c+dx)a+a)^{7/2}d\sec(c+dx)-\frac{2(a\sec(c+dx)+a)^{9/2}}{9a}}{ad}$$

$$\downarrow 60$$

$$-\frac{a\int \cos(c+dx)(\sec(c+dx)a+a)^{5/2}d\sec(c+dx)-\frac{2(a\sec(c+dx)+a)^{9/2}}{9a}+\frac{2}{7}(a\sec(c+dx)+a)^{7/2}}{ad}$$

$$\downarrow 60$$

$$-\frac{a\left(a\int \cos(c+dx)(\sec(c+dx)a+a)^{3/2}d\sec(c+dx)+\frac{2}{5}(a\sec(c+dx)+a)^{5/2}\right)-\frac{2(a\sec(c+dx)+a)^{9/2}}{9a}+\frac{2}{7}(a\sec(c+$$
$$ad$$

$$\downarrow 60$$

$$-\frac{a\left(a\left(a\int \cos(c+dx)\sqrt{\sec(c+dx)a+a}d\sec(c+dx)+\frac{2}{3}(a\sec(c+dx)+a)^{3/2}\right)+\frac{2}{5}(a\sec(c+dx)+a)^{5/2}\right)-\frac{2($$
$$ad$$

$$\downarrow 60$$

$$-\frac{a\left(a\left(a\left(a\int \frac{\cos(c+dx)}{\sqrt{\sec(c+dx)a+a}}d\sec(c+dx)+2\sqrt{a\sec(c+dx)+a}\right)+\frac{2}{3}(a\sec(c+dx)+a)^{3/2}\right)+\frac{2}{5}(a\sec(c+dx)+$$
$$ad$$

$$\downarrow 73$$

$$-\frac{a\left(a\left(a\left(2\int \frac{1}{\frac{\sec(c+dx)a+a}{a}-1}d\sqrt{\sec(c+dx)a+a}+2\sqrt{a\sec(c+dx)+a}\right)+\frac{2}{3}(a\sec(c+dx)+a)^{3/2}\right)+\frac{2}{5}(a\sec(c$$
$$ad$$

$$\downarrow 220$$

$$-\frac{a\left(a\left(a\left(2\sqrt{a\sec(c+dx)+a}-2\sqrt{a}\operatorname{arctanh}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{a}}\right)\right)+\frac{2}{3}(a\sec(c+dx)+a)^{3/2}\right)+\frac{2}{5}(a\sec(c+dx)+a$$
$$ad$$

input
```
Int[(a + a*Sec[c + d*x])^(5/2)*Tan[c + d*x]^3,x]
```

output
```
-(((2*(a + a*Sec[c + d*x])^(7/2))/7 - (2*(a + a*Sec[c + d*x])^(9/2))/(9*a)
 + a*((2*(a + a*Sec[c + d*x])^(5/2))/5 + a*((2*(a + a*Sec[c + d*x])^(3/2))
/3 + a*(-2*Sqrt[a]*ArcTanh[Sqrt[a + a*Sec[c + d*x]]/Sqrt[a]] + 2*Sqrt[a +
a*Sec[c + d*x]])))))/(a*d))
```

3.160.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4368

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Simp[-(d*b^(m - 1))^(-1)   Subst[Int[(-a + b*x)^((m - 1)/2
)*((a + b*x)^((m - 1)/2 + n)/x), x], x, Csc[c + d*x]], x] /; FreeQ[{a, b, c
, d, n}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[n]
```

3.160.4 Maple [A] (verified)

Time = 55.97 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.72

method	result
default	$-\frac{2a^2\sqrt{a(1+\sec(dx+c))}\left(315\arctan\left(\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}\right)\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}+493+226\sec(dx+c)-12\sec(dx+c)^2-95\sec(dx+c)^3-35\sec$
	$315d$

input

```
int((a+a*sec(d*x+c))^(5/2)*tan(d*x+c)^3,x,method=_RETURNVERBOSE)
```

output

```
-2/315/d*a^2*(a*(1+sec(d*x+c)))^(1/2)*(315*arctan((-cos(d*x+c)/(cos(d*x+c)
+1))^(1/2))*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+493+226*sec(d*x+c)-12*sec(d
*x+c)^2-95*sec(d*x+c)^3-35*sec(d*x+c)^4)
```

3.160.5 Fricas [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 334, normalized size of antiderivative = 2.30

$$\int (a + a\sec(c + dx))^{5/2}\tan^3(c + dx)\,dx = \left[\frac{315\,a^{\frac{5}{2}}\cos(dx+c)^4\log\left(-8\,a\cos(dx+c)^2 - 4\left(2\cos(dx+c)^2+\cos(dx+c)\right)\sqrt{a}\sqrt{\frac{a\cos(dx+c)+}{\cos(dx+c)}}\right.\right.$$

$$\left.\left.-\frac{315\sqrt{-a}a^2\arctan\left(\frac{2\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)}{2\,a\cos(dx+c)+a}\right)\cos(dx+c)^4 + 2\left(493\,a^2\cos(dx+c)^4 + 226\,a^2\cos(dx+c)\right.}{315\,d\cos(dx+c)^4}\right.$$

input

```
integrate((a+a*sec(d*x+c))^(5/2)*tan(d*x+c)^3,x, algorithm="fricas")
```

output
```
[1/630*(315*a^(5/2)*cos(d*x + c)^4*log(-8*a*cos(d*x + c)^2 - 4*(2*cos(d*x
+ c)^2 + cos(d*x + c))*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)) - 8
*a*cos(d*x + c) - a) - 4*(493*a^2*cos(d*x + c)^4 + 226*a^2*cos(d*x + c)^3
- 12*a^2*cos(d*x + c)^2 - 95*a^2*cos(d*x + c) - 35*a^2)*sqrt((a*cos(d*x +
c) + a)/cos(d*x + c)))/(d*cos(d*x + c)^4), -1/315*(315*sqrt(-a)*a^2*arctan
(2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(2*a*cos(
d*x + c) + a))*cos(d*x + c)^4 + 2*(493*a^2*cos(d*x + c)^4 + 226*a^2*cos(d*
x + c)^3 - 12*a^2*cos(d*x + c)^2 - 95*a^2*cos(d*x + c) - 35*a^2)*sqrt((a*c
os(d*x + c) + a)/cos(d*x + c)))/(d*cos(d*x + c)^4)]
```

3.160.6 Sympy [F]

$$\int (a + a\sec(c+dx))^{5/2}\tan^3(c+dx)\,dx = \int (a(\sec(c+dx)+1))^{\frac{5}{2}}\tan^3(c+dx)\,dx$$

input
```
integrate((a+a*sec(d*x+c))**(5/2)*tan(d*x+c)**3,x)
```

output
```
Integral((a*(sec(c + d*x) + 1))**(5/2)*tan(c + d*x)**3, x)
```

3.160.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.99

$$\int (a + a\sec(c+dx))^{5/2}\tan^3(c+dx)\,dx =$$

$$-\frac{315\,a^{\frac{5}{2}}\log\left(\frac{\sqrt{a+\frac{a}{\cos(dx+c)}}-\sqrt{a}}{\sqrt{a+\frac{a}{\cos(dx+c)}}+\sqrt{a}}\right)+126\left(a+\frac{a}{\cos(dx+c)}\right)^{\frac{5}{2}}-\frac{70\left(a+\frac{a}{\cos(dx+c)}\right)^{\frac{9}{2}}}{a^2}+\frac{90\left(a+\frac{a}{\cos(dx+c)}\right)^{\frac{7}{2}}}{a}+210\left(a+\frac{a}{\cos(dx}\right.}{315\,d}$$

input
```
integrate((a+a*sec(d*x+c))^(5/2)*tan(d*x+c)^3,x, algorithm="maxima")
```

output
```
-1/315*(315*a^(5/2)*log((sqrt(a + a/cos(d*x + c)) - sqrt(a))/(sqrt(a + a/c
os(d*x + c)) + sqrt(a))) + 126*(a + a/cos(d*x + c))^(5/2) - 70*(a + a/cos(
d*x + c))^(9/2)/a^2 + 90*(a + a/cos(d*x + c))^(7/2)/a + 210*(a + a/cos(d*x
 + c))^(3/2)*a + 630*sqrt(a + a/cos(d*x + c))*a^2)/d
```

3.160.8 Giac [F]

$$\int (a + a\sec(c + dx))^{5/2} \tan^3(c + dx)\, dx = \int (a\sec(dx + c) + a)^{\frac{5}{2}} \tan(dx + c)^3\, dx$$

input
```
integrate((a+a*sec(d*x+c))^(5/2)*tan(d*x+c)^3,x, algorithm="giac")
```

output
```
sage0*x
```

3.160.9 Mupad [F(-1)]

Timed out.

$$\int (a + a\sec(c + dx))^{5/2} \tan^3(c + dx)\, dx = \int \tan(c + d\,x)^3 \left(a + \frac{a}{\cos(c + d\,x)}\right)^{5/2} dx$$

input
```
int(tan(c + d*x)^3*(a + a/cos(c + d*x))^(5/2),x)
```

output
```
int(tan(c + d*x)^3*(a + a/cos(c + d*x))^(5/2), x)
```

3.161 $\int (a + a\sec(c+dx))^{5/2}\tan(c+dx)\,dx$

3.161.1 Optimal result 1204
3.161.2 Mathematica [A] (verified) 1204
3.161.3 Rubi [A] (verified) 1205
3.161.4 Maple [A] (verified) 1207
3.161.5 Fricas [A] (verification not implemented) 1207
3.161.6 Sympy [F] 1208
3.161.7 Maxima [A] (verification not implemented) 1208
3.161.8 Giac [F] 1209
3.161.9 Mupad [B] (verification not implemented) 1209

3.161.1 Optimal result

Integrand size = 21, antiderivative size = 97

$$\int (a + a\sec(c+dx))^{5/2}\tan(c+dx)\,dx = -\frac{2a^{5/2}\operatorname{arctanh}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{a}}\right)}{d} + \frac{2a^2\sqrt{a+a\sec(c+dx)}}{d} + \frac{2a(a+a\sec(c+dx))^{3/2}}{3d} + \frac{2(a+a\sec(c+dx))^{5/2}}{5d}$$

output
```
-2*a^(5/2)*arctanh((a+a*sec(d*x+c))^(1/2)/a^(1/2))/d+2/3*a*(a+a*sec(d*x+c)
)^(3/2)/d+2/5*(a+a*sec(d*x+c))^(5/2)/d+2*a^2*(a+a*sec(d*x+c))^(1/2)/d
```

3.161.2 Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.85

$$\int (a + a\sec(c+dx))^{5/2}\tan(c+dx)\,dx = \frac{2(a(1+\sec(c+dx)))^{5/2}\left(-15\operatorname{arctanh}\left(\sqrt{1+\sec(c+dx)}\right) + \sqrt{1+\sec(c+dx)}(23+11\sec(c+dx)+3\sec(c+dx)^2)\right)}{15d(1+\sec(c+dx))^{5/2}}$$

input
```
Integrate[(a + a*Sec[c + d*x])^(5/2)*Tan[c + d*x],x]
```

output
```
(2*(a*(1 + Sec[c + d*x]))^(5/2)*(-15*ArcTanh[Sqrt[1 + Sec[c + d*x]]] + Sqr
t[1 + Sec[c + d*x]]*(23 + 11*Sec[c + d*x] + 3*Sec[c + d*x]^2)))/(15*d*(1 +
 Sec[c + d*x])^(5/2))
```

3.161.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.94, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {3042, 25, 4368, 60, 60, 60, 73, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan(c+dx)(a\sec(c+dx)+a)^{5/2}\,dx$$

$$\downarrow 3042$$

$$\int -\cot\left(c+dx+\frac{\pi}{2}\right)\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^{5/2}dx$$

$$\downarrow 25$$

$$-\int \cot\left(\frac{1}{2}(2c+\pi)+dx\right)\left(\csc\left(\frac{1}{2}(2c+\pi)+dx\right)a+a\right)^{5/2}dx$$

$$\downarrow 4368$$

$$\frac{\int \cos(c+dx)(\sec(c+dx)a+a)^{5/2}d\sec(c+dx)}{d}$$

$$\downarrow 60$$

$$\frac{a\int \cos(c+dx)(\sec(c+dx)a+a)^{3/2}d\sec(c+dx)+\frac{2}{5}(a\sec(c+dx)+a)^{5/2}}{d}$$

$$\downarrow 60$$

$$\frac{a\left(a\int \cos(c+dx)\sqrt{\sec(c+dx)a+a}d\sec(c+dx)+\frac{2}{3}(a\sec(c+dx)+a)^{3/2}\right)+\frac{2}{5}(a\sec(c+dx)+a)^{5/2}}{d}$$

$$\downarrow 60$$

$$\frac{a\left(a\left(a\int \frac{\cos(c+dx)}{\sqrt{\sec(c+dx)a+a}}d\sec(c+dx)+2\sqrt{a\sec(c+dx)+a}\right)+\frac{2}{3}(a\sec(c+dx)+a)^{3/2}\right)+\frac{2}{5}(a\sec(c+dx)+a)^{5/2}}{d}$$

$$\downarrow 73$$

$$\frac{a\left(a\left(2\int \frac{1}{\frac{\sec(c+dx)a+a}{a}-1}d\sqrt{\sec(c+dx)a+a}+2\sqrt{a\sec(c+dx)+a}\right)+\frac{2}{3}(a\sec(c+dx)+a)^{3/2}\right)+\frac{2}{5}(a\sec(c+dx)+a)^{5/2}}{d}$$

$$\downarrow 220$$

$$\frac{a\left(a\left(2\sqrt{a\sec(c+dx)+a}-2\sqrt{a}\operatorname{arctanh}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{a}}\right)\right)+\frac{2}{3}(a\sec(c+dx)+a)^{3/2}\right)+\frac{2}{5}(a\sec(c+dx)+a)^{5/2}}{d}$$

input
```
Int[(a + a*Sec[c + d*x])^(5/2)*Tan[c + d*x],x]
```

output
```
((2*(a + a*Sec[c + d*x])^(5/2))/5 + a*((2*(a + a*Sec[c + d*x])^(3/2))/3 +
a*(-2*Sqrt[a]*ArcTanh[Sqrt[a + a*Sec[c + d*x]]/Sqrt[a]] + 2*Sqrt[a + a*Sec
[c + d*x]])))/d
```

3.161.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4368
```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Simp[-(d*b^(m - 1))^(-1)   Subst[Int[(-a + b*x)^((m - 1)/2
)*((a + b*x)^((m - 1)/2 + n)/x), x], x, Csc[c + d*x]], x] /; FreeQ[{a, b, c
, d, n}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[n]
```

3.161.4 Maple [A] (verified)

Time = 7.18 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.76

method	result	size
derivativedivides	$\frac{\frac{2(a+a\sec(dx+c))^{\frac{5}{2}}}{5}+\frac{2a(a+a\sec(dx+c))^{\frac{3}{2}}}{3}+2\sqrt{a+a\sec(dx+c)}\,a^2-2a^{\frac{5}{2}}\operatorname{arctanh}\left(\frac{\sqrt{a+a\sec(dx+c)}}{\sqrt{a}}\right)}{d}$	74
default	$\frac{\frac{2(a+a\sec(dx+c))^{\frac{5}{2}}}{5}+\frac{2a(a+a\sec(dx+c))^{\frac{3}{2}}}{3}+2\sqrt{a+a\sec(dx+c)}\,a^2-2a^{\frac{5}{2}}\operatorname{arctanh}\left(\frac{\sqrt{a+a\sec(dx+c)}}{\sqrt{a}}\right)}{d}$	74

input
```
int((a+a*sec(d*x+c))^(5/2)*tan(d*x+c),x,method=_RETURNVERBOSE)
```

output
```
1/d*(2/5*(a+a*sec(d*x+c))^(5/2)+2/3*a*(a+a*sec(d*x+c))^(3/2)+2*(a+a*sec(d*
x+c))^(1/2)*a^2-2*a^(5/2)*arctanh((a+a*sec(d*x+c))^(1/2)/a^(1/2)))
```

3.161.5 Fricas [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 282, normalized size of antiderivative = 2.91

$$\int (a+a\sec(c+dx))^{5/2}\tan(c+dx)\,dx = \left[\frac{15\,a^{\frac{5}{2}}\cos(dx+c)^2\log\left(-8\,a\cos(dx+c)^2+4\left(2\cos(dx+c)^2+\cos(dx+c)\right)\sqrt{a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\right.\right.}{30\,d\cos(dx$$

input
```
integrate((a+a*sec(d*x+c))^(5/2)*tan(d*x+c),x, algorithm="fricas")
```

output
```
[1/30*(15*a^(5/2)*cos(d*x + c)^2*log(-8*a*cos(d*x + c)^2 + 4*(2*cos(d*x +
c)^2 + cos(d*x + c))*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)) - 8*a
*cos(d*x + c) - a) + 4*(23*a^2*cos(d*x + c)^2 + 11*a^2*cos(d*x + c) + 3*a^
2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)))/(d*cos(d*x + c)^2), 1/15*(15*s
qrt(-a)*a^2*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(
d*x + c)/(2*a*cos(d*x + c) + a))*cos(d*x + c)^2 + 2*(23*a^2*cos(d*x + c)^2
 + 11*a^2*cos(d*x + c) + 3*a^2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)))/(
d*cos(d*x + c)^2)]
```

3.161.6 Sympy [F]

$$\int (a + a\sec(c + dx))^{5/2} \tan(c + dx)\, dx = \int (a(\sec(c + dx) + 1))^{\frac{5}{2}} \tan(c + dx)\, dx$$

input
```
integrate((a+a*sec(d*x+c))**(5/2)*tan(d*x+c),x)
```

output
```
Integral((a*(sec(c + d*x) + 1))**(5/2)*tan(c + d*x), x)
```

3.161.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.08

$$\int (a + a\sec(c + dx))^{5/2} \tan(c + dx)\, dx = \frac{15\, a^{\frac{5}{2}} \log\left(\frac{\sqrt{a + \frac{a}{\cos(dx+c)}} - \sqrt{a}}{\sqrt{a + \frac{a}{\cos(dx+c)}} + \sqrt{a}}\right) + 6\left(a + \frac{a}{\cos(dx+c)}\right)^{\frac{5}{2}} + 10\left(a + \frac{a}{\cos(dx+c)}\right)^{\frac{3}{2}} a + 30\sqrt{a + \frac{a}{\cos(dx+c)}}\, a^2}{15\, d}$$

input
```
integrate((a+a*sec(d*x+c))^(5/2)*tan(d*x+c),x, algorithm="maxima")
```

output
```
1/15*(15*a^(5/2)*log((sqrt(a + a/cos(d*x + c)) - sqrt(a))/(sqrt(a + a/cos(
d*x + c)) + sqrt(a))) + 6*(a + a/cos(d*x + c))^(5/2) + 10*(a + a/cos(d*x +
 c))^(3/2)*a + 30*sqrt(a + a/cos(d*x + c))*a^2)/d
```

3.161.8 Giac [F]

$$\int (a + a\sec(c+dx))^{5/2}\tan(c+dx)\,dx = \int (a\sec(dx+c)+a)^{\frac{5}{2}}\tan(dx+c)\,dx$$

input
```
integrate((a+a*sec(d*x+c))^(5/2)*tan(d*x+c),x, algorithm="giac")
```

output
```
sage0*x
```

3.161.9 Mupad [B] (verification not implemented)

Time = 13.96 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.95

$$\int (a + a\sec(c+dx))^{5/2}\tan(c+dx)\,dx = \frac{2\left(a+\frac{a}{\cos(c+dx)}\right)^{5/2}}{5\,d} + \frac{2\,a\left(a+\frac{a}{\cos(c+dx)}\right)^{3/2}}{3\,d} + \frac{2\,a^2\sqrt{a+\frac{a}{\cos(c+dx)}}}{d} + \frac{a^{5/2}\,\mathrm{atan}\left(\frac{\sqrt{a+\frac{a}{\cos(c+dx)}}\,1\mathrm{i}}{\sqrt{a}}\right)\,2\mathrm{i}}{d}$$

input
```
int(tan(c + d*x)*(a + a/cos(c + d*x))^(5/2),x)
```

output
```
(2*(a + a/cos(c + d*x))^(5/2))/(5*d) + (a^(5/2)*atan(((a + a/cos(c + d*x))
^(1/2)*1i)/a^(1/2))*2i)/d + (2*a*(a + a/cos(c + d*x))^(3/2))/(3*d) + (2*a^
2*(a + a/cos(c + d*x))^(1/2))/d
```

3.162 $\int \cot(c+dx)(a+a\sec(c+dx))^{5/2}\,dx$

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3.162.9 Mupad [F(-1)] 1216

3.162.1 Optimal result

Integrand size = 21, antiderivative size = 95

$$\int \cot(c+dx)(a+a\sec(c+dx))^{5/2}\,dx = \frac{2a^{5/2}\operatorname{arctanh}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{a}}\right)}{d} - \frac{4\sqrt{2}a^{5/2}\operatorname{arctanh}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{d} + \frac{2a^2\sqrt{a+a\sec(c+dx)}}{d}$$

output
```
2*a^(5/2)*arctanh((a+a*sec(d*x+c))^(1/2)/a^(1/2))/d-4*a^(5/2)*arctanh(1/2*
(a+a*sec(d*x+c))^(1/2)*2^(1/2)/a^(1/2))*2^(1/2)/d+2*a^2*(a+a*sec(d*x+c))^(
1/2)/d
```

3.162.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.87

$$\int \cot(c+dx)(a+a\sec(c+dx))^{5/2}\,dx = \frac{2(a(1+\sec(c+dx)))^{5/2}\left(\operatorname{arctanh}\left(\sqrt{1+\sec(c+dx)}\right) - 2\sqrt{2}\operatorname{arctanh}\left(\frac{\sqrt{1+\sec(c+dx)}}{\sqrt{2}}\right) + \sqrt{1+}\right.}{d(1+\sec(c+dx))^{5/2}}$$

input
```
Integrate[Cot[c + d*x]*(a + a*Sec[c + d*x])^(5/2),x]
```

output
```
(2*(a*(1 + Sec[c + d*x]))^(5/2)*(ArcTanh[Sqrt[1 + Sec[c + d*x]]] - 2*Sqrt[
2]*ArcTanh[Sqrt[1 + Sec[c + d*x]]/Sqrt[2]] + Sqrt[1 + Sec[c + d*x]]))/(d*(
1 + Sec[c + d*x])^(5/2))
```

3.162.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {3042, 25, 4368, 25, 27, 95, 25, 27, 174, 73, 219, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot(c+dx)(a\sec(c+dx)+a)^{5/2}\,dx$$

$$\downarrow 3042$$

$$\int -\frac{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^{5/2}}{\cot\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\downarrow 25$$

$$-\int \frac{\left(\csc\left(\frac{1}{2}(2c+\pi)+dx\right)a+a\right)^{5/2}}{\cot\left(\frac{1}{2}(2c+\pi)+dx\right)}dx$$

$$\downarrow 4368$$

$$\frac{a^2\int -\frac{\cos(c+dx)(\sec(c+dx)a+a)^{3/2}}{a(1-\sec(c+dx))}d\sec(c+dx)}{d}$$

$$\downarrow 25$$

$$-\frac{a^2\int \frac{\cos(c+dx)(\sec(c+dx)a+a)^{3/2}}{a(1-\sec(c+dx))}d\sec(c+dx)}{d}$$

$$\downarrow 27$$

$$-\frac{a\int \frac{\cos(c+dx)(\sec(c+dx)a+a)^{3/2}}{1-\sec(c+dx)}d\sec(c+dx)}{d}$$

$$\downarrow 95$$

$$-\frac{a\left(-\int -\frac{a^2\cos(c+dx)(3\sec(c+dx)+1)}{(1-\sec(c+dx))\sqrt{\sec(c+dx)a+a}}d\sec(c+dx)-2a\sqrt{a\sec(c+dx)+a}\right)}{d}$$

$$\downarrow 25$$

$$-\frac{a\left(\int \frac{a^2\cos(c+dx)(3\sec(c+dx)+1)}{(1-\sec(c+dx))\sqrt{\sec(c+dx)a+a}}d\sec(c+dx) - 2a\sqrt{a\sec(c+dx)+a}\right)}{d}$$

$$\downarrow 27$$

$$-\frac{a\left(a^2\int \frac{\cos(c+dx)(3\sec(c+dx)+1)}{(1-\sec(c+dx))\sqrt{\sec(c+dx)a+a}}d\sec(c+dx) - 2a\sqrt{a\sec(c+dx)+a}\right)}{d}$$

$$\downarrow 174$$

$$-\frac{a\left(a^2\left(4\int \frac{1}{(1-\sec(c+dx))\sqrt{\sec(c+dx)a+a}}d\sec(c+dx) + \int \frac{\cos(c+dx)}{\sqrt{\sec(c+dx)a+a}}d\sec(c+dx)\right) - 2a\sqrt{a\sec(c+dx)+a}\right)}{d}$$

$$\downarrow 73$$

$$-\frac{a\left(a^2\left(\frac{8\int \frac{1}{2-\frac{\sec(c+dx)a+a}{a}}d\sqrt{\sec(c+dx)a+a}}{a} + \frac{2\int \frac{1}{\frac{\sec(c+dx)a+a}{a}-1}d\sqrt{\sec(c+dx)a+a}}{a}\right) - 2a\sqrt{a\sec(c+dx)+a}\right)}{d}$$

$$\downarrow 219$$

$$-\frac{a\left(a^2\left(\frac{2\int \frac{1}{\frac{\sec(c+dx)a+a}{a}-1}d\sqrt{\sec(c+dx)a+a}}{a} + \frac{4\sqrt{2}\text{arctanh}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{2}\sqrt{a}}\right)}{\sqrt{a}}\right) - 2a\sqrt{a\sec(c+dx)+a}\right)}{d}$$

$$\downarrow 220$$

$$-\frac{a\left(a^2\left(\frac{4\sqrt{2}\text{arctanh}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{2}\sqrt{a}}\right)}{\sqrt{a}} - \frac{2\text{arctanh}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{a}}\right)}{\sqrt{a}}\right) - 2a\sqrt{a\sec(c+dx)+a}\right)}{d}$$

input
```
Int[Cot[c + d*x]*(a + a*Sec[c + d*x])^(5/2),x]
```

output
```
-((a*(a^2*((-2*ArcTanh[Sqrt[a + a*Sec[c + d*x]]/Sqrt[a]])/Sqrt[a] + (4*Sqr
t[2]*ArcTanh[Sqrt[a + a*Sec[c + d*x]]/(Sqrt[2]*Sqrt[a])])/Sqrt[a]) - 2*a*S
qrt[a + a*Sec[c + d*x]]))/d)
```

3.162.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 95
```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_] :> Simp[f*((e + f*x)^(p - 1)/(b*d*(p - 1))), x] + Simp[1/(b*d)   Int[(b
*d*e^2 - a*c*f^2 + f*(2*b*d*e - b*c*f - a*d*f)*x)*((e + f*x)^(p - 2)/((a +
b*x)*(c + d*x))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 1]
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4368
```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Simp[-(d*b^(m - 1))^(-1)   Subst[Int[(-a + b*x)^((m - 1)/2
)*((a + b*x)^((m - 1)/2 + n)/x), x], x, Csc[c + d*x]], x] /; FreeQ[{a, b, c
, d, n}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[n]
```

3.162.4 Maple [A] (verified)

Time = 5.42 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.22

method	result	s
default	$-\frac{2a^2\sqrt{a(1+\sec(dx+c))}\left(2\sqrt{2}\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}\arctan\left(\frac{\sqrt{2}}{2\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}}\right)+\arctan\left(\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}\right)\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}-1\right)}{d}$	1

input
```
int(cot(d*x+c)*(a+a*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-2/d*a^2*(a*(1+sec(d*x+c)))^(1/2)*(2*2^(1/2)*(-cos(d*x+c)/(cos(d*x+c)+1))^
(1/2)*arctan(1/2*2^(1/2)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))+arctan((-cos(
d*x+c)/(cos(d*x+c)+1))^(1/2))*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)-1)
```

3.162.5 Fricas [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 300, normalized size of antiderivative = 3.16

$$\int \cot(c+dx)(a+a\sec(c+dx))^{5/2}\,dx = \left[\frac{2\sqrt{2}a^{\frac{5}{2}}\log\left(-\frac{2\sqrt{2}\sqrt{a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)-3a\cos(dx+c)-a}{\cos(dx+c)-1}\right)+a^{\frac{5}{2}}\log\left(-2a\cos(dx+c)-2\sqrt{a}\right.}{d}\right.$$

input
```
integrate(cot(d*x+c)*(a+a*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
[(2*sqrt(2)*a^(5/2)*log(-(2*sqrt(2)*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(
d*x + c))*cos(d*x + c) - 3*a*cos(d*x + c) - a)/(cos(d*x + c) - 1)) + a^(5/
2)*log(-2*a*cos(d*x + c) - 2*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c
))*cos(d*x + c) - a) + 2*a^2*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)))/d, 2
*(2*sqrt(2)*sqrt(-a)*a^2*arctan(sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)
/cos(d*x + c))*cos(d*x + c)/(a*cos(d*x + c) + a)) - sqrt(-a)*a^2*arctan(sq
rt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(a*cos(d*x + c
) + a)) + a^2*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)))/d]
```

3.162.6 Sympy [F(-1)]

Timed out.

$$\int \cot(c+dx)(a+a\sec(c+dx))^{5/2}\,dx = \text{Timed out}$$

input
```
integrate(cot(d*x+c)*(a+a*sec(d*x+c))**(5/2),x)
```

output
```
Timed out
```

3.162.7 Maxima [F]

$$\int \cot(c+dx)(a+a\sec(c+dx))^{5/2}\,dx = \int (a\sec(dx+c)+a)^{\frac{5}{2}}\cot(dx+c)\,dx$$

input
```
integrate(cot(d*x+c)*(a+a*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
integrate((a*sec(d*x + c) + a)^(5/2)*cot(d*x + c), x)
```

3.162.8 Giac [F]

$$\int \cot(c+dx)(a+a\sec(c+dx))^{5/2}\,dx = \int (a\sec(dx+c)+a)^{\frac{5}{2}}\cot(dx+c)\,dx$$

input
```
integrate(cot(d*x+c)*(a+a*sec(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.162.9 Mupad [F(-1)]

Timed out.

$$\int \cot(c+dx)(a+a\sec(c+dx))^{5/2}\,dx = \int \cot(c+d\,x)\left(a+\frac{a}{\cos(c+d\,x)}\right)^{5/2}dx$$

input
```
int(cot(c + d*x)*(a + a/cos(c + d*x))^(5/2),x)
```

output
```
int(cot(c + d*x)*(a + a/cos(c + d*x))^(5/2), x)
```

3.163 $\int \cot^3(c+dx)(a+a\sec(c+dx))^{5/2}\,dx$

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3.163.1 Optimal result

Integrand size = 23, antiderivative size = 106

$$\int \cot^3(c+dx)(a+a\sec(c+dx))^{5/2}\,dx = -\frac{2a^{5/2}\operatorname{arctanh}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{a}}\right)}{d} + \frac{3a^{5/2}\operatorname{arctanh}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{\sqrt{2}d} + \frac{a^2\sqrt{a+a\sec(c+dx)}}{d(1-\sec(c+dx))}$$

output
```
-2*a^(5/2)*arctanh((a+a*sec(d*x+c))^(1/2)/a^(1/2))/d+3/2*a^(5/2)*arctanh(1
/2*(a+a*sec(d*x+c))^(1/2)*2^(1/2)/a^(1/2))*2^(1/2)/d+a^2*(a+a*sec(d*x+c))^
(1/2)/d/(1-sec(d*x+c))
```

3.163.2 Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.91

$$\int \cot^3(c+dx)(a+a\sec(c+dx))^{5/2}\,dx = \frac{(a(1+\sec(c+dx)))^{5/2}\left(-2\operatorname{arctanh}\left(\sqrt{1+\sec(c+dx)}\right) + \frac{3\operatorname{arctanh}\left(\frac{\sqrt{1+\sec(c+dx)}}{\sqrt{2}}\right)}{\sqrt{2}} - \frac{\sqrt{1+\sec(c+dx)}}{-1+\sec(c+dx)}\right)}{d(1+\sec(c+dx))^{5/2}}$$

input
```
Integrate[Cot[c + d*x]^3*(a + a*Sec[c + d*x])^(5/2),x]
```

output
```
((a*(1 + Sec[c + d*x]))^(5/2)*(-2*ArcTanh[Sqrt[1 + Sec[c + d*x]]] + (3*Arc
Tanh[Sqrt[1 + Sec[c + d*x]]/Sqrt[2]])/Sqrt[2] - Sqrt[1 + Sec[c + d*x]]/(-1
 + Sec[c + d*x])))/(d*(1 + Sec[c + d*x])^(5/2))
```

3.163.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.01, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.435$, Rules used = {3042, 25, 4368, 27, 110, 27, 174, 73, 219, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^3(c+dx)(a\sec(c+dx)+a)^{5/2}\,dx$$

$$\downarrow 3042$$

$$\int -\frac{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^{5/2}}{\cot\left(c+dx+\frac{\pi}{2}\right)^3}dx$$

$$\downarrow 25$$

$$-\int \frac{\left(\csc\left(\frac{1}{2}(2c+\pi)+dx\right)a+a\right)^{5/2}}{\cot\left(\frac{1}{2}(2c+\pi)+dx\right)^3}dx$$

$$\downarrow 4368$$

$$\frac{a^4\int \frac{\cos(c+dx)\sqrt{\sec(c+dx)a+a}}{a^2(1-\sec(c+dx))^2}d\sec(c+dx)}{d}$$

$$\downarrow 27$$

$$\frac{a^2\int \frac{\cos(c+dx)\sqrt{\sec(c+dx)a+a}}{(1-\sec(c+dx))^2}d\sec(c+dx)}{d}$$

$$\downarrow 110$$

$$\frac{a^2\left(\frac{\sqrt{a\sec(c+dx)+a}}{1-\sec(c+dx)}-\int -\frac{a\cos(c+dx)(\sec(c+dx)+2)}{2(1-\sec(c+dx))\sqrt{\sec(c+dx)a+a}}d\sec(c+dx)\right)}{d}$$

$$\downarrow 27$$

$$\frac{a^2\left(\frac{1}{2}a\int \frac{\cos(c+dx)(\sec(c+dx)+2)}{(1-\sec(c+dx))\sqrt{\sec(c+dx)a+a}}d\sec(c+dx)+\frac{\sqrt{a\sec(c+dx)+a}}{1-\sec(c+dx)}\right)}{d}$$

$$\downarrow 174$$

$$\frac{a^2\left(\frac{1}{2}a\left(3\int\frac{1}{(1-\sec(c+dx))\sqrt{\sec(c+dx)a+a}}d\sec(c+dx)+2\int\frac{\cos(c+dx)}{\sqrt{\sec(c+dx)a+a}}d\sec(c+dx)\right)+\frac{\sqrt{a\sec(c+dx)+a}}{1-\sec(c+dx)}\right)}{d}$$

$$\downarrow 73$$

$$\frac{a^2\left(\frac{1}{2}a\left(\frac{6\int\frac{1}{2-\frac{\sec(c+dx)a+a}{a}}d\sqrt{\sec(c+dx)a+a}}{a}+\frac{4\int\frac{1}{\frac{\sec(c+dx)a+a}{a}-1}d\sqrt{\sec(c+dx)a+a}}{a}\right)+\frac{\sqrt{a\sec(c+dx)+a}}{1-\sec(c+dx)}\right)}{d}$$

$$\downarrow 219$$

$$\frac{a^2\left(\frac{1}{2}a\left(\frac{4\int\frac{1}{\frac{\sec(c+dx)a+a}{a}-1}d\sqrt{\sec(c+dx)a+a}}{a}+\frac{3\sqrt{2}\operatorname{arctanh}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{2}\sqrt{a}}\right)}{\sqrt{a}}\right)+\frac{\sqrt{a\sec(c+dx)+a}}{1-\sec(c+dx)}\right)}{d}$$

$$\downarrow 220$$

$$\frac{a^2\left(\frac{1}{2}a\left(\frac{3\sqrt{2}\operatorname{arctanh}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{2}\sqrt{a}}\right)}{\sqrt{a}}-\frac{4\operatorname{arctanh}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{a}}\right)}{\sqrt{a}}\right)+\frac{\sqrt{a\sec(c+dx)+a}}{1-\sec(c+dx)}\right)}{d}$$

input
```
Int[Cot[c + d*x]^3*(a + a*Sec[c + d*x])^(5/2),x]
```

output
```
(a^2*((a*((-4*ArcTanh[Sqrt[a + a*Sec[c + d*x]]/Sqrt[a]])/Sqrt[a] + (3*Sqrt
[2]*ArcTanh[Sqrt[a + a*Sec[c + d*x]]/(Sqrt[2]*Sqrt[a])])/Sqrt[a]))/2 + Sqr
t[a + a*Sec[c + d*x]]/(1 - Sec[c + d*x])))/d
```

3.163.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 110
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)/((m +
1)*(b*e - a*f))), x] - Simp[1/((m + 1)*(b*e - a*f))   Int[(a + b*x)^(m + 1)
*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m + n +
p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1] && Gt
Q[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p,
 m + n])
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4368
```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Simp[-(d*b^(m - 1))^(-1)   Subst[Int[(-a + b*x)^((m - 1)/2
)*((a + b*x)^((m - 1)/2 + n)/x), x], x, Csc[c + d*x]], x] /; FreeQ[{a, b, c
, d, n}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[n]
```

3.163.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 239 vs. $2(89) = 178$.

Time = 15.21 (sec) , antiderivative size = 240, normalized size of antiderivative = 2.26

method	result
default	$\frac{a^2\sqrt{a(1+\sec(dx+c))}\left(3\arctan\left(\frac{\sqrt{2}}{2\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}}\right)\cos(dx+c)\sqrt{2}\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}+4\arctan\left(\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}\right)\cos(dx+c)\sqrt{-\frac{\cos}{\cos(}}\right.}{2d(\cos(dx+c)}$

input
```
int(cot(d*x+c)^3*(a+a*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/2/d*a^2*(a*(1+sec(d*x+c)))^(1/2)*(3*arctan(1/2*2^(1/2)/(-cos(d*x+c)/(cos
(d*x+c)+1))^(1/2))*cos(d*x+c)*2^(1/2)*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+4
*arctan((-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*cos(d*x+c)*(-cos(d*x+c)/(cos(d
*x+c)+1))^(1/2)-3*2^(1/2)*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctan(1/2*2^
(1/2)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))-4*arctan((-cos(d*x+c)/(cos(d*x+c
)+1))^(1/2))*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+2*cos(d*x+c))/(cos(d*x+c)-
1)
```

3.163.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 196 vs. $2(88) = 176$.

Time = 0.34 (sec) , antiderivative size = 398, normalized size of antiderivative = 3.75

$$\int \cot^3(c+dx)(a+a\sec(c+dx))^{5/2}\,dx = \left[\frac{4\,a^2\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)+4\,(a^2\cos(dx+c)-a^2)\sqrt{a}\log\left(-2\,a\cos(dx+c)+2\sqrt{a}\sqrt{\right.}\right.}{}$$

input
```
integrate(cot(d*x+c)^3*(a+a*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
[1/4*(4*a^2*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c) + 4*(a^2*
cos(d*x + c) - a^2)*sqrt(a)*log(-2*a*cos(d*x + c) + 2*sqrt(a)*sqrt((a*cos(
d*x + c) + a)/cos(d*x + c))*cos(d*x + c) - a) + 3*(sqrt(2)*a^2*cos(d*x + c
) - sqrt(2)*a^2)*sqrt(a)*log((2*sqrt(2)*sqrt(a)*sqrt((a*cos(d*x + c) + a)/
cos(d*x + c))*cos(d*x + c) + 3*a*cos(d*x + c) + a)/(cos(d*x + c) - 1)))/(d
*cos(d*x + c) - d), 1/2*(2*a^2*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos
(d*x + c) - 3*(sqrt(2)*a^2*cos(d*x + c) - sqrt(2)*a^2)*sqrt(-a)*arctan(sqr
t(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(a*cos(
d*x + c) + a)) + 4*(a^2*cos(d*x + c) - a^2)*sqrt(-a)*arctan(sqrt(-a)*sqrt(
(a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(a*cos(d*x + c) + a)))/(d*
cos(d*x + c) - d)]
```

3.163.6 Sympy [F(-1)]

Timed out.

$$\int \cot^3(c+dx)(a+a\sec(c+dx))^{5/2}\,dx = \text{Timed out}$$

input
```
integrate(cot(d*x+c)**3*(a+a*sec(d*x+c))**(5/2),x)
```

output
```
Timed out
```

3.163.7 Maxima [F]

$$\int \cot^3(c+dx)(a+a\sec(c+dx))^{5/2}\,dx = \int (a\sec(dx+c)+a)^{\frac{5}{2}}\cot(dx+c)^3\,dx$$

input
```
integrate(cot(d*x+c)^3*(a+a*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
integrate((a*sec(d*x + c) + a)^(5/2)*cot(d*x + c)^3, x)
```

3.163.8 Giac [F]

$$\int \cot^3(c+dx)(a+a\sec(c+dx))^{5/2}\,dx = \int (a\sec(dx+c)+a)^{\frac{5}{2}}\cot(dx+c)^3\,dx$$

input
```
integrate(cot(d*x+c)^3*(a+a*sec(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.163.9 Mupad [F(-1)]

Timed out.

$$\int \cot^3(c+dx)(a+a\sec(c+dx))^{5/2}\,dx = \int \cot(c+d\,x)^3\left(a+\frac{a}{\cos(c+d\,x)}\right)^{5/2}dx$$

input
```
int(cot(c + d*x)^3*(a + a/cos(c + d*x))^(5/2),x)
```

output
```
int(cot(c + d*x)^3*(a + a/cos(c + d*x))^(5/2), x)
```

3.164 $\int \cot^5(c+dx)(a+a\sec(c+dx))^{5/2}\,dx$

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3.164.2 Mathematica [A] (verified) 1224
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3.164.8 Giac [F] 1230
3.164.9 Mupad [F(-1)] 1231

3.164.1 Optimal result

Integrand size = 23, antiderivative size = 147

$$\int \cot^5(c+dx)(a+a\sec(c+dx))^{5/2}\,dx = \frac{2a^{5/2}\operatorname{arctanh}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{a}}\right)}{d} - \frac{43a^{5/2}\operatorname{arctanh}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{16\sqrt{2}d} - \frac{a^2\sqrt{a+a\sec(c+dx)}}{4d(1-\sec(c+dx))^2} - \frac{11a^2\sqrt{a+a\sec(c+dx)}}{16d(1-\sec(c+dx))}$$

output

```
2*a^(5/2)*arctanh((a+a*sec(d*x+c))^(1/2)/a^(1/2))/d-43/32*a^(5/2)*arctanh(
1/2*(a+a*sec(d*x+c))^(1/2)*2^(1/2)/a^(1/2))*2^(1/2)/d-1/4*a^2*(a+a*sec(d*x
+c))^(1/2)/d/(1-sec(d*x+c))^2-11/16*a^2*(a+a*sec(d*x+c))^(1/2)/d/(1-sec(d*
x+c))
```

3.164.2 Mathematica [A] (verified)

Time = 0.69 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.74

$$\int \cot^5(c+dx)(a+a\sec(c+dx))^{5/2}\,dx = \frac{(a(1+\sec(c+dx)))^{5/2}\left(64\operatorname{arctanh}\left(\sqrt{1+\sec(c+dx)}\right) - 43\sqrt{2}\operatorname{arctanh}\left(\frac{\sqrt{1+\sec(c+dx)}}{\sqrt{2}}\right) + \frac{2\sqrt{1}}{32d(1+\sec(c+dx))^{5/2}}\right.$$

input

```
Integrate[Cot[c + d*x]^5*(a + a*Sec[c + d*x])^(5/2),x]
```

output
```
((a*(1 + Sec[c + d*x]))^(5/2)*(64*ArcTanh[Sqrt[1 + Sec[c + d*x]]] - 43*Sqr
t[2]*ArcTanh[Sqrt[1 + Sec[c + d*x]]/Sqrt[2]] + (2*Sqrt[1 + Sec[c + d*x]]*(
-15 + 11*Sec[c + d*x]))/(-1 + Sec[c + d*x])^2))/(32*d*(1 + Sec[c + d*x])^(
5/2))
```

3.164.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.03, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.565$, Rules used = {3042, 25, 4368, 25, 27, 114, 27, 168, 27, 174, 73, 219, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^5(c+dx)(a\sec(c+dx)+a)^{5/2}\,dx$$

$$\downarrow 3042$$

$$\int -\frac{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^{5/2}}{\cot\left(c+dx+\frac{\pi}{2}\right)^5}dx$$

$$\downarrow 25$$

$$-\int \frac{\left(\csc\left(\frac{1}{2}(2c+\pi)+dx\right)a+a\right)^{5/2}}{\cot\left(\frac{1}{2}(2c+\pi)+dx\right)^5}dx$$

$$\downarrow 4368$$

$$\frac{a^6\int -\frac{\cos(c+dx)}{a^3(1-\sec(c+dx))^3\sqrt{\sec(c+dx)a+a}}d\sec(c+dx)}{d}$$

$$\downarrow 25$$

$$-\frac{a^6\int \frac{\cos(c+dx)}{a^3(1-\sec(c+dx))^3\sqrt{\sec(c+dx)a+a}}d\sec(c+dx)}{d}$$

$$\downarrow 27$$

$$-\frac{a^3\int \frac{\cos(c+dx)}{(1-\sec(c+dx))^3\sqrt{\sec(c+dx)a+a}}d\sec(c+dx)}{d}$$

$$\downarrow 114$$

$$-\frac{a^3\left(\frac{\sqrt{a\sec(c+dx)+a}}{4a(1-\sec(c+dx))^2}-\frac{\int -\frac{a\cos(c+dx)(3\sec(c+dx)+8)}{2(1-\sec(c+dx))^2\sqrt{\sec(c+dx)a+a}}d\sec(c+dx)}{4a}\right)}{d}$$

$$\downarrow 27$$

$$-\frac{a^3\left(\frac{1}{8}\int\frac{\cos(c+dx)(3\sec(c+dx)+8)}{(1-\sec(c+dx))^2\sqrt{\sec(c+dx)a+a}}d\sec(c+dx)+\frac{\sqrt{a\sec(c+dx)+a}}{4a(1-\sec(c+dx))^2}\right)}{d}$$

$$\downarrow 168$$

$$-\frac{a^3\left(\frac{1}{8}\left(\frac{11\sqrt{a\sec(c+dx)+a}}{2a(1-\sec(c+dx))}-\frac{\int -\frac{a\cos(c+dx)(11\sec(c+dx)+32)}{2(1-\sec(c+dx))\sqrt{\sec(c+dx)a+a}}d\sec(c+dx)}{2a}\right)+\frac{\sqrt{a\sec(c+dx)+a}}{4a(1-\sec(c+dx))^2}\right)}{d}$$

$$\downarrow 27$$

$$-\frac{a^3\left(\frac{1}{8}\left(\frac{1}{4}\int\frac{\cos(c+dx)(11\sec(c+dx)+32)}{(1-\sec(c+dx))\sqrt{\sec(c+dx)a+a}}d\sec(c+dx)+\frac{11\sqrt{a\sec(c+dx)+a}}{2a(1-\sec(c+dx))}\right)+\frac{\sqrt{a\sec(c+dx)+a}}{4a(1-\sec(c+dx))^2}\right)}{d}$$

$$\downarrow 174$$

$$-\frac{a^3\left(\frac{1}{8}\left(\frac{1}{4}\left(43\int\frac{1}{(1-\sec(c+dx))\sqrt{\sec(c+dx)a+a}}d\sec(c+dx)+32\int\frac{\cos(c+dx)}{\sqrt{\sec(c+dx)a+a}}d\sec(c+dx)\right)+\frac{11\sqrt{a\sec(c+dx)+a}}{2a(1-\sec(c+dx))}\right)+\right.}{d}$$

$$\downarrow 73$$

$$-\frac{a^3\left(\frac{1}{8}\left(\frac{1}{4}\left(\frac{86\int\frac{1}{2-\frac{\sec(c+dx)a+a}{a}}d\sqrt{\sec(c+dx)a+a}}{a}+\frac{64\int\frac{1}{\frac{\sec(c+dx)a+a}{a}-1}d\sqrt{\sec(c+dx)a+a}}{a}\right)+\frac{11\sqrt{a\sec(c+dx)+a}}{2a(1-\sec(c+dx))}\right)+\frac{\sqrt{a\sec(c+dx)+a}}{4a(1-\sec(c+\right.}{d}$$

$$\downarrow 219$$

$$-\frac{a^3\left(\frac{1}{8}\left(\frac{1}{4}\left(\frac{64\int\frac{1}{\frac{\sec(c+dx)a+a}{a}-1}d\sqrt{\sec(c+dx)a+a}}{a}+\frac{43\sqrt{2}\operatorname{arctanh}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{2}\sqrt{a}}\right)}{\sqrt{a}}\right)+\frac{11\sqrt{a\sec(c+dx)+a}}{2a(1-\sec(c+dx))}\right)+\frac{\sqrt{a\sec(c+dx)+}}{4a(1-\sec(c+dx)}\right.}{d}$$

$$\downarrow 220$$

$$-\frac{a^3\left(\frac{1}{8}\left(\frac{1}{4}\left(\frac{43\sqrt{2}\operatorname{arctanh}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{2}\sqrt{a}}\right)}{\sqrt{a}}-\frac{64\operatorname{arctanh}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{a}}\right)}{\sqrt{a}}\right)+\frac{11\sqrt{a\sec(c+dx)+a}}{2a(1-\sec(c+dx))}\right)+\frac{\sqrt{a\sec(c+dx)+a}}{4a(1-\sec(c+dx))^2}\right)}{d}$$

input
```
Int[Cot[c + d*x]^5*(a + a*Sec[c + d*x])^(5/2),x]
```

output
```
-((a^3*(Sqrt[a + a*Sec[c + d*x]]/(4*a*(1 - Sec[c + d*x])^2) + (((-64*ArcTa
nh[Sqrt[a + a*Sec[c + d*x]]/Sqrt[a]])/Sqrt[a] + (43*Sqrt[2]*ArcTanh[Sqrt[a
 + a*Sec[c + d*x]]/(Sqrt[2]*Sqrt[a])])/Sqrt[a])/4 + (11*Sqrt[a + a*Sec[c +
 d*x]])/(2*a*(1 - Sec[c + d*x])))/8))/d)
```

3.164.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 114
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[1/((m + 1)*(b*c - a*d)*(b*e
 - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1)
 - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (IntegerQ[n] ||
 IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

rule 168
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4368
```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Simp[-(d*b^(m - 1))^(-1)   Subst[Int[(-a + b*x)^((m - 1)/2
)*((a + b*x)^((m - 1)/2 + n)/x), x], x, Csc[c + d*x]], x] /; FreeQ[{a, b, c
, d, n}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[n]
```

3.164.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 271 vs. $2(122) = 244$.

Time = 88.52 (sec) , antiderivative size = 272, normalized size of antiderivative = 1.85

method	result
default	$\frac{a^2\sqrt{a(1+\sec(dx+c))}\left(-43\arctan\left(\frac{\sqrt{2}}{2\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}}\right)\cos(dx+c)\sqrt{2}\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}-64\arctan\left(\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}\right)\cos(dx+c)\sqrt{-}\right.$

input
```
int(cot(d*x+c)^5*(a+a*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/32/d*a^2*(a*(1+sec(d*x+c)))^(1/2)/(cos(d*x+c)-1)*(-43*arctan(1/2*2^(1/2)
/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*cos(d*x+c)*2^(1/2)*(-cos(d*x+c)/(cos(
d*x+c)+1))^(1/2)-64*arctan((-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*cos(d*x+c)*
(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+43*2^(1/2)*(-cos(d*x+c)/(cos(d*x+c)+1))
^(1/2)*arctan(1/2*2^(1/2)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))+64*arctan((-
cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+30*co
s(d*x+c)*cot(d*x+c)^2+8*cot(d*x+c)^2-22*cot(d*x+c)*csc(d*x+c))
```

3.164.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 247 vs. $2(118) = 236$.

Time $= 0.33$ (sec) , antiderivative size $= 503$, normalized size of antiderivative $= 3.42$

$$\int \cot^5(c+dx)(a+a\sec(c+dx))^{5/2}\,dx = \left[\frac{64\left(a^2\cos(dx+c)^2-2\,a^2\cos(dx+c)+a^2\right)\sqrt{a}\log\left(-2\,a\cos(dx+c)-2\sqrt{a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\right.}{}\right.$$

input
```
integrate(cot(d*x+c)^5*(a+a*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
[1/64*(64*(a^2*cos(d*x + c)^2 - 2*a^2*cos(d*x + c) + a^2)*sqrt(a)*log(-2*a
*cos(d*x + c) - 2*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x
+ c) - a) + 43*(sqrt(2)*a^2*cos(d*x + c)^2 - 2*sqrt(2)*a^2*cos(d*x + c) +
sqrt(2)*a^2)*sqrt(a)*log(-(2*sqrt(2)*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos
(d*x + c))*cos(d*x + c) - 3*a*cos(d*x + c) - a)/(cos(d*x + c) - 1)) - 4*(1
5*a^2*cos(d*x + c)^2 - 11*a^2*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(
d*x + c)))/(d*cos(d*x + c)^2 - 2*d*cos(d*x + c) + d), 1/32*(43*(sqrt(2)*a^
2*cos(d*x + c)^2 - 2*sqrt(2)*a^2*cos(d*x + c) + sqrt(2)*a^2)*sqrt(-a)*arct
an(sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(
a*cos(d*x + c) + a)) - 64*(a^2*cos(d*x + c)^2 - 2*a^2*cos(d*x + c) + a^2)*
sqrt(-a)*arctan(sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x +
 c)/(a*cos(d*x + c) + a)) - 2*(15*a^2*cos(d*x + c)^2 - 11*a^2*cos(d*x + c)
)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)))/(d*cos(d*x + c)^2 - 2*d*cos(d*x
 + c) + d)]
```

3.164.6 Sympy [F(-1)]

Timed out.

$$\int \cot^5(c+dx)(a+a\sec(c+dx))^{5/2}\,dx = \text{Timed out}$$

input
```
integrate(cot(d*x+c)**5*(a+a*sec(d*x+c))**(5/2),x)
```

output
```
Timed out
```

3.164.7 Maxima [F(-1)]

Timed out.

$$\int \cot^5(c+dx)(a+a\sec(c+dx))^{5/2}\,dx = \text{Timed out}$$

input
```
integrate(cot(d*x+c)^5*(a+a*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
Timed out
```

3.164.8 Giac [F]

$$\int \cot^5(c+dx)(a+a\sec(c+dx))^{5/2}\,dx = \int (a\sec(dx+c)+a)^{\frac{5}{2}}\cot(dx+c)^5\,dx$$

input
```
integrate(cot(d*x+c)^5*(a+a*sec(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.164.9 Mupad [F(-1)]

Timed out.

$$\int \cot^5(c+dx)(a+a\sec(c+dx))^{5/2}\,dx = \int \cot(c+d\,x)^5\left(a+\frac{a}{\cos(c+d\,x)}\right)^{5/2}dx$$

input
```
int(cot(c + d*x)^5*(a + a/cos(c + d*x))^(5/2),x)
```

output
```
int(cot(c + d*x)^5*(a + a/cos(c + d*x))^(5/2), x)
```

3.165 $\int (a + a\sec(c+dx))^{5/2}\tan^6(c+dx)\,dx$

3.165.1 Optimal result . . . 1232
3.165.2 Mathematica [A] (verified) . . . 1233
3.165.3 Rubi [A] (verified) . . . 1233
3.165.4 Maple [A] (verified) . . . 1235
3.165.5 Fricas [A] (verification not implemented) . . . 1235
3.165.6 Sympy [F(-1)] . . . 1236
3.165.7 Maxima [F(-1)] . . . 1236
3.165.8 Giac [F] . . . 1237
3.165.9 Mupad [F(-1)] . . . 1237

3.165.1 Optimal result

Integrand size = 23, antiderivative size = 290

$$\int (a + a\sec(c+dx))^{5/2}\tan^6(c+dx)\,dx = -\frac{2a^{5/2}\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d} + \frac{2a^3\tan(c+dx)}{d\sqrt{a+a\sec(c+dx)}} - \frac{2a^4\tan^3(c+dx)}{3d(a+a\sec(c+dx))^{3/2}} + \frac{2a^5\tan^5(c+dx)}{5d(a+a\sec(c+dx))^{5/2}} + \frac{62a^6\tan^7(c+dx)}{7d(a+a\sec(c+dx))^{7/2}} + \frac{98a^7\tan^9(c+dx)}{9d(a+a\sec(c+dx))^{9/2}} + \frac{62a^8\tan^{11}(c+dx)}{11d(a+a\sec(c+dx))^{11/2}} + \frac{18a^9\tan^{13}(c+dx)}{13d(a+a\sec(c+dx))^{13/2}} + \frac{2a^{10}\tan^{15}(c+dx)}{15d(a+a\sec(c+dx))^{15/2}}$$

output

```
-2*a^(5/2)*arctan(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))/d+2*a^3*tan(d
*x+c)/d/(a+a*sec(d*x+c))^(1/2)-2/3*a^4*tan(d*x+c)^3/d/(a+a*sec(d*x+c))^(3/
2)+2/5*a^5*tan(d*x+c)^5/d/(a+a*sec(d*x+c))^(5/2)+62/7*a^6*tan(d*x+c)^7/d/(
a+a*sec(d*x+c))^(7/2)+98/9*a^7*tan(d*x+c)^9/d/(a+a*sec(d*x+c))^(9/2)+62/11
*a^8*tan(d*x+c)^11/d/(a+a*sec(d*x+c))^(11/2)+18/13*a^9*tan(d*x+c)^13/d/(a+
a*sec(d*x+c))^(13/2)+2/15*a^10*tan(d*x+c)^15/d/(a+a*sec(d*x+c))^(15/2)
```

3.165.2 Mathematica [A] (verified)

Time = 8.76 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.60

$$\int (a + a\sec(c+dx))^{5/2}\tan^6(c$$

$$+dx)\,dx = \frac{a^2\sec\left(\frac{1}{2}(c+dx)\right)\sec^7(c+dx)\sqrt{a(1+\sec(c+dx))}\Big(-2882880\sqrt{2}\arcsin\left(\sqrt{2}\sin\left(\frac{1}{2}(c+dx)\right)\right)\cos$$

input
```
Integrate[(a + a*Sec[c + d*x])^(5/2)*Tan[c + d*x]^6,x]
```

output
```
(a^2*Sec[(c + d*x)/2]*Sec[c + d*x]^7*Sqrt[a*(1 + Sec[c + d*x])]*(-2882880*
Sqrt[2]*ArcSin[Sqrt[2]*Sin[(c + d*x)/2]]*Cos[c + d*x]^(15/2) + 604890*Sin[
(c + d*x)/2] - 87230*Sin[(3*(c + d*x))/2] + 450450*Sin[(5*(c + d*x))/2] -
137670*Sin[(7*(c + d*x))/2] + 210210*Sin[(9*(c + d*x))/2] + 75450*Sin[(11*
(c + d*x))/2] + 90090*Sin[(13*(c + d*x))/2] + 16066*Sin[(15*(c + d*x))/2])
)/(2882880*d)
```

3.165.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 265, normalized size of antiderivative = 0.91, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3042, 4375, 364, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^6(c+dx)(a\sec(c+dx)+a)^{5/2}\,dx$$

$$\downarrow 3042$$

$$\int \cot\left(c+dx+\frac{\pi}{2}\right)^6\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^{5/2}dx$$

$$\downarrow 4375$$

$$-\frac{2a^6\int \frac{\tan^6(c+dx)\left(\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+2\right)^5}{(\sec(c+dx)a+a)^3\left(\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+1\right)}d\left(-\frac{\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)}{d}$$

$$\downarrow 364$$

$$-\frac{2a^6\int\left(\frac{a^4\tan^{14}(c+dx)}{(\sec(c+dx)a+a)^7}+\frac{9a^3\tan^{12}(c+dx)}{(\sec(c+dx)a+a)^6}+\frac{31a^2\tan^{10}(c+dx)}{(\sec(c+dx)a+a)^5}+\frac{49a\tan^8(c+dx)}{(\sec(c+dx)a+a)^4}+\frac{31\tan^6(c+dx)}{(\sec(c+dx)a+a)^3}+\frac{\tan^4(c+dx)}{a(\sec(c+dx)a+a)^2}-\right.}{d}$$

$$\downarrow 2009$$

$$-\frac{2a^6\left(\frac{\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{a^{7/2}}-\frac{a^4\tan^{15}(c+dx)}{15(a\sec(c+dx)+a)^{15/2}}-\frac{9a^3\tan^{13}(c+dx)}{13(a\sec(c+dx)+a)^{13/2}}-\frac{\tan(c+dx)}{a^3\sqrt{a\sec(c+dx)+a}}-\frac{31a^2\tan^{11}(c+dx)}{11(a\sec(c+dx)+a)^{11/2}}+\right.}{d}$$

input
```
Int[(a + a*Sec[c + d*x])^(5/2)*Tan[c + d*x]^6,x]
```

output
```
(-2*a^6*(ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]]/a^(7/2) -
 Tan[c + d*x]/(a^3*Sqrt[a + a*Sec[c + d*x]]) + Tan[c + d*x]^3/(3*a^2*(a +
a*Sec[c + d*x])^(3/2)) - Tan[c + d*x]^5/(5*a*(a + a*Sec[c + d*x])^(5/2)) -
 (31*Tan[c + d*x]^7)/(7*(a + a*Sec[c + d*x])^(7/2)) - (49*a*Tan[c + d*x]^9
)/(9*(a + a*Sec[c + d*x])^(9/2)) - (31*a^2*Tan[c + d*x]^11)/(11*(a + a*Sec
[c + d*x])^(11/2)) - (9*a^3*Tan[c + d*x]^13)/(13*(a + a*Sec[c + d*x])^(13/
2)) - (a^4*Tan[c + d*x]^15)/(15*(a + a*Sec[c + d*x])^(15/2))))/d
```

3.165.3.1 Defintions of rubi rules used

rule 364
```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_))/((c_) + (d_.)*(x_)^2),
x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*((a + b*x^2)^p/(c + d*x^2)), x], x
] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && (In
tegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4375

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Simp[-2*(a^(m/2 + n + 1/2)/d)   Subst[Int[x^m*((2 + a*x^2
)^(m/2 + n - 1/2)/(1 + a*x^2)), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]
]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && I
ntegerQ[n - 1/2]
```

3.165.4 Maple [A] (verified)

Time = 1.25 (sec) , antiderivative size = 268, normalized size of antiderivative = 0.92

$$-\frac{2a^2\sqrt{a\left(1+\sec\left(dx+c\right)\right)}\left(45045\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,\operatorname{arctanh}\left(\frac{\sin(dx+c)}{(\cos(dx+c)+1)\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}}\right)\cos\left(dx+c\right)+45045\right.}{}$$

input

```
int((a+a*sec(d*x+c))^(5/2)*tan(d*x+c)^6,x)
```

output

```
-2/45045/d*a^2*(a*(1+sec(d*x+c)))^(1/2)/(cos(d*x+c)+1)*(45045*(-cos(d*x+c)
/(cos(d*x+c)+1))^(1/2)*arctanh(sin(d*x+c)/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos
(d*x+c)+1))^(1/2))*cos(d*x+c)+45045*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arc
tanh(sin(d*x+c)/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))-16066*s
in(d*x+c)-53078*tan(d*x+c)-17286*sec(d*x+c)*tan(d*x+c)+30640*tan(d*x+c)*se
c(d*x+c)^2+26810*tan(d*x+c)*sec(d*x+c)^3-2898*sec(d*x+c)^4*tan(d*x+c)-1016
4*tan(d*x+c)*sec(d*x+c)^5-3003*tan(d*x+c)*sec(d*x+c)^6)
```

3.165.5 Fricas [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 477, normalized size of antiderivative = 1.64

$$\int (a+a\sec(c+dx))^{5/2}\tan^6(c+dx)\,dx = \left[\frac{45045\left(a^2\cos\left(dx+c\right)^8+a^2\cos\left(dx+c\right)^7\right)\sqrt{-a}\log\left(\frac{2\,a\cos(dx+c)^2+2\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)\sin(dx+c)}{\cos(dx+c)+1}\right.\right.$$

input

```
integrate((a+a*sec(d*x+c))^(5/2)*tan(d*x+c)^6,x, algorithm="fricas")
```

output
```
[1/45045*(45045*(a^2*cos(d*x + c)^8 + a^2*cos(d*x + c)^7)*sqrt(-a)*log((2*
a*cos(d*x + c)^2 + 2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(
d*x + c)*sin(d*x + c) + a*cos(d*x + c) - a)/(cos(d*x + c) + 1)) + 2*(16066
*a^2*cos(d*x + c)^7 + 53078*a^2*cos(d*x + c)^6 + 17286*a^2*cos(d*x + c)^5
 - 30640*a^2*cos(d*x + c)^4 - 26810*a^2*cos(d*x + c)^3 + 2898*a^2*cos(d*x +
 c)^2 + 10164*a^2*cos(d*x + c) + 3003*a^2)*sqrt((a*cos(d*x + c) + a)/cos(d
*x + c))*sin(d*x + c))/(d*cos(d*x + c)^8 + d*cos(d*x + c)^7), 2/45045*(450
45*(a^2*cos(d*x + c)^8 + a^2*cos(d*x + c)^7)*sqrt(a)*arctan(sqrt((a*cos(d*
x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c))) + (16066*a^
2*cos(d*x + c)^7 + 53078*a^2*cos(d*x + c)^6 + 17286*a^2*cos(d*x + c)^5 - 3
0640*a^2*cos(d*x + c)^4 - 26810*a^2*cos(d*x + c)^3 + 2898*a^2*cos(d*x + c)
^2 + 10164*a^2*cos(d*x + c) + 3003*a^2)*sqrt((a*cos(d*x + c) + a)/cos(d*x
+ c))*sin(d*x + c))/(d*cos(d*x + c)^8 + d*cos(d*x + c)^7)]
```

3.165.6 Sympy [F(-1)]

Timed out.

$$\int (a + a\sec(c + dx))^{5/2} \tan^6(c + dx)\, dx = \text{Timed out}$$

input
```
integrate((a+a*sec(d*x+c))**(5/2)*tan(d*x+c)**6,x)
```

output
```
Timed out
```

3.165.7 Maxima [F(-1)]

Timed out.

$$\int (a + a\sec(c + dx))^{5/2} \tan^6(c + dx)\, dx = \text{Timed out}$$

input
```
integrate((a+a*sec(d*x+c))^(5/2)*tan(d*x+c)^6,x, algorithm="maxima")
```

output
```
Timed out
```

3.165.8 Giac [F]

$$\int (a + a\sec(c + dx))^{5/2} \tan^6(c + dx)\, dx = \int (a\sec(dx + c) + a)^{\frac{5}{2}} \tan(dx + c)^6\, dx$$

input
```
integrate((a+a*sec(d*x+c))^(5/2)*tan(d*x+c)^6,x, algorithm="giac")
```

output
```
sage0*x
```

3.165.9 Mupad [F(-1)]

Timed out.

$$\int (a + a\sec(c + dx))^{5/2} \tan^6(c + dx)\, dx = \int \tan(c + d\,x)^6 \left(a + \frac{a}{\cos(c + d\,x)}\right)^{5/2} dx$$

input
```
int(tan(c + d*x)^6*(a + a/cos(c + d*x))^(5/2),x)
```

output
```
int(tan(c + d*x)^6*(a + a/cos(c + d*x))^(5/2), x)
```

3.166 $\int (a + a\sec(c+dx))^{5/2}\tan^4(c+dx)\,dx$

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3.166.1 Optimal result

Integrand size = 23, antiderivative size = 224

$$\int (a + a\sec(c+dx))^{5/2}\tan^4(c+dx)\,dx = \frac{2a^{5/2}\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d} - \frac{2a^3\tan(c+dx)}{d\sqrt{a+a\sec(c+dx)}} + \frac{2a^4\tan^3(c+dx)}{3d(a+a\sec(c+dx))^{3/2}} + \frac{6a^5\tan^5(c+dx)}{d(a+a\sec(c+dx))^{5/2}} + \frac{34a^6\tan^7(c+dx)}{7d(a+a\sec(c+dx))^{7/2}} + \frac{14a^7\tan^9(c+dx)}{9d(a+a\sec(c+dx))^{9/2}} + \frac{2a^8\tan^{11}(c+dx)}{11d(a+a\sec(c+dx))^{11/2}}$$

output

```
2*a^(5/2)*arctan(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))/d-2*a^3*tan(d*
x+c)/d/(a+a*sec(d*x+c))^(1/2)+2/3*a^4*tan(d*x+c)^3/d/(a+a*sec(d*x+c))^(3/2
)+6*a^5*tan(d*x+c)^5/d/(a+a*sec(d*x+c))^(5/2)+34/7*a^6*tan(d*x+c)^7/d/(a+a
*sec(d*x+c))^(7/2)+14/9*a^7*tan(d*x+c)^9/d/(a+a*sec(d*x+c))^(9/2)+2/11*a^8
*tan(d*x+c)^11/d/(a+a*sec(d*x+c))^(11/2)
```

3.166.2 Mathematica [A] (verified)

Time = 7.81 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.67

$$\int (a + a\sec(c+dx))^{5/2}\tan^4(c+dx)\,dx = \frac{a^2\sec\left(\frac{1}{2}(c+dx)\right)\sec^5(c+dx)\sqrt{a(1+\sec(c+dx))}\Big(5544\sqrt{2}\arcsin\left(\sqrt{2}\sin\left(\frac{1}{2}(c+dx)\right)\right)\cos^{\frac{11}{2}}(c$$

input
```
Integrate[(a + a*Sec[c + d*x])^(5/2)*Tan[c + d*x]^4,x]
```

output
```
(a^2*Sec[(c + d*x)/2]*Sec[c + d*x]^5*Sqrt[a*(1 + Sec[c + d*x])]*(5544*Sqrt
[2]*ArcSin[Sqrt[2]*Sin[(c + d*x)/2]]*Cos[c + d*x]^(11/2) - 1386*Sin[(c + d
*x)/2] + 1584*Sin[(3*(c + d*x))/2] - 1386*Sin[(5*(c + d*x))/2] - 143*Sin[(
7*(c + d*x))/2] - 693*Sin[(9*(c + d*x))/2] - 26*Sin[(11*(c + d*x))/2]))/(5
544*d)
```

3.166.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 205, normalized size of antiderivative = 0.92, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3042, 4375, 364, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^4(c+dx)(a\sec(c+dx)+a)^{5/2}\,dx$$

$$\downarrow 3042$$

$$\int \cot\left(c+dx+\frac{\pi}{2}\right)^4\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^{5/2}dx$$

$$\downarrow 4375$$

$$-\frac{2a^5\int \frac{\tan^4(c+dx)\left(\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+2\right)^4}{(\sec(c+dx)a+a)^2\left(\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+1\right)}d\left(-\frac{\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)}{d}$$

$$\downarrow 364$$

$$-\frac{2a^5\int\left(\frac{a^3\tan^{10}(c+dx)}{(\sec(c+dx)a+a)^5}+\frac{7a^2\tan^8(c+dx)}{(\sec(c+dx)a+a)^4}+\frac{17a\tan^6(c+dx)}{(\sec(c+dx)a+a)^3}+\frac{15\tan^4(c+dx)}{(\sec(c+dx)a+a)^2}+\frac{\tan^2(c+dx)}{a(\sec(c+dx)a+a)}+\frac{1}{a^2\left(\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+1\right)}-\right.}{d}$$

$$\downarrow 2009$$

$$-\frac{2a^5\left(-\frac{\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{a^{5/2}}-\frac{a^3\tan^{11}(c+dx)}{11(a\sec(c+dx)+a)^{11/2}}-\frac{7a^2\tan^9(c+dx)}{9(a\sec(c+dx)+a)^{9/2}}+\frac{\tan(c+dx)}{a^2\sqrt{a\sec(c+dx)+a}}-\frac{17a\tan^7(c+dx)}{7(a\sec(c+dx)+a)^{7/2}}-\right.}{d}$$

input
```
Int[(a + a*Sec[c + d*x])^(5/2)*Tan[c + d*x]^4,x]
```

output
```
(-2*a^5*(-(ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]]/a^(5/2)
) + Tan[c + d*x]/(a^2*Sqrt[a + a*Sec[c + d*x]]) - Tan[c + d*x]^3/(3*a*(a +
 a*Sec[c + d*x])^(3/2)) - (3*Tan[c + d*x]^5)/(a + a*Sec[c + d*x])^(5/2) -
(17*a*Tan[c + d*x]^7)/(7*(a + a*Sec[c + d*x])^(7/2)) - (7*a^2*Tan[c + d*x]
^9)/(9*(a + a*Sec[c + d*x])^(9/2)) - (a^3*Tan[c + d*x]^11)/(11*(a + a*Sec[
c + d*x])^(11/2))))/d
```

3.166.3.1 Defintions of rubi rules used

rule 364
```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_))/((c_) + (d_.)*(x_)^2),
x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*((a + b*x^2)^p/(c + d*x^2)), x], x
] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && (In
tegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4375
```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Simp[-2*(a^(m/2 + n + 1/2)/d)   Subst[Int[x^m*((2 + a*x^2
)^(m/2 + n - 1/2)/(1 + a*x^2)), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]
]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && I
ntegerQ[n - 1/2]
```

3.166.4 Maple [A] (verified)

Time = 129.41 (sec) , antiderivative size = 236, normalized size of antiderivative = 1.05

method	result
default	$\frac{2a^2\sqrt{a(1+\sec(dx+c))}\left(693\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,\operatorname{arctanh}\left(\frac{\sin(dx+c)}{(\cos(dx+c)+1)\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}}\right)\cos(dx+c)+693\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,\operatorname{arctanh}\left(\right.$

input
```
int((a+a*sec(d*x+c))^(5/2)*tan(d*x+c)^4,x,method=_RETURNVERBOSE)
```

output
```
2/693/d*a^2*(a*(1+sec(d*x+c)))^(1/2)/(cos(d*x+c)+1)*(693*(-cos(d*x+c)/(cos
(d*x+c)+1))^(1/2)*arctanh(sin(d*x+c)/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+
c)+1))^(1/2))*cos(d*x+c)+693*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctanh(si
n(d*x+c)/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))-52*sin(d*x+c)-
719*tan(d*x+c)-366*sec(d*x+c)*tan(d*x+c)+157*tan(d*x+c)*sec(d*x+c)^2+224*t
an(d*x+c)*sec(d*x+c)^3+63*sec(d*x+c)^4*tan(d*x+c))
```

3.166.5 Fricas [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 425, normalized size of antiderivative = 1.90

$$\int (a + a\sec(c+dx))^{5/2}\tan^4(c + dx)\,dx = \left[\frac{693\left(a^2\cos(dx+c)^6 + a^2\cos(dx+c)^5\right)\sqrt{-a}\log\left(\frac{2a\cos(dx+c)^2 - 2\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)\sin(dx+c)}{\cos(dx+c)+1}\right.}{}\right.$$

$$\left.\left. - \frac{2\left(693\left(a^2\cos(dx+c)^6 + a^2\cos(dx+c)^5\right)\sqrt{a}\arctan\left(\frac{\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)}{\sqrt{a}\sin(dx+c)}\right) + \left(52a^2\cos(dx+c)^5 + 719\right.\right.\right.}{693\left(d\cos(d\right.}\right.$$

input
```
integrate((a+a*sec(d*x+c))^(5/2)*tan(d*x+c)^4,x, algorithm="fricas")
```

output
```
[1/693*(693*(a^2*cos(d*x + c)^6 + a^2*cos(d*x + c)^5)*sqrt(-a)*log((2*a*co
s(d*x + c)^2 - 2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x
+ c)*sin(d*x + c) + a*cos(d*x + c) - a)/(cos(d*x + c) + 1)) - 2*(52*a^2*co
s(d*x + c)^5 + 719*a^2*cos(d*x + c)^4 + 366*a^2*cos(d*x + c)^3 - 157*a^2*c
os(d*x + c)^2 - 224*a^2*cos(d*x + c) - 63*a^2)*sqrt((a*cos(d*x + c) + a)/c
os(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^6 + d*cos(d*x + c)^5), -2/693*(
693*(a^2*cos(d*x + c)^6 + a^2*cos(d*x + c)^5)*sqrt(a)*arctan(sqrt((a*cos(d
*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c))) + (52*a^2*
cos(d*x + c)^5 + 719*a^2*cos(d*x + c)^4 + 366*a^2*cos(d*x + c)^3 - 157*a^2
*cos(d*x + c)^2 - 224*a^2*cos(d*x + c) - 63*a^2)*sqrt((a*cos(d*x + c) + a)
/cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^6 + d*cos(d*x + c)^5)]
```

3.166.6 Sympy [F(-1)]

Timed out.

$$\int (a + a\sec(c + dx))^{5/2} \tan^4(c + dx)\, dx = \text{Timed out}$$

input
```
integrate((a+a*sec(d*x+c))**(5/2)*tan(d*x+c)**4,x)
```

output
```
Timed out
```

3.166.7 Maxima [F(-1)]

Timed out.

$$\int (a + a\sec(c + dx))^{5/2} \tan^4(c + dx)\, dx = \text{Timed out}$$

input
```
integrate((a+a*sec(d*x+c))^(5/2)*tan(d*x+c)^4,x, algorithm="maxima")
```

output
```
Timed out
```

3.166.8 Giac [F]

$$\int (a + a\sec(c + dx))^{5/2} \tan^4(c + dx)\, dx = \int (a\sec(dx + c) + a)^{\frac{5}{2}} \tan(dx + c)^4\, dx$$

input
```
integrate((a+a*sec(d*x+c))^(5/2)*tan(d*x+c)^4,x, algorithm="giac")
```

output
```
sage0*x
```

3.166.9 Mupad [F(-1)]

Timed out.

$$\int (a + a\sec(c + dx))^{5/2} \tan^4(c + dx)\, dx = \int \tan(c + d\,x)^4 \left(a + \frac{a}{\cos(c + d\,x)}\right)^{5/2} dx$$

input
```
int(tan(c + d*x)^4*(a + a/cos(c + d*x))^(5/2),x)
```

output
```
int(tan(c + d*x)^4*(a + a/cos(c + d*x))^(5/2), x)
```

3.167 $\int (a + a\sec(c+dx))^{5/2}\tan^2(c+dx)\,dx$

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3.167.1 Optimal result

Integrand size = 23, antiderivative size = 160

$$\int (a + a\sec(c+dx))^{5/2}\tan^2(c+dx)\,dx = -\frac{2a^{5/2}\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d} + \frac{2a^3\tan(c+dx)}{d\sqrt{a+a\sec(c+dx)}} + \frac{14a^4\tan^3(c+dx)}{3d(a+a\sec(c+dx))^{3/2}} + \frac{2a^5\tan^5(c+dx)}{d(a+a\sec(c+dx))^{5/2}} + \frac{2a^6\tan^7(c+dx)}{7d(a+a\sec(c+dx))^{7/2}}$$

output

```
-2*a^(5/2)*arctan(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))/d+2*a^3*tan(d
*x+c)/d/(a+a*sec(d*x+c))^(1/2)+14/3*a^4*tan(d*x+c)^3/d/(a+a*sec(d*x+c))^(3
/2)+2*a^5*tan(d*x+c)^5/d/(a+a*sec(d*x+c))^(5/2)+2/7*a^6*tan(d*x+c)^7/d/(a+
a*sec(d*x+c))^(7/2)
```

3.167.2 Mathematica [A] (verified)

Time = 6.42 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.78

$$\int (a + a\sec(c+dx))^{5/2}\tan^2(c+dx)\,dx = -\frac{a^2\sec\left(\frac{1}{2}(c+dx)\right)\sec^3(c+dx)\sqrt{a(1+\sec(c+dx))}\left(42\sqrt{2}\arcsin\left(\sqrt{2}\sin\left(\frac{1}{2}(c+dx)\right)\right)\cos^{\frac{7}{2}}(c+dx) - 35\right.}{42d}$$

input
```
Integrate[(a + a*Sec[c + d*x])^(5/2)*Tan[c + d*x]^2,x]
```

output
```
-1/42*(a^2*Sec[(c + d*x)/2]*Sec[c + d*x]^3*Sqrt[a*(1 + Sec[c + d*x])]*(42*
Sqrt[2]*ArcSin[Sqrt[2]*Sin[(c + d*x)/2]]*Cos[c + d*x]^(7/2) - 35*Sin[(c +
d*x)/2] + 7*Sin[(3*(c + d*x))/2] - 21*Sin[(5*(c + d*x))/2] + 5*Sin[(7*(c +
 d*x))/2]))/d
```

3.167.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.92, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3042, 4375, 364, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^2(c+dx)(a\sec(c+dx)+a)^{5/2}\,dx$$

$$\downarrow 3042$$

$$\int \cot\left(c+dx+\frac{\pi}{2}\right)^2\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^{5/2}dx$$

$$\downarrow 4375$$

$$-\frac{2a^4\int \frac{\tan^2(c+dx)\left(\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+2\right)^3}{(\sec(c+dx)a+a)\left(\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+1\right)}d\left(-\frac{\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)}{d}$$

$$\downarrow 364$$

$$-\frac{2a^4\int\left(\frac{a^2\tan^6(c+dx)}{(\sec(c+dx)a+a)^3}+\frac{5a\tan^4(c+dx)}{(\sec(c+dx)a+a)^2}+\frac{7\tan^2(c+dx)}{\sec(c+dx)a+a}+\frac{1}{a}-\frac{1}{a\left(\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+1\right)}\right)d\left(-\frac{\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)}{d}$$

$$\downarrow 2009$$

$$-\frac{2a^4\left(\frac{\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{a^{3/2}}-\frac{a^2\tan^7(c+dx)}{7(a\sec(c+dx)+a)^{7/2}}-\frac{a\tan^5(c+dx)}{(a\sec(c+dx)+a)^{5/2}}-\frac{7\tan^3(c+dx)}{3(a\sec(c+dx)+a)^{3/2}}-\frac{\tan(c+dx)}{a\sqrt{a\sec(c+dx)+a}}\right)}{d}$$

input
```
Int[(a + a*Sec[c + d*x])^(5/2)*Tan[c + d*x]^2,x]
```

output
```
(-2*a^4*(ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]]/a^(3/2) -
 Tan[c + d*x]/(a*Sqrt[a + a*Sec[c + d*x]]) - (7*Tan[c + d*x]^3)/(3*(a + a*
Sec[c + d*x])^(3/2)) - (a*Tan[c + d*x]^5)/(a + a*Sec[c + d*x])^(5/2) - (a^
2*Tan[c + d*x]^7)/(7*(a + a*Sec[c + d*x])^(7/2))))/d
```

3.167.3.1 Defintions of rubi rules used

rule 364
```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_))/((c_) + (d_.)*(x_)^2),
x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*((a + b*x^2)^p/(c + d*x^2)), x], x
] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && (In
tegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4375
```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Simp[-2*(a^(m/2 + n + 1/2)/d)   Subst[Int[x^m*((2 + a*x^2
)^(m/2 + n - 1/2)/(1 + a*x^2)), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]
]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && I
ntegerQ[n - 1/2]
```

3.167.4 Maple [A] (warning: unable to verify)

Time = 13.54 (sec) , antiderivative size = 233, normalized size of antiderivative = 1.46

method	result
default	$-\frac{a^2\left(21\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{2}\,(-\cot(dx+c)+\csc(dx+c))}{\sqrt{(1-\cos(dx+c))^2\csc(dx+c)^2-1}}\right)\left((1-\cos(dx+c))^2\csc(dx+c)^2-1\right)^{\frac{7}{2}}+34(1-\cos(dx+c))^7\csc(dx+c)^7-98(1-c}{21d(\csc(dx+c)-\cot(dx+c)+1)^3(-c}$

input
```
int((a+a*sec(d*x+c))^(5/2)*tan(d*x+c)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/21/d*a^2*(21*2^(1/2)*arctanh(2^(1/2)/((1-cos(d*x+c))^2*csc(d*x+c)^2-1)^
(1/2)*(-cot(d*x+c)+csc(d*x+c)))*((1-cos(d*x+c))^2*csc(d*x+c)^2-1)^(7/2)+34
*(1-cos(d*x+c))^7*csc(d*x+c)^7-98*(1-cos(d*x+c))^5*csc(d*x+c)^5+70*(1-cos(
d*x+c))^3*csc(d*x+c)^3+42*csc(d*x+c)-42*cot(d*x+c))*(-2*a/((1-cos(d*x+c))^
2*csc(d*x+c)^2-1))^(1/2)/(csc(d*x+c)-cot(d*x+c)+1)^3/(-cot(d*x+c)+csc(d*x+
c)-1)^3
```

3.167.5 Fricas [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 374, normalized size of antiderivative = 2.34

$$\int (a + a\sec(c+dx))^{5/2}\tan^2(c + dx)\,dx = \left[\frac{21\left(a^2\cos(dx+c)^4 + a^2\cos(dx+c)^3\right)\sqrt{-a}\log\left(\frac{2\,a\cos(dx+c)^2+2\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)\sin(dx+c)+}{\cos(dx+c)+1}\right.}{21\left(d\cos(\right.}\right.$$

input
```
integrate((a+a*sec(d*x+c))^(5/2)*tan(d*x+c)^2,x, algorithm="fricas")
```

output
```
[1/21*(21*(a^2*cos(d*x + c)^4 + a^2*cos(d*x + c)^3)*sqrt(-a)*log((2*a*cos(
d*x + c)^2 + 2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x +
c)*sin(d*x + c) + a*cos(d*x + c) - a)/(cos(d*x + c) + 1)) - 2*(10*a^2*cos(
d*x + c)^3 - 16*a^2*cos(d*x + c)^2 - 12*a^2*cos(d*x + c) - 3*a^2)*sqrt((a*
cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^4 + d*cos(d*
x + c)^3), 2/21*(21*(a^2*cos(d*x + c)^4 + a^2*cos(d*x + c)^3)*sqrt(a)*arct
an(sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x +
 c))) - (10*a^2*cos(d*x + c)^3 - 16*a^2*cos(d*x + c)^2 - 12*a^2*cos(d*x +
c) - 3*a^2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(d*cos(d
*x + c)^4 + d*cos(d*x + c)^3)]
```

3.167.6 Sympy [F]

$$\int (a + a\sec(c+dx))^{5/2}\tan^2(c+dx)\,dx = \int (a(\sec(c+dx)+1))^{\frac{5}{2}}\tan^2(c+dx)\,dx$$

input
```
integrate((a+a*sec(d*x+c))**(5/2)*tan(d*x+c)**2,x)
```

output
```
Integral((a*(sec(c + d*x) + 1))**(5/2)*tan(c + d*x)**2, x)
```

3.167.7 Maxima [F]

$$\int (a + a\sec(c+dx))^{5/2}\tan^2(c+dx)\,dx = \int (a\sec(dx+c)+a)^{\frac{5}{2}}\tan(dx+c)^2\,dx$$

input
```
integrate((a+a*sec(d*x+c))^(5/2)*tan(d*x+c)^2,x, algorithm="maxima")
```

output
```
1/42*(21*((a^2*cos(2*d*x + 2*c)^2 + a^2*sin(2*d*x + 2*c)^2 + 2*a^2*cos(2*d
*x + 2*c) + a^2)*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(
2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
 + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)
^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + 1) - (a^
2*cos(2*d*x + 2*c)^2 + a^2*sin(2*d*x + 2*c)^2 + 2*a^2*cos(2*d*x + 2*c) + a
^2)*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c)
 + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)), (cos(
2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/
2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - 1) - 2*(a^2*d*cos(2*d
*x + 2*c)^2 + a^2*d*sin(2*d*x + 2*c)^2 + 2*a^2*d*cos(2*d*x + 2*c) + a^2*d)
*integrate((((cos(6*d*x + 6*c)*cos(2*d*x + 2*c) + 2*cos(4*d*x + 4*c)*cos(2
*d*x + 2*c) + cos(2*d*x + 2*c)^2 + sin(6*d*x + 6*c)*sin(2*d*x + 2*c) + 2*s
in(4*d*x + 4*c)*sin(2*d*x + 2*c) + sin(2*d*x + 2*c)^2)*cos(9/2*arctan2(sin
(2*d*x + 2*c), cos(2*d*x + 2*c))) + (cos(2*d*x + 2*c)*sin(6*d*x + 6*c) + 2
*cos(2*d*x + 2*c)*sin(4*d*x + 4*c) - cos(6*d*x + 6*c)*sin(2*d*x + 2*c) - 2
*cos(4*d*x + 4*c)*sin(2*d*x + 2*c))*sin(9/2*arctan2(sin(2*d*x + 2*c), cos(
2*d*x + 2*c))))*cos(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) -
 ((cos(2*d*x + 2*c)*sin(6*d*x + 6*c) + 2*cos(2*d*x + 2*c)*sin(4*d*x + 4*c)
 - cos(6*d*x + 6*c)*sin(2*d*x + 2*c) - 2*cos(4*d*x + 4*c)*sin(2*d*x + 2...
```

3.167.8 Giac [F]

$$\int (a + a\sec(c + dx))^{5/2} \tan^2(c + dx)\, dx = \int (a\sec(dx + c) + a)^{\frac{5}{2}} \tan(dx + c)^2\, dx$$

input
```
integrate((a+a*sec(d*x+c))^(5/2)*tan(d*x+c)^2,x, algorithm="giac")
```

output
```
sage0*x
```

3.167.9 Mupad [F(-1)]

Timed out.

$$\int (a + a\sec(c + dx))^{5/2} \tan^2(c + dx)\, dx = \int \tan(c + d\,x)^2 \left(a + \frac{a}{\cos(c + d\,x)}\right)^{5/2} dx$$

input
```
int(tan(c + d*x)^2*(a + a/cos(c + d*x))^(5/2),x)
```

output
```
int(tan(c + d*x)^2*(a + a/cos(c + d*x))^(5/2), x)
```

3.168 $\int \cot^2(c+dx)(a+a\sec(c+dx))^{5/2}\,dx$

3.168.1 Optimal result 1250
3.168.2 Mathematica [A] (verified) 1250
3.168.3 Rubi [A] (verified) 1251
3.168.4 Maple [A] (verified) 1252
3.168.5 Fricas [B] (verification not implemented) 1253
3.168.6 Sympy [F(-1)] 1253
3.168.7 Maxima [F] 1254
3.168.8 Giac [F] 1254
3.168.9 Mupad [F(-1)] 1254

3.168.1 Optimal result

Integrand size = 23, antiderivative size = 66

$$\int \cot^2(c+dx)(a+a\sec(c+dx))^{5/2}\,dx = -\frac{2a^{5/2}\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d} - \frac{4a^2\cot(c+dx)\sqrt{a+a\sec(c+dx)}}{d}$$

output
```
-2*a^(5/2)*arctan(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))/d-4*a^2*cot(d
*x+c)*(a+a*sec(d*x+c))^(1/2)/d
```

3.168.2 Mathematica [A] (verified)

Time = 0.58 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.88

$$\int \cot^2(c+dx)(a+a\sec(c+dx))^{5/2}\,dx = -\frac{\sqrt{2}\cot(c+dx)\sec^2\left(\frac{1}{2}(c+dx)\right)\left(2\cos(c+dx)-\frac{\operatorname{arctanh}\left(\sqrt{1-\sec(c+dx)}\right)(-1+\cos(c+dx))}{\sqrt{1-\sec(c+dx)}}\right)\left(\frac{1}{1+\sec(c+dx)}\right)^{3/2}(a($$
$$d\sqrt{1-\tan^2\left(\frac{1}{2}(c+dx)\right)}$$

input
```
Integrate[Cot[c + d*x]^2*(a + a*Sec[c + d*x])^(5/2),x]
```

output
```
-((Sqrt[2]*Cot[c + d*x]*Sec[(c + d*x)/2]^2*(2*Cos[c + d*x] - (ArcTanh[Sqrt
[1 - Sec[c + d*x]]]*(-1 + Cos[c + d*x]))/Sqrt[1 - Sec[c + d*x]])*((1 + Sec
[c + d*x])^(-1))^(3/2)*(a*(1 + Sec[c + d*x]))^(5/2))/(d*Sqrt[1 - Tan[(c +
d*x)/2]^2]))
```

3.168.3 Rubi [A] (verified)

Time $= 0.25$ (sec) , antiderivative size $= 64$, normalized size of antiderivative $= 0.97$, number of steps used $= 5$, number of rules used $= 4$, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used $= \{3042, 4375, 359, 216\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^2(c+dx)(a\sec(c+dx)+a)^{5/2}\,dx$$

$$\downarrow 3042$$

$$\int \frac{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^{5/2}}{\cot\left(c+dx+\frac{\pi}{2}\right)^2}dx$$

$$\downarrow 4375$$

$$-\frac{2a^2\int \frac{\cot^2(c+dx)(\sec(c+dx)a+a)\left(\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+2\right)}{\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+1}d\left(-\frac{\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)}{d}$$

$$\downarrow 359$$

$$-\frac{2a^2\left(2\cot(c+dx)\sqrt{a\sec(c+dx)+a}-a\int \frac{1}{\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+1}d\left(-\frac{\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)\right)}{d}$$

$$\downarrow 216$$

$$-\frac{2a^2\left(\sqrt{a}\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)+2\cot(c+dx)\sqrt{a\sec(c+dx)+a}\right)}{d}$$

input
```
Int[Cot[c + d*x]^2*(a + a*Sec[c + d*x])^(5/2),x]
```

output
```
(-2*a^2*(Sqrt[a]*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]] +
 2*Cot[c + d*x]*Sqrt[a + a*Sec[c + d*x]]))/d
```

3.168.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4375
```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Simp[-2*(a^(m/2 + n + 1/2)/d)   Subst[Int[x^m*((2 + a*x^2
)^(m/2 + n - 1/2)/(1 + a*x^2)), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]
]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && I
ntegerQ[n - 1/2]
```

3.168.4 Maple [A] (verified)

Time = 6.62 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.35

method	result	size
default	$-\frac{2a^2\sqrt{a(1+\sec(dx+c))}\left(\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,\operatorname{arctanh}\left(\frac{\sin(dx+c)}{(\cos(dx+c)+1)\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}}\right)+2\cot(dx+c)\right)}{d}$	89

input
```
int(cot(d*x+c)^2*(a+a*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-2/d*a^2*(a*(1+sec(d*x+c)))^(1/2)*((-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arct
anh(sin(d*x+c)/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))+2*cot(d*
x+c))
```

3.168.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 120 vs. $2(58) = 116$.

Time = 0.35 (sec) , antiderivative size = 270, normalized size of antiderivative = 4.09

$$\int \cot^2(c+dx)(a+a\sec(c+dx))^{5/2}\,dx = \left[\frac{\sqrt{-a}a^2\log\left(-\frac{8\,a\cos(dx+c)^3+4\left(2\cos(dx+c)^2-\cos(dx+c)\right)\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sin(dx+c)-7\,a\cos(dx+c)+a}{\cos(dx+c)+1}\right)\sin(}{2\,d\sin(dx+c)}\right.$$

$$\left.-\frac{a^{\frac{5}{2}}\arctan\left(\frac{2\sqrt{a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)\sin(dx+c)}{2\,a\cos(dx+c)^2+a\cos(dx+c)-a}\right)\sin(dx+c)+4\,a^2\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)}{d\sin(dx+c)}\right]$$

input
```
integrate(cot(d*x+c)^2*(a+a*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
[1/2*(sqrt(-a)*a^2*log(-(8*a*cos(d*x + c)^3 + 4*(2*cos(d*x + c)^2 - cos(d*
x + c))*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c) - 7*
a*cos(d*x + c) + a)/(cos(d*x + c) + 1))*sin(d*x + c) - 8*a^2*sqrt((a*cos(d
*x + c) + a)/cos(d*x + c))*cos(d*x + c))/(d*sin(d*x + c)), -(a^(5/2)*arcta
n(2*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x +
 c)/(2*a*cos(d*x + c)^2 + a*cos(d*x + c) - a))*sin(d*x + c) + 4*a^2*sqrt((
a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c))/(d*sin(d*x + c))]
```

3.168.6 Sympy [F(-1)]

Timed out.

$$\int \cot^2(c+dx)(a+a\sec(c+dx))^{5/2}\,dx = \text{Timed out}$$

input
```
integrate(cot(d*x+c)**2*(a+a*sec(d*x+c))**(5/2),x)
```

output
```
Timed out
```

3.168.7 Maxima [F]

$$\int \cot^2(c+dx)(a+a\sec(c+dx))^{5/2}\,dx = \int (a\sec(dx+c)+a)^{\frac{5}{2}}\cot(dx+c)^2\,dx$$

input
```
integrate(cot(d*x+c)^2*(a+a*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
integrate((a*sec(d*x + c) + a)^(5/2)*cot(d*x + c)^2, x)
```

3.168.8 Giac [F]

$$\int \cot^2(c+dx)(a+a\sec(c+dx))^{5/2}\,dx = \int (a\sec(dx+c)+a)^{\frac{5}{2}}\cot(dx+c)^2\,dx$$

input
```
integrate(cot(d*x+c)^2*(a+a*sec(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.168.9 Mupad [F(-1)]

Timed out.

$$\int \cot^2(c+dx)(a+a\sec(c+dx))^{5/2}\,dx = \int \cot(c+d\,x)^2\left(a+\frac{a}{\cos(c+d\,x)}\right)^{5/2}dx$$

input
```
int(cot(c + d*x)^2*(a + a/cos(c + d*x))^(5/2),x)
```

output
```
int(cot(c + d*x)^2*(a + a/cos(c + d*x))^(5/2), x)
```

3.169 $\int \cot^4(c+dx)(a+a\sec(c+dx))^{5/2}\,dx$

3.169.1 Optimal result 1255
3.169.2 Mathematica [C] (warning: unable to verify) 1255
3.169.3 Rubi [A] (verified) 1256
3.169.4 Maple [B] (verified) 1257
3.169.5 Fricas [A] (verification not implemented) 1258
3.169.6 Sympy [F(-1)] 1259
3.169.7 Maxima [F] 1259
3.169.8 Giac [F] 1259
3.169.9 Mupad [F(-1)] 1260

3.169.1 Optimal result

Integrand size = 23, antiderivative size = 96

$$\int \cot^4(c+dx)(a+a\sec(c+dx))^{5/2}\,dx = \frac{2a^{5/2}\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d} + \frac{2a^2\cot(c+dx)\sqrt{a+a\sec(c+dx)}}{d} - \frac{2a\cot^3(c+dx)(a+a\sec(c+dx))^{3/2}}{3d}$$

output
```
2*a^(5/2)*arctan(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))/d-2/3*a*cot(d*
x+c)^3*(a+a*sec(d*x+c))^(3/2)/d+2*a^2*cot(d*x+c)*(a+a*sec(d*x+c))^(1/2)/d
```

3.169.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.20 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.84

$$\int \cot^4(c+dx)(a+a\sec(c+dx))^{5/2}\,dx = -\frac{2\left(\frac{1}{1+\cos(c+dx)}\right)^{3/2}\cot^3(c+dx)\,\text{Hypergeometric2F1}\left(-\frac{3}{2},-\frac{3}{2},-\frac{1}{2},2\sin^2\left(\frac{1}{2}(c+dx)\right)\right)(a(1+\sec(c+dx)))^{5}}{3d\sqrt{\frac{1}{1+\sec(c+dx)}}}$$

input
```
Integrate[Cot[c + d*x]^4*(a + a*Sec[c + d*x])^(5/2),x]
```

output
```
(-2*((1 + Cos[c + d*x])^(-1))^(3/2)*Cot[c + d*x]^3*Hypergeometric2F1[-3/2,
 -3/2, -1/2, 2*Sin[(c + d*x)/2]^2]*(a*(1 + Sec[c + d*x]))^(5/2))/(3*d*Sqrt
[(1 + Sec[c + d*x])^(-1)])
```

3.169.3 Rubi [A] (verified)

Time $= 0.25$ (sec) , antiderivative size $= 91$, normalized size of antiderivative $= 0.95$, number of steps used $= 6$, number of rules used $= 5$, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used $= \{3042, 4375, 264, 264, 216\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^4(c+dx)(a\sec(c+dx)+a)^{5/2}\,dx$$

$$\downarrow 3042$$

$$\int \frac{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^{5/2}}{\cot\left(c+dx+\frac{\pi}{2}\right)^4}dx$$

$$\downarrow 4375$$

$$-\frac{2a\int \frac{\cot^4(c+dx)(\sec(c+dx)a+a)^2}{\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+1}d\left(-\frac{\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)}{d}$$

$$\downarrow 264$$

$$-\frac{2a\left(\frac{1}{3}\cot^3(c+dx)(a\sec(c+dx)+a)^{3/2}-a\int \frac{\cot^2(c+dx)(\sec(c+dx)a+a)}{\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+1}d\left(-\frac{\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)\right)}{d}$$

$$\downarrow 264$$

$$-\frac{2a\left(\frac{1}{3}\cot^3(c+dx)(a\sec(c+dx)+a)^{3/2}-a\left(\cot(c+dx)\sqrt{a\sec(c+dx)+a}-a\int \frac{1}{\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+1}d\left(-\frac{\tan(c+dx)}{\sqrt{\sec(c+dx)}}\right.\right.\right.}{d}$$

$$\downarrow 216$$

$$-\frac{2a\left(\frac{1}{3}\cot^3(c+dx)(a\sec(c+dx)+a)^{3/2}-a\left(\sqrt{a}\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)+\cot(c+dx)\sqrt{a\sec(c+dx)+a}\right)\right)}{d}$$

input
```
Int[Cot[c + d*x]^4*(a + a*Sec[c + d*x])^(5/2),x]
```

output
```
(-2*a*((Cot[c + d*x]^3*(a + a*Sec[c + d*x])^(3/2))/3 - a*(Sqrt[a]*ArcTan[(
Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]] + Cot[c + d*x]*Sqrt[a + a*
Sec[c + d*x]])))/d
```

3.169.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4375
```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Simp[-2*(a^(m/2 + n + 1/2)/d)   Subst[Int[x^m*((2 + a*x^2
)^(m/2 + n - 1/2)/(1 + a*x^2)), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]
]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && I
ntegerQ[n - 1/2]
```

3.169.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 179 vs. $2(84) = 168$.

Time = 40.39 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.88

method	result
default	$\frac{2a^2\sqrt{a(1+\sec(dx+c))}\left(3\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,\operatorname{arctanh}\left(\frac{\sin(dx+c)}{(\cos(dx+c)+1)\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}}\right)\cos(dx+c)-3\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,\operatorname{arctanh}\left(\frac{}{(\cos(d}\right.}{3d(\cos(dx+c)-1)}$

input
```
int(cot(d*x+c)^4*(a+a*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/3/d*a^2*(a*(1+sec(d*x+c)))^(1/2)/(cos(d*x+c)-1)*(3*(-cos(d*x+c)/(cos(d*x
+c)+1))^(1/2)*arctanh(sin(d*x+c)/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1
))^(1/2))*cos(d*x+c)-3*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctanh(sin(d*x+
c)/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))+4*cot(d*x+c)*cos(d*x
+c)-3*cot(d*x+c))
```

3.169.5 Fricas [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 355, normalized size of antiderivative = 3.70

$$\int \cot^4(c+dx)(a+a\sec(c+dx))^{5/2}\,dx = \left[\frac{3\left(a^2\cos\left(dx+c\right)-a^2\right)\sqrt{-a}\log\left(-\frac{8\,a\cos(dx+c)^3-4\left(2\,\cos(dx+c)^2-\cos(dx+c)\right)\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sin(dx+c}{\cos(dx+c)+1}\right.}{6\left(d\cos\left(dx+c\right)-d\right.}\right.$$

input
```
integrate(cot(d*x+c)^4*(a+a*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
[1/6*(3*(a^2*cos(d*x + c) - a^2)*sqrt(-a)*log(-(8*a*cos(d*x + c)^3 - 4*(2*
cos(d*x + c)^2 - cos(d*x + c))*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x
+ c))*sin(d*x + c) - 7*a*cos(d*x + c) + a)/(cos(d*x + c) + 1))*sin(d*x + c
) + 4*(4*a^2*cos(d*x + c)^2 - 3*a^2*cos(d*x + c))*sqrt((a*cos(d*x + c) + a
)/cos(d*x + c)))/((d*cos(d*x + c) - d)*sin(d*x + c)), 1/3*(3*(a^2*cos(d*x
+ c) - a^2)*sqrt(a)*arctan(2*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c
))*cos(d*x + c)*sin(d*x + c)/(2*a*cos(d*x + c)^2 + a*cos(d*x + c) - a))*si
n(d*x + c) + 2*(4*a^2*cos(d*x + c)^2 - 3*a^2*cos(d*x + c))*sqrt((a*cos(d*x
 + c) + a)/cos(d*x + c)))/((d*cos(d*x + c) - d)*sin(d*x + c))]
```

3.169.6 Sympy [F(-1)]

Timed out.

$$\int \cot^4(c+dx)(a+a\sec(c+dx))^{5/2}\,dx = \text{Timed out}$$

input
```
integrate(cot(d*x+c)**4*(a+a*sec(d*x+c))**(5/2),x)
```

output
```
Timed out
```

3.169.7 Maxima [F]

$$\int \cot^4(c+dx)(a+a\sec(c+dx))^{5/2}\,dx = \int (a\sec(dx+c)+a)^{\frac{5}{2}}\cot(dx+c)^4\,dx$$

input
```
integrate(cot(d*x+c)^4*(a+a*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
integrate((a*sec(d*x + c) + a)^(5/2)*cot(d*x + c)^4, x)
```

3.169.8 Giac [F]

$$\int \cot^4(c+dx)(a+a\sec(c+dx))^{5/2}\,dx = \int (a\sec(dx+c)+a)^{\frac{5}{2}}\cot(dx+c)^4\,dx$$

input
```
integrate(cot(d*x+c)^4*(a+a*sec(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.169.9 Mupad [F(-1)]

Timed out.

$$\int \cot^4(c+dx)(a+a\sec(c+dx))^{5/2}\,dx = \int \cot(c+d\,x)^4\left(a+\frac{a}{\cos(c+d\,x)}\right)^{5/2}dx$$

input
```
int(cot(c + d*x)^4*(a + a/cos(c + d*x))^(5/2),x)
```

output
```
int(cot(c + d*x)^4*(a + a/cos(c + d*x))^(5/2), x)
```

3.170 $\int \cot^6(c+dx)(a+a\sec(c+dx))^{5/2}\,dx$

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3.170.1 Optimal result

Integrand size = 23, antiderivative size = 176

$$\int \cot^6(c+dx)(a+a\sec(c+dx))^{5/2}\,dx = -\frac{2a^{5/2}\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d} + \frac{a^{5/2}\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{4\sqrt{2}d} - \frac{7a^2\cot(c+dx)\sqrt{a+a\sec(c+dx)}}{4d} + \frac{a\cot^3(c+dx)(a+a\sec(c+dx))^{3/2}}{2d} - \frac{\cot^5(c+dx)(a+a\sec(c+dx))^{5/2}}{5d}$$

output

```
-2*a^(5/2)*arctan(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))/d+1/2*a*cot(d
*x+c)^3*(a+a*sec(d*x+c))^(3/2)/d-1/5*cot(d*x+c)^5*(a+a*sec(d*x+c))^(5/2)/d
+1/8*a^(5/2)*arctan(1/2*a^(1/2)*tan(d*x+c)*2^(1/2)/(a+a*sec(d*x+c))^(1/2))
/d*2^(1/2)-7/4*a^2*cot(d*x+c)*(a+a*sec(d*x+c))^(1/2)/d
```

3.170.2 Mathematica [A] (verified)

Time = 6.40 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.15

$$\int \cot^6(c+dx)(a+a\sec(c+dx))^{5/2}\,dx = \sec^5\left(\tfrac{1}{2}(c+dx)\right)(a(1+\sec(c+dx)))^{5/2}\left(\tfrac{1}{40}\cos^3(c+dx)(160\cos(c+dx) - 7(17+7\cos(2(c+dx))\right.$$

input
```
Integrate[Cot[c + d*x]^6*(a + a*Sec[c + d*x])^(5/2),x]
```

output
```
(Sec[(c + d*x)/2]^5*(a*(1 + Sec[c + d*x]))^(5/2)*((Cos[c + d*x]^3*(160*Cos
[c + d*x] - 7*(17 + 7*Cos[2*(c + d*x)]))*Csc[(c + d*x)/2]^5)/40 + ((ArcSin
[Tan[(c + d*x)/2]] - 8*Sqrt[2]*ArcTan[Tan[(c + d*x)/2]/Sqrt[Cos[c + d*x]/(
1 + Cos[c + d*x])]])*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sec[(c + d*x)/2
]*Sqrt[1 + Sec[c + d*x]])/(Sqrt[Sec[(c + d*x)/2]^2]*Sec[c + d*x]^(5/2))))/
(32*d)
```

3.170.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.01, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.435$, Rules used = {3042, 4375, 382, 27, 445, 27, 445, 27, 397, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^6(c+dx)(a\sec(c+dx)+a)^{5/2}\,dx$$

$$\downarrow 3042$$

$$\int \frac{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^{5/2}}{\cot\left(c+dx+\frac{\pi}{2}\right)^6}dx$$

$$\downarrow 4375$$

$$-\frac{2\int \frac{\cot^6(c+dx)(\sec(c+dx)a+a)^3}{\left(\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+1\right)\left(\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+2\right)}d\left(-\frac{\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)}{d}$$

$$\downarrow 382$$

$$-\frac{2\left(\frac{1}{10}\int -\frac{5a\cot^4(c+dx)(\sec(c+dx)a+a)^2\left(\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+3\right)}{\left(\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+1\right)\left(\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+2\right)}d\left(-\frac{\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)+\frac{1}{10}\cot^5(c+dx)(a\sec(c+dx)+a)^{5/2}\right)}{d}$$

$$\downarrow 27$$

$$-\frac{2\left(\frac{1}{10}\cot^5(c+dx)(a\sec(c+dx)+a)^{5/2}-\frac{1}{2}a\int \frac{\cot^4(c+dx)(\sec(c+dx)a+a)^2\left(\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+3\right)}{\left(\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+1\right)\left(\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+2\right)}d\left(-\frac{\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)\right)}{d}$$

$$\downarrow 445$$

$$-\frac{2\left(\frac{1}{10}\cot^5(c+dx)(a\sec(c+dx)+a)^{5/2}-\frac{1}{2}a\left(\frac{1}{2}\cot^3(c+dx)(a\sec(c+dx)+a)^{3/2}-\frac{1}{6}\int\frac{3a\cot^2(c+dx)(\sec(c+dx)a}{\left(\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+1\right)(}\right.\right.}{d}$$

$$\downarrow 27$$

$$-\frac{2\left(\frac{1}{10}\cot^5(c+dx)(a\sec(c+dx)+a)^{5/2}-\frac{1}{2}a\left(\frac{1}{2}\cot^3(c+dx)(a\sec(c+dx)+a)^{3/2}-\frac{1}{2}a\int\frac{\cot^2(c+dx)(\sec(c+dx)a+}{\left(\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+1\right)(}\right.\right.}{d}$$

$$\downarrow 445$$

$$-\frac{2\left(\frac{1}{10}\cot^5(c+dx)(a\sec(c+dx)+a)^{5/2}-\frac{1}{2}a\left(\frac{1}{2}\cot^3(c+dx)(a\sec(c+dx)+a)^{3/2}-\frac{1}{2}a\left(\frac{7}{2}\cot(c+dx)\sqrt{a\sec\right.\right.\right.}{d}$$

$$\downarrow 27$$

$$-\frac{2\left(\frac{1}{10}\cot^5(c+dx)(a\sec(c+dx)+a)^{5/2}-\frac{1}{2}a\left(\frac{1}{2}\cot^3(c+dx)(a\sec(c+dx)+a)^{3/2}-\frac{1}{2}a\left(\frac{7}{2}\cot(c+dx)\sqrt{a\sec(\right.\right.\right.}{d}$$

$$\downarrow 397$$

$$-\frac{2\left(\frac{1}{10}\cot^5(c+dx)(a\sec(c+dx)+a)^{5/2}-\frac{1}{2}a\left(\frac{1}{2}\cot^3(c+dx)(a\sec(c+dx)+a)^{3/2}-\frac{1}{2}a\left(\frac{7}{2}\cot(c+dx)\sqrt{a\sec(\right.\right.\right.}{d}$$

$$\downarrow 216$$

$$-\frac{2\left(\frac{1}{10}\cot^5(c+dx)(a\sec(c+dx)+a)^{5/2}-\frac{1}{2}a\left(\frac{1}{2}\cot^3(c+dx)(a\sec(c+dx)+a)^{3/2}-\frac{1}{2}a\left(\frac{7}{2}\cot(c+dx)\sqrt{a\sec\right.\right.\right.}{d}$$

input
```
Int[Cot[c + d*x]^6*(a + a*Sec[c + d*x])^(5/2),x]
```

3.170. $\int \cot^6(c+dx)(a+a\sec(c+dx))^{5/2}\,dx$

output
```
(-2*((Cot[c + d*x]^5*(a + a*Sec[c + d*x])^(5/2))/10 - (a*((Cot[c + d*x]^3*
(a + a*Sec[c + d*x])^(3/2))/2 - (a*(-1/2*(a*((-8*ArcTan[(Sqrt[a]*Tan[c + d
*x])/Sqrt[a + a*Sec[c + d*x]]])/Sqrt[a] + ArcTan[(Sqrt[a]*Tan[c + d*x])/(S
qrt[2]*Sqrt[a + a*Sec[c + d*x]])]/(Sqrt[2]*Sqrt[a])) + (7*Cot[c + d*x]*Sq
rt[a + a*Sec[c + d*x]])/2))/2))/2))/d
```

3.170.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 382
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)
, x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/
(a*c*e*(m + 1))), x] - Simp[1/(a*c*e^2*(m + 1))   Int[(e*x)^(m + 2)*(a + b*
x^2)^p*(c + d*x^2)^q*Simp[(b*c + a*d)*(m + 3) + 2*(b*c*p + a*d*q) + b*d*(m
+ 2*p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[
b*c - a*d, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, d, e, m, 2, p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 445
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^2*(m + 1))
 Int[(g*x)^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*f*c*(m + 1) - e*(b*c
+ a*d)*(m + 2 + 1) - e*2*(b*c*p + a*d*q) - b*e*d*(m + 2*(p + q + 2) + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && LtQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4375

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Simp[-2*(a^(m/2 + n + 1/2)/d)   Subst[Int[x^m*((2 + a*x^2
)^(m/2 + n - 1/2)/(1 + a*x^2)), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]
]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && I
ntegerQ[n - 1/2]
```

3.170.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 371 vs. $2(147) = 294$.

Time = 194.44 (sec) , antiderivative size = 372, normalized size of antiderivative = 2.11

method	result
default	$\frac{a^2\sqrt{a(1+\sec(dx+c))}\left(5\sqrt{2}\,\ln\left(\csc(dx+c)-\cot(dx+c)+\sqrt{\cot(dx+c)^2-2\cot(dx+c)\csc(dx+c)+\csc(dx+c)^2-1}\right)\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}\ \cos$

input

```
int(cot(d*x+c)^6*(a+a*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output

```
1/40/d*a^2*(a*(1+sec(d*x+c)))^(1/2)/(cos(d*x+c)-1)*(5*2^(1/2)*ln(csc(d*x+c
)-cot(d*x+c)+(cot(d*x+c)^2-2*cot(d*x+c)*csc(d*x+c)+csc(d*x+c)^2-1)^(1/2))*
(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)-80*(-cos(d*x+c)/(cos(d*x+c)+
1))^(1/2)*arctanh(sin(d*x+c)/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(
1/2))*cos(d*x+c)-5*ln(csc(d*x+c)-cot(d*x+c)+(cot(d*x+c)^2-2*cot(d*x+c)*csc
(d*x+c)+csc(d*x+c)^2-1)^(1/2))*2^(1/2)*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+
80*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctanh(sin(d*x+c)/(cos(d*x+c)+1)/(-
cos(d*x+c)/(cos(d*x+c)+1))^(1/2))+98*cos(d*x+c)*cot(d*x+c)^3-62*cot(d*x+c)
^3-90*cot(d*x+c)^2*csc(d*x+c)+70*cot(d*x+c)*csc(d*x+c)^2)
```

3.170.5 Fricas [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 650, normalized size of antiderivative = 3.69

$$\int \cot^6(c+dx)(a+a\sec(c+dx))^{5/2}\,dx = \left[\frac{5\left(\sqrt{2}a^2\cos(dx+c)^2-2\sqrt{2}a^2\cos(dx+c)+\sqrt{2}a^2\right)\sqrt{-a}\log\left(-\frac{2\sqrt{2}\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)}{\cos(dx+c)}\right.}{}\right.$$

$$\left.\left.-\frac{40\left(a^2\cos(dx+c)^2-2a^2\cos(dx+c)+a^2\right)\sqrt{a}\arctan\left(\frac{2\sqrt{a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)\sin(dx+c)}{2a\cos(dx+c)^2+a\cos(dx+c)-a}\right)\sin(dx+c)+5}{}\right.\right]$$

input
```
integrate(cot(d*x+c)^6*(a+a*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
[1/80*(5*(sqrt(2)*a^2*cos(d*x + c)^2 - 2*sqrt(2)*a^2*cos(d*x + c) + sqrt(2
)*a^2)*sqrt(-a)*log(-(2*sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x
 + c))*cos(d*x + c)*sin(d*x + c) - 3*a*cos(d*x + c)^2 - 2*a*cos(d*x + c) +
 a)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1))*sin(d*x + c) + 40*(a^2*cos(d*x
+ c)^2 - 2*a^2*cos(d*x + c) + a^2)*sqrt(-a)*log(-(8*a*cos(d*x + c)^3 + 4*(
2*cos(d*x + c)^2 - cos(d*x + c))*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*
x + c))*sin(d*x + c) - 7*a*cos(d*x + c) + a)/(cos(d*x + c) + 1))*sin(d*x +
 c) - 4*(49*a^2*cos(d*x + c)^3 - 80*a^2*cos(d*x + c)^2 + 35*a^2*cos(d*x +
c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)))/((d*cos(d*x + c)^2 - 2*d*cos(
d*x + c) + d)*sin(d*x + c)), -1/40*(40*(a^2*cos(d*x + c)^2 - 2*a^2*cos(d*x
 + c) + a^2)*sqrt(a)*arctan(2*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x +
c))*cos(d*x + c)*sin(d*x + c)/(2*a*cos(d*x + c)^2 + a*cos(d*x + c) - a))*s
in(d*x + c) + 5*(sqrt(2)*a^2*cos(d*x + c)^2 - 2*sqrt(2)*a^2*cos(d*x + c) +
 sqrt(2)*a^2)*sqrt(a)*arctan(sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c
))*cos(d*x + c)/(sqrt(a)*sin(d*x + c)))*sin(d*x + c) + 2*(49*a^2*cos(d*x +
 c)^3 - 80*a^2*cos(d*x + c)^2 + 35*a^2*cos(d*x + c))*sqrt((a*cos(d*x + c)
+ a)/cos(d*x + c)))/((d*cos(d*x + c)^2 - 2*d*cos(d*x + c) + d)*sin(d*x + c
))]
```

3.170.6 Sympy [F(-1)]

Timed out.

$$\int \cot^6(c+dx)(a+a\sec(c+dx))^{5/2}\,dx = \text{Timed out}$$

input
```
integrate(cot(d*x+c)**6*(a+a*sec(d*x+c))**(5/2),x)
```

output
```
Timed out
```

3.170.7 Maxima [F(-1)]

Timed out.

$$\int \cot^6(c+dx)(a+a\sec(c+dx))^{5/2}\,dx = \text{Timed out}$$

input
```
integrate(cot(d*x+c)^6*(a+a*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
Timed out
```

3.170.8 Giac [F]

$$\int \cot^6(c+dx)(a+a\sec(c+dx))^{5/2}\,dx = \int (a\sec(dx+c)+a)^{\frac{5}{2}}\cot(dx+c)^6\,dx$$

input
```
integrate(cot(d*x+c)^6*(a+a*sec(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.170.9 Mupad [F(-1)]

Timed out.

$$\int \cot^6(c+dx)(a+a\sec(c+dx))^{5/2}\,dx = \int \cot(c+d\,x)^6\left(a+\frac{a}{\cos(c+d\,x)}\right)^{5/2}dx$$

input
```
int(cot(c + d*x)^6*(a + a/cos(c + d*x))^(5/2),x)
```

output
```
int(cot(c + d*x)^6*(a + a/cos(c + d*x))^(5/2), x)
```

3.171 $\int \frac{\tan^5(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx$

3.171.1 Optimal result 1269
3.171.2 Mathematica [A] (verified) 1269
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3.171.1 Optimal result

Integrand size = 23, antiderivative size = 126

$$\int \frac{\tan^5(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx = -\frac{2\operatorname{arctanh}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}d} + \frac{2\sqrt{a+a\sec(c+dx)}}{ad} + \frac{2(a+a\sec(c+dx))^{3/2}}{3a^2d} - \frac{6(a+a\sec(c+dx))^{5/2}}{5a^3d} + \frac{2(a+a\sec(c+dx))^{7/2}}{7a^4d}$$

output
```
2/3*(a+a*sec(d*x+c))^(3/2)/a^2/d-6/5*(a+a*sec(d*x+c))^(5/2)/a^3/d+2/7*(a+a
*sec(d*x+c))^(7/2)/a^4/d-2*arctanh((a+a*sec(d*x+c))^(1/2)/a^(1/2))/d/a^(1/
2)+2*(a+a*sec(d*x+c))^(1/2)/a/d
```

3.171.2 Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.70

$$\int \frac{\tan^5(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx = \frac{2\Big(92+46\sec(c+dx)-64\sec^2(c+dx)-3\sec^3(c+dx)+15\sec^4(c+dx)-105\operatorname{arctanh}\Big(\sqrt{1+\sec(c+$$

$$105d\sqrt{a(1+\sec(c+dx))}$$

input
```
Integrate[Tan[c + d*x]^5/Sqrt[a + a*Sec[c + d*x]],x]
```

output
```
(2*(92 + 46*Sec[c + d*x] - 64*Sec[c + d*x]^2 - 3*Sec[c + d*x]^3 + 15*Sec[c
 + d*x]^4 - 105*ArcTanh[Sqrt[1 + Sec[c + d*x]]]*Sqrt[1 + Sec[c + d*x]]))/(
105*d*Sqrt[a*(1 + Sec[c + d*x])])
```

3.171.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.90, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3042, 25, 4368, 27, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^5(c+dx)}{\sqrt{a\sec(c+dx)+a}}\,dx$$

$$\downarrow 3042$$

$$\int -\frac{\cot\left(c+dx+\frac{\pi}{2}\right)^5}{\sqrt{a\csc\left(c+dx+\frac{\pi}{2}\right)+a}}dx$$

$$\downarrow 25$$

$$-\int \frac{\cot\left(\frac{1}{2}(2c+\pi)+dx\right)^5}{\sqrt{\csc\left(\frac{1}{2}(2c+\pi)+dx\right)a+a}}dx$$

$$\downarrow 4368$$

$$\frac{\int a^2\cos(c+dx)(1-\sec(c+dx))^2(\sec(c+dx)a+a)^{3/2}d\sec(c+dx)}{a^4d}$$

$$\downarrow 27$$

$$\frac{\int \cos(c+dx)(1-\sec(c+dx))^2(\sec(c+dx)a+a)^{3/2}d\sec(c+dx)}{a^2d}$$

$$\downarrow 99$$

$$\frac{\int \left(\frac{(\sec(c+dx)a+a)^{5/2}}{a}+\cos(c+dx)(\sec(c+dx)a+a)^{3/2}-3(\sec(c+dx)a+a)^{3/2}\right)d\sec(c+dx)}{a^2d}$$

$$\downarrow 2009$$

$$\frac{-2a^{3/2}\text{arctanh}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{a}}\right)+\frac{2(a\sec(c+dx)+a)^{7/2}}{7a^2}-\frac{6(a\sec(c+dx)+a)^{5/2}}{5a}+\frac{2}{3}(a\sec(c+dx)+a)^{3/2}+2a\sqrt{a\sec(c+}}{a^2d}$$

input
```
Int[Tan[c + d*x]^5/Sqrt[a + a*Sec[c + d*x]],x]
```

output
```
(-2*a^(3/2)*ArcTanh[Sqrt[a + a*Sec[c + d*x]]/Sqrt[a]] + 2*a*Sqrt[a + a*Sec
[c + d*x]] + (2*(a + a*Sec[c + d*x])^(3/2))/3 - (6*(a + a*Sec[c + d*x])^(5
/2))/(5*a) + (2*(a + a*Sec[c + d*x])^(7/2))/(7*a^2))/(a^2*d)
```

3.171.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4368
```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Simp[-(d*b^(m - 1))^(-1)   Subst[Int[(-a + b*x)^((m - 1)/2
)*((a + b*x)^((m - 1)/2 + n)/x), x], x, Csc[c + d*x]], x] /; FreeQ[{a, b, c
, d, n}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[n]
```

3.171.4 Maple [A] (verified)

Time = 3.78 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.75

method	result	size
default	$\frac{2\sqrt{a(1+\sec(dx+c))}\left(105\arctan\left(\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}\right)\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}+92-46\sec(dx+c)-18\sec(dx+c)^2+15\sec(dx+c)^3\right)}{105da}$	94

input
```
int(tan(d*x+c)^5/(a+a*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/105/d/a*(a*(1+sec(d*x+c)))^(1/2)*(105*arctan((-cos(d*x+c)/(cos(d*x+c)+1)
)^(1/2))*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+92-46*sec(d*x+c)-18*sec(d*x+c)
^2+15*sec(d*x+c)^3)
```

3.171.5 Fricas [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 285, normalized size of antiderivative = 2.26

$$\int \frac{\tan^5(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx$$
$$= \left[\frac{105\sqrt{a}\cos(dx+c)^3\log\left(-8\,a\cos(dx+c)^2+4\left(2\cos(dx+c)^2+\cos(dx+c)\right)\sqrt{a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}-8\,a\right.\right.}{210\,ad\cos(dx}$$

input
```
integrate(tan(d*x+c)^5/(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
[1/210*(105*sqrt(a)*cos(d*x + c)^3*log(-8*a*cos(d*x + c)^2 + 4*(2*cos(d*x
+ c)^2 + cos(d*x + c))*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)) - 8
*a*cos(d*x + c) - a) + 4*(92*cos(d*x + c)^3 - 46*cos(d*x + c)^2 - 18*cos(d
*x + c) + 15)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)))/(a*d*cos(d*x + c)^3
), 1/105*(105*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x
 + c))*cos(d*x + c)/(2*a*cos(d*x + c) + a))*cos(d*x + c)^3 + 2*(92*cos(d*x
 + c)^3 - 46*cos(d*x + c)^2 - 18*cos(d*x + c) + 15)*sqrt((a*cos(d*x + c) +
 a)/cos(d*x + c)))/(a*d*cos(d*x + c)^3)]
```

3.171.6 Sympy [F]

$$\int \frac{\tan^5(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx = \int \frac{\tan^5(c+dx)}{\sqrt{a\left(\sec(c+dx)+1\right)}}\,dx$$

input
```
integrate(tan(d*x+c)**5/(a+a*sec(d*x+c))**(1/2),x)
```

output
```
Integral(tan(c + d*x)**5/sqrt(a*(sec(c + d*x) + 1)), x)
```

3.171.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.02

$$\int \frac{\tan^5(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx = \frac{\frac{105\log\left(\frac{\sqrt{a+\frac{a}{\cos(dx+c)}}-\sqrt{a}}{\sqrt{a+\frac{a}{\cos(dx+c)}}+\sqrt{a}}\right)}{\sqrt{a}} + \frac{30\left(a+\frac{a}{\cos(dx+c)}\right)^{\frac{7}{2}}}{a^4} - \frac{126\left(a+\frac{a}{\cos(dx+c)}\right)^{\frac{5}{2}}}{a^3} + \frac{70\left(a+\frac{a}{\cos(dx+c)}\right)^{\frac{3}{2}}}{a^2} + \frac{210\sqrt{a+\frac{a}{\cos(dx+c)}}}{a}}{105\,d}$$

input
```
integrate(tan(d*x+c)^5/(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
1/105*(105*log((sqrt(a + a/cos(d*x + c)) - sqrt(a))/(sqrt(a + a/cos(d*x +
c)) + sqrt(a)))/sqrt(a) + 30*(a + a/cos(d*x + c))^(7/2)/a^4 - 126*(a + a/c
os(d*x + c))^(5/2)/a^3 + 70*(a + a/cos(d*x + c))^(3/2)/a^2 + 210*sqrt(a +
a/cos(d*x + c))/a)/d
```

3.171.8 Giac [A] (verification not implemented)

Time = 2.04 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.33

$$\int \frac{\tan^5(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx = \frac{\sqrt{2}\left(\frac{105\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}}{2\sqrt{-a}}\right)}{\sqrt{-a}} + \frac{2\left(105\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-a\right)^3-70\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-a\right)^2a-252\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2\right.\right.\right.}{\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-a\right)^3\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}}$$

$$= \frac{}{105\,d\,\mathrm{sgn}\left(\cos(dx+c)\right)}$$

input
```
integrate(tan(d*x+c)^5/(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
1/105*sqrt(2)*(105*sqrt(2)*arctan(1/2*sqrt(2)*sqrt(-a*tan(1/2*d*x + 1/2*c)
^2 + a)/sqrt(-a))/sqrt(-a) + 2*(105*(a*tan(1/2*d*x + 1/2*c)^2 - a)^3 - 70*
(a*tan(1/2*d*x + 1/2*c)^2 - a)^2*a - 252*(a*tan(1/2*d*x + 1/2*c)^2 - a)*a^
2 - 120*a^3)/((a*tan(1/2*d*x + 1/2*c)^2 - a)^3*sqrt(-a*tan(1/2*d*x + 1/2*c
)^2 + a)))/(d*sgn(cos(d*x + c)))
```

3.171.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\tan^5(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx = \int \frac{\tan(c+d\,x)^5}{\sqrt{a+\frac{a}{\cos(c+d\,x)}}}\,dx$$

input
```
int(tan(c + d*x)^5/(a + a/cos(c + d*x))^(1/2),x)
```

output
```
int(tan(c + d*x)^5/(a + a/cos(c + d*x))^(1/2), x)
```

3.172 $\int \frac{\tan^3(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx$

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3.172.2 Mathematica [A] (verified) 1275
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3.172.6 Sympy [F] 1279
3.172.7 Maxima [A] (verification not implemented) 1280
3.172.8 Giac [A] (verification not implemented) 1280
3.172.9 Mupad [F(-1)] 1281

3.172.1 Optimal result

Integrand size = 23, antiderivative size = 78

$$\int \frac{\tan^3(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx = \frac{2\operatorname{arctanh}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}d} - \frac{2\sqrt{a+a\sec(c+dx)}}{ad} + \frac{2(a+a\sec(c+dx))^{3/2}}{3a^2d}$$

output
```
2/3*(a+a*sec(d*x+c))^(3/2)/a^2/d+2*arctanh((a+a*sec(d*x+c))^(1/2)/a^(1/2))
/d/a^(1/2)-2*(a+a*sec(d*x+c))^(1/2)/a/d
```

3.172.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.85

$$\int \frac{\tan^3(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx = \frac{2\left(-2-\sec(c+dx)+\sec^2(c+dx)+3\operatorname{arctanh}\left(\sqrt{1+\sec(c+dx)}\right)\sqrt{1+\sec(c+dx)}\right)}{3d\sqrt{a(1+\sec(c+dx))}}$$

input
```
Integrate[Tan[c + d*x]^3/Sqrt[a + a*Sec[c + d*x]],x]
```

output
```
(2*(-2 - Sec[c + d*x] + Sec[c + d*x]^2 + 3*ArcTanh[Sqrt[1 + Sec[c + d*x]]]
*Sqrt[1 + Sec[c + d*x]]))/(3*d*Sqrt[a*(1 + Sec[c + d*x])])
```

3.172.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.95, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {3042, 25, 4368, 25, 27, 90, 60, 73, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^3(c+dx)}{\sqrt{a\sec(c+dx)+a}}\,dx$$

$$\downarrow 3042$$

$$\int -\frac{\cot\left(c+dx+\frac{\pi}{2}\right)^3}{\sqrt{a\csc\left(c+dx+\frac{\pi}{2}\right)+a}}dx$$

$$\downarrow 25$$

$$-\int \frac{\cot\left(\frac{1}{2}(2c+\pi)+dx\right)^3}{\sqrt{\csc\left(\frac{1}{2}(2c+\pi)+dx\right)a+a}}dx$$

$$\downarrow 4368$$

$$\frac{\int -a\cos(c+dx)(1-\sec(c+dx))\sqrt{\sec(c+dx)a+a}d\sec(c+dx)}{a^2d}$$

$$\downarrow 25$$

$$-\frac{\int a\cos(c+dx)(1-\sec(c+dx))\sqrt{\sec(c+dx)a+a}d\sec(c+dx)}{a^2d}$$

$$\downarrow 27$$

$$-\frac{\int \cos(c+dx)(1-\sec(c+dx))\sqrt{\sec(c+dx)a+a}d\sec(c+dx)}{ad}$$

$$\downarrow 90$$

$$-\frac{\int \cos(c+dx)\sqrt{\sec(c+dx)a+a}d\sec(c+dx)-\frac{2(a\sec(c+dx)+a)^{3/2}}{3a}}{ad}$$

$$\downarrow 60$$

$$-\frac{a\int \frac{\cos(c+dx)}{\sqrt{\sec(c+dx)a+a}}d\sec(c+dx)-\frac{2(a\sec(c+dx)+a)^{3/2}}{3a}+2\sqrt{a\sec(c+dx)+a}}{ad}$$

$$\downarrow 73$$

$$-\frac{2\int \frac{1}{\frac{\sec(c+dx)a+a}{a}-1}d\sqrt{\sec(c+dx)a+a}-\frac{2(a\sec(c+dx)+a)^{3/2}}{3a}+2\sqrt{a\sec(c+dx)+a}}{ad}$$

$$\downarrow 220$$

$$-\frac{-2\sqrt{a}\operatorname{arctanh}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{a}}\right)-\frac{2(a\sec(c+dx)+a)^{3/2}}{3a}+2\sqrt{a\sec(c+dx)+a}}{ad}$$

input
```
Int[Tan[c + d*x]^3/Sqrt[a + a*Sec[c + d*x]],x]
```

output
```
-((-2*Sqrt[a]*ArcTanh[Sqrt[a + a*Sec[c + d*x]]/Sqrt[a]] + 2*Sqrt[a + a*Sec
[c + d*x]] - (2*(a + a*Sec[c + d*x])^(3/2))/(3*a))/(a*d))
```

3.172.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4368
```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Simp[-(d*b^(m - 1))^(-1)   Subst[Int[(-a + b*x)^((m - 1)/2
)*((a + b*x)^((m - 1)/2 + n)/x), x], x, Csc[c + d*x]], x] /; FreeQ[{a, b, c
, d, n}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[n]
```

3.172.4 Maple [A] (verified)

Time = 3.14 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.95

method	result	size
default	$-\frac{2\sqrt{a(1+\sec(dx+c))}\left(3\arctan\left(\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}\right)\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}+2-\sec(dx+c)\right)}{3da}$	74

input
```
int(tan(d*x+c)^3/(a+a*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3/d/a*(a*(1+sec(d*x+c)))^(1/2)*(3*arctan((-cos(d*x+c)/(cos(d*x+c)+1))^(
1/2))*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+2-sec(d*x+c))
```

3.172.5 Fricas [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 241, normalized size of antiderivative = 3.09

$$\int \frac{\tan^3(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx$$

$$= \left[\frac{3\sqrt{a}\cos(dx+c)\log\left(-8\,a\cos(dx+c)^2 - 4\left(2\cos(dx+c)^2+\cos(dx+c)\right)\sqrt{a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}} - 8\,a\cos\right.}{6\,ad\cos(dx+c)} \right.$$

$$\left. - \frac{3\sqrt{-a}\arctan\left(\frac{2\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)}{2\,a\cos(dx+c)+a}\right)\cos(dx+c) + 2\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}(2\cos(dx+c)-1)}{3\,ad\cos(dx+c)} \right]$$

input
```
integrate(tan(d*x+c)^3/(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
[1/6*(3*sqrt(a)*cos(d*x + c)*log(-8*a*cos(d*x + c)^2 - 4*(2*cos(d*x + c)^2
 + cos(d*x + c))*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)) - 8*a*cos
(d*x + c) - a) - 4*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*(2*cos(d*x + c)
 - 1))/(a*d*cos(d*x + c)), -1/3*(3*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(
d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(2*a*cos(d*x + c) + a))*cos(d*x +
 c) + 2*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*(2*cos(d*x + c) - 1))/(a*d
*cos(d*x + c))]
```

3.172.6 Sympy [F]

$$\int \frac{\tan^3(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx = \int \frac{\tan^3(c+dx)}{\sqrt{a\left(\sec(c+dx)+1\right)}}\,dx$$

input
```
integrate(tan(d*x+c)**3/(a+a*sec(d*x+c))**(1/2),x)
```

output
```
Integral(tan(c + d*x)**3/sqrt(a*(sec(c + d*x) + 1)), x)
```

3.172.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.17

$$\int \frac{\tan^3(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx = -\frac{\frac{3\log\left(\frac{\sqrt{a+\frac{a}{\cos(dx+c)}}-\sqrt{a}}{\sqrt{a+\frac{a}{\cos(dx+c)}}+\sqrt{a}}\right)}{\sqrt{a}} - \frac{2\left(a+\frac{a}{\cos(dx+c)}\right)^{\frac{3}{2}}}{a^2} + \frac{6\sqrt{a+\frac{a}{\cos(dx+c)}}}{a}}{3\,d}$$

input
```
integrate(tan(d*x+c)^3/(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
-1/3*(3*log((sqrt(a + a/cos(d*x + c)) - sqrt(a))/(sqrt(a + a/cos(d*x + c))
 + sqrt(a)))/sqrt(a) - 2*(a + a/cos(d*x + c))^(3/2)/a^2 + 6*sqrt(a + a/cos
(d*x + c))/a)/d
```

3.172.8 Giac [A] (verification not implemented)

Time = 1.09 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.62

$$\int \frac{\tan^3(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx$$
$$= -\frac{\sqrt{2}\left(\frac{3\sqrt{2}a\arctan\left(\frac{\sqrt{2}\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}}{2\sqrt{-a}}\right)}{\sqrt{-a}} + \frac{2\left(3\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-a\right)a+2a^2\right)}{\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-a\right)\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}}\right)}{3\,ad\mathrm{sgn}\left(\cos\left(dx+c\right)\right)}$$

input
```
integrate(tan(d*x+c)^3/(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
-1/3*sqrt(2)*(3*sqrt(2)*a*arctan(1/2*sqrt(2)*sqrt(-a*tan(1/2*d*x + 1/2*c)^
2 + a)/sqrt(-a))/sqrt(-a) + 2*(3*(a*tan(1/2*d*x + 1/2*c)^2 - a)*a + 2*a^2)
/((a*tan(1/2*d*x + 1/2*c)^2 - a)*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)))/(a*
d*sgn(cos(d*x + c)))
```

3.172.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\tan^3(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx = \int \frac{\tan(c+d\,x)^3}{\sqrt{a+\frac{a}{\cos(c+d\,x)}}}\,dx$$

input
```
int(tan(c + d*x)^3/(a + a/cos(c + d*x))^(1/2),x)
```

output
```
int(tan(c + d*x)^3/(a + a/cos(c + d*x))^(1/2), x)
```

3.173 $\int \frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx$

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3.173.2 Mathematica [A] (verified) 1282
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3.173.8 Giac [A] (verification not implemented) 1286
3.173.9 Mupad [B] (verification not implemented) 1286

3.173.1 Optimal result

Integrand size = 21, antiderivative size = 31

$$\int \frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx = -\frac{2\operatorname{arctanh}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}d}$$

output
```
-2*arctanh((a+a*sec(d*x+c))^(1/2)/a^(1/2))/d/a^(1/2)
```

3.173.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.42

$$\int \frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx = -\frac{2\operatorname{arctanh}\left(\sqrt{1+\sec(c+dx)}\right)\sqrt{1+\sec(c+dx)}}{d\sqrt{a(1+\sec(c+dx))}}$$

input
```
Integrate[Tan[c + d*x]/Sqrt[a + a*Sec[c + d*x]],x]
```

output
```
(-2*ArcTanh[Sqrt[1 + Sec[c + d*x]]]*Sqrt[1 + Sec[c + d*x]])/(d*Sqrt[a*(1 +
 Sec[c + d*x])])
```

3.173.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3042, 25, 4368, 73, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\,dx$$

$$\downarrow 3042$$

$$\int -\frac{\cot\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{a\csc\left(c+dx+\frac{\pi}{2}\right)+a}}dx$$

$$\downarrow 25$$

$$-\int \frac{\cot\left(\frac{1}{2}(2c+\pi)+dx\right)}{\sqrt{\csc\left(\frac{1}{2}(2c+\pi)+dx\right)a+a}}dx$$

$$\downarrow 4368$$

$$\frac{\int \frac{\cos(c+dx)}{\sqrt{\sec(c+dx)a+a}}d\sec(c+dx)}{d}$$

$$\downarrow 73$$

$$\frac{2\int \frac{1}{\frac{\sec(c+dx)a+a}{a}-1}d\sqrt{\sec(c+dx)a+a}}{ad}$$

$$\downarrow 220$$

$$-\frac{2\text{arctanh}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{a}}\right)}{\sqrt{a}d}$$

input
```
Int[Tan[c + d*x]/Sqrt[a + a*Sec[c + d*x]],x]
```

output
```
(-2*ArcTanh[Sqrt[a + a*Sec[c + d*x]]/Sqrt[a]])/(Sqrt[a]*d)
```

3.173.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4368
```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Simp[-(d*b^(m - 1))^(-1)   Subst[Int[(-a + b*x)^((m - 1)/2
)*((a + b*x)^((m - 1)/2 + n)/x), x], x, Csc[c + d*x]], x] /; FreeQ[{a, b, c
, d, n}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[n]
```

3.173.4 Maple [A] (verified)

Time = 0.69 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.84

method	result	size
derivativedivides	$-\frac{2 \operatorname{arctanh}\left(\frac{\sqrt{a+a \sec(dx+c)}}{\sqrt{a}}\right)}{d\sqrt{a}}$	26
default	$-\frac{2 \operatorname{arctanh}\left(\frac{\sqrt{a+a \sec(dx+c)}}{\sqrt{a}}\right)}{d\sqrt{a}}$	26

input
```
int(tan(d*x+c)/(a+a*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2*arctanh((a+a*sec(d*x+c))^(1/2)/a^(1/2))/d/a^(1/2)
```

3.173.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 60 vs. $2(25) = 50$.

Time = 0.32 (sec) , antiderivative size = 137, normalized size of antiderivative = 4.42

$$\int \frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx$$

$$= \left[\frac{\log\left(-8\,a\cos\left(dx+c\right)^2+4\left(2\,\cos\left(dx+c\right)^2+\cos\left(dx+c\right)\right)\sqrt{a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}-8\,a\cos\left(dx+c\right)-a\right)}{2\,\sqrt{a}d}, \frac{\sqrt{}\right.$$

input
```
integrate(tan(d*x+c)/(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
[1/2*log(-8*a*cos(d*x + c)^2 + 4*(2*cos(d*x + c)^2 + cos(d*x + c))*sqrt(a)
*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)) - 8*a*cos(d*x + c) - a)/(sqrt(a)*
d), sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos
(d*x + c)/(2*a*cos(d*x + c) + a))/(a*d)]
```

3.173.6 Sympy [F]

$$\int \frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx = \int \frac{\tan\left(c+dx\right)}{\sqrt{a\left(\sec\left(c+dx\right)+1\right)}}\,dx$$

input
```
integrate(tan(d*x+c)/(a+a*sec(d*x+c))**(1/2),x)
```

output
```
Integral(tan(c + d*x)/sqrt(a*(sec(c + d*x) + 1)), x)
```

3.173.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.58

$$\int \frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx = \frac{\log\left(\frac{\sqrt{a+\frac{a}{\cos(dx+c)}}-\sqrt{a}}{\sqrt{a+\frac{a}{\cos(dx+c)}}+\sqrt{a}}\right)}{\sqrt{a}d}$$

input
```
integrate(tan(d*x+c)/(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
log((sqrt(a + a/cos(d*x + c)) - sqrt(a))/(sqrt(a + a/cos(d*x + c)) + sqrt(
a)))/(sqrt(a)*d)
```

3.173.8 Giac [A] (verification not implemented)

Time = 0.78 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.55

$$\int \frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx = \frac{2\arctan\left(\frac{\sqrt{2}\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}}{2\sqrt{-a}}\right)}{\sqrt{-a}d\operatorname{sgn}\left(\cos\left(dx+c\right)\right)}$$

input
```
integrate(tan(d*x+c)/(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
2*arctan(1/2*sqrt(2)*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)/sqrt(-a))/(sqrt(-
a)*d*sgn(cos(d*x + c)))
```

3.173.9 Mupad [B] (verification not implemented)

Time = 15.18 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.87

$$\int \frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx = -\frac{2\,\mathrm{atanh}\left(\frac{\sqrt{a+\frac{a}{\cos(c+d\,x)}}}{\sqrt{a}}\right)}{\sqrt{a}\,d}$$

input
```
int(tan(c + d*x)/(a + a/cos(c + d*x))^(1/2),x)
```

output
```
-(2*atanh((a + a/cos(c + d*x))^(1/2)/a^(1/2)))/(a^(1/2)*d)
```

3.174 $\int \frac{\cot(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx$

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3.174.7 Maxima [F] 1293
3.174.8 Giac [A] (verification not implemented) 1293
3.174.9 Mupad [F(-1)] 1293

3.174.1 Optimal result

Integrand size = 21, antiderivative size = 92

$$\int \frac{\cot(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx = \frac{2\operatorname{arctanh}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}d} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{\sqrt{2}\sqrt{a}d} - \frac{1}{d\sqrt{a+a\sec(c+dx)}}$$

output

```
2*arctanh((a+a*sec(d*x+c))^(1/2)/a^(1/2))/d/a^(1/2)-1/2*arctanh(1/2*(a+a*s
ec(d*x+c))^(1/2)*2^(1/2)/a^(1/2))/d*2^(1/2)/a^(1/2)-1/d/(a+a*sec(d*x+c))^(
1/2)
```

3.174.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.06 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.62

$$\int \frac{\cot(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx = \frac{\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},\frac{1}{2}(1+\sec(c+dx))\right) - 2\,\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},1+\sec(c+dx)\right)}{d\sqrt{a(1+\sec(c+dx))}}$$

input
```
Integrate[Cot[c + d*x]/Sqrt[a + a*Sec[c + d*x]],x]
```

output
```
(Hypergeometric2F1[-1/2, 1, 1/2, (1 + Sec[c + d*x])/2] - 2*Hypergeometric2
F1[-1/2, 1, 1/2, 1 + Sec[c + d*x]])/(d*Sqrt[a*(1 + Sec[c + d*x])])
```

3.174.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.07, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {3042, 25, 4368, 25, 27, 96, 25, 27, 174, 73, 219, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot(c+dx)}{\sqrt{a\sec(c+dx)+a}}\,dx$$

$$\downarrow 3042$$

$$\int -\frac{1}{\cot\left(c+dx+\frac{\pi}{2}\right)\sqrt{a\csc\left(c+dx+\frac{\pi}{2}\right)+a}}dx$$

$$\downarrow 25$$

$$-\int \frac{1}{\cot\left(\frac{1}{2}(2c+\pi)+dx\right)\sqrt{\csc\left(\frac{1}{2}(2c+\pi)+dx\right)a+a}}dx$$

$$\downarrow 4368$$

$$\frac{a^2\int -\frac{\cos(c+dx)}{a(1-\sec(c+dx))(\sec(c+dx)a+a)^{3/2}}d\sec(c+dx)}{d}$$

$$\downarrow 25$$

$$-\frac{a^2\int \frac{\cos(c+dx)}{a(1-\sec(c+dx))(\sec(c+dx)a+a)^{3/2}}d\sec(c+dx)}{d}$$

$$\downarrow 27$$

$$-\frac{a\int \frac{\cos(c+dx)}{(1-\sec(c+dx))(\sec(c+dx)a+a)^{3/2}}d\sec(c+dx)}{d}$$

$$\downarrow 96$$

$$-\frac{a\left(\frac{1}{a\sqrt{a\sec(c+dx)+a}}-\frac{\int -\frac{a\cos(c+dx)(2-\sec(c+dx))}{(1-\sec(c+dx))\sqrt{\sec(c+dx)a+a}}d\sec(c+dx)}{2a^2}\right)}{d}$$

$$\downarrow 25$$

$$-\frac{a\left(\frac{\int \frac{a\cos(c+dx)(2-\sec(c+dx))}{(1-\sec(c+dx))\sqrt{\sec(c+dx)a+a}}d\sec(c+dx)}{2a^2}+\frac{1}{a\sqrt{a\sec(c+dx)+a}}\right)}{d}$$

$$\downarrow 27$$

$$-\frac{a\left(\frac{\int \frac{\cos(c+dx)(2-\sec(c+dx))}{(1-\sec(c+dx))\sqrt{\sec(c+dx)a+a}}d\sec(c+dx)}{2a}+\frac{1}{a\sqrt{a\sec(c+dx)+a}}\right)}{d}$$

$$\downarrow 174$$

$$-\frac{a\left(\frac{\int \frac{1}{(1-\sec(c+dx))\sqrt{\sec(c+dx)a+a}}d\sec(c+dx)+2\int \frac{\cos(c+dx)}{\sqrt{\sec(c+dx)a+a}}d\sec(c+dx)}{2a}+\frac{1}{a\sqrt{a\sec(c+dx)+a}}\right)}{d}$$

$$\downarrow 73$$

$$-\frac{a\left(\frac{2\int \frac{1}{2-\frac{\sec(c+dx)a+a}{a}}d\sqrt{\sec(c+dx)a+a}}{a}+\frac{4\int \frac{1}{\frac{\sec(c+dx)a+a}{a}-1}d\sqrt{\sec(c+dx)a+a}}{a}}{2a}+\frac{1}{a\sqrt{a\sec(c+dx)+a}}\right)}{d}$$

$$\downarrow 219$$

$$-\frac{a\left(\frac{4\int \frac{1}{\frac{\sec(c+dx)a+a}{a}-1}d\sqrt{\sec(c+dx)a+a}}{a}+\frac{\sqrt{2}\operatorname{arctanh}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{2}\sqrt{a}}\right)}{\sqrt{a}}}{2a}+\frac{1}{a\sqrt{a\sec(c+dx)+a}}\right)}{d}$$

$$\downarrow 220$$

$$-\frac{a\left(\frac{\sqrt{2}\operatorname{arctanh}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{2}\sqrt{a}}\right)}{\sqrt{a}}-\frac{4\operatorname{arctanh}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{a}}\right)}{\sqrt{a}}}{2a}+\frac{1}{a\sqrt{a\sec(c+dx)+a}}\right)}{d}$$

input
```
Int[Cot[c + d*x]/Sqrt[a + a*Sec[c + d*x]],x]
```

output
```
-((a*(((-4*ArcTanh[Sqrt[a + a*Sec[c + d*x]]/Sqrt[a]])/Sqrt[a] + (Sqrt[2]*A
rcTanh[Sqrt[a + a*Sec[c + d*x]]/(Sqrt[2]*Sqrt[a])])/Sqrt[a])/(2*a) + 1/(a*
Sqrt[a + a*Sec[c + d*x]])))/d)
```

3.174.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 96
```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_] :> Simp[f*((e + f*x)^(p + 1)/((p + 1)*(b*e - a*f)*(d*e - c*f))), x] + S
imp[1/((b*e - a*f)*(d*e - c*f))   Int[(b*d*e - b*c*f - a*d*f - b*d*f*x)*((e
 + f*x)^(p + 1)/((a + b*x)*(c + d*x))), x], x] /; FreeQ[{a, b, c, d, e, f},
 x] && LtQ[p, -1]
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4368
```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Simp[-(d*b^(m - 1))^(-1)   Subst[Int[(-a + b*x)^((m - 1)/2
)*((a + b*x)^((m - 1)/2 + n)/x), x], x, Csc[c + d*x]], x] /; FreeQ[{a, b, c
, d, n}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[n]
```

3.174.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 228 vs. $2(75) = 150$.

Time = 1.80 (sec) , antiderivative size = 229, normalized size of antiderivative = 2.49

method	result
default	$-\frac{\sqrt{-\frac{2a}{(1-\cos(dx+c))^2\csc(dx+c)^2-1}}\sqrt{(1-\cos(dx+c))^2\csc(dx+c)^2-1}\left(-\left((1-\cos(dx+c))^2\csc(dx+c)^2-1\right)^{\frac{3}{2}}+(1-\cos(dx+c))^2\sqrt{(1-$

input
```
int(cot(d*x+c)/(a+a*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/6/d/a*(-2*a/((1-cos(d*x+c))^2*csc(d*x+c)^2-1))^(1/2)*((1-cos(d*x+c))^2*
csc(d*x+c)^2-1)^(1/2)*(-((1-cos(d*x+c))^2*csc(d*x+c)^2-1)^(3/2)+(1-cos(d*x
+c))^2*((1-cos(d*x+c))^2*csc(d*x+c)^2-1)^(1/2)*csc(d*x+c)^2+6*2^(1/2)*arct
an(1/2*2^(1/2)*((1-cos(d*x+c))^2*csc(d*x+c)^2-1)^(1/2))+3*arctan(1/((1-cos
(d*x+c))^2*csc(d*x+c)^2-1)^(1/2))-4*((1-cos(d*x+c))^2*csc(d*x+c)^2-1)^(1/2
))
```

3.174.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 180 vs. $2(75) = 150$.

Time = 0.35 (sec) , antiderivative size = 384, normalized size of antiderivative = 4.17

$$\int \frac{\cot(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx$$

$$= \left[\frac{2\sqrt{a}(\cos(dx+c)+1)\log\left(-8\,a\cos(dx+c)^2 - 4\left(2\cos(dx+c)^2+\cos(dx+c)\right)\sqrt{a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}} - 8\right.}{4\,(ad\cos(} \right.$$

input
```
integrate(cot(d*x+c)/(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
[1/4*(2*sqrt(a)*(cos(d*x + c) + 1)*log(-8*a*cos(d*x + c)^2 - 4*(2*cos(d*x
+ c)^2 + cos(d*x + c))*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)) - 8
*a*cos(d*x + c) - a) + sqrt(2)*(a*cos(d*x + c) + a)*log(-(2*sqrt(2)*sqrt((
a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/sqrt(a) - 3*cos(d*x + c) -
1)/(cos(d*x + c) - 1))/sqrt(a) - 4*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))
*cos(d*x + c))/(a*d*cos(d*x + c) + a*d), 1/2*(sqrt(2)*(a*cos(d*x + c) + a)
*sqrt(-1/a)*arctan(sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(-1
/a)*cos(d*x + c)/(cos(d*x + c) + 1)) - 2*sqrt(-a)*(cos(d*x + c) + 1)*arcta
n(2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(2*a*cos
(d*x + c) + a)) - 2*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c))/
(a*d*cos(d*x + c) + a*d)]
```

3.174.6 Sympy [F]

$$\int \frac{\cot(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx = \int \frac{\cot(c+dx)}{\sqrt{a\,(\sec(c+dx)+1)}}\,dx$$

input
```
integrate(cot(d*x+c)/(a+a*sec(d*x+c))**(1/2),x)
```

output
```
Integral(cot(c + d*x)/sqrt(a*(sec(c + d*x) + 1)), x)
```

3.174.7 Maxima [F]

$$\int \frac{\cot(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx = \int \frac{\cot(dx+c)}{\sqrt{a\sec(dx+c)+a}}\,dx$$

input
```
integrate(cot(d*x+c)/(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(cot(d*x + c)/sqrt(a*sec(d*x + c) + a), x)
```

3.174.8 Giac [A] (verification not implemented)

Time = 0.75 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.21

$$\int \frac{\cot(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx = -\frac{\sqrt{2}\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}}{2\sqrt{-a}}\right)}{\sqrt{-a}} - \frac{\arctan\left(\frac{\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}} + \frac{\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}}{a}\right)}{2\,d\operatorname{sgn}(\cos(dx+c))}$$

input
```
integrate(cot(d*x+c)/(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
-1/2*sqrt(2)*(2*sqrt(2)*arctan(1/2*sqrt(2)*sqrt(-a*tan(1/2*d*x + 1/2*c)^2
+ a)/sqrt(-a))/sqrt(-a) - arctan(sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)/sqrt(
-a))/sqrt(-a) + sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)/a)/(d*sgn(cos(d*x + c)
))
```

3.174.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cot(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx = \int \frac{\cot(c+d\,x)}{\sqrt{a+\frac{a}{\cos(c+d\,x)}}}\,dx$$

input
```
int(cot(c + d*x)/(a + a/cos(c + d*x))^(1/2),x)
```

output
```
int(cot(c + d*x)/(a + a/cos(c + d*x))^(1/2), x)
```

3.175 $\int \frac{\cot^3(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx$

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3.175.2 Mathematica [C] (verified) 1295
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3.175.1 Optimal result

Integrand size = 23, antiderivative size = 152

$$\int \frac{\cot^3(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx = -\frac{2\operatorname{arctanh}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}d} + \frac{9\operatorname{arctanh}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{8\sqrt{2}\sqrt{a}d} - \frac{a}{12d(a+a\sec(c+dx))^{3/2}} + \frac{a}{2d(1-\sec(c+dx))(a+a\sec(c+dx))^{3/2}} + \frac{7}{8d\sqrt{a+a\sec(c+dx)}}$$

output
```
-1/12*a/d/(a+a*sec(d*x+c))^(3/2)+1/2*a/d/(1-sec(d*x+c))/(a+a*sec(d*x+c))^(
3/2)-2*arctanh((a+a*sec(d*x+c))^(1/2)/a^(1/2))/d/a^(1/2)+9/16*arctanh(1/2*
(a+a*sec(d*x+c))^(1/2)*2^(1/2)/a^(1/2))/d*2^(1/2)/a^(1/2)+7/8/d/(a+a*sec(d
*x+c))^(1/2)
```

3.175.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.16 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.59

$$\int \frac{\cot^3(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx$$
$$= \frac{a\left(-6-9\,\text{Hypergeometric2F1}\left(-\frac{3}{2},1,-\frac{1}{2},\frac{1}{2}(1+\sec(c+dx))\right)(-1+\sec(c+dx))+8\,\text{Hypergeometric2F}\right.}{12d(-1+\sec(c+dx))(a(1+\sec(c+dx)))^{3/2}}$$

input
```
Integrate[Cot[c + d*x]^3/Sqrt[a + a*Sec[c + d*x]],x]
```

output
```
(a*(-6 - 9*Hypergeometric2F1[-3/2, 1, -1/2, (1 + Sec[c + d*x])/2]*(-1 + Se
c[c + d*x]) + 8*Hypergeometric2F1[-3/2, 1, -1/2, 1 + Sec[c + d*x]]*(-1 + S
ec[c + d*x])))/(12*d*(-1 + Sec[c + d*x])*(a*(1 + Sec[c + d*x]))^(3/2))
```

3.175.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.11, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.609$, Rules used = {3042, 25, 4368, 27, 114, 27, 169, 27, 169, 27, 174, 73, 219, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^3(c+dx)}{\sqrt{a\sec(c+dx)+a}}\,dx$$
$$\downarrow 3042$$
$$\int -\frac{1}{\cot\left(c+dx+\frac{\pi}{2}\right)^3\sqrt{a\csc\left(c+dx+\frac{\pi}{2}\right)+a}}\,dx$$
$$\downarrow 25$$
$$-\int \frac{1}{\cot\left(\frac{1}{2}(2c+\pi)+dx\right)^3\sqrt{\csc\left(\frac{1}{2}(2c+\pi)+dx\right)a+a}}\,dx$$
$$\downarrow 4368$$
$$\frac{a^4\int \frac{\cos(c+dx)}{a^2(1-\sec(c+dx))^2(\sec(c+dx)a+a)^{5/2}}\,d\sec(c+dx)}{d}$$

$$\downarrow 27$$

$$\frac{a^2 \int \frac{\cos(c+dx)}{(1-\sec(c+dx))^2(\sec(c+dx)a+a)^{5/2}} d\sec(c+dx)}{d}$$

$$\downarrow 114$$

$$\frac{a^2\left(\frac{1}{2a(1-\sec(c+dx))(a\sec(c+dx)+a)^{3/2}} - \frac{\int -\frac{a\cos(c+dx)(5\sec(c+dx)+4)}{2(1-\sec(c+dx))(\sec(c+dx)a+a)^{5/2}} d\sec(c+dx)}{2a}\right)}{d}$$

$$\downarrow 27$$

$$\frac{a^2\left(\frac{1}{4}\int \frac{\cos(c+dx)(5\sec(c+dx)+4)}{(1-\sec(c+dx))(\sec(c+dx)a+a)^{5/2}} d\sec(c+dx) + \frac{1}{2a(1-\sec(c+dx))(a\sec(c+dx)+a)^{3/2}}\right)}{d}$$

$$\downarrow 169$$

$$\frac{a^2\left(\frac{1}{4}\left(\frac{\int \frac{3a\cos(c+dx)(\sec(c+dx)+8)}{2(1-\sec(c+dx))(\sec(c+dx)a+a)^{3/2}} d\sec(c+dx)}{3a^2} - \frac{1}{3a(a\sec(c+dx)+a)^{3/2}}\right) + \frac{1}{2a(1-\sec(c+dx))(a\sec(c+dx)+a)^{3/2}}\right)}{d}$$

$$\downarrow 27$$

$$\frac{a^2\left(\frac{1}{4}\left(\frac{\int \frac{\cos(c+dx)(\sec(c+dx)+8)}{(1-\sec(c+dx))(\sec(c+dx)a+a)^{3/2}} d\sec(c+dx)}{2a} - \frac{1}{3a(a\sec(c+dx)+a)^{3/2}}\right) + \frac{1}{2a(1-\sec(c+dx))(a\sec(c+dx)+a)^{3/2}}\right)}{d}$$

$$\downarrow 169$$

$$\frac{a^2\left(\frac{1}{4}\left(\frac{\int \frac{a\cos(c+dx)(16-7\sec(c+dx))}{2(1-\sec(c+dx))\sqrt{\sec(c+dx)a+a}} d\sec(c+dx)}{a^2} + \frac{7}{a\sqrt{a\sec(c+dx)+a}}}{2a} - \frac{1}{3a(a\sec(c+dx)+a)^{3/2}}\right) + \frac{1}{2a(1-\sec(c+dx))(a\sec(c+dx)+a)^{3/2}}\right)}{d}$$

$$\downarrow 27$$

$$\frac{a^2\left(\frac{1}{4}\left(\frac{\int \frac{\cos(c+dx)(16-7\sec(c+dx))}{(1-\sec(c+dx))\sqrt{\sec(c+dx)a+a}} d\sec(c+dx)}{2a} + \frac{7}{a\sqrt{a\sec(c+dx)+a}}}{2a} - \frac{1}{3a(a\sec(c+dx)+a)^{3/2}}\right) + \frac{1}{2a(1-\sec(c+dx))(a\sec(c+dx)+a)^{3/2}}\right)}{d}$$

$$\downarrow 174$$

3.175. $\int \frac{\cot^3(c+dx)}{\sqrt{a+a\sec(c+dx)}} dx$

$$a^2\left(\frac{1}{4}\left(\frac{\frac{9\int\frac{1}{(1-\sec(c+dx))\sqrt{\sec(c+dx)a+a}}d\sec(c+dx)+16\int\frac{\cos(c+dx)}{\sqrt{\sec(c+dx)a+a}}d\sec(c+dx)}{2a}+\frac{7}{a\sqrt{a\sec(c+dx)+a}}}{2a}-\frac{1}{3a(a\sec(c+dx)+a)^{3/2}}\right)+\frac{}{2a(1-\sec($$

$$\Big\downarrow 73$$

$$a^2\left(\frac{1}{4}\left(\frac{\frac{18\int\frac{1}{2-\frac{\sec(c+dx)a+a}{a}}d\sqrt{\sec(c+dx)a+a}}{a}+\frac{32\int\frac{1}{\frac{\sec(c+dx)a+a}{a}-1}d\sqrt{\sec(c+dx)a+a}}{a}}{2a}+\frac{7}{a\sqrt{a\sec(c+dx)+a}}}{2a}-\frac{1}{3a(a\sec(c+dx)+a)^{3/2}}\right)+\frac{}{2a(1-s}$$

$$\Big\downarrow 219$$

$$a^2\left(\frac{1}{4}\left(\frac{\frac{32\int\frac{1}{\frac{\sec(c+dx)a+a}{a}-1}d\sqrt{\sec(c+dx)a+a}}{a}+\frac{9\sqrt{2}\operatorname{arctanh}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{2}\sqrt{a}}\right)}{\sqrt{a}}}{2a}+\frac{7}{a\sqrt{a\sec(c+dx)+a}}}{2a}-\frac{1}{3a(a\sec(c+dx)+a)^{3/2}}\right)+\frac{}{2a(1-\sec(c}$$

$$\Big\downarrow 220$$

$$a^2\left(\frac{1}{4}\left(\frac{\frac{9\sqrt{2}\operatorname{arctanh}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{2}\sqrt{a}}\right)}{\sqrt{a}}-\frac{32\operatorname{arctanh}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{a}}\right)}{\sqrt{a}}}{2a}+\frac{7}{a\sqrt{a\sec(c+dx)+a}}}{2a}-\frac{1}{3a(a\sec(c+dx)+a)^{3/2}}\right)+\frac{}{2a(1-\sec(c+dx)}$$

input
```
Int[Cot[c + d*x]^3/Sqrt[a + a*Sec[c + d*x]],x]
```

output
```
(a^2*(1/(2*a*(1 - Sec[c + d*x])*(a + a*Sec[c + d*x])^(3/2)) + (-1/3*1/(a*(
a + a*Sec[c + d*x])^(3/2)) + (((-32*ArcTanh[Sqrt[a + a*Sec[c + d*x]]/Sqrt[
a]])/Sqrt[a] + (9*Sqrt[2]*ArcTanh[Sqrt[a + a*Sec[c + d*x]]/(Sqrt[2]*Sqrt[a
])])/Sqrt[a])/(2*a) + 7/(a*Sqrt[a + a*Sec[c + d*x]]))/(2*a))/4))/d
```

3.175. $\int \frac{\cot^3(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx$

3.175.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 114
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[1/((m + 1)*(b*c - a*d)*(b*e
 - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1)
 - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (IntegerQ[n] ||
 IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

rule 169
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && IntegersQ[
2*m, 2*n, 2*p]
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4368
```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Simp[-(d*b^(m - 1))^(-1)   Subst[Int[(-a + b*x)^((m - 1)/2
)*((a + b*x)^((m - 1)/2 + n)/x), x], x, Csc[c + d*x]], x] /; FreeQ[{a, b, c
, d, n}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[n]
```

3.175.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 271 vs. $2(123) = 246$.

Time = 1.75 (sec) , antiderivative size = 272, normalized size of antiderivative = 1.79

method	result
default	$\frac{\sqrt{a(1+\sec(dx+c))}\left(27\arctan\left(\frac{\sqrt{2}}{2\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}}\right)\cos(dx+c)\sqrt{2}\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}+27\sqrt{2}\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}\arctan\left(\frac{\sqrt{2}}{2\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+}}}\right.\right.$

input
```
int(cot(d*x+c)^3/(a+a*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/48/d/a*(a*(1+sec(d*x+c)))^(1/2)/(cos(d*x+c)+1)*(27*arctan(1/2*2^(1/2)/(-
cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*cos(d*x+c)*2^(1/2)*(-cos(d*x+c)/(cos(d*x
+c)+1))^(1/2)+27*2^(1/2)*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctan(1/2*2^(
1/2)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))+96*arctan((-cos(d*x+c)/(cos(d*x+c
)+1))^(1/2))*cos(d*x+c)*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+96*arctan((-cos
(d*x+c)/(cos(d*x+c)+1))^(1/2))*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)-62*cos(d
*x+c)*cot(d*x+c)^2-4*cot(d*x+c)^2+42*cot(d*x+c)*csc(d*x+c))
```

3.175.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 260 vs. $2(121) = 242$.

Time = 0.35 (sec) , antiderivative size = 546, normalized size of antiderivative = 3.59

$$\int \frac{\cot^3(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx$$

$$= \left[\frac{27\sqrt{2}\left(\cos(dx+c)^3+\cos(dx+c)^2-\cos(dx+c)-1\right)\sqrt{a}\log\left(\frac{2\sqrt{2}\sqrt{a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)+3\,a\cos(dx+c)+a}{\cos(dx+c)-1}\right.}{} \right.$$

$$\left. \left. -\frac{27\sqrt{2}\left(\cos(dx+c)^3+\cos(dx+c)^2-\cos(dx+c)-1\right)\sqrt{-a}\arctan\left(\frac{\sqrt{2}\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)}{a\cos(dx+c)+a}\right)-48}{} \right] \right.$$

input
```
integrate(cot(d*x+c)^3/(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
[1/96*(27*sqrt(2)*(cos(d*x + c)^3 + cos(d*x + c)^2 - cos(d*x + c) - 1)*sqr
t(a)*log((2*sqrt(2)*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*
x + c) + 3*a*cos(d*x + c) + a)/(cos(d*x + c) - 1)) + 48*(cos(d*x + c)^3 +
cos(d*x + c)^2 - cos(d*x + c) - 1)*sqrt(a)*log(-8*a*cos(d*x + c)^2 + 4*(2*
cos(d*x + c)^2 + cos(d*x + c))*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x +
 c)) - 8*a*cos(d*x + c) - a) + 4*(31*cos(d*x + c)^3 + 2*cos(d*x + c)^2 - 2
1*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)))/(a*d*cos(d*x + c)
^3 + a*d*cos(d*x + c)^2 - a*d*cos(d*x + c) - a*d), -1/48*(27*sqrt(2)*(cos(
d*x + c)^3 + cos(d*x + c)^2 - cos(d*x + c) - 1)*sqrt(-a)*arctan(sqrt(2)*sq
rt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(a*cos(d*x + c
) + a)) - 48*(cos(d*x + c)^3 + cos(d*x + c)^2 - cos(d*x + c) - 1)*sqrt(-a)
*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(2
*a*cos(d*x + c) + a)) - 2*(31*cos(d*x + c)^3 + 2*cos(d*x + c)^2 - 21*cos(d
*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)))/(a*d*cos(d*x + c)^3 + a*
d*cos(d*x + c)^2 - a*d*cos(d*x + c) - a*d)]
```

3.175.6 Sympy [F]

$$\int \frac{\cot^3(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx = \int \frac{\cot^3(c+dx)}{\sqrt{a\left(\sec(c+dx)+1\right)}}\,dx$$

input
```
integrate(cot(d*x+c)**3/(a+a*sec(d*x+c))**(1/2),x)
```

output
```
Integral(cot(c + d*x)**3/sqrt(a*(sec(c + d*x) + 1)), x)
```

3.175.7 Maxima [F]

$$\int \frac{\cot^3(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx = \int \frac{\cot(dx+c)^3}{\sqrt{a\sec(dx+c)+a}}\,dx$$

input
```
integrate(cot(d*x+c)^3/(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(cot(d*x + c)^3/sqrt(a*sec(d*x + c) + a), x)
```

3.175.8 Giac [A] (verification not implemented)

Time = 0.84 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.14

$$\int \frac{\cot^3(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx$$
$$= \frac{\sqrt{2}\left(\frac{48\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}}{2\sqrt{-a}}\right)}{\sqrt{-a}} - \frac{27\arctan\left(\frac{\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}} - \frac{3\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}}{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2} + \frac{2\left(\left(-a\tan\left(\frac{1}{2}d\right.\right.\right.\right.}{48\,d\,\mathrm{sgn}\left(\cos\left(dx+c\right)\right)}$$

input
```
integrate(cot(d*x+c)^3/(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
1/48*sqrt(2)*(48*sqrt(2)*arctan(1/2*sqrt(2)*sqrt(-a*tan(1/2*d*x + 1/2*c)^2
 + a)/sqrt(-a))/sqrt(-a) - 27*arctan(sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)/s
qrt(-a))/sqrt(-a) - 3*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)/(a*tan(1/2*d*x +
 1/2*c)^2) + 2*((-a*tan(1/2*d*x + 1/2*c)^2 + a)^(3/2)*a^4 + 12*sqrt(-a*tan
(1/2*d*x + 1/2*c)^2 + a)*a^5)/a^6)/(d*sgn(cos(d*x + c)))
```

3.175.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cot^3(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx = \int \frac{\cot(c+d\,x)^3}{\sqrt{a+\frac{a}{\cos(c+d\,x)}}}\,dx$$

input
```
int(cot(c + d*x)^3/(a + a/cos(c + d*x))^(1/2),x)
```

output
```
int(cot(c + d*x)^3/(a + a/cos(c + d*x))^(1/2), x)
```

3.176 $\int \frac{\cot^5(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx$

3.176.1 Optimal result 1303
3.176.2 Mathematica [C] (verified) 1304
3.176.3 Rubi [A] (verified) 1304
3.176.4 Maple [A] (verified) 1309
3.176.5 Fricas [A] (verification not implemented) 1310
3.176.6 Sympy [F] 1311
3.176.7 Maxima [F] 1312
3.176.8 Giac [A] (verification not implemented) 1312
3.176.9 Mupad [F(-1)] 1313

3.176.1 Optimal result

Integrand size = 23, antiderivative size = 214

$$\begin{aligned}\int \frac{\cot^5(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx = {}& \frac{2\operatorname{arctanh}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}d} - \frac{151\operatorname{arctanh}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{128\sqrt{2}\sqrt{a}d} \\ & + \frac{87a^2}{160d(a+a\sec(c+dx))^{5/2}} \\ & - \frac{a^2}{4d(1-\sec(c+dx))^2(a+a\sec(c+dx))^{5/2}} \\ & - \frac{17a^2}{16d(1-\sec(c+dx))(a+a\sec(c+dx))^{5/2}} \\ & + \frac{23a}{192d(a+a\sec(c+dx))^{3/2}} - \frac{105}{128d\sqrt{a+a\sec(c+dx)}}\end{aligned}$$

output

```
87/160*a^2/d/(a+a*sec(d*x+c))^(5/2)-1/4*a^2/d/(1-sec(d*x+c))^2/(a+a*sec(d*
x+c))^(5/2)-17/16*a^2/d/(1-sec(d*x+c))/(a+a*sec(d*x+c))^(5/2)+23/192*a/d/(
a+a*sec(d*x+c))^(3/2)+2*arctanh((a+a*sec(d*x+c))^(1/2)/a^(1/2))/d/a^(1/2)-
151/256*arctanh(1/2*(a+a*sec(d*x+c))^(1/2)*2^(1/2)/a^(1/2))/d*2^(1/2)/a^(1
/2)-105/128/d/(a+a*sec(d*x+c))^(1/2)
```

3.176.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.26 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.48

$$\int \frac{\cot^5(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx$$
$$= \frac{\cot^4(c+dx)\left(-2\left(105+32\,\text{Hypergeometric2F1}\left(-\frac{5}{2},1,-\frac{3}{2},1+\sec(c+dx)\right)\right)(-1+\sec(c+dx))^2-85\sec(c+dx)\right)}{160d\sqrt{a(1+\sec(c+dx))}}$$

input
```
Integrate[Cot[c + d*x]^5/Sqrt[a + a*Sec[c + d*x]],x]
```

output
```
(Cot[c + d*x]^4*(-2*(105 + 32*Hypergeometric2F1[-5/2, 1, -3/2, 1 + Sec[c +
 d*x]]*(-1 + Sec[c + d*x])^2 - 85*Sec[c + d*x]) + 151*Hypergeometric2F1[-5
/2, 1, -3/2, (1 + Sec[c + d*x])/2]*(-1 + Sec[c + d*x])^2))/(160*d*Sqrt[a*(
1 + Sec[c + d*x])])
```

3.176.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 236, normalized size of antiderivative = 1.10, number of steps used = 20, number of rules used = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.826$, Rules used = {3042, 25, 4368, 25, 27, 114, 27, 168, 27, 169, 27, 169, 27, 169, 27, 174, 73, 219, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^5(c+dx)}{\sqrt{a\sec(c+dx)+a}}\,dx$$
$$\downarrow 3042$$
$$\int -\frac{1}{\cot\left(c+dx+\frac{\pi}{2}\right)^5\sqrt{a\csc\left(c+dx+\frac{\pi}{2}\right)+a}}\,dx$$
$$\downarrow 25$$
$$-\int \frac{1}{\cot\left(\frac{1}{2}(2c+\pi)+dx\right)^5\sqrt{\csc\left(\frac{1}{2}(2c+\pi)+dx\right)a+a}}\,dx$$
$$\downarrow 4368$$

$$\frac{a^6 \int -\frac{\cos(c+dx)}{a^3(1-\sec(c+dx))^3(\sec(c+dx)a+a)^{7/2}}\,d\sec(c+dx)}{d}$$

$$\downarrow 25$$

$$-\frac{a^6 \int \frac{\cos(c+dx)}{a^3(1-\sec(c+dx))^3(\sec(c+dx)a+a)^{7/2}}\,d\sec(c+dx)}{d}$$

$$\downarrow 27$$

$$-\frac{a^3 \int \frac{\cos(c+dx)}{(1-\sec(c+dx))^3(\sec(c+dx)a+a)^{7/2}}\,d\sec(c+dx)}{d}$$

$$\downarrow 114$$

$$-\frac{a^3\left(\frac{1}{4a(1-\sec(c+dx))^2(a\sec(c+dx)+a)^{5/2}} - \frac{\int -\frac{a\cos(c+dx)(9\sec(c+dx)+8)}{2(1-\sec(c+dx))^2(\sec(c+dx)a+a)^{7/2}}\,d\sec(c+dx)}{4a}\right)}{d}$$

$$\downarrow 27$$

$$-\frac{a^3\left(\frac{1}{8}\int \frac{\cos(c+dx)(9\sec(c+dx)+8)}{(1-\sec(c+dx))^2(\sec(c+dx)a+a)^{7/2}}\,d\sec(c+dx) + \frac{1}{4a(1-\sec(c+dx))^2(a\sec(c+dx)+a)^{5/2}}\right)}{d}$$

$$\downarrow 168$$

$$-\frac{a^3\left(\frac{1}{8}\left(\frac{17}{2a(1-\sec(c+dx))(a\sec(c+dx)+a)^{5/2}} - \frac{\int -\frac{a\cos(c+dx)(119\sec(c+dx)+32)}{2(1-\sec(c+dx))(\sec(c+dx)a+a)^{7/2}}\,d\sec(c+dx)}{2a}\right) + \frac{1}{4a(1-\sec(c+dx))^2(a\sec(c+dx)+a)^{5/}}\right)}{d}$$

$$\downarrow 27$$

$$-\frac{a^3\left(\frac{1}{8}\left(\frac{1}{4}\int \frac{\cos(c+dx)(119\sec(c+dx)+32)}{(1-\sec(c+dx))(\sec(c+dx)a+a)^{7/2}}\,d\sec(c+dx) + \frac{17}{2a(1-\sec(c+dx))(a\sec(c+dx)+a)^{5/2}}\right) + \frac{1}{4a(1-\sec(c+dx))^2(a\sec(c+dx)}\right)}{d}$$

$$\downarrow 169$$

$$-\frac{a^3\left(\frac{1}{8}\left(\frac{1}{4}\left(\frac{\int \frac{5a\cos(c+dx)(87\sec(c+dx)+64)}{2(1-\sec(c+dx))(\sec(c+dx)a+a)^{5/2}}\,d\sec(c+dx)}{5a^2} - \frac{87}{5a(a\sec(c+dx)+a)^{5/2}}\right) + \frac{17}{2a(1-\sec(c+dx))(a\sec(c+dx)+a)^{5/2}}\right) + \frac{}{4a(1-}\right)}{d}$$

$$\downarrow 27$$

$$-\frac{a^3\left(\frac{1}{8}\left(\frac{1}{4}\left(\frac{\int \frac{\cos(c+dx)(87\sec(c+dx)+64)}{(1-\sec(c+dx))(\sec(c+dx)a+a)^{5/2}}\,d\sec(c+dx)}{2a} - \frac{87}{5a(a\sec(c+dx)+a)^{5/2}}\right) + \frac{17}{2a(1-\sec(c+dx))(a\sec(c+dx)+a)^{5/2}}\right) + \frac{}{4a(1-}\right)}{d}$$

3.176. $\int \frac{\cot^5(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx$

$$\downarrow 169$$

$$-\frac{a^3\left(\frac{1}{8}\left(\frac{1}{4}\left(\frac{\frac{\int \frac{3a\cos(c+dx)(23\sec(c+dx)+128)}{2(1-\sec(c+dx))(\sec(c+dx)a+a)^{3/2}}d\sec(c+dx)}{3a^2}-\frac{23}{3a(a\sec(c+dx)+a)^{3/2}}}{2a}-\frac{87}{5a(a\sec(c+dx)+a)^{5/2}}\right)+\frac{17}{2a(1-\sec(c+dx))(a\sec(c+}\right.}{d}$$

$$\downarrow 27$$

$$-\frac{a^3\left(\frac{1}{8}\left(\frac{1}{4}\left(\frac{\frac{\int \frac{\cos(c+dx)(23\sec(c+dx)+128)}{(1-\sec(c+dx))(\sec(c+dx)a+a)^{3/2}}d\sec(c+dx)}{2a}-\frac{23}{3a(a\sec(c+dx)+a)^{3/2}}}{2a}-\frac{87}{5a(a\sec(c+dx)+a)^{5/2}}\right)+\frac{17}{2a(1-\sec(c+dx))(a\sec(c+d}\right.}{d}$$

$$\downarrow 169$$

$$-\frac{a^3\left(\frac{1}{8}\left(\frac{1}{4}\left(\frac{\frac{\int \frac{a\cos(c+dx)(256-105\sec(c+dx))}{2(1-\sec(c+dx))\sqrt{\sec(c+dx)a+a}}d\sec(c+dx)}{a^2}+\frac{105}{a\sqrt{a\sec(c+dx)+a}}}{2a}-\frac{23}{3a(a\sec(c+dx)+a)^{3/2}}}{2a}-\frac{87}{5a(a\sec(c+dx)+a)^{5/2}}\right)+\frac{}{2a(1-\sec(c}\right.}{d}$$

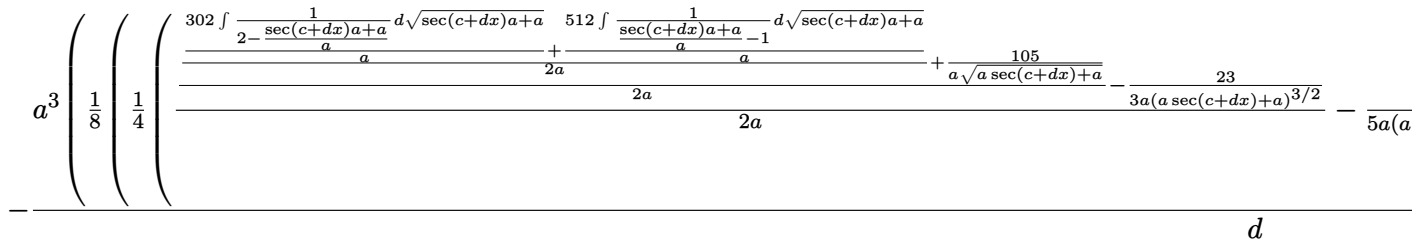
$$\downarrow 27$$

$$-\frac{a^3\left(\frac{1}{8}\left(\frac{1}{4}\left(\frac{\frac{\int \frac{\cos(c+dx)(256-105\sec(c+dx))}{(1-\sec(c+dx))\sqrt{\sec(c+dx)a+a}}d\sec(c+dx)}{2a}+\frac{105}{a\sqrt{a\sec(c+dx)+a}}}{2a}-\frac{23}{3a(a\sec(c+dx)+a)^{3/2}}}{2a}-\frac{87}{5a(a\sec(c+dx)+a)^{5/2}}\right)+\frac{}{2a(1-\sec(c+}\right.}{d}$$

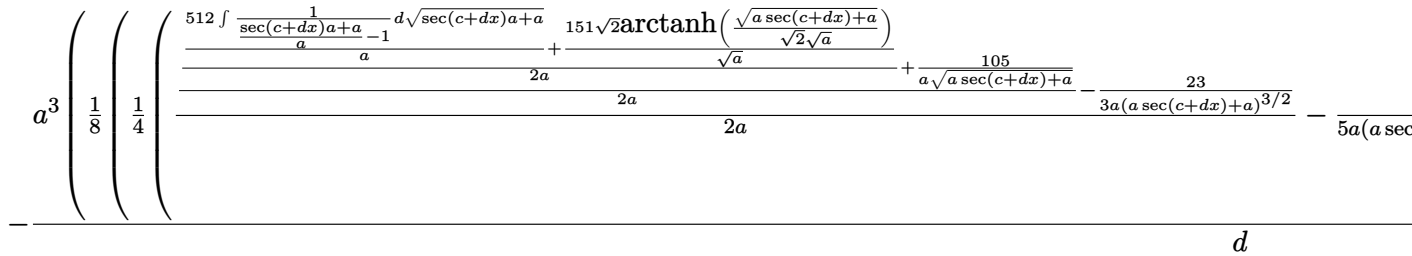
$$\downarrow 174$$

$$-\frac{a^3\left(\frac{1}{8}\left(\frac{1}{4}\left(\frac{\frac{151\int \frac{1}{(1-\sec(c+dx))\sqrt{\sec(c+dx)a+a}}d\sec(c+dx)+256\int \frac{\cos(c+dx)}{\sqrt{\sec(c+dx)a+a}}d\sec(c+dx)}{2a}+\frac{105}{a\sqrt{a\sec(c+dx)+a}}}{2a}-\frac{23}{3a(a\sec(c+dx)+a)^{3/2}}}{2a}-\frac{}{5a(a\sec}\right.}{d}$$

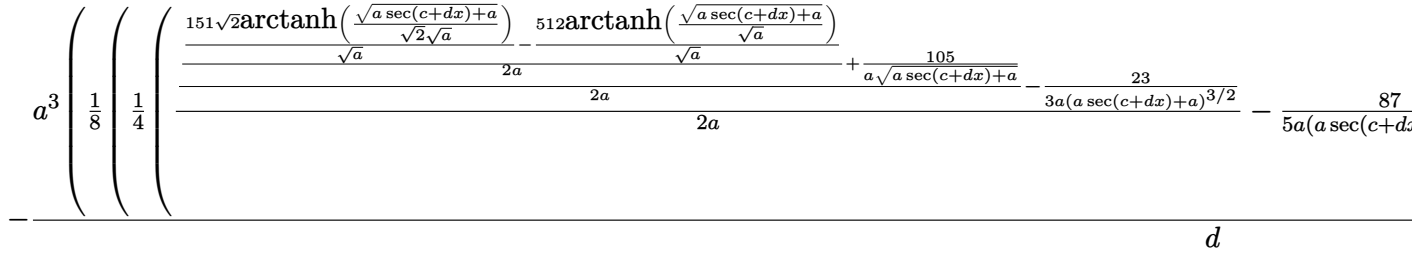
$$\downarrow 73$$



$$\downarrow 219$$



$$\downarrow 220$$



input
```
Int[Cot[c + d*x]^5/Sqrt[a + a*Sec[c + d*x]],x]
```

output
```
-((a^3*(1/(4*a*(1 - Sec[c + d*x])^2*(a + a*Sec[c + d*x])^(5/2)) + (17/(2*a
*(1 - Sec[c + d*x])*(a + a*Sec[c + d*x])^(5/2)) + (-87/(5*a*(a + a*Sec[c +
 d*x])^(5/2)) + (-23/(3*a*(a + a*Sec[c + d*x])^(3/2)) + (((-512*ArcTanh[Sq
rt[a + a*Sec[c + d*x]]/Sqrt[a]])/Sqrt[a] + (151*Sqrt[2]*ArcTanh[Sqrt[a + a
*Sec[c + d*x]]/(Sqrt[2]*Sqrt[a])])/Sqrt[a])/(2*a) + 105/(a*Sqrt[a + a*Sec[
c + d*x]]))/(2*a))/(2*a))/4)/8))/d)
```

3.176.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 114
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[1/((m + 1)*(b*c - a*d)*(b*e
 - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1)
 - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (IntegerQ[n] ||
 IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

rule 168
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

rule 169
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && IntegersQ[
2*m, 2*n, 2*p]
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4368
```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Simp[-(d*b^(m - 1))^(-1)   Subst[Int[(-a + b*x)^((m - 1)/2
)*((a + b*x)^((m - 1)/2 + n)/x), x], x, Csc[c + d*x]], x] /; FreeQ[{a, b, c
, d, n}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[n]
```

3.176.4 Maple [A] (verified)

Time = 1.58 (sec) , antiderivative size = 308, normalized size of antiderivative = 1.44

method	result
default	$-\frac{\sqrt{a(1+\sec(dx+c))}\left(2265\arctan\left(\frac{\sqrt{2}}{2\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}}\right)\cos(dx+c)\sqrt{2}\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}+7680\arctan\left(\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}\right)\cos(dx+c)\right.$

input
```
int(cot(d*x+c)^5/(a+a*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3840/d/a*(a*(1+sec(d*x+c)))^(1/2)/(cos(d*x+c)+1)*(2265*arctan(1/2*2^(1/
2)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*cos(d*x+c)*2^(1/2)*(-cos(d*x+c)/(co
s(d*x+c)+1))^(1/2)+7680*arctan((-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*cos(d*x
+c)*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+2265*2^(1/2)*(-cos(d*x+c)/(cos(d*x+
c)+1))^(1/2)*arctan(1/2*2^(1/2)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))+7680*a
rctan((-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/
2)+5642*cos(d*x+c)*cot(d*x+c)^4+556*cot(d*x+c)^4-7928*cot(d*x+c)^3*csc(d*x
+c)-460*cot(d*x+c)^2*csc(d*x+c)^2+3150*csc(d*x+c)^3*cot(d*x+c))
```

3.176.5 Fricas [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 705, normalized size of antiderivative = 3.29

$$\int \frac{\cot^5(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx$$

$$= \left[\frac{2265\sqrt{2}\left(\cos(dx+c)^5+\cos(dx+c)^4-2\cos(dx+c)^3-2\cos(dx+c)^2+\cos(dx+c)+1\right)\sqrt{a}\log\left(-\right.}{} \right.$$

input
```
integrate(cot(d*x+c)^5/(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
[1/7680*(2265*sqrt(2)*(cos(d*x + c)^5 + cos(d*x + c)^4 - 2*cos(d*x + c)^3
- 2*cos(d*x + c)^2 + cos(d*x + c) + 1)*sqrt(a)*log(-(2*sqrt(2)*sqrt(a)*sqr
t((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c) - 3*a*cos(d*x + c) - a)/
(cos(d*x + c) - 1)) + 3840*(cos(d*x + c)^5 + cos(d*x + c)^4 - 2*cos(d*x +
c)^3 - 2*cos(d*x + c)^2 + cos(d*x + c) + 1)*sqrt(a)*log(-8*a*cos(d*x + c)^
2 - 4*(2*cos(d*x + c)^2 + cos(d*x + c))*sqrt(a)*sqrt((a*cos(d*x + c) + a)/
cos(d*x + c)) - 8*a*cos(d*x + c) - a) - 4*(2821*cos(d*x + c)^5 + 278*cos(d
*x + c)^4 - 3964*cos(d*x + c)^3 - 230*cos(d*x + c)^2 + 1575*cos(d*x + c))*
sqrt((a*cos(d*x + c) + a)/cos(d*x + c)))/(a*d*cos(d*x + c)^5 + a*d*cos(d*x
 + c)^4 - 2*a*d*cos(d*x + c)^3 - 2*a*d*cos(d*x + c)^2 + a*d*cos(d*x + c) +
 a*d), 1/3840*(2265*sqrt(2)*(cos(d*x + c)^5 + cos(d*x + c)^4 - 2*cos(d*x +
 c)^3 - 2*cos(d*x + c)^2 + cos(d*x + c) + 1)*sqrt(-a)*arctan(sqrt(2)*sqrt(
-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(a*cos(d*x + c) +
 a)) - 3840*(cos(d*x + c)^5 + cos(d*x + c)^4 - 2*cos(d*x + c)^3 - 2*cos(d*
x + c)^2 + cos(d*x + c) + 1)*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x +
c) + a)/cos(d*x + c))*cos(d*x + c)/(2*a*cos(d*x + c) + a)) - 2*(2821*cos(d
*x + c)^5 + 278*cos(d*x + c)^4 - 3964*cos(d*x + c)^3 - 230*cos(d*x + c)^2
+ 1575*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)))/(a*d*cos(d*x
 + c)^5 + a*d*cos(d*x + c)^4 - 2*a*d*cos(d*x + c)^3 - 2*a*d*cos(d*x + c)^2
 + a*d*cos(d*x + c) + a*d)]
```

3.176.6 Sympy [F]

$$\int \frac{\cot^5(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx = \int \frac{\cot^5(c+dx)}{\sqrt{a\left(\sec(c+dx)+1\right)}}\,dx$$

input
```
integrate(cot(d*x+c)**5/(a+a*sec(d*x+c))**(1/2),x)
```

output
```
Integral(cot(c + d*x)**5/sqrt(a*(sec(c + d*x) + 1)), x)
```

3.176.7 Maxima [F]

$$\int \frac{\cot^5(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx = \int \frac{\cot\left(dx+c\right)^5}{\sqrt{a\sec\left(dx+c\right)+a}}\,dx$$

input
```
integrate(cot(d*x+c)^5/(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(cot(d*x + c)^5/sqrt(a*sec(d*x + c) + a), x)
```

3.176.8 Giac [A] (verification not implemented)

Time = 0.89 (sec) , antiderivative size = 241, normalized size of antiderivative = 1.13

$$\int \frac{\cot^5(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx =$$
$$\sqrt{2}\left(\frac{3840\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a}}{2\sqrt{-a}}\right)}{\sqrt{-a}} - \frac{2265\arctan\left(\frac{\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}} + \frac{15\left(25\left(-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a\right)^{\frac{3}{2}}-23\right.}{a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\right.}\right.$$
$$-\frac{}{384}$$

input
```
integrate(cot(d*x+c)^5/(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
-1/3840*sqrt(2)*(3840*sqrt(2)*arctan(1/2*sqrt(2)*sqrt(-a*tan(1/2*d*x + 1/2
*c)^2 + a)/sqrt(-a))/sqrt(-a) - 2265*arctan(sqrt(-a*tan(1/2*d*x + 1/2*c)^2
 + a)/sqrt(-a))/sqrt(-a) + 15*(25*(-a*tan(1/2*d*x + 1/2*c)^2 + a)^(3/2) -
23*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)*a)/(a^2*tan(1/2*d*x + 1/2*c)^4) + 8
*(3*(a*tan(1/2*d*x + 1/2*c)^2 - a)^2*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)*a
^12 + 25*(-a*tan(1/2*d*x + 1/2*c)^2 + a)^(3/2)*a^13 + 240*sqrt(-a*tan(1/2*
d*x + 1/2*c)^2 + a)*a^14)/a^15)/(d*sgn(cos(d*x + c)))
```

3.176.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cot^5(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx = \int \frac{\cot(c+d\,x)^5}{\sqrt{a+\frac{a}{\cos(c+d\,x)}}}\,dx$$

input
```
int(cot(c + d*x)^5/(a + a/cos(c + d*x))^(1/2),x)
```

output
```
int(cot(c + d*x)^5/(a + a/cos(c + d*x))^(1/2), x)
```

3.177 $\int \frac{\tan^6(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx$

3.177.1 Optimal result 1314
3.177.2 Mathematica [A] (warning: unable to verify) 1314
3.177.3 Rubi [A] (verified) 1315
3.177.4 Maple [A] (verified) 1316
3.177.5 Fricas [A] (verification not implemented) 1317
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3.177.7 Maxima [F] 1318
3.177.8 Giac [A] (verification not implemented) 1319
3.177.9 Mupad [F(-1)] 1319

3.177.1 Optimal result

Integrand size = 23, antiderivative size = 189

$$\begin{aligned}\int \frac{\tan^6(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx &= -\frac{2\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{\sqrt{a}d}+\frac{2\tan(c+dx)}{d\sqrt{a+a\sec(c+dx)}}\\ &\quad-\frac{2a\tan^3(c+dx)}{3d(a+a\sec(c+dx))^{3/2}}+\frac{2a^2\tan^5(c+dx)}{5d(a+a\sec(c+dx))^{5/2}}\\ &\quad+\frac{6a^3\tan^7(c+dx)}{7d(a+a\sec(c+dx))^{7/2}}+\frac{2a^4\tan^9(c+dx)}{9d(a+a\sec(c+dx))^{9/2}}\end{aligned}$$

output
```
-2*arctan(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))/d/a^(1/2)+2*tan(d*x+c
)/d/(a+a*sec(d*x+c))^(1/2)-2/3*a*tan(d*x+c)^3/d/(a+a*sec(d*x+c))^(3/2)+2/5
*a^2*tan(d*x+c)^5/d/(a+a*sec(d*x+c))^(5/2)+6/7*a^3*tan(d*x+c)^7/d/(a+a*sec
(d*x+c))^(7/2)+2/9*a^4*tan(d*x+c)^9/d/(a+a*sec(d*x+c))^(9/2)
```

3.177.2 Mathematica [A] (warning: unable to verify)

Time = 14.08 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.88

$$\int \frac{\tan^6(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx$$
$$= \frac{-5040\arctan\left(\frac{\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{\frac{\cos(c+dx)}{1+\cos(c+dx)}}}\right)\cos^2\left(\frac{1}{2}(c+dx)\right)\sqrt{\sec(c+dx)}\sqrt{\frac{\sec(c+dx)}{(1+\sec(c+dx))^2}}\sqrt{1+\sec(c+dx)}+(901+164}{1260d\sqrt{a(1+}$$

input
```
Integrate[Tan[c + d*x]^6/Sqrt[a + a*Sec[c + d*x]],x]
```

output
```
(-5040*ArcTan[Tan[(c + d*x)/2]/Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]]*Cos[
(c + d*x)/2]^2*Sqrt[Sec[c + d*x]]*Sqrt[Sec[c + d*x]/(1 + Sec[c + d*x])^2]*
Sqrt[1 + Sec[c + d*x]] + (901 + 164*Cos[c + d*x] + 1004*Cos[2*(c + d*x)] +
 68*Cos[3*(c + d*x)] + 383*Cos[4*(c + d*x)])*Sec[c + d*x]^4*Tan[c + d*x])/
(1260*d*Sqrt[a*(1 + Sec[c + d*x])])
```

3.177.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 178, normalized size of antiderivative = 0.94, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3042, 4375, 364, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^6(c+dx)}{\sqrt{a\sec(c+dx)+a}}\,dx$$

$$\downarrow 3042$$

$$\int \frac{\cot\left(c+dx+\frac{\pi}{2}\right)^6}{\sqrt{a\csc\left(c+dx+\frac{\pi}{2}\right)+a}}dx$$

$$\downarrow 4375$$

$$-\frac{2a^3\int \frac{\tan^6(c+dx)\left(\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+2\right)^2}{(\sec(c+dx)a+a)^3\left(\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+1\right)}d\left(-\frac{\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)}{d}$$

$$\downarrow 364$$

$$-\frac{2a^3\int\left(\frac{a\tan^8(c+dx)}{(\sec(c+dx)a+a)^4}+\frac{3\tan^6(c+dx)}{(\sec(c+dx)a+a)^3}+\frac{\tan^4(c+dx)}{a(\sec(c+dx)a+a)^2}-\frac{\tan^2(c+dx)}{a^2(\sec(c+dx)a+a)}-\frac{1}{a^3\left(\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+1\right)}+\frac{1}{a^3}\right)d\left(-\frac{\tan}{\sqrt{\sec}}\right.}{d}$$

$$\downarrow 2009$$

$$-\frac{2a^3\left(\frac{\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{a^{7/2}}-\frac{\tan(c+dx)}{a^3\sqrt{a\sec(c+dx)+a}}+\frac{\tan^3(c+dx)}{3a^2(a\sec(c+dx)+a)^{3/2}}-\frac{a\tan^9(c+dx)}{9(a\sec(c+dx)+a)^{9/2}}-\frac{3\tan^7(c+dx)}{7(a\sec(c+dx)+a)^{7/2}}-\frac{}{5a(}\right.}{d}$$

input
```
Int[Tan[c + d*x]^6/Sqrt[a + a*Sec[c + d*x]],x]
```

output
```
(-2*a^3*(ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]]/a^(7/2) -
 Tan[c + d*x]/(a^3*Sqrt[a + a*Sec[c + d*x]]) + Tan[c + d*x]^3/(3*a^2*(a +
a*Sec[c + d*x])^(3/2)) - Tan[c + d*x]^5/(5*a*(a + a*Sec[c + d*x])^(5/2)) -
 (3*Tan[c + d*x]^7)/(7*(a + a*Sec[c + d*x])^(7/2)) - (a*Tan[c + d*x]^9)/(9
*(a + a*Sec[c + d*x])^(9/2))))/d
```

3.177.3.1 Defintions of rubi rules used

rule 364
```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_))/((c_) + (d_.)*(x_)^2),
x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*((a + b*x^2)^p/(c + d*x^2)), x], x
] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && (In
tegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4375
```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Simp[-2*(a^(m/2 + n + 1/2)/d)   Subst[Int[x^m*((2 + a*x^2
)^(m/2 + n - 1/2)/(1 + a*x^2)), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]
]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && I
ntegerQ[n - 1/2]
```

3.177.4 Maple [A] (verified)

Time = 3.72 (sec) , antiderivative size = 220, normalized size of antiderivative = 1.16

method	result
default	$-\frac{2\sqrt{a(1+\sec(dx+c))}\left(315\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,\operatorname{arctanh}\left(\frac{\sin(dx+c)}{(\cos(dx+c)+1)\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}}\right)\cos(dx+c)+315\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,\operatorname{arctanh}\left($

input
```
int(tan(d*x+c)^6/(a+a*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/315/d/a*(a*(1+sec(d*x+c)))^(1/2)/(cos(d*x+c)+1)*(315*(-cos(d*x+c)/(cos(
d*x+c)+1))^(1/2)*arctanh(sin(d*x+c)/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c
)+1))^(1/2))*cos(d*x+c)+315*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctanh(sin
(d*x+c)/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))-383*sin(d*x+c)-
34*tan(d*x+c)+132*sec(d*x+c)*tan(d*x+c)+5*tan(d*x+c)*sec(d*x+c)^2-35*tan(d
*x+c)*sec(d*x+c)^3)
```

3.177.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 355, normalized size of antiderivative = 1.88

$$\int \frac{\tan^6(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx$$

$$= \left[-\frac{315\left(\cos\left(dx+c\right)^5+\cos\left(dx+c\right)^4\right)\sqrt{-a}\log\left(\frac{2\,a\cos(dx+c)^2-2\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)\sin(dx+c)+a\cos(dx+c)-a}{\cos(dx+c)+1}\right.}{315\left(ad\cos\left(dx\right.\right.} \right.$$

input
```
integrate(tan(d*x+c)^6/(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
[-1/315*(315*(cos(d*x + c)^5 + cos(d*x + c)^4)*sqrt(-a)*log((2*a*cos(d*x +
 c)^2 - 2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*si
n(d*x + c) + a*cos(d*x + c) - a)/(cos(d*x + c) + 1)) - 2*(383*cos(d*x + c)
^4 + 34*cos(d*x + c)^3 - 132*cos(d*x + c)^2 - 5*cos(d*x + c) + 35)*sqrt((a
*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(a*d*cos(d*x + c)^5 + a*d*c
os(d*x + c)^4), 2/315*(315*(cos(d*x + c)^5 + cos(d*x + c)^4)*sqrt(a)*arcta
n(sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x +
c))) + (383*cos(d*x + c)^4 + 34*cos(d*x + c)^3 - 132*cos(d*x + c)^2 - 5*co
s(d*x + c) + 35)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(a*
d*cos(d*x + c)^5 + a*d*cos(d*x + c)^4)]
```

3.177.6 Sympy [F]

$$\int \frac{\tan^6(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx = \int \frac{\tan^6(c+dx)}{\sqrt{a\left(\sec(c+dx)+1\right)}}\,dx$$

input
```
integrate(tan(d*x+c)**6/(a+a*sec(d*x+c))**(1/2),x)
```

output
```
Integral(tan(c + d*x)**6/sqrt(a*(sec(c + d*x) + 1)), x)
```

3.177.7 Maxima [F]

$$\int \frac{\tan^6(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx = \int \frac{\tan(dx+c)^6}{\sqrt{a\sec(dx+c)+a}}\,dx$$

input
```
integrate(tan(d*x+c)^6/(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
-1/630*(12*(105*sin(8*d*x + 8*c) + 280*sin(6*d*x + 6*c) + 546*sin(4*d*x +
4*c) + 312*sin(2*d*x + 2*c))*cos(9/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
 2*c) + 1)) - 4*(315*cos(8*d*x + 8*c) + 840*cos(6*d*x + 6*c) + 1638*cos(4*
d*x + 4*c) + 936*cos(2*d*x + 2*c) + 383)*sin(9/2*arctan2(sin(2*d*x + 2*c),
 cos(2*d*x + 2*c) + 1)) - 315*((cos(2*d*x + 2*c)^4 + sin(2*d*x + 2*c)^4 +
4*cos(2*d*x + 2*c)^3 + 2*(cos(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sin
(2*d*x + 2*c)^2 + 6*cos(2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c) + 1)*arctan2((
cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*si
n(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^
2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin
(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + 1) - (cos(2*d*x + 2*c)^4 + sin(2*d
*x + 2*c)^4 + 4*cos(2*d*x + 2*c)^3 + 2*(cos(2*d*x + 2*c)^2 + 2*cos(2*d*x +
 2*c) + 1)*sin(2*d*x + 2*c)^2 + 6*cos(2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c)
+ 1)*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c)
 + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)), (cos
(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1
/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - 1) + 2*(a*d*cos(2*d*
x + 2*c)^4 + a*d*sin(2*d*x + 2*c)^4 + 4*a*d*cos(2*d*x + 2*c)^3 + 6*a*d*cos
(2*d*x + 2*c)^2 + 4*a*d*cos(2*d*x + 2*c) + 2*(a*d*cos(2*d*x + 2*c)^2 + 2*a
*d*cos(2*d*x + 2*c) + a*d)*sin(2*d*x + 2*c)^2 + a*d)*integrate(-(cos(2*...
```

3.177.8 Giac [A] (verification not implemented)

Time = 2.84 (sec) , antiderivative size = 266, normalized size of antiderivative = 1.41

$$\int \frac{\tan^6(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx$$

$$= \frac{\sqrt{2}\left(\frac{315\sqrt{2}\sqrt{-a}\log\left(\frac{\left|2\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^2-4\sqrt{2}|a|-6a\right|}{\left|2\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^2+4\sqrt{2}|a|-6a\right|}\right)}{|a|} + \frac{4\left(315a^4-\left(1470a^4-\left(2772a^4+\left(257a^4\tan\left(\frac{1}{2}dx\right.\right.\right.\right.\right.}{\left.\left.\left.\left.\left.\left(a\tan\right.\right.\right.\right.\right.}\right.$$

$$630\,d\,\mathrm{sgn}\left(\cos\left(dx+c\right)\right)$$

input
```
integrate(tan(d*x+c)^6/(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
1/630*sqrt(2)*(315*sqrt(2)*sqrt(-a)*log(abs(2*(sqrt(-a)*tan(1/2*d*x + 1/2*
c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2 - 4*sqrt(2)*abs(a) - 6*a)/abs(
2*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2
+ 4*sqrt(2)*abs(a) - 6*a))/abs(a) + 4*(315*a^4 - (1470*a^4 - (2772*a^4 + (
257*a^4*tan(1/2*d*x + 1/2*c)^2 - 1314*a^4)*tan(1/2*d*x + 1/2*c)^2)*tan(1/2
*d*x + 1/2*c)^2)*tan(1/2*d*x + 1/2*c)^2)*tan(1/2*d*x + 1/2*c)/((a*tan(1/2*
d*x + 1/2*c)^2 - a)^4*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)))/(d*sgn(cos(d*x
 + c)))
```

3.177.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\tan^6(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx = \int \frac{\tan(c+d\,x)^6}{\sqrt{a+\frac{a}{\cos(c+d\,x)}}}\,dx$$

input
```
int(tan(c + d*x)^6/(a + a/cos(c + d*x))^(1/2),x)
```

output
```
int(tan(c + d*x)^6/(a + a/cos(c + d*x))^(1/2), x)
```

3.178 $\int \frac{\tan^4(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx$

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3.178.1 Optimal result

Integrand size = 23, antiderivative size = 125

$$\int \frac{\tan^4(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx = \frac{2\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{\sqrt{a}d} - \frac{2\tan(c+dx)}{d\sqrt{a+a\sec(c+dx)}} + \frac{2a\tan^3(c+dx)}{3d(a+a\sec(c+dx))^{3/2}} + \frac{2a^2\tan^5(c+dx)}{5d(a+a\sec(c+dx))^{5/2}}$$

output
```
2*arctan(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))/d/a^(1/2)-2*tan(d*x+c)
/d/(a+a*sec(d*x+c))^(1/2)+2/3*a*tan(d*x+c)^3/d/(a+a*sec(d*x+c))^(3/2)+2/5*
a^2*tan(d*x+c)^5/d/(a+a*sec(d*x+c))^(5/2)
```

3.178.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 2.47 (sec) , antiderivative size = 238, normalized size of antiderivative = 1.90

$$\int \frac{\tan^4(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx = \frac{16\sqrt{2}\left(\frac{1}{1+\sec(c+dx)}\right)^{9/2}\left(-\frac{\cos(c+dx)(9+5\cos(c+dx))\csc^6\left(\frac{1}{2}(c+dx)\right)\sec^2\left(\frac{1}{2}(c+dx)\right)\left(30\operatorname{arctanh}\left(\sqrt{1-\sec(c+dx)}\right)\cos^2(c+dx)+(-29\right.}{480\sqrt{1-\sec(c+dx)}}\right.}{5d\sqrt{}}$$

input
```
Integrate[Tan[c + d*x]^4/Sqrt[a + a*Sec[c + d*x]],x]
```

output
```
(16*Sqrt[2]*((1 + Sec[c + d*x])^(-1))^(9/2)*(-1/480*(Cos[c + d*x]*(9 + 5*C
os[c + d*x])*Csc[(c + d*x)/2]^6*Sec[(c + d*x)/2]^2*(30*ArcTanh[Sqrt[1 - Se
c[c + d*x]]]*Cos[c + d*x]^2 + (-29 + 22*Cos[c + d*x] - 23*Cos[2*(c + d*x)]
)*Sqrt[1 - Sec[c + d*x]]))/Sqrt[1 - Sec[c + d*x]] - (4*Hypergeometric2F1[2
, 9/2, 11/2, -2*Sec[c + d*x]*Sin[(c + d*x)/2]^2]*Sec[c + d*x]*Tan[(c + d*x
)/2]^2)/9)*Tan[c + d*x]^5)/(5*d*Sqrt[a*(1 + Sec[c + d*x])]*(1 - Tan[(c + d
*x)/2]^2)^(7/2))
```

3.178.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.98, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3042, 4375, 363, 254, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^4(c+dx)}{\sqrt{a\sec(c+dx)+a}}\,dx$$

$$\downarrow 3042$$

$$\int \frac{\cot\left(c+dx+\frac{\pi}{2}\right)^4}{\sqrt{a\csc\left(c+dx+\frac{\pi}{2}\right)+a}}dx$$

$$\downarrow 4375$$

$$-\frac{2a^2\int \frac{\tan^4(c+dx)\left(\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+2\right)}{(\sec(c+dx)a+a)^2\left(\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+1\right)}d\left(-\frac{\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)}{d}$$

$$\downarrow 363$$

$$-\frac{2a^2\left(\int \frac{\tan^4(c+dx)}{(\sec(c+dx)a+a)^2\left(\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+1\right)}d\left(-\frac{\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)-\frac{\tan^5(c+dx)}{5(a\sec(c+dx)+a)^{5/2}}\right)}{d}$$

$$\downarrow 254$$

$$-\frac{2a^2\left(\int\left(\frac{\tan^2(c+dx)}{a(\sec(c+dx)a+a)}+\frac{1}{a^2\left(\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+1\right)}-\frac{1}{a^2}\right)d\left(-\frac{\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)-\frac{\tan^5(c+dx)}{5(a\sec(c+dx)+a)^{5/2}}\right)}{d}$$

$$\downarrow 2009$$

$$-\frac{2a^2\left(-\frac{\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{a^{5/2}}+\frac{\tan(c+dx)}{a^2\sqrt{a\sec(c+dx)+a}}-\frac{\tan^5(c+dx)}{5(a\sec(c+dx)+a)^{5/2}}-\frac{\tan^3(c+dx)}{3a(a\sec(c+dx)+a)^{3/2}}\right)}{d}$$

input
```
Int[Tan[c + d*x]^4/Sqrt[a + a*Sec[c + d*x]],x]
```

output
```
(-2*a^2*(-(ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]]/a^(5/2)
) + Tan[c + d*x]/(a^2*Sqrt[a + a*Sec[c + d*x]]) - Tan[c + d*x]^3/(3*a*(a +
 a*Sec[c + d*x])^(3/2)) - Tan[c + d*x]^5/(5*(a + a*Sec[c + d*x])^(5/2))))/
d
```

3.178.3.1 Defintions of rubi rules used

rule 254
```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Int[PolynomialDivide[x^m,
 a + b*x^2, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 3]
```

rule 363
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(b*e*(m + 2*p + 3))),
 x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(b*(m + 2*p + 3))  Int[(e*x)^
m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a*d
, 0] && NeQ[m + 2*p + 3, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4375
```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Simp[-2*(a^(m/2 + n + 1/2)/d)  Subst[Int[x^m*((2 + a*x^2
)^(m/2 + n - 1/2)/(1 + a*x^2)), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]
]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && I
ntegerQ[n - 1/2]
```

3.178.4 Maple [A] (verified)

Time = 3.69 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.50

method	result
default	$\frac{2\sqrt{a(1+\sec(dx+c))}\left(15\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,\operatorname{arctanh}\left(\frac{\sin(dx+c)}{(\cos(dx+c)+1)\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}}\right)\cos(dx+c)+15\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,\operatorname{arctanh}\left(\frac{}{(\cos(dx}\right.}{15da(\cos(dx+c)+1)}$

input
```
int(tan(d*x+c)^4/(a+a*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/15/d/a*(a*(1+sec(d*x+c)))^(1/2)/(cos(d*x+c)+1)*(15*(-cos(d*x+c)/(cos(d*x
+c)+1))^(1/2)*arctanh(sin(d*x+c)/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1
))^(1/2))*cos(d*x+c)+15*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctanh(sin(d*x
+c)/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))-17*sin(d*x+c)-tan(d
*x+c)+3*sec(d*x+c)*tan(d*x+c))
```

3.178.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 311, normalized size of antiderivative = 2.49

$$\int \frac{\tan^4(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx$$

$$= \left[-\frac{15\left(\cos(dx+c)^3+\cos(dx+c)^2\right)\sqrt{-a}\log\left(\frac{2\,a\cos(dx+c)^2+2\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)\sin(dx+c)+a\cos(dx+c)-a}{\cos(dx+c)+1}\right.}{15\left(ad\cos(dx+c)^3+ad\cos(dx+c)\right.}\right.$$

$$\left. -\frac{2\left(15\left(\cos(dx+c)^3+\cos(dx+c)^2\right)\sqrt{a}\arctan\left(\frac{\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)}{\sqrt{a}\sin(dx+c)}\right)+\left(17\cos(dx+c)^2+\cos(dx\right.\right.}{15\left(ad\cos(dx+c)^3+ad\cos(dx+c)^2\right)}$$

input
```
integrate(tan(d*x+c)^4/(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
[-1/15*(15*(cos(d*x + c)^3 + cos(d*x + c)^2)*sqrt(-a)*log((2*a*cos(d*x + c
)^2 + 2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(
d*x + c) + a*cos(d*x + c) - a)/(cos(d*x + c) + 1)) + 2*(17*cos(d*x + c)^2
+ cos(d*x + c) - 3)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/
(a*d*cos(d*x + c)^3 + a*d*cos(d*x + c)^2), -2/15*(15*(cos(d*x + c)^3 + cos
(d*x + c)^2)*sqrt(a)*arctan(sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*
x + c)/(sqrt(a)*sin(d*x + c))) + (17*cos(d*x + c)^2 + cos(d*x + c) - 3)*sq
rt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(a*d*cos(d*x + c)^3 +
a*d*cos(d*x + c)^2)]
```

3.178.6 Sympy [F]

$$\int \frac{\tan^4(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx = \int \frac{\tan^4(c+dx)}{\sqrt{a\left(\sec(c+dx)+1\right)}}\,dx$$

input
```
integrate(tan(d*x+c)**4/(a+a*sec(d*x+c))**(1/2),x)
```

output
```
Integral(tan(c + d*x)**4/sqrt(a*(sec(c + d*x) + 1)), x)
```

3.178.7 Maxima [F]

$$\int \frac{\tan^4(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx = \int \frac{\tan(dx+c)^4}{\sqrt{a\sec(dx+c)+a}}\,dx$$

input
```
integrate(tan(d*x+c)^4/(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

output

```
1/30*(20*(3*sin(4*d*x + 4*c) + 4*sin(2*d*x + 2*c))*cos(5/2*arctan2(sin(2*d
*x + 2*c), cos(2*d*x + 2*c) + 1)) - 4*(15*cos(4*d*x + 4*c) + 20*cos(2*d*x
+ 2*c) + 17)*sin(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - 15
*((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*arcta
n2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4
)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)), (cos(2*d*x + 2
*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2
(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + 1) - (cos(2*d*x + 2*c)^2 + sin
(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*arctan2((cos(2*d*x + 2*c)^2 + si
n(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x
 + 2*c), cos(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2
+ 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*
x + 2*c) + 1)) - 1) + 2*(a*d*cos(2*d*x + 2*c)^2 + a*d*sin(2*d*x + 2*c)^2 +
 2*a*d*cos(2*d*x + 2*c) + a*d)*integrate(-(cos(2*d*x + 2*c)^2 + sin(2*d*x
+ 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(((cos(10*d*x + 10*c)*cos(2*d*x +
 2*c) + 4*cos(8*d*x + 8*c)*cos(2*d*x + 2*c) + 6*cos(6*d*x + 6*c)*cos(2*d*x
 + 2*c) + 4*cos(4*d*x + 4*c)*cos(2*d*x + 2*c) + cos(2*d*x + 2*c)^2 + sin(1
0*d*x + 10*c)*sin(2*d*x + 2*c) + 4*sin(8*d*x + 8*c)*sin(2*d*x + 2*c) + 6*s
in(6*d*x + 6*c)*sin(2*d*x + 2*c) + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + s
in(2*d*x + 2*c)^2)*cos(7/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))...
```

3.178.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 228 vs. $2(109) = 218$.

Time $= 1.60$ (sec) , antiderivative size $= 228$, normalized size of antiderivative $= 1.82$

$$\int \frac{\tan^4(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx =$$

$$\sqrt{2}\left(\frac{15\sqrt{2}\sqrt{-a}\log\left(\left|\frac{2\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^2-4\sqrt{2}|a|-6a}{2\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^2+4\sqrt{2}|a|-6a}\right|\right)}{|a|}+\frac{4\left(\left(13a^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-40a^2\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}\right.\right.}{\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-a\right)^2\sqrt{-a\tan(}}\right.$$

$$-\frac{\left.\left.\right.\right.}{30\,d\operatorname{sgn}\left(\cos\left(dx+c\right)\right)}$$

input

```
integrate(tan(d*x+c)^4/(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
-1/30*sqrt(2)*(15*sqrt(2)*sqrt(-a)*log(abs(2*(sqrt(-a)*tan(1/2*d*x + 1/2*c
) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2 - 4*sqrt(2)*abs(a) - 6*a)/abs(2
*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2 +
 4*sqrt(2)*abs(a) - 6*a))/abs(a) + 4*((13*a^2*tan(1/2*d*x + 1/2*c)^2 - 40*
a^2)*tan(1/2*d*x + 1/2*c)^2 + 15*a^2)*tan(1/2*d*x + 1/2*c)/((a*tan(1/2*d*x
 + 1/2*c)^2 - a)^2*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)))/(d*sgn(cos(d*x +
c)))
```

3.178.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\tan^4(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx = \int \frac{\tan(c+d\,x)^4}{\sqrt{a+\frac{a}{\cos(c+d\,x)}}}\,dx$$

input
```
int(tan(c + d*x)^4/(a + a/cos(c + d*x))^(1/2),x)
```

output
```
int(tan(c + d*x)^4/(a + a/cos(c + d*x))^(1/2), x)
```

3.179 $\int \frac{\tan^2(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx$

3.179.1 Optimal result 1327
3.179.2 Mathematica [A] (warning: unable to verify) 1327
3.179.3 Rubi [A] (verified) 1328
3.179.4 Maple [A] (verified) 1329
3.179.5 Fricas [A] (verification not implemented) 1330
3.179.6 Sympy [F] 1330
3.179.7 Maxima [F] 1331
3.179.8 Giac [B] (verification not implemented) 1332
3.179.9 Mupad [F(-1)] 1332

3.179.1 Optimal result

Integrand size = 23, antiderivative size = 63

$$\int \frac{\tan^2(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx = -\frac{2\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{\sqrt{a}d} + \frac{2\tan(c+dx)}{d\sqrt{a+a\sec(c+dx)}}$$

output
```
-2*arctan(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))/d/a^(1/2)+2*tan(d*x+c
)/d/(a+a*sec(d*x+c))^(1/2)
```

3.179.2 Mathematica [A] (warning: unable to verify)

Time = 0.59 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.89

$$\int \frac{\tan^2(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx$$
$$= \frac{16\cos^6\left(\frac{1}{2}(c+dx)\right)\sec^4(c+dx)\left(\frac{1}{1+\sec(c+dx)}\right)^{5/2}\left(-\arcsin\left(\frac{\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{\frac{1}{1+\cos(c+dx)}}}\right)\cos(c+dx)+\sqrt{\cos(c+dx)}\sqrt{}\right.}{d\sqrt{a(1+\sec(c+dx))}}$$

input
```
Integrate[Tan[c + d*x]^2/Sqrt[a + a*Sec[c + d*x]],x]
```

output
```
(16*Cos[(c + d*x)/2]^6*Sec[c + d*x]^4*((1 + Sec[c + d*x])^(-1))^(5/2)*(-(A
rcSin[Tan[(c + d*x)/2]/Sqrt[(1 + Cos[c + d*x])^(-1)]]*Cos[c + d*x]) + Sqrt
[Cos[c + d*x]]*Sqrt[(1 + Cos[c + d*x])^(-1)]*Sin[c + d*x]))/(d*Sqrt[a*(1 +
 Sec[c + d*x])])
```

3.179.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.03, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3042, 4375, 262, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^2(c+dx)}{\sqrt{a\sec(c+dx)+a}}\,dx$$

$$\downarrow 3042$$

$$\int \frac{\cot\left(c+dx+\frac{\pi}{2}\right)^2}{\sqrt{a\csc\left(c+dx+\frac{\pi}{2}\right)+a}}dx$$

$$\downarrow 4375$$

$$-\frac{2a\int \frac{\tan^2(c+dx)}{(\sec(c+dx)a+a)\left(\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+1\right)}d\left(-\frac{\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)}{d}$$

$$\downarrow 262$$

$$-\frac{2a\left(-\frac{\int \frac{1}{\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+1}d\left(-\frac{\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)}{a}-\frac{\tan(c+dx)}{a\sqrt{a\sec(c+dx)+a}}\right)}{d}$$

$$\downarrow 216$$

$$-\frac{2a\left(\frac{\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{a^{3/2}}-\frac{\tan(c+dx)}{a\sqrt{a\sec(c+dx)+a}}\right)}{d}$$

input
```
Int[Tan[c + d*x]^2/Sqrt[a + a*Sec[c + d*x]],x]
```

output
```
(-2*a*(ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]]/a^(3/2) - T
an[c + d*x]/(a*Sqrt[a + a*Sec[c + d*x]])))/d
```

3.179.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))  Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4375
```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Simp[-2*(a^(m/2 + n + 1/2)/d)  Subst[Int[x^m*((2 + a*x^2
)^(m/2 + n - 1/2)/(1 + a*x^2)), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]
]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && I
ntegerQ[n - 1/2]
```

3.179.4 Maple [A] (verified)

Time = 3.16 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.51

method	result	size
default	$-\frac{2\sqrt{a(1+\sec(dx+c))}\left(\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,\operatorname{arctanh}\left(\frac{\sin(dx+c)}{(\cos(dx+c)+1)\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}}\right)+\cot(dx+c)-\csc(dx+c)\right)}{da}$	95

input
```
int(tan(d*x+c)^2/(a+a*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/d/a*(a*(1+sec(d*x+c)))^(1/2)*((-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctan
h(sin(d*x+c)/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))+cot(d*x+c)
-csc(d*x+c))
```

3.179.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 235, normalized size of antiderivative = 3.73

$$\int \frac{\tan^2(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx$$
$$= \left[-\frac{\sqrt{-a}(\cos(dx+c)+1)\log\left(\frac{2\,a\cos(dx+c)^2-2\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)\sin(dx+c)+a\cos(dx+c)-a}{\cos(dx+c)+1}\right)-2\sqrt{\frac{a\cos(dx+}{\cos(dx+}}}{ad\cos(dx+c)+ad}\right.$$

input
```
integrate(tan(d*x+c)^2/(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
[-(sqrt(-a)*(cos(d*x + c) + 1)*log((2*a*cos(d*x + c)^2 - 2*sqrt(-a)*sqrt((
a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x + c) + a*cos(d*x +
c) - a)/(cos(d*x + c) + 1)) - 2*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*si
n(d*x + c))/(a*d*cos(d*x + c) + a*d), 2*(sqrt(a)*(cos(d*x + c) + 1)*arctan
(sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c
))) + sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(a*d*cos(d*x +
 c) + a*d)]
```

3.179.6 Sympy [F]

$$\int \frac{\tan^2(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx = \int \frac{\tan^2(c+dx)}{\sqrt{a\left(\sec(c+dx)+1\right)}}\,dx$$

input
```
integrate(tan(d*x+c)**2/(a+a*sec(d*x+c))**(1/2),x)
```

output
```
Integral(tan(c + d*x)**2/sqrt(a*(sec(c + d*x) + 1)), x)
```

3.179.7 Maxima [F]

$$\int \frac{\tan^2(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx = \int \frac{\tan\left(dx+c\right)^2}{\sqrt{a\sec\left(dx+c\right)+a}}\,dx$$

input
```
integrate(tan(d*x+c)^2/(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
1/2*((2*a*d*integrate(-(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*
d*x + 2*c) + 1)^(1/4)*(((cos(6*d*x + 6*c)*cos(2*d*x + 2*c) + 2*cos(4*d*x +
 4*c)*cos(2*d*x + 2*c) + cos(2*d*x + 2*c)^2 + sin(6*d*x + 6*c)*sin(2*d*x +
 2*c) + 2*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + sin(2*d*x + 2*c)^2)*cos(3/2*
arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + (cos(2*d*x + 2*c)*sin(6*d*x
 + 6*c) + 2*cos(2*d*x + 2*c)*sin(4*d*x + 4*c) - cos(6*d*x + 6*c)*sin(2*d*x
 + 2*c) - 2*cos(4*d*x + 4*c)*sin(2*d*x + 2*c))*sin(3/2*arctan2(sin(2*d*x +
 2*c), cos(2*d*x + 2*c))))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2
*c) + 1)) + ((cos(2*d*x + 2*c)*sin(6*d*x + 6*c) + 2*cos(2*d*x + 2*c)*sin(4
*d*x + 4*c) - cos(6*d*x + 6*c)*sin(2*d*x + 2*c) - 2*cos(4*d*x + 4*c)*sin(2
*d*x + 2*c))*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - (cos(6
*d*x + 6*c)*cos(2*d*x + 2*c) + 2*cos(4*d*x + 4*c)*cos(2*d*x + 2*c) + cos(2
*d*x + 2*c)^2 + sin(6*d*x + 6*c)*sin(2*d*x + 2*c) + 2*sin(4*d*x + 4*c)*sin
(2*d*x + 2*c) + sin(2*d*x + 2*c)^2)*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(
2*d*x + 2*c))))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)))/
(a*cos(6*d*x + 6*c)^2 + 4*a*cos(4*d*x + 4*c)^2 + 4*a*cos(4*d*x + 4*c)*cos(
2*d*x + 2*c) + a*cos(2*d*x + 2*c)^2 + a*sin(6*d*x + 6*c)^2 + 4*a*sin(4*d*x
 + 4*c)^2 + 4*a*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + a*sin(2*d*x + 2*c)^2 +
 2*(2*a*cos(4*d*x + 4*c) + a*cos(2*d*x + 2*c))*cos(6*d*x + 6*c) + 2*(2*a*s
in(4*d*x + 4*c) + a*sin(2*d*x + 2*c))*sin(6*d*x + 6*c)), x) - 2*a*d*int...
```

3.179.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 187 vs. $2(55) = 110$.

Time = 1.14 (sec) , antiderivative size = 187, normalized size of antiderivative = 2.97

$$\int \frac{\tan^2(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx$$

$$= \frac{\sqrt{2}\left(\frac{\sqrt{2}\sqrt{-a}\log\left(\frac{\left|2\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^2-4\sqrt{2}|a|-6a\right|}{\left|2\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^2+4\sqrt{2}|a|-6a\right|}\right)}{|a|}-\frac{4\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-a}\right)}{2\,d\,\mathrm{sgn}\left(\cos\left(dx+c\right)\right)}$$

input
```
integrate(tan(d*x+c)^2/(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
1/2*sqrt(2)*(sqrt(2)*sqrt(-a)*log(abs(2*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - s
qrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2 - 4*sqrt(2)*abs(a) - 6*a)/abs(2*(sqr
t(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2 + 4*sq
rt(2)*abs(a) - 6*a))/abs(a) - 4*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)*tan(1/
2*d*x + 1/2*c)/(a*tan(1/2*d*x + 1/2*c)^2 - a))/(d*sgn(cos(d*x + c)))
```

3.179.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\tan^2(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx = \int \frac{\tan(c+d\,x)^2}{\sqrt{a+\frac{a}{\cos(c+d\,x)}}}\,dx$$

input
```
int(tan(c + d*x)^2/(a + a/cos(c + d*x))^(1/2),x)
```

output
```
int(tan(c + d*x)^2/(a + a/cos(c + d*x))^(1/2), x)
```

3.180 $\int \frac{\cot^2(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx$

3.180.1 Optimal result 1333
3.180.2 Mathematica [A] (verified) 1333
3.180.3 Rubi [A] (verified) 1334
3.180.4 Maple [B] (verified) 1337
3.180.5 Fricas [A] (verification not implemented) 1337
3.180.6 Sympy [F] 1338
3.180.7 Maxima [F] 1338
3.180.8 Giac [A] (verification not implemented) 1339
3.180.9 Mupad [F(-1)] 1339

3.180.1 Optimal result

Integrand size = 23, antiderivative size = 165

$$\int \frac{\cot^2(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx = -\frac{2\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{\sqrt{a}d} + \frac{7\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{4\sqrt{2}\sqrt{a}d} - \frac{\cot(c+dx)\sqrt{a+a\sec(c+dx)}}{4ad} - \frac{\cos(c+dx)\cot(c+dx)\sec^2\left(\frac{1}{2}(c+dx)\right)\sqrt{a+a\sec(c+dx)}}{4ad}$$

output
```
-2*arctan(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))/d/a^(1/2)+7/8*arctan(
1/2*a^(1/2)*tan(d*x+c)*2^(1/2)/(a+a*sec(d*x+c))^(1/2))/d*2^(1/2)/a^(1/2)-1
/4*cot(d*x+c)*(a+a*sec(d*x+c))^(1/2)/a/d-1/4*cos(d*x+c)*cot(d*x+c)*sec(1/2
*d*x+1/2*c)^2*(a+a*sec(d*x+c))^(1/2)/a/d
```

3.180.2 Mathematica [A] (verified)

Time = 5.05 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.13

$$\int \frac{\cot^2(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx = \frac{-2(3\cot(c+dx)+\csc(c+dx)) - 32\arctan\left(\frac{\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{\frac{1}{1+\sec(c+dx)}}}\right)\cos^2\left(\frac{1}{2}(c+dx)\right)\sqrt{\sec(c+dx)}\sqrt{\frac{\sec(c+dx)}{(1+\sec(c+dx))^2}}}{8d\sqrt{a(1+\sec(c+dx))}}$$

input
```
Integrate[Cot[c + d*x]^2/Sqrt[a + a*Sec[c + d*x]],x]
```

output
```
(-2*(3*Cot[c + d*x] + Csc[c + d*x]) - 32*ArcTan[Tan[(c + d*x)/2]/Sqrt[(1 +
 Sec[c + d*x])^(-1)]]*Cos[(c + d*x)/2]^2*Sqrt[Sec[c + d*x]]*Sqrt[Sec[c + d
*x]/(1 + Sec[c + d*x])^2]*Sqrt[1 + Sec[c + d*x]] + (14*ArcSin[Tan[(c + d*x
)/2]]*Sqrt[Sec[c + d*x]]*Sqrt[(1 + Sec[c + d*x])^(-1)]*Sqrt[1 + Sec[c + d*
x]])/Sqrt[Sec[(c + d*x)/2]^2])/(8*d*Sqrt[a*(1 + Sec[c + d*x])])
```

3.180.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.04, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {3042, 4375, 374, 27, 445, 27, 397, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^2(c+dx)}{\sqrt{a\sec(c+dx)+a}}\,dx$$

$$\downarrow 3042$$

$$\int \frac{1}{\cot\left(c+dx+\frac{\pi}{2}\right)^2\sqrt{a\csc\left(c+dx+\frac{\pi}{2}\right)+a}}dx$$

$$\downarrow 4375$$

$$-\frac{2\int \frac{\cot^2(c+dx)(\sec(c+dx)a+a)}{\left(\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+1\right)\left(\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+2\right)^2}d\left(-\frac{\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)}{ad}$$

$$\downarrow 374$$

$$-\frac{2\left(\int \frac{a\cot^2(c+dx)(\sec(c+dx)a+a)\left(1-\frac{3a\tan^2(c+dx)}{\sec(c+dx)a+a}\right)}{\left(\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+1\right)\left(\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+2\right)}d\left(-\frac{\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)}{4a}+\frac{\cot(c+dx)\sqrt{a\sec(c+dx)+a}}{4\left(\frac{a\tan^2(c+dx)}{a\sec(c+dx)+a}+2\right)}\right)}{ad}$$

$$\downarrow 27$$

$$-\frac{2\left(\frac{1}{4}\int \frac{\cot^2(c+dx)(\sec(c+dx)a+a)\left(1-\frac{3a\tan^2(c+dx)}{\sec(c+dx)a+a}\right)}{\left(\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+1\right)\left(\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+2\right)}d\left(-\frac{\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)+\frac{\cot(c+dx)\sqrt{a\sec(c+dx)+a}}{4\left(\frac{a\tan^2(c+dx)}{a\sec(c+dx)+a}+2\right)}\right)}{ad}$$

$$\downarrow 445$$

$$-\frac{2\left(\frac{1}{4}\left(\frac{1}{2}\cot(c+dx)\sqrt{a\sec(c+dx)+a}-\frac{1}{2}\int\frac{a\left(\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+9\right)}{\left(\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+1\right)\left(\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+2\right)}d\left(-\frac{\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)\right)+\frac{\cot(c+dx)\sqrt{a}}{4\left(\frac{a\tan^2(}{a\sec(c+}\right.\right.}{ad}$$

$$\downarrow 27$$

$$-\frac{2\left(\frac{1}{4}\left(\frac{1}{2}\cot(c+dx)\sqrt{a\sec(c+dx)+a}-\frac{1}{2}a\int\frac{\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+9}{\left(\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+1\right)\left(\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+2\right)}d\left(-\frac{\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)\right)+\frac{\cot(c+dx)\sqrt{}}{4\left(\frac{a\tan^2}{a\sec(c-}\right.\right.}{ad}$$

$$\downarrow 397$$

$$-\frac{2\left(\frac{1}{4}\left(\frac{1}{2}\cot(c+dx)\sqrt{a\sec(c+dx)+a}-\frac{1}{2}a\left(8\int\frac{1}{\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+1}d\left(-\frac{\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)-7\int\frac{1}{\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+2}d\left(-\frac{\tan}{\sqrt{\sec}}\right.\right.\right.\right.}{ad}$$

$$\downarrow 216$$

$$-\frac{2\left(\frac{1}{4}\left(\frac{1}{2}\cot(c+dx)\sqrt{a\sec(c+dx)+a}-\frac{1}{2}a\left(\frac{7\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{2}\sqrt{a}}-\frac{8\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}}\right)\right)+\frac{\cot(c+dx)\sqrt{}}{4\left(\frac{a\tan^2}{a\sec(}\right.\right.}{ad}$$

input
```
Int[Cot[c + d*x]^2/Sqrt[a + a*Sec[c + d*x]],x]
```

output
```
(-2*((-1/2*(a*((-8*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]]
)/Sqrt[a] + (7*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d
*x]])])/(Sqrt[2]*Sqrt[a]))) + (Cot[c + d*x]*Sqrt[a + a*Sec[c + d*x]])/2)/4
 + (Cot[c + d*x]*Sqrt[a + a*Sec[c + d*x]])/(4*(2 + (a*Tan[c + d*x]^2)/(a +
 a*Sec[c + d*x])))))/(a*d)
```

3.180.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 374
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q
 + 1)/(a*e*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(e*x)^m*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[b*c*(m + 1) + 2*(b*c -
a*d)*(p + 1) + d*b*(m + 2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b,
c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IntBinomialQ[a, b,
 c, d, e, m, 2, p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 445
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^2*(m + 1))
 Int[(g*x)^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*f*c*(m + 1) - e*(b*c
+ a*d)*(m + 2 + 1) - e*2*(b*c*p + a*d*q) - b*e*d*(m + 2*(p + q + 2) + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && LtQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4375
```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Simp[-2*(a^(m/2 + n + 1/2)/d)   Subst[Int[x^m*((2 + a*x^2
)^(m/2 + n - 1/2)/(1 + a*x^2)), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]
]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && I
ntegerQ[n - 1/2]
```

3.180.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 335 vs. $2(139) = 278$.

Time = 2.01 (sec) , antiderivative size = 336, normalized size of antiderivative = 2.04

method	result
default	$\frac{\sqrt{a(1+\sec(dx+c))}\left(7\sqrt{2}\,\ln\left(\csc(dx+c)-\cot(dx+c)+\sqrt{\cot(dx+c)^2-2\cot(dx+c)\csc(dx+c)+\csc(dx+c)^2-1}\right)\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,\cos($

input
```
int(cot(d*x+c)^2/(a+a*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/8/d/a*(a*(1+sec(d*x+c)))^(1/2)/(cos(d*x+c)+1)*(7*2^(1/2)*ln(csc(d*x+c)-c
ot(d*x+c)+(cot(d*x+c)^2-2*cot(d*x+c)*csc(d*x+c)+csc(d*x+c)^2-1)^(1/2))*(-c
os(d*x+c)/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)+7*ln(csc(d*x+c)-cot(d*x+c)+(cot
(d*x+c)^2-2*cot(d*x+c)*csc(d*x+c)+csc(d*x+c)^2-1)^(1/2))*2^(1/2)*(-cos(d*x
+c)/(cos(d*x+c)+1))^(1/2)-16*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctanh(si
n(d*x+c)/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*cos(d*x+c)-16*
(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctanh(sin(d*x+c)/(cos(d*x+c)+1)/(-cos
(d*x+c)/(cos(d*x+c)+1))^(1/2))-6*cot(d*x+c)*cos(d*x+c)-2*cot(d*x+c))
```

3.180.5 Fricas [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 503, normalized size of antiderivative = 3.05

$$\int \frac{\cot^2(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx$$

$$= \left[-\frac{7\sqrt{2}\sqrt{-a}(\cos(dx+c)+1)\log\left(\frac{2\sqrt{2}\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)\sin(dx+c)+3a\cos(dx+c)^2+2a\cos(dx+c)-a}{\cos(dx+c)^2+2\cos(dx+c)+1}\right)\sin(d\right.$$

$$\left. -\frac{7\sqrt{2}\sqrt{a}(\cos(dx+c)+1)\arctan\left(\frac{\sqrt{2}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)}{\sqrt{a}\sin(dx+c)}\right)\sin(dx+c)+8\sqrt{a}(\cos(dx+c)+1)\arcta}{8(ad\cos(dx+c)+} \right]$$

input
```
integrate(cot(d*x+c)^2/(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
[-1/16*(7*sqrt(2)*sqrt(-a)*(cos(d*x + c) + 1)*log((2*sqrt(2)*sqrt(-a)*sqrt
((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x + c) + 3*a*cos(d*
x + c)^2 + 2*a*cos(d*x + c) - a)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1))*si
n(d*x + c) + 8*sqrt(-a)*(cos(d*x + c) + 1)*log(-(8*a*cos(d*x + c)^3 - 4*(2
*cos(d*x + c)^2 - cos(d*x + c))*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x
 + c))*sin(d*x + c) - 7*a*cos(d*x + c) + a)/(cos(d*x + c) + 1))*sin(d*x +
c) + 4*(3*cos(d*x + c)^2 + cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x
 + c)))/((a*d*cos(d*x + c) + a*d)*sin(d*x + c)), -1/8*(7*sqrt(2)*sqrt(a)*(
cos(d*x + c) + 1)*arctan(sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*c
os(d*x + c)/(sqrt(a)*sin(d*x + c)))*sin(d*x + c) + 8*sqrt(a)*(cos(d*x + c)
 + 1)*arctan(2*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c
)*sin(d*x + c)/(2*a*cos(d*x + c)^2 + a*cos(d*x + c) - a))*sin(d*x + c) + 2
*(3*cos(d*x + c)^2 + cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))
)/((a*d*cos(d*x + c) + a*d)*sin(d*x + c))]
```

3.180.6 Sympy [F]

$$\int \frac{\cot^2(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx = \int \frac{\cot^2(c+dx)}{\sqrt{a\left(\sec(c+dx)+1\right)}}\,dx$$

input
```
integrate(cot(d*x+c)**2/(a+a*sec(d*x+c))**(1/2),x)
```

output
```
Integral(cot(c + d*x)**2/sqrt(a*(sec(c + d*x) + 1)), x)
```

3.180.7 Maxima [F]

$$\int \frac{\cot^2(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx = \int \frac{\cot(dx+c)^2}{\sqrt{a\sec(dx+c)+a}}\,dx$$

input
```
integrate(cot(d*x+c)^2/(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(cot(d*x + c)^2/sqrt(a*sec(d*x + c) + a), x)
```

3.180.8 Giac [A] (verification not implemented)

Time = 1.09 (sec) , antiderivative size = 269, normalized size of antiderivative = 1.63

$$\int \frac{\cot^2(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx$$

$$= \frac{\sqrt{2}\left(\frac{8\sqrt{2}\sqrt{-a}\log\left(\left|\frac{2\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^2-4\sqrt{2}|a|-6a}{2\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^2+4\sqrt{2}|a|-6a}\right|\right)}{|a|} - \frac{7\log\left(\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^2\right)}{\sqrt{-a}} + \frac{8\sqrt{2}}{\sqrt{-a}\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)} - 6\sqrt{2}\log\left(|a|\right)\right)}{16\,d\,\mathrm{sgn}\left(\cos\left(dx+c\right)\right)}$$

input
```
integrate(cot(d*x+c)^2/(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
1/16*sqrt(2)*(8*sqrt(2)*sqrt(-a)*log(abs(2*(sqrt(-a)*tan(1/2*d*x + 1/2*c)
- sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2 - 4*sqrt(2)*abs(a) - 6*a)/abs(2*(
sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2 + 4
*sqrt(2)*abs(a) - 6*a))/abs(a) - 7*log((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sq
rt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2)/sqrt(-a) + 2*sqrt(-a*tan(1/2*d*x + 1
/2*c)^2 + a)*tan(1/2*d*x + 1/2*c)/a + 8*sqrt(-a)/((sqrt(-a)*tan(1/2*d*x +
1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2 - a))/(d*sgn(cos(d*x + c))
)
```

3.180.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cot^2(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx = \int \frac{\cot(c+d\,x)^2}{\sqrt{a+\frac{a}{\cos(c+d\,x)}}}\,dx$$

input
```
int(cot(c + d*x)^2/(a + a/cos(c + d*x))^(1/2),x)
```

output
```
int(cot(c + d*x)^2/(a + a/cos(c + d*x))^(1/2), x)
```

3.181 $\int \frac{\cot^4(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx$

3.181.1 Optimal result 1340
3.181.2 Mathematica [A] (verified) 1341
3.181.3 Rubi [A] (verified) 1341
3.181.4 Maple [A] (verified) 1345
3.181.5 Fricas [A] (verification not implemented) 1346
3.181.6 Sympy [F] 1347
3.181.7 Maxima [F] 1347
3.181.8 Giac [A] (verification not implemented) 1348
3.181.9 Mupad [F(-1)] 1348

3.181.1 Optimal result

Integrand size = 23, antiderivative size = 251

$$\begin{aligned}
&\int \frac{\cot^4(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx \\
&= \frac{2\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{\sqrt{a}d} - \frac{107\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{64\sqrt{2}\sqrt{a}d} \\
&\quad + \frac{21\cot(c+dx)\sqrt{a+a\sec(c+dx)}}{64ad} + \frac{43\cot^3(c+dx)(a+a\sec(c+dx))^{3/2}}{96a^2d} \\
&\quad - \frac{15\cos(c+dx)\cot^3(c+dx)\sec^2\left(\frac{1}{2}(c+dx)\right)(a+a\sec(c+dx))^{3/2}}{32a^2d} \\
&\quad - \frac{\cos^2(c+dx)\cot^3(c+dx)\sec^4\left(\frac{1}{2}(c+dx)\right)(a+a\sec(c+dx))^{3/2}}{16a^2d}
\end{aligned}$$

output

```
43/96*cot(d*x+c)^3*(a+a*sec(d*x+c))^(3/2)/a^2/d-15/32*cos(d*x+c)*cot(d*x+c
)^3*sec(1/2*d*x+1/2*c)^2*(a+a*sec(d*x+c))^(3/2)/a^2/d-1/16*cos(d*x+c)^2*co
t(d*x+c)^3*sec(1/2*d*x+1/2*c)^4*(a+a*sec(d*x+c))^(3/2)/a^2/d+2*arctan(a^(1
/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))/d/a^(1/2)-107/128*arctan(1/2*a^(1/2
)*tan(d*x+c)*2^(1/2)/(a+a*sec(d*x+c))^(1/2))/d*2^(1/2)/a^(1/2)+21/64*cot(d
*x+c)*(a+a*sec(d*x+c))^(1/2)/a/d
```

3.181.2 Mathematica [A] (verified)

Time = 1.66 (sec) , antiderivative size = 229, normalized size of antiderivative = 0.91

$$\int \frac{\cot^4(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx$$

$$= \frac{-321\arcsin\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)\sqrt{\sec(c+dx)}\sqrt{\frac{1}{1+\sec(c+dx)}}\sqrt{1+\sec(c+dx)} + \frac{-\left(\frac{1}{1+\cos(c+dx)}\right)^{3/2}(-110+19\cos(c+}{}}{192d\sqrt{\sec^2\left(\frac{1}{2}(c+dx\right.}}$$

input
```
Integrate[Cot[c + d*x]^4/Sqrt[a + a*Sec[c + d*x]],x]
```

output
```
(-321*ArcSin[Tan[(c + d*x)/2]]*Sqrt[Sec[c + d*x]]*Sqrt[(1 + Sec[c + d*x])^
(-1)]*Sqrt[1 + Sec[c + d*x]] + (-(((1 + Cos[c + d*x])^(-1))^(3/2)*(-110 +
19*Cos[c + d*x] + 142*Cos[2*(c + d*x)] + 205*Cos[3*(c + d*x)])*Csc[(c + d*
x)/2]^3*Sec[(c + d*x)/2]) + 6144*ArcTan[Tan[(c + d*x)/2]/Sqrt[(1 + Sec[c +
 d*x])^(-1)]]*Sqrt[Sec[c + d*x]]*Sqrt[(1 + Sec[c + d*x])^(-1)]*Sqrt[1 + Se
c[c + d*x]])/(8*Sqrt[2]))/(192*d*Sqrt[Sec[(c + d*x)/2]^2]*Sqrt[a*(1 + Sec[
c + d*x])])
```

3.181.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 263, normalized size of antiderivative = 1.05, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.565$, Rules used = {3042, 4375, 374, 27, 441, 25, 27, 445, 27, 445, 27, 397, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^4(c+dx)}{\sqrt{a\sec(c+dx)+a}}\,dx$$

$$\downarrow 3042$$

$$\int \frac{1}{\cot\left(c+dx+\frac{\pi}{2}\right)^4\sqrt{a\csc\left(c+dx+\frac{\pi}{2}\right)+a}}\,dx$$

$$\downarrow 4375$$

$$-\frac{2\int \frac{\cot^4(c+dx)(\sec(c+dx)a+a)^2}{\left(\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+1\right)\left(\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+2\right)^3} d\left(-\frac{\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)}{a^2 d}$$

$$\downarrow 374$$

$$-\frac{2\left(\frac{\int \frac{a\cot^4(c+dx)(\sec(c+dx)a+a)^2\left(1-\frac{7a\tan^2(c+dx)}{\sec(c+dx)a+a}\right)}{\left(\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+1\right)\left(\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+2\right)^2} d\left(-\frac{\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)}{8a}+\frac{\cot^3(c+dx)(a\sec(c+dx)+a)^{3/2}}{8\left(\frac{a\tan^2(c+dx)}{a\sec(c+dx)+a}+2\right)^2}\right)}{a^2 d}$$

$$\downarrow 27$$

$$-\frac{2\left(\frac{1}{8}\int \frac{\cot^4(c+dx)(\sec(c+dx)a+a)^2\left(1-\frac{7a\tan^2(c+dx)}{\sec(c+dx)a+a}\right)}{\left(\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+1\right)\left(\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+2\right)^2} d\left(-\frac{\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)+\frac{\cot^3(c+dx)(a\sec(c+dx)+a)^{3/2}}{8\left(\frac{a\tan^2(c+dx)}{a\sec(c+dx)+a}+2\right)^2}\right)}{a^2 d}$$

$$\downarrow 441$$

$$-\frac{2\left(\frac{1}{8}\left(\frac{\int -\frac{a\cot^4(c+dx)(\sec(c+dx)a+a)^2\left(\frac{75a\tan^2(c+dx)}{\sec(c+dx)a+a}+43\right)}{\left(\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+1\right)\left(\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+2\right)} d\left(-\frac{\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)}{4a}+\frac{15\cot^3(c+dx)(a\sec(c+dx)+a)^{3/2}}{4\left(\frac{a\tan^2(c+dx)}{a\sec(c+dx)+a}+2\right)}\right)+\frac{\cot^3(c+dx)(a\sec(c+dx)+a)^{3/2}}{8\left(\frac{a\tan^2(c+dx)}{a\sec(c+dx)+a}+2\right)^2}\right)}{a^2 d}$$

$$\downarrow 25$$

$$-\frac{2\left(\frac{1}{8}\left(\frac{15\cot^3(c+dx)(a\sec(c+dx)+a)^{3/2}}{4\left(\frac{a\tan^2(c+dx)}{a\sec(c+dx)+a}+2\right)}-\frac{\int \frac{a\cot^4(c+dx)(\sec(c+dx)a+a)^2\left(\frac{75a\tan^2(c+dx)}{\sec(c+dx)a+a}+43\right)}{\left(\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+1\right)\left(\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+2\right)} d\left(-\frac{\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)}{4a}\right)+\frac{\cot^3(c+dx)(a\sec(c+dx)+a)^{3/2}}{8\left(\frac{a\tan^2(c+dx)}{a\sec(c+dx)+a}+2\right)^2}\right)}{a^2 d}$$

$$\downarrow 27$$

$$-\frac{2\left(\frac{1}{8}\left(\frac{15\cot^3(c+dx)(a\sec(c+dx)+a)^{3/2}}{4\left(\frac{a\tan^2(c+dx)}{a\sec(c+dx)+a}+2\right)}-\frac{1}{4}\int \frac{\cot^4(c+dx)(\sec(c+dx)a+a)^2\left(\frac{75a\tan^2(c+dx)}{\sec(c+dx)a+a}+43\right)}{\left(\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+1\right)\left(\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+2\right)} d\left(-\frac{\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)\right)+\frac{\cot^3(c+dx)(a\sec(c+dx)+a)^{3/2}}{8\left(\frac{a\tan^2(c+dx)}{a\sec(c+dx)+a}+2\right)^2}\right)}{a^2 d}$$

$$\downarrow 445$$

$$-\frac{2\left(\frac{1}{8}\left(\frac{1}{4}\left(\frac{1}{6}\int -\frac{3a\cot^2(c+dx)(\sec(c+dx)a+a)\left(21-\frac{43a\tan^2(c+dx)}{\sec(c+dx)a+a}\right)}{\left(\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+1\right)\left(\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+2\right)}d\left(-\frac{\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)-\frac{43}{6}\cot^3(c+dx)(a\sec(c+dx)+\right.}{a^2d}$$

$$\downarrow 27$$

$$-\frac{2\left(\frac{1}{8}\left(\frac{1}{4}\left(-\frac{1}{2}a\int \frac{\cot^2(c+dx)(\sec(c+dx)a+a)\left(21-\frac{43a\tan^2(c+dx)}{\sec(c+dx)a+a}\right)}{\left(\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+1\right)\left(\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+2\right)}d\left(-\frac{\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)-\frac{43}{6}\cot^3(c+dx)(a\sec(c+dx)+a\right.}{a^2d}$$

$$\downarrow 445$$

$$-\frac{2\left(\frac{1}{8}\left(\frac{1}{4}\left(-\frac{1}{2}a\left(\frac{21}{2}\cot(c+dx)\sqrt{a\sec(c+dx)+a}-\frac{1}{2}\int \frac{a\left(\frac{21a\tan^2(c+dx)}{\sec(c+dx)a+a}+149\right)}{\left(\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+1\right)\left(\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+2\right)}d\left(-\frac{\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)\right)-\right.}{a^2d}$$

$$\downarrow 27$$

$$-\frac{2\left(\frac{1}{8}\left(\frac{1}{4}\left(-\frac{1}{2}a\left(\frac{21}{2}\cot(c+dx)\sqrt{a\sec(c+dx)+a}-\frac{1}{2}a\int \frac{\frac{21a\tan^2(c+dx)}{\sec(c+dx)a+a}+149}{\left(\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+1\right)\left(\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+2\right)}d\left(-\frac{\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)\right)-\right.}{a^2d}$$

$$\downarrow 397$$

$$-\frac{2\left(\frac{1}{8}\left(\frac{1}{4}\left(-\frac{1}{2}a\left(\frac{21}{2}\cot(c+dx)\sqrt{a\sec(c+dx)+a}-\frac{1}{2}a\left(128\int \frac{1}{\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+1}d\left(-\frac{\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)-107\int \frac{1}{\frac{a\tan^2(c+dx)}{\sec(c+dx)}}\right.\right.\right.\right.}{a^2d}$$

$$\downarrow 216$$

$$-\frac{2\left(\frac{1}{8}\left(\frac{1}{4}\left(-\frac{1}{2}a\left(\frac{21}{2}\cot(c+dx)\sqrt{a\sec(c+dx)+a}-\frac{1}{2}a\left(\frac{107\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{2}\sqrt{a}}-\frac{128\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}}\right)\right)\right)\right)}{a^2d}$$

input
```
Int[Cot[c + d*x]^4/Sqrt[a + a*Sec[c + d*x]],x]
```

output
```
(-2*((Cot[c + d*x]^3*(a + a*Sec[c + d*x])^(3/2))/(8*(2 + (a*Tan[c + d*x]^2
)/(a + a*Sec[c + d*x]))^2) + (((-43*Cot[c + d*x]^3*(a + a*Sec[c + d*x])^(3
/2))/6 - (a*(-1/2*(a*((-128*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c
 + d*x]]])/Sqrt[a] + (107*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a +
a*Sec[c + d*x]])])/(Sqrt[2]*Sqrt[a])))  + (21*Cot[c + d*x]*Sqrt[a + a*Sec[c
 + d*x]])/2))/2)/4 + (15*Cot[c + d*x]^3*(a + a*Sec[c + d*x])^(3/2))/(4*(2
+ (a*Tan[c + d*x]^2)/(a + a*Sec[c + d*x]))))/8))/(a^2*d)
```

3.181.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 374
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q
 + 1)/(a*e*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(e*x)^m*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[b*c*(m + 1) + 2*(b*c -
a*d)*(p + 1) + d*b*(m + 2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b,
c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IntBinomialQ[a, b,
 c, d, e, m, 2, p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 441
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*(g*x)^(m + 1)*(a
+ b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(a*g*2*(b*c - a*d)*(p + 1))), x] + Si
mp[1/(a*2*(b*c - a*d)*(p + 1))   Int[(g*x)^m*(a + b*x^2)^(p + 1)*(c + d*x^2
)^q*Simp[c*(b*e - a*f)*(m + 1) + e*2*(b*c - a*d)*(p + 1) + d*(b*e - a*f)*(m
 + 2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m, q},
 x] && LtQ[p, -1]
```

rule 445
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^2*(m + 1))
 Int[(g*x)^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*f*c*(m + 1) - e*(b*c
+ a*d)*(m + 2 + 1) - e*2*(b*c*p + a*d*q) - b*e*d*(m + 2*(p + q + 2) + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && LtQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4375
```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Simp[-2*(a^(m/2 + n + 1/2)/d)   Subst[Int[x^m*((2 + a*x^2
)^(m/2 + n - 1/2)/(1 + a*x^2)), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]
]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && I
ntegerQ[n - 1/2]
```

3.181.4 Maple [A] (verified)

Time = 2.10 (sec) , antiderivative size = 372, normalized size of antiderivative = 1.48

method	result
default	$-\frac{\sqrt{a(1+\sec(dx+c))}\left(321\sqrt{2}\,\ln\left(\csc(dx+c)-\cot(dx+c)+\sqrt{\cot(dx+c)^2-2\cot(dx+c)\csc(dx+c)+\csc(dx+c)^2-1}\right)\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}\right.$

input
```
int(cot(d*x+c)^4/(a+a*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/384/d/a*(a*(1+sec(d*x+c)))^(1/2)/(cos(d*x+c)+1)*(321*2^(1/2)*ln(csc(d*x
+c)-cot(d*x+c)+(cot(d*x+c)^2-2*cot(d*x+c)*csc(d*x+c)+csc(d*x+c)^2-1)^(1/2)
)*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)+321*ln(csc(d*x+c)-cot(d*x+
c)+(cot(d*x+c)^2-2*cot(d*x+c)*csc(d*x+c)+csc(d*x+c)^2-1)^(1/2))*2^(1/2)*(-
cos(d*x+c)/(cos(d*x+c)+1))^(1/2)-768*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*ar
ctanh(sin(d*x+c)/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*cos(d*
x+c)-768*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctanh(sin(d*x+c)/(cos(d*x+c)
+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))+410*cos(d*x+c)*cot(d*x+c)^3+142*co
t(d*x+c)^3-298*cot(d*x+c)^2*csc(d*x+c)-126*cot(d*x+c)*csc(d*x+c)^2)
```

3.181.5 Fricas [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 666, normalized size of antiderivative = 2.65

$$\int \frac{\cot^4(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx$$

$$= \left[-\frac{321\sqrt{2}\left(\cos(dx+c)^3+\cos(dx+c)^2-\cos(dx+c)-1\right)\sqrt{-a}\log\left(-\frac{2\sqrt{2}\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)\sin(dx+\right.\right.}{\cos(dx+c)^2+2\,\mathrm{co}}$$

input
```
integrate(cot(d*x+c)^4/(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
[-1/768*(321*sqrt(2)*(cos(d*x + c)^3 + cos(d*x + c)^2 - cos(d*x + c) - 1)*
sqrt(-a)*log(-(2*sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*
cos(d*x + c)*sin(d*x + c) - 3*a*cos(d*x + c)^2 - 2*a*cos(d*x + c) + a)/(co
s(d*x + c)^2 + 2*cos(d*x + c) + 1))*sin(d*x + c) + 384*(cos(d*x + c)^3 + c
os(d*x + c)^2 - cos(d*x + c) - 1)*sqrt(-a)*log(-(8*a*cos(d*x + c)^3 + 4*(2
*cos(d*x + c)^2 - cos(d*x + c))*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x
 + c))*sin(d*x + c) - 7*a*cos(d*x + c) + a)/(cos(d*x + c) + 1))*sin(d*x +
c) - 4*(205*cos(d*x + c)^4 + 71*cos(d*x + c)^3 - 149*cos(d*x + c)^2 - 63*c
os(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)))/((a*d*cos(d*x + c)^3
 + a*d*cos(d*x + c)^2 - a*d*cos(d*x + c) - a*d)*sin(d*x + c)), 1/384*(321*
sqrt(2)*(cos(d*x + c)^3 + cos(d*x + c)^2 - cos(d*x + c) - 1)*sqrt(a)*arcta
n(sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*si
n(d*x + c)))*sin(d*x + c) + 384*(cos(d*x + c)^3 + cos(d*x + c)^2 - cos(d*x
 + c) - 1)*sqrt(a)*arctan(2*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)
)*cos(d*x + c)*sin(d*x + c)/(2*a*cos(d*x + c)^2 + a*cos(d*x + c) - a))*sin
(d*x + c) + 2*(205*cos(d*x + c)^4 + 71*cos(d*x + c)^3 - 149*cos(d*x + c)^2
 - 63*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)))/((a*d*cos(d*x
 + c)^3 + a*d*cos(d*x + c)^2 - a*d*cos(d*x + c) - a*d)*sin(d*x + c))]
```

3.181.6 Sympy [F]

$$\int \frac{\cot^4(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx = \int \frac{\cot^4(c+dx)}{\sqrt{a\left(\sec(c+dx)+1\right)}}\,dx$$

input
```
integrate(cot(d*x+c)**4/(a+a*sec(d*x+c))**(1/2),x)
```

output
```
Integral(cot(c + d*x)**4/sqrt(a*(sec(c + d*x) + 1)), x)
```

3.181.7 Maxima [F]

$$\int \frac{\cot^4(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx = \int \frac{\cot(dx+c)^4}{\sqrt{a\sec(dx+c)+a}}\,dx$$

input
```
integrate(cot(d*x+c)^4/(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(cot(d*x + c)^4/sqrt(a*sec(d*x + c) + a), x)
```

3.181.8 Giac [A] (verification not implemented)

Time = 1.21 (sec) , antiderivative size = 385, normalized size of antiderivative = 1.53

$$\int \frac{\cot^4(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx$$

$$= \frac{\sqrt{2}\left(6\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a}\left(\frac{2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2}{a}-\frac{21}{a}\right)\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\frac{384\sqrt{2}\sqrt{-a}\log\left(\left|\frac{2\left(\sqrt{-a}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right.\right.\right.}{2\left(\sqrt{-a}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right.\right.}\right.}{}\right.$$

input
```
integrate(cot(d*x+c)^4/(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
1/768*sqrt(2)*(6*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)*(2*tan(1/2*d*x + 1/2*
c)^2/a - 21/a)*tan(1/2*d*x + 1/2*c) - 384*sqrt(2)*sqrt(-a)*log(abs(2*(sqrt
(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2 - 4*sqr
t(2)*abs(a) - 6*a)/abs(2*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*
d*x + 1/2*c)^2 + a))^2 + 4*sqrt(2)*abs(a) - 6*a))/abs(a) + 321*log((sqrt(-
a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2)/sqrt(-a)
 - 64*(9*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 +
 a))^4*sqrt(-a) - 15*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x
+ 1/2*c)^2 + a))^2*sqrt(-a)*a + 8*sqrt(-a)*a^2)/((sqrt(-a)*tan(1/2*d*x + 1
/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2 - a)^3)/(d*sgn(cos(d*x + c)
))
```

3.181.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cot^4(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx = \int \frac{\cot(c+d\,x)^4}{\sqrt{a+\frac{a}{\cos(c+d\,x)}}}\,dx$$

input
```
int(cot(c + d*x)^4/(a + a/cos(c + d*x))^(1/2),x)
```

output
```
int(cot(c + d*x)^4/(a + a/cos(c + d*x))^(1/2), x)
```

3.182 $\int \frac{\cot^6(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx$

3.182.1 Optimal result 1349
3.182.2 Mathematica [A] (verified) 1350
3.182.3 Rubi [A] (verified) 1351
3.182.4 Maple [A] (verified) 1355
3.182.5 Fricas [A] (verification not implemented) 1356
3.182.6 Sympy [F] 1357
3.182.7 Maxima [F] 1357
3.182.8 Giac [A] (verification not implemented) 1357
3.182.9 Mupad [F(-1)] 1358

3.182.1 Optimal result

Integrand size = 23, antiderivative size = 335

$$
\begin{aligned}
&\int \frac{\cot^6(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx \\
&= -\frac{2\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{\sqrt{a}d} + \frac{835\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{512\sqrt{2}\sqrt{a}d} \\
&\quad - \frac{189\cot(c+dx)\sqrt{a+a\sec(c+dx)}}{512ad} - \frac{323\cot^3(c+dx)(a+a\sec(c+dx))^{3/2}}{768a^2d} \\
&\quad + \frac{579\cot^5(c+dx)(a+a\sec(c+dx))^{5/2}}{640a^3d} \\
&\quad - \frac{101\cos(c+dx)\cot^5(c+dx)\sec^2\left(\frac{1}{2}(c+dx)\right)(a+a\sec(c+dx))^{5/2}}{128a^3d} \\
&\quad - \frac{23\cos^2(c+dx)\cot^5(c+dx)\sec^4\left(\frac{1}{2}(c+dx)\right)(a+a\sec(c+dx))^{5/2}}{192a^3d} \\
&\quad - \frac{\cos^3(c+dx)\cot^5(c+dx)\sec^6\left(\frac{1}{2}(c+dx)\right)(a+a\sec(c+dx))^{5/2}}{48a^3d}
\end{aligned}
$$

output
```
-323/768*cot(d*x+c)^3*(a+a*sec(d*x+c))^(3/2)/a^2/d+579/640*cot(d*x+c)^5*(a
+a*sec(d*x+c))^(5/2)/a^3/d-101/128*cos(d*x+c)*cot(d*x+c)^5*sec(1/2*d*x+1/2
*c)^2*(a+a*sec(d*x+c))^(5/2)/a^3/d-23/192*cos(d*x+c)^2*cot(d*x+c)^5*sec(1/
2*d*x+1/2*c)^4*(a+a*sec(d*x+c))^(5/2)/a^3/d-1/48*cos(d*x+c)^3*cot(d*x+c)^5
*sec(1/2*d*x+1/2*c)^6*(a+a*sec(d*x+c))^(5/2)/a^3/d-2*arctan(a^(1/2)*tan(d*
x+c)/(a+a*sec(d*x+c))^(1/2))/d/a^(1/2)+835/1024*arctan(1/2*a^(1/2)*tan(d*x
+c)*2^(1/2)/(a+a*sec(d*x+c))^(1/2))/d*2^(1/2)/a^(1/2)-189/512*cot(d*x+c)*(
a+a*sec(d*x+c))^(1/2)/a/d
```

3.182.2 Mathematica [A] (verified)

Time = 3.25 (sec) , antiderivative size = 263, normalized size of antiderivative = 0.79

$$\int \frac{\cot^6(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx$$
$$= \frac{\sqrt{\sec(c+dx)}\left(-\frac{\left(\frac{1}{1+\cos(c+dx)}\right)^{3/2}(21682+11948\cos(c+dx)+12791\cos(2(c+dx))-14754\cos(3(c+dx))+846\cos(4(c+dx))+6902\cos(5(c+dx)+9737\cos(6(c+dx))\csc^5\left(\frac{c+dx}{2}\right)\sec^3\left(\frac{c+dx}{2}\right)}{256\sqrt{2}}\right.}{}$$

input
```
Integrate[Cot[c + d*x]^6/Sqrt[a + a*Sec[c + d*x]],x]
```

output
```
(Sqrt[Sec[c + d*x]]*(-1/256*(((1 + Cos[c + d*x])^(-1))^(3/2)*(21682 + 1194
8*Cos[c + d*x] + 12791*Cos[2*(c + d*x)] - 14754*Cos[3*(c + d*x)] + 846*Cos
[4*(c + d*x)] + 6902*Cos[5*(c + d*x)] + 9737*Cos[6*(c + d*x)])*Csc[(c + d*
x)/2]^5*Sec[(c + d*x)/2]^3*Sqrt[Sec[c + d*x]])/Sqrt[2] + 12525*ArcSin[Tan[
(c + d*x)/2]]*Sqrt[(1 + Sec[c + d*x])^(-1)]*Sqrt[1 + Sec[c + d*x]] - 15360
*Sqrt[2]*ArcTan[Tan[(c + d*x)/2]/Sqrt[(1 + Sec[c + d*x])^(-1)]]*Sqrt[(1 +
Sec[c + d*x])^(-1)]*Sqrt[1 + Sec[c + d*x]]))/(7680*d*Sqrt[Sec[(c + d*x)/2]
^2]*Sqrt[a*(1 + Sec[c + d*x])])
```

3.182.3 Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 352, normalized size of antiderivative = 1.05, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.696$, Rules used = {3042, 4375, 374, 27, 441, 27, 441, 27, 445, 27, 445, 27, 445, 27, 397, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^6(c+dx)}{\sqrt{a\sec(c+dx)+a}}\,dx$$

$$\downarrow 3042$$

$$\int \frac{1}{\cot\left(c+dx+\frac{\pi}{2}\right)^6\sqrt{a\csc\left(c+dx+\frac{\pi}{2}\right)+a}}dx$$

$$\downarrow 4375$$

$$-\frac{2\int \frac{\cot^6(c+dx)(\sec(c+dx)a+a)^3}{\left(\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+1\right)\left(\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+2\right)^4}d\left(-\frac{\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)}{a^3d}$$

$$\downarrow 374$$

$$-\frac{2\left(\frac{\int \frac{a\cot^6(c+dx)(\sec(c+dx)a+a)^3\left(1-\frac{11a\tan^2(c+dx)}{\sec(c+dx)a+a}\right)}{\left(\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+1\right)\left(\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+2\right)^3}d\left(-\frac{\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)}{12a}+\frac{\cot^5(c+dx)(a\sec(c+dx)+a)^{5/2}}{12\left(\frac{a\tan^2(c+dx)}{a\sec(c+dx)+a}+2\right)^3}\right)}{a^3d}$$

$$\downarrow 27$$

$$-\frac{2\left(\frac{1}{12}\int \frac{\cot^6(c+dx)(\sec(c+dx)a+a)^3\left(1-\frac{11a\tan^2(c+dx)}{\sec(c+dx)a+a}\right)}{\left(\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+1\right)\left(\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+2\right)^3}d\left(-\frac{\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)+\frac{\cot^5(c+dx)(a\sec(c+dx)+a)^{5/2}}{12\left(\frac{a\tan^2(c+dx)}{a\sec(c+dx)+a}+2\right)^3}\right)}{a^3d}$$

$$\downarrow 441$$

$$-\frac{2\left(\frac{1}{12}\left(\frac{\int -\frac{3a\cot^6(c+dx)(\sec(c+dx)a+a)^3\left(\frac{69a\tan^2(c+dx)}{\sec(c+dx)a+a}+37\right)}{\left(\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+1\right)\left(\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+2\right)^2}d\left(-\frac{\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)}{8a}+\frac{23\cot^5(c+dx)(a\sec(c+dx)+a)^{5/2}}{8\left(\frac{a\tan^2(c+dx)}{a\sec(c+dx)+a}+2\right)^2}\right)+\frac{\cot^5(c+dx)(}{12\left(\frac{a\tan}{a\sec}\right.}\right)}{a^3d}$$

$$\downarrow 27$$

$$-\frac{2\left(\frac{1}{12}\left(\frac{23\cot^5(c+dx)(a\sec(c+dx)+a)^{5/2}}{8\left(\frac{a\tan^2(c+dx)}{a\sec(c+dx)+a}+2\right)^2}-\frac{3}{8}\int\frac{\cot^6(c+dx)(\sec(c+dx)a+a)^3\left(\frac{69a\tan^2(c+dx)}{\sec(c+dx)a+a}+37\right)}{\left(\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+1\right)\left(\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+2\right)^2}d\left(-\frac{\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)\right)+\frac{\cot^5(c}{12}}{a^3d}$$

$$\downarrow 441$$

$$-\frac{2\left(\frac{1}{12}\left(\frac{23\cot^5(c+dx)(a\sec(c+dx)+a)^{5/2}}{8\left(\frac{a\tan^2(c+dx)}{a\sec(c+dx)+a}+2\right)^2}-\frac{3}{8}\left(\frac{\int\frac{a\cot^6(c+dx)(\sec(c+dx)a+a)^3\left(\frac{707a\tan^2(c+dx)}{\sec(c+dx)a+a}+579\right)}{\left(\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+1\right)\left(\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+2\right)}d\left(-\frac{\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)}{4a}-\frac{101\cot^5(c+}{4\left(\frac{}{a}\right.}\right.\right.}{a^3d}$$

$$\downarrow 27$$

$$-\frac{2\left(\frac{1}{12}\left(\frac{23\cot^5(c+dx)(a\sec(c+dx)+a)^{5/2}}{8\left(\frac{a\tan^2(c+dx)}{a\sec(c+dx)+a}+2\right)^2}-\frac{3}{8}\left(\frac{1}{4}\int\frac{\cot^6(c+dx)(\sec(c+dx)a+a)^3\left(\frac{707a\tan^2(c+dx)}{\sec(c+dx)a+a}+579\right)}{\left(\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+1\right)\left(\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+2\right)}d\left(-\frac{\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)-\frac{10}{}\right.\right.\right.}{a^3d}$$

$$\downarrow 445$$

$$-\frac{2\left(\frac{1}{12}\left(\frac{23\cot^5(c+dx)(a\sec(c+dx)+a)^{5/2}}{8\left(\frac{a\tan^2(c+dx)}{a\sec(c+dx)+a}+2\right)^2}-\frac{3}{8}\left(\frac{1}{4}\left(\frac{579}{10}\cot^5(c+dx)(a\sec(c+dx)+a)^{5/2}-\frac{1}{10}\int\frac{5a\cot^4(c+dx)(\sec(c+dx)a-}{\left(\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+1\right.}\right.\right.\right.}{a^3c}$$

$$\downarrow 27$$

$$-\frac{2\left(\frac{1}{12}\left(\frac{23\cot^5(c+dx)(a\sec(c+dx)+a)^{5/2}}{8\left(\frac{a\tan^2(c+dx)}{a\sec(c+dx)+a}+2\right)^2}-\frac{3}{8}\left(\frac{1}{4}\left(\frac{579}{10}\cot^5(c+dx)(a\sec(c+dx)+a)^{5/2}-\frac{1}{2}a\int\frac{\cot^4(c+dx)(\sec(c+dx)a+a}{\left(\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+1\right)}\right.\right.\right.}{a^3d}$$

$$\downarrow 445$$

$$-\frac{2\left(\frac{1}{12}\left(\frac{23\cot^5(c+dx)(a\sec(c+dx)+a)^{5/2}}{8\left(\frac{a\tan^2(c+dx)}{a\sec(c+dx)+a}+2\right)^2}-\frac{3}{8}\left(\frac{1}{4}\left(\frac{579}{10}\cot^5(c+dx)(a\sec(c+dx)+a)^{5/2}-\frac{1}{2}a\left(\frac{323}{6}\cot^3(c+dx)(a\sec\right.\right.\right.\right.}{a^3d}$$

$$\downarrow 27$$

$$-2\left(\frac{1}{12}\left(\frac{23\cot^5(c+dx)(a\sec(c+dx)+a)^{5/2}}{8\left(\frac{a\tan^2(c+dx)}{a\sec(c+dx)+a}+2\right)^2}-\frac{3}{8}\left(\frac{1}{4}\left(\frac{579}{10}\cot^5(c+dx)(a\sec(c+dx)+a)^{5/2}-\frac{1}{2}a\left(\frac{1}{2}a\int\frac{\cot^2(c+dx)(\sec(c+dx)}{\left(\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}\right.}\right.\right.\right.\right.$$

$$\downarrow 445$$

$$-2\left(\frac{1}{12}\left(\frac{23\cot^5(c+dx)(a\sec(c+dx)+a)^{5/2}}{8\left(\frac{a\tan^2(c+dx)}{a\sec(c+dx)+a}+2\right)^2}-\frac{3}{8}\left(\frac{1}{4}\left(\frac{579}{10}\cot^5(c+dx)(a\sec(c+dx)+a)^{5/2}-\frac{1}{2}a\left(\frac{1}{2}a\left(\frac{189}{2}\cot(c+dx)\sqrt{}\right.\right.\right.\right.\right.$$

$$\downarrow 27$$

$$-2\left(\frac{1}{12}\left(\frac{23\cot^5(c+dx)(a\sec(c+dx)+a)^{5/2}}{8\left(\frac{a\tan^2(c+dx)}{a\sec(c+dx)+a}+2\right)^2}-\frac{3}{8}\left(\frac{1}{4}\left(\frac{579}{10}\cot^5(c+dx)(a\sec(c+dx)+a)^{5/2}-\frac{1}{2}a\left(\frac{1}{2}a\left(\frac{189}{2}\cot(c+dx)\sqrt{}\right.\right.\right.\right.\right.$$

$$\downarrow 397$$

$$-2\left(\frac{1}{12}\left(\frac{23\cot^5(c+dx)(a\sec(c+dx)+a)^{5/2}}{8\left(\frac{a\tan^2(c+dx)}{a\sec(c+dx)+a}+2\right)^2}-\frac{3}{8}\left(\frac{1}{4}\left(\frac{579}{10}\cot^5(c+dx)(a\sec(c+dx)+a)^{5/2}-\frac{1}{2}a\left(\frac{1}{2}a\left(\frac{189}{2}\cot(c+dx)\sqrt{}\right.\right.\right.\right.\right.$$

$$\downarrow 216$$

$$-2\left(\frac{1}{12}\left(\frac{23\cot^5(c+dx)(a\sec(c+dx)+a)^{5/2}}{8\left(\frac{a\tan^2(c+dx)}{a\sec(c+dx)+a}+2\right)^2}-\frac{3}{8}\left(\frac{1}{4}\left(\frac{579}{10}\cot^5(c+dx)(a\sec(c+dx)+a)^{5/2}-\frac{1}{2}a\left(\frac{1}{2}a\left(\frac{189}{2}\cot(c+dx)\sqrt{}\right.\right.\right.\right.\right.$$

input
```
Int[Cot[c + d*x]^6/Sqrt[a + a*Sec[c + d*x]],x]
```

output
```
(-2*((Cot[c + d*x]^5*(a + a*Sec[c + d*x])^(5/2))/(12*(2 + (a*Tan[c + d*x]^
2)/(a + a*Sec[c + d*x]))^3) + ((23*Cot[c + d*x]^5*(a + a*Sec[c + d*x])^(5/
2))/(8*(2 + (a*Tan[c + d*x]^2)/(a + a*Sec[c + d*x]))^2) - (3*(((579*Cot[c
+ d*x]^5*(a + a*Sec[c + d*x])^(5/2))/10 - (a*((323*Cot[c + d*x]^3*(a + a*S
ec[c + d*x])^(3/2))/6 + (a*(-1/2*(a*((-1024*ArcTan[(Sqrt[a]*Tan[c + d*x])/
Sqrt[a + a*Sec[c + d*x]]])/Sqrt[a] + (835*ArcTan[(Sqrt[a]*Tan[c + d*x])/(S
qrt[2]*Sqrt[a + a*Sec[c + d*x]])])/(Sqrt[2]*Sqrt[a])) + (189*Cot[c + d*x]
*Sqrt[a + a*Sec[c + d*x]])/2))/2))/2)/4 - (101*Cot[c + d*x]^5*(a + a*Sec[c
 + d*x])^(5/2))/(4*(2 + (a*Tan[c + d*x]^2)/(a + a*Sec[c + d*x])))))/8)/12)
)/(a^3*d)
```

3.182.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 374
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q
 + 1)/(a*e*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(e*x)^m*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[b*c*(m + 1) + 2*(b*c -
a*d)*(p + 1) + d*b*(m + 2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b,
c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IntBinomialQ[a, b,
 c, d, e, m, 2, p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 441
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*(g*x)^(m + 1)*(a
+ b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(a*g*2*(b*c - a*d)*(p + 1))), x] + Si
mp[1/(a*2*(b*c - a*d)*(p + 1))   Int[(g*x)^m*(a + b*x^2)^(p + 1)*(c + d*x^2
)^q*Simp[c*(b*e - a*f)*(m + 1) + e*2*(b*c - a*d)*(p + 1) + d*(b*e - a*f)*(m
 + 2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m, q},
 x] && LtQ[p, -1]
```

rule 445
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^2*(m + 1))
 Int[(g*x)^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*f*c*(m + 1) - e*(b*c
+ a*d)*(m + 2 + 1) - e*2*(b*c*p + a*d*q) - b*e*d*(m + 2*(p + q + 2) + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && LtQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4375
```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Simp[-2*(a^(m/2 + n + 1/2)/d)   Subst[Int[x^m*((2 + a*x^2
)^(m/2 + n - 1/2)/(1 + a*x^2)), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]
]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && I
ntegerQ[n - 1/2]
```

3.182.4 Maple [A] (verified)

Time = 1.95 (sec) , antiderivative size = 408, normalized size of antiderivative = 1.22

method	result
default	$-\frac{\sqrt{a(1+\sec(dx+c))}\left(-12525\sqrt{2}\,\ln\left(\csc(dx+c)-\cot(dx+c)+\sqrt{\cot(dx+c)^2-2\cot(dx+c)\csc(dx+c)+\csc(dx+c)^2-1}\right)\sqrt{-\frac{\cos(dx+}{\cos(dx+c}\right.}$

input
```
int(cot(d*x+c)^6/(a+a*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/15360/d/a*(a*(1+sec(d*x+c)))^(1/2)/(cos(d*x+c)+1)*(-12525*2^(1/2)*ln(cs
c(d*x+c)-cot(d*x+c)+(cot(d*x+c)^2-2*cot(d*x+c)*csc(d*x+c)+csc(d*x+c)^2-1)^
(1/2))*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)+30720*(-cos(d*x+c)/(c
os(d*x+c)+1))^(1/2)*arctanh(sin(d*x+c)/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*
x+c)+1))^(1/2))*cos(d*x+c)-12525*ln(csc(d*x+c)-cot(d*x+c)+(cot(d*x+c)^2-2*
cot(d*x+c)*csc(d*x+c)+csc(d*x+c)^2-1)^(1/2))*2^(1/2)*(-cos(d*x+c)/(cos(d*x
+c)+1))^(1/2)+30720*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctanh(sin(d*x+c)/
(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))+19474*cos(d*x+c)*cot(d*
x+c)^5+6902*cot(d*x+c)^5-28788*cot(d*x+c)^4*csc(d*x+c)-12316*cot(d*x+c)^3*
csc(d*x+c)^2+12130*cot(d*x+c)^2*csc(d*x+c)^3+5670*cot(d*x+c)*csc(d*x+c)^4)
```

3.182.5 Fricas [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 823, normalized size of antiderivative = 2.46

$$\int \frac{\cot^6(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx = \text{Too large to display}$$

input
```
integrate(cot(d*x+c)^6/(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
[-1/30720*(12525*sqrt(2)*(cos(d*x + c)^5 + cos(d*x + c)^4 - 2*cos(d*x + c)
^3 - 2*cos(d*x + c)^2 + cos(d*x + c) + 1)*sqrt(-a)*log((2*sqrt(2)*sqrt(-a)
*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x + c) + 3*a*c
os(d*x + c)^2 + 2*a*cos(d*x + c) - a)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1
))*sin(d*x + c) + 15360*(cos(d*x + c)^5 + cos(d*x + c)^4 - 2*cos(d*x + c)^
3 - 2*cos(d*x + c)^2 + cos(d*x + c) + 1)*sqrt(-a)*log(-(8*a*cos(d*x + c)^3
 - 4*(2*cos(d*x + c)^2 - cos(d*x + c))*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/
cos(d*x + c))*sin(d*x + c) - 7*a*cos(d*x + c) + a)/(cos(d*x + c) + 1))*sin
(d*x + c) + 4*(9737*cos(d*x + c)^6 + 3451*cos(d*x + c)^5 - 14394*cos(d*x +
 c)^4 - 6158*cos(d*x + c)^3 + 6065*cos(d*x + c)^2 + 2835*cos(d*x + c))*sqr
t((a*cos(d*x + c) + a)/cos(d*x + c)))/((a*d*cos(d*x + c)^5 + a*d*cos(d*x +
 c)^4 - 2*a*d*cos(d*x + c)^3 - 2*a*d*cos(d*x + c)^2 + a*d*cos(d*x + c) + a
*d)*sin(d*x + c)), -1/15360*(12525*sqrt(2)*(cos(d*x + c)^5 + cos(d*x + c)^
4 - 2*cos(d*x + c)^3 - 2*cos(d*x + c)^2 + cos(d*x + c) + 1)*sqrt(a)*arctan
(sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin
(d*x + c)))*sin(d*x + c) + 15360*(cos(d*x + c)^5 + cos(d*x + c)^4 - 2*cos(
d*x + c)^3 - 2*cos(d*x + c)^2 + cos(d*x + c) + 1)*sqrt(a)*arctan(2*sqrt(a)
*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x + c)/(2*a*co
s(d*x + c)^2 + a*cos(d*x + c) - a))*sin(d*x + c) + 2*(9737*cos(d*x + c)^6
+ 3451*cos(d*x + c)^5 - 14394*cos(d*x + c)^4 - 6158*cos(d*x + c)^3 + 60...
```

3.182.6 Sympy [F]

$$\int \frac{\cot^6(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx = \int \frac{\cot^6(c+dx)}{\sqrt{a\left(\sec(c+dx)+1\right)}}\,dx$$

input
```
integrate(cot(d*x+c)**6/(a+a*sec(d*x+c))**(1/2),x)
```

output
```
Integral(cot(c + d*x)**6/sqrt(a*(sec(c + d*x) + 1)), x)
```

3.182.7 Maxima [F]

$$\int \frac{\cot^6(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx = \int \frac{\cot(dx+c)^6}{\sqrt{a\sec(dx+c)+a}}\,dx$$

input
```
integrate(cot(d*x+c)^6/(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(cot(d*x + c)^6/sqrt(a*sec(d*x + c) + a), x)
```

3.182.8 Giac [A] (verification not implemented)

Time = 1.32 (sec) , antiderivative size = 500, normalized size of antiderivative = 1.49

$$\int \frac{\cot^6(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx$$

$$= \frac{\sqrt{2}\left(10\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a}\left(2\left(\frac{4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2}{a}-\frac{43}{a}\right)\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+\frac{567}{a}\right)\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+\right.}{}$$

input
```
integrate(cot(d*x+c)^6/(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
1/30720*sqrt(2)*(10*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)*(2*(4*tan(1/2*d*x
+ 1/2*c)^2/a - 43/a)*tan(1/2*d*x + 1/2*c)^2 + 567/a)*tan(1/2*d*x + 1/2*c)
+ 15360*sqrt(2)*sqrt(-a)*log(abs(2*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-
a*tan(1/2*d*x + 1/2*c)^2 + a))^2 - 4*sqrt(2)*abs(a) - 6*a)/abs(2*(sqrt(-a)
*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2 + 4*sqrt(2)
*abs(a) - 6*a))/abs(a) - 12525*log((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-
a*tan(1/2*d*x + 1/2*c)^2 + a))^2)/sqrt(-a) + 192*(145*(sqrt(-a)*tan(1/2*d*
x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^8*sqrt(-a) - 500*(sqrt(-
a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^6*sqrt(-a)*
a + 710*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 +
a))^4*sqrt(-a)*a^2 - 460*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*
d*x + 1/2*c)^2 + a))^2*sqrt(-a)*a^3 + 121*sqrt(-a)*a^4)/((sqrt(-a)*tan(1/2
*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2 - a)^5)/(d*sgn(cos(
d*x + c)))
```

3.182.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cot^6(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx = \int \frac{\cot(c+d\,x)^6}{\sqrt{a+\frac{a}{\cos(c+d\,x)}}}\,dx$$

input
```
int(cot(c + d*x)^6/(a + a/cos(c + d*x))^(1/2),x)
```

output
```
int(cot(c + d*x)^6/(a + a/cos(c + d*x))^(1/2), x)
```

3.183 $\int \frac{\tan^5(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx$

3.183.1 Optimal result 1359
3.183.2 Mathematica [A] (verified) 1359
3.183.3 Rubi [A] (verified) 1360
3.183.4 Maple [A] (verified) 1361
3.183.5 Fricas [A] (verification not implemented) 1362
3.183.6 Sympy [F] 1362
3.183.7 Maxima [A] (verification not implemented) 1362
3.183.8 Giac [A] (verification not implemented) 1363
3.183.9 Mupad [F(-1)] 1363

3.183.1 Optimal result

Integrand size = 23, antiderivative size = 100

$$\int \frac{\tan^5(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = -\frac{2\operatorname{arctanh}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{a}}\right)}{a^{3/2}d} + \frac{2\sqrt{a+a\sec(c+dx)}}{a^2 d} - \frac{2(a+a\sec(c+dx))^{3/2}}{a^3 d} + \frac{2(a+a\sec(c+dx))^{5/2}}{5a^4 d}$$

output
```
-2*arctanh((a+a*sec(d*x+c))^(1/2)/a^(1/2))/a^(3/2)/d-2*(a+a*sec(d*x+c))^(3
/2)/a^3/d+2/5*(a+a*sec(d*x+c))^(5/2)/a^4/d+2*(a+a*sec(d*x+c))^(1/2)/a^2/d
```

3.183.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.79

$$\int \frac{\tan^5(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = \frac{2\left(1-2\sec(c+dx)-2\sec^2(c+dx)+\sec^3(c+dx)-5\operatorname{arctanh}\left(\sqrt{1+\sec(c\right.\right.}{5ad\sqrt{a(1+\sec(c+dx))}}$$

input
```
Integrate[Tan[c + d*x]^5/(a + a*Sec[c + d*x])^(3/2),x]
```

output
```
(2*(1 - 2*Sec[c + d*x] - 2*Sec[c + d*x]^2 + Sec[c + d*x]^3 - 5*ArcTanh[Sqr
t[1 + Sec[c + d*x]]]*Sqrt[1 + Sec[c + d*x]]))/(5*a*d*Sqrt[a*(1 + Sec[c + d
*x])])
```

3.183.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.92, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3042, 25, 4368, 27, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^5(c+dx)}{(a\sec(c+dx)+a)^{3/2}}\,dx$$

$$\downarrow 3042$$

$$\int -\frac{\cot\left(c+dx+\frac{\pi}{2}\right)^5}{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^{3/2}}dx$$

$$\downarrow 25$$

$$-\int \frac{\cot\left(\frac{1}{2}(2c+\pi)+dx\right)^5}{\left(\csc\left(\frac{1}{2}(2c+\pi)+dx\right)a+a\right)^{3/2}}dx$$

$$\downarrow 4368$$

$$\frac{\int a^2\cos(c+dx)(1-\sec(c+dx))^2\sqrt{\sec(c+dx)a+a}d\sec(c+dx)}{a^4d}$$

$$\downarrow 27$$

$$\frac{\int \cos(c+dx)(1-\sec(c+dx))^2\sqrt{\sec(c+dx)a+a}d\sec(c+dx)}{a^2d}$$

$$\downarrow 99$$

$$\frac{\int\left(\frac{(\sec(c+dx)a+a)^{3/2}}{a}+\cos(c+dx)\sqrt{\sec(c+dx)a+a}-3\sqrt{\sec(c+dx)a+a}\right)d\sec(c+dx)}{a^2d}$$

$$\downarrow 2009$$

$$\frac{\frac{2(a\sec(c+dx)+a)^{5/2}}{5a^2}-2\sqrt{a}\operatorname{arctanh}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{a}}\right)-\frac{2(a\sec(c+dx)+a)^{3/2}}{a}+2\sqrt{a\sec(c+dx)+a}}{a^2d}$$

input
```
Int[Tan[c + d*x]^5/(a + a*Sec[c + d*x])^(3/2),x]
```

output
```
(-2*Sqrt[a]*ArcTanh[Sqrt[a + a*Sec[c + d*x]]/Sqrt[a]] + 2*Sqrt[a + a*Sec[c
 + d*x]] - (2*(a + a*Sec[c + d*x])^(3/2))/a + (2*(a + a*Sec[c + d*x])^(5/2
))/(5*a^2))/(a^2*d)
```

3.183.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4368
```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Simp[-(d*b^(m - 1))^(-1)   Subst[Int[(-a + b*x)^((m - 1)/2
)*((a + b*x)^((m - 1)/2 + n)/x), x], x, Csc[c + d*x]], x] /; FreeQ[{a, b, c
, d, n}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[n]
```

3.183.4 Maple [A] (verified)

Time = 3.59 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.82

method	result	size
default	$\frac{2\sqrt{a(1+\sec(dx+c))}\left(5\arctan\left(\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}\right)\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}+1-3\sec(dx+c)+\sec(dx+c)^2\right)}{5da^2}$	82

input
```
int(tan(d*x+c)^5/(a+a*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/5/d/a^2*(a*(1+sec(d*x+c)))^(1/2)*(5*arctan((-cos(d*x+c)/(cos(d*x+c)+1))^
(1/2))*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+1-3*sec(d*x+c)+sec(d*x+c)^2)
```

3.183.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 261, normalized size of antiderivative = 2.61

$$\int \frac{\tan^5(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = \left[\frac{5\sqrt{a}\cos(dx+c)^2\log\left(-8\,a\cos(dx+c)^2+4\left(2\cos(dx+c)^2+\cos(dx+c\right.\right.}{}$$

input
```
integrate(tan(d*x+c)^5/(a+a*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
[1/10*(5*sqrt(a)*cos(d*x + c)^2*log(-8*a*cos(d*x + c)^2 + 4*(2*cos(d*x + c
)^2 + cos(d*x + c))*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)) - 8*a*
cos(d*x + c) - a) + 4*(cos(d*x + c)^2 - 3*cos(d*x + c) + 1)*sqrt((a*cos(d*
x + c) + a)/cos(d*x + c)))/(a^2*d*cos(d*x + c)^2), 1/5*(5*sqrt(-a)*arctan(
2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(2*a*cos(d
*x + c) + a))*cos(d*x + c)^2 + 2*(cos(d*x + c)^2 - 3*cos(d*x + c) + 1)*sqr
t((a*cos(d*x + c) + a)/cos(d*x + c)))/(a^2*d*cos(d*x + c)^2)]
```

3.183.6 Sympy [F]

$$\int \frac{\tan^5(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = \int \frac{\tan^5(c+dx)}{(a(\sec(c+dx)+1))^{\frac{3}{2}}}\,dx$$

input
```
integrate(tan(d*x+c)**5/(a+a*sec(d*x+c))**(3/2),x)
```

output
```
Integral(tan(c + d*x)**5/(a*(sec(c + d*x) + 1))**(3/2), x)
```

3.183.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.10

$$\int \frac{\tan^5(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = \frac{\frac{5\log\left(\frac{\sqrt{a+\frac{a}{\cos(dx+c)}}-\sqrt{a}}{\sqrt{a+\frac{a}{\cos(dx+c)}}+\sqrt{a}}\right)}{a^{\frac{3}{2}}}+\frac{2\left(a+\frac{a}{\cos(dx+c)}\right)^{\frac{5}{2}}}{a^4}-\frac{10\left(a+\frac{a}{\cos(dx+c)}\right)^{\frac{3}{2}}}{a^3}+\frac{10\sqrt{a+\frac{a}{\cos(dx+c)}}}{a^2}}{5\,d}$$

input
```
integrate(tan(d*x+c)^5/(a+a*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
1/5*(5*log((sqrt(a + a/cos(d*x + c)) - sqrt(a))/(sqrt(a + a/cos(d*x + c))
+ sqrt(a)))/a^(3/2) + 2*(a + a/cos(d*x + c))^(5/2)/a^4 - 10*(a + a/cos(d*x
 + c))^(3/2)/a^3 + 10*sqrt(a + a/cos(d*x + c))/a^2)/d
```

3.183.8 Giac [A] (verification not implemented)

Time = 2.29 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.54

$$\int \frac{\tan^5(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = \frac{2\left(\frac{5\arctan\left(\frac{\sqrt{2}\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}}{2\sqrt{-a}}\right)}{\sqrt{-a}a\operatorname{sgn}(\cos(dx+c))} + \frac{\sqrt{2}\left(5\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-a\right)^2+10\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-\right.\right.}{\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-a\right)^2\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}a\operatorname{sgn}(}\right.}{5\,d}$$

input
```
integrate(tan(d*x+c)^5/(a+a*sec(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
2/5*(5*arctan(1/2*sqrt(2)*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)/sqrt(-a))/(s
qrt(-a)*a*sgn(cos(d*x + c))) + sqrt(2)*(5*(a*tan(1/2*d*x + 1/2*c)^2 - a)^2
 + 10*(a*tan(1/2*d*x + 1/2*c)^2 - a)*a + 4*a^2)/((a*tan(1/2*d*x + 1/2*c)^2
 - a)^2*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)*a*sgn(cos(d*x + c))))/d
```

3.183.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\tan^5(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = \int \frac{\tan(c+d\,x)^5}{\left(a+\frac{a}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

input
```
int(tan(c + d*x)^5/(a + a/cos(c + d*x))^(3/2),x)
```

output
```
int(tan(c + d*x)^5/(a + a/cos(c + d*x))^(3/2), x)
```

3.184 $\int \frac{\tan^3(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx$

3.184.1 Optimal result 1364
3.184.2 Mathematica [A] (verified) 1364
3.184.3 Rubi [A] (verified) 1365
3.184.4 Maple [A] (verified) 1367
3.184.5 Fricas [A] (verification not implemented) 1367
3.184.6 Sympy [F] 1368
3.184.7 Maxima [A] (verification not implemented) 1368
3.184.8 Giac [A] (verification not implemented) 1368
3.184.9 Mupad [F(-1)] 1369

3.184.1 Optimal result

Integrand size = 23, antiderivative size = 54

$$\int \frac{\tan^3(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = \frac{2\operatorname{arctanh}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{a}}\right)}{a^{3/2}d} + \frac{2\sqrt{a+a\sec(c+dx)}}{a^2 d}$$

output
```
2*arctanh((a+a*sec(d*x+c))^(1/2)/a^(1/2))/a^(3/2)/d+2*(a+a*sec(d*x+c))^(1/
2)/a^2/d
```

3.184.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.04

$$\int \frac{\tan^3(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = \frac{2\left(1+\sec(c+dx)+\operatorname{arctanh}\left(\sqrt{1+\sec(c+dx)}\right)\sqrt{1+\sec(c+dx)}\right)}{ad\sqrt{a(1+\sec(c+dx))}}$$

input
```
Integrate[Tan[c + d*x]^3/(a + a*Sec[c + d*x])^(3/2),x]
```

output
```
(2*(1 + Sec[c + d*x] + ArcTanh[Sqrt[1 + Sec[c + d*x]]]*Sqrt[1 + Sec[c + d*
x]]))/(a*d*Sqrt[a*(1 + Sec[c + d*x])])
```

3.184.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.04, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {3042, 25, 4368, 25, 27, 90, 73, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^3(c+dx)}{(a\sec(c+dx)+a)^{3/2}}\,dx$$

$$\downarrow 3042$$

$$\int -\frac{\cot\left(c+dx+\frac{\pi}{2}\right)^3}{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^{3/2}}dx$$

$$\downarrow 25$$

$$-\int \frac{\cot\left(\frac{1}{2}(2c+\pi)+dx\right)^3}{\left(\csc\left(\frac{1}{2}(2c+\pi)+dx\right)a+a\right)^{3/2}}dx$$

$$\downarrow 4368$$

$$\frac{\int -\frac{a\cos(c+dx)(1-\sec(c+dx))}{\sqrt{\sec(c+dx)a+a}}d\sec(c+dx)}{a^2d}$$

$$\downarrow 25$$

$$-\frac{\int \frac{a\cos(c+dx)(1-\sec(c+dx))}{\sqrt{\sec(c+dx)a+a}}d\sec(c+dx)}{a^2d}$$

$$\downarrow 27$$

$$-\frac{\int \frac{\cos(c+dx)(1-\sec(c+dx))}{\sqrt{\sec(c+dx)a+a}}d\sec(c+dx)}{ad}$$

$$\downarrow 90$$

$$-\frac{\int \frac{\cos(c+dx)}{\sqrt{\sec(c+dx)a+a}}d\sec(c+dx)-\frac{2\sqrt{a\sec(c+dx)+a}}{a}}{ad}$$

$$\downarrow 73$$

$$-\frac{\frac{2\int \frac{1}{\frac{\sec(c+dx)a+a}{a}-1}d\sqrt{\sec(c+dx)a+a}}{a}-\frac{2\sqrt{a\sec(c+dx)+a}}{a}}{ad}$$

$$\downarrow 220$$

$$\frac{-\frac{2\text{arctanh}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{a}}\right)}{\sqrt{a}}-\frac{2\sqrt{a\sec(c+dx)+a}}{a}}{ad}$$

input
```
Int[Tan[c + d*x]^3/(a + a*Sec[c + d*x])^(3/2),x]
```

output
```
-(((-2*ArcTanh[Sqrt[a + a*Sec[c + d*x]]/Sqrt[a]])/Sqrt[a] - (2*Sqrt[a + a*
Sec[c + d*x]])/a)/(a*d))
```

3.184.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4368
```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Simp[-(d*b^(m - 1))^(-1)   Subst[Int[(-a + b*x)^((m - 1)/2
)*((a + b*x)^((m - 1)/2 + n)/x), x], x, Csc[c + d*x]], x] /; FreeQ[{a, b, c
, d, n}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[n]
```

3.184.4 Maple [A] (verified)

Time = 3.29 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.20

method	result	size
default	$-\frac{2\sqrt{a(1+\sec(dx+c))}\left(\arctan\left(\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}\right)\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}-1\right)}{d\,a^2}$	65

input
```
int(tan(d*x+c)^3/(a+a*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/d/a^2*(a*(1+sec(d*x+c)))^(1/2)*(arctan((-cos(d*x+c)/(cos(d*x+c)+1))^(1/
2))*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)-1)
```

3.184.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 191, normalized size of antiderivative = 3.54

$$\int \frac{\tan^3(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = \left[\frac{\sqrt{a}\log\left(-8\,a\cos(dx+c)^2-4\left(2\cos(dx+c)^2+\cos(dx+c)\right)\sqrt{a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}+8\,a\cos(dx+c)+3\,a\right)}{2\,a^2 d}\right.$$
$$\left.-\frac{\sqrt{-a}\arctan\left(\frac{2\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)}{2\,a\cos(dx+c)+a}\right)-2\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}}{a^2 d}\right]$$

input
```
integrate(tan(d*x+c)^3/(a+a*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
[1/2*(sqrt(a)*log(-8*a*cos(d*x + c)^2 - 4*(2*cos(d*x + c)^2 + cos(d*x + c)
)*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)) - 8*a*cos(d*x + c) - a)
+ 4*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)))/(a^2*d), -(sqrt(-a)*arctan(2*
sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(2*a*cos(d*x
 + c) + a)) - 2*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)))/(a^2*d)]
```

3.184.6 Sympy [F]

$$\int \frac{\tan^3(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = \int \frac{\tan^3(c+dx)}{(a(\sec(c+dx)+1))^{\frac{3}{2}}}\,dx$$

input
```
integrate(tan(d*x+c)**3/(a+a*sec(d*x+c))**(3/2),x)
```

output
```
Integral(tan(c + d*x)**3/(a*(sec(c + d*x) + 1))**(3/2), x)
```

3.184.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.31

$$\int \frac{\tan^3(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = -\frac{\frac{\log\left(\frac{\sqrt{a+\frac{a}{\cos(dx+c)}}-\sqrt{a}}{\sqrt{a+\frac{a}{\cos(dx+c)}}+\sqrt{a}}\right)}{a^{\frac{3}{2}}} - \frac{2\sqrt{a+\frac{a}{\cos(dx+c)}}}{a^2}}{d}$$

input
```
integrate(tan(d*x+c)^3/(a+a*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
-(log((sqrt(a + a/cos(d*x + c)) - sqrt(a))/(sqrt(a + a/cos(d*x + c)) + sqr
t(a)))/a^(3/2) - 2*sqrt(a + a/cos(d*x + c))/a^2)/d
```

3.184.8 Giac [A] (verification not implemented)

Time = 1.25 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.57

$$\int \frac{\tan^3(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = -\frac{2\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}}{2\sqrt{-a}}\right)}{\sqrt{-a}\operatorname{sgn}(\cos(dx+c))} - \frac{\sqrt{2}}{\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\operatorname{sgn}(\cos(dx+c))}\right)}{ad}$$

input
```
integrate(tan(d*x+c)^3/(a+a*sec(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
-2*(arctan(1/2*sqrt(2)*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)/sqrt(-a))/(sqrt
(-a)*sgn(cos(d*x + c))) - sqrt(2)/(sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)*sgn
(cos(d*x + c))))/(a*d)
```

3.184.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\tan^3(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = \int \frac{\tan(c+d\,x)^3}{\left(a+\frac{a}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

input
```
int(tan(c + d*x)^3/(a + a/cos(c + d*x))^(3/2),x)
```

output
```
int(tan(c + d*x)^3/(a + a/cos(c + d*x))^(3/2), x)
```

3.185 $\int \frac{\tan(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx$

3.185.1 Optimal result 1370
3.185.2 Mathematica [C] (verified) 1370
3.185.3 Rubi [A] (verified) 1371
3.185.4 Maple [A] (verified) 1373
3.185.5 Fricas [B] (verification not implemented) 1373
3.185.6 Sympy [F] 1374
3.185.7 Maxima [A] (verification not implemented) 1374
3.185.8 Giac [A] (verification not implemented) 1374
3.185.9 Mupad [B] (verification not implemented) 1375

3.185.1 Optimal result

Integrand size = 21, antiderivative size = 54

$$\int \frac{\tan(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = -\frac{2\operatorname{arctanh}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{a}}\right)}{a^{3/2}d} + \frac{2}{ad\sqrt{a+a\sec(c+dx)}}$$

output
```
-2*arctanh((a+a*sec(d*x+c))^(1/2)/a^(1/2))/a^(3/2)/d+2/a/d/(a+a*sec(d*x+c)
)^(1/2)
```

3.185.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.03 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.70

$$\int \frac{\tan(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = \frac{2\,\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},1+\sec(c+dx)\right)}{ad\sqrt{a(1+\sec(c+dx))}}$$

input
```
Integrate[Tan[c + d*x]/(a + a*Sec[c + d*x])^(3/2),x]
```

output
```
(2*Hypergeometric2F1[-1/2, 1, 1/2, 1 + Sec[c + d*x]])/(a*d*Sqrt[a*(1 + Sec
[c + d*x])])
```

3.185.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.96, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 25, 4368, 61, 73, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan(c+dx)}{(a\sec(c+dx)+a)^{3/2}}\,dx$$

$$\downarrow 3042$$

$$\int -\frac{\cot\left(c+dx+\frac{\pi}{2}\right)}{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^{3/2}}dx$$

$$\downarrow 25$$

$$-\int \frac{\cot\left(\frac{1}{2}(2c+\pi)+dx\right)}{\left(\csc\left(\frac{1}{2}(2c+\pi)+dx\right)a+a\right)^{3/2}}dx$$

$$\downarrow 4368$$

$$\frac{\int \frac{\cos(c+dx)}{(\sec(c+dx)a+a)^{3/2}}d\sec(c+dx)}{d}$$

$$\downarrow 61$$

$$\frac{\frac{\int \frac{\cos(c+dx)}{\sqrt{\sec(c+dx)a+a}}d\sec(c+dx)}{a}+\frac{2}{a\sqrt{a\sec(c+dx)+a}}}{d}$$

$$\downarrow 73$$

$$\frac{\frac{2\int \frac{1}{\frac{\sec(c+dx)a+a}{a}-1}d\sqrt{\sec(c+dx)a+a}}{a^2}+\frac{2}{a\sqrt{a\sec(c+dx)+a}}}{d}$$

$$\downarrow 220$$

$$\frac{\frac{2}{a\sqrt{a\sec(c+dx)+a}}-\frac{2\operatorname{arctanh}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{a}}\right)}{a^{3/2}}}{d}$$

input
```
Int[Tan[c + d*x]/(a + a*Sec[c + d*x])^(3/2),x]
```

output
```
((-2*ArcTanh[Sqrt[a + a*Sec[c + d*x]]/Sqrt[a]])/a^(3/2) + 2/(a*Sqrt[a + a*
Sec[c + d*x]]))/d
```

3.185.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4368
```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Simp[-(d*b^(m - 1))^(-1)   Subst[Int[(-a + b*x)^((m - 1)/2
)*((a + b*x)^((m - 1)/2 + n)/x), x], x, Csc[c + d*x]], x] /; FreeQ[{a, b, c
, d, n}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[n]
```

3.185.4 Maple [A] (verified)

Time = 0.87 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.83

method	result	size
derivativedivides	$\frac{-\frac{2\operatorname{arctanh}\left(\frac{\sqrt{a+a\sec(dx+c)}}{\sqrt{a}}\right)}{a^{\frac{3}{2}}}+\frac{2}{a\sqrt{a+a\sec(dx+c)}}}{d}$	45
default	$\frac{-\frac{2\operatorname{arctanh}\left(\frac{\sqrt{a+a\sec(dx+c)}}{\sqrt{a}}\right)}{a^{\frac{3}{2}}}+\frac{2}{a\sqrt{a+a\sec(dx+c)}}}{d}$	45

input
```
int(tan(d*x+c)/(a+a*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/d*(-2/a^(3/2)*arctanh((a+a*sec(d*x+c))^(1/2)/a^(1/2))+2/a/(a+a*sec(d*x+c
))^(1/2))
```

3.185.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 112 vs. $2(46) = 92$.

Time = 0.31 (sec) , antiderivative size = 244, normalized size of antiderivative = 4.52

$$\int \frac{\tan(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = \left[\frac{\sqrt{a}(\cos(dx+c)+1)\log\left(-8\,a\cos(dx+c)^2+4\left(2\cos(dx+c)^2+\cos(dx\right.\right.}{2\left(a^2d\cos(d\right.}$$

input
```
integrate(tan(d*x+c)/(a+a*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
[1/2*(sqrt(a)*(cos(d*x + c) + 1)*log(-8*a*cos(d*x + c)^2 + 4*(2*cos(d*x +
c)^2 + cos(d*x + c))*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)) - 8*a
*cos(d*x + c) - a) + 4*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c
))/(a^2*d*cos(d*x + c) + a^2*d), (sqrt(-a)*(cos(d*x + c) + 1)*arctan(2*sqr
t(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(2*a*cos(d*x +
c) + a)) + 2*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c))/(a^2*d*
cos(d*x + c) + a^2*d)]
```

3.185.6 Sympy [F]

$$\int \frac{\tan(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = \int \frac{\tan\left(c+dx\right)}{\left(a\left(\sec\left(c+dx\right)+1\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(tan(d*x+c)/(a+a*sec(d*x+c))**(3/2),x)
```

output
```
Integral(tan(c + d*x)/(a*(sec(c + d*x) + 1))**(3/2), x)
```

3.185.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.30

$$\int \frac{\tan(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = \frac{\frac{\log\left(\frac{\sqrt{a+\frac{a}{\cos(dx+c)}}-\sqrt{a}}{\sqrt{a+\frac{a}{\cos(dx+c)}}+\sqrt{a}}\right)}{a^{\frac{3}{2}}} + \frac{2}{\sqrt{a+\frac{a}{\cos(dx+c)}}a}}{d}$$

input
```
integrate(tan(d*x+c)/(a+a*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
(log((sqrt(a + a/cos(d*x + c)) - sqrt(a))/(sqrt(a + a/cos(d*x + c)) + sqrt
(a)))/a^(3/2) + 2/(sqrt(a + a/cos(d*x + c))*a))/d
```

3.185.8 Giac [A] (verification not implemented)

Time = 0.93 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.61

$$\int \frac{\tan(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = \frac{\frac{2\arctan\left(\frac{\sqrt{2}\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a}}{2\sqrt{-a}}\right)}{\sqrt{-a}a\mathrm{sgn}(\cos(dx+c))} + \frac{\sqrt{2}\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a}}{a^2\mathrm{sgn}(\cos(dx+c))}}{d}$$

input
```
integrate(tan(d*x+c)/(a+a*sec(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
(2*arctan(1/2*sqrt(2)*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)/sqrt(-a))/(sqrt(
-a)*a*sgn(cos(d*x + c))) + sqrt(2)*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)/(a^
2*sgn(cos(d*x + c))))/d
```

3.185.9 Mupad [B] (verification not implemented)

Time = 14.47 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.93

$$\int \frac{\tan(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = \frac{2}{a\,d\,\sqrt{a+\frac{a}{\cos(c+d\,x)}}} - \frac{2\,\mathrm{atanh}\left(\frac{\sqrt{a+\frac{a}{\cos(c+d\,x)}}}{\sqrt{a}}\right)}{a^{3/2}\,d}$$

input
```
int(tan(c + d*x)/(a + a/cos(c + d*x))^(3/2),x)
```

output
```
2/(a*d*(a + a/cos(c + d*x))^(1/2)) - (2*atanh((a + a/cos(c + d*x))^(1/2)/a
^(1/2)))/(a^(3/2)*d)
```

3.186 $\int \frac{\cot(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx$

3.186.1 Optimal result 1376
3.186.2 Mathematica [C] (verified) 1376
3.186.3 Rubi [A] (verified) 1377
3.186.4 Maple [B] (verified) 1380
3.186.5 Fricas [B] (verification not implemented) 1381
3.186.6 Sympy [F] 1382
3.186.7 Maxima [F] 1382
3.186.8 Giac [A] (verification not implemented) 1382
3.186.9 Mupad [F(-1)] 1383

3.186.1 Optimal result

Integrand size = 21, antiderivative size = 120

$$\int \frac{\cot(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = \frac{2\operatorname{arctanh}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{a}}\right)}{a^{3/2}d} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{2\sqrt{2}a^{3/2}d} - \frac{1}{3d(a+a\sec(c+dx))^{3/2}} - \frac{3}{2ad\sqrt{a+a\sec(c+dx)}}$$

output
```
2*arctanh((a+a*sec(d*x+c))^(1/2)/a^(1/2))/a^(3/2)/d-1/3/d/(a+a*sec(d*x+c))
^(3/2)-1/4*arctanh(1/2*(a+a*sec(d*x+c))^(1/2)*2^(1/2)/a^(1/2))/a^(3/2)/d*2
^(1/2)-3/2/a/d/(a+a*sec(d*x+c))^(1/2)
```

3.186.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.05 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.50

$$\int \frac{\cot(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = \frac{\text{Hypergeometric2F1}\left(-\frac{3}{2},1,-\frac{1}{2},\frac{1}{2}(1+\sec(c+dx))\right) - 2\,\text{Hypergeometric2F1}}{3d(a(1+\sec(c+dx)))^{3/2}}$$

input
```
Integrate[Cot[c + d*x]/(a + a*Sec[c + d*x])^(3/2),x]
```

output
```
(Hypergeometric2F1[-3/2, 1, -1/2, (1 + Sec[c + d*x])/2] - 2*Hypergeometric
2F1[-3/2, 1, -1/2, 1 + Sec[c + d*x]])/(3*d*(a*(1 + Sec[c + d*x]))^(3/2))
```

3.186.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.07, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {3042, 25, 4368, 25, 27, 96, 25, 27, 169, 27, 174, 73, 219, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot(c+dx)}{(a\sec(c+dx)+a)^{3/2}}\,dx$$

$$\downarrow 3042$$

$$\int -\frac{1}{\cot\left(c+dx+\frac{\pi}{2}\right)\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^{3/2}}dx$$

$$\downarrow 25$$

$$-\int \frac{1}{\cot\left(\frac{1}{2}(2c+\pi)+dx\right)\left(\csc\left(\frac{1}{2}(2c+\pi)+dx\right)a+a\right)^{3/2}}dx$$

$$\downarrow 4368$$

$$\frac{a^2\int -\frac{\cos(c+dx)}{a(1-\sec(c+dx))(\sec(c+dx)a+a)^{5/2}}d\sec(c+dx)}{d}$$

$$\downarrow 25$$

$$-\frac{a^2\int \frac{\cos(c+dx)}{a(1-\sec(c+dx))(\sec(c+dx)a+a)^{5/2}}d\sec(c+dx)}{d}$$

$$\downarrow 27$$

$$-\frac{a\int \frac{\cos(c+dx)}{(1-\sec(c+dx))(\sec(c+dx)a+a)^{5/2}}d\sec(c+dx)}{d}$$

$$\downarrow 96$$

$$-\frac{a\left(\frac{1}{3a(a\sec(c+dx)+a)^{3/2}}-\frac{\int -\frac{a\cos(c+dx)(2-\sec(c+dx))}{(1-\sec(c+dx))(\sec(c+dx)a+a)^{3/2}}d\sec(c+dx)}{2a^2}\right)}{d}$$

$$\downarrow 25$$

$$-\frac{a\left(\frac{\int \frac{a\cos(c+dx)(2-\sec(c+dx))}{(1-\sec(c+dx))(\sec(c+dx)a+a)^{3/2}}d\sec(c+dx)}{2a^2}+\frac{1}{3a(a\sec(c+dx)+a)^{3/2}}\right)}{d}$$

$$\downarrow 27$$

$$-\frac{a\left(\frac{\int \frac{\cos(c+dx)(2-\sec(c+dx))}{(1-\sec(c+dx))(\sec(c+dx)a+a)^{3/2}}\,d\sec(c+dx)}{2a}+\frac{1}{3a(a\sec(c+dx)+a)^{3/2}}\right)}{d}$$

$$\downarrow 169$$

$$-\frac{a\left(\frac{\int \frac{a\cos(c+dx)(4-3\sec(c+dx))}{2(1-\sec(c+dx))\sqrt{\sec(c+dx)a+a}}\,d\sec(c+dx)}{a^2}+\frac{3}{a\sqrt{a\sec(c+dx)+a}}}{2a}+\frac{1}{3a(a\sec(c+dx)+a)^{3/2}}\right)}{d}$$

$$\downarrow 27$$

$$-\frac{a\left(\frac{\int \frac{\cos(c+dx)(4-3\sec(c+dx))}{(1-\sec(c+dx))\sqrt{\sec(c+dx)a+a}}\,d\sec(c+dx)}{2a}+\frac{3}{a\sqrt{a\sec(c+dx)+a}}}{2a}+\frac{1}{3a(a\sec(c+dx)+a)^{3/2}}\right)}{d}$$

$$\downarrow 174$$

$$-\frac{a\left(\frac{\int \frac{1}{(1-\sec(c+dx))\sqrt{\sec(c+dx)a+a}}\,d\sec(c+dx)+4\int \frac{\cos(c+dx)}{\sqrt{\sec(c+dx)a+a}}\,d\sec(c+dx)}{2a}+\frac{3}{a\sqrt{a\sec(c+dx)+a}}}{2a}+\frac{1}{3a(a\sec(c+dx)+a)^{3/2}}\right)}{d}$$

$$\downarrow 73$$

$$-\frac{a\left(\frac{\frac{2\int \frac{1}{2-\frac{\sec(c+dx)a+a}{a}}\,d\sqrt{\sec(c+dx)a+a}}{a}+\frac{8\int \frac{1}{\frac{\sec(c+dx)a+a}{a}-1}\,d\sqrt{\sec(c+dx)a+a}}{a}}{2a}+\frac{3}{a\sqrt{a\sec(c+dx)+a}}}{2a}+\frac{1}{3a(a\sec(c+dx)+a)^{3/2}}\right)}{d}$$

$$\downarrow 219$$

$$-\frac{a\left(\frac{\frac{8\int \frac{1}{\frac{\sec(c+dx)a+a}{a}-1}\,d\sqrt{\sec(c+dx)a+a}}{a}+\frac{\sqrt{2}\operatorname{arctanh}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{2}\sqrt{a}}\right)}{\sqrt{a}}}{2a}+\frac{3}{a\sqrt{a\sec(c+dx)+a}}}{2a}+\frac{1}{3a(a\sec(c+dx)+a)^{3/2}}\right)}{d}$$

$$\downarrow 220$$

$$-\frac{a\left(\frac{\frac{\sqrt{2}\operatorname{arctanh}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{2}\sqrt{a}}\right)}{\sqrt{a}}-\frac{8\operatorname{arctanh}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{a}}\right)}{\sqrt{a}}}{2a}+\frac{3}{a\sqrt{a\sec(c+dx)+a}}}{2a}+\frac{1}{3a(a\sec(c+dx)+a)^{3/2}}\right)}{d}$$

input
```
Int[Cot[c + d*x]/(a + a*Sec[c + d*x])^(3/2),x]
```

output
```
-((a*(1/(3*a*(a + a*Sec[c + d*x])^(3/2)) + (((-8*ArcTanh[Sqrt[a + a*Sec[c
+ d*x]]/Sqrt[a]])/Sqrt[a] + (Sqrt[2]*ArcTanh[Sqrt[a + a*Sec[c + d*x]]/(Sqr
t[2]*Sqrt[a])])/Sqrt[a])/(2*a) + 3/(a*Sqrt[a + a*Sec[c + d*x]]))/(2*a)))/d
)
```

3.186.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 96
```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_] :> Simp[f*((e + f*x)^(p + 1)/((p + 1)*(b*e - a*f)*(d*e - c*f))), x] + S
imp[1/((b*e - a*f)*(d*e - c*f))   Int[(b*d*e - b*c*f - a*d*f - b*d*f*x)*((e
 + f*x)^(p + 1)/((a + b*x)*(c + d*x))), x], x] /; FreeQ[{a, b, c, d, e, f},
 x] && LtQ[p, -1]
```

rule 169
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && IntegersQ[
2*m, 2*n, 2*p]
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4368
```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Simp[-(d*b^(m - 1))^(-1)   Subst[Int[(-a + b*x)^((m - 1)/2
)*((a + b*x)^((m - 1)/2 + n)/x), x], x, Csc[c + d*x]], x] /; FreeQ[{a, b, c
, d, n}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[n]
```

3.186.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 303 vs. $2(95) = 190$.

Time = 1.58 (sec) , antiderivative size = 304, normalized size of antiderivative = 2.53

method	result
default	$-\frac{\sqrt{-\frac{2a}{(1-\cos(dx+c))^2\csc(dx+c)^2-1}}\sqrt{(1-\cos(dx+c))^2\csc(dx+c)^2-1}\left(3\left((1-\cos(dx+c))^2\csc(dx+c)^2-1\right)^{\frac{5}{2}}-3(1-\cos(dx+c))^4\sqrt{(1$

input
```
int(cot(d*x+c)/(a+a*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/60/d/a^2*(-2*a/((1-cos(d*x+c))^2*csc(d*x+c)^2-1))^(1/2)*((1-cos(d*x+c))
^2*csc(d*x+c)^2-1)^(1/2)*(3*((1-cos(d*x+c))^2*csc(d*x+c)^2-1)^(5/2)-3*(1-c
os(d*x+c))^4*((1-cos(d*x+c))^2*csc(d*x+c)^2-1)^(1/2)*csc(d*x+c)^4-5*((1-co
s(d*x+c))^2*csc(d*x+c)^2-1)^(3/2)+16*(1-cos(d*x+c))^2*((1-cos(d*x+c))^2*cs
c(d*x+c)^2-1)^(1/2)*csc(d*x+c)^2+60*2^(1/2)*arctan(1/2*2^(1/2)*((1-cos(d*x
+c))^2*csc(d*x+c)^2-1)^(1/2))+15*arctan(1/((1-cos(d*x+c))^2*csc(d*x+c)^2-1
)^(1/2))-58*((1-cos(d*x+c))^2*csc(d*x+c)^2-1)^(1/2))
```

3.186.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 228 vs. $2(95) = 190$.

Time $= 0.36$ (sec) , antiderivative size $= 485$, normalized size of antiderivative $= 4.04$

$$\int \frac{\cot(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = \left[\frac{3\sqrt{2}\left(\cos(dx+c)^2+2\cos(dx+c)+1\right)\sqrt{a}\log\left(-\frac{2\sqrt{2}\sqrt{a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+\right.}{\cos(dx+c)-}\right.$$

input
```
integrate(cot(d*x+c)/(a+a*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
[1/24*(3*sqrt(2)*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(a)*log(-(2*sqr
t(2)*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c) - 3*a*co
s(d*x + c) - a)/(cos(d*x + c) - 1)) + 12*(cos(d*x + c)^2 + 2*cos(d*x + c)
+ 1)*sqrt(a)*log(-8*a*cos(d*x + c)^2 - 4*(2*cos(d*x + c)^2 + cos(d*x + c))
*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)) - 8*a*cos(d*x + c) - a) -
 4*(11*cos(d*x + c)^2 + 9*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x
+ c)))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d), 1/12*(3*sqrt
(2)*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(-a)*arctan(sqrt(2)*sqrt(-a)
*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(a*cos(d*x + c) + a)
) - 12*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(-a)*arctan(2*sqrt(-a)*sq
rt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(2*a*cos(d*x + c) + a))
 - 2*(11*cos(d*x + c)^2 + 9*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*
x + c)))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)]
```

3.186.6 Sympy [F]

$$\int \frac{\cot(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = \int \frac{\cot(c+dx)}{(a(\sec(c+dx)+1))^{\frac{3}{2}}}\,dx$$

input
```
integrate(cot(d*x+c)/(a+a*sec(d*x+c))**(3/2),x)
```

output
```
Integral(cot(c + d*x)/(a*(sec(c + d*x) + 1))**(3/2), x)
```

3.186.7 Maxima [F]

$$\int \frac{\cot(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = \int \frac{\cot(dx+c)}{(a\sec(dx+c)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(cot(d*x+c)/(a+a*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate(cot(d*x + c)/(a*sec(d*x + c) + a)^(3/2), x)
```

3.186.8 Giac [A] (verification not implemented)

Time = 0.92 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.37

$$\int \frac{\cot(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = \frac{\frac{3\sqrt{2}\arctan\left(\frac{\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a\operatorname{sgn}(\cos(dx+c))} - \frac{24\arctan\left(\frac{\sqrt{2}\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}}{2\sqrt{-a}}\right)}{\sqrt{-a}a\operatorname{sgn}(\cos(dx+c))} - \frac{\sqrt{2}\left(\left(-a\tan\left(\frac{1}{2}\right.\right.\right.}{12\,d}$$

input
```
integrate(cot(d*x+c)/(a+a*sec(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
1/12*(3*sqrt(2)*arctan(sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)/sqrt(-a))/(sqrt
(-a)*a*sgn(cos(d*x + c))) - 24*arctan(1/2*sqrt(2)*sqrt(-a*tan(1/2*d*x + 1/
2*c)^2 + a)/sqrt(-a))/(sqrt(-a)*a*sgn(cos(d*x + c))) - sqrt(2)*((-a*tan(1/
2*d*x + 1/2*c)^2 + a)^(3/2)*a^6 + 9*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)*a^
7)/(a^9*sgn(cos(d*x + c))))/d
```

3.186.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cot(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = \int \frac{\cot(c+d\,x)}{\left(a+\frac{a}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

input
```
int(cot(c + d*x)/(a + a/cos(c + d*x))^(3/2),x)
```

output
```
int(cot(c + d*x)/(a + a/cos(c + d*x))^(3/2), x)
```

3.187 $\int \frac{\cot^3(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx$

3.187.1 Optimal result 1384
3.187.2 Mathematica [C] (verified) 1385
3.187.3 Rubi [A] (verified) 1385
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3.187.9 Mupad [F(-1)] 1392

3.187.1 Optimal result

Integrand size = 23, antiderivative size = 176

$$\int \frac{\cot^3(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = -\frac{2\operatorname{arctanh}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{a}}\right)}{a^{3/2}d} + \frac{11\operatorname{arctanh}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{16\sqrt{2}a^{3/2}d} - \frac{3a}{20d(a+a\sec(c+dx))^{5/2}} + \frac{a}{2d(1-\sec(c+dx))(a+a\sec(c+dx))^{5/2}} + \frac{5}{24d(a+a\sec(c+dx))^{3/2}} + \frac{21}{16ad\sqrt{a+a\sec(c+dx)}}$$

output

```
-2*arctanh((a+a*sec(d*x+c))^(1/2)/a^(1/2))/a^(3/2)/d-3/20*a/d/(a+a*sec(d*x
+c))^(5/2)+1/2*a/d/(1-sec(d*x+c))/(a+a*sec(d*x+c))^(5/2)+5/24/d/(a+a*sec(d
*x+c))^(3/2)+11/32*arctanh(1/2*(a+a*sec(d*x+c))^(1/2)*2^(1/2)/a^(1/2))/a^(
3/2)/d*2^(1/2)+21/16/a/d/(a+a*sec(d*x+c))^(1/2)
```

3.187.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.13 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.51

$$\int \frac{\cot^3(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = \frac{a\left(-10-11\,\text{Hypergeometric2F1}\left(-\frac{5}{2},1,-\frac{3}{2},\frac{1}{2}(1+\sec(c+dx))\right)\right)(-1+\sec(c}{20d(-1+\sec(c+dx}$$

input
```
Integrate[Cot[c + d*x]^3/(a + a*Sec[c + d*x])^(3/2),x]
```

output
```
(a*(-10 - 11*Hypergeometric2F1[-5/2, 1, -3/2, (1 + Sec[c + d*x])/2]*(-1 +
Sec[c + d*x]) + 8*Hypergeometric2F1[-5/2, 1, -3/2, 1 + Sec[c + d*x]]*(-1 +
 Sec[c + d*x])))/(20*d*(-1 + Sec[c + d*x])*(a*(1 + Sec[c + d*x]))^(5/2))
```

3.187.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.12, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.696$, Rules used = {3042, 25, 4368, 27, 114, 27, 169, 27, 169, 27, 169, 27, 174, 73, 219, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^3(c+dx)}{(a\sec(c+dx)+a)^{3/2}}\,dx$$
$$\downarrow 3042$$
$$\int -\frac{1}{\cot\left(c+dx+\frac{\pi}{2}\right)^3\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^{3/2}}dx$$
$$\downarrow 25$$
$$-\int \frac{1}{\cot\left(\frac{1}{2}(2c+\pi)+dx\right)^3\left(\csc\left(\frac{1}{2}(2c+\pi)+dx\right)a+a\right)^{3/2}}dx$$
$$\downarrow 4368$$
$$\frac{a^4\int \frac{\cos(c+dx)}{a^2(1-\sec(c+dx))^2(\sec(c+dx)a+a)^{7/2}}d\sec(c+dx)}{d}$$
$$\downarrow 27$$

$$\frac{a^2 \int \frac{\cos(c+dx)}{(1-\sec(c+dx))^2(\sec(c+dx)a+a)^{7/2}}\,d\sec(c+dx)}{d}$$

$$\downarrow 114$$

$$\frac{a^2\left(\frac{1}{2a(1-\sec(c+dx))(a\sec(c+dx)+a)^{5/2}} - \frac{\int -\frac{a\cos(c+dx)(7\sec(c+dx)+4)}{2(1-\sec(c+dx))(\sec(c+dx)a+a)^{7/2}}\,d\sec(c+dx)}{2a}\right)}{d}$$

$$\downarrow 27$$

$$\frac{a^2\left(\frac{1}{4}\int \frac{\cos(c+dx)(7\sec(c+dx)+4)}{(1-\sec(c+dx))(\sec(c+dx)a+a)^{7/2}}\,d\sec(c+dx) + \frac{1}{2a(1-\sec(c+dx))(a\sec(c+dx)+a)^{5/2}}\right)}{d}$$

$$\downarrow 169$$

$$\frac{a^2\left(\frac{1}{4}\left(\frac{\int \frac{5a\cos(c+dx)(3\sec(c+dx)+8)}{2(1-\sec(c+dx))(\sec(c+dx)a+a)^{5/2}}\,d\sec(c+dx)}{5a^2} - \frac{3}{5a(a\sec(c+dx)+a)^{5/2}}\right) + \frac{1}{2a(1-\sec(c+dx))(a\sec(c+dx)+a)^{5/2}}\right)}{d}$$

$$\downarrow 27$$

$$\frac{a^2\left(\frac{1}{4}\left(\frac{\int \frac{\cos(c+dx)(3\sec(c+dx)+8)}{(1-\sec(c+dx))(\sec(c+dx)a+a)^{5/2}}\,d\sec(c+dx)}{2a} - \frac{3}{5a(a\sec(c+dx)+a)^{5/2}}\right) + \frac{1}{2a(1-\sec(c+dx))(a\sec(c+dx)+a)^{5/2}}\right)}{d}$$

$$\downarrow 169$$

$$\frac{a^2\left(\frac{1}{4}\left(\frac{\int \frac{3a\cos(c+dx)(16-5\sec(c+dx))}{2(1-\sec(c+dx))(\sec(c+dx)a+a)^{3/2}}\,d\sec(c+dx)}{3a^2} + \frac{5}{3a(a\sec(c+dx)+a)^{3/2}}}{2a} - \frac{3}{5a(a\sec(c+dx)+a)^{5/2}}\right) + \frac{1}{2a(1-\sec(c+dx))(a\sec(c+dx)+a)}\right.}{d}$$

$$\downarrow 27$$

$$\frac{a^2\left(\frac{1}{4}\left(\frac{\int \frac{\cos(c+dx)(16-5\sec(c+dx))}{(1-\sec(c+dx))(\sec(c+dx)a+a)^{3/2}}\,d\sec(c+dx)}{2a} + \frac{5}{3a(a\sec(c+dx)+a)^{3/2}}}{2a} - \frac{3}{5a(a\sec(c+dx)+a)^{5/2}}\right) + \frac{1}{2a(1-\sec(c+dx))(a\sec(c+dx)+a)^5}\right.}{d}$$

$$\downarrow 169$$

$$a^2\left(\frac{1}{4}\left(\frac{\frac{\int \frac{a\cos(c+dx)(32-21\sec(c+dx))}{2(1-\sec(c+dx))\sqrt{\sec(c+dx)a+a}}d\sec(c+dx)}{a^2}}{2a}+\frac{\frac{21}{a\sqrt{a\sec(c+dx)+a}}}{2a}+\frac{5}{3a(a\sec(c+dx)+a)^{3/2}}\right)-\frac{3}{5a(a\sec(c+dx)+a)^{5/2}}\right)+\frac{}{2a(1-\sec(c+dx))(}$$
$$d$$

$$\downarrow 27$$

$$a^2\left(\frac{1}{4}\left(\frac{\frac{\int \frac{\cos(c+dx)(32-21\sec(c+dx))}{(1-\sec(c+dx))\sqrt{\sec(c+dx)a+a}}d\sec(c+dx)}{2a}}{2a}+\frac{\frac{21}{a\sqrt{a\sec(c+dx)+a}}}{2a}+\frac{5}{3a(a\sec(c+dx)+a)^{3/2}}\right)-\frac{3}{5a(a\sec(c+dx)+a)^{5/2}}\right)+\frac{1}{2a(1-\sec(c+dx))(a}$$
$$d$$

$$\downarrow 174$$

$$a^2\left(\frac{1}{4}\left(\frac{\frac{11\int \frac{1}{(1-\sec(c+dx))\sqrt{\sec(c+dx)a+a}}d\sec(c+dx)+32\int \frac{\cos(c+dx)}{\sqrt{\sec(c+dx)a+a}}d\sec(c+dx)}{2a}}{2a}+\frac{\frac{21}{a\sqrt{a\sec(c+dx)+a}}}{2a}+\frac{5}{3a(a\sec(c+dx)+a)^{3/2}}\right)-\frac{3}{5a(a\sec(c+dx)}\right.$$
$$d$$

$$\downarrow 73$$

$$a^2\left(\frac{1}{4}\left(\frac{\frac{22\int \frac{1}{2-\frac{\sec(c+dx)a+a}{a}}d\sqrt{\sec(c+dx)a+a}}{a}+\frac{64\int \frac{1}{\frac{\sec(c+dx)a+a}{a}-1}d\sqrt{\sec(c+dx)a+a}}{a}}{2a}+\frac{\frac{21}{a\sqrt{a\sec(c+dx)+a}}}{2a}+\frac{5}{3a(a\sec(c+dx)+a)^{3/2}}\right)-\frac{3}{5a(a\sec(c+dx}\right.$$
$$d$$

$$\downarrow 219$$

$$a^2\left(\frac{1}{4}\left(\frac{\frac{64\int \frac{1}{\frac{\sec(c+dx)a+a}{a}-1}d\sqrt{\sec(c+dx)a+a}}{a}+\frac{11\sqrt{2}\operatorname{arctanh}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{2}\sqrt{a}}\right)}{\sqrt{a}}}{2a}+\frac{\frac{21}{a\sqrt{a\sec(c+dx)+a}}}{2a}+\frac{5}{3a(a\sec(c+dx)+a)^{3/2}}\right)-\frac{3}{5a(a\sec(c+dx)+}\right.$$
$$d$$

$$\downarrow 220$$

3.187. $\int \frac{\cot^3(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx$

$$a^2\left(\frac{\frac{1}{4}\left(\frac{\frac{11\sqrt{2}\operatorname{arctanh}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{2}\sqrt{a}}\right)}{\sqrt{a}}-\frac{64\operatorname{arctanh}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{a}}\right)}{\sqrt{a}}}{2a}+\frac{21}{a\sqrt{a\sec(c+dx)+a}}}{2a}+\frac{5}{3a(a\sec(c+dx)+a)^{3/2}}\right)-\frac{3}{5a(a\sec(c+dx)+a)^{5/2}}}{d}$$

input
```
Int[Cot[c + d*x]^3/(a + a*Sec[c + d*x])^(3/2),x]
```

output
```
(a^2*(1/(2*a*(1 - Sec[c + d*x])*(a + a*Sec[c + d*x])^(5/2)) + (-3/(5*a*(a
+ a*Sec[c + d*x])^(5/2)) + (5/(3*a*(a + a*Sec[c + d*x])^(3/2)) + (((-64*Ar
cTanh[Sqrt[a + a*Sec[c + d*x]]/Sqrt[a]])/Sqrt[a] + (11*Sqrt[2]*ArcTanh[Sqr
t[a + a*Sec[c + d*x]]/(Sqrt[2]*Sqrt[a])])/Sqrt[a])/(2*a) + 21/(a*Sqrt[a +
a*Sec[c + d*x]]))/(2*a))/(2*a))/4))/d
```

3.187.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 114
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[1/((m + 1)*(b*c - a*d)*(b*e
 - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1)
 - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (IntegerQ[n] ||
 IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

rule 169
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && IntegersQ[
2*m, 2*n, 2*p]
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4368
```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Simp[-(d*b^(m - 1))^(-1)   Subst[Int[(-a + b*x)^((m - 1)/2
)*((a + b*x)^((m - 1)/2 + n)/x), x], x, Csc[c + d*x]], x] /; FreeQ[{a, b, c
, d, n}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[n]
```

3.187.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 399 vs. $2(143) = 286$.

Time = 1.92 (sec) , antiderivative size = 400, normalized size of antiderivative = 2.27

method	result
default	$\frac{\sqrt{a(1+\sec(dx+c))}\left(165\cos(dx+c)^2\arctan\left(\frac{\sqrt{2}}{2\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}}\right)\sqrt{2}\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}+330\arctan\left(\frac{\sqrt{2}}{2\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}}\right)\cos(dx+c)\sqrt{}\right.}{}$

input
```
int(cot(d*x+c)^3/(a+a*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/480/d/a^2*(a*(1+sec(d*x+c)))^(1/2)/(cos(d*x+c)+1)^2*(165*cos(d*x+c)^2*ar
ctan(1/2*2^(1/2)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*2^(1/2)*(-cos(d*x+c)/
(cos(d*x+c)+1))^(1/2)+330*arctan(1/2*2^(1/2)/(-cos(d*x+c)/(cos(d*x+c)+1))^
(1/2))*cos(d*x+c)*2^(1/2)*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+960*cos(d*x+c
)^2*arctan((-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*(-cos(d*x+c)/(cos(d*x+c)+1)
)^(1/2)+165*2^(1/2)*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctan(1/2*2^(1/2)/
(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))+1920*arctan((-cos(d*x+c)/(cos(d*x+c)+1
))^(1/2))*cos(d*x+c)*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+960*arctan((-cos(d
*x+c)/(cos(d*x+c)+1))^(1/2))*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)-898*cos(d*
x+c)^2*cot(d*x+c)^2-702*cos(d*x+c)*cot(d*x+c)^2+730*cot(d*x+c)^2+630*cot(d
*x+c)*csc(d*x+c))
```

3.187.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 283 vs. $2(141) = 282$.

Time = 0.38 (sec) , antiderivative size = 592, normalized size of antiderivative = 3.36

$$\int \frac{\cot^3(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = \left[\frac{165\sqrt{2}\left(\cos(dx+c)^4+2\cos(dx+c)^3-2\cos(dx+c)-1\right)\sqrt{a}\log\left(\frac{2\sqrt{2}\sqrt{}\right.}{}\right.$$

$$-\frac{165\sqrt{2}\left(\cos(dx+c)^4+2\cos(dx+c)^3-2\cos(dx+c)-1\right)\sqrt{-a}\arctan\left(\frac{\sqrt{2}\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)}{a\cos(dx+c)+a}\right)-}{}$$

input
```
integrate(cot(d*x+c)^3/(a+a*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
[1/960*(165*sqrt(2)*(cos(d*x + c)^4 + 2*cos(d*x + c)^3 - 2*cos(d*x + c) -
1)*sqrt(a)*log((2*sqrt(2)*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*
cos(d*x + c) + 3*a*cos(d*x + c) + a)/(cos(d*x + c) - 1)) + 480*(cos(d*x +
c)^4 + 2*cos(d*x + c)^3 - 2*cos(d*x + c) - 1)*sqrt(a)*log(-8*a*cos(d*x + c
)^2 + 4*(2*cos(d*x + c)^2 + cos(d*x + c))*sqrt(a)*sqrt((a*cos(d*x + c) + a
)/cos(d*x + c)) - 8*a*cos(d*x + c) - a) + 4*(449*cos(d*x + c)^4 + 351*cos(
d*x + c)^3 - 365*cos(d*x + c)^2 - 315*cos(d*x + c))*sqrt((a*cos(d*x + c) +
 a)/cos(d*x + c)))/(a^2*d*cos(d*x + c)^4 + 2*a^2*d*cos(d*x + c)^3 - 2*a^2*
d*cos(d*x + c) - a^2*d), -1/480*(165*sqrt(2)*(cos(d*x + c)^4 + 2*cos(d*x +
 c)^3 - 2*cos(d*x + c) - 1)*sqrt(-a)*arctan(sqrt(2)*sqrt(-a)*sqrt((a*cos(d
*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(a*cos(d*x + c) + a)) - 480*(cos(d
*x + c)^4 + 2*cos(d*x + c)^3 - 2*cos(d*x + c) - 1)*sqrt(-a)*arctan(2*sqrt(
-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(2*a*cos(d*x + c)
 + a)) - 2*(449*cos(d*x + c)^4 + 351*cos(d*x + c)^3 - 365*cos(d*x + c)^2 -
 315*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)))/(a^2*d*cos(d*x
 + c)^4 + 2*a^2*d*cos(d*x + c)^3 - 2*a^2*d*cos(d*x + c) - a^2*d)]
```

3.187.6 Sympy [F]

$$\int \frac{\cot^3(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = \int \frac{\cot^3(c+dx)}{(a(\sec(c+dx)+1))^{\frac{3}{2}}}\,dx$$

input
```
integrate(cot(d*x+c)**3/(a+a*sec(d*x+c))**(3/2),x)
```

output
```
Integral(cot(c + d*x)**3/(a*(sec(c + d*x) + 1))**(3/2), x)
```

3.187.7 Maxima [F]

$$\int \frac{\cot^3(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = \int \frac{\cot(dx+c)^3}{(a\sec(dx+c)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(cot(d*x+c)^3/(a+a*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate(cot(d*x + c)^3/(a*sec(d*x + c) + a)^(3/2), x)
```

3.187.8 Giac [A] (verification not implemented)

Time = 1.09 (sec) , antiderivative size = 253, normalized size of antiderivative = 1.44

$$\int \frac{\cot^3(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx =$$

$$\frac{165\sqrt{2}\arctan\left(\frac{\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a\operatorname{sgn}(\cos(dx+c))} - \frac{960\arctan\left(\frac{\sqrt{2}\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}}{2\sqrt{-a}}\right)}{\sqrt{-a}a\operatorname{sgn}(\cos(dx+c))} + \frac{15\sqrt{2}\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}}{a^2\operatorname{sgn}(\cos(dx+c))\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2} - \frac{2\sqrt{2}\left(3\left(a\tan$$

$$-\qquad\qquad 480\,d$$

input
```
integrate(cot(d*x+c)^3/(a+a*sec(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
-1/480*(165*sqrt(2)*arctan(sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)/sqrt(-a))/(
sqrt(-a)*a*sgn(cos(d*x + c))) - 960*arctan(1/2*sqrt(2)*sqrt(-a*tan(1/2*d*x
 + 1/2*c)^2 + a)/sqrt(-a))/(sqrt(-a)*a*sgn(cos(d*x + c))) + 15*sqrt(2)*sqr
t(-a*tan(1/2*d*x + 1/2*c)^2 + a)/(a^2*sgn(cos(d*x + c))*tan(1/2*d*x + 1/2*
c)^2) - 2*sqrt(2)*(3*(a*tan(1/2*d*x + 1/2*c)^2 - a)^2*sqrt(-a*tan(1/2*d*x
+ 1/2*c)^2 + a)*a^16 + 20*(-a*tan(1/2*d*x + 1/2*c)^2 + a)^(3/2)*a^17 + 165
*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)*a^18)/(a^20*sgn(cos(d*x + c))))/d
```

3.187.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cot^3(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = \int \frac{\cot(c+d\,x)^3}{\left(a+\frac{a}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

input
```
int(cot(c + d*x)^3/(a + a/cos(c + d*x))^(3/2),x)
```

output
```
int(cot(c + d*x)^3/(a + a/cos(c + d*x))^(3/2), x)
```

3.188 $\int \frac{\cot^5(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx$

3.188.1 Optimal result 1393
3.188.2 Mathematica [C] (verified) 1394
3.188.3 Rubi [A] (verified) 1394
3.188.4 Maple [B] (verified) 1400
3.188.5 Fricas [B] (verification not implemented) 1401
3.188.6 Sympy [F] 1402
3.188.7 Maxima [F] 1402
3.188.8 Giac [A] (verification not implemented) 1402
3.188.9 Mupad [F(-1)] 1403

3.188.1 Optimal result

Integrand size = 23, antiderivative size = 238

$$\int \frac{\cot^5(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = \frac{2\operatorname{arctanh}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{a}}\right)}{a^{3/2}d} - \frac{203\operatorname{arctanh}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{256\sqrt{2}a^{3/2}d} + \frac{139a^2}{224d(a+a\sec(c+dx))^{7/2}} - \frac{a^2}{4d(1-\sec(c+dx))^2(a+a\sec(c+dx))^{7/2}} - \frac{19a^2}{16d(1-\sec(c+dx))(a+a\sec(c+dx))^{7/2}} + \frac{15a}{64d(a+a\sec(c+dx))^{5/2}} - \frac{53}{384d(a+a\sec(c+dx))^{3/2}} - \frac{309}{256ad\sqrt{a+a\sec(c+dx)}}$$

output

```
2*arctanh((a+a*sec(d*x+c))^(1/2)/a^(1/2))/a^(3/2)/d+139/224*a^2/d/(a+a*sec
(d*x+c))^(7/2)-1/4*a^2/d/(1-sec(d*x+c))^2/(a+a*sec(d*x+c))^(7/2)-19/16*a^2
/d/(1-sec(d*x+c))/(a+a*sec(d*x+c))^(7/2)+15/64*a/d/(a+a*sec(d*x+c))^(5/2)-
53/384/d/(a+a*sec(d*x+c))^(3/2)-203/512*arctanh(1/2*(a+a*sec(d*x+c))^(1/2)
*2^(1/2)/a^(1/2))/a^(3/2)/d*2^(1/2)-309/256/a/d/(a+a*sec(d*x+c))^(1/2)
```

3.188.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.21 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.42

$$\int \frac{\cot^5(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = \frac{\cot^4(c+dx)\left(-322+203\,\text{Hypergeometric2F1}\left(-\frac{7}{2},1,-\frac{5}{2},\frac{1}{2}(1+\sec(c+dx)\right.\right.$$

input
```
Integrate[Cot[c + d*x]^5/(a + a*Sec[c + d*x])^(3/2),x]
```

output
```
(Cot[c + d*x]^4*(-322 + 203*Hypergeometric2F1[-7/2, 1, -5/2, (1 + Sec[c +
d*x])/2]*(-1 + Sec[c + d*x])^2 - 64*Hypergeometric2F1[-7/2, 1, -5/2, 1 + S
ec[c + d*x]]*(-1 + Sec[c + d*x])^2 + 266*Sec[c + d*x]))/(224*d*(a*(1 + Sec
[c + d*x]))^(3/2))
```

3.188.3 Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 263, normalized size of antiderivative = 1.11, number of steps used = 22, number of rules used = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.913$, Rules used = {3042, 25, 4368, 25, 27, 114, 27, 168, 27, 169, 27, 169, 27, 169, 27, 169, 27, 174, 73, 219, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^5(c+dx)}{(a\sec(c+dx)+a)^{3/2}}\,dx$$

$$\downarrow 3042$$

$$\int -\frac{1}{\cot\left(c+dx+\frac{\pi}{2}\right)^5\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^{3/2}}dx$$

$$\downarrow 25$$

$$-\int \frac{1}{\cot\left(\frac{1}{2}(2c+\pi)+dx\right)^5\left(\csc\left(\frac{1}{2}(2c+\pi)+dx\right)a+a\right)^{3/2}}dx$$

$$\downarrow 4368$$

$$\frac{a^6\int -\frac{\cos(c+dx)}{a^3(1-\sec(c+dx))^3(\sec(c+dx)a+a)^{9/2}}d\sec(c+dx)}{d}$$

$$\downarrow 25$$

$$-\frac{a^6\int \frac{\cos(c+dx)}{a^3(1-\sec(c+dx))^3(\sec(c+dx)a+a)^{9/2}}d\sec(c+dx)}{d}$$

$$\downarrow 27$$

$$-\frac{a^3\int \frac{\cos(c+dx)}{(1-\sec(c+dx))^3(\sec(c+dx)a+a)^{9/2}}d\sec(c+dx)}{d}$$

$$\downarrow 114$$

$$-\frac{a^3\left(\frac{1}{4a(1-\sec(c+dx))^2(a\sec(c+dx)+a)^{7/2}}-\frac{\int -\frac{a\cos(c+dx)(11\sec(c+dx)+8)}{2(1-\sec(c+dx))^2(\sec(c+dx)a+a)^{9/2}}d\sec(c+dx)}{4a}\right)}{d}$$

$$\downarrow 27$$

$$-\frac{a^3\left(\frac{1}{8}\int \frac{\cos(c+dx)(11\sec(c+dx)+8)}{(1-\sec(c+dx))^2(\sec(c+dx)a+a)^{9/2}}d\sec(c+dx)+\frac{1}{4a(1-\sec(c+dx))^2(a\sec(c+dx)+a)^{7/2}}\right)}{d}$$

$$\downarrow 168$$

$$-\frac{a^3\left(\frac{1}{8}\left(\frac{19}{2a(1-\sec(c+dx))(a\sec(c+dx)+a)^{7/2}}-\frac{\int -\frac{a\cos(c+dx)(171\sec(c+dx)+32)}{2(1-\sec(c+dx))(\sec(c+dx)a+a)^{9/2}}d\sec(c+dx)}{2a}\right)+\frac{1}{4a(1-\sec(c+dx))^2(a\sec(c+dx)+a)^{7/}\right.}{d}$$

$$\downarrow 27$$

$$-\frac{a^3\left(\frac{1}{8}\left(\frac{1}{4}\int \frac{\cos(c+dx)(171\sec(c+dx)+32)}{(1-\sec(c+dx))(\sec(c+dx)a+a)^{9/2}}d\sec(c+dx)+\frac{19}{2a(1-\sec(c+dx))(a\sec(c+dx)+a)^{7/2}}\right)+\frac{1}{4a(1-\sec(c+dx))^2(a\sec(c+dx}\right.}{d}$$

$$\downarrow 169$$

$$-\frac{a^3\left(\frac{1}{8}\left(\frac{1}{4}\left(\frac{\int \frac{7a\cos(c+dx)(139\sec(c+dx)+64)}{2(1-\sec(c+dx))(\sec(c+dx)a+a)^{7/2}}d\sec(c+dx)}{7a^2}-\frac{139}{7a(a\sec(c+dx)+a)^{7/2}}\right)+\frac{19}{2a(1-\sec(c+dx))(a\sec(c+dx)+a)^{7/2}}\right)+\frac{}{4a(1-}\right.}{d}$$

$$\downarrow 27$$

$$-\frac{a^3\left(\frac{1}{8}\left(\frac{1}{4}\left(\frac{\int \frac{\cos(c+dx)(139\sec(c+dx)+64)}{(1-\sec(c+dx))(\sec(c+dx)a+a)^{7/2}}d\sec(c+dx)}{2a}-\frac{139}{7a(a\sec(c+dx)+a)^{7/2}}\right)+\frac{19}{2a(1-\sec(c+dx))(a\sec(c+dx)+a)^{7/2}}\right)+\frac{}{4a(1-}\right.}{d}$$

$$\downarrow 169$$

$$-\frac{a^3\left(\frac{1}{8}\left(\frac{1}{4}\left(\frac{\int \frac{5a\cos(c+dx)(75\sec(c+dx)+128)}{2(1-\sec(c+dx))(\sec(c+dx)a+a)^{5/2}}\,d\sec(c+dx)}{5a^2}-\frac{15}{a(a\sec(c+dx)+a)^{5/2}}\right)-\frac{139}{7a(a\sec(c+dx)+a)^{7/2}}\right)+\frac{19}{2a(1-\sec(c+dx))(a\sec(c+dx}\right)}{d}$$

$$\downarrow 27$$

$$-\frac{a^3\left(\frac{1}{8}\left(\frac{1}{4}\left(\frac{\int \frac{\cos(c+dx)(75\sec(c+dx)+128)}{(1-\sec(c+dx))(\sec(c+dx)a+a)^{5/2}}\,d\sec(c+dx)}{2a}-\frac{15}{a(a\sec(c+dx)+a)^{5/2}}\right)-\frac{139}{7a(a\sec(c+dx)+a)^{7/2}}\right)+\frac{19}{2a(1-\sec(c+dx))(a\sec(c+dx}\right)}{d}$$

$$\downarrow 169$$

$$-\frac{a^3\left(\frac{1}{8}\left(\frac{1}{4}\left(\frac{\int \frac{3a\cos(c+dx)(256-53\sec(c+dx))}{2(1-\sec(c+dx))(\sec(c+dx)a+a)^{3/2}}\,d\sec(c+dx)}{3a^2}+\frac{53}{3a(a\sec(c+dx)+a)^{3/2}}\right)-\frac{15}{a(a\sec(c+dx)+a)^{5/2}}\right)-\frac{139}{7a(a\sec(c+dx)+a)^{7/2}}\right)+\frac{}{2a(1-}}{d}$$

$$\downarrow 27$$

$$-\frac{a^3\left(\frac{1}{8}\left(\frac{1}{4}\left(\frac{\int \frac{\cos(c+dx)(256-53\sec(c+dx))}{(1-\sec(c+dx))(\sec(c+dx)a+a)^{3/2}}\,d\sec(c+dx)}{2a}+\frac{53}{3a(a\sec(c+dx)+a)^{3/2}}\right)-\frac{15}{a(a\sec(c+dx)+a)^{5/2}}\right)-\frac{139}{7a(a\sec(c+dx)+a)^{7/2}}\right)+\frac{}{2a(1-}}{d}$$

$$\downarrow 169$$

$$-\frac{a^3\left(\frac{1}{8}\left(\frac{1}{4}\left(\frac{\int \frac{a\cos(c+dx)(512-309\sec(c+dx))}{2(1-\sec(c+dx))\sqrt{\sec(c+dx)a+a}}\,d\sec(c+dx)}{a^2}+\frac{309}{a\sqrt{a\sec(c+dx)+a}}\right)+\frac{53}{3a(a\sec(c+dx)+a)^{3/2}}\right)-\frac{15}{a(a\sec(c+dx)+a)^{5/2}}\right)-\frac{139}{7a(a\sec(c+dx)+a}}{d}$$

$$\downarrow 27$$

$$-\frac{a^3\left(\frac{1}{8}\left(\frac{1}{4}\left(\frac{\frac{\int \frac{\cos(c+dx)(512-309\sec(c+dx))}{(1-\sec(c+dx))\sqrt{\sec(c+dx)a+a}}\,d\sec(c+dx)}{2a}+\frac{309}{a\sqrt{a\sec(c+dx)+a}}}{2a}+\frac{53}{3a(a\sec(c+dx)+a)^{3/2}}\right)-\frac{15}{a(a\sec(c+dx)+a)^{5/2}}\right)-\frac{139}{7a(a\sec(c+dx)+a)^{7}}}{2a}}{d}$$

$$\downarrow 174$$

$$-\frac{a^3\left(\frac{1}{8}\left(\frac{1}{4}\left(\frac{\frac{203\int \frac{1}{(1-\sec(c+dx))\sqrt{\sec(c+dx)a+a}}\,d\sec(c+dx)+512\int \frac{\cos(c+dx)}{\sqrt{\sec(c+dx)a+a}}\,d\sec(c+dx)}{2a}+\frac{309}{a\sqrt{a\sec(c+dx)+a}}}{2a}+\frac{53}{3a(a\sec(c+dx)+a)^{3/2}}\right)-\frac{}{a(a\sec(c+}\right)}{2a}}{d}$$

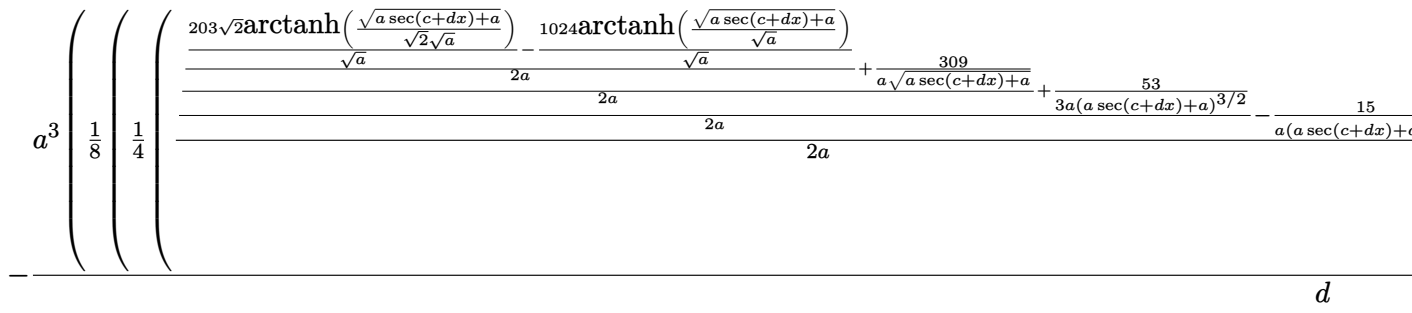
$$\downarrow 73$$

$$-\frac{a^3\left(\frac{1}{8}\left(\frac{1}{4}\left(\frac{\frac{406\int \frac{1}{2-\frac{\sec(c+dx)a+a}{a}}\,d\sqrt{\sec(c+dx)a+a}}{a}+\frac{1024\int \frac{1}{\frac{\sec(c+dx)a+a}{a}-1}\,d\sqrt{\sec(c+dx)a+a}}{a}}{2a}+\frac{309}{a\sqrt{a\sec(c+dx)+a}}}{2a}+\frac{53}{3a(a\sec(c+dx)+a)^{3/2}}\right)-\frac{}{a(a\sec}\right)}{2a}}{d}$$

$$\downarrow 219$$

$$-\frac{a^3\left(\frac{1}{8}\left(\frac{1}{4}\left(\frac{\frac{1024\int \frac{1}{\frac{\sec(c+dx)a+a}{a}-1}\,d\sqrt{\sec(c+dx)a+a}}{a}+\frac{203\sqrt{2}\operatorname{arctanh}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{2}\sqrt{a}}\right)}{\sqrt{a}}}{2a}+\frac{309}{a\sqrt{a\sec(c+dx)+a}}}{2a}+\frac{53}{3a(a\sec(c+dx)+a)^{3/2}}\right)-\frac{}{a(a\sec(c+}\right)}{2a}}{d}$$

$$\downarrow 220$$



input
```
Int[Cot[c + d*x]^5/(a + a*Sec[c + d*x])^(3/2),x]
```

output
```
-((a^3*(1/(4*a*(1 - Sec[c + d*x])^2*(a + a*Sec[c + d*x])^(7/2)) + (19/(2*a
*(1 - Sec[c + d*x])*(a + a*Sec[c + d*x])^(7/2)) + (-139/(7*a*(a + a*Sec[c
+ d*x])^(7/2)) + (-15/(a*(a + a*Sec[c + d*x])^(5/2)) + (53/(3*a*(a + a*Sec
[c + d*x])^(3/2)) + (((-1024*ArcTanh[Sqrt[a + a*Sec[c + d*x]]/Sqrt[a]])/Sq
rt[a] + (203*Sqrt[2]*ArcTanh[Sqrt[a + a*Sec[c + d*x]]/(Sqrt[2]*Sqrt[a])])/
Sqrt[a])/(2*a) + 309/(a*Sqrt[a + a*Sec[c + d*x]]))/(2*a))/(2*a))/(2*a))/4)
/8))/d)
```

3.188.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 114
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[1/((m + 1)*(b*c - a*d)*(b*e
 - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1)
 - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (IntegerQ[n] ||
 IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

rule 168
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

rule 169
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && IntegersQ[
2*m, 2*n, 2*p]
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

3.188. $\int \frac{\cot^5(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx$

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4368
```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Simp[-(d*b^(m - 1))^(-1)   Subst[Int[(-a + b*x)^((m - 1)/2
)*((a + b*x)^((m - 1)/2 + n)/x), x], x, Csc[c + d*x]], x] /; FreeQ[{a, b, c
, d, n}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[n]
```

3.188.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 435 vs. 2(197) = 394.

Time = 1.82 (sec) , antiderivative size = 436, normalized size of antiderivative = 1.83

method	result
default	$-\frac{\sqrt{a(1+\sec(dx+c))}\left(4263\cos(dx+c)^2\arctan\left(\frac{\sqrt{2}}{2\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}}\right)\sqrt{2}\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}+8526\arctan\left(\frac{\sqrt{2}}{2\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}}\right)\cos(dx+$

input
```
int(cot(d*x+c)^5/(a+a*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/10752/d/a^2*(a*(1+sec(d*x+c)))^(1/2)/(cos(d*x+c)+1)^2*(4263*cos(d*x+c)^
2*arctan(1/2*2^(1/2)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*2^(1/2)*(-cos(d*x
+c)/(cos(d*x+c)+1))^(1/2)+8526*arctan(1/2*2^(1/2)/(-cos(d*x+c)/(cos(d*x+c)
+1))^(1/2))*cos(d*x+c)*2^(1/2)*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+21504*co
s(d*x+c)^2*arctan((-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*(-cos(d*x+c)/(cos(d*
x+c)+1))^(1/2)+4263*2^(1/2)*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctan(1/2*
2^(1/2)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))+43008*arctan((-cos(d*x+c)/(cos
(d*x+c)+1))^(1/2))*cos(d*x+c)*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+21504*arc
tan((-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)
+20726*cos(d*x+c)^2*cot(d*x+c)^4+16074*cos(d*x+c)*cot(d*x+c)^4-33076*cot(d
*x+c)^4-28476*cot(d*x+c)^3*csc(d*x+c)+14462*cot(d*x+c)^2*csc(d*x+c)^2+1297
8*csc(d*x+c)^3*cot(d*x+c))
```

3.188.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 404 vs. $2(193) = 386$.

Time $= 0.40$ (sec) , antiderivative size $= 837$, normalized size of antiderivative $= 3.52$

$$\int \frac{\cot^5(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = \left[\frac{4263\sqrt{2}\left(\cos(dx+c)^6+2\cos(dx+c)^5-\cos(dx+c)^4-4\cos(dx+c)^3-\right.}{}\right.$$

input
```
integrate(cot(d*x+c)^5/(a+a*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
[1/21504*(4263*sqrt(2)*(cos(d*x + c)^6 + 2*cos(d*x + c)^5 - cos(d*x + c)^4
 - 4*cos(d*x + c)^3 - cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(a)*log(-(2
*sqrt(2)*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c) - 3*
a*cos(d*x + c) - a)/(cos(d*x + c) - 1)) + 10752*(cos(d*x + c)^6 + 2*cos(d*
x + c)^5 - cos(d*x + c)^4 - 4*cos(d*x + c)^3 - cos(d*x + c)^2 + 2*cos(d*x
+ c) + 1)*sqrt(a)*log(-8*a*cos(d*x + c)^2 - 4*(2*cos(d*x + c)^2 + cos(d*x
+ c))*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)) - 8*a*cos(d*x + c) -
 a) - 4*(10363*cos(d*x + c)^6 + 8037*cos(d*x + c)^5 - 16538*cos(d*x + c)^4
 - 14238*cos(d*x + c)^3 + 7231*cos(d*x + c)^2 + 6489*cos(d*x + c))*sqrt((a
*cos(d*x + c) + a)/cos(d*x + c)))/(a^2*d*cos(d*x + c)^6 + 2*a^2*d*cos(d*x
+ c)^5 - a^2*d*cos(d*x + c)^4 - 4*a^2*d*cos(d*x + c)^3 - a^2*d*cos(d*x + c
)^2 + 2*a^2*d*cos(d*x + c) + a^2*d), 1/10752*(4263*sqrt(2)*(cos(d*x + c)^6
 + 2*cos(d*x + c)^5 - cos(d*x + c)^4 - 4*cos(d*x + c)^3 - cos(d*x + c)^2 +
 2*cos(d*x + c) + 1)*sqrt(-a)*arctan(sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x + c)
 + a)/cos(d*x + c))*cos(d*x + c)/(a*cos(d*x + c) + a)) - 10752*(cos(d*x +
c)^6 + 2*cos(d*x + c)^5 - cos(d*x + c)^4 - 4*cos(d*x + c)^3 - cos(d*x + c)
^2 + 2*cos(d*x + c) + 1)*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) +
 a)/cos(d*x + c))*cos(d*x + c)/(2*a*cos(d*x + c) + a)) - 2*(10363*cos(d*x
+ c)^6 + 8037*cos(d*x + c)^5 - 16538*cos(d*x + c)^4 - 14238*cos(d*x + c)^3
 + 7231*cos(d*x + c)^2 + 6489*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/c...
```

3.188.6 Sympy [F]

$$\int \frac{\cot^5(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = \int \frac{\cot^5(c+dx)}{(a(\sec(c+dx)+1))^{\frac{3}{2}}}\,dx$$

input
```
integrate(cot(d*x+c)**5/(a+a*sec(d*x+c))**(3/2),x)
```

output
```
Integral(cot(c + d*x)**5/(a*(sec(c + d*x) + 1))**(3/2), x)
```

3.188.7 Maxima [F]

$$\int \frac{\cot^5(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = \int \frac{\cot(dx+c)^5}{(a\sec(dx+c)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(cot(d*x+c)^5/(a+a*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate(cot(d*x + c)^5/(a*sec(d*x + c) + a)^(3/2), x)
```

3.188.8 Giac [A] (verification not implemented)

Time = 1.01 (sec) , antiderivative size = 322, normalized size of antiderivative = 1.35

$$\int \frac{\cot^5(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = \frac{4263\sqrt{2}\arctan\left(\frac{\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a\operatorname{sgn}(\cos(dx+c))} - \frac{21504\arctan\left(\frac{\sqrt{2}\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}}{2\sqrt{-a}}\right)}{\sqrt{-a}a\operatorname{sgn}(\cos(dx+c))} - \frac{21\left(29\sqrt{2}$$

input
```
integrate(cot(d*x+c)^5/(a+a*sec(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
1/10752*(4263*sqrt(2)*arctan(sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)/sqrt(-a))
/(sqrt(-a)*a*sgn(cos(d*x + c))) - 21504*arctan(1/2*sqrt(2)*sqrt(-a*tan(1/2
*d*x + 1/2*c)^2 + a)/sqrt(-a))/(sqrt(-a)*a*sgn(cos(d*x + c))) - 21*(29*sqr
t(2)*(-a*tan(1/2*d*x + 1/2*c)^2 + a)^(3/2) - 27*sqrt(2)*sqrt(-a*tan(1/2*d*
x + 1/2*c)^2 + a)*a)/(a^3*sgn(cos(d*x + c))*tan(1/2*d*x + 1/2*c)^4) + 8*sq
rt(2)*(3*(a*tan(1/2*d*x + 1/2*c)^2 - a)^3*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 +
 a)*a^30 - 21*(a*tan(1/2*d*x + 1/2*c)^2 - a)^2*sqrt(-a*tan(1/2*d*x + 1/2*c
)^2 + a)*a^31 - 112*(-a*tan(1/2*d*x + 1/2*c)^2 + a)^(3/2)*a^32 - 882*sqrt(
-a*tan(1/2*d*x + 1/2*c)^2 + a)*a^33)/(a^35*sgn(cos(d*x + c))))/d
```

3.188.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cot^5(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = \int \frac{\cot(c+d\,x)^5}{\left(a+\frac{a}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

input
```
int(cot(c + d*x)^5/(a + a/cos(c + d*x))^(3/2),x)
```

output
```
int(cot(c + d*x)^5/(a + a/cos(c + d*x))^(3/2), x)
```

3.189 $\int \frac{\tan^6(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx$

3.189.1 Optimal result 1404
3.189.2 Mathematica [C] (warning: unable to verify) 1404
3.189.3 Rubi [A] (verified) 1405
3.189.4 Maple [A] (warning: unable to verify) 1407
3.189.5 Fricas [A] (verification not implemented) 1407
3.189.6 Sympy [F] 1408
3.189.7 Maxima [F] 1408
3.189.8 Giac [B] (verification not implemented) 1409
3.189.9 Mupad [F(-1)] 1409

3.189.1 Optimal result

Integrand size = 23, antiderivative size = 157

$$\int \frac{\tan^6(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = -\frac{2\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{a^{3/2}d} + \frac{2\tan(c+dx)}{ad\sqrt{a+a\sec(c+dx)}} - \frac{2\tan^3(c+dx)}{3d(a+a\sec(c+dx))^{3/2}} + \frac{2a\tan^5(c+dx)}{5d(a+a\sec(c+dx))^{5/2}} + \frac{2a^2\tan^7(c+dx)}{7d(a+a\sec(c+dx))^{7/2}}$$

output
```
-2*arctan(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))/a^(3/2)/d+2*tan(d*x+c
)/a/d/(a+a*sec(d*x+c))^(1/2)-2/3*tan(d*x+c)^3/d/(a+a*sec(d*x+c))^(3/2)+2/5
*a*tan(d*x+c)^5/d/(a+a*sec(d*x+c))^(5/2)+2/7*a^2*tan(d*x+c)^7/d/(a+a*sec(d
*x+c))^(7/2)
```

3.189.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 1.93 (sec) , antiderivative size = 248, normalized size of antiderivative = 1.58

$$\int \frac{\tan^6(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = \frac{32\sqrt{2}\left(\frac{1}{1+\sec(c+dx)}\right)^{11/2}\left(\frac{\cos(c+dx)(11+7\cos(c+dx))\csc^8\left(\frac{1}{2}(c+dx)\right)\sec^2\left(\frac{1}{2}(c+dx)\right)\left(105\arctan\right.\right.}{}$$

input
```
Integrate[Tan[c + d*x]^6/(a + a*Sec[c + d*x])^(3/2),x]
```

output

```
(32*Sqrt[2]*((1 + Sec[c + d*x])^(-1))^(11/2)*((Cos[c + d*x]*(11 + 7*Cos[c
+ d*x])*Csc[(c + d*x)/2]^8*Sec[(c + d*x)/2]^2*(105*ArcTanh[Sqrt[1 - Sec[c
+ d*x]]]*Cos[c + d*x]^3 + (76 - 198*Cos[c + d*x] + 61*Cos[2*(c + d*x)] - 4
4*Cos[3*(c + d*x)])*Sqrt[1 - Sec[c + d*x]]))/(3360*Sqrt[1 - Sec[c + d*x]])
 - (4*Hypergeometric2F1[2, 11/2, 13/2, -2*Sec[c + d*x]*Sin[(c + d*x)/2]^2]
*Sec[c + d*x]*Tan[(c + d*x)/2]^2)/11)*Tan[c + d*x]^7)/(7*d*(a*(1 + Sec[c +
 d*x]))^(3/2)*(1 - Tan[(c + d*x)/2]^2)^(9/2))
```

3.189.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.96, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3042, 4375, 363, 254, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^6(c+dx)}{(a\sec(c+dx)+a)^{3/2}}\,dx$$

$$\downarrow 3042$$

$$\int \frac{\cot\left(c+dx+\frac{\pi}{2}\right)^6}{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^{3/2}}dx$$

$$\downarrow 4375$$

$$-\frac{2a^2\int \frac{\tan^6(c+dx)\left(\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+2\right)}{(\sec(c+dx)a+a)^3\left(\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+1\right)}d\left(-\frac{\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)}{d}$$

$$\downarrow 363$$

$$-\frac{2a^2\left(\int \frac{\tan^6(c+dx)}{(\sec(c+dx)a+a)^3\left(\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+1\right)}d\left(-\frac{\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)-\frac{\tan^7(c+dx)}{7(a\sec(c+dx)+a)^{7/2}}\right)}{d}$$

$$\downarrow 254$$

$$-\frac{2a^2\left(\int\left(\frac{\tan^4(c+dx)}{a(\sec(c+dx)a+a)^2}-\frac{\tan^2(c+dx)}{a^2(\sec(c+dx)a+a)}-\frac{1}{a^3\left(\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+1\right)}+\frac{1}{a^3}\right)d\left(-\frac{\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)-\frac{\tan^7(c+dx)}{7(a\sec(c+dx)+a)^{7/2}}\right)}{d}$$

$$\downarrow 2009$$

$$-\frac{2a^2\left(\frac{\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{a^{7/2}}-\frac{\tan(c+dx)}{a^3\sqrt{a\sec(c+dx)+a}}+\frac{\tan^3(c+dx)}{3a^2(a\sec(c+dx)+a)^{3/2}}-\frac{\tan^7(c+dx)}{7(a\sec(c+dx)+a)^{7/2}}-\frac{\tan^5(c+dx)}{5a(a\sec(c+dx)+a)^{5/2}}\right)}{d}$$

input
```
Int[Tan[c + d*x]^6/(a + a*Sec[c + d*x])^(3/2),x]
```

output
```
(-2*a^2*(ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]]/a^(7/2) -
 Tan[c + d*x]/(a^3*Sqrt[a + a*Sec[c + d*x]]) + Tan[c + d*x]^3/(3*a^2*(a +
a*Sec[c + d*x])^(3/2)) - Tan[c + d*x]^5/(5*a*(a + a*Sec[c + d*x])^(5/2)) -
 Tan[c + d*x]^7/(7*(a + a*Sec[c + d*x])^(7/2))))/d
```

3.189.3.1 Defintions of rubi rules used

rule 254
```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Int[PolynomialDivide[x^m,
 a + b*x^2, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 3]
```

rule 363
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(b*e*(m + 2*p + 3))),
 x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(b*(m + 2*p + 3))   Int[(e*x)^
m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a*d
, 0] && NeQ[m + 2*p + 3, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4375
```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Simp[-2*(a^(m/2 + n + 1/2)/d)   Subst[Int[x^m*((2 + a*x^2
)^(m/2 + n - 1/2)/(1 + a*x^2)), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]
]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && I
ntegerQ[n - 1/2]
```

3.189.4 Maple [A] (warning: unable to verify)

Time = 3.74 (sec) , antiderivative size = 233, normalized size of antiderivative = 1.48

method	result
default	$-\frac{\left(105\sqrt{2}\arctanh\left(\frac{\sqrt{2}(-\cot(dx+c)+\csc(dx+c))}{\sqrt{(1-\cos(dx+c))^2\csc(dx+c)^2-1}}\right)\left((1-\cos(dx+c))^2\csc(dx+c)^2-1\right)^{\frac{7}{2}}-278(1-\cos(dx+c))^7\csc(dx+c)^7+1078(1-\right.}{105d\,a^2(-\cot(dx+c)+\csc(dx+c)-}$

input
```
int(tan(d*x+c)^6/(a+a*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/105/d/a^2*(105*2^(1/2)*arctanh(2^(1/2)/((1-cos(d*x+c))^2*csc(d*x+c)^2-1
)^(1/2)*(-cot(d*x+c)+csc(d*x+c)))*((1-cos(d*x+c))^2*csc(d*x+c)^2-1)^(7/2)-
278*(1-cos(d*x+c))^7*csc(d*x+c)^7+1078*(1-cos(d*x+c))^5*csc(d*x+c)^5-770*(
1-cos(d*x+c))^3*csc(d*x+c)^3+210*csc(d*x+c)-210*cot(d*x+c))*(-2*a/((1-cos(
d*x+c))^2*csc(d*x+c)^2-1))^(1/2)/(-cot(d*x+c)+csc(d*x+c)-1)^3/(csc(d*x+c)-
cot(d*x+c)+1)^3
```

3.189.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 343, normalized size of antiderivative = 2.18

$$\int \frac{\tan^6(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = \left[-\frac{105\left(\cos(dx+c)^4+\cos(dx+c)^3\right)\sqrt{-a}\log\left(\frac{2\,a\cos(dx+c)^2-2\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+}{\cos(dx+c)}}}{\cos(d}\right.\right.$$

input
```
integrate(tan(d*x+c)^6/(a+a*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
[-1/105*(105*(cos(d*x + c)^4 + cos(d*x + c)^3)*sqrt(-a)*log((2*a*cos(d*x +
 c)^2 - 2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*si
n(d*x + c) + a*cos(d*x + c) - a)/(cos(d*x + c) + 1)) - 2*(146*cos(d*x + c)
^3 - 32*cos(d*x + c)^2 - 24*cos(d*x + c) + 15)*sqrt((a*cos(d*x + c) + a)/c
os(d*x + c))*sin(d*x + c))/(a^2*d*cos(d*x + c)^4 + a^2*d*cos(d*x + c)^3),
2/105*(105*(cos(d*x + c)^4 + cos(d*x + c)^3)*sqrt(a)*arctan(sqrt((a*cos(d*
x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c))) + (146*cos(
d*x + c)^3 - 32*cos(d*x + c)^2 - 24*cos(d*x + c) + 15)*sqrt((a*cos(d*x + c
) + a)/cos(d*x + c))*sin(d*x + c))/(a^2*d*cos(d*x + c)^4 + a^2*d*cos(d*x +
 c)^3)]
```

3.189.6 Sympy [F]

$$\int \frac{\tan^6(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = \int \frac{\tan^6(c+dx)}{(a(\sec(c+dx)+1))^{\frac{3}{2}}}\,dx$$

input
```
integrate(tan(d*x+c)**6/(a+a*sec(d*x+c))**(3/2),x)
```

output
```
Integral(tan(c + d*x)**6/(a*(sec(c + d*x) + 1))**(3/2), x)
```

3.189.7 Maxima [F]

$$\int \frac{\tan^6(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = \int \frac{\tan(dx+c)^6}{(a\sec(dx+c)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(tan(d*x+c)^6/(a+a*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
1/210*(105*((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c)
+ 1)*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c)
 + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)), (cos
(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1
/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + 1) - (cos(2*d*x + 2*
c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*arctan2((cos(2*d*x + 2
*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2
(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x
 + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c)
, cos(2*d*x + 2*c) + 1)) - 1) + 2*(a^2*d*cos(2*d*x + 2*c)^2 + a^2*d*sin(2*
d*x + 2*c)^2 + 2*a^2*d*cos(2*d*x + 2*c) + a^2*d)*integrate(-(cos(2*d*x + 2
*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(3/4)*(((cos(14*d*x +
 14*c)*cos(2*d*x + 2*c) + 6*cos(12*d*x + 12*c)*cos(2*d*x + 2*c) + 15*cos(1
0*d*x + 10*c)*cos(2*d*x + 2*c) + 20*cos(8*d*x + 8*c)*cos(2*d*x + 2*c) + 15
*cos(6*d*x + 6*c)*cos(2*d*x + 2*c) + 6*cos(4*d*x + 4*c)*cos(2*d*x + 2*c) +
 cos(2*d*x + 2*c)^2 + sin(14*d*x + 14*c)*sin(2*d*x + 2*c) + 6*sin(12*d*x +
 12*c)*sin(2*d*x + 2*c) + 15*sin(10*d*x + 10*c)*sin(2*d*x + 2*c) + 20*sin(
8*d*x + 8*c)*sin(2*d*x + 2*c) + 15*sin(6*d*x + 6*c)*sin(2*d*x + 2*c) + 6*s
in(4*d*x + 4*c)*sin(2*d*x + 2*c) + sin(2*d*x + 2*c)^2)*cos(9/2*arctan2(sin
(2*d*x + 2*c), cos(2*d*x + 2*c))) + (cos(2*d*x + 2*c)*sin(14*d*x + 14*c...
```

3.189.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 296 vs. $2(137) = 274$.

Time = 3.75 (sec) , antiderivative size = 296, normalized size of antiderivative = 1.89

$$\int \frac{\tan^6(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = \frac{105\sqrt{-a}\left(\frac{\log\left(\left|\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^2-a\left(2\sqrt{2}+3\right)\right|\right)}{a^2\mathrm{sgn}(\cos(dx+c))} - \frac{\log\left(\left|\left(\sqrt{-}\right.\right.\right.\right.}$$

input
```
integrate(tan(d*x+c)^6/(a+a*sec(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
1/105*(105*sqrt(-a)*(log(abs((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(
1/2*d*x + 1/2*c)^2 + a))^2 - a*(2*sqrt(2) + 3)))/(a^2*sgn(cos(d*x + c))) -
 log(abs((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 +
 a))^2 + a*(2*sqrt(2) - 3)))/(a^2*sgn(cos(d*x + c)))) + 2*(((139*sqrt(2)*a
^2*tan(1/2*d*x + 1/2*c)^2/sgn(cos(d*x + c)) - 539*sqrt(2)*a^2/sgn(cos(d*x
+ c)))*tan(1/2*d*x + 1/2*c)^2 + 385*sqrt(2)*a^2/sgn(cos(d*x + c)))*tan(1/2
*d*x + 1/2*c)^2 - 105*sqrt(2)*a^2/sgn(cos(d*x + c)))*tan(1/2*d*x + 1/2*c)/
((a*tan(1/2*d*x + 1/2*c)^2 - a)^3*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)))/d
```

3.189.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\tan^6(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = \int \frac{\tan(c+d\,x)^6}{\left(a+\frac{a}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

input
```
int(tan(c + d*x)^6/(a + a/cos(c + d*x))^(3/2),x)
```

output
```
int(tan(c + d*x)^6/(a + a/cos(c + d*x))^(3/2), x)
```

3.190 $\int \frac{\tan^4(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx$

3.190.1 Optimal result 1410
3.190.2 Mathematica [A] (warning: unable to verify) 1410
3.190.3 Rubi [A] (verified) 1411
3.190.4 Maple [B] (verified) 1412
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3.190.8 Giac [B] (verification not implemented) 1415
3.190.9 Mupad [F(-1)] 1415

3.190.1 Optimal result

Integrand size = 23, antiderivative size = 95

$$\int \frac{\tan^4(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = \frac{2\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{a^{3/2}d} - \frac{2\tan(c+dx)}{ad\sqrt{a+a\sec(c+dx)}} + \frac{2\tan^3(c+dx)}{3d(a+a\sec(c+dx))^{3/2}}$$

output
```
2*arctan(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))/a^(3/2)/d-2*tan(d*x+c)
/a/d/(a+a*sec(d*x+c))^(1/2)+2/3*tan(d*x+c)^3/d/(a+a*sec(d*x+c))^(3/2)
```

3.190.2 Mathematica [A] (warning: unable to verify)

Time = 3.32 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.71

$$\int \frac{\tan^4(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = \frac{64\cos^6\left(\frac{1}{2}(c+dx)\right)\cot^4\left(\frac{1}{2}(c+dx)\right)\sec^5(c+dx)\left(\frac{1}{1+\sec(c+dx)}\right)^{7/2}\left(3\arcsin\left(\right.\right.}{3d\left(-1+\cot^2\left(\frac{1}{2}(c-\right.\right.}$$

input
```
Integrate[Tan[c + d*x]^4/(a + a*Sec[c + d*x])^(3/2),x]
```

output
```
(64*Cos[(c + d*x)/2]^6*Cot[(c + d*x)/2]^4*Sec[c + d*x]^5*((1 + Sec[c + d*x
])^(-1))^(7/2)*(3*ArcSin[Tan[(c + d*x)/2]/Sqrt[(1 + Cos[c + d*x])^(-1)]]*C
os[c + d*x]^2 + Sqrt[Cos[c + d*x]]*Sqrt[(1 + Cos[c + d*x])^(-1)]*(Sin[c +
d*x] - 2*Sin[2*(c + d*x)])))/(3*d*(-1 + Cot[(c + d*x)/2]^2)^2*(a*(1 + Sec[
c + d*x]))^(3/2))
```

3.190.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.99, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3042, 4375, 254, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^4(c+dx)}{(a\sec(c+dx)+a)^{3/2}}\,dx$$

$$\downarrow 3042$$

$$\int \frac{\cot\left(c+dx+\frac{\pi}{2}\right)^4}{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^{3/2}}dx$$

$$\downarrow 4375$$

$$-\frac{2a\int \frac{\tan^4(c+dx)}{(\sec(c+dx)a+a)^2\left(\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+1\right)}d\left(-\frac{\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)}{d}$$

$$\downarrow 254$$

$$-\frac{2a\int \left(\frac{\tan^2(c+dx)}{a(\sec(c+dx)a+a)}+\frac{1}{a^2\left(\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+1\right)}-\frac{1}{a^2}\right)d\left(-\frac{\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)}{d}$$

$$\downarrow 2009$$

$$-\frac{2a\left(-\frac{\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{a^{5/2}}+\frac{\tan(c+dx)}{a^2\sqrt{a\sec(c+dx)+a}}-\frac{\tan^3(c+dx)}{3a(a\sec(c+dx)+a)^{3/2}}\right)}{d}$$

input
```
Int[Tan[c + d*x]^4/(a + a*Sec[c + d*x])^(3/2),x]
```

output
```
(-2*a*(-(ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]]/a^(5/2))
+ Tan[c + d*x]/(a^2*Sqrt[a + a*Sec[c + d*x]]) - Tan[c + d*x]^3/(3*a*(a + a
*Sec[c + d*x])^(3/2))))/d
```

3.190.3.1 Defintions of rubi rules used

rule 254
```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Int[PolynomialDivide[x^m,
 a + b*x^2, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 3]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4375
```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Simp[-2*(a^(m/2 + n + 1/2)/d)   Subst[Int[x^m*((2 + a*x^2
)^(m/2 + n - 1/2)/(1 + a*x^2)), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]
]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && I
ntegerQ[n - 1/2]
```

3.190.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 173 vs. $2(83) = 166$.

Time = 3.76 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.83

method	result
default	$-\frac{2\sqrt{a(1+\sec(dx+c))}\left(-3\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,\operatorname{arctanh}\left(\frac{\sin(dx+c)}{(\cos(dx+c)+1)\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}}\right)\cos(dx+c)-3\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,\operatorname{arctanh}\left(\frac{}{(\cos$
	$3d\,a^2(\cos(dx+c)+1)$

input
```
int(tan(d*x+c)^4/(a+a*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3/d/a^2*(a*(1+sec(d*x+c)))^(1/2)/(cos(d*x+c)+1)*(-3*(-cos(d*x+c)/(cos(d
*x+c)+1))^(1/2)*arctanh(sin(d*x+c)/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)
+1))^(1/2))*cos(d*x+c)-3*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctanh(sin(d*
x+c)/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))+4*sin(d*x+c)-tan(d
*x+c))
```

3.190.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 295, normalized size of antiderivative = 3.11

$$\int \frac{\tan^4(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = \left[-\frac{3\left(\cos\left(dx+c\right)^2+\cos\left(dx+c\right)\right)\sqrt{-a}\log\left(\frac{2\,a\cos(dx+c)^2+2\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos}{\cos(dx+c)}\right.}{3\left(a^2d\cos\left(dx+c\right)^2+a^2d\cos\left(dx+c\right)\right)} \right.$$

$$\left. -\frac{2\left(3\left(\cos\left(dx+c\right)^2+\cos\left(dx+c\right)\right)\sqrt{a}\arctan\left(\frac{\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)}{\sqrt{a}\sin(dx+c)}\right)+\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}(4\cos\left(dx+c\right)-1)\right.}{3\left(a^2d\cos\left(dx+c\right)^2+a^2d\cos\left(dx+c\right)\right)} \right]$$

input
```
integrate(tan(d*x+c)^4/(a+a*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
[-1/3*(3*(cos(d*x + c)^2 + cos(d*x + c))*sqrt(-a)*log((2*a*cos(d*x + c)^2
+ 2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x
+ c) + a*cos(d*x + c) - a)/(cos(d*x + c) + 1)) + 2*sqrt((a*cos(d*x + c) +
a)/cos(d*x + c))*(4*cos(d*x + c) - 1)*sin(d*x + c))/(a^2*d*cos(d*x + c)^2
+ a^2*d*cos(d*x + c)), -2/3*(3*(cos(d*x + c)^2 + cos(d*x + c))*sqrt(a)*arc
tan(sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x
+ c))) + sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*(4*cos(d*x + c) - 1)*sin(
d*x + c))/(a^2*d*cos(d*x + c)^2 + a^2*d*cos(d*x + c))]
```

3.190.6 Sympy [F]

$$\int \frac{\tan^4(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = \int \frac{\tan^4(c+dx)}{(a(\sec(c+dx)+1))^{\frac{3}{2}}}\,dx$$

input
```
integrate(tan(d*x+c)**4/(a+a*sec(d*x+c))**(3/2),x)
```

output
```
Integral(tan(c + d*x)**4/(a*(sec(c + d*x) + 1))**(3/2), x)
```

3.190.7 Maxima [F]

$$\int \frac{\tan^4(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = \int \frac{\tan(dx+c)^4}{(a\sec(dx+c)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(tan(d*x+c)^4/(a+a*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
-1/6*(3*(2*a^2*d*integrate(-(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*c
os(2*d*x + 2*c) + 1)^(3/4)*(((cos(10*d*x + 10*c)*cos(2*d*x + 2*c) + 4*cos(
8*d*x + 8*c)*cos(2*d*x + 2*c) + 6*cos(6*d*x + 6*c)*cos(2*d*x + 2*c) + 4*co
s(4*d*x + 4*c)*cos(2*d*x + 2*c) + cos(2*d*x + 2*c)^2 + sin(10*d*x + 10*c)*
sin(2*d*x + 2*c) + 4*sin(8*d*x + 8*c)*sin(2*d*x + 2*c) + 6*sin(6*d*x + 6*c
)*sin(2*d*x + 2*c) + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + sin(2*d*x + 2*c
)^2)*cos(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + (cos(2*d*x + 2
*c)*sin(10*d*x + 10*c) + 4*cos(2*d*x + 2*c)*sin(8*d*x + 8*c) + 6*cos(2*d*x
 + 2*c)*sin(6*d*x + 6*c) + 4*cos(2*d*x + 2*c)*sin(4*d*x + 4*c) - cos(10*d*
x + 10*c)*sin(2*d*x + 2*c) - 4*cos(8*d*x + 8*c)*sin(2*d*x + 2*c) - 6*cos(6
*d*x + 6*c)*sin(2*d*x + 2*c) - 4*cos(4*d*x + 4*c)*sin(2*d*x + 2*c))*sin(5/
2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*cos(3/2*arctan2(sin(2*d*x
 + 2*c), cos(2*d*x + 2*c) + 1)) + ((cos(2*d*x + 2*c)*sin(10*d*x + 10*c) + 4
*cos(2*d*x + 2*c)*sin(8*d*x + 8*c) + 6*cos(2*d*x + 2*c)*sin(6*d*x + 6*c) +
 4*cos(2*d*x + 2*c)*sin(4*d*x + 4*c) - cos(10*d*x + 10*c)*sin(2*d*x + 2*c)
 - 4*cos(8*d*x + 8*c)*sin(2*d*x + 2*c) - 6*cos(6*d*x + 6*c)*sin(2*d*x + 2*
c) - 4*cos(4*d*x + 4*c)*sin(2*d*x + 2*c))*cos(5/2*arctan2(sin(2*d*x + 2*c)
, cos(2*d*x + 2*c))) - (cos(10*d*x + 10*c)*cos(2*d*x + 2*c) + 4*cos(8*d*x
 + 8*c)*cos(2*d*x + 2*c) + 6*cos(6*d*x + 6*c)*cos(2*d*x + 2*c) + 4*cos(4*d*
x + 4*c)*cos(2*d*x + 2*c) + cos(2*d*x + 2*c)^2 + sin(10*d*x + 10*c)*sin...
```

3.190.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 230 vs. $2(83) = 166$.

Time $= 2.44$ (sec) , antiderivative size $= 230$, normalized size of antiderivative $= 2.42$

$$\int \frac{\tan^4(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx =$$

$$-\frac{3\sqrt{-a}\left(\frac{\log\left(\left|\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^2-a\left(2\sqrt{2}+3\right)\right|\right)}{a^2\operatorname{sgn}(\cos(dx+c))} - \frac{\log\left(\left|\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+}\right.\right.\right.}{a^2\operatorname{sgn}(\cos(dx+c))}}{3d}$$

input
```
integrate(tan(d*x+c)^4/(a+a*sec(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
-1/3*(3*sqrt(-a)*(log(abs((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2
*d*x + 1/2*c)^2 + a))^2 - a*(2*sqrt(2) + 3)))/(a^2*sgn(cos(d*x + c))) - lo
g(abs((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)
)^2 + a*(2*sqrt(2) - 3)))/(a^2*sgn(cos(d*x + c)))) + 2*(5*sqrt(2)*tan(1/2*
d*x + 1/2*c)^2/sgn(cos(d*x + c)) - 3*sqrt(2)/sgn(cos(d*x + c)))*tan(1/2*d*
x + 1/2*c)/((a*tan(1/2*d*x + 1/2*c)^2 - a)*sqrt(-a*tan(1/2*d*x + 1/2*c)^2
+ a)))/d
```

3.190.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\tan^4(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = \int \frac{\tan(c+d\,x)^4}{\left(a+\frac{a}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

input
```
int(tan(c + d*x)^4/(a + a/cos(c + d*x))^(3/2),x)
```

output
```
int(tan(c + d*x)^4/(a + a/cos(c + d*x))^(3/2), x)
```

3.191 $\int \frac{\tan^2(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx$

3.191.1 Optimal result 1416
3.191.2 Mathematica [A] (verified) 1416
3.191.3 Rubi [A] (verified) 1417
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3.191.7 Maxima [F] 1420
3.191.8 Giac [A] (verification not implemented) 1420
3.191.9 Mupad [F(-1)] 1420

3.191.1 Optimal result

Integrand size = 23, antiderivative size = 85

$$\int \frac{\tan^2(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = -\frac{2\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{a^{3/2}d} + \frac{2\sqrt{2}\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{a^{3/2}d}$$

output
```
-2*arctan(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))/a^(3/2)/d+2*arctan(1/
2*a^(1/2)*tan(d*x+c)*2^(1/2)/(a+a*sec(d*x+c))^(1/2))*2^(1/2)/a^(3/2)/d
```

3.191.2 Mathematica [A] (verified)

Time = 0.78 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.55

$$\int \frac{\tan^2(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = \frac{4\left(2\arcsin\left(\tan\left(\frac{1}{2}(c+dx)\right)\right) - \sqrt{2}\arctan\left(\frac{\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{\frac{\cos(c+dx)}{1+\cos(c+dx)}}}\right)\right)\sqrt{\frac{\cos(c+dx)}{1+\cos(c+dx)}}\sec^{\frac{3}{2}}}{d\sec^2\left(\frac{1}{2}(c+dx)\right)^{3/2}(a(1+\sec(c+dx)))^{3/2}}$$

input
```
Integrate[Tan[c + d*x]^2/(a + a*Sec[c + d*x])^(3/2),x]
```

output
```
(4*(2*ArcSin[Tan[(c + d*x)/2]] - Sqrt[2]*ArcTan[Tan[(c + d*x)/2]/Sqrt[Cos[
c + d*x]/(1 + Cos[c + d*x])]])*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sec[c
 + d*x]^(3/2)*Sqrt[1 + Sec[c + d*x]])/(d*(Sec[(c + d*x)/2]^2)^(3/2)*(a*(1
+ Sec[c + d*x]))^(3/2))
```

3.191.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.98, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3042, 4375, 383, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^2(c+dx)}{(a\sec(c+dx)+a)^{3/2}}\,dx$$

$$\downarrow 3042$$

$$\int \frac{\cot\left(c+dx+\frac{\pi}{2}\right)^2}{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^{3/2}}dx$$

$$\downarrow 4375$$

$$-\frac{2\int \frac{\tan^2(c+dx)}{(\sec(c+dx)a+a)\left(\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+1\right)\left(\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+2\right)}d\left(-\frac{\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)}{d}$$

$$\downarrow 383$$

$$-\frac{2\left(\frac{2\int \frac{1}{\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+2}d\left(-\frac{\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)}{a}-\frac{\int \frac{1}{\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+1}d\left(-\frac{\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)}{a}\right)}{d}$$

$$\downarrow 216$$

$$-\frac{2\left(\frac{\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{a^{3/2}}-\frac{\sqrt{2}\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{a^{3/2}}\right)}{d}$$

input
```
Int[Tan[c + d*x]^2/(a + a*Sec[c + d*x])^(3/2),x]
```

output
```
(-2*(ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]]/a^(3/2) - (Sq
rt[2]*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])])/a
^(3/2)))/d
```

3.191.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 383
```
Int[((e_.)*(x_))^(m_.)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_Sym
bol] :> Simp[(-a)*(e^2/(b*c - a*d))   Int[(e*x)^(m - 2)/(a + b*x^2), x], x]
 + Simp[c*(e^2/(b*c - a*d))   Int[(e*x)^(m - 2)/(c + d*x^2), x], x] /; Free
Q[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && LeQ[2, m, 3]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4375
```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Simp[-2*(a^(m/2 + n + 1/2)/d)   Subst[Int[x^m*((2 + a*x^2
)^(m/2 + n - 1/2)/(1 + a*x^2)), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]
]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && I
ntegerQ[n - 1/2]
```

3.191.4 Maple [A] (verified)

Time = 1.82 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.60

method	result
default	$\frac{2\sqrt{a(1+\sec(dx+c))}\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}\left(-\operatorname{arctanh}\left(\frac{\sin(dx+c)}{(\cos(dx+c)+1)\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}}\right)+\ln\left(\csc(dx+c)-\cot(dx+c)+\sqrt{\cot(dx+c)^2-2\,\mathrm{co}\right.\right.}{d\,a^2}$

input
```
int(tan(d*x+c)^2/(a+a*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/d/a^2*(a*(1+sec(d*x+c)))^(1/2)*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(-arct
anh(sin(d*x+c)/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))+ln(csc(d
*x+c)-cot(d*x+c)+(cot(d*x+c)^2-2*cot(d*x+c)*csc(d*x+c)+csc(d*x+c)^2-1)^(1/
2))*2^(1/2))
```

3.191.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 295, normalized size of antiderivative = 3.47

$$\int \frac{\tan^2(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = \left[\frac{\sqrt{2}a\sqrt{-\frac{1}{a}}\log\left(-\frac{2\sqrt{2}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{-\frac{1}{a}}\cos(dx+c)\sin(dx+c)-3\cos(dx+c)^2-2\cos(dx+c)+}{\cos(dx+c)^2+2\cos(dx+c)+1}\right.}{a^2}\right.$$

$$\left.-\frac{2\left(\sqrt{2}\sqrt{a}\arctan\left(\frac{\sqrt{2}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)}{\sqrt{a}\sin(dx+c)}\right)-\sqrt{a}\arctan\left(\frac{\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)}{\sqrt{a}\sin(dx+c)}\right)\right)}{a^2 d}\right]$$

input
```
integrate(tan(d*x+c)^2/(a+a*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
[(sqrt(2)*a*sqrt(-1/a)*log(-(2*sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x +
 c))*sqrt(-1/a)*cos(d*x + c)*sin(d*x + c) - 3*cos(d*x + c)^2 - 2*cos(d*x +
 c) + 1)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) - sqrt(-a)*log((2*a*cos(d*
x + c)^2 - 2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)
*sin(d*x + c) + a*cos(d*x + c) - a)/(cos(d*x + c) + 1)))/(a^2*d), -2*(sqrt
(2)*sqrt(a)*arctan(sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x
 + c)/(sqrt(a)*sin(d*x + c))) - sqrt(a)*arctan(sqrt((a*cos(d*x + c) + a)/c
os(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c))))/(a^2*d)]
```

3.191.6 Sympy [F]

$$\int \frac{\tan^2(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = \int \frac{\tan^2(c+dx)}{\left(a\left(\sec(c+dx)+1\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(tan(d*x+c)**2/(a+a*sec(d*x+c))**(3/2),x)
```

output
```
Integral(tan(c + d*x)**2/(a*(sec(c + d*x) + 1))**(3/2), x)
```

3.191.7 Maxima [F]

$$\int \frac{\tan^2(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = \int \frac{\tan\left(dx+c\right)^2}{\left(a\sec\left(dx+c\right)+a\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(tan(d*x+c)^2/(a+a*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate(tan(d*x + c)^2/(a*sec(d*x + c) + a)^(3/2), x)
```

3.191.8 Giac [A] (verification not implemented)

Time = 1.14 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.85

$$\int \frac{\tan^2(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = \frac{\sqrt{2}\left(\frac{\sqrt{2}\arctan\left(\frac{\sqrt{-a+\frac{a}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2}}}{2\sqrt{a}}\right)}{a^{\frac{3}{2}}} - \frac{2\arctan\left(\frac{\sqrt{-a+\frac{a}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2}}}{\sqrt{a}}\right)}{a^{\frac{3}{2}}}\right)}{d}$$

input
```
integrate(tan(d*x+c)^2/(a+a*sec(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
sqrt(2)*(sqrt(2)*arctan(1/2*sqrt(2)*sqrt(-a + a/tan(1/2*d*x + 1/2*c)^2)/sq
rt(a))/a^(3/2) - 2*arctan(sqrt(-a + a/tan(1/2*d*x + 1/2*c)^2)/sqrt(a))/a^(
3/2))/d
```

3.191.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\tan^2(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = \int \frac{\tan(c+d\,x)^2}{\left(a+\frac{a}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

input
```
int(tan(c + d*x)^2/(a + a/cos(c + d*x))^(3/2),x)
```

output
```
int(tan(c + d*x)^2/(a + a/cos(c + d*x))^(3/2), x)
```

3.192 $\int \frac{\cot^2(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx$

3.192.1 Optimal result 1421
3.192.2 Mathematica [A] (warning: unable to verify) 1422
3.192.3 Rubi [A] (verified) 1422
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3.192.7 Maxima [F] 1428
3.192.8 Giac [A] (verification not implemented) 1429
3.192.9 Mupad [F(-1)] 1429

3.192.1 Optimal result

Integrand size = 23, antiderivative size = 215

$$\int \frac{\cot^2(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = -\frac{2\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{a^{3/2}d} + \frac{71\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{32\sqrt{2}a^{3/2}d} + \frac{7\cot(c+dx)\sqrt{a+a\sec(c+dx)}}{32a^2d} - \frac{13\cos(c+dx)\cot(c+dx)\sec^2\left(\frac{1}{2}(c+dx)\right)\sqrt{a+a\sec(c+dx)}}{32a^2d} - \frac{\cos^2(c+dx)\cot(c+dx)\sec^4\left(\frac{1}{2}(c+dx)\right)\sqrt{a+a\sec(c+dx)}}{16a^2d}$$

output

```
-2*arctan(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))/a^(3/2)/d+71/64*arcta
n(1/2*a^(1/2)*tan(d*x+c)*2^(1/2)/(a+a*sec(d*x+c))^(1/2))*2^(1/2)/a^(3/2)/d
+7/32*cot(d*x+c)*(a+a*sec(d*x+c))^(1/2)/a^2/d-13/32*cos(d*x+c)*cot(d*x+c)*
sec(1/2*d*x+1/2*c)^2*(a+a*sec(d*x+c))^(1/2)/a^2/d-1/16*cos(d*x+c)^2*cot(d*
x+c)*sec(1/2*d*x+1/2*c)^4*(a+a*sec(d*x+c))^(1/2)/a^2/d
```

3.192.2 Mathematica [A] (warning: unable to verify)

Time = 6.51 (sec) , antiderivative size = 210, normalized size of antiderivative = 0.98

$$\int \frac{\cot^2(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx =$$

$$-\frac{27\cot(2(c+dx))+12\csc(c+dx)+13\csc(2(c+dx))+256\arctan\left(\frac{\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{\frac{1}{1+\sec(c+dx)}}}\right)\cos^4\left(\frac{1}{2}(c+dx)\right)\sec^{\frac{3}{2}}(}{}$$

input
```
Integrate[Cot[c + d*x]^2/(a + a*Sec[c + d*x])^(3/2),x]
```

output
```
-1/32*(27*Cot[2*(c + d*x)] + 12*Csc[c + d*x] + 13*Csc[2*(c + d*x)] + 256*A
rcTan[Tan[(c + d*x)/2]/Sqrt[(1 + Sec[c + d*x])^(-1)]]*Cos[(c + d*x)/2]^4*S
ec[c + d*x]^(3/2)*Sqrt[Sec[c + d*x]/(1 + Sec[c + d*x])^2]*Sqrt[1 + Sec[c +
 d*x]] - 142*ArcSin[Tan[(c + d*x)/2]]*Cos[(c + d*x)/2]^4*Sqrt[Sec[(c + d*x
)/2]^2]*Sec[c + d*x]^(3/2)*Sqrt[(1 + Sec[c + d*x])^(-1)]*Sqrt[1 + Sec[c +
d*x]])/(d*(a*(1 + Sec[c + d*x]))^(3/2))
```

3.192.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.06, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.522$, Rules used = {3042, 4375, 374, 27, 441, 25, 27, 445, 25, 27, 397, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^2(c+dx)}{(a\sec(c+dx)+a)^{3/2}}\,dx$$

$$\downarrow 3042$$

$$\int \frac{1}{\cot\left(c+dx+\frac{\pi}{2}\right)^2\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^{3/2}}dx$$

$$\downarrow 4375$$

$$-\frac{2\int \frac{\cot^2(c+dx)(\sec(c+dx)a+a)}{\left(\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+1\right)\left(\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+2\right)^3}d\left(-\frac{\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)}{a^2d}$$

$$\downarrow 374$$

$$-\frac{2\left(\frac{\int \frac{a\cot^2(c+dx)(\sec(c+dx)a+a)\left(3-\frac{5a\tan^2(c+dx)}{\sec(c+dx)a+a}\right)}{\left(\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+1\right)\left(\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+2\right)^2}d\left(-\frac{\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)}{8a}+\frac{\cot(c+dx)\sqrt{a\sec(c+dx)+a}}{8\left(\frac{a\tan^2(c+dx)}{a\sec(c+dx)+a}+2\right)^2}\right)}{a^2d}$$

$$\downarrow 27$$

$$-\frac{2\left(\frac{1}{8}\int \frac{\cot^2(c+dx)(\sec(c+dx)a+a)\left(3-\frac{5a\tan^2(c+dx)}{\sec(c+dx)a+a}\right)}{\left(\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+1\right)\left(\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+2\right)^2}d\left(-\frac{\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)+\frac{\cot(c+dx)\sqrt{a\sec(c+dx)+a}}{8\left(\frac{a\tan^2(c+dx)}{a\sec(c+dx)+a}+2\right)^2}\right)}{a^2d}$$

$$\downarrow 441$$

$$-\frac{2\left(\frac{1}{8}\left(\frac{\int -\frac{a\cot^2(c+dx)(\sec(c+dx)a+a)\left(\frac{39a\tan^2(c+dx)}{\sec(c+dx)a+a}+7\right)}{\left(\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+1\right)\left(\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+2\right)}d\left(-\frac{\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)}{4a}+\frac{13\cot(c+dx)\sqrt{a\sec(c+dx)+a}}{4\left(\frac{a\tan^2(c+dx)}{a\sec(c+dx)+a}+2\right)}\right)+\frac{\cot(c+dx)\sqrt{a\sec(c+}}{8\left(\frac{a\tan^2(c+dx)}{a\sec(c+dx)+a}+\right.}\right)}{a^2d}$$

$$\downarrow 25$$

$$-\frac{2\left(\frac{1}{8}\left(\frac{13\cot(c+dx)\sqrt{a\sec(c+dx)+a}}{4\left(\frac{a\tan^2(c+dx)}{a\sec(c+dx)+a}+2\right)}-\frac{\int \frac{a\cot^2(c+dx)(\sec(c+dx)a+a)\left(\frac{39a\tan^2(c+dx)}{\sec(c+dx)a+a}+7\right)}{\left(\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+1\right)\left(\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+2\right)}d\left(-\frac{\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)}{4a}\right)+\frac{\cot(c+dx)\sqrt{a\sec(c+d}}{8\left(\frac{a\tan^2(c+dx)}{a\sec(c+dx)+a}+2\right.}\right)}{a^2d}$$

$$\downarrow 27$$

$$-\frac{2\left(\frac{1}{8}\left(\frac{13\cot(c+dx)\sqrt{a\sec(c+dx)+a}}{4\left(\frac{a\tan^2(c+dx)}{a\sec(c+dx)+a}+2\right)}-\frac{1}{4}\int \frac{\cot^2(c+dx)(\sec(c+dx)a+a)\left(\frac{39a\tan^2(c+dx)}{\sec(c+dx)a+a}+7\right)}{\left(\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+1\right)\left(\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+2\right)}d\left(-\frac{\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)\right)+\frac{\cot(c+dx)\sqrt{a}}{8\left(\frac{a\tan^2(c+}{a\sec(c+a}\right.}\right)}{a^2d}$$

$$\downarrow 445$$

$$-\frac{2\left(\frac{1}{8}\left(\frac{1}{4}\left(\frac{1}{2}\int -\frac{a\left(57-\frac{7a\tan^2(c+dx)}{\sec(c+dx)a+a}\right)}{\left(\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+1\right)\left(\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+2\right)}d\left(-\frac{\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)-\frac{7}{2}\cot(c+dx)\sqrt{a\sec(c+dx)+a}\right)+\frac{13\cot(}{4(}\right)\right)}{a^2d}$$

$$\downarrow 25$$

3.192. $\int \frac{\cot^2(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx$

$$-\frac{2\left(\frac{1}{8}\left(\frac{1}{4}\left(-\frac{1}{2}\int \frac{a\left(57-\frac{7a\tan^2(c+dx)}{\sec(c+dx)a+a}\right)}{\left(\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+1\right)\left(\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+2\right)}d\left(-\frac{\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)-\frac{7}{2}\cot(c+dx)\sqrt{a\sec(c+dx)+a}\right)+\frac{13\cot(}{4(}\right.\right.\right.}{a^2d}$$

$$\downarrow 27$$

$$-\frac{2\left(\frac{1}{8}\left(\frac{1}{4}\left(-\frac{1}{2}a\int \frac{57-\frac{7a\tan^2(c+dx)}{\sec(c+dx)a+a}}{\left(\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+1\right)\left(\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+2\right)}d\left(-\frac{\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)-\frac{7}{2}\cot(c+dx)\sqrt{a\sec(c+dx)+a}\right)+\frac{13\cot}{4}\right.\right.\right.}{a^2d}$$

$$\downarrow 397$$

$$-\frac{2\left(\frac{1}{8}\left(\frac{1}{4}\left(-\frac{1}{2}a\left(64\int \frac{1}{\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+1}d\left(-\frac{\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)-71\int \frac{1}{\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+2}d\left(-\frac{\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)\right)-\frac{7}{2}\cot(c+dx)\right.\right.\right.}{a^2d}$$

$$\downarrow 216$$

$$-\frac{2\left(\frac{1}{8}\left(\frac{1}{4}\left(-\frac{1}{2}a\left(\frac{71\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{2}\sqrt{a}}-\frac{64\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}}\right)-\frac{7}{2}\cot(c+dx)\sqrt{a\sec(c+dx)+a}\right)+\frac{13}{}\right.\right.\right.}{a^2d}$$

input
```
Int[Cot[c + d*x]^2/(a + a*Sec[c + d*x])^(3/2),x]
```

output
```
(-2*((Cot[c + d*x]*Sqrt[a + a*Sec[c + d*x]])/(8*(2 + (a*Tan[c + d*x]^2)/(a
 + a*Sec[c + d*x]))^2) + ((-1/2*(a*((-64*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqr
t[a + a*Sec[c + d*x]]])/Sqrt[a] + (71*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[
2]*Sqrt[a + a*Sec[c + d*x]])])/(Sqrt[2]*Sqrt[a])) - (7*Cot[c + d*x]*Sqrt[
a + a*Sec[c + d*x]])/2)/4 + (13*Cot[c + d*x]*Sqrt[a + a*Sec[c + d*x]])/(4*
(2 + (a*Tan[c + d*x]^2)/(a + a*Sec[c + d*x]))))/8))/(a^2*d)
```

3.192.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 374
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q
 + 1)/(a*e*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(e*x)^m*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[b*c*(m + 1) + 2*(b*c -
a*d)*(p + 1) + d*b*(m + 2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b,
c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IntBinomialQ[a, b,
 c, d, e, m, 2, p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 441
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*(g*x)^(m + 1)*(a
+ b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(a*g*2*(b*c - a*d)*(p + 1))), x] + Si
mp[1/(a*2*(b*c - a*d)*(p + 1))   Int[(g*x)^m*(a + b*x^2)^(p + 1)*(c + d*x^2
)^q*Simp[c*(b*e - a*f)*(m + 1) + e*2*(b*c - a*d)*(p + 1) + d*(b*e - a*f)*(m
 + 2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m, q},
 x] && LtQ[p, -1]
```

rule 445
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^2*(m + 1))
 Int[(g*x)^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*f*c*(m + 1) - e*(b*c
+ a*d)*(m + 2 + 1) - e*2*(b*c*p + a*d*q) - b*e*d*(m + 2*(p + q + 2) + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && LtQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4375
```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Simp[-2*(a^(m/2 + n + 1/2)/d)   Subst[Int[x^m*((2 + a*x^2
)^(m/2 + n - 1/2)/(1 + a*x^2)), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]
]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && I
ntegerQ[n - 1/2]
```

3.192.4 Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 496 vs. $2(184) = 368$.

Time = 2.00 (sec) , antiderivative size = 497, normalized size of antiderivative = 2.31

method	result
default	$\frac{\sqrt{-\frac{2a}{(1-\cos(dx+c))^2\csc(dx+c)^2-1}}\sqrt{(1-\cos(dx+c))^2\csc(dx+c)^2-1}\left(24\left((1-\cos(dx+c))^2\csc(dx+c)^2-1\right)^{\frac{9}{2}}\sin(dx+c)-24(1-\cos(dx$

input
```
int(cot(d*x+c)^2/(a+a*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/192/d/a^2*(-2*a/((1-cos(d*x+c))^2*csc(d*x+c)^2-1))^(1/2)*((1-cos(d*x+c))
^2*csc(d*x+c)^2-1)^(1/2)/(1-cos(d*x+c))*(24*((1-cos(d*x+c))^2*csc(d*x+c)^2
-1)^(9/2)*sin(d*x+c)-24*(1-cos(d*x+c))^2*((1-cos(d*x+c))^2*csc(d*x+c)^2-1)
^(7/2)*csc(d*x+c)+28*(1-cos(d*x+c))^2*((1-cos(d*x+c))^2*csc(d*x+c)^2-1)^(5
/2)*csc(d*x+c)-4*(1-cos(d*x+c))^6*((1-cos(d*x+c))^2*csc(d*x+c)^2-1)^(1/2)*
csc(d*x+c)^5-35*(1-cos(d*x+c))^2*((1-cos(d*x+c))^2*csc(d*x+c)^2-1)^(3/2)*c
sc(d*x+c)+25*(1-cos(d*x+c))^4*((1-cos(d*x+c))^2*csc(d*x+c)^2-1)^(1/2)*csc(
d*x+c)^3-192*2^(1/2)*arctanh(2^(1/2)/((1-cos(d*x+c))^2*csc(d*x+c)^2-1)^(1/
2)*(-cot(d*x+c)+csc(d*x+c)))*(1-cos(d*x+c))-42*(1-cos(d*x+c))^2*((1-cos(d*
x+c))^2*csc(d*x+c)^2-1)^(1/2)*csc(d*x+c)+213*ln(csc(d*x+c)-cot(d*x+c)+((1-
cos(d*x+c))^2*csc(d*x+c)^2-1)^(1/2))*(1-cos(d*x+c)))
```

3.192.5 Fricas [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 603, normalized size of antiderivative = 2.80

$$\int \frac{\cot^2(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = \left[-\frac{71\sqrt{2}\left(\cos(dx+c)^2+2\cos(dx+c)+1\right)\sqrt{-a}\log\left(\frac{2\sqrt{2}\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\,\cos(dx+c)}{c}\right.\right.$$

$$\left.\left. -\frac{71\sqrt{2}\left(\cos(dx+c)^2+2\cos(dx+c)+1\right)\sqrt{a}\arctan\left(\frac{\sqrt{2}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)}{\sqrt{a}\sin(dx+c)}\right)\sin(dx+c)+64\left(\cos(dx+\right.\right.}{64\left(a\right.}\right]$$

input
```
integrate(cot(d*x+c)^2/(a+a*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
[-1/128*(71*sqrt(2)*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(-a)*log((2*
sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(
d*x + c) + 3*a*cos(d*x + c)^2 + 2*a*cos(d*x + c) - a)/(cos(d*x + c)^2 + 2*
cos(d*x + c) + 1))*sin(d*x + c) + 64*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)
*sqrt(-a)*log(-(8*a*cos(d*x + c)^3 - 4*(2*cos(d*x + c)^2 - cos(d*x + c))*s
qrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c) - 7*a*cos(d*x
 + c) + a)/(cos(d*x + c) + 1))*sin(d*x + c) + 4*(27*cos(d*x + c)^3 + 12*co
s(d*x + c)^2 - 7*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)))/((
a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)*sin(d*x + c)), -1/64*
(71*sqrt(2)*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(a)*arctan(sqrt(2)*s
qrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c))
)*sin(d*x + c) + 64*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(a)*arctan(2
*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x + c)
/(2*a*cos(d*x + c)^2 + a*cos(d*x + c) - a))*sin(d*x + c) + 2*(27*cos(d*x +
 c)^3 + 12*cos(d*x + c)^2 - 7*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(
d*x + c)))/((a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)*sin(d*x
+ c))]
```

3.192.6 Sympy [F]

$$\int \frac{\cot^2(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = \int \frac{\cot^2(c+dx)}{(a(\sec(c+dx)+1))^{\frac{3}{2}}}\,dx$$

input
```
integrate(cot(d*x+c)**2/(a+a*sec(d*x+c))**(3/2),x)
```

output
```
Integral(cot(c + d*x)**2/(a*(sec(c + d*x) + 1))**(3/2), x)
```

3.192.7 Maxima [F]

$$\int \frac{\cot^2(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = \int \frac{\cot(dx+c)^2}{(a\sec(dx+c)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(cot(d*x+c)^2/(a+a*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate(cot(d*x + c)^2/(a*sec(d*x + c) + a)^(3/2), x)
```

3.192.8 Giac [A] (verification not implemented)

Time = 1.09 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.67

$$\int \frac{\cot^2(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx =$$

$$\sqrt{-a\tan\left(\tfrac{1}{2}\,dx+\tfrac{1}{2}\,c\right)^2+a}\left(\frac{2\sqrt{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2}{a^2\mathrm{sgn}(\cos(dx+c))}-\frac{17\sqrt{2}}{a^2\mathrm{sgn}(\cos(dx+c))}\right)\tan\left(\tfrac{1}{2}\,dx+\tfrac{1}{2}\,c\right)+\frac{}{\left(\left(\sqrt{-a}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\sqrt{-a}\right.\right.}$$

$$-\frac{}{64\,d}$$

input
```
integrate(cot(d*x+c)^2/(a+a*sec(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
-1/64*(sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)*(2*sqrt(2)*tan(1/2*d*x + 1/2*c)
^2/(a^2*sgn(cos(d*x + c))) - 17*sqrt(2)/(a^2*sgn(cos(d*x + c))))*tan(1/2*d
*x + 1/2*c) + 16*sqrt(2)/(((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/
2*d*x + 1/2*c)^2 + a))^2 - a)*sqrt(-a)*sgn(cos(d*x + c))))/d
```

3.192.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cot^2(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = \int \frac{\cot(c+d\,x)^2}{\left(a+\frac{a}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

input
```
int(cot(c + d*x)^2/(a + a/cos(c + d*x))^(3/2),x)
```

output
```
int(cot(c + d*x)^2/(a + a/cos(c + d*x))^(3/2), x)
```

3.193 $\int \frac{\cot^4(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx$

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3.193.1 Optimal result

Integrand size = 23, antiderivative size = 303

$$\int \frac{\cot^4(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = \frac{2\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{a^{3/2}d} - \frac{533\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{256\sqrt{2}a^{3/2}d} - \frac{21\cot(c+dx)\sqrt{a+a\sec(c+dx)}}{256a^2d} + \frac{277\cot^3(c+dx)(a+a\sec(c+dx))^{3/2}}{384a^3d} - \frac{81\cos(c+dx)\cot^3(c+dx)\sec^2\left(\frac{1}{2}(c+dx)\right)(a+a\sec(c+dx))^{3/2}}{128a^3d} - \frac{7\cos^2(c+dx)\cot^3(c+dx)\sec^4\left(\frac{1}{2}(c+dx)\right)(a+a\sec(c+dx))^{3/2}}{64a^3d} - \frac{\cos^3(c+dx)\cot^3(c+dx)\sec^6\left(\frac{1}{2}(c+dx)\right)(a+a\sec(c+dx))^{3/2}}{48a^3d}$$

output

```
2*arctan(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))/a^(3/2)/d+277/384*cot(
d*x+c)^3*(a+a*sec(d*x+c))^(3/2)/a^3/d-81/128*cos(d*x+c)*cot(d*x+c)^3*sec(1
/2*d*x+1/2*c)^2*(a+a*sec(d*x+c))^(3/2)/a^3/d-7/64*cos(d*x+c)^2*cot(d*x+c)^
3*sec(1/2*d*x+1/2*c)^4*(a+a*sec(d*x+c))^(3/2)/a^3/d-1/48*cos(d*x+c)^3*cot(
d*x+c)^3*sec(1/2*d*x+1/2*c)^6*(a+a*sec(d*x+c))^(3/2)/a^3/d-533/512*arctan(
1/2*a^(1/2)*tan(d*x+c)*2^(1/2)/(a+a*sec(d*x+c))^(1/2))*2^(1/2)/a^(3/2)/d-2
1/256*cot(d*x+c)*(a+a*sec(d*x+c))^(1/2)/a^2/d
```

3.193.2 Mathematica [A] (warning: unable to verify)

Time = 2.52 (sec) , antiderivative size = 240, normalized size of antiderivative = 0.79

$$\int \frac{\cot^4(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = \frac{-\frac{1}{16}(201-472\cos(c+dx)+516\cos(2(c+dx))+984\cos(3(c+dx))+819($$

input
```
Integrate[Cot[c + d*x]^4/(a + a*Sec[c + d*x])^(3/2),x]
```

output
```
(-1/16*((201 - 472*Cos[c + d*x] + 516*Cos[2*(c + d*x)] + 984*Cos[3*(c + d*
x)] + 819*Cos[4*(c + d*x)])*Csc[c + d*x]^3*Sec[c + d*x]) + 3072*ArcTan[Tan
[(c + d*x)/2]/Sqrt[(1 + Sec[c + d*x])^(-1)]]*Cos[(c + d*x)/2]^4*Sec[c + d*
x]^(3/2)*Sqrt[Sec[c + d*x]/(1 + Sec[c + d*x])^2]*Sqrt[1 + Sec[c + d*x]] -
1599*ArcSin[Tan[(c + d*x)/2]]*Cos[(c + d*x)/2]^4*Sqrt[Sec[(c + d*x)/2]^2]*
Sec[c + d*x]^(3/2)*Sqrt[(1 + Sec[c + d*x])^(-1)]*Sqrt[1 + Sec[c + d*x]])/(
384*d*(a*(1 + Sec[c + d*x]))^(3/2))
```

3.193.3 Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 320, normalized size of antiderivative = 1.06, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.696$, Rules used = {3042, 4375, 374, 27, 441, 25, 27, 441, 27, 445, 27, 445, 25, 27, 397, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^4(c+dx)}{(a\sec(c+dx)+a)^{3/2}}\,dx$$

$$\downarrow 3042$$

$$\int \frac{1}{\cot\left(c+dx+\frac{\pi}{2}\right)^4\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^{3/2}}dx$$

$$\downarrow 4375$$

$$-\frac{2\int \frac{\cot^4(c+dx)(\sec(c+dx)a+a)^2}{\left(\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+1\right)\left(\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+2\right)^4}d\left(-\frac{\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)}{a^3d}$$

$$\downarrow 374$$

$$-\frac{2\left(\frac{\int \frac{3a\cot^4(c+dx)(\sec(c+dx)a+a)^2\left(1-\frac{3a\tan^2(c+dx)}{\sec(c+dx)a+a}\right)}{\left(\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+1\right)\left(\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+2\right)^3}d\left(-\frac{\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)}{12a}+\frac{\cot^3(c+dx)(a\sec(c+dx)+a)^{3/2}}{12\left(\frac{a\tan^2(c+dx)}{a\sec(c+dx)+a}+2\right)^3}\right)}{a^3d}$$

$$\downarrow 27$$

$$-\frac{2\left(\frac{1}{4}\int \frac{\cot^4(c+dx)(\sec(c+dx)a+a)^2\left(1-\frac{3a\tan^2(c+dx)}{\sec(c+dx)a+a}\right)}{\left(\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+1\right)\left(\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+2\right)^3}d\left(-\frac{\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)+\frac{\cot^3(c+dx)(a\sec(c+dx)+a)^{3/2}}{12\left(\frac{a\tan^2(c+dx)}{a\sec(c+dx)+a}+2\right)^3}\right)}{a^3d}$$

$$\downarrow 441$$

$$-\frac{2\left(\frac{1}{4}\left(\frac{\int -\frac{a\cot^4(c+dx)(\sec(c+dx)a+a)^2\left(\frac{49a\tan^2(c+dx)}{\sec(c+dx)a+a}+17\right)}{\left(\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+1\right)\left(\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+2\right)^2}d\left(-\frac{\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)}{8a}+\frac{7\cot^3(c+dx)(a\sec(c+dx)+a)^{3/2}}{8\left(\frac{a\tan^2(c+dx)}{a\sec(c+dx)+a}+2\right)^2}\right)+\frac{\cot^3(c+dx)(a\sec}{12\left(\frac{a\tan^2(c+}{a\sec(c+}\right.\right.}{a^3d}$$

$$\downarrow 25$$

$$-\frac{2\left(\frac{1}{4}\left(\frac{7\cot^3(c+dx)(a\sec(c+dx)+a)^{3/2}}{8\left(\frac{a\tan^2(c+dx)}{a\sec(c+dx)+a}+2\right)^2}-\frac{\int \frac{a\cot^4(c+dx)(\sec(c+dx)a+a)^2\left(\frac{49a\tan^2(c+dx)}{\sec(c+dx)a+a}+17\right)}{\left(\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+1\right)\left(\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+2\right)^2}d\left(-\frac{\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)}{8a}\right)+\frac{\cot^3(c+dx)(a\sec(}{12\left(\frac{a\tan^2(c+}{a\sec(c+d}\right.\right.}{a^3d}$$

$$\downarrow 27$$

$$-\frac{2\left(\frac{1}{4}\left(\frac{7\cot^3(c+dx)(a\sec(c+dx)+a)^{3/2}}{8\left(\frac{a\tan^2(c+dx)}{a\sec(c+dx)+a}+2\right)^2}-\frac{1}{8}\int \frac{\cot^4(c+dx)(\sec(c+dx)a+a)^2\left(\frac{49a\tan^2(c+dx)}{\sec(c+dx)a+a}+17\right)}{\left(\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+1\right)\left(\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+2\right)^2}d\left(-\frac{\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)\right)+\frac{\cot^3(c+d}{12\left(\frac{}{}\right.}{a^3d}$$

$$\downarrow 441$$

$$-\frac{2\left(\frac{1}{4}\left(\frac{1}{8}\left(\frac{81\cot^3(c+dx)(a\sec(c+dx)+a)^{3/2}}{4\left(\frac{a\tan^2(c+dx)}{a\sec(c+dx)+a}+2\right)}-\frac{\int \frac{a\cot^4(c+dx)(\sec(c+dx)a+a)^2\left(\frac{405a\tan^2(c+dx)}{\sec(c+dx)a+a}+277\right)}{\left(\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+1\right)\left(\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+2\right)}d\left(-\frac{\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)}{4a}\right)+\frac{7\cot^3(c+}{8\left(\frac{}{}\right.}{a^3d}$$

3.193. $\int \frac{\cot^4(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx$

$$\downarrow 27$$

$$-\frac{2\left(\frac{1}{4}\left(\frac{1}{8}\left(\frac{81\cot^3(c+dx)(a\sec(c+dx)+a)^{3/2}}{4\left(\frac{a\tan^2(c+dx)}{a\sec(c+dx)+a}+2\right)}-\frac{1}{4}\int\frac{\cot^4(c+dx)(\sec(c+dx)a+a)^2\left(\frac{405a\tan^2(c+dx)}{\sec(c+dx)a+a}+277\right)}{\left(\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+1\right)\left(\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+2\right)}d\left(-\frac{\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)\right)+\frac{7}{}\right.\right.\right.}{a^3d}$$

$$\downarrow 445$$

$$-\frac{2\left(\frac{1}{4}\left(\frac{1}{8}\left(\frac{1}{4}\left(\frac{1}{6}\int\frac{3a\cot^2(c+dx)(\sec(c+dx)a+a)\left(\frac{277a\tan^2(c+dx)}{\sec(c+dx)a+a}+21\right)}{\left(\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+1\right)\left(\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+2\right)}d\left(-\frac{\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)-\frac{277}{6}\cot^3(c+dx)(a\sec(c+dx)\right.\right.\right.\right.}{a^3d}$$

$$\downarrow 27$$

$$-\frac{2\left(\frac{1}{4}\left(\frac{1}{8}\left(\frac{1}{4}\left(\frac{1}{2}a\int\frac{\cot^2(c+dx)(\sec(c+dx)a+a)\left(\frac{277a\tan^2(c+dx)}{\sec(c+dx)a+a}+21\right)}{\left(\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+1\right)\left(\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+2\right)}d\left(-\frac{\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)-\frac{277}{6}\cot^3(c+dx)(a\sec(c+dx)\right.\right.\right.\right.}{a^3d}$$

$$\downarrow 445$$

$$-\frac{2\left(\frac{1}{4}\left(\frac{1}{8}\left(\frac{1}{4}\left(\frac{1}{2}a\left(\frac{21}{2}\cot(c+dx)\sqrt{a\sec(c+dx)+a}-\frac{1}{2}\int-\frac{a\left(491-\frac{21a\tan^2(c+dx)}{\sec(c+dx)a+a}\right)}{\left(\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+1\right)\left(\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+2\right)}d\left(-\frac{\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)\right)\right.\right.\right.\right.}{}$$

$$\downarrow 25$$

$$-\frac{2\left(\frac{1}{4}\left(\frac{1}{8}\left(\frac{1}{4}\left(\frac{1}{2}a\left(\frac{1}{2}\int\frac{a\left(491-\frac{21a\tan^2(c+dx)}{\sec(c+dx)a+a}\right)}{\left(\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+1\right)\left(\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+2\right)}d\left(-\frac{\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)+\frac{21}{2}\cot(c+dx)\sqrt{a\sec(c+dx)+a}\right)\right.\right.\right.\right.}{}$$

$$\downarrow 27$$

$$-\frac{2\left(\frac{1}{4}\left(\frac{1}{8}\left(\frac{1}{4}\left(\frac{1}{2}a\left(\frac{1}{2}a\int\frac{491-\frac{21a\tan^2(c+dx)}{\sec(c+dx)a+a}}{\left(\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+1\right)\left(\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+2\right)}d\left(-\frac{\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)+\frac{21}{2}\cot(c+dx)\sqrt{a\sec(c+dx)+a}\right)\right.\right.\right.\right.}{}$$

$$\downarrow 397$$

$$-\frac{2\left(\frac{1}{4}\left(\frac{1}{8}\left(\frac{1}{4}\left(\frac{1}{2}a\left(\frac{1}{2}a\left(512\int\frac{1}{\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+1}d\left(-\frac{\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)-533\int\frac{1}{\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+2}d\left(-\frac{\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)\right)+\frac{21}{2}\mathrm{co}\right.\right.\right.\right.}{}$$

$$\downarrow 216$$

$$-\frac{2\left(\frac{1}{4}\left(\frac{1}{8}\left(\frac{1}{4}\left(\frac{1}{2}a\left(\frac{1}{2}a\left(\frac{533\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{2}\sqrt{a}}-\frac{512\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}}\right)\right)+\frac{21}{2}\cot(c+dx)\sqrt{a\sec(c+dx)+}\right.\right.\right.\right.\right.}{}$$

input
```
Int[Cot[c + d*x]^4/(a + a*Sec[c + d*x])^(3/2),x]
```

output
```
(-2*((Cot[c + d*x]^3*(a + a*Sec[c + d*x])^(3/2))/(12*(2 + (a*Tan[c + d*x]^
2)/(a + a*Sec[c + d*x]))^3) + ((7*Cot[c + d*x]^3*(a + a*Sec[c + d*x])^(3/2
))/(8*(2 + (a*Tan[c + d*x]^2)/(a + a*Sec[c + d*x]))^2) + (((-277*Cot[c + d
*x]^3*(a + a*Sec[c + d*x])^(3/2))/6 + (a*((a*((-512*ArcTan[(Sqrt[a]*Tan[c
+ d*x])/Sqrt[a + a*Sec[c + d*x]]])/Sqrt[a] + (533*ArcTan[(Sqrt[a]*Tan[c +
d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])])/(Sqrt[2]*Sqrt[a])))/2 + (21*Cot
[c + d*x]*Sqrt[a + a*Sec[c + d*x]])/2))/2)/4 + (81*Cot[c + d*x]^3*(a + a*S
ec[c + d*x])^(3/2))/(4*(2 + (a*Tan[c + d*x]^2)/(a + a*Sec[c + d*x])))))/8)/
4))/(a^3*d)
```

3.193.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 374
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q
 + 1)/(a*e*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(e*x)^m*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[b*c*(m + 1) + 2*(b*c -
a*d)*(p + 1) + d*b*(m + 2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b,
c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IntBinomialQ[a, b,
 c, d, e, m, 2, p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 441
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*(g*x)^(m + 1)*(a
+ b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(a*g*2*(b*c - a*d)*(p + 1))), x] + Si
mp[1/(a*2*(b*c - a*d)*(p + 1))   Int[(g*x)^m*(a + b*x^2)^(p + 1)*(c + d*x^2
)^q*Simp[c*(b*e - a*f)*(m + 1) + e*2*(b*c - a*d)*(p + 1) + d*(b*e - a*f)*(m
 + 2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m, q},
 x] && LtQ[p, -1]
```

rule 445
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^2*(m + 1))
 Int[(g*x)^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*f*c*(m + 1) - e*(b*c
+ a*d)*(m + 2 + 1) - e*2*(b*c*p + a*d*q) - b*e*d*(m + 2*(p + q + 2) + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && LtQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4375
```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Simp[-2*(a^(m/2 + n + 1/2)/d)   Subst[Int[x^m*((2 + a*x^2
)^(m/2 + n - 1/2)/(1 + a*x^2)), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]
]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && I
ntegerQ[n - 1/2]
```

3.193.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 540 vs. $2(263) = 526$.

Time = 1.87 (sec) , antiderivative size = 541, normalized size of antiderivative = 1.79

method	result
default	$-\frac{\sqrt{a(1+\sec(dx+c))}\left(1599\sqrt{2}\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}\ \ln\left(\csc(dx+c)-\cot(dx+c)+\sqrt{\cot(dx+c)^2-2\cot(dx+c)\csc(dx+c)+\csc(dx+c)^2-1}\right.\right.$

input
```
int(cot(d*x+c)^4/(a+a*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/1536/d/a^2*(a*(1+sec(d*x+c)))^(1/2)/(cos(d*x+c)+1)^2*(1599*2^(1/2)*(-co
s(d*x+c)/(cos(d*x+c)+1))^(1/2)*ln(csc(d*x+c)-cot(d*x+c)+(cot(d*x+c)^2-2*co
t(d*x+c)*csc(d*x+c)+csc(d*x+c)^2-1)^(1/2))*cos(d*x+c)^2-3072*(-cos(d*x+c)/
(cos(d*x+c)+1))^(1/2)*arctanh(sin(d*x+c)/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(
d*x+c)+1))^(1/2))*cos(d*x+c)^2+3198*2^(1/2)*ln(csc(d*x+c)-cot(d*x+c)+(cot(
d*x+c)^2-2*cot(d*x+c)*csc(d*x+c)+csc(d*x+c)^2-1)^(1/2))*(-cos(d*x+c)/(cos(
d*x+c)+1))^(1/2)*cos(d*x+c)-6144*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctan
h(sin(d*x+c)/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*cos(d*x+c)
+1599*ln(csc(d*x+c)-cot(d*x+c)+(cot(d*x+c)^2-2*cot(d*x+c)*csc(d*x+c)+csc(d
*x+c)^2-1)^(1/2))*2^(1/2)*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+1638*cot(d*x+
c)^3*cos(d*x+c)^2-3072*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctanh(sin(d*x+
c)/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))+984*cos(d*x+c)*cot(d
*x+c)^3-1380*cot(d*x+c)^3-856*cot(d*x+c)^2*csc(d*x+c)+126*cot(d*x+c)*csc(d
*x+c)^2)
```

3.193.5 Fricas [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 712, normalized size of antiderivative = 2.35

$$\int \frac{\cot^4(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = \left[-\frac{1599\sqrt{2}\left(\cos(dx+c)^4+2\cos(dx+c)^3-2\cos(dx+c)-1\right)\sqrt{-a}\log\left(\right.\right.$$

input
```
integrate(cot(d*x+c)^4/(a+a*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
[-1/3072*(1599*sqrt(2)*(cos(d*x + c)^4 + 2*cos(d*x + c)^3 - 2*cos(d*x + c)
 - 1)*sqrt(-a)*log(-(2*sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x
+ c))*cos(d*x + c)*sin(d*x + c) - 3*a*cos(d*x + c)^2 - 2*a*cos(d*x + c) +
a)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1))*sin(d*x + c) + 1536*(cos(d*x + c
)^4 + 2*cos(d*x + c)^3 - 2*cos(d*x + c) - 1)*sqrt(-a)*log(-(8*a*cos(d*x +
c)^3 + 4*(2*cos(d*x + c)^2 - cos(d*x + c))*sqrt(-a)*sqrt((a*cos(d*x + c) +
 a)/cos(d*x + c))*sin(d*x + c) - 7*a*cos(d*x + c) + a)/(cos(d*x + c) + 1))
*sin(d*x + c) - 4*(819*cos(d*x + c)^5 + 492*cos(d*x + c)^4 - 690*cos(d*x +
 c)^3 - 428*cos(d*x + c)^2 + 63*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/co
s(d*x + c)))/((a^2*d*cos(d*x + c)^4 + 2*a^2*d*cos(d*x + c)^3 - 2*a^2*d*cos
(d*x + c) - a^2*d)*sin(d*x + c)), 1/1536*(1599*sqrt(2)*(cos(d*x + c)^4 + 2
*cos(d*x + c)^3 - 2*cos(d*x + c) - 1)*sqrt(a)*arctan(sqrt(2)*sqrt((a*cos(d
*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c)))*sin(d*x +
c) + 1536*(cos(d*x + c)^4 + 2*cos(d*x + c)^3 - 2*cos(d*x + c) - 1)*sqrt(a)
*arctan(2*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin
(d*x + c)/(2*a*cos(d*x + c)^2 + a*cos(d*x + c) - a))*sin(d*x + c) + 2*(819
*cos(d*x + c)^5 + 492*cos(d*x + c)^4 - 690*cos(d*x + c)^3 - 428*cos(d*x +
c)^2 + 63*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)))/((a^2*d*c
os(d*x + c)^4 + 2*a^2*d*cos(d*x + c)^3 - 2*a^2*d*cos(d*x + c) - a^2*d)*sin
(d*x + c))]
```

3.193.6 Sympy [F]

$$\int \frac{\cot^4(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = \int \frac{\cot^4(c+dx)}{(a(\sec(c+dx)+1))^{\frac{3}{2}}}\,dx$$

input
```
integrate(cot(d*x+c)**4/(a+a*sec(d*x+c))**(3/2),x)
```

output
```
Integral(cot(c + d*x)**4/(a*(sec(c + d*x) + 1))**(3/2), x)
```

3.193.7 Maxima [F]

$$\int \frac{\cot^4(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = \int \frac{\cot\left(dx+c\right)^4}{\left(a\sec\left(dx+c\right)+a\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(cot(d*x+c)^4/(a+a*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate(cot(d*x + c)^4/(a*sec(d*x + c) + a)^(3/2), x)
```

3.193.8 Giac [A] (verification not implemented)

Time = 1.09 (sec) , antiderivative size = 261, normalized size of antiderivative = 0.86

$$\int \frac{\cot^4(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx =$$

$$-\frac{\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a}\left(2\left(\frac{4\sqrt{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2}{a^2\mathrm{sgn}(\cos(dx+c))}-\frac{37\sqrt{2}}{a^2\mathrm{sgn}(\cos(dx+c))}\right)\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+\frac{417\sqrt{2}}{a^2\mathrm{sgn}(\cos(dx+c))}\right)\tan$$

input
```
integrate(cot(d*x+c)^4/(a+a*sec(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
-1/1536*(sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)*(2*(4*sqrt(2)*tan(1/2*d*x + 1
/2*c)^2/(a^2*sgn(cos(d*x + c))) - 37*sqrt(2)/(a^2*sgn(cos(d*x + c))))*tan(
1/2*d*x + 1/2*c)^2 + 417*sqrt(2)/(a^2*sgn(cos(d*x + c))))*tan(1/2*d*x + 1/
2*c) - 32*sqrt(2)*(21*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x
 + 1/2*c)^2 + a))^4 - 36*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*
d*x + 1/2*c)^2 + a))^2*a + 19*a^2)/(((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt
(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2 - a)^3*sqrt(-a)*sgn(cos(d*x + c))))/d
```

3.193.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cot^4(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = \int \frac{\cot(c+d\,x)^4}{\left(a+\frac{a}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

input
```
int(cot(c + d*x)^4/(a + a/cos(c + d*x))^(3/2),x)
```

output
```
int(cot(c + d*x)^4/(a + a/cos(c + d*x))^(3/2), x)
```

3.194 $\int \frac{\cot^6(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx$

3.194.1 Optimal result 1440
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3.194.9 Mupad [F(-1)] 1450

3.194.1 Optimal result

Integrand size = 23, antiderivative size = 387

$$\int \frac{\cot^6(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = -\frac{2\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{a^{3/2}d} + \frac{16363\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{8192\sqrt{2}a^{3/2}d} - \frac{21\cot(c+dx)\sqrt{a+a\sec(c+dx)}}{8192a^2d} - \frac{8171\cot^3(c+dx)(a+a\sec(c+dx))^{3/2}}{12288a^3d} + \frac{12267\cot^5(c+dx)(a+a\sec(c+dx))^{5/2}}{10240a^4d} - \frac{2045\cos(c+dx)\cot^5(c+dx)\sec^2\left(\frac{1}{2}(c+dx)\right)(a+a\sec(c+dx))^{5/2}}{2048a^4d} - \frac{511\cos^2(c+dx)\cot^5(c+dx)\sec^4\left(\frac{1}{2}(c+dx)\right)(a+a\sec(c+dx))^{5/2}}{3072a^4d} - \frac{29\cos^3(c+dx)\cot^5(c+dx)\sec^6\left(\frac{1}{2}(c+dx)\right)(a+a\sec(c+dx))^{5/2}}{768a^4d} - \frac{\cos^4(c+dx)\cot^5(c+dx)\sec^8\left(\frac{1}{2}(c+dx)\right)(a+a\sec(c+dx))^{5/2}}{128a^4d}$$

output
```
-2*arctan(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))/a^(3/2)/d-8171/12288*
cot(d*x+c)^3*(a+a*sec(d*x+c))^(3/2)/a^3/d+12267/10240*cot(d*x+c)^5*(a+a*se
c(d*x+c))^(5/2)/a^4/d-2045/2048*cos(d*x+c)*cot(d*x+c)^5*sec(1/2*d*x+1/2*c)
^2*(a+a*sec(d*x+c))^(5/2)/a^4/d-511/3072*cos(d*x+c)^2*cot(d*x+c)^5*sec(1/2
*d*x+1/2*c)^4*(a+a*sec(d*x+c))^(5/2)/a^4/d-29/768*cos(d*x+c)^3*cot(d*x+c)^
5*sec(1/2*d*x+1/2*c)^6*(a+a*sec(d*x+c))^(5/2)/a^4/d-1/128*cos(d*x+c)^4*cot
(d*x+c)^5*sec(1/2*d*x+1/2*c)^8*(a+a*sec(d*x+c))^(5/2)/a^4/d+16363/16384*ar
ctan(1/2*a^(1/2)*tan(d*x+c)*2^(1/2)/(a+a*sec(d*x+c))^(1/2))*2^(1/2)/a^(3/2
)/d-21/8192*cot(d*x+c)*(a+a*sec(d*x+c))^(1/2)/a^2/d
```

3.194.2 Mathematica [A] (verified)

Time = 5.13 (sec) , antiderivative size = 264, normalized size of antiderivative = 0.68

$$\int \frac{\cot^6(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = \frac{\sec^{\frac{3}{2}}(c+dx)\left(245445\arcsin\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)\sqrt{\frac{1}{1+\sec(c+dx)}}\sqrt{1+\sec(c+dx}\right.}{}$$

input
```
Integrate[Cot[c + d*x]^6/(a + a*Sec[c + d*x])^(3/2),x]
```

output
```
(Sec[c + d*x]^(3/2)*(245445*ArcSin[Tan[(c + d*x)/2]]*Sqrt[(1 + Sec[c + d*x
])^(-1)]*Sqrt[1 + Sec[c + d*x]] - 245760*Sqrt[2]*ArcTan[Tan[(c + d*x)/2]/S
qrt[(1 + Sec[c + d*x])^(-1)]]*Sqrt[(1 + Sec[c + d*x])^(-1)]*Sqrt[1 + Sec[c
 + d*x]] - (Sqrt[(2 + 2*Cos[c + d*x])^(-1)]*(355216 + 831963*Cos[c + d*x]
- 59592*Cos[2*(c + d*x)] + 153657*Cos[3*(c + d*x)] - 207760*Cos[4*(c + d*x
)] + 141291*Cos[5*(c + d*x)] + 207048*Cos[6*(c + d*x)] + 151041*Cos[7*(c +
 d*x)])*Csc[c + d*x]^6*Sqrt[Sec[c + d*x]]*Tan[(c + d*x)/2])/32))/(61440*d*
(Sec[(c + d*x)/2]^2)^(3/2)*(a*(1 + Sec[c + d*x]))^(3/2))
```

3.194.3 Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 409, normalized size of antiderivative = 1.06, number of steps used = 20, number of rules used = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.826$, Rules used = {3042, 4375, 374, 27, 441, 25, 27, 441, 27, 441, 27, 445, 27, 445, 27, 445, 27, 397, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^6(c+dx)}{(a\sec(c+dx)+a)^{3/2}}\,dx$$

$$\downarrow 3042$$

$$\int \frac{1}{\cot\left(c+dx+\frac{\pi}{2}\right)^6\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^{3/2}}dx$$

$$\downarrow 4375$$

$$-\frac{2\int \frac{\cot^6(c+dx)(\sec(c+dx)a+a)^3}{\left(\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+1\right)\left(\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+2\right)^5}d\left(-\frac{\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)}{a^4d}$$

$$\downarrow 374$$

$$-\frac{2\left(\int \frac{a\cot^6(c+dx)(\sec(c+dx)a+a)^3\left(3-\frac{13a\tan^2(c+dx)}{\sec(c+dx)a+a}\right)}{\left(\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+1\right)\left(\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+2\right)^4}d\left(-\frac{\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)}{16a}+\frac{\cot^5(c+dx)(a\sec(c+dx)+a)^{5/2}}{16\left(\frac{a\tan^2(c+dx)}{a\sec(c+dx)+a}+2\right)^4}\right)}{a^4d}$$

$$\downarrow 27$$

$$-\frac{2\left(\frac{1}{16}\int \frac{\cot^6(c+dx)(\sec(c+dx)a+a)^3\left(3-\frac{13a\tan^2(c+dx)}{\sec(c+dx)a+a}\right)}{\left(\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+1\right)\left(\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+2\right)^4}d\left(-\frac{\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)+\frac{\cot^5(c+dx)(a\sec(c+dx)+a)^{5/2}}{16\left(\frac{a\tan^2(c+dx)}{a\sec(c+dx)+a}+2\right)^4}\right)}{a^4d}$$

$$\downarrow 441$$

$$-\frac{2\left(\frac{1}{16}\left(\frac{\int -\frac{a\cot^6(c+dx)(\sec(c+dx)a+a)^3\left(\frac{319a\tan^2(c+dx)}{\sec(c+dx)a+a}+127\right)}{\left(\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+1\right)\left(\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+2\right)^3}d\left(-\frac{\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)}{12a}+\frac{29\cot^5(c+dx)(a\sec(c+dx)+a)^{5/2}}{12\left(\frac{a\tan^2(c+dx)}{a\sec(c+dx)+a}+2\right)^3}\right)+\frac{\cot^5(c+dx)}{16\left(\frac{a}{a}\right.}\right.}{a^4d}$$

$$\downarrow 25$$

$$-\frac{2\left(\frac{1}{16}\left(\frac{29\cot^5(c+dx)(a\sec(c+dx)+a)^{5/2}}{12\left(\frac{a\tan^2(c+dx)}{a\sec(c+dx)+a}+2\right)^3}-\frac{\int \frac{a\cot^6(c+dx)(\sec(c+dx)a+a)^3\left(\frac{319a\tan^2(c+dx)}{\sec(c+dx)a+a}+127\right)}{\left(\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+1\right)\left(\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+2\right)^3}d\left(-\frac{\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)}{12a}\right)+\frac{\cot^5(c+dx)(}{16\left(\frac{a\tan}{a\sec(}\right.}\right.}{a^4d}$$

$$\downarrow 27$$

$$-\frac{2\left(\frac{1}{16}\left(\frac{29\cot^5(c+dx)(a\sec(c+dx)+a)^{5/2}}{12\left(\frac{a\tan^2(c+dx)}{a\sec(c+dx)+a}+2\right)^3}-\frac{1}{12}\int\frac{\cot^6(c+dx)(\sec(c+dx)a+a)^3\left(\frac{319a\tan^2(c+dx)}{\sec(c+dx)a+a}+127\right)}{\left(\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+1\right)\left(\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+2\right)^3}d\left(-\frac{\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)\right)+\cot}{a^4d}$$

$$\downarrow 441$$

$$-\frac{2\left(\frac{1}{16}\left(\frac{1}{12}\left(\frac{511\cot^5(c+dx)(a\sec(c+dx)+a)^{5/2}}{8\left(\frac{a\tan^2(c+dx)}{a\sec(c+dx)+a}+2\right)^2}-\frac{\int\frac{3a\cot^6(c+dx)(\sec(c+dx)a+a)^3\left(\frac{1533a\tan^2(c+dx)}{\sec(c+dx)a+a}+1021\right)}{\left(\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+1\right)\left(\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+2\right)^2}d\left(-\frac{\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)}{8a}\right)+\frac{29c}{a^4d}$$

$$\downarrow 27$$

$$-\frac{2\left(\frac{1}{16}\left(\frac{1}{12}\left(\frac{511\cot^5(c+dx)(a\sec(c+dx)+a)^{5/2}}{8\left(\frac{a\tan^2(c+dx)}{a\sec(c+dx)+a}+2\right)^2}-\frac{3}{8}\int\frac{\cot^6(c+dx)(\sec(c+dx)a+a)^3\left(\frac{1533a\tan^2(c+dx)}{\sec(c+dx)a+a}+1021\right)}{\left(\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+1\right)\left(\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+2\right)^2}d\left(-\frac{\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)\right)\right)}{a^4d}$$

$$\downarrow 441$$

$$-\frac{2\left(\frac{1}{16}\left(\frac{1}{12}\left(\frac{511\cot^5(c+dx)(a\sec(c+dx)+a)^{5/2}}{8\left(\frac{a\tan^2(c+dx)}{a\sec(c+dx)+a}+2\right)^2}-\frac{3}{8}\left(\frac{\int\frac{a\cot^6(c+dx)(\sec(c+dx)a+a)^3\left(\frac{14315a\tan^2(c+dx)}{\sec(c+dx)a+a}+12267\right)}{\left(\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+1\right)\left(\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+2\right)}d\left(-\frac{\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)}{4a}-\right.\right.\right.\right.}{a^4d}$$

$$\downarrow 27$$

$$-\frac{2\left(\frac{1}{16}\left(\frac{1}{12}\left(\frac{511\cot^5(c+dx)(a\sec(c+dx)+a)^{5/2}}{8\left(\frac{a\tan^2(c+dx)}{a\sec(c+dx)+a}+2\right)^2}-\frac{3}{8}\left(\frac{1}{4}\int\frac{\cot^6(c+dx)(\sec(c+dx)a+a)^3\left(\frac{14315a\tan^2(c+dx)}{\sec(c+dx)a+a}+12267\right)}{\left(\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+1\right)\left(\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+2\right)}d\left(-\frac{\tan(c+dx)}{\sqrt{\sec(c+dx)a-}\right.\right.\right.\right.}{a^4d}$$

$$\downarrow 445$$

$$-\frac{2\left(\frac{1}{16}\left(\frac{1}{12}\left(\frac{511\cot^5(c+dx)(a\sec(c+dx)+a)^{5/2}}{8\left(\frac{a\tan^2(c+dx)}{a\sec(c+dx)+a}+2\right)^2}-\frac{3}{8}\left(\frac{1}{4}\left(\frac{12267}{10}\cot^5(c+dx)(a\sec(c+dx)+a)^{5/2}-\frac{1}{10}\int\frac{5a\cot^4(c+dx)(\sec\right.\right.\right.\right.}{\left(\frac{a\tan^2}{\sec(c+\right.}$$

$$\downarrow 27$$

$$-2\left(\frac{1}{16}\left(\frac{1}{12}\left(\frac{511\cot^5(c+dx)(a\sec(c+dx)+a)^{5/2}}{8\left(\frac{a\tan^2(c+dx)}{a\sec(c+dx)+a}+2\right)^2}-\frac{3}{8}\left(\frac{1}{4}\left(\frac{12267}{10}\cot^5(c+dx)(a\sec(c+dx)+a)^{5/2}-\frac{1}{2}a\int\frac{\cot^4(c+dx)(\sec(c+dx)}{\left(\frac{a\tan^2(c+dx)}{\sec(c+dx)}\right.}\right.\right.\right.\right.\right.$$

$$\downarrow 445$$

$$-2\left(\frac{1}{16}\left(\frac{1}{12}\left(\frac{511\cot^5(c+dx)(a\sec(c+dx)+a)^{5/2}}{8\left(\frac{a\tan^2(c+dx)}{a\sec(c+dx)+a}+2\right)^2}-\frac{3}{8}\left(\frac{1}{4}\left(\frac{12267}{10}\cot^5(c+dx)(a\sec(c+dx)+a)^{5/2}-\frac{1}{2}a\left(\frac{8171}{6}\cot^3(c+d\right.\right.\right.\right.\right.\right.$$

$$\downarrow 27$$

$$-2\left(\frac{1}{16}\left(\frac{1}{12}\left(\frac{511\cot^5(c+dx)(a\sec(c+dx)+a)^{5/2}}{8\left(\frac{a\tan^2(c+dx)}{a\sec(c+dx)+a}+2\right)^2}-\frac{3}{8}\left(\frac{1}{4}\left(\frac{12267}{10}\cot^5(c+dx)(a\sec(c+dx)+a)^{5/2}-\frac{1}{2}a\left(\frac{1}{2}a\int\frac{\cot^2(c+dx)}{\left(\frac{a\tan}{\sec(}\right.\right.\right.\right.\right.\right.\right.$$

$$\downarrow 445$$

$$-2\left(\frac{1}{16}\left(\frac{1}{12}\left(\frac{511\cot^5(c+dx)(a\sec(c+dx)+a)^{5/2}}{8\left(\frac{a\tan^2(c+dx)}{a\sec(c+dx)+a}+2\right)^2}-\frac{3}{8}\left(\frac{1}{4}\left(\frac{12267}{10}\cot^5(c+dx)(a\sec(c+dx)+a)^{5/2}-\frac{1}{2}a\left(\frac{1}{2}a\left(\frac{21}{2}\cot(c+\right.\right.\right.\right.\right.\right.\right.$$

$$\downarrow 27$$

$$-2\left(\frac{1}{16}\left(\frac{1}{12}\left(\frac{511\cot^5(c+dx)(a\sec(c+dx)+a)^{5/2}}{8\left(\frac{a\tan^2(c+dx)}{a\sec(c+dx)+a}+2\right)^2}-\frac{3}{8}\left(\frac{1}{4}\left(\frac{12267}{10}\cot^5(c+dx)(a\sec(c+dx)+a)^{5/2}-\frac{1}{2}a\left(\frac{1}{2}a\left(\frac{21}{2}\cot(c+\right.\right.\right.\right.\right.\right.\right.$$

$$\downarrow 397$$

$$-2\left(\frac{1}{16}\left(\frac{1}{12}\left(\frac{511\cot^5(c+dx)(a\sec(c+dx)+a)^{5/2}}{8\left(\frac{a\tan^2(c+dx)}{a\sec(c+dx)+a}+2\right)^2}-\frac{3}{8}\left(\frac{1}{4}\left(\frac{12267}{10}\cot^5(c+dx)(a\sec(c+dx)+a)^{5/2}-\frac{1}{2}a\left(\frac{1}{2}a\left(\frac{21}{2}\cot(c+\right.\right.\right.\right.\right.\right.\right.$$

$$\downarrow 216$$

$$-2\left(\frac{1}{16}\left(\frac{1}{12}\left(\frac{511\cot^5(c+dx)(a\sec(c+dx)+a)^{5/2}}{8\left(\frac{a\tan^2(c+dx)}{a\sec(c+dx)+a}+2\right)^2}-\frac{3}{8}\left(\frac{1}{4}\left(\frac{12267}{10}\cot^5(c+dx)(a\sec(c+dx)+a)^{5/2}-\frac{1}{2}a\left(\frac{1}{2}a\left(\frac{21}{2}\cot(c+\right.\right.\right.\right.\right.\right.\right.$$

input
```
Int[Cot[c + d*x]^6/(a + a*Sec[c + d*x])^(3/2),x]
```

output
```
(-2*((Cot[c + d*x]^5*(a + a*Sec[c + d*x])^(5/2))/(16*(2 + (a*Tan[c + d*x]^
2)/(a + a*Sec[c + d*x]))^4) + ((29*Cot[c + d*x]^5*(a + a*Sec[c + d*x])^(5/
2))/(12*(2 + (a*Tan[c + d*x]^2)/(a + a*Sec[c + d*x]))^3) + ((511*Cot[c + d
*x]^5*(a + a*Sec[c + d*x])^(5/2))/(8*(2 + (a*Tan[c + d*x]^2)/(a + a*Sec[c
+ d*x]))^2) - (3*(((12267*Cot[c + d*x]^5*(a + a*Sec[c + d*x])^(5/2))/10 -
(a*((8171*Cot[c + d*x]^3*(a + a*Sec[c + d*x])^(3/2))/6 + (a*(-1/2*(a*((-16
384*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/Sqrt[a] + (16
363*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])])/(Sq
rt[2]*Sqrt[a]))) + (21*Cot[c + d*x]*Sqrt[a + a*Sec[c + d*x]])/2))/2))/2)/4
 - (2045*Cot[c + d*x]^5*(a + a*Sec[c + d*x])^(5/2))/(4*(2 + (a*Tan[c + d*x
]^2)/(a + a*Sec[c + d*x])))))/8)/12)/16))/(a^4*d)
```

3.194.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 374
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q
 + 1)/(a*e*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(e*x)^m*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[b*c*(m + 1) + 2*(b*c -
a*d)*(p + 1) + d*b*(m + 2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b,
c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IntBinomialQ[a, b,
 c, d, e, m, 2, p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 441
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*(g*x)^(m + 1)*(a
+ b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(a*g*2*(b*c - a*d)*(p + 1))), x] + Si
mp[1/(a*2*(b*c - a*d)*(p + 1))   Int[(g*x)^m*(a + b*x^2)^(p + 1)*(c + d*x^2
)^q*Simp[c*(b*e - a*f)*(m + 1) + e*2*(b*c - a*d)*(p + 1) + d*(b*e - a*f)*(m
 + 2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m, q},
 x] && LtQ[p, -1]
```

rule 445
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^2*(m + 1))
 Int[(g*x)^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*f*c*(m + 1) - e*(b*c
+ a*d)*(m + 2 + 1) - e*2*(b*c*p + a*d*q) - b*e*d*(m + 2*(p + q + 2) + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && LtQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4375
```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Simp[-2*(a^(m/2 + n + 1/2)/d)   Subst[Int[x^m*((2 + a*x^2
)^(m/2 + n - 1/2)/(1 + a*x^2)), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]
]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && I
ntegerQ[n - 1/2]
```

3.194.4 Maple [A] (verified)

Time = 1.59 (sec) , antiderivative size = 577, normalized size of antiderivative = 1.49

method	result
default	$-\frac{\sqrt{a(1+\sec(dx+c))}\left(-245445\sqrt{2}\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}\ \ln\left(\csc(dx+c)-\cot(dx+c)+\sqrt{\cot(dx+c)^2-2\cot(dx+c)\csc(dx+c)+\csc(dx+c)}\right.\right.$

input
```
int(cot(d*x+c)^6/(a+a*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/245760/d/a^2*(a*(1+sec(d*x+c)))^(1/2)/(cos(d*x+c)+1)^2*(-245445*2^(1/2)
*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*ln(csc(d*x+c)-cot(d*x+c)+(cot(d*x+c)^2
-2*cot(d*x+c)*csc(d*x+c)+csc(d*x+c)^2-1)^(1/2))*cos(d*x+c)^2+491520*(-cos(
d*x+c)/(cos(d*x+c)+1))^(1/2)*arctanh(sin(d*x+c)/(cos(d*x+c)+1)/(-cos(d*x+c
)/(cos(d*x+c)+1))^(1/2))*cos(d*x+c)^2-490890*2^(1/2)*ln(csc(d*x+c)-cot(d*x
+c)+(cot(d*x+c)^2-2*cot(d*x+c)*csc(d*x+c)+csc(d*x+c)^2-1)^(1/2))*(-cos(d*x
+c)/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)+983040*(-cos(d*x+c)/(cos(d*x+c)+1))^(
1/2)*arctanh(sin(d*x+c)/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))
*cos(d*x+c)-245445*ln(csc(d*x+c)-cot(d*x+c)+(cot(d*x+c)^2-2*cot(d*x+c)*csc
(d*x+c)+csc(d*x+c)^2-1)^(1/2))*2^(1/2)*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+
491520*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctanh(sin(d*x+c)/(cos(d*x+c)+1
)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))+302082*cos(d*x+c)^2*cot(d*x+c)^5+207
048*cos(d*x+c)*cot(d*x+c)^5-457998*cot(d*x+c)^5-362512*cot(d*x+c)^4*csc(d*
x+c)+195222*cot(d*x+c)^3*csc(d*x+c)^2+164680*cot(d*x+c)^2*csc(d*x+c)^3+630
*cot(d*x+c)*csc(d*x+c)^4)
```

3.194.5 Fricas [A] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 955, normalized size of antiderivative = 2.47

$$\int \frac{\cot^6(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(cot(d*x+c)^6/(a+a*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
[-1/491520*(245445*sqrt(2)*(cos(d*x + c)^6 + 2*cos(d*x + c)^5 - cos(d*x +
c)^4 - 4*cos(d*x + c)^3 - cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(-a)*lo
g((2*sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)
*sin(d*x + c) + 3*a*cos(d*x + c)^2 + 2*a*cos(d*x + c) - a)/(cos(d*x + c)^2
 + 2*cos(d*x + c) + 1))*sin(d*x + c) + 245760*(cos(d*x + c)^6 + 2*cos(d*x
+ c)^5 - cos(d*x + c)^4 - 4*cos(d*x + c)^3 - cos(d*x + c)^2 + 2*cos(d*x +
c) + 1)*sqrt(-a)*log(-(8*a*cos(d*x + c)^3 - 4*(2*cos(d*x + c)^2 - cos(d*x
+ c))*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c) - 7*a*
cos(d*x + c) + a)/(cos(d*x + c) + 1))*sin(d*x + c) + 4*(151041*cos(d*x + c
)^7 + 103524*cos(d*x + c)^6 - 228999*cos(d*x + c)^5 - 181256*cos(d*x + c)^
4 + 97611*cos(d*x + c)^3 + 82340*cos(d*x + c)^2 + 315*cos(d*x + c))*sqrt((
a*cos(d*x + c) + a)/cos(d*x + c)))/((a^2*d*cos(d*x + c)^6 + 2*a^2*d*cos(d*
x + c)^5 - a^2*d*cos(d*x + c)^4 - 4*a^2*d*cos(d*x + c)^3 - a^2*d*cos(d*x +
 c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)*sin(d*x + c)), -1/245760*(245445*sqr
t(2)*(cos(d*x + c)^6 + 2*cos(d*x + c)^5 - cos(d*x + c)^4 - 4*cos(d*x + c)^
3 - cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(a)*arctan(sqrt(2)*sqrt((a*co
s(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c)))*sin(d*x
 + c) + 245760*(cos(d*x + c)^6 + 2*cos(d*x + c)^5 - cos(d*x + c)^4 - 4*cos
(d*x + c)^3 - cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(a)*arctan(2*sqrt(a
)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x + c)/(2*...
```

3.194.6 Sympy [F]

$$\int \frac{\cot^6(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = \int \frac{\cot^6(c+dx)}{(a(\sec(c+dx)+1))^{\frac{3}{2}}}\,dx$$

input
```
integrate(cot(d*x+c)**6/(a+a*sec(d*x+c))**(3/2),x)
```

output
```
Integral(cot(c + d*x)**6/(a*(sec(c + d*x) + 1))**(3/2), x)
```

3.194.7 Maxima [F(-1)]

Timed out.

$$\int \frac{\cot^6(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(cot(d*x+c)^6/(a+a*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
Timed out
```

3.194.8 Giac [A] (verification not implemented)

Time = 1.33 (sec) , antiderivative size = 379, normalized size of antiderivative = 0.98

$$\int \frac{\cot^6(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx =$$

$$5\left(2\left(4\left(\frac{6\sqrt{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2}{a^2\mathrm{sgn}(\cos(dx+c))}-\frac{65\sqrt{2}}{a^2\mathrm{sgn}(\cos(dx+c))}\right)\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+\frac{1451\sqrt{2}}{a^2\mathrm{sgn}(\cos(dx+c))}\right)\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-\frac{13}{a^2\mathrm{sgn}(}$$

$$-$$

input
```
integrate(cot(d*x+c)^6/(a+a*sec(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
-1/245760*(5*(2*(4*(6*sqrt(2)*tan(1/2*d*x + 1/2*c)^2/(a^2*sgn(cos(d*x + c)
)) - 65*sqrt(2)/(a^2*sgn(cos(d*x + c))))*tan(1/2*d*x + 1/2*c)^2 + 1451*sqr
t(2)/(a^2*sgn(cos(d*x + c))))*tan(1/2*d*x + 1/2*c)^2 - 13503*sqrt(2)/(a^2*
sgn(cos(d*x + c))))*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)*tan(1/2*d*x + 1/2*
c) + 256*sqrt(2)*(555*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x
 + 1/2*c)^2 + a))^8 - 1950*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/
2*d*x + 1/2*c)^2 + a))^6*a + 2780*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a
*tan(1/2*d*x + 1/2*c)^2 + a))^4*a^2 - 1810*(sqrt(-a)*tan(1/2*d*x + 1/2*c)
- sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2*a^3 + 473*a^4)/(((sqrt(-a)*tan(1/
2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2 - a)^5*sqrt(-a)*sg
n(cos(d*x + c))))/d
```

3.194.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cot^6(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = \int \frac{\cot(c+d\,x)^6}{\left(a+\frac{a}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

input
```
int(cot(c + d*x)^6/(a + a/cos(c + d*x))^(3/2),x)
```

output
```
int(cot(c + d*x)^6/(a + a/cos(c + d*x))^(3/2), x)
```

3.195 $\int \frac{\tan^5(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx$

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3.195.2 Mathematica [A] (verified) 1451
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3.195.1 Optimal result

Integrand size = 23, antiderivative size = 78

$$\int \frac{\tan^5(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = -\frac{2\operatorname{arctanh}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{a}}\right)}{a^{5/2}d} - \frac{6\sqrt{a+a\sec(c+dx)}}{a^3 d} + \frac{2(a+a\sec(c+dx))^{3/2}}{3a^4 d}$$

output
```
-2*arctanh((a+a*sec(d*x+c))^(1/2)/a^(1/2))/a^(5/2)/d+2/3*(a+a*sec(d*x+c))^
(3/2)/a^4/d-6*(a+a*sec(d*x+c))^(1/2)/a^3/d
```

3.195.2 Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.88

$$\int \frac{\tan^5(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = \frac{2\left(-8-7\sec(c+dx)+\sec^2(c+dx)-3\operatorname{arctanh}\left(\sqrt{1+\sec(c+dx)}\right)\sqrt{1+\sec(c+dx)}\right)}{3a^2 d\sqrt{a(1+\sec(c+dx))}}$$

input
```
Integrate[Tan[c + d*x]^5/(a + a*Sec[c + d*x])^(5/2),x]
```

output
```
(2*(-8 - 7*Sec[c + d*x] + Sec[c + d*x]^2 - 3*ArcTanh[Sqrt[1 + Sec[c + d*x]
]]*Sqrt[1 + Sec[c + d*x]]))/(3*a^2*d*Sqrt[a*(1 + Sec[c + d*x])])
```

3.195.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.97, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3042, 25, 4368, 27, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^5(c+dx)}{(a\sec(c+dx)+a)^{5/2}}\,dx$$

$$\downarrow 3042$$

$$\int -\frac{\cot\left(c+dx+\frac{\pi}{2}\right)^5}{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^{5/2}}dx$$

$$\downarrow 25$$

$$-\int \frac{\cot\left(\frac{1}{2}(2c+\pi)+dx\right)^5}{\left(\csc\left(\frac{1}{2}(2c+\pi)+dx\right)a+a\right)^{5/2}}dx$$

$$\downarrow 4368$$

$$\frac{\int \frac{a^2\cos(c+dx)(1-\sec(c+dx))^2}{\sqrt{\sec(c+dx)a+a}}d\sec(c+dx)}{a^4 d}$$

$$\downarrow 27$$

$$\frac{\int \frac{\cos(c+dx)(1-\sec(c+dx))^2}{\sqrt{\sec(c+dx)a+a}}d\sec(c+dx)}{a^2 d}$$

$$\downarrow 99$$

$$\frac{\int \left(\frac{\cos(c+dx)}{\sqrt{\sec(c+dx)a+a}}+\frac{\sqrt{\sec(c+dx)a+a}}{a}-\frac{3}{\sqrt{\sec(c+dx)a+a}}\right)d\sec(c+dx)}{a^2 d}$$

$$\downarrow 2009$$

$$\frac{\frac{2(a\sec(c+dx)+a)^{3/2}}{3a^2}-\frac{2\operatorname{arctanh}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{a}}\right)}{\sqrt{a}}-\frac{6\sqrt{a\sec(c+dx)+a}}{a}}{a^2 d}$$

input
```
Int[Tan[c + d*x]^5/(a + a*Sec[c + d*x])^(5/2),x]
```

output
```
((-2*ArcTanh[Sqrt[a + a*Sec[c + d*x]]/Sqrt[a]])/Sqrt[a] - (6*Sqrt[a + a*Se
c[c + d*x]])/a + (2*(a + a*Sec[c + d*x])^(3/2))/(3*a^2))/(a^2*d)
```

3.195.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4368
```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Simp[-(d*b^(m - 1))^(-1)   Subst[Int[(-a + b*x)^((m - 1)/2
)*((a + b*x)^((m - 1)/2 + n)/x), x], x, Csc[c + d*x]], x] /; FreeQ[{a, b, c
, d, n}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[n]
```

3.195.4 Maple [A] (verified)

Time = 3.75 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.92

method	result	size
default	$\frac{2\sqrt{a(1+\sec(dx+c))}\left(3\arctan\left(\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}\right)\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}-8+\sec(dx+c)\right)}{3da^3}$	72

input
```
int(tan(d*x+c)^5/(a+a*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/3/d/a^3*(a*(1+sec(d*x+c)))^(1/2)*(3*arctan((-cos(d*x+c)/(cos(d*x+c)+1))^
(1/2))*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)-8+sec(d*x+c))
```

3.195.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 241, normalized size of antiderivative = 3.09

$$\int \frac{\tan^5(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = \left[\frac{3\sqrt{a}\cos(dx+c)\log\left(-8\,a\cos(dx+c)^2+4\left(2\cos(dx+c)^2+\cos(dx+c)\right.\right.}{6\,a^3d}\right.$$

input
```
integrate(tan(d*x+c)^5/(a+a*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
[1/6*(3*sqrt(a)*cos(d*x + c)*log(-8*a*cos(d*x + c)^2 + 4*(2*cos(d*x + c)^2
 + cos(d*x + c))*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)) - 8*a*cos
(d*x + c) - a) - 4*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*(8*cos(d*x + c)
 - 1))/(a^3*d*cos(d*x + c)), 1/3*(3*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos
(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(2*a*cos(d*x + c) + a))*cos(d*x
+ c) - 2*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*(8*cos(d*x + c) - 1))/(a^
3*d*cos(d*x + c))]
```

3.195.6 Sympy [F]

$$\int \frac{\tan^5(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = \int \frac{\tan^5(c+dx)}{(a(\sec(c+dx)+1))^{\frac{5}{2}}}\,dx$$

input
```
integrate(tan(d*x+c)**5/(a+a*sec(d*x+c))**(5/2),x)
```

output
```
Integral(tan(c + d*x)**5/(a*(sec(c + d*x) + 1))**(5/2), x)
```

3.195.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 163 vs. 2(66) = 132.

Time = 0.28 (sec) , antiderivative size = 163, normalized size of antiderivative = 2.09

$$\int \frac{\tan^5(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = \frac{\frac{3\log\left(\frac{\sqrt{a+\frac{a}{\cos(dx+c)}}-\sqrt{a}}{\sqrt{a+\frac{a}{\cos(dx+c)}}+\sqrt{a}}\right)}{a^{\frac{5}{2}}} + \frac{2\left(a+\frac{a}{\cos(dx+c)}\right)^{\frac{3}{2}}}{a^4} - \frac{18\sqrt{a+\frac{a}{\cos(dx+c)}}}{a^3} + \frac{2\left(4a+\frac{3a}{\cos(dx+c)}\right)}{\left(a+\frac{a}{\cos(dx+c)}\right)^{\frac{3}{2}}a^2} - \frac{}{\sqrt{}}}{3d}$$

input
```
integrate(tan(d*x+c)^5/(a+a*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
1/3*(3*log((sqrt(a + a/cos(d*x + c)) - sqrt(a))/(sqrt(a + a/cos(d*x + c))
+ sqrt(a)))/a^(5/2) + 2*(a + a/cos(d*x + c))^(3/2)/a^4 - 18*sqrt(a + a/cos
(d*x + c))/a^3 + 2*(4*a + 3*a/cos(d*x + c))/((a + a/cos(d*x + c))^(3/2)*a^
2) - 6/(sqrt(a + a/cos(d*x + c))*a^2) - 2/((a + a/cos(d*x + c))^(3/2)*a))/
d
```

3.195.8 Giac [A] (verification not implemented)

Time = 2.79 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.62

$$\int \frac{\tan^5(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = \frac{2\left(\frac{3\arctan\left(\frac{\sqrt{2}\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}}{2\sqrt{-a}}\right)}{\sqrt{-a}a^2\operatorname{sgn}(\cos(dx+c))} - \frac{\sqrt{2}\left(9a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-7a\right)}{\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-a\right)\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}a^2\operatorname{sgn}(c}\right.}{3d}$$

input
```
integrate(tan(d*x+c)^5/(a+a*sec(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
2/3*(3*arctan(1/2*sqrt(2)*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)/sqrt(-a))/(s
qrt(-a)*a^2*sgn(cos(d*x + c))) - sqrt(2)*(9*a*tan(1/2*d*x + 1/2*c)^2 - 7*a
)/((a*tan(1/2*d*x + 1/2*c)^2 - a)*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)*a^2*
sgn(cos(d*x + c))))/d
```

3.195.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\tan^5(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = \int \frac{\tan(c+d\,x)^5}{\left(a+\frac{a}{\cos(c+d\,x)}\right)^{5/2}}\,dx$$

input
```
int(tan(c + d*x)^5/(a + a/cos(c + d*x))^(5/2),x)
```

output
```
int(tan(c + d*x)^5/(a + a/cos(c + d*x))^(5/2), x)
```

3.196 $\int \frac{\tan^3(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx$

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3.196.1 Optimal result

Integrand size = 23, antiderivative size = 54

$$\int \frac{\tan^3(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = \frac{2\operatorname{arctanh}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{a}}\right)}{a^{5/2}d} - \frac{4}{a^2 d\sqrt{a+a\sec(c+dx)}}$$

output
```
2*arctanh((a+a*sec(d*x+c))^(1/2)/a^(1/2))/a^(5/2)/d-4/a^2/d/(a+a*sec(d*x+c
))^(1/2)
```

3.196.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.93

$$\int \frac{\tan^3(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = \frac{2\left(-2+\operatorname{arctanh}\left(\sqrt{1+\sec(c+dx)}\right)\sqrt{1+\sec(c+dx)}\right)}{a^2 d\sqrt{a(1+\sec(c+dx))}}$$

input
```
Integrate[Tan[c + d*x]^3/(a + a*Sec[c + d*x])^(5/2),x]
```

output
```
(2*(-2 + ArcTanh[Sqrt[1 + Sec[c + d*x]]]*Sqrt[1 + Sec[c + d*x]]))/(a^2*d*S
qrt[a*(1 + Sec[c + d*x])])
```

3.196.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.04, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {3042, 25, 4368, 25, 27, 87, 73, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^3(c+dx)}{(a\sec(c+dx)+a)^{5/2}}\,dx$$

$$\downarrow 3042$$

$$\int -\frac{\cot\left(c+dx+\frac{\pi}{2}\right)^3}{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^{5/2}}dx$$

$$\downarrow 25$$

$$-\int \frac{\cot\left(\frac{1}{2}(2c+\pi)+dx\right)^3}{\left(\csc\left(\frac{1}{2}(2c+\pi)+dx\right)a+a\right)^{5/2}}dx$$

$$\downarrow 4368$$

$$\frac{\int -\frac{a\cos(c+dx)(1-\sec(c+dx))}{(\sec(c+dx)a+a)^{3/2}}d\sec(c+dx)}{a^2d}$$

$$\downarrow 25$$

$$-\frac{\int \frac{a\cos(c+dx)(1-\sec(c+dx))}{(\sec(c+dx)a+a)^{3/2}}d\sec(c+dx)}{a^2d}$$

$$\downarrow 27$$

$$-\frac{\int \frac{\cos(c+dx)(1-\sec(c+dx))}{(\sec(c+dx)a+a)^{3/2}}d\sec(c+dx)}{ad}$$

$$\downarrow 87$$

$$-\frac{\frac{\int \frac{\cos(c+dx)}{\sqrt{\sec(c+dx)a+a}}d\sec(c+dx)}{a}+\frac{4}{a\sqrt{a\sec(c+dx)+a}}}{ad}$$

$$\downarrow 73$$

$$-\frac{\frac{2\int \frac{1}{\frac{\sec(c+dx)a+a}{a}-1}d\sqrt{\sec(c+dx)a+a}}{a^2}+\frac{4}{a\sqrt{a\sec(c+dx)+a}}}{ad}$$

$$\downarrow 220$$

$$\frac{-\frac{4}{a\sqrt{a\sec(c+dx)+a}}-\frac{2\operatorname{arctanh}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{a}}\right)}{a^{3/2}}}{ad}$$

input
```
Int[Tan[c + d*x]^3/(a + a*Sec[c + d*x])^(5/2),x]
```

output
```
-(((-2*ArcTanh[Sqrt[a + a*Sec[c + d*x]]/Sqrt[a]])/a^(3/2) + 4/(a*Sqrt[a +
a*Sec[c + d*x]]))/(a*d))
```

3.196.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4368
```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Simp[-(d*b^(m - 1))^(-1)   Subst[Int[(-a + b*x)^((m - 1)/2
)*((a + b*x)^((m - 1)/2 + n)/x), x], x, Csc[c + d*x]], x] /; FreeQ[{a, b, c
, d, n}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[n]
```

3.196.4 Maple [A] (verified)

Time = 1.78 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.61

method	result	size
default	$\frac{2\sqrt{a(1+\sec(dx+c))}\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}\left(2\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}-\arctan\left(\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}\right)\right)}{d\,a^3}$	87

input
```
int(tan(d*x+c)^3/(a+a*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/d/a^3*(a*(1+sec(d*x+c)))^(1/2)*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(2*(-c
os(d*x+c)/(cos(d*x+c)+1))^(1/2)-arctan((-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))
)
```

3.196.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 113 vs. 2(46) = 92.

Time = 0.31 (sec) , antiderivative size = 245, normalized size of antiderivative = 4.54

$$\int \frac{\tan^3(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = \left[\frac{\sqrt{a}(\cos(dx+c)+1)\log\left(-8\,a\cos(dx+c)^2-4\left(2\cos(dx+c)^2+\cos(dx\right.\right.}{2\left(a^3d\cos(d\right.}$$

$$\left.\left.-\frac{\sqrt{-a}(\cos(dx+c)+1)\arctan\left(\frac{2\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)}{2\,a\cos(dx+c)+a}\right)+4\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)}{a^3d\cos(dx+c)+a^3d}\right]\right.$$

input
```
integrate(tan(d*x+c)^3/(a+a*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
[1/2*(sqrt(a)*(cos(d*x + c) + 1)*log(-8*a*cos(d*x + c)^2 - 4*(2*cos(d*x +
c)^2 + cos(d*x + c))*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)) - 8*a
*cos(d*x + c) - a) - 8*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c
))/(a^3*d*cos(d*x + c) + a^3*d), -(sqrt(-a)*(cos(d*x + c) + 1)*arctan(2*sq
rt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(2*a*cos(d*x +
 c) + a)) + 4*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c))/(a^3*d
*cos(d*x + c) + a^3*d)]
```

3.196.6 Sympy [F]

$$\int \frac{\tan^3(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = \int \frac{\tan^3(c+dx)}{(a(\sec(c+dx)+1))^{\frac{5}{2}}}\,dx$$

input
```
integrate(tan(d*x+c)**3/(a+a*sec(d*x+c))**(5/2),x)
```

output
```
Integral(tan(c + d*x)**3/(a*(sec(c + d*x) + 1))**(5/2), x)
```

3.196.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 125 vs. $2(46) = 92$.

Time $= 0.28$ (sec) , antiderivative size $= 125$, normalized size of antiderivative $= 2.31$

$$\int \frac{\tan^3(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = -\frac{\frac{3\log\left(\frac{\sqrt{a+\frac{a}{\cos(dx+c)}}-\sqrt{a}}{\sqrt{a+\frac{a}{\cos(dx+c)}}+\sqrt{a}}\right)}{a^{\frac{5}{2}}} + \frac{2\left(4a+\frac{3a}{\cos(dx+c)}\right)}{\left(a+\frac{a}{\cos(dx+c)}\right)^{\frac{3}{2}}a^2} + \frac{6}{\sqrt{a+\frac{a}{\cos(dx+c)}}a^2} - \frac{2}{\left(a+\frac{a}{\cos(dx+c)}\right)^{\frac{3}{2}}a}}{3d}$$

input
```
integrate(tan(d*x+c)^3/(a+a*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
-1/3*(3*log((sqrt(a + a/cos(d*x + c)) - sqrt(a))/(sqrt(a + a/cos(d*x + c))
 + sqrt(a)))/a^(5/2) + 2*(4*a + 3*a/cos(d*x + c))/((a + a/cos(d*x + c))^(3
/2)*a^2) + 6/(sqrt(a + a/cos(d*x + c))*a^2) - 2/((a + a/cos(d*x + c))^(3/2
)*a))/d
```

3.196.8 Giac [A] (verification not implemented)

Time = 1.71 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.67

$$\int \frac{\tan^3(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = -\frac{2\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a}}{2\sqrt{-a}}\right)}{\sqrt{-a}a\mathrm{sgn}(\cos(dx+c))}+\frac{\sqrt{2}\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a}}{a^2\mathrm{sgn}(\cos(dx+c))}\right)}{ad}$$

input
```
integrate(tan(d*x+c)^3/(a+a*sec(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
-2*(arctan(1/2*sqrt(2)*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)/sqrt(-a))/(sqrt
(-a)*a*sgn(cos(d*x + c))) + sqrt(2)*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)/(a
^2*sgn(cos(d*x + c))))/(a*d)
```

3.196.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\tan^3(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = \int \frac{\tan(c+d\,x)^3}{\left(a+\frac{a}{\cos(c+d\,x)}\right)^{5/2}}\,dx$$

input
```
int(tan(c + d*x)^3/(a + a/cos(c + d*x))^(5/2),x)
```

output
```
int(tan(c + d*x)^3/(a + a/cos(c + d*x))^(5/2), x)
```

3.197 $\int \frac{\tan(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx$

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3.197.1 Optimal result

Integrand size = 21, antiderivative size = 78

$$\int \frac{\tan(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = -\frac{2\operatorname{arctanh}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{a}}\right)}{a^{5/2}d} + \frac{2}{3ad(a+a\sec(c+dx))^{3/2}} + \frac{2}{a^2d\sqrt{a+a\sec(c+dx)}}$$

output
```
-2*arctanh((a+a*sec(d*x+c))^(1/2)/a^(1/2))/a^(5/2)/d+2/3/a/d/(a+a*sec(d*x+
c))^(3/2)+2/a^2/d/(a+a*sec(d*x+c))^(1/2)
```

3.197.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.08 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.51

$$\int \frac{\tan(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = \frac{2\,\text{Hypergeometric2F1}\left(-\frac{3}{2},1,-\frac{1}{2},1+\sec(c+dx)\right)}{3ad(a(1+\sec(c+dx)))^{3/2}}$$

input
```
Integrate[Tan[c + d*x]/(a + a*Sec[c + d*x])^(5/2),x]
```

output
```
(2*Hypergeometric2F1[-3/2, 1, -1/2, 1 + Sec[c + d*x]])/(3*a*d*(a*(1 + Sec[
c + d*x]))^(3/2))
```

3.197.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 25, 4368, 61, 61, 73, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan(c+dx)}{(a\sec(c+dx)+a)^{5/2}}\,dx$$

$$\downarrow 3042$$

$$\int -\frac{\cot\left(c+dx+\frac{\pi}{2}\right)}{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^{5/2}}dx$$

$$\downarrow 25$$

$$-\int \frac{\cot\left(\frac{1}{2}(2c+\pi)+dx\right)}{\left(\csc\left(\frac{1}{2}(2c+\pi)+dx\right)a+a\right)^{5/2}}dx$$

$$\downarrow 4368$$

$$\frac{\int \frac{\cos(c+dx)}{(\sec(c+dx)a+a)^{5/2}}\,d\sec(c+dx)}{d}$$

$$\downarrow 61$$

$$\frac{\frac{\int \frac{\cos(c+dx)}{(\sec(c+dx)a+a)^{3/2}}\,d\sec(c+dx)}{a}+\frac{2}{3a(a\sec(c+dx)+a)^{3/2}}}{d}$$

$$\downarrow 61$$

$$\frac{\frac{\frac{\int \frac{\cos(c+dx)}{\sqrt{\sec(c+dx)a+a}}\,d\sec(c+dx)}{a}+\frac{2}{a\sqrt{a\sec(c+dx)+a}}}{a}+\frac{2}{3a(a\sec(c+dx)+a)^{3/2}}}{d}$$

$$\downarrow 73$$

$$\frac{\frac{\frac{2\int \frac{1}{\frac{\sec(c+dx)a+a}{a}-1}\,d\sqrt{\sec(c+dx)a+a}}{a^2}+\frac{2}{a\sqrt{a\sec(c+dx)+a}}}{a}+\frac{2}{3a(a\sec(c+dx)+a)^{3/2}}}{d}$$

$$\downarrow 220$$

$$\frac{\frac{\frac{2}{a\sqrt{a\sec(c+dx)+a}}-\frac{2\operatorname{arctanh}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{a}}\right)}{a^{3/2}}}{a}+\frac{2}{3a(a\sec(c+dx)+a)^{3/2}}}{d}$$

input
```
Int[Tan[c + d*x]/(a + a*Sec[c + d*x])^(5/2),x]
```

output
```
(2/(3*a*(a + a*Sec[c + d*x])^(3/2)) + ((-2*ArcTanh[Sqrt[a + a*Sec[c + d*x]
]/Sqrt[a]])/a^(3/2) + 2/(a*Sqrt[a + a*Sec[c + d*x]]))/a)/d
```

3.197.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4368
```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Simp[-(d*b^(m - 1))^(-1)   Subst[Int[(-a + b*x)^((m - 1)/2
)*((a + b*x)^((m - 1)/2 + n)/x), x], x, Csc[c + d*x]], x] /; FreeQ[{a, b, c
, d, n}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[n]
```

3.197.4 Maple [A] (verified)

Time = 0.88 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.79

method	result	size
derivativedivides	$\frac{-\frac{2\operatorname{arctanh}\left(\frac{\sqrt{a+a\sec(dx+c)}}{\sqrt{a}}\right)}{a^{\frac{5}{2}}}+\frac{2}{a^2\sqrt{a+a\sec(dx+c)}}+\frac{2}{3a(a+a\sec(dx+c))^{\frac{3}{2}}}}{d}$	62
default	$\frac{-\frac{2\operatorname{arctanh}\left(\frac{\sqrt{a+a\sec(dx+c)}}{\sqrt{a}}\right)}{a^{\frac{5}{2}}}+\frac{2}{a^2\sqrt{a+a\sec(dx+c)}}+\frac{2}{3a(a+a\sec(dx+c))^{\frac{3}{2}}}}{d}$	62

input
```
int(tan(d*x+c)/(a+a*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/d*(-2/a^(5/2)*arctanh((a+a*sec(d*x+c))^(1/2)/a^(1/2))+2/a^2/(a+a*sec(d*x
+c))^(1/2)+2/3/a/(a+a*sec(d*x+c))^(3/2))
```

3.197.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 151 vs. $2(66) = 132$.

Time = 0.30 (sec) , antiderivative size = 321, normalized size of antiderivative = 4.12

$$\int \frac{\tan(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = \left[\frac{3\left(\cos\left(dx+c\right)^2+2\,\cos\left(dx+c\right)+1\right)\sqrt{a}\log\left(-8\,a\cos\left(dx+c\right)^2+4\left(2\,\cos\right.\right.}{6}\right.$$

input
```
integrate(tan(d*x+c)/(a+a*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
[1/6*(3*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(a)*log(-8*a*cos(d*x + c
)^2 + 4*(2*cos(d*x + c)^2 + cos(d*x + c))*sqrt(a)*sqrt((a*cos(d*x + c) + a
)/cos(d*x + c)) - 8*a*cos(d*x + c) - a) + 4*(4*cos(d*x + c)^2 + 3*cos(d*x
+ c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)))/(a^3*d*cos(d*x + c)^2 + 2*a
^3*d*cos(d*x + c) + a^3*d), 1/3*(3*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*s
qrt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x
+ c)/(2*a*cos(d*x + c) + a)) + 2*(4*cos(d*x + c)^2 + 3*cos(d*x + c))*sqrt(
(a*cos(d*x + c) + a)/cos(d*x + c)))/(a^3*d*cos(d*x + c)^2 + 2*a^3*d*cos(d*
x + c) + a^3*d)]
```

3.197.6 Sympy [F]

$$\int \frac{\tan(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = \int \frac{\tan(c+dx)}{(a(\sec(c+dx)+1))^{\frac{5}{2}}}\,dx$$

input
```
integrate(tan(d*x+c)/(a+a*sec(d*x+c))**(5/2),x)
```

output
```
Integral(tan(c + d*x)/(a*(sec(c + d*x) + 1))**(5/2), x)
```

3.197.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.12

$$\int \frac{\tan(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = \frac{\frac{3\log\left(\frac{\sqrt{a+\frac{a}{\cos(dx+c)}}-\sqrt{a}}{\sqrt{a+\frac{a}{\cos(dx+c)}}+\sqrt{a}}\right)}{a^{\frac{5}{2}}} + \frac{2\left(4a+\frac{3a}{\cos(dx+c)}\right)}{\left(a+\frac{a}{\cos(dx+c)}\right)^{\frac{3}{2}}a^2}}{3d}$$

input
```
integrate(tan(d*x+c)/(a+a*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
1/3*(3*log((sqrt(a + a/cos(d*x + c)) - sqrt(a))/(sqrt(a + a/cos(d*x + c))
+ sqrt(a)))/a^(5/2) + 2*(4*a + 3*a/cos(d*x + c))/((a + a/cos(d*x + c))^(3/
2)*a^2))/d
```

3.197.8 Giac [A] (verification not implemented)

Time = 1.23 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.49

$$\int \frac{\tan(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = \frac{\frac{12\arctan\left(\frac{\sqrt{2}\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}}{2\sqrt{-a}}\right)}{\sqrt{-a}a^2\mathrm{sgn}(\cos(dx+c))} + \frac{\sqrt{2}\left(\left(-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a\right)^{\frac{3}{2}}a^8+6\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right.}{a^{12}\mathrm{sgn}(\cos(dx+c))}}{6d}$$

input
```
integrate(tan(d*x+c)/(a+a*sec(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
1/6*(12*arctan(1/2*sqrt(2)*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)/sqrt(-a))/(
sqrt(-a)*a^2*sgn(cos(d*x + c))) + sqrt(2)*((-a*tan(1/2*d*x + 1/2*c)^2 + a)
^(3/2)*a^8 + 6*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)*a^9)/(a^12*sgn(cos(d*x
+ c))))/d
```

3.197.9 Mupad [B] (verification not implemented)

Time = 14.76 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.88

$$\int \frac{\tan(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = \frac{\frac{2\left(a+\frac{a}{\cos(c+d\,x)}\right)}{a^2}+\frac{2}{3\,a}}{d\left(a+\frac{a}{\cos(c+d\,x)}\right)^{3/2}} - \frac{2\,\mathrm{atanh}\left(\frac{\sqrt{a+\frac{a}{\cos(c+d\,x)}}}{\sqrt{a}}\right)}{a^{5/2}\,d}$$

input
```
int(tan(c + d*x)/(a + a/cos(c + d*x))^(5/2),x)
```

output
```
((2*(a + a/cos(c + d*x)))/a^2 + 2/(3*a))/(d*(a + a/cos(c + d*x))^(3/2)) -
(2*atanh((a + a/cos(c + d*x))^(1/2)/a^(1/2)))/(a^(5/2)*d)
```

3.198 $\int \frac{\cot(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx$

3.198.1 Optimal result 1469
3.198.2 Mathematica [C] (verified) 1469
3.198.3 Rubi [A] (verified) 1470
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3.198.8 Giac [A] (verification not implemented) 1476
3.198.9 Mupad [F(-1)] 1476

3.198.1 Optimal result

Integrand size = 21, antiderivative size = 144

$$\int \frac{\cot(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = \frac{2\operatorname{arctanh}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{a}}\right)}{a^{5/2}d} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{4\sqrt{2}a^{5/2}d} - \frac{1}{5d(a+a\sec(c+dx))^{5/2}} - \frac{1}{2ad(a+a\sec(c+dx))^{3/2}} - \frac{7}{4a^2d\sqrt{a+a\sec(c+dx)}}$$

output
```
2*arctanh((a+a*sec(d*x+c))^(1/2)/a^(1/2))/a^(5/2)/d-1/5/d/(a+a*sec(d*x+c))
^(5/2)-1/2/a/d/(a+a*sec(d*x+c))^(3/2)-1/8*arctanh(1/2*(a+a*sec(d*x+c))^(1/
2)*2^(1/2)/a^(1/2))/a^(5/2)/d*2^(1/2)-7/4/a^2/d/(a+a*sec(d*x+c))^(1/2)
```

3.198.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.14 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.42

$$\int \frac{\cot(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = \frac{\text{Hypergeometric2F1}\left(-\frac{5}{2},1,-\frac{3}{2},\frac{1}{2}(1+\sec(c+dx))\right) - 2\,\text{Hypergeometric2F1}}{5d(a(1+\sec(c+dx)))^{5/2}}$$

input
```
Integrate[Cot[c + d*x]/(a + a*Sec[c + d*x])^(5/2),x]
```

output
```
(Hypergeometric2F1[-5/2, 1, -3/2, (1 + Sec[c + d*x])/2] - 2*Hypergeometric
2F1[-5/2, 1, -3/2, 1 + Sec[c + d*x]])/(5*d*(a*(1 + Sec[c + d*x]))^(5/2))
```

3.198.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.07, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.762$, Rules used = {3042, 25, 4368, 25, 27, 96, 25, 27, 169, 27, 169, 27, 174, 73, 219, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot(c+dx)}{(a\sec(c+dx)+a)^{5/2}}\,dx$$

$$\downarrow 3042$$

$$\int -\frac{1}{\cot\left(c+dx+\frac{\pi}{2}\right)\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^{5/2}}dx$$

$$\downarrow 25$$

$$-\int \frac{1}{\cot\left(\frac{1}{2}(2c+\pi)+dx\right)\left(\csc\left(\frac{1}{2}(2c+\pi)+dx\right)a+a\right)^{5/2}}dx$$

$$\downarrow 4368$$

$$\frac{a^2\int -\frac{\cos(c+dx)}{a(1-\sec(c+dx))(\sec(c+dx)a+a)^{7/2}}d\sec(c+dx)}{d}$$

$$\downarrow 25$$

$$-\frac{a^2\int \frac{\cos(c+dx)}{a(1-\sec(c+dx))(\sec(c+dx)a+a)^{7/2}}d\sec(c+dx)}{d}$$

$$\downarrow 27$$

$$-\frac{a\int \frac{\cos(c+dx)}{(1-\sec(c+dx))(\sec(c+dx)a+a)^{7/2}}d\sec(c+dx)}{d}$$

$$\downarrow 96$$

$$-\frac{a\left(\frac{1}{5a(a\sec(c+dx)+a)^{5/2}}-\frac{\int -\frac{a\cos(c+dx)(2-\sec(c+dx))}{(1-\sec(c+dx))(\sec(c+dx)a+a)^{5/2}}d\sec(c+dx)}{2a^2}\right)}{d}$$

$$\downarrow 25$$

$$-\frac{a\left(\frac{\int \frac{a\cos(c+dx)(2-\sec(c+dx))}{(1-\sec(c+dx))(\sec(c+dx)a+a)^{5/2}}d\sec(c+dx)}{2a^2}+\frac{1}{5a(a\sec(c+dx)+a)^{5/2}}\right)}{d}$$

$$\downarrow 27$$

$$-\frac{a\left(\frac{\int \frac{\cos(c+dx)(2-\sec(c+dx))}{(1-\sec(c+dx))(\sec(c+dx)a+a)^{5/2}}\,d\sec(c+dx)}{2a}+\frac{1}{5a(a\sec(c+dx)+a)^{5/2}}\right)}{d}$$

$$\downarrow 169$$

$$-\frac{a\left(\frac{\frac{\int \frac{3a\cos(c+dx)(4-3\sec(c+dx))}{2(1-\sec(c+dx))(\sec(c+dx)a+a)^{3/2}}\,d\sec(c+dx)}{3a^2}+\frac{1}{a(a\sec(c+dx)+a)^{3/2}}}{2a}+\frac{1}{5a(a\sec(c+dx)+a)^{5/2}}\right)}{d}$$

$$\downarrow 27$$

$$-\frac{a\left(\frac{\frac{\int \frac{\cos(c+dx)(4-3\sec(c+dx))}{(1-\sec(c+dx))(\sec(c+dx)a+a)^{3/2}}\,d\sec(c+dx)}{2a}+\frac{1}{a(a\sec(c+dx)+a)^{3/2}}}{2a}+\frac{1}{5a(a\sec(c+dx)+a)^{5/2}}\right)}{d}$$

$$\downarrow 169$$

$$-\frac{a\left(\frac{\frac{\int \frac{a\cos(c+dx)(8-7\sec(c+dx))}{2(1-\sec(c+dx))\sqrt{\sec(c+dx)a+a}}\,d\sec(c+dx)}{a^2}+\frac{7}{a\sqrt{a\sec(c+dx)+a}}}{2a}+\frac{1}{a(a\sec(c+dx)+a)^{3/2}}+\frac{1}{5a(a\sec(c+dx)+a)^{5/2}}\right)}{d}$$

$$\downarrow 27$$

$$-\frac{a\left(\frac{\frac{\int \frac{\cos(c+dx)(8-7\sec(c+dx))}{(1-\sec(c+dx))\sqrt{\sec(c+dx)a+a}}\,d\sec(c+dx)}{2a}+\frac{7}{a\sqrt{a\sec(c+dx)+a}}}{2a}+\frac{1}{a(a\sec(c+dx)+a)^{3/2}}+\frac{1}{5a(a\sec(c+dx)+a)^{5/2}}\right)}{d}$$

$$\downarrow 174$$

$$-\frac{a\left(\frac{\frac{\int \frac{1}{(1-\sec(c+dx))\sqrt{\sec(c+dx)a+a}}\,d\sec(c+dx)+8\int \frac{\cos(c+dx)}{\sqrt{\sec(c+dx)a+a}}\,d\sec(c+dx)}{2a}+\frac{7}{a\sqrt{a\sec(c+dx)+a}}}{2a}+\frac{1}{a(a\sec(c+dx)+a)^{3/2}}+\frac{1}{5a(a\sec(c+dx)+a)^{5/2}}\right)}{d}$$

$$\downarrow 73$$

$$-\frac{a\left(\frac{2\int \frac{1}{2-\frac{\sec(c+dx)a+a}{a}}d\sqrt{\sec(c+dx)a+a}}{a}+\frac{16\int \frac{1}{\frac{\sec(c+dx)a+a}{a}-1}d\sqrt{\sec(c+dx)a+a}}{a}}{2a}+\frac{7}{a\sqrt{a\sec(c+dx)+a}}}{2a}+\frac{1}{a(a\sec(c+dx)+a)^{3/2}}+\frac{1}{5a(a\sec(c+dx)+a)}}{d}$$

$$\downarrow 219$$

$$-\frac{a\left(\frac{\frac{16\int \frac{1}{\frac{\sec(c+dx)a+a}{a}-1}d\sqrt{\sec(c+dx)a+a}}{a}+\frac{\sqrt{2}\operatorname{arctanh}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{2}\sqrt{a}}\right)}{\sqrt{a}}}{2a}+\frac{7}{a\sqrt{a\sec(c+dx)+a}}}{2a}+\frac{1}{a(a\sec(c+dx)+a)^{3/2}}+\frac{1}{5a(a\sec(c+dx)+a)^{5/2}}}{d}$$

$$\downarrow 220$$

$$-\frac{a\left(\frac{\frac{\sqrt{2}\operatorname{arctanh}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{2}\sqrt{a}}\right)}{\sqrt{a}}-\frac{16\operatorname{arctanh}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{a}}\right)}{\sqrt{a}}}{2a}+\frac{7}{a\sqrt{a\sec(c+dx)+a}}}{2a}+\frac{1}{a(a\sec(c+dx)+a)^{3/2}}+\frac{1}{5a(a\sec(c+dx)+a)^{5/2}}\right)}{d}$$

input
```
Int[Cot[c + d*x]/(a + a*Sec[c + d*x])^(5/2),x]
```

output
```
-((a*(1/(5*a*(a + a*Sec[c + d*x])^(5/2)) + (1/(a*(a + a*Sec[c + d*x])^(3/2
)) + (((-16*ArcTanh[Sqrt[a + a*Sec[c + d*x]]/Sqrt[a]])/Sqrt[a] + (Sqrt[2]*
ArcTanh[Sqrt[a + a*Sec[c + d*x]]/(Sqrt[2]*Sqrt[a])])/Sqrt[a])/(2*a) + 7/(a
*Sqrt[a + a*Sec[c + d*x]]))/(2*a))/(2*a)))/d)
```

3.198.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 96
```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_] :> Simp[f*((e + f*x)^(p + 1)/((p + 1)*(b*e - a*f)*(d*e - c*f))), x] + S
imp[1/((b*e - a*f)*(d*e - c*f))   Int[(b*d*e - b*c*f - a*d*f - b*d*f*x)*((e
 + f*x)^(p + 1)/((a + b*x)*(c + d*x))), x], x] /; FreeQ[{a, b, c, d, e, f},
 x] && LtQ[p, -1]
```

rule 169
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && IntegersQ[
2*m, 2*n, 2*p]
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4368
```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Simp[-(d*b^(m - 1))^(-1)   Subst[Int[(-a + b*x)^((m - 1)/2
)*((a + b*x)^((m - 1)/2 + n)/x), x], x, Csc[c + d*x]], x] /; FreeQ[{a, b, c
, d, n}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[n]
```

3.198.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 377 vs. $2(115) = 230$.

Time = 1.73 (sec) , antiderivative size = 378, normalized size of antiderivative = 2.62

method	result
default	$-\frac{\sqrt{-\frac{2a}{(1-\cos(dx+c))^2\csc(dx+c)^2-1}}\sqrt{(1-\cos(dx+c))^2\csc(dx+c)^2-1}\left(-15\left((1-\cos(dx+c))^2\csc(dx+c)^2-1\right)^{\frac{7}{2}}+15(1-\cos(dx+c))^6$

input
```
int(cot(d*x+c)/(a+a*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/840/d/a^3*(-2*a/((1-cos(d*x+c))^2*csc(d*x+c)^2-1))^(1/2)*((1-cos(d*x+c)
)^2*csc(d*x+c)^2-1)^(1/2)*(-15*((1-cos(d*x+c))^2*csc(d*x+c)^2-1)^(7/2)+15*
(1-cos(d*x+c))^6*((1-cos(d*x+c))^2*csc(d*x+c)^2-1)^(1/2)*csc(d*x+c)^6+21*(
(1-cos(d*x+c))^2*csc(d*x+c)^2-1)^(5/2)-87*(1-cos(d*x+c))^4*((1-cos(d*x+c))
^2*csc(d*x+c)^2-1)^(1/2)*csc(d*x+c)^4-35*((1-cos(d*x+c))^2*csc(d*x+c)^2-1)
^(3/2)+269*(1-cos(d*x+c))^2*((1-cos(d*x+c))^2*csc(d*x+c)^2-1)^(1/2)*csc(d*
x+c)^2+840*2^(1/2)*arctan(1/2*2^(1/2)*((1-cos(d*x+c))^2*csc(d*x+c)^2-1)^(1
/2))+105*arctan(1/((1-cos(d*x+c))^2*csc(d*x+c)^2-1)^(1/2))-932*((1-cos(d*x
+c))^2*csc(d*x+c)^2-1)^(1/2))
```

3.198.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 272 vs. $2(115) = 230$.

Time = 0.36 (sec) , antiderivative size = 573, normalized size of antiderivative = 3.98

$$\int \frac{\cot(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = \left[\frac{5\sqrt{2}\left(\cos\left(dx+c\right)^3+3\cos\left(dx+c\right)^2+3\cos\left(dx+c\right)+1\right)\sqrt{a}\log\left(-\frac{2\sqrt{2}\sqrt{a}}{}\right.\right.$$

input
```
integrate(cot(d*x+c)/(a+a*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
[1/80*(5*sqrt(2)*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + 3*cos(d*x + c) + 1)*
sqrt(a)*log(-(2*sqrt(2)*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*co
s(d*x + c) - 3*a*cos(d*x + c) - a)/(cos(d*x + c) - 1)) + 40*(cos(d*x + c)^
3 + 3*cos(d*x + c)^2 + 3*cos(d*x + c) + 1)*sqrt(a)*log(-8*a*cos(d*x + c)^2
 - 4*(2*cos(d*x + c)^2 + cos(d*x + c))*sqrt(a)*sqrt((a*cos(d*x + c) + a)/c
os(d*x + c)) - 8*a*cos(d*x + c) - a) - 4*(49*cos(d*x + c)^3 + 80*cos(d*x +
 c)^2 + 35*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)))/(a^3*d*c
os(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d), 1/
40*(5*sqrt(2)*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + 3*cos(d*x + c) + 1)*sqr
t(-a)*arctan(sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(
d*x + c)/(a*cos(d*x + c) + a)) - 40*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + 3
*cos(d*x + c) + 1)*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/co
s(d*x + c))*cos(d*x + c)/(2*a*cos(d*x + c) + a)) - 2*(49*cos(d*x + c)^3 +
80*cos(d*x + c)^2 + 35*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c
)))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c)
+ a^3*d)]
```

3.198.6 Sympy [F]

$$\int \frac{\cot(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = \int \frac{\cot\left(c+dx\right)}{\left(a\left(\sec\left(c+dx\right)+1\right)\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(cot(d*x+c)/(a+a*sec(d*x+c))**(5/2),x)
```

output
```
Integral(cot(c + d*x)/(a*(sec(c + d*x) + 1))**(5/2), x)
```

3.198.7 Maxima [F]

$$\int \frac{\cot(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = \int \frac{\cot(dx+c)}{(a\sec(dx+c)+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(cot(d*x+c)/(a+a*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
integrate(cot(d*x + c)/(a*sec(d*x + c) + a)^(5/2), x)
```

3.198.8 Giac [A] (verification not implemented)

Time = 0.93 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.43

$$\int \frac{\cot(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = \frac{5\sqrt{2}\arctan\left(\frac{\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a^2\operatorname{sgn}(\cos(dx+c))} - \frac{80\arctan\left(\frac{\sqrt{2}\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}}{2\sqrt{-a}}\right)}{\sqrt{-a}a^2\operatorname{sgn}(\cos(dx+c))} - \frac{\sqrt{2}\left(\left(a\tan\left(\frac{1}{2}dx\right.\right.\right.}{}$$

input
```
integrate(cot(d*x+c)/(a+a*sec(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
1/40*(5*sqrt(2)*arctan(sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)/sqrt(-a))/(sqrt
(-a)*a^2*sgn(cos(d*x + c))) - 80*arctan(1/2*sqrt(2)*sqrt(-a*tan(1/2*d*x +
1/2*c)^2 + a)/sqrt(-a))/(sqrt(-a)*a^2*sgn(cos(d*x + c))) - sqrt(2)*((a*tan
(1/2*d*x + 1/2*c)^2 - a)^2*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)*a^20 + 5*(-
a*tan(1/2*d*x + 1/2*c)^2 + a)^(3/2)*a^21 + 35*sqrt(-a*tan(1/2*d*x + 1/2*c)
^2 + a)*a^22)/(a^25*sgn(cos(d*x + c))))/d
```

3.198.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cot(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = \int \frac{\cot(c+d\,x)}{\left(a+\frac{a}{\cos(c+d\,x)}\right)^{5/2}}\,dx$$

input
```
int(cot(c + d*x)/(a + a/cos(c + d*x))^(5/2),x)
```

output
```
int(cot(c + d*x)/(a + a/cos(c + d*x))^(5/2), x)
```

3.199 $\int \frac{\cot^3(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx$

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3.199.1 Optimal result

Integrand size = 23, antiderivative size = 200

$$\int \frac{\cot^3(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = -\frac{2\operatorname{arctanh}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{a}}\right)}{a^{5/2}d} + \frac{13\operatorname{arctanh}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{32\sqrt{2}a^{5/2}d} - \frac{5a}{28d(a+a\sec(c+dx))^{7/2}} + \frac{a}{2d(1-\sec(c+dx))(a+a\sec(c+dx))^{7/2}} + \frac{3}{40d(a+a\sec(c+dx))^{5/2}} + \frac{19}{48ad(a+a\sec(c+dx))^{3/2}} + \frac{51}{32a^2d\sqrt{a+a\sec(c+dx)}}$$

output

```
-2*arctanh((a+a*sec(d*x+c))^(1/2)/a^(1/2))/a^(5/2)/d-5/28*a/d/(a+a*sec(d*x
+c))^(7/2)+1/2*a/d/(1-sec(d*x+c))/(a+a*sec(d*x+c))^(7/2)+3/40/d/(a+a*sec(d
*x+c))^(5/2)+19/48/a/d/(a+a*sec(d*x+c))^(3/2)+13/64*arctanh(1/2*(a+a*sec(d
*x+c))^(1/2)*2^(1/2)/a^(1/2))/a^(5/2)/d*2^(1/2)+51/32/a^2/d/(a+a*sec(d*x+c
))^(1/2)
```

3.199.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.29 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.45

$$\int \frac{\cot^3(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = \frac{a\left(-14-13\,\text{Hypergeometric2F1}\left(-\frac{7}{2},1,-\frac{5}{2},\frac{1}{2}(1+\sec(c+dx))\right)\right)(-1+\sec(c}{28d(-1+\sec(c+dx}$$

input
```
Integrate[Cot[c + d*x]^3/(a + a*Sec[c + d*x])^(5/2),x]
```

output
```
(a*(-14 - 13*Hypergeometric2F1[-7/2, 1, -5/2, (1 + Sec[c + d*x])/2]*(-1 +
Sec[c + d*x]) + 8*Hypergeometric2F1[-7/2, 1, -5/2, 1 + Sec[c + d*x]]*(-1 +
 Sec[c + d*x])))/(28*d*(-1 + Sec[c + d*x])*(a*(1 + Sec[c + d*x]))^(7/2))
```

3.199.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.13, number of steps used = 19, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.783$, Rules used = {3042, 25, 4368, 27, 114, 27, 169, 27, 169, 27, 169, 27, 169, 27, 174, 73, 219, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^3(c+dx)}{(a\sec(c+dx)+a)^{5/2}}\,dx$$
$$\downarrow 3042$$
$$\int -\frac{1}{\cot\left(c+dx+\frac{\pi}{2}\right)^3\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^{5/2}}dx$$
$$\downarrow 25$$
$$-\int \frac{1}{\cot\left(\frac{1}{2}(2c+\pi)+dx\right)^3\left(\csc\left(\frac{1}{2}(2c+\pi)+dx\right)a+a\right)^{5/2}}dx$$
$$\downarrow 4368$$
$$\frac{a^4\int \frac{\cos(c+dx)}{a^2(1-\sec(c+dx))^2(\sec(c+dx)a+a)^{9/2}}d\sec(c+dx)}{d}$$
$$\downarrow 27$$

$$\frac{a^2 \int \frac{\cos(c+dx)}{(1-\sec(c+dx))^2(\sec(c+dx)a+a)^{9/2}}\, d\sec(c+dx)}{d}$$

$$\downarrow 114$$

$$\frac{a^2\left(\frac{1}{2a(1-\sec(c+dx))(a\sec(c+dx)+a)^{7/2}} - \frac{\int -\frac{a\cos(c+dx)(9\sec(c+dx)+4)}{2(1-\sec(c+dx))(\sec(c+dx)a+a)^{9/2}}\, d\sec(c+dx)}{2a}\right)}{d}$$

$$\downarrow 27$$

$$\frac{a^2\left(\frac{1}{4}\int \frac{\cos(c+dx)(9\sec(c+dx)+4)}{(1-\sec(c+dx))(\sec(c+dx)a+a)^{9/2}}\, d\sec(c+dx) + \frac{1}{2a(1-\sec(c+dx))(a\sec(c+dx)+a)^{7/2}}\right)}{d}$$

$$\downarrow 169$$

$$\frac{a^2\left(\frac{1}{4}\left(\frac{\int \frac{7a\cos(c+dx)(5\sec(c+dx)+8)}{2(1-\sec(c+dx))(\sec(c+dx)a+a)^{7/2}}\, d\sec(c+dx)}{7a^2} - \frac{5}{7a(a\sec(c+dx)+a)^{7/2}}\right) + \frac{1}{2a(1-\sec(c+dx))(a\sec(c+dx)+a)^{7/2}}\right)}{d}$$

$$\downarrow 27$$

$$\frac{a^2\left(\frac{1}{4}\left(\frac{\int \frac{\cos(c+dx)(5\sec(c+dx)+8)}{(1-\sec(c+dx))(\sec(c+dx)a+a)^{7/2}}\, d\sec(c+dx)}{2a} - \frac{5}{7a(a\sec(c+dx)+a)^{7/2}}\right) + \frac{1}{2a(1-\sec(c+dx))(a\sec(c+dx)+a)^{7/2}}\right)}{d}$$

$$\downarrow 169$$

$$\frac{a^2\left(\frac{1}{4}\left(\frac{\int \frac{5a\cos(c+dx)(16-3\sec(c+dx))}{2(1-\sec(c+dx))(\sec(c+dx)a+a)^{5/2}}\, d\sec(c+dx)}{5a^2} + \frac{3}{5a(a\sec(c+dx)+a)^{5/2}}}{2a} - \frac{5}{7a(a\sec(c+dx)+a)^{7/2}}\right) + \frac{1}{2a(1-\sec(c+dx))(a\sec(c+dx)+a)}\right.}{d}$$

$$\downarrow 27$$

$$\frac{a^2\left(\frac{1}{4}\left(\frac{\int \frac{\cos(c+dx)(16-3\sec(c+dx))}{(1-\sec(c+dx))(\sec(c+dx)a+a)^{5/2}}\, d\sec(c+dx)}{2a} + \frac{3}{5a(a\sec(c+dx)+a)^{5/2}}}{2a} - \frac{5}{7a(a\sec(c+dx)+a)^{7/2}}\right) + \frac{1}{2a(1-\sec(c+dx))(a\sec(c+dx)+a)^{7}}\right.}{d}$$

$$\downarrow 169$$

$$a^2\left(\frac{1}{4}\left(\frac{\frac{\int \frac{3a\cos(c+dx)(32-19\sec(c+dx))}{2(1-\sec(c+dx))(\sec(c+dx)a+a)^{3/2}}\,d\sec(c+dx)}{3a^2}+\frac{19}{3a(a\sec(c+dx)+a)^{3/2}}}{2a}+\frac{3}{5a(a\sec(c+dx)+a)^{5/2}}\right)-\frac{5}{7a(a\sec(c+dx)+a)^{7/2}}\right)+\frac{}{2a(1-\sec(c}$$

$$\Big\downarrow 27$$

$$a^2\left(\frac{1}{4}\left(\frac{\frac{\int \frac{\cos(c+dx)(32-19\sec(c+dx))}{(1-\sec(c+dx))(\sec(c+dx)a+a)^{3/2}}\,d\sec(c+dx)}{2a}+\frac{19}{3a(a\sec(c+dx)+a)^{3/2}}}{2a}+\frac{3}{5a(a\sec(c+dx)+a)^{5/2}}\right)-\frac{5}{7a(a\sec(c+dx)+a)^{7/2}}\right)+\frac{}{2a(1-\sec(c+}$$

$$\Big\downarrow 169$$

$$a^2\left(\frac{1}{4}\left(\frac{\frac{\int \frac{a\cos(c+dx)(64-51\sec(c+dx))}{2(1-\sec(c+dx))\sqrt{\sec(c+dx)a+a}}\,d\sec(c+dx)}{a^2}+\frac{51}{a\sqrt{a\sec(c+dx)+a}}}{2a}+\frac{19}{3a(a\sec(c+dx)+a)^{3/2}}}{2a}+\frac{3}{5a(a\sec(c+dx)+a)^{5/2}}\right)-\frac{5}{7a(a\sec(c+dx)+a)^{7/2}}\right)$$

$$\Big\downarrow 27$$

$$a^2\left(\frac{1}{4}\left(\frac{\frac{\int \frac{\cos(c+dx)(64-51\sec(c+dx))}{(1-\sec(c+dx))\sqrt{\sec(c+dx)a+a}}\,d\sec(c+dx)}{2a}+\frac{51}{a\sqrt{a\sec(c+dx)+a}}}{2a}+\frac{19}{3a(a\sec(c+dx)+a)^{3/2}}}{2a}+\frac{3}{5a(a\sec(c+dx)+a)^{5/2}}\right)-\frac{5}{7a(a\sec(c+dx)+a)^{7/2}}\right)$$

$$\Big\downarrow 174$$

$$a^2\left(\frac{1}{4}\left(\frac{\frac{13\int \frac{1}{(1-\sec(c+dx))\sqrt{\sec(c+dx)a+a}}\,d\sec(c+dx)+64\int \frac{\cos(c+dx)}{\sqrt{\sec(c+dx)a+a}}\,d\sec(c+dx)}{2a}+\frac{51}{a\sqrt{a\sec(c+dx)+a}}}{2a}+\frac{19}{3a(a\sec(c+dx)+a)^{3/2}}}{2a}+\frac{3}{5a(a\sec(c+dx)+a)}\right)$$

3.199. $\int \frac{\cot^3(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx$

$$\downarrow 73$$

$$a^2\left(\frac{1}{4}\left(\frac{\frac{26\int \frac{1}{2-\frac{\sec(c+dx)a+a}{a}}d\sqrt{\sec(c+dx)a+a}}{a}+\frac{\frac{128\int \frac{1}{\frac{\sec(c+dx)a+a}{a}-1}d\sqrt{\sec(c+dx)a+a}}{a}}{2a}+\frac{51}{a\sqrt{a\sec(c+dx)+a}}}{2a}+\frac{19}{3a(a\sec(c+dx)+a)^{3/2}}}{2a}+\frac{3}{5a(a\sec(c+dx)+\right.\right.$$

$$d$$

$$\downarrow 219$$

$$a^2\left(\frac{1}{4}\left(\frac{\frac{128\int \frac{1}{\frac{\sec(c+dx)a+a}{a}-1}d\sqrt{\sec(c+dx)a+a}}{a}+\frac{13\sqrt{2}\operatorname{arctanh}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{2}\sqrt{a}}\right)}{\sqrt{a}}}{2a}+\frac{51}{a\sqrt{a\sec(c+dx)+a}}}{2a}+\frac{19}{3a(a\sec(c+dx)+a)^{3/2}}}{2a}+\frac{3}{5a(a\sec(c+dx)+a)^{5}}\right.\right.$$

$$d$$

$$\downarrow 220$$

$$a^2\left(\frac{1}{4}\left(\frac{\frac{13\sqrt{2}\operatorname{arctanh}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{2}\sqrt{a}}\right)}{\sqrt{a}}-\frac{128\operatorname{arctanh}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{a}}\right)}{\sqrt{a}}}{2a}+\frac{51}{a\sqrt{a\sec(c+dx)+a}}}{2a}+\frac{19}{3a(a\sec(c+dx)+a)^{3/2}}}{2a}+\frac{3}{5a(a\sec(c+dx)+a)^{5/2}}-\right.\right.$$

$$d$$

input
```
Int[Cot[c + d*x]^3/(a + a*Sec[c + d*x])^(5/2),x]
```

output
```
(a^2*(1/(2*a*(1 - Sec[c + d*x])*(a + a*Sec[c + d*x])^(7/2)) + (-5/(7*a*(a
+ a*Sec[c + d*x])^(7/2)) + (3/(5*a*(a + a*Sec[c + d*x])^(5/2)) + (19/(3*a*
(a + a*Sec[c + d*x])^(3/2)) + (((-128*ArcTanh[Sqrt[a + a*Sec[c + d*x]]/Sqr
t[a]])/Sqrt[a] + (13*Sqrt[2]*ArcTanh[Sqrt[a + a*Sec[c + d*x]]/(Sqrt[2]*Sqr
t[a])])/Sqrt[a])/(2*a) + 51/(a*Sqrt[a + a*Sec[c + d*x]]))/(2*a))/(2*a))/(2
*a))/4))/d
```

3.199. $\int \frac{\cot^3(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx$

3.199.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 114
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[1/((m + 1)*(b*c - a*d)*(b*e
 - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1)
 - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (IntegerQ[n] ||
 IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

rule 169
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && IntegersQ[
2*m, 2*n, 2*p]
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4368
```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Simp[-(d*b^(m - 1))^(-1)   Subst[Int[(-a + b*x)^((m - 1)/2
)*((a + b*x)^((m - 1)/2 + n)/x), x], x, Csc[c + d*x]], x] /; FreeQ[{a, b, c
, d, n}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[n]
```

3.199.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 527 vs. $2(163) = 326$.

Time = 1.76 (sec) , antiderivative size = 528, normalized size of antiderivative = 2.64

method	result
default	$\frac{\sqrt{a(1+\sec(dx+c))}\left(1365\cos(dx+c)^3\sqrt{2}\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}\arctan\left(\frac{\sqrt{2}}{2\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}}\right)+13440\cos(dx+c)^3\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}\arctan($

input
```
int(cot(d*x+c)^3/(a+a*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/6720/d/a^3*(a*(1+sec(d*x+c)))^(1/2)/(cos(d*x+c)+1)^3*(1365*cos(d*x+c)^3*
2^(1/2)*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctan(1/2*2^(1/2)/(-cos(d*x+c)
/(cos(d*x+c)+1))^(1/2))+13440*cos(d*x+c)^3*(-cos(d*x+c)/(cos(d*x+c)+1))^(1
/2)*arctan((-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))+4095*cos(d*x+c)^2*arctan(1/
2*2^(1/2)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*2^(1/2)*(-cos(d*x+c)/(cos(d*
x+c)+1))^(1/2)+40320*cos(d*x+c)^2*arctan((-cos(d*x+c)/(cos(d*x+c)+1))^(1/2
))*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+4095*arctan(1/2*2^(1/2)/(-cos(d*x+c)
/(cos(d*x+c)+1))^(1/2))*cos(d*x+c)*2^(1/2)*(-cos(d*x+c)/(cos(d*x+c)+1))^(1
/2)+40320*arctan((-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*cos(d*x+c)*(-cos(d*x+
c)/(cos(d*x+c)+1))^(1/2)+1365*2^(1/2)*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*a
rctan(1/2*2^(1/2)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))-16034*cos(d*x+c)^3*c
ot(d*x+c)^2+13440*arctan((-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*(-cos(d*x+c)/
(cos(d*x+c)+1))^(1/2)-25280*cos(d*x+c)^2*cot(d*x+c)^2+3164*cos(d*x+c)*cot(
d*x+c)^2+24080*cot(d*x+c)^2+10710*cot(d*x+c)*csc(d*x+c))
```

3.199.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 361 vs. $2(161) = 322$.

Time $= 0.38$ (sec) , antiderivative size $= 748$, normalized size of antiderivative $= 3.74$

$$\int \frac{\cot^3(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = \left[\frac{1365\sqrt{2}\left(\cos(dx+c)^5+3\cos(dx+c)^4+2\cos(dx+c)^3-2\cos(dx+c)^2\right.}{\right.}$$

$$-\frac{1365\sqrt{2}\left(\cos(dx+c)^5+3\cos(dx+c)^4+2\cos(dx+c)^3-2\cos(dx+c)^2-3\cos(dx+c)-1\right)\sqrt{-a}\,\mathrm{ar}}{}$$

input
```
integrate(cot(d*x+c)^3/(a+a*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

output

```
[1/13440*(1365*sqrt(2)*(cos(d*x + c)^5 + 3*cos(d*x + c)^4 + 2*cos(d*x + c)
^3 - 2*cos(d*x + c)^2 - 3*cos(d*x + c) - 1)*sqrt(a)*log((2*sqrt(2)*sqrt(a)
*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c) + 3*a*cos(d*x + c) +
 a)/(cos(d*x + c) - 1)) + 6720*(cos(d*x + c)^5 + 3*cos(d*x + c)^4 + 2*cos(
d*x + c)^3 - 2*cos(d*x + c)^2 - 3*cos(d*x + c) - 1)*sqrt(a)*log(-8*a*cos(d
*x + c)^2 + 4*(2*cos(d*x + c)^2 + cos(d*x + c))*sqrt(a)*sqrt((a*cos(d*x +
c) + a)/cos(d*x + c)) - 8*a*cos(d*x + c) - a) + 4*(8017*cos(d*x + c)^5 + 1
2640*cos(d*x + c)^4 - 1582*cos(d*x + c)^3 - 12040*cos(d*x + c)^2 - 5355*co
s(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)))/(a^3*d*cos(d*x + c)^5
 + 3*a^3*d*cos(d*x + c)^4 + 2*a^3*d*cos(d*x + c)^3 - 2*a^3*d*cos(d*x + c)^
2 - 3*a^3*d*cos(d*x + c) - a^3*d), -1/6720*(1365*sqrt(2)*(cos(d*x + c)^5 +
 3*cos(d*x + c)^4 + 2*cos(d*x + c)^3 - 2*cos(d*x + c)^2 - 3*cos(d*x + c) -
 1)*sqrt(-a)*arctan(sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c
))*cos(d*x + c)/(a*cos(d*x + c) + a)) - 6720*(cos(d*x + c)^5 + 3*cos(d*x +
 c)^4 + 2*cos(d*x + c)^3 - 2*cos(d*x + c)^2 - 3*cos(d*x + c) - 1)*sqrt(-a)
*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(2
*a*cos(d*x + c) + a)) - 2*(8017*cos(d*x + c)^5 + 12640*cos(d*x + c)^4 - 15
82*cos(d*x + c)^3 - 12040*cos(d*x + c)^2 - 5355*cos(d*x + c))*sqrt((a*cos(
d*x + c) + a)/cos(d*x + c)))/(a^3*d*cos(d*x + c)^5 + 3*a^3*d*cos(d*x + c)^
4 + 2*a^3*d*cos(d*x + c)^3 - 2*a^3*d*cos(d*x + c)^2 - 3*a^3*d*cos(d*x +...
```

3.199.6 Sympy [F]

$$\int \frac{\cot^3(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = \int \frac{\cot^3(c+dx)}{(a(\sec(c+dx)+1))^{\frac{5}{2}}}\,dx$$

input

```
integrate(cot(d*x+c)**3/(a+a*sec(d*x+c))**(5/2),x)
```

output

```
Integral(cot(c + d*x)**3/(a*(sec(c + d*x) + 1))**(5/2), x)
```

3.199.7 Maxima [F]

$$\int \frac{\cot^3(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = \int \frac{\cot\left(dx+c\right)^3}{\left(a\sec\left(dx+c\right)+a\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(cot(d*x+c)^3/(a+a*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
integrate(cot(d*x + c)^3/(a*sec(d*x + c) + a)^(5/2), x)
```

3.199.8 Giac [A] (verification not implemented)

Time = 1.08 (sec) , antiderivative size = 295, normalized size of antiderivative = 1.48

$$\int \frac{\cot^3(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx =$$
$$-\frac{1365\sqrt{2}\arctan\left(\frac{\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a^2\mathrm{sgn}(\cos(dx+c))} - \frac{13440\arctan\left(\frac{\sqrt{2}\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a}}{2\sqrt{-a}}\right)}{\sqrt{-a}a^2\mathrm{sgn}(\cos(dx+c))} + \frac{105\sqrt{2}\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a}}{a^3\mathrm{sgn}(\cos(dx+c))\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2} + \frac{2\sqrt{2}\left(15\right.}{\qquad}$$

input
```
integrate(cot(d*x+c)^3/(a+a*sec(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
-1/6720*(1365*sqrt(2)*arctan(sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)/sqrt(-a))
/(sqrt(-a)*a^2*sgn(cos(d*x + c))) - 13440*arctan(1/2*sqrt(2)*sqrt(-a*tan(1
/2*d*x + 1/2*c)^2 + a)/sqrt(-a))/(sqrt(-a)*a^2*sgn(cos(d*x + c))) + 105*sq
rt(2)*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)/(a^3*sgn(cos(d*x + c))*tan(1/2*d
*x + 1/2*c)^2) + 2*sqrt(2)*(15*(a*tan(1/2*d*x + 1/2*c)^2 - a)^3*sqrt(-a*ta
n(1/2*d*x + 1/2*c)^2 + a)*a^36 - 84*(a*tan(1/2*d*x + 1/2*c)^2 - a)^2*sqrt(
-a*tan(1/2*d*x + 1/2*c)^2 + a)*a^37 - 385*(-a*tan(1/2*d*x + 1/2*c)^2 + a)^
(3/2)*a^38 - 2730*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)*a^39)/(a^42*sgn(cos(
d*x + c))))/d
```

3.199.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cot^3(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = \int \frac{\cot(c+d\,x)^3}{\left(a+\frac{a}{\cos(c+d\,x)}\right)^{5/2}}\,dx$$

input
```
int(cot(c + d*x)^3/(a + a/cos(c + d*x))^(5/2),x)
```

output
```
int(cot(c + d*x)^3/(a + a/cos(c + d*x))^(5/2), x)
```

3.200 $\int \frac{\cot^5(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx$

3.200.1 Optimal result . . . 1488
3.200.2 Mathematica [C] (verified) . . . 1489
3.200.3 Rubi [A] (verified) . . . 1489
3.200.4 Maple [B] (verified) . . . 1496
3.200.5 Fricas [B] (verification not implemented) . . . 1496
3.200.6 Sympy [F] . . . 1497
3.200.7 Maxima [F(-1)] . . . 1498
3.200.8 Giac [A] (verification not implemented) . . . 1498
3.200.9 Mupad [F(-1)] . . . 1499

3.200.1 Optimal result

Integrand size = 23, antiderivative size = 262

$$\int \frac{\cot^5(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = \frac{2\operatorname{arctanh}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{a}}\right)}{a^{5/2}d} - \frac{263\operatorname{arctanh}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{512\sqrt{2}a^{5/2}d} + \frac{199a^2}{288d(a+a\sec(c+dx))^{9/2}} - \frac{a^2}{4d(1-\sec(c+dx))^2(a+a\sec(c+dx))^{9/2}} - \frac{21a^2}{16d(1-\sec(c+dx))(a+a\sec(c+dx))^{9/2}} + \frac{135a}{448d(a+a\sec(c+dx))^{7/2}} + \frac{7}{640d(a+a\sec(c+dx))^{5/2}} - \frac{83}{256ad(a+a\sec(c+dx))^{3/2}} - \frac{761}{512a^2d\sqrt{a+a\sec(c+dx)}}$$

output

```
2*arctanh((a+a*sec(d*x+c))^(1/2)/a^(1/2))/a^(5/2)/d+199/288*a^2/d/(a+a*sec
(d*x+c))^(9/2)-1/4*a^2/d/(1-sec(d*x+c))^2/(a+a*sec(d*x+c))^(9/2)-21/16*a^2
/d/(1-sec(d*x+c))/(a+a*sec(d*x+c))^(9/2)+135/448*a/d/(a+a*sec(d*x+c))^(7/2
)+7/640/d/(a+a*sec(d*x+c))^(5/2)-83/256/a/d/(a+a*sec(d*x+c))^(3/2)-263/102
4*arctanh(1/2*(a+a*sec(d*x+c))^(1/2)*2^(1/2)/a^(1/2))/a^(5/2)/d*2^(1/2)-76
1/512/a^2/d/(a+a*sec(d*x+c))^(1/2)
```

3.200.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.35 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.38

$$\int \frac{\cot^5(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = \frac{\cot^4(c+dx)\left(-450+263\,\text{Hypergeometric2F1}\left(-\frac{9}{2},1,-\frac{7}{2},\frac{1}{2}(1+\sec(c+dx)\right.\right.$$

input
```
Integrate[Cot[c + d*x]^5/(a + a*Sec[c + d*x])^(5/2),x]
```

output
```
(Cot[c + d*x]^4*(-450 + 263*Hypergeometric2F1[-9/2, 1, -7/2, (1 + Sec[c +
d*x])/2]*(-1 + Sec[c + d*x])^2 - 64*Hypergeometric2F1[-9/2, 1, -7/2, 1 + S
ec[c + d*x]]*(-1 + Sec[c + d*x])^2 + 378*Sec[c + d*x]))/(288*d*(a*(1 + Sec
[c + d*x]))^(5/2))
```

3.200.3 Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 292, normalized size of antiderivative = 1.11, number of steps used = 24, number of rules used = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 1.000$, Rules used = {3042, 25, 4368, 25, 27, 114, 27, 168, 27, 169, 27, 169, 27, 169, 27, 169, 27, 174, 73, 219, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^5(c+dx)}{(a\sec(c+dx)+a)^{5/2}}\,dx$$

$$\downarrow 3042$$

$$\int -\frac{1}{\cot\left(c+dx+\frac{\pi}{2}\right)^5\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^{5/2}}dx$$

$$\downarrow 25$$

$$-\int \frac{1}{\cot\left(\frac{1}{2}(2c+\pi)+dx\right)^5\left(\csc\left(\frac{1}{2}(2c+\pi)+dx\right)a+a\right)^{5/2}}dx$$

$$\downarrow 4368$$

$$\frac{a^6\int -\frac{\cos(c+dx)}{a^3(1-\sec(c+dx))^3(\sec(c+dx)a+a)^{11/2}}\,d\sec(c+dx)}{d}$$

$$\downarrow 25$$

$$-\frac{a^6 \int \frac{\cos(c+dx)}{a^3(1-\sec(c+dx))^3(\sec(c+dx)a+a)^{11/2}}\,d\sec(c+dx)}{d}$$

$$\downarrow 27$$

$$-\frac{a^3 \int \frac{\cos(c+dx)}{(1-\sec(c+dx))^3(\sec(c+dx)a+a)^{11/2}}\,d\sec(c+dx)}{d}$$

$$\downarrow 114$$

$$-\frac{a^3\left(\frac{1}{4a(1-\sec(c+dx))^2(a\sec(c+dx)+a)^{9/2}} - \frac{\int -\frac{a\cos(c+dx)(13\sec(c+dx)+8)}{2(1-\sec(c+dx))^2(\sec(c+dx)a+a)^{11/2}}\,d\sec(c+dx)}{4a}\right)}{d}$$

$$\downarrow 27$$

$$-\frac{a^3\left(\frac{1}{8}\int \frac{\cos(c+dx)(13\sec(c+dx)+8)}{(1-\sec(c+dx))^2(\sec(c+dx)a+a)^{11/2}}\,d\sec(c+dx) + \frac{1}{4a(1-\sec(c+dx))^2(a\sec(c+dx)+a)^{9/2}}\right)}{d}$$

$$\downarrow 168$$

$$-\frac{a^3\left(\frac{1}{8}\left(\frac{21}{2a(1-\sec(c+dx))(a\sec(c+dx)+a)^{9/2}} - \frac{\int -\frac{a\cos(c+dx)(231\sec(c+dx)+32)}{2(1-\sec(c+dx))(\sec(c+dx)a+a)^{11/2}}\,d\sec(c+dx)}{2a}\right) + \frac{1}{4a(1-\sec(c+dx))^2(a\sec(c+dx)+a)^{9}}\right.}{d}$$

$$\downarrow 27$$

$$-\frac{a^3\left(\frac{1}{8}\left(\frac{1}{4}\int \frac{\cos(c+dx)(231\sec(c+dx)+32)}{(1-\sec(c+dx))(\sec(c+dx)a+a)^{11/2}}\,d\sec(c+dx) + \frac{21}{2a(1-\sec(c+dx))(a\sec(c+dx)+a)^{9/2}}\right) + \frac{1}{4a(1-\sec(c+dx))^2(a\sec(c+d}\right.}{d}$$

$$\downarrow 169$$

$$-\frac{a^3\left(\frac{1}{8}\left(\frac{1}{4}\left(\frac{\int \frac{9a\cos(c+dx)(199\sec(c+dx)+64)}{2(1-\sec(c+dx))(\sec(c+dx)a+a)^{9/2}}\,d\sec(c+dx)}{9a^2} - \frac{199}{9a(a\sec(c+dx)+a)^{9/2}}\right) + \frac{21}{2a(1-\sec(c+dx))(a\sec(c+dx)+a)^{9/2}}\right) + \frac{}{4a(1-}\right.}{d}$$

$$\downarrow 27$$

$$-\frac{a^3\left(\frac{1}{8}\left(\frac{1}{4}\left(\frac{\int \frac{\cos(c+dx)(199\sec(c+dx)+64)}{(1-\sec(c+dx))(\sec(c+dx)a+a)^{9/2}}\,d\sec(c+dx)}{2a} - \frac{199}{9a(a\sec(c+dx)+a)^{9/2}}\right) + \frac{21}{2a(1-\sec(c+dx))(a\sec(c+dx)+a)^{9/2}}\right) + \frac{}{4a(1-}\right.}{d}$$

$$\downarrow 169$$

$$-\frac{a^3\left(\frac{1}{8}\left(\frac{1}{4}\left(\frac{\frac{\int \frac{7a\cos(c+dx)(135\sec(c+dx)+128)}{2(1-\sec(c+dx))(\sec(c+dx)a+a)^{7/2}}\,d\sec(c+dx)}{7a^2}-\frac{135}{7a(a\sec(c+dx)+a)^{7/2}}}{2a}-\frac{199}{9a(a\sec(c+dx)+a)^{9/2}}\right)+\frac{21}{2a(1-\sec(c+dx))(a\sec(c+}\right.}{d}$$

$$\downarrow 27$$

$$-\frac{a^3\left(\frac{1}{8}\left(\frac{1}{4}\left(\frac{\frac{\int \frac{\cos(c+dx)(135\sec(c+dx)+128)}{(1-\sec(c+dx))(\sec(c+dx)a+a)^{7/2}}\,d\sec(c+dx)}{2a}-\frac{135}{7a(a\sec(c+dx)+a)^{7/2}}}{2a}-\frac{199}{9a(a\sec(c+dx)+a)^{9/2}}\right)+\frac{21}{2a(1-\sec(c+dx))(a\sec(c+dx}\right.}{d}$$

$$\downarrow 169$$

$$-\frac{a^3\left(\frac{1}{8}\left(\frac{1}{4}\left(\frac{\frac{\int \frac{5a\cos(c+dx)(7\sec(c+dx)+256)}{2(1-\sec(c+dx))(\sec(c+dx)a+a)^{5/2}}\,d\sec(c+dx)}{5a^2}-\frac{7}{5a(a\sec(c+dx)+a)^{5/2}}}{2a}-\frac{135}{7a(a\sec(c+dx)+a)^{7/2}}\right)-\frac{199}{9a(a\sec(c+dx)+a)^{9/2}}\right)+\frac{}{2a(1}\right.}{d}$$

$$\downarrow 27$$

$$-\frac{a^3\left(\frac{1}{8}\left(\frac{1}{4}\left(\frac{\frac{\int \frac{\cos(c+dx)(7\sec(c+dx)+256)}{(1-\sec(c+dx))(\sec(c+dx)a+a)^{5/2}}\,d\sec(c+dx)}{2a}-\frac{7}{5a(a\sec(c+dx)+a)^{5/2}}}{2a}-\frac{135}{7a(a\sec(c+dx)+a)^{7/2}}\right)-\frac{199}{9a(a\sec(c+dx)+a)^{9/2}}\right)+\frac{}{2a(1-}\right.}{d}$$

$$\downarrow 169$$

$$-\frac{a^3\left(\frac{1}{8}\left(\frac{1}{4}\left(\frac{\frac{\int \frac{3a\cos(c+dx)(512-249\sec(c+dx))}{2(1-\sec(c+dx))(\sec(c+dx)a+a)^{3/2}}\,d\sec(c+dx)}{3a^2}+\frac{83}{a(a\sec(c+dx)+a)^{3/2}}}{2a}-\frac{7}{5a(a\sec(c+dx)+a)^{5/2}}\right)-\frac{135}{7a(a\sec(c+dx)+a)^{7/2}}\right)-\frac{19}{9a(a\sec(c+}\right.}{d}$$

$$\downarrow 27$$

3.200. $\int \frac{\cot^5(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx$

$$a^3\left(\frac{1}{8}\left(\frac{1}{4}\left(\frac{\frac{\int \frac{\cos(c+dx)(512-249\sec(c+dx))}{(1-\sec(c+dx))(\sec(c+dx)a+a)^{3/2}}\,d\sec(c+dx)}{2a}+\frac{83}{a(a\sec(c+dx)+a)^{3/2}}}{2a}-\frac{7}{5a(a\sec(c+dx)+a)^{5/2}}}{2a}-\frac{135}{7a(a\sec(c+dx)+a)^{7/2}}\right)-\frac{199}{9a(a\sec(c+d\right.\right.\right.$$

$$\left.\left.\left.\right.\right.\right)\Bigg)\Bigg/d$$

$$\downarrow 169$$

$$a^3\left(\frac{1}{8}\left(\frac{1}{4}\left(\frac{\frac{\int \frac{a\cos(c+dx)(1024-761\sec(c+dx))}{2(1-\sec(c+dx))\sqrt{\sec(c+dx)a+a}}\,d\sec(c+dx)}{a^2}+\frac{761}{a\sqrt{a\sec(c+dx)+a}}}{2a}+\frac{83}{a(a\sec(c+dx)+a)^{3/2}}}{2a}-\frac{7}{5a(a\sec(c+dx)+a)^{5/2}}}{2a}-\frac{135}{7a(a\sec(c+dx)+a)^{7/2}}\right)\Bigg)\Bigg/d$$

$$\downarrow 27$$

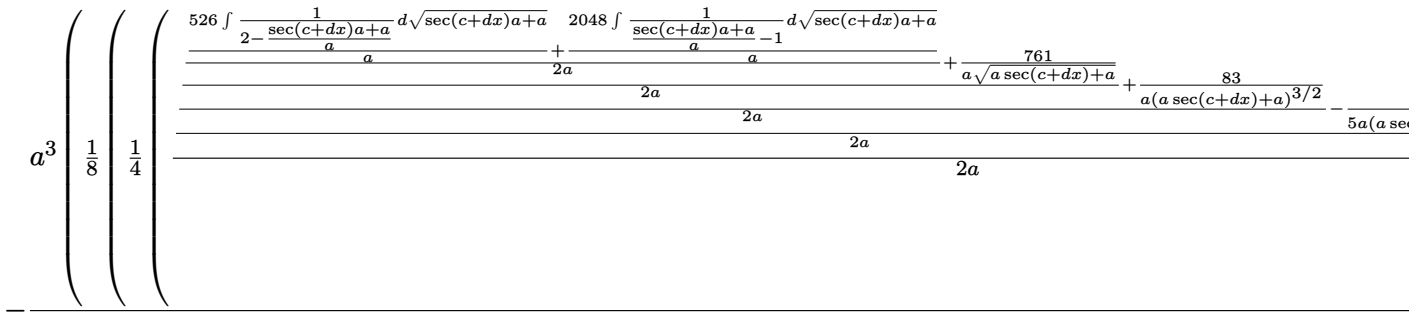
$$a^3\left(\frac{1}{8}\left(\frac{1}{4}\left(\frac{\frac{\int \frac{\cos(c+dx)(1024-761\sec(c+dx))}{(1-\sec(c+dx))\sqrt{\sec(c+dx)a+a}}\,d\sec(c+dx)}{2a}+\frac{761}{a\sqrt{a\sec(c+dx)+a}}}{2a}+\frac{83}{a(a\sec(c+dx)+a)^{3/2}}}{2a}-\frac{7}{5a(a\sec(c+dx)+a)^{5/2}}}{2a}-\frac{135}{7a(a\sec(c+dx)+a)^{7/2}}\right)\Bigg)\Bigg/d$$

$$\downarrow 174$$

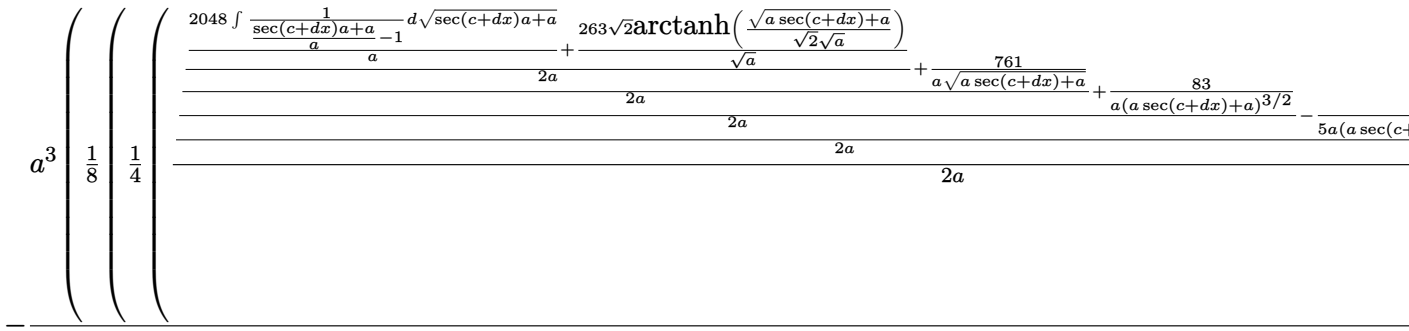
$$a^3\left(\frac{1}{8}\left(\frac{1}{4}\left(\frac{\frac{263\int \frac{1}{(1-\sec(c+dx))\sqrt{\sec(c+dx)a+a}}\,d\sec(c+dx)+1024\int \frac{\cos(c+dx)}{\sqrt{\sec(c+dx)a+a}}\,d\sec(c+dx)}{2a}+\frac{761}{a\sqrt{a\sec(c+dx)+a}}}{2a}+\frac{83}{a(a\sec(c+dx)+a)^{3/2}}}{2a}-\frac{\right.\right.\right.$$

$$\left.\left.\left.\right.\right.\right)\Bigg)\Bigg/2a$$

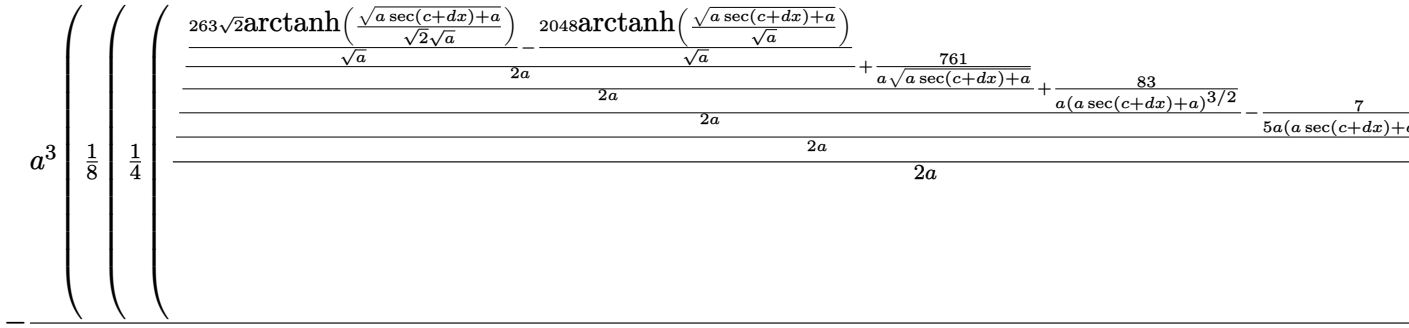
$$\downarrow 73$$



↓ 219



↓ 220



input
```
Int[Cot[c + d*x]^5/(a + a*Sec[c + d*x])^(5/2),x]
```

3.200. $\int \frac{\cot^5(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx$

output
```
-((a^3*(1/(4*a*(1 - Sec[c + d*x])^2*(a + a*Sec[c + d*x])^(9/2)) + (21/(2*a
*(1 - Sec[c + d*x])*(a + a*Sec[c + d*x])^(9/2)) + (-199/(9*a*(a + a*Sec[c
+ d*x])^(9/2)) + (-135/(7*a*(a + a*Sec[c + d*x])^(7/2)) + (-7/(5*a*(a + a*
Sec[c + d*x])^(5/2)) + (83/(a*(a + a*Sec[c + d*x])^(3/2)) + (((-2048*ArcTa
nh[Sqrt[a + a*Sec[c + d*x]]/Sqrt[a]])/Sqrt[a] + (263*Sqrt[2]*ArcTanh[Sqrt[
a + a*Sec[c + d*x]]/(Sqrt[2]*Sqrt[a])])/Sqrt[a])/(2*a) + 761/(a*Sqrt[a + a
*Sec[c + d*x]]))/(2*a))/(2*a))/(2*a))/(2*a))/4)/8))/d)
```

3.200.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 114
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[1/((m + 1)*(b*c - a*d)*(b*e
 - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1)
 - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (IntegerQ[n] ||
 IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

rule 168
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

rule 169
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && IntegersQ[
2*m, 2*n, 2*p]
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4368
```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Simp[-(d*b^(m - 1))^(-1)   Subst[Int[(-a + b*x)^((m - 1)/2
)*((a + b*x)^((m - 1)/2 + n)/x), x], x, Csc[c + d*x]], x] /; FreeQ[{a, b, c
, d, n}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[n]
```

3.200.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 563 vs. 2(217) = 434.

Time = 1.68 (sec) , antiderivative size = 564, normalized size of antiderivative = 2.15

method	result
default	$-\frac{\sqrt{a(1+\sec(dx+c))}\left(82845\cos(dx+c)^3\sqrt{2}\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}\arctan\left(\frac{\sqrt{2}}{2\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}}\right)+645120\cos(dx+c)^3\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}\arc$

input
```
int(cot(d*x+c)^5/(a+a*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/322560/d/a^3*(a*(1+sec(d*x+c)))^(1/2)/(cos(d*x+c)+1)^3*(82845*cos(d*x+c
)^3*2^(1/2)*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctan(1/2*2^(1/2)/(-cos(d*
x+c)/(cos(d*x+c)+1))^(1/2))+645120*cos(d*x+c)^3*(-cos(d*x+c)/(cos(d*x+c)+1
))^(1/2)*arctan((-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))+248535*cos(d*x+c)^2*ar
ctan(1/2*2^(1/2)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*2^(1/2)*(-cos(d*x+c)/
(cos(d*x+c)+1))^(1/2)+1935360*cos(d*x+c)^2*arctan((-cos(d*x+c)/(cos(d*x+c)
+1))^(1/2))*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+248535*arctan(1/2*2^(1/2)/(
-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*cos(d*x+c)*2^(1/2)*(-cos(d*x+c)/(cos(d*
x+c)+1))^(1/2)+1935360*arctan((-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*cos(d*x+
c)*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+82845*2^(1/2)*(-cos(d*x+c)/(cos(d*x+
c)+1))^(1/2)*arctan(1/2*2^(1/2)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))+764402
*cos(d*x+c)^3*cot(d*x+c)^4+645120*arctan((-cos(d*x+c)/(cos(d*x+c)+1))^(1/2
))*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+1183040*cos(d*x+c)^2*cot(d*x+c)^4-80
7214*cos(d*x+c)*cot(d*x+c)^4-2224080*cot(d*x+c)^4-378378*cot(d*x+c)^3*csc(
d*x+c)+1063440*cot(d*x+c)^2*csc(d*x+c)^2+479430*csc(d*x+c)^3*cot(d*x+c))
```

3.200.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 438 vs. 2(213) = 426.

Time = 0.39 (sec) , antiderivative size = 905, normalized size of antiderivative = 3.45

$$\int \frac{\cot^5(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(cot(d*x+c)^5/(a+a*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
[1/645120*(82845*sqrt(2)*(cos(d*x + c)^7 + 3*cos(d*x + c)^6 + cos(d*x + c)
^5 - 5*cos(d*x + c)^4 - 5*cos(d*x + c)^3 + cos(d*x + c)^2 + 3*cos(d*x + c)
 + 1)*sqrt(a)*log(-(2*sqrt(2)*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x +
c))*cos(d*x + c) - 3*a*cos(d*x + c) - a)/(cos(d*x + c) - 1)) + 322560*(cos
(d*x + c)^7 + 3*cos(d*x + c)^6 + cos(d*x + c)^5 - 5*cos(d*x + c)^4 - 5*cos
(d*x + c)^3 + cos(d*x + c)^2 + 3*cos(d*x + c) + 1)*sqrt(a)*log(-8*a*cos(d*
x + c)^2 - 4*(2*cos(d*x + c)^2 + cos(d*x + c))*sqrt(a)*sqrt((a*cos(d*x + c
) + a)/cos(d*x + c)) - 8*a*cos(d*x + c) - a) - 4*(382201*cos(d*x + c)^7 +
591520*cos(d*x + c)^6 - 403607*cos(d*x + c)^5 - 1112040*cos(d*x + c)^4 - 1
89189*cos(d*x + c)^3 + 531720*cos(d*x + c)^2 + 239715*cos(d*x + c))*sqrt((
a*cos(d*x + c) + a)/cos(d*x + c)))/(a^3*d*cos(d*x + c)^7 + 3*a^3*d*cos(d*x
 + c)^6 + a^3*d*cos(d*x + c)^5 - 5*a^3*d*cos(d*x + c)^4 - 5*a^3*d*cos(d*x
+ c)^3 + a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d), 1/322560*(8
2845*sqrt(2)*(cos(d*x + c)^7 + 3*cos(d*x + c)^6 + cos(d*x + c)^5 - 5*cos(d
*x + c)^4 - 5*cos(d*x + c)^3 + cos(d*x + c)^2 + 3*cos(d*x + c) + 1)*sqrt(-
a)*arctan(sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x
 + c)/(a*cos(d*x + c) + a)) - 322560*(cos(d*x + c)^7 + 3*cos(d*x + c)^6 +
cos(d*x + c)^5 - 5*cos(d*x + c)^4 - 5*cos(d*x + c)^3 + cos(d*x + c)^2 + 3*
cos(d*x + c) + 1)*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos
(d*x + c))*cos(d*x + c)/(2*a*cos(d*x + c) + a)) - 2*(382201*cos(d*x + c...
```

3.200.6 Sympy [F]

$$\int \frac{\cot^5(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = \int \frac{\cot^5(c+dx)}{(a(\sec(c+dx)+1))^{\frac{5}{2}}}\,dx$$

input
```
integrate(cot(d*x+c)**5/(a+a*sec(d*x+c))**(5/2),x)
```

output
```
Integral(cot(c + d*x)**5/(a*(sec(c + d*x) + 1))**(5/2), x)
```

3.200.7 Maxima [F(-1)]

Timed out.

$$\int \frac{\cot^5(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(cot(d*x+c)^5/(a+a*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
Timed out
```

3.200.8 Giac [A] (verification not implemented)

Time = 1.27 (sec) , antiderivative size = 364, normalized size of antiderivative = 1.39

$$\int \frac{\cot^5(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = \frac{82845\sqrt{2}\arctan\left(\frac{\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a^2\mathrm{sgn}(\cos(dx+c))} - \frac{645120\arctan\left(\frac{\sqrt{2}\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}}{2\sqrt{-a}}\right)}{\sqrt{-a}a^2\mathrm{sgn}(\cos(dx+c))} - \frac{315\left(33}{}$$

input
```
integrate(cot(d*x+c)^5/(a+a*sec(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
1/322560*(82845*sqrt(2)*arctan(sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)/sqrt(-a
))/(sqrt(-a)*a^2*sgn(cos(d*x + c))) - 645120*arctan(1/2*sqrt(2)*sqrt(-a*ta
n(1/2*d*x + 1/2*c)^2 + a)/sqrt(-a))/(sqrt(-a)*a^2*sgn(cos(d*x + c))) - 315
*(33*sqrt(2)*(-a*tan(1/2*d*x + 1/2*c)^2 + a)^(3/2) - 31*sqrt(2)*sqrt(-a*ta
n(1/2*d*x + 1/2*c)^2 + a)*a)/(a^4*sgn(cos(d*x + c))*tan(1/2*d*x + 1/2*c)^4
) - 8*sqrt(2)*(35*(a*tan(1/2*d*x + 1/2*c)^2 - a)^4*sqrt(-a*tan(1/2*d*x + 1
/2*c)^2 + a)*a^56 - 225*(a*tan(1/2*d*x + 1/2*c)^2 - a)^3*sqrt(-a*tan(1/2*d
*x + 1/2*c)^2 + a)*a^57 + 1008*(a*tan(1/2*d*x + 1/2*c)^2 - a)^2*sqrt(-a*ta
n(1/2*d*x + 1/2*c)^2 + a)*a^58 + 4410*(-a*tan(1/2*d*x + 1/2*c)^2 + a)^(3/2
)*a^59 + 31185*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)*a^60)/(a^63*sgn(cos(d*x
 + c))))/d
```

3.200.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cot^5(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = \int \frac{\cot(c+d\,x)^5}{\left(a+\frac{a}{\cos(c+d\,x)}\right)^{5/2}}\,dx$$

input
```
int(cot(c + d*x)^5/(a + a/cos(c + d*x))^(5/2),x)
```

output
```
int(cot(c + d*x)^5/(a + a/cos(c + d*x))^(5/2), x)
```

3.201 $\int \frac{\tan^6(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx$

3.201.1 Optimal result 1500
3.201.2 Mathematica [B] (warning: unable to verify) 1500
3.201.3 Rubi [A] (verified) 1501
3.201.4 Maple [A] (warning: unable to verify) 1502
3.201.5 Fricas [A] (verification not implemented) 1503
3.201.6 Sympy [F] 1503
3.201.7 Maxima [F] 1504
3.201.8 Giac [B] (verification not implemented) 1504
3.201.9 Mupad [F(-1)] 1505

3.201.1 Optimal result

Integrand size = 23, antiderivative size = 127

$$\int \frac{\tan^6(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = -\frac{2\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{a^{5/2}d} + \frac{2\tan(c+dx)}{a^2d\sqrt{a+a\sec(c+dx)}} - \frac{2\tan^3(c+dx)}{3ad(a+a\sec(c+dx))^{3/2}} + \frac{2\tan^5(c+dx)}{5d(a+a\sec(c+dx))^{5/2}}$$

output

```
-2*arctan(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))/a^(5/2)/d+2*tan(d*x+c
)/a^2/d/(a+a*sec(d*x+c))^(1/2)-2/3*tan(d*x+c)^3/a/d/(a+a*sec(d*x+c))^(3/2)
+2/5*tan(d*x+c)^5/d/(a+a*sec(d*x+c))^(5/2)
```

3.201.2 Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 447 vs. 2(127) = 254.

Time = 6.09 (sec) , antiderivative size = 447, normalized size of antiderivative = 3.52

$$\int \frac{\tan^6(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = \frac{\sqrt{2}\cot^8\left(\frac{1}{2}(c+dx)\right)\left(\frac{1}{1+\sec(c+dx)}\right)^{9/2}\sqrt{1+\tan^2\left(\frac{1}{2}(c+dx)\right)}\left(-1+\frac{2\tan^2\left(\frac{1}{2}(c+\right.\right.}{1+\tan^2\left(\frac{1}{2}(c-\right.}$$

input
```
Integrate[Tan[c + d*x]^6/(a + a*Sec[c + d*x])^(5/2),x]
```

output
```
(Sqrt[2]*Cot[(c + d*x)/2]^8*((1 + Sec[c + d*x])^(-1))^(9/2)*Sqrt[1 + Tan[(
c + d*x)/2]^2]*(-1 + (2*Tan[(c + d*x)/2]^2)/(1 + Tan[(c + d*x)/2]^2))^3*((
Sqrt[2]*ArcSin[(Sqrt[2]*Tan[(c + d*x)/2])/Sqrt[1 + Tan[(c + d*x)/2]^2]]*Ta
n[(c + d*x)/2])/(Sqrt[1 + Tan[(c + d*x)/2]^2]*Sqrt[1 - (2*Tan[(c + d*x)/2]
^2)/(1 + Tan[(c + d*x)/2]^2)]) + (8*Tan[(c + d*x)/2]^6)/(5*(1 + Tan[(c + d
*x)/2]^2)^3*(-1 + (2*Tan[(c + d*x)/2]^2)/(1 + Tan[(c + d*x)/2]^2))^3) + (4
*Tan[(c + d*x)/2]^4)/(3*(1 + Tan[(c + d*x)/2]^2)^2*(-1 + (2*Tan[(c + d*x)/
2]^2)/(1 + Tan[(c + d*x)/2]^2))^2) + (2*Tan[(c + d*x)/2]^2)/((1 + Tan[(c +
 d*x)/2]^2)*(-1 + (2*Tan[(c + d*x)/2]^2)/(1 + Tan[(c + d*x)/2]^2))))*Tan[c
 + d*x]^7)/(d*(a*(1 + Sec[c + d*x]))^(5/2)*(1 - (2*Tan[(c + d*x)/2]^2)/(1
+ Tan[(c + d*x)/2]^2))^(5/2))
```

3.201.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.97, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3042, 4375, 254, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^6(c+dx)}{(a\sec(c+dx)+a)^{5/2}}\,dx$$

$$\downarrow 3042$$

$$\int \frac{\cot\left(c+dx+\frac{\pi}{2}\right)^6}{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^{5/2}}dx$$

$$\downarrow 4375$$

$$-\frac{2a\int \frac{\tan^6(c+dx)}{(\sec(c+dx)a+a)^3\left(\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+1\right)}d\left(-\frac{\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)}{d}$$

$$\downarrow 254$$

$$-\frac{2a\int \left(\frac{\tan^4(c+dx)}{a(\sec(c+dx)a+a)^2}-\frac{\tan^2(c+dx)}{a^2(\sec(c+dx)a+a)}-\frac{1}{a^3\left(\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+1\right)}+\frac{1}{a^3}\right)d\left(-\frac{\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)}{d}$$

$$\downarrow 2009$$

$$-\frac{2a\left(\frac{\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{a^{7/2}}-\frac{\tan(c+dx)}{a^3\sqrt{a\sec(c+dx)+a}}+\frac{\tan^3(c+dx)}{3a^2(a\sec(c+dx)+a)^{3/2}}-\frac{\tan^5(c+dx)}{5a(a\sec(c+dx)+a)^{5/2}}\right)}{d}$$

input
```
Int[Tan[c + d*x]^6/(a + a*Sec[c + d*x])^(5/2),x]
```

output
```
(-2*a*(ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]]/a^(7/2) - T
an[c + d*x]/(a^3*Sqrt[a + a*Sec[c + d*x]]) + Tan[c + d*x]^3/(3*a^2*(a + a*
Sec[c + d*x])^(3/2)) - Tan[c + d*x]^5/(5*a*(a + a*Sec[c + d*x])^(5/2))))/d
```

3.201.3.1 Defintions of rubi rules used

rule 254
```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Int[PolynomialDivide[x^m,
 a + b*x^2, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 3]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4375
```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Simp[-2*(a^(m/2 + n + 1/2)/d)   Subst[Int[x^m*((2 + a*x^2
)^(m/2 + n - 1/2)/(1 + a*x^2)), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]
]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && I
ntegerQ[n - 1/2]
```

3.201.4 Maple [A] (warning: unable to verify)

Time = 3.70 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.66

method	result
default	$-\frac{\left(15\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{2}\,(-\cot(dx+c)+\csc(dx+c))}{\sqrt{(1-\cos(dx+c))^2\csc(dx+c)^2-1}}\right)\left((1-\cos(dx+c))^2\csc(dx+c)^2-1\right)^{\frac{5}{2}}-74(1-\cos(dx+c))^5\csc(dx+c)^5+80(1-\cos$ $15d\,a^3(\csc(dx+c)-\cot(dx+c)+1)^2(-\cot(dx+c)+\csc(d$

input
```
int(tan(d*x+c)^6/(a+a*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/15/d/a^3*(15*2^(1/2)*arctanh(2^(1/2)/((1-cos(d*x+c))^2*csc(d*x+c)^2-1)^
(1/2)*(-cot(d*x+c)+csc(d*x+c)))*((1-cos(d*x+c))^2*csc(d*x+c)^2-1)^(5/2)-74
*(1-cos(d*x+c))^5*csc(d*x+c)^5+80*(1-cos(d*x+c))^3*csc(d*x+c)^3-30*csc(d*x
+c)+30*cot(d*x+c))*(-2*a/((1-cos(d*x+c))^2*csc(d*x+c)^2-1))^(1/2)/(csc(d*x
+c)-cot(d*x+c)+1)^2/(-cot(d*x+c)+csc(d*x+c)-1)^2
```

3.201.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 323, normalized size of antiderivative = 2.54

$$\int \frac{\tan^6(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = \left[-\frac{15\left(\cos\left(dx+c\right)^3+\cos\left(dx+c\right)^2\right)\sqrt{-a}\log\left(\frac{2\,a\cos(dx+c)^2-2\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}}{\cos(dx-}\right.\right.$$
$$\left.\left.\qquad\qquad 15\left(a^3d\right.\right.\right.$$

input
```
integrate(tan(d*x+c)^6/(a+a*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
[-1/15*(15*(cos(d*x + c)^3 + cos(d*x + c)^2)*sqrt(-a)*log((2*a*cos(d*x + c
)^2 - 2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(
d*x + c) + a*cos(d*x + c) - a)/(cos(d*x + c) + 1)) - 2*(23*cos(d*x + c)^2
- 11*cos(d*x + c) + 3)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c
))/(a^3*d*cos(d*x + c)^3 + a^3*d*cos(d*x + c)^2), 2/15*(15*(cos(d*x + c)^3
 + cos(d*x + c)^2)*sqrt(a)*arctan(sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*
cos(d*x + c)/(sqrt(a)*sin(d*x + c))) + (23*cos(d*x + c)^2 - 11*cos(d*x + c
) + 3)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(a^3*d*cos(d*
x + c)^3 + a^3*d*cos(d*x + c)^2)]
```

3.201.6 Sympy [F]

$$\int \frac{\tan^6(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = \int \frac{\tan^6\left(c+dx\right)}{\left(a\left(\sec\left(c+dx\right)+1\right)\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(tan(d*x+c)**6/(a+a*sec(d*x+c))**(5/2),x)
```

output
```
Integral(tan(c + d*x)**6/(a*(sec(c + d*x) + 1))**(5/2), x)
```

3.201.7 Maxima [F]

$$\int \frac{\tan^6(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = \int \frac{\tan\left(dx+c\right)^6}{\left(a\sec\left(dx+c\right)+a\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(tan(d*x+c)^6/(a+a*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
-1/30*(20*(9*sin(4*d*x + 4*c) + 16*sin(2*d*x + 2*c))*cos(5/2*arctan2(sin(2
*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - 4*(45*cos(4*d*x + 4*c) + 80*cos(2*d*
x + 2*c) + 23)*sin(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) -
15*((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*arc
tan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1
/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)), (cos(2*d*x +
 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arcta
n2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + 1) - (cos(2*d*x + 2*c)^2 + s
in(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*arctan2((cos(2*d*x + 2*c)^2 +
sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d
*x + 2*c), cos(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^
2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*
d*x + 2*c) + 1)) - 1) + 2*(a^3*d*cos(2*d*x + 2*c)^2 + a^3*d*sin(2*d*x + 2*
c)^2 + 2*a^3*d*cos(2*d*x + 2*c) + a^3*d)*integrate(-(cos(2*d*x + 2*c)^2 +
sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(((cos(2*d*x + 2*c)^4 +
 sin(2*d*x + 2*c)^4 + 2*cos(2*d*x + 2*c)^3 + (2*cos(2*d*x + 2*c)^2 + 2*cos
(2*d*x + 2*c) + 1)*sin(2*d*x + 2*c)^2 + (cos(2*d*x + 2*c)^3 + cos(2*d*x +
2*c)*sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c)^2 + cos(2*d*x + 2*c))*cos(14*
d*x + 14*c) + 6*(cos(2*d*x + 2*c)^3 + cos(2*d*x + 2*c)*sin(2*d*x + 2*c)^2
+ 2*cos(2*d*x + 2*c)^2 + cos(2*d*x + 2*c))*cos(12*d*x + 12*c) + 15*(cos...
```

3.201.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 257 vs. $2(111) = 222$.

Time $= 4.29$ (sec) , antiderivative size $= 257$, normalized size of antiderivative $= 2.02$

$$\int \frac{\tan^6(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = 15\sqrt{-a}\left(\frac{\log\left(\left|\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^2-a\left(2\sqrt{2}+3\right)\right|\right)}{a^3\operatorname{sgn}(\cos(dx+c))} - \frac{\log\left(\left|\left(\sqrt{-a}\right.\right.\right.\right.$$

input
```
integrate(tan(d*x+c)^6/(a+a*sec(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
1/15*(15*sqrt(-a)*(log(abs((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/
2*d*x + 1/2*c)^2 + a))^2 - a*(2*sqrt(2) + 3)))/(a^3*sgn(cos(d*x + c))) - l
og(abs((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a
))^2 + a*(2*sqrt(2) - 3)))/(a^3*sgn(cos(d*x + c)))) + 2*((37*sqrt(2)*tan(1
/2*d*x + 1/2*c)^2/sgn(cos(d*x + c)) - 40*sqrt(2)/sgn(cos(d*x + c)))*tan(1/
2*d*x + 1/2*c)^2 + 15*sqrt(2)/sgn(cos(d*x + c)))*tan(1/2*d*x + 1/2*c)/((a*
tan(1/2*d*x + 1/2*c)^2 - a)^2*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)))/d
```

3.201.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\tan^6(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = \int \frac{\tan(c+d\,x)^6}{\left(a+\frac{a}{\cos(c+d\,x)}\right)^{5/2}}\,dx$$

input
```
int(tan(c + d*x)^6/(a + a/cos(c + d*x))^(5/2),x)
```

output
```
int(tan(c + d*x)^6/(a + a/cos(c + d*x))^(5/2), x)
```

3.202 $\int \frac{\tan^4(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx$

3.202.1 Optimal result 1506
3.202.2 Mathematica [A] (verified) 1506
3.202.3 Rubi [A] (verified) 1507
3.202.4 Maple [B] (verified) 1509
3.202.5 Fricas [A] (verification not implemented) 1510
3.202.6 Sympy [F] 1511
3.202.7 Maxima [F] 1511
3.202.8 Giac [A] (verification not implemented) 1511
3.202.9 Mupad [F(-1)] 1512

3.202.1 Optimal result

Integrand size = 23, antiderivative size = 113

$$\int \frac{\tan^4(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = \frac{2\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{a^{5/2}d} - \frac{4\sqrt{2}\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{a^{5/2}d} + \frac{2\tan(c+dx)}{a^2 d\sqrt{a+a\sec(c+dx)}}$$

output
```
2*arctan(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))/a^(5/2)/d-4*arctan(1/2
*a^(1/2)*tan(d*x+c)*2^(1/2)/(a+a*sec(d*x+c))^(1/2))*2^(1/2)/a^(5/2)/d+2*ta
n(d*x+c)/a^2/d/(a+a*sec(d*x+c))^(1/2)
```

3.202.2 Mathematica [A] (verified)

Time = 0.98 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.66

$$\int \frac{\tan^4(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = \frac{8\cos^4\left(\frac{1}{2}(c+dx)\right)\sec^{\frac{5}{2}}(c+dx)\left(-4\arcsin\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)\sqrt{\frac{1}{1+\sec(c+dx)}}\sqrt{}\right.}{}$$

input
```
Integrate[Tan[c + d*x]^4/(a + a*Sec[c + d*x])^(5/2),x]
```

output

```
(8*Cos[(c + d*x)/2]^4*Sec[c + d*x]^(5/2)*(-4*ArcSin[Tan[(c + d*x)/2]]*Sqrt
[(1 + Sec[c + d*x])^(-1)]*Sqrt[1 + Sec[c + d*x]] + Sqrt[2]*(ArcTan[Tan[(c
+ d*x)/2]/Sqrt[(1 + Sec[c + d*x])^(-1)]]*Sqrt[(1 + Sec[c + d*x])^(-1)]*Sqr
t[1 + Sec[c + d*x]] + Sqrt[(1 + Cos[c + d*x])^(-1)]*Sqrt[Sec[c + d*x]]*Sin
[c + d*x])))/(d*Sqrt[Sec[(c + d*x)/2]^2]*(a*(1 + Sec[c + d*x]))^(5/2))
```

3.202.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.01, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3042, 4375, 381, 397, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^4(c+dx)}{(a\sec(c+dx)+a)^{5/2}}\,dx$$

$$\downarrow 3042$$

$$\int \frac{\cot\left(c+dx+\frac{\pi}{2}\right)^4}{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^{5/2}}dx$$

$$\downarrow 4375$$

$$-\frac{2\int \frac{\tan^4(c+dx)}{(\sec(c+dx)a+a)^2\left(\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+1\right)\left(\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+2\right)}d\left(-\frac{\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)}{d}$$

$$\downarrow 381$$

$$-\frac{2\left(-\frac{\int \frac{\frac{3a\tan^2(c+dx)}{\sec(c+dx)a+a}+2}{\left(\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+1\right)\left(\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+2\right)}d\left(-\frac{\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)}{a^2}-\frac{\tan(c+dx)}{a^2\sqrt{a\sec(c+dx)+a}}\right)}{d}$$

$$\downarrow 397$$

$$-\frac{2\left(-\frac{4\int \frac{1}{\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+2}d\left(-\frac{\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)-\int \frac{1}{\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+1}d\left(-\frac{\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)}{a^2}-\frac{\tan(c+dx)}{a^2\sqrt{a\sec(c+dx)+a}}\right)}{d}$$

$$\downarrow 216$$

$$-\frac{2\left(-\frac{\frac{\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}}-\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}}}{a^2}-\frac{\tan(c+dx)}{a^2\sqrt{a\sec(c+dx)+a}}\right)}{d}$$

input
```
Int[Tan[c + d*x]^4/(a + a*Sec[c + d*x])^(5/2),x]
```

output
```
(-2*(-((ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]]/Sqrt[a] -
(2*Sqrt[2]*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]]
)])/Sqrt[a])/a^2) - Tan[c + d*x]/(a^2*Sqrt[a + a*Sec[c + d*x]])))/d
```

3.202.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 381
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[e^3*(e*x)^(m - 3)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q
+ 1)/(b*d*(m + 2*(p + q) + 1))), x] - Simp[e^4/(b*d*(m + 2*(p + q) + 1))
Int[(e*x)^(m - 4)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*c*(m - 3) + (a*d*(m +
2*q - 1) + b*c*(m + 2*p - 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
}, x] && NeQ[b*c - a*d, 0] && GtQ[m, 3] && IntBinomialQ[a, b, c, d, e, m, 2
, p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4375

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Simp[-2*(a^(m/2 + n + 1/2)/d)   Subst[Int[x^m*((2 + a*x^2
)^(m/2 + n - 1/2)/(1 + a*x^2)), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]
]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && I
ntegerQ[n - 1/2]
```

3.202.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 317 vs. $2(96) = 192$.

Time = 3.73 (sec) , antiderivative size = 318, normalized size of antiderivative = 2.81

method	result
default	$\frac{2\sqrt{a(1+\sec(dx+c))}\left(-2\sqrt{2}\,\ln\left(\csc(dx+c)-\cot(dx+c)+\sqrt{\cot(dx+c)^2-2\cot(dx+c)\csc(dx+c)+\csc(dx+c)^2-1}\right)\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,c\right.$

input

```
int(tan(d*x+c)^4/(a+a*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output

```
2/d/a^3*(a*(1+sec(d*x+c)))^(1/2)*(-2*2^(1/2)*ln(csc(d*x+c)-cot(d*x+c)+(cot
(d*x+c)^2-2*cot(d*x+c)*csc(d*x+c)+csc(d*x+c)^2-1)^(1/2))*(-cos(d*x+c)/(cos
(d*x+c)+1))^(1/2)*cos(d*x+c)+(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctanh(si
n(d*x+c)/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*cos(d*x+c)-2*l
n(csc(d*x+c)-cot(d*x+c)+(cot(d*x+c)^2-2*cot(d*x+c)*csc(d*x+c)+csc(d*x+c)^2
-1)^(1/2))*2^(1/2)*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+(-cos(d*x+c)/(cos(d*
x+c)+1))^(1/2)*arctanh(sin(d*x+c)/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+
1))^(1/2))+sin(d*x+c))/(cos(d*x+c)+1)
```

3.202.5 Fricas [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 414, normalized size of antiderivative = 3.66

$$\int \frac{\tan^4(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = \left[\frac{2\sqrt{2}(a\cos(dx+c)+a)\sqrt{-\frac{1}{a}}\log\left(\frac{2\sqrt{2}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{-\frac{1}{a}}\cos(dx+c)\sin(dx+c)+3\cos(dx+c)^2+2\cos(dx+c)-1}{\cos(dx+c)^2+2\cos(dx+c)+1}\right)}{2\left(\sqrt{a}(\cos(dx+c)+1)\arctan\left(\frac{\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)}{\sqrt{a}\sin(dx+c)}\right)-\frac{2\sqrt{2}(a\cos(dx+c)+a)\arctan\left(\frac{\sqrt{2}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)}{\sqrt{a}\sin(dx+c)}\right)}{\sqrt{a}}-\right.}\right.$$

$$\left.\left.-\frac{}{a^3d\cos(dx+c)+a^3d}\right)\right.$$

input
```
integrate(tan(d*x+c)^4/(a+a*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
[(2*sqrt(2)*(a*cos(d*x + c) + a)*sqrt(-1/a)*log((2*sqrt(2)*sqrt((a*cos(d*x
 + c) + a)/cos(d*x + c))*sqrt(-1/a)*cos(d*x + c)*sin(d*x + c) + 3*cos(d*x
+ c)^2 + 2*cos(d*x + c) - 1)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) - sqrt
(-a)*(cos(d*x + c) + 1)*log((2*a*cos(d*x + c)^2 + 2*sqrt(-a)*sqrt((a*cos(d
*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x + c) + a*cos(d*x + c) - a)
/(cos(d*x + c) + 1)) + 2*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x +
 c))/(a^3*d*cos(d*x + c) + a^3*d), -2*(sqrt(a)*(cos(d*x + c) + 1)*arctan(s
qrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c))
) - 2*sqrt(2)*(a*cos(d*x + c) + a)*arctan(sqrt(2)*sqrt((a*cos(d*x + c) + a
)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c)))/sqrt(a) - sqrt((a*cos
(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(a^3*d*cos(d*x + c) + a^3*d)]
```

3.202.6 Sympy [F]

$$\int \frac{\tan^4(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = \int \frac{\tan^4(c+dx)}{\left(a\left(\sec(c+dx)+1\right)\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(tan(d*x+c)**4/(a+a*sec(d*x+c))**(5/2),x)
```

output
```
Integral(tan(c + d*x)**4/(a*(sec(c + d*x) + 1))**(5/2), x)
```

3.202.7 Maxima [F]

$$\int \frac{\tan^4(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = \int \frac{\tan(dx+c)^4}{(a\sec(dx+c)+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(tan(d*x+c)^4/(a+a*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
integrate(tan(d*x + c)^4/(a*sec(d*x + c) + a)^(5/2), x)
```

3.202.8 Giac [A] (verification not implemented)

Time = 2.08 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.58

$$\int \frac{\tan^4(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = -\frac{2\sqrt{2}\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-a\right)a^2d\operatorname{sgn}\left(\cos(dx+c)\right)}$$

input
```
integrate(tan(d*x+c)^4/(a+a*sec(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
-2*sqrt(2)*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)*tan(1/2*d*x + 1/2*c)/((a*ta
n(1/2*d*x + 1/2*c)^2 - a)*a^2*d*sgn(cos(d*x + c)))
```

3.202.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\tan^4(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = \int \frac{\tan(c+d\,x)^4}{\left(a+\frac{a}{\cos(c+d\,x)}\right)^{5/2}}\,dx$$

input
```
int(tan(c + d*x)^4/(a + a/cos(c + d*x))^(5/2),x)
```

output
```
int(tan(c + d*x)^4/(a + a/cos(c + d*x))^(5/2), x)
```

3.203 $\int \frac{\tan^2(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx$

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3.203.1 Optimal result

Integrand size = 23, antiderivative size = 127

$$\int \frac{\tan^2(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = -\frac{2\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{a^{5/2}d} + \frac{3\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{\sqrt{2}a^{5/2}d} + \frac{\sec^2\left(\frac{1}{2}(c+dx)\right)\sin(c+dx)}{2a^2d\sqrt{a+a\sec(c+dx)}}$$

output
```
-2*arctan(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))/a^(5/2)/d+3/2*arctan(
1/2*a^(1/2)*tan(d*x+c)*2^(1/2)/(a+a*sec(d*x+c))^(1/2))*2^(1/2)/a^(5/2)/d+1
/2*sec(1/2*d*x+1/2*c)^2*sin(d*x+c)/a^2/d/(a+a*sec(d*x+c))^(1/2)
```

3.203.2 Mathematica [A] (verified)

Time = 2.35 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.52

$$\int \frac{\tan^2(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = \frac{4\cos^4\left(\frac{1}{2}(c+dx)\right)\sec^{\frac{5}{2}}(c+dx)\left(3\arcsin\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)\sqrt{\frac{1}{1+\sec(c+dx)}}\sqrt{1-}\right.}{d\sqrt{\sec^2}}$$

input
```
Integrate[Tan[c + d*x]^2/(a + a*Sec[c + d*x])^(5/2),x]
```

output
```
(4*Cos[(c + d*x)/2]^4*Sec[c + d*x]^(5/2)*(3*ArcSin[Tan[(c + d*x)/2]]*Sqrt[
(1 + Sec[c + d*x])^(-1)]*Sqrt[1 + Sec[c + d*x]] + Sqrt[2]*(-2*ArcTan[Tan[(
c + d*x)/2]/Sqrt[(1 + Sec[c + d*x])^(-1)]]*Sqrt[(1 + Sec[c + d*x])^(-1)]*S
qrt[1 + Sec[c + d*x]] + (Sqrt[(1 + Cos[c + d*x])^(-1)]*Tan[(c + d*x)/2])/S
qrt[Sec[c + d*x]])))/(d*Sqrt[Sec[(c + d*x)/2]^2]*(a*(1 + Sec[c + d*x]))^(5
/2))
```

3.203.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.17, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3042, 4375, 373, 397, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^2(c+dx)}{(a\sec(c+dx)+a)^{5/2}}\,dx$$

$$\downarrow 3042$$

$$\int \frac{\cot\left(c+dx+\frac{\pi}{2}\right)^2}{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^{5/2}}dx$$

$$\downarrow 4375$$

$$-\frac{2\int \frac{\tan^2(c+dx)}{(\sec(c+dx)a+a)\left(\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+1\right)\left(\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+2\right)^2} d\left(-\frac{\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)}{ad}$$

$$\downarrow 373$$

$$-\frac{2\left(-\frac{\int \frac{1-\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}}{\left(\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+1\right)\left(\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+2\right)} d\left(-\frac{\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)}{2a} - \frac{\tan(c+dx)}{2a\sqrt{a\sec(c+dx)+a}\left(\frac{a\tan^2(c+dx)}{a\sec(c+dx)+a}+2\right)}\right)}{ad}$$

$$\downarrow 397$$

$$-\frac{2\left(-\frac{2\int \frac{1}{\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+1} d\left(-\frac{\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right) - 3\int \frac{1}{\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+2} d\left(-\frac{\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)}{2a} - \frac{\tan(c+dx)}{2a\sqrt{a\sec(c+dx)+a}\left(\frac{a\tan^2(c+dx)}{a\sec(c+dx)+a}+2\right)}\right)}{ad}$$

$$\downarrow 216$$

$$-\frac{2\left(-\frac{\frac{3\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{2}\sqrt{a}}}{2a}-\frac{\frac{2\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}}}{2a}-\frac{\tan(c+dx)}{2a\sqrt{a\sec(c+dx)+a}\left(\frac{a\tan^2(c+dx)}{a\sec(c+dx)+a}+2\right)}\right)}{ad}$$

input
```
Int[Tan[c + d*x]^2/(a + a*Sec[c + d*x])^(5/2),x]
```

output
```
(-2*(-1/2*((-2*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/Sq
rt[a] + (3*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]]
)])/(Sqrt[2]*Sqrt[a]))/a - Tan[c + d*x]/(2*a*Sqrt[a + a*Sec[c + d*x]]*(2 +
 (a*Tan[c + d*x]^2)/(a + a*Sec[c + d*x])))))/(a*d)
```

3.203.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 373
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[e*(e*x)^(m - 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q +
1)/(2*(b*c - a*d)*(p + 1))), x] - Simp[e^2/(2*(b*c - a*d)*(p + 1))   Int[(e
*x)^(m - 2)*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(m - 1) + d*(m + 2*p +
 2*q + 3)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d,
 0] && LtQ[p, -1] && GtQ[m, 1] && LeQ[m, 3] && IntBinomialQ[a, b, c, d, e,
m, 2, p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4375

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Simp[-2*(a^(m/2 + n + 1/2)/d)   Subst[Int[x^m*((2 + a*x^2
)^(m/2 + n - 1/2)/(1 + a*x^2)), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]
]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && I
ntegerQ[n - 1/2]
```

3.203.4 Maple [A] (verified)

Time = 2.10 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.57

method	result
default	$-\frac{\sqrt{-\frac{2a}{(1-\cos(dx+c))^2\csc(dx+c)^2-1}}\sqrt{(1-\cos(dx+c))^2\csc(dx+c)^2-1}\left(\sqrt{(1-\cos(dx+c))^2\csc(dx+c)^2-1}(-\cot(dx+c)+\csc(dx+c))+\right.}{2da^3}$

input

```
int(tan(d*x+c)^2/(a+a*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output

```
-1/2/d/a^3*(-2*a/((1-cos(d*x+c))^2*csc(d*x+c)^2-1))^(1/2)*((1-cos(d*x+c))^
2*csc(d*x+c)^2-1)^(1/2)*(((1-cos(d*x+c))^2*csc(d*x+c)^2-1)^(1/2)*(-cot(d*x
+c)+csc(d*x+c))+2*2^(1/2)*arctanh(2^(1/2)/((1-cos(d*x+c))^2*csc(d*x+c)^2-1
)^(1/2)*(-cot(d*x+c)+csc(d*x+c)))-3*ln(csc(d*x+c)-cot(d*x+c)+((1-cos(d*x+c
))^2*csc(d*x+c)^2-1)^(1/2)))
```

3.203.5 Fricas [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 492, normalized size of antiderivative = 3.87

$$\int \frac{\tan^2(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = \left[-\frac{3\sqrt{2}\left(\cos(dx+c)^2+2\cos(dx+c)+1\right)\sqrt{-a}\log\left(\frac{2\sqrt{2}\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos\right.}{\cos}\right.$$

$$\left. -\frac{3\sqrt{2}\left(\cos(dx+c)^2+2\cos(dx+c)+1\right)\sqrt{a}\arctan\left(\frac{\sqrt{2}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)}{\sqrt{a}\sin(dx+c)}\right)-4\left(\cos(dx+c)^2+2\cos(\right.}{2\left(a^3d\cos(dx+c)^2+2a^3d\cos(\right.}$$

input

```
integrate(tan(d*x+c)^2/(a+a*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
[-1/4*(3*sqrt(2)*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(-a)*log((2*sqr
t(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x
 + c) + 3*a*cos(d*x + c)^2 + 2*a*cos(d*x + c) - a)/(cos(d*x + c)^2 + 2*cos
(d*x + c) + 1)) + 4*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(-a)*log((2*
a*cos(d*x + c)^2 - 2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(
d*x + c)*sin(d*x + c) + a*cos(d*x + c) - a)/(cos(d*x + c) + 1)) - 4*sqrt((
a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x + c))/(a^3*d*cos(d*
x + c)^2 + 2*a^3*d*cos(d*x + c) + a^3*d), -1/2*(3*sqrt(2)*(cos(d*x + c)^2
 + 2*cos(d*x + c) + 1)*sqrt(a)*arctan(sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos
(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c))) - 4*(cos(d*x + c)^2 + 2*co
s(d*x + c) + 1)*sqrt(a)*arctan(sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos
(d*x + c)/(sqrt(a)*sin(d*x + c))) - 2*sqrt((a*cos(d*x + c) + a)/cos(d*x +
c))*cos(d*x + c)*sin(d*x + c))/(a^3*d*cos(d*x + c)^2 + 2*a^3*d*cos(d*x + c
) + a^3*d)]
```

3.203.6 Sympy [F]

$$\int \frac{\tan^2(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = \int \frac{\tan^2(c+dx)}{(a(\sec(c+dx)+1))^{\frac{5}{2}}}\,dx$$

input
```
integrate(tan(d*x+c)**2/(a+a*sec(d*x+c))**(5/2),x)
```

output
```
Integral(tan(c + d*x)**2/(a*(sec(c + d*x) + 1))**(5/2), x)
```

3.203.7 Maxima [F]

$$\int \frac{\tan^2(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = \int \frac{\tan(dx+c)^2}{(a\sec(dx+c)+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(tan(d*x+c)^2/(a+a*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
integrate(tan(d*x + c)^2/(a*sec(d*x + c) + a)^(5/2), x)
```

3.203.8 Giac [A] (verification not implemented)

Time = 1.50 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.37

$$\int \frac{\tan^2(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = \frac{\sqrt{2}\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{2\,a^3 d\operatorname{sgn}\left(\cos\left(dx+c\right)\right)}$$

input
```
integrate(tan(d*x+c)^2/(a+a*sec(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
1/2*sqrt(2)*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)*tan(1/2*d*x + 1/2*c)/(a^3*
d*sgn(cos(d*x + c)))
```

3.203.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\tan^2(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = \int \frac{\tan(c+d\,x)^2}{\left(a+\frac{a}{\cos(c+d\,x)}\right)^{5/2}}\,dx$$

input
```
int(tan(c + d*x)^2/(a + a/cos(c + d*x))^(5/2),x)
```

output
```
int(tan(c + d*x)^2/(a + a/cos(c + d*x))^(5/2), x)
```

3.204 $\int \frac{\cot^2(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx$

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3.204.2 Mathematica [A] (warning: unable to verify) . . . 1520
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3.204.1 Optimal result

Integrand size = 23, antiderivative size = 265

$$\int \frac{\cot^2(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = -\frac{2\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{a^{5/2}d} + \frac{319\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{128\sqrt{2}a^{5/2}d} + \frac{63\cot(c+dx)\sqrt{a+a\sec(c+dx)}}{128a^3d} - \frac{191\cos(c+dx)\cot(c+dx)\sec^2\left(\frac{1}{2}(c+dx)\right)\sqrt{a+a\sec(c+dx)}}{384a^3d} - \frac{19\cos^2(c+dx)\cot(c+dx)\sec^4\left(\frac{1}{2}(c+dx)\right)\sqrt{a+a\sec(c+dx)}}{192a^3d} - \frac{\cos^3(c+dx)\cot(c+dx)\sec^6\left(\frac{1}{2}(c+dx)\right)\sqrt{a+a\sec(c+dx)}}{48a^3d}$$

output

```
-2*arctan(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))/a^(5/2)/d+319/256*arc
tan(1/2*a^(1/2)*tan(d*x+c)*2^(1/2)/(a+a*sec(d*x+c))^(1/2))*2^(1/2)/a^(5/2)
/d+63/128*cot(d*x+c)*(a+a*sec(d*x+c))^(1/2)/a^3/d-191/384*cos(d*x+c)*cot(d
*x+c)*sec(1/2*d*x+1/2*c)^2*(a+a*sec(d*x+c))^(1/2)/a^3/d-19/192*cos(d*x+c)^
2*cot(d*x+c)*sec(1/2*d*x+1/2*c)^4*(a+a*sec(d*x+c))^(1/2)/a^3/d-1/48*cos(d*
x+c)^3*cot(d*x+c)*sec(1/2*d*x+1/2*c)^6*(a+a*sec(d*x+c))^(1/2)/a^3/d
```

3.204.2 Mathematica [A] (warning: unable to verify)

Time = 5.56 (sec) , antiderivative size = 232, normalized size of antiderivative = 0.88

$$\int \frac{\cot^2(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = \frac{\cos^6\left(\frac{1}{2}(c+dx)\right)\sec^2(c+dx)\left(-((-58+487\cos(c+dx)+698\cos(2(c+dx)$$

input
```
Integrate[Cot[c + d*x]^2/(a + a*Sec[c + d*x])^(5/2),x]
```

output
```
(Cos[(c + d*x)/2]^6*Sec[c + d*x]^2*(-((-58 + 487*Cos[c + d*x] + 698*Cos[2*
(c + d*x)] + 409*Cos[3*(c + d*x)])*Csc[(c + d*x)/2]*Sec[(c + d*x)/2]^7) -
49152*ArcTan[Tan[(c + d*x)/2]/Sqrt[(1 + Sec[c + d*x])^(-1)]]*Sqrt[Sec[c +
d*x]]*Sqrt[Sec[c + d*x]/(1 + Sec[c + d*x])^2]*Sqrt[1 + Sec[c + d*x]] + 306
24*ArcSin[Tan[(c + d*x)/2]]*Sqrt[Sec[(c + d*x)/2]^2]*Sqrt[Sec[c + d*x]]*Sq
rt[(1 + Sec[c + d*x])^(-1)]*Sqrt[1 + Sec[c + d*x]]))/(3072*d*(a*(1 + Sec[c
 + d*x]))^(5/2))
```

3.204.3 Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 282, normalized size of antiderivative = 1.06, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.565$, Rules used = {3042, 4375, 374, 27, 441, 27, 441, 27, 445, 25, 27, 397, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^2(c+dx)}{(a\sec(c+dx)+a)^{5/2}}\,dx$$

$$\downarrow 3042$$

$$\int \frac{1}{\cot\left(c+dx+\frac{\pi}{2}\right)^2\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^{5/2}}dx$$

$$\downarrow 4375$$

$$-\frac{2\int \frac{\cot^2(c+dx)(\sec(c+dx)a+a)}{\left(\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+1\right)\left(\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+2\right)^4}d\left(-\frac{\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)}{a^3d}$$

$$\downarrow 374$$

$$-\frac{2\left(\frac{\int \frac{a\cot^2(c+dx)(\sec(c+dx)a+a)\left(5-\frac{7a\tan^2(c+dx)}{\sec(c+dx)a+a}\right)}{\left(\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+1\right)\left(\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+2\right)^3}d\left(-\frac{\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)}{12a}+\frac{\cot(c+dx)\sqrt{a\sec(c+dx)+a}}{12\left(\frac{a\tan^2(c+dx)}{a\sec(c+dx)+a}+2\right)^3}\right)}{a^3d}$$

$$\downarrow 27$$

$$-\frac{2\left(\frac{1}{12}\int \frac{\cot^2(c+dx)(\sec(c+dx)a+a)\left(5-\frac{7a\tan^2(c+dx)}{\sec(c+dx)a+a}\right)}{\left(\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+1\right)\left(\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+2\right)^3}d\left(-\frac{\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)+\frac{\cot(c+dx)\sqrt{a\sec(c+dx)+a}}{12\left(\frac{a\tan^2(c+dx)}{a\sec(c+dx)+a}+2\right)^3}\right)}{a^3d}$$

$$\downarrow 441$$

$$-\frac{2\left(\frac{1}{12}\left(\frac{\int \frac{a\cot^2(c+dx)(\sec(c+dx)a+a)\left(1-\frac{95a\tan^2(c+dx)}{\sec(c+dx)a+a}\right)}{\left(\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+1\right)\left(\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+2\right)^2}d\left(-\frac{\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)}{8a}+\frac{19\cot(c+dx)\sqrt{a\sec(c+dx)+a}}{8\left(\frac{a\tan^2(c+dx)}{a\sec(c+dx)+a}+2\right)^2}\right)+\frac{\cot(c+dx)\sqrt{a\sec(c+}}{12\left(\frac{a\tan^2(c+dx)}{a\sec(c+dx)+a}+\right.}\right)}{a^3d}$$

$$\downarrow 27$$

$$-\frac{2\left(\frac{1}{12}\left(\frac{1}{8}\int \frac{\cot^2(c+dx)(\sec(c+dx)a+a)\left(1-\frac{95a\tan^2(c+dx)}{\sec(c+dx)a+a}\right)}{\left(\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+1\right)\left(\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+2\right)^2}d\left(-\frac{\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)+\frac{19\cot(c+dx)\sqrt{a\sec(c+dx)+a}}{8\left(\frac{a\tan^2(c+dx)}{a\sec(c+dx)+a}+2\right)^2}\right)+\frac{\cot(c+dx)\sqrt{}}{12\left(\frac{a\tan^2}{a\sec(c-}\right.}\right)}{a^3d}$$

$$\downarrow 441$$

$$-\frac{2\left(\frac{1}{12}\left(\frac{1}{8}\left(\frac{\int -\frac{3a\cot^2(c+dx)(\sec(c+dx)a+a)\left(\frac{191a\tan^2(c+dx)}{\sec(c+dx)a+a}+63\right)}{\left(\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+1\right)\left(\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+2\right)}d\left(-\frac{\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)}{4a}+\frac{191\cot(c+dx)\sqrt{a\sec(c+dx)+a}}{4\left(\frac{a\tan^2(c+dx)}{a\sec(c+dx)+a}+2\right)}\right)+\frac{19\cot(c+d}{8\left(\frac{a\tan}{a\sec}\right.}\right)}{a^3d}$$

$$\downarrow 27$$

$$-\frac{2\left(\frac{1}{12}\left(\frac{1}{8}\left(\frac{191\cot(c+dx)\sqrt{a\sec(c+dx)+a}}{4\left(\frac{a\tan^2(c+dx)}{a\sec(c+dx)+a}+2\right)}-\frac{3}{4}\int \frac{\cot^2(c+dx)(\sec(c+dx)a+a)\left(\frac{191a\tan^2(c+dx)}{\sec(c+dx)a+a}+63\right)}{\left(\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+1\right)\left(\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+2\right)}d\left(-\frac{\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)\right)+\frac{19\text{co}}{8}\right)}{a^3d}$$

$$\downarrow 445$$

3.204. $\int \frac{\cot^2(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx$

$$-\frac{2\left(\frac{1}{12}\left(\frac{1}{8}\left(\frac{191\cot(c+dx)\sqrt{a\sec(c+dx)+a}}{4\left(\frac{a\tan^2(c+dx)}{a\sec(c+dx)+a}+2\right)}-\frac{3}{4}\left(\frac{63}{2}\cot(c+dx)\sqrt{a\sec(c+dx)+a}-\frac{1}{2}\int -\frac{a\left(193-\frac{63a\tan^2(c+dx)}{\sec(c+dx)a+a}\right)}{\left(\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+1\right)\left(\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}\right.}\right.\right.\right.\right.}{a^3d}$$

$$\downarrow 25$$

$$-\frac{2\left(\frac{1}{12}\left(\frac{1}{8}\left(\frac{191\cot(c+dx)\sqrt{a\sec(c+dx)+a}}{4\left(\frac{a\tan^2(c+dx)}{a\sec(c+dx)+a}+2\right)}-\frac{3}{4}\left(\frac{1}{2}\int \frac{a\left(193-\frac{63a\tan^2(c+dx)}{\sec(c+dx)a+a}\right)}{\left(\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+1\right)\left(\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+2\right)}d\left(-\frac{\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)+\frac{63}{2}\cot(c+dx\right.\right.\right.\right.\right.}{a^3d}$$

$$\downarrow 27$$

$$-\frac{2\left(\frac{1}{12}\left(\frac{1}{8}\left(\frac{191\cot(c+dx)\sqrt{a\sec(c+dx)+a}}{4\left(\frac{a\tan^2(c+dx)}{a\sec(c+dx)+a}+2\right)}-\frac{3}{4}\left(\frac{1}{2}a\int \frac{193-\frac{63a\tan^2(c+dx)}{\sec(c+dx)a+a}}{\left(\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+1\right)\left(\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+2\right)}d\left(-\frac{\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)+\frac{63}{2}\cot(c+d\right.\right.\right.\right.\right.}{a^3d}$$

$$\downarrow 397$$

$$-\frac{2\left(\frac{1}{12}\left(\frac{1}{8}\left(\frac{191\cot(c+dx)\sqrt{a\sec(c+dx)+a}}{4\left(\frac{a\tan^2(c+dx)}{a\sec(c+dx)+a}+2\right)}-\frac{3}{4}\left(\frac{1}{2}a\left(256\int \frac{1}{\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+1}d\left(-\frac{\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)-319\int \frac{1}{\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+2}d\left(-\right.\right.\right.\right.\right.}{a}$$

$$\downarrow 216$$

$$-\frac{2\left(\frac{1}{12}\left(\frac{1}{8}\left(\frac{191\cot(c+dx)\sqrt{a\sec(c+dx)+a}}{4\left(\frac{a\tan^2(c+dx)}{a\sec(c+dx)+a}+2\right)}-\frac{3}{4}\left(\frac{1}{2}a\left(\frac{319\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{2}\sqrt{a}}-\frac{256\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}}\right)+\frac{63}{2}\cot(c\right.\right.\right.\right.\right.}{a^3d}$$

input
```
Int[Cot[c + d*x]^2/(a + a*Sec[c + d*x])^(5/2),x]
```

output
```
(-2*((Cot[c + d*x]*Sqrt[a + a*Sec[c + d*x]])/(12*(2 + (a*Tan[c + d*x]^2)/(
a + a*Sec[c + d*x]))^3) + ((19*Cot[c + d*x]*Sqrt[a + a*Sec[c + d*x]])/(8*(
2 + (a*Tan[c + d*x]^2)/(a + a*Sec[c + d*x]))^2) + ((-3*((a*((-256*ArcTan[(
Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/Sqrt[a] + (319*ArcTan[(Sq
rt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])])/(Sqrt[2]*Sqrt[a])
))/2 + (63*Cot[c + d*x]*Sqrt[a + a*Sec[c + d*x]])/2))/4 + (191*Cot[c + d*x
]*Sqrt[a + a*Sec[c + d*x]])/(4*(2 + (a*Tan[c + d*x]^2)/(a + a*Sec[c + d*x]
))))/8)/12))/(a^3*d)
```

3.204. $\int \frac{\cot^2(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx$

3.204.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 374
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q
 + 1)/(a*e*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(e*x)^m*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[b*c*(m + 1) + 2*(b*c -
a*d)*(p + 1) + d*b*(m + 2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b,
c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IntBinomialQ[a, b,
 c, d, e, m, 2, p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 441
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*(g*x)^(m + 1)*(a
+ b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(a*g*2*(b*c - a*d)*(p + 1))), x] + Si
mp[1/(a*2*(b*c - a*d)*(p + 1))   Int[(g*x)^m*(a + b*x^2)^(p + 1)*(c + d*x^2
)^q*Simp[c*(b*e - a*f)*(m + 1) + e*2*(b*c - a*d)*(p + 1) + d*(b*e - a*f)*(m
 + 2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m, q},
 x] && LtQ[p, -1]
```

rule 445
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^2*(m + 1))
 Int[(g*x)^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*f*c*(m + 1) - e*(b*c
+ a*d)*(m + 2 + 1) - e*2*(b*c*p + a*d*q) - b*e*d*(m + 2*(p + q + 2) + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && LtQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4375
```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Simp[-2*(a^(m/2 + n + 1/2)/d)   Subst[Int[x^m*((2 + a*x^2
)^(m/2 + n - 1/2)/(1 + a*x^2)), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]
]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && I
ntegerQ[n - 1/2]
```

3.204.4 Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 588 vs. $2(229) = 458$.

Time = 1.88 (sec) , antiderivative size = 589, normalized size of antiderivative = 2.22

method	result
default	$\frac{\sqrt{-\frac{2a}{(1-\cos(dx+c))^2\csc(dx+c)^2-1}}\sqrt{(1-\cos(dx+c))^2\csc(dx+c)^2-1}\left(-192\left((1-\cos(dx+c))^2\csc(dx+c)^2-1\right)^{\frac{11}{2}}\sin(dx+c)+192(1-c\right.}$

input
```
int(cot(d*x+c)^2/(a+a*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/3072/d/a^3*(-2*a/((1-cos(d*x+c))^2*csc(d*x+c)^2-1))^(1/2)*((1-cos(d*x+c)
)^2*csc(d*x+c)^2-1)^(1/2)/(1-cos(d*x+c))*(-192*((1-cos(d*x+c))^2*csc(d*x+c
)^2-1)^(11/2)*sin(d*x+c)+192*(1-cos(d*x+c))^2*((1-cos(d*x+c))^2*csc(d*x+c)
^2-1)^(9/2)*csc(d*x+c)-216*(1-cos(d*x+c))^2*((1-cos(d*x+c))^2*csc(d*x+c)^2
-1)^(7/2)*csc(d*x+c)+24*(1-cos(d*x+c))^8*((1-cos(d*x+c))^2*csc(d*x+c)^2-1)
^(1/2)*csc(d*x+c)^7+252*(1-cos(d*x+c))^2*((1-cos(d*x+c))^2*csc(d*x+c)^2-1)
^(5/2)*csc(d*x+c)-164*(1-cos(d*x+c))^6*((1-cos(d*x+c))^2*csc(d*x+c)^2-1)^(
1/2)*csc(d*x+c)^5-315*(1-cos(d*x+c))^2*((1-cos(d*x+c))^2*csc(d*x+c)^2-1)^(
3/2)*csc(d*x+c)+563*(1-cos(d*x+c))^4*((1-cos(d*x+c))^2*csc(d*x+c)^2-1)^(1/
2)*csc(d*x+c)^3-3072*2^(1/2)*arctanh(2^(1/2)/((1-cos(d*x+c))^2*csc(d*x+c)^
2-1)^(1/2)*(-cot(d*x+c)+csc(d*x+c)))*(1-cos(d*x+c))-1179*(1-cos(d*x+c))^2*
((1-cos(d*x+c))^2*csc(d*x+c)^2-1)^(1/2)*csc(d*x+c)+3828*ln(csc(d*x+c)-cot(
d*x+c)+((1-cos(d*x+c))^2*csc(d*x+c)^2-1)^(1/2))*(1-cos(d*x+c)))
```

3.204.5 Fricas [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 691, normalized size of antiderivative = 2.61

$$\int \frac{\cot^2(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = \left[-\frac{957\sqrt{2}\left(\cos(dx+c)^3+3\cos(dx+c)^2+3\cos(dx+c)+1\right)\sqrt{-a}\log\left(\frac{2}{-}\right.}{} \right.$$

$$\left. -\frac{957\sqrt{2}\left(\cos(dx+c)^3+3\cos(dx+c)^2+3\cos(dx+c)+1\right)\sqrt{a}\arctan\left(\frac{\sqrt{2}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)}{\sqrt{a}\sin(dx+c)}\right)\sin(dx+}{} \right]$$

input
```
integrate(cot(d*x+c)^2/(a+a*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
[-1/1536*(957*sqrt(2)*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + 3*cos(d*x + c)
+ 1)*sqrt(-a)*log((2*sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x +
c))*cos(d*x + c)*sin(d*x + c) + 3*a*cos(d*x + c)^2 + 2*a*cos(d*x + c) - a)
/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1))*sin(d*x + c) + 768*(cos(d*x + c)^3
 + 3*cos(d*x + c)^2 + 3*cos(d*x + c) + 1)*sqrt(-a)*log(-(8*a*cos(d*x + c)^
3 - 4*(2*cos(d*x + c)^2 - cos(d*x + c))*sqrt(-a)*sqrt((a*cos(d*x + c) + a)
/cos(d*x + c))*sin(d*x + c) - 7*a*cos(d*x + c) + a)/(cos(d*x + c) + 1))*si
n(d*x + c) + 4*(409*cos(d*x + c)^4 + 349*cos(d*x + c)^3 - 185*cos(d*x + c)
^2 - 189*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)))/((a^3*d*co
s(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)*sin(
d*x + c)), -1/768*(957*sqrt(2)*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + 3*cos(
d*x + c) + 1)*sqrt(a)*arctan(sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c
))*cos(d*x + c)/(sqrt(a)*sin(d*x + c)))*sin(d*x + c) + 768*(cos(d*x + c)^3
 + 3*cos(d*x + c)^2 + 3*cos(d*x + c) + 1)*sqrt(a)*arctan(2*sqrt(a)*sqrt((a
*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x + c)/(2*a*cos(d*x +
c)^2 + a*cos(d*x + c) - a))*sin(d*x + c) + 2*(409*cos(d*x + c)^4 + 349*cos
(d*x + c)^3 - 185*cos(d*x + c)^2 - 189*cos(d*x + c))*sqrt((a*cos(d*x + c)
+ a)/cos(d*x + c)))/((a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^
3*d*cos(d*x + c) + a^3*d)*sin(d*x + c))]
```

3.204.6 Sympy [F]

$$\int \frac{\cot^2(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = \int \frac{\cot^2(c+dx)}{(a(\sec(c+dx)+1))^{\frac{5}{2}}}\,dx$$

input
```
integrate(cot(d*x+c)**2/(a+a*sec(d*x+c))**(5/2),x)
```

output
```
Integral(cot(c + d*x)**2/(a*(sec(c + d*x) + 1))**(5/2), x)
```

3.204.7 Maxima [F]

$$\int \frac{\cot^2(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = \int \frac{\cot(dx+c)^2}{(a\sec(dx+c)+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(cot(d*x+c)^2/(a+a*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
integrate(cot(d*x + c)^2/(a*sec(d*x + c) + a)^(5/2), x)
```

3.204.8 Giac [A] (verification not implemented)

Time = 1.08 (sec) , antiderivative size = 177, normalized size of antiderivative = 0.67

$$\int \frac{\cot^2(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = \frac{\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a}\left(2\left(\frac{4\sqrt{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2}{a^3\mathrm{sgn}(\cos(dx+c))}-\frac{31\sqrt{2}}{a^3\mathrm{sgn}(\cos(dx+c))}\right)\tan\left(\frac{1}{2}\,dx\right.\right.}{}$$

input
```
integrate(cot(d*x+c)^2/(a+a*sec(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
1/768*(sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)*(2*(4*sqrt(2)*tan(1/2*d*x + 1/2
*c)^2/(a^3*sgn(cos(d*x + c))) - 31*sqrt(2)/(a^3*sgn(cos(d*x + c))))*tan(1/
2*d*x + 1/2*c)^2 + 291*sqrt(2)/(a^3*sgn(cos(d*x + c))))*tan(1/2*d*x + 1/2*
c) - 96*sqrt(2)/(((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1
/2*c)^2 + a))^2 - a)*sqrt(-a)*a*sgn(cos(d*x + c))))/d
```

3.204.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cot^2(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = \int \frac{\cot(c+d\,x)^2}{\left(a+\frac{a}{\cos(c+d\,x)}\right)^{5/2}}\,dx$$

input
```
int(cot(c + d*x)^2/(a + a/cos(c + d*x))^(5/2),x)
```

output
```
int(cot(c + d*x)^2/(a + a/cos(c + d*x))^(5/2), x)
```

3.205 $\int \frac{\cot^4(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx$

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3.205.1 Optimal result

Integrand size = 23, antiderivative size = 355

$$\int \frac{\cot^4(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = \frac{2\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{a^{5/2}d} - \frac{9683\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{4096\sqrt{2}a^{5/2}d} - \frac{1491\cot(c+dx)\sqrt{a+a\sec(c+dx)}}{4096a^3d} + \frac{5587\cot^3(c+dx)(a+a\sec(c+dx))^{3/2}}{6144a^4d} - \frac{1527\cos(c+dx)\cot^3(c+dx)\sec^2\left(\frac{1}{2}(c+dx)\right)(a+a\sec(c+dx))^{3/2}}{2048a^4d} - \frac{145\cos^2(c+dx)\cot^3(c+dx)\sec^4\left(\frac{1}{2}(c+dx)\right)(a+a\sec(c+dx))^{3/2}}{1024a^4d} - \frac{9\cos^3(c+dx)\cot^3(c+dx)\sec^6\left(\frac{1}{2}(c+dx)\right)(a+a\sec(c+dx))^{3/2}}{256a^4d} - \frac{\cos^4(c+dx)\cot^3(c+dx)\sec^8\left(\frac{1}{2}(c+dx)\right)(a+a\sec(c+dx))^{3/2}}{128a^4d}$$

output
```
2*arctan(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))/a^(5/2)/d+5587/6144*co
t(d*x+c)^3*(a+a*sec(d*x+c))^(3/2)/a^4/d-1527/2048*cos(d*x+c)*cot(d*x+c)^3*
sec(1/2*d*x+1/2*c)^2*(a+a*sec(d*x+c))^(3/2)/a^4/d-145/1024*cos(d*x+c)^2*co
t(d*x+c)^3*sec(1/2*d*x+1/2*c)^4*(a+a*sec(d*x+c))^(3/2)/a^4/d-9/256*cos(d*x
+c)^3*cot(d*x+c)^3*sec(1/2*d*x+1/2*c)^6*(a+a*sec(d*x+c))^(3/2)/a^4/d-1/128
*cos(d*x+c)^4*cot(d*x+c)^3*sec(1/2*d*x+1/2*c)^8*(a+a*sec(d*x+c))^(3/2)/a^4
/d-9683/8192*arctan(1/2*a^(1/2)*tan(d*x+c)*2^(1/2)/(a+a*sec(d*x+c))^(1/2))
*2^(1/2)/a^(5/2)/d-1491/4096*cot(d*x+c)*(a+a*sec(d*x+c))^(1/2)/a^3/d
```

3.205.2 Mathematica [A] (warning: unable to verify)

Time = 3.44 (sec) , antiderivative size = 287, normalized size of antiderivative = 0.81

$$\int \frac{\cot^4(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = \frac{\cos^5\left(\frac{1}{2}(c+dx)\right)\sec^{\frac{5}{2}}(c+dx)\left(\left(-29258+3200\csc^2\left(\frac{1}{2}(c+dx)\right)-128\csc^4(\right.\right.}{}$$

input
```
Integrate[Cot[c + d*x]^4/(a + a*Sec[c + d*x])^(5/2),x]
```

output
```
(Cos[(c + d*x)/2]^5*Sec[c + d*x]^(5/2)*((-29258 + 3200*Csc[(c + d*x)/2]^2
- 128*Csc[(c + d*x)/2]^4 + 18225*Sec[(c + d*x)/2]^2 - 4470*Sec[(c + d*x)/2
]^4 + 696*Sec[(c + d*x)/2]^6 - 48*Sec[(c + d*x)/2]^8)*Sqrt[Sec[c + d*x]] +
 49152*ArcTan[Tan[(c + d*x)/2]/Sqrt[(1 + Sec[c + d*x])^(-1)]]*Cot[(c + d*x
)/2]*Sqrt[Sec[c + d*x]/(1 + Sec[c + d*x])^2]*Sqrt[1 + Sec[c + d*x]] - 2904
9*ArcSin[Tan[(c + d*x)/2]]*Cot[(c + d*x)/2]*Sqrt[Sec[(c + d*x)/2]^2]*Sqrt[
(1 + Sec[c + d*x])^(-1)]*Sqrt[1 + Sec[c + d*x]])*Sin[(c + d*x)/2])/(3072*d
*(a*(1 + Sec[c + d*x]))^(5/2))
```

3.205.3 Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 377, normalized size of antiderivative = 1.06, number of steps used = 18, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.739$, Rules used = {3042, 4375, 374, 27, 441, 27, 441, 27, 441, 27, 445, 27, 445, 25, 27, 397, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^4(c+dx)}{(a\sec(c+dx)+a)^{5/2}}\,dx$$

$$\downarrow 3042$$

$$\int \frac{1}{\cot\left(c+dx+\frac{\pi}{2}\right)^4\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^{5/2}}dx$$

$$\downarrow 4375$$

$$-\frac{2\int \frac{\cot^4(c+dx)(\sec(c+dx)a+a)^2}{\left(\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+1\right)\left(\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+2\right)^5}d\left(-\frac{\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)}{a^4d}$$

$$\downarrow 374$$

$$-\frac{2\left(\frac{\int \frac{a\cot^4(c+dx)(\sec(c+dx)a+a)^2\left(5-\frac{11a\tan^2(c+dx)}{\sec(c+dx)a+a}\right)}{\left(\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+1\right)\left(\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+2\right)^4}d\left(-\frac{\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)}{16a}+\frac{\cot^3(c+dx)(a\sec(c+dx)+a)^{3/2}}{16\left(\frac{a\tan^2(c+dx)}{a\sec(c+dx)+a}+2\right)^4}\right)}{a^4d}$$

$$\downarrow 27$$

$$-\frac{2\left(\frac{1}{16}\int \frac{\cot^4(c+dx)(\sec(c+dx)a+a)^2\left(5-\frac{11a\tan^2(c+dx)}{\sec(c+dx)a+a}\right)}{\left(\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+1\right)\left(\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+2\right)^4}d\left(-\frac{\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)+\frac{\cot^3(c+dx)(a\sec(c+dx)+a)^{3/2}}{16\left(\frac{a\tan^2(c+dx)}{a\sec(c+dx)+a}+2\right)^4}\right)}{a^4d}$$

$$\downarrow 441$$

$$-\frac{2\left(\frac{1}{16}\left(\frac{\int -\frac{3a\cot^4(c+dx)(\sec(c+dx)a+a)^2\left(\frac{81a\tan^2(c+dx)}{\sec(c+dx)a+a}+17\right)}{\left(\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+1\right)\left(\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+2\right)^3}d\left(-\frac{\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)}{12a}+\frac{9\cot^3(c+dx)(a\sec(c+dx)+a)^{3/2}}{4\left(\frac{a\tan^2(c+dx)}{a\sec(c+dx)+a}+2\right)^3}\right)+\frac{\cot^3(c+dx)(a}{16\left(\frac{a\tan}{a\sec(}\right.}{a^4d}$$

$$\downarrow 27$$

$$-\frac{2\left(\frac{1}{16}\left(\frac{9\cot^3(c+dx)(a\sec(c+dx)+a)^{3/2}}{4\left(\frac{a\tan^2(c+dx)}{a\sec(c+dx)+a}+2\right)^3}-\frac{1}{4}\int \frac{\cot^4(c+dx)(\sec(c+dx)a+a)^2\left(\frac{81a\tan^2(c+dx)}{\sec(c+dx)a+a}+17\right)}{\left(\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+1\right)\left(\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+2\right)^3}d\left(-\frac{\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)\right)+\frac{\cot^3(c+}{16\left(}\right.}{a^4d}$$

$$\downarrow 441$$

$$-\frac{2\left(\frac{1}{16}\left(\frac{1}{4}\left(\frac{145\cot^3(c+dx)(a\sec(c+dx)+a)^{3/2}}{8\left(\frac{a\tan^2(c+dx)}{a\sec(c+dx)+a}+2\right)^2}-\frac{\int\frac{a\cot^4(c+dx)(\sec(c+dx)a+a)^2\left(\frac{1015a\tan^2(c+dx)}{\sec(c+dx)a+a}+503\right)}{\left(\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+1\right)\left(\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+2\right)^2}d\left(-\frac{\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)}{8a}\right)+\frac{9\cot^3(}{4}\right.\right.\right.}{a^4d}$$

$$\downarrow 27$$

$$-\frac{2\left(\frac{1}{16}\left(\frac{1}{4}\left(\frac{145\cot^3(c+dx)(a\sec(c+dx)+a)^{3/2}}{8\left(\frac{a\tan^2(c+dx)}{a\sec(c+dx)+a}+2\right)^2}-\frac{1}{8}\int\frac{\cot^4(c+dx)(\sec(c+dx)a+a)^2\left(\frac{1015a\tan^2(c+dx)}{\sec(c+dx)a+a}+503\right)}{\left(\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+1\right)\left(\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+2\right)^2}d\left(-\frac{\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)\right)-\right.\right.\right.}{a^4d}$$

$$\downarrow 441$$

$$-\frac{2\left(\frac{1}{16}\left(\frac{1}{4}\left(\frac{1}{8}\left(\frac{1527\cot^3(c+dx)(a\sec(c+dx)+a)^{3/2}}{4\left(\frac{a\tan^2(c+dx)}{a\sec(c+dx)+a}+2\right)}-\frac{\int\frac{a\cot^4(c+dx)(\sec(c+dx)a+a)^2\left(\frac{7635a\tan^2(c+dx)}{\sec(c+dx)a+a}+5587\right)}{\left(\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+1\right)\left(\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+2\right)}d\left(-\frac{\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)}{4a}\right)+\right.\right.\right.}{a^4d}$$

$$\downarrow 27$$

$$-\frac{2\left(\frac{1}{16}\left(\frac{1}{4}\left(\frac{1}{8}\left(\frac{1527\cot^3(c+dx)(a\sec(c+dx)+a)^{3/2}}{4\left(\frac{a\tan^2(c+dx)}{a\sec(c+dx)+a}+2\right)}-\frac{1}{4}\int\frac{\cot^4(c+dx)(\sec(c+dx)a+a)^2\left(\frac{7635a\tan^2(c+dx)}{\sec(c+dx)a+a}+5587\right)}{\left(\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+1\right)\left(\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+2\right)}d\left(-\frac{\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right.\right.\right.\right.}{a^4d}$$

$$\downarrow 445$$

$$-\frac{2\left(\frac{1}{16}\left(\frac{1}{4}\left(\frac{1}{8}\left(\frac{1}{4}\left(\frac{1}{6}\int\frac{3a\cot^2(c+dx)(\sec(c+dx)a+a)\left(\frac{5587a\tan^2(c+dx)}{\sec(c+dx)a+a}+1491\right)}{\left(\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+1\right)\left(\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+2\right)}d\left(-\frac{\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)-\frac{5587}{6}\cot^3(c+dx)(a\sec\right.\right.\right.\right.\right.}{a^4d}$$

$$\downarrow 27$$

$$-\frac{2\left(\frac{1}{16}\left(\frac{1}{4}\left(\frac{1}{8}\left(\frac{1}{4}\left(\frac{1}{2}a\int\frac{\cot^2(c+dx)(\sec(c+dx)a+a)\left(\frac{5587a\tan^2(c+dx)}{\sec(c+dx)a+a}+1491\right)}{\left(\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+1\right)\left(\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+2\right)}d\left(-\frac{\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)-\frac{5587}{6}\cot^3(c+dx)(a\sec(\right.\right.\right.\right.\right.}{a^4d}$$

$$\downarrow 445$$

$$-2\left(\frac{1}{16}\left(\frac{1}{4}\left(\frac{1}{8}\left(\frac{1}{4}\left(\frac{1}{2}a\left(\frac{1491}{2}\cot(c+dx)\sqrt{a\sec(c+dx)+a}-\frac{1}{2}\int -\frac{a\left(6701-\frac{1491a\tan^2(c+dx)}{\sec(c+dx)a+a}\right)}{\left(\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+1\right)\left(\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+2\right)}d\left(-\frac{\tan(c+dx)}{\sqrt{\sec(c+dx)}}\right.\right.\right.\right.\right.\right.\right.$$

$$\downarrow 25$$

$$-2\left(\frac{1}{16}\left(\frac{1}{4}\left(\frac{1}{8}\left(\frac{1}{4}\left(\frac{1}{2}a\left(\frac{1}{2}\int \frac{a\left(6701-\frac{1491a\tan^2(c+dx)}{\sec(c+dx)a+a}\right)}{\left(\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+1\right)\left(\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+2\right)}d\left(-\frac{\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)+\frac{1491}{2}\cot(c+dx)\sqrt{a\sec(c+dx)}\right.\right.\right.\right.\right.\right.\right.$$

$$\downarrow 27$$

$$-2\left(\frac{1}{16}\left(\frac{1}{4}\left(\frac{1}{8}\left(\frac{1}{4}\left(\frac{1}{2}a\left(\frac{1}{2}a\int \frac{6701-\frac{1491a\tan^2(c+dx)}{\sec(c+dx)a+a}}{\left(\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+1\right)\left(\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+2\right)}d\left(-\frac{\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)+\frac{1491}{2}\cot(c+dx)\sqrt{a\sec(c+dx)}\right.\right.\right.\right.\right.\right.\right.$$

$$\downarrow 397$$

$$-2\left(\frac{1}{16}\left(\frac{1}{4}\left(\frac{1}{8}\left(\frac{1}{4}\left(\frac{1}{2}a\left(\frac{1}{2}a\left(8192\int \frac{1}{\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+1}d\left(-\frac{\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)-9683\int \frac{1}{\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+2}d\left(-\frac{\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)\right)\right.\right.\right.\right.\right.\right.\right.$$

$$\downarrow 216$$

$$-2\left(\frac{1}{16}\left(\frac{1}{4}\left(\frac{1}{8}\left(\frac{1}{4}\left(\frac{1}{2}a\left(\frac{1}{2}a\left(\frac{9683\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{2}\sqrt{a}}-\frac{8192\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}}\right)+\frac{1491}{2}\cot(c+dx)\sqrt{a\sec(c\right.\right.\right.\right.\right.\right.\right.$$

input
```
Int[Cot[c + d*x]^4/(a + a*Sec[c + d*x])^(5/2),x]
```

3.205. $\int \frac{\cot^4(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx$

output
```
(-2*((Cot[c + d*x]^3*(a + a*Sec[c + d*x])^(3/2))/(16*(2 + (a*Tan[c + d*x]^
2)/(a + a*Sec[c + d*x]))^4) + ((9*Cot[c + d*x]^3*(a + a*Sec[c + d*x])^(3/2
))/(4*(2 + (a*Tan[c + d*x]^2)/(a + a*Sec[c + d*x]))^3) + ((145*Cot[c + d*x
]^3*(a + a*Sec[c + d*x])^(3/2))/(8*(2 + (a*Tan[c + d*x]^2)/(a + a*Sec[c +
d*x]))^2) + (((-5587*Cot[c + d*x]^3*(a + a*Sec[c + d*x])^(3/2))/6 + (a*((a
*((-8192*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/Sqrt[a]
+ (9683*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])])
/(Sqrt[2]*Sqrt[a])))/2 + (1491*Cot[c + d*x]*Sqrt[a + a*Sec[c + d*x]])/2))/
2)/4 + (1527*Cot[c + d*x]^3*(a + a*Sec[c + d*x])^(3/2))/(4*(2 + (a*Tan[c +
 d*x]^2)/(a + a*Sec[c + d*x])))/8)/4)/16))/(a^4*d)
```

3.205.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 374
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q
 + 1)/(a*e*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(e*x)^m*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[b*c*(m + 1) + 2*(b*c -
a*d)*(p + 1) + d*b*(m + 2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b,
c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IntBinomialQ[a, b,
 c, d, e, m, 2, p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 441
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*(g*x)^(m + 1)*(a
+ b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(a*g*2*(b*c - a*d)*(p + 1))), x] + Si
mp[1/(a*2*(b*c - a*d)*(p + 1))   Int[(g*x)^m*(a + b*x^2)^(p + 1)*(c + d*x^2
)^q*Simp[c*(b*e - a*f)*(m + 1) + e*2*(b*c - a*d)*(p + 1) + d*(b*e - a*f)*(m
 + 2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m, q},
 x] && LtQ[p, -1]
```

rule 445
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^2*(m + 1))
 Int[(g*x)^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*f*c*(m + 1) - e*(b*c
+ a*d)*(m + 2 + 1) - e*2*(b*c*p + a*d*q) - b*e*d*(m + 2*(p + q + 2) + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && LtQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4375
```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Simp[-2*(a^(m/2 + n + 1/2)/d)   Subst[Int[x^m*((2 + a*x^2
)^(m/2 + n - 1/2)/(1 + a*x^2)), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]
]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && I
ntegerQ[n - 1/2]
```

3.205.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 709 vs. $2(310) = 620$.

Time = 1.93 (sec) , antiderivative size = 710, normalized size of antiderivative = 2.00

method	result
default	$-\frac{\sqrt{a(1+\sec(dx+c))}\left(29049\cos(dx+c)^3\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}\sqrt{2}\ln\left(\csc(dx+c)-\cot(dx+c)+\sqrt{\cot(dx+c)^2-2\cot(dx+c)\csc(dx+c)+cs\right.\right.}{}$

input
```
int(cot(d*x+c)^4/(a+a*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/24576/d/a^3*(a*(1+sec(d*x+c)))^(1/2)/(cos(d*x+c)+1)^3*(29049*cos(d*x+c)
^3*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*2^(1/2)*ln(csc(d*x+c)-cot(d*x+c)+(co
t(d*x+c)^2-2*cot(d*x+c)*csc(d*x+c)+csc(d*x+c)^2-1)^(1/2))-49152*cos(d*x+c)
^3*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctanh(sin(d*x+c)/(cos(d*x+c)+1)/(-
cos(d*x+c)/(cos(d*x+c)+1))^(1/2))+87147*2^(1/2)*(-cos(d*x+c)/(cos(d*x+c)+1
))^(1/2)*ln(csc(d*x+c)-cot(d*x+c)+(cot(d*x+c)^2-2*cot(d*x+c)*csc(d*x+c)+cs
c(d*x+c)^2-1)^(1/2))*cos(d*x+c)^2-147456*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2
)*arctanh(sin(d*x+c)/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*co
s(d*x+c)^2+87147*2^(1/2)*ln(csc(d*x+c)-cot(d*x+c)+(cot(d*x+c)^2-2*cot(d*x+
c)*csc(d*x+c)+csc(d*x+c)^2-1)^(1/2))*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*co
s(d*x+c)-147456*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctanh(sin(d*x+c)/(cos
(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*cos(d*x+c)+29049*ln(csc(d*x
+c)-cot(d*x+c)+(cot(d*x+c)^2-2*cot(d*x+c)*csc(d*x+c)+csc(d*x+c)^2-1)^(1/2)
)*2^(1/2)*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+29258*cos(d*x+c)^3*cot(d*x+c)
^3-49152*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctanh(sin(d*x+c)/(cos(d*x+c)
+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))+28466*cot(d*x+c)^3*cos(d*x+c)^2-28
116*cos(d*x+c)*cot(d*x+c)^3-34852*cot(d*x+c)^3+4490*cot(d*x+c)^2*csc(d*x+c
)+8946*cot(d*x+c)*csc(d*x+c)^2)
```

3.205.5 Fricas [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 868, normalized size of antiderivative = 2.45

$$\int \frac{\cot^4(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(cot(d*x+c)^4/(a+a*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

output

```
[-1/49152*(29049*sqrt(2)*(cos(d*x + c)^5 + 3*cos(d*x + c)^4 + 2*cos(d*x +
c)^3 - 2*cos(d*x + c)^2 - 3*cos(d*x + c) - 1)*sqrt(-a)*log(-(2*sqrt(2)*sqr
t(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x + c) -
3*a*cos(d*x + c)^2 - 2*a*cos(d*x + c) + a)/(cos(d*x + c)^2 + 2*cos(d*x + c
) + 1))*sin(d*x + c) + 24576*(cos(d*x + c)^5 + 3*cos(d*x + c)^4 + 2*cos(d*
x + c)^3 - 2*cos(d*x + c)^2 - 3*cos(d*x + c) - 1)*sqrt(-a)*log(-(8*a*cos(d
*x + c)^3 + 4*(2*cos(d*x + c)^2 - cos(d*x + c))*sqrt(-a)*sqrt((a*cos(d*x +
 c) + a)/cos(d*x + c))*sin(d*x + c) - 7*a*cos(d*x + c) + a)/(cos(d*x + c)
+ 1))*sin(d*x + c) - 4*(14629*cos(d*x + c)^6 + 14233*cos(d*x + c)^5 - 1405
8*cos(d*x + c)^4 - 17426*cos(d*x + c)^3 + 2245*cos(d*x + c)^2 + 4473*cos(d
*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)))/((a^3*d*cos(d*x + c)^5 +
 3*a^3*d*cos(d*x + c)^4 + 2*a^3*d*cos(d*x + c)^3 - 2*a^3*d*cos(d*x + c)^2
- 3*a^3*d*cos(d*x + c) - a^3*d)*sin(d*x + c)), 1/24576*(29049*sqrt(2)*(cos
(d*x + c)^5 + 3*cos(d*x + c)^4 + 2*cos(d*x + c)^3 - 2*cos(d*x + c)^2 - 3*c
os(d*x + c) - 1)*sqrt(a)*arctan(sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x
+ c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c)))*sin(d*x + c) + 24576*(cos(d*x +
 c)^5 + 3*cos(d*x + c)^4 + 2*cos(d*x + c)^3 - 2*cos(d*x + c)^2 - 3*cos(d*x
 + c) - 1)*sqrt(a)*arctan(2*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)
)*cos(d*x + c)*sin(d*x + c)/(2*a*cos(d*x + c)^2 + a*cos(d*x + c) - a))*sin
(d*x + c) + 2*(14629*cos(d*x + c)^6 + 14233*cos(d*x + c)^5 - 14058*cos(...
```

3.205.6 Sympy [F]

$$\int \frac{\cot^4(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = \int \frac{\cot^4(c+dx)}{(a(\sec(c+dx)+1))^{\frac{5}{2}}}\,dx$$

input

```
integrate(cot(d*x+c)**4/(a+a*sec(d*x+c))**(5/2),x)
```

output

```
Integral(cot(c + d*x)**4/(a*(sec(c + d*x) + 1))**(5/2), x)
```

3.205.7 Maxima [F]

$$\int \frac{\cot^4(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = \int \frac{\cot\left(dx+c\right)^4}{\left(a\sec\left(dx+c\right)+a\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(cot(d*x+c)^4/(a+a*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
integrate(cot(d*x + c)^4/(a*sec(d*x + c) + a)^(5/2), x)
```

3.205.8 Giac [A] (verification not implemented)

Time = 1.39 (sec) , antiderivative size = 296, normalized size of antiderivative = 0.83

$$\int \frac{\cot^4(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = \frac{3\left(2\left(4\left(\frac{2\sqrt{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2}{a^3\mathrm{sgn}(\cos(dx+c))}-\frac{19\sqrt{2}}{a^3\mathrm{sgn}(\cos(dx+c))}\right)\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+\frac{369\sqrt{2}}{a^3\mathrm{sgn}(\cos(dx+c}\right.\right.}{}$$

input
```
integrate(cot(d*x+c)^4/(a+a*sec(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
1/24576*(3*(2*(4*(2*sqrt(2)*tan(1/2*d*x + 1/2*c)^2/(a^3*sgn(cos(d*x + c)))
 - 19*sqrt(2)/(a^3*sgn(cos(d*x + c))))*tan(1/2*d*x + 1/2*c)^2 + 369*sqrt(2
)/(a^3*sgn(cos(d*x + c))))*tan(1/2*d*x + 1/2*c)^2 - 2989*sqrt(2)/(a^3*sgn(
cos(d*x + c))))*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)*tan(1/2*d*x + 1/2*c) +
 512*sqrt(2)*(12*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/
2*c)^2 + a))^4 - 21*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x +
 1/2*c)^2 + a))^2*a + 11*a^2)/(((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*t
an(1/2*d*x + 1/2*c)^2 + a))^2 - a)^3*sqrt(-a)*a*sgn(cos(d*x + c))))/d
```

3.205.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cot^4(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = \int \frac{\cot(c+d\,x)^4}{\left(a+\frac{a}{\cos(c+d\,x)}\right)^{5/2}}\,dx$$

input
```
int(cot(c + d*x)^4/(a + a/cos(c + d*x))^(5/2),x)
```

output
```
int(cot(c + d*x)^4/(a + a/cos(c + d*x))^(5/2), x)
```

3.206 $\int \frac{\cot^6(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx$

3.206.1 Optimal result . . . 1539
3.206.2 Mathematica [A] (verified) . . . 1540
3.206.3 Rubi [A] (verified) . . . 1541
3.206.4 Maple [A] (verified) . . . 1546
3.206.5 Fricas [A] (verification not implemented) . . . 1547
3.206.6 Sympy [F] . . . 1548
3.206.7 Maxima [F(-1)] . . . 1549
3.206.8 Giac [A] (verification not implemented) . . . 1549
3.206.9 Mupad [F(-1)] . . . 1550

3.206.1 Optimal result

Integrand size = 23, antiderivative size = 439

$$\begin{aligned}
\int \frac{\cot^6(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx &= -\frac{2\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{a^{5/2}d} \\
&+ \frac{74461\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{32768\sqrt{2}a^{5/2}d} + \frac{8925\cot(c+dx)\sqrt{a+a\sec(c+dx)}}{32768a^3d} \\
&- \frac{41693\cot^3(c+dx)(a+a\sec(c+dx))^{3/2}}{49152a^4d} \\
&+ \frac{58077\cot^5(c+dx)(a+a\sec(c+dx))^{5/2}}{40960a^5d} \\
&- \frac{9467\cos(c+dx)\cot^5(c+dx)\sec^2\left(\frac{1}{2}(c+dx)\right)(a+a\sec(c+dx))^{5/2}}{8192a^5d} \\
&- \frac{2473\cos^2(c+dx)\cot^5(c+dx)\sec^4\left(\frac{1}{2}(c+dx)\right)(a+a\sec(c+dx))^{5/2}}{12288a^5d} \\
&- \frac{155\cos^3(c+dx)\cot^5(c+dx)\sec^6\left(\frac{1}{2}(c+dx)\right)(a+a\sec(c+dx))^{5/2}}{3072a^5d} \\
&- \frac{7\cos^4(c+dx)\cot^5(c+dx)\sec^8\left(\frac{1}{2}(c+dx)\right)(a+a\sec(c+dx))^{5/2}}{512a^5d} \\
&- \frac{\cos^5(c+dx)\cot^5(c+dx)\sec^{10}\left(\frac{1}{2}(c+dx)\right)(a+a\sec(c+dx))^{5/2}}{320a^5d}
\end{aligned}$$

output
```
-2*arctan(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))/a^(5/2)/d-41693/49152
*cot(d*x+c)^3*(a+a*sec(d*x+c))^(3/2)/a^4/d+58077/40960*cot(d*x+c)^5*(a+a*s
ec(d*x+c))^(5/2)/a^5/d-9467/8192*cos(d*x+c)*cot(d*x+c)^5*sec(1/2*d*x+1/2*c
)^2*(a+a*sec(d*x+c))^(5/2)/a^5/d-2473/12288*cos(d*x+c)^2*cot(d*x+c)^5*sec(
1/2*d*x+1/2*c)^4*(a+a*sec(d*x+c))^(5/2)/a^5/d-155/3072*cos(d*x+c)^3*cot(d*
x+c)^5*sec(1/2*d*x+1/2*c)^6*(a+a*sec(d*x+c))^(5/2)/a^5/d-7/512*cos(d*x+c)^
4*cot(d*x+c)^5*sec(1/2*d*x+1/2*c)^8*(a+a*sec(d*x+c))^(5/2)/a^5/d-1/320*cos
(d*x+c)^5*cot(d*x+c)^5*sec(1/2*d*x+1/2*c)^10*(a+a*sec(d*x+c))^(5/2)/a^5/d+
74461/65536*arctan(1/2*a^(1/2)*tan(d*x+c)*2^(1/2)/(a+a*sec(d*x+c))^(1/2))*
2^(1/2)/a^(5/2)/d+8925/32768*cot(d*x+c)*(a+a*sec(d*x+c))^(1/2)/a^3/d
```

3.206.2 Mathematica [A] (verified)

Time = 5.53 (sec) , antiderivative size = 292, normalized size of antiderivative = 0.67

$$\int \frac{\cot^6(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = \frac{\cos^4\left(\frac{1}{2}(c+dx)\right)\sec^{\frac{5}{2}}(c+dx)\left(-\frac{\sqrt{\frac{1}{2+2\cos(c+dx)}}(3364685+2115266\cos(c+dx)+3550428\cos$$

input
```
Integrate[Cot[c + d*x]^6/(a + a*Sec[c + d*x])^(5/2),x]
```

output
```
(Cos[(c + d*x)/2]^4*Sec[c + d*x]^(5/2)*(-1/8192*(Sqrt[(2 + 2*Cos[c + d*x])
^(-1)]*(3364685 + 2115266*Cos[c + d*x] + 3550428*Cos[2*(c + d*x)] + 100578
2*Cos[3*(c + d*x)] + 714844*Cos[4*(c + d*x)] - 1338430*Cos[5*(c + d*x)] +
1168164*Cos[6*(c + d*x)] + 1363110*Cos[7*(c + d*x)] + 639063*Cos[8*(c + d*
x)])*Csc[(c + d*x)/2]^5*Sec[(c + d*x)/2]^9*Sqrt[Sec[c + d*x]]) + 1116915*A
rcSin[Tan[(c + d*x)/2]]*Sqrt[(1 + Sec[c + d*x])^(-1)]*Sqrt[1 + Sec[c + d*x
]] - 983040*Sqrt[2]*ArcTan[Tan[(c + d*x)/2]/Sqrt[(1 + Sec[c + d*x])^(-1)]]
*Sqrt[(1 + Sec[c + d*x])^(-1)]*Sqrt[1 + Sec[c + d*x]]))/(122880*d*Sqrt[Sec
[(c + d*x)/2]^2]*(a*(1 + Sec[c + d*x]))^(5/2))
```

3.206.3 Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 466, normalized size of antiderivative = 1.06, number of steps used = 23, number of rules used = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.957$, Rules used = {3042, 4375, 374, 27, 441, 25, 27, 441, 27, 441, 27, 441, 27, 445, 27, 445, 27, 445, 25, 27, 397, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^6(c+dx)}{(a\sec(c+dx)+a)^{5/2}}\,dx$$

$$\downarrow 3042$$

$$\int \frac{1}{\cot\left(c+dx+\frac{\pi}{2}\right)^6\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^{5/2}}dx$$

$$\downarrow 4375$$

$$-\frac{2\int \frac{\cot^6(c+dx)(\sec(c+dx)a+a)^3}{\left(\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+1\right)\left(\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+2\right)^6}d\left(-\frac{\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)}{a^5d}$$

$$\downarrow 374$$

$$-\frac{2\left(\frac{\int \frac{5a\cot^6(c+dx)(\sec(c+dx)a+a)^3\left(1-\frac{3a\tan^2(c+dx)}{\sec(c+dx)a+a}\right)}{\left(\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+1\right)\left(\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+2\right)^5}d\left(-\frac{\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)}{20a}+\frac{\cot^5(c+dx)(a\sec(c+dx)+a)^{5/2}}{20\left(\frac{a\tan^2(c+dx)}{a\sec(c+dx)+a}+2\right)^5}\right)}{a^5d}$$

$$\downarrow 27$$

$$-\frac{2\left(\frac{1}{4}\int \frac{\cot^6(c+dx)(\sec(c+dx)a+a)^3\left(1-\frac{3a\tan^2(c+dx)}{\sec(c+dx)a+a}\right)}{\left(\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+1\right)\left(\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+2\right)^5}d\left(-\frac{\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)+\frac{\cot^5(c+dx)(a\sec(c+dx)+a)^{5/2}}{20\left(\frac{a\tan^2(c+dx)}{a\sec(c+dx)+a}+2\right)^5}\right)}{a^5d}$$

$$\downarrow 441$$

$$-\frac{2\left(\frac{1}{4}\left(\frac{\int -\frac{a\cot^6(c+dx)(\sec(c+dx)a+a)^3\left(\frac{91a\tan^2(c+dx)}{\sec(c+dx)a+a}+27\right)}{\left(\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+1\right)\left(\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+2\right)^4}d\left(-\frac{\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)}{16a}+\frac{7\cot^5(c+dx)(a\sec(c+dx)+a)^{5/2}}{16\left(\frac{a\tan^2(c+dx)}{a\sec(c+dx)+a}+2\right)^4}\right)+\frac{\cot^5(c+dx)(a\sec}{20\left(\frac{a\tan^2(}{a\sec(c+}\right.\right.}{a^5d}$$

$$\downarrow 25$$

$$-\frac{2\left(\frac{1}{4}\left(\frac{7\cot^5(c+dx)(a\sec(c+dx)+a)^{5/2}}{16\left(\frac{a\tan^2(c+dx)}{a\sec(c+dx)+a}+2\right)^4}-\frac{\int\frac{a\cot^6(c+dx)(\sec(c+dx)a+a)^3\left(\frac{91a\tan^2(c+dx)}{\sec(c+dx)a+a}+27\right)}{\left(\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+1\right)\left(\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+2\right)^4}d\left(-\frac{\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)}{16a}\right)+\frac{\cot^5(c+dx)(a\sec(}{20\left(\frac{a\tan^2(c+dx)}{a\sec(c+dx)}\right.}\right.}{a^5d}$$

$$\downarrow 27$$

$$-\frac{2\left(\frac{1}{4}\left(\frac{7\cot^5(c+dx)(a\sec(c+dx)+a)^{5/2}}{16\left(\frac{a\tan^2(c+dx)}{a\sec(c+dx)+a}+2\right)^4}-\frac{1}{16}\int\frac{\cot^6(c+dx)(\sec(c+dx)a+a)^3\left(\frac{91a\tan^2(c+dx)}{\sec(c+dx)a+a}+27\right)}{\left(\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+1\right)\left(\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+2\right)^4}d\left(-\frac{\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)\right)+\frac{\cot^5(c+}{20\left(\right.}\right.}{a^5d}$$

$$\downarrow 441$$

$$-\frac{2\left(\frac{1}{4}\left(\frac{1}{16}\left(\frac{155\cot^5(c+dx)(a\sec(c+dx)+a)^{5/2}}{12\left(\frac{a\tan^2(c+dx)}{a\sec(c+dx)+a}+2\right)^3}-\frac{\int\frac{a\cot^6(c+dx)(\sec(c+dx)a+a)^3\left(\frac{1705a\tan^2(c+dx)}{\sec(c+dx)a+a}+937\right)}{\left(\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+1\right)\left(\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+2\right)^3}d\left(-\frac{\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)}{12a}\right)+\frac{7\cot^5(}{1}\right.}{a^5d}$$

$$\downarrow 27$$

$$-\frac{2\left(\frac{1}{4}\left(\frac{1}{16}\left(\frac{155\cot^5(c+dx)(a\sec(c+dx)+a)^{5/2}}{12\left(\frac{a\tan^2(c+dx)}{a\sec(c+dx)+a}+2\right)^3}-\frac{1}{12}\int\frac{\cot^6(c+dx)(\sec(c+dx)a+a)^3\left(\frac{1705a\tan^2(c+dx)}{\sec(c+dx)a+a}+937\right)}{\left(\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+1\right)\left(\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+2\right)^3}d\left(-\frac{\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)\right)\right.}{a^5d}$$

$$\downarrow 441$$

$$-\frac{2\left(\frac{1}{4}\left(\frac{1}{16}\left(\frac{1}{12}\left(\frac{2473\cot^5(c+dx)(a\sec(c+dx)+a)^{5/2}}{8\left(\frac{a\tan^2(c+dx)}{a\sec(c+dx)+a}+2\right)^2}-\frac{\int\frac{3a\cot^6(c+dx)(\sec(c+dx)a+a)^3\left(\frac{7419a\tan^2(c+dx)}{\sec(c+dx)a+a}+5371\right)}{\left(\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+1\right)\left(\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+2\right)^2}d\left(-\frac{\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)}{8a}\right)-\right.\right.}{a^5d}$$

$$\downarrow 27$$

$$-\frac{2\left(\frac{1}{4}\left(\frac{1}{16}\left(\frac{1}{12}\left(\frac{2473\cot^5(c+dx)(a\sec(c+dx)+a)^{5/2}}{8\left(\frac{a\tan^2(c+dx)}{a\sec(c+dx)+a}+2\right)^2}-\frac{3}{8}\int\frac{\cot^6(c+dx)(\sec(c+dx)a+a)^3\left(\frac{7419a\tan^2(c+dx)}{\sec(c+dx)a+a}+5371\right)}{\left(\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+1\right)\left(\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+2\right)^2}d\left(-\frac{\tan(c+dx)}{\sqrt{\sec(c+dx)a+}}\right.\right.\right.\right.\right.}{a^5d}$$

$$\downarrow 441$$

$$-\frac{2\left(\frac{1}{4}\left(\frac{1}{16}\left(\frac{1}{12}\left(\frac{2473\cot^5(c+dx)(a\sec(c+dx)+a)^{5/2}}{8\left(\frac{a\tan^2(c+dx)}{a\sec(c+dx)+a}+2\right)^2}-\frac{3}{8}\left(\frac{\int\frac{a\cot^6(c+dx)(\sec(c+dx)a+a)^3\left(\frac{66269a\tan^2(c+dx)}{\sec(c+dx)a+a}+58077\right)}{\left(\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+1\right)\left(\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+2\right)}d\left(-\frac{\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right.\right.}{4a}\right.\right.\right.\right.\right.}$$

$$\downarrow 27$$

$$-\frac{2\left(\frac{1}{4}\left(\frac{1}{16}\left(\frac{1}{12}\left(\frac{2473\cot^5(c+dx)(a\sec(c+dx)+a)^{5/2}}{8\left(\frac{a\tan^2(c+dx)}{a\sec(c+dx)+a}+2\right)^2}-\frac{3}{8}\left(\frac{1}{4}\int\frac{\cot^6(c+dx)(\sec(c+dx)a+a)^3\left(\frac{66269a\tan^2(c+dx)}{\sec(c+dx)a+a}+58077\right)}{\left(\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+1\right)\left(\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+2\right)}d\left(-\frac{\tan(c\right.\right.\right.\right.\right.}$$

$$\downarrow 445$$

$$-\frac{2\left(\frac{1}{4}\left(\frac{1}{16}\left(\frac{1}{12}\left(\frac{2473\cot^5(c+dx)(a\sec(c+dx)+a)^{5/2}}{8\left(\frac{a\tan^2(c+dx)}{a\sec(c+dx)+a}+2\right)^2}-\frac{3}{8}\left(\frac{1}{4}\left(\frac{58077}{10}\cot^5(c+dx)(a\sec(c+dx)+a)^{5/2}-\frac{1}{10}\int\frac{5a\cot^4(c+d\right.\right.\right.\right.\right.}$$

$$\downarrow 27$$

$$-\frac{2\left(\frac{1}{4}\left(\frac{1}{16}\left(\frac{1}{12}\left(\frac{2473\cot^5(c+dx)(a\sec(c+dx)+a)^{5/2}}{8\left(\frac{a\tan^2(c+dx)}{a\sec(c+dx)+a}+2\right)^2}-\frac{3}{8}\left(\frac{1}{4}\left(\frac{58077}{10}\cot^5(c+dx)(a\sec(c+dx)+a)^{5/2}-\frac{1}{2}a\int\frac{\cot^4(c+dx)(\right.\right.\right.\right.\right.}$$

$$\downarrow 445$$

$$-\frac{2\left(\frac{1}{4}\left(\frac{1}{16}\left(\frac{1}{12}\left(\frac{2473\cot^5(c+dx)(a\sec(c+dx)+a)^{5/2}}{8\left(\frac{a\tan^2(c+dx)}{a\sec(c+dx)+a}+2\right)^2}-\frac{3}{8}\left(\frac{1}{4}\left(\frac{58077}{10}\cot^5(c+dx)(a\sec(c+dx)+a)^{5/2}-\frac{1}{2}a\left(\frac{41693}{6}\cot^3(\right.\right.\right.\right.\right.}$$

$$\downarrow 27$$

$$-2\left(\frac{1}{4}\left(\frac{1}{16}\left(\frac{1}{12}\left(\frac{2473\cot^5(c+dx)(a\sec(c+dx)+a)^{5/2}}{8\left(\frac{a\tan^2(c+dx)}{a\sec(c+dx)+a}+2\right)^2}-\frac{3}{8}\left(\frac{1}{4}\left(\frac{58077}{10}\cot^5(c+dx)(a\sec(c+dx)+a)^{5/2}-\frac{1}{2}a\left(\frac{41693}{6}\cot^3(\right.\right.\right.\right.\right.\right.\right.$$

$$\downarrow 445$$

$$-2\left(\frac{1}{4}\left(\frac{1}{16}\left(\frac{1}{12}\left(\frac{2473\cot^5(c+dx)(a\sec(c+dx)+a)^{5/2}}{8\left(\frac{a\tan^2(c+dx)}{a\sec(c+dx)+a}+2\right)^2}-\frac{3}{8}\left(\frac{1}{4}\left(\frac{58077}{10}\cot^5(c+dx)(a\sec(c+dx)+a)^{5/2}-\frac{1}{2}a\left(\frac{41693}{6}\cot^3(\right.\right.\right.\right.\right.\right.\right.$$

$$\downarrow 25$$

$$-2\left(\frac{1}{4}\left(\frac{1}{16}\left(\frac{1}{12}\left(\frac{2473\cot^5(c+dx)(a\sec(c+dx)+a)^{5/2}}{8\left(\frac{a\tan^2(c+dx)}{a\sec(c+dx)+a}+2\right)^2}-\frac{3}{8}\left(\frac{1}{4}\left(\frac{58077}{10}\cot^5(c+dx)(a\sec(c+dx)+a)^{5/2}-\frac{1}{2}a\left(\frac{41693}{6}\cot^3(\right.\right.\right.\right.\right.\right.\right.$$

$$\downarrow 27$$

$$-2\left(\frac{1}{4}\left(\frac{1}{16}\left(\frac{1}{12}\left(\frac{2473\cot^5(c+dx)(a\sec(c+dx)+a)^{5/2}}{8\left(\frac{a\tan^2(c+dx)}{a\sec(c+dx)+a}+2\right)^2}-\frac{3}{8}\left(\frac{1}{4}\left(\frac{58077}{10}\cot^5(c+dx)(a\sec(c+dx)+a)^{5/2}-\frac{1}{2}a\left(\frac{41693}{6}\cot^3(c\right.\right.\right.\right.\right.\right.\right.$$

$$\downarrow 397$$

$$-2\left(\frac{1}{4}\left(\frac{1}{16}\left(\frac{1}{12}\left(\frac{2473\cot^5(c+dx)(a\sec(c+dx)+a)^{5/2}}{8\left(\frac{a\tan^2(c+dx)}{a\sec(c+dx)+a}+2\right)^2}-\frac{3}{8}\left(\frac{1}{4}\left(\frac{58077}{10}\cot^5(c+dx)(a\sec(c+dx)+a)^{5/2}-\frac{1}{2}a\left(\frac{41693}{6}\cot^3(c\right.\right.\right.\right.\right.\right.\right.$$

$$\downarrow 216$$

$$-2\left(\frac{1}{4}\left(\frac{1}{16}\left(\frac{1}{12}\left(\frac{2473\cot^5(c+dx)(a\sec(c+dx)+a)^{5/2}}{8\left(\frac{a\tan^2(c+dx)}{a\sec(c+dx)+a}+2\right)^2}-\frac{3}{8}\left(\frac{1}{4}\left(\frac{58077}{10}\cot^5(c+dx)(a\sec(c+dx)+a)^{5/2}-\frac{1}{2}a\left(\frac{41693}{6}\cot^3(\right.\right.\right.\right.\right.\right.\right.$$

input
```
Int[Cot[c + d*x]^6/(a + a*Sec[c + d*x])^(5/2),x]
```

3.206. $\int \frac{\cot^6(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx$

output
```
(-2*((Cot[c + d*x]^5*(a + a*Sec[c + d*x])^(5/2))/(20*(2 + (a*Tan[c + d*x]^
2)/(a + a*Sec[c + d*x]))^5) + ((7*Cot[c + d*x]^5*(a + a*Sec[c + d*x])^(5/2
))/(16*(2 + (a*Tan[c + d*x]^2)/(a + a*Sec[c + d*x]))^4) + ((155*Cot[c + d*
x]^5*(a + a*Sec[c + d*x])^(5/2))/(12*(2 + (a*Tan[c + d*x]^2)/(a + a*Sec[c
+ d*x]))^3) + ((2473*Cot[c + d*x]^5*(a + a*Sec[c + d*x])^(5/2))/(8*(2 + (a
*Tan[c + d*x]^2)/(a + a*Sec[c + d*x]))^2) - (3*(((58077*Cot[c + d*x]^5*(a
+ a*Sec[c + d*x])^(5/2))/10 - (a*((41693*Cot[c + d*x]^3*(a + a*Sec[c + d*x
])^(3/2))/6 - (a*((a*((-65536*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec
[c + d*x]]])/Sqrt[a] + (74461*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[
a + a*Sec[c + d*x]])])/(Sqrt[2]*Sqrt[a]))/2 + (8925*Cot[c + d*x]*Sqrt[a +
 a*Sec[c + d*x]])/2))/2))/2)/4 - (9467*Cot[c + d*x]^5*(a + a*Sec[c + d*x])
^(5/2))/(4*(2 + (a*Tan[c + d*x]^2)/(a + a*Sec[c + d*x])))))/8)/12)/16)/4))
/(a^5*d)
```

3.206.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 374
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q
 + 1)/(a*e*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(e*x)^m*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[b*c*(m + 1) + 2*(b*c -
a*d)*(p + 1) + d*b*(m + 2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b,
c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IntBinomialQ[a, b,
 c, d, e, m, 2, p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 441
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*(g*x)^(m + 1)*(a
+ b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(a*g*2*(b*c - a*d)*(p + 1))), x] + Si
mp[1/(a*2*(b*c - a*d)*(p + 1))   Int[(g*x)^m*(a + b*x^2)^(p + 1)*(c + d*x^2
)^q*Simp[c*(b*e - a*f)*(m + 1) + e*2*(b*c - a*d)*(p + 1) + d*(b*e - a*f)*(m
 + 2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m, q},
 x] && LtQ[p, -1]
```

rule 445
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^2*(m + 1))
 Int[(g*x)^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*f*c*(m + 1) - e*(b*c
+ a*d)*(m + 2 + 1) - e*2*(b*c*p + a*d*q) - b*e*d*(m + 2*(p + q + 2) + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && LtQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4375
```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Simp[-2*(a^(m/2 + n + 1/2)/d)   Subst[Int[x^m*((2 + a*x^2
)^(m/2 + n - 1/2)/(1 + a*x^2)), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]
]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && I
ntegerQ[n - 1/2]
```

3.206.4 Maple [A] (verified)

Time = 1.74 (sec) , antiderivative size = 746, normalized size of antiderivative = 1.70

method	result
default	$\frac{\sqrt{a(1+\sec(dx+c))}\left(1116915\cos(dx+c)^3\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}\sqrt{2}\ln\left(\csc(dx+c)-\cot(dx+c)+\sqrt{\cot(dx+c)^2-2\cot(dx+c)\csc(dx+c)+cs\right.\right.}{}$

input
```
int(cot(d*x+c)^6/(a+a*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/983040/d/a^3*(a*(1+sec(d*x+c)))^(1/2)/(cos(d*x+c)+1)^3*(1116915*cos(d*x+
c)^3*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*2^(1/2)*ln(csc(d*x+c)-cot(d*x+c)+(
cot(d*x+c)^2-2*cot(d*x+c)*csc(d*x+c)+csc(d*x+c)^2-1)^(1/2))+3350745*2^(1/2
)*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*ln(csc(d*x+c)-cot(d*x+c)+(cot(d*x+c)^
2-2*cot(d*x+c)*csc(d*x+c)+csc(d*x+c)^2-1)^(1/2))*cos(d*x+c)^2-1966080*cos(
d*x+c)^3*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctanh(sin(d*x+c)/(cos(d*x+c)
+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))+3350745*2^(1/2)*ln(csc(d*x+c)-cot(
d*x+c)+(cot(d*x+c)^2-2*cot(d*x+c)*csc(d*x+c)+csc(d*x+c)^2-1)^(1/2))*(-cos(
d*x+c)/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)-5898240*(-cos(d*x+c)/(cos(d*x+c)+1
))^(1/2)*arctanh(sin(d*x+c)/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1
/2))*cos(d*x+c)^2+1116915*ln(csc(d*x+c)-cot(d*x+c)+(cot(d*x+c)^2-2*cot(d*x
+c)*csc(d*x+c)+csc(d*x+c)^2-1)^(1/2))*2^(1/2)*(-cos(d*x+c)/(cos(d*x+c)+1))
^(1/2)-1278126*cos(d*x+c)^3*cot(d*x+c)^5-5898240*(-cos(d*x+c)/(cos(d*x+c)+
1))^(1/2)*arctanh(sin(d*x+c)/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(
1/2))*cos(d*x+c)-1363110*cos(d*x+c)^2*cot(d*x+c)^5-1966080*(-cos(d*x+c)/(c
os(d*x+c)+1))^(1/2)*arctanh(sin(d*x+c)/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*
x+c)+1))^(1/2))+1972170*cos(d*x+c)*cot(d*x+c)^5+2720050*cot(d*x+c)^5-81089
0*cot(d*x+c)^4*csc(d*x+c)-1673842*cot(d*x+c)^3*csc(d*x+c)^2-30610*cot(d*x+
c)^2*csc(d*x+c)^3+267750*cot(d*x+c)*csc(d*x+c)^4)
```

3.206.5 Fricas [A] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 1023, normalized size of antiderivative = 2.33

$$\int \frac{\cot^6(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(cot(d*x+c)^6/(a+a*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
[-1/1966080*(1116915*sqrt(2)*(cos(d*x + c)^7 + 3*cos(d*x + c)^6 + cos(d*x
+ c)^5 - 5*cos(d*x + c)^4 - 5*cos(d*x + c)^3 + cos(d*x + c)^2 + 3*cos(d*x
+ c) + 1)*sqrt(-a)*log((2*sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d
*x + c))*cos(d*x + c)*sin(d*x + c) + 3*a*cos(d*x + c)^2 + 2*a*cos(d*x + c)
 - a)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1))*sin(d*x + c) + 983040*(cos(d*
x + c)^7 + 3*cos(d*x + c)^6 + cos(d*x + c)^5 - 5*cos(d*x + c)^4 - 5*cos(d*
x + c)^3 + cos(d*x + c)^2 + 3*cos(d*x + c) + 1)*sqrt(-a)*log(-(8*a*cos(d*x
 + c)^3 - 4*(2*cos(d*x + c)^2 - cos(d*x + c))*sqrt(-a)*sqrt((a*cos(d*x + c
) + a)/cos(d*x + c))*sin(d*x + c) - 7*a*cos(d*x + c) + a)/(cos(d*x + c) +
1))*sin(d*x + c) + 4*(639063*cos(d*x + c)^8 + 681555*cos(d*x + c)^7 - 9860
85*cos(d*x + c)^6 - 1360025*cos(d*x + c)^5 + 405445*cos(d*x + c)^4 + 83692
1*cos(d*x + c)^3 + 15305*cos(d*x + c)^2 - 133875*cos(d*x + c))*sqrt((a*cos
(d*x + c) + a)/cos(d*x + c)))/((a^3*d*cos(d*x + c)^7 + 3*a^3*d*cos(d*x + c
)^6 + a^3*d*cos(d*x + c)^5 - 5*a^3*d*cos(d*x + c)^4 - 5*a^3*d*cos(d*x + c)
^3 + a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)*sin(d*x + c)), -
1/983040*(1116915*sqrt(2)*(cos(d*x + c)^7 + 3*cos(d*x + c)^6 + cos(d*x + c
)^5 - 5*cos(d*x + c)^4 - 5*cos(d*x + c)^3 + cos(d*x + c)^2 + 3*cos(d*x + c
) + 1)*sqrt(a)*arctan(sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(
d*x + c)/(sqrt(a)*sin(d*x + c)))*sin(d*x + c) + 983040*(cos(d*x + c)^7 + 3
*cos(d*x + c)^6 + cos(d*x + c)^5 - 5*cos(d*x + c)^4 - 5*cos(d*x + c)^3 ...
```

3.206.6 Sympy [F]

$$\int \frac{\cot^6(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = \int \frac{\cot^6(c+dx)}{(a(\sec(c+dx)+1))^{\frac{5}{2}}}\,dx$$

input
```
integrate(cot(d*x+c)**6/(a+a*sec(d*x+c))**(5/2),x)
```

output
```
Integral(cot(c + d*x)**6/(a*(sec(c + d*x) + 1))**(5/2), x)
```

3.206.7 Maxima [F(-1)]

Timed out.

$$\int \frac{\cot^6(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(cot(d*x+c)^6/(a+a*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
Timed out
```

3.206.8 Giac [A] (verification not implemented)

Time = 1.62 (sec) , antiderivative size = 412, normalized size of antiderivative = 0.94

$$\int \frac{\cot^6(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = \frac{\left(2\left(4\left(6\left(\frac{8\sqrt{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2}{a^3\mathrm{sgn}(\cos(dx+c))}-\frac{91\sqrt{2}}{a^3\mathrm{sgn}(\cos(dx+c))}\right)\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+\frac{3043\sqrt{2}}{a^3\mathrm{sgn}(\cos(dx-}\right.\right.\right.}{}$$

input
```
integrate(cot(d*x+c)^6/(a+a*sec(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
1/983040*((2*(4*(6*(8*sqrt(2)*tan(1/2*d*x + 1/2*c)^2/(a^3*sgn(cos(d*x + c)
)) - 91*sqrt(2)/(a^3*sgn(cos(d*x + c))))*tan(1/2*d*x + 1/2*c)^2 + 3043*sqr
t(2)/(a^3*sgn(cos(d*x + c))))*tan(1/2*d*x + 1/2*c)^2 - 47185*sqrt(2)/(a^3*
sgn(cos(d*x + c))))*tan(1/2*d*x + 1/2*c)^2 + 349965*sqrt(2)/(a^3*sgn(cos(d
*x + c))))*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)*tan(1/2*d*x + 1/2*c) - 1024
*sqrt(2)*(345*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c
)^2 + a))^8 - 1230*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x +
1/2*c)^2 + a))^6*a + 1760*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2
*d*x + 1/2*c)^2 + a))^4*a^2 - 1150*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-
a*tan(1/2*d*x + 1/2*c)^2 + a))^2*a^3 + 299*a^4)/(((sqrt(-a)*tan(1/2*d*x +
1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2 - a)^5*sqrt(-a)*a*sgn(cos(
d*x + c))))/d
```

3.206.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cot^6(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = \int \frac{\cot(c+d\,x)^6}{\left(a+\frac{a}{\cos(c+d\,x)}\right)^{5/2}}\,dx$$

input
```
int(cot(c + d*x)^6/(a + a/cos(c + d*x))^(5/2),x)
```

output
```
int(cot(c + d*x)^6/(a + a/cos(c + d*x))^(5/2), x)
```

3.207 $\int \frac{\tan^2(e+fx)}{(a+a\sec(e+fx))^{9/2}}\,dx$

3.207.1 Optimal result 1551
3.207.2 Mathematica [A] (warning: unable to verify) 1551
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3.207.8 Giac [A] (verification not implemented) 1558
3.207.9 Mupad [F(-1)] 1558

3.207.1 Optimal result

Integrand size = 23, antiderivative size = 177

$$\int \frac{\tan^2(e+fx)}{(a+a\sec(e+fx))^{9/2}}\,dx = -\frac{2\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{a^{9/2}f} + \frac{91\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a+a\sec(e+fx)}}\right)}{32\sqrt{2}a^{9/2}f} + \frac{\tan(e+fx)}{3af(a+a\sec(e+fx))^{7/2}} + \frac{11\tan(e+fx)}{24a^2f(a+a\sec(e+fx))^{5/2}} + \frac{27\tan(e+fx)}{32a^3f(a+a\sec(e+fx))^{3/2}}$$

output
```
-2*arctan(a^(1/2)*tan(f*x+e)/(a+a*sec(f*x+e))^(1/2))/a^(9/2)/f+91/64*arcta
n(1/2*a^(1/2)*tan(f*x+e)*2^(1/2)/(a+a*sec(f*x+e))^(1/2))/a^(9/2)/f*2^(1/2)
+1/3*tan(f*x+e)/a/f/(a+a*sec(f*x+e))^(7/2)+11/24*tan(f*x+e)/a^2/f/(a+a*sec
(f*x+e))^(5/2)+27/32*tan(f*x+e)/a^3/f/(a+a*sec(f*x+e))^(3/2)
```

3.207.2 Mathematica [A] (warning: unable to verify)

Time = 6.36 (sec) , antiderivative size = 241, normalized size of antiderivative = 1.36

$$\int \frac{\tan^2(e+fx)}{(a+a\sec(e+fx))^{9/2}}\,dx = \frac{\cos^3\left(\frac{1}{2}(e+fx)\right)\sec^{\frac{9}{2}}(e+fx)\sqrt{a(1+\sec(e+fx))}\left(-384\arctan\left(\frac{\tan\left(\frac{1}{2}(e+fx\right.\right.}{\sqrt{\frac{1}{1+\sec(e+f}}}\right.$$

input
```
Integrate[Tan[e + f*x]^2/(a + a*Sec[e + f*x])^(9/2),x]
```

output
```
(Cos[(e + f*x)/2]^3*Sec[e + f*x]^(9/2)*Sqrt[a*(1 + Sec[e + f*x])]*(-384*Ar
cTan[Tan[(e + f*x)/2]/Sqrt[(1 + Sec[e + f*x])^(-1)]]*Cos[(e + f*x)/2]^7*Sq
rt[Cos[e + f*x]/(1 + Cos[e + f*x])^2]*Sqrt[1 + Sec[e + f*x]] + 273*ArcSin[
Tan[(e + f*x)/2]]*Cos[(e + f*x)/2]^7*Sqrt[Sec[(e + f*x)/2]^2]*Sqrt[(1 + Se
c[e + f*x])^(-1)]*Sqrt[1 + Sec[e + f*x]] + ((319 + 412*Cos[e + f*x] + 157*
Cos[2*(e + f*x)])*Sin[(e + f*x)/2])/(8*Sqrt[Sec[e + f*x]])))/(6*a^5*f*(1 +
 Sec[e + f*x])^5)
```

3.207.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 258, normalized size of antiderivative = 1.46, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {3042, 4375, 373, 402, 27, 402, 27, 397, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^2(e+fx)}{(a\sec(e+fx)+a)^{9/2}}\,dx$$

$$\downarrow 3042$$

$$\int \frac{\cot\left(e+fx+\frac{\pi}{2}\right)^2}{\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^{9/2}}dx$$

$$\downarrow 4375$$

$$-\frac{2\int \frac{\tan^2(e+fx)}{(\sec(e+fx)a+a)\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1\right)\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+2\right)^4} d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)}{a^3 f}$$

$$\downarrow 373$$

$$-\frac{2\left(-\frac{\int \frac{1-\frac{5a\tan^2(e+fx)}{\sec(e+fx)a+a}}{\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1\right)\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+2\right)^3} d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)}{6a} - \frac{\tan(e+fx)}{6a\sqrt{a\sec(e+fx)+a}\left(\frac{a\tan^2(e+fx)}{a\sec(e+fx)+a}+2\right)^3}\right)}{a^3 f}$$

$$\downarrow 402$$

$$-\frac{2\left(-\frac{\int \frac{3a\left(5-\frac{11a\tan^2(e+fx)}{\sec(e+fx)a+a}\right)}{\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1\right)\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+2\right)^2}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)}{8a}+\frac{11\tan(e+fx)}{8\sqrt{a\sec(e+fx)+a}\left(\frac{a\tan^2(e+fx)}{a\sec(e+fx)+a}+2\right)^2}\right)}{6a}-\frac{\tan(e+fx)}{6a\sqrt{a\sec(e+fx)+a}\left(\frac{a\tan^2(e+fx)}{a\sec(e+fx)+a}+2\right)}}{a^3f}$$

$$\downarrow 27$$

$$-\frac{2\left(-\frac{\frac{3}{8}\int \frac{5-\frac{11a\tan^2(e+fx)}{\sec(e+fx)a+a}}{\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1\right)\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+2\right)^2}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)+\frac{11\tan(e+fx)}{8\sqrt{a\sec(e+fx)+a}\left(\frac{a\tan^2(e+fx)}{a\sec(e+fx)+a}+2\right)^2}}{6a}-\frac{\tan(e+fx)}{6a\sqrt{a\sec(e+fx)+a}\left(\frac{a\tan^2(e+fx)}{a\sec(e+fx)+a}+2\right)}}{a^3f}$$

$$\downarrow 402$$

$$-\frac{2\left(-\frac{\frac{3}{8}\left(\int \frac{a\left(37-\frac{27a\tan^2(e+fx)}{\sec(e+fx)a+a}\right)}{\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1\right)\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+2\right)^2}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)+\frac{27\tan(e+fx)}{4\sqrt{a\sec(e+fx)+a}\left(\frac{a\tan^2(e+fx)}{a\sec(e+fx)+a}+2\right)}\right)}{6a}+\frac{11\tan(e+fx)}{8\sqrt{a\sec(e+fx)+a}\left(\frac{a\tan^2(e+fx)}{a\sec(e+fx)+a}+2\right)}}{a^3f}$$

$$\downarrow 27$$

$$-\frac{2\left(-\frac{\frac{3}{8}\left(\frac{1}{4}\int \frac{37-\frac{27a\tan^2(e+fx)}{\sec(e+fx)a+a}}{\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1\right)\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+2\right)}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)+\frac{27\tan(e+fx)}{4\sqrt{a\sec(e+fx)+a}\left(\frac{a\tan^2(e+fx)}{a\sec(e+fx)+a}+2\right)}\right)}{6a}+\frac{11\tan(e+fx)}{8\sqrt{a\sec(e+fx)+a}\left(\frac{a\tan^2(e+fx)}{a\sec(e+fx)+a}+2\right)}}{a^3f}$$

$$\downarrow 397$$

$$-\frac{2\left(-\frac{\frac{3}{8}\left(\frac{1}{4}\left(64\int\frac{1}{\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)-91\int\frac{1}{\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+2}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)\right)+\frac{27\tan(e+fx)}{4\sqrt{a\sec(e+fx)+a}\left(\frac{a\tan^2(e+fx)}{a\sec(e+fx)+a}+2\right)}\right.}{6a}\right.}{a^3f}$$

$$\downarrow 216$$

$$-\frac{2\left(-\frac{\frac{3}{8}\left(\frac{1}{4}\left(\frac{91\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a\sec(e+fx)+a}}\right)}{\sqrt{2}\sqrt{a}}-\frac{64\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{\sqrt{a}}\right)+\frac{27\tan(e+fx)}{4\sqrt{a\sec(e+fx)+a}\left(\frac{a\tan^2(e+fx)}{a\sec(e+fx)+a}+2\right)}\right)+\frac{11\tan(e+fx)}{8\sqrt{a\sec(e+fx)+a}\left(\frac{a\tan^2(e+f}\right.}{6a}\right.}{a^3f}$$

input
```
Int[Tan[e + f*x]^2/(a + a*Sec[e + f*x])^(9/2),x]
```

output
```
(-2*(-1/6*Tan[e + f*x]/(a*Sqrt[a + a*Sec[e + f*x]]*(2 + (a*Tan[e + f*x]^2)
/(a + a*Sec[e + f*x]))^3) - ((11*Tan[e + f*x])/(8*Sqrt[a + a*Sec[e + f*x]]
*(2 + (a*Tan[e + f*x]^2)/(a + a*Sec[e + f*x]))^2) + (3*(((-64*ArcTan[(Sqrt
[a]*Tan[e + f*x])/Sqrt[a + a*Sec[e + f*x]]])/Sqrt[a] + (91*ArcTan[(Sqrt[a]
*Tan[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sec[e + f*x]])])/(Sqrt[2]*Sqrt[a]))/4 +
 (27*Tan[e + f*x])/(4*Sqrt[a + a*Sec[e + f*x]]*(2 + (a*Tan[e + f*x]^2)/(a
+ a*Sec[e + f*x])))))/8)/(6*a)))/(a^3*f)
```

3.207.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 373
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[e*(e*x)^(m - 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q +
1)/(2*(b*c - a*d)*(p + 1))), x] - Simp[e^2/(2*(b*c - a*d)*(p + 1))   Int[(e
*x)^(m - 2)*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(m - 1) + d*(m + 2*p +
 2*q + 3)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d,
 0] && LtQ[p, -1] && GtQ[m, 1] && LeQ[m, 3] && IntBinomialQ[a, b, c, d, e,
m, 2, p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4375
```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Simp[-2*(a^(m/2 + n + 1/2)/d)   Subst[Int[x^m*((2 + a*x^2
)^(m/2 + n - 1/2)/(1 + a*x^2)), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]
]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && I
ntegerQ[n - 1/2]
```

3.207.4 Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 330 vs. $2(148) = 296$.

Time = 3.07 (sec) , antiderivative size = 331, normalized size of antiderivative = 1.87

method	result
default	$\frac{\sqrt{-\frac{2a}{(1-\cos(fx+e))^2\csc(fx+e)^2-1}}\sqrt{(1-\cos(fx+e))^2\csc(fx+e)^2-1}\left(-8\left((1-\cos(fx+e))^2\csc(fx+e)^2-1\right)^{\frac{5}{2}}(-\cot(fx+e)+\csc(fx+$

input
```
int(tan(f*x+e)^2/(a+a*sec(f*x+e))^(9/2),x,method=_RETURNVERBOSE)
```

output
```
1/192/f/a^5*(-2*a/((1-cos(f*x+e))^2*csc(f*x+e)^2-1))^(1/2)*((1-cos(f*x+e))
^2*csc(f*x+e)^2-1)^(1/2)*(-8*((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^(5/2)*(-cot
(f*x+e)+csc(f*x+e))+10*((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^(3/2)*(-cot(f*x+e
)+csc(f*x+e))+12*(1-cos(f*x+e))^3*((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^(1/2)*
csc(f*x+e)^3-192*2^(1/2)*arctanh(2^(1/2)/((1-cos(f*x+e))^2*csc(f*x+e)^2-1)
^(1/2)*(-cot(f*x+e)+csc(f*x+e)))-93*((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^(1/2
)*(-cot(f*x+e)+csc(f*x+e))+273*ln(csc(f*x+e)-cot(f*x+e)+((1-cos(f*x+e))^2*
csc(f*x+e)^2-1)^(1/2)))
```

3.207.5 Fricas [A] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 674, normalized size of antiderivative = 3.81

$$\int \frac{\tan^2(e+fx)}{(a+a\sec(e+fx))^{9/2}}\,dx = \left[-\frac{273\sqrt{2}\left(\cos(fx+e)^4+4\cos(fx+e)^3+6\cos(fx+e)^2+4\cos(fx+e\right.\right.$$

$$\left.\left. -\frac{273\sqrt{2}\left(\cos(fx+e)^4+4\cos(fx+e)^3+6\cos(fx+e)^2+4\cos(fx+e)+1\right)\sqrt{a}\arctan\left(\frac{\sqrt{2}\sqrt{\frac{a\cos(fx+e)+}{\cos(fx+e)}}}{\sqrt{a}\sin(fx\right.\right.\right.$$

input
```
integrate(tan(f*x+e)^2/(a+a*sec(f*x+e))^(9/2),x, algorithm="fricas")
```

output
```
[-1/384*(273*sqrt(2)*(cos(f*x + e)^4 + 4*cos(f*x + e)^3 + 6*cos(f*x + e)^2
 + 4*cos(f*x + e) + 1)*sqrt(-a)*log((2*sqrt(2)*sqrt(-a)*sqrt((a*cos(f*x +
e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) + 3*a*cos(f*x + e)^2 + 2*a
*cos(f*x + e) - a)/(cos(f*x + e)^2 + 2*cos(f*x + e) + 1)) + 384*(cos(f*x +
 e)^4 + 4*cos(f*x + e)^3 + 6*cos(f*x + e)^2 + 4*cos(f*x + e) + 1)*sqrt(-a)
*log((2*a*cos(f*x + e)^2 - 2*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x +
e))*cos(f*x + e)*sin(f*x + e) + a*cos(f*x + e) - a)/(cos(f*x + e) + 1)) -
4*(157*cos(f*x + e)^3 + 206*cos(f*x + e)^2 + 81*cos(f*x + e))*sqrt((a*cos(
f*x + e) + a)/cos(f*x + e))*sin(f*x + e))/(a^5*f*cos(f*x + e)^4 + 4*a^5*f*
cos(f*x + e)^3 + 6*a^5*f*cos(f*x + e)^2 + 4*a^5*f*cos(f*x + e) + a^5*f), -
1/192*(273*sqrt(2)*(cos(f*x + e)^4 + 4*cos(f*x + e)^3 + 6*cos(f*x + e)^2 +
 4*cos(f*x + e) + 1)*sqrt(a)*arctan(sqrt(2)*sqrt((a*cos(f*x + e) + a)/cos(
f*x + e))*cos(f*x + e)/(sqrt(a)*sin(f*x + e))) - 384*(cos(f*x + e)^4 + 4*c
os(f*x + e)^3 + 6*cos(f*x + e)^2 + 4*cos(f*x + e) + 1)*sqrt(a)*arctan(sqrt
((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)/(sqrt(a)*sin(f*x + e))) -
 2*(157*cos(f*x + e)^3 + 206*cos(f*x + e)^2 + 81*cos(f*x + e))*sqrt((a*cos
(f*x + e) + a)/cos(f*x + e))*sin(f*x + e))/(a^5*f*cos(f*x + e)^4 + 4*a^5*f
*cos(f*x + e)^3 + 6*a^5*f*cos(f*x + e)^2 + 4*a^5*f*cos(f*x + e) + a^5*f)]
```

3.207.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\tan^2(e+fx)}{(a+a\sec(e+fx))^{9/2}}\,dx = \text{Timed out}$$

input
```
integrate(tan(f*x+e)**2/(a+a*sec(f*x+e))**(9/2),x)
```

output
```
Timed out
```

3.207.7 Maxima [F]

$$\int \frac{\tan^2(e+fx)}{(a+a\sec(e+fx))^{9/2}}\,dx = \int \frac{\tan\left(fx+e\right)^2}{\left(a\sec\left(fx+e\right)+a\right)^{\frac{9}{2}}}\,dx$$

input
```
integrate(tan(f*x+e)^2/(a+a*sec(f*x+e))^(9/2),x, algorithm="maxima")
```

output
```
integrate(tan(f*x + e)^2/(a*sec(f*x + e) + a)^(9/2), x)
```

3.207.8 Giac [A] (verification not implemented)

Time = 1.34 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.62

$$\int \frac{\tan^2(e+fx)}{(a+a\sec(e+fx))^{9/2}}\,dx = \frac{\sqrt{-a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+a}\left(2\left(\frac{4\sqrt{2}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2}{a^5\mathrm{sgn}(\cos(fx+e))}-\frac{19\sqrt{2}}{a^5\mathrm{sgn}(\cos(fx+e))}\right)\tan\left(\frac{1}{2}fx\right.\right.}{192\,f}$$

input
```
integrate(tan(f*x+e)^2/(a+a*sec(f*x+e))^(9/2),x, algorithm="giac")
```

output
```
1/192*sqrt(-a*tan(1/2*f*x + 1/2*e)^2 + a)*(2*(4*sqrt(2)*tan(1/2*f*x + 1/2*
e)^2/(a^5*sgn(cos(f*x + e))) - 19*sqrt(2)/(a^5*sgn(cos(f*x + e))))*tan(1/2
*f*x + 1/2*e)^2 + 111*sqrt(2)/(a^5*sgn(cos(f*x + e))))*tan(1/2*f*x + 1/2*e
)/f
```

3.207.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\tan^2(e+fx)}{(a+a\sec(e+fx))^{9/2}}\,dx = \int \frac{\tan(e+f\,x)^2}{\left(a+\frac{a}{\cos(e+f\,x)}\right)^{9/2}}\,dx$$

input
```
int(tan(e + f*x)^2/(a + a/cos(e + f*x))^(9/2),x)
```

output
```
int(tan(e + f*x)^2/(a + a/cos(e + f*x))^(9/2), x)
```

3.208 $\int (a + a\sec(c+dx))^n (e\tan(c+dx))^m \, dx$

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3.208.9 Mupad [F(-1)] 1562

3.208.1 Optimal result

Integrand size = 23, antiderivative size = 125

$$\int (a + a\sec(c+dx))^n (e\tan(c+dx))^m \, dx$$
$$= \frac{2^{1+m+n}\,\text{AppellF1}\left(\frac{1+m}{2}, m+n, 1, \frac{3+m}{2}, -\frac{a-a\sec(c+dx)}{a+a\sec(c+dx)}, \frac{a-a\sec(c+dx)}{a+a\sec(c+dx)}\right)\left(\frac{1}{1+\sec(c+dx)}\right)^{1+m+n}(a+a\sec(c+dx))}{de(1+m)}$$

output
```
2^(1+m+n)*AppellF1(1/2+1/2*m,m+n,1,3/2+1/2*m,(-a+a*sec(d*x+c))/(a+a*sec(d*
x+c)),(a-a*sec(d*x+c))/(a+a*sec(d*x+c)))*(1/(1+sec(d*x+c)))^(1+m+n)*(a+a*s
ec(d*x+c))^n*(e*tan(d*x+c))^(1+m)/d/e/(1+m)
```

3.208.2 Mathematica [F]

$$\int (a + a\sec(c+dx))^n (e\tan(c+dx))^m \, dx = \int (a + a\sec(c+dx))^n (e\tan(c+dx))^m \, dx$$

input
```
Integrate[(a + a*Sec[c + d*x])^n*(e*Tan[c + d*x])^m,x]
```

output
```
Integrate[(a + a*Sec[c + d*x])^n*(e*Tan[c + d*x])^m, x]
```

3.208.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {3042, 4377}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sec(c+dx)+a)^n (e\tan(c+dx))^m\,dx$$

$$\downarrow 3042$$

$$\int \left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^n \left(-e\cot\left(c+dx+\frac{\pi}{2}\right)\right)^m dx$$

$$\downarrow 4377$$

$$\frac{2^{m+n+1}(a\sec(c+dx)+a)^n(e\tan(c+dx))^{m+1}\left(\frac{1}{\sec(c+dx)+1}\right)^{m+n+1}\text{AppellF1}\left(\frac{m+1}{2},m+n,1,\frac{m+3}{2},-\frac{a-a\sec(c+dx)}{\sec(c+dx)a+a}\right.}{de(m+1)}$$

input
```
Int[(a + a*Sec[c + d*x])^n*(e*Tan[c + d*x])^m,x]
```

output
```
(2^(1 + m + n)*AppellF1[(1 + m)/2, m + n, 1, (3 + m)/2, -((a - a*Sec[c + d
*x])/(a + a*Sec[c + d*x])), (a - a*Sec[c + d*x])/(a + a*Sec[c + d*x])]*((1
 + Sec[c + d*x])^(-1))^(1 + m + n)*(a + a*Sec[c + d*x])^n*(e*Tan[c + d*x])
^(1 + m))/(d*e*(1 + m))
```

3.208.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4377
```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Simp[(-2^(m + n + 1))*(e*Cot[c + d*x])^(m + 1)*((a
+ b*Csc[c + d*x])^n/(d*e*(m + 1)))*(a/(a + b*Csc[c + d*x]))^(m + n + 1)*App
ellF1[(m + 1)/2, m + n, 1, (m + 3)/2, -(a - b*Csc[c + d*x])/(a + b*Csc[c +
d*x]), (a - b*Csc[c + d*x])/(a + b*Csc[c + d*x])], x] /; FreeQ[{a, b, c, d,
 e, m, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[n]
```

3.208.4 Maple [F]

$$\int (a + a \sec (dx + c))^n (e \tan (dx + c))^m \, dx$$

input
```
int((a+a*sec(d*x+c))^n*(e*tan(d*x+c))^m,x)
```

output
```
int((a+a*sec(d*x+c))^n*(e*tan(d*x+c))^m,x)
```

3.208.5 Fricas [F]

$$\int (a + a \sec(c + dx))^n (e \tan(c + dx))^m \, dx = \int (a \sec (dx + c) + a)^n (e \tan (dx + c))^m \, dx$$

input
```
integrate((a+a*sec(d*x+c))^n*(e*tan(d*x+c))^m,x, algorithm="fricas")
```

output
```
integral((a*sec(d*x + c) + a)^n*(e*tan(d*x + c))^m, x)
```

3.208.6 Sympy [F]

$$\int (a + a \sec(c + dx))^n (e \tan(c + dx))^m \, dx = \int (a(\sec (c + dx) + 1))^n (e \tan (c + dx))^m \, dx$$

input
```
integrate((a+a*sec(d*x+c))**n*(e*tan(d*x+c))**m,x)
```

output
```
Integral((a*(sec(c + d*x) + 1))**n*(e*tan(c + d*x))**m, x)
```

3.208.7 Maxima [F]

$$\int (a + a\sec(c + dx))^n (e\tan(c + dx))^m \, dx = \int (a\sec(dx + c) + a)^n (e\tan(dx + c))^m \, dx$$

input
```
integrate((a+a*sec(d*x+c))^n*(e*tan(d*x+c))^m,x, algorithm="maxima")
```

output
```
integrate((a*sec(d*x + c) + a)^n*(e*tan(d*x + c))^m, x)
```

3.208.8 Giac [F]

$$\int (a + a\sec(c + dx))^n (e\tan(c + dx))^m \, dx = \int (a\sec(dx + c) + a)^n (e\tan(dx + c))^m \, dx$$

input
```
integrate((a+a*sec(d*x+c))^n*(e*tan(d*x+c))^m,x, algorithm="giac")
```

output
```
integrate((a*sec(d*x + c) + a)^n*(e*tan(d*x + c))^m, x)
```

3.208.9 Mupad [F(-1)]

Timed out.

$$\int (a + a\sec(c + dx))^n (e\tan(c + dx))^m \, dx = \int (e\tan(c + d\,x))^m \left(a + \frac{a}{\cos(c + d\,x)}\right)^n dx$$

input
```
int((e*tan(c + d*x))^m*(a + a/cos(c + d*x))^n,x)
```

output
```
int((e*tan(c + d*x))^m*(a + a/cos(c + d*x))^n, x)
```

3.209 $\int (a + a\sec(c+dx))^3 (e\tan(c+dx))^m \, dx$

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3.209.2 Mathematica [A] (verified) 1564
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3.209.1 Optimal result

Integrand size = 23, antiderivative size = 243

$$\int (a + a\sec(c+dx))^3 (e\tan(c+dx))^m \, dx = \frac{3a^3 (e\tan(c+dx))^{1+m}}{de(1+m)}$$
$$+ \frac{a^3 \,\text{Hypergeometric2F1}\left(1, \frac{1+m}{2}, \frac{3+m}{2}, -\tan^2(c+dx)\right) (e\tan(c+dx))^{1+m}}{de(1+m)}$$
$$+ \frac{3a^3 \cos^2(c+dx)^{\frac{2+m}{2}} \,\text{Hypergeometric2F1}\left(\frac{1+m}{2}, \frac{2+m}{2}, \frac{3+m}{2}, \sin^2(c+dx)\right) \sec(c+dx) (e\tan(c+dx))^{1+m}}{de(1+m)}$$
$$+ \frac{a^3 \cos^2(c+dx)^{\frac{4+m}{2}} \,\text{Hypergeometric2F1}\left(\frac{1+m}{2}, \frac{4+m}{2}, \frac{3+m}{2}, \sin^2(c+dx)\right) \sec^3(c+dx) (e\tan(c+dx))^{1+m}}{de(1+m)}$$

output

```
3*a^3*(e*tan(d*x+c))^(1+m)/d/e/(1+m)+a^3*hypergeom([1, 1/2+1/2*m],[3/2+1/2
*m],-tan(d*x+c)^2)*(e*tan(d*x+c))^(1+m)/d/e/(1+m)+3*a^3*(cos(d*x+c)^2)^(1+
1/2*m)*hypergeom([1+1/2*m, 1/2+1/2*m],[3/2+1/2*m],sin(d*x+c)^2)*sec(d*x+c)
*(e*tan(d*x+c))^(1+m)/d/e/(1+m)+a^3*(cos(d*x+c)^2)^(2+1/2*m)*hypergeom([2+
1/2*m, 1/2+1/2*m],[3/2+1/2*m],sin(d*x+c)^2)*sec(d*x+c)^3*(e*tan(d*x+c))^(1
+m)/d/e/(1+m)
```

3.209.2 Mathematica [A] (verified)

Time = 2.10 (sec) , antiderivative size = 226, normalized size of antiderivative = 0.93

$$\int (a + a\sec(c+dx))^3 (e\tan(c+dx))^m \, dx$$
$$= \frac{a^3 e (e\tan(c+dx))^{-1+m} \left(-\tan^2(c+dx)\right)^{-m/2} \left(9\sqrt{-\tan^2(c+dx)} + 9(1+m)\,\text{Hypergeometric2F1}\left(\frac{1}{2}, \frac{1-}{2}\right.\right.}{}$$

input
```
Integrate[(a + a*Sec[c + d*x])^3*(e*Tan[c + d*x])^m,x]
```

output
```
(a^3*e*(e*Tan[c + d*x])^(-1 + m)*(9*Sqrt[-Tan[c + d*x]^2] + 9*(1 + m)*Hype
rgeometric2F1[1/2, (1 - m)/2, 3/2, Sec[c + d*x]^2]*Sec[c + d*x]*Sqrt[-Tan[
c + d*x]^2] + (1 + m)*Hypergeometric2F1[3/2, (1 - m)/2, 5/2, Sec[c + d*x]^
2]*Sec[c + d*x]^3*Sqrt[-Tan[c + d*x]^2] - 9*(-Tan[c + d*x]^2)^((2 + m)/2)
- 3*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, -Tan[c + d*x]^2]*(-Tan[c +
d*x]^2)^((2 + m)/2)))/(3*d*(1 + m)*(-Tan[c + d*x]^2)^(m/2))
```

3.209.3 Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3042, 4374, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sec(c+dx)+a)^3 (e\tan(c+dx))^m \, dx$$
$$\downarrow 3042$$
$$\int \left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^3 \left(-e\cot\left(c+dx+\frac{\pi}{2}\right)\right)^m dx$$
$$\downarrow 4374$$
$$\int (a^3(e\tan(c+dx))^m + a^3\sec^3(c+dx)(e\tan(c+dx))^m + 3a^3\sec^2(c+dx)(e\tan(c+dx))^m + 3a^3\sec(c+dx)(e$$
$$\downarrow 2009$$

$$\frac{a^3(e\tan(c+dx))^{m+1}\,\text{Hypergeometric2F1}\left(1,\frac{m+1}{2},\frac{m+3}{2},-\tan^2(c+dx)\right)}{de(m+1)}+$$
$$\frac{a^3\sec^3(c+dx)\cos^2(c+dx)^{\frac{m+4}{2}}(e\tan(c+dx))^{m+1}\,\text{Hypergeometric2F1}\left(\frac{m+1}{2},\frac{m+4}{2},\frac{m+3}{2},\sin^2(c+dx)\right)}{de(m+1)}+$$
$$\frac{3a^3\sec(c+dx)\cos^2(c+dx)^{\frac{m+2}{2}}(e\tan(c+dx))^{m+1}\,\text{Hypergeometric2F1}\left(\frac{m+1}{2},\frac{m+2}{2},\frac{m+3}{2},\sin^2(c+dx)\right)}{de(m+1)}+$$
$$\frac{3a^3(e\tan(c+dx))^{m+1}}{de(m+1)}$$

input
```
Int[(a + a*Sec[c + d*x])^3*(e*Tan[c + d*x])^m,x]
```

output
```
(3*a^3*(e*Tan[c + d*x])^(1 + m))/(d*e*(1 + m)) + (a^3*Hypergeometric2F1[1,
 (1 + m)/2, (3 + m)/2, -Tan[c + d*x]^2]*(e*Tan[c + d*x])^(1 + m))/(d*e*(1
+ m)) + (3*a^3*(Cos[c + d*x]^2)^((2 + m)/2)*Hypergeometric2F1[(1 + m)/2, (
2 + m)/2, (3 + m)/2, Sin[c + d*x]^2]*Sec[c + d*x]*(e*Tan[c + d*x])^(1 + m)
)/(d*e*(1 + m)) + (a^3*(Cos[c + d*x]^2)^((4 + m)/2)*Hypergeometric2F1[(1 +
 m)/2, (4 + m)/2, (3 + m)/2, Sin[c + d*x]^2]*Sec[c + d*x]^3*(e*Tan[c + d*x
])^(1 + m))/(d*e*(1 + m))
```

3.209.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4374
```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

3.209.4 Maple [F]

$$\int (a + a\sec(dx + c))^3 (e\tan(dx + c))^m \, dx$$

input
```
int((a+a*sec(d*x+c))^3*(e*tan(d*x+c))^m,x)
```

output
```
int((a+a*sec(d*x+c))^3*(e*tan(d*x+c))^m,x)
```

3.209.5 Fricas [F]

$$\int (a + a\sec(c + dx))^3 (e\tan(c + dx))^m \, dx = \int (a\sec(dx + c) + a)^3 (e\tan(dx + c))^m \, dx$$

input
```
integrate((a+a*sec(d*x+c))^3*(e*tan(d*x+c))^m,x, algorithm="fricas")
```

output
```
integral((a^3*sec(d*x + c)^3 + 3*a^3*sec(d*x + c)^2 + 3*a^3*sec(d*x + c) +
 a^3)*(e*tan(d*x + c))^m, x)
```

3.209.6 Sympy [F]

$$\begin{aligned}\int (a + a\sec(c + dx))^3 (e\tan(c + dx))^m \, dx = a^3 \Bigg(& \int (e\tan(c + dx))^m \, dx \\ & + \int 3(e\tan(c + dx))^m \sec(c + dx) \, dx \\ & + \int 3(e\tan(c + dx))^m \sec^2(c + dx) \, dx \\ & + \int (e\tan(c + dx))^m \sec^3(c + dx) \, dx \Bigg)\end{aligned}$$

input
```
integrate((a+a*sec(d*x+c))**3*(e*tan(d*x+c))**m,x)
```

output
```
a**3*(Integral((e*tan(c + d*x))**m, x) + Integral(3*(e*tan(c + d*x))**m*se
c(c + d*x), x) + Integral(3*(e*tan(c + d*x))**m*sec(c + d*x)**2, x) + Inte
gral((e*tan(c + d*x))**m*sec(c + d*x)**3, x))
```

3.209.7 Maxima [F]

$$\int (a + a\sec(c + dx))^3 (e\tan(c + dx))^m \, dx = \int (a\sec(dx + c) + a)^3 (e\tan(dx + c))^m \, dx$$

input
```
integrate((a+a*sec(d*x+c))^3*(e*tan(d*x+c))^m,x, algorithm="maxima")
```

output
```
integrate((a*sec(d*x + c) + a)^3*(e*tan(d*x + c))^m, x)
```

3.209.8 Giac [F]

$$\int (a + a\sec(c + dx))^3 (e\tan(c + dx))^m \, dx = \int (a\sec(dx + c) + a)^3 (e\tan(dx + c))^m \, dx$$

input
```
integrate((a+a*sec(d*x+c))^3*(e*tan(d*x+c))^m,x, algorithm="giac")
```

output
```
integrate((a*sec(d*x + c) + a)^3*(e*tan(d*x + c))^m, x)
```

3.209.9 Mupad [F(-1)]

Timed out.

$$\int (a + a\sec(c + dx))^3 (e\tan(c + dx))^m \, dx = \int (e\tan(c + dx))^m \left(a + \frac{a}{\cos(c + dx)}\right)^3 dx$$

input
```
int((e*tan(c + d*x))^m*(a + a/cos(c + d*x))^3,x)
```

output
```
int((e*tan(c + d*x))^m*(a + a/cos(c + d*x))^3, x)
```

3.210 $\int (a + a\sec(c+dx))^2 (e\tan(c+dx))^m \, dx$

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3.210.7 Maxima [F] 1571
3.210.8 Giac [F] 1571
3.210.9 Mupad [F(-1)] 1572

3.210.1 Optimal result

Integrand size = 23, antiderivative size = 161

$$\int (a + a\sec(c+dx))^2 (e\tan(c+dx))^m \, dx = \frac{a^2 (e\tan(c+dx))^{1+m}}{de(1+m)} + \frac{a^2 \text{Hypergeometric2F1}\left(1, \frac{1+m}{2}, \frac{3+m}{2}, -\tan^2(c+dx)\right) (e\tan(c+dx))^{1+m}}{de(1+m)} + \frac{2a^2 \cos^2(c+dx)^{\frac{2+m}{2}} \text{Hypergeometric2F1}\left(\frac{1+m}{2}, \frac{2+m}{2}, \frac{3+m}{2}, \sin^2(c+dx)\right) \sec(c+dx) (e\tan(c+dx))^{1+m}}{de(1+m)}$$

output

```
a^2*(e*tan(d*x+c))^(1+m)/d/e/(1+m)+a^2*hypergeom([1, 1/2+1/2*m],[3/2+1/2*m
],-tan(d*x+c)^2)*(e*tan(d*x+c))^(1+m)/d/e/(1+m)+2*a^2*(cos(d*x+c)^2)^(1+1/
2*m)*hypergeom([1+1/2*m, 1/2+1/2*m],[3/2+1/2*m],sin(d*x+c)^2)*sec(d*x+c)*(
e*tan(d*x+c))^(1+m)/d/e/(1+m)
```

3.210.2 Mathematica [A] (verified)

Time = 1.23 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.06

$$\int (a + a\sec(c+dx))^2 (e\tan(c+dx))^m \, dx = \frac{a^2 e (e\tan(c+dx))^{-1+m} (-\tan^2(c+dx))^{-m/2} \left(\sqrt{-\tan^2(c+dx)} + 2(1+m)\,\text{Hypergeometric2F1}\left(\frac{1}{2}, \frac{1-m}{2}\right.\right.$$

input
```
Integrate[(a + a*Sec[c + d*x])^2*(e*Tan[c + d*x])^m,x]
```

output
```
(a^2*e*(e*Tan[c + d*x])^(-1 + m)*(Sqrt[-Tan[c + d*x]^2] + 2*(1 + m)*Hyperg
eometric2F1[1/2, (1 - m)/2, 3/2, Sec[c + d*x]^2]*Sec[c + d*x]*Sqrt[-Tan[c
+ d*x]^2] - (-Tan[c + d*x]^2)^((2 + m)/2) - Hypergeometric2F1[1, (1 + m)/2
, (3 + m)/2, -Tan[c + d*x]^2]*(-Tan[c + d*x]^2)^((2 + m)/2)))/(d*(1 + m)*(
-Tan[c + d*x]^2)^(m/2))
```

3.210.3 Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3042, 4374, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sec(c+dx)+a)^2(e\tan(c+dx))^m\,dx$$

$$\downarrow 3042$$

$$\int \left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^2\left(-e\cot\left(c+dx+\frac{\pi}{2}\right)\right)^m dx$$

$$\downarrow 4374$$

$$\int \left(a^2(e\tan(c+dx))^m+a^2\sec^2(c+dx)(e\tan(c+dx))^m+2a^2\sec(c+dx)(e\tan(c+dx))^m\right)dx$$

$$\downarrow 2009$$

$$\frac{a^2(e\tan(c+dx))^{m+1}\,\text{Hypergeometric2F1}\left(1,\frac{m+1}{2},\frac{m+3}{2},-\tan^2(c+dx)\right)}{de(m+1)}+$$
$$\frac{2a^2\sec(c+dx)\cos^2(c+dx)^{\frac{m+2}{2}}(e\tan(c+dx))^{m+1}\,\text{Hypergeometric2F1}\left(\frac{m+1}{2},\frac{m+2}{2},\frac{m+3}{2},\sin^2(c+dx)\right)}{de(m+1)}+$$
$$\frac{a^2(e\tan(c+dx))^{m+1}}{de(m+1)}$$

input
```
Int[(a + a*Sec[c + d*x])^2*(e*Tan[c + d*x])^m,x]
```

output
```
(a^2*(e*Tan[c + d*x])^(1 + m))/(d*e*(1 + m)) + (a^2*Hypergeometric2F1[1, (
1 + m)/2, (3 + m)/2, -Tan[c + d*x]^2]*(e*Tan[c + d*x])^(1 + m))/(d*e*(1 +
m)) + (2*a^2*(Cos[c + d*x]^2)^((2 + m)/2)*Hypergeometric2F1[(1 + m)/2, (2
+ m)/2, (3 + m)/2, Sin[c + d*x]^2]*Sec[c + d*x]*(e*Tan[c + d*x])^(1 + m))/
(d*e*(1 + m))
```

3.210.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4374
```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

3.210.4 Maple [F]

$$\int (a + a \sec(dx + c))^2 (e \tan(dx + c))^m \, dx$$

input
```
int((a+a*sec(d*x+c))^2*(e*tan(d*x+c))^m,x)
```

output
```
int((a+a*sec(d*x+c))^2*(e*tan(d*x+c))^m,x)
```

3.210.5 Fricas [F]

$$\int (a + a \sec(c + dx))^2 (e \tan(c + dx))^m \, dx = \int (a \sec(dx + c) + a)^2 (e \tan(dx + c))^m \, dx$$

input
```
integrate((a+a*sec(d*x+c))^2*(e*tan(d*x+c))^m,x, algorithm="fricas")
```

output
```
integral((a^2*sec(d*x + c)^2 + 2*a^2*sec(d*x + c) + a^2)*(e*tan(d*x + c))^
m, x)
```

3.210.6 Sympy [F]

$$\int (a + a\sec(c+dx))^2 (e\tan(c+dx))^m \, dx = a^2 \left(\int (e\tan(c+dx))^m \, dx + \int 2(e\tan(c+dx))^m \sec(c+dx) \, dx + \int (e\tan(c+dx))^m \sec^2(c+dx) \, dx \right)$$

input
```
integrate((a+a*sec(d*x+c))**2*(e*tan(d*x+c))**m,x)
```

output
```
a**2*(Integral((e*tan(c + d*x))**m, x) + Integral(2*(e*tan(c + d*x))**m*se
c(c + d*x), x) + Integral((e*tan(c + d*x))**m*sec(c + d*x)**2, x))
```

3.210.7 Maxima [F]

$$\int (a + a\sec(c+dx))^2 (e\tan(c+dx))^m \, dx = \int (a\sec(dx+c) + a)^2 (e\tan(dx+c))^m \, dx$$

input
```
integrate((a+a*sec(d*x+c))^2*(e*tan(d*x+c))^m,x, algorithm="maxima")
```

output
```
integrate((a*sec(d*x + c) + a)^2*(e*tan(d*x + c))^m, x)
```

3.210.8 Giac [F]

$$\int (a + a\sec(c+dx))^2 (e\tan(c+dx))^m \, dx = \int (a\sec(dx+c) + a)^2 (e\tan(dx+c))^m \, dx$$

input
```
integrate((a+a*sec(d*x+c))^2*(e*tan(d*x+c))^m,x, algorithm="giac")
```

output
```
integrate((a*sec(d*x + c) + a)^2*(e*tan(d*x + c))^m, x)
```

3.210.9 Mupad [F(-1)]

Timed out.

$$\int (a + a\sec(c + dx))^2 (e\tan(c + dx))^m \, dx = \int (e\tan(c + d\,x))^m \left(a + \frac{a}{\cos(c + d\,x)}\right)^2 dx$$

input
```
int((e*tan(c + d*x))^m*(a + a/cos(c + d*x))^2,x)
```

output
```
int((e*tan(c + d*x))^m*(a + a/cos(c + d*x))^2, x)
```

3.211 $\int (a + a\sec(c+dx))(e\tan(c+dx))^m\,dx$

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3.211.1 Optimal result

Integrand size = 21, antiderivative size = 129

$$\int (a + a\sec(c+dx))(e\tan(c+dx))^m\,dx$$
$$= \frac{a\,\text{Hypergeometric2F1}\left(1, \frac{1+m}{2}, \frac{3+m}{2}, -\tan^2(c+dx)\right)(e\tan(c+dx))^{1+m}}{de(1+m)}$$
$$+ \frac{a\cos^2(c+dx)^{\frac{2+m}{2}}\,\text{Hypergeometric2F1}\left(\frac{1+m}{2}, \frac{2+m}{2}, \frac{3+m}{2}, \sin^2(c+dx)\right)\sec(c+dx)(e\tan(c+dx))^{1+m}}{de(1+m)}$$

output
```
a*hypergeom([1, 1/2+1/2*m],[3/2+1/2*m],-tan(d*x+c)^2)*(e*tan(d*x+c))^(1+m)
/d/e/(1+m)+a*(cos(d*x+c)^2)^(1+1/2*m)*hypergeom([1+1/2*m, 1/2+1/2*m],[3/2+
1/2*m],sin(d*x+c)^2)*sec(d*x+c)*(e*tan(d*x+c))^(1+m)/d/e/(1+m)
```

3.211.2 Mathematica [A] (verified)

Time = 0.43 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.81

$$\int (a + a\sec(c+dx))(e\tan(c+dx))^m\,dx$$
$$= \frac{a(e\tan(c+dx))^m\left(\frac{\text{Hypergeometric2F1}\left(1, \frac{1+m}{2}, \frac{3+m}{2}, -\tan^2(c+dx)\right)\tan(c+dx)}{1+m} + \csc(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2}, \frac{1-}{2}\right.\right.}{d}$$

input
```
Integrate[(a + a*Sec[c + d*x])*(e*Tan[c + d*x])^m,x]
```

output
```
(a*(e*Tan[c + d*x])^m*((Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, -Tan[c
+ d*x]^2]*Tan[c + d*x])/(1 + m) + Csc[c + d*x]*Hypergeometric2F1[1/2, (1 -
 m)/2, 3/2, Sec[c + d*x]^2]*(-Tan[c + d*x]^2)^((1 - m)/2)))/d
```

3.211.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 4372, 3042, 3097, 3957, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sec(c+dx)+a)(e\tan(c+dx))^m\,dx$$

$$\downarrow 3042$$

$$\int \left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)\left(-e\cot\left(c+dx+\frac{\pi}{2}\right)\right)^m dx$$

$$\downarrow 4372$$

$$a\int (e\tan(c+dx))^m dx + a\int \sec(c+dx)(e\tan(c+dx))^m dx$$

$$\downarrow 3042$$

$$a\int (e\tan(c+dx))^m dx + a\int \sec(c+dx)(e\tan(c+dx))^m dx$$

$$\downarrow 3097$$

$$a\int (e\tan(c+dx))^m dx + \frac{a\sec(c+dx)\cos^2(c+dx)^{\frac{m+2}{2}}(e\tan(c+dx))^{m+1}\,\text{Hypergeometric2F1}\left(\frac{m+1}{2},\frac{m+2}{2},\frac{m+3}{2},\sin^2(c+dx)\right)}{de(m+1)}$$

$$\downarrow 3957$$

$$\frac{ae\int \frac{(e\tan(c+dx))^m}{\tan^2(c+dx)e^2+e^2}d(e\tan(c+dx))}{d} + \frac{a\sec(c+dx)\cos^2(c+dx)^{\frac{m+2}{2}}(e\tan(c+dx))^{m+1}\,\text{Hypergeometric2F1}\left(\frac{m+1}{2},\frac{m+2}{2},\frac{m+3}{2},\sin^2(c+dx)\right)}{de(m+1)}$$

$$\downarrow 278$$

$$\frac{a(e\tan(c+dx))^{m+1}\,\text{Hypergeometric2F1}\left(1,\frac{m+1}{2},\frac{m+3}{2},-\tan^2(c+dx)\right)}{de(m+1)}+$$
$$\frac{a\sec(c+dx)\cos^2(c+dx)^{\frac{m+2}{2}}(e\tan(c+dx))^{m+1}\,\text{Hypergeometric2F1}\left(\frac{m+1}{2},\frac{m+2}{2},\frac{m+3}{2},\sin^2(c+dx)\right)}{de(m+1)}$$

input
```
Int[(a + a*Sec[c + d*x])*(e*Tan[c + d*x])^m,x]
```

output
```
(a*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, -Tan[c + d*x]^2]*(e*Tan[c +
d*x])^(1 + m))/(d*e*(1 + m)) + (a*(Cos[c + d*x]^2)^((2 + m)/2)*Hypergeomet
ric2F1[(1 + m)/2, (2 + m)/2, (3 + m)/2, Sin[c + d*x]^2]*Sec[c + d*x]*(e*Ta
n[c + d*x])^(1 + m))/(d*e*(1 + m))
```

3.211.3.1 Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3097
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1)*((Cos[e
+ f*x]^2)^((m + n + 1)/2)/(b*f*(n + 1)))*Hypergeometric2F1[(n + 1)/2, (m +
n + 1)/2, (n + 3)/2, Sin[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] &&
  !IntegerQ[(n - 1)/2] &&  !IntegerQ[m/2]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

rule 4372
```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(e*Cot[c + d*x])^m, x], x] + Simp[b   Int[
(e*Cot[c + d*x])^m*Csc[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, m}, x]
```

3.211.4 Maple [F]

$$\int (a + a \sec(dx + c)) (e \tan(dx + c))^m \, dx$$

input
```
int((a+a*sec(d*x+c))*(e*tan(d*x+c))^m,x)
```

output
```
int((a+a*sec(d*x+c))*(e*tan(d*x+c))^m,x)
```

3.211.5 Fricas [F]

$$\int (a + a \sec(c + dx))(e \tan(c + dx))^m \, dx = \int (a \sec(dx + c) + a)(e \tan(dx + c))^m \, dx$$

input
```
integrate((a+a*sec(d*x+c))*(e*tan(d*x+c))^m,x, algorithm="fricas")
```

output
```
integral((a*sec(d*x + c) + a)*(e*tan(d*x + c))^m, x)
```

3.211.6 Sympy [F]

$$\int (a + a \sec(c + dx))(e \tan(c + dx))^m \, dx = a\left(\int (e \tan(c + dx))^m \, dx + \int (e \tan(c + dx))^m \sec(c + dx) \, dx\right)$$

input
```
integrate((a+a*sec(d*x+c))*(e*tan(d*x+c))**m,x)
```

output
```
a*(Integral((e*tan(c + d*x))**m, x) + Integral((e*tan(c + d*x))**m*sec(c +
 d*x), x))
```

3.211.7 Maxima [F]

$$\int (a + a\sec(c + dx))(e\tan(c + dx))^m \, dx = \int (a\sec(dx + c) + a)(e\tan(dx + c))^m \, dx$$

input
```
integrate((a+a*sec(d*x+c))*(e*tan(d*x+c))^m,x, algorithm="maxima")
```

output
```
integrate((a*sec(d*x + c) + a)*(e*tan(d*x + c))^m, x)
```

3.211.8 Giac [F]

$$\int (a + a\sec(c + dx))(e\tan(c + dx))^m \, dx = \int (a\sec(dx + c) + a)(e\tan(dx + c))^m \, dx$$

input
```
integrate((a+a*sec(d*x+c))*(e*tan(d*x+c))^m,x, algorithm="giac")
```

output
```
integrate((a*sec(d*x + c) + a)*(e*tan(d*x + c))^m, x)
```

3.211.9 Mupad [F(-1)]

Timed out.

$$\int (a + a\sec(c + dx))(e\tan(c + dx))^m \, dx = \int (e\tan(c + d\,x))^m \left(a + \frac{a}{\cos(c + d\,x)}\right) dx$$

input
```
int((e*tan(c + d*x))^m*(a + a/cos(c + d*x)),x)
```

output
```
int((e*tan(c + d*x))^m*(a + a/cos(c + d*x)), x)
```

3.212 $\int \frac{(e\tan(c+dx))^m}{a+a\sec(c+dx)}\,dx$

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3.212.1 Optimal result

Integrand size = 23, antiderivative size = 130

$$\int \frac{(e\tan(c+dx))^m}{a+a\sec(c+dx)}\,dx$$
$$= \frac{e\,\text{Hypergeometric2F1}\left(1,\frac{1}{2}(-1+m),\frac{1+m}{2},-\tan^2(c+dx)\right)(e\tan(c+dx))^{-1+m}}{ad(1-m)}$$
$$-\frac{e\cos^2(c+dx)^{m/2}\,\text{Hypergeometric2F1}\left(\frac{1}{2}(-1+m),\frac{m}{2},\frac{1+m}{2},\sin^2(c+dx)\right)\sec(c+dx)(e\tan(c+dx))^{-1+}}{ad(1-m)}$$

output
```
e*hypergeom([1, -1/2+1/2*m],[1/2+1/2*m],-tan(d*x+c)^2)*(e*tan(d*x+c))^(-1+
m)/a/d/(1-m)-e*(cos(d*x+c)^2)^(1/2*m)*hypergeom([1/2*m, -1/2+1/2*m],[1/2+1
/2*m],sin(d*x+c)^2)*sec(d*x+c)*(e*tan(d*x+c))^(-1+m)/a/d/(1-m)
```

3.212.2 Mathematica [F]

$$\int \frac{(e\tan(c+dx))^m}{a+a\sec(c+dx)}\,dx = \int \frac{(e\tan(c+dx))^m}{a+a\sec(c+dx)}\,dx$$

input
```
Integrate[(e*Tan[c + d*x])^m/(a + a*Sec[c + d*x]),x]
```

output
```
Integrate[(e*Tan[c + d*x])^m/(a + a*Sec[c + d*x]), x]
```

3.212.3 Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.06, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {3042, 4376, 25, 3042, 4372, 3042, 3097, 3957, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e\tan(c+dx))^m}{a\sec(c+dx)+a}\,dx$$

$$\downarrow 3042$$

$$\int \frac{\left(-e\cot\left(c+dx+\frac{\pi}{2}\right)\right)^m}{a\csc\left(c+dx+\frac{\pi}{2}\right)+a}dx$$

$$\downarrow 4376$$

$$\frac{e^2\int -\left((a-a\sec(c+dx))(e\tan(c+dx))^{m-2}\right)dx}{a^2}$$

$$\downarrow 25$$

$$-\frac{e^2\int (a-a\sec(c+dx))(e\tan(c+dx))^{m-2}dx}{a^2}$$

$$\downarrow 3042$$

$$-\frac{e^2\int \left(-e\cot\left(c+dx+\frac{\pi}{2}\right)\right)^{m-2}\left(a-a\csc\left(c+dx+\frac{\pi}{2}\right)\right)dx}{a^2}$$

$$\downarrow 4372$$

$$-\frac{e^2\left(a\int (e\tan(c+dx))^{m-2}dx-a\int \sec(c+dx)(e\tan(c+dx))^{m-2}dx\right)}{a^2}$$

$$\downarrow 3042$$

$$-\frac{e^2\left(a\int (e\tan(c+dx))^{m-2}dx-a\int \sec(c+dx)(e\tan(c+dx))^{m-2}dx\right)}{a^2}$$

$$\downarrow 3097$$

$$-\frac{e^2\left(a\int (e\tan(c+dx))^{m-2}dx+\frac{a\sec(c+dx)\cos^2(c+dx)^{m/2}(e\tan(c+dx))^{m-1}\,\text{Hypergeometric2F1}\left(\frac{m-1}{2},\frac{m}{2},\frac{m+1}{2},\sin^2(c+dx)\right)}{de(1-m)}\right)}{a^2}$$

$$\downarrow 3957$$

$$-\frac{e^2\left(\frac{ae\int\frac{(e\tan(c+dx))^{m-2}}{\tan^2(c+dx)e^2+e^2}d(e\tan(c+dx))}{d}+\frac{a\sec(c+dx)\cos^2(c+dx)^{m/2}(e\tan(c+dx))^{m-1}\,\text{Hypergeometric2F1}\left(\frac{m-1}{2},\frac{m}{2},\frac{m+1}{2},\sin^2(c+dx)\right)}{de(1-m)}\right)}{a^2}$$

$$\downarrow 278$$

$$-\frac{e^2\left(\frac{a\sec(c+dx)\cos^2(c+dx)^{m/2}(e\tan(c+dx))^{m-1}\,\text{Hypergeometric2F1}\left(\frac{m-1}{2},\frac{m}{2},\frac{m+1}{2},\sin^2(c+dx)\right)}{de(1-m)}-\frac{a(e\tan(c+dx))^{m-1}\,\text{Hypergeometric2F}}{de(1-m)}\right)}{a^2}$$

input
```
Int[(e*Tan[c + d*x])^m/(a + a*Sec[c + d*x]),x]
```

output
```
-((e^2*(-((a*Hypergeometric2F1[1, (-1 + m)/2, (1 + m)/2, -Tan[c + d*x]^2]*
(e*Tan[c + d*x])^(-1 + m))/(d*e*(1 - m))) + (a*(Cos[c + d*x]^2)^(m/2)*Hype
rgeometric2F1[(-1 + m)/2, m/2, (1 + m)/2, Sin[c + d*x]^2]*Sec[c + d*x]*(e*
Tan[c + d*x])^(-1 + m))/(d*e*(1 - m))))/a^2)
```

3.212.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3097
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1)*((Cos[e
+ f*x]^2)^((m + n + 1)/2)/(b*f*(n + 1)))*Hypergeometric2F1[(n + 1)/2, (m +
n + 1)/2, (n + 3)/2, Sin[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] &&
  !IntegerQ[(n - 1)/2] &&  !IntegerQ[m/2]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

rule 4372
```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(e*Cot[c + d*x])^m, x], x] + Simp[b   Int[
(e*Cot[c + d*x])^m*Csc[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, m}, x]
```

rule 4376
```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Simp[a^(2*n)/e^(2*n)   Int[(e*Cot[c + d*x])^(m + 2*
n)/(-a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[a
^2 - b^2, 0] && ILtQ[n, 0]
```

3.212.4 Maple [F]

$$\int \frac{(e \tan (dx+c))^m}{a+a \sec (dx+c)} dx$$

input
```
int((e*tan(d*x+c))^m/(a+a*sec(d*x+c)),x)
```

output
```
int((e*tan(d*x+c))^m/(a+a*sec(d*x+c)),x)
```

3.212.5 Fricas [F]

$$\int \frac{(e \tan(c+dx))^m}{a+a \sec(c+dx)} dx = \int \frac{(e \tan (dx+c))^m}{a \sec (dx+c)+a} dx$$

input
```
integrate((e*tan(d*x+c))^m/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

output
```
integral((e*tan(d*x + c))^m/(a*sec(d*x + c) + a), x)
```

3.212.6 Sympy [F]

$$\int \frac{(e\tan(c+dx))^m}{a+a\sec(c+dx)}\,dx = \frac{\int \frac{(e\tan(c+dx))^m}{\sec(c+dx)+1}\,dx}{a}$$

input
```
integrate((e*tan(d*x+c))**m/(a+a*sec(d*x+c)),x)
```

output
```
Integral((e*tan(c + d*x))**m/(sec(c + d*x) + 1), x)/a
```

3.212.7 Maxima [F]

$$\int \frac{(e\tan(c+dx))^m}{a+a\sec(c+dx)}\,dx = \int \frac{(e\tan(dx+c))^m}{a\sec(dx+c)+a}\,dx$$

input
```
integrate((e*tan(d*x+c))^m/(a+a*sec(d*x+c)),x, algorithm="maxima")
```

output
```
integrate((e*tan(d*x + c))^m/(a*sec(d*x + c) + a), x)
```

3.212.8 Giac [F]

$$\int \frac{(e\tan(c+dx))^m}{a+a\sec(c+dx)}\,dx = \int \frac{(e\tan(dx+c))^m}{a\sec(dx+c)+a}\,dx$$

input
```
integrate((e*tan(d*x+c))^m/(a+a*sec(d*x+c)),x, algorithm="giac")
```

output
```
integrate((e*tan(d*x + c))^m/(a*sec(d*x + c) + a), x)
```

3.212.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(e\tan(c+dx))^m}{a+a\sec(c+dx)}\,dx = \int \frac{\cos(c+d\,x)\,(e\tan(c+d\,x))^m}{a\,(\cos(c+d\,x)+1)}\,dx$$

input
```
int((e*tan(c + d*x))^m/(a + a/cos(c + d*x)),x)
```

output
```
int((cos(c + d*x)*(e*tan(c + d*x))^m)/(a*(cos(c + d*x) + 1)), x)
```

3.213 $\int \frac{(e\tan(c+dx))^m}{(a+a\sec(c+dx))^2}\,dx$

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3.213.1 Optimal result

Integrand size = 23, antiderivative size = 169

$$\int \frac{(e\tan(c+dx))^m}{(a+a\sec(c+dx))^2}\,dx = -\frac{e^3(e\tan(c+dx))^{-3+m}}{a^2d(3-m)} - \frac{e^3\,\text{Hypergeometric2F1}\left(1,\frac{1}{2}(-3+m),\frac{1}{2}(-1+m),-\tan^2(c+dx)\right)(e\tan(c+dx))^{-3+m}}{a^2d(3-m)} + \frac{2e^3\cos^2(c+dx)^{\frac{1}{2}(-2+m)}\,\text{Hypergeometric2F1}\left(\frac{1}{2}(-3+m),\frac{1}{2}(-2+m),\frac{1}{2}(-1+m),\sin^2(c+dx)\right)\sec(c+$$

output
```
-e^3*(e*tan(d*x+c))^(-3+m)/a^2/d/(3-m)-e^3*hypergeom([1, -3/2+1/2*m],[-1/2
+1/2*m],-tan(d*x+c)^2)*(e*tan(d*x+c))^(-3+m)/a^2/d/(3-m)+2*e^3*(cos(d*x+c)
^2)^(-1+1/2*m)*hypergeom([-1+1/2*m, -3/2+1/2*m],[-1/2+1/2*m],sin(d*x+c)^2)
*sec(d*x+c)*(e*tan(d*x+c))^(-3+m)/a^2/d/(3-m)
```

3.213.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.84 (sec) , antiderivative size = 329, normalized size of antiderivative = 1.95

$$\int \frac{(e\tan(c+dx))^m}{(a+a\sec(c+dx))^2}\,dx = \frac{\left(\cos(c+dx)\sec^2\left(\frac{1}{2}(c+dx)\right)\right)^m\tan\left(\frac{1}{2}(c+dx)\right)\left(-3(3+m)\,\text{Hypergeometric2F1}\left(m,\frac{1+m}{2},\frac{3+m}{2},\tan^2\left(\frac{1}{2}(c\right.\right.\right.}{2a^2d(1+} + \frac{i2^{1-m}\left(-\frac{i\left(-1+e^{2i(c+dx)}\right)}{1+e^{2i(c+dx)}}\right)^m\cos^4\left(\frac{c}{2}+\frac{dx}{2}\right)\left(2^m\,\text{Hypergeometric2F1}\left(1,m,1+m,-\frac{-1+e^{2i(c+dx)}}{1+e^{2i(c+dx)}}\right)-(1+e^{2}\right.}{dm(a+a}$$

input
```
Integrate[(e*Tan[c + d*x])^m/(a + a*Sec[c + d*x])^2,x]
```

output
```
((Cos[c + d*x]*Sec[(c + d*x)/2]^2)^m*Tan[(c + d*x)/2]*(-3*(3 + m)*Hypergeo
metric2F1[m, (1 + m)/2, (3 + m)/2, Tan[(c + d*x)/2]^2] + (1 + m)*Hypergeom
etric2F1[m, (3 + m)/2, (5 + m)/2, Tan[(c + d*x)/2]^2]*Tan[(c + d*x)/2]^2)*
(e*Tan[c + d*x])^m)/(2*a^2*d*(1 + m)*(3 + m)) + (I*2^(1 - m)*(((-I)*(-1 +
E^((2*I)*(c + d*x))))/(1 + E^((2*I)*(c + d*x))))^m*Cos[c/2 + (d*x)/2]^4*(2
^m*Hypergeometric2F1[1, m, 1 + m, -((-1 + E^((2*I)*(c + d*x)))/(1 + E^((2*
I)*(c + d*x))))] - (1 + E^((2*I)*(c + d*x)))^m*Hypergeometric2F1[m, m, 1 +
 m, (1 - E^((2*I)*(c + d*x)))/2])*Sec[c + d*x]^2*(e*Tan[c + d*x])^m)/(d*m*
(a + a*Sec[c + d*x])^2*Tan[c + d*x]^m)
```

3.213.3 Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.04, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3042, 4376, 3042, 4374, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e\tan(c+dx))^m}{(a\sec(c+dx)+a)^2}\,dx$$

$$\downarrow 3042$$

$$\int \frac{\left(-e\cot\left(c+dx+\frac{\pi}{2}\right)\right)^m}{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^2}dx$$

$$\downarrow 4376$$

$$\frac{e^4\int(a-a\sec(c+dx))^2(e\tan(c+dx))^{m-4}dx}{a^4}$$

$$\downarrow 3042$$

$$\frac{e^4\int\left(-e\cot\left(c+dx+\frac{\pi}{2}\right)\right)^{m-4}\left(a-a\csc\left(c+dx+\frac{\pi}{2}\right)\right)^2dx}{a^4}$$

$$\downarrow 4374$$

$$\frac{e^4\int\left(a^2(e\tan(c+dx))^{m-4}+a^2\sec^2(c+dx)(e\tan(c+dx))^{m-4}-2a^2\sec(c+dx)(e\tan(c+dx))^{m-4}\right)dx}{a^4}$$

$$\downarrow 2009$$

$$\frac{e^4\left(-\frac{a^2(e\tan(c+dx))^{m-3}\,\text{Hypergeometric2F1}\left(1,\frac{m-3}{2},\frac{m-1}{2},-\tan^2(c+dx)\right)}{de(3-m)}+\frac{2a^2\sec(c+dx)\cos^2(c+dx)^{\frac{m-2}{2}}(e\tan(c+dx))^{m-3}\,\text{Hypergeome}}{de(3-m)}\right)}{a^4}$$

input
```
Int[(e*Tan[c + d*x])^m/(a + a*Sec[c + d*x])^2,x]
```

output
```
(e^4*(-((a^2*(e*Tan[c + d*x])^(-3 + m))/(d*e*(3 - m))) - (a^2*Hypergeometr
ic2F1[1, (-3 + m)/2, (-1 + m)/2, -Tan[c + d*x]^2]*(e*Tan[c + d*x])^(-3 + m
))/(d*e*(3 - m)) + (2*a^2*(Cos[c + d*x]^2)^((-2 + m)/2)*Hypergeometric2F1[
(-3 + m)/2, (-2 + m)/2, (-1 + m)/2, Sin[c + d*x]^2]*Sec[c + d*x]*(e*Tan[c
+ d*x])^(-3 + m))/(d*e*(3 - m))))/a^4
```

3.213.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4374
```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

rule 4376
```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Simp[a^(2*n)/e^(2*n)   Int[(e*Cot[c + d*x])^(m + 2*
n)/(-a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[a
^2 - b^2, 0] && ILtQ[n, 0]
```

3.213.4 Maple [F]

$$\int \frac{(e\tan(dx+c))^m}{(a+a\sec(dx+c))^2}dx$$

input
```
int((e*tan(d*x+c))^m/(a+a*sec(d*x+c))^2,x)
```

output
```
int((e*tan(d*x+c))^m/(a+a*sec(d*x+c))^2,x)
```

3.213.5 Fricas [F]

$$\int \frac{(e\tan(c+dx))^m}{(a+a\sec(c+dx))^2}\,dx = \int \frac{(e\tan(dx+c))^m}{(a\sec(dx+c)+a)^2}\,dx$$

input
```
integrate((e*tan(d*x+c))^m/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

output
```
integral((e*tan(d*x + c))^m/(a^2*sec(d*x + c)^2 + 2*a^2*sec(d*x + c) + a^2
), x)
```

3.213.6 Sympy [F]

$$\int \frac{(e\tan(c+dx))^m}{(a+a\sec(c+dx))^2}\,dx = \frac{\int \frac{(e\tan(c+dx))^m}{\sec^2(c+dx)+2\sec(c+dx)+1}\,dx}{a^2}$$

input
```
integrate((e*tan(d*x+c))**m/(a+a*sec(d*x+c))**2,x)
```

output
```
Integral((e*tan(c + d*x))**m/(sec(c + d*x)**2 + 2*sec(c + d*x) + 1), x)/a*
*2
```

3.213.7 Maxima [F]

$$\int \frac{(e\tan(c+dx))^m}{(a+a\sec(c+dx))^2}\,dx = \int \frac{(e\tan(dx+c))^m}{(a\sec(dx+c)+a)^2}\,dx$$

input
```
integrate((e*tan(d*x+c))^m/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

output
```
integrate((e*tan(d*x + c))^m/(a*sec(d*x + c) + a)^2, x)
```

3.213.8 Giac [F]

$$\int \frac{(e\tan(c+dx))^m}{(a+a\sec(c+dx))^2}\,dx = \int \frac{(e\tan(dx+c))^m}{(a\sec(dx+c)+a)^2}\,dx$$

input
```
integrate((e*tan(d*x+c))^m/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

output
```
integrate((e*tan(d*x + c))^m/(a*sec(d*x + c) + a)^2, x)
```

3.213.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(e\tan(c+dx))^m}{(a+a\sec(c+dx))^2}\,dx = \int \frac{\cos(c+dx)^2\,(e\tan(c+dx))^m}{a^2\,(\cos(c+dx)+1)^2}\,dx$$

input
```
int((e*tan(c + d*x))^m/(a + a/cos(c + d*x))^2,x)
```

output
```
int((cos(c + d*x)^2*(e*tan(c + d*x))^m)/(a^2*(cos(c + d*x) + 1)^2), x)
```

3.214 $\int \frac{(e\tan(c+dx))^m}{(a+a\sec(c+dx))^3}\,dx$

3.214.1 Optimal result 1589
3.214.2 Mathematica [F] 1590
3.214.3 Rubi [A] (verified) 1590
3.214.4 Maple [F] 1592
3.214.5 Fricas [F] 1592
3.214.6 Sympy [F] 1592
3.214.7 Maxima [F] 1593
3.214.8 Giac [F] 1593
3.214.9 Mupad [F(-1)] 1593

3.214.1 Optimal result

Integrand size = 23, antiderivative size = 252

$$\int \frac{(e\tan(c+dx))^m}{(a+a\sec(c+dx))^3}\,dx = \frac{3e^5(e\tan(c+dx))^{-5+m}}{a^3d(5-m)} + \frac{e^5\,\text{Hypergeometric2F1}\left(1,\frac{1}{2}(-5+m),\frac{1}{2}(-3+m),-\tan^2(c+dx)\right)(e\tan(c+dx))^{-5+m}}{a^3d(5-m)} - \frac{3e^5\cos^2(c+dx)^{\frac{1}{2}(-4+m)}\,\text{Hypergeometric2F1}\left(\frac{1}{2}(-5+m),\frac{1}{2}(-4+m),\frac{1}{2}(-3+m),\sin^2(c+dx)\right)\sec(c+$$

$$- \frac{e^5\cos^2(c+dx)^{\frac{1}{2}(-2+m)}\,\text{Hypergeometric2F1}\left(\frac{1}{2}(-5+m),\frac{1}{2}(-2+m),\frac{1}{2}(-3+m),\sin^2(c+dx)\right)\sec^3(c+}{a^3d(5-m)}$$

output

```
3*e^5*(e*tan(d*x+c))^(-5+m)/a^3/d/(5-m)+e^5*hypergeom([1, -5/2+1/2*m],[-3/
2+1/2*m],-tan(d*x+c)^2)*(e*tan(d*x+c))^(-5+m)/a^3/d/(5-m)-3*e^5*(cos(d*x+c
)^2)^(-2+1/2*m)*hypergeom([-2+1/2*m, -5/2+1/2*m],[-3/2+1/2*m],sin(d*x+c)^2
)*sec(d*x+c)*(e*tan(d*x+c))^(-5+m)/a^3/d/(5-m)-e^5*(cos(d*x+c)^2)^(-1+1/2*
m)*hypergeom([-1+1/2*m, -5/2+1/2*m],[-3/2+1/2*m],sin(d*x+c)^2)*sec(d*x+c)^
3*(e*tan(d*x+c))^(-5+m)/a^3/d/(5-m)
```

3.214.2 Mathematica [F]

$$\int \frac{(e\tan(c+dx))^m}{(a+a\sec(c+dx))^3}\,dx = \int \frac{(e\tan(c+dx))^m}{(a+a\sec(c+dx))^3}\,dx$$

input
```
Integrate[(e*Tan[c + d*x])^m/(a + a*Sec[c + d*x])^3,x]
```

output
```
Integrate[(e*Tan[c + d*x])^m/(a + a*Sec[c + d*x])^3, x]
```

3.214.3 Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 260, normalized size of antiderivative = 1.03, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3042, 4376, 25, 3042, 4374, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e\tan(c+dx))^m}{(a\sec(c+dx)+a)^3}\,dx$$
$$\downarrow 3042$$
$$\int \frac{\left(-e\cot\left(c+dx+\frac{\pi}{2}\right)\right)^m}{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^3}dx$$
$$\downarrow 4376$$
$$\frac{e^6\int -(a-a\sec(c+dx))^3(e\tan(c+dx))^{m-6}dx}{a^6}$$
$$\downarrow 25$$
$$-\frac{e^6\int (a-a\sec(c+dx))^3(e\tan(c+dx))^{m-6}dx}{a^6}$$
$$\downarrow 3042$$
$$-\frac{e^6\int \left(-e\cot\left(c+dx+\frac{\pi}{2}\right)\right)^{m-6}\left(a-a\csc\left(c+dx+\frac{\pi}{2}\right)\right)^3 dx}{a^6}$$
$$\downarrow 4374$$
$$-\frac{e^6\int \left(a^3(e\tan(c+dx))^{m-6}-a^3\sec^3(c+dx)(e\tan(c+dx))^{m-6}+3a^3\sec^2(c+dx)(e\tan(c+dx))^{m-6}-3a^3\sec\right.}{a^6}$$

$$\downarrow 2009$$

$$-\frac{e^6\left(-\frac{a^3(e\tan(c+dx))^{m-5}\,\text{Hypergeometric2F1}\left(1,\frac{m-5}{2},\frac{m-3}{2},-\tan^2(c+dx)\right)}{de(5-m)}+\frac{a^3\sec^3(c+dx)\cos^2(c+dx)^{\frac{m-2}{2}}(e\tan(c+dx))^{m-5}\,\text{Hypergeor}}{de(5-m)}\right.}{}$$

input
```
Int[(e*Tan[c + d*x])^m/(a + a*Sec[c + d*x])^3,x]
```

output
```
-((e^6*((-3*a^3*(e*Tan[c + d*x])^(-5 + m))/(d*e*(5 - m)) - (a^3*Hypergeome
tric2F1[1, (-5 + m)/2, (-3 + m)/2, -Tan[c + d*x]^2]*(e*Tan[c + d*x])^(-5 +
 m))/(d*e*(5 - m)) + (3*a^3*(Cos[c + d*x]^2)^((-4 + m)/2)*Hypergeometric2F
1[(-5 + m)/2, (-4 + m)/2, (-3 + m)/2, Sin[c + d*x]^2]*Sec[c + d*x]*(e*Tan[
c + d*x])^(-5 + m))/(d*e*(5 - m)) + (a^3*(Cos[c + d*x]^2)^((-2 + m)/2)*Hyp
ergeometric2F1[(-5 + m)/2, (-2 + m)/2, (-3 + m)/2, Sin[c + d*x]^2]*Sec[c +
 d*x]^3*(e*Tan[c + d*x])^(-5 + m))/(d*e*(5 - m))))/a^6)
```

3.214.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4374
```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

rule 4376
```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Simp[a^(2*n)/e^(2*n)   Int[(e*Cot[c + d*x])^(m + 2*
n)/(-a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[a
^2 - b^2, 0] && ILtQ[n, 0]
```

3.214.4 Maple [F]

$$\int \frac{(e\tan(dx+c))^m}{(a+a\sec(dx+c))^3}dx$$

input
```
int((e*tan(d*x+c))^m/(a+a*sec(d*x+c))^3,x)
```

output
```
int((e*tan(d*x+c))^m/(a+a*sec(d*x+c))^3,x)
```

3.214.5 Fricas [F]

$$\int \frac{(e\tan(c+dx))^m}{(a+a\sec(c+dx))^3}\,dx = \int \frac{(e\tan(dx+c))^m}{(a\sec(dx+c)+a)^3}\,dx$$

input
```
integrate((e*tan(d*x+c))^m/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

output
```
integral((e*tan(d*x + c))^m/(a^3*sec(d*x + c)^3 + 3*a^3*sec(d*x + c)^2 + 3
*a^3*sec(d*x + c) + a^3), x)
```

3.214.6 Sympy [F]

$$\int \frac{(e\tan(c+dx))^m}{(a+a\sec(c+dx))^3}\,dx = \frac{\int \frac{(e\tan(c+dx))^m}{\sec^3(c+dx)+3\sec^2(c+dx)+3\sec(c+dx)+1}\,dx}{a^3}$$

input
```
integrate((e*tan(d*x+c))**m/(a+a*sec(d*x+c))**3,x)
```

output
```
Integral((e*tan(c + d*x))**m/(sec(c + d*x)**3 + 3*sec(c + d*x)**2 + 3*sec(
c + d*x) + 1), x)/a**3
```

3.214.7 Maxima [F]

$$\int \frac{(e\tan(c+dx))^m}{(a+a\sec(c+dx))^3}\,dx = \int \frac{(e\tan(dx+c))^m}{(a\sec(dx+c)+a)^3}\,dx$$

input
```
integrate((e*tan(d*x+c))^m/(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

output
```
integrate((e*tan(d*x + c))^m/(a*sec(d*x + c) + a)^3, x)
```

3.214.8 Giac [F]

$$\int \frac{(e\tan(c+dx))^m}{(a+a\sec(c+dx))^3}\,dx = \int \frac{(e\tan(dx+c))^m}{(a\sec(dx+c)+a)^3}\,dx$$

input
```
integrate((e*tan(d*x+c))^m/(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

output
```
integrate((e*tan(d*x + c))^m/(a*sec(d*x + c) + a)^3, x)
```

3.214.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(e\tan(c+dx))^m}{(a+a\sec(c+dx))^3}\,dx = \int \frac{\cos(c+dx)^3\,(e\tan(c+dx))^m}{a^3\,(\cos(c+dx)+1)^3}\,dx$$

input
```
int((e*tan(c + d*x))^m/(a + a/cos(c + d*x))^3,x)
```

output
```
int((cos(c + d*x)^3*(e*tan(c + d*x))^m)/(a^3*(cos(c + d*x) + 1)^3), x)
```

3.215 $\int (a + a\sec(c+dx))^{3/2}(e\tan(c+dx))^m\,dx$

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3.215.2 Mathematica [F] . . . 1594
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3.215.7 Maxima [F] . . . 1597
3.215.8 Giac [F] . . . 1597
3.215.9 Mupad [F(-1)] . . . 1597

3.215.1 Optimal result

Integrand size = 25, antiderivative size = 131

$$\int (a + a\sec(c+dx))^{3/2}(e\tan(c+dx))^m\,dx = \frac{2^{\frac{5}{2}+m}\,\text{AppellF1}\left(\frac{1+m}{2}, \frac{3}{2}+m, 1, \frac{3+m}{2}, -\frac{a-a\sec(c+dx)}{a+a\sec(c+dx)}, \frac{a-a\sec(c+dx)}{a+a\sec(c+dx)}\right)\left(\frac{1}{1+\sec(c+dx)}\right)^{\frac{5}{2}+m}(a+a\sec(c+dx))^{\frac{5}{2}+m}(e\tan(c+dx))^{1+m}}{de(1+m)}$$

output
```
2^(5/2+m)*AppellF1(1/2+1/2*m,3/2+m,1,3/2+1/2*m,(-a+a*sec(d*x+c))/(a+a*sec(
d*x+c)),(a-a*sec(d*x+c))/(a+a*sec(d*x+c)))*(1/(1+sec(d*x+c)))^(5/2+m)*(a+a
*sec(d*x+c))^(3/2)*(e*tan(d*x+c))^(1+m)/d/e/(1+m)
```

3.215.2 Mathematica [F]

$$\int (a + a\sec(c+dx))^{3/2}(e\tan(c+dx))^m\,dx = \int (a + a\sec(c+dx))^{3/2}(e\tan(c+dx))^m\,dx$$

input
```
Integrate[(a + a*Sec[c + d*x])^(3/2)*(e*Tan[c + d*x])^m,x]
```

output
```
Integrate[(a + a*Sec[c + d*x])^(3/2)*(e*Tan[c + d*x])^m, x]
```

3.215.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {3042, 4377}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sec(c+dx)+a)^{3/2}(e\tan(c+dx))^m\,dx$$

$$\downarrow 3042$$

$$\int \left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^{3/2}\left(-e\cot\left(c+dx+\frac{\pi}{2}\right)\right)^m dx$$

$$\downarrow 4377$$

$$\frac{2^{m+\frac{5}{2}}(a\sec(c+dx)+a)^{3/2}\left(\frac{1}{\sec(c+dx)+1}\right)^{m+\frac{5}{2}}(e\tan(c+dx))^{m+1}\,\text{AppellF1}\left(\frac{m+1}{2},m+\frac{3}{2},1,\frac{m+3}{2},-\frac{a-a\sec(c+dx)}{\sec(c+dx)a+a},\frac{a}{a}\right.}{de(m+1)}$$

input
```
Int[(a + a*Sec[c + d*x])^(3/2)*(e*Tan[c + d*x])^m,x]
```

output
```
(2^(5/2 + m)*AppellF1[(1 + m)/2, 3/2 + m, 1, (3 + m)/2, -((a - a*Sec[c + d
*x])/(a + a*Sec[c + d*x])), (a - a*Sec[c + d*x])/(a + a*Sec[c + d*x])]*((1
 + Sec[c + d*x])^(-1))^(5/2 + m)*(a + a*Sec[c + d*x])^(3/2)*(e*Tan[c + d*x
])^(1 + m))/(d*e*(1 + m))
```

3.215.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4377
```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Simp[(-2^(m + n + 1))*(e*Cot[c + d*x])^(m + 1)*((a
 + b*Csc[c + d*x])^n/(d*e*(m + 1)))*(a/(a + b*Csc[c + d*x]))^(m + n + 1)*App
ellF1[(m + 1)/2, m + n, 1, (m + 3)/2, -(a - b*Csc[c + d*x])/(a + b*Csc[c +
d*x]), (a - b*Csc[c + d*x])/(a + b*Csc[c + d*x])], x] /; FreeQ[{a, b, c, d,
 e, m, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[n]
```

3.215.4 Maple [F]

$$\int (a + a \sec (dx + c))^{\frac{3}{2}} (e \tan (dx + c))^m \, dx$$

input
```
int((a+a*sec(d*x+c))^(3/2)*(e*tan(d*x+c))^m,x)
```

output
```
int((a+a*sec(d*x+c))^(3/2)*(e*tan(d*x+c))^m,x)
```

3.215.5 Fricas [F]

$$\int (a + a \sec(c + dx))^{3/2} (e \tan(c + dx))^m \, dx = \int (a \sec (dx + c) + a)^{\frac{3}{2}} (e \tan (dx + c))^m \, dx$$

input
```
integrate((a+a*sec(d*x+c))^(3/2)*(e*tan(d*x+c))^m,x, algorithm="fricas")
```

output
```
integral((a*sec(d*x + c) + a)^(3/2)*(e*tan(d*x + c))^m, x)
```

3.215.6 Sympy [F]

$$\int (a + a \sec(c + dx))^{3/2} (e \tan(c + dx))^m \, dx = \int (a(\sec (c + dx) + 1))^{\frac{3}{2}} (e \tan (c + dx))^m \, dx$$

input
```
integrate((a+a*sec(d*x+c))**(3/2)*(e*tan(d*x+c))**m,x)
```

output
```
Integral((a*(sec(c + d*x) + 1))**(3/2)*(e*tan(c + d*x))**m, x)
```

3.215.7 Maxima [F]

$$\int (a + a\sec(c + dx))^{3/2}(e\tan(c + dx))^m\,dx = \int (a\sec(dx + c) + a)^{\frac{3}{2}}(e\tan(dx + c))^m\,dx$$

input
```
integrate((a+a*sec(d*x+c))^(3/2)*(e*tan(d*x+c))^m,x, algorithm="maxima")
```

output
```
integrate((a*sec(d*x + c) + a)^(3/2)*(e*tan(d*x + c))^m, x)
```

3.215.8 Giac [F]

$$\int (a + a\sec(c + dx))^{3/2}(e\tan(c + dx))^m\,dx = \int (a\sec(dx + c) + a)^{\frac{3}{2}}(e\tan(dx + c))^m\,dx$$

input
```
integrate((a+a*sec(d*x+c))^(3/2)*(e*tan(d*x+c))^m,x, algorithm="giac")
```

output
```
sage0*x
```

3.215.9 Mupad [F(-1)]

Timed out.

$$\int (a + a\sec(c + dx))^{3/2}(e\tan(c + dx))^m\,dx = \int (e\tan(c + d\,x))^m\left(a + \frac{a}{\cos(c + d\,x)}\right)^{3/2}dx$$

input
```
int((e*tan(c + d*x))^m*(a + a/cos(c + d*x))^(3/2),x)
```

output
```
int((e*tan(c + d*x))^m*(a + a/cos(c + d*x))^(3/2), x)
```

3.216 $\int \sqrt{a + a\sec(c+dx)}(e\tan(c+dx))^m\,dx$

3.216.1 Optimal result 1598
3.216.2 Mathematica [F] 1598
3.216.3 Rubi [A] (verified) 1599
3.216.4 Maple [F] 1600
3.216.5 Fricas [F] 1600
3.216.6 Sympy [F] 1600
3.216.7 Maxima [F] 1601
3.216.8 Giac [F] 1601
3.216.9 Mupad [F(-1)] 1601

3.216.1 Optimal result

Integrand size = 25, antiderivative size = 131

$$\int \sqrt{a + a\sec(c+dx)}(e\tan(c+dx))^m\,dx$$
$$= \frac{2^{\frac{3}{2}+m}\,\text{AppellF1}\left(\frac{1+m}{2}, \frac{1}{2}+m, 1, \frac{3+m}{2}, -\frac{a-a\sec(c+dx)}{a+a\sec(c+dx)}, \frac{a-a\sec(c+dx)}{a+a\sec(c+dx)}\right)\left(\frac{1}{1+\sec(c+dx)}\right)^{\frac{3}{2}+m}\sqrt{a+a\sec(c+dx)}(e\,t}{de(1+m)}$$

output
```
2^(3/2+m)*AppellF1(1/2+1/2*m,1/2+m,1,3/2+1/2*m,(-a+a*sec(d*x+c))/(a+a*sec(
d*x+c)),(a-a*sec(d*x+c))/(a+a*sec(d*x+c)))*(1/(1+sec(d*x+c)))^(3/2+m)*(a+a
*sec(d*x+c))^(1/2)*(e*tan(d*x+c))^(1+m)/d/e/(1+m)
```

3.216.2 Mathematica [F]

$$\int \sqrt{a + a\sec(c+dx)}(e\tan(c+dx))^m\,dx = \int \sqrt{a + a\sec(c+dx)}(e\tan(c+dx))^m\,dx$$

input
```
Integrate[Sqrt[a + a*Sec[c + d*x]]*(e*Tan[c + d*x])^m,x]
```

output
```
Integrate[Sqrt[a + a*Sec[c + d*x]]*(e*Tan[c + d*x])^m, x]
```

3.216.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {3042, 4377}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a\sec(c+dx)+a}(e\tan(c+dx))^m\,dx$$

$$\downarrow 3042$$

$$\int \sqrt{a\csc\left(c+dx+\frac{\pi}{2}\right)+a}\left(-e\cot\left(c+dx+\frac{\pi}{2}\right)\right)^m dx$$

$$\downarrow 4377$$

$$\frac{2^{m+\frac{3}{2}}\sqrt{a\sec(c+dx)+a}\left(\frac{1}{\sec(c+dx)+1}\right)^{m+\frac{3}{2}}(e\tan(c+dx))^{m+1}\,\text{AppellF1}\left(\frac{m+1}{2},m+\frac{1}{2},1,\frac{m+3}{2},-\frac{a-a\sec(c+dx)}{\sec(c+dx)a+a},\frac{a-a}{\sec(}\right.}{de(m+1)}$$

input
```
Int[Sqrt[a + a*Sec[c + d*x]]*(e*Tan[c + d*x])^m,x]
```

output
```
(2^(3/2 + m)*AppellF1[(1 + m)/2, 1/2 + m, 1, (3 + m)/2, -((a - a*Sec[c + d
*x])/(a + a*Sec[c + d*x])), (a - a*Sec[c + d*x])/(a + a*Sec[c + d*x])]*((1
 + Sec[c + d*x])^(-1))^(3/2 + m)*Sqrt[a + a*Sec[c + d*x]]*(e*Tan[c + d*x])
^(1 + m))/(d*e*(1 + m))
```

3.216.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4377
```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Simp[(-2^(m + n + 1))*(e*Cot[c + d*x])^(m + 1)*((a
+ b*Csc[c + d*x])^n/(d*e*(m + 1)))*(a/(a + b*Csc[c + d*x]))^(m + n + 1)*App
ellF1[(m + 1)/2, m + n, 1, (m + 3)/2, -(a - b*Csc[c + d*x])/(a + b*Csc[c +
d*x]), (a - b*Csc[c + d*x])/(a + b*Csc[c + d*x])], x] /; FreeQ[{a, b, c, d,
 e, m, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[n]
```

3.216.4 Maple [F]

$$\int \sqrt{a + a \sec(dx + c)} \, (e \tan(dx + c))^m \, dx$$

input
```
int((a+a*sec(d*x+c))^(1/2)*(e*tan(d*x+c))^m,x)
```

output
```
int((a+a*sec(d*x+c))^(1/2)*(e*tan(d*x+c))^m,x)
```

3.216.5 Fricas [F]

$$\int \sqrt{a + a \sec(c + dx)} (e \tan(c + dx))^m \, dx = \int \sqrt{a \sec(dx + c) + a} (e \tan(dx + c))^m \, dx$$

input
```
integrate((a+a*sec(d*x+c))^(1/2)*(e*tan(d*x+c))^m,x, algorithm="fricas")
```

output
```
integral(sqrt(a*sec(d*x + c) + a)*(e*tan(d*x + c))^m, x)
```

3.216.6 Sympy [F]

$$\int \sqrt{a + a \sec(c + dx)} (e \tan(c + dx))^m \, dx = \int \sqrt{a (\sec(c + dx) + 1)} (e \tan(c + dx))^m \, dx$$

input
```
integrate((a+a*sec(d*x+c))**(1/2)*(e*tan(d*x+c))**m,x)
```

output
```
Integral(sqrt(a*(sec(c + d*x) + 1))*(e*tan(c + d*x))**m, x)
```

3.216.7 Maxima [F]

$$\int \sqrt{a + a\sec(c + dx)}(e\tan(c + dx))^m\,dx = \int \sqrt{a\sec(dx + c) + a}(e\tan(dx + c))^m\,dx$$

input
```
integrate((a+a*sec(d*x+c))^(1/2)*(e*tan(d*x+c))^m,x, algorithm="maxima")
```

output
```
integrate(sqrt(a*sec(d*x + c) + a)*(e*tan(d*x + c))^m, x)
```

3.216.8 Giac [F]

$$\int \sqrt{a + a\sec(c + dx)}(e\tan(c + dx))^m\,dx = \int \sqrt{a\sec(dx + c) + a}(e\tan(dx + c))^m\,dx$$

input
```
integrate((a+a*sec(d*x+c))^(1/2)*(e*tan(d*x+c))^m,x, algorithm="giac")
```

output
```
sage0*x
```

3.216.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{a + a\sec(c + dx)}(e\tan(c + dx))^m\,dx = \int (e\tan(c + d\,x))^m \sqrt{a + \frac{a}{\cos(c + d\,x)}}\,dx$$

input
```
int((e*tan(c + d*x))^m*(a + a/cos(c + d*x))^(1/2),x)
```

output
```
int((e*tan(c + d*x))^m*(a + a/cos(c + d*x))^(1/2), x)
```

3.217 $\int \frac{(e\tan(c+dx))^m}{\sqrt{a+a\sec(c+dx)}}\,dx$

3.217.1 Optimal result 1602
3.217.2 Mathematica [F] 1602
3.217.3 Rubi [A] (verified) 1603
3.217.4 Maple [F] 1604
3.217.5 Fricas [F] 1604
3.217.6 Sympy [F] 1605
3.217.7 Maxima [F] 1605
3.217.8 Giac [F] 1605
3.217.9 Mupad [F(-1)] 1606

3.217.1 Optimal result

Integrand size = 25, antiderivative size = 131

$$\int \frac{(e\tan(c+dx))^m}{\sqrt{a+a\sec(c+dx)}}\,dx = \frac{2^{\frac{1}{2}+m}\,\text{AppellF1}\left(\frac{1+m}{2},-\frac{1}{2}+m,1,\frac{3+m}{2},-\frac{a-a\sec(c+dx)}{a+a\sec(c+dx)},\frac{a-a\sec(c+dx)}{a+a\sec(c+dx)}\right)\left(\frac{1}{1+\sec(c+dx)}\right)^{\frac{1}{2}+m}(e\tan(c+dx))^{1+m}}{de(1+m)\sqrt{a+a\sec(c+dx)}}$$

output
```
2^(1/2+m)*AppellF1(1/2+1/2*m,-1/2+m,1,3/2+1/2*m,(-a+a*sec(d*x+c))/(a+a*sec
(d*x+c)),(a-a*sec(d*x+c))/(a+a*sec(d*x+c)))*(1/(1+sec(d*x+c)))^(1/2+m)*(e*
tan(d*x+c))^(1+m)/d/e/(1+m)/(a+a*sec(d*x+c))^(1/2)
```

3.217.2 Mathematica [F]

$$\int \frac{(e\tan(c+dx))^m}{\sqrt{a+a\sec(c+dx)}}\,dx = \int \frac{(e\tan(c+dx))^m}{\sqrt{a+a\sec(c+dx)}}\,dx$$

input
```
Integrate[(e*Tan[c + d*x])^m/Sqrt[a + a*Sec[c + d*x]],x]
```

output
```
Integrate[(e*Tan[c + d*x])^m/Sqrt[a + a*Sec[c + d*x]], x]
```

3.217.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {3042, 4377}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e\tan(c+dx))^m}{\sqrt{a\sec(c+dx)+a}}\,dx$$

$$\downarrow 3042$$

$$\int \frac{\left(-e\cot\left(c+dx+\frac{\pi}{2}\right)\right)^m}{\sqrt{a\csc\left(c+dx+\frac{\pi}{2}\right)+a}}dx$$

$$\downarrow 4377$$

$$\frac{2^{m+\frac{1}{2}}\left(\frac{1}{\sec(c+dx)+1}\right)^{m+\frac{1}{2}}(e\tan(c+dx))^{m+1}\,\text{AppellF1}\left(\frac{m+1}{2},m-\frac{1}{2},1,\frac{m+3}{2},-\frac{a-a\sec(c+dx)}{\sec(c+dx)a+a},\frac{a-a\sec(c+dx)}{\sec(c+dx)a+a}\right)}{de(m+1)\sqrt{a\sec(c+dx)+a}}$$

input
```
Int[(e*Tan[c + d*x])^m/Sqrt[a + a*Sec[c + d*x]],x]
```

output
```
(2^(1/2 + m)*AppellF1[(1 + m)/2, -1/2 + m, 1, (3 + m)/2, -((a - a*Sec[c +
d*x])/(a + a*Sec[c + d*x])), (a - a*Sec[c + d*x])/(a + a*Sec[c + d*x])]*((
1 + Sec[c + d*x])^(-1))^(1/2 + m)*(e*Tan[c + d*x])^(1 + m))/(d*e*(1 + m)*S
qrt[a + a*Sec[c + d*x]])
```

3.217.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4377
```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Simp[(-2^(m + n + 1))*(e*Cot[c + d*x])^(m + 1)*((a
+ b*Csc[c + d*x])^n/(d*e*(m + 1)))*(a/(a + b*Csc[c + d*x]))^(m + n + 1)*App
ellF1[(m + 1)/2, m + n, 1, (m + 3)/2, -(a - b*Csc[c + d*x])/(a + b*Csc[c +
d*x]), (a - b*Csc[c + d*x])/(a + b*Csc[c + d*x])], x] /; FreeQ[{a, b, c, d,
 e, m, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[n]
```

3.217.4 Maple [F]

$$\int \frac{(e\tan(dx+c))^m}{\sqrt{a+a\sec(dx+c)}}dx$$

input
```
int((e*tan(d*x+c))^m/(a+a*sec(d*x+c))^(1/2),x)
```

output
```
int((e*tan(d*x+c))^m/(a+a*sec(d*x+c))^(1/2),x)
```

3.217.5 Fricas [F]

$$\int \frac{(e\tan(c+dx))^m}{\sqrt{a+a\sec(c+dx)}}\,dx = \int \frac{(e\tan(dx+c))^m}{\sqrt{a\sec(dx+c)+a}}\,dx$$

input
```
integrate((e*tan(d*x+c))^m/(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
integral((e*tan(d*x + c))^m/sqrt(a*sec(d*x + c) + a), x)
```

3.217.6 Sympy [F]

$$\int \frac{(e\tan(c+dx))^m}{\sqrt{a+a\sec(c+dx)}}\,dx = \int \frac{(e\tan(c+dx))^m}{\sqrt{a\left(\sec(c+dx)+1\right)}}\,dx$$

input
```
integrate((e*tan(d*x+c))**m/(a+a*sec(d*x+c))**(1/2),x)
```

output
```
Integral((e*tan(c + d*x))**m/sqrt(a*(sec(c + d*x) + 1)), x)
```

3.217.7 Maxima [F]

$$\int \frac{(e\tan(c+dx))^m}{\sqrt{a+a\sec(c+dx)}}\,dx = \int \frac{(e\tan(dx+c))^m}{\sqrt{a\sec(dx+c)+a}}\,dx$$

input
```
integrate((e*tan(d*x+c))^m/(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate((e*tan(d*x + c))^m/sqrt(a*sec(d*x + c) + a), x)
```

3.217.8 Giac [F]

$$\int \frac{(e\tan(c+dx))^m}{\sqrt{a+a\sec(c+dx)}}\,dx = \int \frac{(e\tan(dx+c))^m}{\sqrt{a\sec(dx+c)+a}}\,dx$$

input
```
integrate((e*tan(d*x+c))^m/(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate((e*tan(d*x + c))^m/sqrt(a*sec(d*x + c) + a), x)
```

3.217.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(e\tan(c+dx))^m}{\sqrt{a+a\sec(c+dx)}}\,dx = \int \frac{(e\tan(c+d\,x))^m}{\sqrt{a+\frac{a}{\cos(c+d\,x)}}}\,dx$$

input
```
int((e*tan(c + d*x))^m/(a + a/cos(c + d*x))^(1/2),x)
```

output
```
int((e*tan(c + d*x))^m/(a + a/cos(c + d*x))^(1/2), x)
```

3.218 $\int \frac{(e\tan(c+dx))^m}{(a+a\sec(c+dx))^{3/2}}\,dx$

3.218.1 Optimal result . . . 1607
3.218.2 Mathematica [F] . . . 1607
3.218.3 Rubi [A] (verified) . . . 1608
3.218.4 Maple [F] . . . 1609
3.218.5 Fricas [F] . . . 1609
3.218.6 Sympy [F] . . . 1609
3.218.7 Maxima [F] . . . 1610
3.218.8 Giac [F] . . . 1610
3.218.9 Mupad [F(-1)] . . . 1610

3.218.1 Optimal result

Integrand size = 25, antiderivative size = 131

$$\int \frac{(e\tan(c+dx))^m}{(a+a\sec(c+dx))^{3/2}}\,dx = \frac{2^{-\frac{1}{2}+m}\,\text{AppellF1}\left(\frac{1+m}{2},-\frac{3}{2}+m,1,\frac{3+m}{2},-\frac{a-a\sec(c+dx)}{a+a\sec(c+dx)},\frac{a-a\sec(c+dx)}{a+a\sec(c+dx)}\right)\left(\frac{1}{1+\sec(c+}\right.}{de(1+m)(a+a\sec(c+dx))^{3/2}}$$

output
```
2^(-1/2+m)*AppellF1(1/2+1/2*m,-3/2+m,1,3/2+1/2*m,(-a+a*sec(d*x+c))/(a+a*se
c(d*x+c)),(a-a*sec(d*x+c))/(a+a*sec(d*x+c)))*(1/(1+sec(d*x+c)))^(-1/2+m)*(
e*tan(d*x+c))^(1+m)/d/e/(1+m)/(a+a*sec(d*x+c))^(3/2)
```

3.218.2 Mathematica [F]

$$\int \frac{(e\tan(c+dx))^m}{(a+a\sec(c+dx))^{3/2}}\,dx = \int \frac{(e\tan(c+dx))^m}{(a+a\sec(c+dx))^{3/2}}\,dx$$

input
```
Integrate[(e*Tan[c + d*x])^m/(a + a*Sec[c + d*x])^(3/2),x]
```

output
```
Integrate[(e*Tan[c + d*x])^m/(a + a*Sec[c + d*x])^(3/2), x]
```

3.218.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {3042, 4377}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e\tan(c+dx))^m}{(a\sec(c+dx)+a)^{3/2}}\,dx$$

$$\downarrow 3042$$

$$\int \frac{\left(-e\cot\left(c+dx+\frac{\pi}{2}\right)\right)^m}{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^{3/2}}dx$$

$$\downarrow 4377$$

$$\frac{2^{m-\frac{1}{2}}\left(\frac{1}{\sec(c+dx)+1}\right)^{m-\frac{1}{2}}(e\tan(c+dx))^{m+1}\,\text{AppellF1}\left(\frac{m+1}{2},m-\frac{3}{2},1,\frac{m+3}{2},-\frac{a-a\sec(c+dx)}{\sec(c+dx)a+a},\frac{a-a\sec(c+dx)}{\sec(c+dx)a+a}\right)}{de(m+1)(a\sec(c+dx)+a)^{3/2}}$$

input
```
Int[(e*Tan[c + d*x])^m/(a + a*Sec[c + d*x])^(3/2),x]
```

output
```
(2^(-1/2 + m)*AppellF1[(1 + m)/2, -3/2 + m, 1, (3 + m)/2, -((a - a*Sec[c +
 d*x])/(a + a*Sec[c + d*x])), (a - a*Sec[c + d*x])/(a + a*Sec[c + d*x])]*(
(1 + Sec[c + d*x])^(-1))^(-1/2 + m)*(e*Tan[c + d*x])^(1 + m))/(d*e*(1 + m)
*(a + a*Sec[c + d*x])^(3/2))
```

3.218.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4377
```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Simp[(-2^(m + n + 1))*(e*Cot[c + d*x])^(m + 1)*((a
+ b*Csc[c + d*x])^n/(d*e*(m + 1)))*(a/(a + b*Csc[c + d*x]))^(m + n + 1)*App
ellF1[(m + 1)/2, m + n, 1, (m + 3)/2, -(a - b*Csc[c + d*x])/(a + b*Csc[c +
d*x]), (a - b*Csc[c + d*x])/(a + b*Csc[c + d*x])], x] /; FreeQ[{a, b, c, d,
 e, m, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[n]
```

3.218.4 Maple [F]

$$\int \frac{(e\tan(dx+c))^m}{(a+a\sec(dx+c))^{\frac{3}{2}}}dx$$

input
```
int((e*tan(d*x+c))^m/(a+a*sec(d*x+c))^(3/2),x)
```

output
```
int((e*tan(d*x+c))^m/(a+a*sec(d*x+c))^(3/2),x)
```

3.218.5 Fricas [F]

$$\int \frac{(e\tan(c+dx))^m}{(a+a\sec(c+dx))^{3/2}}\,dx = \int \frac{(e\tan(dx+c))^m}{(a\sec(dx+c)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*tan(d*x+c))^m/(a+a*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(a*sec(d*x + c) + a)*(e*tan(d*x + c))^m/(a^2*sec(d*x + c)^2 +
 2*a^2*sec(d*x + c) + a^2), x)
```

3.218.6 Sympy [F]

$$\int \frac{(e\tan(c+dx))^m}{(a+a\sec(c+dx))^{3/2}}\,dx = \int \frac{(e\tan(c+dx))^m}{(a(\sec(c+dx)+1))^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*tan(d*x+c))**m/(a+a*sec(d*x+c))**(3/2),x)
```

output
```
Integral((e*tan(c + d*x))**m/(a*(sec(c + d*x) + 1))**(3/2), x)
```

3.218.7 Maxima [F]

$$\int \frac{(e\tan(c+dx))^m}{(a+a\sec(c+dx))^{3/2}}\,dx = \int \frac{(e\tan(dx+c))^m}{(a\sec(dx+c)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*tan(d*x+c))^m/(a+a*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate((e*tan(d*x + c))^m/(a*sec(d*x + c) + a)^(3/2), x)
```

3.218.8 Giac [F]

$$\int \frac{(e\tan(c+dx))^m}{(a+a\sec(c+dx))^{3/2}}\,dx = \int \frac{(e\tan(dx+c))^m}{(a\sec(dx+c)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*tan(d*x+c))^m/(a+a*sec(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
integrate((e*tan(d*x + c))^m/(a*sec(d*x + c) + a)^(3/2), x)
```

3.218.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(e\tan(c+dx))^m}{(a+a\sec(c+dx))^{3/2}}\,dx = \int \frac{(e\tan(c+d\,x))^m}{\left(a+\frac{a}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

input
```
int((e*tan(c + d*x))^m/(a + a/cos(c + d*x))^(3/2),x)
```

output
```
int((e*tan(c + d*x))^m/(a + a/cos(c + d*x))^(3/2), x)
```

3.219 $\int (a + a\sec(c+dx))^n \tan^7(c+dx)\,dx$

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3.219.2 Mathematica [A] (verified) 1611
3.219.3 Rubi [A] (verified) 1612
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3.219.8 Giac [F] 1615
3.219.9 Mupad [F(-1)] 1615

3.219.1 Optimal result

Integrand size = 21, antiderivative size = 123

$$\begin{aligned}
&\int (a + a\sec(c+dx))^n \tan^7(c+dx)\,dx \\
&= \frac{7(a + a\sec(c+dx))^{4+n}}{a^4 d(4+n)} \\
&\quad + \frac{\text{Hypergeometric2F1}(1, 4+n, 5+n, 1+\sec(c+dx))(a + a\sec(c+dx))^{4+n}}{a^4 d(4+n)} \\
&\quad - \frac{5(a + a\sec(c+dx))^{5+n}}{a^5 d(5+n)} + \frac{(a + a\sec(c+dx))^{6+n}}{a^6 d(6+n)}
\end{aligned}$$

output
```
7*(a+a*sec(d*x+c))^(4+n)/a^4/d/(4+n)+hypergeom([1, 4+n],[5+n],1+sec(d*x+c)
)*(a+a*sec(d*x+c))^(4+n)/a^4/d/(4+n)-5*(a+a*sec(d*x+c))^(5+n)/a^5/d/(5+n)+
(a+a*sec(d*x+c))^(6+n)/a^6/d/(6+n)
```

3.219.2 Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.71

$$\begin{aligned}
&\int (a + a\sec(c+dx))^n \tan^7(c+dx)\,dx \\
&= \frac{(1+\sec(c+dx))^4 (a(1+\sec(c+dx)))^n \left(\frac{7}{4+n} + \frac{\text{Hypergeometric2F1}(1,4+n,5+n,1+\sec(c+dx))}{4+n} - \frac{5(1+\sec(c+dx))}{5+n} + \frac{(1+\sec(c+dx))^2}{6+n}\right)}{d}
\end{aligned}$$

input
```
Integrate[(a + a*Sec[c + d*x])^n*Tan[c + d*x]^7,x]
```

output
```
((1 + Sec[c + d*x])^4*(a*(1 + Sec[c + d*x]))^n*(7/(4 + n) + Hypergeometric
2F1[1, 4 + n, 5 + n, 1 + Sec[c + d*x]]/(4 + n) - (5*(1 + Sec[c + d*x]))/(5
 + n) + (1 + Sec[c + d*x])^2/(6 + n)))/d
```

3.219.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.98, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 25, 4368, 25, 27, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^7(c+dx)(a\sec(c+dx)+a)^n\,dx$$

$$\downarrow 3042$$

$$\int -\cot\left(c+dx+\frac{\pi}{2}\right)^7\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^n dx$$

$$\downarrow 25$$

$$-\int \cot\left(\frac{1}{2}(2c+\pi)+dx\right)^7\left(\csc\left(\frac{1}{2}(2c+\pi)+dx\right)a+a\right)^n dx$$

$$\downarrow 4368$$

$$\frac{\int -a^3\cos(c+dx)(1-\sec(c+dx))^3(\sec(c+dx)a+a)^{n+3}d\sec(c+dx)}{a^6d}$$

$$\downarrow 25$$

$$-\frac{\int a^3\cos(c+dx)(1-\sec(c+dx))^3(\sec(c+dx)a+a)^{n+3}d\sec(c+dx)}{a^6d}$$

$$\downarrow 27$$

$$-\frac{\int \cos(c+dx)(1-\sec(c+dx))^3(\sec(c+dx)a+a)^{n+3}d\sec(c+dx)}{a^3d}$$

$$\downarrow 99$$

$$-\frac{\int\left(\cos(c+dx)(\sec(c+dx)a+a)^{n+3}-7(\sec(c+dx)a+a)^{n+3}+\frac{5(\sec(c+dx)a+a)^{n+4}}{a}-\frac{(\sec(c+dx)a+a)^{n+5}}{a^2}\right)d\sec(c+$$

$$a^3d$$

$$\downarrow 2009$$

$$-\frac{-\frac{(a\sec(c+dx)+a)^{n+6}}{a^3(n+6)}+\frac{5(a\sec(c+dx)+a)^{n+5}}{a^2(n+5)}-\frac{(a\sec(c+dx)+a)^{n+4}\,\text{Hypergeometric2F1}(1,n+4,n+5,\sec(c+dx)+1)}{a(n+4)}-\frac{7(a\sec(c+dx)+a}{a(n+4)}}{a^3d}$$

input
```
Int[(a + a*Sec[c + d*x])^n*Tan[c + d*x]^7,x]
```

output
```
-(((-7*(a + a*Sec[c + d*x])^(4 + n))/(a*(4 + n)) - (Hypergeometric2F1[1, 4
 + n, 5 + n, 1 + Sec[c + d*x]]*(a + a*Sec[c + d*x])^(4 + n))/(a*(4 + n)) +
 (5*(a + a*Sec[c + d*x])^(5 + n))/(a^2*(5 + n)) - (a + a*Sec[c + d*x])^(6
+ n)/(a^3*(6 + n)))/(a^3*d))
```

3.219.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4368
```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Simp[-(d*b^(m - 1))^(-1)   Subst[Int[(-a + b*x)^((m - 1)/2
)*((a + b*x)^((m - 1)/2 + n)/x), x], x, Csc[c + d*x]], x] /; FreeQ[{a, b, c
, d, n}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[n]
```

3.219.4 Maple [F]

$$\int (a + a \sec (dx + c))^n \tan (dx + c)^7 \, dx$$

input
```
int((a+a*sec(d*x+c))^n*tan(d*x+c)^7,x)
```

output
```
int((a+a*sec(d*x+c))^n*tan(d*x+c)^7,x)
```

3.219.5 Fricas [F]

$$\int (a + a \sec(c + dx))^n \tan^7(c + dx) \, dx = \int (a \sec (dx + c) + a)^n \tan (dx + c)^7 \, dx$$

input
```
integrate((a+a*sec(d*x+c))^n*tan(d*x+c)^7,x, algorithm="fricas")
```

output
```
integral((a*sec(d*x + c) + a)^n*tan(d*x + c)^7, x)
```

3.219.6 Sympy [F]

$$\int (a + a \sec(c + dx))^n \tan^7(c + dx) \, dx = \int (a(\sec (c + dx) + 1))^n \tan^7 (c + dx) \, dx$$

input
```
integrate((a+a*sec(d*x+c))**n*tan(d*x+c)**7,x)
```

output
```
Integral((a*(sec(c + d*x) + 1))**n*tan(c + d*x)**7, x)
```

3.219.7 Maxima [F]

$$\int (a + a\sec(c + dx))^n \tan^7(c + dx)\,dx = \int (a\sec(dx + c) + a)^n \tan(dx + c)^7\,dx$$

input
```
integrate((a+a*sec(d*x+c))^n*tan(d*x+c)^7,x, algorithm="maxima")
```

output
```
integrate((a*sec(d*x + c) + a)^n*tan(d*x + c)^7, x)
```

3.219.8 Giac [F]

$$\int (a + a\sec(c + dx))^n \tan^7(c + dx)\,dx = \int (a\sec(dx + c) + a)^n \tan(dx + c)^7\,dx$$

input
```
integrate((a+a*sec(d*x+c))^n*tan(d*x+c)^7,x, algorithm="giac")
```

output
```
integrate((a*sec(d*x + c) + a)^n*tan(d*x + c)^7, x)
```

3.219.9 Mupad [F(-1)]

Timed out.

$$\int (a + a\sec(c + dx))^n \tan^7(c + dx)\,dx = \int \tan(c + d\,x)^7 \left(a + \frac{a}{\cos(c + d\,x)}\right)^n dx$$

input
```
int(tan(c + d*x)^7*(a + a/cos(c + d*x))^n,x)
```

output
```
int(tan(c + d*x)^7*(a + a/cos(c + d*x))^n, x)
```

3.220 $\int (a + a\sec(c+dx))^n \tan^5(c+dx)\,dx$

3.220.1 Optimal result 1616
3.220.2 Mathematica [A] (verified) 1616
3.220.3 Rubi [A] (verified) 1617
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3.220.7 Maxima [F] 1620
3.220.8 Giac [F] 1620
3.220.9 Mupad [F(-1)] 1620

3.220.1 Optimal result

Integrand size = 21, antiderivative size = 97

$$\begin{aligned}
&\int (a + a\sec(c+dx))^n \tan^5(c+dx)\,dx \\
&= -\frac{3(a + a\sec(c+dx))^{3+n}}{a^3 d(3+n)} \\
&\quad - \frac{\text{Hypergeometric2F1}(1, 3+n, 4+n, 1+\sec(c+dx))(a + a\sec(c+dx))^{3+n}}{a^3 d(3+n)} \\
&\quad + \frac{(a + a\sec(c+dx))^{4+n}}{a^4 d(4+n)}
\end{aligned}$$

output
```
-3*(a+a*sec(d*x+c))^(3+n)/a^3/d/(3+n)-hypergeom([1, 3+n],[4+n],1+sec(d*x+c
))*(a+a*sec(d*x+c))^(3+n)/a^3/d/(3+n)+(a+a*sec(d*x+c))^(4+n)/a^4/d/(4+n)
```

3.220.2 Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.74

$$\begin{aligned}
&\int (a + a\sec(c+dx))^n \tan^5(c+dx)\,dx \\
&= \frac{(1+\sec(c+dx))^3 (a(1+\sec(c+dx)))^n (-9-2n-(4+n)\,\text{Hypergeometric2F1}(1, 3+n, 4+n, 1+\sec(c
\end{aligned}$$

input
```
Integrate[(a + a*Sec[c + d*x])^n*Tan[c + d*x]^5,x]
```

output
```
((1 + Sec[c + d*x])^3*(a*(1 + Sec[c + d*x]))^n*(-9 - 2*n - (4 + n)*Hyperge
ometric2F1[1, 3 + n, 4 + n, 1 + Sec[c + d*x]] + (3 + n)*Sec[c + d*x]))/(d*
(3 + n)*(4 + n))
```

3.220.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.98, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 25, 4368, 27, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^5(c+dx)(a\sec(c+dx)+a)^n\,dx$$

$$\downarrow 3042$$

$$\int -\cot\left(c+dx+\frac{\pi}{2}\right)^5\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^n dx$$

$$\downarrow 25$$

$$-\int \cot\left(\frac{1}{2}(2c+\pi)+dx\right)^5\left(\csc\left(\frac{1}{2}(2c+\pi)+dx\right)a+a\right)^n dx$$

$$\downarrow 4368$$

$$\frac{\int a^2\cos(c+dx)(1-\sec(c+dx))^2(\sec(c+dx)a+a)^{n+2}d\sec(c+dx)}{a^4d}$$

$$\downarrow 27$$

$$\frac{\int \cos(c+dx)(1-\sec(c+dx))^2(\sec(c+dx)a+a)^{n+2}d\sec(c+dx)}{a^2d}$$

$$\downarrow 99$$

$$\frac{\int\left(\cos(c+dx)(\sec(c+dx)a+a)^{n+2}-3(\sec(c+dx)a+a)^{n+2}+\frac{(\sec(c+dx)a+a)^{n+3}}{a}\right)d\sec(c+dx)}{a^2d}$$

$$\downarrow 2009$$

$$\frac{\frac{(a\sec(c+dx)+a)^{n+4}}{a^2(n+4)}-\frac{(a\sec(c+dx)+a)^{n+3}\,\text{Hypergeometric2F1}(1,n+3,n+4,\sec(c+dx)+1)}{a(n+3)}-\frac{3(a\sec(c+dx)+a)^{n+3}}{a(n+3)}}{a^2d}$$

input
```
Int[(a + a*Sec[c + d*x])^n*Tan[c + d*x]^5,x]
```

output
```
((-3*(a + a*Sec[c + d*x])^(3 + n))/(a*(3 + n)) - (Hypergeometric2F1[1, 3 +
 n, 4 + n, 1 + Sec[c + d*x]]*(a + a*Sec[c + d*x])^(3 + n))/(a*(3 + n)) + (
a + a*Sec[c + d*x])^(4 + n)/(a^2*(4 + n)))/(a^2*d)
```

3.220.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4368
```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Simp[-(d*b^(m - 1))^(-1)   Subst[Int[(-a + b*x)^((m - 1)/2
)*((a + b*x)^((m - 1)/2 + n)/x), x], x, Csc[c + d*x]], x] /; FreeQ[{a, b, c
, d, n}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[n]
```

3.220.4 Maple [F]

$$\int (a + a \sec (dx + c))^n \tan (dx + c)^5 \, dx$$

input
```
int((a+a*sec(d*x+c))^n*tan(d*x+c)^5,x)
```

output
```
int((a+a*sec(d*x+c))^n*tan(d*x+c)^5,x)
```

3.220.5 Fricas [F]

$$\int (a + a \sec(c + dx))^n \tan^5(c + dx) \, dx = \int (a \sec (dx + c) + a)^n \tan (dx + c)^5 \, dx$$

input
```
integrate((a+a*sec(d*x+c))^n*tan(d*x+c)^5,x, algorithm="fricas")
```

output
```
integral((a*sec(d*x + c) + a)^n*tan(d*x + c)^5, x)
```

3.220.6 Sympy [F]

$$\int (a + a \sec(c + dx))^n \tan^5(c + dx) \, dx = \int (a(\sec (c + dx) + 1))^n \tan^5 (c + dx) \, dx$$

input
```
integrate((a+a*sec(d*x+c))**n*tan(d*x+c)**5,x)
```

output
```
Integral((a*(sec(c + d*x) + 1))**n*tan(c + d*x)**5, x)
```

3.220.7 Maxima [F]

$$\int (a + a\sec(c + dx))^n \tan^5(c + dx)\,dx = \int (a\sec(dx + c) + a)^n \tan(dx + c)^5\,dx$$

input
```
integrate((a+a*sec(d*x+c))^n*tan(d*x+c)^5,x, algorithm="maxima")
```

output
```
integrate((a*sec(d*x + c) + a)^n*tan(d*x + c)^5, x)
```

3.220.8 Giac [F]

$$\int (a + a\sec(c + dx))^n \tan^5(c + dx)\,dx = \int (a\sec(dx + c) + a)^n \tan(dx + c)^5\,dx$$

input
```
integrate((a+a*sec(d*x+c))^n*tan(d*x+c)^5,x, algorithm="giac")
```

output
```
integrate((a*sec(d*x + c) + a)^n*tan(d*x + c)^5, x)
```

3.220.9 Mupad [F(-1)]

Timed out.

$$\int (a + a\sec(c + dx))^n \tan^5(c + dx)\,dx = \int \tan(c + d\,x)^5 \left(a + \frac{a}{\cos(c + d\,x)}\right)^n dx$$

input
```
int(tan(c + d*x)^5*(a + a/cos(c + d*x))^n,x)
```

output
```
int(tan(c + d*x)^5*(a + a/cos(c + d*x))^n, x)
```

3.221 $\int (a + a\sec(c+dx))^n \tan^3(c+dx)\,dx$

3.221.1 Optimal result 1621
3.221.2 Mathematica [A] (verified) 1621
3.221.3 Rubi [A] (verified) 1622
3.221.4 Maple [F] 1624
3.221.5 Fricas [F] 1624
3.221.6 Sympy [F] 1624
3.221.7 Maxima [F] 1625
3.221.8 Giac [F] 1625
3.221.9 Mupad [F(-1)] 1625

3.221.1 Optimal result

Integrand size = 21, antiderivative size = 69

$$\int (a + a\sec(c+dx))^n \tan^3(c+dx)\,dx$$
$$= \frac{(a + a\sec(c+dx))^{2+n}}{a^2 d(2+n)} + \frac{\text{Hypergeometric2F1}(1, 2+n, 3+n, 1+\sec(c+dx))(a + a\sec(c+dx))^{2+n}}{a^2 d(2+n)}$$

output
```
(a+a*sec(d*x+c))^(2+n)/a^2/d/(2+n)+hypergeom([1, 2+n],[3+n],1+sec(d*x+c))*
(a+a*sec(d*x+c))^(2+n)/a^2/d/(2+n)
```

3.221.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.71

$$\int (a + a\sec(c+dx))^n \tan^3(c+dx)\,dx$$
$$= \frac{(1 + \text{Hypergeometric2F1}(1, 2+n, 3+n, 1+\sec(c+dx)))(1+\sec(c+dx))^2 (a(1+\sec(c+dx)))^n}{d(2+n)}$$

input
```
Integrate[(a + a*Sec[c + d*x])^n*Tan[c + d*x]^3,x]
```

output
```
((1 + Hypergeometric2F1[1, 2 + n, 3 + n, 1 + Sec[c + d*x]])*(1 + Sec[c + d
*x])^2*(a*(1 + Sec[c + d*x]))^n)/(d*(2 + n))
```

3.221.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.06, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 25, 4368, 25, 27, 90, 75}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^3(c+dx)(a\sec(c+dx)+a)^n\,dx$$

$$\downarrow 3042$$

$$\int -\cot\left(c+dx+\frac{\pi}{2}\right)^3\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^n dx$$

$$\downarrow 25$$

$$-\int \cot\left(\frac{1}{2}(2c+\pi)+dx\right)^3\left(\csc\left(\frac{1}{2}(2c+\pi)+dx\right)a+a\right)^n dx$$

$$\downarrow 4368$$

$$\frac{\int -a\cos(c+dx)(1-\sec(c+dx))(\sec(c+dx)a+a)^{n+1}d\sec(c+dx)}{a^2d}$$

$$\downarrow 25$$

$$-\frac{\int a\cos(c+dx)(1-\sec(c+dx))(\sec(c+dx)a+a)^{n+1}d\sec(c+dx)}{a^2d}$$

$$\downarrow 27$$

$$-\frac{\int \cos(c+dx)(1-\sec(c+dx))(\sec(c+dx)a+a)^{n+1}d\sec(c+dx)}{ad}$$

$$\downarrow 90$$

$$-\frac{\int \cos(c+dx)(\sec(c+dx)a+a)^{n+1}d\sec(c+dx)-\frac{(a\sec(c+dx)+a)^{n+2}}{a(n+2)}}{ad}$$

$$\downarrow 75$$

$$-\frac{-\frac{(a\sec(c+dx)+a)^{n+2}\,\text{Hypergeometric2F1}(1,n+2,n+3,\sec(c+dx)+1)}{a(n+2)}-\frac{(a\sec(c+dx)+a)^{n+2}}{a(n+2)}}{ad}$$

input
```
Int[(a + a*Sec[c + d*x])^n*Tan[c + d*x]^3,x]
```

output
```
-((-((a + a*Sec[c + d*x])^(2 + n)/(a*(2 + n))) - (Hypergeometric2F1[1, 2 +
 n, 3 + n, 1 + Sec[c + d*x]]*(a + a*Sec[c + d*x])^(2 + n))/(a*(2 + n)))/(a
*d))
```

3.221.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 75
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
 + 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4368
```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Simp[-(d*b^(m - 1))^(-1)   Subst[Int[(-a + b*x)^((m - 1)/2
)*((a + b*x)^((m - 1)/2 + n)/x), x], x, Csc[c + d*x]], x] /; FreeQ[{a, b, c
, d, n}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[n]
```

3.221.4 Maple [F]

$$\int (a + a \sec (dx + c))^n \tan (dx + c)^3 \, dx$$

input
```
int((a+a*sec(d*x+c))^n*tan(d*x+c)^3,x)
```

output
```
int((a+a*sec(d*x+c))^n*tan(d*x+c)^3,x)
```

3.221.5 Fricas [F]

$$\int (a + a \sec(c + dx))^n \tan^3(c + dx) \, dx = \int (a \sec (dx + c) + a)^n \tan (dx + c)^3 \, dx$$

input
```
integrate((a+a*sec(d*x+c))^n*tan(d*x+c)^3,x, algorithm="fricas")
```

output
```
integral((a*sec(d*x + c) + a)^n*tan(d*x + c)^3, x)
```

3.221.6 Sympy [F]

$$\int (a + a \sec(c + dx))^n \tan^3(c + dx) \, dx = \int (a(\sec (c + dx) + 1))^n \tan^3 (c + dx) \, dx$$

input
```
integrate((a+a*sec(d*x+c))**n*tan(d*x+c)**3,x)
```

output
```
Integral((a*(sec(c + d*x) + 1))**n*tan(c + d*x)**3, x)
```

3.221.7 Maxima [F]

$$\int (a + a\sec(c + dx))^n \tan^3(c + dx)\,dx = \int (a\sec(dx + c) + a)^n \tan(dx + c)^3\,dx$$

input
```
integrate((a+a*sec(d*x+c))^n*tan(d*x+c)^3,x, algorithm="maxima")
```

output
```
integrate((a*sec(d*x + c) + a)^n*tan(d*x + c)^3, x)
```

3.221.8 Giac [F]

$$\int (a + a\sec(c + dx))^n \tan^3(c + dx)\,dx = \int (a\sec(dx + c) + a)^n \tan(dx + c)^3\,dx$$

input
```
integrate((a+a*sec(d*x+c))^n*tan(d*x+c)^3,x, algorithm="giac")
```

output
```
integrate((a*sec(d*x + c) + a)^n*tan(d*x + c)^3, x)
```

3.221.9 Mupad [F(-1)]

Timed out.

$$\int (a + a\sec(c + dx))^n \tan^3(c + dx)\,dx = \int \tan(c + d\,x)^3 \left(a + \frac{a}{\cos(c + d\,x)}\right)^n dx$$

input
```
int(tan(c + d*x)^3*(a + a/cos(c + d*x))^n,x)
```

output
```
int(tan(c + d*x)^3*(a + a/cos(c + d*x))^n, x)
```

3.222 $\int (a + a\sec(c+dx))^n \tan(c+dx)\,dx$

3.222.1 Optimal result . . . 1626
3.222.2 Mathematica [A] (verified) . . . 1626
3.222.3 Rubi [A] (verified) . . . 1627
3.222.4 Maple [F] . . . 1628
3.222.5 Fricas [F] . . . 1628
3.222.6 Sympy [F] . . . 1628
3.222.7 Maxima [F] . . . 1629
3.222.8 Giac [F] . . . 1629
3.222.9 Mupad [F(-1)] . . . 1629

3.222.1 Optimal result

Integrand size = 19, antiderivative size = 43

$$\int (a + a\sec(c+dx))^n \tan(c+dx)\,dx = -\frac{\text{Hypergeometric2F1}(1, 1+n, 2+n, 1+\sec(c+dx))(a + a\sec(c+dx))^{1+n}}{ad(1+n)}$$

output
```
-hypergeom([1, 1+n],[2+n],1+sec(d*x+c))*(a+a*sec(d*x+c))^(1+n)/a/d/(1+n)
```

3.222.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00

$$\int (a + a\sec(c+dx))^n \tan(c+dx)\,dx = -\frac{\text{Hypergeometric2F1}(1, 1+n, 2+n, 1+\sec(c+dx))(a(1+\sec(c+dx)))^{1+n}}{ad(1+n)}$$

input
```
Integrate[(a + a*Sec[c + d*x])^n*Tan[c + d*x],x]
```

output
```
-((Hypergeometric2F1[1, 1 + n, 2 + n, 1 + Sec[c + d*x]]*(a*(1 + Sec[c + d*
x]))^(1 + n))/(a*d*(1 + n)))
```

3.222.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {3042, 25, 4368, 75}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan(c+dx)(a\sec(c+dx)+a)^n\,dx$$

$$\downarrow 3042$$

$$\int -\cot\left(c+dx+\frac{\pi}{2}\right)\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^n dx$$

$$\downarrow 25$$

$$-\int \cot\left(\frac{1}{2}(2c+\pi)+dx\right)\left(\csc\left(\frac{1}{2}(2c+\pi)+dx\right)a+a\right)^n dx$$

$$\downarrow 4368$$

$$\frac{\int \cos(c+dx)(\sec(c+dx)a+a)^n d\sec(c+dx)}{d}$$

$$\downarrow 75$$

$$-\frac{(a\sec(c+dx)+a)^{n+1}\,\text{Hypergeometric2F1}(1,n+1,n+2,\sec(c+dx)+1)}{ad(n+1)}$$

input
```
Int[(a + a*Sec[c + d*x])^n*Tan[c + d*x],x]
```

output
```
-((Hypergeometric2F1[1, 1 + n, 2 + n, 1 + Sec[c + d*x]]*(a + a*Sec[c + d*x
])^(1 + n))/(a*d*(1 + n)))
```

3.222.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 75
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4368
```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Simp[-(d*b^(m - 1))^(-1)   Subst[Int[(-a + b*x)^((m - 1)/2
)*((a + b*x)^((m - 1)/2 + n)/x), x], x, Csc[c + d*x]], x] /; FreeQ[{a, b, c
, d, n}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[n]
```

3.222.4 Maple [F]

$$\int (a + a \sec (dx + c))^n \tan (dx + c) \, dx$$

input
```
int((a+a*sec(d*x+c))^n*tan(d*x+c),x)
```

output
```
int((a+a*sec(d*x+c))^n*tan(d*x+c),x)
```

3.222.5 Fricas [F]

$$\int (a + a \sec(c + dx))^n \tan(c + dx) \, dx = \int (a \sec (dx + c) + a)^n \tan (dx + c) \, dx$$

input
```
integrate((a+a*sec(d*x+c))^n*tan(d*x+c),x, algorithm="fricas")
```

output
```
integral((a*sec(d*x + c) + a)^n*tan(d*x + c), x)
```

3.222.6 Sympy [F]

$$\int (a + a \sec(c + dx))^n \tan(c + dx) \, dx = \int (a(\sec (c + dx) + 1))^n \tan (c + dx) \, dx$$

input
```
integrate((a+a*sec(d*x+c))**n*tan(d*x+c),x)
```

output
```
Integral((a*(sec(c + d*x) + 1))**n*tan(c + d*x), x)
```

3.222.7 Maxima [F]

$$\int (a + a\sec(c + dx))^n \tan(c + dx)\,dx = \int (a\sec(dx + c) + a)^n \tan(dx + c)\,dx$$

input
```
integrate((a+a*sec(d*x+c))^n*tan(d*x+c),x, algorithm="maxima")
```

output
```
integrate((a*sec(d*x + c) + a)^n*tan(d*x + c), x)
```

3.222.8 Giac [F]

$$\int (a + a\sec(c + dx))^n \tan(c + dx)\,dx = \int (a\sec(dx + c) + a)^n \tan(dx + c)\,dx$$

input
```
integrate((a+a*sec(d*x+c))^n*tan(d*x+c),x, algorithm="giac")
```

output
```
integrate((a*sec(d*x + c) + a)^n*tan(d*x + c), x)
```

3.222.9 Mupad [F(-1)]

Timed out.

$$\int (a + a\sec(c + dx))^n \tan(c + dx)\,dx = \int \tan(c + d\,x)\left(a + \frac{a}{\cos(c + d\,x)}\right)^n dx$$

input
```
int(tan(c + d*x)*(a + a/cos(c + d*x))^n,x)
```

output
```
int(tan(c + d*x)*(a + a/cos(c + d*x))^n, x)
```

3.223 $\int \cot(c+dx)(a+a\sec(c+dx))^n\,dx$

3.223.1 Optimal result 1630
3.223.2 Mathematica [A] (verified) 1630
3.223.3 Rubi [A] (verified) 1631
3.223.4 Maple [F] 1633
3.223.5 Fricas [F] 1633
3.223.6 Sympy [F] 1633
3.223.7 Maxima [F] 1634
3.223.8 Giac [F] 1634
3.223.9 Mupad [F(-1)] 1634

3.223.1 Optimal result

Integrand size = 19, antiderivative size = 74

$$\int \cot(c+dx)(a+a\sec(c+dx))^n\,dx = -\frac{\text{Hypergeometric2F1}\left(1,n,1+n,\frac{1}{2}(1+\sec(c+dx))\right)(a+a\sec(c+dx))^n}{2dn} + \frac{\text{Hypergeometric2F1}(1,n,1+n,1+\sec(c+dx))(a+a\sec(c+dx))^n}{dn}$$

output
```
-1/2*hypergeom([1, n],[1+n],1/2+1/2*sec(d*x+c))*(a+a*sec(d*x+c))^n/d/n+hyp
ergeom([1, n],[1+n],1+sec(d*x+c))*(a+a*sec(d*x+c))^n/d/n
```

3.223.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.77

$$\int \cot(c+dx)(a+a\sec(c+dx))^n\,dx = -\frac{\left(\text{Hypergeometric2F1}\left(1,n,1+n,\frac{1}{2}(1+\sec(c+dx))\right) - 2\,\text{Hypergeometric2F1}(1,n,1+n,1+\sec(c+\right.}{2dn}$$

input
```
Integrate[Cot[c + d*x]*(a + a*Sec[c + d*x])^n,x]
```

output
```
-1/2*((Hypergeometric2F1[1, n, 1 + n, (1 + Sec[c + d*x])/2] - 2*Hypergeome
tric2F1[1, n, 1 + n, 1 + Sec[c + d*x]])*(a*(1 + Sec[c + d*x]))^n)/(d*n)
```

3.223.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.09, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$, Rules used = {3042, 25, 4368, 25, 27, 97, 75, 78}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot(c+dx)(a\sec(c+dx)+a)^n\,dx$$

$$\downarrow 3042$$

$$\int -\frac{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^n}{\cot\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\downarrow 25$$

$$-\int \frac{\left(\csc\left(\frac{1}{2}(2c+\pi)+dx\right)a+a\right)^n}{\cot\left(\frac{1}{2}(2c+\pi)+dx\right)}dx$$

$$\downarrow 4368$$

$$\frac{a^2\int -\frac{\cos(c+dx)(\sec(c+dx)a+a)^{n-1}}{a(1-\sec(c+dx))}d\sec(c+dx)}{d}$$

$$\downarrow 25$$

$$-\frac{a^2\int \frac{\cos(c+dx)(\sec(c+dx)a+a)^{n-1}}{a(1-\sec(c+dx))}d\sec(c+dx)}{d}$$

$$\downarrow 27$$

$$-\frac{a\int \frac{\cos(c+dx)(\sec(c+dx)a+a)^{n-1}}{1-\sec(c+dx)}d\sec(c+dx)}{d}$$

$$\downarrow 97$$

$$-\frac{a\left(\int \frac{(\sec(c+dx)a+a)^{n-1}}{1-\sec(c+dx)}d\sec(c+dx)+\int \cos(c+dx)(\sec(c+dx)a+a)^{n-1}d\sec(c+dx)\right)}{d}$$

$$\downarrow 75$$

$$-\frac{a\left(\int \frac{(\sec(c+dx)a+a)^{n-1}}{1-\sec(c+dx)}d\sec(c+dx)-\frac{(a\sec(c+dx)+a)^n\,\text{Hypergeometric2F1}(1,n,n+1,\sec(c+dx)+1)}{an}\right)}{d}$$

$$\downarrow 78$$

$$-\frac{a\left(\frac{(a\sec(c+dx)+a)^n\,\text{Hypergeometric2F1}\left(1,n,n+1,\frac{1}{2}(\sec(c+dx)+1)\right)}{2an}-\frac{(a\sec(c+dx)+a)^n\,\text{Hypergeometric2F1}(1,n,n+1,\sec(c+dx)+1)}{an}\right)}{d}$$

input
```
Int[Cot[c + d*x]*(a + a*Sec[c + d*x])^n,x]
```

output
```
-((a*((Hypergeometric2F1[1, n, 1 + n, (1 + Sec[c + d*x])/2]*(a + a*Sec[c +
 d*x])^n)/(2*a*n) - (Hypergeometric2F1[1, n, 1 + n, 1 + Sec[c + d*x]]*(a +
 a*Sec[c + d*x])^n)/(a*n)))/d)
```

3.223.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 75
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

rule 78
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 &&  !IntegerQ[m] && IntegerQ[n]
```

rule 97
```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_] :> Simp[b/(b*c - a*d)   Int[(e + f*x)^p/(a + b*x), x], x] - Simp[d/(b*c
 - a*d)   Int[(e + f*x)^p/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, p},
 x] &&  !IntegerQ[p]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4368
```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Simp[-(d*b^(m - 1))^(-1)   Subst[Int[(-a + b*x)^((m - 1)/2
)*((a + b*x)^((m - 1)/2 + n)/x), x], x, Csc[c + d*x]], x] /; FreeQ[{a, b, c
, d, n}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[n]
```

3.223.4 Maple [F]

$$\int \cot(dx+c)\,(a+a\sec(dx+c))^n\,dx$$

input
```
int(cot(d*x+c)*(a+a*sec(d*x+c))^n,x)
```

output
```
int(cot(d*x+c)*(a+a*sec(d*x+c))^n,x)
```

3.223.5 Fricas [F]

$$\int \cot(c+dx)(a+a\sec(c+dx))^n\,dx = \int (a\sec(dx+c)+a)^n\cot(dx+c)\,dx$$

input
```
integrate(cot(d*x+c)*(a+a*sec(d*x+c))^n,x, algorithm="fricas")
```

output
```
integral((a*sec(d*x + c) + a)^n*cot(d*x + c), x)
```

3.223.6 Sympy [F]

$$\int \cot(c+dx)(a+a\sec(c+dx))^n\,dx = \int (a(\sec(c+dx)+1))^n\cot(c+dx)\,dx$$

input
```
integrate(cot(d*x+c)*(a+a*sec(d*x+c))**n,x)
```

output
```
Integral((a*(sec(c + d*x) + 1))**n*cot(c + d*x), x)
```

3.223.7 Maxima [F]

$$\int \cot(c+dx)(a+a\sec(c+dx))^n\,dx = \int (a\sec(dx+c)+a)^n\cot(dx+c)\,dx$$

input
```
integrate(cot(d*x+c)*(a+a*sec(d*x+c))^n,x, algorithm="maxima")
```

output
```
integrate((a*sec(d*x + c) + a)^n*cot(d*x + c), x)
```

3.223.8 Giac [F]

$$\int \cot(c+dx)(a+a\sec(c+dx))^n\,dx = \int (a\sec(dx+c)+a)^n\cot(dx+c)\,dx$$

input
```
integrate(cot(d*x+c)*(a+a*sec(d*x+c))^n,x, algorithm="giac")
```

output
```
integrate((a*sec(d*x + c) + a)^n*cot(d*x + c), x)
```

3.223.9 Mupad [F(-1)]

Timed out.

$$\int \cot(c+dx)(a+a\sec(c+dx))^n\,dx = \int \cot(c+d\,x)\left(a+\frac{a}{\cos(c+d\,x)}\right)^n dx$$

input
```
int(cot(c + d*x)*(a + a/cos(c + d*x))^n,x)
```

output
```
int(cot(c + d*x)*(a + a/cos(c + d*x))^n, x)
```

3.224 $\int \cot^3(c+dx)(a+a\sec(c+dx))^n\,dx$

3.224.1 Optimal result . . . 1635
3.224.2 Mathematica [A] (verified) . . . 1635
3.224.3 Rubi [A] (verified) . . . 1636
3.224.4 Maple [F] . . . 1639
3.224.5 Fricas [F] . . . 1639
3.224.6 Sympy [F] . . . 1639
3.224.7 Maxima [F] . . . 1640
3.224.8 Giac [F] . . . 1640
3.224.9 Mupad [F(-1)] . . . 1640

3.224.1 Optimal result

Integrand size = 21, antiderivative size = 127

$$\int \cot^3(c+dx)(a+a\sec(c+dx))^n\,dx =$$
$$-\frac{a(4-n)\,\text{Hypergeometric2F1}\left(1,-1+n,n,\frac{1}{2}(1+\sec(c+dx))\right)(a+a\sec(c+dx))^{-1+n}}{4d(1-n)}$$
$$+\frac{a\,\text{Hypergeometric2F1}(1,-1+n,n,1+\sec(c+dx))(a+a\sec(c+dx))^{-1+n}}{d(1-n)}$$
$$+\frac{a(a+a\sec(c+dx))^{-1+n}}{2d(1-\sec(c+dx))}$$

output
```
-1/4*a*(4-n)*hypergeom([1, -1+n],[n],1/2+1/2*sec(d*x+c))*(a+a*sec(d*x+c))^
(-1+n)/d/(1-n)+a*hypergeom([1, -1+n],[n],1+sec(d*x+c))*(a+a*sec(d*x+c))^(-
1+n)/d/(1-n)+1/2*a*(a+a*sec(d*x+c))^(-1+n)/d/(1-sec(d*x+c))
```

3.224.2 Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.76

$$\int \cot^3(c+dx)(a+a\sec(c+dx))^n\,dx =$$
$$-\frac{a\left(-2+2n+(-4+n)\,\text{Hypergeometric2F1}\left(1,-1+n,n,\frac{1}{2}(1+\sec(c+dx))\right)(-1+\sec(c+dx))+4\right.}{4d(-1+n)(-1+$$

input
```
Integrate[Cot[c + d*x]^3*(a + a*Sec[c + d*x])^n,x]
```

output
```
-1/4*(a*(-2 + 2*n + (-4 + n)*Hypergeometric2F1[1, -1 + n, n, (1 + Sec[c +
d*x])/2]*(-1 + Sec[c + d*x]) + 4*Hypergeometric2F1[1, -1 + n, n, 1 + Sec[c
 + d*x]]*(-1 + Sec[c + d*x]))*(a*(1 + Sec[c + d*x]))^(-1 + n))/(d*(-1 + n)
*(-1 + Sec[c + d*x]))
```

3.224.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.08, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$, Rules used = {3042, 25, 4368, 27, 114, 25, 27, 174, 75, 78}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^3(c+dx)(a\sec(c+dx)+a)^n\,dx$$

$$\downarrow 3042$$

$$\int -\frac{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^n}{\cot\left(c+dx+\frac{\pi}{2}\right)^3}dx$$

$$\downarrow 25$$

$$-\int \frac{\left(\csc\left(\frac{1}{2}(2c+\pi)+dx\right)a+a\right)^n}{\cot\left(\frac{1}{2}(2c+\pi)+dx\right)^3}dx$$

$$\downarrow 4368$$

$$\frac{a^4\int \frac{\cos(c+dx)(\sec(c+dx)a+a)^{n-2}}{a^2(1-\sec(c+dx))^2}d\sec(c+dx)}{d}$$

$$\downarrow 27$$

$$\frac{a^2\int \frac{\cos(c+dx)(\sec(c+dx)a+a)^{n-2}}{(1-\sec(c+dx))^2}d\sec(c+dx)}{d}$$

$$\downarrow 114$$

$$\frac{a^2\left(\frac{(a\sec(c+dx)+a)^{n-1}}{2a(1-\sec(c+dx))}-\frac{\int -\frac{a\cos(c+dx)(\sec(c+dx)a+a)^{n-2}((2-n)\sec(c+dx)+2)}{1-\sec(c+dx)}d\sec(c+dx)}{2a}\right)}{d}$$

$$\downarrow 25$$

$$\frac{a^2\left(\frac{\int \frac{a\cos(c+dx)(\sec(c+dx)a+a)^{n-2}((2-n)\sec(c+dx)+2)}{1-\sec(c+dx)}d\sec(c+dx)}{2a}+\frac{(a\sec(c+dx)+a)^{n-1}}{2a(1-\sec(c+dx))}\right)}{d}$$

$$\downarrow 27$$

$$\frac{a^2\left(\frac{1}{2}\int \frac{\cos(c+dx)(\sec(c+dx)a+a)^{n-2}((2-n)\sec(c+dx)+2)}{1-\sec(c+dx)}d\sec(c+dx)+\frac{(a\sec(c+dx)+a)^{n-1}}{2a(1-\sec(c+dx))}\right)}{d}$$

$$\downarrow 174$$

$$\frac{a^2\left(\frac{1}{2}\left((4-n)\int \frac{(\sec(c+dx)a+a)^{n-2}}{1-\sec(c+dx)}d\sec(c+dx)+2\int \cos(c+dx)(\sec(c+dx)a+a)^{n-2}d\sec(c+dx)\right)+\frac{(a\sec(c+dx)-}{2a(1-\sec(c-}\right.}{d}$$

$$\downarrow 75$$

$$\frac{a^2\left(\frac{1}{2}\left((4-n)\int \frac{(\sec(c+dx)a+a)^{n-2}}{1-\sec(c+dx)}d\sec(c+dx)+\frac{2(a\sec(c+dx)+a)^{n-1}\,\text{Hypergeometric2F1}(1,n-1,n,\sec(c+dx)+1)}{a(1-n)}\right)+\frac{(a\sec(c+d}{2a(1-sec}\right.}{d}$$

$$\downarrow 78$$

$$\frac{a^2\left(\frac{1}{2}\left(\frac{2(a\sec(c+dx)+a)^{n-1}\,\text{Hypergeometric2F1}(1,n-1,n,\sec(c+dx)+1)}{a(1-n)}-\frac{(4-n)(a\sec(c+dx)+a)^{n-1}\,\text{Hypergeometric2F1}\left(1,n-1,n,\frac{1}{2}(\sec(c+\right.\right.\right.}{2a(1-n)}}{d}$$

input
```
Int[Cot[c + d*x]^3*(a + a*Sec[c + d*x])^n,x]
```

output
```
(a^2*((a + a*Sec[c + d*x])^(-1 + n)/(2*a*(1 - Sec[c + d*x])) + (-1/2*((4 -
 n)*Hypergeometric2F1[1, -1 + n, n, (1 + Sec[c + d*x])/2]*(a + a*Sec[c + d
*x])^(-1 + n))/(a*(1 - n)) + (2*Hypergeometric2F1[1, -1 + n, n, 1 + Sec[c
+ d*x]]*(a + a*Sec[c + d*x])^(-1 + n))/(a*(1 - n)))/2))/d
```

3.224.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 75
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

rule 78
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 &&  !IntegerQ[m] && IntegerQ[n]
```

rule 114
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[1/((m + 1)*(b*c - a*d)*(b*e
 - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1)
 - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (IntegerQ[n] ||
 IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4368
```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Simp[-(d*b^(m - 1))^(-1)   Subst[Int[(-a + b*x)^((m - 1)/2
)*((a + b*x)^((m - 1)/2 + n)/x), x], x, Csc[c + d*x]], x] /; FreeQ[{a, b, c
, d, n}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[n]
```

3.224.4 Maple [F]

$$\int \cot(dx+c)^3 (a+a\sec(dx+c))^n \, dx$$

input
```
int(cot(d*x+c)^3*(a+a*sec(d*x+c))^n,x)
```

output
```
int(cot(d*x+c)^3*(a+a*sec(d*x+c))^n,x)
```

3.224.5 Fricas [F]

$$\int \cot^3(c+dx)(a+a\sec(c+dx))^n \, dx = \int (a\sec(dx+c)+a)^n \cot(dx+c)^3 \, dx$$

input
```
integrate(cot(d*x+c)^3*(a+a*sec(d*x+c))^n,x, algorithm="fricas")
```

output
```
integral((a*sec(d*x + c) + a)^n*cot(d*x + c)^3, x)
```

3.224.6 Sympy [F]

$$\int \cot^3(c+dx)(a+a\sec(c+dx))^n \, dx = \int (a(\sec(c+dx)+1))^n \cot^3(c+dx) \, dx$$

input
```
integrate(cot(d*x+c)**3*(a+a*sec(d*x+c))**n,x)
```

output
```
Integral((a*(sec(c + d*x) + 1))**n*cot(c + d*x)**3, x)
```

3.224.7 Maxima [F]

$$\int \cot^3(c+dx)(a+a\sec(c+dx))^n\,dx = \int (a\sec(dx+c)+a)^n \cot(dx+c)^3\,dx$$

input
```
integrate(cot(d*x+c)^3*(a+a*sec(d*x+c))^n,x, algorithm="maxima")
```

output
```
integrate((a*sec(d*x + c) + a)^n*cot(d*x + c)^3, x)
```

3.224.8 Giac [F]

$$\int \cot^3(c+dx)(a+a\sec(c+dx))^n\,dx = \int (a\sec(dx+c)+a)^n \cot(dx+c)^3\,dx$$

input
```
integrate(cot(d*x+c)^3*(a+a*sec(d*x+c))^n,x, algorithm="giac")
```

output
```
integrate((a*sec(d*x + c) + a)^n*cot(d*x + c)^3, x)
```

3.224.9 Mupad [F(-1)]

Timed out.

$$\int \cot^3(c+dx)(a+a\sec(c+dx))^n\,dx = \int \cot(c+d\,x)^3 \left(a+\frac{a}{\cos(c+d\,x)}\right)^n dx$$

input
```
int(cot(c + d*x)^3*(a + a/cos(c + d*x))^n,x)
```

output
```
int(cot(c + d*x)^3*(a + a/cos(c + d*x))^n, x)
```

3.225 $\int (a + a\sec(c+dx))^n \tan^4(c+dx)\,dx$

3.225.1 Optimal result 1641
3.225.2 Mathematica [F] 1641
3.225.3 Rubi [A] (verified) 1642
3.225.4 Maple [F] 1643
3.225.5 Fricas [F] 1643
3.225.6 Sympy [F] 1643
3.225.7 Maxima [F] 1644
3.225.8 Giac [F] 1644
3.225.9 Mupad [F(-1)] 1644

3.225.1 Optimal result

Integrand size = 21, antiderivative size = 106

$$\int (a + a\sec(c+dx))^n \tan^4(c+dx)\,dx$$
$$= \frac{2^{5+n}\,\text{AppellF1}\left(\frac{5}{2}, 4+n, 1, \frac{7}{2}, -\frac{a-a\sec(c+dx)}{a+a\sec(c+dx)}, \frac{a-a\sec(c+dx)}{a+a\sec(c+dx)}\right)\left(\frac{1}{1+\sec(c+dx)}\right)^{5+n}(a+a\sec(c+dx))^n \tan^5(c+d}{5d}$$

output
```
1/5*2^(5+n)*AppellF1(5/2,4+n,1,7/2,(-a+a*sec(d*x+c))/(a+a*sec(d*x+c)),(a-a
*sec(d*x+c))/(a+a*sec(d*x+c)))*(1/(1+sec(d*x+c)))^(5+n)*(a+a*sec(d*x+c))^n
*tan(d*x+c)^5/d
```

3.225.2 Mathematica [F]

$$\int (a + a\sec(c+dx))^n \tan^4(c+dx)\,dx = \int (a + a\sec(c+dx))^n \tan^4(c+dx)\,dx$$

input
```
Integrate[(a + a*Sec[c + d*x])^n*Tan[c + d*x]^4,x]
```

output
```
Integrate[(a + a*Sec[c + d*x])^n*Tan[c + d*x]^4, x]
```

3.225.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {3042, 4377}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^4(c+dx)(a\sec(c+dx)+a)^n\,dx$$

$$\downarrow 3042$$

$$\int \cot\left(c+dx+\frac{\pi}{2}\right)^4\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^n dx$$

$$\downarrow 4377$$

$$\frac{2^{n+5}\tan^5(c+dx)\left(\frac{1}{\sec(c+dx)+1}\right)^{n+5}(a\sec(c+dx)+a)^n\,\text{AppellF1}\left(\frac{5}{2},n+4,1,\frac{7}{2},-\frac{a-a\sec(c+dx)}{\sec(c+dx)a+a},\frac{a-a\sec(c+dx)}{\sec(c+dx)a+a}\right)}{5d}$$

input
```
Int[(a + a*Sec[c + d*x])^n*Tan[c + d*x]^4,x]
```

output
```
(2^(5 + n)*AppellF1[5/2, 4 + n, 1, 7/2, -((a - a*Sec[c + d*x])/(a + a*Sec[
c + d*x])), (a - a*Sec[c + d*x])/(a + a*Sec[c + d*x])]*((1 + Sec[c + d*x])
^(-1))^(5 + n)*(a + a*Sec[c + d*x])^n*Tan[c + d*x]^5)/(5*d)
```

3.225.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4377
```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Simp[(-2^(m + n + 1))*(e*Cot[c + d*x])^(m + 1)*((a
+ b*Csc[c + d*x])^n/(d*e*(m + 1)))*(a/(a + b*Csc[c + d*x]))^(m + n + 1)*App
ellF1[(m + 1)/2, m + n, 1, (m + 3)/2, -(a - b*Csc[c + d*x])/(a + b*Csc[c +
d*x]), (a - b*Csc[c + d*x])/(a + b*Csc[c + d*x])], x] /; FreeQ[{a, b, c, d,
 e, m, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[n]
```

3.225.4 Maple [F]

$$\int (a + a \sec (dx + c))^n \tan (dx + c)^4 \, dx$$

input
```
int((a+a*sec(d*x+c))^n*tan(d*x+c)^4,x)
```

output
```
int((a+a*sec(d*x+c))^n*tan(d*x+c)^4,x)
```

3.225.5 Fricas [F]

$$\int (a + a \sec(c + dx))^n \tan^4(c + dx) \, dx = \int (a \sec (dx + c) + a)^n \tan (dx + c)^4 \, dx$$

input
```
integrate((a+a*sec(d*x+c))^n*tan(d*x+c)^4,x, algorithm="fricas")
```

output
```
integral((a*sec(d*x + c) + a)^n*tan(d*x + c)^4, x)
```

3.225.6 Sympy [F]

$$\int (a + a \sec(c + dx))^n \tan^4(c + dx) \, dx = \int (a(\sec (c + dx) + 1))^n \tan^4 (c + dx) \, dx$$

input
```
integrate((a+a*sec(d*x+c))**n*tan(d*x+c)**4,x)
```

output
```
Integral((a*(sec(c + d*x) + 1))**n*tan(c + d*x)**4, x)
```

3.225.7 Maxima [F]

$$\int (a + a\sec(c + dx))^n \tan^4(c + dx)\,dx = \int (a\sec(dx + c) + a)^n \tan(dx + c)^4\,dx$$

input
```
integrate((a+a*sec(d*x+c))^n*tan(d*x+c)^4,x, algorithm="maxima")
```

output
```
integrate((a*sec(d*x + c) + a)^n*tan(d*x + c)^4, x)
```

3.225.8 Giac [F]

$$\int (a + a\sec(c + dx))^n \tan^4(c + dx)\,dx = \int (a\sec(dx + c) + a)^n \tan(dx + c)^4\,dx$$

input
```
integrate((a+a*sec(d*x+c))^n*tan(d*x+c)^4,x, algorithm="giac")
```

output
```
integrate((a*sec(d*x + c) + a)^n*tan(d*x + c)^4, x)
```

3.225.9 Mupad [F(-1)]

Timed out.

$$\int (a + a\sec(c + dx))^n \tan^4(c + dx)\,dx = \int \tan(c + d\,x)^4 \left(a + \frac{a}{\cos(c + d\,x)}\right)^n dx$$

input
```
int(tan(c + d*x)^4*(a + a/cos(c + d*x))^n,x)
```

output
```
int(tan(c + d*x)^4*(a + a/cos(c + d*x))^n, x)
```

3.226 $\int (a + a\sec(c+dx))^n \tan^2(c+dx)\,dx$

3.226.1 Optimal result . . . 1645
3.226.2 Mathematica [B] (warning: unable to verify) . . . 1645
3.226.3 Rubi [A] (verified) . . . 1646
3.226.4 Maple [F] . . . 1647
3.226.5 Fricas [F] . . . 1647
3.226.6 Sympy [F] . . . 1648
3.226.7 Maxima [F] . . . 1648
3.226.8 Giac [F] . . . 1648
3.226.9 Mupad [F(-1)] . . . 1649

3.226.1 Optimal result

Integrand size = 21, antiderivative size = 106

$$\int (a + a\sec(c+dx))^n \tan^2(c+dx)\,dx$$
$$= \frac{2^{3+n}\,\text{AppellF1}\left(\frac{3}{2}, 2+n, 1, \frac{5}{2}, -\frac{a-a\sec(c+dx)}{a+a\sec(c+dx)}, \frac{a-a\sec(c+dx)}{a+a\sec(c+dx)}\right)\left(\frac{1}{1+\sec(c+dx)}\right)^{3+n}(a+a\sec(c+dx))^n \tan^3(c+d}{3d}$$

output
```
1/3*2^(3+n)*AppellF1(3/2,2+n,1,5/2,(-a+a*sec(d*x+c))/(a+a*sec(d*x+c)),(a-a
*sec(d*x+c))/(a+a*sec(d*x+c)))*(1/(1+sec(d*x+c)))^(3+n)*(a+a*sec(d*x+c))^n
*tan(d*x+c)^3/d
```

3.226.2 Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 910 vs. 2(106) = 212.

Time = 6.33 (sec) , antiderivative size = 910, normalized size of antiderivative = 8.58

$$\int (a + a\sec(c+dx))^n \tan^2(c+dx)\,dx = \text{Too large to display}$$

input
```
Integrate[(a + a*Sec[c + d*x])^n*Tan[c + d*x]^2,x]
```

output

```
((a*(1 + Sec[c + d*x]))^n*((-4*Hypergeometric2F1[-1 - n, n, -n, (1 - Tan[(
c + d*x)/2])/2]*(Cos[(c + d*x)/2]^2*Sec[c + d*x])^n*(1 + Tan[(c + d*x)/2])
^n)/((1 + n)*(1 + Sec[c + d*x])^n*(-1 + Tan[(c + d*x)/2])) - (Hypergeometr
ic2F1[1 - n, 2 + n, 2 - n, (1 - Tan[(c + d*x)/2])/2]*(Cos[(c + d*x)/2]^2*S
ec[c + d*x])^n*(-1 + Tan[(c + d*x)/2])*(1 + Tan[(c + d*x)/2])^n)/((-1 + n)
*(1 + Sec[c + d*x])^n) - (120*AppellF1[1/2, n, 1, 3/2, Tan[(c + d*x)/2]^2,
 -Tan[(c + d*x)/2]^2]*Cos[(c + d*x)/2]^2*Cos[c + d*x]*Sin[c + d*x]*(3*Appe
llF1[1/2, n, 1, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] - 2*(AppellF
1[3/2, n, 2, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] - n*AppellF1[3/
2, 1 + n, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*Tan[(c + d*x)/
2]^2))/(45*AppellF1[1/2, n, 1, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^
2]^2*Cos[(c + d*x)/2]^2*(1 + 2*n - 2*n*Cos[c + d*x] + Cos[2*(c + d*x)]) +
6*AppellF1[1/2, n, 1, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sin[(c
 + d*x)/2]^2*(-5*AppellF1[3/2, n, 2, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*
x)/2]^2]*(1 + 2*n - 2*(2 + n)*Cos[c + d*x] + Cos[2*(c + d*x)]) + 5*n*Appel
lF1[3/2, 1 + n, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*(1 + 2*n
- 2*(2 + n)*Cos[c + d*x] + Cos[2*(c + d*x)]) - 48*(2*AppellF1[5/2, n, 3, 7
/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] - 2*n*AppellF1[5/2, 1 + n, 2,
 7/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + n*(1 + n)*AppellF1[5/2, 2
 + n, 1, 7/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*Cot[c + d*x]*Cs...
```

3.226.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {3042, 4377}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^2(c+dx)(a\sec(c+dx)+a)^n\,dx$$

$$\downarrow 3042$$

$$\int \cot\left(c+dx+\frac{\pi}{2}\right)^2\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^n dx$$

$$\downarrow 4377$$

$$\frac{2^{n+3}\tan^3(c+dx)\left(\frac{1}{\sec(c+dx)+1}\right)^{n+3}(a\sec(c+dx)+a)^n\,\text{AppellF1}\left(\frac{3}{2},n+2,1,\frac{5}{2},-\frac{a-a\sec(c+dx)}{\sec(c+dx)a+a},\frac{a-a\sec(c+dx)}{\sec(c+dx)a+a}\right)}{3d}$$

input
```
Int[(a + a*Sec[c + d*x])^n*Tan[c + d*x]^2,x]
```

output
```
(2^(3 + n)*AppellF1[3/2, 2 + n, 1, 5/2, -((a - a*Sec[c + d*x])/(a + a*Sec[
c + d*x])), (a - a*Sec[c + d*x])/(a + a*Sec[c + d*x])]*((1 + Sec[c + d*x])
^(-1))^(3 + n)*(a + a*Sec[c + d*x])^n*Tan[c + d*x]^3)/(3*d)
```

3.226.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4377
```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Simp[(-2^(m + n + 1))*(e*Cot[c + d*x])^(m + 1)*((a
+ b*Csc[c + d*x])^n/(d*e*(m + 1)))*(a/(a + b*Csc[c + d*x]))^(m + n + 1)*App
ellF1[(m + 1)/2, m + n, 1, (m + 3)/2, -(a - b*Csc[c + d*x])/(a + b*Csc[c +
d*x]), (a - b*Csc[c + d*x])/(a + b*Csc[c + d*x])], x] /; FreeQ[{a, b, c, d,
 e, m, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[n]
```

3.226.4 Maple [F]

$$\int (a + a \sec (dx + c))^n \tan (dx + c)^2 \, dx$$

input
```
int((a+a*sec(d*x+c))^n*tan(d*x+c)^2,x)
```

output
```
int((a+a*sec(d*x+c))^n*tan(d*x+c)^2,x)
```

3.226.5 Fricas [F]

$$\int (a + a \sec(c + dx))^n \tan^2(c + dx) \, dx = \int (a \sec (dx + c) + a)^n \tan (dx + c)^2 \, dx$$

input
```
integrate((a+a*sec(d*x+c))^n*tan(d*x+c)^2,x, algorithm="fricas")
```

output
```
integral((a*sec(d*x + c) + a)^n*tan(d*x + c)^2, x)
```

3.226.6 Sympy [F]

$$\int (a + a\sec(c + dx))^n \tan^2(c + dx)\,dx = \int (a(\sec(c + dx) + 1))^n \tan^2(c + dx)\,dx$$

input
```
integrate((a+a*sec(d*x+c))**n*tan(d*x+c)**2,x)
```

output
```
Integral((a*(sec(c + d*x) + 1))**n*tan(c + d*x)**2, x)
```

3.226.7 Maxima [F]

$$\int (a + a\sec(c + dx))^n \tan^2(c + dx)\,dx = \int (a\sec(dx + c) + a)^n \tan(dx + c)^2\,dx$$

input
```
integrate((a+a*sec(d*x+c))^n*tan(d*x+c)^2,x, algorithm="maxima")
```

output
```
integrate((a*sec(d*x + c) + a)^n*tan(d*x + c)^2, x)
```

3.226.8 Giac [F]

$$\int (a + a\sec(c + dx))^n \tan^2(c + dx)\,dx = \int (a\sec(dx + c) + a)^n \tan(dx + c)^2\,dx$$

input
```
integrate((a+a*sec(d*x+c))^n*tan(d*x+c)^2,x, algorithm="giac")
```

output
```
integrate((a*sec(d*x + c) + a)^n*tan(d*x + c)^2, x)
```

3.226.9 Mupad [F(-1)]

Timed out.

$$\int (a + a\sec(c + dx))^n \tan^2(c + dx)\,dx = \int \tan(c + d\,x)^2 \left(a + \frac{a}{\cos(c + d\,x)}\right)^n dx$$

input
```
int(tan(c + d*x)^2*(a + a/cos(c + d*x))^n,x)
```

output
```
int(tan(c + d*x)^2*(a + a/cos(c + d*x))^n, x)
```

3.227 $\int \cot^2(c+dx)(a+a\sec(c+dx))^n\,dx$

3.227.1 Optimal result . . . 1650
3.227.2 Mathematica [B] (warning: unable to verify) . . . 1650
3.227.3 Rubi [A] (verified) . . . 1651
3.227.4 Maple [F] . . . 1652
3.227.5 Fricas [F] . . . 1652
3.227.6 Sympy [F] . . . 1653
3.227.7 Maxima [F] . . . 1653
3.227.8 Giac [F] . . . 1653
3.227.9 Mupad [F(-1)] . . . 1654

3.227.1 Optimal result

Integrand size = 21, antiderivative size = 102

$$\int \cot^2(c+dx)(a+a\sec(c+dx))^n\,dx = -\frac{2^{-1+n}\,\text{AppellF1}\left(-\frac{1}{2},-2+n,1,\frac{1}{2},-\frac{a-a\sec(c+dx)}{a+a\sec(c+dx)},\frac{a-a\sec(c+dx)}{a+a\sec(c+dx)}\right)\cot(c+dx)\left(\frac{1}{1+\sec(c+dx)}\right)^{-1+n}(a+a\sec($$

output
```
-2^(-1+n)*AppellF1(-1/2,-2+n,1,1/2,(-a+a*sec(d*x+c))/(a+a*sec(d*x+c)),(a-a
*sec(d*x+c))/(a+a*sec(d*x+c)))*cot(d*x+c)*(1/(1+sec(d*x+c)))^(-1+n)*(a+a*s
ec(d*x+c))^n/d
```

3.227.2 Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 893 vs. 2(102) = 204.

Time = 3.28 (sec) , antiderivative size = 893, normalized size of antiderivative = 8.75

$$\int \cot^2(c+dx)(a+a\sec(c+dx))^n\,dx = \text{Too large to display}$$

input
```
Integrate[Cot[c + d*x]^2*(a + a*Sec[c + d*x])^n,x]
```

output

```
((a*(1 + Sec[c + d*x]))^n*(-((2^n*Cot[(c + d*x)/2]*Hypergeometric2F1[-1/2,
 n, 1/2, Tan[(c + d*x)/2]^2]*(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^n*(Cos[(c +
 d*x)/2]^2*Sec[c + d*x])^n)/(1 + Sec[c + d*x])^n) + (2^n*Hypergeometric2F1
[1/2, n, 3/2, Tan[(c + d*x)/2]^2]*(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^n*(Cos
[(c + d*x)/2]^2*Sec[c + d*x])^n*Tan[(c + d*x)/2])/(1 + Sec[c + d*x])^n - (
60*AppellF1[1/2, n, 1, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Cos[(
c + d*x)/2]^2*Cos[c + d*x]*Sin[c + d*x]*(3*AppellF1[1/2, n, 1, 3/2, Tan[(c
 + d*x)/2]^2, -Tan[(c + d*x)/2]^2] - 2*(AppellF1[3/2, n, 2, 5/2, Tan[(c +
d*x)/2]^2, -Tan[(c + d*x)/2]^2] - n*AppellF1[3/2, 1 + n, 1, 5/2, Tan[(c +
d*x)/2]^2, -Tan[(c + d*x)/2]^2])*Tan[(c + d*x)/2]^2))/(45*AppellF1[1/2, n,
 1, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]^2*Cos[(c + d*x)/2]^2*(1
+ 2*n - 2*n*Cos[c + d*x] + Cos[2*(c + d*x)]) + 6*AppellF1[1/2, n, 1, 3/2,
Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sin[(c + d*x)/2]^2*(-5*AppellF1[3
/2, n, 2, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*(1 + 2*n - 2*(2 +
n)*Cos[c + d*x] + Cos[2*(c + d*x)]) + 5*n*AppellF1[3/2, 1 + n, 1, 5/2, Tan
[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*(1 + 2*n - 2*(2 + n)*Cos[c + d*x] +
Cos[2*(c + d*x)]) - 48*(2*AppellF1[5/2, n, 3, 7/2, Tan[(c + d*x)/2]^2, -Ta
n[(c + d*x)/2]^2] - 2*n*AppellF1[5/2, 1 + n, 2, 7/2, Tan[(c + d*x)/2]^2, -
Tan[(c + d*x)/2]^2] + n*(1 + n)*AppellF1[5/2, 2 + n, 1, 7/2, Tan[(c + d*x)
/2]^2, -Tan[(c + d*x)/2]^2])*Cot[c + d*x]*Csc[c + d*x]*Sin[(c + d*x)/2]...
```

3.227.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {3042, 4377}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^2(c+dx)(a\sec(c+dx)+a)^n\,dx$$

$$\downarrow 3042$$

$$\int \frac{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^n}{\cot\left(c+dx+\frac{\pi}{2}\right)^2}dx$$

$$\downarrow 4377$$

$$-\frac{2^{n-1}\cot(c+dx)\left(\frac{1}{\sec(c+dx)+1}\right)^{n-1}(a\sec(c+dx)+a)^n\,\text{AppellF1}\left(-\frac{1}{2},n-2,1,\frac{1}{2},-\frac{a-a\sec(c+dx)}{\sec(c+dx)a+a},\frac{a-a\sec(c+dx)}{\sec(c+dx)a+a}\right)}{d}$$

input
```
Int[Cot[c + d*x]^2*(a + a*Sec[c + d*x])^n,x]
```

output
```
-((2^(-1 + n)*AppellF1[-1/2, -2 + n, 1, 1/2, -((a - a*Sec[c + d*x])/(a + a
*Sec[c + d*x])), (a - a*Sec[c + d*x])/(a + a*Sec[c + d*x])]*Cot[c + d*x]*(
(1 + Sec[c + d*x])^(-1))^(-1 + n)*(a + a*Sec[c + d*x])^n)/d)
```

3.227.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4377
```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Simp[(-2^(m + n + 1))*(e*Cot[c + d*x])^(m + 1)*((a
+ b*Csc[c + d*x])^n/(d*e*(m + 1)))*(a/(a + b*Csc[c + d*x]))^(m + n + 1)*App
ellF1[(m + 1)/2, m + n, 1, (m + 3)/2, -(a - b*Csc[c + d*x])/(a + b*Csc[c +
d*x]), (a - b*Csc[c + d*x])/(a + b*Csc[c + d*x])], x] /; FreeQ[{a, b, c, d,
 e, m, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[n]
```

3.227.4 Maple [F]

$$\int \cot (dx+c)^2 (a+a\sec (dx+c))^n \, dx$$

input
```
int(cot(d*x+c)^2*(a+a*sec(d*x+c))^n,x)
```

output
```
int(cot(d*x+c)^2*(a+a*sec(d*x+c))^n,x)
```

3.227.5 Fricas [F]

$$\int \cot^2(c+dx)(a+a\sec(c+dx))^n \, dx = \int (a\sec (dx+c)+a)^n \cot (dx+c)^2 \, dx$$

input
```
integrate(cot(d*x+c)^2*(a+a*sec(d*x+c))^n,x, algorithm="fricas")
```

output
```
integral((a*sec(d*x + c) + a)^n*cot(d*x + c)^2, x)
```

3.227.6 Sympy [F]

$$\int \cot^2(c+dx)(a+a\sec(c+dx))^n\,dx = \int (a(\sec(c+dx)+1))^n \cot^2(c+dx)\,dx$$

input
```
integrate(cot(d*x+c)**2*(a+a*sec(d*x+c))**n,x)
```

output
```
Integral((a*(sec(c + d*x) + 1))**n*cot(c + d*x)**2, x)
```

3.227.7 Maxima [F]

$$\int \cot^2(c+dx)(a+a\sec(c+dx))^n\,dx = \int (a\sec(dx+c)+a)^n \cot(dx+c)^2\,dx$$

input
```
integrate(cot(d*x+c)^2*(a+a*sec(d*x+c))^n,x, algorithm="maxima")
```

output
```
integrate((a*sec(d*x + c) + a)^n*cot(d*x + c)^2, x)
```

3.227.8 Giac [F]

$$\int \cot^2(c+dx)(a+a\sec(c+dx))^n\,dx = \int (a\sec(dx+c)+a)^n \cot(dx+c)^2\,dx$$

input
```
integrate(cot(d*x+c)^2*(a+a*sec(d*x+c))^n,x, algorithm="giac")
```

output
```
integrate((a*sec(d*x + c) + a)^n*cot(d*x + c)^2, x)
```

3.227.9 Mupad [F(-1)]

Timed out.

$$\int \cot^2(c+dx)(a+a\sec(c+dx))^n\,dx = \int \cot(c+d\,x)^2\left(a+\frac{a}{\cos(c+d\,x)}\right)^n dx$$

input
```
int(cot(c + d*x)^2*(a + a/cos(c + d*x))^n,x)
```

output
```
int(cot(c + d*x)^2*(a + a/cos(c + d*x))^n, x)
```

3.228 $\int \cot^4(c+dx)(a+a\sec(c+dx))^n\,dx$

3.228.1 Optimal result . . . 1655
3.228.2 Mathematica [F] . . . 1655
3.228.3 Rubi [A] (verified) . . . 1656
3.228.4 Maple [F] . . . 1657
3.228.5 Fricas [F] . . . 1657
3.228.6 Sympy [F] . . . 1657
3.228.7 Maxima [F] . . . 1658
3.228.8 Giac [F] . . . 1658
3.228.9 Mupad [F(-1)] . . . 1658

3.228.1 Optimal result

Integrand size = 21, antiderivative size = 106

$$\int \cot^4(c+dx)(a+a\sec(c+dx))^n\,dx = -\frac{2^{-3+n}\,\text{AppellF1}\left(-\frac{3}{2},-4+n,1,-\frac{1}{2},-\frac{a-a\sec(c+dx)}{a+a\sec(c+dx)},\frac{a-a\sec(c+dx)}{a+a\sec(c+dx)}\right)\cot^3(c+dx)\left(\frac{1}{1+\sec(c+dx)}\right)^{-3+n}(a+a\s}{3d}$$

output
```
-1/3*2^(-3+n)*AppellF1(-3/2,-4+n,1,-1/2,(-a+a*sec(d*x+c))/(a+a*sec(d*x+c))
,(a-a*sec(d*x+c))/(a+a*sec(d*x+c)))*cot(d*x+c)^3*(1/(1+sec(d*x+c)))^(-3+n)
*(a+a*sec(d*x+c))^n/d
```

3.228.2 Mathematica [F]

$$\int \cot^4(c+dx)(a+a\sec(c+dx))^n\,dx = \int \cot^4(c+dx)(a+a\sec(c+dx))^n\,dx$$

input
```
Integrate[Cot[c + d*x]^4*(a + a*Sec[c + d*x])^n,x]
```

output
```
Integrate[Cot[c + d*x]^4*(a + a*Sec[c + d*x])^n, x]
```

3.228.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {3042, 4377}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^4(c+dx)(a\sec(c+dx)+a)^n\,dx$$

$$\downarrow 3042$$

$$\int \frac{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^n}{\cot\left(c+dx+\frac{\pi}{2}\right)^4}dx$$

$$\downarrow 4377$$

$$-\frac{2^{n-3}\cot^3(c+dx)\left(\frac{1}{\sec(c+dx)+1}\right)^{n-3}(a\sec(c+dx)+a)^n\,\text{AppellF1}\left(-\frac{3}{2},n-4,1,-\frac{1}{2},-\frac{a-a\sec(c+dx)}{\sec(c+dx)a+a},\frac{a-a\sec(c+dx)}{\sec(c+dx)a+a}\right.}{3d}$$

input
```
Int[Cot[c + d*x]^4*(a + a*Sec[c + d*x])^n,x]
```

output
```
-1/3*(2^(-3 + n)*AppellF1[-3/2, -4 + n, 1, -1/2, -((a - a*Sec[c + d*x])/(a
 + a*Sec[c + d*x])), (a - a*Sec[c + d*x])/(a + a*Sec[c + d*x])]*Cot[c + d*
x]^3*((1 + Sec[c + d*x])^(-1))^(-3 + n)*(a + a*Sec[c + d*x])^n)/d
```

3.228.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4377
```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Simp[(-2^(m + n + 1))*(e*Cot[c + d*x])^(m + 1)*((a
+ b*Csc[c + d*x])^n/(d*e*(m + 1)))*(a/(a + b*Csc[c + d*x]))^(m + n + 1)*App
ellF1[(m + 1)/2, m + n, 1, (m + 3)/2, -(a - b*Csc[c + d*x])/(a + b*Csc[c +
d*x]), (a - b*Csc[c + d*x])/(a + b*Csc[c + d*x])], x] /; FreeQ[{a, b, c, d,
 e, m, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[n]
```

3.228.4 Maple [F]

$$\int \cot(dx+c)^4 (a+a\sec(dx+c))^n \, dx$$

input
```
int(cot(d*x+c)^4*(a+a*sec(d*x+c))^n,x)
```

output
```
int(cot(d*x+c)^4*(a+a*sec(d*x+c))^n,x)
```

3.228.5 Fricas [F]

$$\int \cot^4(c+dx)(a+a\sec(c+dx))^n \, dx = \int (a\sec(dx+c)+a)^n \cot(dx+c)^4 \, dx$$

input
```
integrate(cot(d*x+c)^4*(a+a*sec(d*x+c))^n,x, algorithm="fricas")
```

output
```
integral((a*sec(d*x + c) + a)^n*cot(d*x + c)^4, x)
```

3.228.6 Sympy [F]

$$\int \cot^4(c+dx)(a+a\sec(c+dx))^n \, dx = \int (a(\sec(c+dx)+1))^n \cot^4(c+dx) \, dx$$

input
```
integrate(cot(d*x+c)**4*(a+a*sec(d*x+c))**n,x)
```

output
```
Integral((a*(sec(c + d*x) + 1))**n*cot(c + d*x)**4, x)
```

3.228.7 Maxima [F]

$$\int \cot^4(c+dx)(a+a\sec(c+dx))^n\,dx = \int (a\sec(dx+c)+a)^n \cot(dx+c)^4\,dx$$

input
```
integrate(cot(d*x+c)^4*(a+a*sec(d*x+c))^n,x, algorithm="maxima")
```

output
```
integrate((a*sec(d*x + c) + a)^n*cot(d*x + c)^4, x)
```

3.228.8 Giac [F]

$$\int \cot^4(c+dx)(a+a\sec(c+dx))^n\,dx = \int (a\sec(dx+c)+a)^n \cot(dx+c)^4\,dx$$

input
```
integrate(cot(d*x+c)^4*(a+a*sec(d*x+c))^n,x, algorithm="giac")
```

output
```
integrate((a*sec(d*x + c) + a)^n*cot(d*x + c)^4, x)
```

3.228.9 Mupad [F(-1)]

Timed out.

$$\int \cot^4(c+dx)(a+a\sec(c+dx))^n\,dx = \int \cot(c+d\,x)^4 \left(a+\frac{a}{\cos(c+d\,x)}\right)^n dx$$

input
```
int(cot(c + d*x)^4*(a + a/cos(c + d*x))^n,x)
```

output
```
int(cot(c + d*x)^4*(a + a/cos(c + d*x))^n, x)
```

3.229 $\int (a + a\sec(c+dx))^n \tan^{\frac{3}{2}}(c+dx)\,dx$

3.229.1 Optimal result 1659
3.229.2 Mathematica [B] (warning: unable to verify) 1659
3.229.3 Rubi [A] (verified) 1660
3.229.4 Maple [F] 1661
3.229.5 Fricas [F] 1661
3.229.6 Sympy [F(-1)] 1662
3.229.7 Maxima [F] 1662
3.229.8 Giac [F] 1662
3.229.9 Mupad [F(-1)] 1663

3.229.1 Optimal result

Integrand size = 23, antiderivative size = 114

$$\int (a + a\sec(c+dx))^n \tan^{\frac{3}{2}}(c+dx)\,dx$$
$$= \frac{2^{\frac{7}{2}+n}\,\text{AppellF1}\left(\frac{5}{4}, \frac{3}{2}+n, 1, \frac{9}{4}, -\frac{a-a\sec(c+dx)}{a+a\sec(c+dx)}, \frac{a-a\sec(c+dx)}{a+a\sec(c+dx)}\right)\left(\frac{1}{1+\sec(c+dx)}\right)^{\frac{5}{2}+n}(a+a\sec(c+dx))^n \tan^{\frac{5}{2}}(c+}{5d}$$

output
```
1/5*2^(7/2+n)*AppellF1(5/4,3/2+n,1,9/4,(-a+a*sec(d*x+c))/(a+a*sec(d*x+c)),
(a-a*sec(d*x+c))/(a+a*sec(d*x+c)))*(1/(1+sec(d*x+c)))^(5/2+n)*(a+a*sec(d*x
+c))^n*tan(d*x+c)^(5/2)/d
```

3.229.2 Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 2072 vs. 2(114) = 228.

Time = 27.34 (sec) , antiderivative size = 2072, normalized size of antiderivative = 18.18

$$\int (a + a\sec(c+dx))^n \tan^{\frac{3}{2}}(c+dx)\,dx = \text{Result too large to show}$$

input
```
Integrate[(a + a*Sec[c + d*x])^n*Tan[c + d*x]^(3/2),x]
```

output

```
(2^(1 + n)*(Cos[(c + d*x)/2]^2*Sec[c + d*x])^n*(a*(1 + Sec[c + d*x]))^n*(-
1 + Tan[(c + d*x)/2])^(-1/2 - n)*(-2*AppellF1[1/4, 1/2 + n, 1, 5/4, Tan[(c
 + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^(1/2
+ n)*(-1 + Tan[(c + d*x)/2])^(1/2 + n) + (AppellF1[1/2, 1/2 + n, 3/2 + n,
3/2, Tan[(c + d*x)/2], -Tan[(c + d*x)/2]] + AppellF1[1/2, 3/2 + n, 1/2 + n
, 3/2, Tan[(c + d*x)/2], -Tan[(c + d*x)/2]])*(1 - Tan[(c + d*x)/2])^(1/2 +
 n)*(-1 + Tan[(c + d*x)/2]^2)^(1/2 + n))*Tan[c + d*x]^2)/(d*((2^n*Sec[c +
d*x]^2*(Cos[(c + d*x)/2]^2*Sec[c + d*x])^n*(-1 + Tan[(c + d*x)/2])^(-1/2 -
 n)*(-2*AppellF1[1/4, 1/2 + n, 1, 5/4, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/
2]^2]*(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^(1/2 + n)*(-1 + Tan[(c + d*x)/2])^
(1/2 + n) + (AppellF1[1/2, 1/2 + n, 3/2 + n, 3/2, Tan[(c + d*x)/2], -Tan[(
c + d*x)/2]] + AppellF1[1/2, 3/2 + n, 1/2 + n, 3/2, Tan[(c + d*x)/2], -Tan
[(c + d*x)/2]])*(1 - Tan[(c + d*x)/2])^(1/2 + n)*(-1 + Tan[(c + d*x)/2]^2)
^(1/2 + n)))/Sqrt[Tan[c + d*x]] + 2^n*(-1/2 - n)*Sec[(c + d*x)/2]^2*(Cos[(
c + d*x)/2]^2*Sec[c + d*x])^n*(-1 + Tan[(c + d*x)/2])^(-3/2 - n)*(-2*Appel
lF1[1/4, 1/2 + n, 1, 5/4, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*(Cos[c
+ d*x]*Sec[(c + d*x)/2]^2)^(1/2 + n)*(-1 + Tan[(c + d*x)/2])^(1/2 + n) + (
AppellF1[1/2, 1/2 + n, 3/2 + n, 3/2, Tan[(c + d*x)/2], -Tan[(c + d*x)/2]]
+ AppellF1[1/2, 3/2 + n, 1/2 + n, 3/2, Tan[(c + d*x)/2], -Tan[(c + d*x)/2]
])*(1 - Tan[(c + d*x)/2])^(1/2 + n)*(-1 + Tan[(c + d*x)/2]^2)^(1/2 + n)...
```

3.229.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {3042, 4377}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)^n\,dx$$

$$\downarrow 3042$$

$$\int \left(-\cot\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^n dx$$

$$\downarrow 4377$$

$$\frac{2^{n+\frac{7}{2}}\tan^{\frac{5}{2}}(c+dx)\left(\frac{1}{\sec(c+dx)+1}\right)^{n+\frac{5}{2}}(a\sec(c+dx)+a)^n\,\text{AppellF1}\left(\frac{5}{4},n+\frac{3}{2},1,\frac{9}{4},-\frac{a-a\sec(c+dx)}{\sec(c+dx)a+a},\frac{a-a\sec(c+dx)}{\sec(c+dx)a+a}\right)}{5d}$$

input
```
Int[(a + a*Sec[c + d*x])^n*Tan[c + d*x]^(3/2),x]
```

output
```
(2^(7/2 + n)*AppellF1[5/4, 3/2 + n, 1, 9/4, -((a - a*Sec[c + d*x])/(a + a*
Sec[c + d*x])), (a - a*Sec[c + d*x])/(a + a*Sec[c + d*x])]*((1 + Sec[c + d
*x])^(-1))^(5/2 + n)*(a + a*Sec[c + d*x])^n*Tan[c + d*x]^(5/2))/(5*d)
```

3.229.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4377
```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Simp[(-2^(m + n + 1))*(e*Cot[c + d*x])^(m + 1)*((a
+ b*Csc[c + d*x])^n/(d*e*(m + 1)))*(a/(a + b*Csc[c + d*x]))^(m + n + 1)*App
ellF1[(m + 1)/2, m + n, 1, (m + 3)/2, -(a - b*Csc[c + d*x])/(a + b*Csc[c +
d*x]), (a - b*Csc[c + d*x])/(a + b*Csc[c + d*x])], x] /; FreeQ[{a, b, c, d,
 e, m, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[n]
```

3.229.4 Maple [F]

$$\int (a + a \sec (dx + c))^n \tan (dx + c)^{\frac{3}{2}} \, dx$$

input
```
int((a+a*sec(d*x+c))^n*tan(d*x+c)^(3/2),x)
```

output
```
int((a+a*sec(d*x+c))^n*tan(d*x+c)^(3/2),x)
```

3.229.5 Fricas [F]

$$\int (a + a \sec(c + dx))^n \tan^{\frac{3}{2}}(c + dx) \, dx = \int (a \sec (dx + c) + a)^n \tan (dx + c)^{\frac{3}{2}} \, dx$$

input
```
integrate((a+a*sec(d*x+c))^n*tan(d*x+c)^(3/2),x, algorithm="fricas")
```

output
```
integral((a*sec(d*x + c) + a)^n*tan(d*x + c)^(3/2), x)
```

3.229.6 Sympy [F(-1)]

Timed out.

$$\int (a + a\sec(c + dx))^n \tan^{\frac{3}{2}}(c + dx)\, dx = \text{Timed out}$$

input
```
integrate((a+a*sec(d*x+c))**n*tan(d*x+c)**(3/2),x)
```

output
```
Timed out
```

3.229.7 Maxima [F]

$$\int (a + a\sec(c + dx))^n \tan^{\frac{3}{2}}(c + dx)\, dx = \int (a\sec(dx + c) + a)^n \tan(dx + c)^{\frac{3}{2}}\, dx$$

input
```
integrate((a+a*sec(d*x+c))^n*tan(d*x+c)^(3/2),x, algorithm="maxima")
```

output
```
integrate((a*sec(d*x + c) + a)^n*tan(d*x + c)^(3/2), x)
```

3.229.8 Giac [F]

$$\int (a + a\sec(c + dx))^n \tan^{\frac{3}{2}}(c + dx)\, dx = \int (a\sec(dx + c) + a)^n \tan(dx + c)^{\frac{3}{2}}\, dx$$

input
```
integrate((a+a*sec(d*x+c))^n*tan(d*x+c)^(3/2),x, algorithm="giac")
```

output
```
integrate((a*sec(d*x + c) + a)^n*tan(d*x + c)^(3/2), x)
```

3.229.9 Mupad [F(-1)]

Timed out.

$$\int (a + a\sec(c + dx))^n \tan^{\frac{3}{2}}(c + dx)\, dx = \int \tan(c + d\,x)^{3/2} \left(a + \frac{a}{\cos(c + d\,x)}\right)^n dx$$

input
```
int(tan(c + d*x)^(3/2)*(a + a/cos(c + d*x))^n,x)
```

output
```
int(tan(c + d*x)^(3/2)*(a + a/cos(c + d*x))^n, x)
```

3.230 $\int (a + a\sec(c+dx))^n \sqrt{\tan(c+dx)}\, dx$

3.230.1 Optimal result . . . 1664
3.230.2 Mathematica [B] (warning: unable to verify) . . . 1664
3.230.3 Rubi [A] (verified) . . . 1665
3.230.4 Maple [F] . . . 1666
3.230.5 Fricas [F] . . . 1666
3.230.6 Sympy [F] . . . 1667
3.230.7 Maxima [F] . . . 1667
3.230.8 Giac [F] . . . 1667
3.230.9 Mupad [F(-1)] . . . 1668

3.230.1 Optimal result

Integrand size = 23, antiderivative size = 114

$$\int (a + a\sec(c+dx))^n \sqrt{\tan(c+dx)}\, dx$$
$$= \frac{2^{\frac{5}{2}+n}\,\text{AppellF1}\left(\frac{3}{4}, \frac{1}{2}+n, 1, \frac{7}{4}, -\frac{a-a\sec(c+dx)}{a+a\sec(c+dx)}, \frac{a-a\sec(c+dx)}{a+a\sec(c+dx)}\right)\left(\frac{1}{1+\sec(c+dx)}\right)^{\frac{3}{2}+n}(a+a\sec(c+dx))^n \tan^{\frac{3}{2}}(c+}{3d}$$

output
```
1/3*2^(5/2+n)*AppellF1(3/4,1/2+n,1,7/4,(-a+a*sec(d*x+c))/(a+a*sec(d*x+c)),
(a-a*sec(d*x+c))/(a+a*sec(d*x+c)))*(1/(1+sec(d*x+c)))^(3/2+n)*(a+a*sec(d*x
+c))^n*tan(d*x+c)^(3/2)/d
```

3.230.2 Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 238 vs. 2(114) = 228.

Time = 4.88 (sec) , antiderivative size = 238, normalized size of antiderivative = 2.09

$$\int (a + a\sec(c+dx))^n \sqrt{\tan(c+dx)}\, dx$$
$$= \frac{56\,\text{AppellF1}\left(\frac{3}{4}, \frac{1}{2}+n, 1, \frac{7}{4}, \tan^2\left(\frac{1}{2}(c+dx)\right)\right.}{d\left(6\left(2\,\text{AppellF1}\left(\frac{7}{4}, \frac{1}{2}+n, 2, \frac{11}{4}, \tan^2\left(\frac{1}{2}(c+dx)\right), -\tan^2\left(\frac{1}{2}(c+dx)\right)\right)\right) - (1+2n)\,\text{AppellF1}\left(\frac{7}{4}, \frac{3}{2}+n, 1,\right.}$$

input
```
Integrate[(a + a*Sec[c + d*x])^n*Sqrt[Tan[c + d*x]],x]
```

output
```
(56*AppellF1[3/4, 1/2 + n, 1, 7/4, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2
]*Cos[(c + d*x)/2]^3*(a*(1 + Sec[c + d*x]))^n*Sin[(c + d*x)/2]*Sqrt[Tan[c
+ d*x]])/(d*(6*(2*AppellF1[7/4, 1/2 + n, 2, 11/4, Tan[(c + d*x)/2]^2, -Tan
[(c + d*x)/2]^2] - (1 + 2*n)*AppellF1[7/4, 3/2 + n, 1, 11/4, Tan[(c + d*x)
/2]^2, -Tan[(c + d*x)/2]^2])*(-1 + Cos[c + d*x]) + 21*AppellF1[3/4, 1/2 +
n, 1, 7/4, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*(1 + Cos[c + d*x])))
```

3.230.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {3042, 4377}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\tan(c+dx)}(a\sec(c+dx)+a)^n\,dx$$

$$\downarrow 3042$$

$$\int \sqrt{-\cot\left(c+dx+\frac{\pi}{2}\right)}\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^n dx$$

$$\downarrow 4377$$

$$\frac{2^{n+\frac{5}{2}}\tan^{\frac{3}{2}}(c+dx)\left(\frac{1}{\sec(c+dx)+1}\right)^{n+\frac{3}{2}}(a\sec(c+dx)+a)^n\,\text{AppellF1}\left(\frac{3}{4},n+\frac{1}{2},1,\frac{7}{4},-\frac{a-a\sec(c+dx)}{\sec(c+dx)a+a},\frac{a-a\sec(c+dx)}{\sec(c+dx)a+a}\right)}{3d}$$

input
```
Int[(a + a*Sec[c + d*x])^n*Sqrt[Tan[c + d*x]],x]
```

output
```
(2^(5/2 + n)*AppellF1[3/4, 1/2 + n, 1, 7/4, -((a - a*Sec[c + d*x])/(a + a*
Sec[c + d*x])), (a - a*Sec[c + d*x])/(a + a*Sec[c + d*x])]*((1 + Sec[c + d
*x])^(-1))^(3/2 + n)*(a + a*Sec[c + d*x])^n*Tan[c + d*x]^(3/2))/(3*d)
```

3.230.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4377
```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Simp[(-2^(m + n + 1))*(e*Cot[c + d*x])^(m + 1)*((a
+ b*Csc[c + d*x])^n/(d*e*(m + 1)))*(a/(a + b*Csc[c + d*x]))^(m + n + 1)*App
ellF1[(m + 1)/2, m + n, 1, (m + 3)/2, -(a - b*Csc[c + d*x])/(a + b*Csc[c +
d*x]), (a - b*Csc[c + d*x])/(a + b*Csc[c + d*x])], x] /; FreeQ[{a, b, c, d,
 e, m, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[n]
```

3.230.4 Maple [F]

$$\int (a + a \sec (dx + c))^n \sqrt{\tan (dx + c)} dx$$

input
```
int((a+a*sec(d*x+c))^n*tan(d*x+c)^(1/2),x)
```

output
```
int((a+a*sec(d*x+c))^n*tan(d*x+c)^(1/2),x)
```

3.230.5 Fricas [F]

$$\int (a + a \sec(c + dx))^n \sqrt{\tan(c + dx)}\, dx = \int (a \sec (dx + c) + a)^n \sqrt{\tan (dx + c)}\, dx$$

input
```
integrate((a+a*sec(d*x+c))^n*tan(d*x+c)^(1/2),x, algorithm="fricas")
```

output
```
integral((a*sec(d*x + c) + a)^n*sqrt(tan(d*x + c)), x)
```

3.230.6 Sympy [F]

$$\int (a + a\sec(c + dx))^n \sqrt{\tan(c + dx)}\, dx = \int (a(\sec(c + dx) + 1))^n \sqrt{\tan(c + dx)}\, dx$$

input
```
integrate((a+a*sec(d*x+c))**n*tan(d*x+c)**(1/2),x)
```

output
```
Integral((a*(sec(c + d*x) + 1))**n*sqrt(tan(c + d*x)), x)
```

3.230.7 Maxima [F]

$$\int (a + a\sec(c + dx))^n \sqrt{\tan(c + dx)}\, dx = \int (a\sec(dx + c) + a)^n \sqrt{\tan(dx + c)}\, dx$$

input
```
integrate((a+a*sec(d*x+c))^n*tan(d*x+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate((a*sec(d*x + c) + a)^n*sqrt(tan(d*x + c)), x)
```

3.230.8 Giac [F]

$$\int (a + a\sec(c + dx))^n \sqrt{\tan(c + dx)}\, dx = \int (a\sec(dx + c) + a)^n \sqrt{\tan(dx + c)}\, dx$$

input
```
integrate((a+a*sec(d*x+c))^n*tan(d*x+c)^(1/2),x, algorithm="giac")
```

output
```
integrate((a*sec(d*x + c) + a)^n*sqrt(tan(d*x + c)), x)
```

3.230.9 Mupad [F(-1)]

Timed out.

$$\int (a + a\sec(c + dx))^n \sqrt{\tan(c + dx)}\, dx = \int \sqrt{\tan(c + d\,x)} \left(a + \frac{a}{\cos(c + d\,x)}\right)^n dx$$

input
```
int(tan(c + d*x)^(1/2)*(a + a/cos(c + d*x))^n,x)
```

output
```
int(tan(c + d*x)^(1/2)*(a + a/cos(c + d*x))^n, x)
```

3.231 $\int \frac{(a+a\sec(c+dx))^n}{\sqrt{\tan(c+dx)}}\,dx$

3.231.1 Optimal result 1669
3.231.2 Mathematica [B] (warning: unable to verify) 1669
3.231.3 Rubi [A] (verified) 1670
3.231.4 Maple [F] 1671
3.231.5 Fricas [F] 1671
3.231.6 Sympy [F] 1672
3.231.7 Maxima [F] 1672
3.231.8 Giac [F] 1672
3.231.9 Mupad [F(-1)] 1673

3.231.1 Optimal result

Integrand size = 23, antiderivative size = 111

$$\int \frac{(a+a\sec(c+dx))^n}{\sqrt{\tan(c+dx)}}\,dx$$
$$= \frac{2^{\frac{3}{2}+n}\,\text{AppellF1}\left(\frac{1}{4},-\frac{1}{2}+n,1,\frac{5}{4},-\frac{a-a\sec(c+dx)}{a+a\sec(c+dx)},\frac{a-a\sec(c+dx)}{a+a\sec(c+dx)}\right)\left(\frac{1}{1+\sec(c+dx)}\right)^{\frac{1}{2}+n}(a+a\sec(c+dx))^n\sqrt{\tan(c}}{d}$$

output
```
2^(3/2+n)*AppellF1(1/4,-1/2+n,1,5/4,(-a+a*sec(d*x+c))/(a+a*sec(d*x+c)),(a-
a*sec(d*x+c))/(a+a*sec(d*x+c)))*(1/(1+sec(d*x+c)))^(1/2+n)*(a+a*sec(d*x+c)
)^n*tan(d*x+c)^(1/2)/d
```

3.231.2 Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 229 vs. 2(111) = 222.

Time = 9.02 (sec) , antiderivative size = 229, normalized size of antiderivative = 2.06

$$\int \frac{(a+a\sec(c+dx))^n}{\sqrt{\tan(c+dx)}}\,dx$$
$$= \frac{10\,\text{AppellF1}\left(\frac{1}{4},-\frac{1}{2}+n,1,\frac{5}{4},\tan^2\left(\frac{1}{2}(c+dx\right)\right.}{d\left(2\left(2\,\text{AppellF1}\left(\frac{5}{4},-\frac{1}{2}+n,2,\frac{9}{4},\tan^2\left(\frac{1}{2}(c+dx)\right),-\tan^2\left(\frac{1}{2}(c+dx)\right)\right)+(1-2n)\,\text{AppellF1}\left(\frac{5}{4},\frac{1}{2}+n,1\right.\right.\right.}$$

input
```
Integrate[(a + a*Sec[c + d*x])^n/Sqrt[Tan[c + d*x]],x]
```

output
```
(10*AppellF1[1/4, -1/2 + n, 1, 5/4, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^
2]*Cos[c + d*x]*(1 + Cos[c + d*x])*(a*(1 + Sec[c + d*x]))^n*Sqrt[Tan[c + d
*x]])/(d*(2*(2*AppellF1[5/4, -1/2 + n, 2, 9/4, Tan[(c + d*x)/2]^2, -Tan[(c
 + d*x)/2]^2] + (1 - 2*n)*AppellF1[5/4, 1/2 + n, 1, 9/4, Tan[(c + d*x)/2]^
2, -Tan[(c + d*x)/2]^2])*(-1 + Cos[c + d*x]) + 5*AppellF1[1/4, -1/2 + n, 1
, 5/4, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*(1 + Cos[c + d*x])))
```

3.231.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {3042, 4377}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sec(c+dx)+a)^n}{\sqrt{\tan(c+dx)}}\,dx$$

$$\downarrow 3042$$

$$\int \frac{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^n}{\sqrt{-\cot\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\downarrow 4377$$

$$\frac{2^{n+\frac{3}{2}}\sqrt{\tan(c+dx)}\left(\frac{1}{\sec(c+dx)+1}\right)^{n+\frac{1}{2}}(a\sec(c+dx)+a)^n\,\text{AppellF1}\left(\frac{1}{4},n-\frac{1}{2},1,\frac{5}{4},-\frac{a-a\sec(c+dx)}{\sec(c+dx)a+a},\frac{a-a\sec(c+dx)}{\sec(c+dx)a+a}\right)}{d}$$

input
```
Int[(a + a*Sec[c + d*x])^n/Sqrt[Tan[c + d*x]],x]
```

output
```
(2^(3/2 + n)*AppellF1[1/4, -1/2 + n, 1, 5/4, -((a - a*Sec[c + d*x])/(a + a
*Sec[c + d*x])), (a - a*Sec[c + d*x])/(a + a*Sec[c + d*x])]*((1 + Sec[c + 
d*x])^(-1))^(1/2 + n)*(a + a*Sec[c + d*x])^n*Sqrt[Tan[c + d*x]])/d
```

3.231.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4377
```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Simp[(-2^(m + n + 1))*(e*Cot[c + d*x])^(m + 1)*((a
+ b*Csc[c + d*x])^n/(d*e*(m + 1)))*(a/(a + b*Csc[c + d*x]))^(m + n + 1)*App
ellF1[(m + 1)/2, m + n, 1, (m + 3)/2, -(a - b*Csc[c + d*x])/(a + b*Csc[c +
d*x]), (a - b*Csc[c + d*x])/(a + b*Csc[c + d*x])], x] /; FreeQ[{a, b, c, d,
 e, m, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[n]
```

3.231.4 Maple [F]

$$\int \frac{(a + a \sec(dx + c))^n}{\sqrt{\tan(dx + c)}} dx$$

input
```
int((a+a*sec(d*x+c))^n/tan(d*x+c)^(1/2),x)
```

output
```
int((a+a*sec(d*x+c))^n/tan(d*x+c)^(1/2),x)
```

3.231.5 Fricas [F]

$$\int \frac{(a + a \sec(c + dx))^n}{\sqrt{\tan(c + dx)}} dx = \int \frac{(a \sec(dx + c) + a)^n}{\sqrt{\tan(dx + c)}} dx$$

input
```
integrate((a+a*sec(d*x+c))^n/tan(d*x+c)^(1/2),x, algorithm="fricas")
```

output
```
integral((a*sec(d*x + c) + a)^n/sqrt(tan(d*x + c)), x)
```

3.231.6 Sympy [F]

$$\int \frac{(a + a\sec(c + dx))^n}{\sqrt{\tan(c + dx)}}\,dx = \int \frac{(a(\sec(c + dx) + 1))^n}{\sqrt{\tan(c + dx)}}\,dx$$

input
```
integrate((a+a*sec(d*x+c))**n/tan(d*x+c)**(1/2),x)
```

output
```
Integral((a*(sec(c + d*x) + 1))**n/sqrt(tan(c + d*x)), x)
```

3.231.7 Maxima [F]

$$\int \frac{(a + a\sec(c + dx))^n}{\sqrt{\tan(c + dx)}}\,dx = \int \frac{(a\sec(dx + c) + a)^n}{\sqrt{\tan(dx + c)}}\,dx$$

input
```
integrate((a+a*sec(d*x+c))^n/tan(d*x+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate((a*sec(d*x + c) + a)^n/sqrt(tan(d*x + c)), x)
```

3.231.8 Giac [F]

$$\int \frac{(a + a\sec(c + dx))^n}{\sqrt{\tan(c + dx)}}\,dx = \int \frac{(a\sec(dx + c) + a)^n}{\sqrt{\tan(dx + c)}}\,dx$$

input
```
integrate((a+a*sec(d*x+c))^n/tan(d*x+c)^(1/2),x, algorithm="giac")
```

output
```
integrate((a*sec(d*x + c) + a)^n/sqrt(tan(d*x + c)), x)
```

3.231.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+a\sec(c+dx))^n}{\sqrt{\tan(c+dx)}}\,dx = \int \frac{\left(a+\frac{a}{\cos(c+d\,x)}\right)^n}{\sqrt{\tan(c+d\,x)}}\,dx$$

input
```
int((a + a/cos(c + d*x))^n/tan(c + d*x)^(1/2),x)
```

output
```
int((a + a/cos(c + d*x))^n/tan(c + d*x)^(1/2), x)
```

3.232 $\int \frac{(a+a\sec(c+dx))^n}{\tan^{\frac{3}{2}}(c+dx)}\,dx$

3.232.1 Optimal result 1674
3.232.2 Mathematica [B] (warning: unable to verify) 1674
3.232.3 Rubi [A] (verified) 1675
3.232.4 Maple [F] 1676
3.232.5 Fricas [F] 1676
3.232.6 Sympy [F] 1677
3.232.7 Maxima [F] 1677
3.232.8 Giac [F] 1677
3.232.9 Mupad [F(-1)] 1678

3.232.1 Optimal result

Integrand size = 23, antiderivative size = 112

$$\int \frac{(a+a\sec(c+dx))^n}{\tan^{\frac{3}{2}}(c+dx)}\,dx = -\frac{2^{\frac{1}{2}+n}\,\text{AppellF1}\left(-\frac{1}{4},-\frac{3}{2}+n,1,\frac{3}{4},-\frac{a-a\sec(c+dx)}{a+a\sec(c+dx)},\frac{a-a\sec(c+dx)}{a+a\sec(c+dx)}\right)\left(\frac{1}{1+\sec(c+dx)}\right)^{-\frac{1}{2}+n}(a+a\sec(c+dx))^n}{d\sqrt{\tan(c+dx)}}$$

output
```
-2^(1/2+n)*AppellF1(-1/4,-3/2+n,1,3/4,(-a+a*sec(d*x+c))/(a+a*sec(d*x+c)),(
a-a*sec(d*x+c))/(a+a*sec(d*x+c)))*(1/(1+sec(d*x+c)))^(-1/2+n)*(a+a*sec(d*x
+c))^n/d/tan(d*x+c)^(1/2)
```

3.232.2 Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 2164 vs. 2(112) = 224.

Time = 17.52 (sec) , antiderivative size = 2164, normalized size of antiderivative = 19.32

$$\int \frac{(a+a\sec(c+dx))^n}{\tan^{\frac{3}{2}}(c+dx)}\,dx = \text{Result too large to show}$$

input
```
Integrate[(a + a*Sec[c + d*x])^n/Tan[c + d*x]^(3/2),x]
```

output

```
-1/21*(2^(1/2 + n)*Cot[c + d*x]^2*(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^n*(Cos
[(c + d*x)/2]^2*Sec[c + d*x])^n*(a*(1 + Sec[c + d*x]))^n*(21*Hypergeometri
c2F1[-1/4, -1/2 + n, 3/4, Tan[(c + d*x)/2]^2] + 7*AppellF1[3/4, -1/2 + n,
1, 7/4, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Tan[(c + d*x)/2]^2 + 7*Hy
pergeometric2F1[3/4, -1/2 + n, 7/4, Tan[(c + d*x)/2]^2]*Tan[(c + d*x)/2]^2
 - 3*AppellF1[7/4, -1/2 + n, 1, 11/4, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2
]^2]*Tan[(c + d*x)/2]^4))/(d*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*((2^(-1
/2 + n)*(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^n*Sec[c + d*x]^2*(Cos[(c + d*x)/
2]^2*Sec[c + d*x])^n*(21*Hypergeometric2F1[-1/4, -1/2 + n, 3/4, Tan[(c + d
*x)/2]^2] + 7*AppellF1[3/4, -1/2 + n, 1, 7/4, Tan[(c + d*x)/2]^2, -Tan[(c
+ d*x)/2]^2]*Tan[(c + d*x)/2]^2 + 7*Hypergeometric2F1[3/4, -1/2 + n, 7/4,
Tan[(c + d*x)/2]^2]*Tan[(c + d*x)/2]^2 - 3*AppellF1[7/4, -1/2 + n, 1, 11/4
, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Tan[(c + d*x)/2]^4))/(21*Sqrt[C
os[c + d*x]/(1 + Cos[c + d*x])]*Tan[c + d*x]^(3/2)) + (2^(-1/2 + n)*(Cos[c
 + d*x]*Sec[(c + d*x)/2]^2)^n*(Cos[(c + d*x)/2]^2*Sec[c + d*x])^n*((Cos[c
+ d*x]*Sin[c + d*x])/(1 + Cos[c + d*x])^2 - Sin[c + d*x]/(1 + Cos[c + d*x]
))*(21*Hypergeometric2F1[-1/4, -1/2 + n, 3/4, Tan[(c + d*x)/2]^2] + 7*Appe
llF1[3/4, -1/2 + n, 1, 7/4, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Tan[(
c + d*x)/2]^2 + 7*Hypergeometric2F1[3/4, -1/2 + n, 7/4, Tan[(c + d*x)/2]^2
]*Tan[(c + d*x)/2]^2 - 3*AppellF1[7/4, -1/2 + n, 1, 11/4, Tan[(c + d*x)...
```

3.232.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {3042, 4377}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sec(c+dx)+a)^n}{\tan^{\frac{3}{2}}(c+dx)}\,dx$$

$$\downarrow 3042$$

$$\int \frac{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^n}{\left(-\cot\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx$$

$$\downarrow 4377$$

$$-\frac{2^{n+\frac{1}{2}}\left(\frac{1}{\sec(c+dx)+1}\right)^{n-\frac{1}{2}}(a\sec(c+dx)+a)^n\,\text{AppellF1}\left(-\frac{1}{4},n-\frac{3}{2},1,\frac{3}{4},-\frac{a-a\sec(c+dx)}{\sec(c+dx)a+a},\frac{a-a\sec(c+dx)}{\sec(c+dx)a+a}\right)}{d\sqrt{\tan(c+dx)}}$$

input
```
Int[(a + a*Sec[c + d*x])^n/Tan[c + d*x]^(3/2),x]
```

output
```
-((2^(1/2 + n)*AppellF1[-1/4, -3/2 + n, 1, 3/4, -((a - a*Sec[c + d*x])/(a
+ a*Sec[c + d*x])), (a - a*Sec[c + d*x])/(a + a*Sec[c + d*x])]*((1 + Sec[c
 + d*x])^(-1))^(-1/2 + n)*(a + a*Sec[c + d*x])^n)/(d*Sqrt[Tan[c + d*x]]))
```

3.232.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4377
```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Simp[(-2^(m + n + 1))*(e*Cot[c + d*x])^(m + 1)*((a
+ b*Csc[c + d*x])^n/(d*e*(m + 1)))*(a/(a + b*Csc[c + d*x]))^(m + n + 1)*App
ellF1[(m + 1)/2, m + n, 1, (m + 3)/2, -(a - b*Csc[c + d*x])/(a + b*Csc[c +
d*x]), (a - b*Csc[c + d*x])/(a + b*Csc[c + d*x])], x] /; FreeQ[{a, b, c, d,
 e, m, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[n]
```

3.232.4 Maple [F]

$$\int \frac{(a + a \sec (dx + c))^n}{\tan (dx + c)^{\frac{3}{2}}} dx$$

input
```
int((a+a*sec(d*x+c))^n/tan(d*x+c)^(3/2),x)
```

output
```
int((a+a*sec(d*x+c))^n/tan(d*x+c)^(3/2),x)
```

3.232.5 Fricas [F]

$$\int \frac{(a + a \sec(c + dx))^n}{\tan^{\frac{3}{2}}(c + dx)} dx = \int \frac{(a \sec (dx + c) + a)^n}{\tan (dx + c)^{\frac{3}{2}}} dx$$

input
```
integrate((a+a*sec(d*x+c))^n/tan(d*x+c)^(3/2),x, algorithm="fricas")
```

output
```
integral((a*sec(d*x + c) + a)^n/tan(d*x + c)^(3/2), x)
```

3.232.6 Sympy [F]

$$\int \frac{(a + a\sec(c + dx))^n}{\tan^{\frac{3}{2}}(c + dx)}\,dx = \int \frac{(a(\sec(c + dx) + 1))^n}{\tan^{\frac{3}{2}}(c + dx)}\,dx$$

input
```
integrate((a+a*sec(d*x+c))**n/tan(d*x+c)**(3/2),x)
```

output
```
Integral((a*(sec(c + d*x) + 1))**n/tan(c + d*x)**(3/2), x)
```

3.232.7 Maxima [F]

$$\int \frac{(a + a\sec(c + dx))^n}{\tan^{\frac{3}{2}}(c + dx)}\,dx = \int \frac{(a\sec(dx + c) + a)^n}{\tan(dx + c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+a*sec(d*x+c))^n/tan(d*x+c)^(3/2),x, algorithm="maxima")
```

output
```
integrate((a*sec(d*x + c) + a)^n/tan(d*x + c)^(3/2), x)
```

3.232.8 Giac [F]

$$\int \frac{(a + a\sec(c + dx))^n}{\tan^{\frac{3}{2}}(c + dx)}\,dx = \int \frac{(a\sec(dx + c) + a)^n}{\tan(dx + c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+a*sec(d*x+c))^n/tan(d*x+c)^(3/2),x, algorithm="giac")
```

output
```
integrate((a*sec(d*x + c) + a)^n/tan(d*x + c)^(3/2), x)
```

3.232.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+a\sec(c+dx))^n}{\tan^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{\left(a+\frac{a}{\cos(c+d\,x)}\right)^n}{\tan\left(c+d\,x\right)^{3/2}}\,dx$$

input
```
int((a + a/cos(c + d*x))^n/tan(c + d*x)^(3/2),x)
```

output
```
int((a + a/cos(c + d*x))^n/tan(c + d*x)^(3/2), x)
```

3.233 $\int (e\cot(c+dx))^{5/2}(a+a\sec(c+dx))\,dx$

3.233.1 Optimal result 1679
3.233.2 Mathematica [C] (verified) 1680
3.233.3 Rubi [A] (verified) 1680
3.233.4 Maple [C] (verified) 1687
3.233.5 Fricas [F(-1)] 1687
3.233.6 Sympy [F(-1)] 1688
3.233.7 Maxima [F(-2)] 1688
3.233.8 Giac [F] 1688
3.233.9 Mupad [F(-1)] 1689

3.233.1 Optimal result

Integrand size = 23, antiderivative size = 320

$$\begin{aligned}
&\int (e\cot(c+dx))^{5/2}(a+a\sec(c+dx))\,dx = \\
&-\frac{2(e\cot(c+dx))^{5/2}(a+a\sec(c+dx))\tan(c+dx)}{3d} \\
&-\frac{a(e\cot(c+dx))^{5/2}\,\text{EllipticF}\left(c-\frac{\pi}{4}+dx,2\right)\sec(c+dx)\sqrt{\sin(2c+2dx)}\tan^2(c+dx)}{3d} \\
&+\frac{a\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)(e\cot(c+dx))^{5/2}\tan^{\frac{5}{2}}(c+dx)}{\sqrt{2}d} \\
&-\frac{a\arctan\left(1+\sqrt{2}\sqrt{\tan(c+dx)}\right)(e\cot(c+dx))^{5/2}\tan^{\frac{5}{2}}(c+dx)}{\sqrt{2}d} \\
&+\frac{a(e\cot(c+dx))^{5/2}\log\left(1-\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+dx)\right)\tan^{\frac{5}{2}}(c+dx)}{2\sqrt{2}d} \\
&-\frac{a(e\cot(c+dx))^{5/2}\log\left(1+\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+dx)\right)\tan^{\frac{5}{2}}(c+dx)}{2\sqrt{2}d}
\end{aligned}$$

output
```
-2/3*(e*cot(d*x+c))^(5/2)*(a+a*sec(d*x+c))*tan(d*x+c)/d+1/3*a*(e*cot(d*x+c
))^(5/2)*(sin(c+1/4*Pi+d*x)^2)^(1/2)/sin(c+1/4*Pi+d*x)*EllipticF(cos(c+1/4
*Pi+d*x),2^(1/2))*sec(d*x+c)*sin(2*d*x+2*c)^(1/2)*tan(d*x+c)^2/d-1/2*a*arc
tan(-1+2^(1/2)*tan(d*x+c)^(1/2))*(e*cot(d*x+c))^(5/2)*tan(d*x+c)^(5/2)/d*2
^(1/2)-1/2*a*arctan(1+2^(1/2)*tan(d*x+c)^(1/2))*(e*cot(d*x+c))^(5/2)*tan(d
*x+c)^(5/2)/d*2^(1/2)+1/4*a*(e*cot(d*x+c))^(5/2)*ln(1-2^(1/2)*tan(d*x+c)^(
1/2)+tan(d*x+c))*tan(d*x+c)^(5/2)/d*2^(1/2)-1/4*a*(e*cot(d*x+c))^(5/2)*ln(
1+2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c))*tan(d*x+c)^(5/2)/d*2^(1/2)
```

3.233.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 14.43 (sec) , antiderivative size = 185, normalized size of antiderivative = 0.58

$$\int (e\cot(c+dx))^{5/2}(a+a\sec(c+dx))\,dx =$$

$$-\frac{a(e\cot(c+dx))^{5/2}\sec(c+dx)\left(\sqrt{\cot(c+dx)}\Big(4(1+\cos(c+dx))\cot(c+dx)-3\arcsin(\cos(c+dx)-\sin(c+dx)\right)}{}$$

input
```
Integrate[(e*Cot[c + d*x])^(5/2)*(a + a*Sec[c + d*x]),x]
```

output
```
-1/6*(a*(e*Cot[c + d*x])^(5/2)*Sec[c + d*x]*(Sqrt[Cot[c + d*x]]*(4*(1 + Co
s[c + d*x])*Cot[c + d*x] - 3*ArcSin[Cos[c + d*x] - Sin[c + d*x]]*Sqrt[Sin[
2*(c + d*x)]] + 3*Log[Cos[c + d*x] + Sin[c + d*x] + Sqrt[Sin[2*(c + d*x)]]
]*Sqrt[Sin[2*(c + d*x)]]) + 2*(-1)^(1/4)*Sqrt[Csc[c + d*x]^2]*EllipticF[I*
ArcSinh[(-1)^(1/4)*Sqrt[Cot[c + d*x]]], -1]*Sin[2*(c + d*x)]))/(d*Cot[c +
d*x]^(5/2))
```

3.233.3 Rubi [A] (verified)

Time = 1.01 (sec) , antiderivative size = 241, normalized size of antiderivative = 0.75, number of steps used = 24, number of rules used = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 1.000$, Rules used = {3042, 4388, 3042, 4370, 27, 3042, 4372, 3042, 3094, 3042, 3053, 3042, 3120, 3957, 266, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sec(c+dx)+a)(e\cot(c+dx))^{5/2}\,dx$$

$$\downarrow 3042$$

$$\int (a\sec(c+dx)+a)(e\cot(c+dx))^{5/2}dx$$

$$\downarrow 4388$$

$$\tan^{\frac{5}{2}}(c+dx)(e\cot(c+dx))^{5/2}\int \frac{\sec(c+dx)a+a}{\tan^{\frac{5}{2}}(c+dx)}dx$$

$$\downarrow 3042$$

$$\tan^{\frac{5}{2}}(c+dx)(e\cot(c+dx))^{5/2}\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}{\left(-\cot\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}dx$$

$$\downarrow 4370$$

$$\tan^{\frac{5}{2}}(c+dx)(e\cot(c+dx))^{5/2}\left(\frac{2}{3}\int -\frac{\sec(c+dx)a+3a}{2\sqrt{\tan(c+dx)}}dx-\frac{2(a\sec(c+dx)+a)}{3d\tan^{\frac{3}{2}}(c+dx)}\right)$$

$$\downarrow 27$$

$$\tan^{\frac{5}{2}}(c+dx)(e\cot(c+dx))^{5/2}\left(-\frac{1}{3}\int \frac{\sec(c+dx)a+3a}{\sqrt{\tan(c+dx)}}dx-\frac{2(a\sec(c+dx)+a)}{3d\tan^{\frac{3}{2}}(c+dx)}\right)$$

$$\downarrow 3042$$

$$\tan^{\frac{5}{2}}(c+dx)(e\cot(c+dx))^{5/2}\left(-\frac{1}{3}\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)a+3a}{\sqrt{-\cot\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{2(a\sec(c+dx)+a)}{3d\tan^{\frac{3}{2}}(c+dx)}\right)$$

$$\downarrow 4372$$

$$\tan^{\frac{5}{2}}(c+dx)(e\cot(c+dx))^{5/2}\left(\frac{1}{3}\left(-3a\int \frac{1}{\sqrt{\tan(c+dx)}}dx-a\int \frac{\sec(c+dx)}{\sqrt{\tan(c+dx)}}dx\right)-\frac{2(a\sec(c+dx)+a)}{3d\tan^{\frac{3}{2}}(c+dx)}\right)$$

$$\downarrow 3042$$

$$\tan^{\frac{5}{2}}(c+dx)(e\cot(c+dx))^{5/2}\left(\frac{1}{3}\left(-3a\int \frac{1}{\sqrt{\tan(c+dx)}}dx-a\int \frac{\sec(c+dx)}{\sqrt{\tan(c+dx)}}dx\right)-\frac{2(a\sec(c+dx)+a)}{3d\tan^{\frac{3}{2}}(c+dx)}\right)$$

$$\downarrow 3094$$

$$dx))^{5/2}\left(\frac{1}{3}\left(-3a\int\frac{1}{\sqrt{\tan(c+dx)}}dx-\frac{\tan^{\frac{5}{2}}(c+dx)(e\cot(c+dx))^{5/2}\left(a\sqrt{\sin(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}\sqrt{\sin(c+dx)}}dx\right)}{\sqrt{\cos(c+dx)}\sqrt{\tan(c+dx)}}\right)-\frac{2(a\sec(c+dx)+a)}{3d\tan^{\frac{3}{2}}(c+dx)}\right)$$

$$\downarrow 3042$$

$$dx))^{5/2}\left(\frac{1}{3}\left(-3a\int\frac{1}{\sqrt{\tan(c+dx)}}dx-\frac{\tan^{\frac{5}{2}}(c+dx)(e\cot(c+dx))^{5/2}\left(a\sqrt{\sin(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}\sqrt{\sin(c+dx)}}dx\right)}{\sqrt{\cos(c+dx)}\sqrt{\tan(c+dx)}}\right)-\frac{2(a\sec(c+dx)+a)}{3d\tan^{\frac{3}{2}}(c+dx)}\right)$$

$$\downarrow 3053$$

$$dx))^{5/2}\left(\frac{1}{3}\left(-3a\int\frac{1}{\sqrt{\tan(c+dx)}}dx-\frac{\tan^{\frac{5}{2}}(c+dx)(e\cot(c+dx))^{5/2}\left(a\sqrt{\sin(2c+2dx)}\sec(c+dx)\int\frac{1}{\sqrt{\sin(2c+2dx)}}dx\right)}{\sqrt{\tan(c+dx)}}\right)-\frac{2(a\sec(c+dx)+a)}{3d\tan^{\frac{3}{2}}(c+dx)}\right)$$

$$\downarrow 3042$$

$$dx))^{5/2}\left(\frac{1}{3}\left(-3a\int\frac{1}{\sqrt{\tan(c+dx)}}dx-\frac{\tan^{\frac{5}{2}}(c+dx)(e\cot(c+dx))^{5/2}\left(a\sqrt{\sin(2c+2dx)}\sec(c+dx)\int\frac{1}{\sqrt{\sin(2c+2dx)}}dx\right)}{\sqrt{\tan(c+dx)}}\right)-\frac{2(a\sec(c+dx)+a)}{3d\tan^{\frac{3}{2}}(c+dx)}\right)$$

$$\downarrow 3120$$

$$dx))^{5/2}\left(\frac{1}{3}\left(-3a\int\frac{1}{\sqrt{\tan(c+dx)}}dx-\frac{\tan^{\frac{5}{2}}(c+dx)(e\cot(c+dx))^{5/2}\left(a\sqrt{\sin(2c+2dx)}\sec(c+dx)\,\text{EllipticF}\left(c+dx-\frac{\pi}{4},2\right)\right)}{d\sqrt{\tan(c+dx)}}\right)-\frac{2(a\sec(c+dx)+a)}{3d\tan^{\frac{3}{2}}(c+dx)}\right)$$

$$\downarrow 3957$$

$$dx))^{5/2}\left(\frac{1}{3}\left(-\frac{3a\int\frac{1}{\sqrt{\tan(c+dx)}(\tan^2(c+dx)+1)}d\tan(c+dx)}{d}-\frac{\tan^{\frac{5}{2}}(c+dx)(e\cot(c+dx))^{5/2}\left(a\sqrt{\sin(2c+2dx)}\sec(c+dx)\,\text{EllipticF}\left(c+dx-\frac{\pi}{4},2\right)\right)}{d\sqrt{\tan(c+dx)}}\right)-\frac{2(a\sec(c+dx)+a)}{3d\tan^{\frac{3}{2}}(c+dx)}\right)$$

$$\downarrow 266$$

$$dx))^{5/2}\left(\frac{1}{3}\left(-\frac{6a\int\frac{1}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}}{d}-\frac{\tan^{\frac{5}{2}}(c+dx)(e\cot(c+dx))^{5/2}\left(a\sqrt{\sin(2c+2dx)}\sec(c+dx)\,\text{EllipticF}\left(c+dx-\frac{\pi}{4},2\right)\right)}{d\sqrt{\tan(c+dx)}}\right)-\frac{2(a\sec(c+dx)+a)}{3d\tan^{\frac{3}{2}}(c+dx)}\right)$$

$$\downarrow 755$$

$$\tan^{\frac{5}{2}}(c+dx)(e\cot(c+dx))^{5/2}\left(\frac{1}{3}\left(-\frac{6a\left(\frac{1}{2}\int\frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}+\frac{1}{2}\int\frac{\tan(c+dx)+1}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}\right)}{d}-\frac{a\sqrt{\sin(2c+2dx)}\sec(c+$$

$$\downarrow 1476$$

$$\tan^{\frac{5}{2}}(c+dx)(e\cot(c+dx))^{5/2}\left(\frac{1}{3}\left(-\frac{6a\left(\frac{1}{2}\int\frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}+\frac{1}{2}\left(\frac{1}{2}\int\frac{1}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}+\frac{1}{2}\int\frac{}{\tan(c+d}\right.\right.\right.}{d}$$

$$\downarrow 1082$$

$$\tan^{\frac{5}{2}}(c+dx)(e\cot(c+dx))^{5/2}\left(\frac{1}{3}\left(-\frac{6a\left(\frac{1}{2}\int\frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}+\frac{1}{2}\left(\frac{\int\frac{1}{-\tan(c+dx)-1}d\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}}-\frac{\int\frac{1}{-\tan(c+dx)-1}d\left(\sqrt{2}\sqrt{\tan(}\right.\right.\right.\right.}{d}$$

$$\downarrow 217$$

$$\tan^{\frac{5}{2}}(c+dx)(e\cot(c+dx))^{5/2}\left(\frac{1}{3}\left(-\frac{6a\left(\frac{1}{2}\int\frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}}\right)\right)}{d}-\frac{a}{}\right.\right.\right.$$

$$\downarrow 1479$$

$$\tan^{\frac{5}{2}}(c+dx)(e\cot(c+dx))^{5/2}\left(\frac{1}{3}\left(-\frac{6a\left(\frac{1}{2}\left(-\frac{\int-\frac{\sqrt{2}-2\sqrt{\tan(c+dx)}}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}-\frac{\int-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\arctan\right.\right.}{d}\right.\right.\right.$$

$$\downarrow 25$$

$$\tan^{\frac{5}{2}}(c+dx)(e\cot(c+dx))^{5/2}\left(\frac{1}{3}\left(-\frac{6a\left(\frac{1}{2}\left(\frac{\int\frac{\sqrt{2}-2\sqrt{\tan(c+dx)}}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}+\frac{\int\frac{\sqrt{2}\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\arctan\left(\sqrt{2}\sqrt{\right.\right.\right.\right.}{d}\right.\right.\right.$$

$$\downarrow 27$$

$$\tan^{\frac{5}{2}}(c+dx)(e\cot(c+dx))^{5/2}\left(\frac{1}{3}\left(-\frac{6a\left(\frac{1}{2}\left(\frac{\int\frac{\sqrt{2}-2\sqrt{\tan(c+dx)}}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}+\frac{1}{2}\int\frac{\sqrt{2}\sqrt{\tan(c+dx)}+1}{\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}\right)\right)+\frac{1}{2}\left(\text{a}\right.}{d}\right.\right.$$

$$\downarrow 1103$$

$$\tan^{\frac{5}{2}}(c+dx)(e\cot(c+dx))^{5/2}\left(\frac{1}{3}\left(-\frac{6a\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}}\right)\right)+\frac{1}{2}\left(\frac{\log\left(\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}}-\text{log}\right.}{d}\right.\right.$$

input
```
Int[(e*Cot[c + d*x])^(5/2)*(a + a*Sec[c + d*x]),x]
```

output
```
(e*Cot[c + d*x])^(5/2)*(((-6*a*((-(ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]]/
Sqrt[2]) + ArcTan[1 + Sqrt[2]*Sqrt[Tan[c + d*x]]]/Sqrt[2])/2 + (-1/2*Log[1
 - Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]]/Sqrt[2] + Log[1 + Sqrt[2]*Sq
rt[Tan[c + d*x]] + Tan[c + d*x]]/(2*Sqrt[2]))/2))/d - (a*EllipticF[c - Pi/
4 + d*x, 2]*Sec[c + d*x]*Sqrt[Sin[2*c + 2*d*x]])/(d*Sqrt[Tan[c + d*x]]))/3
 - (2*(a + a*Sec[c + d*x]))/(3*d*Tan[c + d*x]^(3/2)))*Tan[c + d*x]^(5/2)
```

3.233.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3053
```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Simp[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]])   Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f
}, x]
```

rule 3094
```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]], x_Symbol]
:> Simp[Sqrt[Sin[e + f*x]]/(Sqrt[Cos[e + f*x]]*Sqrt[b*Tan[e + f*x]])   Int[
1/(Sqrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]]), x], x] /; FreeQ[{b, e, f}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

rule 4370
```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(-(e*Cot[c + d*x])^(m + 1))*((a + b*Csc[c + d*x])/(
d*e*(m + 1))), x] - Simp[1/(e^2*(m + 1))   Int[(e*Cot[c + d*x])^(m + 2)*(a*
(m + 1) + b*(m + 2)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && L
tQ[m, -1]
```

rule 4372
```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(e*Cot[c + d*x])^m, x], x] + Simp[b   Int[
(e*Cot[c + d*x])^m*Csc[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, m}, x]
```

rule 4388
```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*((a_) + (b_.)*sec[(c_.) + (d_.)*(x
_)])^(n_.), x_Symbol] :> Simp[(e*Cot[c + d*x])^m*Tan[c + d*x]^m   Int[(a +
b*Sec[c + d*x])^n/Tan[c + d*x]^m, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x]
 &&  !IntegerQ[m]
```

3.233.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 10.22 (sec) , antiderivative size = 494, normalized size of antiderivative = 1.54

method	result
default	$-\frac{e^2 a\sqrt{2}\,(\cos(dx+c)+1)\left(3i\sin(dx+c)\sqrt{\csc(dx+c)-\cot(dx+c)+1}\,\sqrt{\cot(dx+c)-\csc(dx+c)+1}\,\sqrt{\cot(dx+c)-\csc(dx+c)}\ \text{EllipticPi}\left($
parts	$2ae\left(\frac{(e\cot(dx+c))^{\frac{3}{2}}}{3}-\frac{e^2\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)-\left(e^2\right)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\,\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)+\left(e^2\right)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\,\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{e\cot(dx+c)}}{\left(e^2\right)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\frac{\sqrt{2}\,\sqrt{e\cot(dx+}\right.\right.}{8\left(e^2\right)^{\frac{1}{4}}}\right.$
	$-\frac{\left.\left.\right)\right)}{d}$

input
```
int((e*cot(d*x+c))^(5/2)*(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
-1/6*e^2*a/d*2^(1/2)*(cos(d*x+c)+1)*(3*I*sin(d*x+c)*(csc(d*x+c)-cot(d*x+c)
+1)^(1/2)*(cot(d*x+c)-csc(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)*El
lipticPi((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2-1/2*I,1/2*2^(1/2))-3*I*sin(d*
x+c)*(csc(d*x+c)-cot(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c)+1)^(1/2)*(cot(
d*x+c)-csc(d*x+c))^(1/2)*EllipticPi((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2+1/
2*I,1/2*2^(1/2))+3*sin(d*x+c)*(cot(d*x+c)-csc(d*x+c))^(1/2)*(csc(d*x+c)-co
t(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c)+1)^(1/2)*EllipticPi((csc(d*x+c)-c
ot(d*x+c)+1)^(1/2),1/2-1/2*I,1/2*2^(1/2))+3*sin(d*x+c)*(cot(d*x+c)-csc(d*x
+c))^(1/2)*(csc(d*x+c)-cot(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c)+1)^(1/2)
*EllipticPi((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2+1/2*I,1/2*2^(1/2))-4*sin(d
*x+c)*(cot(d*x+c)-csc(d*x+c))^(1/2)*(csc(d*x+c)-cot(d*x+c)+1)^(1/2)*(cot(d
*x+c)-csc(d*x+c)+1)^(1/2)*EllipticF((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2*2^
(1/2))+2*2^(1/2)*cos(d*x+c))*(e*cot(d*x+c))^(1/2)*sec(d*x+c)*csc(d*x+c)
```

3.233.5 Fricas [F(-1)]

Timed out.

$$\int (e\cot(c+dx))^{5/2}(a+a\sec(c+dx))\,dx = \text{Timed out}$$

input
```
integrate((e*cot(d*x+c))^(5/2)*(a+a*sec(d*x+c)),x, algorithm="fricas")
```

output
```
Timed out
```

3.233.6 Sympy [F(-1)]

Timed out.

$$\int (e\cot(c+dx))^{5/2}(a+a\sec(c+dx))\,dx = \text{Timed out}$$

input
```
integrate((e*cot(d*x+c))**(5/2)*(a+a*sec(d*x+c)),x)
```

output
```
Timed out
```

3.233.7 Maxima [F(-2)]

Exception generated.

$$\int (e\cot(c+dx))^{5/2}(a+a\sec(c+dx))\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((e*cot(d*x+c))^(5/2)*(a+a*sec(d*x+c)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.233.8 Giac [F]

$$\int (e\cot(c+dx))^{5/2}(a+a\sec(c+dx))\,dx = \int (e\cot(dx+c))^{\frac{5}{2}}(a\sec(dx+c)+a)\,dx$$

input
```
integrate((e*cot(d*x+c))^(5/2)*(a+a*sec(d*x+c)),x, algorithm="giac")
```

output
```
integrate((e*cot(d*x + c))^(5/2)*(a*sec(d*x + c) + a), x)
```

3.233.9 Mupad [F(-1)]

Timed out.

$$\int (e\cot(c+dx))^{5/2}(a+a\sec(c+dx))\,dx = \int (e\cot(c+d\,x))^{5/2}\left(a+\frac{a}{\cos(c+d\,x)}\right)\,dx$$

input
```
int((e*cot(c + d*x))^(5/2)*(a + a/cos(c + d*x)),x)
```

output
```
int((e*cot(c + d*x))^(5/2)*(a + a/cos(c + d*x)), x)
```

3.234 $\int (e\cot(c+dx))^{3/2}(a+a\sec(c+dx))\,dx$

3.234.1 Optimal result 1690
3.234.2 Mathematica [C] (verified) 1691
3.234.3 Rubi [A] (verified) 1691
3.234.4 Maple [A] (verified) 1698
3.234.5 Fricas [F(-1)] 1699
3.234.6 Sympy [F] 1699
3.234.7 Maxima [F(-2)] 1699
3.234.8 Giac [F] 1700
3.234.9 Mupad [F(-1)] 1700

3.234.1 Optimal result

Integrand size = 23, antiderivative size = 346

$$
\begin{aligned}
&\int (e\cot(c+dx))^{3/2}(a+a\sec(c+dx))\,dx = \\
&-\frac{2(e\cot(c+dx))^{3/2}(a+a\sec(c+dx))\tan(c+dx)}{d} \\
&-\frac{2a(e\cot(c+dx))^{3/2}E\left(c-\frac{\pi}{4}+dx\middle|2\right)\sin(c+dx)\tan(c+dx)}{d\sqrt{\sin(2c+2dx)}} \\
&+\frac{a\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)(e\cot(c+dx))^{3/2}\tan^{\frac{3}{2}}(c+dx)}{\sqrt{2}d} \\
&-\frac{a\arctan\left(1+\sqrt{2}\sqrt{\tan(c+dx)}\right)(e\cot(c+dx))^{3/2}\tan^{\frac{3}{2}}(c+dx)}{\sqrt{2}d} \\
&-\frac{a(e\cot(c+dx))^{3/2}\log\left(1-\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+dx)\right)\tan^{\frac{3}{2}}(c+dx)}{2\sqrt{2}d} \\
&+\frac{a(e\cot(c+dx))^{3/2}\log\left(1+\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+dx)\right)\tan^{\frac{3}{2}}(c+dx)}{2\sqrt{2}d} \\
&+\frac{2a(e\cot(c+dx))^{3/2}\sin(c+dx)\tan^2(c+dx)}{d}
\end{aligned}
$$

output
```
-2*(e*cot(d*x+c))^(3/2)*(a+a*sec(d*x+c))*tan(d*x+c)/d+2*a*(e*cot(d*x+c))^(
3/2)*(sin(c+1/4*Pi+d*x)^2)^(1/2)/sin(c+1/4*Pi+d*x)*EllipticE(cos(c+1/4*Pi+
d*x),2^(1/2))*sin(d*x+c)*tan(d*x+c)/d/sin(2*d*x+2*c)^(1/2)-1/2*a*arctan(-1
+2^(1/2)*tan(d*x+c)^(1/2))*(e*cot(d*x+c))^(3/2)*tan(d*x+c)^(3/2)/d*2^(1/2)
-1/2*a*arctan(1+2^(1/2)*tan(d*x+c)^(1/2))*(e*cot(d*x+c))^(3/2)*tan(d*x+c)^
(3/2)/d*2^(1/2)-1/4*a*(e*cot(d*x+c))^(3/2)*ln(1-2^(1/2)*tan(d*x+c)^(1/2)+t
an(d*x+c))*tan(d*x+c)^(3/2)/d*2^(1/2)+1/4*a*(e*cot(d*x+c))^(3/2)*ln(1+2^(1
/2)*tan(d*x+c)^(1/2)+tan(d*x+c))*tan(d*x+c)^(3/2)/d*2^(1/2)+2*a*(e*cot(d*x
+c))^(3/2)*sin(d*x+c)*tan(d*x+c)^2/d
```

3.234.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 1.24 (sec) , antiderivative size = 191, normalized size of antiderivative = 0.55

$$\int (e\cot(c+dx))^{3/2}(a + a\sec(c+dx))\,dx = \frac{ae(1+\cos(c+dx))\sqrt{e\cot(c+dx)}\sec^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\left(8\cot^2(c+dx)\,\text{Hyperg}\right.$$

input
```
Integrate[(e*Cot[c + d*x])^(3/2)*(a + a*Sec[c + d*x]),x]
```

output
```
(a*e*(1 + Cos[c + d*x])*Sqrt[e*Cot[c + d*x]]*Sec[(c + d*x)/2]^2*Sec[c + d*
x]*(8*Cot[c + d*x]^2*Hypergeometric2F1[3/4, 3/2, 7/4, -Cot[c + d*x]^2] + 3
*Sqrt[Csc[c + d*x]^2]*(-4*Cos[c + d*x] - 4*Cos[c + d*x]^2 + ArcSin[Cos[c +
 d*x] - Sin[c + d*x]]*Sqrt[Sin[2*(c + d*x)]] + Log[Cos[c + d*x] + Sin[c +
d*x] + Sqrt[Sin[2*(c + d*x)]]]*Sqrt[Sin[2*(c + d*x)]])))/(12*d*Sqrt[Csc[c
+ d*x]^2])
```

3.234.3 Rubi [A] (verified)

Time = 1.12 (sec) , antiderivative size = 257, normalized size of antiderivative = 0.74, number of steps used = 26, number of rules used = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 1.087$, Rules used = {3042, 4388, 3042, 4370, 27, 3042, 4372, 3042, 3093, 3042, 3095, 3042, 3052, 3042, 3119, 3957, 266, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sec(c+dx)+a)(e\cot(c+dx))^{3/2}\,dx$$

$$\downarrow 3042$$

$$\int (a\sec(c+dx)+a)(e\cot(c+dx))^{3/2}dx$$

$$\downarrow 4388$$

$$\tan^{\frac{3}{2}}(c+dx)(e\cot(c+dx))^{3/2}\int \frac{\sec(c+dx)a+a}{\tan^{\frac{3}{2}}(c+dx)}dx$$

$$\downarrow 3042$$

$$\tan^{\frac{3}{2}}(c+dx)(e\cot(c+dx))^{3/2}\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}{\left(-\cot\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx$$

$$\downarrow 4370$$

$$\tan^{\frac{3}{2}}(c+dx)(e\cot(c+dx))^{3/2}\left(2\int -\frac{1}{2}(a-a\sec(c+dx))\sqrt{\tan(c+dx)}dx-\frac{2(a\sec(c+dx)+a)}{d\sqrt{\tan(c+dx)}}\right)$$

$$\downarrow 27$$

$$\tan^{\frac{3}{2}}(c+dx)(e\cot(c+dx))^{3/2}\left(-\int (a-a\sec(c+dx))\sqrt{\tan(c+dx)}dx-\frac{2(a\sec(c+dx)+a)}{d\sqrt{\tan(c+dx)}}\right)$$

$$\downarrow 3042$$

$$\tan^{\frac{3}{2}}(c+dx)(e\cot(c+dx))^{3/2}\left(-\int \sqrt{-\cot\left(c+dx+\frac{\pi}{2}\right)}\left(a-a\csc\left(c+dx+\frac{\pi}{2}\right)\right)dx-\frac{2(a\sec(c+dx)+a)}{d\sqrt{\tan(c+dx)}}\right)$$

$$\downarrow 4372$$

$$\tan^{\frac{3}{2}}(c+dx)(e\cot(c+dx))^{3/2}\left(-a\int \sqrt{\tan(c+dx)}dx+a\int \sec(c+dx)\sqrt{\tan(c+dx)}dx-\frac{2(a\sec(c+dx)+a)}{d\sqrt{\tan(c+dx)}}\right)$$

$$\downarrow 3042$$

$$\tan^{\frac{3}{2}}(c+dx)(e\cot(c+dx))^{3/2}\left(-a\int \sqrt{\tan(c+dx)}dx+a\int \sec(c+dx)\sqrt{\tan(c+dx)}dx-\frac{2(a\sec(c+dx)+a)}{d\sqrt{\tan(c+dx)}}\right)$$

$$\downarrow 3093$$

$$\tan^{\frac{3}{2}}(c+dx)(e\cot(c+dx))^{3/2}\left(-a\int\sqrt{\tan(c+dx)}dx+a\left(\frac{2\cos(c+dx)\tan^{\frac{3}{2}}(c+dx)}{d}-2\int\cos(c+dx)\sqrt{\tan(c+dx)}dx\right)-\frac{2(a\sec(c+dx)+a)}{d\sqrt{\tan(c+dx)}}\right)$$

$$\downarrow 3042$$

$$\tan^{\frac{3}{2}}(c+dx)(e\cot(c+dx))^{3/2}\left(-a\int\sqrt{\tan(c+dx)}dx+a\left(\frac{2\cos(c+dx)\tan^{\frac{3}{2}}(c+dx)}{d}-2\int\frac{\sqrt{\tan(c+dx)}}{\sec(c+dx)}dx\right)-\frac{2(a\sec(c+dx)+a)}{d\sqrt{\tan(c+dx)}}\right)$$

$$\downarrow 3095$$

$$\tan^{\frac{3}{2}}(c+dx)(e\cot(c+dx))^{3/2}\left(-a\int\sqrt{\tan(c+dx)}dx+a\left(\frac{2\cos(c+dx)\tan^{\frac{3}{2}}(c+dx)}{d}-\frac{2\sqrt{\cos(c+dx)}\sqrt{\tan(c+dx)}\int\sqrt{\cos(c+dx)}}{\sqrt{\sin(c+dx)}}\right)\right.$$

$$\downarrow 3042$$

$$\tan^{\frac{3}{2}}(c+dx)(e\cot(c+dx))^{3/2}\left(-a\int\sqrt{\tan(c+dx)}dx+a\left(\frac{2\cos(c+dx)\tan^{\frac{3}{2}}(c+dx)}{d}-\frac{2\sqrt{\cos(c+dx)}\sqrt{\tan(c+dx)}\int\sqrt{\cos(c+dx)}}{\sqrt{\sin(c+dx)}}\right)\right.$$

$$\downarrow 3052$$

$$\tan^{\frac{3}{2}}(c+dx)(e\cot(c+dx))^{3/2}\left(-a\int\sqrt{\tan(c+dx)}dx+a\left(\frac{2\cos(c+dx)\tan^{\frac{3}{2}}(c+dx)}{d}-\frac{2\cos(c+dx)\sqrt{\tan(c+dx)}\int\sqrt{\sin(2c+2dx)}}{\sqrt{\sin(2c+2dx)}}\right)\right.$$

$$\downarrow 3042$$

$$\tan^{\frac{3}{2}}(c+dx)(e\cot(c+dx))^{3/2}\left(-a\int\sqrt{\tan(c+dx)}dx+a\left(\frac{2\cos(c+dx)\tan^{\frac{3}{2}}(c+dx)}{d}-\frac{2\cos(c+dx)\sqrt{\tan(c+dx)}\int\sqrt{\sin(2c+2dx)}}{\sqrt{\sin(2c+2dx)}}\right)\right.$$

$$\downarrow 3119$$

$$\tan^{\frac{3}{2}}(c+dx)(e\cot(c+dx))^{3/2}\left(-a\int\sqrt{\tan(c+dx)}dx-\frac{2(a\sec(c+dx)+a)}{d\sqrt{\tan(c+dx)}}+a\left(\frac{2\cos(c+dx)\tan^{\frac{3}{2}}(c+dx)}{d}-\frac{2\cos(c+dx)\sqrt{\tan(c+dx)}}{d\sqrt{\sin(2}}\right.\right.$$

$$\downarrow 3957$$

$$\tan^{\frac{3}{2}}(c+dx)(e\cot(c+dx))^{3/2}\left(-\frac{a\int\frac{\sqrt{\tan(c+dx)}}{\tan^2(c+dx)+1}d\tan(c+dx)}{d}-\frac{2(a\sec(c+dx)+a)}{d\sqrt{\tan(c+dx)}}+a\left(\frac{2\cos(c+dx)\tan^{\frac{3}{2}}(c+dx)}{d}-\frac{2\cos(c+dx)}{}\right.\right.$$

$$\downarrow 266$$

$$\tan^{\frac{3}{2}}(c+dx)(e\cot(c+dx))^{3/2}\left(-\frac{2a\int\frac{\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}}{d}-\frac{2(a\sec(c+dx)+a)}{d\sqrt{\tan(c+dx)}}+a\left(\frac{2\cos(c+dx)\tan^{\frac{3}{2}}(c+dx)}{d}-\frac{2\cos(c+$$

$$\downarrow 826$$

$$\tan^{\frac{3}{2}}(c+dx)(e\cot(c+dx))^{3/2}\left(-\frac{2a\left(\frac{1}{2}\int\frac{\tan(c+dx)+1}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}-\frac{1}{2}\int\frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}\right)}{d}-\frac{2(a\sec(c+dx)+a)}{d\sqrt{\tan(c+dx)}}+a\left(\frac{2}{}\right.\right.$$

$$\downarrow 1476$$

$$\tan^{\frac{3}{2}}(c+dx)(e\cot(c+dx))^{3/2}\left(-\frac{2a\left(\frac{1}{2}\left(\frac{1}{2}\int\frac{1}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}+\frac{1}{2}\int\frac{1}{\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}\right)-\right.}{d}$$

$$\downarrow 1082$$

$$\tan^{\frac{3}{2}}(c+dx)(e\cot(c+dx))^{3/2}\left(-\frac{2a\left(\frac{1}{2}\left(\frac{\int\frac{1}{-\tan(c+dx)-1}d\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}}-\frac{\int\frac{1}{-\tan(c+dx)-1}d\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}}\right)-\frac{1}{2}\int\frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(}\right.}{d}$$

$$\downarrow 217$$

$$\tan^{\frac{3}{2}}(c+dx)(e\cot(c+dx))^{3/2}\left(-\frac{2a\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}}\right)-\frac{1}{2}\int\frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}\right)}{d}-\frac{2(a\,s}{d\sqrt{}}\right.$$

$$\downarrow 1479$$

$$\tan^{\frac{3}{2}}(c+dx)(e\cot(c+dx))^{3/2}\left(-\frac{2a\left(\frac{1}{2}\left(\frac{\int-\frac{\sqrt{2}-2\sqrt{\tan(c+dx)}}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}+\frac{\int-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{}\right.\right.\right.}{d}$$

$$\downarrow 25$$

$$\tan^{\frac{3}{2}}(c+dx)(e\cot(c+dx))^{3/2}\left(-\frac{2a\left(\frac{1}{2}\left(-\frac{\int\frac{\sqrt{2}-2\sqrt{\tan(c+dx)}}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}-\frac{\int\frac{\sqrt{2}\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{ta}\right.\right.}{\sqrt{}}\right.}{d}$$

$$\downarrow 27$$

$$\tan^{\frac{3}{2}}(c+dx)(e\cot(c+dx))^{3/2}\left(-\frac{2a\left(\frac{1}{2}\left(-\frac{\int\frac{\sqrt{2}-2\sqrt{\tan(c+dx)}}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}-\frac{1}{2}\int\frac{\sqrt{2}\sqrt{\tan(c+dx)}+1}{\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}\right)+\frac{1}{2}\left(\frac{\text{arc}}{}\right.\right.}{d}$$

$$\downarrow 1103$$

$$\tan^{\frac{3}{2}}(c+dx)(e\cot(c+dx))^{3/2}\left(-\frac{2a\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\log\left(\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}}-\frac{\log\left(\text{tan}\right.\right.}{}\right.}{d}$$

input
```
Int[(e*Cot[c + d*x])^(3/2)*(a + a*Sec[c + d*x]),x]
```

output
```
(e*Cot[c + d*x])^(3/2)*Tan[c + d*x]^(3/2)*((-2*a*((-(ArcTan[1 - Sqrt[2]*Sq
rt[Tan[c + d*x]]]/Sqrt[2]) + ArcTan[1 + Sqrt[2]*Sqrt[Tan[c + d*x]]]/Sqrt[2
])/2 + (Log[1 - Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]]/(2*Sqrt[2]) - L
og[1 + Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]]/(2*Sqrt[2]))/2))/d - (2*
(a + a*Sec[c + d*x]))/(d*Sqrt[Tan[c + d*x]]) + a*((-2*Cos[c + d*x]*Ellipti
cE[c - Pi/4 + d*x, 2]*Sqrt[Tan[c + d*x]])/(d*Sqrt[Sin[2*c + 2*d*x]]) + (2*
Cos[c + d*x]*Tan[c + d*x]^(3/2))/d))
```

3.234.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3052
```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Simp[Sqrt[a*Sin[e + f*x]]*(Sqrt[b*Cos[e + f*x]]/Sqrt[Sin[2*e
 + 2*f*x]])   Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

rule 3093
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[a^2*(a*Sec[e + f*x])^(m - 2)*((b*Tan[e + f*x])^(n +
1)/(b*f*(m + n - 1))), x] + Simp[a^2*((m - 2)/(m + n - 1))   Int[(a*Sec[e +
 f*x])^(m - 2)*(b*Tan[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (
GtQ[m, 1] || (EqQ[m, 1] && EqQ[n, 1/2])) && NeQ[m + n - 1, 0] && IntegersQ[
2*m, 2*n]
```

rule 3095
```
Int[Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]]/sec[(e_.) + (f_.)*(x_)], x_Symbol]
:> Simp[Sqrt[Cos[e + f*x]]*(Sqrt[b*Tan[e + f*x]]/Sqrt[Sin[e + f*x]])   Int[
Sqrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]], x], x] /; FreeQ[{b, e, f}, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

rule 4370
```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(-(e*Cot[c + d*x])^(m + 1))*((a + b*Csc[c + d*x])/(
d*e*(m + 1))), x] - Simp[1/(e^2*(m + 1))   Int[(e*Cot[c + d*x])^(m + 2)*(a*
(m + 1) + b*(m + 2)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && L
tQ[m, -1]
```

rule 4372
```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(e*Cot[c + d*x])^m, x], x] + Simp[b   Int[
(e*Cot[c + d*x])^m*Csc[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, m}, x]
```

rule 4388
```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*((a_) + (b_.)*sec[(c_.) + (d_.)*(x
_)])^(n_.), x_Symbol] :> Simp[(e*Cot[c + d*x])^m*Tan[c + d*x]^m   Int[(a +
b*Sec[c + d*x])^n/Tan[c + d*x]^m, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x]
 &&  !IntegerQ[m]
```

3.234.4 Maple [A] (verified)

Time = 10.76 (sec) , antiderivative size = 501, normalized size of antiderivative = 1.45

method	result
parts	$-\frac{2ae\left(\sqrt{e\cot(dx+c)}-\frac{\left(e^2\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+\left(e^2\right)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)-\left(e^2\right)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(dx+c)}}{\left(e^2\right)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\frac{\sqrt{2}\sqrt{e\cot}}{\left(e^2\right)}\right.\right.}{8}\right.}{d}$
default	Expression too large to display

input
```
int((e*cot(d*x+c))^(3/2)*(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
-2*a/d*e*((e*cot(d*x+c))^(1/2)-1/8*(e^2)^(1/4)*2^(1/2)*(ln((e*cot(d*x+c)+(
e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2))/(e*cot(d*x+c)-(e^2)^(
1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2)))+2*arctan(2^(1/2)/(e^2)^(1/
4)*(e*cot(d*x+c))^(1/2)+1)-2*arctan(-2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1
/2)+1)))-a/d*2^(1/2)*e*(e*cot(d*x+c))^(1/2)*(-2*(csc(d*x+c)-cot(d*x+c)+1)^
(1/2)*(cot(d*x+c)-csc(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)*Ellipt
icE((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2*2^(1/2))+(csc(d*x+c)-cot(d*x+c)+1)
^(1/2)*(cot(d*x+c)-csc(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)*Ellip
ticF((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2*2^(1/2))-2*(csc(d*x+c)-cot(d*x+c)
+1)^(1/2)*(cot(d*x+c)-csc(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)*El
lipticE((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2*2^(1/2))*sec(d*x+c)+(csc(d*x+c
)-cot(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+
c))^(1/2)*EllipticF((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2*2^(1/2))*sec(d*x+c
)+2^(1/2))
```

3.234.5 Fricas [F(-1)]

Timed out.

$$\int (e\cot(c+dx))^{3/2}(a+a\sec(c+dx))\,dx = \text{Timed out}$$

input
```
integrate((e*cot(d*x+c))^(3/2)*(a+a*sec(d*x+c)),x, algorithm="fricas")
```

output
```
Timed out
```

3.234.6 Sympy [F]

$$\int (e\cot(c+dx))^{3/2}(a+a\sec(c+dx))\,dx = a\left(\int (e\cot(c+dx))^{\frac{3}{2}}\,dx + \int (e\cot(c+dx))^{\frac{3}{2}}\sec(c+dx)\,dx\right)$$

input
```
integrate((e*cot(d*x+c))**(3/2)*(a+a*sec(d*x+c)),x)
```

output
```
a*(Integral((e*cot(c + d*x))**(3/2), x) + Integral((e*cot(c + d*x))**(3/2)
*sec(c + d*x), x))
```

3.234.7 Maxima [F(-2)]

Exception generated.

$$\int (e\cot(c+dx))^{3/2}(a+a\sec(c+dx))\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((e*cot(d*x+c))^(3/2)*(a+a*sec(d*x+c)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.234.8 Giac [F]

$$\int (e\cot(c+dx))^{3/2}(a+a\sec(c+dx))\,dx = \int (e\cot(dx+c))^{\frac{3}{2}}(a\sec(dx+c)+a)\,dx$$

input
```
integrate((e*cot(d*x+c))^(3/2)*(a+a*sec(d*x+c)),x, algorithm="giac")
```

output
```
integrate((e*cot(d*x + c))^(3/2)*(a*sec(d*x + c) + a), x)
```

3.234.9 Mupad [F(-1)]

Timed out.

$$\int (e\cot(c+dx))^{3/2}(a+a\sec(c+dx))\,dx = \int (e\cot(c+dx))^{3/2}\left(a+\frac{a}{\cos(c+dx)}\right)dx$$

input
```
int((e*cot(c + d*x))^(3/2)*(a + a/cos(c + d*x)),x)
```

output
```
int((e*cot(c + d*x))^(3/2)*(a + a/cos(c + d*x)), x)
```

3.235 $\int \sqrt{e\cot(c+dx)}(a+a\sec(c+dx))\,dx$

3.235.1 Optimal result 1701
3.235.2 Mathematica [C] (verified) 1702
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3.235.1 Optimal result

Integrand size = 23, antiderivative size = 274

$$\begin{aligned}
&\int \sqrt{e\cot(c+dx)}(a+a\sec(c+dx))\,dx \\
&= \frac{a\sqrt{e\cot(c+dx)}\,\text{EllipticF}\left(c-\frac{\pi}{4}+dx,2\right)\sec(c+dx)\sqrt{\sin(2c+2dx)}}{d} \\
&\quad - \frac{a\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)\sqrt{e\cot(c+dx)}\sqrt{\tan(c+dx)}}{\sqrt{2}d} \\
&\quad + \frac{a\arctan\left(1+\sqrt{2}\sqrt{\tan(c+dx)}\right)\sqrt{e\cot(c+dx)}\sqrt{\tan(c+dx)}}{\sqrt{2}d} \\
&\quad - \frac{a\sqrt{e\cot(c+dx)}\log\left(1-\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+dx)\right)\sqrt{\tan(c+dx)}}{2\sqrt{2}d} \\
&\quad + \frac{a\sqrt{e\cot(c+dx)}\log\left(1+\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+dx)\right)\sqrt{\tan(c+dx)}}{2\sqrt{2}d}
\end{aligned}$$

output

```
-a*(sin(c+1/4*Pi+d*x)^2)^(1/2)/sin(c+1/4*Pi+d*x)*EllipticF(cos(c+1/4*Pi+d*
x),2^(1/2))*sec(d*x+c)*(e*cot(d*x+c))^(1/2)*sin(2*d*x+2*c)^(1/2)/d+1/2*a*a
rctan(-1+2^(1/2)*tan(d*x+c)^(1/2))*(e*cot(d*x+c))^(1/2)*tan(d*x+c)^(1/2)/d
*2^(1/2)+1/2*a*arctan(1+2^(1/2)*tan(d*x+c)^(1/2))*(e*cot(d*x+c))^(1/2)*tan
(d*x+c)^(1/2)/d*2^(1/2)-1/4*a*ln(1-2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c))*(e
*cot(d*x+c))^(1/2)*tan(d*x+c)^(1/2)/d*2^(1/2)+1/4*a*ln(1+2^(1/2)*tan(d*x+c
)^(1/2)+tan(d*x+c))*(e*cot(d*x+c))^(1/2)*tan(d*x+c)^(1/2)/d*2^(1/2)
```

3.235.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.83 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.62

$$\int \sqrt{e\cot(c+dx)}(a+a\sec(c+dx))\,dx$$

$$= \frac{a(1+\cos(c+dx))\sqrt{e\cot(c+dx)}\sec^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\left(4\sqrt[4]{-1}\sqrt{\cot(c+dx)}\,\text{EllipticF}\left(i\text{arcsinh}\right.\right.}{}$$

input
```
Integrate[Sqrt[e*Cot[c + d*x]]*(a + a*Sec[c + d*x]),x]
```

output
```
(a*(1 + Cos[c + d*x])*Sqrt[e*Cot[c + d*x]]*Sec[(c + d*x)/2]^2*Sec[c + d*x]
*(4*(-1)^(1/4)*Sqrt[Cot[c + d*x]]*EllipticF[I*ArcSinh[(-1)^(1/4)*Sqrt[Cot[
c + d*x]]], -1] + Sqrt[Csc[c + d*x]^2]*(-ArcSin[Cos[c + d*x] - Sin[c + d*x
]] + Log[Cos[c + d*x] + Sin[c + d*x] + Sqrt[Sin[2*(c + d*x)]]])*Sqrt[Sin[2
*(c + d*x)]]))/(4*d*Sqrt[Csc[c + d*x]^2])
```

3.235.3 Rubi [A] (verified)

Time = 0.77 (sec) , antiderivative size = 208, normalized size of antiderivative = 0.76, number of steps used = 21, number of rules used = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.870$, Rules used = {3042, 4388, 3042, 4372, 3042, 3094, 3042, 3053, 3042, 3120, 3957, 266, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sec(c+dx)+a)\sqrt{e\cot(c+dx)}\,dx$$

$$\downarrow 3042$$

$$\int (a\sec(c+dx)+a)\sqrt{e\cot(c+dx)}dx$$

$$\downarrow 4388$$

$$\sqrt{\tan(c+dx)}\sqrt{e\cot(c+dx)}\int \frac{\sec(c+dx)a+a}{\sqrt{\tan(c+dx)}}dx$$

$$\downarrow 3042$$

$$\sqrt{\tan(c+dx)}\sqrt{e\cot(c+dx)}\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}{\sqrt{-\cot\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\downarrow 4372$$

$$\sqrt{\tan(c+dx)}\sqrt{e\cot(c+dx)}\left(a\int\frac{1}{\sqrt{\tan(c+dx)}}dx+a\int\frac{\sec(c+dx)}{\sqrt{\tan(c+dx)}}dx\right)$$

$$\downarrow 3042$$

$$\sqrt{\tan(c+dx)}\sqrt{e\cot(c+dx)}\left(a\int\frac{1}{\sqrt{\tan(c+dx)}}dx+a\int\frac{\sec(c+dx)}{\sqrt{\tan(c+dx)}}dx\right)$$

$$\downarrow 3094$$

$$\sqrt{\tan(c+dx)}\sqrt{e\cot(c+dx)}\left(a\int\frac{1}{\sqrt{\tan(c+dx)}}dx+\frac{a\sqrt{\sin(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}\sqrt{\sin(c+dx)}}dx}{\sqrt{\cos(c+dx)}\sqrt{\tan(c+dx)}}\right)$$

$$\downarrow 3042$$

$$\sqrt{\tan(c+dx)}\sqrt{e\cot(c+dx)}\left(a\int\frac{1}{\sqrt{\tan(c+dx)}}dx+\frac{a\sqrt{\sin(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}\sqrt{\sin(c+dx)}}dx}{\sqrt{\cos(c+dx)}\sqrt{\tan(c+dx)}}\right)$$

$$\downarrow 3053$$

$$\sqrt{\tan(c+dx)}\sqrt{e\cot(c+dx)}\left(a\int\frac{1}{\sqrt{\tan(c+dx)}}dx+\frac{a\sqrt{\sin(2c+2dx)}\sec(c+dx)\int\frac{1}{\sqrt{\sin(2c+2dx)}}dx}{\sqrt{\tan(c+dx)}}\right)$$

$$\downarrow 3042$$

$$\sqrt{\tan(c+dx)}\sqrt{e\cot(c+dx)}\left(a\int\frac{1}{\sqrt{\tan(c+dx)}}dx+\frac{a\sqrt{\sin(2c+2dx)}\sec(c+dx)\int\frac{1}{\sqrt{\sin(2c+2dx)}}dx}{\sqrt{\tan(c+dx)}}\right)$$

$$\downarrow 3120$$

$$\sqrt{\tan(c+dx)}\sqrt{e\cot(c+dx)}\left(a\int\frac{1}{\sqrt{\tan(c+dx)}}dx+\frac{a\sqrt{\sin(2c+2dx)}\sec(c+dx)\,\text{EllipticF}\left(c+dx-\frac{\pi}{4},2\right)}{d\sqrt{\tan(c+dx)}}\right)$$

$$\downarrow 3957$$

$$\sqrt{\tan(c+dx)}\sqrt{e\cot(c+dx)}\left(\frac{a\int\frac{1}{\sqrt{\tan(c+dx)}(\tan^2(c+dx)+1)}d\tan(c+dx)}{d}+\frac{a\sqrt{\sin(2c+2dx)}\sec(c+dx)\,\text{EllipticF}\right.$$

$$\downarrow 266$$

$$\sqrt{\tan(c+dx)}\sqrt{e\cot(c+dx)}\left(\frac{2a\int\frac{1}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}}{d}+\frac{a\sqrt{\sin(2c+2dx)}\sec(c+dx)\,\text{EllipticF}\left(c+dx\right.}{d\sqrt{\tan(c+dx)}}\right.$$

$$\downarrow 755$$

$$\sqrt{\tan(c+dx)}\sqrt{e\cot(c+dx)}\left(\frac{2a\left(\frac{1}{2}\int\frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}+\frac{1}{2}\int\frac{\tan(c+dx)+1}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}\right)}{d}+\frac{a\sqrt{\sin(2}}{}\right.$$

$$\downarrow 1476$$

$$\sqrt{\tan(c+dx)}\sqrt{e\cot(c+dx)}\left(\frac{2a\left(\frac{1}{2}\int\frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}+\frac{1}{2}\left(\frac{1}{2}\int\frac{1}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+d}\right.\right.\right.$$

$$\downarrow 1082$$

$$\sqrt{\tan(c+dx)}\sqrt{e\cot(c+dx)}\left(\frac{2a\left(\frac{1}{2}\int\frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}+\frac{1}{2}\left(\frac{\int\frac{1}{-\tan(c+dx)-1}d\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}}-\frac{\int\frac{}{-\tan(}}{}\right.\right.\right.\right.$$

$$\downarrow 217$$

$$\sqrt{\tan(c+dx)}\sqrt{e\cot(c+dx)}\left(\frac{2a\left(\frac{1}{2}\int\frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\tan}\right.}{\sqrt{2}}\right.\right.\right.\right.$$

$$\downarrow 1479$$

$$\sqrt{\tan(c+dx)}\sqrt{e\cot(c+dx)}\left(\frac{2a\left(\frac{1}{2}\left(-\frac{\int-\frac{\sqrt{2}-2\sqrt{\tan(c+dx)}}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}-\frac{\int-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+}}{2\sqrt{2}}\right.\right.\right.\right.\right.$$

$$\downarrow 25$$

$$\sqrt{\tan(c+dx)}\sqrt{e\cot(c+dx)}\left(\frac{2a\left(\frac{1}{2}\left(\frac{\int\frac{\sqrt{2}-2\sqrt{\tan(c+dx)}}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}+\frac{\int\frac{\sqrt{2}\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}\right)\right.}{d}\right.$$

$$\downarrow 27$$

$$\sqrt{\tan(c+dx)}\sqrt{e\cot(c+dx)}\left(\frac{2a\left(\frac{1}{2}\left(\frac{\int\frac{\sqrt{2}-2\sqrt{\tan(c+dx)}}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}+\frac{1}{2}\int\frac{\sqrt{2}\sqrt{\tan(c+dx)}+1}{\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(\right.\right.\right.}{d}\right.$$

$$\downarrow 1103$$

$$\sqrt{\tan(c+dx)}\sqrt{e\cot(c+dx)}\left(\frac{2a\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\log\left(\tan(c+dx)+\sqrt{2}\sqrt{t}\right.\right.\right.}{2\sqrt{2}}\right.}{d}\right.$$

input
```
Int[Sqrt[e*Cot[c + d*x]]*(a + a*Sec[c + d*x]),x]
```

output
```
Sqrt[e*Cot[c + d*x]]*((2*a*((-(ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]]/Sqrt
[2]) + ArcTan[1 + Sqrt[2]*Sqrt[Tan[c + d*x]]]/Sqrt[2])/2 + (-1/2*Log[1 - S
qrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]]/Sqrt[2] + Log[1 + Sqrt[2]*Sqrt[T
an[c + d*x]] + Tan[c + d*x]]/(2*Sqrt[2]))/2))/d + (a*EllipticF[c - Pi/4 +
d*x, 2]*Sec[c + d*x]*Sqrt[Sin[2*c + 2*d*x]])/(d*Sqrt[Tan[c + d*x]]))*Sqrt[
Tan[c + d*x]]
```

3.235.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3053
```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Simp[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]])   Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f
}, x]
```

rule 3094
```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]], x_Symbol]
:> Simp[Sqrt[Sin[e + f*x]]/(Sqrt[Cos[e + f*x]]*Sqrt[b*Tan[e + f*x]])   Int[
1/(Sqrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]]), x], x] /; FreeQ[{b, e, f}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

rule 4372
```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(e*Cot[c + d*x])^m, x], x] + Simp[b   Int[
(e*Cot[c + d*x])^m*Csc[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, m}, x]
```

rule 4388
```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*((a_) + (b_.)*sec[(c_.) + (d_.)*(x
_)])^(n_.), x_Symbol] :> Simp[(e*Cot[c + d*x])^m*Tan[c + d*x]^m   Int[(a +
b*Sec[c + d*x])^n/Tan[c + d*x]^m, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x]
 &&  !IntegerQ[m]
```

3.235.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 9.35 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.51

method	result
default	$\frac{(\frac{1}{2}-\frac{i}{2})a\sqrt{2}\left(i\,\text{EllipticPi}\left(\sqrt{\csc(dx+c)-\cot(dx+c)+1},\frac{1}{2}-\frac{i}{2},\frac{\sqrt{2}}{2}\right)+\text{EllipticPi}\left(\sqrt{\csc(dx+c)-\cot(dx+c)+1},\frac{1}{2}+\frac{i}{2},\frac{\sqrt{2}}{2}\right)\right)\sqrt{e\cot(dx+c}}{d}$
parts	$-\frac{ae\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)-\left(e^2\right)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\,\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)+\left(e^2\right)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\,\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(dx+c)}}{\left(e^2\right)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\frac{\sqrt{2}\sqrt{e\cot(dx+c)}}{\left(e^2\right)^{\frac{1}{4}}}+1\right)\right)}{4d\left(e^2\right)^{\frac{1}{4}}}+\frac{a}{-}$

input
```
int((a+a*sec(d*x+c))*(e*cot(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(1/2-1/2*I)*a/d*2^(1/2)*(I*EllipticPi((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2-
1/2*I,1/2*2^(1/2))+EllipticPi((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2+1/2*I,1/
2*2^(1/2)))*(e*cot(d*x+c))^(1/2)*(csc(d*x+c)-cot(d*x+c)+1)^(1/2)*(cot(d*x+
c)-csc(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)*(1+sec(d*x+c))
```

3.235.5 Fricas [F(-1)]

Timed out.

$$\int \sqrt{e\cot(c+dx)}(a+a\sec(c+dx))\,dx = \text{Timed out}$$

input
```
integrate((a+a*sec(d*x+c))*(e*cot(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.235.6 Sympy [F]

$$\int \sqrt{e\cot(c+dx)}(a+a\sec(c+dx))\,dx = a\left(\int \sqrt{e\cot(c+dx)}\,dx + \int \sqrt{e\cot(c+dx)}\sec(c+dx)\,dx\right)$$

input
```
integrate((a+a*sec(d*x+c))*(e*cot(d*x+c))**(1/2),x)
```

output
```
a*(Integral(sqrt(e*cot(c + d*x)), x) + Integral(sqrt(e*cot(c + d*x))*sec(c
 + d*x), x))
```

3.235.7 Maxima [F(-2)]

Exception generated.

$$\int \sqrt{e\cot(c+dx)}(a+a\sec(c+dx))\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+a*sec(d*x+c))*(e*cot(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.235.8 Giac [F]

$$\int \sqrt{e\cot(c+dx)}(a+a\sec(c+dx))\,dx = \int \sqrt{e\cot(dx+c)}(a\sec(dx+c)+a)\,dx$$

input
```
integrate((a+a*sec(d*x+c))*(e*cot(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(e*cot(d*x + c))*(a*sec(d*x + c) + a), x)
```

3.235.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{e\cot(c+dx)}(a+a\sec(c+dx))\,dx = \int \sqrt{e\cot(c+dx)}\left(a+\frac{a}{\cos(c+dx)}\right)\,dx$$

input
```
int((e*cot(c + d*x))^(1/2)*(a + a/cos(c + d*x)),x)
```

output
```
int((e*cot(c + d*x))^(1/2)*(a + a/cos(c + d*x)), x)
```

3.236 $\int \frac{a+a\sec(c+dx)}{\sqrt{e\cot(c+dx)}}\,dx$

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3.236.1 Optimal result

Integrand size = 23, antiderivative size = 299

$$\begin{aligned}\int \frac{a+a\sec(c+dx)}{\sqrt{e\cot(c+dx)}}\,dx &= \frac{2a\sin(c+dx)}{d\sqrt{e\cot(c+dx)}} - \frac{2a\cos(c+dx)E\left(c-\frac{\pi}{4}+dx\middle|2\right)}{d\sqrt{e\cot(c+dx)}\sqrt{\sin(2c+2dx)}} \\ &\quad - \frac{a\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}d\sqrt{e\cot(c+dx)}\sqrt{\tan(c+dx)}} \\ &\quad + \frac{a\arctan\left(1+\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}d\sqrt{e\cot(c+dx)}\sqrt{\tan(c+dx)}} \\ &\quad + \frac{a\log\left(1-\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+dx)\right)}{2\sqrt{2}d\sqrt{e\cot(c+dx)}\sqrt{\tan(c+dx)}} \\ &\quad - \frac{a\log\left(1+\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+dx)\right)}{2\sqrt{2}d\sqrt{e\cot(c+dx)}\sqrt{\tan(c+dx)}}\end{aligned}$$

output

```
2*a*sin(d*x+c)/d/(e*cot(d*x+c))^(1/2)+2*a*cos(d*x+c)*(sin(c+1/4*Pi+d*x)^2)
^(1/2)/sin(c+1/4*Pi+d*x)*EllipticE(cos(c+1/4*Pi+d*x),2^(1/2))/d/(e*cot(d*x
+c))^(1/2)/sin(2*d*x+2*c)^(1/2)+1/2*a*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2))/
d*2^(1/2)/(e*cot(d*x+c))^(1/2)/tan(d*x+c)^(1/2)+1/2*a*arctan(1+2^(1/2)*tan
(d*x+c)^(1/2))/d*2^(1/2)/(e*cot(d*x+c))^(1/2)/tan(d*x+c)^(1/2)+1/4*a*ln(1-
2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c))/d*2^(1/2)/(e*cot(d*x+c))^(1/2)/tan(d*
x+c)^(1/2)-1/4*a*ln(1+2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c))/d*2^(1/2)/(e*co
t(d*x+c))^(1/2)/tan(d*x+c)^(1/2)
```

3.236.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 1.42 (sec) , antiderivative size = 189, normalized size of antiderivative = 0.63

$$\int \frac{a + a\sec(c+dx)}{\sqrt{e\cot(c+dx)}}\,dx$$
$$= \frac{a(1+\cos(c+dx))\sec^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\left(8\cot^3(c+dx)\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{3}{2},\frac{7}{4},-\cot^2(c+d\right.\right.}{}$$

input
```
Integrate[(a + a*Sec[c + d*x])/Sqrt[e*Cot[c + d*x]],x]
```

output
```
(a*(1 + Cos[c + d*x])*Sec[(c + d*x)/2]^2*Sec[c + d*x]*(8*Cot[c + d*x]^3*Hy
pergeometric2F1[3/4, 3/2, 7/4, -Cot[c + d*x]^2] - 3*Cot[c + d*x]*Sqrt[Csc[
c + d*x]^2]*(-2 + 2*Cos[2*(c + d*x)] + ArcSin[Cos[c + d*x] - Sin[c + d*x]]
*Sqrt[Sin[2*(c + d*x)]] + Log[Cos[c + d*x] + Sin[c + d*x] + Sqrt[Sin[2*(c
+ d*x)]]]*Sqrt[Sin[2*(c + d*x)]])))/(12*d*Sqrt[e*Cot[c + d*x]]*Sqrt[Csc[c
+ d*x]^2])
```

3.236.3 Rubi [A] (verified)

Time = 0.93 (sec) , antiderivative size = 232, normalized size of antiderivative = 0.78, number of steps used = 23, number of rules used = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.957$, Rules used = {3042, 4388, 3042, 4372, 3042, 3093, 3042, 3095, 3042, 3052, 3042, 3119, 3957, 266, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a\sec(c+dx)+a}{\sqrt{e\cot(c+dx)}}\,dx$$
$$\downarrow 3042$$
$$\int \frac{a\sec(c+dx)+a}{\sqrt{e\cot(c+dx)}}dx$$
$$\downarrow 4388$$
$$\frac{\int(\sec(c+dx)a+a)\sqrt{\tan(c+dx)}dx}{\sqrt{\tan(c+dx)}\sqrt{e\cot(c+dx)}}$$

$$\downarrow 3042$$

$$\frac{\int \sqrt{-\cot\left(c+dx+\frac{\pi}{2}\right)}\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)dx}{\sqrt{\tan(c+dx)}\sqrt{e\cot(c+dx)}}$$

$$\downarrow 4372$$

$$\frac{a\int \sqrt{\tan(c+dx)}dx+a\int \sec(c+dx)\sqrt{\tan(c+dx)}dx}{\sqrt{\tan(c+dx)}\sqrt{e\cot(c+dx)}}$$

$$\downarrow 3042$$

$$\frac{a\int \sqrt{\tan(c+dx)}dx+a\int \sec(c+dx)\sqrt{\tan(c+dx)}dx}{\sqrt{\tan(c+dx)}\sqrt{e\cot(c+dx)}}$$

$$\downarrow 3093$$

$$\frac{a\int \sqrt{\tan(c+dx)}dx+a\left(\frac{2\cos(c+dx)\tan^{\frac{3}{2}}(c+dx)}{d}-2\int \cos(c+dx)\sqrt{\tan(c+dx)}dx\right)}{\sqrt{\tan(c+dx)}\sqrt{e\cot(c+dx)}}$$

$$\downarrow 3042$$

$$\frac{a\int \sqrt{\tan(c+dx)}dx+a\left(\frac{2\cos(c+dx)\tan^{\frac{3}{2}}(c+dx)}{d}-2\int \frac{\sqrt{\tan(c+dx)}}{\sec(c+dx)}dx\right)}{\sqrt{\tan(c+dx)}\sqrt{e\cot(c+dx)}}$$

$$\downarrow 3095$$

$$\frac{a\int \sqrt{\tan(c+dx)}dx+a\left(\frac{2\cos(c+dx)\tan^{\frac{3}{2}}(c+dx)}{d}-\frac{2\sqrt{\cos(c+dx)}\sqrt{\tan(c+dx)}\int \sqrt{\cos(c+dx)}\sqrt{\sin(c+dx)}dx}{\sqrt{\sin(c+dx)}}\right)}{\sqrt{\tan(c+dx)}\sqrt{e\cot(c+dx)}}$$

$$\downarrow 3042$$

$$\frac{a\int \sqrt{\tan(c+dx)}dx+a\left(\frac{2\cos(c+dx)\tan^{\frac{3}{2}}(c+dx)}{d}-\frac{2\sqrt{\cos(c+dx)}\sqrt{\tan(c+dx)}\int \sqrt{\cos(c+dx)}\sqrt{\sin(c+dx)}dx}{\sqrt{\sin(c+dx)}}\right)}{\sqrt{\tan(c+dx)}\sqrt{e\cot(c+dx)}}$$

$$\downarrow 3052$$

$$\frac{a\int \sqrt{\tan(c+dx)}dx+a\left(\frac{2\cos(c+dx)\tan^{\frac{3}{2}}(c+dx)}{d}-\frac{2\cos(c+dx)\sqrt{\tan(c+dx)}\int \sqrt{\sin(2c+2dx)}dx}{\sqrt{\sin(2c+2dx)}}\right)}{\sqrt{\tan(c+dx)}\sqrt{e\cot(c+dx)}}$$

$$\downarrow 3042$$

$$\frac{a\int \sqrt{\tan(c+dx)}dx+a\left(\frac{2\cos(c+dx)\tan^{\frac{3}{2}}(c+dx)}{d}-\frac{2\cos(c+dx)\sqrt{\tan(c+dx)}\int \sqrt{\sin(2c+2dx)}dx}{\sqrt{\sin(2c+2dx)}}\right)}{\sqrt{\tan(c+dx)}\sqrt{e\cot(c+dx)}}$$

$$\downarrow 3119$$

$$\frac{a\int \sqrt{\tan(c+dx)}dx + a\left(\frac{2\cos(c+dx)\tan^{\frac{3}{2}}(c+dx)}{d} - \frac{2\cos(c+dx)\sqrt{\tan(c+dx)}E\left(c+dx-\frac{\pi}{4}|2\right)}{d\sqrt{\sin(2c+2dx)}}\right)}{\sqrt{\tan(c+dx)}\sqrt{e\cot(c+dx)}}$$

$$\downarrow 3957$$

$$\frac{\frac{a\int \frac{\sqrt{\tan(c+dx)}}{\tan^2(c+dx)+1}d\tan(c+dx)}{d} + a\left(\frac{2\cos(c+dx)\tan^{\frac{3}{2}}(c+dx)}{d} - \frac{2\cos(c+dx)\sqrt{\tan(c+dx)}E\left(c+dx-\frac{\pi}{4}|2\right)}{d\sqrt{\sin(2c+2dx)}}\right)}{\sqrt{\tan(c+dx)}\sqrt{e\cot(c+dx)}}$$

$$\downarrow 266$$

$$\frac{\frac{2a\int \frac{\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}}{d} + a\left(\frac{2\cos(c+dx)\tan^{\frac{3}{2}}(c+dx)}{d} - \frac{2\cos(c+dx)\sqrt{\tan(c+dx)}E\left(c+dx-\frac{\pi}{4}|2\right)}{d\sqrt{\sin(2c+2dx)}}\right)}{\sqrt{\tan(c+dx)}\sqrt{e\cot(c+dx)}}$$

$$\downarrow 826$$

$$\frac{\frac{2a\left(\frac{1}{2}\int \frac{\tan(c+dx)+1}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)} - \frac{1}{2}\int \frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}\right)}{d} + a\left(\frac{2\cos(c+dx)\tan^{\frac{3}{2}}(c+dx)}{d} - \frac{2\cos(c+dx)\sqrt{\tan(c+dx)}E\left(c+dx-\frac{\pi}{4}\right.}{d\sqrt{\sin(2c+2dx)}}\right.}{\sqrt{\tan(c+dx)}\sqrt{e\cot(c+dx)}}$$

$$\downarrow 1476$$

$$\frac{\frac{2a\left(\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)} + \frac{1}{2}\int \frac{1}{\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}\right) - \frac{1}{2}\int \frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}\right)}{d} + a\left(}{\sqrt{\tan(c+dx)}\sqrt{e\cot(c+dx)}}$$

$$\downarrow 1082$$

$$\frac{\frac{2a\left(\frac{1}{2}\left(\frac{\int \frac{1}{-\tan(c+dx)-1}d\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}} - \frac{\int \frac{1}{-\tan(c+dx)-1}d\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}}\right) - \frac{1}{2}\int \frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}\right)}{d} + a\left(\frac{2\cos(c+dx)\tan^{\frac{3}{2}}}{d}\right.}{\sqrt{\tan(c+dx)}\sqrt{e\cot(c+dx)}}$$

$$\downarrow 217$$

$$\frac{\frac{2a\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}} - \frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}}\right) - \frac{1}{2}\int \frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}\right)}{d} + a\left(\frac{2\cos(c+dx)\tan^{\frac{3}{2}}(c+dx)}{d} - \frac{2\cos(c+dx)}{}\right.}{\sqrt{\tan(c+dx)}\sqrt{e\cot(c+dx)}}$$

$$\downarrow 1479$$

$$\frac{2a\left(\frac{1}{2}\left(\int -\frac{\sqrt{2}-2\sqrt{\tan(c+dx)}}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1}\,d\sqrt{\tan(c+dx)}+\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1}\,d\sqrt{\tan(c+dx)}\right)\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{}\right.}{\sqrt{2}}\right.}{d}}{\sqrt{\tan(c+dx)}\sqrt{e\cot(c+dx)}}$$

$$\downarrow 25$$

$$\frac{2a\left(\frac{1}{2}\left(-\int \frac{\sqrt{2}-2\sqrt{\tan(c+dx)}}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1}\,d\sqrt{\tan(c+dx)}-\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1}\,d\sqrt{\tan(c+dx)}\right)\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{t}\right.}{\sqrt{2}}\right.}{d}}{\sqrt{\tan(c+dx)}\sqrt{e\cot(c+dx)}}$$

$$\downarrow 27$$

$$\frac{2a\left(\frac{1}{2}\left(-\int \frac{\sqrt{2}-2\sqrt{\tan(c+dx)}}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1}\,d\sqrt{\tan(c+dx)}-\frac{1}{2}\int \frac{\sqrt{2}\sqrt{\tan(c+dx)}+1}{\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1}\,d\sqrt{\tan(c+dx)}\right)\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{}\right.}{\sqrt{2}}\right.}{d}}{\sqrt{\tan(c+dx)}\sqrt{e\cot(c+dx)}}$$

$$\downarrow 1103$$

$$\frac{2a\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\log\left(\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}}-\frac{\log\left(\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}}\right)\right)}{d}+a\left(}{\sqrt{\tan(c+dx)}\sqrt{e\cot(c+dx)}}$$

input
```
Int[(a + a*Sec[c + d*x])/Sqrt[e*Cot[c + d*x]],x]
```

output
```
((2*a*((-(ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]]/Sqrt[2]) + ArcTan[1 + Sqr
t[2]*Sqrt[Tan[c + d*x]]]/Sqrt[2])/2 + (Log[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]
+ Tan[c + d*x]]/(2*Sqrt[2]) - Log[1 + Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c +
 d*x]]/(2*Sqrt[2]))/2))/d + a*((-2*Cos[c + d*x]*EllipticE[c - Pi/4 + d*x,
2]*Sqrt[Tan[c + d*x]])/(d*Sqrt[Sin[2*c + 2*d*x]]) + (2*Cos[c + d*x]*Tan[c
+ d*x]^(3/2))/d))/(Sqrt[e*Cot[c + d*x]]*Sqrt[Tan[c + d*x]])
```

3.236. $\int \frac{a+a\sec(c+dx)}{\sqrt{e\cot(c+dx)}}\,dx$

3.236.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3052
```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Simp[Sqrt[a*Sin[e + f*x]]*(Sqrt[b*Cos[e + f*x]]/Sqrt[Sin[2*e
 + 2*f*x]])   Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

rule 3093
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[a^2*(a*Sec[e + f*x])^(m - 2)*((b*Tan[e + f*x])^(n +
1)/(b*f*(m + n - 1))), x] + Simp[a^2*((m - 2)/(m + n - 1))   Int[(a*Sec[e +
 f*x])^(m - 2)*(b*Tan[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (
GtQ[m, 1] || (EqQ[m, 1] && EqQ[n, 1/2])) && NeQ[m + n - 1, 0] && IntegersQ[
2*m, 2*n]
```

rule 3095
```
Int[Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]]/sec[(e_.) + (f_.)*(x_)], x_Symbol]
:> Simp[Sqrt[Cos[e + f*x]]*(Sqrt[b*Tan[e + f*x]]/Sqrt[Sin[e + f*x]])   Int[
Sqrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]], x], x] /; FreeQ[{b, e, f}, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

rule 4372
```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(e*Cot[c + d*x])^m, x], x] + Simp[b   Int[
(e*Cot[c + d*x])^m*Csc[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, m}, x]
```

rule 4388
```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*((a_) + (b_.)*sec[(c_.) + (d_.)*(x
_)])^(n_.), x_Symbol] :> Simp[(e*Cot[c + d*x])^m*Tan[c + d*x]^m   Int[(a +
b*Sec[c + d*x])^n/Tan[c + d*x]^m, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x]
 &&  !IntegerQ[m]
```

3.236.4 Maple [A] (verified)

Time = 10.38 (sec) , antiderivative size = 507, normalized size of antiderivative = 1.70

method	result
parts	$-\frac{a(e^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)-(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(dx+c)}}{(e^2)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\frac{\sqrt{2}\sqrt{e\cot(dx+c)}}{(e^2)^{\frac{1}{4}}}+1\right)\right)}{4de}$
default	Expression too large to display

input
```
int((a+a*sec(d*x+c))/(e*cot(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/4*a/d/e*(e^2)^(1/4)*2^(1/2)*(ln((e*cot(d*x+c)+(e^2)^(1/4)*(e*cot(d*x+c)
)^(1/2)*2^(1/2)+(e^2)^(1/2))/(e*cot(d*x+c)-(e^2)^(1/4)*(e*cot(d*x+c))^(1/2
)*2^(1/2)+(e^2)^(1/2)))+2*arctan(2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+
1)-2*arctan(-2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1))-a/d*2^(1/2)*(-2*
(csc(d*x+c)-cot(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c)+1)^(1/2)*(cot(d*x+c
)-csc(d*x+c))^(1/2)*EllipticE((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2*2^(1/2))
*cos(d*x+c)+(csc(d*x+c)-cot(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c)+1)^(1/2
)*(cot(d*x+c)-csc(d*x+c))^(1/2)*EllipticF((csc(d*x+c)-cot(d*x+c)+1)^(1/2),
1/2*2^(1/2))*cos(d*x+c)-2*(csc(d*x+c)-cot(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(
d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)*EllipticE((csc(d*x+c)-cot(d*
x+c)+1)^(1/2),1/2*2^(1/2))+(csc(d*x+c)-cot(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc
(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)*EllipticF((csc(d*x+c)-cot(d
*x+c)+1)^(1/2),1/2*2^(1/2))+2^(1/2)*cos(d*x+c)-2^(1/2))/(e*cot(d*x+c))^(1/
2)*csc(d*x+c)
```

3.236.5 Fricas [F(-1)]

Timed out.

$$\int \frac{a + a\sec(c + dx)}{\sqrt{e\cot(c + dx)}}\,dx = \text{Timed out}$$

input
```
integrate((a+a*sec(d*x+c))/(e*cot(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.236.6 Sympy [F]

$$\int \frac{a + a\sec(c + dx)}{\sqrt{e\cot(c + dx)}}\,dx = a\left(\int \frac{1}{\sqrt{e\cot(c + dx)}}\,dx + \int \frac{\sec(c + dx)}{\sqrt{e\cot(c + dx)}}\,dx\right)$$

input
```
integrate((a+a*sec(d*x+c))/(e*cot(d*x+c))**(1/2),x)
```

output
```
a*(Integral(1/sqrt(e*cot(c + d*x)), x) + Integral(sec(c + d*x)/sqrt(e*cot(
c + d*x)), x))
```

3.236.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{a + a\sec(c + dx)}{\sqrt{e\cot(c + dx)}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+a*sec(d*x+c))/(e*cot(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.236.8 Giac [F]

$$\int \frac{a + a\sec(c + dx)}{\sqrt{e\cot(c + dx)}}\,dx = \int \frac{a\sec(dx + c) + a}{\sqrt{e\cot(dx + c)}}\,dx$$

input
```
integrate((a+a*sec(d*x+c))/(e*cot(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate((a*sec(d*x + c) + a)/sqrt(e*cot(d*x + c)), x)
```

3.236.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a + a\sec(c + dx)}{\sqrt{e\cot(c + dx)}}\,dx = \int \frac{a + \frac{a}{\cos(c+d\,x)}}{\sqrt{e\cot(c + d\,x)}}\,dx$$

input
```
int((a + a/cos(c + d*x))/(e*cot(c + d*x))^(1/2),x)
```

output
```
int((a + a/cos(c + d*x))/(e*cot(c + d*x))^(1/2), x)
```

3.237 $\int \frac{a+a\sec(c+dx)}{(e\cot(c+dx))^{3/2}}\,dx$

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3.237.1 Optimal result

Integrand size = 23, antiderivative size = 320

$$\begin{aligned}\int \frac{a+a\sec(c+dx)}{(e\cot(c+dx))^{3/2}}\,dx &= \frac{2\cot(c+dx)(3a+a\sec(c+dx))}{3d(e\cot(c+dx))^{3/2}} \\ &- \frac{a\cot(c+dx)\csc(c+dx)\,\text{EllipticF}\left(c-\frac{\pi}{4}+dx,2\right)\sqrt{\sin(2c+2dx)}}{3d(e\cot(c+dx))^{3/2}} \\ &+ \frac{a\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}d(e\cot(c+dx))^{3/2}\tan^{\frac{3}{2}}(c+dx)} - \frac{a\arctan\left(1+\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}d(e\cot(c+dx))^{3/2}\tan^{\frac{3}{2}}(c+dx)} \\ &+ \frac{a\log\left(1-\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+dx)\right)}{2\sqrt{2}d(e\cot(c+dx))^{3/2}\tan^{\frac{3}{2}}(c+dx)} \\ &- \frac{a\log\left(1+\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+dx)\right)}{2\sqrt{2}d(e\cot(c+dx))^{3/2}\tan^{\frac{3}{2}}(c+dx)}\end{aligned}$$

output

```
2/3*cot(d*x+c)*(3*a+a*sec(d*x+c))/d/(e*cot(d*x+c))^(3/2)+1/3*a*cot(d*x+c)*
csc(d*x+c)*(sin(c+1/4*Pi+d*x)^2)^(1/2)/sin(c+1/4*Pi+d*x)*EllipticF(cos(c+1
/4*Pi+d*x),2^(1/2))*sin(2*d*x+2*c)^(1/2)/d/(e*cot(d*x+c))^(3/2)-1/2*a*arct
an(-1+2^(1/2)*tan(d*x+c)^(1/2))/d/(e*cot(d*x+c))^(3/2)*2^(1/2)/tan(d*x+c)^
(3/2)-1/2*a*arctan(1+2^(1/2)*tan(d*x+c)^(1/2))/d/(e*cot(d*x+c))^(3/2)*2^(1
/2)/tan(d*x+c)^(3/2)+1/4*a*ln(1-2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c))/d/(e*
cot(d*x+c))^(3/2)*2^(1/2)/tan(d*x+c)^(3/2)-1/4*a*ln(1+2^(1/2)*tan(d*x+c)^(
1/2)+tan(d*x+c))/d/(e*cot(d*x+c))^(3/2)*2^(1/2)/tan(d*x+c)^(3/2)
```

3.237.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.13 (sec) , antiderivative size = 224, normalized size of antiderivative = 0.70

$$\int \frac{a + a\sec(c+dx)}{(e\cot(c+dx))^{3/2}}\,dx = \frac{a(1+\cos(c+dx))\cos(2(c+dx))\csc(c+dx)\sqrt{\csc^2(c+dx)}\sec^2\left(\frac{1}{2}(c+dx)\right)\left(-4\right.}$$

input
```
Integrate[(a + a*Sec[c + d*x])/(e*Cot[c + d*x])^(3/2),x]
```

output
```
(a*(1 + Cos[c + d*x])*Cos[2*(c + d*x)]*Csc[c + d*x]*Sqrt[Csc[c + d*x]^2]*S
ec[(c + d*x)/2]^2*(-4*(-1)^(1/4)*Cot[c + d*x]^(3/2)*EllipticF[I*ArcSinh[(-
1)^(1/4)*Sqrt[Cot[c + d*x]]], -1] + Sqrt[Csc[c + d*x]^2]*(4 + 12*Cos[c + d
*x] + 3*ArcSin[Cos[c + d*x] - Sin[c + d*x]]*Cot[c + d*x]*Sqrt[Sin[2*(c + d
*x)]] - 3*Cot[c + d*x]*Log[Cos[c + d*x] + Sin[c + d*x] + Sqrt[Sin[2*(c + d
*x)]]]*Sqrt[Sin[2*(c + d*x)]])))/(12*d*(e*Cot[c + d*x])^(3/2)*(-1 + Cot[c
+ d*x]^2))
```

3.237.3 Rubi [A] (verified)

Time = 0.99 (sec) , antiderivative size = 243, normalized size of antiderivative = 0.76, number of steps used = 24, number of rules used = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 1.000$, Rules used = {3042, 4388, 3042, 4369, 27, 3042, 4372, 3042, 3094, 3042, 3053, 3042, 3120, 3957, 266, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a\sec(c+dx)+a}{(e\cot(c+dx))^{3/2}}\,dx$$

$$\downarrow 3042$$

$$\int \frac{a\sec(c+dx)+a}{(e\cot(c+dx))^{3/2}}dx$$

$$\downarrow 4388$$

$$\frac{\int(\sec(c+dx)a+a)\tan^{\frac{3}{2}}(c+dx)dx}{\tan^{\frac{3}{2}}(c+dx)(e\cot(c+dx))^{3/2}}$$

$$\downarrow 3042$$

$$\frac{\int \left(-\cot\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)dx}{\tan^{\frac{3}{2}}(c+dx)(e\cot(c+dx))^{3/2}}$$

$$\downarrow 4369$$

$$\frac{\frac{2\sqrt{\tan(c+dx)}(a\sec(c+dx)+3a)}{3d}-\frac{2}{3}\int\frac{\sec(c+dx)a+3a}{2\sqrt{\tan(c+dx)}}dx}{\tan^{\frac{3}{2}}(c+dx)(e\cot(c+dx))^{3/2}}$$

$$\downarrow 27$$

$$\frac{\frac{2\sqrt{\tan(c+dx)}(a\sec(c+dx)+3a)}{3d}-\frac{1}{3}\int\frac{\sec(c+dx)a+3a}{\sqrt{\tan(c+dx)}}dx}{\tan^{\frac{3}{2}}(c+dx)(e\cot(c+dx))^{3/2}}$$

$$\downarrow 3042$$

$$\frac{\frac{2\sqrt{\tan(c+dx)}(a\sec(c+dx)+3a)}{3d}-\frac{1}{3}\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)a+3a}{\sqrt{-\cot\left(c+dx+\frac{\pi}{2}\right)}}dx}{\tan^{\frac{3}{2}}(c+dx)(e\cot(c+dx))^{3/2}}$$

$$\downarrow 4372$$

$$\frac{\frac{1}{3}\left(-3a\int\frac{1}{\sqrt{\tan(c+dx)}}dx-a\int\frac{\sec(c+dx)}{\sqrt{\tan(c+dx)}}dx\right)+\frac{2\sqrt{\tan(c+dx)}(a\sec(c+dx)+3a)}{3d}}{\tan^{\frac{3}{2}}(c+dx)(e\cot(c+dx))^{3/2}}$$

$$\downarrow 3042$$

$$\frac{\frac{1}{3}\left(-3a\int\frac{1}{\sqrt{\tan(c+dx)}}dx-a\int\frac{\sec(c+dx)}{\sqrt{\tan(c+dx)}}dx\right)+\frac{2\sqrt{\tan(c+dx)}(a\sec(c+dx)+3a)}{3d}}{\tan^{\frac{3}{2}}(c+dx)(e\cot(c+dx))^{3/2}}$$

$$\downarrow 3094$$

$$\frac{\frac{1}{3}\left(-3a\int\frac{1}{\sqrt{\tan(c+dx)}}dx-\frac{a\sqrt{\sin(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}\sqrt{\sin(c+dx)}}dx}{\sqrt{\cos(c+dx)}\sqrt{\tan(c+dx)}}\right)+\frac{2\sqrt{\tan(c+dx)}(a\sec(c+dx)+3a)}{3d}}{\tan^{\frac{3}{2}}(c+dx)(e\cot(c+dx))^{3/2}}$$

$$\downarrow 3042$$

$$\frac{\frac{1}{3}\left(-3a\int\frac{1}{\sqrt{\tan(c+dx)}}dx-\frac{a\sqrt{\sin(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}\sqrt{\sin(c+dx)}}dx}{\sqrt{\cos(c+dx)}\sqrt{\tan(c+dx)}}\right)+\frac{2\sqrt{\tan(c+dx)}(a\sec(c+dx)+3a)}{3d}}{\tan^{\frac{3}{2}}(c+dx)(e\cot(c+dx))^{3/2}}$$

$$\downarrow 3053$$

$$\frac{\frac{1}{3}\left(-3a\int\frac{1}{\sqrt{\tan(c+dx)}}dx-\frac{a\sqrt{\sin(2c+2dx)}\sec(c+dx)\int\frac{1}{\sqrt{\sin(2c+2dx)}}dx}{\sqrt{\tan(c+dx)}}\right)+\frac{2\sqrt{\tan(c+dx)}(a\sec(c+dx)+3a)}{3d}}{\tan^{\frac{3}{2}}(c+dx)(e\cot(c+dx))^{3/2}}$$

$$\downarrow 3042$$

$$\frac{\frac{1}{3}\left(-3a\int\frac{1}{\sqrt{\tan(c+dx)}}dx-\frac{a\sqrt{\sin(2c+2dx)}\sec(c+dx)\int\frac{1}{\sqrt{\sin(2c+2dx)}}dx}{\sqrt{\tan(c+dx)}}\right)+\frac{2\sqrt{\tan(c+dx)}(a\sec(c+dx)+3a)}{3d}}{\tan^{\frac{3}{2}}(c+dx)(e\cot(c+dx))^{3/2}}$$

$$\downarrow 3120$$

$$\frac{\frac{1}{3}\left(-3a\int\frac{1}{\sqrt{\tan(c+dx)}}dx-\frac{a\sqrt{\sin(2c+2dx)}\sec(c+dx)\,\text{EllipticF}\left(c+dx-\frac{\pi}{4},2\right)}{d\sqrt{\tan(c+dx)}}\right)+\frac{2\sqrt{\tan(c+dx)}(a\sec(c+dx)+3a)}{3d}}{\tan^{\frac{3}{2}}(c+dx)(e\cot(c+dx))^{3/2}}$$

$$\downarrow 3957$$

$$\frac{\frac{1}{3}\left(-\frac{3a\int\frac{1}{\sqrt{\tan(c+dx)}\left(\tan^2(c+dx)+1\right)}d\tan(c+dx)}{d}-\frac{a\sqrt{\sin(2c+2dx)}\sec(c+dx)\,\text{EllipticF}\left(c+dx-\frac{\pi}{4},2\right)}{d\sqrt{\tan(c+dx)}}\right)+\frac{2\sqrt{\tan(c+dx)}(a\sec(c+dx)+3a)}{3d}}{\tan^{\frac{3}{2}}(c+dx)(e\cot(c+dx))^{3/2}}$$

$$\downarrow 266$$

$$\frac{\frac{1}{3}\left(-\frac{6a\int\frac{1}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}}{d}-\frac{a\sqrt{\sin(2c+2dx)}\sec(c+dx)\,\text{EllipticF}\left(c+dx-\frac{\pi}{4},2\right)}{d\sqrt{\tan(c+dx)}}\right)+\frac{2\sqrt{\tan(c+dx)}(a\sec(c+dx)+3a)}{3d}}{\tan^{\frac{3}{2}}(c+dx)(e\cot(c+dx))^{3/2}}$$

$$\downarrow 755$$

$$\frac{\frac{1}{3}\left(-\frac{6a\left(\frac{1}{2}\int\frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}+\frac{1}{2}\int\frac{\tan(c+dx)+1}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}\right)}{d}-\frac{a\sqrt{\sin(2c+2dx)}\sec(c+dx)\,\text{EllipticF}\left(c+dx-\frac{\pi}{4},2\right)}{d\sqrt{\tan(c+dx)}}\right)+\frac{2\sqrt{\tan(}}{}}{\tan^{\frac{3}{2}}(c+dx)(e\cot(c+dx))^{3/2}}$$

$$\downarrow 1476$$

$$\frac{\frac{1}{3}\left(-\frac{6a\left(\frac{1}{2}\int\frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}+\frac{1}{2}\left(\frac{1}{2}\int\frac{1}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}+\frac{1}{2}\int\frac{1}{\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}\right)\right)}{d}}{\tan^{\frac{3}{2}}(c+dx)(e\cot(c+dx))^{3/2}}$$

$$\downarrow 1082$$

$$\frac{\frac{1}{3}\left(-\frac{6a\left(\frac{1}{2}\int\frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}+\frac{1}{2}\left(\frac{\int\frac{1}{-\tan(c+dx)-1}d\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}}-\frac{\int\frac{1}{-\tan(c+dx)-1}d\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}}\right)\right)}{d}-\frac{a\sqrt{\sin(2c+2d}}{}}{\tan^{\frac{3}{2}}(c+dx)(e\cot(c+dx))^{3/2}}$$

$$\downarrow 217$$

$$\frac{1}{3}\left(-\frac{6a\left(\frac{1}{2}\int\frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}}\right)\right)}{d}-\frac{a\sqrt{\sin(2c+2dx)}\sec(c+dx)\,\text{EllipticF}(}{d\sqrt{\tan(c+dx)}}\right)$$

$$\tan^{\frac{3}{2}}(c+dx)(e\cot(c+dx))^{3/2}$$

$$\downarrow 1479$$

$$\frac{1}{3}\left(-\frac{6a\left(\frac{1}{2}\left(-\frac{\int-\frac{\sqrt{2}-2\sqrt{\tan(c+dx)}}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}-\frac{\int-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}}-\frac{\arcta}{}\right.\right.}{d}\right)$$

$$\tan^{\frac{3}{2}}(c+dx)(e\cot(c+dx)$$

$$\downarrow 25$$

$$\frac{1}{3}\left(-\frac{6a\left(\frac{1}{2}\left(\frac{\int\frac{\sqrt{2}-2\sqrt{\tan(c+dx)}}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}+\frac{\int\frac{\sqrt{2}\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\right.}{}\right.\right.}{d}\right)$$

$$\tan^{\frac{3}{2}}(c+dx)(e\cot(c+dx))^{3}$$

$$\downarrow 27$$

$$\frac{1}{3}\left(-\frac{6a\left(\frac{1}{2}\left(\frac{\int\frac{\sqrt{2}-2\sqrt{\tan(c+dx)}}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}+\frac{1}{2}\int\frac{\sqrt{2}\sqrt{\tan(c+dx)}+1}{\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}}-\frac{\arctan}{}\right.\right.}{d}\right)$$

$$\tan^{\frac{3}{2}}(c+dx)(e\cot(c+dx)$$

$$\downarrow 1103$$

$$\frac{1}{3}\left(-\frac{6a\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\log\left(\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}}-\frac{\log\left(\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}}\right)\right)}{d}\right)$$

$$\tan^{\frac{3}{2}}(c+dx)(e\cot(c+dx))^{3/2}$$

input
```
Int[(a + a*Sec[c + d*x])/(e*Cot[c + d*x])^(3/2),x]
```

output
```
(((-6*a*((-(ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]]/Sqrt[2]) + ArcTan[1 + S
qrt[2]*Sqrt[Tan[c + d*x]]]/Sqrt[2])/2 + (-1/2*Log[1 - Sqrt[2]*Sqrt[Tan[c +
 d*x]] + Tan[c + d*x]]/Sqrt[2] + Log[1 + Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[
c + d*x]]/(2*Sqrt[2]))/2))/d - (a*EllipticF[c - Pi/4 + d*x, 2]*Sec[c + d*x
]*Sqrt[Sin[2*c + 2*d*x]])/(d*Sqrt[Tan[c + d*x]]))/3 + (2*(3*a + a*Sec[c +
d*x])*Sqrt[Tan[c + d*x]])/(3*d))/((e*Cot[c + d*x])^(3/2)*Tan[c + d*x]^(3/2
))
```

3.237.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3053
```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Simp[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]])   Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f
}, x]
```

rule 3094
```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]], x_Symbol]
:> Simp[Sqrt[Sin[e + f*x]]/(Sqrt[Cos[e + f*x]]*Sqrt[b*Tan[e + f*x]])   Int[
1/(Sqrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]]), x], x] /; FreeQ[{b, e, f}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

rule 4369
```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(-e)*(e*Cot[c + d*x])^(m - 1)*((a*m + b*(m - 1)*Csc
[c + d*x])/(d*m*(m - 1))), x] - Simp[e^2/m   Int[(e*Cot[c + d*x])^(m - 2)*(
a*m + b*(m - 1)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && GtQ[m
, 1]
```

rule 4372
```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(e*Cot[c + d*x])^m, x], x] + Simp[b   Int[
(e*Cot[c + d*x])^m*Csc[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, m}, x]
```

rule 4388
```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*((a_) + (b_.)*sec[(c_.) + (d_.)*(x
_)])^(n_.), x_Symbol] :> Simp[(e*Cot[c + d*x])^m*Tan[c + d*x]^m   Int[(a +
b*Sec[c + d*x])^n/Tan[c + d*x]^m, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x]
 &&  !IntegerQ[m]
```

3.237.4 Maple [A] (verified)

Time = 10.12 (sec) , antiderivative size = 382, normalized size of antiderivative = 1.19

method	result
parts	$-\dfrac{2ae\left(-\frac{1}{e^2\sqrt{e\cot(dx+c)}}-\frac{\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)-\left(e^2\right)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\,\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)+\left(e^2\right)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\,\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(dx+c)}}{\left(e^2\right)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\frac{\sqrt{2}\sqrt{e\cot(dx+c)}}{\left(e^2\right)^{\frac{1}{4}}}\right.\right.}{8e^2\left(e^2\right)^{\frac{1}{4}}}\right.}{d}$
default	Expression too large to display

input
```
int((a+a*sec(d*x+c))/(e*cot(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2*a/d*e*(-1/e^2/(e*cot(d*x+c))^(1/2)-1/8/e^2/(e^2)^(1/4)*2^(1/2)*(ln((e*c
ot(d*x+c)-(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2))/(e*cot(d*x
+c)+(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2)))+2*arctan(2^(1/2
)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)-2*arctan(-2^(1/2)/(e^2)^(1/4)*(e*cot
(d*x+c))^(1/2)+1)))-1/3*a/d*2^(1/2)*(-(csc(d*x+c)-cot(d*x+c)+1)^(1/2)*(cot
(d*x+c)-csc(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)*EllipticF((csc(d
*x+c)-cot(d*x+c)+1)^(1/2),1/2*2^(1/2))*cos(d*x+c)^2-(csc(d*x+c)-cot(d*x+c)
+1)^(1/2)*(cot(d*x+c)-csc(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)*El
lipticF((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2*2^(1/2))*cos(d*x+c)+2^(1/2)*si
n(d*x+c))/e/(e*cot(d*x+c))^(1/2)/(cos(d*x+c)^2-1)*tan(d*x+c)
```

3.237.5 Fricas [F(-1)]

Timed out.

$$\int \frac{a + a\sec(c+dx)}{(e\cot(c+dx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((a+a*sec(d*x+c))/(e*cot(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.237.6 Sympy [F]

$$\int \frac{a + a\sec(c+dx)}{(e\cot(c+dx))^{3/2}}\,dx = a\left(\int \frac{1}{(e\cot(c+dx))^{\frac{3}{2}}}\,dx + \int \frac{\sec(c+dx)}{(e\cot(c+dx))^{\frac{3}{2}}}\,dx\right)$$

input
```
integrate((a+a*sec(d*x+c))/(e*cot(d*x+c))**(3/2),x)
```

output
```
a*(Integral((e*cot(c + d*x))**(-3/2), x) + Integral(sec(c + d*x)/(e*cot(c
+ d*x))**(3/2), x))
```

3.237.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{a + a\sec(c + dx)}{(e\cot(c + dx))^{3/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+a*sec(d*x+c))/(e*cot(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.237.8 Giac [F]

$$\int \frac{a + a\sec(c + dx)}{(e\cot(c + dx))^{3/2}}\,dx = \int \frac{a\sec(dx + c) + a}{(e\cot(dx + c))^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+a*sec(d*x+c))/(e*cot(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
integrate((a*sec(d*x + c) + a)/(e*cot(d*x + c))^(3/2), x)
```

3.237.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a + a\sec(c + dx)}{(e\cot(c + dx))^{3/2}}\,dx = \int \frac{a + \frac{a}{\cos(c+d\,x)}}{(e\cot(c + d\,x))^{3/2}}\,dx$$

input
```
int((a + a/cos(c + d*x))/(e*cot(c + d*x))^(3/2),x)
```

output
```
int((a + a/cos(c + d*x))/(e*cot(c + d*x))^(3/2), x)
```

3.238 $\int (e\cot(c+dx))^{5/2}(a+a\sec(c+dx))^2\,dx$

3.238.1 Optimal result 1731
3.238.2 Mathematica [C] (warning: unable to verify) 1732
3.238.3 Rubi [A] (verified) 1732
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3.238.8 Giac [F] 1736
3.238.9 Mupad [F(-1)] 1736

3.238.1 Optimal result

Integrand size = 25, antiderivative size = 357

$$\int (e\cot(c+dx))^{5/2}(a+a\sec(c+dx))^2\,dx = -\frac{4a^2(e\cot(c+dx))^{5/2}\tan(c+dx)}{3d}$$
$$-\frac{4a^2(e\cot(c+dx))^{5/2}\sec(c+dx)\tan(c+dx)}{3d}$$
$$-\frac{2a^2(e\cot(c+dx))^{5/2}\,\text{EllipticF}\left(c-\frac{\pi}{4}+dx,2\right)\sec(c+dx)\sqrt{\sin(2c+2dx)}\tan^2(c+dx)}{3d}$$
$$+\frac{a^2\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)(e\cot(c+dx))^{5/2}\tan^{\frac{5}{2}}(c+dx)}{\sqrt{2}d}$$
$$-\frac{a^2\arctan\left(1+\sqrt{2}\sqrt{\tan(c+dx)}\right)(e\cot(c+dx))^{5/2}\tan^{\frac{5}{2}}(c+dx)}{\sqrt{2}d}$$
$$+\frac{a^2(e\cot(c+dx))^{5/2}\log\left(1-\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+dx)\right)\tan^{\frac{5}{2}}(c+dx)}{2\sqrt{2}d}$$
$$-\frac{a^2(e\cot(c+dx))^{5/2}\log\left(1+\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+dx)\right)\tan^{\frac{5}{2}}(c+dx)}{2\sqrt{2}d}$$

output
```
-4/3*a^2*(e*cot(d*x+c))^(5/2)*tan(d*x+c)/d-4/3*a^2*(e*cot(d*x+c))^(5/2)*se
c(d*x+c)*tan(d*x+c)/d+2/3*a^2*(e*cot(d*x+c))^(5/2)*(sin(c+1/4*Pi+d*x)^2)^(
1/2)/sin(c+1/4*Pi+d*x)*EllipticF(cos(c+1/4*Pi+d*x),2^(1/2))*sec(d*x+c)*sin
(2*d*x+2*c)^(1/2)*tan(d*x+c)^2/d-1/2*a^2*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2
))*(e*cot(d*x+c))^(5/2)*tan(d*x+c)^(5/2)/d*2^(1/2)-1/2*a^2*arctan(1+2^(1/2
)*tan(d*x+c)^(1/2))*(e*cot(d*x+c))^(5/2)*tan(d*x+c)^(5/2)/d*2^(1/2)+1/4*a^
2*(e*cot(d*x+c))^(5/2)*ln(1-2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c))*tan(d*x+c
)^(5/2)/d*2^(1/2)-1/4*a^2*(e*cot(d*x+c))^(5/2)*ln(1+2^(1/2)*tan(d*x+c)^(1/
2)+tan(d*x+c))*tan(d*x+c)^(5/2)/d*2^(1/2)
```

3.238.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 12.83 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.26

$$\int (e\cot(c+dx))^{5/2}(a+a\sec(c+dx))^2\,dx =$$
$$-\frac{2a^2e\cos^4\left(\frac{1}{2}(c+dx)\right)(e\cot(c+dx))^{3/2}\left(2+2\,\text{Hypergeometric2F1}\left(-\frac{3}{4},\frac{1}{2},\frac{1}{4},-\tan^2(c+dx)\right)-\text{Hyperge}\right.}{3d}$$

input
```
Integrate[(e*Cot[c + d*x])^(5/2)*(a + a*Sec[c + d*x])^2,x]
```

output
```
(-2*a^2*e*Cos[(c + d*x)/2]^4*(e*Cot[c + d*x])^(3/2)*(2 + 2*Hypergeometric2
F1[-3/4, 1/2, 1/4, -Tan[c + d*x]^2] - Hypergeometric2F1[3/4, 1, 7/4, -Cot[
c + d*x]^2])*Sec[ArcCot[Cot[c + d*x]]/2]^4)/(3*d)
```

3.238.3 Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 266, normalized size of antiderivative = 0.75, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3042, 4388, 3042, 4374, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sec(c+dx)+a)^2(e\cot(c+dx))^{5/2}\,dx$$
$$\downarrow \text{3042}$$

$$\int (a\sec(c+dx)+a)^2(e\cot(c+dx))^{5/2}dx$$

$$\downarrow 4388$$

$$\tan^{\frac{5}{2}}(c+dx)(e\cot(c+dx))^{5/2}\int \frac{(\sec(c+dx)a+a)^2}{\tan^{\frac{5}{2}}(c+dx)}dx$$

$$\downarrow 3042$$

$$\tan^{\frac{5}{2}}(c+dx)(e\cot(c+dx))^{5/2}\int \frac{\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^2}{\left(-\cot\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}dx$$

$$\downarrow 4374$$

$$\tan^{\frac{5}{2}}(c+dx)(e\cot(c+dx))^{5/2}\int \left(\frac{\sec^2(c+dx)a^2}{\tan^{\frac{5}{2}}(c+dx)}+\frac{2\sec(c+dx)a^2}{\tan^{\frac{5}{2}}(c+dx)}+\frac{a^2}{\tan^{\frac{5}{2}}(c+dx)}\right)dx$$

$$\downarrow 2009$$

$$\tan^{\frac{5}{2}}(c+dx)(e\cot(c+dx))^{5/2}\left(\frac{a^2\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}d}-\frac{a^2\arctan\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}d}-\frac{4a^2}{3d\tan^{\frac{3}{2}}(c+dx)}+\frac{a^2\log\left(\text{tan}\right.}{}\right.$$

input
```
Int[(e*Cot[c + d*x])^(5/2)*(a + a*Sec[c + d*x])^2,x]
```

output
```
(e*Cot[c + d*x])^(5/2)*((a^2*ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]])/(Sqrt
[2]*d) - (a^2*ArcTan[1 + Sqrt[2]*Sqrt[Tan[c + d*x]]])/(Sqrt[2]*d) + (a^2*L
og[1 - Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]])/(2*Sqrt[2]*d) - (a^2*Lo
g[1 + Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]])/(2*Sqrt[2]*d) - (4*a^2)/
(3*d*Tan[c + d*x]^(3/2)) - (4*a^2*Sec[c + d*x])/(3*d*Tan[c + d*x]^(3/2)) -
 (2*a^2*EllipticF[c - Pi/4 + d*x, 2]*Sec[c + d*x]*Sqrt[Sin[2*c + 2*d*x]])/
(3*d*Sqrt[Tan[c + d*x]]))*Tan[c + d*x]^(5/2)
```

3.238.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4374
```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

rule 4388
```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*((a_) + (b_.)*sec[(c_.) + (d_.)*(x
_)])^(n_.), x_Symbol] :> Simp[(e*Cot[c + d*x])^m*Tan[c + d*x]^m   Int[(a +
b*Sec[c + d*x])^n/Tan[c + d*x]^m, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x]
 &&  !IntegerQ[m]
```

3.238.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 17.61 (sec) , antiderivative size = 496, normalized size of antiderivative = 1.39

method	result
default	$-\frac{a^2e^2\sqrt{2}\,(\cos(dx+c)+1)\left(-3i\sin(dx+c)\sqrt{\csc(dx+c)-\cot(dx+c)+1}\,\sqrt{\cot(dx+c)-\csc(dx+c)+1}\,\sqrt{\cot(dx+c)-\csc(dx+c)}\ \text{EllipticF}\right.$
parts	$2a^2e\left(\frac{(e\cot(dx+c))^{\frac{3}{2}}}{3}-\frac{e^2\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)-\left(e^2\right)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\,\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)+\left(e^2\right)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\,\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{e\cot(dx+c)}}{\left(e^2\right)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\frac{\sqrt{2}\,\sqrt{e\cot(dx}{\left(e^2\right)^{\frac{1}{4}}}\right.\right.}{8\left(e^2\right)^{\frac{1}{4}}}\right.$
	$\left.\left.-\frac{}{d}\right.\right.$

input
```
int((e*cot(d*x+c))^(5/2)*(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
-1/6*a^2*e^2/d*2^(1/2)*(cos(d*x+c)+1)*(-3*I*sin(d*x+c)*(csc(d*x+c)-cot(d*x
+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)
*EllipticPi((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2+1/2*I,1/2*2^(1/2))+3*I*sin
(d*x+c)*(csc(d*x+c)-cot(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c)+1)^(1/2)*(c
ot(d*x+c)-csc(d*x+c))^(1/2)*EllipticPi((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2
-1/2*I,1/2*2^(1/2))+3*sin(d*x+c)*(cot(d*x+c)-csc(d*x+c))^(1/2)*(csc(d*x+c)
-cot(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c)+1)^(1/2)*EllipticPi((csc(d*x+c
)-cot(d*x+c)+1)^(1/2),1/2+1/2*I,1/2*2^(1/2))+3*sin(d*x+c)*(cot(d*x+c)-csc(
d*x+c))^(1/2)*(csc(d*x+c)-cot(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c)+1)^(1
/2)*EllipticPi((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2-1/2*I,1/2*2^(1/2))-2*si
n(d*x+c)*(cot(d*x+c)-csc(d*x+c))^(1/2)*(csc(d*x+c)-cot(d*x+c)+1)^(1/2)*(co
t(d*x+c)-csc(d*x+c)+1)^(1/2)*EllipticF((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2
*2^(1/2))+4*2^(1/2)*cos(d*x+c))*(e*cot(d*x+c))^(1/2)*sec(d*x+c)*csc(d*x+c)
```

3.238.5 Fricas [F(-1)]

Timed out.

$$\int (e\cot(c+dx))^{5/2}(a+a\sec(c+dx))^2\,dx = \text{Timed out}$$

input
```
integrate((e*cot(d*x+c))^(5/2)*(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

output
```
Timed out
```

3.238.6 Sympy [F(-1)]

Timed out.

$$\int (e\cot(c+dx))^{5/2}(a+a\sec(c+dx))^2\,dx = \text{Timed out}$$

input
```
integrate((e*cot(d*x+c))**(5/2)*(a+a*sec(d*x+c))**2,x)
```

output
```
Timed out
```

3.238.7 Maxima [F(-2)]

Exception generated.

$$\int (e\cot(c+dx))^{5/2}(a+a\sec(c+dx))^2\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((e*cot(d*x+c))^(5/2)*(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.238.8 Giac [F]

$$\int (e\cot(c+dx))^{5/2}(a+a\sec(c+dx))^2\,dx = \int (e\cot(dx+c))^{\frac{5}{2}}(a\sec(dx+c)+a)^2\,dx$$

input
```
integrate((e*cot(d*x+c))^(5/2)*(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

output
```
integrate((e*cot(d*x + c))^(5/2)*(a*sec(d*x + c) + a)^2, x)
```

3.238.9 Mupad [F(-1)]

Timed out.

$$\int (e\cot(c+dx))^{5/2}(a+a\sec(c+dx))^2\,dx = \int (e\cot(c+dx))^{5/2}\left(a+\frac{a}{\cos(c+dx)}\right)^2 dx$$

input
```
int((e*cot(c + d*x))^(5/2)*(a + a/cos(c + d*x))^2,x)
```

output
```
int((e*cot(c + d*x))^(5/2)*(a + a/cos(c + d*x))^2, x)
```

3.239 $\int (e\cot(c+dx))^{3/2}(a+a\sec(c+dx))^2\,dx$

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3.239.1 Optimal result

Integrand size = 25, antiderivative size = 343

$$\begin{aligned}
&\int (e\cot(c+dx))^{3/2}(a+a\sec(c+dx))^2\,dx = \\
&-\frac{4a^2(e\cot(c+dx))^{3/2}\sin(c+dx)}{d} - \frac{4a^2(e\cot(c+dx))^{3/2}\tan(c+dx)}{d} \\
&-\frac{4a^2(e\cot(c+dx))^{3/2}E\left(c-\frac{\pi}{4}+dx\middle|2\right)\sin(c+dx)\tan(c+dx)}{d\sqrt{\sin(2c+2dx)}} \\
&+\frac{a^2\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)(e\cot(c+dx))^{3/2}\tan^{\frac{3}{2}}(c+dx)}{\sqrt{2}d} \\
&-\frac{a^2\arctan\left(1+\sqrt{2}\sqrt{\tan(c+dx)}\right)(e\cot(c+dx))^{3/2}\tan^{\frac{3}{2}}(c+dx)}{\sqrt{2}d} \\
&-\frac{a^2(e\cot(c+dx))^{3/2}\log\left(1-\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+dx)\right)\tan^{\frac{3}{2}}(c+dx)}{2\sqrt{2}d} \\
&+\frac{a^2(e\cot(c+dx))^{3/2}\log\left(1+\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+dx)\right)\tan^{\frac{3}{2}}(c+dx)}{2\sqrt{2}d}
\end{aligned}$$

output
```
-4*a^2*(e*cot(d*x+c))^(3/2)*sin(d*x+c)/d-4*a^2*(e*cot(d*x+c))^(3/2)*tan(d*
x+c)/d+4*a^2*(e*cot(d*x+c))^(3/2)*(sin(c+1/4*Pi+d*x)^2)^(1/2)/sin(c+1/4*Pi
+d*x)*EllipticE(cos(c+1/4*Pi+d*x),2^(1/2))*sin(d*x+c)*tan(d*x+c)/d/sin(2*d
*x+2*c)^(1/2)-1/2*a^2*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2))*(e*cot(d*x+c))^(
3/2)*tan(d*x+c)^(3/2)/d*2^(1/2)-1/2*a^2*arctan(1+2^(1/2)*tan(d*x+c)^(1/2))
*(e*cot(d*x+c))^(3/2)*tan(d*x+c)^(3/2)/d*2^(1/2)-1/4*a^2*(e*cot(d*x+c))^(3
/2)*ln(1-2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c))*tan(d*x+c)^(3/2)/d*2^(1/2)+1
/4*a^2*(e*cot(d*x+c))^(3/2)*ln(1+2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c))*tan(
d*x+c)^(3/2)/d*2^(1/2)
```

3.239.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 14.44 (sec) , antiderivative size = 220, normalized size of antiderivative = 0.64

$$\int (e\cot(c+dx))^{3/2}(a+a\sec(c+dx))^2\,dx =$$

$$-\frac{a^2\cos^4\left(\frac{1}{2}(c+dx)\right)(e\cot(c+dx))^{3/2}\left(2\sqrt{2}\arctan\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)-2\sqrt{2}\arctan\left(1+\sqrt{2}\sqrt{\cot(c\right.\right.}$$

input
```
Integrate[(e*Cot[c + d*x])^(3/2)*(a + a*Sec[c + d*x])^2,x]
```

output
```
-1/4*(a^2*Cos[(c + d*x)/2]^4*(e*Cot[c + d*x])^(3/2)*(2*Sqrt[2]*ArcTan[1 -
Sqrt[2]*Sqrt[Cot[c + d*x]]] - 2*Sqrt[2]*ArcTan[1 + Sqrt[2]*Sqrt[Cot[c + d*
x]]] + 16*Sqrt[Cot[c + d*x]] + 16*Sqrt[Cot[c + d*x]]*Hypergeometric2F1[-1/
4, 1/2, 3/4, -Tan[c + d*x]^2] + Sqrt[2]*Log[1 - Sqrt[2]*Sqrt[Cot[c + d*x]]
 + Cot[c + d*x]] - Sqrt[2]*Log[1 + Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*
x]])*Sec[ArcCot[Cot[c + d*x]]/2]^4)/(d*Cot[c + d*x]^(3/2))
```

3.239.3 Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 260, normalized size of antiderivative = 0.76, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3042, 4388, 3042, 4374, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sec(c+dx)+a)^2(e\cot(c+dx))^{3/2}\,dx$$

$$\downarrow 3042$$

$$\int (a\sec(c+dx)+a)^2(e\cot(c+dx))^{3/2}dx$$

$$\downarrow 4388$$

$$\tan^{\frac{3}{2}}(c+dx)(e\cot(c+dx))^{3/2}\int \frac{(\sec(c+dx)a+a)^2}{\tan^{\frac{3}{2}}(c+dx)}dx$$

$$\downarrow 3042$$

$$\tan^{\frac{3}{2}}(c+dx)(e\cot(c+dx))^{3/2}\int \frac{\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^2}{\left(-\cot\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx$$

$$\downarrow 4374$$

$$\tan^{\frac{3}{2}}(c+dx)(e\cot(c+dx))^{3/2}\int \left(\frac{\sec^2(c+dx)a^2}{\tan^{\frac{3}{2}}(c+dx)}+\frac{2\sec(c+dx)a^2}{\tan^{\frac{3}{2}}(c+dx)}+\frac{a^2}{\tan^{\frac{3}{2}}(c+dx)}\right)dx$$

$$\downarrow 2009$$

$$\tan^{\frac{3}{2}}(c+dx)(e\cot(c+dx))^{3/2}\left(\frac{a^2\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}d}-\frac{a^2\arctan\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}d}-\frac{4a^2}{d\sqrt{\tan(c+dx)}}-\frac{a^2\log\left(\tan\right.}{}\right.$$

input
```
Int[(e*Cot[c + d*x])^(3/2)*(a + a*Sec[c + d*x])^2,x]
```

output
```
(e*Cot[c + d*x])^(3/2)*((a^2*ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]])/(Sqrt
[2]*d) - (a^2*ArcTan[1 + Sqrt[2]*Sqrt[Tan[c + d*x]]])/(Sqrt[2]*d) - (a^2*L
og[1 - Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]])/(2*Sqrt[2]*d) + (a^2*Lo
g[1 + Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]])/(2*Sqrt[2]*d) - (4*a^2)/
(d*Sqrt[Tan[c + d*x]]) - (4*a^2*Cos[c + d*x])/(d*Sqrt[Tan[c + d*x]]) - (4*
a^2*Cos[c + d*x]*EllipticE[c - Pi/4 + d*x, 2]*Sqrt[Tan[c + d*x]])/(d*Sqrt[
Sin[2*c + 2*d*x]]))*Tan[c + d*x]^(3/2)
```

3.239.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4374
```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

rule 4388
```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*((a_) + (b_.)*sec[(c_.) + (d_.)*(x
_)])^(n_.), x_Symbol] :> Simp[(e*Cot[c + d*x])^m*Tan[c + d*x]^m   Int[(a +
b*Sec[c + d*x])^n/Tan[c + d*x]^m, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x]
 &&  !IntegerQ[m]
```

3.239.4 Maple [A] (verified)

Time = 10.33 (sec) , antiderivative size = 524, normalized size of antiderivative = 1.53

method	result
parts	$2a^2e\left(\sqrt{e\cot(dx+c)}-\frac{\left(e^2\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+\left(e^2\right)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)-\left(e^2\right)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(dx+c)}}{\left(e^2\right)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\frac{\sqrt{2}\sqrt{e\cot(dx+c)}}{\left(e^2\right)^{\frac{1}{4}}}\right.\right.}{8}\right.$ $\left.\left.\right)\right)$ $-\frac{}{d}$
default	Expression too large to display

input
```
int((e*cot(d*x+c))^(3/2)*(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
-2*a^2/d*e*((e*cot(d*x+c))^(1/2)-1/8*(e^2)^(1/4)*2^(1/2)*(ln((e*cot(d*x+c)
+(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2))/(e*cot(d*x+c)-(e^2)
^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2)))+2*arctan(2^(1/2)/(e^2)^(
1/4)*(e*cot(d*x+c))^(1/2)+1)-2*arctan(-2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^
(1/2)+1)))-2*a^2/d*e*(e*cot(d*x+c))^(1/2)-2*a^2/d*2^(1/2)*e*(e*cot(d*x+c))
^(1/2)*(-2*(csc(d*x+c)-cot(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c)+1)^(1/2)
*(cot(d*x+c)-csc(d*x+c))^(1/2)*EllipticE((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1
/2*2^(1/2))+(csc(d*x+c)-cot(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c)+1)^(1/2
)*(cot(d*x+c)-csc(d*x+c))^(1/2)*EllipticF((csc(d*x+c)-cot(d*x+c)+1)^(1/2),
1/2*2^(1/2))-2*(csc(d*x+c)-cot(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c)+1)^(
1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)*EllipticE((csc(d*x+c)-cot(d*x+c)+1)^(1/
2),1/2*2^(1/2))*sec(d*x+c)+(csc(d*x+c)-cot(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc
(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)*EllipticF((csc(d*x+c)-cot(d
*x+c)+1)^(1/2),1/2*2^(1/2))*sec(d*x+c)+2^(1/2))
```

3.239.5 Fricas [F(-1)]

Timed out.

$$\int (e\cot(c+dx))^{3/2}(a+a\sec(c+dx))^2\,dx = \text{Timed out}$$

input
```
integrate((e*cot(d*x+c))^(3/2)*(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

output
```
Timed out
```

3.239.6 Sympy [F(-1)]

Timed out.

$$\int (e\cot(c+dx))^{3/2}(a+a\sec(c+dx))^2\,dx = \text{Timed out}$$

input
```
integrate((e*cot(d*x+c))**(3/2)*(a+a*sec(d*x+c))**2,x)
```

output
```
Timed out
```

3.239.7 Maxima [F(-2)]

Exception generated.

$$\int (e\cot(c+dx))^{3/2}(a+a\sec(c+dx))^2\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((e*cot(d*x+c))^(3/2)*(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.239.8 Giac [F]

$$\int (e\cot(c+dx))^{3/2}(a+a\sec(c+dx))^2\,dx = \int (e\cot(dx+c))^{\frac{3}{2}}(a\sec(dx+c)+a)^2\,dx$$

input
```
integrate((e*cot(d*x+c))^(3/2)*(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

output
```
integrate((e*cot(d*x + c))^(3/2)*(a*sec(d*x + c) + a)^2, x)
```

3.239.9 Mupad [F(-1)]

Timed out.

$$\int (e\cot(c+dx))^{3/2}(a+a\sec(c+dx))^2\,dx = \int (e\cot(c+dx))^{3/2}\left(a+\frac{a}{\cos(c+dx)}\right)^2 dx$$

input
```
int((e*cot(c + d*x))^(3/2)*(a + a/cos(c + d*x))^2,x)
```

output
```
int((e*cot(c + d*x))^(3/2)*(a + a/cos(c + d*x))^2, x)
```

3.240 $\int \sqrt{e\cot(c+dx)}(a+a\sec(c+dx))^2\,dx$

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3.240.2 Mathematica [C] (warning: unable to verify) . . . 1744
3.240.3 Rubi [A] (verified) . . . 1744
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3.240.8 Giac [F] . . . 1748
3.240.9 Mupad [F(-1)] . . . 1748

3.240.1 Optimal result

Integrand size = 25, antiderivative size = 311

$$
\begin{aligned}
&\int \sqrt{e\cot(c+dx)}(a+a\sec(c+dx))^2\,dx \\
&= \frac{2a^2\sqrt{e\cot(c+dx)}\,\text{EllipticF}\left(c-\frac{\pi}{4}+dx,2\right)\sec(c+dx)\sqrt{\sin(2c+2dx)}}{d} \\
&\quad - \frac{a^2\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)\sqrt{e\cot(c+dx)}\sqrt{\tan(c+dx)}}{\sqrt{2}d} \\
&\quad + \frac{a^2\arctan\left(1+\sqrt{2}\sqrt{\tan(c+dx)}\right)\sqrt{e\cot(c+dx)}\sqrt{\tan(c+dx)}}{\sqrt{2}d} \\
&\quad - \frac{a^2\sqrt{e\cot(c+dx)}\log\left(1-\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+dx)\right)\sqrt{\tan(c+dx)}}{2\sqrt{2}d} \\
&\quad + \frac{a^2\sqrt{e\cot(c+dx)}\log\left(1+\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+dx)\right)\sqrt{\tan(c+dx)}}{2\sqrt{2}d} \\
&\quad + \frac{2a^2\sqrt{e\cot(c+dx)}\tan(c+dx)}{d}
\end{aligned}
$$

output
```
-2*a^2*(sin(c+1/4*Pi+d*x)^2)^(1/2)/sin(c+1/4*Pi+d*x)*EllipticF(cos(c+1/4*P
i+d*x),2^(1/2))*sec(d*x+c)*(e*cot(d*x+c))^(1/2)*sin(2*d*x+2*c)^(1/2)/d+1/2
*a^2*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2))*(e*cot(d*x+c))^(1/2)*tan(d*x+c)^(
1/2)/d*2^(1/2)+1/2*a^2*arctan(1+2^(1/2)*tan(d*x+c)^(1/2))*(e*cot(d*x+c))^(
1/2)*tan(d*x+c)^(1/2)/d*2^(1/2)-1/4*a^2*ln(1-2^(1/2)*tan(d*x+c)^(1/2)+tan(
d*x+c))*(e*cot(d*x+c))^(1/2)*tan(d*x+c)^(1/2)/d*2^(1/2)+1/4*a^2*ln(1+2^(1/
2)*tan(d*x+c)^(1/2)+tan(d*x+c))*(e*cot(d*x+c))^(1/2)*tan(d*x+c)^(1/2)/d*2^
(1/2)+2*a^2*(e*cot(d*x+c))^(1/2)*tan(d*x+c)/d
```

3.240.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 12.01 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.38

$$\int \sqrt{e\cot(c+dx)}(a+a\sec(c+dx))^2\,dx$$
$$= \frac{a^2e(1+\cos(c+dx))^2\left(3\,\text{Hypergeometric2F1}\left(-\frac{1}{4},1,\frac{3}{4},-\cot^2(c+dx)\right)+6\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},-\right.\right.}{6d\sqrt{e\,\text{co}}}$$

input
```
Integrate[Sqrt[e*Cot[c + d*x]]*(a + a*Sec[c + d*x])^2,x]
```

output
```
(a^2*e*(1 + Cos[c + d*x])^2*(3*Hypergeometric2F1[-1/4, 1, 3/4, -Cot[c + d*
x]^2] + 6*Hypergeometric2F1[1/4, 1/2, 5/4, -Tan[c + d*x]^2] - 2*Cot[c + d*
x]^2*Hypergeometric2F1[3/4, 1, 7/4, -Cot[c + d*x]^2])*Sec[ArcCot[Cot[c + d
*x]]/2]^4)/(6*d*Sqrt[e*Cot[c + d*x]])
```

3.240.3 Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 236, normalized size of antiderivative = 0.76, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3042, 4388, 3042, 4374, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sec(c+dx)+a)^2\sqrt{e\cot(c+dx)}\,dx$$
$$\downarrow \text{3042}$$

$$\int (a\sec(c+dx)+a)^2\sqrt{e\cot(c+dx)}dx$$

$$\downarrow 4388$$

$$\sqrt{\tan(c+dx)}\sqrt{e\cot(c+dx)}\int \frac{(\sec(c+dx)a+a)^2}{\sqrt{\tan(c+dx)}}dx$$

$$\downarrow 3042$$

$$\sqrt{\tan(c+dx)}\sqrt{e\cot(c+dx)}\int \frac{\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^2}{\sqrt{-\cot\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\downarrow 4374$$

$$\sqrt{\tan(c+dx)}\sqrt{e\cot(c+dx)}\int \left(\frac{\sec^2(c+dx)a^2}{\sqrt{\tan(c+dx)}}+\frac{2\sec(c+dx)a^2}{\sqrt{\tan(c+dx)}}+\frac{a^2}{\sqrt{\tan(c+dx)}}\right)dx$$

$$\downarrow 2009$$

$$\sqrt{\tan(c+dx)}\sqrt{e\cot(c+dx)}\left(-\frac{a^2\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}d}+\frac{a^2\arctan\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}d}+\frac{2a^2\sqrt{}}{}\right.$$

input
```
Int[Sqrt[e*Cot[c + d*x]]*(a + a*Sec[c + d*x])^2,x]
```

output
```
Sqrt[e*Cot[c + d*x]]*(-((a^2*ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]])/(Sqrt
[2]*d)) + (a^2*ArcTan[1 + Sqrt[2]*Sqrt[Tan[c + d*x]]])/(Sqrt[2]*d) - (a^2*
Log[1 - Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]])/(2*Sqrt[2]*d) + (a^2*L
og[1 + Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]])/(2*Sqrt[2]*d) + (2*a^2*
EllipticF[c - Pi/4 + d*x, 2]*Sec[c + d*x]*Sqrt[Sin[2*c + 2*d*x]])/(d*Sqrt[
Tan[c + d*x]]) + (2*a^2*Sqrt[Tan[c + d*x]])/d)*Sqrt[Tan[c + d*x]]
```

3.240.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4374
```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

rule 4388
```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*((a_) + (b_.)*sec[(c_.) + (d_.)*(x
_)])^(n_.), x_Symbol] :> Simp[(e*Cot[c + d*x])^m*Tan[c + d*x]^m   Int[(a +
b*Sec[c + d*x])^n/Tan[c + d*x]^m, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x]
 &&  !IntegerQ[m]
```

3.240.4 Maple [A] (verified)

Time = 11.14 (sec) , antiderivative size = 265, normalized size of antiderivative = 0.85

method	result
parts	$-\frac{a^2 e\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)-\left(e^2\right)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)+\left(e^2\right)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(dx+c)}}{\left(e^2\right)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\frac{\sqrt{2}\sqrt{e\cot(dx+c)}}{\left(e^2\right)^{\frac{1}{4}}}+1\right)\right)}{4d\left(e^2\right)^{\frac{1}{4}}}+$
default	Expression too large to display

input
```
int((a+a*sec(d*x+c))^2*(e*cot(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/4*a^2/d*e/(e^2)^(1/4)*2^(1/2)*(ln((e*cot(d*x+c)-(e^2)^(1/4)*(e*cot(d*x+
c))^(1/2)*2^(1/2)+(e^2)^(1/2))/(e*cot(d*x+c)+(e^2)^(1/4)*(e*cot(d*x+c))^(1
/2)*2^(1/2)+(e^2)^(1/2)))+2*arctan(2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2
)+1)-2*arctan(-2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1))+2*a^2/d*e/(e*c
ot(d*x+c))^(1/2)+2*a^2/d*2^(1/2)*(e*cot(d*x+c))^(1/2)*(csc(d*x+c)-cot(d*x+
c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)*
EllipticF((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2*2^(1/2))*(1+sec(d*x+c))
```

3.240.5 Fricas [F(-1)]

Timed out.

$$\int \sqrt{e\cot(c+dx)}(a+a\sec(c+dx))^2\,dx = \text{Timed out}$$

input
```
integrate((a+a*sec(d*x+c))^2*(e*cot(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.240.6 Sympy [F]

$$\int \sqrt{e\cot(c+dx)}(a+a\sec(c+dx))^2\,dx = a^2\left(\int \sqrt{e\cot(c+dx)}\,dx + \int 2\sqrt{e\cot(c+dx)}\sec(c+dx)\,dx + \int \sqrt{e\cot(c+dx)}\sec^2(c+dx)\,dx\right)$$

input
```
integrate((a+a*sec(d*x+c))**2*(e*cot(d*x+c))**(1/2),x)
```

output
```
a**2*(Integral(sqrt(e*cot(c + d*x)), x) + Integral(2*sqrt(e*cot(c + d*x))*
sec(c + d*x), x) + Integral(sqrt(e*cot(c + d*x))*sec(c + d*x)**2, x))
```

3.240.7 Maxima [F(-2)]

Exception generated.

$$\int \sqrt{e\cot(c+dx)}(a+a\sec(c+dx))^2\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+a*sec(d*x+c))^2*(e*cot(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.240.8 Giac [F]

$$\int \sqrt{e\cot(c+dx)}(a+a\sec(c+dx))^2\,dx = \int \sqrt{e\cot(dx+c)}(a\sec(dx+c)+a)^2\,dx$$

input
```
integrate((a+a*sec(d*x+c))^2*(e*cot(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(e*cot(d*x + c))*(a*sec(d*x + c) + a)^2, x)
```

3.240.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{e\cot(c+dx)}(a+a\sec(c+dx))^2\,dx = \int \sqrt{e\cot(c+dx)}\left(a+\frac{a}{\cos(c+dx)}\right)^2 dx$$

input
```
int((e*cot(c + d*x))^(1/2)*(a + a/cos(c + d*x))^2,x)
```

output
```
int((e*cot(c + d*x))^(1/2)*(a + a/cos(c + d*x))^2, x)
```

3.241 $\int \frac{(a+a\sec(c+dx))^2}{\sqrt{e\cot(c+dx)}}\,dx$

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3.241.1 Optimal result

Integrand size = 25, antiderivative size = 339

$$\begin{aligned}\int \frac{(a+a\sec(c+dx))^2}{\sqrt{e\cot(c+dx)}}\,dx &= \frac{4a^2\sin(c+dx)}{d\sqrt{e\cot(c+dx)}} - \frac{4a^2\cos(c+dx)E\left(c-\frac{\pi}{4}+dx\middle|2\right)}{d\sqrt{e\cot(c+dx)}\sqrt{\sin(2c+2dx)}} \\ &- \frac{a^2\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}d\sqrt{e\cot(c+dx)}\sqrt{\tan(c+dx)}} \\ &+ \frac{a^2\arctan\left(1+\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}d\sqrt{e\cot(c+dx)}\sqrt{\tan(c+dx)}} \\ &+ \frac{a^2\log\left(1-\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+dx)\right)}{2\sqrt{2}d\sqrt{e\cot(c+dx)}\sqrt{\tan(c+dx)}} \\ &- \frac{a^2\log\left(1+\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+dx)\right)}{2\sqrt{2}d\sqrt{e\cot(c+dx)}\sqrt{\tan(c+dx)}} \\ &+ \frac{2a^2\tan(c+dx)}{3d\sqrt{e\cot(c+dx)}}\end{aligned}$$

output
```
4*a^2*sin(d*x+c)/d/(e*cot(d*x+c))^(1/2)+4*a^2*cos(d*x+c)*(sin(c+1/4*Pi+d*x
)^2)^(1/2)/sin(c+1/4*Pi+d*x)*EllipticE(cos(c+1/4*Pi+d*x),2^(1/2))/d/(e*cot
(d*x+c))^(1/2)/sin(2*d*x+2*c)^(1/2)+1/2*a^2*arctan(-1+2^(1/2)*tan(d*x+c)^(
1/2))/d*2^(1/2)/(e*cot(d*x+c))^(1/2)/tan(d*x+c)^(1/2)+1/2*a^2*arctan(1+2^(
1/2)*tan(d*x+c)^(1/2))/d*2^(1/2)/(e*cot(d*x+c))^(1/2)/tan(d*x+c)^(1/2)+1/4
*a^2*ln(1-2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c))/d*2^(1/2)/(e*cot(d*x+c))^(1
/2)/tan(d*x+c)^(1/2)-1/4*a^2*ln(1+2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c))/d*2
^(1/2)/(e*cot(d*x+c))^(1/2)/tan(d*x+c)^(1/2)+2/3*a^2*tan(d*x+c)/d/(e*cot(d
*x+c))^(1/2)
```

3.241.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 25.08 (sec) , antiderivative size = 221, normalized size of antiderivative = 0.65

$$\int \frac{(a+a\sec(c+dx))^2}{\sqrt{e\cot(c+dx)}}\,dx$$
$$= \frac{a^2\cos^5\left(\frac{1}{2}(c+dx)\right)\left(4\,\text{Hypergeometric2F1}\left(-\frac{3}{4},1,\frac{1}{4},-\cot^2(c+dx)\right)+8\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},-t\right.\right.}{}$$

input
```
Integrate[(a + a*Sec[c + d*x])^2/Sqrt[e*Cot[c + d*x]],x]
```

output
```
(a^2*Cos[(c + d*x)/2]^5*(4*Hypergeometric2F1[-3/4, 1, 1/4, -Cot[c + d*x]^2
] + 8*Hypergeometric2F1[1/2, 3/4, 7/4, -Tan[c + d*x]^2] + 3*Sqrt[2]*Cot[c
+ d*x]^(3/2)*(2*ArcTan[1 - Sqrt[2]*Sqrt[Cot[c + d*x]]] - 2*ArcTan[1 + Sqrt
[2]*Sqrt[Cot[c + d*x]]] + Log[1 - Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x
]] - Log[1 + Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]]))*Sec[c + d*x]*Sec
[ArcCot[Cot[c + d*x]]/2]^4*Sin[(c + d*x)/2])/(3*d*Sqrt[e*Cot[c + d*x]])
```

3.241.3 Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 262, normalized size of antiderivative = 0.77, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3042, 4388, 3042, 4374, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sec(c+dx)+a)^2}{\sqrt{e\cot(c+dx)}}\,dx$$

$$\downarrow 3042$$

$$\int \frac{(a\sec(c+dx)+a)^2}{\sqrt{e\cot(c+dx)}}dx$$

$$\downarrow 4388$$

$$\frac{\int(\sec(c+dx)a+a)^2\sqrt{\tan(c+dx)}dx}{\sqrt{\tan(c+dx)}\sqrt{e\cot(c+dx)}}$$

$$\downarrow 3042$$

$$\frac{\int\sqrt{-\cot\left(c+dx+\frac{\pi}{2}\right)}\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^2dx}{\sqrt{\tan(c+dx)}\sqrt{e\cot(c+dx)}}$$

$$\downarrow 4374$$

$$\frac{\int\left(\sec^2(c+dx)\sqrt{\tan(c+dx)}a^2+2\sec(c+dx)\sqrt{\tan(c+dx)}a^2+\sqrt{\tan(c+dx)}a^2\right)dx}{\sqrt{\tan(c+dx)}\sqrt{e\cot(c+dx)}}$$

$$\downarrow 2009$$

$$\frac{-\frac{a^2\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}d}+\frac{a^2\arctan\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}d}+\frac{2a^2\tan^{\frac{3}{2}}(c+dx)}{3d}+\frac{a^2\log\left(\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}d}-\frac{a^2\log(}{}}{\sqrt{\tan(c+dx)}\sqrt{e\cot(c+dx)}}$$

input
```
Int[(a + a*Sec[c + d*x])^2/Sqrt[e*Cot[c + d*x]],x]
```

output
```
(-((a^2*ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]])/(Sqrt[2]*d)) + (a^2*ArcTan
[1 + Sqrt[2]*Sqrt[Tan[c + d*x]]])/(Sqrt[2]*d) + (a^2*Log[1 - Sqrt[2]*Sqrt[
Tan[c + d*x]] + Tan[c + d*x]])/(2*Sqrt[2]*d) - (a^2*Log[1 + Sqrt[2]*Sqrt[T
an[c + d*x]] + Tan[c + d*x]])/(2*Sqrt[2]*d) - (4*a^2*Cos[c + d*x]*Elliptic
E[c - Pi/4 + d*x, 2]*Sqrt[Tan[c + d*x]])/(d*Sqrt[Sin[2*c + 2*d*x]]) + (2*a
^2*Tan[c + d*x]^(3/2))/(3*d) + (4*a^2*Cos[c + d*x]*Tan[c + d*x]^(3/2))/d)/
(Sqrt[e*Cot[c + d*x]]*Sqrt[Tan[c + d*x]])
```

3.241.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4374
```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

rule 4388
```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*((a_) + (b_.)*sec[(c_.) + (d_.)*(x
_)])^(n_.), x_Symbol] :> Simp[(e*Cot[c + d*x])^m*Tan[c + d*x]^m   Int[(a +
b*Sec[c + d*x])^n/Tan[c + d*x]^m, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x]
 &&  !IntegerQ[m]
```

3.241.4 Maple [A] (verified)

Time = 11.51 (sec) , antiderivative size = 531, normalized size of antiderivative = 1.57

method	result
parts	$-\frac{a^2 \left(e^2\right)^{\frac{1}{4}} \sqrt{2} \left(\ln\left(\frac{e\cot(dx+c)+\left(e^2\right)^{\frac{1}{4}} \sqrt{e\cot(dx+c)}\, \sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)-\left(e^2\right)^{\frac{1}{4}} \sqrt{e\cot(dx+c)}\, \sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\frac{\sqrt{2}\, \sqrt{e\cot(dx+c)}}{\left(e^2\right)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\frac{\sqrt{2}\, \sqrt{e\cot(dx+c)}}{\left(e^2\right)^{\frac{1}{4}}}+1\right)\right)}{4de}$
default	Expression too large to display

input
```
int((a+a*sec(d*x+c))^2/(e*cot(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/4*a^2/d/e*(e^2)^(1/4)*2^(1/2)*(ln((e*cot(d*x+c)+(e^2)^(1/4)*(e*cot(d*x+
c))^(1/2)*2^(1/2)+(e^2)^(1/2))/(e*cot(d*x+c)-(e^2)^(1/4)*(e*cot(d*x+c))^(1
/2)*2^(1/2)+(e^2)^(1/2)))+2*arctan(2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2
)+1)-2*arctan(-2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1))+2/3*a^2/d*e/(e
*cot(d*x+c))^(3/2)+2*a^2/d*2^(1/2)*(2*(csc(d*x+c)-cot(d*x+c)+1)^(1/2)*(cot
(d*x+c)-csc(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)*EllipticE((csc(d
*x+c)-cot(d*x+c)+1)^(1/2),1/2*2^(1/2))*cos(d*x+c)-(csc(d*x+c)-cot(d*x+c)+1
)^(1/2)*(cot(d*x+c)-csc(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)*Elli
pticF((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2*2^(1/2))*cos(d*x+c)+2*(csc(d*x+c
)-cot(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+
c))^(1/2)*EllipticE((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2*2^(1/2))-(csc(d*x+
c)-cot(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x
+c))^(1/2)*EllipticF((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2*2^(1/2))-2^(1/2)*
cos(d*x+c)+2^(1/2))/(e*cot(d*x+c))^(1/2)*csc(d*x+c)
```

3.241.5 Fricas [F(-1)]

Timed out.

$$\int \frac{(a+a\sec(c+dx))^2}{\sqrt{e\cot(c+dx)}}\,dx = \text{Timed out}$$

input
```
integrate((a+a*sec(d*x+c))^2/(e*cot(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.241.6 Sympy [F]

$$\int \frac{(a+a\sec(c+dx))^2}{\sqrt{e\cot(c+dx)}}\,dx = a^2\left(\int \frac{1}{\sqrt{e\cot(c+dx)}}\,dx + \int \frac{2\sec(c+dx)}{\sqrt{e\cot(c+dx)}}\,dx + \int \frac{\sec^2(c+dx)}{\sqrt{e\cot(c+dx)}}\,dx\right)$$

input
```
integrate((a+a*sec(d*x+c))**2/(e*cot(d*x+c))**(1/2),x)
```

output
```
a**2*(Integral(1/sqrt(e*cot(c + d*x)), x) + Integral(2*sec(c + d*x)/sqrt(e
*cot(c + d*x)), x) + Integral(sec(c + d*x)**2/sqrt(e*cot(c + d*x)), x))
```

3.241.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+a\sec(c+dx))^2}{\sqrt{e\cot(c+dx)}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+a*sec(d*x+c))^2/(e*cot(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.241.8 Giac [F]

$$\int \frac{(a+a\sec(c+dx))^2}{\sqrt{e\cot(c+dx)}}\,dx = \int \frac{(a\sec(dx+c)+a)^2}{\sqrt{e\cot(dx+c)}}\,dx$$

input
```
integrate((a+a*sec(d*x+c))^2/(e*cot(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate((a*sec(d*x + c) + a)^2/sqrt(e*cot(d*x + c)), x)
```

3.241.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+a\sec(c+dx))^2}{\sqrt{e\cot(c+dx)}}\,dx = \int \frac{\left(a+\frac{a}{\cos(c+dx)}\right)^2}{\sqrt{e\cot(c+dx)}}\,dx$$

input
```
int((a + a/cos(c + d*x))^2/(e*cot(c + d*x))^(1/2),x)
```

output
```
int((a + a/cos(c + d*x))^2/(e*cot(c + d*x))^(1/2), x)
```

3.242 $\int \frac{(a+a\sec(c+dx))^2}{(e\cot(c+dx))^{3/2}}\,dx$

3.242.1 Optimal result . . . 1755
3.242.2 Mathematica [C] (warning: unable to verify) . . . 1756
3.242.3 Rubi [A] (verified) . . . 1756
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3.242.7 Maxima [F(-2)] . . . 1759
3.242.8 Giac [F] . . . 1759
3.242.9 Mupad [F(-1)] . . . 1760

3.242.1 Optimal result

Integrand size = 25, antiderivative size = 375

$$\begin{aligned}\int \frac{(a+a\sec(c+dx))^2}{(e\cot(c+dx))^{3/2}}\,dx &= \frac{2a^2\cot(c+dx)}{d(e\cot(c+dx))^{3/2}} + \frac{4a^2\csc(c+dx)}{3d(e\cot(c+dx))^{3/2}} \\ &- \frac{2a^2\cot(c+dx)\csc(c+dx)\,\text{EllipticF}\left(c-\frac{\pi}{4}+dx,2\right)\sqrt{\sin(2c+2dx)}}{3d(e\cot(c+dx))^{3/2}} \\ &+ \frac{a^2\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}d(e\cot(c+dx))^{3/2}\tan^{\frac{3}{2}}(c+dx)} - \frac{a^2\arctan\left(1+\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}d(e\cot(c+dx))^{3/2}\tan^{\frac{3}{2}}(c+dx)} \\ &+ \frac{a^2\log\left(1-\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+dx)\right)}{2\sqrt{2}d(e\cot(c+dx))^{3/2}\tan^{\frac{3}{2}}(c+dx)} \\ &- \frac{a^2\log\left(1+\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+dx)\right)}{2\sqrt{2}d(e\cot(c+dx))^{3/2}\tan^{\frac{3}{2}}(c+dx)} + \frac{2a^2\tan(c+dx)}{5d(e\cot(c+dx))^{3/2}}\end{aligned}$$

output

```
2*a^2*cot(d*x+c)/d/(e*cot(d*x+c))^(3/2)+4/3*a^2*csc(d*x+c)/d/(e*cot(d*x+c)
)^(3/2)+2/3*a^2*cot(d*x+c)*csc(d*x+c)*(sin(c+1/4*Pi+d*x)^2)^(1/2)/sin(c+1/
4*Pi+d*x)*EllipticF(cos(c+1/4*Pi+d*x),2^(1/2))*sin(2*d*x+2*c)^(1/2)/d/(e*c
ot(d*x+c))^(3/2)-1/2*a^2*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2))/d/(e*cot(d*x+
c))^(3/2)*2^(1/2)/tan(d*x+c)^(3/2)-1/2*a^2*arctan(1+2^(1/2)*tan(d*x+c)^(1/
2))/d/(e*cot(d*x+c))^(3/2)*2^(1/2)/tan(d*x+c)^(3/2)+1/4*a^2*ln(1-2^(1/2)*t
an(d*x+c)^(1/2)+tan(d*x+c))/d/(e*cot(d*x+c))^(3/2)*2^(1/2)/tan(d*x+c)^(3/2
)-1/4*a^2*ln(1+2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c))/d/(e*cot(d*x+c))^(3/2)
*2^(1/2)/tan(d*x+c)^(3/2)+2/5*a^2*tan(d*x+c)/d/(e*cot(d*x+c))^(3/2)
```

3.242.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 13.32 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.34

$$\int \frac{(a+a\sec(c+dx))^2}{(e\cot(c+dx))^{3/2}}\,dx = \frac{a^2\left(\text{Hypergeometric2F1}\left(-\frac{5}{4},1,-\frac{1}{4},-\cot^2(c+dx)\right)+2\left(5\cot^2(c+dx)\,\text{Hyperg}\right.\right.}$$

input
```
Integrate[(a + a*Sec[c + d*x])^2/(e*Cot[c + d*x])^(3/2),x]
```

output
```
(a^2*(Hypergeometric2F1[-5/4, 1, -1/4, -Cot[c + d*x]^2] + 2*(5*Cot[c + d*x
]^2*Hypergeometric2F1[-1/4, 1, 3/4, -Cot[c + d*x]^2] + Hypergeometric2F1[1
/2, 5/4, 9/4, -Tan[c + d*x]^2]))*(1 + Sec[c + d*x])^2*Sec[ArcCot[Cot[c + d
*x]]/2]^4*Sin[c + d*x]^2)/(10*d*e*Sqrt[e*Cot[c + d*x]])
```

3.242.3 Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 284, normalized size of antiderivative = 0.76, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3042, 4388, 3042, 4374, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sec(c+dx)+a)^2}{(e\cot(c+dx))^{3/2}}\,dx$$

$$\downarrow 3042$$

$$\int \frac{(a\sec(c+dx)+a)^2}{(e\cot(c+dx))^{3/2}}dx$$

$$\downarrow 4388$$

$$\frac{\int (\sec(c+dx)a+a)^2\tan^{\frac{3}{2}}(c+dx)dx}{\tan^{\frac{3}{2}}(c+dx)(e\cot(c+dx))^{3/2}}$$

$$\downarrow 3042$$

$$\frac{\int \left(-\cot\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^2 dx}{\tan^{\frac{3}{2}}(c+dx)(e\cot(c+dx))^{3/2}}$$

$$\downarrow 4374$$

$$\frac{\int \left(\sec^2(c+dx)\tan^{\frac{3}{2}}(c+dx)a^2+2\sec(c+dx)\tan^{\frac{3}{2}}(c+dx)a^2+\tan^{\frac{3}{2}}(c+dx)a^2\right)dx}{\tan^{\frac{3}{2}}(c+dx)(e\cot(c+dx))^{3/2}}$$

$$\downarrow \text{2009}$$

$$\frac{\frac{a^2\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}d}-\frac{a^2\arctan\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}d}+\frac{2a^2\tan^{\frac{5}{2}}(c+dx)}{5d}+\frac{2a^2\sqrt{\tan(c+dx)}}{d}+\frac{a^2\log\left(\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+d}\right.}{2\sqrt{2}d}}{\tan^{\frac{3}{2}}(c+dx)(e\,co}$$

input
```
Int[(a + a*Sec[c + d*x])^2/(e*Cot[c + d*x])^(3/2),x]
```

output
```
((a^2*ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]])/(Sqrt[2]*d) - (a^2*ArcTan[1
+ Sqrt[2]*Sqrt[Tan[c + d*x]]])/(Sqrt[2]*d) + (a^2*Log[1 - Sqrt[2]*Sqrt[Tan
[c + d*x]] + Tan[c + d*x]])/(2*Sqrt[2]*d) - (a^2*Log[1 + Sqrt[2]*Sqrt[Tan[
c + d*x]] + Tan[c + d*x]])/(2*Sqrt[2]*d) - (2*a^2*EllipticF[c - Pi/4 + d*x
, 2]*Sec[c + d*x]*Sqrt[Sin[2*c + 2*d*x]])/(3*d*Sqrt[Tan[c + d*x]]) + (2*a^
2*Sqrt[Tan[c + d*x]])/d + (4*a^2*Sec[c + d*x]*Sqrt[Tan[c + d*x]])/(3*d) +
(2*a^2*Tan[c + d*x]^(5/2))/(5*d))/((e*Cot[c + d*x])^(3/2)*Tan[c + d*x]^(3/
2))
```

3.242.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4374
```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

rule 4388
```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*((a_) + (b_.)*sec[(c_.) + (d_.)*(x
_)])^(n_.), x_Symbol] :> Simp[(e*Cot[c + d*x])^m*Tan[c + d*x]^m   Int[(a +
b*Sec[c + d*x])^n/Tan[c + d*x]^m, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x]
 &&  !IntegerQ[m]
```

3.242.4 Maple [A] (verified)

Time = 11.11 (sec) , antiderivative size = 405, normalized size of antiderivative = 1.08

method	result
parts	$-\dfrac{2a^2e\left(-\frac{1}{e^2\sqrt{e\cot(dx+c)}}-\frac{\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)-\left(e^2\right)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)+\left(e^2\right)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(dx+c)}}{\left(e^2\right)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\frac{\sqrt{2}\sqrt{e\cot(dx}}{\left(e^2\right)^{\frac{1}{4}}}\right.\right.\right.}{8e^2\left(e^2\right)^{\frac{1}{4}}}\right.\right.}{d}$
default	Expression too large to display

input
```
int((a+a*sec(d*x+c))^2/(e*cot(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2*a^2/d*e*(-1/e^2/(e*cot(d*x+c))^(1/2)-1/8/e^2/(e^2)^(1/4)*2^(1/2)*(ln((e
*cot(d*x+c)-(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2))/(e*cot(d
*x+c)+(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2)))+2*arctan(2^(1
/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)-2*arctan(-2^(1/2)/(e^2)^(1/4)*(e*c
ot(d*x+c))^(1/2)+1)))+2/5*a^2*e/d/(e*cot(d*x+c))^(5/2)-2/3*a^2/d*2^(1/2)*(
-(csc(d*x+c)-cot(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c)+1)^(1/2)*(cot(d*x+
c)-csc(d*x+c))^(1/2)*EllipticF((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2*2^(1/2)
)*cos(d*x+c)^2-(csc(d*x+c)-cot(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c)+1)^(
1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)*EllipticF((csc(d*x+c)-cot(d*x+c)+1)^(1/
2),1/2*2^(1/2))*cos(d*x+c)+2^(1/2)*sin(d*x+c))/e/(e*cot(d*x+c))^(1/2)/(cos
(d*x+c)^2-1)*tan(d*x+c)
```

3.242.5 Fricas [F(-1)]

Timed out.

$$\int \frac{(a+a\sec(c+dx))^2}{(e\cot(c+dx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((a+a*sec(d*x+c))^2/(e*cot(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.242.6 Sympy [F]

$$\int \frac{(a + a\sec(c + dx))^2}{(e\cot(c + dx))^{3/2}}\,dx = a^2\left(\int \frac{1}{(e\cot(c + dx))^{\frac{3}{2}}}\,dx + \int \frac{2\sec(c + dx)}{(e\cot(c + dx))^{\frac{3}{2}}}\,dx + \int \frac{\sec^2(c + dx)}{(e\cot(c + dx))^{\frac{3}{2}}}\,dx\right)$$

input
```
integrate((a+a*sec(d*x+c))**2/(e*cot(d*x+c))**(3/2),x)
```

output
```
a**2*(Integral((e*cot(c + d*x))**(-3/2), x) + Integral(2*sec(c + d*x)/(e*c
ot(c + d*x))**(3/2), x) + Integral(sec(c + d*x)**2/(e*cot(c + d*x))**(3/2)
, x))
```

3.242.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(a + a\sec(c + dx))^2}{(e\cot(c + dx))^{3/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+a*sec(d*x+c))^2/(e*cot(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.242.8 Giac [F]

$$\int \frac{(a + a\sec(c + dx))^2}{(e\cot(c + dx))^{3/2}}\,dx = \int \frac{(a\sec(dx + c) + a)^2}{(e\cot(dx + c))^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+a*sec(d*x+c))^2/(e*cot(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
integrate((a*sec(d*x + c) + a)^2/(e*cot(d*x + c))^(3/2), x)
```

3.242.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a + a\sec(c + dx))^2}{(e\cot(c + dx))^{3/2}}\,dx = \int \frac{\left(a + \frac{a}{\cos(c+d\,x)}\right)^2}{(e\cot(c + d\,x))^{3/2}}\,dx$$

input
```
int((a + a/cos(c + d*x))^2/(e*cot(c + d*x))^(3/2),x)
```

output
```
int((a + a/cos(c + d*x))^2/(e*cot(c + d*x))^(3/2), x)
```

3.243 $\int \frac{(e\cot(c+dx))^{3/2}}{a+a\sec(c+dx)}\,dx$

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3.243.1 Optimal result

Integrand size = 25, antiderivative size = 405

$$\begin{aligned}
\int \frac{(e\cot(c+dx))^{3/2}}{a+a\sec(c+dx)}\,dx &= \frac{2\cot(c+dx)(e\cot(c+dx))^{3/2}(1-\sec(c+dx))}{5ad} \\
&- \frac{2(e\cot(c+dx))^{3/2}(5-3\sec(c+dx))\tan(c+dx)}{5ad} \\
&+ \frac{6(e\cot(c+dx))^{3/2}E\left(c-\frac{\pi}{4}+dx\middle|2\right)\sin(c+dx)\tan(c+dx)}{5ad\sqrt{\sin(2c+2dx)}} \\
&+ \frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)(e\cot(c+dx))^{3/2}\tan^{\frac{3}{2}}(c+dx)}{\sqrt{2}ad} \\
&- \frac{\arctan\left(1+\sqrt{2}\sqrt{\tan(c+dx)}\right)(e\cot(c+dx))^{3/2}\tan^{\frac{3}{2}}(c+dx)}{\sqrt{2}ad} \\
&- \frac{(e\cot(c+dx))^{3/2}\log\left(1-\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+dx)\right)\tan^{\frac{3}{2}}(c+dx)}{2\sqrt{2}ad} \\
&+ \frac{(e\cot(c+dx))^{3/2}\log\left(1+\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+dx)\right)\tan^{\frac{3}{2}}(c+dx)}{2\sqrt{2}ad} \\
&- \frac{6(e\cot(c+dx))^{3/2}\sin(c+dx)\tan^2(c+dx)}{5ad}
\end{aligned}$$

output
```
2/5*cot(d*x+c)*(e*cot(d*x+c))^(3/2)*(1-sec(d*x+c))/a/d-2/5*(e*cot(d*x+c))^
(3/2)*(5-3*sec(d*x+c))*tan(d*x+c)/a/d-6/5*(e*cot(d*x+c))^(3/2)*(sin(c+1/4*
Pi+d*x)^2)^(1/2)/sin(c+1/4*Pi+d*x)*EllipticE(cos(c+1/4*Pi+d*x),2^(1/2))*si
n(d*x+c)*tan(d*x+c)/a/d/sin(2*d*x+2*c)^(1/2)-1/2*arctan(-1+2^(1/2)*tan(d*x
+c)^(1/2))*(e*cot(d*x+c))^(3/2)*tan(d*x+c)^(3/2)/a/d*2^(1/2)-1/2*arctan(1+
2^(1/2)*tan(d*x+c)^(1/2))*(e*cot(d*x+c))^(3/2)*tan(d*x+c)^(3/2)/a/d*2^(1/2
)-1/4*(e*cot(d*x+c))^(3/2)*ln(1-2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c))*tan(d
*x+c)^(3/2)/a/d*2^(1/2)+1/4*(e*cot(d*x+c))^(3/2)*ln(1+2^(1/2)*tan(d*x+c)^(
1/2)+tan(d*x+c))*tan(d*x+c)^(3/2)/a/d*2^(1/2)-6/5*(e*cot(d*x+c))^(3/2)*sin
(d*x+c)*tan(d*x+c)^2/a/d
```

3.243.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 14.10 (sec) , antiderivative size = 316, normalized size of antiderivative = 0.78

$$\int \frac{(e\cot(c+dx))^{3/2}}{a+a\sec(c+dx)}\,dx =$$
$$-\frac{e\sqrt{e\cot(c+dx)}\left(30\sqrt{2}\arctan\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)\cot^{\frac{3}{2}}(c+dx)-30\sqrt{2}\arctan\left(1+\sqrt{2}\sqrt{\cot(c+dx)}\right.\right.}{}$$

input
```
Integrate[(e*Cot[c + d*x])^(3/2)/(a + a*Sec[c + d*x]),x]
```

output
```
-1/30*(e*Sqrt[e*Cot[c + d*x]]*(30*Sqrt[2]*ArcTan[1 - Sqrt[2]*Sqrt[Cot[c +
d*x]]]*Cot[c + d*x]^(3/2) - 30*Sqrt[2]*ArcTan[1 + Sqrt[2]*Sqrt[Cot[c + d*x
]]]*Cot[c + d*x]^(3/2) + 120*Cot[c + d*x]^2 - 24*Cot[c + d*x]^4 + 24*Cot[c
 + d*x]^4*Hypergeometric2F1[-5/4, -1/2, -1/4, -Tan[c + d*x]^2] - 120*Cot[c
 + d*x]^2*Hypergeometric2F1[-1/2, -1/4, 3/4, -Tan[c + d*x]^2] - 40*Hyperge
ometric2F1[1/2, 3/4, 7/4, -Tan[c + d*x]^2] + 15*Sqrt[2]*Cot[c + d*x]^(3/2)
*Log[1 - Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]] - 15*Sqrt[2]*Cot[c + d
*x]^(3/2)*Log[1 + Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]])*Sec[c + d*x]
*(1 + Sqrt[Sec[c + d*x]^2])*Sin[(c + d*x)/2]^2)/(a*d)
```

3.243.3 Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e\cot(c+dx))^{3/2}}{a\sec(c+dx)+a}\,dx$$

$$\downarrow 3042$$

$$\int \frac{(e\cot(c+dx))^{3/2}}{a\sec(c+dx)+a}dx$$

$$\downarrow 4388$$

$$\tan^{\frac{3}{2}}(c+dx)(e\cot(c+dx))^{3/2}\int \frac{1}{(\sec(c+dx)a+a)\tan^{\frac{3}{2}}(c+dx)}dx$$

$$\downarrow 3042$$

$$\tan^{\frac{3}{2}}(c+dx)(e\cot(c+dx))^{3/2}\int \frac{1}{\left(-\cot\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)}dx$$

$$\downarrow 4376$$

$$\frac{\tan^{\frac{3}{2}}(c+dx)(e\cot(c+dx))^{3/2}\int -\frac{a-a\sec(c+dx)}{\tan^{\frac{7}{2}}(c+dx)}dx}{a^2}$$

$$\downarrow 25$$

$$-\frac{\tan^{\frac{3}{2}}(c+dx)(e\cot(c+dx))^{3/2}\int \frac{a-a\sec(c+dx)}{\tan^{\frac{7}{2}}(c+dx)}dx}{a^2}$$

$$\downarrow 3042$$

$$-\frac{\tan^{\frac{3}{2}}(c+dx)(e\cot(c+dx))^{3/2}\int \frac{a-a\csc\left(c+dx+\frac{\pi}{2}\right)}{\left(-\cot\left(c+dx+\frac{\pi}{2}\right)\right)^{7/2}}dx}{a^2}$$

$$\downarrow 4370$$

$$-\frac{\tan^{\frac{3}{2}}(c+dx)(e\cot(c+dx))^{3/2}\left(\frac{2}{5}\int -\frac{5a-3a\sec(c+dx)}{2\tan^{\frac{3}{2}}(c+dx)}dx-\frac{2(a-a\sec(c+dx))}{5d\tan^{\frac{5}{2}}(c+dx)}\right)}{a^2}$$

$$\downarrow 27$$

$$-\frac{\tan^{\frac{3}{2}}(c+dx)(e\cot(c+dx))^{3/2}\left(-\frac{1}{5}\int \frac{5a-3a\sec(c+dx)}{\tan^{\frac{3}{2}}(c+dx)}dx-\frac{2(a-a\sec(c+dx))}{5d\tan^{\frac{5}{2}}(c+dx)}\right)}{a^2}$$

$$\downarrow 3042$$

$$-\frac{\tan^{\frac{3}{2}}(c+dx)(e\cot(c+dx))^{3/2}\left(-\frac{1}{5}\int\frac{5a-3a\csc\left(c+dx+\frac{\pi}{2}\right)}{\left(-\cot\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx-\frac{2(a-a\sec(c+dx))}{5d\tan^{\frac{5}{2}}(c+dx)}\right)}{a^2}$$

$$\downarrow 4370$$

$$-\frac{\tan^{\frac{3}{2}}(c+dx)(e\cot(c+dx))^{3/2}\left(\frac{1}{5}\left(\frac{2(5a-3a\sec(c+dx))}{d\sqrt{\tan(c+dx)}}-2\int-\frac{1}{2}(3\sec(c+dx)a+5a)\sqrt{\tan(c+dx)}dx\right)-\frac{2(a-a\sec}{5d\tan^{\frac{5}{2}}}\right.}{a^2}$$

$$\downarrow 27$$

$$-\frac{\tan^{\frac{3}{2}}(c+dx)(e\cot(c+dx))^{3/2}\left(\frac{1}{5}\left(\int(3\sec(c+dx)a+5a)\sqrt{\tan(c+dx)}dx+\frac{2(5a-3a\sec(c+dx))}{d\sqrt{\tan(c+dx)}}\right)-\frac{2(a-a\sec(c+dx)}{5d\tan^{\frac{5}{2}}(c+dx)}\right.}{a^2}$$

$$\downarrow 3042$$

$$-\frac{\tan^{\frac{3}{2}}(c+dx)(e\cot(c+dx))^{3/2}\left(\frac{1}{5}\left(\int\sqrt{-\cot\left(c+dx+\frac{\pi}{2}\right)}\left(3\csc\left(c+dx+\frac{\pi}{2}\right)a+5a\right)dx+\frac{2(5a-3a\sec(c+dx))}{d\sqrt{\tan(c+dx)}}\right)-\right.}{a^2}$$

$$\downarrow 4372$$

$$-\frac{\tan^{\frac{3}{2}}(c+dx)(e\cot(c+dx))^{3/2}\left(\frac{1}{5}\left(5a\int\sqrt{\tan(c+dx)}dx+3a\int\sec(c+dx)\sqrt{\tan(c+dx)}dx+\frac{2(5a-3a\sec(c+dx))}{d\sqrt{\tan(c+dx)}}\right.\right.}{a^2}$$

$$\downarrow 3042$$

$$-\frac{\tan^{\frac{3}{2}}(c+dx)(e\cot(c+dx))^{3/2}\left(\frac{1}{5}\left(5a\int\sqrt{\tan(c+dx)}dx+3a\int\sec(c+dx)\sqrt{\tan(c+dx)}dx+\frac{2(5a-3a\sec(c+dx))}{d\sqrt{\tan(c+dx)}}\right.\right.}{a^2}$$

$$\downarrow 3093$$

$$-\frac{\tan^{\frac{3}{2}}(c+dx)(e\cot(c+dx))^{3/2}\left(\frac{1}{5}\left(5a\int\sqrt{\tan(c+dx)}dx+3a\left(\frac{2\cos(c+dx)\tan^{\frac{3}{2}}(c+dx)}{d}-2\int\cos(c+dx)\sqrt{\tan(c+\right.\right.\right.}{a^2}$$

$$\downarrow 3042$$

$$-\frac{\tan^{\frac{3}{2}}(c+dx)(e\cot(c+dx))^{3/2}\left(\frac{1}{5}\left(5a\int\sqrt{\tan(c+dx)}dx+3a\left(\frac{2\cos(c+dx)\tan^{\frac{3}{2}}(c+dx)}{d}-2\int\frac{\sqrt{\tan(c+dx)}}{\sec(c+dx)}dx\right)+\frac{2(5}{a^2}\right.\right.\right.$$

$$\downarrow 3095$$

$$-\frac{\tan^{\frac{3}{2}}(c+dx)(e\cot(c+dx))^{3/2}\left(\frac{1}{5}\left(5a\int\sqrt{\tan(c+dx)}dx+3a\left(\frac{2\cos(c+dx)\tan^{\frac{3}{2}}(c+dx)}{d}-\frac{2\sqrt{\cos(c+dx)}\sqrt{\tan(c+dx)}\int\sqrt{}}{\sqrt{\sin(c+}}\right.\right.\right.}{a^2}$$

$$\downarrow 3042$$

$$-\frac{\tan^{\frac{3}{2}}(c+dx)(e\cot(c+dx))^{3/2}\left(\frac{1}{5}\left(5a\int\sqrt{\tan(c+dx)}dx+3a\left(\frac{2\cos(c+dx)\tan^{\frac{3}{2}}(c+dx)}{d}-\frac{2\sqrt{\cos(c+dx)}\sqrt{\tan(c+dx)}\int\sqrt{}}{\sqrt{\sin(c+}}\right.\right.\right.}{a^2}$$

$$\downarrow 3052$$

$$-\frac{\tan^{\frac{3}{2}}(c+dx)(e\cot(c+dx))^{3/2}\left(\frac{1}{5}\left(5a\int\sqrt{\tan(c+dx)}dx+3a\left(\frac{2\cos(c+dx)\tan^{\frac{3}{2}}(c+dx)}{d}-\frac{2\cos(c+dx)\sqrt{\tan(c+dx)}\int\sqrt{si}}{\sqrt{\sin(2c+2dx)}}\right.\right.\right.}{a^2}$$

$$\downarrow 3042$$

$$-\frac{\tan^{\frac{3}{2}}(c+dx)(e\cot(c+dx))^{3/2}\left(\frac{1}{5}\left(5a\int\sqrt{\tan(c+dx)}dx+3a\left(\frac{2\cos(c+dx)\tan^{\frac{3}{2}}(c+dx)}{d}-\frac{2\cos(c+dx)\sqrt{\tan(c+dx)}\int\sqrt{si}}{\sqrt{\sin(2c+2dx)}}\right.\right.\right.}{a^2}$$

$$\downarrow 3119$$

$$-\frac{\tan^{\frac{3}{2}}(c+dx)(e\cot(c+dx))^{3/2}\left(\frac{1}{5}\left(5a\int\sqrt{\tan(c+dx)}dx+\frac{2(5a-3a\sec(c+dx))}{d\sqrt{\tan(c+dx)}}+3a\left(\frac{2\cos(c+dx)\tan^{\frac{3}{2}}(c+dx)}{d}-\frac{2\cos(c}{}\right.\right.\right.}{a^2}$$

$$\downarrow 3957$$

$$-\frac{\tan^{\frac{3}{2}}(c+dx)(e\cot(c+dx))^{3/2}\left(\frac{1}{5}\left(\frac{5a\int\frac{\sqrt{\tan(c+dx)}}{\tan^2(c+dx)+1}d\tan(c+dx)}{d}+\frac{2(5a-3a\sec(c+dx))}{d\sqrt{\tan(c+dx)}}+3a\left(\frac{2\cos(c+dx)\tan^{\frac{3}{2}}(c+dx)}{d}-\frac{2c}{}\right.\right.\right.}{a^2}$$

$$\downarrow 266$$

$$-\frac{\tan^{\frac{3}{2}}(c+dx)(e\cot(c+dx))^{3/2}\left(\frac{1}{5}\left(\frac{10a\int\frac{\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}}{d}+\frac{2(5a-3a\sec(c+dx))}{d\sqrt{\tan(c+dx)}}+3a\left(\frac{2\cos(c+dx)\tan^{\frac{3}{2}}(c+dx)}{d}-\right.\right.\right.}{a^2}$$

$$\downarrow 826$$

$$-\frac{\tan^{\frac{3}{2}}(c+dx)(e\cot(c+dx))^{3/2}\left(\frac{1}{5}\left(\frac{10a\left(\frac{1}{2}\int\frac{\tan(c+dx)+1}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}-\frac{1}{2}\int\frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}\right)}{d}+\frac{2(5a-3a\sec(c+dx))}{d\sqrt{\tan(c+dx)}}\right.\right.\right.}{a^2}$$

$$\downarrow 1476$$

$$-\frac{\tan^{\frac{3}{2}}(c+dx)(e\cot(c+dx))^{3/2}\left(\frac{1}{5}\left(\frac{10a\left(\frac{1}{2}\left(\frac{1}{2}\int\frac{1}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}+\frac{1}{2}\int\frac{1}{\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(}\right.\right.\right.}{d}$$

$$\downarrow 1082$$

$$-\frac{\tan^{\frac{3}{2}}(c+dx)(e\cot(c+dx))^{3/2}\left(\frac{1}{5}\left(\frac{10a\left(\frac{1}{2}\left(\frac{\int\frac{1}{-\tan(c+dx)-1}d\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}}-\frac{\int\frac{1}{-\tan(c+dx)-1}d\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}}\right)\right)-\frac{1}{2}\int\frac{1-t}{\tan^2}\right.\right.\right.}{d}$$

$$\downarrow 217$$

$$-\frac{\tan^{\frac{3}{2}}(c+dx)(e\cot(c+dx))^{3/2}\left(\frac{1}{5}\left(\frac{10a\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}}\right)\right)-\frac{1}{2}\int\frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}\right.\right.\right.}{d}$$

$$a^2$$

$$\downarrow 1479$$

$$-\frac{\tan^{\frac{3}{2}}(c+dx)(e\cot(c+dx))^{3/2}\left(\frac{1}{5}\left(\frac{10a\left(\frac{1}{2}\left(\frac{\int-\frac{\sqrt{2}-2\sqrt{\tan(c+dx)}}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}+\frac{\int-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+}}{2\sqrt{2}}\right.\right.\right.\right.}{d}$$

$$\downarrow 25$$

$$-\frac{\tan^{\frac{3}{2}}(c+dx)(e\cot(c+dx))^{3/2}\left(\frac{1}{5}\left(\frac{10a\left(\frac{1}{2}\left(-\frac{\int\frac{\sqrt{2}-2\sqrt{\tan(c+dx)}}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}-\frac{\int\frac{\sqrt{2}\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx}}{2\sqrt{2}}\right.\right.\right.\right.}{d}$$

$$\downarrow 27$$

$$-\frac{\tan^{\frac{3}{2}}(c+dx)(e\cot(c+dx))^{3/2}\left(\frac{1}{5}\left(\frac{10a\left(\frac{1}{2}\left(-\frac{\int\frac{\sqrt{2}-2\sqrt{\tan(c+dx)}}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}-\frac{1}{2}\int\frac{\sqrt{2}\sqrt{\tan(c+dx)}+1}{\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c\right.\right.\right.\right.\right.\right.\right.}{d}$$

input
```
Int[(e*Cot[c + d*x])^(3/2)/(a + a*Sec[c + d*x]),x]
```

output
```
$Aborted
```

3.243.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3052
```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Simp[Sqrt[a*Sin[e + f*x]]*(Sqrt[b*Cos[e + f*x]]/Sqrt[Sin[2*e
 + 2*f*x]])   Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

rule 3093
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[a^2*(a*Sec[e + f*x])^(m - 2)*((b*Tan[e + f*x])^(n +
1)/(b*f*(m + n - 1))), x] + Simp[a^2*((m - 2)/(m + n - 1))   Int[(a*Sec[e +
 f*x])^(m - 2)*(b*Tan[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (
GtQ[m, 1] || (EqQ[m, 1] && EqQ[n, 1/2])) && NeQ[m + n - 1, 0] && IntegersQ[
2*m, 2*n]
```

rule 3095
```
Int[Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]]/sec[(e_.) + (f_.)*(x_)], x_Symbol]
:> Simp[Sqrt[Cos[e + f*x]]*(Sqrt[b*Tan[e + f*x]]/Sqrt[Sin[e + f*x]])   Int[
Sqrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]], x], x] /; FreeQ[{b, e, f}, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

rule 4370
```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(-(e*Cot[c + d*x])^(m + 1))*((a + b*Csc[c + d*x])/(
d*e*(m + 1))), x] - Simp[1/(e^2*(m + 1))   Int[(e*Cot[c + d*x])^(m + 2)*(a*
(m + 1) + b*(m + 2)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && L
tQ[m, -1]
```

rule 4372
```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(e*Cot[c + d*x])^m, x], x] + Simp[b   Int[
(e*Cot[c + d*x])^m*Csc[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, m}, x]
```

rule 4376
```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Simp[a^(2*n)/e^(2*n)   Int[(e*Cot[c + d*x])^(m + 2*
n)/(-a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[a
^2 - b^2, 0] && ILtQ[n, 0]
```

rule 4388
```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*((a_) + (b_.)*sec[(c_.) + (d_.)*(x
_)])^(n_.), x_Symbol] :> Simp[(e*Cot[c + d*x])^m*Tan[c + d*x]^m   Int[(a +
b*Sec[c + d*x])^n/Tan[c + d*x]^m, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x]
 &&  !IntegerQ[m]
```

3.243.4 Maple [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 8.23 (sec) , antiderivative size = 1116, normalized size of antiderivative = 2.76

method	result	size
default	Expression too large to display	1116

input
```
int((e*cot(d*x+c))^(3/2)/(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
-1/10/a/d*2^(1/2)*(-e/(1-cos(d*x+c))*((1-cos(d*x+c))^2*csc(d*x+c)-sin(d*x+
c)))^(3/2)*(1-cos(d*x+c))*(5*I*(csc(d*x+c)-cot(d*x+c)+1)^(1/2)*(2-2*csc(d*
x+c)+2*cot(d*x+c))^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)*EllipticPi((csc(d*x
+c)-cot(d*x+c)+1)^(1/2),1/2-1/2*I,1/2*2^(1/2))*((1-cos(d*x+c))*((1-cos(d*x
+c))^2*csc(d*x+c)^2-1)*csc(d*x+c))^(1/2)-5*I*(csc(d*x+c)-cot(d*x+c)+1)^(1/
2)*(2-2*csc(d*x+c)+2*cot(d*x+c))^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)*Ellip
ticPi((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2+1/2*I,1/2*2^(1/2))*((1-cos(d*x+c
))*((1-cos(d*x+c))^2*csc(d*x+c)^2-1)*csc(d*x+c))^(1/2)+12*(csc(d*x+c)-cot(
d*x+c)+1)^(1/2)*(2-2*csc(d*x+c)+2*cot(d*x+c))^(1/2)*(cot(d*x+c)-csc(d*x+c)
)^(1/2)*EllipticE((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2*2^(1/2))*((1-cos(d*x
+c))*((1-cos(d*x+c))^2*csc(d*x+c)^2-1)*csc(d*x+c))^(1/2)-6*(csc(d*x+c)-cot
(d*x+c)+1)^(1/2)*(2-2*csc(d*x+c)+2*cot(d*x+c))^(1/2)*(cot(d*x+c)-csc(d*x+c
))^(1/2)*EllipticF((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2*2^(1/2))*((1-cos(d*
x+c))*((1-cos(d*x+c))^2*csc(d*x+c)^2-1)*csc(d*x+c))^(1/2)-5*(csc(d*x+c)-co
t(d*x+c)+1)^(1/2)*(2-2*csc(d*x+c)+2*cot(d*x+c))^(1/2)*(cot(d*x+c)-csc(d*x+
c))^(1/2)*EllipticPi((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2-1/2*I,1/2*2^(1/2)
)*((1-cos(d*x+c))*((1-cos(d*x+c))^2*csc(d*x+c)^2-1)*csc(d*x+c))^(1/2)-5*(c
sc(d*x+c)-cot(d*x+c)+1)^(1/2)*(2-2*csc(d*x+c)+2*cot(d*x+c))^(1/2)*(cot(d*x
+c)-csc(d*x+c))^(1/2)*EllipticPi((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2+1/2*I
,1/2*2^(1/2))*((1-cos(d*x+c))*((1-cos(d*x+c))^2*csc(d*x+c)^2-1)*csc(d*x...
```

3.243.5 Fricas [F(-1)]

Timed out.

$$\int \frac{(e\cot(c+dx))^{3/2}}{a+a\sec(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((e*cot(d*x+c))^(3/2)/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

output
```
Timed out
```

3.243.6 Sympy [F]

$$\int \frac{(e\cot(c+dx))^{3/2}}{a+a\sec(c+dx)}\,dx = \frac{\int \frac{(e\cot(c+dx))^{\frac{3}{2}}}{\sec(c+dx)+1}\,dx}{a}$$

input
```
integrate((e*cot(d*x+c))**(3/2)/(a+a*sec(d*x+c)),x)
```

output
```
Integral((e*cot(c + d*x))**(3/2)/(sec(c + d*x) + 1), x)/a
```

3.243.7 Maxima [F(-1)]

Timed out.

$$\int \frac{(e\cot(c+dx))^{3/2}}{a+a\sec(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((e*cot(d*x+c))^(3/2)/(a+a*sec(d*x+c)),x, algorithm="maxima")
```

output
```
Timed out
```

3.243.8 Giac [F]

$$\int \frac{(e\cot(c+dx))^{3/2}}{a+a\sec(c+dx)}\,dx = \int \frac{(e\cot(dx+c))^{\frac{3}{2}}}{a\sec(dx+c)+a}\,dx$$

input
```
integrate((e*cot(d*x+c))^(3/2)/(a+a*sec(d*x+c)),x, algorithm="giac")
```

output
```
integrate((e*cot(d*x + c))^(3/2)/(a*sec(d*x + c) + a), x)
```

3.243.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(e\cot(c+dx))^{3/2}}{a+a\sec(c+dx)}\,dx = \int \frac{\cos(c+d\,x)\,(e\cot(c+d\,x))^{3/2}}{a\,(\cos(c+d\,x)+1)}\,dx$$

input
```
int((e*cot(c + d*x))^(3/2)/(a + a/cos(c + d*x)),x)
```

output
```
int((cos(c + d*x)*(e*cot(c + d*x))^(3/2))/(a*(cos(c + d*x) + 1)), x)
```

3.244 $\int \frac{\sqrt{e\cot(c+dx)}}{a+a\sec(c+dx)}\,dx$

3.244.1 Optimal result 1773
3.244.2 Mathematica [C] (warning: unable to verify) 1774
3.244.3 Rubi [A] (verified) 1774
3.244.4 Maple [C] (verified) 1781
3.244.5 Fricas [F(-1)] 1782
3.244.6 Sympy [F] 1782
3.244.7 Maxima [F] 1782
3.244.8 Giac [F] 1783
3.244.9 Mupad [F(-1)] 1783

3.244.1 Optimal result

Integrand size = 25, antiderivative size = 325

$$
\begin{aligned}
&\int \frac{\sqrt{e\cot(c+dx)}}{a+a\sec(c+dx)}\,dx \\
&= \frac{2\cot(c+dx)\sqrt{e\cot(c+dx)}(1-\sec(c+dx))}{3ad} \\
&\quad - \frac{\sqrt{e\cot(c+dx)}\,\text{EllipticF}\left(c-\tfrac{\pi}{4}+dx,2\right)\sec(c+dx)\sqrt{\sin(2c+2dx)}}{3ad} \\
&\quad - \frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)\sqrt{e\cot(c+dx)}\sqrt{\tan(c+dx)}}{\sqrt{2}ad} \\
&\quad + \frac{\arctan\left(1+\sqrt{2}\sqrt{\tan(c+dx)}\right)\sqrt{e\cot(c+dx)}\sqrt{\tan(c+dx)}}{\sqrt{2}ad} \\
&\quad - \frac{\sqrt{e\cot(c+dx)}\log\left(1-\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+dx)\right)\sqrt{\tan(c+dx)}}{2\sqrt{2}ad} \\
&\quad + \frac{\sqrt{e\cot(c+dx)}\log\left(1+\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+dx)\right)\sqrt{\tan(c+dx)}}{2\sqrt{2}ad}
\end{aligned}
$$

output
```
2/3*cot(d*x+c)*(1-sec(d*x+c))*(e*cot(d*x+c))^(1/2)/a/d+1/3*(sin(c+1/4*Pi+d
*x)^2)^(1/2)/sin(c+1/4*Pi+d*x)*EllipticF(cos(c+1/4*Pi+d*x),2^(1/2))*sec(d*
x+c)*(e*cot(d*x+c))^(1/2)*sin(2*d*x+2*c)^(1/2)/a/d+1/2*arctan(-1+2^(1/2)*t
an(d*x+c)^(1/2))*(e*cot(d*x+c))^(1/2)*tan(d*x+c)^(1/2)/a/d*2^(1/2)+1/2*arc
tan(1+2^(1/2)*tan(d*x+c)^(1/2))*(e*cot(d*x+c))^(1/2)*tan(d*x+c)^(1/2)/a/d*
2^(1/2)-1/4*ln(1-2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c))*(e*cot(d*x+c))^(1/2)
*tan(d*x+c)^(1/2)/a/d*2^(1/2)+1/4*ln(1+2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c)
)*(e*cot(d*x+c))^(1/2)*tan(d*x+c)^(1/2)/a/d*2^(1/2)
```

3.244.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 12.31 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.42

$$\int \frac{\sqrt{e\cot(c+dx)}}{a+a\sec(c+dx)}\,dx =$$
$$-\frac{4\sqrt{e\cot(c+dx)}\csc(c+dx)\left(\cot^2(c+dx)\,\text{Hypergeometric2F1}\left(-\frac{3}{4},-\frac{1}{2},\frac{1}{4},-\tan^2(c+dx)\right)+3\,\text{Hyperg}\right.}{}$$

input
```
Integrate[Sqrt[e*Cot[c + d*x]]/(a + a*Sec[c + d*x]),x]
```

output
```
(-4*Sqrt[e*Cot[c + d*x]]*Csc[c + d*x]*(Cot[c + d*x]^2*Hypergeometric2F1[-3
/4, -1/2, 1/4, -Tan[c + d*x]^2] + 3*Hypergeometric2F1[1/4, 1/2, 5/4, -Tan[
c + d*x]^2] + Cot[c + d*x]^2*(-1 + Hypergeometric2F1[3/4, 1, 7/4, -Cot[c +
 d*x]^2]))*(1 + Sqrt[Sec[c + d*x]^2])*Sin[(c + d*x)/2]^2)/(3*a*d)
```

3.244.3 Rubi [A] (verified)

Time = 1.08 (sec) , antiderivative size = 245, normalized size of antiderivative = 0.75, number of steps used = 27, number of rules used = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 1.040$, Rules used = {3042, 4388, 3042, 4376, 25, 3042, 4370, 27, 3042, 4372, 3042, 3094, 3042, 3053, 3042, 3120, 3957, 266, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{e\cot(c+dx)}}{a\sec(c+dx)+a}\,dx$$

$$\downarrow 3042$$

$$\int \frac{\sqrt{e\cot(c+dx)}}{a\sec(c+dx)+a}dx$$

$$\downarrow 4388$$

$$\sqrt{\tan(c+dx)}\sqrt{e\cot(c+dx)}\int \frac{1}{(\sec(c+dx)a+a)\sqrt{\tan(c+dx)}}dx$$

$$\downarrow 3042$$

$$\sqrt{\tan(c+dx)}\sqrt{e\cot(c+dx)}\int \frac{1}{\sqrt{-\cot\left(c+dx+\frac{\pi}{2}\right)}\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)}dx$$

$$\downarrow 4376$$

$$\frac{\sqrt{\tan(c+dx)}\sqrt{e\cot(c+dx)}\int -\frac{a-a\sec(c+dx)}{\tan^{\frac{5}{2}}(c+dx)}dx}{a^2}$$

$$\downarrow 25$$

$$-\frac{\sqrt{\tan(c+dx)}\sqrt{e\cot(c+dx)}\int \frac{a-a\sec(c+dx)}{\tan^{\frac{5}{2}}(c+dx)}dx}{a^2}$$

$$\downarrow 3042$$

$$-\frac{\sqrt{\tan(c+dx)}\sqrt{e\cot(c+dx)}\int \frac{a-a\csc\left(c+dx+\frac{\pi}{2}\right)}{\left(-\cot\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}dx}{a^2}$$

$$\downarrow 4370$$

$$-\frac{\sqrt{\tan(c+dx)}\sqrt{e\cot(c+dx)}\left(\frac{2}{3}\int -\frac{3a-a\sec(c+dx)}{2\sqrt{\tan(c+dx)}}dx-\frac{2(a-a\sec(c+dx))}{3d\tan^{\frac{3}{2}}(c+dx)}\right)}{a^2}$$

$$\downarrow 27$$

$$-\frac{\sqrt{\tan(c+dx)}\sqrt{e\cot(c+dx)}\left(-\frac{1}{3}\int \frac{3a-a\sec(c+dx)}{\sqrt{\tan(c+dx)}}dx-\frac{2(a-a\sec(c+dx))}{3d\tan^{\frac{3}{2}}(c+dx)}\right)}{a^2}$$

$$\downarrow 3042$$

$$-\frac{\sqrt{\tan(c+dx)}\sqrt{e\cot(c+dx)}\left(-\frac{1}{3}\int \frac{3a-a\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{-\cot\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{2(a-a\sec(c+dx))}{3d\tan^{\frac{3}{2}}(c+dx)}\right)}{a^2}$$

$$\downarrow 4372$$

$$-\frac{\sqrt{\tan(c+dx)}\sqrt{e\cot(c+dx)}\left(\frac{1}{3}\left(a\int\frac{\sec(c+dx)}{\sqrt{\tan(c+dx)}}dx-3a\int\frac{1}{\sqrt{\tan(c+dx)}}dx\right)-\frac{2(a-a\sec(c+dx))}{3d\tan^{\frac{3}{2}}(c+dx)}\right)}{a^2}$$

$$\downarrow 3042$$

$$-\frac{\sqrt{\tan(c+dx)}\sqrt{e\cot(c+dx)}\left(\frac{1}{3}\left(a\int\frac{\sec(c+dx)}{\sqrt{\tan(c+dx)}}dx-3a\int\frac{1}{\sqrt{\tan(c+dx)}}dx\right)-\frac{2(a-a\sec(c+dx))}{3d\tan^{\frac{3}{2}}(c+dx)}\right)}{a^2}$$

$$\downarrow 3094$$

$$-\frac{\sqrt{\tan(c+dx)}\sqrt{e\cot(c+dx)}\left(\frac{1}{3}\left(\frac{a\sqrt{\sin(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}\sqrt{\sin(c+dx)}}dx}{\sqrt{\cos(c+dx)}\sqrt{\tan(c+dx)}}-3a\int\frac{1}{\sqrt{\tan(c+dx)}}dx\right)-\frac{2(a-a\sec(c+dx))}{3d\tan^{\frac{3}{2}}(c+dx)}\right)}{a^2}$$

$$\downarrow 3042$$

$$-\frac{\sqrt{\tan(c+dx)}\sqrt{e\cot(c+dx)}\left(\frac{1}{3}\left(\frac{a\sqrt{\sin(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}\sqrt{\sin(c+dx)}}dx}{\sqrt{\cos(c+dx)}\sqrt{\tan(c+dx)}}-3a\int\frac{1}{\sqrt{\tan(c+dx)}}dx\right)-\frac{2(a-a\sec(c+dx))}{3d\tan^{\frac{3}{2}}(c+dx)}\right)}{a^2}$$

$$\downarrow 3053$$

$$-\frac{\sqrt{\tan(c+dx)}\sqrt{e\cot(c+dx)}\left(\frac{1}{3}\left(\frac{a\sqrt{\sin(2c+2dx)}\sec(c+dx)\int\frac{1}{\sqrt{\sin(2c+2dx)}}dx}{\sqrt{\tan(c+dx)}}-3a\int\frac{1}{\sqrt{\tan(c+dx)}}dx\right)-\frac{2(a-a\sec(c+dx))}{3d\tan^{\frac{3}{2}}(c+dx)}\right)}{a^2}$$

$$\downarrow 3042$$

$$-\frac{\sqrt{\tan(c+dx)}\sqrt{e\cot(c+dx)}\left(\frac{1}{3}\left(\frac{a\sqrt{\sin(2c+2dx)}\sec(c+dx)\int\frac{1}{\sqrt{\sin(2c+2dx)}}dx}{\sqrt{\tan(c+dx)}}-3a\int\frac{1}{\sqrt{\tan(c+dx)}}dx\right)-\frac{2(a-a\sec(c+dx))}{3d\tan^{\frac{3}{2}}(c+dx)}\right)}{a^2}$$

$$\downarrow 3120$$

$$-\frac{\sqrt{\tan(c+dx)}\sqrt{e\cot(c+dx)}\left(\frac{1}{3}\left(\frac{a\sqrt{\sin(2c+2dx)}\sec(c+dx)\,\text{EllipticF}\left(c+dx-\frac{\pi}{4},2\right)}{d\sqrt{\tan(c+dx)}}-3a\int\frac{1}{\sqrt{\tan(c+dx)}}dx\right)-\frac{2(a-a\sec(c+dx)}{3d\tan^{\frac{3}{2}}(c+dx)}\right.}{a^2}$$

$$\downarrow 3957$$

$$-\frac{\sqrt{\tan(c+dx)}\sqrt{e\cot(c+dx)}\left(\frac{1}{3}\left(\frac{a\sqrt{\sin(2c+2dx)}\sec(c+dx)\,\text{EllipticF}\left(c+dx-\frac{\pi}{4},2\right)}{d\sqrt{\tan(c+dx)}}-\frac{3a\int\frac{1}{\sqrt{\tan(c+dx)}\left(\tan^2(c+dx)+1\right)}d\tan(c+dx)}{d}\right.\right.}{a^2}$$

$$\downarrow 266$$

$$-\frac{\sqrt{\tan(c+dx)}\sqrt{e\cot(c+dx)}\left(\frac{1}{3}\left(\frac{a\sqrt{\sin(2c+2dx)}\sec(c+dx)\,\text{EllipticF}\left(c+dx-\frac{\pi}{4},2\right)}{d\sqrt{\tan(c+dx)}}-\frac{6a\int\frac{1}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}}{d}\right)-\frac{2(a-a}{3d\tan}\right.}{a^2}$$

$$\downarrow 755$$

$$-\frac{\sqrt{\tan(c+dx)}\sqrt{e\cot(c+dx)}\left(\frac{1}{3}\left(\frac{a\sqrt{\sin(2c+2dx)}\sec(c+dx)\,\text{EllipticF}\left(c+dx-\frac{\pi}{4},2\right)}{d\sqrt{\tan(c+dx)}}-\frac{6a\left(\frac{1}{2}\int\frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}+\frac{1}{2}\int\frac{\tan}{\tan^2}\right.\right.\right.}{a^2}$$

$$\downarrow 1476$$

$$-\frac{\sqrt{\tan(c+dx)}\sqrt{e\cot(c+dx)}\left(\frac{1}{3}\left(\frac{a\sqrt{\sin(2c+2dx)}\sec(c+dx)\,\text{EllipticF}\left(c+dx-\frac{\pi}{4},2\right)}{d\sqrt{\tan(c+dx)}}-\frac{6a\left(\frac{1}{2}\int\frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}+\frac{1}{2}\left(\frac{1}{2}\int\right.\right.\right.\right.}{a^2}$$

$$\downarrow 1082$$

$$-\frac{\sqrt{\tan(c+dx)}\sqrt{e\cot(c+dx)}\left(\frac{1}{3}\left(\frac{a\sqrt{\sin(2c+2dx)}\sec(c+dx)\,\text{EllipticF}\left(c+dx-\frac{\pi}{4},2\right)}{d\sqrt{\tan(c+dx)}}-\frac{6a\left(\frac{1}{2}\int\frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}+\frac{1}{2}\left(\frac{\int-}{}\right.\right.\right.\right.}{a^2}$$

$$\downarrow 217$$

$$-\frac{\sqrt{\tan(c+dx)}\sqrt{e\cot(c+dx)}\left(\frac{1}{3}\left(\frac{a\sqrt{\sin(2c+2dx)}\sec(c+dx)\,\text{EllipticF}\left(c+dx-\frac{\pi}{4},2\right)}{d\sqrt{\tan(c+dx)}}-\frac{6a\left(\frac{1}{2}\int\frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}+\frac{1}{2}\left(\frac{\text{arc}}{}\right.\right.\right.\right.}{a^2}$$

$$\downarrow 1479$$

$$-\frac{\sqrt{\tan(c+dx)}\sqrt{e\cot(c+dx)}\left(\frac{1}{3}\left(\frac{a\sqrt{\sin(2c+2dx)}\sec(c+dx)\,\text{EllipticF}\left(c+dx-\frac{\pi}{4},2\right)}{d\sqrt{\tan(c+dx)}}-\frac{6a\left(\frac{1}{2}\left(-\frac{\int-\frac{\sqrt{2}-2\sqrt{\tan(c+dx)}}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{t}}{2\sqrt{2}}\right.\right.\right.\right.}{}$$

$$\downarrow 25$$

$$-\frac{\sqrt{\tan(c+dx)}\sqrt{e\cot(c+dx)}\left(\frac{1}{3}\left(\frac{a\sqrt{\sin(2c+2dx)}\sec(c+dx)\,\text{EllipticF}\left(c+dx-\frac{\pi}{4},2\right)}{d\sqrt{\tan(c+dx)}}-\frac{6a\left(\frac{1}{2}\left(\int\frac{\sqrt{2}-2\sqrt{\tan(c+dx)}}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1}\frac{d\sqrt{\tan(}}{2\sqrt{2}}\right.\right.\right.}{}}{a^2}$$

$$\downarrow 27$$

$$-\frac{\sqrt{\tan(c+dx)}\sqrt{e\cot(c+dx)}\left(\frac{1}{3}\left(\frac{a\sqrt{\sin(2c+2dx)}\sec(c+dx)\,\text{EllipticF}\left(c+dx-\frac{\pi}{4},2\right)}{d\sqrt{\tan(c+dx)}}-\frac{6a\left(\frac{1}{2}\left(\int\frac{\sqrt{2}-2\sqrt{\tan(c+dx)}}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1}\frac{d\sqrt{\tan(}}{2\sqrt{2}}\right.\right.\right.}{}}{a}$$

$$\downarrow 1103$$

$$-\frac{\sqrt{\tan(c+dx)}\sqrt{e\cot(c+dx)}\left(\frac{1}{3}\left(\frac{a\sqrt{\sin(2c+2dx)}\sec(c+dx)\,\text{EllipticF}\left(c+dx-\frac{\pi}{4},2\right)}{d\sqrt{\tan(c+dx)}}-\frac{6a\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\right.\right.\right.\right.}{}}{a^2}$$

input
```
Int[Sqrt[e*Cot[c + d*x]]/(a + a*Sec[c + d*x]),x]
```

output
```
-((Sqrt[e*Cot[c + d*x]]*(((-6*a*((-(ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]]
/Sqrt[2]) + ArcTan[1 + Sqrt[2]*Sqrt[Tan[c + d*x]]]/Sqrt[2])/2 + (-1/2*Log[
1 - Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]]/Sqrt[2] + Log[1 + Sqrt[2]*S
qrt[Tan[c + d*x]] + Tan[c + d*x]]/(2*Sqrt[2]))/2))/d + (a*EllipticF[c - Pi
/4 + d*x, 2]*Sec[c + d*x]*Sqrt[Sin[2*c + 2*d*x]])/(d*Sqrt[Tan[c + d*x]]))/
3 - (2*(a - a*Sec[c + d*x]))/(3*d*Tan[c + d*x]^(3/2)))*Sqrt[Tan[c + d*x]])
/a^2)
```

3.244.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3053
```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Simp[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]])   Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f
}, x]
```

rule 3094
```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]], x_Symbol]
:> Simp[Sqrt[Sin[e + f*x]]/(Sqrt[Cos[e + f*x]]*Sqrt[b*Tan[e + f*x]])   Int[
1/(Sqrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]]), x], x] /; FreeQ[{b, e, f}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

rule 4370
```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(-(e*Cot[c + d*x])^(m + 1))*((a + b*Csc[c + d*x])/(
d*e*(m + 1))), x] - Simp[1/(e^2*(m + 1))   Int[(e*Cot[c + d*x])^(m + 2)*(a*
(m + 1) + b*(m + 2)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && L
tQ[m, -1]
```

rule 4372
```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(e*Cot[c + d*x])^m, x], x] + Simp[b   Int[
(e*Cot[c + d*x])^m*Csc[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, m}, x]
```

rule 4376
```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Simp[a^(2*n)/e^(2*n)   Int[(e*Cot[c + d*x])^(m + 2*
n)/(-a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[a
^2 - b^2, 0] && ILtQ[n, 0]
```

rule 4388
```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*((a_) + (b_.)*sec[(c_.) + (d_.)*(x
_)])^(n_.), x_Symbol] :> Simp[(e*Cot[c + d*x])^m*Tan[c + d*x]^m   Int[(a +
b*Sec[c + d*x])^n/Tan[c + d*x]^m, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x]
 &&  !IntegerQ[m]
```

3.244.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 8.16 (sec) , antiderivative size = 611, normalized size of antiderivative = 1.88

method	result
default	$-\frac{\sqrt{2}\sqrt{-\frac{e\left((1-\cos(dx+c))^2\csc(dx+c)-\sin(dx+c)\right)}{1-\cos(dx+c)}}\,(1-\cos(dx+c))\left(3i\sqrt{\csc(dx+c)-\cot(dx+c)+1}\,\sqrt{2-2\csc(dx+c)+2\cot(dx+c)}\,\sqrt{\cot}\right.$

input
```
int((e*cot(d*x+c))^(1/2)/(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
-1/6/a/d*2^(1/2)*(-e/(1-cos(d*x+c))*((1-cos(d*x+c))^2*csc(d*x+c)-sin(d*x+c
)))^(1/2)*(1-cos(d*x+c))*(3*I*(csc(d*x+c)-cot(d*x+c)+1)^(1/2)*(2-2*csc(d*x
+c)+2*cot(d*x+c))^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)*EllipticPi((csc(d*x+
c)-cot(d*x+c)+1)^(1/2),1/2-1/2*I,1/2*2^(1/2))-3*I*(csc(d*x+c)-cot(d*x+c)+1
)^(1/2)*(2-2*csc(d*x+c)+2*cot(d*x+c))^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)*
EllipticPi((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2+1/2*I,1/2*2^(1/2))-8*(csc(d
*x+c)-cot(d*x+c)+1)^(1/2)*(2-2*csc(d*x+c)+2*cot(d*x+c))^(1/2)*(cot(d*x+c)-
csc(d*x+c))^(1/2)*EllipticF((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2*2^(1/2))+3
*(csc(d*x+c)-cot(d*x+c)+1)^(1/2)*(2-2*csc(d*x+c)+2*cot(d*x+c))^(1/2)*(cot(
d*x+c)-csc(d*x+c))^(1/2)*EllipticPi((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2-1/
2*I,1/2*2^(1/2))+3*(csc(d*x+c)-cot(d*x+c)+1)^(1/2)*(2-2*csc(d*x+c)+2*cot(d
*x+c))^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)*EllipticPi((csc(d*x+c)-cot(d*x+
c)+1)^(1/2),1/2+1/2*I,1/2*2^(1/2))+2*(1-cos(d*x+c))^3*csc(d*x+c)^3-2*csc(d
*x+c)+2*cot(d*x+c))/((1-cos(d*x+c))*((1-cos(d*x+c))^2*csc(d*x+c)^2-1)*csc(
d*x+c))^(1/2)/((1-cos(d*x+c))^3*csc(d*x+c)^3+cot(d*x+c)-csc(d*x+c))^(1/2)*
csc(d*x+c)
```

3.244.5 Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{e\cot(c+dx)}}{a+a\sec(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((e*cot(d*x+c))^(1/2)/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

output
```
Timed out
```

3.244.6 Sympy [F]

$$\int \frac{\sqrt{e\cot(c+dx)}}{a+a\sec(c+dx)}\,dx = \frac{\int \frac{\sqrt{e\cot(c+dx)}}{\sec(c+dx)+1}\,dx}{a}$$

input
```
integrate((e*cot(d*x+c))**(1/2)/(a+a*sec(d*x+c)),x)
```

output
```
Integral(sqrt(e*cot(c + d*x))/(sec(c + d*x) + 1), x)/a
```

3.244.7 Maxima [F]

$$\int \frac{\sqrt{e\cot(c+dx)}}{a+a\sec(c+dx)}\,dx = \int \frac{\sqrt{e\cot(dx+c)}}{a\sec(dx+c)+a}\,dx$$

input
```
integrate((e*cot(d*x+c))^(1/2)/(a+a*sec(d*x+c)),x, algorithm="maxima")
```

output
```
integrate(sqrt(e*cot(d*x + c))/(a*sec(d*x + c) + a), x)
```

3.244.8 Giac [F]

$$\int \frac{\sqrt{e\cot(c+dx)}}{a+a\sec(c+dx)}\,dx = \int \frac{\sqrt{e\cot(dx+c)}}{a\sec(dx+c)+a}\,dx$$

input
```
integrate((e*cot(d*x+c))^(1/2)/(a+a*sec(d*x+c)),x, algorithm="giac")
```

output
```
integrate(sqrt(e*cot(d*x + c))/(a*sec(d*x + c) + a), x)
```

3.244.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{e\cot(c+dx)}}{a+a\sec(c+dx)}\,dx = \int \frac{\cos(c+dx)\,\sqrt{e\cot(c+dx)}}{a\,(\cos(c+dx)+1)}\,dx$$

input
```
int((e*cot(c + d*x))^(1/2)/(a + a/cos(c + d*x)),x)
```

output
```
int((cos(c + d*x)*(e*cot(c + d*x))^(1/2))/(a*(cos(c + d*x) + 1)), x)
```

3.245 $\int \frac{1}{\sqrt{e\cot(c+dx)}(a+a\sec(c+dx))}\,dx$

3.245.1 Optimal result . . . 1784
3.245.2 Mathematica [C] (warning: unable to verify) . . . 1785
3.245.3 Rubi [A] (verified) . . . 1786
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3.245.5 Fricas [F(-1)] . . . 1793
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3.245.1 Optimal result

Integrand size = 25, antiderivative size = 347

$$\begin{aligned}\int \frac{1}{\sqrt{e\cot(c+dx)}(a+a\sec(c+dx))}\,dx &= \frac{2\cot(c+dx)(1-\sec(c+dx))}{ad\sqrt{e\cot(c+dx)}} \\ &+ \frac{2\sin(c+dx)}{ad\sqrt{e\cot(c+dx)}} \\ &- \frac{2\cos(c+dx)E\left(c-\frac{\pi}{4}+dx\middle|2\right)}{ad\sqrt{e\cot(c+dx)}\sqrt{\sin(2c+2dx)}} \\ &- \frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}ad\sqrt{e\cot(c+dx)}\sqrt{\tan(c+dx)}} \\ &+ \frac{\arctan\left(1+\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}ad\sqrt{e\cot(c+dx)}\sqrt{\tan(c+dx)}} \\ &+ \frac{\log\left(1-\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+dx)\right)}{2\sqrt{2}ad\sqrt{e\cot(c+dx)}\sqrt{\tan(c+dx)}} \\ &- \frac{\log\left(1+\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+dx)\right)}{2\sqrt{2}ad\sqrt{e\cot(c+dx)}\sqrt{\tan(c+dx)}}\end{aligned}$$

output
```
2*cot(d*x+c)*(1-sec(d*x+c))/a/d/(e*cot(d*x+c))^(1/2)+2*sin(d*x+c)/a/d/(e*c
ot(d*x+c))^(1/2)+2*cos(d*x+c)*(sin(c+1/4*Pi+d*x)^2)^(1/2)/sin(c+1/4*Pi+d*x
)*EllipticE(cos(c+1/4*Pi+d*x),2^(1/2))/a/d/(e*cot(d*x+c))^(1/2)/sin(2*d*x+
2*c)^(1/2)+1/2*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2))/a/d*2^(1/2)/(e*cot(d*x+
c))^(1/2)/tan(d*x+c)^(1/2)+1/2*arctan(1+2^(1/2)*tan(d*x+c)^(1/2))/a/d*2^(1
/2)/(e*cot(d*x+c))^(1/2)/tan(d*x+c)^(1/2)+1/4*ln(1-2^(1/2)*tan(d*x+c)^(1/2
)+tan(d*x+c))/a/d*2^(1/2)/(e*cot(d*x+c))^(1/2)/tan(d*x+c)^(1/2)-1/4*ln(1+2
^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c))/a/d*2^(1/2)/(e*cot(d*x+c))^(1/2)/tan(d
*x+c)^(1/2)
```

3.245.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 13.94 (sec) , antiderivative size = 249, normalized size of antiderivative = 0.72

$$\int \frac{1}{\sqrt{e\cot(c+dx)}(a+a\sec(c+dx))}\,dx =$$
$$-\frac{\csc(c+dx)\left(24\cot^2(c+dx)\,\text{Hypergeometric2F1}\left(-\frac{1}{2},-\frac{1}{4},\frac{3}{4},-\tan^2(c+dx)\right)+8\,\text{Hypergeometric2F1}\right.}{}$$

input
```
Integrate[1/(Sqrt[e*Cot[c + d*x]]*(a + a*Sec[c + d*x])),x]
```

output
```
-1/6*(Csc[c + d*x]*(24*Cot[c + d*x]^2*Hypergeometric2F1[-1/2, -1/4, 3/4, -
Tan[c + d*x]^2] + 8*Hypergeometric2F1[1/2, 3/4, 7/4, -Tan[c + d*x]^2] - 3*
Cot[c + d*x]^(3/2)*(2*Sqrt[2]*ArcTan[1 - Sqrt[2]*Sqrt[Cot[c + d*x]]] - 2*S
qrt[2]*ArcTan[1 + Sqrt[2]*Sqrt[Cot[c + d*x]]] + 8*Sqrt[Cot[c + d*x]] + Sqr
t[2]*Log[1 - Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]] - Sqrt[2]*Log[1 +
Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]]))*(1 + Sqrt[Sec[c + d*x]^2])*Si
n[(c + d*x)/2]^2)/(a*d*Sqrt[e*Cot[c + d*x]])
```

3.245.3 Rubi [A] (verified)

Time = 1.20 (sec) , antiderivative size = 263, normalized size of antiderivative = 0.76, number of steps used = 29, number of rules used = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 1.120$, Rules used = {3042, 4388, 3042, 4376, 25, 3042, 4370, 27, 3042, 4372, 3042, 3093, 3042, 3095, 3042, 3052, 3042, 3119, 3957, 266, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a\sec(c+dx)+a)\sqrt{e\cot(c+dx)}}\,dx$$

$$\downarrow 3042$$

$$\int \frac{1}{(a\sec(c+dx)+a)\sqrt{e\cot(c+dx)}}dx$$

$$\downarrow 4388$$

$$\frac{\int \frac{\sqrt{\tan(c+dx)}}{\sec(c+dx)a+a}dx}{\sqrt{\tan(c+dx)}\sqrt{e\cot(c+dx)}}$$

$$\downarrow 3042$$

$$\frac{\int \frac{\sqrt{-\cot\left(c+dx+\frac{\pi}{2}\right)}}{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx}{\sqrt{\tan(c+dx)}\sqrt{e\cot(c+dx)}}$$

$$\downarrow 4376$$

$$\frac{\int -\frac{a-a\sec(c+dx)}{\tan^{\frac{3}{2}}(c+dx)}dx}{a^2\sqrt{\tan(c+dx)}\sqrt{e\cot(c+dx)}}$$

$$\downarrow 25$$

$$-\frac{\int \frac{a-a\sec(c+dx)}{\tan^{\frac{3}{2}}(c+dx)}dx}{a^2\sqrt{\tan(c+dx)}\sqrt{e\cot(c+dx)}}$$

$$\downarrow 3042$$

$$-\frac{\int \frac{a-a\csc\left(c+dx+\frac{\pi}{2}\right)}{\left(-\cot\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{a^2\sqrt{\tan(c+dx)}\sqrt{e\cot(c+dx)}}$$

$$\downarrow 4370$$

$$-\frac{2\int -\frac{1}{2}(\sec(c+dx)a+a)\sqrt{\tan(c+dx)}dx-\frac{2(a-a\sec(c+dx))}{d\sqrt{\tan(c+dx)}}}{a^2\sqrt{\tan(c+dx)}\sqrt{e\cot(c+dx)}}$$

$$\downarrow 27$$

$$-\frac{-\int(\sec(c+dx)a+a)\sqrt{\tan(c+dx)}dx-\frac{2(a-a\sec(c+dx))}{d\sqrt{\tan(c+dx)}}}{a^2\sqrt{\tan(c+dx)}\sqrt{e\cot(c+dx)}}$$

$$\downarrow 3042$$

$$-\frac{-\int\sqrt{-\cot\left(c+dx+\frac{\pi}{2}\right)}\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)dx-\frac{2(a-a\sec(c+dx))}{d\sqrt{\tan(c+dx)}}}{a^2\sqrt{\tan(c+dx)}\sqrt{e\cot(c+dx)}}$$

$$\downarrow 4372$$

$$-\frac{-a\int\sqrt{\tan(c+dx)}dx-a\int\sec(c+dx)\sqrt{\tan(c+dx)}dx-\frac{2(a-a\sec(c+dx))}{d\sqrt{\tan(c+dx)}}}{a^2\sqrt{\tan(c+dx)}\sqrt{e\cot(c+dx)}}$$

$$\downarrow 3042$$

$$-\frac{-a\int\sqrt{\tan(c+dx)}dx-a\int\sec(c+dx)\sqrt{\tan(c+dx)}dx-\frac{2(a-a\sec(c+dx))}{d\sqrt{\tan(c+dx)}}}{a^2\sqrt{\tan(c+dx)}\sqrt{e\cot(c+dx)}}$$

$$\downarrow 3093$$

$$-\frac{-a\int\sqrt{\tan(c+dx)}dx-a\left(\frac{2\cos(c+dx)\tan^{\frac{3}{2}}(c+dx)}{d}-2\int\cos(c+dx)\sqrt{\tan(c+dx)}dx\right)-\frac{2(a-a\sec(c+dx))}{d\sqrt{\tan(c+dx)}}}{a^2\sqrt{\tan(c+dx)}\sqrt{e\cot(c+dx)}}$$

$$\downarrow 3042$$

$$-\frac{-a\int\sqrt{\tan(c+dx)}dx-a\left(\frac{2\cos(c+dx)\tan^{\frac{3}{2}}(c+dx)}{d}-2\int\frac{\sqrt{\tan(c+dx)}}{\sec(c+dx)}dx\right)-\frac{2(a-a\sec(c+dx))}{d\sqrt{\tan(c+dx)}}}{a^2\sqrt{\tan(c+dx)}\sqrt{e\cot(c+dx)}}$$

$$\downarrow 3095$$

$$-\frac{-a\int\sqrt{\tan(c+dx)}dx-a\left(\frac{2\cos(c+dx)\tan^{\frac{3}{2}}(c+dx)}{d}-\frac{2\sqrt{\cos(c+dx)}\sqrt{\tan(c+dx)}\int\sqrt{\cos(c+dx)}\sqrt{\sin(c+dx)}dx}{\sqrt{\sin(c+dx)}}\right)-\frac{2(a-a\sec(c+d}{d\sqrt{\tan(c+dx}}}{a^2\sqrt{\tan(c+dx)}\sqrt{e\cot(c+dx)}}$$

$$\downarrow 3042$$

$$-\frac{-a\int\sqrt{\tan(c+dx)}dx-a\left(\frac{2\cos(c+dx)\tan^{\frac{3}{2}}(c+dx)}{d}-\frac{2\sqrt{\cos(c+dx)}\sqrt{\tan(c+dx)}\int\sqrt{\cos(c+dx)}\sqrt{\sin(c+dx)}dx}{\sqrt{\sin(c+dx)}}\right)-\frac{2(a-a\sec(c+d}{d\sqrt{\tan(c+dx}}}{a^2\sqrt{\tan(c+dx)}\sqrt{e\cot(c+dx)}}$$

3.245. $\int\frac{1}{\sqrt{e\cot(c+dx)}(a+a\sec(c+dx))}dx$

$$\downarrow 3052$$

$$-\frac{-a\int\sqrt{\tan(c+dx)}dx-a\left(\frac{2\cos(c+dx)\tan^{\frac{3}{2}}(c+dx)}{d}-\frac{2\cos(c+dx)\sqrt{\tan(c+dx)}\int\sqrt{\sin(2c+2dx)}dx}{\sqrt{\sin(2c+2dx)}}\right)-\frac{2(a-a\sec(c+dx))}{d\sqrt{\tan(c+dx)}}}{a^2\sqrt{\tan(c+dx)}\sqrt{e\cot(c+dx)}}$$

$$\downarrow 3042$$

$$-\frac{-a\int\sqrt{\tan(c+dx)}dx-a\left(\frac{2\cos(c+dx)\tan^{\frac{3}{2}}(c+dx)}{d}-\frac{2\cos(c+dx)\sqrt{\tan(c+dx)}\int\sqrt{\sin(2c+2dx)}dx}{\sqrt{\sin(2c+2dx)}}\right)-\frac{2(a-a\sec(c+dx))}{d\sqrt{\tan(c+dx)}}}{a^2\sqrt{\tan(c+dx)}\sqrt{e\cot(c+dx)}}$$

$$\downarrow 3119$$

$$-\frac{-a\int\sqrt{\tan(c+dx)}dx-\frac{2(a-a\sec(c+dx))}{d\sqrt{\tan(c+dx)}}-a\left(\frac{2\cos(c+dx)\tan^{\frac{3}{2}}(c+dx)}{d}-\frac{2\cos(c+dx)\sqrt{\tan(c+dx)}E\left(c+dx-\frac{\pi}{4}|2\right)}{d\sqrt{\sin(2c+2dx)}}\right)}{a^2\sqrt{\tan(c+dx)}\sqrt{e\cot(c+dx)}}$$

$$\downarrow 3957$$

$$-\frac{-\frac{a\int\frac{\sqrt{\tan(c+dx)}}{\tan^2(c+dx)+1}d\tan(c+dx)}{d}-\frac{2(a-a\sec(c+dx))}{d\sqrt{\tan(c+dx)}}-a\left(\frac{2\cos(c+dx)\tan^{\frac{3}{2}}(c+dx)}{d}-\frac{2\cos(c+dx)\sqrt{\tan(c+dx)}E\left(c+dx-\frac{\pi}{4}|2\right)}{d\sqrt{\sin(2c+2dx)}}\right)}{a^2\sqrt{\tan(c+dx)}\sqrt{e\cot(c+dx)}}$$

$$\downarrow 266$$

$$-\frac{-\frac{2a\int\frac{\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}}{d}-\frac{2(a-a\sec(c+dx))}{d\sqrt{\tan(c+dx)}}-a\left(\frac{2\cos(c+dx)\tan^{\frac{3}{2}}(c+dx)}{d}-\frac{2\cos(c+dx)\sqrt{\tan(c+dx)}E\left(c+dx-\frac{\pi}{4}|2\right)}{d\sqrt{\sin(2c+2dx)}}\right)}{a^2\sqrt{\tan(c+dx)}\sqrt{e\cot(c+dx)}}$$

$$\downarrow 826$$

$$-\frac{-\frac{2a\left(\frac{1}{2}\int\frac{\tan(c+dx)+1}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}-\frac{1}{2}\int\frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}\right)}{d}-\frac{2(a-a\sec(c+dx))}{d\sqrt{\tan(c+dx)}}-a\left(\frac{2\cos(c+dx)\tan^{\frac{3}{2}}(c+dx)}{d}-\frac{2\cos(c+dx}{}\right.}{a^2\sqrt{\tan(c+dx)}\sqrt{e\cot(c+dx)}}$$

$$\downarrow 1476$$

$$-\frac{-\frac{2a\left(\frac{1}{2}\left(\frac{1}{2}\int\frac{1}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}+\frac{1}{2}\int\frac{1}{\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}\right)-\frac{1}{2}\int\frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}\right)}{d}-}{a^2\sqrt{\tan(c+dx)}\sqrt{e\cot(c+dx)}}$$

$$\downarrow 1082$$

$$-\frac{2a\left(\frac{1}{2}\left(\frac{\int\frac{1}{-\tan(c+dx)-1}d\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}}-\frac{\int\frac{1}{-\tan(c+dx)-1}d\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}}\right)-\frac{1}{2}\int\frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}\right)}{d}-\frac{2(a-a\sec(c+dx)}{d\sqrt{\tan(c+dx)}}}{a^2\sqrt{\tan(c+dx)}\sqrt{e\cot(c+dx)}}$$

$$\downarrow 217$$

$$-\frac{2a\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}}\right)-\frac{1}{2}\int\frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}\right)}{d}-\frac{2(a-a\sec(c+dx))}{d\sqrt{\tan(c+dx)}}-a\left(\frac{2\cos(c+dx)\,t}{d}\right.}{a^2\sqrt{\tan(c+dx)}\sqrt{e\cot(c+dx)}}$$

$$\downarrow 1479$$

$$-\frac{2a\left(\frac{1}{2}\left(\frac{\int-\frac{\sqrt{2}-2\sqrt{\tan(c+dx)}}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}+\frac{\int-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\right.}{d}\right.}{a^2\sqrt{\tan(c+dx)}\sqrt{e\cot(}$$

$$\downarrow 25$$

$$-\frac{2a\left(\frac{1}{2}\left(-\frac{\int\frac{\sqrt{2}-2\sqrt{\tan(c+dx)}}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}-\frac{\int\frac{\sqrt{2}\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\right.}{d}\right.}{a^2\sqrt{\tan(c+dx)}\sqrt{e\cot(}$$

$$\downarrow 27$$

$$-\frac{2a\left(\frac{1}{2}\left(-\frac{\int\frac{\sqrt{2}-2\sqrt{\tan(c+dx)}}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}-\frac{1}{2}\int\frac{\sqrt{2}\sqrt{\tan(c+dx)}+1}{\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}}-\frac{\arctan(}{d}\right.}{a^2\sqrt{\tan(c+dx)}\sqrt{e\cot}$$

$$\downarrow 1103$$

$$-\frac{2a\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\log\left(\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}}-\frac{\log\left(\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}}\right)\right)}{d}-}{a^2\sqrt{\tan(c+dx)}\sqrt{e\cot(c+dx)}}$$

input
```
Int[1/(Sqrt[e*Cot[c + d*x]]*(a + a*Sec[c + d*x])),x]
```

output
```
-(((-2*a*((-(ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]]/Sqrt[2]) + ArcTan[1 +
Sqrt[2]*Sqrt[Tan[c + d*x]]]/Sqrt[2])/2 + (Log[1 - Sqrt[2]*Sqrt[Tan[c + d*x
]] + Tan[c + d*x]]/(2*Sqrt[2]) - Log[1 + Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[
c + d*x]]/(2*Sqrt[2]))/2))/d - (2*(a - a*Sec[c + d*x]))/(d*Sqrt[Tan[c + d*
x]]) - a*((-2*Cos[c + d*x]*EllipticE[c - Pi/4 + d*x, 2]*Sqrt[Tan[c + d*x]]
)/(d*Sqrt[Sin[2*c + 2*d*x]]) + (2*Cos[c + d*x]*Tan[c + d*x]^(3/2))/d))/(a^
2*Sqrt[e*Cot[c + d*x]]*Sqrt[Tan[c + d*x]]))
```

3.245.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3052
```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Simp[Sqrt[a*Sin[e + f*x]]*(Sqrt[b*Cos[e + f*x]]/Sqrt[Sin[2*e
 + 2*f*x]])   Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

rule 3093
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[a^2*(a*Sec[e + f*x])^(m - 2)*((b*Tan[e + f*x])^(n +
1)/(b*f*(m + n - 1))), x] + Simp[a^2*((m - 2)/(m + n - 1))   Int[(a*Sec[e +
 f*x])^(m - 2)*(b*Tan[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (
GtQ[m, 1] || (EqQ[m, 1] && EqQ[n, 1/2])) && NeQ[m + n - 1, 0] && IntegersQ[
2*m, 2*n]
```

rule 3095
```
Int[Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]]/sec[(e_.) + (f_.)*(x_)], x_Symbol]
:> Simp[Sqrt[Cos[e + f*x]]*(Sqrt[b*Tan[e + f*x]]/Sqrt[Sin[e + f*x]])   Int[
Sqrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]], x], x] /; FreeQ[{b, e, f}, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

rule 4370
```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(-(e*Cot[c + d*x])^(m + 1))*((a + b*Csc[c + d*x])/(
d*e*(m + 1))), x] - Simp[1/(e^2*(m + 1))   Int[(e*Cot[c + d*x])^(m + 2)*(a*
(m + 1) + b*(m + 2)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && L
tQ[m, -1]
```

rule 4372
```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(e*Cot[c + d*x])^m, x], x] + Simp[b   Int[
(e*Cot[c + d*x])^m*Csc[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, m}, x]
```

rule 4376
```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Simp[a^(2*n)/e^(2*n)   Int[(e*Cot[c + d*x])^(m + 2*
n)/(-a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[a
^2 - b^2, 0] && ILtQ[n, 0]
```

rule 4388
```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*((a_) + (b_.)*sec[(c_.) + (d_.)*(x
_)])^(n_.), x_Symbol] :> Simp[(e*Cot[c + d*x])^m*Tan[c + d*x]^m   Int[(a +
b*Sec[c + d*x])^n/Tan[c + d*x]^m, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x]
 &&  !IntegerQ[m]
```

3.245.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 8.32 (sec) , antiderivative size = 252, normalized size of antiderivative = 0.73

method	result
default	$\frac{(-\frac{1}{2}-\frac{i}{2})\sqrt{2}\left(\text{EllipticPi}\left(\sqrt{\csc(dx+c)-\cot(dx+c)+1},\frac{1}{2}+\frac{i}{2},\frac{\sqrt{2}}{2}\right)-i\,\text{EllipticPi}\left(\sqrt{\csc(dx+c)-\cot(dx+c)+1},\frac{1}{2}-\frac{i}{2},\frac{\sqrt{2}}{2}\right)+2i\,\text{EllipticE}\right.$

input
```
int(1/(a+a*sec(d*x+c))/(e*cot(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(-1/2-1/2*I)/a/d*2^(1/2)*(EllipticPi((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2+1
/2*I,1/2*2^(1/2))-I*EllipticPi((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2-1/2*I,1
/2*2^(1/2))+2*I*EllipticE((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2*2^(1/2))-I*E
llipticF((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2*2^(1/2))-2*EllipticE((csc(d*x
+c)-cot(d*x+c)+1)^(1/2),1/2*2^(1/2))+EllipticF((csc(d*x+c)-cot(d*x+c)+1)^(
1/2),1/2*2^(1/2)))*(csc(d*x+c)-cot(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c)+
1)^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)/(e*cot(d*x+c))^(1/2)*(cot(d*x+c)+cs
c(d*x+c))
```

3.245.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{e\cot(c+dx)}(a+a\sec(c+dx))}\,dx = \text{Timed out}$$

input
```
integrate(1/(a+a*sec(d*x+c))/(e*cot(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.245.6 Sympy [F]

$$\int \frac{1}{\sqrt{e\cot(c+dx)}(a+a\sec(c+dx))}\,dx = \frac{\int \frac{1}{\sqrt{e\cot(c+dx)}\sec(c+dx)+\sqrt{e\cot(c+dx)}}\,dx}{a}$$

input
```
integrate(1/(a+a*sec(d*x+c))/(e*cot(d*x+c))**(1/2),x)
```

output
```
Integral(1/(sqrt(e*cot(c + d*x))*sec(c + d*x) + sqrt(e*cot(c + d*x))), x)/
a
```

3.245.7 Maxima [F]

$$\int \frac{1}{\sqrt{e\cot(c+dx)}(a+a\sec(c+dx))}\,dx = \int \frac{1}{\sqrt{e\cot(dx+c)}(a\sec(dx+c)+a)}\,dx$$

input
```
integrate(1/(a+a*sec(d*x+c))/(e*cot(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(e*cot(d*x + c))*(a*sec(d*x + c) + a)), x)
```

3.245.8 Giac [F]

$$\int \frac{1}{\sqrt{e\cot(c+dx)}(a+a\sec(c+dx))}\,dx = \int \frac{1}{\sqrt{e\cot(dx+c)}(a\sec(dx+c)+a)}\,dx$$

input
```
integrate(1/(a+a*sec(d*x+c))/(e*cot(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(e*cot(d*x + c))*(a*sec(d*x + c) + a)), x)
```

3.245.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{e\cot(c+dx)}(a+a\sec(c+dx))}\,dx = \int \frac{\cos(c+dx)}{a\sqrt{e\cot(c+dx)}\,(\cos(c+dx)+1)}\,dx$$

input
```
int(1/((e*cot(c + d*x))^(1/2)*(a + a/cos(c + d*x))),x)
```

output
```
int(cos(c + d*x)/(a*(e*cot(c + d*x))^(1/2)*(cos(c + d*x) + 1)), x)
```

3.246 $\int \frac{1}{(e\cot(c+dx))^{3/2}(a+a\sec(c+dx))}\,dx$

3.246.1 Optimal result . . . 1796
3.246.2 Mathematica [C] (warning: unable to verify) . . . 1797
3.246.3 Rubi [A] (verified) . . . 1797
3.246.4 Maple [C] (verified) . . . 1803
3.246.5 Fricas [F(-1)] . . . 1803
3.246.6 Sympy [F] . . . 1804
3.246.7 Maxima [F] . . . 1804
3.246.8 Giac [F] . . . 1804
3.246.9 Mupad [F(-1)] . . . 1805

3.246.1 Optimal result

Integrand size = 25, antiderivative size = 290

$$\int \frac{1}{(e\cot(c+dx))^{3/2}(a+a\sec(c+dx))}\,dx = \frac{\cot(c+dx)\csc(c+dx)\,\text{EllipticF}\left(c-\frac{\pi}{4}+dx,2\right)\sqrt{\sin(2c+2d}}{ad(e\cot(c+dx))^{3/2}}$$
$$+\frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}ad(e\cot(c+dx))^{3/2}\tan^{\frac{3}{2}}(c+dx)}-\frac{\arctan\left(1+\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}ad(e\cot(c+dx))^{3/2}\tan^{\frac{3}{2}}(c+dx)}$$
$$+\frac{\log\left(1-\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+dx)\right)}{2\sqrt{2}ad(e\cot(c+dx))^{3/2}\tan^{\frac{3}{2}}(c+dx)}-\frac{\log\left(1+\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+dx)\right)}{2\sqrt{2}ad(e\cot(c+dx))^{3/2}\tan^{\frac{3}{2}}(c+dx)}$$

output

```
-cot(d*x+c)*csc(d*x+c)*(sin(c+1/4*Pi+d*x)^2)^(1/2)/sin(c+1/4*Pi+d*x)*Ellip
ticF(cos(c+1/4*Pi+d*x),2^(1/2))*sin(2*d*x+2*c)^(1/2)/a/d/(e*cot(d*x+c))^(3
/2)-1/2*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2))/a/d/(e*cot(d*x+c))^(3/2)*2^(1/
2)/tan(d*x+c)^(3/2)-1/2*arctan(1+2^(1/2)*tan(d*x+c)^(1/2))/a/d/(e*cot(d*x+
c))^(3/2)*2^(1/2)/tan(d*x+c)^(3/2)+1/4*ln(1-2^(1/2)*tan(d*x+c)^(1/2)+tan(d
*x+c))/a/d/(e*cot(d*x+c))^(3/2)*2^(1/2)/tan(d*x+c)^(3/2)-1/4*ln(1+2^(1/2)*
tan(d*x+c)^(1/2)+tan(d*x+c))/a/d/(e*cot(d*x+c))^(3/2)*2^(1/2)/tan(d*x+c)^(
3/2)
```

3.246.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 13.29 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.39

$$\int \frac{1}{(e\cot(c+dx))^{3/2}(a+a\sec(c+dx))}\,dx = \frac{4\cot^2(c+dx)\csc(c+dx)\left(3\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},-\text{ta}\right.\right.}{}$$

input
```
Integrate[1/((e*Cot[c + d*x])^(3/2)*(a + a*Sec[c + d*x])),x]
```

output
```
(4*Cot[c + d*x]^2*Csc[c + d*x]*(3*Hypergeometric2F1[1/4, 1/2, 5/4, -Tan[c
+ d*x]^2] + Cot[c + d*x]^2*Hypergeometric2F1[3/4, 1, 7/4, -Cot[c + d*x]^2]
)*(1 + Sqrt[Sec[c + d*x]^2])*Sin[(c + d*x)/2]^2)/(3*a*d*(e*Cot[c + d*x])^(
3/2))
```

3.246.3 Rubi [A] (verified)

Time = 0.93 (sec) , antiderivative size = 213, normalized size of antiderivative = 0.73, number of steps used = 24, number of rules used = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.920$, Rules used = {3042, 4388, 3042, 4376, 25, 3042, 4372, 3042, 3094, 3042, 3053, 3042, 3120, 3957, 266, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a\sec(c+dx)+a)(e\cot(c+dx))^{3/2}}\,dx$$
$$\downarrow \text{3042}$$
$$\int \frac{1}{(a\sec(c+dx)+a)(e\cot(c+dx))^{3/2}}dx$$
$$\downarrow \text{4388}$$
$$\frac{\int \frac{\tan^{\frac{3}{2}}(c+dx)}{\sec(c+dx)a+a}dx}{\tan^{\frac{3}{2}}(c+dx)(e\cot(c+dx))^{3/2}}$$
$$\downarrow \text{3042}$$
$$\frac{\int \frac{\left(-\cot\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx}{\tan^{\frac{3}{2}}(c+dx)(e\cot(c+dx))^{3/2}}$$

$$\downarrow 4376$$

$$\frac{\int -\frac{a-a\sec(c+dx)}{\sqrt{\tan(c+dx)}}dx}{a^2\tan^{\frac{3}{2}}(c+dx)(e\cot(c+dx))^{3/2}}$$

$$\downarrow 25$$

$$-\frac{\int \frac{a-a\sec(c+dx)}{\sqrt{\tan(c+dx)}}dx}{a^2\tan^{\frac{3}{2}}(c+dx)(e\cot(c+dx))^{3/2}}$$

$$\downarrow 3042$$

$$-\frac{\int \frac{a-a\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{-\cot\left(c+dx+\frac{\pi}{2}\right)}}dx}{a^2\tan^{\frac{3}{2}}(c+dx)(e\cot(c+dx))^{3/2}}$$

$$\downarrow 4372$$

$$-\frac{a\int \frac{1}{\sqrt{\tan(c+dx)}}dx - a\int \frac{\sec(c+dx)}{\sqrt{\tan(c+dx)}}dx}{a^2\tan^{\frac{3}{2}}(c+dx)(e\cot(c+dx))^{3/2}}$$

$$\downarrow 3042$$

$$-\frac{a\int \frac{1}{\sqrt{\tan(c+dx)}}dx - a\int \frac{\sec(c+dx)}{\sqrt{\tan(c+dx)}}dx}{a^2\tan^{\frac{3}{2}}(c+dx)(e\cot(c+dx))^{3/2}}$$

$$\downarrow 3094$$

$$-\frac{a\int \frac{1}{\sqrt{\tan(c+dx)}}dx - \frac{a\sqrt{\sin(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{\sin(c+dx)}}dx}{\sqrt{\cos(c+dx)}\sqrt{\tan(c+dx)}}}{a^2\tan^{\frac{3}{2}}(c+dx)(e\cot(c+dx))^{3/2}}$$

$$\downarrow 3042$$

$$-\frac{a\int \frac{1}{\sqrt{\tan(c+dx)}}dx - \frac{a\sqrt{\sin(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{\sin(c+dx)}}dx}{\sqrt{\cos(c+dx)}\sqrt{\tan(c+dx)}}}{a^2\tan^{\frac{3}{2}}(c+dx)(e\cot(c+dx))^{3/2}}$$

$$\downarrow 3053$$

$$-\frac{a\int \frac{1}{\sqrt{\tan(c+dx)}}dx - \frac{a\sqrt{\sin(2c+2dx)}\sec(c+dx)\int \frac{1}{\sqrt{\sin(2c+2dx)}}dx}{\sqrt{\tan(c+dx)}}}{a^2\tan^{\frac{3}{2}}(c+dx)(e\cot(c+dx))^{3/2}}$$

$$\downarrow 3042$$

$$-\frac{a\int \frac{1}{\sqrt{\tan(c+dx)}}dx - \frac{a\sqrt{\sin(2c+2dx)}\sec(c+dx)\int \frac{1}{\sqrt{\sin(2c+2dx)}}dx}{\sqrt{\tan(c+dx)}}}{a^2\tan^{\frac{3}{2}}(c+dx)(e\cot(c+dx))^{3/2}}$$

$$\downarrow 3120$$

$$-\frac{a\int \frac{1}{\sqrt{\tan(c+dx)}}dx - \frac{a\sqrt{\sin(2c+2dx)}\sec(c+dx)\,\text{EllipticF}\left(c+dx-\frac{\pi}{4},2\right)}{d\sqrt{\tan(c+dx)}}}{a^2\tan^{\frac{3}{2}}(c+dx)(e\cot(c+dx))^{3/2}}$$

$$\downarrow 3957$$

$$-\frac{\frac{a\int \frac{1}{\sqrt{\tan(c+dx)}\left(\tan^2(c+dx)+1\right)}d\tan(c+dx)}{d} - \frac{a\sqrt{\sin(2c+2dx)}\sec(c+dx)\,\text{EllipticF}\left(c+dx-\frac{\pi}{4},2\right)}{d\sqrt{\tan(c+dx)}}}{a^2\tan^{\frac{3}{2}}(c+dx)(e\cot(c+dx))^{3/2}}$$

$$\downarrow 266$$

$$-\frac{\frac{2a\int \frac{1}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}}{d} - \frac{a\sqrt{\sin(2c+2dx)}\sec(c+dx)\,\text{EllipticF}\left(c+dx-\frac{\pi}{4},2\right)}{d\sqrt{\tan(c+dx)}}}{a^2\tan^{\frac{3}{2}}(c+dx)(e\cot(c+dx))^{3/2}}$$

$$\downarrow 755$$

$$-\frac{\frac{2a\left(\frac{1}{2}\int \frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}+\frac{1}{2}\int \frac{\tan(c+dx)+1}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}\right)}{d} - \frac{a\sqrt{\sin(2c+2dx)}\sec(c+dx)\,\text{EllipticF}\left(c+dx-\frac{\pi}{4},2\right)}{d\sqrt{\tan(c+dx)}}}{a^2\tan^{\frac{3}{2}}(c+dx)(e\cot(c+dx))^{3/2}}$$

$$\downarrow 1476$$

$$-\frac{\frac{2a\left(\frac{1}{2}\int \frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}+\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}+\frac{1}{2}\int \frac{1}{\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}\right)\right)}{d} - \cdots}{a^2\tan^{\frac{3}{2}}(c+dx)(e\cot(c+dx))^{3/2}}$$

$$\downarrow 1082$$

$$-\frac{\frac{2a\left(\frac{1}{2}\int \frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}+\frac{1}{2}\left(\frac{\int \frac{1}{-\tan(c+dx)-1}d\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}}-\frac{\int \frac{1}{-\tan(c+dx)-1}d\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}}\right)\right)}{d} - \frac{a\sqrt{\sin(2c+2dx)}\sec}{d}}{a^2\tan^{\frac{3}{2}}(c+dx)(e\cot(c+dx))^{3/2}}$$

$$\downarrow 217$$

$$-\frac{\frac{2a\left(\frac{1}{2}\int \frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}}\right)\right)}{d} - \frac{a\sqrt{\sin(2c+2dx)}\sec(c+dx)\,\text{EllipticF}\left(c+dx}{d\sqrt{\tan(c+dx)}}}{a^2\tan^{\frac{3}{2}}(c+dx)(e\cot(c+dx))^{3/2}}$$

$$\downarrow 1479$$

$$-\frac{2a\left(\frac{1}{2}\left(-\frac{\int -\frac{\sqrt{2}-2\sqrt{\tan(c+dx)}}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\right.}{}\right.}{d}}{a^2\tan^{\frac{3}{2}}(c+dx)(e\cot(c+dx))^{3/2}}$$

$$\downarrow 25$$

$$-\frac{2a\left(\frac{1}{2}\left(\frac{\int \frac{\sqrt{2}-2\sqrt{\tan(c+dx)}}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\right.}}{}\right.}{d}}{a^2\tan^{\frac{3}{2}}(c+dx)(e\cot(c+dx))^{3/2}}$$

$$\downarrow 27$$

$$-\frac{2a\left(\frac{1}{2}\left(\frac{\int \frac{\sqrt{2}-2\sqrt{\tan(c+dx)}}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}+\frac{1}{2}\int \frac{\sqrt{2}\sqrt{\tan(c+dx)}+1}{\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{\right.}}{}\right.}{d}}{a^2\tan^{\frac{3}{2}}(c+dx)(e\cot(c+dx))^{3/2}}$$

$$\downarrow 1103$$

$$-\frac{2a\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\log\left(\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}}-\frac{\log\left(\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}}\right)\right)}{d}}{a^2\tan^{\frac{3}{2}}(c+dx)(e\cot(c+dx))^{3/2}}$$

input
```
Int[1/((e*Cot[c + d*x])^(3/2)*(a + a*Sec[c + d*x])),x]
```

output
```
-(((2*a*((-(ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]]/Sqrt[2]) + ArcTan[1 + S
qrt[2]*Sqrt[Tan[c + d*x]]]/Sqrt[2])/2 + (-1/2*Log[1 - Sqrt[2]*Sqrt[Tan[c +
 d*x]] + Tan[c + d*x]]/Sqrt[2] + Log[1 + Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[
c + d*x]]/(2*Sqrt[2]))/2))/d - (a*EllipticF[c - Pi/4 + d*x, 2]*Sec[c + d*x
]*Sqrt[Sin[2*c + 2*d*x]])/(d*Sqrt[Tan[c + d*x]]))/(a^2*(e*Cot[c + d*x])^(3
/2)*Tan[c + d*x]^(3/2)))
```

3.246.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3053
```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Simp[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]])   Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f
}, x]
```

rule 3094
```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]], x_Symbol]
:> Simp[Sqrt[Sin[e + f*x]]/(Sqrt[Cos[e + f*x]]*Sqrt[b*Tan[e + f*x]])   Int[
1/(Sqrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]]), x], x] /; FreeQ[{b, e, f}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

rule 4372
```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(e*Cot[c + d*x])^m, x], x] + Simp[b   Int[
(e*Cot[c + d*x])^m*Csc[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, m}, x]
```

rule 4376
```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Simp[a^(2*n)/e^(2*n)   Int[(e*Cot[c + d*x])^(m + 2*
n)/(-a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[a
^2 - b^2, 0] && ILtQ[n, 0]
```

rule 4388
```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*((a_) + (b_.)*sec[(c_.) + (d_.)*(x
_)])^(n_.), x_Symbol] :> Simp[(e*Cot[c + d*x])^m*Tan[c + d*x]^m   Int[(a +
b*Sec[c + d*x])^n/Tan[c + d*x]^m, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x]
 &&  !IntegerQ[m]
```

3.246.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 7.97 (sec) , antiderivative size = 207, normalized size of antiderivative = 0.71

method	result
default	$\frac{(\frac{1}{2}-\frac{i}{2})\sqrt{2}\left(i\,\text{EllipticPi}\left(\sqrt{\csc(dx+c)-\cot(dx+c)+1},\frac{1}{2}-\frac{i}{2},\frac{\sqrt{2}}{2}\right)+\text{EllipticPi}\left(\sqrt{\csc(dx+c)-\cot(dx+c)+1},\frac{1}{2}+\frac{i}{2},\frac{\sqrt{2}}{2}\right)-2\,\text{EllipticF}\left(\sqrt{}\right.\right.$

input
```
int(1/(e*cot(d*x+c))^(3/2)/(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
(1/2-1/2*I)/a/d*2^(1/2)*(I*EllipticPi((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2-
1/2*I,1/2*2^(1/2))+EllipticPi((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2+1/2*I,1/
2*2^(1/2))-2*EllipticF((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2*2^(1/2))-2*I*El
lipticF((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2*2^(1/2)))*(csc(d*x+c)-cot(d*x+
c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)/
(cos(d*x+c)-1)/e/(e*cot(d*x+c))^(1/2)*sin(d*x+c)
```

3.246.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(e\cot(c+dx))^{3/2}(a+a\sec(c+dx))}\,dx = \text{Timed out}$$

input
```
integrate(1/(e*cot(d*x+c))^(3/2)/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

output
```
Timed out
```

3.246.6 Sympy [F]

$$\int \frac{1}{(e\cot(c+dx))^{3/2}(a+a\sec(c+dx))}\,dx = \frac{\int \frac{1}{(e\cot(c+dx))^{\frac{3}{2}}\sec(c+dx)+(e\cot(c+dx))^{\frac{3}{2}}}\,dx}{a}$$

input
```
integrate(1/(e*cot(d*x+c))**(3/2)/(a+a*sec(d*x+c)),x)
```

output
```
Integral(1/((e*cot(c + d*x))**(3/2)*sec(c + d*x) + (e*cot(c + d*x))**(3/2)
), x)/a
```

3.246.7 Maxima [F]

$$\int \frac{1}{(e\cot(c+dx))^{3/2}(a+a\sec(c+dx))}\,dx = \int \frac{1}{(e\cot(dx+c))^{\frac{3}{2}}(a\sec(dx+c)+a)}\,dx$$

input
```
integrate(1/(e*cot(d*x+c))^(3/2)/(a+a*sec(d*x+c)),x, algorithm="maxima")
```

output
```
integrate(1/((e*cot(d*x + c))^(3/2)*(a*sec(d*x + c) + a)), x)
```

3.246.8 Giac [F]

$$\int \frac{1}{(e\cot(c+dx))^{3/2}(a+a\sec(c+dx))}\,dx = \int \frac{1}{(e\cot(dx+c))^{\frac{3}{2}}(a\sec(dx+c)+a)}\,dx$$

input
```
integrate(1/(e*cot(d*x+c))^(3/2)/(a+a*sec(d*x+c)),x, algorithm="giac")
```

output
```
integrate(1/((e*cot(d*x + c))^(3/2)*(a*sec(d*x + c) + a)), x)
```

3.246.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(e\cot(c+dx))^{3/2}(a+a\sec(c+dx))}\,dx = \int \frac{\cos(c+dx)}{a\,(e\cot(c+dx))^{3/2}\,(\cos(c+dx)+1)}\,dx$$

input
```
int(1/((e*cot(c + d*x))^(3/2)*(a + a/cos(c + d*x))),x)
```

output
```
int(cos(c + d*x)/(a*(e*cot(c + d*x))^(3/2)*(cos(c + d*x) + 1)), x)
```

3.247 $\int \frac{1}{(e\cot(c+dx))^{5/2}(a+a\sec(c+dx))}\,dx$

3.247.1 Optimal result . . . 1806
3.247.2 Mathematica [C] (warning: unable to verify) . . . 1807
3.247.3 Rubi [A] (verified) . . . 1807
3.247.4 Maple [C] (warning: unable to verify) . . . 1813
3.247.5 Fricas [F(-1)] . . . 1814
3.247.6 Sympy [F] . . . 1815
3.247.7 Maxima [F] . . . 1815
3.247.8 Giac [F] . . . 1815
3.247.9 Mupad [F(-1)] . . . 1816

3.247.1 Optimal result

Integrand size = 25, antiderivative size = 325

$$\int \frac{1}{(e\cot(c+dx))^{5/2}(a+a\sec(c+dx))}\,dx = \frac{2\cos(c+dx)\cot(c+dx)}{ad(e\cot(c+dx))^{5/2}} - \frac{2\cos(c+dx)\cot^2(c+dx)E\left(c-\frac{\pi}{4}+dx\middle|2\right)}{ad(e\cot(c+dx))^{5/2}\sqrt{\sin(2c+2dx)}} + \frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}ad(e\cot(c+dx))^{5/2}\tan^{\frac{5}{2}}(c+dx)} - \frac{\arctan\left(1+\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}ad(e\cot(c+dx))^{5/2}\tan^{\frac{5}{2}}(c+dx)} - \frac{\log\left(1-\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+dx)\right)}{2\sqrt{2}ad(e\cot(c+dx))^{5/2}\tan^{\frac{5}{2}}(c+dx)} + \frac{\log\left(1+\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+dx)\right)}{2\sqrt{2}ad(e\cot(c+dx))^{5/2}\tan^{\frac{5}{2}}(c+dx)}$$

output

```
2*cos(d*x+c)*cot(d*x+c)/a/d/(e*cot(d*x+c))^(5/2)+2*cos(d*x+c)*cot(d*x+c)^2
*(sin(c+1/4*Pi+d*x)^2)^(1/2)/sin(c+1/4*Pi+d*x)*EllipticE(cos(c+1/4*Pi+d*x)
,2^(1/2))/a/d/(e*cot(d*x+c))^(5/2)/sin(2*d*x+2*c)^(1/2)-1/2*arctan(-1+2^(1
/2)*tan(d*x+c)^(1/2))/a/d/(e*cot(d*x+c))^(5/2)*2^(1/2)/tan(d*x+c)^(5/2)-1/
2*arctan(1+2^(1/2)*tan(d*x+c)^(1/2))/a/d/(e*cot(d*x+c))^(5/2)*2^(1/2)/tan(
d*x+c)^(5/2)-1/4*ln(1-2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c))/a/d/(e*cot(d*x+
c))^(5/2)*2^(1/2)/tan(d*x+c)^(5/2)+1/4*ln(1+2^(1/2)*tan(d*x+c)^(1/2)+tan(d
*x+c))/a/d/(e*cot(d*x+c))^(5/2)*2^(1/2)/tan(d*x+c)^(5/2)
```

3.247.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 23.23 (sec) , antiderivative size = 194, normalized size of antiderivative = 0.60

$$\int \frac{1}{(e\cot(c+dx))^{5/2}(a+a\sec(c+dx))}\,dx =$$
$$-\frac{\sqrt{e\cot(c+dx)}\Big(-8\,\text{Hypergeometric2F1}\left(\tfrac{1}{2},\tfrac{3}{4},\tfrac{7}{4},-\tan^2(c+dx)\right)+3\sqrt{2}\cot^{\frac{3}{2}}(c+dx)\left(2\arctan\left(1-\sqrt{2}\right.\right.}{}$$

input
```
Integrate[1/((e*Cot[c + d*x])^(5/2)*(a + a*Sec[c + d*x])),x]
```

output
```
-1/6*(Sqrt[e*Cot[c + d*x]]*(-8*Hypergeometric2F1[1/2, 3/4, 7/4, -Tan[c + d
*x]^2] + 3*Sqrt[2]*Cot[c + d*x]^(3/2)*(2*ArcTan[1 - Sqrt[2]*Sqrt[Cot[c + d
*x]]] - 2*ArcTan[1 + Sqrt[2]*Sqrt[Cot[c + d*x]]] + Log[1 - Sqrt[2]*Sqrt[Co
t[c + d*x]] + Cot[c + d*x]] - Log[1 + Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c +
 d*x]]))*Sec[c + d*x]*(1 + Sqrt[Sec[c + d*x]^2])*Sin[(c + d*x)/2]^2)/(a*d*
e^3)
```

3.247.3 Rubi [A] (verified)

Time = 1.09 (sec) , antiderivative size = 237, normalized size of antiderivative = 0.73, number of steps used = 26, number of rules used = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 1.000$, Rules used = {3042, 4388, 3042, 4376, 25, 3042, 4372, 3042, 3093, 3042, 3095, 3042, 3052, 3042, 3119, 3957, 266, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a\sec(c+dx)+a)(e\cot(c+dx))^{5/2}}\,dx$$
$$\downarrow \text{3042}$$
$$\int \frac{1}{(a\sec(c+dx)+a)(e\cot(c+dx))^{5/2}}dx$$
$$\downarrow \text{4388}$$
$$\frac{\int \frac{\tan^{\frac{5}{2}}(c+dx)}{\sec(c+dx)a+a}dx}{\tan^{\frac{5}{2}}(c+dx)(e\cot(c+dx))^{5/2}}$$

$$\downarrow 3042$$

$$\frac{\int \frac{(-\cot(c+dx+\frac{\pi}{2}))^{5/2}}{\csc(c+dx+\frac{\pi}{2})a+a}dx}{\tan^{\frac{5}{2}}(c+dx)(e\cot(c+dx))^{5/2}}$$

$$\downarrow 4376$$

$$\frac{\int -\left((a-a\sec(c+dx))\sqrt{\tan(c+dx)}\right)dx}{a^2\tan^{\frac{5}{2}}(c+dx)(e\cot(c+dx))^{5/2}}$$

$$\downarrow 25$$

$$-\frac{\int (a-a\sec(c+dx))\sqrt{\tan(c+dx)}dx}{a^2\tan^{\frac{5}{2}}(c+dx)(e\cot(c+dx))^{5/2}}$$

$$\downarrow 3042$$

$$-\frac{\int \sqrt{-\cot\left(c+dx+\frac{\pi}{2}\right)}\left(a-a\csc\left(c+dx+\frac{\pi}{2}\right)\right)dx}{a^2\tan^{\frac{5}{2}}(c+dx)(e\cot(c+dx))^{5/2}}$$

$$\downarrow 4372$$

$$-\frac{a\int \sqrt{\tan(c+dx)}dx - a\int \sec(c+dx)\sqrt{\tan(c+dx)}dx}{a^2\tan^{\frac{5}{2}}(c+dx)(e\cot(c+dx))^{5/2}}$$

$$\downarrow 3042$$

$$-\frac{a\int \sqrt{\tan(c+dx)}dx - a\int \sec(c+dx)\sqrt{\tan(c+dx)}dx}{a^2\tan^{\frac{5}{2}}(c+dx)(e\cot(c+dx))^{5/2}}$$

$$\downarrow 3093$$

$$-\frac{a\int \sqrt{\tan(c+dx)}dx - a\left(\frac{2\cos(c+dx)\tan^{\frac{3}{2}}(c+dx)}{d} - 2\int \cos(c+dx)\sqrt{\tan(c+dx)}dx\right)}{a^2\tan^{\frac{5}{2}}(c+dx)(e\cot(c+dx))^{5/2}}$$

$$\downarrow 3042$$

$$-\frac{a\int \sqrt{\tan(c+dx)}dx - a\left(\frac{2\cos(c+dx)\tan^{\frac{3}{2}}(c+dx)}{d} - 2\int \frac{\sqrt{\tan(c+dx)}}{\sec(c+dx)}dx\right)}{a^2\tan^{\frac{5}{2}}(c+dx)(e\cot(c+dx))^{5/2}}$$

$$\downarrow 3095$$

$$-\frac{a\int \sqrt{\tan(c+dx)}dx - a\left(\frac{2\cos(c+dx)\tan^{\frac{3}{2}}(c+dx)}{d} - \frac{2\sqrt{\cos(c+dx)}\sqrt{\tan(c+dx)}\int \sqrt{\cos(c+dx)}\sqrt{\sin(c+dx)}dx}{\sqrt{\sin(c+dx)}}\right)}{a^2\tan^{\frac{5}{2}}(c+dx)(e\cot(c+dx))^{5/2}}$$

$$\downarrow 3042$$

$$-\frac{a\int\sqrt{\tan(c+dx)}dx-a\left(\frac{2\cos(c+dx)\tan^{\frac{3}{2}}(c+dx)}{d}-\frac{2\sqrt{\cos(c+dx)}\sqrt{\tan(c+dx)}\int\sqrt{\cos(c+dx)}\sqrt{\sin(c+dx)}dx}{\sqrt{\sin(c+dx)}}\right)}{a^2\tan^{\frac{5}{2}}(c+dx)(e\cot(c+dx))^{5/2}}$$

$$\downarrow 3052$$

$$-\frac{a\int\sqrt{\tan(c+dx)}dx-a\left(\frac{2\cos(c+dx)\tan^{\frac{3}{2}}(c+dx)}{d}-\frac{2\cos(c+dx)\sqrt{\tan(c+dx)}\int\sqrt{\sin(2c+2dx)}dx}{\sqrt{\sin(2c+2dx)}}\right)}{a^2\tan^{\frac{5}{2}}(c+dx)(e\cot(c+dx))^{5/2}}$$

$$\downarrow 3042$$

$$-\frac{a\int\sqrt{\tan(c+dx)}dx-a\left(\frac{2\cos(c+dx)\tan^{\frac{3}{2}}(c+dx)}{d}-\frac{2\cos(c+dx)\sqrt{\tan(c+dx)}\int\sqrt{\sin(2c+2dx)}dx}{\sqrt{\sin(2c+2dx)}}\right)}{a^2\tan^{\frac{5}{2}}(c+dx)(e\cot(c+dx))^{5/2}}$$

$$\downarrow 3119$$

$$-\frac{a\int\sqrt{\tan(c+dx)}dx-a\left(\frac{2\cos(c+dx)\tan^{\frac{3}{2}}(c+dx)}{d}-\frac{2\cos(c+dx)\sqrt{\tan(c+dx)}E\left(c+dx-\frac{\pi}{4}|2\right)}{d\sqrt{\sin(2c+2dx)}}\right)}{a^2\tan^{\frac{5}{2}}(c+dx)(e\cot(c+dx))^{5/2}}$$

$$\downarrow 3957$$

$$-\frac{\frac{a\int\frac{\sqrt{\tan(c+dx)}}{\tan^2(c+dx)+1}d\tan(c+dx)}{d}-a\left(\frac{2\cos(c+dx)\tan^{\frac{3}{2}}(c+dx)}{d}-\frac{2\cos(c+dx)\sqrt{\tan(c+dx)}E\left(c+dx-\frac{\pi}{4}|2\right)}{d\sqrt{\sin(2c+2dx)}}\right)}{a^2\tan^{\frac{5}{2}}(c+dx)(e\cot(c+dx))^{5/2}}$$

$$\downarrow 266$$

$$-\frac{\frac{2a\int\frac{\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}}{d}-a\left(\frac{2\cos(c+dx)\tan^{\frac{3}{2}}(c+dx)}{d}-\frac{2\cos(c+dx)\sqrt{\tan(c+dx)}E\left(c+dx-\frac{\pi}{4}|2\right)}{d\sqrt{\sin(2c+2dx)}}\right)}{a^2\tan^{\frac{5}{2}}(c+dx)(e\cot(c+dx))^{5/2}}$$

$$\downarrow 826$$

$$-\frac{\frac{2a\left(\frac{1}{2}\int\frac{\tan(c+dx)+1}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}-\frac{1}{2}\int\frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}\right)}{d}-a\left(\frac{2\cos(c+dx)\tan^{\frac{3}{2}}(c+dx)}{d}-\frac{2\cos(c+dx)\sqrt{\tan(c+dx)}E\left(c+dx-\right.\right.}{d\sqrt{\sin(2c+2dx)}}$$

$$a^2\tan^{\frac{5}{2}}(c+dx)(e\cot(c+dx))^{5/2}$$

$$\downarrow 1476$$

$$-\frac{\frac{2a\left(\frac{1}{2}\left(\frac{1}{2}\int\frac{1}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}+\frac{1}{2}\int\frac{1}{\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}\right)-\frac{1}{2}\int\frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}\right)}{d}-$$

$$a^2\tan^{\frac{5}{2}}(c+dx)(e\cot(c+dx))^{5/2}$$

$$\downarrow 1082$$

$$-\frac{2a\left(\frac{1}{2}\left(\frac{\int \frac{1}{-\tan(c+dx)-1}d\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}}-\frac{\int \frac{1}{-\tan(c+dx)-1}d\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}}\right)-\frac{1}{2}\int \frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}\right)}{d}-a\left(\frac{2\cos(c+dx)\tan}{d}\right.}{a^2\tan^{\frac{5}{2}}(c+dx)(e\cot(c+dx))^{5/2}}$$

$$\downarrow 217$$

$$-\frac{2a\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}}\right)-\frac{1}{2}\int \frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}\right)}{d}-a\left(\frac{2\cos(c+dx)\tan^{\frac{3}{2}}(c+dx)}{d}-\frac{2\cos(c+}{}\right.}{a^2\tan^{\frac{5}{2}}(c+dx)(e\cot(c+dx))^{5/2}}$$

$$\downarrow 1479$$

$$-\frac{2a\left(\frac{1}{2}\left(\int -\frac{\sqrt{2}-2\sqrt{\tan(c+dx)}}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}+\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}\right)\right)}{d}+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{}\right.}{}\right)}{a^2\tan^{\frac{5}{2}}(c+dx)(e\cot(c+dx))^{5/2}}$$

$$\downarrow 25$$

$$-\frac{2a\left(\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}-2\sqrt{\tan(c+dx)}}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}-\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}\right)\right)}{d}+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\right.}{\sqrt{}}\right)}{a^2\tan^{\frac{5}{2}}(c+dx)(e\cot(c+dx))^{5/2}}$$

$$\downarrow 27$$

$$-\frac{2a\left(\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}-2\sqrt{\tan(c+dx)}}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}-\frac{1}{2}\int \frac{\sqrt{2}\sqrt{\tan(c+dx)}+1}{\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}\right)\right)}{d}+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\right.}{}\right)}{a^2\tan^{\frac{5}{2}}(c+dx)(e\cot(c+dx))^{5/2}}$$

$$\downarrow 1103$$

$$-\frac{2a\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\log\left(\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}}-\frac{\log\left(\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}}\right)\right)}{d}-a$$

$$a^2\tan^{\frac{5}{2}}(c+dx)(e\cot(c+dx))^{5/2}$$

input
```
Int[1/((e*Cot[c + d*x])^(5/2)*(a + a*Sec[c + d*x])),x]
```

output
```
-(((2*a*((-(ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]]/Sqrt[2]) + ArcTan[1 + S
qrt[2]*Sqrt[Tan[c + d*x]]]/Sqrt[2])/2 + (Log[1 - Sqrt[2]*Sqrt[Tan[c + d*x]
] + Tan[c + d*x]]/(2*Sqrt[2]) - Log[1 + Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c
 + d*x]]/(2*Sqrt[2]))/2))/d - a*((-2*Cos[c + d*x]*EllipticE[c - Pi/4 + d*x
, 2]*Sqrt[Tan[c + d*x]])/(d*Sqrt[Sin[2*c + 2*d*x]]) + (2*Cos[c + d*x]*Tan[
c + d*x]^(3/2))/d))/(a^2*(e*Cot[c + d*x])^(5/2)*Tan[c + d*x]^(5/2)))
```

3.247.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3052
```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Simp[Sqrt[a*Sin[e + f*x]]*(Sqrt[b*Cos[e + f*x]]/Sqrt[Sin[2*e
 + 2*f*x]])   Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

rule 3093
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[a^2*(a*Sec[e + f*x])^(m - 2)*((b*Tan[e + f*x])^(n +
1)/(b*f*(m + n - 1))), x] + Simp[a^2*((m - 2)/(m + n - 1))   Int[(a*Sec[e +
 f*x])^(m - 2)*(b*Tan[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (
GtQ[m, 1] || (EqQ[m, 1] && EqQ[n, 1/2])) && NeQ[m + n - 1, 0] && IntegersQ[
2*m, 2*n]
```

rule 3095
```
Int[Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]]/sec[(e_.) + (f_.)*(x_)], x_Symbol]
:> Simp[Sqrt[Cos[e + f*x]]*(Sqrt[b*Tan[e + f*x]]/Sqrt[Sin[e + f*x]])   Int[
Sqrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]], x], x] /; FreeQ[{b, e, f}, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

rule 4372
```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(e*Cot[c + d*x])^m, x], x] + Simp[b   Int[
(e*Cot[c + d*x])^m*Csc[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, m}, x]
```

rule 4376
```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Simp[a^(2*n)/e^(2*n)   Int[(e*Cot[c + d*x])^(m + 2*
n)/(-a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[a
^2 - b^2, 0] && ILtQ[n, 0]
```

rule 4388
```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*((a_) + (b_.)*sec[(c_.) + (d_.)*(x
_)])^(n_.), x_Symbol] :> Simp[(e*Cot[c + d*x])^m*Tan[c + d*x]^m   Int[(a +
b*Sec[c + d*x])^n/Tan[c + d*x]^m, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x]
 &&  !IntegerQ[m]
```

3.247.4 Maple [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 9.77 (sec) , antiderivative size = 1100, normalized size of antiderivative = 3.38

method	result	size
default	Expression too large to display	1100

input
```
int(1/(e*cot(d*x+c))^(5/2)/(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
-1/2/a/d*2^(1/2)*((1-cos(d*x+c))^2*csc(d*x+c)^2-1)^3*(I*(csc(d*x+c)-cot(d*
x+c)+1)^(1/2)*(2-2*csc(d*x+c)+2*cot(d*x+c))^(1/2)*(cot(d*x+c)-csc(d*x+c))^
(1/2)*EllipticPi((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2-1/2*I,1/2*2^(1/2))*((
1-cos(d*x+c))*(-cot(d*x+c)+csc(d*x+c)-1)*(csc(d*x+c)-cot(d*x+c)+1)*csc(d*x
+c))^(1/2)-I*(csc(d*x+c)-cot(d*x+c)+1)^(1/2)*(2-2*csc(d*x+c)+2*cot(d*x+c))
^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)*EllipticPi((csc(d*x+c)-cot(d*x+c)+1)^
(1/2),1/2+1/2*I,1/2*2^(1/2))*((1-cos(d*x+c))*(-cot(d*x+c)+csc(d*x+c)-1)*(c
sc(d*x+c)-cot(d*x+c)+1)*csc(d*x+c))^(1/2)-4*(csc(d*x+c)-cot(d*x+c)+1)^(1/2
)*(2-2*csc(d*x+c)+2*cot(d*x+c))^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)*Ellipt
icE((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2*2^(1/2))*((1-cos(d*x+c))*(-cot(d*x
+c)+csc(d*x+c)-1)*(csc(d*x+c)-cot(d*x+c)+1)*csc(d*x+c))^(1/2)+2*(csc(d*x+c
)-cot(d*x+c)+1)^(1/2)*(2-2*csc(d*x+c)+2*cot(d*x+c))^(1/2)*(cot(d*x+c)-csc(
d*x+c))^(1/2)*EllipticF((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2*2^(1/2))*((1-c
os(d*x+c))*(-cot(d*x+c)+csc(d*x+c)-1)*(csc(d*x+c)-cot(d*x+c)+1)*csc(d*x+c)
)^(1/2)-(csc(d*x+c)-cot(d*x+c)+1)^(1/2)*(2-2*csc(d*x+c)+2*cot(d*x+c))^(1/2
)*(cot(d*x+c)-csc(d*x+c))^(1/2)*EllipticPi((csc(d*x+c)-cot(d*x+c)+1)^(1/2)
,1/2-1/2*I,1/2*2^(1/2))*((1-cos(d*x+c))*(-cot(d*x+c)+csc(d*x+c)-1)*(csc(d*
x+c)-cot(d*x+c)+1)*csc(d*x+c))^(1/2)-(csc(d*x+c)-cot(d*x+c)+1)^(1/2)*(2-2*
csc(d*x+c)+2*cot(d*x+c))^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)*EllipticPi((c
sc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2+1/2*I,1/2*2^(1/2))*((1-cos(d*x+c))*(-...
```

3.247.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(e\cot(c+dx))^{5/2}(a+a\sec(c+dx))}\,dx = \text{Timed out}$$

input
```
integrate(1/(e*cot(d*x+c))^(5/2)/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

output
```
Timed out
```

3.247.6 Sympy [F]

$$\int \frac{1}{(e\cot(c+dx))^{5/2}(a+a\sec(c+dx))}\,dx = \frac{\int \frac{1}{(e\cot(c+dx))^{\frac{5}{2}}\sec(c+dx)+(e\cot(c+dx))^{\frac{5}{2}}}\,dx}{a}$$

input
```
integrate(1/(e*cot(d*x+c))**(5/2)/(a+a*sec(d*x+c)),x)
```

output
```
Integral(1/((e*cot(c + d*x))**(5/2)*sec(c + d*x) + (e*cot(c + d*x))**(5/2)
), x)/a
```

3.247.7 Maxima [F]

$$\int \frac{1}{(e\cot(c+dx))^{5/2}(a+a\sec(c+dx))}\,dx = \int \frac{1}{(e\cot(dx+c))^{\frac{5}{2}}(a\sec(dx+c)+a)}\,dx$$

input
```
integrate(1/(e*cot(d*x+c))^(5/2)/(a+a*sec(d*x+c)),x, algorithm="maxima")
```

output
```
integrate(1/((e*cot(d*x + c))^(5/2)*(a*sec(d*x + c) + a)), x)
```

3.247.8 Giac [F]

$$\int \frac{1}{(e\cot(c+dx))^{5/2}(a+a\sec(c+dx))}\,dx = \int \frac{1}{(e\cot(dx+c))^{\frac{5}{2}}(a\sec(dx+c)+a)}\,dx$$

input
```
integrate(1/(e*cot(d*x+c))^(5/2)/(a+a*sec(d*x+c)),x, algorithm="giac")
```

output
```
integrate(1/((e*cot(d*x + c))^(5/2)*(a*sec(d*x + c) + a)), x)
```

3.247.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(e\cot(c+dx))^{5/2}(a+a\sec(c+dx))}\,dx = \int \frac{\cos(c+dx)}{a\,(e\cot(c+dx))^{5/2}\,(\cos(c+dx)+1)}\,dx$$

input
```
int(1/((e*cot(c + d*x))^(5/2)*(a + a/cos(c + d*x))),x)
```

output
```
int(cos(c + d*x)/(a*(e*cot(c + d*x))^(5/2)*(cos(c + d*x) + 1)), x)
```

3.248 $\int \frac{1}{(e\cot(c+dx))^{7/2}(a+a\sec(c+dx))}\,dx$

3.248.1 Optimal result 1817
3.248.2 Mathematica [C] (warning: unable to verify) 1818
3.248.3 Rubi [A] (verified) 1818
3.248.4 Maple [C] (warning: unable to verify) 1825
3.248.5 Fricas [F(-1)] 1825
3.248.6 Sympy [F(-1)] 1826
3.248.7 Maxima [F] 1826
3.248.8 Giac [F] 1826
3.248.9 Mupad [F(-1)] 1827

3.248.1 Optimal result

Integrand size = 25, antiderivative size = 335

$$\int \frac{1}{(e\cot(c+dx))^{7/2}(a+a\sec(c+dx))}\,dx = -\frac{2\cot^3(c+dx)(3-\sec(c+dx))}{3ad(e\cot(c+dx))^{7/2}}$$
$$-\frac{\cot^3(c+dx)\csc(c+dx)\,\text{EllipticF}\left(c-\frac{\pi}{4}+dx,2\right)\sqrt{\sin(2c+2dx)}}{3ad(e\cot(c+dx))^{7/2}}$$
$$-\frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}ad(e\cot(c+dx))^{7/2}\tan^{\frac{7}{2}}(c+dx)}+\frac{\arctan\left(1+\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}ad(e\cot(c+dx))^{7/2}\tan^{\frac{7}{2}}(c+dx)}$$
$$-\frac{\log\left(1-\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+dx)\right)}{2\sqrt{2}ad(e\cot(c+dx))^{7/2}\tan^{\frac{7}{2}}(c+dx)}$$
$$+\frac{\log\left(1+\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+dx)\right)}{2\sqrt{2}ad(e\cot(c+dx))^{7/2}\tan^{\frac{7}{2}}(c+dx)}$$

output

```
-2/3*cot(d*x+c)^3*(3-sec(d*x+c))/a/d/(e*cot(d*x+c))^(7/2)+1/3*cot(d*x+c)^3
*csc(d*x+c)*(sin(c+1/4*Pi+d*x)^2)^(1/2)/sin(c+1/4*Pi+d*x)*EllipticF(cos(c+
1/4*Pi+d*x),2^(1/2))*sin(2*d*x+2*c)^(1/2)/a/d/(e*cot(d*x+c))^(7/2)+1/2*arc
tan(-1+2^(1/2)*tan(d*x+c)^(1/2))/a/d/(e*cot(d*x+c))^(7/2)*2^(1/2)/tan(d*x+
c)^(7/2)+1/2*arctan(1+2^(1/2)*tan(d*x+c)^(1/2))/a/d/(e*cot(d*x+c))^(7/2)*2
^(1/2)/tan(d*x+c)^(7/2)-1/4*ln(1-2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c))/a/d/
(e*cot(d*x+c))^(7/2)*2^(1/2)/tan(d*x+c)^(7/2)+1/4*ln(1+2^(1/2)*tan(d*x+c)^
(1/2)+tan(d*x+c))/a/d/(e*cot(d*x+c))^(7/2)*2^(1/2)/tan(d*x+c)^(7/2)
```

3.248.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 12.94 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.39

$$\int \frac{1}{(e\cot(c+dx))^{7/2}(a+a\sec(c+dx))}\,dx =$$
$$-\frac{4\sqrt{e\cot(c+dx)}\csc(c+dx)\left(3-3\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1}{4},\frac{5}{4},-\tan^2(c+dx)\right)+3\,\text{Hypergeometric2F}\right.}{}$$

input
```
Integrate[1/((e*Cot[c + d*x])^(7/2)*(a + a*Sec[c + d*x])),x]
```

output
```
(-4*Sqrt[e*Cot[c + d*x]]*Csc[c + d*x]*(3 - 3*Hypergeometric2F1[-1/2, 1/4,
5/4, -Tan[c + d*x]^2] + 3*Hypergeometric2F1[1/4, 1/2, 5/4, -Tan[c + d*x]^2
] + Cot[c + d*x]^2*Hypergeometric2F1[3/4, 1, 7/4, -Cot[c + d*x]^2])*(1 + S
qrt[Sec[c + d*x]^2])*Sin[(c + d*x)/2]^2)/(3*a*d*e^4)
```

3.248.3 Rubi [A] (verified)

Time = 1.11 (sec) , antiderivative size = 247, normalized size of antiderivative = 0.74, number of steps used = 27, number of rules used = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 1.040$, Rules used = {3042, 4388, 3042, 4376, 25, 3042, 4369, 27, 3042, 4372, 3042, 3094, 3042, 3053, 3042, 3120, 3957, 266, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a\sec(c+dx)+a)(e\cot(c+dx))^{7/2}}\,dx$$
$$\downarrow 3042$$
$$\int \frac{1}{(a\sec(c+dx)+a)(e\cot(c+dx))^{7/2}}dx$$
$$\downarrow 4388$$
$$\frac{\int \frac{\tan^{\frac{7}{2}}(c+dx)}{\sec(c+dx)a+a}dx}{\tan^{\frac{7}{2}}(c+dx)(e\cot(c+dx))^{7/2}}$$
$$\downarrow 3042$$

$$\frac{\int \frac{\left(-\cot\left(c+dx+\frac{\pi}{2}\right)\right)^{7/2}}{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx}{\tan^{\frac{7}{2}}(c+dx)(e\cot(c+dx))^{7/2}}$$

$$\downarrow 4376$$

$$\frac{\int -\left((a-a\sec(c+dx))\tan^{\frac{3}{2}}(c+dx)\right)dx}{a^2\tan^{\frac{7}{2}}(c+dx)(e\cot(c+dx))^{7/2}}$$

$$\downarrow 25$$

$$-\frac{\int (a-a\sec(c+dx))\tan^{\frac{3}{2}}(c+dx)dx}{a^2\tan^{\frac{7}{2}}(c+dx)(e\cot(c+dx))^{7/2}}$$

$$\downarrow 3042$$

$$-\frac{\int \left(-\cot\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}\left(a-a\csc\left(c+dx+\frac{\pi}{2}\right)\right)dx}{a^2\tan^{\frac{7}{2}}(c+dx)(e\cot(c+dx))^{7/2}}$$

$$\downarrow 4369$$

$$-\frac{\frac{2\sqrt{\tan(c+dx)}(3a-a\sec(c+dx))}{3d}-\frac{2}{3}\int \frac{3a-a\sec(c+dx)}{2\sqrt{\tan(c+dx)}}dx}{a^2\tan^{\frac{7}{2}}(c+dx)(e\cot(c+dx))^{7/2}}$$

$$\downarrow 27$$

$$-\frac{\frac{2\sqrt{\tan(c+dx)}(3a-a\sec(c+dx))}{3d}-\frac{1}{3}\int \frac{3a-a\sec(c+dx)}{\sqrt{\tan(c+dx)}}dx}{a^2\tan^{\frac{7}{2}}(c+dx)(e\cot(c+dx))^{7/2}}$$

$$\downarrow 3042$$

$$-\frac{\frac{2\sqrt{\tan(c+dx)}(3a-a\sec(c+dx))}{3d}-\frac{1}{3}\int \frac{3a-a\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{-\cot\left(c+dx+\frac{\pi}{2}\right)}}dx}{a^2\tan^{\frac{7}{2}}(c+dx)(e\cot(c+dx))^{7/2}}$$

$$\downarrow 4372$$

$$-\frac{\frac{1}{3}\left(a\int \frac{\sec(c+dx)}{\sqrt{\tan(c+dx)}}dx-3a\int \frac{1}{\sqrt{\tan(c+dx)}}dx\right)+\frac{2\sqrt{\tan(c+dx)}(3a-a\sec(c+dx))}{3d}}{a^2\tan^{\frac{7}{2}}(c+dx)(e\cot(c+dx))^{7/2}}$$

$$\downarrow 3042$$

$$-\frac{\frac{1}{3}\left(a\int \frac{\sec(c+dx)}{\sqrt{\tan(c+dx)}}dx-3a\int \frac{1}{\sqrt{\tan(c+dx)}}dx\right)+\frac{2\sqrt{\tan(c+dx)}(3a-a\sec(c+dx))}{3d}}{a^2\tan^{\frac{7}{2}}(c+dx)(e\cot(c+dx))^{7/2}}$$

$$\downarrow 3094$$

$$-\frac{\frac{1}{3}\left(\frac{a\sqrt{\sin(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}\sqrt{\sin(c+dx)}}dx}{\sqrt{\cos(c+dx)}\sqrt{\tan(c+dx)}}-3a\int\frac{1}{\sqrt{\tan(c+dx)}}dx\right)+\frac{2\sqrt{\tan(c+dx)}(3a-a\sec(c+dx))}{3d}}{a^2\tan^{\frac{7}{2}}(c+dx)(e\cot(c+dx))^{7/2}}$$

$$\downarrow 3042$$

$$-\frac{\frac{1}{3}\left(\frac{a\sqrt{\sin(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}\sqrt{\sin(c+dx)}}dx}{\sqrt{\cos(c+dx)}\sqrt{\tan(c+dx)}}-3a\int\frac{1}{\sqrt{\tan(c+dx)}}dx\right)+\frac{2\sqrt{\tan(c+dx)}(3a-a\sec(c+dx))}{3d}}{a^2\tan^{\frac{7}{2}}(c+dx)(e\cot(c+dx))^{7/2}}$$

$$\downarrow 3053$$

$$-\frac{\frac{1}{3}\left(\frac{a\sqrt{\sin(2c+2dx)}\sec(c+dx)\int\frac{1}{\sqrt{\sin(2c+2dx)}}dx}{\sqrt{\tan(c+dx)}}-3a\int\frac{1}{\sqrt{\tan(c+dx)}}dx\right)+\frac{2\sqrt{\tan(c+dx)}(3a-a\sec(c+dx))}{3d}}{a^2\tan^{\frac{7}{2}}(c+dx)(e\cot(c+dx))^{7/2}}$$

$$\downarrow 3042$$

$$-\frac{\frac{1}{3}\left(\frac{a\sqrt{\sin(2c+2dx)}\sec(c+dx)\int\frac{1}{\sqrt{\sin(2c+2dx)}}dx}{\sqrt{\tan(c+dx)}}-3a\int\frac{1}{\sqrt{\tan(c+dx)}}dx\right)+\frac{2\sqrt{\tan(c+dx)}(3a-a\sec(c+dx))}{3d}}{a^2\tan^{\frac{7}{2}}(c+dx)(e\cot(c+dx))^{7/2}}$$

$$\downarrow 3120$$

$$-\frac{\frac{1}{3}\left(\frac{a\sqrt{\sin(2c+2dx)}\sec(c+dx)\,\text{EllipticF}\left(c+dx-\frac{\pi}{4},2\right)}{d\sqrt{\tan(c+dx)}}-3a\int\frac{1}{\sqrt{\tan(c+dx)}}dx\right)+\frac{2\sqrt{\tan(c+dx)}(3a-a\sec(c+dx))}{3d}}{a^2\tan^{\frac{7}{2}}(c+dx)(e\cot(c+dx))^{7/2}}$$

$$\downarrow 3957$$

$$-\frac{\frac{1}{3}\left(\frac{a\sqrt{\sin(2c+2dx)}\sec(c+dx)\,\text{EllipticF}\left(c+dx-\frac{\pi}{4},2\right)}{d\sqrt{\tan(c+dx)}}-\frac{3a\int\frac{1}{\sqrt{\tan(c+dx)}\left(\tan^2(c+dx)+1\right)}d\tan(c+dx)}{d}\right)+\frac{2\sqrt{\tan(c+dx)}(3a-a\sec(c+dx))}{3d}}{a^2\tan^{\frac{7}{2}}(c+dx)(e\cot(c+dx))^{7/2}}$$

$$\downarrow 266$$

$$-\frac{\frac{1}{3}\left(\frac{a\sqrt{\sin(2c+2dx)}\sec(c+dx)\,\text{EllipticF}\left(c+dx-\frac{\pi}{4},2\right)}{d\sqrt{\tan(c+dx)}}-\frac{6a\int\frac{1}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}}{d}\right)+\frac{2\sqrt{\tan(c+dx)}(3a-a\sec(c+dx))}{3d}}{a^2\tan^{\frac{7}{2}}(c+dx)(e\cot(c+dx))^{7/2}}$$

$$\downarrow 755$$

$$-\frac{\frac{1}{3}\left(\frac{a\sqrt{\sin(2c+2dx)}\sec(c+dx)\,\text{EllipticF}\left(c+dx-\frac{\pi}{4},2\right)}{d\sqrt{\tan(c+dx)}}-\frac{6a\left(\frac{1}{2}\int\frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}+\frac{1}{2}\int\frac{\tan(c+dx)+1}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}\right)}{d}\right)+\frac{2\sqrt{\tan(}}{}}{a^2\tan^{\frac{7}{2}}(c+dx)(e\cot(c+dx))^{7/2}}$$

$$\downarrow 1476$$

$$-\frac{\frac{1}{3}\left(\frac{a\sqrt{\sin(2c+2dx)}\sec(c+dx)\,\text{EllipticF}\left(c+dx-\frac{\pi}{4},2\right)}{d\sqrt{\tan(c+dx)}}-\frac{6a\left(\frac{1}{2}\int\frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}+\frac{1}{2}\left(\frac{1}{2}\int\frac{1}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+d}\right.\right.\right.}{d}}{a^2\tan^{\frac{7}{2}}(c+dx)(e\cot(c+dx))^{7/2}}$$

$$\downarrow 1082$$

$$-\frac{\frac{1}{3}\left(\frac{a\sqrt{\sin(2c+2dx)}\sec(c+dx)\,\text{EllipticF}\left(c+dx-\frac{\pi}{4},2\right)}{d\sqrt{\tan(c+dx)}}-\frac{6a\left(\frac{1}{2}\int\frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}+\frac{1}{2}\left(\frac{\int\frac{1}{-\tan(c+dx)-1}d\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}}-\frac{\int}{-\ta}\right.\right.\right.}{d}}{a^2\tan^{\frac{7}{2}}(c+dx)(e\cot(c+dx))^{7/2}}$$

$$\downarrow 217$$

$$-\frac{\frac{1}{3}\left(\frac{a\sqrt{\sin(2c+2dx)}\sec(c+dx)\,\text{EllipticF}\left(c+dx-\frac{\pi}{4},2\right)}{d\sqrt{\tan(c+dx)}}-\frac{6a\left(\frac{1}{2}\int\frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\tan}\right.\right.\right.}{d}}{a^2\tan^{\frac{7}{2}}(c+dx)(e\cot(c+dx))^{7/2}}$$

$$\downarrow 1479$$

$$-\frac{\frac{1}{3}\left(\frac{a\sqrt{\sin(2c+2dx)}\sec(c+dx)\,\text{EllipticF}\left(c+dx-\frac{\pi}{4},2\right)}{d\sqrt{\tan(c+dx)}}-\frac{6a\left(\frac{1}{2}\left(-\frac{\int-\frac{\sqrt{2}-2\sqrt{\tan(c+dx)}}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}-\frac{\int-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+}}{2\sqrt{2}}\right.\right.\right.}{d}}{a^2\tan^{\frac{7}{2}}(c+dx)(e\cot(c+d}$$

$$\downarrow 25$$

$$-\frac{\frac{1}{3}\left(\frac{a\sqrt{\sin(2c+2dx)}\sec(c+dx)\,\text{EllipticF}\left(c+dx-\frac{\pi}{4},2\right)}{d\sqrt{\tan(c+dx)}}-\frac{6a\left(\frac{1}{2}\left(\frac{\int\frac{\sqrt{2}-2\sqrt{\tan(c+dx)}}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}+\frac{\int\frac{\sqrt{2}\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{}}{2\sqrt{2}}\right.\right.\right.}{d}}{a^2\tan^{\frac{7}{2}}(c+dx)(e\cot(c+dx)}$$

$$\downarrow 27$$

$$\frac{1}{3}\left(\frac{a\sqrt{\sin(2c+2dx)}\sec(c+dx)\,\text{EllipticF}\left(c+dx-\frac{\pi}{4},2\right)}{d\sqrt{\tan(c+dx)}} - \frac{6a\left(\frac{1}{2}\left(\int \frac{\sqrt{2}-2\sqrt{\tan(c+dx)}}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1}\,d\sqrt{\tan(c+dx)}}{2\sqrt{2}} + \frac{1}{2}\int \frac{\sqrt{2}\sqrt{\tan(c+dx)}+1}{\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1}\,d\right)\right)}{d}\right)$$

$$-\frac{}{a^2\tan^{\frac{7}{2}}(c+dx)(e\cot(c+d$$

$$\downarrow 1103$$

$$\frac{1}{3}\left(\frac{a\sqrt{\sin(2c+2dx)}\sec(c+dx)\,\text{EllipticF}\left(c+dx-\frac{\pi}{4},2\right)}{d\sqrt{\tan(c+dx)}} - \frac{6a\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}} - \frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}}\right) + \frac{1}{2}\left(\frac{\log\left(\tan(c+dx)+\sqrt{2}\sqrt{}\right.\right.}{2\sqrt{2}}\right.}{d}\right)$$

$$-\frac{}{a^2\tan^{\frac{7}{2}}(c+dx)(e\cot(c+dx))^{7/2}}$$

input
```
Int[1/((e*Cot[c + d*x])^(7/2)*(a + a*Sec[c + d*x])),x]
```

output
```
-((((-6*a*((-(ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]]/Sqrt[2]) + ArcTan[1 +
 Sqrt[2]*Sqrt[Tan[c + d*x]]]/Sqrt[2])/2 + (-1/2*Log[1 - Sqrt[2]*Sqrt[Tan[c
 + d*x]] + Tan[c + d*x]]/Sqrt[2] + Log[1 + Sqrt[2]*Sqrt[Tan[c + d*x]] + Ta
n[c + d*x]]/(2*Sqrt[2]))/2))/d + (a*EllipticF[c - Pi/4 + d*x, 2]*Sec[c + d
*x]*Sqrt[Sin[2*c + 2*d*x]])/(d*Sqrt[Tan[c + d*x]]))/3 + (2*(3*a - a*Sec[c
+ d*x])*Sqrt[Tan[c + d*x]])/(3*d))/(a^2*(e*Cot[c + d*x])^(7/2)*Tan[c + d*x
]^(7/2)))
```

3.248.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3053
```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Simp[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]])   Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f
}, x]
```

rule 3094
```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]], x_Symbol]
:> Simp[Sqrt[Sin[e + f*x]]/(Sqrt[Cos[e + f*x]]*Sqrt[b*Tan[e + f*x]])   Int[
1/(Sqrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]]), x], x] /; FreeQ[{b, e, f}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

rule 4369
```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(-e)*(e*Cot[c + d*x])^(m - 1)*((a*m + b*(m - 1)*Csc
[c + d*x])/(d*m*(m - 1))), x] - Simp[e^2/m   Int[(e*Cot[c + d*x])^(m - 2)*(
a*m + b*(m - 1)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && GtQ[m
, 1]
```

rule 4372
```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(e*Cot[c + d*x])^m, x], x] + Simp[b   Int[
(e*Cot[c + d*x])^m*Csc[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, m}, x]
```

rule 4376
```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Simp[a^(2*n)/e^(2*n)   Int[(e*Cot[c + d*x])^(m + 2*
n)/(-a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[a
^2 - b^2, 0] && ILtQ[n, 0]
```

rule 4388
```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*((a_) + (b_.)*sec[(c_.) + (d_.)*(x
_)])^(n_.), x_Symbol] :> Simp[(e*Cot[c + d*x])^m*Tan[c + d*x]^m   Int[(a +
b*Sec[c + d*x])^n/Tan[c + d*x]^m, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x]
 &&  !IntegerQ[m]
```

3.248. $\int \frac{1}{(e \cot(c+dx))^{7/2}(a+a \sec(c+dx))}\,dx$

3.248.4 Maple [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 9.80 (sec) , antiderivative size = 976, normalized size of antiderivative = 2.91

method	result	size
default	Expression too large to display	976

input
```
int(1/(e*cot(d*x+c))^(7/2)/(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
-1/6/a/d*2^(1/2)*(3*I*(cot(d*x+c)-csc(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+
c))^(1/2)*(csc(d*x+c)-cot(d*x+c)+1)^(1/2)*EllipticPi((csc(d*x+c)-cot(d*x+c
)+1)^(1/2),1/2+1/2*I,1/2*2^(1/2))*cos(d*x+c)^2-3*I*(cot(d*x+c)-csc(d*x+c)+
1)^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)*(csc(d*x+c)-cot(d*x+c)+1)^(1/2)*Ell
ipticPi((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2-1/2*I,1/2*2^(1/2))*cos(d*x+c)^
2+3*I*(cot(d*x+c)-csc(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)*(csc(d
*x+c)-cot(d*x+c)+1)^(1/2)*EllipticPi((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2+1
/2*I,1/2*2^(1/2))*cos(d*x+c)-3*I*(cot(d*x+c)-csc(d*x+c)+1)^(1/2)*(cot(d*x+
c)-csc(d*x+c))^(1/2)*(csc(d*x+c)-cot(d*x+c)+1)^(1/2)*EllipticPi((csc(d*x+c
)-cot(d*x+c)+1)^(1/2),1/2-1/2*I,1/2*2^(1/2))*cos(d*x+c)+8*(csc(d*x+c)-cot(
d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1
/2)*EllipticF((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2*2^(1/2))*cos(d*x+c)^2-3*
(cot(d*x+c)-csc(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)*(csc(d*x+c)-
cot(d*x+c)+1)^(1/2)*EllipticPi((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2+1/2*I,1
/2*2^(1/2))*cos(d*x+c)^2-3*(cot(d*x+c)-csc(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc
(d*x+c))^(1/2)*(csc(d*x+c)-cot(d*x+c)+1)^(1/2)*EllipticPi((csc(d*x+c)-cot(
d*x+c)+1)^(1/2),1/2-1/2*I,1/2*2^(1/2))*cos(d*x+c)^2+8*(csc(d*x+c)-cot(d*x+
c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)*
EllipticF((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2*2^(1/2))*cos(d*x+c)-3*(cot(d
*x+c)-csc(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)*EllipticPi((csc...
```

3.248.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(e\cot(c+dx))^{7/2}(a+a\sec(c+dx))}\,dx = \text{Timed out}$$

input
```
integrate(1/(e*cot(d*x+c))^(7/2)/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

output
```
Timed out
```

3.248.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(e\cot(c+dx))^{7/2}(a+a\sec(c+dx))}\,dx = \text{Timed out}$$

input
```
integrate(1/(e*cot(d*x+c))**(7/2)/(a+a*sec(d*x+c)),x)
```

output
```
Timed out
```

3.248.7 Maxima [F]

$$\int \frac{1}{(e\cot(c+dx))^{7/2}(a+a\sec(c+dx))}\,dx = \int \frac{1}{(e\cot(dx+c))^{\frac{7}{2}}(a\sec(dx+c)+a)}\,dx$$

input
```
integrate(1/(e*cot(d*x+c))^(7/2)/(a+a*sec(d*x+c)),x, algorithm="maxima")
```

output
```
integrate(1/((e*cot(d*x + c))^(7/2)*(a*sec(d*x + c) + a)), x)
```

3.248.8 Giac [F]

$$\int \frac{1}{(e\cot(c+dx))^{7/2}(a+a\sec(c+dx))}\,dx = \int \frac{1}{(e\cot(dx+c))^{\frac{7}{2}}(a\sec(dx+c)+a)}\,dx$$

input
```
integrate(1/(e*cot(d*x+c))^(7/2)/(a+a*sec(d*x+c)),x, algorithm="giac")
```

output
```
integrate(1/((e*cot(d*x + c))^(7/2)*(a*sec(d*x + c) + a)), x)
```

3.248.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(e\cot(c+dx))^{7/2}(a+a\sec(c+dx))}\,dx = \int \frac{\cos(c+dx)}{a\,(e\cot(c+dx))^{7/2}\,(\cos(c+dx)+1)}\,dx$$

input
```
int(1/((e*cot(c + d*x))^(7/2)*(a + a/cos(c + d*x))),x)
```

output
```
int(cos(c + d*x)/(a*(e*cot(c + d*x))^(7/2)*(cos(c + d*x) + 1)), x)
```

3.249 $\int \frac{1}{(e\cot(c+dx))^{9/2}(a+a\sec(c+dx))}\,dx$

3.249.1 Optimal result . . . 1828
3.249.2 Mathematica [C] (warning: unable to verify) . . . 1829
3.249.3 Rubi [A] (verified) . . . 1829
3.249.4 Maple [C] (warning: unable to verify) . . . 1836
3.249.5 Fricas [F(-1)] . . . 1837
3.249.6 Sympy [F(-1)] . . . 1838
3.249.7 Maxima [F] . . . 1838
3.249.8 Giac [F] . . . 1838
3.249.9 Mupad [F(-1)] . . . 1839

3.249.1 Optimal result

Integrand size = 25, antiderivative size = 371

$$\int \frac{1}{(e\cot(c+dx))^{9/2}(a+a\sec(c+dx))}\,dx = -\frac{6\cos(c+dx)\cot^3(c+dx)}{5ad(e\cot(c+dx))^{9/2}}$$
$$-\frac{2\cot^3(c+dx)(5-3\sec(c+dx))}{15ad(e\cot(c+dx))^{9/2}}+\frac{6\cos(c+dx)\cot^4(c+dx)E\left(c-\frac{\pi}{4}+dx\middle|2\right)}{5ad(e\cot(c+dx))^{9/2}\sqrt{\sin(2c+2dx)}}$$
$$-\frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}ad(e\cot(c+dx))^{9/2}\tan^{\frac{9}{2}}(c+dx)}+\frac{\arctan\left(1+\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}ad(e\cot(c+dx))^{9/2}\tan^{\frac{9}{2}}(c+dx)}$$
$$+\frac{\log\left(1-\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+dx)\right)}{2\sqrt{2}ad(e\cot(c+dx))^{9/2}\tan^{\frac{9}{2}}(c+dx)}$$
$$-\frac{\log\left(1+\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+dx)\right)}{2\sqrt{2}ad(e\cot(c+dx))^{9/2}\tan^{\frac{9}{2}}(c+dx)}$$

output

```
-6/5*cos(d*x+c)*cot(d*x+c)^3/a/d/(e*cot(d*x+c))^(9/2)-2/15*cot(d*x+c)^3*(5
-3*sec(d*x+c))/a/d/(e*cot(d*x+c))^(9/2)-6/5*cos(d*x+c)*cot(d*x+c)^4*(sin(c
+1/4*Pi+d*x)^2)^(1/2)/sin(c+1/4*Pi+d*x)*EllipticE(cos(c+1/4*Pi+d*x),2^(1/2
))/a/d/(e*cot(d*x+c))^(9/2)/sin(2*d*x+2*c)^(1/2)+1/2*arctan(-1+2^(1/2)*tan
(d*x+c)^(1/2))/a/d/(e*cot(d*x+c))^(9/2)*2^(1/2)/tan(d*x+c)^(9/2)+1/2*arcta
n(1+2^(1/2)*tan(d*x+c)^(1/2))/a/d/(e*cot(d*x+c))^(9/2)*2^(1/2)/tan(d*x+c)^
(9/2)+1/4*ln(1-2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c))/a/d/(e*cot(d*x+c))^(9/
2)*2^(1/2)/tan(d*x+c)^(9/2)-1/4*ln(1+2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c))/
a/d/(e*cot(d*x+c))^(9/2)*2^(1/2)/tan(d*x+c)^(9/2)
```

3.249.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 14.81 (sec) , antiderivative size = 261, normalized size of antiderivative = 0.70

$$\int \frac{1}{(e\cot(c+dx))^{9/2}(a+a\sec(c+dx))}\,dx = \frac{\sqrt{e\cot(c+dx)}\left(-8+6\sqrt{2}\arctan\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)\cot\right.}{}$$

input
```
Integrate[1/((e*Cot[c + d*x])^(9/2)*(a + a*Sec[c + d*x])),x]
```

output
```
(Sqrt[e*Cot[c + d*x]]*(-8 + 6*Sqrt[2]*ArcTan[1 - Sqrt[2]*Sqrt[Cot[c + d*x]
]]*Cot[c + d*x]^(3/2) - 6*Sqrt[2]*ArcTan[1 + Sqrt[2]*Sqrt[Cot[c + d*x]]]*C
ot[c + d*x]^(3/2) + 8*Hypergeometric2F1[-1/2, 3/4, 7/4, -Tan[c + d*x]^2] -
 8*Hypergeometric2F1[1/2, 3/4, 7/4, -Tan[c + d*x]^2] + 3*Sqrt[2]*Cot[c + d
*x]^(3/2)*Log[1 - Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]] - 3*Sqrt[2]*C
ot[c + d*x]^(3/2)*Log[1 + Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]])*Sec[
c + d*x]*(1 + Sqrt[Sec[c + d*x]^2])*Sin[(c + d*x)/2]^2)/(6*a*d*e^5)
```

3.249.3 Rubi [A] (verified)

Time = 1.27 (sec) , antiderivative size = 272, normalized size of antiderivative = 0.73, number of steps used = 29, number of rules used = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 1.120$, Rules used = {3042, 4388, 3042, 4376, 25, 3042, 4369, 27, 3042, 4372, 3042, 3093, 3042, 3095, 3042, 3052, 3042, 3119, 3957, 266, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a\sec(c+dx)+a)(e\cot(c+dx))^{9/2}}\,dx$$
$$\downarrow 3042$$
$$\int \frac{1}{(a\sec(c+dx)+a)(e\cot(c+dx))^{9/2}}dx$$
$$\downarrow 4388$$
$$\frac{\int \frac{\tan^{\frac{9}{2}}(c+dx)}{\sec(c+dx)a+a}dx}{\tan^{\frac{9}{2}}(c+dx)(e\cot(c+dx))^{9/2}}$$
$$\downarrow 3042$$

$$\frac{\int \frac{\left(-\cot\left(c+dx+\frac{\pi}{2}\right)\right)^{9/2}}{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx}{\tan^{\frac{9}{2}}(c+dx)(e\cot(c+dx))^{9/2}}$$

$$\downarrow 4376$$

$$\frac{\int -\Big((a-a\sec(c+dx))\tan^{\frac{5}{2}}(c+dx)\Big)\,dx}{a^2\tan^{\frac{9}{2}}(c+dx)(e\cot(c+dx))^{9/2}}$$

$$\downarrow 25$$

$$-\frac{\int (a-a\sec(c+dx))\tan^{\frac{5}{2}}(c+dx)dx}{a^2\tan^{\frac{9}{2}}(c+dx)(e\cot(c+dx))^{9/2}}$$

$$\downarrow 3042$$

$$-\frac{\int \left(-\cot\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}\left(a-a\csc\left(c+dx+\frac{\pi}{2}\right)\right)dx}{a^2\tan^{\frac{9}{2}}(c+dx)(e\cot(c+dx))^{9/2}}$$

$$\downarrow 4369$$

$$-\frac{\frac{2\tan^{\frac{3}{2}}(c+dx)(5a-3a\sec(c+dx))}{15d}-\frac{2}{5}\int \frac{1}{2}(5a-3a\sec(c+dx))\sqrt{\tan(c+dx)}dx}{a^2\tan^{\frac{9}{2}}(c+dx)(e\cot(c+dx))^{9/2}}$$

$$\downarrow 27$$

$$-\frac{\frac{2\tan^{\frac{3}{2}}(c+dx)(5a-3a\sec(c+dx))}{15d}-\frac{1}{5}\int (5a-3a\sec(c+dx))\sqrt{\tan(c+dx)}dx}{a^2\tan^{\frac{9}{2}}(c+dx)(e\cot(c+dx))^{9/2}}$$

$$\downarrow 3042$$

$$-\frac{\frac{2\tan^{\frac{3}{2}}(c+dx)(5a-3a\sec(c+dx))}{15d}-\frac{1}{5}\int \sqrt{-\cot\left(c+dx+\frac{\pi}{2}\right)}\left(5a-3a\csc\left(c+dx+\frac{\pi}{2}\right)\right)dx}{a^2\tan^{\frac{9}{2}}(c+dx)(e\cot(c+dx))^{9/2}}$$

$$\downarrow 4372$$

$$-\frac{\frac{1}{5}\left(3a\int \sec(c+dx)\sqrt{\tan(c+dx)}dx-5a\int \sqrt{\tan(c+dx)}dx\right)+\frac{2\tan^{\frac{3}{2}}(c+dx)(5a-3a\sec(c+dx))}{15d}}{a^2\tan^{\frac{9}{2}}(c+dx)(e\cot(c+dx))^{9/2}}$$

$$\downarrow 3042$$

$$-\frac{\frac{1}{5}\left(3a\int \sec(c+dx)\sqrt{\tan(c+dx)}dx-5a\int \sqrt{\tan(c+dx)}dx\right)+\frac{2\tan^{\frac{3}{2}}(c+dx)(5a-3a\sec(c+dx))}{15d}}{a^2\tan^{\frac{9}{2}}(c+dx)(e\cot(c+dx))^{9/2}}$$

$$\downarrow 3093$$

$$-\frac{\frac{1}{5}\left(3a\left(\frac{2\cos(c+dx)\tan^{\frac{3}{2}}(c+dx)}{d}-2\int\cos(c+dx)\sqrt{\tan(c+dx)}dx\right)-5a\int\sqrt{\tan(c+dx)}dx\right)+\frac{2\tan^{\frac{3}{2}}(c+dx)(5a-3a\sec}{15d}}{a^2\tan^{\frac{9}{2}}(c+dx)(e\cot(c+dx))^{9/2}}$$

$$\downarrow 3042$$

$$-\frac{\frac{1}{5}\left(3a\left(\frac{2\cos(c+dx)\tan^{\frac{3}{2}}(c+dx)}{d}-2\int\frac{\sqrt{\tan(c+dx)}}{\sec(c+dx)}dx\right)-5a\int\sqrt{\tan(c+dx)}dx\right)+\frac{2\tan^{\frac{3}{2}}(c+dx)(5a-3a\sec(c+dx))}{15d}}{a^2\tan^{\frac{9}{2}}(c+dx)(e\cot(c+dx))^{9/2}}$$

$$\downarrow 3095$$

$$-\frac{\frac{1}{5}\left(3a\left(\frac{2\cos(c+dx)\tan^{\frac{3}{2}}(c+dx)}{d}-\frac{2\sqrt{\cos(c+dx)}\sqrt{\tan(c+dx)}\int\sqrt{\cos(c+dx)}\sqrt{\sin(c+dx)}dx}{\sqrt{\sin(c+dx)}}\right)-5a\int\sqrt{\tan(c+dx)}dx\right)+\frac{2\tan^{\frac{3}{2}}(}{15d}}{a^2\tan^{\frac{9}{2}}(c+dx)(e\cot(c+dx))^{9/2}}$$

$$\downarrow 3042$$

$$-\frac{\frac{1}{5}\left(3a\left(\frac{2\cos(c+dx)\tan^{\frac{3}{2}}(c+dx)}{d}-\frac{2\sqrt{\cos(c+dx)}\sqrt{\tan(c+dx)}\int\sqrt{\cos(c+dx)}\sqrt{\sin(c+dx)}dx}{\sqrt{\sin(c+dx)}}\right)-5a\int\sqrt{\tan(c+dx)}dx\right)+\frac{2\tan^{\frac{3}{2}}(}{15d}}{a^2\tan^{\frac{9}{2}}(c+dx)(e\cot(c+dx))^{9/2}}$$

$$\downarrow 3052$$

$$-\frac{\frac{1}{5}\left(3a\left(\frac{2\cos(c+dx)\tan^{\frac{3}{2}}(c+dx)}{d}-\frac{2\cos(c+dx)\sqrt{\tan(c+dx)}\int\sqrt{\sin(2c+2dx)}dx}{\sqrt{\sin(2c+2dx)}}\right)-5a\int\sqrt{\tan(c+dx)}dx\right)+\frac{2\tan^{\frac{3}{2}}(c+dx)(5a-3}{15d}}{a^2\tan^{\frac{9}{2}}(c+dx)(e\cot(c+dx))^{9/2}}$$

$$\downarrow 3042$$

$$-\frac{\frac{1}{5}\left(3a\left(\frac{2\cos(c+dx)\tan^{\frac{3}{2}}(c+dx)}{d}-\frac{2\cos(c+dx)\sqrt{\tan(c+dx)}\int\sqrt{\sin(2c+2dx)}dx}{\sqrt{\sin(2c+2dx)}}\right)-5a\int\sqrt{\tan(c+dx)}dx\right)+\frac{2\tan^{\frac{3}{2}}(c+dx)(5a-3}{15d}}{a^2\tan^{\frac{9}{2}}(c+dx)(e\cot(c+dx))^{9/2}}$$

$$\downarrow 3119$$

$$-\frac{\frac{1}{5}\left(3a\left(\frac{2\cos(c+dx)\tan^{\frac{3}{2}}(c+dx)}{d}-\frac{2\cos(c+dx)\sqrt{\tan(c+dx)}E\left(c+dx-\frac{\pi}{4}|2\right)}{d\sqrt{\sin(2c+2dx)}}\right)-5a\int\sqrt{\tan(c+dx)}dx\right)+\frac{2\tan^{\frac{3}{2}}(c+dx)(5a-3a\sec}{15d}}{a^2\tan^{\frac{9}{2}}(c+dx)(e\cot(c+dx))^{9/2}}$$

$$\downarrow 3957$$

$$-\frac{\frac{1}{5}\left(3a\left(\frac{2\cos(c+dx)\tan^{\frac{3}{2}}(c+dx)}{d}-\frac{2\cos(c+dx)\sqrt{\tan(c+dx)}E\left(c+dx-\frac{\pi}{4}\middle|2\right)}{d\sqrt{\sin(2c+2dx)}}\right)-\frac{5a\int\frac{\sqrt{\tan(c+dx)}}{\tan^2(c+dx)+1}d\tan(c+dx)}{d}\right)+\frac{2\tan^{\frac{3}{2}}(c+dx)(5a-3}{15d}}{a^2\tan^{\frac{9}{2}}(c+dx)(e\cot(c+dx))^{9/2}}$$

$$\downarrow 266$$

$$-\frac{\frac{1}{5}\left(3a\left(\frac{2\cos(c+dx)\tan^{\frac{3}{2}}(c+dx)}{d}-\frac{2\cos(c+dx)\sqrt{\tan(c+dx)}E\left(c+dx-\frac{\pi}{4}\middle|2\right)}{d\sqrt{\sin(2c+2dx)}}\right)-\frac{10a\int\frac{\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}}{d}\right)+\frac{2\tan^{\frac{3}{2}}(c+dx)(5a}{15}}{a^2\tan^{\frac{9}{2}}(c+dx)(e\cot(c+dx))^{9/2}}$$

$$\downarrow 826$$

$$-\frac{\frac{1}{5}\left(3a\left(\frac{2\cos(c+dx)\tan^{\frac{3}{2}}(c+dx)}{d}-\frac{2\cos(c+dx)\sqrt{\tan(c+dx)}E\left(c+dx-\frac{\pi}{4}\middle|2\right)}{d\sqrt{\sin(2c+2dx)}}\right)-\frac{10a\left(\frac{1}{2}\int\frac{\tan(c+dx)+1}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}-\frac{1}{2}\int\frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\right.}{d}}{a^2\tan^{\frac{9}{2}}(c+dx)(e\cot(c+dx))^{9/2}}$$

$$\downarrow 1476$$

$$-\frac{\frac{1}{5}\left(3a\left(\frac{2\cos(c+dx)\tan^{\frac{3}{2}}(c+dx)}{d}-\frac{2\cos(c+dx)\sqrt{\tan(c+dx)}E\left(c+dx-\frac{\pi}{4}\middle|2\right)}{d\sqrt{\sin(2c+2dx)}}\right)-\frac{10a\left(\frac{1}{2}\left(\frac{1}{2}\int\frac{1}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}+\right.\right.}{a^2\tan^{\frac{9}{2}}(c+dx)(e\cot(c+}$$

$$\downarrow 1082$$

$$-\frac{\frac{1}{5}\left(3a\left(\frac{2\cos(c+dx)\tan^{\frac{3}{2}}(c+dx)}{d}-\frac{2\cos(c+dx)\sqrt{\tan(c+dx)}E\left(c+dx-\frac{\pi}{4}\middle|2\right)}{d\sqrt{\sin(2c+2dx)}}\right)-\frac{10a\left(\frac{1}{2}\left(\frac{\int\frac{1}{-\tan(c+dx)-1}d\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}}-\frac{\int\frac{1}{-\tan(c+}\right.\right.}{a^2\tan^{\frac{9}{2}}(c+dx)(e\cot(c+dx))^{9/}}$$

$$\downarrow 217$$

$$-\frac{\frac{1}{5}\left(3a\left(\frac{2\cos(c+dx)\tan^{\frac{3}{2}}(c+dx)}{d}-\frac{2\cos(c+dx)\sqrt{\tan(c+dx)}E\left(c+dx-\frac{\pi}{4}\middle|2\right)}{d\sqrt{\sin(2c+2dx)}}\right)-\frac{10a\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(c+d}\right.\right.}{d}}{a^2\tan^{\frac{9}{2}}(c+dx)(e\cot(c+dx))^{9/2}}$$

$$\downarrow 1479$$

$$-\frac{\frac{1}{5}\left(3a\left(\frac{2\cos(c+dx)\tan^{\frac{3}{2}}(c+dx)}{d}-\frac{2\cos(c+dx)\sqrt{\tan(c+dx)}E\left(c+dx-\frac{\pi}{4}|2\right)}{d\sqrt{\sin(2c+2dx)}}\right)-\frac{10a\left(\frac{1}{2}\left(\int-\frac{\sqrt{2}-2\sqrt{\tan(c+dx)}}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}+\frac{\int}{}\right.\right.}{a^2\tan^{\frac{9}{2}}(c+dx)(}$$

$$\downarrow 25$$

$$-\frac{\frac{1}{5}\left(3a\left(\frac{2\cos(c+dx)\tan^{\frac{3}{2}}(c+dx)}{d}-\frac{2\cos(c+dx)\sqrt{\tan(c+dx)}E\left(c+dx-\frac{\pi}{4}|2\right)}{d\sqrt{\sin(2c+2dx)}}\right)-\frac{10a\left(\frac{1}{2}\left(-\frac{\int\frac{\sqrt{2}-2\sqrt{\tan(c+dx)}}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}-\frac{\int}{}\right.\right.}{a^2\tan^{\frac{9}{2}}(c+dx)(}$$

$$\downarrow 27$$

$$-\frac{\frac{1}{5}\left(3a\left(\frac{2\cos(c+dx)\tan^{\frac{3}{2}}(c+dx)}{d}-\frac{2\cos(c+dx)\sqrt{\tan(c+dx)}E\left(c+dx-\frac{\pi}{4}|2\right)}{d\sqrt{\sin(2c+2dx)}}\right)-\frac{10a\left(\frac{1}{2}\left(-\frac{\int\frac{\sqrt{2}-2\sqrt{\tan(c+dx)}}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}-\frac{1}{2}\right.\right.}{a^2\tan^{\frac{9}{2}}(c+dx)}$$

$$\downarrow 1103$$

$$-\frac{\frac{1}{5}\left(3a\left(\frac{2\cos(c+dx)\tan^{\frac{3}{2}}(c+dx)}{d}-\frac{2\cos(c+dx)\sqrt{\tan(c+dx)}E\left(c+dx-\frac{\pi}{4}|2\right)}{d\sqrt{\sin(2c+2dx)}}\right)-\frac{10a\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(c+d}\right.\right.}{\sqrt{2}}\right.\right.}{a^2\tan^{\frac{9}{2}}(c+dx)(e\cot(c+}$$

input
```
Int[1/((e*Cot[c + d*x])^(9/2)*(a + a*Sec[c + d*x])),x]
```

3.249. $\int \frac{1}{(e\cot(c+dx))^{9/2}(a+a\sec(c+dx))}\,dx$

output
```
-(((2*(5*a - 3*a*Sec[c + d*x])*Tan[c + d*x]^(3/2))/(15*d) + ((-10*a*((-(Ar
cTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]]/Sqrt[2]) + ArcTan[1 + Sqrt[2]*Sqrt[Ta
n[c + d*x]]]/Sqrt[2])/2 + (Log[1 - Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*
x]]/(2*Sqrt[2]) - Log[1 + Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]]/(2*Sq
rt[2]))/2))/d + 3*a*((-2*Cos[c + d*x]*EllipticE[c - Pi/4 + d*x, 2]*Sqrt[Ta
n[c + d*x]])/(d*Sqrt[Sin[2*c + 2*d*x]]) + (2*Cos[c + d*x]*Tan[c + d*x]^(3/
2))/d))/5)/(a^2*(e*Cot[c + d*x])^(9/2)*Tan[c + d*x]^(9/2)))
```

3.249.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3052
```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Simp[Sqrt[a*Sin[e + f*x]]*(Sqrt[b*Cos[e + f*x]]/Sqrt[Sin[2*e
 + 2*f*x]])   Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

rule 3093
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[a^2*(a*Sec[e + f*x])^(m - 2)*((b*Tan[e + f*x])^(n +
1)/(b*f*(m + n - 1))), x] + Simp[a^2*((m - 2)/(m + n - 1))   Int[(a*Sec[e +
 f*x])^(m - 2)*(b*Tan[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (
GtQ[m, 1] || (EqQ[m, 1] && EqQ[n, 1/2])) && NeQ[m + n - 1, 0] && IntegersQ[
2*m, 2*n]
```

rule 3095
```
Int[Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]]/sec[(e_.) + (f_.)*(x_)], x_Symbol]
:> Simp[Sqrt[Cos[e + f*x]]*(Sqrt[b*Tan[e + f*x]]/Sqrt[Sin[e + f*x]])   Int[
Sqrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]], x], x] /; FreeQ[{b, e, f}, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

3.249. $\int \frac{1}{(e \cot(c+dx))^{9/2}(a+a\sec(c+dx))}\,dx$

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

rule 4369
```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(-e)*(e*Cot[c + d*x])^(m - 1)*((a*m + b*(m - 1)*Csc
[c + d*x])/(d*m*(m - 1))), x] - Simp[e^2/m   Int[(e*Cot[c + d*x])^(m - 2)*(
a*m + b*(m - 1)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && GtQ[m
, 1]
```

rule 4372
```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(e*Cot[c + d*x])^m, x], x] + Simp[b   Int[
(e*Cot[c + d*x])^m*Csc[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, m}, x]
```

rule 4376
```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Simp[a^(2*n)/e^(2*n)   Int[(e*Cot[c + d*x])^(m + 2*
n)/(-a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[a
^2 - b^2, 0] && ILtQ[n, 0]
```

rule 4388
```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*((a_) + (b_.)*sec[(c_.) + (d_.)*(x
_)])^(n_.), x_Symbol] :> Simp[(e*Cot[c + d*x])^m*Tan[c + d*x]^m   Int[(a +
b*Sec[c + d*x])^n/Tan[c + d*x]^m, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x]
 &&  !IntegerQ[m]
```

3.249.4 Maple [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 9.23 (sec) , antiderivative size = 1166, normalized size of antiderivative = 3.14

method	result	size
default	Expression too large to display	1166

input
```
int(1/(e*cot(d*x+c))^(9/2)/(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/30/a/d*2^(1/2)*(-15*I*(cot(d*x+c)-csc(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*
x+c))^(1/2)*(csc(d*x+c)-cot(d*x+c)+1)^(1/2)*EllipticPi((csc(d*x+c)-cot(d*x
+c)+1)^(1/2),1/2+1/2*I,1/2*2^(1/2))*cos(d*x+c)^2-15*I*(cot(d*x+c)-csc(d*x+
c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)*(csc(d*x+c)-cot(d*x+c)+1)^(1/2)*
EllipticPi((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2+1/2*I,1/2*2^(1/2))*cos(d*x+
c)^3+15*I*(cot(d*x+c)-csc(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)*(c
sc(d*x+c)-cot(d*x+c)+1)^(1/2)*EllipticPi((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1
/2-1/2*I,1/2*2^(1/2))*cos(d*x+c)^3+15*I*(cot(d*x+c)-csc(d*x+c)+1)^(1/2)*(c
ot(d*x+c)-csc(d*x+c))^(1/2)*(csc(d*x+c)-cot(d*x+c)+1)^(1/2)*EllipticPi((cs
c(d*x+c)-cot(d*x+c)+1)^(1/2),1/2-1/2*I,1/2*2^(1/2))*cos(d*x+c)^2+18*(csc(d
*x+c)-cot(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(
d*x+c))^(1/2)*EllipticF((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2*2^(1/2))*cos(d
*x+c)^3-15*(cot(d*x+c)-csc(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)*(
csc(d*x+c)-cot(d*x+c)+1)^(1/2)*EllipticPi((csc(d*x+c)-cot(d*x+c)+1)^(1/2),
1/2-1/2*I,1/2*2^(1/2))*cos(d*x+c)^3-15*(cot(d*x+c)-csc(d*x+c)+1)^(1/2)*(co
t(d*x+c)-csc(d*x+c))^(1/2)*EllipticPi((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2+
1/2*I,1/2*2^(1/2))*(csc(d*x+c)-cot(d*x+c)+1)^(1/2)*cos(d*x+c)^3-36*(csc(d*
x+c)-cot(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d
*x+c))^(1/2)*EllipticE((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2*2^(1/2))*cos(d*
x+c)^3+18*(csc(d*x+c)-cot(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c)+1)^(1/...
```

3.249.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(e\cot(c+dx))^{9/2}(a+a\sec(c+dx))}\,dx = \text{Timed out}$$

input
```
integrate(1/(e*cot(d*x+c))^(9/2)/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

output
```
Timed out
```

3.249.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(e\cot(c+dx))^{9/2}(a+a\sec(c+dx))}\,dx = \text{Timed out}$$

input
```
integrate(1/(e*cot(d*x+c))**(9/2)/(a+a*sec(d*x+c)),x)
```

output
```
Timed out
```

3.249.7 Maxima [F]

$$\int \frac{1}{(e\cot(c+dx))^{9/2}(a+a\sec(c+dx))}\,dx = \int \frac{1}{(e\cot(dx+c))^{\frac{9}{2}}(a\sec(dx+c)+a)}\,dx$$

input
```
integrate(1/(e*cot(d*x+c))^(9/2)/(a+a*sec(d*x+c)),x, algorithm="maxima")
```

output
```
integrate(1/((e*cot(d*x + c))^(9/2)*(a*sec(d*x + c) + a)), x)
```

3.249.8 Giac [F]

$$\int \frac{1}{(e\cot(c+dx))^{9/2}(a+a\sec(c+dx))}\,dx = \int \frac{1}{(e\cot(dx+c))^{\frac{9}{2}}(a\sec(dx+c)+a)}\,dx$$

input
```
integrate(1/(e*cot(d*x+c))^(9/2)/(a+a*sec(d*x+c)),x, algorithm="giac")
```

output
```
integrate(1/((e*cot(d*x + c))^(9/2)*(a*sec(d*x + c) + a)), x)
```

3.249.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(e\cot(c+dx))^{9/2}(a+a\sec(c+dx))}\,dx = \int \frac{\cos(c+dx)}{a\,(e\cot(c+dx))^{9/2}\,(\cos(c+dx)+1)}\,dx$$

input
```
int(1/((e*cot(c + d*x))^(9/2)*(a + a/cos(c + d*x))),x)
```

output
```
int(cos(c + d*x)/(a*(e*cot(c + d*x))^(9/2)*(cos(c + d*x) + 1)), x)
```

3.250 $\int \frac{1}{\sqrt{e\cot(c+dx)}(a+a\sec(c+dx))^2}\,dx$

3.250.1 Optimal result 1840
3.250.2 Mathematica [F] 1841
3.250.3 Rubi [A] (verified) 1841
3.250.4 Maple [C] (warning: unable to verify) 1843
3.250.5 Fricas [F(-1)] 1844
3.250.6 Sympy [F] 1844
3.250.7 Maxima [F] 1845
3.250.8 Giac [F] 1845
3.250.9 Mupad [F(-1)] 1845

3.250.1 Optimal result

Integrand size = 25, antiderivative size = 413

$$\begin{aligned}\int \frac{1}{\sqrt{e\cot(c+dx)}(a+a\sec(c+dx))^2}\,dx &= \frac{2\cot(c+dx)}{a^2d\sqrt{e\cot(c+dx)}} - \frac{12\cos(c+dx)\cot(c+dx)}{5a^2d\sqrt{e\cot(c+dx)}} \\ &- \frac{4\cot^3(c+dx)}{5a^2d\sqrt{e\cot(c+dx)}} \\ &+ \frac{4\cot^2(c+dx)\csc(c+dx)}{5a^2d\sqrt{e\cot(c+dx)}} \\ &- \frac{12\cos(c+dx)E\left(c-\frac{\pi}{4}+dx\middle|2\right)}{5a^2d\sqrt{e\cot(c+dx)}\sqrt{\sin(2c+2dx)}} \\ &- \frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}a^2d\sqrt{e\cot(c+dx)}\sqrt{\tan(c+dx)}} \\ &+ \frac{\arctan\left(1+\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}a^2d\sqrt{e\cot(c+dx)}\sqrt{\tan(c+dx)}} \\ &+ \frac{\log\left(1-\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+dx)\right)}{2\sqrt{2}a^2d\sqrt{e\cot(c+dx)}\sqrt{\tan(c+dx)}} \\ &- \frac{\log\left(1+\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+dx)\right)}{2\sqrt{2}a^2d\sqrt{e\cot(c+dx)}\sqrt{\tan(c+dx)}}\end{aligned}$$

output
```
2*cot(d*x+c)/a^2/d/(e*cot(d*x+c))^(1/2)-12/5*cos(d*x+c)*cot(d*x+c)/a^2/d/(
e*cot(d*x+c))^(1/2)-4/5*cot(d*x+c)^3/a^2/d/(e*cot(d*x+c))^(1/2)+4/5*cot(d*
x+c)^2*csc(d*x+c)/a^2/d/(e*cot(d*x+c))^(1/2)+12/5*cos(d*x+c)*(sin(c+1/4*Pi
+d*x)^2)^(1/2)/sin(c+1/4*Pi+d*x)*EllipticE(cos(c+1/4*Pi+d*x),2^(1/2))/a^2/
d/(e*cot(d*x+c))^(1/2)/sin(2*d*x+2*c)^(1/2)+1/2*arctan(-1+2^(1/2)*tan(d*x+
c)^(1/2))/a^2/d*2^(1/2)/(e*cot(d*x+c))^(1/2)/tan(d*x+c)^(1/2)+1/2*arctan(1
+2^(1/2)*tan(d*x+c)^(1/2))/a^2/d*2^(1/2)/(e*cot(d*x+c))^(1/2)/tan(d*x+c)^(
1/2)+1/4*ln(1-2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c))/a^2/d*2^(1/2)/(e*cot(d*
x+c))^(1/2)/tan(d*x+c)^(1/2)-1/4*ln(1+2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c))
/a^2/d*2^(1/2)/(e*cot(d*x+c))^(1/2)/tan(d*x+c)^(1/2)
```

3.250.2 Mathematica [F]

$$\int \frac{1}{\sqrt{e\cot(c+dx)}(a+a\sec(c+dx))^2}\,dx = \int \frac{1}{\sqrt{e\cot(c+dx)}(a+a\sec(c+dx))^2}\,dx$$

input
```
Integrate[1/(Sqrt[e*Cot[c + d*x]]*(a + a*Sec[c + d*x])^2),x]
```

output
```
Integrate[1/(Sqrt[e*Cot[c + d*x]]*(a + a*Sec[c + d*x])^2), x]
```

3.250.3 Rubi [A] (verified)

Time = 0.81 (sec) , antiderivative size = 313, normalized size of antiderivative = 0.76, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {3042, 4388, 3042, 4376, 3042, 4374, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a\sec(c+dx)+a)^2\sqrt{e\cot(c+dx)}}\,dx$$
$$\downarrow 3042$$
$$\int \frac{1}{(a\sec(c+dx)+a)^2\sqrt{e\cot(c+dx)}}dx$$
$$\downarrow 4388$$

$$\frac{\int \frac{\sqrt{\tan(c+dx)}}{(\sec(c+dx)a+a)^2}dx}{\sqrt{\tan(c+dx)}\sqrt{e\cot(c+dx)}}$$

$$\downarrow 3042$$

$$\frac{\int \frac{\sqrt{-\cot\left(c+dx+\frac{\pi}{2}\right)}}{\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^2}dx}{\sqrt{\tan(c+dx)}\sqrt{e\cot(c+dx)}}$$

$$\downarrow 4376$$

$$\frac{\int \frac{(a-a\sec(c+dx))^2}{\tan^{\frac{7}{2}}(c+dx)}dx}{a^4\sqrt{\tan(c+dx)}\sqrt{e\cot(c+dx)}}$$

$$\downarrow 3042$$

$$\frac{\int \frac{\left(a-a\csc\left(c+dx+\frac{\pi}{2}\right)\right)^2}{\left(-\cot\left(c+dx+\frac{\pi}{2}\right)\right)^{7/2}}dx}{a^4\sqrt{\tan(c+dx)}\sqrt{e\cot(c+dx)}}$$

$$\downarrow 4374$$

$$\frac{\int \left(\frac{\sec^2(c+dx)a^2}{\tan^{\frac{7}{2}}(c+dx)} - \frac{2\sec(c+dx)a^2}{\tan^{\frac{7}{2}}(c+dx)} + \frac{a^2}{\tan^{\frac{7}{2}}(c+dx)}\right)dx}{a^4\sqrt{\tan(c+dx)}\sqrt{e\cot(c+dx)}}$$

$$\downarrow 2009$$

$$\frac{-\frac{a^2\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}d} + \frac{a^2\arctan\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}d} - \frac{4a^2}{5d\tan^{\frac{5}{2}}(c+dx)} + \frac{2a^2}{d\sqrt{\tan(c+dx)}} + \frac{a^2\log\left(\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}\right.}{2\sqrt{2}d}}{a^4\sqrt{\tan(c+dx)}\sqrt{}}$$

input
```
Int[1/(Sqrt[e*Cot[c + d*x]]*(a + a*Sec[c + d*x])^2),x]
```

output
```
(-((a^2*ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]])/(Sqrt[2]*d)) + (a^2*ArcTan
[1 + Sqrt[2]*Sqrt[Tan[c + d*x]]])/(Sqrt[2]*d) + (a^2*Log[1 - Sqrt[2]*Sqrt[
Tan[c + d*x]] + Tan[c + d*x]])/(2*Sqrt[2]*d) - (a^2*Log[1 + Sqrt[2]*Sqrt[T
an[c + d*x]] + Tan[c + d*x]])/(2*Sqrt[2]*d) - (4*a^2)/(5*d*Tan[c + d*x]^(5
/2)) + (4*a^2*Sec[c + d*x])/(5*d*Tan[c + d*x]^(5/2)) + (2*a^2)/(d*Sqrt[Tan
[c + d*x]]) - (12*a^2*Cos[c + d*x])/(5*d*Sqrt[Tan[c + d*x]]) - (12*a^2*Cos
[c + d*x]*EllipticE[c - Pi/4 + d*x, 2]*Sqrt[Tan[c + d*x]])/(5*d*Sqrt[Sin[2
*c + 2*d*x]]))/(a^4*Sqrt[e*Cot[c + d*x]]*Sqrt[Tan[c + d*x]])
```

3.250. $\int \frac{1}{\sqrt{e\cot(c+dx)}(a+a\sec(c+dx))^2}dx$

3.250.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4374
```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

rule 4376
```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Simp[a^(2*n)/e^(2*n)   Int[(e*Cot[c + d*x])^(m + 2*
n)/(-a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[a
^2 - b^2, 0] && ILtQ[n, 0]
```

rule 4388
```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*((a_) + (b_.)*sec[(c_.) + (d_.)*(x
_)])^(n_.), x_Symbol] :> Simp[(e*Cot[c + d*x])^m*Tan[c + d*x]^m   Int[(a +
b*Sec[c + d*x])^n/Tan[c + d*x]^m, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x]
 &&  !IntegerQ[m]
```

3.250.4 Maple [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 8.55 (sec) , antiderivative size = 705, normalized size of antiderivative = 1.71

method	result
default	$\frac{\sqrt{2}\left((1-\cos(dx+c))^2\csc(dx+c)^2-1\right)\left(5i\sqrt{\csc(dx+c)-\cot(dx+c)+1}\,\sqrt{2-2\csc(dx+c)+2\cot(dx+c)}\,\sqrt{\cot(dx+c)-\csc(dx+c)}\ \text{Ellipt}\right.}{}$

input
```
int(1/(a+a*sec(d*x+c))^2/(e*cot(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/10/a^2/d*2^(1/2)*((1-cos(d*x+c))^2*csc(d*x+c)^2-1)*(5*I*(csc(d*x+c)-cot(
d*x+c)+1)^(1/2)*(2-2*csc(d*x+c)+2*cot(d*x+c))^(1/2)*(cot(d*x+c)-csc(d*x+c)
)^(1/2)*EllipticPi((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2-1/2*I,1/2*2^(1/2))-
5*I*(csc(d*x+c)-cot(d*x+c)+1)^(1/2)*(2-2*csc(d*x+c)+2*cot(d*x+c))^(1/2)*(c
ot(d*x+c)-csc(d*x+c))^(1/2)*EllipticPi((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2
+1/2*I,1/2*2^(1/2))+24*(csc(d*x+c)-cot(d*x+c)+1)^(1/2)*(2-2*csc(d*x+c)+2*c
ot(d*x+c))^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)*EllipticE((csc(d*x+c)-cot(d
*x+c)+1)^(1/2),1/2*2^(1/2))-12*(csc(d*x+c)-cot(d*x+c)+1)^(1/2)*(2-2*csc(d*
x+c)+2*cot(d*x+c))^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)*EllipticF((csc(d*x+
c)-cot(d*x+c)+1)^(1/2),1/2*2^(1/2))-5*(csc(d*x+c)-cot(d*x+c)+1)^(1/2)*(2-2
*csc(d*x+c)+2*cot(d*x+c))^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)*EllipticPi((
csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2-1/2*I,1/2*2^(1/2))-5*(csc(d*x+c)-cot(d*
x+c)+1)^(1/2)*(2-2*csc(d*x+c)+2*cot(d*x+c))^(1/2)*(cot(d*x+c)-csc(d*x+c))^
(1/2)*EllipticPi((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2+1/2*I,1/2*2^(1/2))+2*
(1-cos(d*x+c))^4*csc(d*x+c)^4-2*(1-cos(d*x+c))^2*csc(d*x+c)^2)/(-e/(1-cos(
d*x+c))*((1-cos(d*x+c))^2*csc(d*x+c)-sin(d*x+c)))^(1/2)/((1-cos(d*x+c))*((
1-cos(d*x+c))^2*csc(d*x+c)^2-1)*csc(d*x+c))^(1/2)/((1-cos(d*x+c))^3*csc(d*
x+c)^3+cot(d*x+c)-csc(d*x+c))^(1/2)
```

3.250.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{e\cot(c+dx)}(a+a\sec(c+dx))^2}\,dx = \text{Timed out}$$

input
```
integrate(1/(a+a*sec(d*x+c))^2/(e*cot(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.250.6 Sympy [F]

$$\int \frac{1}{\sqrt{e\cot(c+dx)}(a+a\sec(c+dx))^2}\,dx = \frac{\int \frac{1}{\sqrt{e\cot(c+dx)}\sec^2(c+dx)+2\sqrt{e\cot(c+dx)}\sec(c+dx)+\sqrt{e\cot(c+dx)}}\,dx}{a^2}$$

input
```
integrate(1/(a+a*sec(d*x+c))**2/(e*cot(d*x+c))**(1/2),x)
```

output
```
Integral(1/(sqrt(e*cot(c + d*x))*sec(c + d*x)**2 + 2*sqrt(e*cot(c + d*x))*
sec(c + d*x) + sqrt(e*cot(c + d*x))), x)/a**2
```

3.250.7 Maxima [F]

$$\int \frac{1}{\sqrt{e\cot(c+dx)}(a+a\sec(c+dx))^2}\,dx = \int \frac{1}{\sqrt{e\cot(dx+c)}(a\sec(dx+c)+a)^2}\,dx$$

input
```
integrate(1/(a+a*sec(d*x+c))^2/(e*cot(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(e*cot(d*x + c))*(a*sec(d*x + c) + a)^2), x)
```

3.250.8 Giac [F]

$$\int \frac{1}{\sqrt{e\cot(c+dx)}(a+a\sec(c+dx))^2}\,dx = \int \frac{1}{\sqrt{e\cot(dx+c)}(a\sec(dx+c)+a)^2}\,dx$$

input
```
integrate(1/(a+a*sec(d*x+c))^2/(e*cot(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(e*cot(d*x + c))*(a*sec(d*x + c) + a)^2), x)
```

3.250.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{e\cot(c+dx)}(a+a\sec(c+dx))^2}\,dx = \int \frac{\cos(c+dx)^2}{a^2\sqrt{e\cot(c+dx)}(\cos(c+dx)+1)^2}\,dx$$

input
```
int(1/((e*cot(c + d*x))^(1/2)*(a + a/cos(c + d*x))^2),x)
```

output
```
int(cos(c + d*x)^2/(a^2*(e*cot(c + d*x))^(1/2)*(cos(c + d*x) + 1)^2), x)
```

3.251 $\int \frac{1}{(e\cot(c+dx))^{3/2}(a+a\sec(c+dx))^2}\,dx$

3.251.1 Optimal result 1846
3.251.2 Mathematica [F] 1847
3.251.3 Rubi [A] (verified) 1847
3.251.4 Maple [C] (warning: unable to verify) 1849
3.251.5 Fricas [F(-1)] 1850
3.251.6 Sympy [F] 1850
3.251.7 Maxima [F(-1)] 1851
3.251.8 Giac [F] 1851
3.251.9 Mupad [F(-1)] 1851

3.251.1 Optimal result

Integrand size = 25, antiderivative size = 359

$$\begin{aligned}
&\int \frac{1}{(e\cot(c+dx))^{3/2}(a+a\sec(c+dx))^2}\,dx = \\
&-\frac{4\cot^3(c+dx)}{3a^2d(e\cot(c+dx))^{3/2}}+\frac{4\cot^2(c+dx)\csc(c+dx)}{3a^2d(e\cot(c+dx))^{3/2}} \\
&+\frac{2\cot(c+dx)\csc(c+dx)\,\text{EllipticF}\left(c-\frac{\pi}{4}+dx,2\right)\sqrt{\sin(2c+2dx)}}{3a^2d(e\cot(c+dx))^{3/2}} \\
&+\frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}a^2d(e\cot(c+dx))^{3/2}\tan^{\frac{3}{2}}(c+dx)}-\frac{\arctan\left(1+\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}a^2d(e\cot(c+dx))^{3/2}\tan^{\frac{3}{2}}(c+dx)} \\
&+\frac{\log\left(1-\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+dx)\right)}{2\sqrt{2}a^2d(e\cot(c+dx))^{3/2}\tan^{\frac{3}{2}}(c+dx)} \\
&-\frac{\log\left(1+\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+dx)\right)}{2\sqrt{2}a^2d(e\cot(c+dx))^{3/2}\tan^{\frac{3}{2}}(c+dx)}
\end{aligned}$$

output
```
-4/3*cot(d*x+c)^3/a^2/d/(e*cot(d*x+c))^(3/2)+4/3*cot(d*x+c)^2*csc(d*x+c)/a
^2/d/(e*cot(d*x+c))^(3/2)-2/3*cot(d*x+c)*csc(d*x+c)*(sin(c+1/4*Pi+d*x)^2)^
(1/2)/sin(c+1/4*Pi+d*x)*EllipticF(cos(c+1/4*Pi+d*x),2^(1/2))*sin(2*d*x+2*c
)^(1/2)/a^2/d/(e*cot(d*x+c))^(3/2)-1/2*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2))
/a^2/d/(e*cot(d*x+c))^(3/2)*2^(1/2)/tan(d*x+c)^(3/2)-1/2*arctan(1+2^(1/2)*
tan(d*x+c)^(1/2))/a^2/d/(e*cot(d*x+c))^(3/2)*2^(1/2)/tan(d*x+c)^(3/2)+1/4*
ln(1-2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c))/a^2/d/(e*cot(d*x+c))^(3/2)*2^(1/
2)/tan(d*x+c)^(3/2)-1/4*ln(1+2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c))/a^2/d/(e
*cot(d*x+c))^(3/2)*2^(1/2)/tan(d*x+c)^(3/2)
```

3.251.2 Mathematica [F]

$$\int \frac{1}{(e\cot(c+dx))^{3/2}(a+a\sec(c+dx))^2}\,dx = \int \frac{1}{(e\cot(c+dx))^{3/2}(a+a\sec(c+dx))^2}\,dx$$

input
```
Integrate[1/((e*Cot[c + d*x])^(3/2)*(a + a*Sec[c + d*x])^2),x]
```

output
```
Integrate[1/((e*Cot[c + d*x])^(3/2)*(a + a*Sec[c + d*x])^2), x]
```

3.251.3 Rubi [A] (verified)

Time = 0.74 (sec) , antiderivative size = 269, normalized size of antiderivative = 0.75, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {3042, 4388, 3042, 4376, 3042, 4374, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a\sec(c+dx)+a)^2(e\cot(c+dx))^{3/2}}\,dx$$
$$\downarrow 3042$$
$$\int \frac{1}{(a\sec(c+dx)+a)^2(e\cot(c+dx))^{3/2}}dx$$
$$\downarrow 4388$$
$$\frac{\int \frac{\tan^{\frac{3}{2}}(c+dx)}{(\sec(c+dx)a+a)^2}dx}{\tan^{\frac{3}{2}}(c+dx)(e\cot(c+dx))^{3/2}}$$

$$\downarrow 3042$$

$$\frac{\int \frac{(-\cot(c+dx+\frac{\pi}{2}))^{3/2}}{(\csc(c+dx+\frac{\pi}{2})a+a)^2}dx}{\tan^{\frac{3}{2}}(c+dx)(e\cot(c+dx))^{3/2}}$$

$$\downarrow 4376$$

$$\frac{\int \frac{(a-a\sec(c+dx))^2}{\tan^{\frac{5}{2}}(c+dx)}dx}{a^4\tan^{\frac{3}{2}}(c+dx)(e\cot(c+dx))^{3/2}}$$

$$\downarrow 3042$$

$$\frac{\int \frac{(a-a\csc(c+dx+\frac{\pi}{2}))^2}{(-\cot(c+dx+\frac{\pi}{2}))^{5/2}}dx}{a^4\tan^{\frac{3}{2}}(c+dx)(e\cot(c+dx))^{3/2}}$$

$$\downarrow 4374$$

$$\frac{\int \left(\frac{\sec^2(c+dx)a^2}{\tan^{\frac{5}{2}}(c+dx)} - \frac{2\sec(c+dx)a^2}{\tan^{\frac{5}{2}}(c+dx)} + \frac{a^2}{\tan^{\frac{5}{2}}(c+dx)}\right)dx}{a^4\tan^{\frac{3}{2}}(c+dx)(e\cot(c+dx))^{3/2}}$$

$$\downarrow 2009$$

$$\frac{a^2\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}d} - \frac{a^2\arctan\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}d} - \frac{4a^2}{3d\tan^{\frac{3}{2}}(c+dx)} + \frac{a^2\log\left(\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}d} - \frac{a^2\log\left(\tan}{a^4\tan^{\frac{3}{2}}(c+dx)(e\cot(c+dx))^{3/}}$$

input
```
Int[1/((e*Cot[c + d*x])^(3/2)*(a + a*Sec[c + d*x])^2),x]
```

output
```
((a^2*ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]])/(Sqrt[2]*d) - (a^2*ArcTan[1
+ Sqrt[2]*Sqrt[Tan[c + d*x]]])/(Sqrt[2]*d) + (a^2*Log[1 - Sqrt[2]*Sqrt[Tan
[c + d*x]] + Tan[c + d*x]])/(2*Sqrt[2]*d) - (a^2*Log[1 + Sqrt[2]*Sqrt[Tan[
c + d*x]] + Tan[c + d*x]])/(2*Sqrt[2]*d) - (4*a^2)/(3*d*Tan[c + d*x]^(3/2)
) + (4*a^2*Sec[c + d*x])/(3*d*Tan[c + d*x]^(3/2)) + (2*a^2*EllipticF[c - P
i/4 + d*x, 2]*Sec[c + d*x]*Sqrt[Sin[2*c + 2*d*x]])/(3*d*Sqrt[Tan[c + d*x]]
))/(a^4*(e*Cot[c + d*x])^(3/2)*Tan[c + d*x]^(3/2))
```

3.251.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4374
```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

rule 4376
```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Simp[a^(2*n)/e^(2*n)   Int[(e*Cot[c + d*x])^(m + 2*
n)/(-a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[a
^2 - b^2, 0] && ILtQ[n, 0]
```

rule 4388
```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*((a_) + (b_.)*sec[(c_.) + (d_.)*(x
_)])^(n_.), x_Symbol] :> Simp[(e*Cot[c + d*x])^m*Tan[c + d*x]^m   Int[(a +
b*Sec[c + d*x])^n/Tan[c + d*x]^m, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x]
 &&  !IntegerQ[m]
```

3.251.4 Maple [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 9.01 (sec) , antiderivative size = 638, normalized size of antiderivative = 1.78

method	result
default	$\frac{\sqrt{2}\left((1-\cos(dx+c))^2\csc(dx+c)^2-1\right)^2\left(3i\sqrt{\csc(dx+c)-\cot(dx+c)+1}\sqrt{2-2\csc(dx+c)+2\cot(dx+c)}\sqrt{\cot(dx+c)-\csc(dx+c)}\ \text{Ellip}$

input
```
int(1/(e*cot(d*x+c))^(3/2)/(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/6/a^2/d*2^(1/2)*((1-cos(d*x+c))^2*csc(d*x+c)^2-1)^2*(3*I*(csc(d*x+c)-cot
(d*x+c)+1)^(1/2)*(2-2*csc(d*x+c)+2*cot(d*x+c))^(1/2)*(cot(d*x+c)-csc(d*x+c
))^(1/2)*EllipticPi((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2-1/2*I,1/2*2^(1/2))
-3*I*(csc(d*x+c)-cot(d*x+c)+1)^(1/2)*(2-2*csc(d*x+c)+2*cot(d*x+c))^(1/2)*(
cot(d*x+c)-csc(d*x+c))^(1/2)*EllipticPi((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/
2+1/2*I,1/2*2^(1/2))-10*(csc(d*x+c)-cot(d*x+c)+1)^(1/2)*(2-2*csc(d*x+c)+2*
cot(d*x+c))^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)*EllipticF((csc(d*x+c)-cot(
d*x+c)+1)^(1/2),1/2*2^(1/2))+3*(csc(d*x+c)-cot(d*x+c)+1)^(1/2)*(2-2*csc(d*
x+c)+2*cot(d*x+c))^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)*EllipticPi((csc(d*x
+c)-cot(d*x+c)+1)^(1/2),1/2-1/2*I,1/2*2^(1/2))+3*(csc(d*x+c)-cot(d*x+c)+1)
^(1/2)*(2-2*csc(d*x+c)+2*cot(d*x+c))^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)*E
llipticPi((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2+1/2*I,1/2*2^(1/2))+4*(1-cos(
d*x+c))^3*csc(d*x+c)^3-4*csc(d*x+c)+4*cot(d*x+c))/(-e/(1-cos(d*x+c))*((1-c
os(d*x+c))^2*csc(d*x+c)-sin(d*x+c)))^(3/2)/(1-cos(d*x+c))*sin(d*x+c)/((1-c
os(d*x+c))*((1-cos(d*x+c))^2*csc(d*x+c)^2-1)*csc(d*x+c))^(1/2)/((1-cos(d*x
+c))^3*csc(d*x+c)^3+cot(d*x+c)-csc(d*x+c))^(1/2)
```

3.251.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(e\cot(c+dx))^{3/2}(a+a\sec(c+dx))^2}\,dx = \text{Timed out}$$

input
```
integrate(1/(e*cot(d*x+c))^(3/2)/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

output
```
Timed out
```

3.251.6 Sympy [F]

$$\int \frac{1}{(e\cot(c+dx))^{3/2}(a+a\sec(c+dx))^2}\,dx = \frac{\int \frac{1}{(e\cot(c+dx))^{\frac{3}{2}}\sec^2(c+dx)+2(e\cot(c+dx))^{\frac{3}{2}}\sec(c+dx)+(e\cot(c+dx))^{\frac{3}{2}}}\,dx}{a^2}$$

input
```
integrate(1/(e*cot(d*x+c))**(3/2)/(a+a*sec(d*x+c))**2,x)
```

output
```
Integral(1/((e*cot(c + d*x))**(3/2)*sec(c + d*x)**2 + 2*(e*cot(c + d*x))**
(3/2)*sec(c + d*x) + (e*cot(c + d*x))**(3/2)), x)/a**2
```

3.251.7 Maxima [F(-1)]

Timed out.

$$\int \frac{1}{(e\cot(c+dx))^{3/2}(a+a\sec(c+dx))^2}\,dx = \text{Timed out}$$

input
```
integrate(1/(e*cot(d*x+c))^(3/2)/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

output
```
Timed out
```

3.251.8 Giac [F]

$$\int \frac{1}{(e\cot(c+dx))^{3/2}(a+a\sec(c+dx))^2}\,dx = \int \frac{1}{(e\cot(dx+c))^{\frac{3}{2}}(a\sec(dx+c)+a)^2}\,dx$$

input
```
integrate(1/(e*cot(d*x+c))^(3/2)/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

output
```
integrate(1/((e*cot(d*x + c))^(3/2)*(a*sec(d*x + c) + a)^2), x)
```

3.251.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(e\cot(c+dx))^{3/2}(a+a\sec(c+dx))^2}\,dx = \int \frac{\cos(c+dx)^2}{a^2(e\cot(c+dx))^{3/2}(\cos(c+dx)+1)^2}\,dx$$

input
```
int(1/((e*cot(c + d*x))^(3/2)*(a + a/cos(c + d*x))^2),x)
```

output
```
int(cos(c + d*x)^2/(a^2*(e*cot(c + d*x))^(3/2)*(cos(c + d*x) + 1)^2), x)
```

3.252 $\int \frac{1}{(e\cot(c+dx))^{5/2}(a+a\sec(c+dx))^2}\,dx$

3.252.1 Optimal result . . . 1852
3.252.2 Mathematica [F] . . . 1853
3.252.3 Rubi [A] (verified) . . . 1853
3.252.4 Maple [C] (warning: unable to verify) . . . 1855
3.252.5 Fricas [F(-1)] . . . 1856
3.252.6 Sympy [F(-1)] . . . 1856
3.252.7 Maxima [F] . . . 1856
3.252.8 Giac [F] . . . 1857
3.252.9 Mupad [F(-1)] . . . 1857

3.252.1 Optimal result

Integrand size = 25, antiderivative size = 355

$$\int \frac{1}{(e\cot(c+dx))^{5/2}(a+a\sec(c+dx))^2}\,dx = -\frac{4\cot^3(c+dx)}{a^2d(e\cot(c+dx))^{5/2}}$$
$$+\frac{4\cos(c+dx)\cot^3(c+dx)}{a^2d(e\cot(c+dx))^{5/2}}+\frac{4\cos(c+dx)\cot^2(c+dx)E\left(c-\frac{\pi}{4}+dx\middle|2\right)}{a^2d(e\cot(c+dx))^{5/2}\sqrt{\sin(2c+2dx)}}$$
$$+\frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}a^2d(e\cot(c+dx))^{5/2}\tan^{\frac{5}{2}}(c+dx)}-\frac{\arctan\left(1+\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}a^2d(e\cot(c+dx))^{5/2}\tan^{\frac{5}{2}}(c+dx)}$$
$$-\frac{\log\left(1-\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+dx)\right)}{2\sqrt{2}a^2d(e\cot(c+dx))^{5/2}\tan^{\frac{5}{2}}(c+dx)}$$
$$+\frac{\log\left(1+\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+dx)\right)}{2\sqrt{2}a^2d(e\cot(c+dx))^{5/2}\tan^{\frac{5}{2}}(c+dx)}$$

output

```
-4*cot(d*x+c)^3/a^2/d/(e*cot(d*x+c))^(5/2)+4*cos(d*x+c)*cot(d*x+c)^3/a^2/d
/(e*cot(d*x+c))^(5/2)-4*cos(d*x+c)*cot(d*x+c)^2*(sin(c+1/4*Pi+d*x)^2)^(1/2
)/sin(c+1/4*Pi+d*x)*EllipticE(cos(c+1/4*Pi+d*x),2^(1/2))/a^2/d/(e*cot(d*x+
c))^(5/2)/sin(2*d*x+2*c)^(1/2)-1/2*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2))/a^2
/d/(e*cot(d*x+c))^(5/2)*2^(1/2)/tan(d*x+c)^(5/2)-1/2*arctan(1+2^(1/2)*tan(
d*x+c)^(1/2))/a^2/d/(e*cot(d*x+c))^(5/2)*2^(1/2)/tan(d*x+c)^(5/2)-1/4*ln(1
-2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c))/a^2/d/(e*cot(d*x+c))^(5/2)*2^(1/2)/t
an(d*x+c)^(5/2)+1/4*ln(1+2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c))/a^2/d/(e*cot
(d*x+c))^(5/2)*2^(1/2)/tan(d*x+c)^(5/2)
```

3.252.2 Mathematica [F]

$$\int \frac{1}{(e\cot(c+dx))^{5/2}(a+a\sec(c+dx))^2}\,dx = \int \frac{1}{(e\cot(c+dx))^{5/2}(a+a\sec(c+dx))^2}\,dx$$

input
```
Integrate[1/((e*Cot[c + d*x])^(5/2)*(a + a*Sec[c + d*x])^2),x]
```

output
```
Integrate[1/((e*Cot[c + d*x])^(5/2)*(a + a*Sec[c + d*x])^2), x]
```

3.252.3 Rubi [A] (verified)

Time = 0.74 (sec) , antiderivative size = 263, normalized size of antiderivative = 0.74, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {3042, 4388, 3042, 4376, 3042, 4374, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a\sec(c+dx)+a)^2(e\cot(c+dx))^{5/2}}\,dx$$
$$\downarrow 3042$$
$$\int \frac{1}{(a\sec(c+dx)+a)^2(e\cot(c+dx))^{5/2}}dx$$
$$\downarrow 4388$$
$$\frac{\int \frac{\tan^{\frac{5}{2}}(c+dx)}{(\sec(c+dx)a+a)^2}dx}{\tan^{\frac{5}{2}}(c+dx)(e\cot(c+dx))^{5/2}}$$
$$\downarrow 3042$$
$$\frac{\int \frac{(-\cot(c+dx+\frac{\pi}{2}))^{5/2}}{\left(\csc(c+dx+\frac{\pi}{2})a+a\right)^2}dx}{\tan^{\frac{5}{2}}(c+dx)(e\cot(c+dx))^{5/2}}$$
$$\downarrow 4376$$
$$\frac{\int \frac{(a-a\sec(c+dx))^2}{\tan^{\frac{3}{2}}(c+dx)}dx}{a^4\tan^{\frac{5}{2}}(c+dx)(e\cot(c+dx))^{5/2}}$$
$$\downarrow 3042$$

$$\frac{\int \frac{(a-a\csc(c+dx+\frac{\pi}{2}))^2}{(-\cot(c+dx+\frac{\pi}{2}))^{3/2}}dx}{a^4\tan^{\frac{5}{2}}(c+dx)(e\cot(c+dx))^{5/2}}$$

$$\downarrow 4374$$

$$\frac{\int\left(\frac{\sec^2(c+dx)a^2}{\tan^{\frac{3}{2}}(c+dx)}-\frac{2\sec(c+dx)a^2}{\tan^{\frac{3}{2}}(c+dx)}+\frac{a^2}{\tan^{\frac{3}{2}}(c+dx)}\right)dx}{a^4\tan^{\frac{5}{2}}(c+dx)(e\cot(c+dx))^{5/2}}$$

$$\downarrow 2009$$

$$\frac{\frac{a^2\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}d}-\frac{a^2\arctan\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}d}-\frac{4a^2}{d\sqrt{\tan(c+dx)}}-\frac{a^2\log\left(\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}d}+\frac{a^2\log\left(\tan(}{}}{a^4\tan^{\frac{5}{2}}(c+dx)(e\cot(c+dx))^{5/2}}$$

input
```
Int[1/((e*Cot[c + d*x])^(5/2)*(a + a*Sec[c + d*x])^2),x]
```

output
```
((a^2*ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]])/(Sqrt[2]*d) - (a^2*ArcTan[1
+ Sqrt[2]*Sqrt[Tan[c + d*x]]])/(Sqrt[2]*d) - (a^2*Log[1 - Sqrt[2]*Sqrt[Tan
[c + d*x]] + Tan[c + d*x]])/(2*Sqrt[2]*d) + (a^2*Log[1 + Sqrt[2]*Sqrt[Tan[
c + d*x]] + Tan[c + d*x]])/(2*Sqrt[2]*d) - (4*a^2)/(d*Sqrt[Tan[c + d*x]])
+ (4*a^2*Cos[c + d*x])/(d*Sqrt[Tan[c + d*x]]) + (4*a^2*Cos[c + d*x]*Ellipt
icE[c - Pi/4 + d*x, 2]*Sqrt[Tan[c + d*x]])/(d*Sqrt[Sin[2*c + 2*d*x]]))/(a^
4*(e*Cot[c + d*x])^(5/2)*Tan[c + d*x]^(5/2))
```

3.252.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4374
```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

rule 4376
```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Simp[a^(2*n)/e^(2*n)   Int[(e*Cot[c + d*x])^(m + 2*
n)/(-a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[a
^2 - b^2, 0] && ILtQ[n, 0]
```

rule 4388
```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*((a_) + (b_.)*sec[(c_.) + (d_.)*(x
_)])^(n_.), x_Symbol] :> Simp[(e*Cot[c + d*x])^m*Tan[c + d*x]^m   Int[(a +
b*Sec[c + d*x])^n/Tan[c + d*x]^m, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x]
 &&  !IntegerQ[m]
```

3.252.4 Maple [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 8.10 (sec) , antiderivative size = 262, normalized size of antiderivative = 0.74

method	result
default	$\frac{(\frac{1}{2}+\frac{i}{2})\sqrt{2}\left(i\,\mathrm{EllipticPi}\left(\sqrt{\csc(dx+c)-\cot(dx+c)+1},\frac{1}{2}-\frac{i}{2},\frac{\sqrt{2}}{2}\right)-4i\,\mathrm{EllipticE}\left(\sqrt{\csc(dx+c)-\cot(dx+c)+1},\frac{\sqrt{2}}{2}\right)+2i\,\mathrm{EllipticF}\left(\sqrt{\csc$

input
```
int(1/(e*cot(d*x+c))^(5/2)/(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
(1/2+1/2*I)/a^2/d*2^(1/2)*(I*EllipticPi((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/
2-1/2*I,1/2*2^(1/2))-4*I*EllipticE((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2*2^(
1/2))+2*I*EllipticF((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2*2^(1/2))-EllipticP
i((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2+1/2*I,1/2*2^(1/2))+4*EllipticE((csc(
d*x+c)-cot(d*x+c)+1)^(1/2),1/2*2^(1/2))-2*EllipticF((csc(d*x+c)-cot(d*x+c)
+1)^(1/2),1/2*2^(1/2)))*(csc(d*x+c)-cot(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*
x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)/(cos(d*x+c)-1)/e^2/(e*cot(d*x+
c))^(1/2)*sin(d*x+c)
```

3.252.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(e\cot(c+dx))^{5/2}(a+a\sec(c+dx))^2}\,dx = \text{Timed out}$$

input
```
integrate(1/(e*cot(d*x+c))^(5/2)/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

output
```
Timed out
```

3.252.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(e\cot(c+dx))^{5/2}(a+a\sec(c+dx))^2}\,dx = \text{Timed out}$$

input
```
integrate(1/(e*cot(d*x+c))**(5/2)/(a+a*sec(d*x+c))**2,x)
```

output
```
Timed out
```

3.252.7 Maxima [F]

$$\int \frac{1}{(e\cot(c+dx))^{5/2}(a+a\sec(c+dx))^2}\,dx = \int \frac{1}{(e\cot(dx+c))^{\frac{5}{2}}(a\sec(dx+c)+a)^2}\,dx$$

input
```
integrate(1/(e*cot(d*x+c))^(5/2)/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

output
```
integrate(1/((e*cot(d*x + c))^(5/2)*(a*sec(d*x + c) + a)^2), x)
```

3.252.8 Giac [F]

$$\int \frac{1}{(e\cot(c+dx))^{5/2}(a+a\sec(c+dx))^2}\,dx = \int \frac{1}{(e\cot(dx+c))^{\frac{5}{2}}(a\sec(dx+c)+a)^2}\,dx$$

input
```
integrate(1/(e*cot(d*x+c))^(5/2)/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

output
```
integrate(1/((e*cot(d*x + c))^(5/2)*(a*sec(d*x + c) + a)^2), x)
```

3.252.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(e\cot(c+dx))^{5/2}(a+a\sec(c+dx))^2}\,dx = \int \frac{\cos(c+dx)^2}{a^2(e\cot(c+dx))^{5/2}(\cos(c+dx)+1)^2}\,dx$$

input
```
int(1/((e*cot(c + d*x))^(5/2)*(a + a/cos(c + d*x))^2),x)
```

output
```
int(cos(c + d*x)^2/(a^2*(e*cot(c + d*x))^(5/2)*(cos(c + d*x) + 1)^2), x)
```

3.253 $\int \frac{1}{(e\cot(c+dx))^{7/2}(a+a\sec(c+dx))^2}\,dx$

3.253.1 Optimal result . . . 1858
3.253.2 Mathematica [F] . . . 1859
3.253.3 Rubi [A] (verified) . . . 1859
3.253.4 Maple [C] (warning: unable to verify) . . . 1861
3.253.5 Fricas [F(-1)] . . . 1862
3.253.6 Sympy [F(-1)] . . . 1863
3.253.7 Maxima [F(-1)] . . . 1863
3.253.8 Giac [F] . . . 1863
3.253.9 Mupad [F(-1)] . . . 1864

3.253.1 Optimal result

Integrand size = 25, antiderivative size = 321

$$\int \frac{1}{(e\cot(c+dx))^{7/2}(a+a\sec(c+dx))^2}\,dx = \frac{2\cot^3(c+dx)}{a^2 d(e\cot(c+dx))^{7/2}} - \frac{2\cot^3(c+dx)\csc(c+dx)\,\text{EllipticF}\left(c-\frac{\pi}{4}+dx,2\right)\sqrt{\sin(2c+2dx)}}{a^2 d(e\cot(c+dx))^{7/2}} - \frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}a^2 d(e\cot(c+dx))^{7/2}\tan^{\frac{7}{2}}(c+dx)} + \frac{\arctan\left(1+\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}a^2 d(e\cot(c+dx))^{7/2}\tan^{\frac{7}{2}}(c+dx)} - \frac{\log\left(1-\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+dx)\right)}{2\sqrt{2}a^2 d(e\cot(c+dx))^{7/2}\tan^{\frac{7}{2}}(c+dx)} + \frac{\log\left(1+\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+dx)\right)}{2\sqrt{2}a^2 d(e\cot(c+dx))^{7/2}\tan^{\frac{7}{2}}(c+dx)}$$

output

```
2*cot(d*x+c)^3/a^2/d/(e*cot(d*x+c))^(7/2)+2*cot(d*x+c)^3*csc(d*x+c)*(sin(c
+1/4*Pi+d*x)^2)^(1/2)/sin(c+1/4*Pi+d*x)*EllipticF(cos(c+1/4*Pi+d*x),2^(1/2
))*sin(2*d*x+2*c)^(1/2)/a^2/d/(e*cot(d*x+c))^(7/2)+1/2*arctan(-1+2^(1/2)*t
an(d*x+c)^(1/2))/a^2/d/(e*cot(d*x+c))^(7/2)*2^(1/2)/tan(d*x+c)^(7/2)+1/2*a
rctan(1+2^(1/2)*tan(d*x+c)^(1/2))/a^2/d/(e*cot(d*x+c))^(7/2)*2^(1/2)/tan(d
*x+c)^(7/2)-1/4*ln(1-2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c))/a^2/d/(e*cot(d*x
+c))^(7/2)*2^(1/2)/tan(d*x+c)^(7/2)+1/4*ln(1+2^(1/2)*tan(d*x+c)^(1/2)+tan(
d*x+c))/a^2/d/(e*cot(d*x+c))^(7/2)*2^(1/2)/tan(d*x+c)^(7/2)
```

3.253.2 Mathematica [F]

$$\int \frac{1}{(e\cot(c+dx))^{7/2}(a+a\sec(c+dx))^2}\,dx = \int \frac{1}{(e\cot(c+dx))^{7/2}(a+a\sec(c+dx))^2}\,dx$$

input
```
Integrate[1/((e*Cot[c + d*x])^(7/2)*(a + a*Sec[c + d*x])^2),x]
```

output
```
Integrate[1/((e*Cot[c + d*x])^(7/2)*(a + a*Sec[c + d*x])^2), x]
```

3.253.3 Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 239, normalized size of antiderivative = 0.74, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {3042, 4388, 3042, 4376, 3042, 4374, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a\sec(c+dx)+a)^2(e\cot(c+dx))^{7/2}}\,dx$$
$$\downarrow 3042$$
$$\int \frac{1}{(a\sec(c+dx)+a)^2(e\cot(c+dx))^{7/2}}dx$$
$$\downarrow 4388$$
$$\frac{\int \frac{\tan^{\frac{7}{2}}(c+dx)}{(\sec(c+dx)a+a)^2}dx}{\tan^{\frac{7}{2}}(c+dx)(e\cot(c+dx))^{7/2}}$$
$$\downarrow 3042$$
$$\frac{\int \frac{(-\cot(c+dx+\frac{\pi}{2}))^{7/2}}{\left(\csc(c+dx+\frac{\pi}{2})a+a\right)^2}dx}{\tan^{\frac{7}{2}}(c+dx)(e\cot(c+dx))^{7/2}}$$
$$\downarrow 4376$$
$$\frac{\int \frac{(a-a\sec(c+dx))^2}{\sqrt{\tan(c+dx)}}dx}{a^4\tan^{\frac{7}{2}}(c+dx)(e\cot(c+dx))^{7/2}}$$
$$\downarrow 3042$$

$$\frac{\int \frac{(a-a\csc(c+dx+\frac{\pi}{2}))^2}{\sqrt{-\cot(c+dx+\frac{\pi}{2})}}dx}{a^4\tan^{\frac{7}{2}}(c+dx)(e\cot(c+dx))^{7/2}}$$

$$\downarrow 4374$$

$$\frac{\int \left(\frac{\sec^2(c+dx)a^2}{\sqrt{\tan(c+dx)}} - \frac{2\sec(c+dx)a^2}{\sqrt{\tan(c+dx)}} + \frac{a^2}{\sqrt{\tan(c+dx)}}\right)dx}{a^4\tan^{\frac{7}{2}}(c+dx)(e\cot(c+dx))^{7/2}}$$

$$\downarrow 2009$$

$$\frac{-\frac{a^2\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}d} + \frac{a^2\arctan\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}d} + \frac{2a^2\sqrt{\tan(c+dx)}}{d} - \frac{a^2\log\left(\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}d} + \frac{a^2\log(}{}}{a^4\tan^{\frac{7}{2}}(c+dx)(e\cot(c+dx))^{7/2}}$$

input
```
Int[1/((e*Cot[c + d*x])^(7/2)*(a + a*Sec[c + d*x])^2),x]
```

output
```
(-((a^2*ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]])/(Sqrt[2]*d)) + (a^2*ArcTan
[1 + Sqrt[2]*Sqrt[Tan[c + d*x]]])/(Sqrt[2]*d) - (a^2*Log[1 - Sqrt[2]*Sqrt[
Tan[c + d*x]] + Tan[c + d*x]])/(2*Sqrt[2]*d) + (a^2*Log[1 + Sqrt[2]*Sqrt[T
an[c + d*x]] + Tan[c + d*x]])/(2*Sqrt[2]*d) - (2*a^2*EllipticF[c - Pi/4 +
d*x, 2]*Sec[c + d*x]*Sqrt[Sin[2*c + 2*d*x]])/(d*Sqrt[Tan[c + d*x]]) + (2*a
^2*Sqrt[Tan[c + d*x]])/d)/(a^4*(e*Cot[c + d*x])^(7/2)*Tan[c + d*x]^(7/2))
```

3.253.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4374
```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

rule 4376
```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Simp[a^(2*n)/e^(2*n)   Int[(e*Cot[c + d*x])^(m + 2*
n)/(-a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[a
^2 - b^2, 0] && ILtQ[n, 0]
```

rule 4388
```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*((a_) + (b_.)*sec[(c_.) + (d_.)*(x
_)])^(n_.), x_Symbol] :> Simp[(e*Cot[c + d*x])^m*Tan[c + d*x]^m   Int[(a +
b*Sec[c + d*x])^n/Tan[c + d*x]^m, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x]
 &&  !IntegerQ[m]
```

3.253.4 Maple [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 10.28 (sec) , antiderivative size = 961, normalized size of antiderivative = 2.99

method	result	size
default	Expression too large to display	961

input
```
int(1/(e*cot(d*x+c))^(7/2)/(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
-1/2/a^2/d*2^(1/2)*((1-cos(d*x+c))^2*csc(d*x+c)^2-1)^4*(I*(csc(d*x+c)-cot(
d*x+c)+1)^(1/2)*(2-2*csc(d*x+c)+2*cot(d*x+c))^(1/2)*(cot(d*x+c)-csc(d*x+c)
)^(1/2)*EllipticPi((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2-1/2*I,1/2*2^(1/2))*
((1-cos(d*x+c))*(-cot(d*x+c)+csc(d*x+c)-1)*(csc(d*x+c)-cot(d*x+c)+1)*csc(d
*x+c))^(1/2)-I*(csc(d*x+c)-cot(d*x+c)+1)^(1/2)*(2-2*csc(d*x+c)+2*cot(d*x+c
))^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)*EllipticPi((csc(d*x+c)-cot(d*x+c)+1
)^(1/2),1/2+1/2*I,1/2*2^(1/2))*((1-cos(d*x+c))*(-cot(d*x+c)+csc(d*x+c)-1)*
(csc(d*x+c)-cot(d*x+c)+1)*csc(d*x+c))^(1/2)-6*(csc(d*x+c)-cot(d*x+c)+1)^(1
/2)*(2-2*csc(d*x+c)+2*cot(d*x+c))^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)*Elli
pticF((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2*2^(1/2))*((1-cos(d*x+c))*(-cot(d
*x+c)+csc(d*x+c)-1)*(csc(d*x+c)-cot(d*x+c)+1)*csc(d*x+c))^(1/2)+(csc(d*x+c
)-cot(d*x+c)+1)^(1/2)*(2-2*csc(d*x+c)+2*cot(d*x+c))^(1/2)*(cot(d*x+c)-csc(
d*x+c))^(1/2)*EllipticPi((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2-1/2*I,1/2*2^(
1/2))*((1-cos(d*x+c))*(-cot(d*x+c)+csc(d*x+c)-1)*(csc(d*x+c)-cot(d*x+c)+1)
*csc(d*x+c))^(1/2)+(csc(d*x+c)-cot(d*x+c)+1)^(1/2)*(2-2*csc(d*x+c)+2*cot(d
*x+c))^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)*EllipticPi((csc(d*x+c)-cot(d*x+
c)+1)^(1/2),1/2+1/2*I,1/2*2^(1/2))*((1-cos(d*x+c))*(-cot(d*x+c)+csc(d*x+c)
-1)*(csc(d*x+c)-cot(d*x+c)+1)*csc(d*x+c))^(1/2)+4*((1-cos(d*x+c))^3*csc(d*
x+c)^3+cot(d*x+c)-csc(d*x+c))^(1/2)*(-cot(d*x+c)+csc(d*x+c)))/(-e/(1-cos(d
*x+c))*((1-cos(d*x+c))^2*csc(d*x+c)-sin(d*x+c)))^(7/2)/(1-cos(d*x+c))^3...
```

3.253.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(e\cot(c+dx))^{7/2}(a+a\sec(c+dx))^2}\,dx = \text{Timed out}$$

input
```
integrate(1/(e*cot(d*x+c))^(7/2)/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

output
```
Timed out
```

3.253.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(e\cot(c+dx))^{7/2}(a+a\sec(c+dx))^2}\,dx = \text{Timed out}$$

input
```
integrate(1/(e*cot(d*x+c))**(7/2)/(a+a*sec(d*x+c))**2,x)
```

output
```
Timed out
```

3.253.7 Maxima [F(-1)]

Timed out.

$$\int \frac{1}{(e\cot(c+dx))^{7/2}(a+a\sec(c+dx))^2}\,dx = \text{Timed out}$$

input
```
integrate(1/(e*cot(d*x+c))^(7/2)/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

output
```
Timed out
```

3.253.8 Giac [F]

$$\int \frac{1}{(e\cot(c+dx))^{7/2}(a+a\sec(c+dx))^2}\,dx = \int \frac{1}{(e\cot(dx+c))^{\frac{7}{2}}(a\sec(dx+c)+a)^2}\,dx$$

input
```
integrate(1/(e*cot(d*x+c))^(7/2)/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

output
```
integrate(1/((e*cot(d*x + c))^(7/2)*(a*sec(d*x + c) + a)^2), x)
```

3.253.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(e\cot(c+dx))^{7/2}(a+a\sec(c+dx))^2}\,dx = \int \frac{\cos(c+dx)^2}{a^2\,(e\cot(c+dx))^{7/2}\,(\cos(c+dx)+1)^2}\,dx$$

input
```
int(1/((e*cot(c + d*x))^(7/2)*(a + a/cos(c + d*x))^2),x)
```

output
```
int(cos(c + d*x)^2/(a^2*(e*cot(c + d*x))^(7/2)*(cos(c + d*x) + 1)^2), x)
```

3.254 $\int \frac{1}{(e\cot(c+dx))^{9/2}(a+a\sec(c+dx))^2}\,dx$

3.254.1 Optimal result . . . 1865
3.254.2 Mathematica [F] . . . 1866
3.254.3 Rubi [A] (verified) . . . 1866
3.254.4 Maple [C] (warning: unable to verify) . . . 1868
3.254.5 Fricas [F(-1)] . . . 1869
3.254.6 Sympy [F(-1)] . . . 1870
3.254.7 Maxima [F] . . . 1870
3.254.8 Giac [F] . . . 1870
3.254.9 Mupad [F(-1)] . . . 1871

3.254.1 Optimal result

Integrand size = 25, antiderivative size = 357

$$\begin{aligned}\int \frac{1}{(e\cot(c+dx))^{9/2}(a+a\sec(c+dx))^2}\,dx &= \frac{2\cot^3(c+dx)}{3a^2d(e\cot(c+dx))^{9/2}} \\ &- \frac{4\cos(c+dx)\cot^3(c+dx)}{a^2d(e\cot(c+dx))^{9/2}} + \frac{4\cos(c+dx)\cot^4(c+dx)E\left(c-\frac{\pi}{4}+dx\middle|2\right)}{a^2d(e\cot(c+dx))^{9/2}\sqrt{\sin(2c+2dx)}} \\ &- \frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}a^2d(e\cot(c+dx))^{9/2}\tan^{\frac{9}{2}}(c+dx)} + \frac{\arctan\left(1+\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}a^2d(e\cot(c+dx))^{9/2}\tan^{\frac{9}{2}}(c+dx)} \\ &+ \frac{\log\left(1-\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+dx)\right)}{2\sqrt{2}a^2d(e\cot(c+dx))^{9/2}\tan^{\frac{9}{2}}(c+dx)} \\ &- \frac{\log\left(1+\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+dx)\right)}{2\sqrt{2}a^2d(e\cot(c+dx))^{9/2}\tan^{\frac{9}{2}}(c+dx)}\end{aligned}$$

output

```
2/3*cot(d*x+c)^3/a^2/d/(e*cot(d*x+c))^(9/2)-4*cos(d*x+c)*cot(d*x+c)^3/a^2/
d/(e*cot(d*x+c))^(9/2)-4*cos(d*x+c)*cot(d*x+c)^4*(sin(c+1/4*Pi+d*x)^2)^(1/
2)/sin(c+1/4*Pi+d*x)*EllipticE(cos(c+1/4*Pi+d*x),2^(1/2))/a^2/d/(e*cot(d*x
+c))^(9/2)/sin(2*d*x+2*c)^(1/2)+1/2*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2))/a^
2/d/(e*cot(d*x+c))^(9/2)*2^(1/2)/tan(d*x+c)^(9/2)+1/2*arctan(1+2^(1/2)*tan
(d*x+c)^(1/2))/a^2/d/(e*cot(d*x+c))^(9/2)*2^(1/2)/tan(d*x+c)^(9/2)+1/4*ln(
1-2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c))/a^2/d/(e*cot(d*x+c))^(9/2)*2^(1/2)/
tan(d*x+c)^(9/2)-1/4*ln(1+2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c))/a^2/d/(e*co
t(d*x+c))^(9/2)*2^(1/2)/tan(d*x+c)^(9/2)
```

3.254.2 Mathematica [F]

$$\int \frac{1}{(e\cot(c+dx))^{9/2}(a+a\sec(c+dx))^2}\,dx = \int \frac{1}{(e\cot(c+dx))^{9/2}(a+a\sec(c+dx))^2}\,dx$$

input
```
Integrate[1/((e*Cot[c + d*x])^(9/2)*(a + a*Sec[c + d*x])^2),x]
```

output
```
Integrate[1/((e*Cot[c + d*x])^(9/2)*(a + a*Sec[c + d*x])^2), x]
```

3.254.3 Rubi [A] (verified)

Time = 0.73 (sec) , antiderivative size = 265, normalized size of antiderivative = 0.74, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {3042, 4388, 3042, 4376, 3042, 4374, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a\sec(c+dx)+a)^2(e\cot(c+dx))^{9/2}}\,dx$$
$$\downarrow 3042$$
$$\int \frac{1}{(a\sec(c+dx)+a)^2(e\cot(c+dx))^{9/2}}dx$$
$$\downarrow 4388$$
$$\frac{\int \frac{\tan^{\frac{9}{2}}(c+dx)}{(\sec(c+dx)a+a)^2}dx}{\tan^{\frac{9}{2}}(c+dx)(e\cot(c+dx))^{9/2}}$$
$$\downarrow 3042$$
$$\frac{\int \frac{(-\cot(c+dx+\frac{\pi}{2}))^{9/2}}{(\csc(c+dx+\frac{\pi}{2})a+a)^2}dx}{\tan^{\frac{9}{2}}(c+dx)(e\cot(c+dx))^{9/2}}$$
$$\downarrow 4376$$
$$\frac{\int (a-a\sec(c+dx))^2\sqrt{\tan(c+dx)}dx}{a^4\tan^{\frac{9}{2}}(c+dx)(e\cot(c+dx))^{9/2}}$$
$$\downarrow 3042$$

$$\frac{\int \sqrt{-\cot\left(c+dx+\frac{\pi}{2}\right)}\left(a-a\csc\left(c+dx+\frac{\pi}{2}\right)\right)^2 dx}{a^4\tan^{\frac{9}{2}}(c+dx)(e\cot(c+dx))^{9/2}}$$

$$\downarrow 4374$$

$$\frac{\int \left(\sec^2(c+dx)\sqrt{\tan(c+dx)}a^2-2\sec(c+dx)\sqrt{\tan(c+dx)}a^2+\sqrt{\tan(c+dx)}a^2\right) dx}{a^4\tan^{\frac{9}{2}}(c+dx)(e\cot(c+dx))^{9/2}}$$

$$\downarrow 2009$$

$$\frac{-\frac{a^2\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}d}+\frac{a^2\arctan\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}d}+\frac{2a^2\tan^{\frac{3}{2}}(c+dx)}{3d}+\frac{a^2\log\left(\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}d}-\frac{a^2\log\left(}{}}{a^4\tan^{\frac{9}{2}}(c+dx)(e\cot(c+dx)}$$

input
```
Int[1/((e*Cot[c + d*x])^(9/2)*(a + a*Sec[c + d*x])^2),x]
```

output
```
(-((a^2*ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]])/(Sqrt[2]*d)) + (a^2*ArcTan
[1 + Sqrt[2]*Sqrt[Tan[c + d*x]]])/(Sqrt[2]*d) + (a^2*Log[1 - Sqrt[2]*Sqrt[
Tan[c + d*x]] + Tan[c + d*x]])/(2*Sqrt[2]*d) - (a^2*Log[1 + Sqrt[2]*Sqrt[T
an[c + d*x]] + Tan[c + d*x]])/(2*Sqrt[2]*d) + (4*a^2*Cos[c + d*x]*Elliptic
E[c - Pi/4 + d*x, 2]*Sqrt[Tan[c + d*x]])/(d*Sqrt[Sin[2*c + 2*d*x]]) + (2*a
^2*Tan[c + d*x]^(3/2))/(3*d) - (4*a^2*Cos[c + d*x]*Tan[c + d*x]^(3/2))/d)/
(a^4*(e*Cot[c + d*x])^(9/2)*Tan[c + d*x]^(9/2))
```

3.254.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4374
```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

rule 4376
```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Simp[a^(2*n)/e^(2*n)   Int[(e*Cot[c + d*x])^(m + 2*
n)/(-a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[a
^2 - b^2, 0] && ILtQ[n, 0]
```

rule 4388
```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*((a_) + (b_.)*sec[(c_.) + (d_.)*(x
_)])^(n_.), x_Symbol] :> Simp[(e*Cot[c + d*x])^m*Tan[c + d*x]^m   Int[(a +
b*Sec[c + d*x])^n/Tan[c + d*x]^m, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x]
 &&  !IntegerQ[m]
```

3.254.4 Maple [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 8.91 (sec) , antiderivative size = 1141, normalized size of antiderivative = 3.20

method	result	size
default	Expression too large to display	1141

input
```
int(1/(e*cot(d*x+c))^(9/2)/(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output

```
-1/6/a^2/d*2^(1/2)*(3*I*(cot(d*x+c)-csc(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*
x+c))^(1/2)*(csc(d*x+c)-cot(d*x+c)+1)^(1/2)*EllipticPi((csc(d*x+c)-cot(d*x
+c)+1)^(1/2),1/2+1/2*I,1/2*2^(1/2))*cos(d*x+c)^2-3*I*(cot(d*x+c)-csc(d*x+c
)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)*(csc(d*x+c)-cot(d*x+c)+1)^(1/2)*E
llipticPi((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2-1/2*I,1/2*2^(1/2))*cos(d*x+c
)^2+3*I*(cot(d*x+c)-csc(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)*(csc
(d*x+c)-cot(d*x+c)+1)^(1/2)*EllipticPi((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2
+1/2*I,1/2*2^(1/2))*cos(d*x+c)-3*I*(cot(d*x+c)-csc(d*x+c)+1)^(1/2)*(cot(d*
x+c)-csc(d*x+c))^(1/2)*(csc(d*x+c)-cot(d*x+c)+1)^(1/2)*EllipticPi((csc(d*x
+c)-cot(d*x+c)+1)^(1/2),1/2-1/2*I,1/2*2^(1/2))*cos(d*x+c)+24*(csc(d*x+c)-c
ot(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c))
^(1/2)*EllipticE((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2*2^(1/2))*cos(d*x+c)^2
-12*(csc(d*x+c)-cot(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c)+1)^(1/2)*(cot(d
*x+c)-csc(d*x+c))^(1/2)*EllipticF((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2*2^(1
/2))*cos(d*x+c)^2+3*(cot(d*x+c)-csc(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c)
)^(1/2)*(csc(d*x+c)-cot(d*x+c)+1)^(1/2)*EllipticPi((csc(d*x+c)-cot(d*x+c)+
1)^(1/2),1/2+1/2*I,1/2*2^(1/2))*cos(d*x+c)^2+3*(cot(d*x+c)-csc(d*x+c)+1)^(
1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)*(csc(d*x+c)-cot(d*x+c)+1)^(1/2)*Ellipti
cPi((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2-1/2*I,1/2*2^(1/2))*cos(d*x+c)^2+24
*(csc(d*x+c)-cot(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c)+1)^(1/2)*(cot(d...
```

3.254.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(e\cot(c+dx))^{9/2}(a+a\sec(c+dx))^2}\,dx = \text{Timed out}$$

input

```
integrate(1/(e*cot(d*x+c))^(9/2)/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

output

```
Timed out
```

3.254.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(e\cot(c+dx))^{9/2}(a+a\sec(c+dx))^2}\,dx = \text{Timed out}$$

input
```
integrate(1/(e*cot(d*x+c))**(9/2)/(a+a*sec(d*x+c))**2,x)
```

output
```
Timed out
```

3.254.7 Maxima [F]

$$\int \frac{1}{(e\cot(c+dx))^{9/2}(a+a\sec(c+dx))^2}\,dx = \int \frac{1}{(e\cot(dx+c))^{\frac{9}{2}}(a\sec(dx+c)+a)^2}\,dx$$

input
```
integrate(1/(e*cot(d*x+c))^(9/2)/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

output
```
integrate(1/((e*cot(d*x + c))^(9/2)*(a*sec(d*x + c) + a)^2), x)
```

3.254.8 Giac [F]

$$\int \frac{1}{(e\cot(c+dx))^{9/2}(a+a\sec(c+dx))^2}\,dx = \int \frac{1}{(e\cot(dx+c))^{\frac{9}{2}}(a\sec(dx+c)+a)^2}\,dx$$

input
```
integrate(1/(e*cot(d*x+c))^(9/2)/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

output
```
integrate(1/((e*cot(d*x + c))^(9/2)*(a*sec(d*x + c) + a)^2), x)
```

3.254.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(e\cot(c+dx))^{9/2}(a+a\sec(c+dx))^2}\,dx = \int \frac{\cos(c+dx)^2}{a^2\,(e\cot(c+dx))^{9/2}\,(\cos(c+dx)+1)^2}\,dx$$

input
```
int(1/((e*cot(c + d*x))^(9/2)*(a + a/cos(c + d*x))^2),x)
```

output
```
int(cos(c + d*x)^2/(a^2*(e*cot(c + d*x))^(9/2)*(cos(c + d*x) + 1)^2), x)
```

3.255 $\int \frac{1}{(e\cot(c+dx))^{11/2}(a+a\sec(c+dx))^2}\,dx$

3.255.1 Optimal result 1872
3.255.2 Mathematica [F] 1873
3.255.3 Rubi [A] (verified) 1873
3.255.4 Maple [C] (warning: unable to verify) 1875
3.255.5 Fricas [F(-1)] 1876
3.255.6 Sympy [F(-1)] 1877
3.255.7 Maxima [F(-1)] 1877
3.255.8 Giac [F] 1877
3.255.9 Mupad [F(-1)] 1878

3.255.1 Optimal result

Integrand size = 25, antiderivative size = 389

$$\int \frac{1}{(e\cot(c+dx))^{11/2}(a+a\sec(c+dx))^2}\,dx = \frac{2\cot^3(c+dx)}{5a^2d(e\cot(c+dx))^{11/2}}$$
$$+\frac{2\cot^5(c+dx)}{a^2d(e\cot(c+dx))^{11/2}}-\frac{4\cot^4(c+dx)\csc(c+dx)}{3a^2d(e\cot(c+dx))^{11/2}}$$
$$+\frac{2\cot^5(c+dx)\csc(c+dx)\,\text{EllipticF}\left(c-\frac{\pi}{4}+dx,2\right)\sqrt{\sin(2c+2dx)}}{3a^2d(e\cot(c+dx))^{11/2}}$$
$$+\frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}a^2d(e\cot(c+dx))^{11/2}\tan^{\frac{11}{2}}(c+dx)}-\frac{\arctan\left(1+\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}a^2d(e\cot(c+dx))^{11/2}\tan^{\frac{11}{2}}(c+dx)}$$
$$+\frac{\log\left(1-\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+dx)\right)}{2\sqrt{2}a^2d(e\cot(c+dx))^{11/2}\tan^{\frac{11}{2}}(c+dx)}$$
$$-\frac{\log\left(1+\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+dx)\right)}{2\sqrt{2}a^2d(e\cot(c+dx))^{11/2}\tan^{\frac{11}{2}}(c+dx)}$$

output
```
2/5*cot(d*x+c)^3/a^2/d/(e*cot(d*x+c))^(11/2)+2*cot(d*x+c)^5/a^2/d/(e*cot(d
*x+c))^(11/2)-4/3*cot(d*x+c)^4*csc(d*x+c)/a^2/d/(e*cot(d*x+c))^(11/2)-2/3*
cot(d*x+c)^5*csc(d*x+c)*(sin(c+1/4*Pi+d*x)^2)^(1/2)/sin(c+1/4*Pi+d*x)*Elli
pticF(cos(c+1/4*Pi+d*x),2^(1/2))*sin(2*d*x+2*c)^(1/2)/a^2/d/(e*cot(d*x+c))
^(11/2)-1/2*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2))/a^2/d/(e*cot(d*x+c))^(11/2
)*2^(1/2)/tan(d*x+c)^(11/2)-1/2*arctan(1+2^(1/2)*tan(d*x+c)^(1/2))/a^2/d/(
e*cot(d*x+c))^(11/2)*2^(1/2)/tan(d*x+c)^(11/2)+1/4*ln(1-2^(1/2)*tan(d*x+c)
^(1/2)+tan(d*x+c))/a^2/d/(e*cot(d*x+c))^(11/2)*2^(1/2)/tan(d*x+c)^(11/2)-1
/4*ln(1+2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c))/a^2/d/(e*cot(d*x+c))^(11/2)*2
^(1/2)/tan(d*x+c)^(11/2)
```

3.255.2 Mathematica [F]

$$\int \frac{1}{(e\cot(c+dx))^{11/2}(a+a\sec(c+dx))^2}\,dx = \int \frac{1}{(e\cot(c+dx))^{11/2}(a+a\sec(c+dx))^2}\,dx$$

input
```
Integrate[1/((e*Cot[c + d*x])^(11/2)*(a + a*Sec[c + d*x])^2),x]
```

output
```
Integrate[1/((e*Cot[c + d*x])^(11/2)*(a + a*Sec[c + d*x])^2), x]
```

3.255.3 Rubi [A] (verified)

Time = 0.75 (sec) , antiderivative size = 287, normalized size of antiderivative = 0.74, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {3042, 4388, 3042, 4376, 3042, 4374, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a\sec(c+dx)+a)^2(e\cot(c+dx))^{11/2}}\,dx$$
$$\downarrow \text{3042}$$
$$\int \frac{1}{(a\sec(c+dx)+a)^2(e\cot(c+dx))^{11/2}}dx$$
$$\downarrow \text{4388}$$

$$\frac{\int \frac{\tan^{\frac{11}{2}}(c+dx)}{(\sec(c+dx)a+a)^2}dx}{\tan^{\frac{11}{2}}(c+dx)(e\cot(c+dx))^{11/2}}$$

$$\downarrow 3042$$

$$\frac{\int \frac{(-\cot(c+dx+\frac{\pi}{2}))^{11/2}}{(\csc(c+dx+\frac{\pi}{2})a+a)^2}dx}{\tan^{\frac{11}{2}}(c+dx)(e\cot(c+dx))^{11/2}}$$

$$\downarrow 4376$$

$$\frac{\int (a-a\sec(c+dx))^2\tan^{\frac{3}{2}}(c+dx)dx}{a^4\tan^{\frac{11}{2}}(c+dx)(e\cot(c+dx))^{11/2}}$$

$$\downarrow 3042$$

$$\frac{\int \left(-\cot\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}\left(a-a\csc\left(c+dx+\frac{\pi}{2}\right)\right)^2 dx}{a^4\tan^{\frac{11}{2}}(c+dx)(e\cot(c+dx))^{11/2}}$$

$$\downarrow 4374$$

$$\frac{\int \left(\sec^2(c+dx)\tan^{\frac{3}{2}}(c+dx)a^2-2\sec(c+dx)\tan^{\frac{3}{2}}(c+dx)a^2+\tan^{\frac{3}{2}}(c+dx)a^2\right)dx}{a^4\tan^{\frac{11}{2}}(c+dx)(e\cot(c+dx))^{11/2}}$$

$$\downarrow 2009$$

$$\frac{\frac{a^2\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}d}-\frac{a^2\arctan\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}d}+\frac{2a^2\tan^{\frac{5}{2}}(c+dx)}{5d}+\frac{2a^2\sqrt{\tan(c+dx)}}{d}+\frac{a^2\log\left(\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+}\right.}{2\sqrt{2}d}}{a^4\tan^{\frac{11}{2}}(c+dx)(e\,c}$$

input
```
Int[1/((e*Cot[c + d*x])^(11/2)*(a + a*Sec[c + d*x])^2),x]
```

output
```
((a^2*ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]])/(Sqrt[2]*d) - (a^2*ArcTan[1
+ Sqrt[2]*Sqrt[Tan[c + d*x]]])/(Sqrt[2]*d) + (a^2*Log[1 - Sqrt[2]*Sqrt[Tan
[c + d*x]] + Tan[c + d*x]])/(2*Sqrt[2]*d) - (a^2*Log[1 + Sqrt[2]*Sqrt[Tan[
c + d*x]] + Tan[c + d*x]])/(2*Sqrt[2]*d) + (2*a^2*EllipticF[c - Pi/4 + d*x
, 2]*Sec[c + d*x]*Sqrt[Sin[2*c + 2*d*x]])/(3*d*Sqrt[Tan[c + d*x]]) + (2*a^
2*Sqrt[Tan[c + d*x]])/d - (4*a^2*Sec[c + d*x]*Sqrt[Tan[c + d*x]])/(3*d) +
(2*a^2*Tan[c + d*x]^(5/2))/(5*d))/(a^4*(e*Cot[c + d*x])^(11/2)*Tan[c + d*x
]^(11/2))
```

3.255.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4374
```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

rule 4376
```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Simp[a^(2*n)/e^(2*n)   Int[(e*Cot[c + d*x])^(m + 2*
n)/(-a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[a
^2 - b^2, 0] && ILtQ[n, 0]
```

rule 4388
```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*((a_) + (b_.)*sec[(c_.) + (d_.)*(x
_)])^(n_.), x_Symbol] :> Simp[(e*Cot[c + d*x])^m*Tan[c + d*x]^m   Int[(a +
b*Sec[c + d*x])^n/Tan[c + d*x]^m, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x]
 &&  !IntegerQ[m]
```

3.255.4 Maple [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 9.69 (sec) , antiderivative size = 1007, normalized size of antiderivative = 2.59

method	result	size
default	Expression too large to display	1007

input
```
int(1/(e*cot(d*x+c))^(11/2)/(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/30/a^2/d*2^(1/2)*(15*I*(cot(d*x+c)-csc(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d
*x+c))^(1/2)*(csc(d*x+c)-cot(d*x+c)+1)^(1/2)*EllipticPi((csc(d*x+c)-cot(d*
x+c)+1)^(1/2),1/2-1/2*I,1/2*2^(1/2))*cos(d*x+c)^3-15*I*(cot(d*x+c)-csc(d*x
+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)*(csc(d*x+c)-cot(d*x+c)+1)^(1/2)
*EllipticPi((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2+1/2*I,1/2*2^(1/2))*cos(d*x
+c)^3+15*I*(cot(d*x+c)-csc(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)*(
csc(d*x+c)-cot(d*x+c)+1)^(1/2)*EllipticPi((csc(d*x+c)-cot(d*x+c)+1)^(1/2),
1/2-1/2*I,1/2*2^(1/2))*cos(d*x+c)^2-15*I*(cot(d*x+c)-csc(d*x+c)+1)^(1/2)*(
cot(d*x+c)-csc(d*x+c))^(1/2)*(csc(d*x+c)-cot(d*x+c)+1)^(1/2)*EllipticPi((c
sc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2+1/2*I,1/2*2^(1/2))*cos(d*x+c)^2+15*(cot(
d*x+c)-csc(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)*(csc(d*x+c)-cot(d
*x+c)+1)^(1/2)*EllipticPi((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2-1/2*I,1/2*2^
(1/2))*cos(d*x+c)^3+15*(cot(d*x+c)-csc(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x
+c))^(1/2)*EllipticPi((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2+1/2*I,1/2*2^(1/2
))*(csc(d*x+c)-cot(d*x+c)+1)^(1/2)*cos(d*x+c)^3-50*(csc(d*x+c)-cot(d*x+c)+
1)^(1/2)*(cot(d*x+c)-csc(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)*Ell
ipticF((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2*2^(1/2))*cos(d*x+c)^3+15*(cot(d
*x+c)-csc(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)*(csc(d*x+c)-cot(d*
x+c)+1)^(1/2)*EllipticPi((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2-1/2*I,1/2*2^(
1/2))*cos(d*x+c)^2+15*(cot(d*x+c)-csc(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d...
```

3.255.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(e\cot(c+dx))^{11/2}(a+a\sec(c+dx))^2}\,dx = \text{Timed out}$$

input
```
integrate(1/(e*cot(d*x+c))^(11/2)/(a+a*sec(d*x+c))^2,x, algorithm="fricas"
)
```

output
```
Timed out
```

3.255.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(e\cot(c+dx))^{11/2}(a+a\sec(c+dx))^2}\,dx = \text{Timed out}$$

input
```
integrate(1/(e*cot(d*x+c))**(11/2)/(a+a*sec(d*x+c))**2,x)
```

output
```
Timed out
```

3.255.7 Maxima [F(-1)]

Timed out.

$$\int \frac{1}{(e\cot(c+dx))^{11/2}(a+a\sec(c+dx))^2}\,dx = \text{Timed out}$$

input
```
integrate(1/(e*cot(d*x+c))^(11/2)/(a+a*sec(d*x+c))^2,x, algorithm="maxima"
)
```

output
```
Timed out
```

3.255.8 Giac [F]

$$\int \frac{1}{(e\cot(c+dx))^{11/2}(a+a\sec(c+dx))^2}\,dx = \int \frac{1}{(e\cot(dx+c))^{\frac{11}{2}}(a\sec(dx+c)+a)^2}\,dx$$

input
```
integrate(1/(e*cot(d*x+c))^(11/2)/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

output
```
integrate(1/((e*cot(d*x + c))^(11/2)*(a*sec(d*x + c) + a)^2), x)
```

3.255.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(e\cot(c+dx))^{11/2}(a+a\sec(c+dx))^2}\,dx = \int \frac{\cos(c+dx)^2}{a^2\,(e\cot(c+dx))^{11/2}\,(\cos(c+dx)+1)^2}\,dx$$

input
```
int(1/((e*cot(c + d*x))^(11/2)*(a + a/cos(c + d*x))^2),x)
```

output
```
int(cos(c + d*x)^2/(a^2*(e*cot(c + d*x))^(11/2)*(cos(c + d*x) + 1)^2), x)
```

3.256 $\int (a + b\sec(c + dx))\tan^7(c + dx)\,dx$

3.256.1 Optimal result 1879
3.256.2 Mathematica [A] (verified) 1879
3.256.3 Rubi [A] (verified) 1880
3.256.4 Maple [A] (verified) 1884
3.256.5 Fricas [A] (verification not implemented) 1884
3.256.6 Sympy [A] (verification not implemented) 1885
3.256.7 Maxima [A] (verification not implemented) 1885
3.256.8 Giac [B] (verification not implemented) 1886
3.256.9 Mupad [B] (verification not implemented) 1886

3.256.1 Optimal result

Integrand size = 19, antiderivative size = 111

$$\int (a + b\sec(c + dx))\tan^7(c + dx)\,dx = \frac{a\log(\cos(c + dx))}{d} - \frac{16b\sec(c + dx)}{35d} + \frac{(35a + 16b\sec(c + dx))\tan^2(c + dx)}{70d} - \frac{(35a + 24b\sec(c + dx))\tan^4(c + dx)}{140d} + \frac{(7a + 6b\sec(c + dx))\tan^6(c + dx)}{42d}$$

output
```
a*ln(cos(d*x+c))/d-16/35*b*sec(d*x+c)/d+1/70*(35*a+16*b*sec(d*x+c))*tan(d*
x+c)^2/d-1/140*(35*a+24*b*sec(d*x+c))*tan(d*x+c)^4/d+1/42*(7*a+6*b*sec(d*x
+c))*tan(d*x+c)^6/d
```

3.256.2 Mathematica [A] (verified)

Time = 0.45 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.95

$$\int (a + b\sec(c + dx))\tan^7(c + dx)\,dx = -\frac{b\sec(c + dx)}{d} + \frac{b\sec^3(c + dx)}{d} - \frac{3b\sec^5(c + dx)}{5d} + \frac{b\sec^7(c + dx)}{7d} + \frac{a(12\log(\cos(c + dx)) + 6\tan^2(c + dx) - 3\tan^4(c + dx) + 2\tan^6(c + dx))}{12d}$$

input
```
Integrate[(a + b*Sec[c + d*x])*Tan[c + d*x]^7,x]
```

output
```
-((b*Sec[c + d*x])/d) + (b*Sec[c + d*x]^3)/d - (3*b*Sec[c + d*x]^5)/(5*d)
+ (b*Sec[c + d*x]^7)/(7*d) + (a*(12*Log[Cos[c + d*x]] + 6*Tan[c + d*x]^2 -
 3*Tan[c + d*x]^4 + 2*Tan[c + d*x]^6))/(12*d)
```

3.256.3 Rubi [A] (verified)

Time = 0.77 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.08, number of steps used = 21, number of rules used = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 1.053$, Rules used = {3042, 25, 4369, 25, 3042, 25, 4369, 25, 3042, 25, 4369, 27, 3042, 25, 4372, 25, 3042, 3086, 24, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^7(c+dx)(a+b\sec(c+dx))\,dx$$

$$\downarrow 3042$$

$$\int -\cot\left(c+dx+\frac{\pi}{2}\right)^7\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)dx$$

$$\downarrow 25$$

$$-\int \cot\left(\frac{1}{2}(2c+\pi)+dx\right)^7\left(a+b\csc\left(\frac{1}{2}(2c+\pi)+dx\right)\right)dx$$

$$\downarrow 4369$$

$$\frac{1}{7}\int -((7a+6b\sec(c+dx))\tan^5(c+dx))\,dx+\frac{\tan^6(c+dx)(7a+6b\sec(c+dx))}{42d}$$

$$\downarrow 25$$

$$\frac{\tan^6(c+dx)(7a+6b\sec(c+dx))}{42d}-\frac{1}{7}\int(7a+6b\sec(c+dx))\tan^5(c+dx)dx$$

$$\downarrow 3042$$

$$\frac{\tan^6(c+dx)(7a+6b\sec(c+dx))}{42d}-\frac{1}{7}\int -\cot\left(c+dx+\frac{\pi}{2}\right)^5\left(7a+6b\csc\left(c+dx+\frac{\pi}{2}\right)\right)dx$$

$$\downarrow 25$$

$$\frac{1}{7}\int \cot\left(\frac{1}{2}(2c+\pi)+dx\right)^5\left(7a+6b\csc\left(\frac{1}{2}(2c+\pi)+dx\right)\right)dx+\frac{\tan^6(c+dx)(7a+6b\sec(c+dx))}{42d}$$

$$\downarrow 4369$$

$$\frac{1}{7}\left(-\frac{1}{5}\int -((35a+24b\sec(c+dx))\tan^3(c+dx))\,dx-\frac{\tan^4(c+dx)(35a+24b\sec(c+dx))}{20d}\right)+\frac{\tan^6(c+dx)(7a+6b\sec(c+dx))}{42d}$$

$$\downarrow 25$$

$$\frac{1}{7}\left(\frac{1}{5}\int (35a+24b\sec(c+dx))\tan^3(c+dx)dx-\frac{\tan^4(c+dx)(35a+24b\sec(c+dx))}{20d}\right)+\frac{\tan^6(c+dx)(7a+6b\sec(c+dx))}{42d}$$

$$\downarrow 3042$$

$$\frac{1}{7}\left(\frac{1}{5}\int -\cot\left(c+dx+\frac{\pi}{2}\right)^3\left(35a+24b\csc\left(c+dx+\frac{\pi}{2}\right)\right)dx-\frac{\tan^4(c+dx)(35a+24b\sec(c+dx))}{20d}\right)+\frac{\tan^6(c+dx)(7a+6b\sec(c+dx))}{42d}$$

$$\downarrow 25$$

$$\frac{1}{7}\left(-\frac{1}{5}\int \cot\left(\frac{1}{2}(2c+\pi)+dx\right)^3\left(35a+24b\csc\left(\frac{1}{2}(2c+\pi)+dx\right)\right)dx-\frac{\tan^4(c+dx)(35a+24b\sec(c+dx))}{20d}\right)+\frac{\tan^6(c+dx)(7a+6b\sec(c+dx))}{42d}$$

$$\downarrow 4369$$

$$\frac{1}{7}\left(\frac{1}{5}\left(\frac{1}{3}\int -3(35a+16b\sec(c+dx))\tan(c+dx)dx+\frac{\tan^2(c+dx)(35a+16b\sec(c+dx))}{2d}\right)-\frac{\tan^4(c+dx)(35a+\cdots}{\cdots}\right)+\frac{\tan^6(c+dx)(7a+6b\sec(c+dx))}{42d}$$

$$\downarrow 27$$

$$\frac{1}{7}\left(\frac{1}{5}\left(\frac{\tan^2(c+dx)(35a+16b\sec(c+dx))}{2d}-\int (35a+16b\sec(c+dx))\tan(c+dx)dx\right)-\frac{\tan^4(c+dx)(35a+24\cdots}{20d}\right)+\frac{\tan^6(c+dx)(7a+6b\sec(c+dx))}{42d}$$

$$\downarrow 3042$$

$$\frac{1}{7}\left(\frac{1}{5}\left(\frac{\tan^2(c+dx)(35a+16b\sec(c+dx))}{2d}-\int -\cot\left(c+dx+\frac{\pi}{2}\right)\left(35a+16b\csc\left(c+dx+\frac{\pi}{2}\right)\right)dx\right)-\frac{\tan^4(c}{}$$

$$\frac{\tan^6(c+dx)(7a+6b\sec(c+dx))}{42d}$$

$$\downarrow 25$$

$$\frac{1}{7}\left(\frac{1}{5}\left(\int \cot\left(\frac{1}{2}(2c+\pi)+dx\right)\left(35a+16b\csc\left(\frac{1}{2}(2c+\pi)+dx\right)\right)dx+\frac{\tan^2(c+dx)(35a+16b\sec(c+dx))}{2d}\right)-\right.$$

$$\frac{\tan^6(c+dx)(7a+6b\sec(c+dx))}{42d}$$

$$\downarrow 4372$$

$$\frac{1}{7}\left(\frac{1}{5}\left(35a\int -\tan(c+dx)dx+16b\int -\sec(c+dx)\tan(c+dx)dx+\frac{\tan^2(c+dx)(35a+16b\sec(c+dx))}{2d}\right)-\frac{\text{ta}}{}\right.$$

$$\frac{\tan^6(c+dx)(7a+6b\sec(c+dx))}{42d}$$

$$\downarrow 25$$

$$\frac{1}{7}\left(\frac{1}{5}\left(-35a\int \tan(c+dx)dx-16b\int \sec(c+dx)\tan(c+dx)dx+\frac{\tan^2(c+dx)(35a+16b\sec(c+dx))}{2d}\right)-\frac{\tan^4}{}\right.$$

$$\frac{\tan^6(c+dx)(7a+6b\sec(c+dx))}{42d}$$

$$\downarrow 3042$$

$$\frac{1}{7}\left(\frac{1}{5}\left(-35a\int \tan(c+dx)dx-16b\int \sec(c+dx)\tan(c+dx)dx+\frac{\tan^2(c+dx)(35a+16b\sec(c+dx))}{2d}\right)-\frac{\tan^4}{}\right.$$

$$\frac{\tan^6(c+dx)(7a+6b\sec(c+dx))}{42d}$$

$$\downarrow 3086$$

$$\frac{1}{7}\left(\frac{1}{5}\left(-35a\int \tan(c+dx)dx-\frac{16b\int 1d\sec(c+dx)}{d}+\frac{\tan^2(c+dx)(35a+16b\sec(c+dx))}{2d}\right)-\frac{\tan^4(c+dx)(35}{}\right.$$

$$\frac{\tan^6(c+dx)(7a+6b\sec(c+dx))}{42d}$$

$$\downarrow 24$$

$$\frac{1}{7}\left(\frac{1}{5}\left(-35a\int \tan(c+dx)dx+\frac{\tan^2(c+dx)(35a+16b\sec(c+dx))}{2d}-\frac{16b\sec(c+dx)}{d}\right)-\frac{\tan^4(c+dx)(35a+2}{20d}\right.$$

$$\frac{\tan^6(c+dx)(7a+6b\sec(c+dx))}{42d}$$

$$\downarrow 3956$$

$$\frac{\tan^6(c+dx)(7a+6b\sec(c+dx))}{42d}+$$

$$\frac{1}{7}\left(\frac{1}{5}\left(\frac{\tan^2(c+dx)(35a+16b\sec(c+dx))}{2d}+\frac{35a\log(\cos(c+dx))}{d}-\frac{16b\sec(c+dx)}{d}\right)-\frac{\tan^4(c+dx)(35a+24b$$

input
```
Int[(a + b*Sec[c + d*x])*Tan[c + d*x]^7,x]
```

output
```
((7*a + 6*b*Sec[c + d*x])*Tan[c + d*x]^6)/(42*d) + (-1/20*((35*a + 24*b*Se
c[c + d*x])*Tan[c + d*x]^4)/d + ((35*a*Log[Cos[c + d*x]])/d - (16*b*Sec[c
+ d*x])/d + ((35*a + 16*b*Sec[c + d*x])*Tan[c + d*x]^2)/(2*d))/5)/7
```

3.256.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3086
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[a/f   Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2
), x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2
] &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4369
```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(-e)*(e*Cot[c + d*x])^(m - 1)*((a*m + b*(m - 1)*Csc
[c + d*x])/(d*m*(m - 1))), x] - Simp[e^2/m   Int[(e*Cot[c + d*x])^(m - 2)*(
a*m + b*(m - 1)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && GtQ[m
, 1]
```

rule 4372
```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(e*Cot[c + d*x])^m, x], x] + Simp[b   Int[
(e*Cot[c + d*x])^m*Csc[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, m}, x]
```

3.256.4 Maple [A] (verified)

Time = 1.41 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.81

method	result
derivativedivides	$\frac{\frac{b\sec(dx+c)^7}{7}+\frac{a\sec(dx+c)^6}{6}-\frac{3b\sec(dx+c)^5}{5}-\frac{3a\sec(dx+c)^4}{4}+b\sec(dx+c)^3+\frac{3a\sec(dx+c)^2}{2}-b\sec(dx+c)-a\ln(\sec(dx+c))}{d}$
default	$\frac{\frac{b\sec(dx+c)^7}{7}+\frac{a\sec(dx+c)^6}{6}-\frac{3b\sec(dx+c)^5}{5}-\frac{3a\sec(dx+c)^4}{4}+b\sec(dx+c)^3+\frac{3a\sec(dx+c)^2}{2}-b\sec(dx+c)-a\ln(\sec(dx+c))}{d}$
parts	$\frac{a\left(\frac{\tan(dx+c)^6}{6}-\frac{\tan(dx+c)^4}{4}+\frac{\tan(dx+c)^2}{2}-\frac{\ln\left(1+\tan(dx+c)^2\right)}{2}\right)}{d}+\frac{b\left(\frac{\sec(dx+c)^7}{7}-\frac{3\sec(dx+c)^5}{5}+\sec(dx+c)^3-\sec(dx+c)\right.}{d}$
risch	$-iax-\frac{2iac}{d}-\frac{2\left(105b\,e^{13i(dx+c)}-315a\,e^{12i(dx+c)}+210b\,e^{11i(dx+c)}-945a\,e^{10i(dx+c)}+903b\,e^{9i(dx+c)}-1820a\,e^{8i(dx}{1}$

input
```
int((a+b*sec(d*x+c))*tan(d*x+c)^7,x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/7*b*sec(d*x+c)^7+1/6*a*sec(d*x+c)^6-3/5*b*sec(d*x+c)^5-3/4*a*sec(d*
x+c)^4+b*sec(d*x+c)^3+3/2*a*sec(d*x+c)^2-b*sec(d*x+c)-a*ln(sec(d*x+c)))
```

3.256.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.91

$$\int (a+b\sec(c+dx))\tan^7(c+dx)\,dx$$
$$=\frac{420\,a\cos(dx+c)^7\log(-\cos(dx+c))-420\,b\cos(dx+c)^6+630\,a\cos(dx+c)^5+420\,b\cos(dx+c)^4-}{420\,d\cos(dx+c)^7}$$

input
```
integrate((a+b*sec(d*x+c))*tan(d*x+c)^7,x, algorithm="fricas")
```

output
```
1/420*(420*a*cos(d*x + c)^7*log(-cos(d*x + c)) - 420*b*cos(d*x + c)^6 + 63
0*a*cos(d*x + c)^5 + 420*b*cos(d*x + c)^4 - 315*a*cos(d*x + c)^3 - 252*b*c
os(d*x + c)^2 + 70*a*cos(d*x + c) + 60*b)/(d*cos(d*x + c)^7)
```

3.256.6 Sympy [A] (verification not implemented)

Time = 0.81 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.33

$$\int (a + b\sec(c + dx))\tan^7(c + dx)\,dx$$

$$= \begin{cases} -\frac{a\log\left(\tan^2(c+dx)+1\right)}{2d} + \frac{a\tan^6(c+dx)}{6d} - \frac{a\tan^4(c+dx)}{4d} + \frac{a\tan^2(c+dx)}{2d} + \frac{b\tan^6(c+dx)\sec(c+dx)}{7d} - \frac{6b\tan^4(c+dx)\sec(c+dx)}{35d} \\ x(a + b\sec(c))\tan^7(c) \end{cases}$$

input
```
integrate((a+b*sec(d*x+c))*tan(d*x+c)**7,x)
```

output
```
Piecewise((-a*log(tan(c + d*x)**2 + 1)/(2*d) + a*tan(c + d*x)**6/(6*d) - a
*tan(c + d*x)**4/(4*d) + a*tan(c + d*x)**2/(2*d) + b*tan(c + d*x)**6*sec(c
 + d*x)/(7*d) - 6*b*tan(c + d*x)**4*sec(c + d*x)/(35*d) + 8*b*tan(c + d*x)
**2*sec(c + d*x)/(35*d) - 16*b*sec(c + d*x)/(35*d), Ne(d, 0)), (x*(a + b*s
ec(c))*tan(c)**7, True))
```

3.256.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.85

$$\int (a + b\sec(c + dx))\tan^7(c + dx)\,dx$$

$$= \frac{420\,a\log\left(\cos\left(dx+c\right)\right) - \frac{420\,b\cos(dx+c)^6 - 630\,a\cos(dx+c)^5 - 420\,b\cos(dx+c)^4 + 315\,a\cos(dx+c)^3 + 252\,b\cos(dx+c)^2 - 70\,a\cos(dx+c)}{\cos(dx+c)^7}}{420\,d}$$

input
```
integrate((a+b*sec(d*x+c))*tan(d*x+c)^7,x, algorithm="maxima")
```

output
```
1/420*(420*a*log(cos(d*x + c)) - (420*b*cos(d*x + c)^6 - 630*a*cos(d*x + c
)^5 - 420*b*cos(d*x + c)^4 + 315*a*cos(d*x + c)^3 + 252*b*cos(d*x + c)^2 -
 70*a*cos(d*x + c) - 60*b)/cos(d*x + c)^7)/d
```

3.256.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 317 vs. $2(103) = 206$.

Time = 3.38 (sec) , antiderivative size = 317, normalized size of antiderivative = 2.86

$$\int (a + b\sec(c+dx))\tan^7(c+dx)\,dx =$$

$$-\frac{420\,a\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right)-420\,a\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right)+\frac{1089\,a+384\,b+\frac{8463\,a(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{2688\,b(\cos(dx+c)}{\cos(dx+c)+1}}{}$$

input
```
integrate((a+b*sec(d*x+c))*tan(d*x+c)^7,x, algorithm="giac")
```

output
```
-1/420*(420*a*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)) - 420*a
*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)) + (1089*a + 384*b +
8463*a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 2688*b*(cos(d*x + c) - 1)/(
cos(d*x + c) + 1) + 28749*a*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 80
64*b*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 56035*a*(cos(d*x + c) - 1
)^3/(cos(d*x + c) + 1)^3 + 13440*b*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)
^3 + 56035*a*(cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4 + 28749*a*(cos(d*x
+ c) - 1)^5/(cos(d*x + c) + 1)^5 + 8463*a*(cos(d*x + c) - 1)^6/(cos(d*x +
c) + 1)^6 + 1089*a*(cos(d*x + c) - 1)^7/(cos(d*x + c) + 1)^7)/((cos(d*x +
c) - 1)/(cos(d*x + c) + 1) + 1)^7)/d
```

3.256.9 Mupad [B] (verification not implemented)

Time = 18.31 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.99

$$\int (a + b\sec(c+dx))\tan^7(c+dx)\,dx$$

$$=\frac{2\,a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{12}-14\,a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}+\frac{128\,a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^8}{3}+\left(-\frac{128\,a}{3}-32\,b\right)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6+\left(14\,a+\frac{96\,b}{5}\right)}{d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{14}-7\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{12}+21\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}-35\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^8+35\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6-\right.}$$

$$-\frac{2\,a\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2\right)}{d}$$

input
```
int(tan(c + d*x)^7*(a + b/cos(c + d*x)),x)
```

output

```
((32*b)/35 - tan(c/2 + (d*x)/2)^2*(2*a + (32*b)/5) + tan(c/2 + (d*x)/2)^4*
(14*a + (96*b)/5) - tan(c/2 + (d*x)/2)^6*((128*a)/3 + 32*b) + (128*a*tan(c
/2 + (d*x)/2)^8)/3 - 14*a*tan(c/2 + (d*x)/2)^10 + 2*a*tan(c/2 + (d*x)/2)^1
2)/(d*(7*tan(c/2 + (d*x)/2)^2 - 21*tan(c/2 + (d*x)/2)^4 + 35*tan(c/2 + (d*
x)/2)^6 - 35*tan(c/2 + (d*x)/2)^8 + 21*tan(c/2 + (d*x)/2)^10 - 7*tan(c/2 +
 (d*x)/2)^12 + tan(c/2 + (d*x)/2)^14 - 1)) - (2*a*atanh(tan(c/2 + (d*x)/2)
^2))/d
```

3.257 $\int (a + b\sec(c + dx))\tan^5(c + dx)\,dx$

3.257.1 Optimal result 1888
3.257.2 Mathematica [A] (verified) 1888
3.257.3 Rubi [A] (verified) 1889
3.257.4 Maple [A] (verified) 1892
3.257.5 Fricas [A] (verification not implemented) 1893
3.257.6 Sympy [A] (verification not implemented) 1893
3.257.7 Maxima [A] (verification not implemented) 1894
3.257.8 Giac [B] (verification not implemented) 1894
3.257.9 Mupad [B] (verification not implemented) 1895

3.257.1 Optimal result

Integrand size = 19, antiderivative size = 84

$$\int (a + b\sec(c + dx))\tan^5(c + dx)\,dx = -\frac{a\log(\cos(c + dx))}{d} + \frac{8b\sec(c + dx)}{15d} - \frac{(15a + 8b\sec(c + dx))\tan^2(c + dx)}{30d} + \frac{(5a + 4b\sec(c + dx))\tan^4(c + dx)}{20d}$$

output
```
-a*ln(cos(d*x+c))/d+8/15*b*sec(d*x+c)/d-1/30*(15*a+8*b*sec(d*x+c))*tan(d*x
+c)^2/d+1/20*(5*a+4*b*sec(d*x+c))*tan(d*x+c)^4/d
```

3.257.2 Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.98

$$\int (a + b\sec(c + dx))\tan^5(c + dx)\,dx = \frac{b\sec(c + dx)}{d} - \frac{2b\sec^3(c + dx)}{3d} + \frac{b\sec^5(c + dx)}{5d} - \frac{a(4\log(\cos(c + dx)) + 2\tan^2(c + dx) - \tan^4(c + dx))}{4d}$$

input
```
Integrate[(a + b*Sec[c + d*x])*Tan[c + d*x]^5,x]
```

output
```
(b*Sec[c + d*x])/d - (2*b*Sec[c + d*x]^3)/(3*d) + (b*Sec[c + d*x]^5)/(5*d)
 - (a*(4*Log[Cos[c + d*x]] + 2*Tan[c + d*x]^2 - Tan[c + d*x]^4))/(4*d)
```

3.257.3 Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.10, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.842$, Rules used = {3042, 25, 4369, 25, 3042, 25, 4369, 25, 3042, 25, 4372, 25, 3042, 3086, 24, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^5(c+dx)(a+b\sec(c+dx))\,dx$$

$$\downarrow 3042$$

$$\int -\cot\left(c+dx+\frac{\pi}{2}\right)^5\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)dx$$

$$\downarrow 25$$

$$-\int \cot\left(\frac{1}{2}(2c+\pi)+dx\right)^5\left(a+b\csc\left(\frac{1}{2}(2c+\pi)+dx\right)\right)dx$$

$$\downarrow 4369$$

$$\frac{1}{5}\int -((5a+4b\sec(c+dx))\tan^3(c+dx))\,dx+\frac{\tan^4(c+dx)(5a+4b\sec(c+dx))}{20d}$$

$$\downarrow 25$$

$$\frac{\tan^4(c+dx)(5a+4b\sec(c+dx))}{20d}-\frac{1}{5}\int (5a+4b\sec(c+dx))\tan^3(c+dx)dx$$

$$\downarrow 3042$$

$$\frac{\tan^4(c+dx)(5a+4b\sec(c+dx))}{20d}-\frac{1}{5}\int -\cot\left(c+dx+\frac{\pi}{2}\right)^3\left(5a+4b\csc\left(c+dx+\frac{\pi}{2}\right)\right)dx$$

$$\downarrow 25$$

$$\frac{1}{5}\int \cot\left(\frac{1}{2}(2c+\pi)+dx\right)^3\left(5a+4b\csc\left(\frac{1}{2}(2c+\pi)+dx\right)\right)dx+\frac{\tan^4(c+dx)(5a+4b\sec(c+dx))}{20d}$$

$$\downarrow 4369$$

$$\frac{1}{5}\left(-\frac{1}{3}\int -((15a+8b\sec(c+dx))\tan(c+dx))dx-\frac{\tan^2(c+dx)(15a+8b\sec(c+dx))}{6d}\right)+\frac{\tan^4(c+dx)(5a+4b\sec(c+dx))}{20d}$$

$$\downarrow 25$$

$$\frac{1}{5}\left(\frac{1}{3}\int (15a+8b\sec(c+dx))\tan(c+dx)dx-\frac{\tan^2(c+dx)(15a+8b\sec(c+dx))}{6d}\right)+\frac{\tan^4(c+dx)(5a+4b\sec(c+dx))}{20d}$$

$$\downarrow 3042$$

$$\frac{1}{5}\left(\frac{1}{3}\int -\cot\left(c+dx+\frac{\pi}{2}\right)\left(15a+8b\csc\left(c+dx+\frac{\pi}{2}\right)\right)dx-\frac{\tan^2(c+dx)(15a+8b\sec(c+dx))}{6d}\right)+\frac{\tan^4(c+dx)(5a+4b\sec(c+dx))}{20d}$$

$$\downarrow 25$$

$$\frac{1}{5}\left(-\frac{1}{3}\int \cot\left(\frac{1}{2}(2c+\pi)+dx\right)\left(15a+8b\csc\left(\frac{1}{2}(2c+\pi)+dx\right)\right)dx-\frac{\tan^2(c+dx)(15a+8b\sec(c+dx))}{6d}\right)+\frac{\tan^4(c+dx)(5a+4b\sec(c+dx))}{20d}$$

$$\downarrow 4372$$

$$\frac{1}{5}\left(\frac{1}{3}(-15a\int -\tan(c+dx)dx-8b\int -\sec(c+dx)\tan(c+dx)dx)-\frac{\tan^2(c+dx)(15a+8b\sec(c+dx))}{6d}\right)+\frac{\tan^4(c+dx)(5a+4b\sec(c+dx))}{20d}$$

$$\downarrow 25$$

$$\frac{1}{5}\left(\frac{1}{3}(15a\int \tan(c+dx)dx+8b\int \sec(c+dx)\tan(c+dx)dx)-\frac{\tan^2(c+dx)(15a+8b\sec(c+dx))}{6d}\right)+\frac{\tan^4(c+dx)(5a+4b\sec(c+dx))}{20d}$$

$$\downarrow 3042$$

$$\frac{1}{5}\left(\frac{1}{3}(15a\int \tan(c+dx)dx+8b\int \sec(c+dx)\tan(c+dx)dx)-\frac{\tan^2(c+dx)(15a+8b\sec(c+dx))}{6d}\right)+\frac{\tan^4(c+dx)(5a+4b\sec(c+dx))}{20d}$$

$$\downarrow 3086$$

$$\frac{1}{5}\left(\frac{1}{3}\left(15a\int\tan(c+dx)dx+\frac{8b\int 1d\sec(c+dx)}{d}\right)-\frac{\tan^2(c+dx)(15a+8b\sec(c+dx))}{6d}\right)+\frac{\tan^4(c+dx)(5a+4b\sec(c+dx))}{20d}$$

$$\downarrow 24$$

$$\frac{1}{5}\left(\frac{1}{3}\left(15a\int\tan(c+dx)dx+\frac{8b\sec(c+dx)}{d}\right)-\frac{\tan^2(c+dx)(15a+8b\sec(c+dx))}{6d}\right)+\frac{\tan^4(c+dx)(5a+4b\sec(c+dx))}{20d}$$

$$\downarrow 3956$$

$$\frac{\tan^4(c+dx)(5a+4b\sec(c+dx))}{20d}+\frac{1}{5}\left(\frac{1}{3}\left(\frac{8b\sec(c+dx)}{d}-\frac{15a\log(\cos(c+dx))}{d}\right)-\frac{\tan^2(c+dx)(15a+8b\sec(c+dx))}{6d}\right)$$

input
```
Int[(a + b*Sec[c + d*x])*Tan[c + d*x]^5,x]
```

output
```
((5*a + 4*b*Sec[c + d*x])*Tan[c + d*x]^4)/(20*d) + (((-15*a*Log[Cos[c + d*
x]])/d + (8*b*Sec[c + d*x])/d)/3 - ((15*a + 8*b*Sec[c + d*x])*Tan[c + d*x]
^2)/(6*d))/5
```

3.257.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3086
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[a/f   Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2
), x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2
] &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4369
```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(-e)*(e*Cot[c + d*x])^(m - 1)*((a*m + b*(m - 1)*Csc
[c + d*x])/(d*m*(m - 1))), x] - Simp[e^2/m   Int[(e*Cot[c + d*x])^(m - 2)*(
a*m + b*(m - 1)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && GtQ[m
, 1]
```

rule 4372
```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(e*Cot[c + d*x])^m, x], x] + Simp[b   Int[
(e*Cot[c + d*x])^m*Csc[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, m}, x]
```

3.257.4 Maple [A] (verified)

Time = 1.10 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.80

method	result
derivativedivides	$\frac{\frac{b\sec(dx+c)^5}{5}+\frac{a\sec(dx+c)^4}{4}-\frac{2b\sec(dx+c)^3}{3}-a\sec(dx+c)^2+b\sec(dx+c)+a\ln(\sec(dx+c))}{d}$
default	$\frac{\frac{b\sec(dx+c)^5}{5}+\frac{a\sec(dx+c)^4}{4}-\frac{2b\sec(dx+c)^3}{3}-a\sec(dx+c)^2+b\sec(dx+c)+a\ln(\sec(dx+c))}{d}$
parts	$\frac{a\left(\frac{\tan(dx+c)^4}{4}-\frac{\tan(dx+c)^2}{2}+\frac{\ln\left(1+\tan(dx+c)^2\right)}{2}\right)}{d}+\frac{b\left(\frac{\sec(dx+c)^5}{5}-\frac{2\sec(dx+c)^3}{3}+\sec(dx+c)\right)}{d}$
risch	$iax+\frac{2iac}{d}+\frac{2b\,e^{9i(dx+c)}-4a\,e^{8i(dx+c)}+\frac{8b\,e^{7i(dx+c)}}{3}-8a\,e^{6i(dx+c)}+\frac{116b\,e^{5i(dx+c)}}{15}-8a\,e^{4i(dx+c)}+\frac{8b\,e^{3i(dx+c)}}{3}-4a$

input
```
int((a+b*sec(d*x+c))*tan(d*x+c)^5,x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/5*b*sec(d*x+c)^5+1/4*a*sec(d*x+c)^4-2/3*b*sec(d*x+c)^3-a*sec(d*x+c)
^2+b*sec(d*x+c)+a*ln(sec(d*x+c)))
```

3.257.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.94

$$\int (a + b\sec(c + dx))\tan^5(c + dx)\,dx =$$

$$-\frac{60\,a\cos(dx+c)^5\log(-\cos(dx+c)) - 60\,b\cos(dx+c)^4 + 60\,a\cos(dx+c)^3 + 40\,b\cos(dx+c)^2 - 15}{60\,d\cos(dx+c)^5}$$

input
```
integrate((a+b*sec(d*x+c))*tan(d*x+c)^5,x, algorithm="fricas")
```

output
```
-1/60*(60*a*cos(d*x + c)^5*log(-cos(d*x + c)) - 60*b*cos(d*x + c)^4 + 60*a
*cos(d*x + c)^3 + 40*b*cos(d*x + c)^2 - 15*a*cos(d*x + c) - 12*b)/(d*cos(d
*x + c)^5)
```

3.257.6 Sympy [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.33

$$\int (a + b\sec(c + dx))\tan^5(c + dx)\,dx$$

$$= \begin{cases} \frac{a\log(\tan^2(c+dx)+1)}{2d} + \frac{a\tan^4(c+dx)}{4d} - \frac{a\tan^2(c+dx)}{2d} + \frac{b\tan^4(c+dx)\sec(c+dx)}{5d} - \frac{4b\tan^2(c+dx)\sec(c+dx)}{15d} + \frac{8b\sec(c+dx)}{15d} & \text{f} \\ x(a + b\sec(c))\tan^5(c) & \text{c} \end{cases}$$

input
```
integrate((a+b*sec(d*x+c))*tan(d*x+c)**5,x)
```

output
```
Piecewise((a*log(tan(c + d*x)**2 + 1)/(2*d) + a*tan(c + d*x)**4/(4*d) - a*
tan(c + d*x)**2/(2*d) + b*tan(c + d*x)**4*sec(c + d*x)/(5*d) - 4*b*tan(c +
 d*x)**2*sec(c + d*x)/(15*d) + 8*b*sec(c + d*x)/(15*d), Ne(d, 0)), (x*(a +
 b*sec(c))*tan(c)**5, True))
```

3.257.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.86

$$\int (a + b\sec(c + dx))\tan^5(c + dx)\,dx$$
$$= -\frac{60\,a\log(\cos(dx + c)) - \frac{60\,b\cos(dx+c)^4 - 60\,a\cos(dx+c)^3 - 40\,b\cos(dx+c)^2 + 15\,a\cos(dx+c) + 12\,b}{\cos(dx+c)^5}}{60\,d}$$

input
```
integrate((a+b*sec(d*x+c))*tan(d*x+c)^5,x, algorithm="maxima")
```

output
```
-1/60*(60*a*log(cos(d*x + c)) - (60*b*cos(d*x + c)^4 - 60*a*cos(d*x + c)^3
 - 40*b*cos(d*x + c)^2 + 15*a*cos(d*x + c) + 12*b)/cos(d*x + c)^5)/d
```

3.257.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 248 vs. 2(78) = 156.

Time = 1.53 (sec) , antiderivative size = 248, normalized size of antiderivative = 2.95

$$\int (a + b\sec(c + dx))\tan^5(c + dx)\,dx$$
$$= \frac{60\,a\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1} + 1\right|\right) - 60\,a\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1} - 1\right|\right) + \frac{137\,a+64\,b+\frac{805\,a(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{320\,b(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{1970}{(c}}{60\,d}$$

input
```
integrate((a+b*sec(d*x+c))*tan(d*x+c)^5,x, algorithm="giac")
```

output
```
1/60*(60*a*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)) - 60*a*log
(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)) + (137*a + 64*b + 805*a*
(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 320*b*(cos(d*x + c) - 1)/(cos(d*x
+ c) + 1) + 1970*a*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 640*b*(cos(
d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 1970*a*(cos(d*x + c) - 1)^3/(cos(d*
x + c) + 1)^3 + 805*a*(cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4 + 137*a*(c
os(d*x + c) - 1)^5/(cos(d*x + c) + 1)^5)/((cos(d*x + c) - 1)/(cos(d*x + c)
 + 1) + 1)^5)/d
```

3.257.9 Mupad [B] (verification not implemented)

Time = 19.03 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.93

$$\int (a+b\sec(c+dx))\tan^5(c+dx)\,dx = \frac{2\,a\,\operatorname{atanh}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2\right)}{d} - \frac{2\,a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^8 - 10\,a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6 + \left(10\,a+\frac{32\,b}{3}\right)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4 + \left(-2\,a-\frac{16\,b}{3}\right)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2 + \frac{16\,b}{15}}{d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{10} - 5\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^8 + 10\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6 - 10\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4 + 5\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2 - 1\right)}$$

input
```
int(tan(c + d*x)^5*(a + b/cos(c + d*x)),x)
```

output
```
(2*a*atanh(tan(c/2 + (d*x)/2)^2))/d - ((16*b)/15 - tan(c/2 + (d*x)/2)^2*(2
*a + (16*b)/3) + tan(c/2 + (d*x)/2)^4*(10*a + (32*b)/3) - 10*a*tan(c/2 + (
d*x)/2)^6 + 2*a*tan(c/2 + (d*x)/2)^8)/(d*(5*tan(c/2 + (d*x)/2)^2 - 10*tan(
c/2 + (d*x)/2)^4 + 10*tan(c/2 + (d*x)/2)^6 - 5*tan(c/2 + (d*x)/2)^8 + tan(
c/2 + (d*x)/2)^10 - 1))
```

3.258 $\int (a + b\sec(c + dx))\tan^3(c + dx)\,dx$

3.258.1 Optimal result 1896
3.258.2 Mathematica [A] (verified) 1896
3.258.3 Rubi [A] (verified) 1897
3.258.4 Maple [A] (verified) 1899
3.258.5 Fricas [A] (verification not implemented) 1900
3.258.6 Sympy [A] (verification not implemented) 1900
3.258.7 Maxima [A] (verification not implemented) 1900
3.258.8 Giac [B] (verification not implemented) 1901
3.258.9 Mupad [B] (verification not implemented) 1901

3.258.1 Optimal result

Integrand size = 19, antiderivative size = 55

$$\int (a + b\sec(c + dx))\tan^3(c + dx)\,dx = \frac{a\log(\cos(c + dx))}{d} - \frac{2b\sec(c + dx)}{3d} + \frac{(3a + 2b\sec(c + dx))\tan^2(c + dx)}{6d}$$

output
```
a*ln(cos(d*x+c))/d-2/3*b*sec(d*x+c)/d+1/6*(3*a+2*b*sec(d*x+c))*tan(d*x+c)^
2/d
```

3.258.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00

$$\int (a + b\sec(c + dx))\tan^3(c + dx)\,dx = -\frac{b\sec(c + dx)}{d} + \frac{b\sec^3(c + dx)}{3d} + \frac{a(2\log(\cos(c + dx)) + \tan^2(c + dx))}{2d}$$

input
```
Integrate[(a + b*Sec[c + d*x])*Tan[c + d*x]^3,x]
```

output
```
-((b*Sec[c + d*x])/d) + (b*Sec[c + d*x]^3)/(3*d) + (a*(2*Log[Cos[c + d*x]]
 + Tan[c + d*x]^2))/(2*d)
```

3.258.3 Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.07, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.632$, Rules used = {3042, 25, 4369, 25, 3042, 25, 4372, 25, 3042, 3086, 24, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^3(c+dx)(a+b\sec(c+dx))\,dx$$

$$\downarrow 3042$$

$$\int -\cot\left(c+dx+\frac{\pi}{2}\right)^3\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)dx$$

$$\downarrow 25$$

$$-\int \cot\left(\frac{1}{2}(2c+\pi)+dx\right)^3\left(a+b\csc\left(\frac{1}{2}(2c+\pi)+dx\right)\right)dx$$

$$\downarrow 4369$$

$$\frac{1}{3}\int -((3a+2b\sec(c+dx))\tan(c+dx))dx+\frac{\tan^2(c+dx)(3a+2b\sec(c+dx))}{6d}$$

$$\downarrow 25$$

$$\frac{\tan^2(c+dx)(3a+2b\sec(c+dx))}{6d}-\frac{1}{3}\int (3a+2b\sec(c+dx))\tan(c+dx)dx$$

$$\downarrow 3042$$

$$\frac{\tan^2(c+dx)(3a+2b\sec(c+dx))}{6d}-\frac{1}{3}\int -\cot\left(c+dx+\frac{\pi}{2}\right)\left(3a+2b\csc\left(c+dx+\frac{\pi}{2}\right)\right)dx$$

$$\downarrow 25$$

$$\frac{1}{3}\int \cot\left(\frac{1}{2}(2c+\pi)+dx\right)\left(3a+2b\csc\left(\frac{1}{2}(2c+\pi)+dx\right)\right)dx+\frac{\tan^2(c+dx)(3a+2b\sec(c+dx))}{6d}$$

$$\downarrow 4372$$

$$\frac{1}{3}(3a\int -\tan(c+dx)dx+2b\int -\sec(c+dx)\tan(c+dx)dx)+\frac{\tan^2(c+dx)(3a+2b\sec(c+dx))}{6d}$$

$$\downarrow 25$$

$$\frac{1}{3}(-3a\int \tan(c+dx)dx-2b\int \sec(c+dx)\tan(c+dx)dx)+\frac{\tan^2(c+dx)(3a+2b\sec(c+dx))}{6d}$$

$$\downarrow 3042$$

$$\frac{1}{3}(-3a\int \tan(c+dx)dx - 2b\int \sec(c+dx)\tan(c+dx)dx) + \frac{\tan^2(c+dx)(3a+2b\sec(c+dx))}{6d}$$

$$\downarrow 3086$$

$$\frac{1}{3}\left(-3a\int \tan(c+dx)dx - \frac{2b\int 1d\sec(c+dx)}{d}\right) + \frac{\tan^2(c+dx)(3a+2b\sec(c+dx))}{6d}$$

$$\downarrow 24$$

$$\frac{1}{3}\left(-3a\int \tan(c+dx)dx - \frac{2b\sec(c+dx)}{d}\right) + \frac{\tan^2(c+dx)(3a+2b\sec(c+dx))}{6d}$$

$$\downarrow 3956$$

$$\frac{\tan^2(c+dx)(3a+2b\sec(c+dx))}{6d} + \frac{1}{3}\left(\frac{3a\log(\cos(c+dx))}{d} - \frac{2b\sec(c+dx)}{d}\right)$$

input
```
Int[(a + b*Sec[c + d*x])*Tan[c + d*x]^3,x]
```

output
```
((3*a*Log[Cos[c + d*x]])/d - (2*b*Sec[c + d*x])/d)/3 + ((3*a + 2*b*Sec[c +
 d*x])*Tan[c + d*x]^2)/(6*d)
```

3.258.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3086
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[a/f   Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2
), x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2
] &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4369
```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(-e)*(e*Cot[c + d*x])^(m - 1)*((a*m + b*(m - 1)*Csc
[c + d*x])/(d*m*(m - 1))), x] - Simp[e^2/m   Int[(e*Cot[c + d*x])^(m - 2)*(
a*m + b*(m - 1)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && GtQ[m
, 1]
```

rule 4372
```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(e*Cot[c + d*x])^m, x], x] + Simp[b   Int[
(e*Cot[c + d*x])^m*Csc[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, m}, x]
```

3.258.4 Maple [A] (verified)

Time = 0.84 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.85

method	result	siz
derivativedivides	$\frac{\frac{b\sec(dx+c)^3}{3}+\frac{a\sec(dx+c)^2}{2}-b\sec(dx+c)-a\ln(\sec(dx+c))}{d}$	47
default	$\frac{\frac{b\sec(dx+c)^3}{3}+\frac{a\sec(dx+c)^2}{2}-b\sec(dx+c)-a\ln(\sec(dx+c))}{d}$	47
parts	$\frac{a\left(\frac{\tan(dx+c)^2}{2}-\frac{\ln\left(1+\tan(dx+c)^2\right)}{2}\right)}{d}+\frac{b\left(\frac{\sec(dx+c)^3}{3}-\sec(dx+c)\right)}{d}$	55
risch	$-iax-\frac{2iac}{d}-\frac{2\left(3b\,e^{5i(dx+c)}-3a\,e^{4i(dx+c)}+2b\,e^{3i(dx+c)}-3a\,e^{2i(dx+c)}+3b\,e^{i(dx+c)}\right)}{3d\left(e^{2i(dx+c)}+1\right)^3}+\frac{a\ln\left(e^{2i(dx+c)}+1\right)}{d}$	11

input
```
int((a+b*sec(d*x+c))*tan(d*x+c)^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/3*b*sec(d*x+c)^3+1/2*a*sec(d*x+c)^2-b*sec(d*x+c)-a*ln(sec(d*x+c)))
```

3.258.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.04

$$\int (a + b\sec(c + dx))\tan^3(c + dx)\,dx$$
$$= \frac{6\,a\cos(dx + c)^3\log(-\cos(dx + c)) - 6\,b\cos(dx + c)^2 + 3\,a\cos(dx + c) + 2\,b}{6\,d\cos(dx + c)^3}$$

input
```
integrate((a+b*sec(d*x+c))*tan(d*x+c)^3,x, algorithm="fricas")
```

output
```
1/6*(6*a*cos(d*x + c)^3*log(-cos(d*x + c)) - 6*b*cos(d*x + c)^2 + 3*a*cos(
d*x + c) + 2*b)/(d*cos(d*x + c)^3)
```

3.258.6 Sympy [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.38

$$\int (a + b\sec(c + dx))\tan^3(c + dx)\,dx$$
$$= \begin{cases} -\frac{a\log(\tan^2(c+dx)+1)}{2d} + \frac{a\tan^2(c+dx)}{2d} + \frac{b\tan^2(c+dx)\sec(c+dx)}{3d} - \frac{2b\sec(c+dx)}{3d} & \text{for } d \neq 0 \\ x(a + b\sec(c))\tan^3(c) & \text{otherwise} \end{cases}$$

input
```
integrate((a+b*sec(d*x+c))*tan(d*x+c)**3,x)
```

output
```
Piecewise((-a*log(tan(c + d*x)**2 + 1)/(2*d) + a*tan(c + d*x)**2/(2*d) + b
*tan(c + d*x)**2*sec(c + d*x)/(3*d) - 2*b*sec(c + d*x)/(3*d), Ne(d, 0)), (
x*(a + b*sec(c))*tan(c)**3, True))
```

3.258.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.91

$$\int (a + b\sec(c + dx))\tan^3(c + dx)\,dx = \frac{6\,a\log(\cos(dx + c)) - \frac{6\,b\cos(dx+c)^2 - 3\,a\cos(dx+c) - 2\,b}{\cos(dx+c)^3}}{6\,d}$$

input
```
integrate((a+b*sec(d*x+c))*tan(d*x+c)^3,x, algorithm="maxima")
```

output
```
1/6*(6*a*log(cos(d*x + c)) - (6*b*cos(d*x + c)^2 - 3*a*cos(d*x + c) - 2*b)
/cos(d*x + c)^3)/d
```

3.258.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 179 vs. $2(51) = 102$.

Time $= 0.62$ (sec) , antiderivative size $= 179$, normalized size of antiderivative $= 3.25$

$$\int (a + b\sec(c+dx))\tan^3(c+dx)\,dx =$$

$$-\frac{6\,a\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right)-6\,a\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right)+\frac{11\,a+8\,b+\frac{45\,a(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{24\,b(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{45\,a(\cos(d}{\left(\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right)^3}}{6\,d}$$

input
```
integrate((a+b*sec(d*x+c))*tan(d*x+c)^3,x, algorithm="giac")
```

output
```
-1/6*(6*a*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)) - 6*a*log(a
bs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)) + (11*a + 8*b + 45*a*(cos(
d*x + c) - 1)/(cos(d*x + c) + 1) + 24*b*(cos(d*x + c) - 1)/(cos(d*x + c) +
 1) + 45*a*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 11*a*(cos(d*x + c)
- 1)^3/(cos(d*x + c) + 1)^3)/((cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)^3
)/d
```

3.258.9 Mupad [B] (verification not implemented)

Time $= 15.54$ (sec) , antiderivative size $= 102$, normalized size of antiderivative $= 1.85$

$$\int (a + b\sec(c+dx))\tan^3(c+dx)\,dx$$

$$= \frac{2\,a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4+(-2\,a-4\,b)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+\frac{4\,b}{3}}{d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6-3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4+3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2-1\right)}-\frac{2\,a\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2\right)}{d}$$

input
```
int(tan(c + d*x)^3*(a + b/cos(c + d*x)),x)
```

output
```
((4*b)/3 - tan(c/2 + (d*x)/2)^2*(2*a + 4*b) + 2*a*tan(c/2 + (d*x)/2)^4)/(d
*(3*tan(c/2 + (d*x)/2)^2 - 3*tan(c/2 + (d*x)/2)^4 + tan(c/2 + (d*x)/2)^6 -
 1)) - (2*a*atanh(tan(c/2 + (d*x)/2)^2))/d
```

3.259 $\int (a + b\sec(c + dx))\tan(c + dx)\,dx$

3.259.1 Optimal result 1903
3.259.2 Mathematica [A] (verified) 1903
3.259.3 Rubi [A] (verified) 1904
3.259.4 Maple [A] (verified) 1906
3.259.5 Fricas [A] (verification not implemented) 1906
3.259.6 Sympy [A] (verification not implemented) 1906
3.259.7 Maxima [A] (verification not implemented) 1907
3.259.8 Giac [B] (verification not implemented) 1907
3.259.9 Mupad [B] (verification not implemented) 1907

3.259.1 Optimal result

Integrand size = 17, antiderivative size = 25

$$\int (a + b\sec(c + dx))\tan(c + dx)\,dx = -\frac{a\log(\cos(c + dx))}{d} + \frac{b\sec(c + dx)}{d}$$

output
```
-a*ln(cos(d*x+c))/d+b*sec(d*x+c)/d
```

3.259.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int (a + b\sec(c + dx))\tan(c + dx)\,dx = -\frac{a\log(\cos(c + dx))}{d} + \frac{b\sec(c + dx)}{d}$$

input
```
Integrate[(a + b*Sec[c + d*x])*Tan[c + d*x],x]
```

output
```
-((a*Log[Cos[c + d*x]])/d) + (b*Sec[c + d*x])/d
```

3.259.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.471$, Rules used = {3042, 25, 4372, 25, 3042, 3086, 24, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan(c+dx)(a+b\sec(c+dx))\,dx$$

$$\downarrow 3042$$

$$\int -\cot\left(c+dx+\frac{\pi}{2}\right)\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)dx$$

$$\downarrow 25$$

$$-\int \cot\left(\frac{1}{2}(2c+\pi)+dx\right)\left(a+b\csc\left(\frac{1}{2}(2c+\pi)+dx\right)\right)dx$$

$$\downarrow 4372$$

$$-a\int -\tan(c+dx)dx - b\int -\sec(c+dx)\tan(c+dx)dx$$

$$\downarrow 25$$

$$a\int \tan(c+dx)dx + b\int \sec(c+dx)\tan(c+dx)dx$$

$$\downarrow 3042$$

$$a\int \tan(c+dx)dx + b\int \sec(c+dx)\tan(c+dx)dx$$

$$\downarrow 3086$$

$$a\int \tan(c+dx)dx + \frac{b\int 1d\sec(c+dx)}{d}$$

$$\downarrow 24$$

$$a\int \tan(c+dx)dx + \frac{b\sec(c+dx)}{d}$$

$$\downarrow 3956$$

$$\frac{b\sec(c+dx)}{d} - \frac{a\log(\cos(c+dx))}{d}$$

input
```
Int[(a + b*Sec[c + d*x])*Tan[c + d*x],x]
```

output
```
-((a*Log[Cos[c + d*x]])/d) + (b*Sec[c + d*x])/d
```

3.259.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3086
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[a/f   Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2
), x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2
] &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4372
```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(e*Cot[c + d*x])^m, x], x] + Simp[b   Int[
(e*Cot[c + d*x])^m*Csc[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, m}, x]
```

3.259.4 Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

method	result	size
derivativedivides	$\frac{b\sec(dx+c)+a\ln(\sec(dx+c))}{d}$	23
default	$\frac{b\sec(dx+c)+a\ln(\sec(dx+c))}{d}$	23
parts	$\frac{a\ln\left(1+\tan(dx+c)^2\right)}{2d}+\frac{b\sec(dx+c)}{d}$	30
risch	$iax+\frac{2iac}{d}+\frac{2b\,e^{i(dx+c)}}{d\left(e^{2i(dx+c)}+1\right)}-\frac{a\ln\left(e^{2i(dx+c)}+1\right)}{d}$	61

input
```
int((a+b*sec(d*x+c))*tan(d*x+c),x,method=_RETURNVERBOSE)
```

output
```
1/d*(b*sec(d*x+c)+a*ln(sec(d*x+c)))
```

3.259.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.36

$$\int (a+b\sec(c+dx))\tan(c+dx)\,dx = -\frac{a\cos(dx+c)\log(-\cos(dx+c))-b}{d\cos(dx+c)}$$

input
```
integrate((a+b*sec(d*x+c))*tan(d*x+c),x, algorithm="fricas")
```

output
```
-(a*cos(d*x + c)*log(-cos(d*x + c)) - b)/(d*cos(d*x + c))
```

3.259.6 Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.48

$$\int (a+b\sec(c+dx))\tan(c+dx)\,dx = \begin{cases} \frac{a\log(\tan^2(c+dx)+1)}{2d}+\frac{b\sec(c+dx)}{d} & \text{for } d\neq 0 \\ x(a+b\sec(c))\tan(c) & \text{otherwise} \end{cases}$$

input
```
integrate((a+b*sec(d*x+c))*tan(d*x+c),x)
```

output
```
Piecewise((a*log(tan(c + d*x)**2 + 1)/(2*d) + b*sec(c + d*x)/d, Ne(d, 0)),
 (x*(a + b*sec(c))*tan(c), True))
```

3.259.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.04

$$\int (a + b\sec(c + dx))\tan(c + dx)\,dx = -\frac{a\log(\cos(dx + c)) - \frac{b}{\cos(dx+c)}}{d}$$

input
```
integrate((a+b*sec(d*x+c))*tan(d*x+c),x, algorithm="maxima")
```

output
```
-(a*log(cos(d*x + c)) - b/cos(d*x + c))/d
```

3.259.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 107 vs. 2(25) = 50.

Time = 0.32 (sec) , antiderivative size = 107, normalized size of antiderivative = 4.28

$$\int (a + b\sec(c + dx))\tan(c + dx)\,dx$$
$$= \frac{a\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right) - a\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right) + \frac{a+2b+\frac{a(\cos(dx+c)-1)}{\cos(dx+c)+1}}{\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1}}{d}$$

input
```
integrate((a+b*sec(d*x+c))*tan(d*x+c),x, algorithm="giac")
```

output
```
(a*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)) - a*log(abs(-(cos(
d*x + c) - 1)/(cos(d*x + c) + 1) - 1)) + (a + 2*b + a*(cos(d*x + c) - 1)/(
cos(d*x + c) + 1))/((cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1))/d
```

3.259.9 Mupad [B] (verification not implemented)

Time = 14.90 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.60

$$\int (a + b\sec(c + dx))\tan(c + dx)\,dx = \frac{2\,a\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2\right)}{d} - \frac{2\,b}{d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2 - 1\right)}$$

input
```
int(tan(c + d*x)*(a + b/cos(c + d*x)),x)
```

output
```
(2*a*atanh(tan(c/2 + (d*x)/2)^2))/d - (2*b)/(d*(tan(c/2 + (d*x)/2)^2 - 1))
```

3.260 $\int \cot(c+dx)(a+b\sec(c+dx))\,dx$

3.260.1 Optimal result 1908
3.260.2 Mathematica [A] (verified) 1908
3.260.3 Rubi [A] (verified) 1909
3.260.4 Maple [A] (verified) 1911
3.260.5 Fricas [A] (verification not implemented) 1911
3.260.6 Sympy [F] 1911
3.260.7 Maxima [A] (verification not implemented) 1912
3.260.8 Giac [A] (verification not implemented) 1912
3.260.9 Mupad [B] (verification not implemented) 1912

3.260.1 Optimal result

Integrand size = 17, antiderivative size = 43

$$\int \cot(c+dx)(a+b\sec(c+dx))\,dx = \frac{(a+b)\log(1-\cos(c+dx))}{2d} + \frac{(a-b)\log(1+\cos(c+dx))}{2d}$$

output
```
1/2*(a+b)*ln(1-cos(d*x+c))/d+1/2*(a-b)*ln(1+cos(d*x+c))/d
```

3.260.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.49

$$\int \cot(c+dx)(a+b\sec(c+dx))\,dx = -\frac{b\log\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{d} + \frac{a\log(\cos(c+dx))}{d} + \frac{b\log\left(\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{d} + \frac{a\log(\tan(c+dx))}{d}$$

input
```
Integrate[Cot[c + d*x]*(a + b*Sec[c + d*x]),x]
```

output
```
-((b*Log[Cos[c/2 + (d*x)/2]])/d) + (a*Log[Cos[c + d*x]])/d + (b*Log[Sin[c/
2 + (d*x)/2]])/d + (a*Log[Tan[c + d*x]])/d
```

3.260.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.95, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.529$, Rules used = {3042, 25, 4371, 3042, 25, 3147, 452, 219, 240}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot(c+dx)(a+b\sec(c+dx))\,dx$$

$$\downarrow 3042$$

$$\int -\frac{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}{\cot\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\downarrow 25$$

$$-\int \frac{a+b\csc\left(\frac{1}{2}(2c+\pi)+dx\right)}{\cot\left(\frac{1}{2}(2c+\pi)+dx\right)}dx$$

$$\downarrow 4371$$

$$-\int \sec\left(\frac{1}{2}(2c+\pi)+dx\right)\left(b+a\sin\left(\frac{1}{2}(2c+\pi)+dx\right)\right)dx$$

$$\downarrow 3042$$

$$-\int -\frac{b-a\sin\left(c+dx-\frac{\pi}{2}\right)}{\cos\left(c+dx-\frac{\pi}{2}\right)}dx$$

$$\downarrow 25$$

$$\int \frac{b-a\sin\left(\frac{1}{2}(2c-\pi)+dx\right)}{\cos\left(\frac{1}{2}(2c-\pi)+dx\right)}dx$$

$$\downarrow 3147$$

$$-\frac{a\int \frac{b+a\cos(c+dx)}{a^2-a^2\cos^2(c+dx)}d(a\cos(c+dx))}{d}$$

$$\downarrow 452$$

$$-\frac{a\left(b\int \frac{1}{a^2-a^2\cos^2(c+dx)}d(a\cos(c+dx))+\int \frac{a\cos(c+dx)}{a^2-a^2\cos^2(c+dx)}d(a\cos(c+dx))\right)}{d}$$

$$\downarrow 219$$

$$-\frac{a\left(\int \frac{a\cos(c+dx)}{a^2-a^2\cos^2(c+dx)}d(a\cos(c+dx))+\frac{b\operatorname{arctanh}(\cos(c+dx))}{a}\right)}{d}$$

$$\downarrow 240$$

$$-\frac{a\left(\frac{b\operatorname{arctanh}(\cos(c+dx))}{a}-\frac{1}{2}\log\left(a^2-a^2\cos^2(c+dx)\right)\right)}{d}$$

input
```
Int[Cot[c + d*x]*(a + b*Sec[c + d*x]),x]
```

output
```
-((a*((b*ArcTanh[Cos[c + d*x]])/a - Log[a^2 - a^2*Cos[c + d*x]^2]/2))/d)
```

3.260.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 240
```
Int[(x_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[Log[RemoveContent[a + b*x
^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]
```

rule 452
```
Int[((c_) + (d_.)*(x_))/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[c   Int[1/
(a + b*x^2), x], x] + Simp[d   Int[x/(a + b*x^2), x], x] /; FreeQ[{a, b, c,
 d}, x] && NeQ[b*c^2 + a*d^2, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3147
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Simp[1/(b^p*f)   Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)
/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
 - 1)/2] && NeQ[a^2 - b^2, 0]
```

rule 4371
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))/cot[(c_.) + (d_.)*(x_)], x_Symbo
l] :> Int[(b + a*Sin[c + d*x])/Cos[c + d*x], x] /; FreeQ[{a, b, c, d}, x]
```

3.260.4 Maple [A] (verified)

Time = 0.47 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.77

method	result	size
derivativedivides	$\frac{a\ln(\sin(dx+c))+b\ln(-\cot(dx+c)+\csc(dx+c))}{d}$	33
default	$\frac{a\ln(\sin(dx+c))+b\ln(-\cot(dx+c)+\csc(dx+c))}{d}$	33
risch	$-iax-\frac{2iac}{d}+\frac{a\ln(e^{i(dx+c)}-1)}{d}+\frac{\ln(e^{i(dx+c)}-1)b}{d}+\frac{a\ln(e^{i(dx+c)}+1)}{d}-\frac{\ln(e^{i(dx+c)}+1)b}{d}$	84

input
```
int(cot(d*x+c)*(a+b*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(a*ln(sin(d*x+c))+b*ln(-cot(d*x+c)+csc(d*x+c)))
```

3.260.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.88

$$\int \cot(c+dx)(a+b\sec(c+dx))\,dx = \frac{(a-b)\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)+(a+b)\log\left(-\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)}{2\,d}$$

input
```
integrate(cot(d*x+c)*(a+b*sec(d*x+c)),x, algorithm="fricas")
```

output
```
1/2*((a - b)*log(1/2*cos(d*x + c) + 1/2) + (a + b)*log(-1/2*cos(d*x + c) +
 1/2))/d
```

3.260.6 Sympy [F]

$$\int \cot(c+dx)(a+b\sec(c+dx))\,dx = \int (a+b\sec(c+dx))\cot(c+dx)\,dx$$

input
```
integrate(cot(d*x+c)*(a+b*sec(d*x+c)),x)
```

output
```
Integral((a + b*sec(c + d*x))*cot(c + d*x), x)
```

3.260.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.79

$$\int \cot(c+dx)(a+b\sec(c+dx))\,dx = \frac{(a-b)\log(\cos(dx+c)+1)+(a+b)\log(\cos(dx+c)-1)}{2\,d}$$

input
```
integrate(cot(d*x+c)*(a+b*sec(d*x+c)),x, algorithm="maxima")
```

output
```
1/2*((a - b)*log(cos(d*x + c) + 1) + (a + b)*log(cos(d*x + c) - 1))/d
```

3.260.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.42

$$\int \cot(c+dx)(a+b\sec(c+dx))\,dx = \frac{(a+b)\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)-2\,a\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right)}{2\,d}$$

input
```
integrate(cot(d*x+c)*(a+b*sec(d*x+c)),x, algorithm="giac")
```

output
```
1/2*((a + b)*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1)) - 2*a*log(a
bs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)))/d
```

3.260.9 Mupad [B] (verification not implemented)

Time = 14.48 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.19

$$\int \cot(c+dx)(a+b\sec(c+dx))\,dx = \frac{a\,\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{d} - \frac{a\,\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+1\right)}{d} + \frac{b\,\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{d}$$

input
```
int(cot(c + d*x)*(a + b/cos(c + d*x)),x)
```

output
```
(a*log(tan(c/2 + (d*x)/2)))/d - (a*log(tan(c/2 + (d*x)/2)^2 + 1))/d + (b*l
og(tan(c/2 + (d*x)/2)))/d
```

3.261 $\int \cot^3(c+dx)(a+b\sec(c+dx))\,dx$

3.261.1 Optimal result 1913
3.261.2 Mathematica [A] (verified) 1913
3.261.3 Rubi [A] (verified) 1914
3.261.4 Maple [A] (verified) 1916
3.261.5 Fricas [A] (verification not implemented) 1917
3.261.6 Sympy [F] 1917
3.261.7 Maxima [A] (verification not implemented) 1918
3.261.8 Giac [B] (verification not implemented) 1918
3.261.9 Mupad [B] (verification not implemented) 1919

3.261.1 Optimal result

Integrand size = 19, antiderivative size = 72

$$\int \cot^3(c+dx)(a+b\sec(c+dx))\,dx = -\frac{(2a+b)\log(1-\cos(c+dx))}{4d} - \frac{(2a-b)\log(1+\cos(c+dx))}{4d} - \frac{\cot^2(c+dx)(a+b\sec(c+dx))}{2d}$$

output
```
-1/4*(2*a+b)*ln(1-cos(d*x+c))/d-1/4*(2*a-b)*ln(1+cos(d*x+c))/d-1/2*cot(d*x
+c)^2*(a+b*sec(d*x+c))/d
```

3.261.2 Mathematica [A] (verified)

Time = 0.79 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.58

$$\int \cot^3(c+dx)(a+b\sec(c+dx))\,dx = -\frac{b\csc^2\left(\frac{1}{2}(c+dx)\right)}{8d} + \frac{b\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{2d} - \frac{b\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{2d} - \frac{a(\cot^2(c+dx)+2\log(\cos(c+dx))+2\log(\tan(c+dx)))}{2d} + \frac{b\sec^2\left(\frac{1}{2}(c+dx)\right)}{8d}$$

input
```
Integrate[Cot[c + d*x]^3*(a + b*Sec[c + d*x]),x]
```

output
```
-1/8*(b*Csc[(c + d*x)/2]^2)/d + (b*Log[Cos[(c + d*x)/2]])/(2*d) - (b*Log[S
in[(c + d*x)/2]])/(2*d) - (a*(Cot[c + d*x]^2 + 2*Log[Cos[c + d*x]] + 2*Log
[Tan[c + d*x]]))/(2*d) + (b*Sec[(c + d*x)/2]^2)/(8*d)
```

3.261.3 Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.99, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.632$, Rules used = {3042, 25, 4370, 3042, 25, 4371, 3042, 25, 3147, 452, 219, 240}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^3(c+dx)(a+b\sec(c+dx))\,dx$$

$$\downarrow 3042$$

$$\int -\frac{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}{\cot\left(c+dx+\frac{\pi}{2}\right)^3}dx$$

$$\downarrow 25$$

$$-\int \frac{a+b\csc\left(\frac{1}{2}(2c+\pi)+dx\right)}{\cot\left(\frac{1}{2}(2c+\pi)+dx\right)^3}dx$$

$$\downarrow 4370$$

$$-\frac{1}{2}\int \cot(c+dx)(2a+b\sec(c+dx))dx - \frac{\cot^2(c+dx)(a+b\sec(c+dx))}{2d}$$

$$\downarrow 3042$$

$$-\frac{1}{2}\int -\frac{2a+b\csc\left(c+dx+\frac{\pi}{2}\right)}{\cot\left(c+dx+\frac{\pi}{2}\right)}dx - \frac{\cot^2(c+dx)(a+b\sec(c+dx))}{2d}$$

$$\downarrow 25$$

$$\frac{1}{2}\int \frac{2a+b\csc\left(\frac{1}{2}(2c+\pi)+dx\right)}{\cot\left(\frac{1}{2}(2c+\pi)+dx\right)}dx - \frac{\cot^2(c+dx)(a+b\sec(c+dx))}{2d}$$

$$\downarrow 4371$$

$$\frac{1}{2}\int \sec\left(\frac{1}{2}(2c+\pi)+dx\right)\left(b+2a\sin\left(\frac{1}{2}(2c+\pi)+dx\right)\right)dx - \frac{\cot^2(c+dx)(a+b\sec(c+dx))}{2d}$$

$$\downarrow 3042$$

$$\frac{1}{2}\int -\frac{b-2a\sin\left(c+dx-\frac{\pi}{2}\right)}{\cos\left(c+dx-\frac{\pi}{2}\right)}dx-\frac{\cot^2(c+dx)(a+b\sec(c+dx))}{2d}$$

$$\downarrow 25$$

$$-\frac{1}{2}\int \frac{b-2a\sin\left(\frac{1}{2}(2c-\pi)+dx\right)}{\cos\left(\frac{1}{2}(2c-\pi)+dx\right)}dx-\frac{\cot^2(c+dx)(a+b\sec(c+dx))}{2d}$$

$$\downarrow 3147$$

$$\frac{a\int \frac{b+2a\cos(c+dx)}{4a^2-4a^2\cos^2(c+dx)}d(2a\cos(c+dx))}{d}-\frac{\cot^2(c+dx)(a+b\sec(c+dx))}{2d}$$

$$\downarrow 452$$

$$\frac{a\left(b\int \frac{1}{4a^2-4a^2\cos^2(c+dx)}d(2a\cos(c+dx))+\int \frac{2a\cos(c+dx)}{4a^2-4a^2\cos^2(c+dx)}d(2a\cos(c+dx))\right)}{d}-\frac{\cot^2(c+dx)(a+b\sec(c+dx))}{2d}$$

$$\downarrow 219$$

$$\frac{a\left(\int \frac{2a\cos(c+dx)}{4a^2-4a^2\cos^2(c+dx)}d(2a\cos(c+dx))+\frac{b\operatorname{arctanh}(\cos(c+dx))}{2a}\right)}{d}-\frac{\cot^2(c+dx)(a+b\sec(c+dx))}{2d}$$

$$\downarrow 240$$

$$\frac{a\left(\frac{b\operatorname{arctanh}(\cos(c+dx))}{2a}-\frac{1}{2}\log\left(4a^2-4a^2\cos^2(c+dx)\right)\right)}{d}-\frac{\cot^2(c+dx)(a+b\sec(c+dx))}{2d}$$

input
```
Int[Cot[c + d*x]^3*(a + b*Sec[c + d*x]),x]
```

output
```
(a*((b*ArcTanh[Cos[c + d*x]])/(2*a) - Log[4*a^2 - 4*a^2*Cos[c + d*x]^2]/2)
)/d - (Cot[c + d*x]^2*(a + b*Sec[c + d*x]))/(2*d)
```

3.261.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 240
```
Int[(x_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[Log[RemoveContent[a + b*x
^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]
```

rule 452
```
Int[((c_) + (d_.)*(x_))/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[c   Int[1/
(a + b*x^2), x], x] + Simp[d   Int[x/(a + b*x^2), x], x] /; FreeQ[{a, b, c,
 d}, x] && NeQ[b*c^2 + a*d^2, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3147
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Simp[1/(b^p*f)   Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)
/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
 - 1)/2] && NeQ[a^2 - b^2, 0]
```

rule 4370
```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(-(e*Cot[c + d*x])^(m + 1))*((a + b*Csc[c + d*x])/(
d*e*(m + 1))), x] - Simp[1/(e^2*(m + 1))   Int[(e*Cot[c + d*x])^(m + 2)*(a*
(m + 1) + b*(m + 2)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && L
tQ[m, -1]
```

rule 4371
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))/cot[(c_.) + (d_.)*(x_)], x_Symbo
l] :> Int[(b + a*Sin[c + d*x])/Cos[c + d*x], x] /; FreeQ[{a, b, c, d}, x]
```

3.261.4 Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.04

method	result
derivativedivides	$\frac{a\left(-\frac{\cot(dx+c)^2}{2}-\ln(\sin(dx+c))\right)+b\left(-\frac{\cos(dx+c)^3}{2\sin(dx+c)^2}-\frac{\cos(dx+c)}{2}-\frac{\ln(-\cot(dx+c)+\csc(dx+c))}{2}\right)}{d}$
default	$\frac{a\left(-\frac{\cot(dx+c)^2}{2}-\ln(\sin(dx+c))\right)+b\left(-\frac{\cos(dx+c)^3}{2\sin(dx+c)^2}-\frac{\cos(dx+c)}{2}-\frac{\ln(-\cot(dx+c)+\csc(dx+c))}{2}\right)}{d}$
risch	$iax+\frac{2iac}{d}+\frac{b\mathrm{e}^{3i(dx+c)}+2a\mathrm{e}^{2i(dx+c)}+b\mathrm{e}^{i(dx+c)}}{d\left(\mathrm{e}^{2i(dx+c)}-1\right)^2}-\frac{a\ln\left(\mathrm{e}^{i(dx+c)}-1\right)}{d}-\frac{\ln\left(\mathrm{e}^{i(dx+c)}-1\right)b}{2d}-\frac{a\ln\left(\mathrm{e}^{i(dx+c)}+1\right)}{d}$

input
```
int(cot(d*x+c)^3*(a+b*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(a*(-1/2*cot(d*x+c)^2-ln(sin(d*x+c)))+b*(-1/2/sin(d*x+c)^2*cos(d*x+c)^
3-1/2*cos(d*x+c)-1/2*ln(-cot(d*x+c)+csc(d*x+c))))
```

3.261.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.38

$$\int \cot^3(c+dx)(a+b\sec(c+dx))\,dx$$
$$= \frac{2\,b\cos(dx+c) - \left((2\,a-b)\cos(dx+c)^2 - 2\,a+b\right)\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right) - \left((2\,a+b)\cos(dx+c)^2 - 2\right.}{4\left(d\cos(dx+c)^2-d\right)}$$

input
```
integrate(cot(d*x+c)^3*(a+b*sec(d*x+c)),x, algorithm="fricas")
```

output
```
1/4*(2*b*cos(d*x + c) - ((2*a - b)*cos(d*x + c)^2 - 2*a + b)*log(1/2*cos(d
*x + c) + 1/2) - ((2*a + b)*cos(d*x + c)^2 - 2*a - b)*log(-1/2*cos(d*x + c
) + 1/2) + 2*a)/(d*cos(d*x + c)^2 - d)
```

3.261.6 Sympy [F]

$$\int \cot^3(c+dx)(a+b\sec(c+dx))\,dx = \int (a+b\sec(c+dx))\cot^3(c+dx)\,dx$$

input
```
integrate(cot(d*x+c)**3*(a+b*sec(d*x+c)),x)
```

output
```
Integral((a + b*sec(c + d*x))*cot(c + d*x)**3, x)
```

3.261.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.86

$$\int \cot^3(c+dx)(a+b\sec(c+dx))\,dx$$
$$= -\frac{(2a-b)\log(\cos(dx+c)+1)+(2a+b)\log(\cos(dx+c)-1)-\frac{2(b\cos(dx+c)+a)}{\cos(dx+c)^2-1}}{4d}$$

input
```
integrate(cot(d*x+c)^3*(a+b*sec(d*x+c)),x, algorithm="maxima")
```

output
```
-1/4*((2*a - b)*log(cos(d*x + c) + 1) + (2*a + b)*log(cos(d*x + c) - 1) -
2*(b*cos(d*x + c) + a)/(cos(d*x + c)^2 - 1))/d
```

3.261.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 170 vs. 2(66) = 132.

Time = 0.30 (sec) , antiderivative size = 170, normalized size of antiderivative = 2.36

$$\int \cot^3(c+dx)(a+b\sec(c+dx))\,dx =$$
$$-\frac{2(2a+b)\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)-8a\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right)-\frac{\left(a+b+\frac{4a(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{2b(\cos(dx+c)-1)}{\cos(dx+c)+1}\right)(\cos(dx+c)}{\cos(dx+c)-1}}{8d}$$

input
```
integrate(cot(d*x+c)^3*(a+b*sec(d*x+c)),x, algorithm="giac")
```

output
```
-1/8*(2*(2*a + b)*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1)) - 8*a*
log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)) - (a + b + 4*a*(cos(d
*x + c) - 1)/(cos(d*x + c) + 1) + 2*b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1
))*(cos(d*x + c) + 1)/(cos(d*x + c) - 1) - a*(cos(d*x + c) - 1)/(cos(d*x +
 c) + 1) + b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1))/d
```

3.261.9 Mupad [B] (verification not implemented)

Time = 14.47 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.19

$$\int \cot^3(c+dx)(a+b\sec(c+dx))\,dx = \frac{a\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+1\right)}{d} - \frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2\left(\frac{a}{8}-\frac{b}{8}\right)}{d} - \frac{\cot\left(\frac{c}{2}+\frac{dx}{2}\right)^2\left(\frac{a}{8}+\frac{b}{8}\right)}{d} - \frac{\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)\left(a+\frac{b}{2}\right)}{d}$$

input
```
int(cot(c + d*x)^3*(a + b/cos(c + d*x)),x)
```

output
```
(a*log(tan(c/2 + (d*x)/2)^2 + 1))/d - (tan(c/2 + (d*x)/2)^2*(a/8 - b/8))/d
 - (cot(c/2 + (d*x)/2)^2*(a/8 + b/8))/d - (log(tan(c/2 + (d*x)/2))*(a + b/
2))/d
```

3.262 $\int \cot^5(c+dx)(a+b\sec(c+dx))\,dx$

3.262.1 Optimal result 1920
3.262.2 Mathematica [A] (verified) 1921
3.262.3 Rubi [A] (verified) 1921
3.262.4 Maple [A] (verified) 1925
3.262.5 Fricas [A] (verification not implemented) 1925
3.262.6 Sympy [F] 1926
3.262.7 Maxima [A] (verification not implemented) 1926
3.262.8 Giac [B] (verification not implemented) 1926
3.262.9 Mupad [B] (verification not implemented) 1927

3.262.1 Optimal result

Integrand size = 19, antiderivative size = 102

$$\begin{aligned}\int \cot^5(c+dx)(a+b\sec(c+dx))\,dx = &\frac{(8a+3b)\log(1-\cos(c+dx))}{16d}\\ &+\frac{(8a-3b)\log(1+\cos(c+dx))}{16d}\\ &-\frac{\cot^4(c+dx)(a+b\sec(c+dx))}{4d}\\ &+\frac{\cot^2(c+dx)(4a+3b\sec(c+dx))}{8d}\end{aligned}$$

output

```
1/16*(8*a+3*b)*ln(1-cos(d*x+c))/d+1/16*(8*a-3*b)*ln(1+cos(d*x+c))/d-1/4*co
t(d*x+c)^4*(a+b*sec(d*x+c))/d+1/8*cot(d*x+c)^2*(4*a+3*b*sec(d*x+c))/d
```

3.262.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.72

$$\begin{aligned}\int \cot^5(c+dx)(a+b\sec(c+dx))\,dx = &\frac{a\cot^2(c+dx)}{2d} - \frac{a\cot^4(c+dx)}{4d} \\ &+ \frac{5b\csc^2\left(\frac{1}{2}(c+dx)\right)}{32d} - \frac{b\csc^4\left(\frac{1}{2}(c+dx)\right)}{64d} \\ &- \frac{3b\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{8d} + \frac{a\log(\cos(c+dx))}{d} \\ &+ \frac{3b\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{8d} + \frac{a\log(\tan(c+dx))}{d} \\ &- \frac{5b\sec^2\left(\frac{1}{2}(c+dx)\right)}{32d} + \frac{b\sec^4\left(\frac{1}{2}(c+dx)\right)}{64d}\end{aligned}$$

input
```
Integrate[Cot[c + d*x]^5*(a + b*Sec[c + d*x]),x]
```

output
```
(a*Cot[c + d*x]^2)/(2*d) - (a*Cot[c + d*x]^4)/(4*d) + (5*b*Csc[(c + d*x)/2
]^2)/(32*d) - (b*Csc[(c + d*x)/2]^4)/(64*d) - (3*b*Log[Cos[(c + d*x)/2]])/
(8*d) + (a*Log[Cos[c + d*x]])/d + (3*b*Log[Sin[(c + d*x)/2]])/(8*d) + (a*L
og[Tan[c + d*x]])/d - (5*b*Sec[(c + d*x)/2]^2)/(32*d) + (b*Sec[(c + d*x)/2
]^4)/(64*d)
```

3.262.3 Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.03, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.789$, Rules used = {3042, 25, 4370, 3042, 25, 4370, 3042, 25, 4371, 3042, 25, 3147, 452, 219, 240}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^5(c+dx)(a+b\sec(c+dx))\,dx$$

$$\downarrow 3042$$

$$\int -\frac{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}{\cot\left(c+dx+\frac{\pi}{2}\right)^5}dx$$

$$\downarrow 25$$

$$-\int \frac{a+b\csc\left(\frac{1}{2}(2c+\pi)+dx\right)}{\cot\left(\frac{1}{2}(2c+\pi)+dx\right)^5}dx$$

$$\downarrow 4370$$

$$-\frac{1}{4}\int \cot^3(c+dx)(4a+3b\sec(c+dx))dx - \frac{\cot^4(c+dx)(a+b\sec(c+dx))}{4d}$$

$$\downarrow 3042$$

$$-\frac{1}{4}\int -\frac{4a+3b\csc\left(c+dx+\frac{\pi}{2}\right)}{\cot\left(c+dx+\frac{\pi}{2}\right)^3}dx - \frac{\cot^4(c+dx)(a+b\sec(c+dx))}{4d}$$

$$\downarrow 25$$

$$\frac{1}{4}\int \frac{4a+3b\csc\left(\frac{1}{2}(2c+\pi)+dx\right)}{\cot\left(\frac{1}{2}(2c+\pi)+dx\right)^3}dx - \frac{\cot^4(c+dx)(a+b\sec(c+dx))}{4d}$$

$$\downarrow 4370$$

$$\frac{1}{4}\left(\frac{1}{2}\int \cot(c+dx)(8a+3b\sec(c+dx))dx + \frac{\cot^2(c+dx)(4a+3b\sec(c+dx))}{2d}\right) - \frac{\cot^4(c+dx)(a+b\sec(c+dx))}{4d}$$

$$\downarrow 3042$$

$$\frac{1}{4}\left(\frac{1}{2}\int -\frac{8a+3b\csc\left(c+dx+\frac{\pi}{2}\right)}{\cot\left(c+dx+\frac{\pi}{2}\right)}dx + \frac{\cot^2(c+dx)(4a+3b\sec(c+dx))}{2d}\right) - \frac{\cot^4(c+dx)(a+b\sec(c+dx))}{4d}$$

$$\downarrow 25$$

$$\frac{1}{4}\left(\frac{\cot^2(c+dx)(4a+3b\sec(c+dx))}{2d} - \frac{1}{2}\int \frac{8a+3b\csc\left(\frac{1}{2}(2c+\pi)+dx\right)}{\cot\left(\frac{1}{2}(2c+\pi)+dx\right)}dx\right) - \frac{\cot^4(c+dx)(a+b\sec(c+dx))}{4d}$$

$$\downarrow 4371$$

$$\frac{1}{4}\left(\frac{\cot^2(c+dx)(4a+3b\sec(c+dx))}{2d} - \frac{1}{2}\int \sec\left(\frac{1}{2}(2c+\pi)+dx\right)\left(3b+8a\sin\left(\frac{1}{2}(2c+\pi)+dx\right)\right)dx\right) - \frac{\cot^4(c+dx)(a+b\sec(c+dx))}{4d}$$

$$\downarrow 3042$$

$$\frac{1}{4}\left(\frac{\cot^2(c+dx)(4a+3b\sec(c+dx))}{2d}-\frac{1}{2}\int-\frac{3b-8a\sin\left(c+dx-\frac{\pi}{2}\right)}{\cos\left(c+dx-\frac{\pi}{2}\right)}dx\right)-\frac{\cot^4(c+dx)(a+b\sec(c+dx))}{4d}$$

$$\downarrow 25$$

$$\frac{1}{4}\left(\frac{1}{2}\int\frac{3b-8a\sin\left(\frac{1}{2}(2c-\pi)+dx\right)}{\cos\left(\frac{1}{2}(2c-\pi)+dx\right)}dx+\frac{\cot^2(c+dx)(4a+3b\sec(c+dx))}{2d}\right)-\frac{\cot^4(c+dx)(a+b\sec(c+dx))}{4d}$$

$$\downarrow 3147$$

$$\frac{1}{4}\left(\frac{\cot^2(c+dx)(4a+3b\sec(c+dx))}{2d}-\frac{4a\int\frac{3b+8a\cos(c+dx)}{64a^2-64a^2\cos^2(c+dx)}d(8a\cos(c+dx))}{d}\right)-\frac{\cot^4(c+dx)(a+b\sec(c+dx))}{4d}$$

$$\downarrow 452$$

$$\frac{1}{4}\left(\frac{\cot^2(c+dx)(4a+3b\sec(c+dx))}{2d}-\frac{4a\left(3b\int\frac{1}{64a^2-64a^2\cos^2(c+dx)}d(8a\cos(c+dx))+\int\frac{8a\cos(c+dx)}{64a^2-64a^2\cos^2(c+dx)}d(8a\right.}{d}\right.$$

$$\frac{\cot^4(c+dx)(a+b\sec(c+dx))}{4d}$$

$$\downarrow 219$$

$$\frac{1}{4}\left(\frac{\cot^2(c+dx)(4a+3b\sec(c+dx))}{2d}-\frac{4a\left(\int\frac{8a\cos(c+dx)}{64a^2-64a^2\cos^2(c+dx)}d(8a\cos(c+dx))+\frac{3b\operatorname{arctanh}(\cos(c+dx))}{8a}\right)}{d}\right)-$$

$$\frac{\cot^4(c+dx)(a+b\sec(c+dx))}{4d}$$

$$\downarrow 240$$

$$\frac{1}{4}\left(\frac{\cot^2(c+dx)(4a+3b\sec(c+dx))}{2d}-\frac{4a\left(\frac{3b\operatorname{arctanh}(\cos(c+dx))}{8a}-\frac{1}{2}\log\left(64a^2-64a^2\cos^2(c+dx)\right)\right)}{d}\right)-$$

$$\frac{\cot^4(c+dx)(a+b\sec(c+dx))}{4d}$$

input
```
Int[Cot[c + d*x]^5*(a + b*Sec[c + d*x]),x]
```

output
```
-1/4*(Cot[c + d*x]^4*(a + b*Sec[c + d*x]))/d + ((-4*a*((3*b*ArcTanh[Cos[c
+ d*x]])/(8*a) - Log[64*a^2 - 64*a^2*Cos[c + d*x]^2]/2))/d + (Cot[c + d*x]
^2*(4*a + 3*b*Sec[c + d*x]))/(2*d))/4
```

3.262.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 240
```
Int[(x_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[Log[RemoveContent[a + b*x
^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]
```

rule 452
```
Int[((c_) + (d_.)*(x_))/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[c   Int[1/
(a + b*x^2), x], x] + Simp[d   Int[x/(a + b*x^2), x], x] /; FreeQ[{a, b, c,
 d}, x] && NeQ[b*c^2 + a*d^2, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3147
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Simp[1/(b^p*f)   Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)
/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
 - 1)/2] && NeQ[a^2 - b^2, 0]
```

rule 4370
```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(-(e*Cot[c + d*x])^(m + 1))*((a + b*Csc[c + d*x])/(
d*e*(m + 1))), x] - Simp[1/(e^2*(m + 1))   Int[(e*Cot[c + d*x])^(m + 2)*(a*
(m + 1) + b*(m + 2)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && L
tQ[m, -1]
```

rule 4371
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))/cot[(c_.) + (d_.)*(x_)], x_Symbo
l] :> Int[(b + a*Sin[c + d*x])/Cos[c + d*x], x] /; FreeQ[{a, b, c, d}, x]
```

3.262.4 Maple [A] (verified)

Time = 0.72 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.09

method	result
derivativedivides	$\frac{a\left(-\frac{\cot(dx+c)^4}{4}+\frac{\cot(dx+c)^2}{2}+\ln(\sin(dx+c))\right)+b\left(-\frac{\cos(dx+c)^5}{4\sin(dx+c)^4}+\frac{\cos(dx+c)^5}{8\sin(dx+c)^2}+\frac{\cos(dx+c)^3}{8}+\frac{3\cos(dx+c)}{8}+\frac{3\ln(-\cot(dx+}{}$
default	$\frac{a\left(-\frac{\cot(dx+c)^4}{4}+\frac{\cot(dx+c)^2}{2}+\ln(\sin(dx+c))\right)+b\left(-\frac{\cos(dx+c)^5}{4\sin(dx+c)^4}+\frac{\cos(dx+c)^5}{8\sin(dx+c)^2}+\frac{\cos(dx+c)^3}{8}+\frac{3\cos(dx+c)}{8}+\frac{3\ln(-\cot(dx+}{}$
risch	$-iax-\frac{2iac}{d}-\frac{5b\,e^{7i(dx+c)}+16a\,e^{6i(dx+c)}+3b\,e^{5i(dx+c)}-16a\,e^{4i(dx+c)}+3b\,e^{3i(dx+c)}+16a\,e^{2i(dx+c)}+5b\,e^{i(dx+c)}}{4d\left(e^{2i(dx+c)}-1\right)^4}$

input
```
int(cot(d*x+c)^5*(a+b*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(a*(-1/4*cot(d*x+c)^4+1/2*cot(d*x+c)^2+ln(sin(d*x+c)))+b*(-1/4/sin(d*x
+c)^4*cos(d*x+c)^5+1/8/sin(d*x+c)^2*cos(d*x+c)^5+1/8*cos(d*x+c)^3+3/8*cos(
d*x+c)+3/8*ln(-cot(d*x+c)+csc(d*x+c))))
```

3.262.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.65

$$\int \cot^5(c+dx)(a+b\sec(c+dx))\,dx =$$

$$-\frac{10\,b\cos(dx+c)^3+16\,a\cos(dx+c)^2-6\,b\cos(dx+c)-\left((8\,a-3\,b)\cos(dx+c)^4-2\,(8\,a-3\,b)\cos(}{}$$

input
```
integrate(cot(d*x+c)^5*(a+b*sec(d*x+c)),x, algorithm="fricas")
```

output
```
-1/16*(10*b*cos(d*x + c)^3 + 16*a*cos(d*x + c)^2 - 6*b*cos(d*x + c) - ((8*
a - 3*b)*cos(d*x + c)^4 - 2*(8*a - 3*b)*cos(d*x + c)^2 + 8*a - 3*b)*log(1/
2*cos(d*x + c) + 1/2) - ((8*a + 3*b)*cos(d*x + c)^4 - 2*(8*a + 3*b)*cos(d*
x + c)^2 + 8*a + 3*b)*log(-1/2*cos(d*x + c) + 1/2) - 12*a)/(d*cos(d*x + c)
^4 - 2*d*cos(d*x + c)^2 + d)
```

3.262.6 Sympy [F]

$$\int \cot^5(c+dx)(a+b\sec(c+dx))\,dx = \int (a+b\sec(c+dx))\cot^5(c+dx)\,dx$$

input
```
integrate(cot(d*x+c)**5*(a+b*sec(d*x+c)),x)
```

output
```
Integral((a + b*sec(c + d*x))*cot(c + d*x)**5, x)
```

3.262.7 Maxima [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.97

$$\int \cot^5(c+dx)(a+b\sec(c+dx))\,dx$$
$$= \frac{(8a-3b)\log(\cos(dx+c)+1)+(8a+3b)\log(\cos(dx+c)-1)-\frac{2\left(5b\cos(dx+c)^3+8a\cos(dx+c)^2-3b\cos(dx+c\right.}{\cos(dx+c)^4-2\cos(dx+c)^2+1}}{16d}$$

input
```
integrate(cot(d*x+c)^5*(a+b*sec(d*x+c)),x, algorithm="maxima")
```

output
```
1/16*((8*a - 3*b)*log(cos(d*x + c) + 1) + (8*a + 3*b)*log(cos(d*x + c) - 1
) - 2*(5*b*cos(d*x + c)^3 + 8*a*cos(d*x + c)^2 - 3*b*cos(d*x + c) - 6*a)/(
cos(d*x + c)^4 - 2*cos(d*x + c)^2 + 1))/d
```

3.262.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 266 vs. 2(94) = 188.

Time = 0.34 (sec) , antiderivative size = 266, normalized size of antiderivative = 2.61

$$\int \cot^5(c+dx)(a+b\sec(c+dx))\,dx$$
$$= \frac{4(8a+3b)\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)-64a\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right)-\frac{\left(a+b+\frac{12a(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{8b(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{48a(\cos(d\right.}{(\cos(dx+c)}}{64d}$$

input
```
integrate(cot(d*x+c)^5*(a+b*sec(d*x+c)),x, algorithm="giac")
```

output
```
1/64*(4*(8*a + 3*b)*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1)) - 64
*a*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)) - (a + b + 12*a*(c
os(d*x + c) - 1)/(cos(d*x + c) + 1) + 8*b*(cos(d*x + c) - 1)/(cos(d*x + c)
 + 1) + 48*a*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 18*b*(cos(d*x + c
) - 1)^2/(cos(d*x + c) + 1)^2)*(cos(d*x + c) + 1)^2/(cos(d*x + c) - 1)^2 -
 12*a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 8*b*(cos(d*x + c) - 1)/(cos(
d*x + c) + 1) - a*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + b*(cos(d*x +
 c) - 1)^2/(cos(d*x + c) + 1)^2)/d
```

3.262.9 Mupad [B] (verification not implemented)

Time = 14.49 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.25

$$\int \cot^5(c+dx)(a+b\sec(c+dx))\,dx$$
$$= \frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2\left(\frac{3a}{16}-\frac{b}{8}\right)}{d} - \frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4\left(\frac{a}{64}-\frac{b}{64}\right)}{d} - \frac{a\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+1\right)}{d}$$
$$- \frac{\cot\left(\frac{c}{2}+\frac{dx}{2}\right)^4\left((-3a-2b)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+\frac{a}{4}+\frac{b}{4}\right)}{16d} + \frac{\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)\left(a+\frac{3b}{8}\right)}{d}$$

input
```
int(cot(c + d*x)^5*(a + b/cos(c + d*x)),x)
```

output
```
(tan(c/2 + (d*x)/2)^2*((3*a)/16 - b/8))/d - (tan(c/2 + (d*x)/2)^4*(a/64 -
b/64))/d - (a*log(tan(c/2 + (d*x)/2)^2 + 1))/d - (cot(c/2 + (d*x)/2)^4*(a/
4 + b/4 - tan(c/2 + (d*x)/2)^2*(3*a + 2*b)))/(16*d) + (log(tan(c/2 + (d*x)
/2))*(a + (3*b)/8))/d
```

3.263 $\int \cot^7(c+dx)(a+b\sec(c+dx))\,dx$

3.263.1 Optimal result 1928
3.263.2 Mathematica [A] (verified) 1929
3.263.3 Rubi [A] (verified) 1929
3.263.4 Maple [A] (verified) 1934
3.263.5 Fricas [A] (verification not implemented) 1934
3.263.6 Sympy [F] 1935
3.263.7 Maxima [A] (verification not implemented) 1935
3.263.8 Giac [B] (verification not implemented) 1935
3.263.9 Mupad [B] (verification not implemented) 1936

3.263.1 Optimal result

Integrand size = 19, antiderivative size = 130

$$\begin{aligned}\int \cot^7(c+dx)(a+b\sec(c+dx))\,dx = &-\frac{(16a+5b)\log(1-\cos(c+dx))}{32d}\\ &-\frac{(16a-5b)\log(1+\cos(c+dx))}{32d}\\ &-\frac{\cot^6(c+dx)(a+b\sec(c+dx))}{6d}\\ &+\frac{\cot^4(c+dx)(6a+5b\sec(c+dx))}{24d}\\ &-\frac{\cot^2(c+dx)(8a+5b\sec(c+dx))}{16d}\end{aligned}$$

output
```
-1/32*(16*a+5*b)*ln(1-cos(d*x+c))/d-1/32*(16*a-5*b)*ln(1+cos(d*x+c))/d-1/6
*cot(d*x+c)^6*(a+b*sec(d*x+c))/d+1/24*cot(d*x+c)^4*(6*a+5*b*sec(d*x+c))/d-
1/16*cot(d*x+c)^2*(8*a+5*b*sec(d*x+c))/d
```

3.263.2 Mathematica [A] (verified)

Time = 0.49 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.66

$$\int \cot^7(c+dx)(a+b\sec(c+dx))\,dx = -\frac{11b\csc^2\left(\frac{1}{2}(c+dx)\right)}{64d} + \frac{b\csc^4\left(\frac{1}{2}(c+dx)\right)}{32d} - \frac{b\csc^6\left(\frac{1}{2}(c+dx)\right)}{384d} + \frac{5b\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{16d} - \frac{5b\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{16d} - \frac{a(6\cot^2(c+dx) - 3\cot^4(c+dx) + 2\cot^6(c+dx) + 12\log(\cos(c+dx)) + 12\log(\tan(c+dx)))}{12d} + \frac{11b\sec^2\left(\frac{1}{2}(c+dx)\right)}{64d} - \frac{b\sec^4\left(\frac{1}{2}(c+dx)\right)}{32d} + \frac{b\sec^6\left(\frac{1}{2}(c+dx)\right)}{384d}$$

input
```
Integrate[Cot[c + d*x]^7*(a + b*Sec[c + d*x]),x]
```

output
```
(-11*b*Csc[(c + d*x)/2]^2)/(64*d) + (b*Csc[(c + d*x)/2]^4)/(32*d) - (b*Csc
[(c + d*x)/2]^6)/(384*d) + (5*b*Log[Cos[(c + d*x)/2]])/(16*d) - (5*b*Log[S
in[(c + d*x)/2]])/(16*d) - (a*(6*Cot[c + d*x]^2 - 3*Cot[c + d*x]^4 + 2*Cot
[c + d*x]^6 + 12*Log[Cos[c + d*x]] + 12*Log[Tan[c + d*x]]))/(12*d) + (11*b
*Sec[(c + d*x)/2]^2)/(64*d) - (b*Sec[(c + d*x)/2]^4)/(32*d) + (b*Sec[(c +
d*x)/2]^6)/(384*d)
```

3.263.3 Rubi [A] (verified)

Time = 0.74 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.06, number of steps used = 20, number of rules used = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 1.000$, Rules used = {3042, 25, 4370, 3042, 25, 4370, 27, 3042, 25, 4370, 3042, 25, 4371, 3042, 25, 3147, 452, 219, 240}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^7(c+dx)(a+b\sec(c+dx))\,dx$$

$$\downarrow 3042$$

$$\int -\frac{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}{\cot\left(c+dx+\frac{\pi}{2}\right)^7}\,dx$$

$$\downarrow 25$$

$$-\int \frac{a+b\csc\left(\frac{1}{2}(2c+\pi)+dx\right)}{\cot\left(\frac{1}{2}(2c+\pi)+dx\right)^7}dx$$

$$\downarrow 4370$$

$$-\frac{1}{6}\int \cot^5(c+dx)(6a+5b\sec(c+dx))dx - \frac{\cot^6(c+dx)(a+b\sec(c+dx))}{6d}$$

$$\downarrow 3042$$

$$-\frac{1}{6}\int -\frac{6a+5b\csc\left(c+dx+\frac{\pi}{2}\right)}{\cot\left(c+dx+\frac{\pi}{2}\right)^5}dx - \frac{\cot^6(c+dx)(a+b\sec(c+dx))}{6d}$$

$$\downarrow 25$$

$$\frac{1}{6}\int \frac{6a+5b\csc\left(\frac{1}{2}(2c+\pi)+dx\right)}{\cot\left(\frac{1}{2}(2c+\pi)+dx\right)^5}dx - \frac{\cot^6(c+dx)(a+b\sec(c+dx))}{6d}$$

$$\downarrow 4370$$

$$\frac{1}{6}\left(\frac{1}{4}\int 3\cot^3(c+dx)(8a+5b\sec(c+dx))dx + \frac{\cot^4(c+dx)(6a+5b\sec(c+dx))}{4d}\right) - \frac{\cot^6(c+dx)(a+b\sec(c+dx))}{6d}$$

$$\downarrow 27$$

$$\frac{1}{6}\left(\frac{3}{4}\int \cot^3(c+dx)(8a+5b\sec(c+dx))dx + \frac{\cot^4(c+dx)(6a+5b\sec(c+dx))}{4d}\right) - \frac{\cot^6(c+dx)(a+b\sec(c+dx))}{6d}$$

$$\downarrow 3042$$

$$\frac{1}{6}\left(\frac{3}{4}\int -\frac{8a+5b\csc\left(c+dx+\frac{\pi}{2}\right)}{\cot\left(c+dx+\frac{\pi}{2}\right)^3}dx + \frac{\cot^4(c+dx)(6a+5b\sec(c+dx))}{4d}\right) - \frac{\cot^6(c+dx)(a+b\sec(c+dx))}{6d}$$

$$\downarrow 25$$

$$\frac{1}{6}\left(\frac{\cot^4(c+dx)(6a+5b\sec(c+dx))}{4d} - \frac{3}{4}\int \frac{8a+5b\csc\left(\frac{1}{2}(2c+\pi)+dx\right)}{\cot\left(\frac{1}{2}(2c+\pi)+dx\right)^3}dx\right) - \frac{\cot^6(c+dx)(a+b\sec(c+dx))}{6d}$$

$$\downarrow 4370$$

$$\frac{1}{6}\left(\frac{\cot^4(c+dx)(6a+5b\sec(c+dx))}{4d}-\frac{3}{4}\left(\frac{1}{2}\int\cot(c+dx)(16a+5b\sec(c+dx))dx+\frac{\cot^2(c+dx)(8a+5b\sec(c\right.\right.$$

$$\frac{\cot^6(c+dx)(a+b\sec(c+dx))}{6d}$$

$$\downarrow 3042$$

$$\frac{1}{6}\left(\frac{\cot^4(c+dx)(6a+5b\sec(c+dx))}{4d}-\frac{3}{4}\left(\frac{1}{2}\int-\frac{16a+5b\csc\left(c+dx+\frac{\pi}{2}\right)}{\cot\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{\cot^2(c+dx)(8a+5b\sec(c+dx)}{2d}\right.\right.$$

$$\frac{\cot^6(c+dx)(a+b\sec(c+dx))}{6d}$$

$$\downarrow 25$$

$$\frac{1}{6}\left(\frac{\cot^4(c+dx)(6a+5b\sec(c+dx))}{4d}-\frac{3}{4}\left(\frac{\cot^2(c+dx)(8a+5b\sec(c+dx))}{2d}-\frac{1}{2}\int\frac{16a+5b\csc\left(\frac{1}{2}(2c+\pi)+dx\right)}{\cot\left(\frac{1}{2}(2c+\pi)+dx\right)}\right.\right.$$

$$\frac{\cot^6(c+dx)(a+b\sec(c+dx))}{6d}$$

$$\downarrow 4371$$

$$\frac{1}{6}\left(\frac{\cot^4(c+dx)(6a+5b\sec(c+dx))}{4d}-\frac{3}{4}\left(\frac{\cot^2(c+dx)(8a+5b\sec(c+dx))}{2d}-\frac{1}{2}\int\sec\left(\frac{1}{2}(2c+\pi)+dx\right)\left(5b+\right.\right.\right.$$

$$\frac{\cot^6(c+dx)(a+b\sec(c+dx))}{6d}$$

$$\downarrow 3042$$

$$\frac{1}{6}\left(\frac{\cot^4(c+dx)(6a+5b\sec(c+dx))}{4d}-\frac{3}{4}\left(\frac{\cot^2(c+dx)(8a+5b\sec(c+dx))}{2d}-\frac{1}{2}\int-\frac{5b-16a\sin\left(c+dx-\frac{\pi}{2}\right)}{\cos\left(c+dx-\frac{\pi}{2}\right)}\right.\right.$$

$$\frac{\cot^6(c+dx)(a+b\sec(c+dx))}{6d}$$

$$\downarrow 25$$

$$\frac{1}{6}\left(\frac{\cot^4(c+dx)(6a+5b\sec(c+dx))}{4d}-\frac{3}{4}\left(\frac{1}{2}\int\frac{5b-16a\sin\left(\frac{1}{2}(2c-\pi)+dx\right)}{\cos\left(\frac{1}{2}(2c-\pi)+dx\right)}dx+\frac{\cot^2(c+dx)(8a+5b\sec(c+\right.\right.\right.$$

$$\frac{\cot^6(c+dx)(a+b\sec(c+dx))}{6d}$$

$$\downarrow 3147$$

$$\frac{1}{6}\left(\frac{\cot^4(c+dx)(6a+5b\sec(c+dx))}{4d}-\frac{3}{4}\left(\frac{\cot^2(c+dx)(8a+5b\sec(c+dx))}{2d}-\frac{8a\int\frac{5b+16a\cos(c+dx)}{256a^2-256a^2\cos^2(c+dx)}d(16a\,c}{d}\right.\right.$$

$$\left.\left.\frac{\cot^6(c+dx)(a+b\sec(c+dx))}{6d}\right)\right)$$

$$\downarrow 452$$

$$\frac{1}{6}\left(\frac{\cot^4(c+dx)(6a+5b\sec(c+dx))}{4d}-\frac{3}{4}\left(\frac{\cot^2(c+dx)(8a+5b\sec(c+dx))}{2d}-\frac{8a\left(5b\int\frac{1}{256a^2-256a^2\cos^2(c+dx)}d(1\right.\right.\right.$$

$$\left.\left.\left.\frac{\cot^6(c+dx)(a+b\sec(c+dx))}{6d}\right)\right)\right)$$

$$\downarrow 219$$

$$\frac{1}{6}\left(\frac{\cot^4(c+dx)(6a+5b\sec(c+dx))}{4d}-\frac{3}{4}\left(\frac{\cot^2(c+dx)(8a+5b\sec(c+dx))}{2d}-\frac{8a\left(\int\frac{16a\cos(c+dx)}{256a^2-256a^2\cos^2(c+dx)}d(16a\right.\right.\right.$$

$$\left.\left.\left.\frac{\cot^6(c+dx)(a+b\sec(c+dx))}{6d}\right)\right)\right)$$

$$\downarrow 240$$

$$\frac{1}{6}\left(\frac{\cot^4(c+dx)(6a+5b\sec(c+dx))}{4d}-\frac{3}{4}\left(\frac{\cot^2(c+dx)(8a+5b\sec(c+dx))}{2d}-\frac{8a\left(\frac{5b\operatorname{arctanh}(\cos(c+dx))}{16a}-\frac{1}{2}\log(\right.\right.\right.$$

$$\left.\left.\left.\frac{\cot^6(c+dx)(a+b\sec(c+dx))}{6d}\right)\right)\right)$$

input
```
Int[Cot[c + d*x]^7*(a + b*Sec[c + d*x]),x]
```

output
```
-1/6*(Cot[c + d*x]^6*(a + b*Sec[c + d*x]))/d + ((Cot[c + d*x]^4*(6*a + 5*b
*Sec[c + d*x]))/(4*d) - (3*((-8*a*((5*b*ArcTanh[Cos[c + d*x]])/(16*a) - Lo
g[256*a^2 - 256*a^2*Cos[c + d*x]^2]/2))/d + (Cot[c + d*x]^2*(8*a + 5*b*Sec
[c + d*x]))/(2*d)))/4)/6
```

3.263.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 240
```
Int[(x_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[Log[RemoveContent[a + b*x
^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]
```

rule 452
```
Int[((c_) + (d_.)*(x_))/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[c   Int[1/
(a + b*x^2), x], x] + Simp[d   Int[x/(a + b*x^2), x], x] /; FreeQ[{a, b, c,
 d}, x] && NeQ[b*c^2 + a*d^2, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3147
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Simp[1/(b^p*f)   Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)
/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
 - 1)/2] && NeQ[a^2 - b^2, 0]
```

rule 4370
```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(-(e*Cot[c + d*x])^(m + 1))*((a + b*Csc[c + d*x])/(
d*e*(m + 1))), x] - Simp[1/(e^2*(m + 1))   Int[(e*Cot[c + d*x])^(m + 2)*(a*
(m + 1) + b*(m + 2)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && L
tQ[m, -1]
```

rule 4371
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))/cot[(c_.) + (d_.)*(x_)], x_Symbo
l] :> Int[(b + a*Sin[c + d*x])/Cos[c + d*x], x] /; FreeQ[{a, b, c, d}, x]
```

3.263.4 Maple [A] (verified)

Time = 0.91 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.16

method	result
derivativedivides	$\frac{a\left(-\frac{\cot(dx+c)^6}{6}+\frac{\cot(dx+c)^4}{4}-\frac{\cot(dx+c)^2}{2}-\ln(\sin(dx+c))\right)+b\left(-\frac{\cos(dx+c)^7}{6\sin(dx+c)^6}+\frac{\cos(dx+c)^7}{24\sin(dx+c)^4}-\frac{\cos(dx+c)^7}{16\sin(dx+c)^2}-\frac{\cos(dx+c)^5}{16}$
default	$\frac{a\left(-\frac{\cot(dx+c)^6}{6}+\frac{\cot(dx+c)^4}{4}-\frac{\cot(dx+c)^2}{2}-\ln(\sin(dx+c))\right)+b\left(-\frac{\cos(dx+c)^7}{6\sin(dx+c)^6}+\frac{\cos(dx+c)^7}{24\sin(dx+c)^4}-\frac{\cos(dx+c)^7}{16\sin(dx+c)^2}-\frac{\cos(dx+c)^5}{16}$
risch	$iax+\frac{2iac}{d}+\frac{e^{i(dx+c)}\left(33be^{10i(dx+c)}+144ae^{9i(dx+c)}+5be^{8i(dx+c)}-288ae^{7i(dx+c)}+90be^{6i(dx+c)}+544ae^{5i(dx+c)}-\right.}{24d\left(e^{2i(dx+c)}-1\right)^6}$

input
```
int(cot(d*x+c)^7*(a+b*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(a*(-1/6*cot(d*x+c)^6+1/4*cot(d*x+c)^4-1/2*cot(d*x+c)^2-ln(sin(d*x+c))
)+b*(-1/6/sin(d*x+c)^6*cos(d*x+c)^7+1/24/sin(d*x+c)^4*cos(d*x+c)^7-1/16/si
n(d*x+c)^2*cos(d*x+c)^7-1/16*cos(d*x+c)^5-5/48*cos(d*x+c)^3-5/16*cos(d*x+c
)-5/16*ln(-cot(d*x+c)+csc(d*x+c))))
```

3.263.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 237, normalized size of antiderivative = 1.82

$$\int \cot^7(c+dx)(a+b\sec(c+dx))\,dx$$
$$= \frac{66\,b\cos(dx+c)^5+144\,a\cos(dx+c)^4-80\,b\cos(dx+c)^3-216\,a\cos(dx+c)^2+30\,b\cos(dx+c)-3\,((}{}$$

input
```
integrate(cot(d*x+c)^7*(a+b*sec(d*x+c)),x, algorithm="fricas")
```

output
```
1/96*(66*b*cos(d*x + c)^5 + 144*a*cos(d*x + c)^4 - 80*b*cos(d*x + c)^3 - 2
16*a*cos(d*x + c)^2 + 30*b*cos(d*x + c) - 3*((16*a - 5*b)*cos(d*x + c)^6 -
 3*(16*a - 5*b)*cos(d*x + c)^4 + 3*(16*a - 5*b)*cos(d*x + c)^2 - 16*a + 5*
b)*log(1/2*cos(d*x + c) + 1/2) - 3*((16*a + 5*b)*cos(d*x + c)^6 - 3*(16*a
+ 5*b)*cos(d*x + c)^4 + 3*(16*a + 5*b)*cos(d*x + c)^2 - 16*a - 5*b)*log(-1
/2*cos(d*x + c) + 1/2) + 88*a)/(d*cos(d*x + c)^6 - 3*d*cos(d*x + c)^4 + 3*
d*cos(d*x + c)^2 - d)
```

3.263.6 Sympy [F]

$$\int \cot^7(c+dx)(a+b\sec(c+dx))\,dx = \int (a+b\sec(c+dx))\cot^7(c+dx)\,dx$$

input
```
integrate(cot(d*x+c)**7*(a+b*sec(d*x+c)),x)
```

output
```
Integral((a + b*sec(c + d*x))*cot(c + d*x)**7, x)
```

3.263.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.02

$$\int \cot^7(c+dx)(a+b\sec(c+dx))\,dx =$$

$$-\frac{3(16a-5b)\log(\cos(dx+c)+1)+3(16a+5b)\log(\cos(dx+c)-1)-\frac{2\left(33b\cos(dx+c)^5+72a\cos(dx+c)^4-\right.}{\cos(dx+c)^6}}{96d}$$

input
```
integrate(cot(d*x+c)^7*(a+b*sec(d*x+c)),x, algorithm="maxima")
```

output
```
-1/96*(3*(16*a - 5*b)*log(cos(d*x + c) + 1) + 3*(16*a + 5*b)*log(cos(d*x +
 c) - 1) - 2*(33*b*cos(d*x + c)^5 + 72*a*cos(d*x + c)^4 - 40*b*cos(d*x + c
)^3 - 108*a*cos(d*x + c)^2 + 15*b*cos(d*x + c) + 44*a)/(cos(d*x + c)^6 - 3
*cos(d*x + c)^4 + 3*cos(d*x + c)^2 - 1))/d
```

3.263.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 358 vs. 2(120) = 240.

Time = 0.36 (sec) , antiderivative size = 358, normalized size of antiderivative = 2.75

$$\int \cot^7(c+dx)(a+b\sec(c+dx))\,dx =$$

$$-\frac{12(16a+5b)\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)-384a\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right)-\frac{\left(a+b+\frac{12a(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{9b(\cos(dx+c)-1)}{\cos(dx+c)+1}+\right.}{}$$

input
```
integrate(cot(d*x+c)^7*(a+b*sec(d*x+c)),x, algorithm="giac")
```

output
```
-1/384*(12*(16*a + 5*b)*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1))
- 384*a*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)) - (a + b + 12
*a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 9*b*(cos(d*x + c) - 1)/(cos(d*x
 + c) + 1) + 87*a*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 45*b*(cos(d*
x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 352*a*(cos(d*x + c) - 1)^3/(cos(d*x +
 c) + 1)^3 + 110*b*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3)*(cos(d*x + c
) + 1)^3/(cos(d*x + c) - 1)^3 - 87*a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1)
 + 45*b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 12*a*(cos(d*x + c) - 1)^2/
(cos(d*x + c) + 1)^2 + 9*b*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - a*(
cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 + b*(cos(d*x + c) - 1)^3/(cos(d*x
 + c) + 1)^3)/d
```

3.263.9 Mupad [B] (verification not implemented)

Time = 14.09 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.31

$$
\begin{aligned}
&\int \cot^7(c+dx)(a+b\sec(c+dx))\,dx \\
&= \frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4\left(\frac{a}{32}-\frac{3b}{128}\right)}{d} \\
&\quad - \frac{\cot\left(\frac{c}{2}+\frac{dx}{2}\right)^6\left(\left(\frac{29a}{2}+\frac{15b}{2}\right)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4+\left(-2a-\frac{3b}{2}\right)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+\frac{a}{6}+\frac{b}{6}\right)}{64\,d} \\
&\quad - \frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2\left(\frac{29a}{128}-\frac{15b}{128}\right)}{d} - \frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6\left(\frac{a}{384}-\frac{b}{384}\right)}{d} \\
&\quad + \frac{a\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+1\right)}{d} - \frac{\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)\left(a+\frac{5b}{16}\right)}{d}
\end{aligned}
$$

input
```
int(cot(c + d*x)^7*(a + b/cos(c + d*x)),x)
```

output
```
(tan(c/2 + (d*x)/2)^4*(a/32 - (3*b)/128))/d - (cot(c/2 + (d*x)/2)^6*(a/6 +
 b/6 - tan(c/2 + (d*x)/2)^2*(2*a + (3*b)/2) + tan(c/2 + (d*x)/2)^4*((29*a)
/2 + (15*b)/2)))/(64*d) - (tan(c/2 + (d*x)/2)^2*((29*a)/128 - (15*b)/128))
/d - (tan(c/2 + (d*x)/2)^6*(a/384 - b/384))/d + (a*log(tan(c/2 + (d*x)/2)^
2 + 1))/d - (log(tan(c/2 + (d*x)/2))*(a + (5*b)/16))/d
```

3.264 $\int (a + b\sec(c + dx))\tan^6(c + dx)\,dx$

3.264.1 Optimal result 1937
3.264.2 Mathematica [A] (verified) 1938
3.264.3 Rubi [A] (verified) 1938
3.264.4 Maple [A] (verified) 1940
3.264.5 Fricas [A] (verification not implemented) 1941
3.264.6 Sympy [F] 1941
3.264.7 Maxima [A] (verification not implemented) 1942
3.264.8 Giac [B] (verification not implemented) 1942
3.264.9 Mupad [B] (verification not implemented) 1943

3.264.1 Optimal result

Integrand size = 19, antiderivative size = 102

$$\begin{aligned}\int (a + b\sec(c + dx))\tan^6(c + dx)\,dx = {} & -ax - \frac{5b\operatorname{arctanh}(\sin(c + dx))}{16d} \\ & + \frac{(16a + 5b\sec(c + dx))\tan(c + dx)}{16d} \\ & - \frac{(8a + 5b\sec(c + dx))\tan^3(c + dx)}{24d} \\ & + \frac{(6a + 5b\sec(c + dx))\tan^5(c + dx)}{30d}\end{aligned}$$

output

```
-a*x-5/16*b*arctanh(sin(d*x+c))/d+1/16*(16*a+5*b*sec(d*x+c))*tan(d*x+c)/d-
1/24*(8*a+5*b*sec(d*x+c))*tan(d*x+c)^3/d+1/30*(6*a+5*b*sec(d*x+c))*tan(d*x
+c)^5/d
```

3.264.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.75

$$\begin{aligned}\int (a+b\sec(c+dx))\tan^6(c+dx)\,dx = &-\frac{a\arctan(\tan(c+dx))}{d}-\frac{5b\operatorname{arctanh}(\sin(c+dx))}{16d}\\&+\frac{a\tan(c+dx)}{d}-\frac{5b\sec(c+dx)\tan(c+dx)}{16d}\\&-\frac{5b\sec^3(c+dx)\tan(c+dx)}{24d}\\&+\frac{5b\sec^5(c+dx)\tan(c+dx)}{6d}-\frac{a\tan^3(c+dx)}{3d}\\&-\frac{5b\sec^3(c+dx)\tan^3(c+dx)}{3d}\\&+\frac{a\tan^5(c+dx)}{5d}+\frac{b\sec(c+dx)\tan^5(c+dx)}{d}\end{aligned}$$

input
```
Integrate[(a + b*Sec[c + d*x])*Tan[c + d*x]^6,x]
```

output
```
-((a*ArcTan[Tan[c + d*x]])/d) - (5*b*ArcTanh[Sin[c + d*x]])/(16*d) + (a*Ta
n[c + d*x])/d - (5*b*Sec[c + d*x]*Tan[c + d*x])/(16*d) - (5*b*Sec[c + d*x]
^3*Tan[c + d*x])/(24*d) + (5*b*Sec[c + d*x]^5*Tan[c + d*x])/(6*d) - (a*Tan
[c + d*x]^3)/(3*d) - (5*b*Sec[c + d*x]^3*Tan[c + d*x]^3)/(3*d) + (a*Tan[c
+ d*x]^5)/(5*d) + (b*Sec[c + d*x]*Tan[c + d*x]^5)/d
```

3.264.3 Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.13, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$, Rules used = {3042, 4369, 3042, 4369, 27, 3042, 4369, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^6(c+dx)(a+b\sec(c+dx))\,dx$$

$$\downarrow 3042$$

$$\int \cot\left(c+dx+\frac{\pi}{2}\right)^6\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)dx$$

$$\downarrow 4369$$

$$\frac{\tan^5(c+dx)(6a+5b\sec(c+dx))}{30d} - \frac{1}{6}\int (6a+5b\sec(c+dx))\tan^4(c+dx)dx$$

$$\downarrow 3042$$

$$\frac{\tan^5(c+dx)(6a+5b\sec(c+dx))}{30d} - \frac{1}{6}\int \cot\left(c+dx+\frac{\pi}{2}\right)^4\left(6a+5b\csc\left(c+dx+\frac{\pi}{2}\right)\right)dx$$

$$\downarrow 4369$$

$$\frac{1}{6}\left(\frac{1}{4}\int 3(8a+5b\sec(c+dx))\tan^2(c+dx)dx - \frac{\tan^3(c+dx)(8a+5b\sec(c+dx))}{4d}\right) + \frac{\tan^5(c+dx)(6a+5b\sec(c+dx))}{30d}$$

$$\downarrow 27$$

$$\frac{1}{6}\left(\frac{3}{4}\int (8a+5b\sec(c+dx))\tan^2(c+dx)dx - \frac{\tan^3(c+dx)(8a+5b\sec(c+dx))}{4d}\right) + \frac{\tan^5(c+dx)(6a+5b\sec(c+dx))}{30d}$$

$$\downarrow 3042$$

$$\frac{1}{6}\left(\frac{3}{4}\int \cot\left(c+dx+\frac{\pi}{2}\right)^2\left(8a+5b\csc\left(c+dx+\frac{\pi}{2}\right)\right)dx - \frac{\tan^3(c+dx)(8a+5b\sec(c+dx))}{4d}\right) + \frac{\tan^5(c+dx)(6a+5b\sec(c+dx))}{30d}$$

$$\downarrow 4369$$

$$\frac{1}{6}\left(\frac{3}{4}\left(\frac{\tan(c+dx)(16a+5b\sec(c+dx))}{2d} - \frac{1}{2}\int (16a+5b\sec(c+dx))dx\right) - \frac{\tan^3(c+dx)(8a+5b\sec(c+dx))}{4d}\right) + \frac{\tan^5(c+dx)(6a+5b\sec(c+dx))}{30d}$$

$$\downarrow 2009$$

$$\frac{1}{6}\left(\frac{3}{4}\left(\frac{1}{2}\left(-16ax - \frac{5b\operatorname{arctanh}(\sin(c+dx))}{d}\right) + \frac{\tan(c+dx)(16a+5b\sec(c+dx))}{2d}\right) - \frac{\tan^3(c+dx)(8a+5b\sec(c+dx))}{4d}\right) + \frac{\tan^5(c+dx)(6a+5b\sec(c+dx))}{30d}$$

input
```
Int[(a + b*Sec[c + d*x])*Tan[c + d*x]^6,x]
```

output
```
((6*a + 5*b*Sec[c + d*x])*Tan[c + d*x]^5)/(30*d) + (-1/4*((8*a + 5*b*Sec[c
 + d*x])*Tan[c + d*x]^3)/d + (3*((-16*a*x - (5*b*ArcTanh[Sin[c + d*x]])/d)
/2 + ((16*a + 5*b*Sec[c + d*x])*Tan[c + d*x])/(2*d)))/4)/6
```

3.264.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4369
```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(-e)*(e*Cot[c + d*x])^(m - 1)*((a*m + b*(m - 1)*Csc
[c + d*x])/(d*m*(m - 1))), x] - Simp[e^2/m   Int[(e*Cot[c + d*x])^(m - 2)*(
a*m + b*(m - 1)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && GtQ[m
, 1]
```

3.264.4 Maple [A] (verified)

Time = 1.91 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.40

method	result
derivativedivides	$\frac{a\left(\frac{\tan(dx+c)^5}{5}-\frac{\tan(dx+c)^3}{3}+\tan(dx+c)-dx-c\right)+b\left(\frac{\sin(dx+c)^7}{6\cos(dx+c)^6}-\frac{\sin(dx+c)^7}{24\cos(dx+c)^4}+\frac{\sin(dx+c)^7}{16\cos(dx+c)^2}+\frac{\sin(dx+c)^5}{16}+\frac{5\sin(dx+c}{48}}{d}$
default	$\frac{a\left(\frac{\tan(dx+c)^5}{5}-\frac{\tan(dx+c)^3}{3}+\tan(dx+c)-dx-c\right)+b\left(\frac{\sin(dx+c)^7}{6\cos(dx+c)^6}-\frac{\sin(dx+c)^7}{24\cos(dx+c)^4}+\frac{\sin(dx+c)^7}{16\cos(dx+c)^2}+\frac{\sin(dx+c)^5}{16}+\frac{5\sin(dx+c}{48}}{d}$
parts	$\frac{a\left(\frac{\tan(dx+c)^5}{5}-\frac{\tan(dx+c)^3}{3}+\tan(dx+c)-\arctan(\tan(dx+c))\right)}{d}+\frac{b\left(\frac{\sin(dx+c)^7}{6\cos(dx+c)^6}-\frac{\sin(dx+c)^7}{24\cos(dx+c)^4}+\frac{\sin(dx+c)^7}{16\cos(dx+c)^2}+\frac{\sin(dx}{1}\right.}{d}$
risch	$-ax-\frac{i\left(165b\,e^{11i(dx+c)}-720a\,e^{10i(dx+c)}-25b\,e^{9i(dx+c)}-2160a\,e^{8i(dx+c)}+450b\,e^{7i(dx+c)}-3680a\,e^{6i(dx+c)}-450b\,e^{5}\right.}{120d\left(e^{2i(dx+c)}+1\right)^6}$

input
```
int((a+b*sec(d*x+c))*tan(d*x+c)^6,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a*(1/5*tan(d*x+c)^5-1/3*tan(d*x+c)^3+tan(d*x+c)-d*x-c)+b*(1/6*sin(d*x
+c)^7/cos(d*x+c)^6-1/24*sin(d*x+c)^7/cos(d*x+c)^4+1/16*sin(d*x+c)^7/cos(d*
x+c)^2+1/16*sin(d*x+c)^5+5/48*sin(d*x+c)^3+5/16*sin(d*x+c)-5/16*ln(sec(d*x
+c)+tan(d*x+c))))
```

3.264.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.31

$$\int (a + b\sec(c + dx))\tan^6(c + dx)\,dx =$$

$$-\frac{480\,adx\cos(dx+c)^6 + 75\,b\cos(dx+c)^6\log(\sin(dx+c)+1) - 75\,b\cos(dx+c)^6\log(-\sin(dx+c)+}{}$$

input
```
integrate((a+b*sec(d*x+c))*tan(d*x+c)^6,x, algorithm="fricas")
```

output
```
-1/480*(480*a*d*x*cos(d*x + c)^6 + 75*b*cos(d*x + c)^6*log(sin(d*x + c) +
1) - 75*b*cos(d*x + c)^6*log(-sin(d*x + c) + 1) - 2*(368*a*cos(d*x + c)^5
+ 165*b*cos(d*x + c)^4 - 176*a*cos(d*x + c)^3 - 130*b*cos(d*x + c)^2 + 48*
a*cos(d*x + c) + 40*b)*sin(d*x + c))/(d*cos(d*x + c)^6)
```

3.264.6 Sympy [F]

$$\int (a + b\sec(c + dx))\tan^6(c + dx)\,dx = \int (a + b\sec(c + dx))\tan^6(c + dx)\,dx$$

input
```
integrate((a+b*sec(d*x+c))*tan(d*x+c)**6,x)
```

output
```
Integral((a + b*sec(c + d*x))*tan(c + d*x)**6, x)
```

3.264.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.31

$$\int (a + b\sec(c + dx))\tan^6(c + dx)\,dx$$

$$= \frac{32\left(3\tan(dx+c)^5 - 5\tan(dx+c)^3 - 15\,dx - 15\,c + 15\tan(dx+c)\right)a - 5\,b\left(\frac{2\left(33\sin(dx+c)^5 - 40\sin(dx+c)^3 + \right.\right.}{\sin(dx+c)^6 - 3\sin(dx+c)^4 + 3\,si}}{480\,d}$$

input
```
integrate((a+b*sec(d*x+c))*tan(d*x+c)^6,x, algorithm="maxima")
```

output
```
1/480*(32*(3*tan(d*x + c)^5 - 5*tan(d*x + c)^3 - 15*d*x - 15*c + 15*tan(d*
x + c))*a - 5*b*(2*(33*sin(d*x + c)^5 - 40*sin(d*x + c)^3 + 15*sin(d*x + c
))/(sin(d*x + c)^6 - 3*sin(d*x + c)^4 + 3*sin(d*x + c)^2 - 1) + 15*log(sin
(d*x + c) + 1) - 15*log(sin(d*x + c) - 1)))/d
```

3.264.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 228 vs. 2(94) = 188.

Time = 2.13 (sec) , antiderivative size = 228, normalized size of antiderivative = 2.24

$$\int (a + b\sec(c + dx))\tan^6(c + dx)\,dx =$$

$$-\frac{240\,(dx+c)a + 75\,b\log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) + 1\right|\right) - 75\,b\log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) - 1\right|\right) + \frac{2\left(240\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right.}{}}{}$$

input
```
integrate((a+b*sec(d*x+c))*tan(d*x+c)^6,x, algorithm="giac")
```

output
```
-1/240*(240*(d*x + c)*a + 75*b*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 75*b*l
og(abs(tan(1/2*d*x + 1/2*c) - 1)) + 2*(240*a*tan(1/2*d*x + 1/2*c)^11 - 75*
b*tan(1/2*d*x + 1/2*c)^11 - 1520*a*tan(1/2*d*x + 1/2*c)^9 + 425*b*tan(1/2*
d*x + 1/2*c)^9 + 4128*a*tan(1/2*d*x + 1/2*c)^7 - 990*b*tan(1/2*d*x + 1/2*c
)^7 - 4128*a*tan(1/2*d*x + 1/2*c)^5 - 990*b*tan(1/2*d*x + 1/2*c)^5 + 1520*
a*tan(1/2*d*x + 1/2*c)^3 + 425*b*tan(1/2*d*x + 1/2*c)^3 - 240*a*tan(1/2*d*
x + 1/2*c) - 75*b*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^6)/d
```

3.264.9 Mupad [B] (verification not implemented)

Time = 15.02 (sec) , antiderivative size = 331, normalized size of antiderivative = 3.25

$$\int (a + b\sec(c + dx))\tan^6(c + dx)\,dx$$

$$= \frac{\left(\frac{5b}{8} - 2a\right)\tan\left(\frac{c}{2} + \frac{dx}{2}\right)^{11} + \left(\frac{38a}{3} - \frac{85b}{24}\right)\tan\left(\frac{c}{2} + \frac{dx}{2}\right)^{9} + \left(\frac{33b}{4} - \frac{172a}{5}\right)\tan\left(\frac{c}{2} + \frac{dx}{2}\right)^{7} + \left(\frac{172a}{5} + \frac{33b}{4}\right)\tan($$
$$d\left(\tan\left(\frac{c}{2} + \frac{dx}{2}\right)^{12} - 6\tan\left(\frac{c}{2} + \frac{dx}{2}\right)^{10} + 15\tan\left(\frac{c}{2} + \frac{dx}{2}\right)^{8} - 20\tan\left(\frac{c}{2} + \frac{dx}{2}\right)^{6} +$$
$$- \frac{5\,b\,\operatorname{atanh}\left(\frac{125\,b^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{64\left(20\,a^2\,b+\frac{125\,b^3}{64}\right)} + \frac{20\,a^2\,b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{20\,a^2\,b+\frac{125\,b^3}{64}}\right)}{8\,d}$$
$$- \frac{2\,a\,\operatorname{atan}\left(\frac{64\,a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{64\,a^3+\frac{25\,a\,b^2}{4}} + \frac{25\,a\,b^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{4\left(64\,a^3+\frac{25\,a\,b^2}{4}\right)}\right)}{d}$$

input
```
int(tan(c + d*x)^6*(a + b/cos(c + d*x)),x)
```

output
```
(tan(c/2 + (d*x)/2)*(2*a + (5*b)/8) - tan(c/2 + (d*x)/2)^11*(2*a - (5*b)/8
) - tan(c/2 + (d*x)/2)^3*((38*a)/3 + (85*b)/24) + tan(c/2 + (d*x)/2)^9*((3
8*a)/3 - (85*b)/24) + tan(c/2 + (d*x)/2)^5*((172*a)/5 + (33*b)/4) - tan(c/
2 + (d*x)/2)^7*((172*a)/5 - (33*b)/4))/(d*(15*tan(c/2 + (d*x)/2)^4 - 6*tan
(c/2 + (d*x)/2)^2 - 20*tan(c/2 + (d*x)/2)^6 + 15*tan(c/2 + (d*x)/2)^8 - 6*
tan(c/2 + (d*x)/2)^10 + tan(c/2 + (d*x)/2)^12 + 1)) - (5*b*atanh((125*b^3*
tan(c/2 + (d*x)/2))/(64*(20*a^2*b + (125*b^3)/64)) + (20*a^2*b*tan(c/2 + (
d*x)/2))/(20*a^2*b + (125*b^3)/64)))/(8*d) - (2*a*atan((64*a^3*tan(c/2 + (
d*x)/2))/((25*a*b^2)/4 + 64*a^3) + (25*a*b^2*tan(c/2 + (d*x)/2))/(4*((25*a
*b^2)/4 + 64*a^3))))/d
```

3.265 $\int (a + b\sec(c+dx))\tan^4(c+dx)\,dx$

3.265.1 Optimal result 1944
3.265.2 Mathematica [A] (verified) 1944
3.265.3 Rubi [A] (verified) 1945
3.265.4 Maple [A] (verified) 1946
3.265.5 Fricas [A] (verification not implemented) 1947
3.265.6 Sympy [F] 1947
3.265.7 Maxima [A] (verification not implemented) 1948
3.265.8 Giac [B] (verification not implemented) 1948
3.265.9 Mupad [B] (verification not implemented) 1949

3.265.1 Optimal result

Integrand size = 19, antiderivative size = 73

$$\int (a + b\sec(c+dx))\tan^4(c+dx)\,dx = ax + \frac{3b\operatorname{arctanh}(\sin(c+dx))}{8d} - \frac{(8a+3b\sec(c+dx))\tan(c+dx)}{8d} + \frac{(4a+3b\sec(c+dx))\tan^3(c+dx)}{12d}$$

output
```
a*x+3/8*b*arctanh(sin(d*x+c))/d-1/8*(8*a+3*b*sec(d*x+c))*tan(d*x+c)/d+1/12
*(4*a+3*b*sec(d*x+c))*tan(d*x+c)^3/d
```

3.265.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.60

$$\int (a + b\sec(c+dx))\tan^4(c+dx)\,dx = \frac{a\arctan(\tan(c+dx))}{d} + \frac{3b\operatorname{arctanh}(\sin(c+dx))}{8d} - \frac{a\tan(c+dx)}{d} + \frac{3b\sec(c+dx)\tan(c+dx)}{8d} - \frac{3b\sec^3(c+dx)\tan(c+dx)}{4d} + \frac{a\tan^3(c+dx)}{3d} + \frac{b\sec(c+dx)\tan^3(c+dx)}{d}$$

input
```
Integrate[(a + b*Sec[c + d*x])*Tan[c + d*x]^4,x]
```

output
```
(a*ArcTan[Tan[c + d*x]])/d + (3*b*ArcTanh[Sin[c + d*x]])/(8*d) - (a*Tan[c
+ d*x])/d + (3*b*Sec[c + d*x]*Tan[c + d*x])/(8*d) - (3*b*Sec[c + d*x]^3*Ta
n[c + d*x])/(4*d) + (a*Tan[c + d*x]^3)/(3*d) + (b*Sec[c + d*x]*Tan[c + d*x
]^3)/d
```

3.265.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.12, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {3042, 4369, 3042, 4369, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^4(c+dx)(a+b\sec(c+dx))\,dx$$

$$\downarrow 3042$$

$$\int \cot\left(c+dx+\frac{\pi}{2}\right)^4\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)dx$$

$$\downarrow 4369$$

$$\frac{\tan^3(c+dx)(4a+3b\sec(c+dx))}{12d}-\frac{1}{4}\int(4a+3b\sec(c+dx))\tan^2(c+dx)dx$$

$$\downarrow 3042$$

$$\frac{\tan^3(c+dx)(4a+3b\sec(c+dx))}{12d}-\frac{1}{4}\int \cot\left(c+dx+\frac{\pi}{2}\right)^2\left(4a+3b\csc\left(c+dx+\frac{\pi}{2}\right)\right)dx$$

$$\downarrow 4369$$

$$\frac{1}{4}\left(\frac{1}{2}\int(8a+3b\sec(c+dx))dx-\frac{\tan(c+dx)(8a+3b\sec(c+dx))}{2d}\right)+\frac{\tan^3(c+dx)(4a+3b\sec(c+dx))}{12d}$$

$$\downarrow 2009$$

$$\frac{1}{4}\left(\frac{1}{2}\left(8ax+\frac{3b\operatorname{arctanh}(\sin(c+dx))}{d}\right)-\frac{\tan(c+dx)(8a+3b\sec(c+dx))}{2d}\right)+\frac{\tan^3(c+dx)(4a+3b\sec(c+dx))}{12d}$$

input
```
Int[(a + b*Sec[c + d*x])*Tan[c + d*x]^4,x]
```

output
```
((4*a + 3*b*Sec[c + d*x])*Tan[c + d*x]^3)/(12*d) + ((8*a*x + (3*b*ArcTanh[
Sin[c + d*x]])/d)/2 - ((8*a + 3*b*Sec[c + d*x])*Tan[c + d*x])/(2*d))/4
```

3.265.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4369
```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(-e)*(e*Cot[c + d*x])^(m - 1)*((a*m + b*(m - 1)*Csc
[c + d*x])/(d*m*(m - 1))), x] - Simp[e^2/m   Int[(e*Cot[c + d*x])^(m - 2)*(
a*m + b*(m - 1)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && GtQ[m
, 1]
```

3.265.4 Maple [A] (verified)

Time = 1.50 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.42

method	result
derivativedivides	$\frac{a\left(\frac{\tan(dx+c)^3}{3}-\tan(dx+c)+dx+c\right)+b\left(\frac{\sin(dx+c)^5}{4\cos(dx+c)^4}-\frac{\sin(dx+c)^5}{8\cos(dx+c)^2}-\frac{\sin(dx+c)^3}{8}-\frac{3\sin(dx+c)}{8}+\frac{3\ln(\sec(dx+c)+\tan(dx+c))}{8}\right.}{d}$
default	$\frac{a\left(\frac{\tan(dx+c)^3}{3}-\tan(dx+c)+dx+c\right)+b\left(\frac{\sin(dx+c)^5}{4\cos(dx+c)^4}-\frac{\sin(dx+c)^5}{8\cos(dx+c)^2}-\frac{\sin(dx+c)^3}{8}-\frac{3\sin(dx+c)}{8}+\frac{3\ln(\sec(dx+c)+\tan(dx+c))}{8}\right.}{d}$
parts	$\frac{a\left(\frac{\tan(dx+c)^3}{3}-\tan(dx+c)+\arctan(\tan(dx+c))\right)}{d}+\frac{b\left(\frac{\sin(dx+c)^5}{4\cos(dx+c)^4}-\frac{\sin(dx+c)^5}{8\cos(dx+c)^2}-\frac{\sin(dx+c)^3}{8}-\frac{3\sin(dx+c)}{8}+\frac{3\ln(\sec(dx$
risch	$ax+\frac{i\left(15b\,e^{7i(dx+c)}-48a\,e^{6i(dx+c)}-9b\,e^{5i(dx+c)}-96a\,e^{4i(dx+c)}+9b\,e^{3i(dx+c)}-80a\,e^{2i(dx+c)}-15b\,e^{i(dx+c)}-32a\right)}{12d\left(e^{2i(dx+c)}+1\right)^4}+$

input
```
int((a+b*sec(d*x+c))*tan(d*x+c)^4,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a*(1/3*tan(d*x+c)^3-tan(d*x+c)+d*x+c)+b*(1/4*sin(d*x+c)^5/cos(d*x+c)^
4-1/8*sin(d*x+c)^5/cos(d*x+c)^2-1/8*sin(d*x+c)^3-3/8*sin(d*x+c)+3/8*ln(sec
(d*x+c)+tan(d*x+c))))
```

3.265.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.53

$$\int (a + b\sec(c + dx))\tan^4(c + dx)\,dx$$

$$= \frac{48\,adx\cos(dx+c)^4 + 9\,b\cos(dx+c)^4\log(\sin(dx+c)+1) - 9\,b\cos(dx+c)^4\log(-\sin(dx+c)+1) -}{48\,d\cos(dx+c)^4}$$

input
```
integrate((a+b*sec(d*x+c))*tan(d*x+c)^4,x, algorithm="fricas")
```

output
```
1/48*(48*a*d*x*cos(d*x + c)^4 + 9*b*cos(d*x + c)^4*log(sin(d*x + c) + 1) -
 9*b*cos(d*x + c)^4*log(-sin(d*x + c) + 1) - 2*(32*a*cos(d*x + c)^3 + 15*b
*cos(d*x + c)^2 - 8*a*cos(d*x + c) - 6*b)*sin(d*x + c))/(d*cos(d*x + c)^4)
```

3.265.6 Sympy [F]

$$\int (a + b\sec(c + dx))\tan^4(c + dx)\,dx = \int (a + b\sec(c + dx))\tan^4(c + dx)\,dx$$

input
```
integrate((a+b*sec(d*x+c))*tan(d*x+c)**4,x)
```

output
```
Integral((a + b*sec(c + d*x))*tan(c + d*x)**4, x)
```

3.265.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.40

$$\int (a + b\sec(c + dx))\tan^4(c + dx)\,dx$$

$$= \frac{16\left(\tan(dx+c)^3 + 3\,dx + 3\,c - 3\,\tan(dx+c)\right)a + 3\,b\left(\frac{2\left(5\sin(dx+c)^3 - 3\sin(dx+c)\right)}{\sin(dx+c)^4 - 2\sin(dx+c)^2 + 1} + 3\log(\sin(dx+c)+1)\right.}{48\,d}$$

input
```
integrate((a+b*sec(d*x+c))*tan(d*x+c)^4,x, algorithm="maxima")
```

output
```
1/48*(16*(tan(d*x + c)^3 + 3*d*x + 3*c - 3*tan(d*x + c))*a + 3*b*(2*(5*sin
(d*x + c)^3 - 3*sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) + 3*
log(sin(d*x + c) + 1) - 3*log(sin(d*x + c) - 1)))/d
```

3.265.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 172 vs. 2(67) = 134.

Time = 0.93 (sec) , antiderivative size = 172, normalized size of antiderivative = 2.36

$$\int (a + b\sec(c + dx))\tan^4(c + dx)\,dx$$

$$= \frac{24\,(dx+c)a + 9\,b\log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) + 1\right|\right) - 9\,b\log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) - 1\right|\right) + \frac{2\left(24\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7 - 9\,b\tan\right.}{2}$$

input
```
integrate((a+b*sec(d*x+c))*tan(d*x+c)^4,x, algorithm="giac")
```

output
```
1/24*(24*(d*x + c)*a + 9*b*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 9*b*log(ab
s(tan(1/2*d*x + 1/2*c) - 1)) + 2*(24*a*tan(1/2*d*x + 1/2*c)^7 - 9*b*tan(1/
2*d*x + 1/2*c)^7 - 104*a*tan(1/2*d*x + 1/2*c)^5 + 33*b*tan(1/2*d*x + 1/2*c
)^5 + 104*a*tan(1/2*d*x + 1/2*c)^3 + 33*b*tan(1/2*d*x + 1/2*c)^3 - 24*a*ta
n(1/2*d*x + 1/2*c) - 9*b*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1
)^4)/d
```

3.265.9 Mupad [B] (verification not implemented)

Time = 15.24 (sec) , antiderivative size = 267, normalized size of antiderivative = 3.66

$$\int (a+b\sec(c+dx))\tan^4(c+dx)\,dx$$

$$= \frac{2\,a\,\mathrm{atan}\left(\frac{64\,a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{64\,a^3+9\,a\,b^2}+\frac{9\,a\,b^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{64\,a^3+9\,a\,b^2}\right)}{d}+\frac{3\,b\,\mathrm{atanh}\left(\frac{27\,b^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{8\left(24\,a^2\,b+\frac{27\,b^3}{8}\right)}+\frac{24\,a^2\,b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{24\,a^2\,b+\frac{27\,b^3}{8}}\right)}{4\,d}$$

$$-\frac{\left(\frac{3b}{4}-2\,a\right)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^7+\left(\frac{26\,a}{3}-\frac{11\,b}{4}\right)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5+\left(-\frac{26\,a}{3}-\frac{11\,b}{4}\right)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3+\left(2\,a+\frac{3b}{4}\right)\tan($$
$$d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^8-4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6+6\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4-4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+1\right)$$

input
```
int(tan(c + d*x)^4*(a + b/cos(c + d*x)),x)
```

output
```
(2*a*atan((64*a^3*tan(c/2 + (d*x)/2))/(9*a*b^2 + 64*a^3) + (9*a*b^2*tan(c/
2 + (d*x)/2))/(9*a*b^2 + 64*a^3)))/d + (3*b*atanh((27*b^3*tan(c/2 + (d*x)/
2))/(8*(24*a^2*b + (27*b^3)/8)) + (24*a^2*b*tan(c/2 + (d*x)/2))/(24*a^2*b
+ (27*b^3)/8)))/(4*d) - (tan(c/2 + (d*x)/2)*(2*a + (3*b)/4) - tan(c/2 + (d
*x)/2)^7*(2*a - (3*b)/4) - tan(c/2 + (d*x)/2)^3*((26*a)/3 + (11*b)/4) + ta
n(c/2 + (d*x)/2)^5*((26*a)/3 - (11*b)/4))/(d*(6*tan(c/2 + (d*x)/2)^4 - 4*t
an(c/2 + (d*x)/2)^2 - 4*tan(c/2 + (d*x)/2)^6 + tan(c/2 + (d*x)/2)^8 + 1))
```

3.266 $\int (a + b\sec(c + dx))\tan^2(c + dx)\,dx$

3.266.1 Optimal result 1950
3.266.2 Mathematica [A] (verified) 1950
3.266.3 Rubi [A] (verified) 1951
3.266.4 Maple [A] (verified) 1952
3.266.5 Fricas [B] (verification not implemented) 1952
3.266.6 Sympy [F] 1953
3.266.7 Maxima [A] (verification not implemented) 1953
3.266.8 Giac [B] (verification not implemented) 1953
3.266.9 Mupad [B] (verification not implemented) 1954

3.266.1 Optimal result

Integrand size = 19, antiderivative size = 45

$$\int (a + b\sec(c + dx))\tan^2(c + dx)\,dx = -ax - \frac{b\operatorname{arctanh}(\sin(c + dx))}{2d} + \frac{(2a + b\sec(c + dx))\tan(c + dx)}{2d}$$

output
```
-a*x-1/2*b*arctanh(sin(d*x+c))/d+1/2*(2*a+b*sec(d*x+c))*tan(d*x+c)/d
```

3.266.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.33

$$\int (a + b\sec(c + dx))\tan^2(c + dx)\,dx = -\frac{a\arctan(\tan(c + dx))}{d} - \frac{b\operatorname{arctanh}(\sin(c + dx))}{2d} + \frac{a\tan(c + dx)}{d} + \frac{b\sec(c + dx)\tan(c + dx)}{2d}$$

input
```
Integrate[(a + b*Sec[c + d*x])*Tan[c + d*x]^2,x]
```

output
```
-((a*ArcTan[Tan[c + d*x]])/d) - (b*ArcTanh[Sin[c + d*x]])/(2*d) + (a*Tan[c
 + d*x])/d + (b*Sec[c + d*x]*Tan[c + d*x])/(2*d)
```

3.266.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.07, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {3042, 4369, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^2(c+dx)(a+b\sec(c+dx))\,dx$$

$$\downarrow 3042$$

$$\int \cot\left(c+dx+\frac{\pi}{2}\right)^2\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)dx$$

$$\downarrow 4369$$

$$\frac{\tan(c+dx)(2a+b\sec(c+dx))}{2d}-\frac{1}{2}\int(2a+b\sec(c+dx))dx$$

$$\downarrow 2009$$

$$\frac{1}{2}\left(-2ax-\frac{b\operatorname{arctanh}(\sin(c+dx))}{d}\right)+\frac{\tan(c+dx)(2a+b\sec(c+dx))}{2d}$$

input
```
Int[(a + b*Sec[c + d*x])*Tan[c + d*x]^2,x]
```

output
```
(-2*a*x - (b*ArcTanh[Sin[c + d*x]])/d)/2 + ((2*a + b*Sec[c + d*x])*Tan[c +
 d*x])/(2*d)
```

3.266.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4369

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(-e)*(e*Cot[c + d*x])^(m - 1)*((a*m + b*(m - 1)*Csc
[c + d*x])/(d*m*(m - 1))), x] - Simp[e^2/m   Int[(e*Cot[c + d*x])^(m - 2)*(
a*m + b*(m - 1)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && GtQ[m
, 1]
```

3.266.4 Maple [A] (verified)

Time = 0.94 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.49

method	result	size
derivativedivides	$\frac{a(\tan(dx+c)-dx-c)+b\left(\frac{\sin(dx+c)^3}{2\cos(dx+c)^2}+\frac{\sin(dx+c)}{2}-\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)}{d}$	67
default	$\frac{a(\tan(dx+c)-dx-c)+b\left(\frac{\sin(dx+c)^3}{2\cos(dx+c)^2}+\frac{\sin(dx+c)}{2}-\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)}{d}$	67
parts	$\frac{a(\tan(dx+c)-\arctan(\tan(dx+c)))}{d}+\frac{b\left(\frac{\sin(dx+c)^3}{2\cos(dx+c)^2}+\frac{\sin(dx+c)}{2}-\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)}{d}$	71
risch	$-ax-\frac{i\left(b\,e^{3i(dx+c)}-2a\,e^{2i(dx+c)}-b\,e^{i(dx+c)}-2a\right)}{d\left(e^{2i(dx+c)}+1\right)^2}-\frac{b\ln\left(e^{i(dx+c)}+i\right)}{2d}+\frac{b\ln\left(e^{i(dx+c)}-i\right)}{2d}$	102

input

```
int((a+b*sec(d*x+c))*tan(d*x+c)^2,x,method=_RETURNVERBOSE)
```

output

```
1/d*(a*(tan(d*x+c)-d*x-c)+b*(1/2*sin(d*x+c)^3/cos(d*x+c)^2+1/2*sin(d*x+c)-
1/2*ln(sec(d*x+c)+tan(d*x+c))))
```

3.266.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 87 vs. 2(41) = 82.

Time = 0.28 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.93

$$\int (a+b\sec(c+dx))\tan^2(c+dx)\,dx =$$

$$-\frac{4\,adx\cos(dx+c)^2+b\cos(dx+c)^2\log(\sin(dx+c)+1)-b\cos(dx+c)^2\log(-\sin(dx+c)+1)-2}{4\,d\cos(dx+c)^2}$$

input

```
integrate((a+b*sec(d*x+c))*tan(d*x+c)^2,x, algorithm="fricas")
```

output
```
-1/4*(4*a*d*x*cos(d*x + c)^2 + b*cos(d*x + c)^2*log(sin(d*x + c) + 1) - b*
cos(d*x + c)^2*log(-sin(d*x + c) + 1) - 2*(2*a*cos(d*x + c) + b)*sin(d*x +
 c))/(d*cos(d*x + c)^2)
```

3.266.6 Sympy [F]

$$\int (a + b\sec(c + dx))\tan^2(c + dx)\,dx = \int (a + b\sec(c + dx))\tan^2(c + dx)\,dx$$

input
```
integrate((a+b*sec(d*x+c))*tan(d*x+c)**2,x)
```

output
```
Integral((a + b*sec(c + d*x))*tan(c + d*x)**2, x)
```

3.266.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.44

$$\int (a + b\sec(c + dx))\tan^2(c + dx)\,dx =$$
$$-\frac{4\,(dx + c - \tan(dx + c))a + b\left(\frac{2\sin(dx+c)}{\sin(dx+c)^2-1} + \log(\sin(dx + c) + 1) - \log(\sin(dx + c) - 1)\right)}{4\,d}$$

input
```
integrate((a+b*sec(d*x+c))*tan(d*x+c)^2,x, algorithm="maxima")
```

output
```
-1/4*(4*(d*x + c - tan(d*x + c))*a + b*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1
) + log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)))/d
```

3.266.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 115 vs. $2(41) = 82$.

Time = 0.48 (sec) , antiderivative size = 115, normalized size of antiderivative = 2.56

$$\int (a + b\sec(c + dx))\tan^2(c + dx)\,dx =$$
$$-\frac{2\,(dx + c)a + b\log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) + 1\right|\right) - b\log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) - 1\right|\right) + \frac{2\left(2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3 - b\tan\left(\frac{1}{2}\,dx\right.\right.}{\left(\tan\right.}}{2\,d}$$

input
```
integrate((a+b*sec(d*x+c))*tan(d*x+c)^2,x, algorithm="giac")
```

output
```
-1/2*(2*(d*x + c)*a + b*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - b*log(abs(tan
(1/2*d*x + 1/2*c) - 1)) + 2*(2*a*tan(1/2*d*x + 1/2*c)^3 - b*tan(1/2*d*x +
1/2*c)^3 - 2*a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x
 + 1/2*c)^2 - 1)^2)/d
```

3.266.9 Mupad [B] (verification not implemented)

Time = 14.61 (sec) , antiderivative size = 96, normalized size of antiderivative = 2.13

$$\int (a + b\sec(c + dx))\tan^2(c + dx)\,dx = \frac{a\,\sin(c + d\,x)}{d\,\cos(c + d\,x)} - \frac{b\,\mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d} - \frac{2\,a\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d} + \frac{b\,\sin(c + d\,x)}{2\,d\cos(c + d\,x)^2}$$

input
```
int(tan(c + d*x)^2*(a + b/cos(c + d*x)),x)
```

output
```
(a*sin(c + d*x))/(d*cos(c + d*x)) - (b*atanh(sin(c/2 + (d*x)/2)/cos(c/2 +
(d*x)/2)))/d - (2*a*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d + (b*si
n(c + d*x))/(2*d*cos(c + d*x)^2)
```

3.267 $\int \cot^2(c+dx)(a+b\sec(c+dx))\,dx$

3.267.1 Optimal result 1955
3.267.2 Mathematica [C] (verified) 1955
3.267.3 Rubi [A] (verified) 1956
3.267.4 Maple [A] (verified) 1957
3.267.5 Fricas [A] (verification not implemented) 1957
3.267.6 Sympy [F] 1958
3.267.7 Maxima [A] (verification not implemented) 1958
3.267.8 Giac [A] (verification not implemented) 1958
3.267.9 Mupad [B] (verification not implemented) 1959

3.267.1 Optimal result

Integrand size = 19, antiderivative size = 26

$$\int \cot^2(c+dx)(a+b\sec(c+dx))\,dx = -ax - \frac{\cot(c+dx)(a+b\sec(c+dx))}{d}$$

output
```
-a*x-cot(d*x+c)*(a+b*sec(d*x+c))/d
```

3.267.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.65

$$\int \cot^2(c+dx)(a+b\sec(c+dx))\,dx = -\frac{b\csc(c+dx)}{d} - \frac{a\cot(c+dx)\,\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},-\tan^2(c+dx)\right)}{d}$$

input
```
Integrate[Cot[c + d*x]^2*(a + b*Sec[c + d*x]),x]
```

output
```
-((b*Csc[c + d*x])/d) - (a*Cot[c + d*x]*Hypergeometric2F1[-1/2, 1, 1/2, -T
an[c + d*x]^2])/d
```

3.267.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {3042, 4370, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^2(c+dx)(a+b\sec(c+dx))\,dx$$

$$\downarrow 3042$$

$$\int \frac{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}{\cot\left(c+dx+\frac{\pi}{2}\right)^2}dx$$

$$\downarrow 4370$$

$$\int -adx - \frac{\cot(c+dx)(a+b\sec(c+dx))}{d}$$

$$\downarrow 24$$

$$-\frac{\cot(c+dx)(a+b\sec(c+dx))}{d} - ax$$

input
```
Int[Cot[c + d*x]^2*(a + b*Sec[c + d*x]),x]
```

output
```
-(a*x) - (Cot[c + d*x]*(a + b*Sec[c + d*x]))/d
```

3.267.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4370
```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(-(e*Cot[c + d*x])^(m + 1))*((a + b*Csc[c + d*x])/(
d*e*(m + 1))), x] - Simp[1/(e^2*(m + 1))   Int[(e*Cot[c + d*x])^(m + 2)*(a*
(m + 1) + b*(m + 2)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && L
tQ[m, -1]
```

3.267.4 Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.35

method	result	size
derivativedivides	$\frac{a(-\cot(dx+c)-dx-c)-\frac{b}{\sin(dx+c)}}{d}$	35
default	$\frac{a(-\cot(dx+c)-dx-c)-\frac{b}{\sin(dx+c)}}{d}$	35
risch	$-ax-\frac{2i\left(b\,\mathrm{e}^{i(dx+c)}+a\right)}{d\left(\mathrm{e}^{2i(dx+c)}-1\right)}$	38

input
```
int(cot(d*x+c)^2*(a+b*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(a*(-cot(d*x+c)-d*x-c)-b/sin(d*x+c))
```

3.267.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.27

$$\int \cot^2(c+dx)(a+b\sec(c+dx))\,dx = -\frac{adx\sin(dx+c)+a\cos(dx+c)+b}{d\sin(dx+c)}$$

input
```
integrate(cot(d*x+c)^2*(a+b*sec(d*x+c)),x, algorithm="fricas")
```

output
```
-(a*d*x*sin(d*x + c) + a*cos(d*x + c) + b)/(d*sin(d*x + c))
```

3.267.6 Sympy [F]

$$\int \cot^2(c+dx)(a+b\sec(c+dx))\,dx = \int (a+b\sec(c+dx))\cot^2(c+dx)\,dx$$

input
```
integrate(cot(d*x+c)**2*(a+b*sec(d*x+c)),x)
```

output
```
Integral((a + b*sec(c + d*x))*cot(c + d*x)**2, x)
```

3.267.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.19

$$\int \cot^2(c+dx)(a+b\sec(c+dx))\,dx = -\frac{\left(dx+c+\frac{1}{\tan(dx+c)}\right)a+\frac{b}{\sin(dx+c)}}{d}$$

input
```
integrate(cot(d*x+c)^2*(a+b*sec(d*x+c)),x, algorithm="maxima")
```

output
```
-((d*x + c + 1/tan(d*x + c))*a + b/sin(d*x + c))/d
```

3.267.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 52, normalized size of antiderivative = 2.00

$$\int \cot^2(c+dx)(a+b\sec(c+dx))\,dx$$
$$= -\frac{2\,(dx+c)a - a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+\frac{a+b}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}}{2\,d}$$

input
```
integrate(cot(d*x+c)^2*(a+b*sec(d*x+c)),x, algorithm="giac")
```

output
```
-1/2*(2*(d*x + c)*a - a*tan(1/2*d*x + 1/2*c) + b*tan(1/2*d*x + 1/2*c) + (a
 + b)/tan(1/2*d*x + 1/2*c))/d
```

3.267.9 Mupad [B] (verification not implemented)

Time = 14.34 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.85

$$\int \cot^2(c+dx)(a+b\sec(c+dx))\,dx = \frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\,\left(\frac{a}{2}-\frac{b}{2}\right)}{d} - \frac{\frac{a}{2}+\frac{b}{2}}{d\tan\left(\frac{c}{2}+\frac{dx}{2}\right)} - a\,x$$

input
```
int(cot(c + d*x)^2*(a + b/cos(c + d*x)),x)
```

output
```
(tan(c/2 + (d*x)/2)*(a/2 - b/2))/d - (a/2 + b/2)/(d*tan(c/2 + (d*x)/2)) -
a*x
```

3.268 $\int \cot^4(c+dx)(a+b\sec(c+dx))\,dx$

3.268.1 Optimal result 1960
3.268.2 Mathematica [C] (verified) 1960
3.268.3 Rubi [A] (verified) 1961
3.268.4 Maple [A] (verified) 1962
3.268.5 Fricas [A] (verification not implemented) 1963
3.268.6 Sympy [F] 1963
3.268.7 Maxima [A] (verification not implemented) 1963
3.268.8 Giac [B] (verification not implemented) 1964
3.268.9 Mupad [B] (verification not implemented) 1964

3.268.1 Optimal result

Integrand size = 19, antiderivative size = 55

$$\int \cot^4(c+dx)(a+b\sec(c+dx))\,dx = ax - \frac{\cot^3(c+dx)(a+b\sec(c+dx))}{3d} + \frac{\cot(c+dx)(3a+2b\sec(c+dx))}{3d}$$

output
```
a*x-1/3*cot(d*x+c)^3*(a+b*sec(d*x+c))/d+1/3*cot(d*x+c)*(3*a+2*b*sec(d*x+c)
)/d
```

3.268.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.13

$$\begin{aligned}&\int \cot^4(c+dx)(a+b\sec(c+dx))\,dx \\ &= \frac{b\csc(c+dx)}{d} - \frac{b\csc^3(c+dx)}{3d} \\ &\quad - \frac{a\cot^3(c+dx)\,\text{Hypergeometric2F1}\left(-\frac{3}{2},1,-\frac{1}{2},-\tan^2(c+dx)\right)}{3d}\end{aligned}$$

input
```
Integrate[Cot[c + d*x]^4*(a + b*Sec[c + d*x]),x]
```

output
```
(b*Csc[c + d*x])/d - (b*Csc[c + d*x]^3)/(3*d) - (a*Cot[c + d*x]^3*Hypergeo
metric2F1[-3/2, 1, -1/2, -Tan[c + d*x]^2])/(3*d)
```

3.268.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.05, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {3042, 4370, 25, 3042, 4370, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^4(c+dx)(a+b\sec(c+dx))\,dx$$

$$\downarrow 3042$$

$$\int \frac{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}{\cot\left(c+dx+\frac{\pi}{2}\right)^4}dx$$

$$\downarrow 4370$$

$$\frac{1}{3}\int -\cot^2(c+dx)(3a+2b\sec(c+dx))dx - \frac{\cot^3(c+dx)(a+b\sec(c+dx))}{3d}$$

$$\downarrow 25$$

$$-\frac{1}{3}\int \cot^2(c+dx)(3a+2b\sec(c+dx))dx - \frac{\cot^3(c+dx)(a+b\sec(c+dx))}{3d}$$

$$\downarrow 3042$$

$$-\frac{1}{3}\int \frac{3a+2b\csc\left(c+dx+\frac{\pi}{2}\right)}{\cot\left(c+dx+\frac{\pi}{2}\right)^2}dx - \frac{\cot^3(c+dx)(a+b\sec(c+dx))}{3d}$$

$$\downarrow 4370$$

$$\frac{1}{3}\left(\frac{\cot(c+dx)(3a+2b\sec(c+dx))}{d} - \int -3a dx\right) - \frac{\cot^3(c+dx)(a+b\sec(c+dx))}{3d}$$

$$\downarrow 24$$

$$\frac{1}{3}\left(\frac{\cot(c+dx)(3a+2b\sec(c+dx))}{d} + 3ax\right) - \frac{\cot^3(c+dx)(a+b\sec(c+dx))}{3d}$$

input
```
Int[Cot[c + d*x]^4*(a + b*Sec[c + d*x]),x]
```

output
```
-1/3*(Cot[c + d*x]^3*(a + b*Sec[c + d*x]))/d + (3*a*x + (Cot[c + d*x]*(3*a
 + 2*b*Sec[c + d*x]))/d)/3
```

3.268.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4370
```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(-(e*Cot[c + d*x])^(m + 1))*((a + b*Csc[c + d*x])/(
d*e*(m + 1))), x] - Simp[1/(e^2*(m + 1))   Int[(e*Cot[c + d*x])^(m + 2)*(a*
(m + 1) + b*(m + 2)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && L
tQ[m, -1]
```

3.268.4 Maple [A] (verified)

Time = 1.23 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.56

method	result	size
derivativedivides	$\frac{a\left(-\frac{\cot(dx+c)^3}{3}+\cot(dx+c)+dx+c\right)+b\left(-\frac{\cos(dx+c)^4}{3\sin(dx+c)^3}+\frac{\cos(dx+c)^4}{3\sin(dx+c)}+\frac{\left(2+\cos(dx+c)^2\right)\sin(dx+c)}{3}\right)}{d}$	86
default	$\frac{a\left(-\frac{\cot(dx+c)^3}{3}+\cot(dx+c)+dx+c\right)+b\left(-\frac{\cos(dx+c)^4}{3\sin(dx+c)^3}+\frac{\cos(dx+c)^4}{3\sin(dx+c)}+\frac{\left(2+\cos(dx+c)^2\right)\sin(dx+c)}{3}\right)}{d}$	86
risch	$ax+\frac{2i\left(3b\,e^{5i(dx+c)}+6a\,e^{4i(dx+c)}-2b\,e^{3i(dx+c)}-6a\,e^{2i(dx+c)}+3b\,e^{i(dx+c)}+4a\right)}{3d\left(e^{2i(dx+c)}-1\right)^3}$	88

input
```
int(cot(d*x+c)^4*(a+b*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(a*(-1/3*cot(d*x+c)^3+cot(d*x+c)+d*x+c)+b*(-1/3/sin(d*x+c)^3*cos(d*x+c
)^4+1/3/sin(d*x+c)*cos(d*x+c)^4+1/3*(2+cos(d*x+c)^2)*sin(d*x+c)))
```

3.268.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.58

$$\int \cot^4(c+dx)(a+b\sec(c+dx))\,dx$$
$$= \frac{4\,a\cos(dx+c)^3 + 3\,b\cos(dx+c)^2 - 3\,a\cos(dx+c) + 3\left(adx\cos(dx+c)^2 - adx\right)\sin(dx+c) - 2\,b}{3\left(d\cos(dx+c)^2 - d\right)\sin(dx+c)}$$

input
```
integrate(cot(d*x+c)^4*(a+b*sec(d*x+c)),x, algorithm="fricas")
```

output
```
1/3*(4*a*cos(d*x + c)^3 + 3*b*cos(d*x + c)^2 - 3*a*cos(d*x + c) + 3*(a*d*x
*cos(d*x + c)^2 - a*d*x)*sin(d*x + c) - 2*b)/((d*cos(d*x + c)^2 - d)*sin(d
*x + c))
```

3.268.6 Sympy [F]

$$\int \cot^4(c+dx)(a+b\sec(c+dx))\,dx = \int (a+b\sec(c+dx))\cot^4(c+dx)\,dx$$

input
```
integrate(cot(d*x+c)**4*(a+b*sec(d*x+c)),x)
```

output
```
Integral((a + b*sec(c + d*x))*cot(c + d*x)**4, x)
```

3.268.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.07

$$\int \cot^4(c+dx)(a+b\sec(c+dx))\,dx = \frac{\left(3\,dx + 3\,c + \frac{3\tan(dx+c)^2-1}{\tan(dx+c)^3}\right)a + \frac{\left(3\sin(dx+c)^2-1\right)b}{\sin(dx+c)^3}}{3\,d}$$

input
```
integrate(cot(d*x+c)^4*(a+b*sec(d*x+c)),x, algorithm="maxima")
```

output
```
1/3*((3*d*x + 3*c + (3*tan(d*x + c)^2 - 1)/tan(d*x + c)^3)*a + (3*sin(d*x
+ c)^2 - 1)*b/sin(d*x + c)^3)/d
```

3.268.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 112 vs. $2(51) = 102$.

Time = 0.33 (sec) , antiderivative size = 112, normalized size of antiderivative = 2.04

$$\int \cot^4(c+dx)(a+b\sec(c+dx))\,dx$$

$$= \frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3 - b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3 + 24\,(dx+c)a - 15\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right) + 9\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right) + \frac{15}{}}{24\,d}$$

input
```
integrate(cot(d*x+c)^4*(a+b*sec(d*x+c)),x, algorithm="giac")
```

output
```
1/24*(a*tan(1/2*d*x + 1/2*c)^3 - b*tan(1/2*d*x + 1/2*c)^3 + 24*(d*x + c)*a
 - 15*a*tan(1/2*d*x + 1/2*c) + 9*b*tan(1/2*d*x + 1/2*c) + (15*a*tan(1/2*d*
x + 1/2*c)^2 + 9*b*tan(1/2*d*x + 1/2*c)^2 - a - b)/tan(1/2*d*x + 1/2*c)^3)
/d
```

3.268.9 Mupad [B] (verification not implemented)

Time = 14.61 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.64

$$\int \cot^4(c+dx)(a+b\sec(c+dx))\,dx$$

$$= a\,x + \frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3\left(\frac{a}{24}-\frac{b}{24}\right)}{d} - \frac{\cot\left(\frac{c}{2}+\frac{dx}{2}\right)^3\left((-5\,a-3\,b)\,\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+\frac{a}{3}+\frac{b}{3}\right)}{8\,d} - \frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\left(\frac{5\,a}{8}-\frac{3\,b}{8}\right)}{d}$$

input
```
int(cot(c + d*x)^4*(a + b/cos(c + d*x)),x)
```

output
```
a*x + (tan(c/2 + (d*x)/2)^3*(a/24 - b/24))/d - (cot(c/2 + (d*x)/2)^3*(a/3
+ b/3 - tan(c/2 + (d*x)/2)^2*(5*a + 3*b)))/(8*d) - (tan(c/2 + (d*x)/2)*((5
*a)/8 - (3*b)/8))/d
```

3.269 $\int \cot^6(c+dx)(a+b\sec(c+dx))\,dx$

3.269.1 Optimal result . . . 1965
3.269.2 Mathematica [C] (verified) . . . 1965
3.269.3 Rubi [A] (verified) . . . 1966
3.269.4 Maple [A] (verified) . . . 1968
3.269.5 Fricas [A] (verification not implemented) . . . 1968
3.269.6 Sympy [F] . . . 1969
3.269.7 Maxima [A] (verification not implemented) . . . 1969
3.269.8 Giac [B] (verification not implemented) . . . 1970
3.269.9 Mupad [B] (verification not implemented) . . . 1970

3.269.1 Optimal result

Integrand size = 19, antiderivative size = 84

$$\int \cot^6(c+dx)(a+b\sec(c+dx))\,dx = -ax - \frac{\cot^5(c+dx)(a+b\sec(c+dx))}{5d} + \frac{\cot^3(c+dx)(5a+4b\sec(c+dx))}{15d} - \frac{\cot(c+dx)(15a+8b\sec(c+dx))}{15d}$$

output
```
-a*x-1/5*cot(d*x+c)^5*(a+b*sec(d*x+c))/d+1/15*cot(d*x+c)^3*(5*a+4*b*sec(d*
x+c))/d-1/15*cot(d*x+c)*(15*a+8*b*sec(d*x+c))/d
```

3.269.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.94

$$\int \cot^6(c+dx)(a+b\sec(c+dx))\,dx = -\frac{b\csc(c+dx)}{d} + \frac{2b\csc^3(c+dx)}{3d} - \frac{b\csc^5(c+dx)}{5d} - \frac{a\cot^5(c+dx)\,\text{Hypergeometric2F1}\left(-\frac{5}{2},1,-\frac{3}{2},-\tan^2(c+dx)\right)}{5d}$$

input
```
Integrate[Cot[c + d*x]^6*(a + b*Sec[c + d*x]),x]
```

output
```
-((b*Csc[c + d*x])/d) + (2*b*Csc[c + d*x]^3)/(3*d) - (b*Csc[c + d*x]^5)/(5
*d) - (a*Cot[c + d*x]^5*Hypergeometric2F1[-5/2, 1, -3/2, -Tan[c + d*x]^2])
/(5*d)
```

3.269.3 Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.10, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.474$, Rules used = {3042, 4370, 25, 3042, 4370, 25, 3042, 4370, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^6(c+dx)(a+b\sec(c+dx))\,dx$$

$$\downarrow 3042$$

$$\int \frac{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}{\cot\left(c+dx+\frac{\pi}{2}\right)^6}dx$$

$$\downarrow 4370$$

$$\frac{1}{5}\int -\cot^4(c+dx)(5a+4b\sec(c+dx))dx - \frac{\cot^5(c+dx)(a+b\sec(c+dx))}{5d}$$

$$\downarrow 25$$

$$-\frac{1}{5}\int \cot^4(c+dx)(5a+4b\sec(c+dx))dx - \frac{\cot^5(c+dx)(a+b\sec(c+dx))}{5d}$$

$$\downarrow 3042$$

$$-\frac{1}{5}\int \frac{5a+4b\csc\left(c+dx+\frac{\pi}{2}\right)}{\cot\left(c+dx+\frac{\pi}{2}\right)^4}dx - \frac{\cot^5(c+dx)(a+b\sec(c+dx))}{5d}$$

$$\downarrow 4370$$

$$\frac{1}{5}\left(\frac{\cot^3(c+dx)(5a+4b\sec(c+dx))}{3d} - \frac{1}{3}\int -\cot^2(c+dx)(15a+8b\sec(c+dx))dx\right) - \frac{\cot^5(c+dx)(a+b\sec(c+dx))}{5d}$$

$$\downarrow 25$$

$$\frac{1}{5}\left(\frac{1}{3}\int \cot^2(c+dx)(15a+8b\sec(c+dx))dx+\frac{\cot^3(c+dx)(5a+4b\sec(c+dx))}{3d}\right)-\frac{\cot^5(c+dx)(a+b\sec(c+dx))}{5d}$$

$$\downarrow 3042$$

$$\frac{1}{5}\left(\frac{1}{3}\int \frac{15a+8b\csc\left(c+dx+\frac{\pi}{2}\right)}{\cot\left(c+dx+\frac{\pi}{2}\right)^2}dx+\frac{\cot^3(c+dx)(5a+4b\sec(c+dx))}{3d}\right)-\frac{\cot^5(c+dx)(a+b\sec(c+dx))}{5d}$$

$$\downarrow 4370$$

$$\frac{1}{5}\left(\frac{1}{3}\left(\int -15adx-\frac{\cot(c+dx)(15a+8b\sec(c+dx))}{d}\right)+\frac{\cot^3(c+dx)(5a+4b\sec(c+dx))}{3d}\right)-\frac{\cot^5(c+dx)(a+b\sec(c+dx))}{5d}$$

$$\downarrow 24$$

$$\frac{1}{5}\left(\frac{\cot^3(c+dx)(5a+4b\sec(c+dx))}{3d}+\frac{1}{3}\left(-\frac{\cot(c+dx)(15a+8b\sec(c+dx))}{d}-15ax\right)\right)-\frac{\cot^5(c+dx)(a+b\sec(c+dx))}{5d}$$

input
```
Int[Cot[c + d*x]^6*(a + b*Sec[c + d*x]),x]
```

output
```
-1/5*(Cot[c + d*x]^5*(a + b*Sec[c + d*x]))/d + ((Cot[c + d*x]^3*(5*a + 4*b
*Sec[c + d*x]))/(3*d) + (-15*a*x - (Cot[c + d*x]*(15*a + 8*b*Sec[c + d*x])
)/d)/3)/5
```

3.269.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4370

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(-(e*Cot[c + d*x])^(m + 1))*((a + b*Csc[c + d*x])/(
d*e*(m + 1))), x] - Simp[1/(e^2*(m + 1))   Int[(e*Cot[c + d*x])^(m + 2)*(a*
(m + 1) + b*(m + 2)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && L
tQ[m, -1]
```

3.269.4 Maple [A] (verified)

Time = 1.22 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.54

method	result
derivativedivides	$\frac{a\left(-\frac{\cot(dx+c)^5}{5}+\frac{\cot(dx+c)^3}{3}-\cot(dx+c)-dx-c\right)+b\left(-\frac{\cos(dx+c)^6}{5\sin(dx+c)^5}+\frac{\cos(dx+c)^6}{15\sin(dx+c)^3}-\frac{\cos(dx+c)^6}{5\sin(dx+c)}-\frac{\left(\frac{8}{3}+\cos(dx+c)^4+\frac{4\cos}{5}\right.}{d}$
default	$\frac{a\left(-\frac{\cot(dx+c)^5}{5}+\frac{\cot(dx+c)^3}{3}-\cot(dx+c)-dx-c\right)+b\left(-\frac{\cos(dx+c)^6}{5\sin(dx+c)^5}+\frac{\cos(dx+c)^6}{15\sin(dx+c)^3}-\frac{\cos(dx+c)^6}{5\sin(dx+c)}-\frac{\left(\frac{8}{3}+\cos(dx+c)^4+\frac{4\cos}{5}\right.}{d}$
risch	$-ax-\frac{2i\left(15b\,e^{9i(dx+c)}+45a\,e^{8i(dx+c)}-20b\,e^{7i(dx+c)}-90a\,e^{6i(dx+c)}+58b\,e^{5i(dx+c)}+140a\,e^{4i(dx+c)}-20b\,e^{3i(dx+c)}-\right.}{15d\left(e^{2i(dx+c)}-1\right)^5}$

input

```
int(cot(d*x+c)^6*(a+b*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

output

```
1/d*(a*(-1/5*cot(d*x+c)^5+1/3*cot(d*x+c)^3-cot(d*x+c)-d*x-c)+b*(-1/5/sin(d
*x+c)^5*cos(d*x+c)^6+1/15/sin(d*x+c)^3*cos(d*x+c)^6-1/5/sin(d*x+c)*cos(d*x
+c)^6-1/5*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c)))
```

3.269.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.55

$$\int \cot^6(c+dx)(a+b\sec(c+dx))\,dx =$$

$$-\frac{23\,a\cos(dx+c)^5+15\,b\cos(dx+c)^4-35\,a\cos(dx+c)^3-20\,b\cos(dx+c)^2+15\,a\cos(dx+c)+15}{15\left(d\cos(dx+c)^4-2\,d\cos(dx+c)^2+d\right)\sin}$$

input

```
integrate(cot(d*x+c)^6*(a+b*sec(d*x+c)),x, algorithm="fricas")
```

output
```
-1/15*(23*a*cos(d*x + c)^5 + 15*b*cos(d*x + c)^4 - 35*a*cos(d*x + c)^3 - 2
0*b*cos(d*x + c)^2 + 15*a*cos(d*x + c) + 15*(a*d*x*cos(d*x + c)^4 - 2*a*d*
x*cos(d*x + c)^2 + a*d*x)*sin(d*x + c) + 8*b)/((d*cos(d*x + c)^4 - 2*d*cos
(d*x + c)^2 + d)*sin(d*x + c))
```

3.269.6 Sympy [F]

$$\int \cot^6(c+dx)(a+b\sec(c+dx))\,dx = \int (a+b\sec(c+dx))\cot^6(c+dx)\,dx$$

input
```
integrate(cot(d*x+c)**6*(a+b*sec(d*x+c)),x)
```

output
```
Integral((a + b*sec(c + d*x))*cot(c + d*x)**6, x)
```

3.269.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.94

$$\int \cot^6(c+dx)(a+b\sec(c+dx))\,dx$$
$$= -\frac{\left(15\,dx+15\,c+\frac{15\tan(dx+c)^4-5\tan(dx+c)^2+3}{\tan(dx+c)^5}\right)a+\frac{\left(15\sin(dx+c)^4-10\sin(dx+c)^2+3\right)b}{\sin(dx+c)^5}}{15\,d}$$

input
```
integrate(cot(d*x+c)^6*(a+b*sec(d*x+c)),x, algorithm="maxima")
```

output
```
-1/15*((15*d*x + 15*c + (15*tan(d*x + c)^4 - 5*tan(d*x + c)^2 + 3)/tan(d*x
 + c)^5)*a + (15*sin(d*x + c)^4 - 10*sin(d*x + c)^2 + 3)*b/sin(d*x + c)^5)
/d
```

3.269.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 170 vs. $2(78) = 156$.

Time = 0.33 (sec) , antiderivative size = 170, normalized size of antiderivative = 2.02

$$\int \cot^6(c+dx)(a+b\sec(c+dx))\,dx$$

$$= \frac{3\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5 - 3\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5 - 35\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3 + 25\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3 - 480\,(dx+c$$

input
```
integrate(cot(d*x+c)^6*(a+b*sec(d*x+c)),x, algorithm="giac")
```

output
```
1/480*(3*a*tan(1/2*d*x + 1/2*c)^5 - 3*b*tan(1/2*d*x + 1/2*c)^5 - 35*a*tan(
1/2*d*x + 1/2*c)^3 + 25*b*tan(1/2*d*x + 1/2*c)^3 - 480*(d*x + c)*a + 330*a
*tan(1/2*d*x + 1/2*c) - 150*b*tan(1/2*d*x + 1/2*c) - (330*a*tan(1/2*d*x +
1/2*c)^4 + 150*b*tan(1/2*d*x + 1/2*c)^4 - 35*a*tan(1/2*d*x + 1/2*c)^2 - 25
*b*tan(1/2*d*x + 1/2*c)^2 + 3*a + 3*b)/tan(1/2*d*x + 1/2*c)^5)/d
```

3.269.9 Mupad [B] (verification not implemented)

Time = 14.83 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.57

$$\int \cot^6(c+dx)(a+b\sec(c+dx))\,dx$$

$$= \frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5\left(\frac{a}{160}-\frac{b}{160}\right)}{d} - \frac{\cot\left(\frac{c}{2}+\frac{dx}{2}\right)^5\left((22\,a+10\,b)\,\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4+\left(-\frac{7\,a}{3}-\frac{5\,b}{3}\right)\,\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+\frac{a}{5}+\frac{b}{5}\right)}{32\,d} - \frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3\left(\frac{7\,a}{96}-\frac{5\,b}{96}\right)}{d} - a\,x + \frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\left(\frac{11\,a}{16}-\frac{5\,b}{16}\right)}{d}$$

input
```
int(cot(c + d*x)^6*(a + b/cos(c + d*x)),x)
```

output
```
(tan(c/2 + (d*x)/2)^5*(a/160 - b/160))/d - (cot(c/2 + (d*x)/2)^5*(a/5 + b/
5 - tan(c/2 + (d*x)/2)^2*((7*a)/3 + (5*b)/3) + tan(c/2 + (d*x)/2)^4*(22*a
+ 10*b)))/(32*d) - (tan(c/2 + (d*x)/2)^3*((7*a)/96 - (5*b)/96))/d - a*x +
(tan(c/2 + (d*x)/2)*((11*a)/16 - (5*b)/16))/d
```

3.270 $\int \cot^8(c+dx)(a+b\sec(c+dx))\,dx$

3.270.1 Optimal result 1971
3.270.2 Mathematica [C] (verified) 1971
3.270.3 Rubi [A] (verified) 1972
3.270.4 Maple [A] (verified) 1975
3.270.5 Fricas [A] (verification not implemented) 1975
3.270.6 Sympy [F] 1976
3.270.7 Maxima [A] (verification not implemented) 1976
3.270.8 Giac [B] (verification not implemented) 1976
3.270.9 Mupad [B] (verification not implemented) 1977

3.270.1 Optimal result

Integrand size = 19, antiderivative size = 111

$$\begin{aligned}\int \cot^8(c+dx)(a+b\sec(c+dx))\,dx = ax &- \frac{\cot^7(c+dx)(a+b\sec(c+dx))}{7d} \\ &+ \frac{\cot^5(c+dx)(7a+6b\sec(c+dx))}{35d} \\ &+ \frac{\cot(c+dx)(35a+16b\sec(c+dx))}{35d} \\ &- \frac{\cot^3(c+dx)(35a+24b\sec(c+dx))}{105d}\end{aligned}$$

output
```
a*x-1/7*cot(d*x+c)^7*(a+b*sec(d*x+c))/d+1/35*cot(d*x+c)^5*(7*a+6*b*sec(d*x
+c))/d+1/35*cot(d*x+c)*(35*a+16*b*sec(d*x+c))/d-1/105*cot(d*x+c)^3*(35*a+2
4*b*sec(d*x+c))/d
```

3.270.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.03 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.83

$$\begin{aligned}&\int \cot^8(c+dx)(a+b\sec(c+dx))\,dx \\ &= \frac{b\csc(c+dx)}{d} - \frac{b\csc^3(c+dx)}{d} + \frac{3b\csc^5(c+dx)}{5d} - \frac{b\csc^7(c+dx)}{7d} \\ &\quad - \frac{a\cot^7(c+dx)\,\text{Hypergeometric2F1}\left(-\frac{7}{2},1,-\frac{5}{2},-\tan^2(c+dx)\right)}{7d}\end{aligned}$$

input
```
Integrate[Cot[c + d*x]^8*(a + b*Sec[c + d*x]),x]
```

output
```
(b*Csc[c + d*x])/d - (b*Csc[c + d*x]^3)/d + (3*b*Csc[c + d*x]^5)/(5*d) - (
b*Csc[c + d*x]^7)/(7*d) - (a*Cot[c + d*x]^7*Hypergeometric2F1[-7/2, 1, -5/
2, -Tan[c + d*x]^2])/(7*d)
```

3.270.3 Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.07, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.632$, Rules used = {3042, 4370, 25, 3042, 4370, 25, 3042, 4370, 27, 3042, 4370, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^8(c+dx)(a+b\sec(c+dx))\,dx$$

$$\downarrow 3042$$

$$\int \frac{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}{\cot\left(c+dx+\frac{\pi}{2}\right)^8}dx$$

$$\downarrow 4370$$

$$\frac{1}{7}\int -\cot^6(c+dx)(7a+6b\sec(c+dx))dx - \frac{\cot^7(c+dx)(a+b\sec(c+dx))}{7d}$$

$$\downarrow 25$$

$$-\frac{1}{7}\int \cot^6(c+dx)(7a+6b\sec(c+dx))dx - \frac{\cot^7(c+dx)(a+b\sec(c+dx))}{7d}$$

$$\downarrow 3042$$

$$-\frac{1}{7}\int \frac{7a+6b\csc\left(c+dx+\frac{\pi}{2}\right)}{\cot\left(c+dx+\frac{\pi}{2}\right)^6}dx - \frac{\cot^7(c+dx)(a+b\sec(c+dx))}{7d}$$

$$\downarrow 4370$$

$$\frac{1}{7}\left(\frac{\cot^5(c+dx)(7a+6b\sec(c+dx))}{5d} - \frac{1}{5}\int -\cot^4(c+dx)(35a+24b\sec(c+dx))dx\right) - \frac{\cot^7(c+dx)(a+b\sec(c+dx))}{7d}$$

$$\downarrow 25$$

$$\frac{1}{7}\left(\frac{1}{5}\int \cot^4(c+dx)(35a+24b\sec(c+dx))dx+\frac{\cot^5(c+dx)(7a+6b\sec(c+dx))}{5d}\right)-\frac{\cot^7(c+dx)(a+b\sec(c+dx))}{7d}$$

$$\downarrow 3042$$

$$\frac{1}{7}\left(\frac{1}{5}\int \frac{35a+24b\csc\left(c+dx+\frac{\pi}{2}\right)}{\cot\left(c+dx+\frac{\pi}{2}\right)^4}dx+\frac{\cot^5(c+dx)(7a+6b\sec(c+dx))}{5d}\right)-\frac{\cot^7(c+dx)(a+b\sec(c+dx))}{7d}$$

$$\downarrow 4370$$

$$\frac{1}{7}\left(\frac{1}{5}\left(\frac{1}{3}\int -3\cot^2(c+dx)(35a+16b\sec(c+dx))dx-\frac{\cot^3(c+dx)(35a+24b\sec(c+dx))}{3d}\right)+\frac{\cot^5(c+dx)(7a\right.$$
$$\left.\frac{}{5}\right)-\frac{\cot^7(c+dx)(a+b\sec(c+dx))}{7d}$$

$$\downarrow 27$$

$$\frac{1}{7}\left(\frac{1}{5}\left(-\int \cot^2(c+dx)(35a+16b\sec(c+dx))dx-\frac{\cot^3(c+dx)(35a+24b\sec(c+dx))}{3d}\right)+\frac{\cot^5(c+dx)(7a+6}{5d}\right.$$
$$\left.\frac{}{}\right)-\frac{\cot^7(c+dx)(a+b\sec(c+dx))}{7d}$$

$$\downarrow 3042$$

$$\frac{1}{7}\left(\frac{1}{5}\left(-\int \frac{35a+16b\csc\left(c+dx+\frac{\pi}{2}\right)}{\cot\left(c+dx+\frac{\pi}{2}\right)^2}dx-\frac{\cot^3(c+dx)(35a+24b\sec(c+dx))}{3d}\right)+\frac{\cot^5(c+dx)(7a+6b\sec(c-}{5d}\right.$$
$$\left.\frac{}{}\right)-\frac{\cot^7(c+dx)(a+b\sec(c+dx))}{7d}$$

$$\downarrow 4370$$

$$\frac{1}{7}\left(\frac{1}{5}\left(-\int -35adx-\frac{\cot^3(c+dx)(35a+24b\sec(c+dx))}{3d}+\frac{\cot(c+dx)(35a+16b\sec(c+dx))}{d}\right)+\frac{\cot^5(c+dx}{}\right.$$
$$\left.\frac{}{}\right)-\frac{\cot^7(c+dx)(a+b\sec(c+dx))}{7d}$$

$$\downarrow 24$$

$$\frac{1}{7}\left(\frac{\cot^5(c+dx)(7a+6b\sec(c+dx))}{5d}+\frac{1}{5}\left(-\frac{\cot^3(c+dx)(35a+24b\sec(c+dx))}{3d}+\frac{\cot(c+dx)(35a+16b\sec(c-}{d}\right.\right.$$
$$\left.\left.\frac{}{}\right)-\frac{\cot^7(c+dx)(a+b\sec(c+dx))}{7d}\right)$$

input
```
Int[Cot[c + d*x]^8*(a + b*Sec[c + d*x]),x]
```

output
```
-1/7*(Cot[c + d*x]^7*(a + b*Sec[c + d*x]))/d + ((Cot[c + d*x]^5*(7*a + 6*b
*Sec[c + d*x]))/(5*d) + (35*a*x + (Cot[c + d*x]*(35*a + 16*b*Sec[c + d*x])
)/d - (Cot[c + d*x]^3*(35*a + 24*b*Sec[c + d*x]))/(3*d))/5)/7
```

3.270.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4370
```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(-(e*Cot[c + d*x])^(m + 1))*((a + b*Csc[c + d*x])/(
d*e*(m + 1))), x] - Simp[1/(e^2*(m + 1))   Int[(e*Cot[c + d*x])^(m + 2)*(a*
(m + 1) + b*(m + 2)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && L
tQ[m, -1]
```

3.270.4 Maple [A] (verified)

Time = 1.69 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.46

method	result
derivativedivides	$\frac{a\left(-\frac{\cot(dx+c)^7}{7}+\frac{\cot(dx+c)^5}{5}-\frac{\cot(dx+c)^3}{3}+\cot(dx+c)+dx+c\right)+b\left(-\frac{\cos(dx+c)^8}{7\sin(dx+c)^7}+\frac{\cos(dx+c)^8}{35\sin(dx+c)^5}-\frac{\cos(dx+c)^8}{35\sin(dx+c)^3}+\frac{\cos(dx}{7\sin(d}$
default	$\frac{a\left(-\frac{\cot(dx+c)^7}{7}+\frac{\cot(dx+c)^5}{5}-\frac{\cot(dx+c)^3}{3}+\cot(dx+c)+dx+c\right)+b\left(-\frac{\cos(dx+c)^8}{7\sin(dx+c)^7}+\frac{\cos(dx+c)^8}{35\sin(dx+c)^5}-\frac{\cos(dx+c)^8}{35\sin(dx+c)^3}+\frac{\cos(dx}{7\sin(d}$
risch	$ax+\frac{2i\left(105b\,e^{13i(dx+c)}+420a\,e^{12i(dx+c)}-210b\,e^{11i(dx+c)}-1260a\,e^{10i(dx+c)}+903b\,e^{9i(dx+c)}+3080a\,e^{8i(dx+c)}-636b\right.}{105d(e^{2i}}$

input
```
int(cot(d*x+c)^8*(a+b*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(a*(-1/7*cot(d*x+c)^7+1/5*cot(d*x+c)^5-1/3*cot(d*x+c)^3+cot(d*x+c)+d*x
+c)+b*(-1/7/sin(d*x+c)^7*cos(d*x+c)^8+1/35/sin(d*x+c)^5*cos(d*x+c)^8-1/35/
sin(d*x+c)^3*cos(d*x+c)^8+1/7/sin(d*x+c)*cos(d*x+c)^8+1/7*(16/5+cos(d*x+c)
^6+6/5*cos(d*x+c)^4+8/5*cos(d*x+c)^2)*sin(d*x+c)))
```

3.270.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.61

$$\int \cot^8(c+dx)(a+b\sec(c+dx))\,dx$$
$$=\frac{176\,a\cos(dx+c)^7+105\,b\cos(dx+c)^6-406\,a\cos(dx+c)^5-210\,b\cos(dx+c)^4+350\,a\cos(dx+c)^3+}{105\left(d\cos(dx+c)^6-\right.}$$

input
```
integrate(cot(d*x+c)^8*(a+b*sec(d*x+c)),x, algorithm="fricas")
```

output
```
1/105*(176*a*cos(d*x + c)^7 + 105*b*cos(d*x + c)^6 - 406*a*cos(d*x + c)^5
- 210*b*cos(d*x + c)^4 + 350*a*cos(d*x + c)^3 + 168*b*cos(d*x + c)^2 - 105
*a*cos(d*x + c) + 105*(a*d*x*cos(d*x + c)^6 - 3*a*d*x*cos(d*x + c)^4 + 3*a
*d*x*cos(d*x + c)^2 - a*d*x)*sin(d*x + c) - 48*b)/((d*cos(d*x + c)^6 - 3*d
*cos(d*x + c)^4 + 3*d*cos(d*x + c)^2 - d)*sin(d*x + c))
```

3.270.6 Sympy [F]

$$\int \cot^8(c+dx)(a+b\sec(c+dx))\,dx = \int (a+b\sec(c+dx))\cot^8(c+dx)\,dx$$

input
```
integrate(cot(d*x+c)**8*(a+b*sec(d*x+c)),x)
```

output
```
Integral((a + b*sec(c + d*x))*cot(c + d*x)**8, x)
```

3.270.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.90

$$\int \cot^8(c+dx)(a+b\sec(c+dx))\,dx$$
$$= \frac{\left(105\,dx + 105\,c + \frac{105\tan(dx+c)^6 - 35\tan(dx+c)^4 + 21\tan(dx+c)^2 - 15}{\tan(dx+c)^7}\right)a + \frac{3\left(35\sin(dx+c)^6 - 35\sin(dx+c)^4 + 21\sin(dx+c)^2 - 5\right)b}{\sin(dx+c)^7}}{105\,d}$$

input
```
integrate(cot(d*x+c)^8*(a+b*sec(d*x+c)),x, algorithm="maxima")
```

output
```
1/105*((105*d*x + 105*c + (105*tan(d*x + c)^6 - 35*tan(d*x + c)^4 + 21*tan
(d*x + c)^2 - 15)/tan(d*x + c)^7)*a + 3*(35*sin(d*x + c)^6 - 35*sin(d*x +
c)^4 + 21*sin(d*x + c)^2 - 5)*b/sin(d*x + c)^7)/d
```

3.270.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 225 vs. 2(103) = 206.

Time = 0.37 (sec) , antiderivative size = 225, normalized size of antiderivative = 2.03

$$\int \cot^8(c+dx)(a+b\sec(c+dx))\,dx$$
$$= \frac{15\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7 - 15\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7 - 189\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5 + 147\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5 + 1295\,a}{}$$

input
```
integrate(cot(d*x+c)^8*(a+b*sec(d*x+c)),x, algorithm="giac")
```

output
```
1/13440*(15*a*tan(1/2*d*x + 1/2*c)^7 - 15*b*tan(1/2*d*x + 1/2*c)^7 - 189*a
*tan(1/2*d*x + 1/2*c)^5 + 147*b*tan(1/2*d*x + 1/2*c)^5 + 1295*a*tan(1/2*d*
x + 1/2*c)^3 - 735*b*tan(1/2*d*x + 1/2*c)^3 + 13440*(d*x + c)*a - 9765*a*t
an(1/2*d*x + 1/2*c) + 3675*b*tan(1/2*d*x + 1/2*c) + (9765*a*tan(1/2*d*x +
1/2*c)^6 + 3675*b*tan(1/2*d*x + 1/2*c)^6 - 1295*a*tan(1/2*d*x + 1/2*c)^4 -
 735*b*tan(1/2*d*x + 1/2*c)^4 + 189*a*tan(1/2*d*x + 1/2*c)^2 + 147*b*tan(1
/2*d*x + 1/2*c)^2 - 15*a - 15*b)/tan(1/2*d*x + 1/2*c)^7)/d
```

3.270.9 Mupad [B] (verification not implemented)

Time = 14.66 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.57

$$\int \cot^8(c+dx)(a+b\sec(c+dx))\,dx$$
$$= a\,x + \frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3\left(\frac{37\,a}{384}-\frac{7\,b}{128}\right)}{d} - \frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5\left(\frac{9\,a}{640}-\frac{7\,b}{640}\right)}{d} + \frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^7\left(\frac{a}{896}-\frac{b}{896}\right)}{d}$$
$$- \frac{\cot\left(\frac{c}{2}+\frac{dx}{2}\right)^7\left((-93\,a-35\,b)\,\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6+\left(\frac{37\,a}{3}+7\,b\right)\,\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4+\left(-\frac{9\,a}{5}-\frac{7\,b}{5}\right)\,\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+\right.}{128\,d}$$
$$- \frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\,\left(\frac{93\,a}{128}-\frac{35\,b}{128}\right)}{d}$$

input
```
int(cot(c + d*x)^8*(a + b/cos(c + d*x)),x)
```

output
```
a*x + (tan(c/2 + (d*x)/2)^3*((37*a)/384 - (7*b)/128))/d - (tan(c/2 + (d*x)
/2)^5*((9*a)/640 - (7*b)/640))/d + (tan(c/2 + (d*x)/2)^7*(a/896 - b/896))/
d - (cot(c/2 + (d*x)/2)^7*(a/7 + b/7 - tan(c/2 + (d*x)/2)^2*((9*a)/5 + (7*
b)/5) + tan(c/2 + (d*x)/2)^4*((37*a)/3 + 7*b) - tan(c/2 + (d*x)/2)^6*(93*a
 + 35*b)))/(128*d) - (tan(c/2 + (d*x)/2)*((93*a)/128 - (35*b)/128))/d
```

3.271 $\int (a + b\sec(c + dx))^2 \tan^9(c + dx)\, dx$

3.271.1 Optimal result 1978
3.271.2 Mathematica [A] (verified) 1979
3.271.3 Rubi [A] (verified) 1979
3.271.4 Maple [A] (verified) 1981
3.271.5 Fricas [A] (verification not implemented) 1981
3.271.6 Sympy [A] (verification not implemented) 1982
3.271.7 Maxima [A] (verification not implemented) 1982
3.271.8 Giac [B] (verification not implemented) 1983
3.271.9 Mupad [B] (verification not implemented) 1983

3.271.1 Optimal result

Integrand size = 21, antiderivative size = 185

$$\begin{aligned}\int (a + b\sec(c + dx))^2 \tan^9(c + dx)\, dx = &-\frac{a^2 \log(\cos(c + dx))}{d} \\ &+ \frac{2ab\sec(c + dx)}{d} - \frac{2a^2\sec^2(c + dx)}{d} \\ &- \frac{8ab\sec^3(c + dx)}{3d} + \frac{3a^2\sec^4(c + dx)}{2d} \\ &+ \frac{12ab\sec^5(c + dx)}{5d} - \frac{2a^2\sec^6(c + dx)}{3d} \\ &- \frac{8ab\sec^7(c + dx)}{7d} + \frac{a^2\sec^8(c + dx)}{8d} \\ &+ \frac{2ab\sec^9(c + dx)}{9d} + \frac{b^2\tan^{10}(c + dx)}{10d}\end{aligned}$$

output

```
-a^2*ln(cos(d*x+c))/d+2*a*b*sec(d*x+c)/d-2*a^2*sec(d*x+c)^2/d-8/3*a*b*sec(
d*x+c)^3/d+3/2*a^2*sec(d*x+c)^4/d+12/5*a*b*sec(d*x+c)^5/d-2/3*a^2*sec(d*x+
c)^6/d-8/7*a*b*sec(d*x+c)^7/d+1/8*a^2*sec(d*x+c)^8/d+2/9*a*b*sec(d*x+c)^9/
d+1/10*b^2*tan(d*x+c)^10/d
```

3.271.2 Mathematica [A] (verified)

Time = 1.17 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.94

$$\int (a+b\sec(c+dx))^2\tan^9(c+dx)\,dx$$
$$=\frac{-2520a^2\log(\cos(c+dx))+5040ab\sec(c+dx)-1260(4a^2-b^2)\sec^2(c+dx)-6720ab\sec^3(c+dx)+12}{}$$

input
```
Integrate[(a + b*Sec[c + d*x])^2*Tan[c + d*x]^9,x]
```

output
```
(-2520*a^2*Log[Cos[c + d*x]] + 5040*a*b*Sec[c + d*x] - 1260*(4*a^2 - b^2)*
Sec[c + d*x]^2 - 6720*a*b*Sec[c + d*x]^3 + 1260*(3*a^2 - 2*b^2)*Sec[c + d*
x]^4 + 6048*a*b*Sec[c + d*x]^5 - 840*(2*a^2 - 3*b^2)*Sec[c + d*x]^6 - 2880
*a*b*Sec[c + d*x]^7 + 315*(a^2 - 4*b^2)*Sec[c + d*x]^8 + 560*a*b*Sec[c + d
*x]^9 + 252*b^2*Sec[c + d*x]^10)/(2520*d)
```

3.271.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.17, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3042, 25, 4373, 522, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^9(c+dx)(a+b\sec(c+dx))^2\,dx$$
$$\downarrow 3042$$
$$\int -\cot\left(c+dx+\frac{\pi}{2}\right)^9\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^2 dx$$
$$\downarrow 25$$
$$-\int \cot\left(\frac{1}{2}(2c+\pi)+dx\right)^9\left(a+b\csc\left(\frac{1}{2}(2c+\pi)+dx\right)\right)^2 dx$$
$$\downarrow 4373$$
$$\frac{\int \frac{\cos(c+dx)(a+b\sec(c+dx))^2\left(b^2-b^2\sec^2(c+dx)\right)^4}{b}d(b\sec(c+dx))}{b^8 d}$$
$$\downarrow 522$$

$$\int \left(b^9 \sec^9(c+dx) + 2ab^8 \sec^8(c+dx) + b^7\left(a^2-4b^2\right)\sec^7(c+dx) - 8ab^8 \sec^6(c+dx) - 2b^7\left(2a^2-3b^2\right)\sec^5(c+$$

$$\downarrow \text{2009}$$

$$a^2b^8 \log(b\sec(c+dx)) + \tfrac{1}{8}b^8\left(a^2-4b^2\right)\sec^8(c+dx) - \tfrac{1}{3}b^8\left(2a^2-3b^2\right)\sec^6(c+dx) + \tfrac{1}{2}b^8\left(3a^2-2b^2\right)\sec^4(c+dx)$$

input
```
Int[(a + b*Sec[c + d*x])^2*Tan[c + d*x]^9,x]
```

output
```
(a^2*b^8*Log[b*Sec[c + d*x]] + 2*a*b^9*Sec[c + d*x] - (b^8*(4*a^2 - b^2)*S
ec[c + d*x]^2)/2 - (8*a*b^9*Sec[c + d*x]^3)/3 + (b^8*(3*a^2 - 2*b^2)*Sec[c
 + d*x]^4)/2 + (12*a*b^9*Sec[c + d*x]^5)/5 - (b^8*(2*a^2 - 3*b^2)*Sec[c +
d*x]^6)/3 - (8*a*b^9*Sec[c + d*x]^7)/7 + (b^8*(a^2 - 4*b^2)*Sec[c + d*x]^8
)/8 + (2*a*b^9*Sec[c + d*x]^9)/9 + (b^10*Sec[c + d*x]^10)/10)/(b^8*d)
```

3.271.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 522
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_.
), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(c + d*x)^n*(a + b*x^2)^p, x],
x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4373
```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Simp[-(-1)^((m - 1)/2)/(d*b^(m - 1))   Subst[Int[(b^2 - x^
2)^((m - 1)/2)*((a + x)^n/x), x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
 d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

3.271.4 Maple [A] (verified)

Time = 6.16 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.72

method	result
parts	$\frac{a^2\left(\frac{\tan(dx+c)^8}{8}-\frac{\tan(dx+c)^6}{6}+\frac{\tan(dx+c)^4}{4}-\frac{\tan(dx+c)^2}{2}+\frac{\ln\left(1+\tan(dx+c)^2\right)}{2}\right)}{d}+\frac{b^2\tan(dx+c)^{10}}{10d}+\frac{2ab\left(\frac{\sec(dx+c)^9}{9}-\frac{4}{}\right.}{}$
derivativedivides	$\frac{b^2\sec(dx+c)^{10}}{10}+\frac{2ab\sec(dx+c)^9}{9}+\frac{a^2\sec(dx+c)^8}{8}-\frac{b^2\sec(dx+c)^8}{2}-\frac{8ab\sec(dx+c)^7}{7}-\frac{2a^2\sec(dx+c)^6}{3}+b^2\sec(dx+c)^6+\frac{12ab\sec(}{5}$
default	$\frac{b^2\sec(dx+c)^{10}}{10}+\frac{2ab\sec(dx+c)^9}{9}+\frac{a^2\sec(dx+c)^8}{8}-\frac{b^2\sec(dx+c)^8}{2}-\frac{8ab\sec(dx+c)^7}{7}-\frac{2a^2\sec(dx+c)^6}{3}+b^2\sec(dx+c)^6+\frac{12ab\sec(}{5}$
risch	$ia^2x+\frac{2ia^2c}{d}+\frac{4ab\,e^{19i(dx+c)}-8a^2e^{18i(dx+c)}+2b^2e^{18i(dx+c)}+\frac{44ab\,e^{17i(dx+c)}}{3}-40a^2e^{16i(dx+c)}+\frac{1072ab\,e^{15i(dx+c)}}{15}-}{}$

input
```
int((a+b*sec(d*x+c))^2*tan(d*x+c)^9,x,method=_RETURNVERBOSE)
```

output
```
a^2/d*(1/8*tan(d*x+c)^8-1/6*tan(d*x+c)^6+1/4*tan(d*x+c)^4-1/2*tan(d*x+c)^2
+1/2*ln(1+tan(d*x+c)^2))+1/10*b^2*tan(d*x+c)^10/d+2*a*b/d*(1/9*sec(d*x+c)^
9-4/7*sec(d*x+c)^7+6/5*sec(d*x+c)^5-4/3*sec(d*x+c)^3+sec(d*x+c))
```

3.271.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 181, normalized size of antiderivative = 0.98

$$\int (a+b\sec(c+dx))^2\tan^9(c+dx)\,dx =$$
$$-\frac{2520\,a^2\cos(dx+c)^{10}\log(-\cos(dx+c))-5040\,ab\cos(dx+c)^9+6720\,ab\cos(dx+c)^7+1260\,(4\,a^2-}{}$$

input
```
integrate((a+b*sec(d*x+c))^2*tan(d*x+c)^9,x, algorithm="fricas")
```

output
```
-1/2520*(2520*a^2*cos(d*x + c)^10*log(-cos(d*x + c)) - 5040*a*b*cos(d*x +
c)^9 + 6720*a*b*cos(d*x + c)^7 + 1260*(4*a^2 - b^2)*cos(d*x + c)^8 - 6048*
a*b*cos(d*x + c)^5 - 1260*(3*a^2 - 2*b^2)*cos(d*x + c)^6 + 2880*a*b*cos(d*
x + c)^3 + 840*(2*a^2 - 3*b^2)*cos(d*x + c)^4 - 560*a*b*cos(d*x + c) - 315
*(a^2 - 4*b^2)*cos(d*x + c)^2 - 252*b^2)/(d*cos(d*x + c)^10)
```

3.271.6 Sympy [A] (verification not implemented)

Time = 2.48 (sec) , antiderivative size = 314, normalized size of antiderivative = 1.70

$$\int (a + b\sec(c + dx))^2 \tan^9(c + dx)\, dx$$

$$= \begin{cases} \frac{a^2 \log\left(\tan^2(c+dx)+1\right)}{2d} + \frac{a^2 \tan^8(c+dx)}{8d} - \frac{a^2 \tan^6(c+dx)}{6d} + \frac{a^2 \tan^4(c+dx)}{4d} - \frac{a^2 \tan^2(c+dx)}{2d} + \frac{2ab \tan^8(c+dx)\sec(c+dx)}{9d} - \frac{16d}{} \\ x(a + b\sec(c))^2 \tan^9(c) \end{cases}$$

input
```
integrate((a+b*sec(d*x+c))**2*tan(d*x+c)**9,x)
```

output
```
Piecewise((a**2*log(tan(c + d*x)**2 + 1)/(2*d) + a**2*tan(c + d*x)**8/(8*d
) - a**2*tan(c + d*x)**6/(6*d) + a**2*tan(c + d*x)**4/(4*d) - a**2*tan(c +
 d*x)**2/(2*d) + 2*a*b*tan(c + d*x)**8*sec(c + d*x)/(9*d) - 16*a*b*tan(c +
 d*x)**6*sec(c + d*x)/(63*d) + 32*a*b*tan(c + d*x)**4*sec(c + d*x)/(105*d)
 - 128*a*b*tan(c + d*x)**2*sec(c + d*x)/(315*d) + 256*a*b*sec(c + d*x)/(31
5*d) + b**2*tan(c + d*x)**8*sec(c + d*x)**2/(10*d) - b**2*tan(c + d*x)**6*
sec(c + d*x)**2/(10*d) + b**2*tan(c + d*x)**4*sec(c + d*x)**2/(10*d) - b**
2*tan(c + d*x)**2*sec(c + d*x)**2/(10*d) + b**2*sec(c + d*x)**2/(10*d), Ne
(d, 0)), (x*(a + b*sec(c))**2*tan(c)**9, True))
```

3.271.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 174, normalized size of antiderivative = 0.94

$$\int (a + b\sec(c + dx))^2 \tan^9(c + dx)\, dx =$$

$$-\frac{2520\, a^2 \log\left(\cos(dx + c)\right) - \frac{5040\, ab\cos(dx+c)^9 - 6720\, ab\cos(dx+c)^7 - 1260\left(4a^2 - b^2\right)\cos(dx+c)^8 + 6048\, ab\cos(dx+c)^5 + 1260\left(3a^2}{}}{2520\, d}$$

input
```
integrate((a+b*sec(d*x+c))^2*tan(d*x+c)^9,x, algorithm="maxima")
```

output
```
-1/2520*(2520*a^2*log(cos(d*x + c)) - (5040*a*b*cos(d*x + c)^9 - 6720*a*b*
cos(d*x + c)^7 - 1260*(4*a^2 - b^2)*cos(d*x + c)^8 + 6048*a*b*cos(d*x + c)
^5 + 1260*(3*a^2 - 2*b^2)*cos(d*x + c)^6 - 2880*a*b*cos(d*x + c)^3 - 840*(
2*a^2 - 3*b^2)*cos(d*x + c)^4 + 560*a*b*cos(d*x + c) + 315*(a^2 - 4*b^2)*c
os(d*x + c)^2 + 252*b^2)/cos(d*x + c)^10)/d
```

3.271.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 489 vs. 2(169) = 338.

Time = 5.92 (sec) , antiderivative size = 489, normalized size of antiderivative = 2.64

$$\int (a+b\sec(c+dx))^2\tan^9(c+dx)\,dx$$

$$= \frac{2520\,a^2\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right)-2520\,a^2\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right)+\frac{7381\,a^2+4096\,ab+\frac{78850\,a^2(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{40960\,a}{\cos(dx+c)+1}}{\cos(dx+c)+1}}{\cos(dx+c)+1}

input
```
int(tan(c + d*x)^9*(a + b/cos(c + d*x))^2,x)
```

output
```
((512*a*b)/315 + tan(c/2 + (d*x)/2)^4*((512*a*b)/7 + 20*a^2) - tan(c/2 + (
d*x)/2)^2*((1024*a*b)/63 + 2*a^2) + tan(c/2 + (d*x)/2)^8*((1024*a*b)/3 + (
740*a^2)/3) - tan(c/2 + (d*x)/2)^6*((4096*a*b)/21 + (272*a^2)/3) + (740*a^
2*tan(c/2 + (d*x)/2)^12)/3 - (272*a^2*tan(c/2 + (d*x)/2)^14)/3 + 20*a^2*ta
n(c/2 + (d*x)/2)^16 - 2*a^2*tan(c/2 + (d*x)/2)^18 - tan(c/2 + (d*x)/2)^10*
((1024*a*b)/5 + 348*a^2 - (512*b^2)/5))/(d*(45*tan(c/2 + (d*x)/2)^4 - 10*t
an(c/2 + (d*x)/2)^2 - 120*tan(c/2 + (d*x)/2)^6 + 210*tan(c/2 + (d*x)/2)^8
- 252*tan(c/2 + (d*x)/2)^10 + 210*tan(c/2 + (d*x)/2)^12 - 120*tan(c/2 + (d
*x)/2)^14 + 45*tan(c/2 + (d*x)/2)^16 - 10*tan(c/2 + (d*x)/2)^18 + tan(c/2
+ (d*x)/2)^20 + 1)) + (2*a^2*atanh(tan(c/2 + (d*x)/2)^2))/d
```

3.272 $\int (a + b\sec(c + dx))^2 \tan^7(c + dx)\, dx$

3.272.1 Optimal result 1985
3.272.2 Mathematica [A] (verified) 1986
3.272.3 Rubi [A] (verified) 1986
3.272.4 Maple [A] (verified) 1988
3.272.5 Fricas [A] (verification not implemented) 1988
3.272.6 Sympy [A] (verification not implemented) 1989
3.272.7 Maxima [A] (verification not implemented) 1989
3.272.8 Giac [B] (verification not implemented) 1990
3.272.9 Mupad [B] (verification not implemented) 1990

3.272.1 Optimal result

Integrand size = 21, antiderivative size = 149

$$\begin{aligned}\int (a + b\sec(c + dx))^2 \tan^7(c + dx)\, dx = {} & \frac{a^2 \log(\cos(c + dx))}{d} \\ & - \frac{2ab\sec(c + dx)}{d} + \frac{3a^2\sec^2(c + dx)}{2d} \\ & + \frac{2ab\sec^3(c + dx)}{d} - \frac{3a^2\sec^4(c + dx)}{4d} \\ & - \frac{6ab\sec^5(c + dx)}{5d} + \frac{a^2\sec^6(c + dx)}{6d} \\ & + \frac{2ab\sec^7(c + dx)}{7d} + \frac{b^2\tan^8(c + dx)}{8d}\end{aligned}$$

output

```
a^2*ln(cos(d*x+c))/d-2*a*b*sec(d*x+c)/d+3/2*a^2*sec(d*x+c)^2/d+2*a*b*sec(d
*x+c)^3/d-3/4*a^2*sec(d*x+c)^4/d-6/5*a*b*sec(d*x+c)^5/d+1/6*a^2*sec(d*x+c)
^6/d+2/7*a*b*sec(d*x+c)^7/d+1/8*b^2*tan(d*x+c)^8/d
```

3.272.2 Mathematica [A] (verified)

Time = 0.67 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.93

$$\int (a+b\sec(c+dx))^2\tan^7(c+dx)\,dx$$
$$=\frac{840a^2\log(\cos(c+dx))-1680ab\sec(c+dx)+420(3a^2-b^2)\sec^2(c+dx)+1680ab\sec^3(c+dx)-630(a^2$$

input
```
Integrate[(a + b*Sec[c + d*x])^2*Tan[c + d*x]^7,x]
```

output
```
(840*a^2*Log[Cos[c + d*x]] - 1680*a*b*Sec[c + d*x] + 420*(3*a^2 - b^2)*Sec
[c + d*x]^2 + 1680*a*b*Sec[c + d*x]^3 - 630*(a^2 - b^2)*Sec[c + d*x]^4 - 1
008*a*b*Sec[c + d*x]^5 + 140*(a^2 - 3*b^2)*Sec[c + d*x]^6 + 240*a*b*Sec[c
+ d*x]^7 + 105*b^2*Sec[c + d*x]^8)/(840*d)
```

3.272.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.15, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3042, 25, 4373, 522, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^7(c+dx)(a+b\sec(c+dx))^2\,dx$$
$$\downarrow 3042$$
$$\int -\cot\left(c+dx+\frac{\pi}{2}\right)^7\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^2 dx$$
$$\downarrow 25$$
$$-\int \cot\left(\frac{1}{2}(2c+\pi)+dx\right)^7\left(a+b\csc\left(\frac{1}{2}(2c+\pi)+dx\right)\right)^2 dx$$
$$\downarrow 4373$$
$$-\frac{\int \frac{\cos(c+dx)(a+b\sec(c+dx))^2\left(b^2-b^2\sec^2(c+dx)\right)^3}{b}d(b\sec(c+dx))}{b^6 d}$$
$$\downarrow 522$$

$$-\frac{\int \left(-b^7 \sec^7(c+dx) - 2ab^6 \sec^6(c+dx) - b^5\left(a^2-3b^2\right)\sec^5(c+dx) + 6ab^6 \sec^4(c+dx) + 3b^5\left(a^2-b^2\right)\sec^3(c+\right.}{b^6 d}$$

$$\downarrow \text{2009}$$

$$-\frac{a^2b^6\log(b\sec(c+dx)) - \frac{1}{6}b^6\left(a^2-3b^2\right)\sec^6(c+dx) + \frac{3}{4}b^6\left(a^2-b^2\right)\sec^4(c+dx) - \frac{1}{2}b^6\left(3a^2-b^2\right)\sec^2(c+dx) -}{b^6 d}$$

input
```
Int[(a + b*Sec[c + d*x])^2*Tan[c + d*x]^7,x]
```

output
```
-((a^2*b^6*Log[b*Sec[c + d*x]] + 2*a*b^7*Sec[c + d*x] - (b^6*(3*a^2 - b^2)
*Sec[c + d*x]^2)/2 - 2*a*b^7*Sec[c + d*x]^3 + (3*b^6*(a^2 - b^2)*Sec[c + d
*x]^4)/4 + (6*a*b^7*Sec[c + d*x]^5)/5 - (b^6*(a^2 - 3*b^2)*Sec[c + d*x]^6)
/6 - (2*a*b^7*Sec[c + d*x]^7)/7 - (b^8*Sec[c + d*x]^8)/8)/(b^6*d))
```

3.272.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 522
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_.
), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(c + d*x)^n*(a + b*x^2)^p, x],
x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4373
```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Simp[-(-1)^((m - 1)/2)/(d*b^(m - 1))   Subst[Int[(b^2 - x^
2)^((m - 1)/2)*((a + x)^n/x), x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
 d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

3.272.4 Maple [A] (verified)

Time = 3.74 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.76

method	result
parts	$\frac{a^2\left(\frac{\tan(dx+c)^6}{6}-\frac{\tan(dx+c)^4}{4}+\frac{\tan(dx+c)^2}{2}-\frac{\ln\left(1+\tan(dx+c)^2\right)}{2}\right)}{d}+\frac{b^2\tan(dx+c)^8}{8d}+\frac{2ab\left(\frac{\sec(dx+c)^7}{7}-\frac{3\sec(dx+c)^5}{5}+s}{d}$
derivativedivides	$\frac{\frac{b^2\sec(dx+c)^8}{8}+\frac{2ab\sec(dx+c)^7}{7}+\frac{a^2\sec(dx+c)^6}{6}-\frac{b^2\sec(dx+c)^6}{2}-\frac{6ab\sec(dx+c)^5}{5}-\frac{3a^2\sec(dx+c)^4}{4}+\frac{3b^2\sec(dx+c)^4}{4}+2ab\sec(d}{d}$
default	$\frac{\frac{b^2\sec(dx+c)^8}{8}+\frac{2ab\sec(dx+c)^7}{7}+\frac{a^2\sec(dx+c)^6}{6}-\frac{b^2\sec(dx+c)^6}{2}-\frac{6ab\sec(dx+c)^5}{5}-\frac{3a^2\sec(dx+c)^4}{4}+\frac{3b^2\sec(dx+c)^4}{4}+2ab\sec(d}{d}$
risch	$-ia^2x-\frac{2ia^2c}{d}-\frac{2\left(210ab\,e^{15i(dx+c)}-315a^2e^{14i(dx+c)}+105b^2e^{14i(dx+c)}+630ab\,e^{13i(dx+c)}-1260a^2e^{12i(dx+c)}+22}{}$

input
```
int((a+b*sec(d*x+c))^2*tan(d*x+c)^7,x,method=_RETURNVERBOSE)
```

output
```
a^2/d*(1/6*tan(d*x+c)^6-1/4*tan(d*x+c)^4+1/2*tan(d*x+c)^2-1/2*ln(1+tan(d*x
+c)^2))+1/8*b^2*tan(d*x+c)^8/d+2*a*b/d*(1/7*sec(d*x+c)^7-3/5*sec(d*x+c)^5+
sec(d*x+c)^3-sec(d*x+c))
```

3.272.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.98

$$\int (a+b\sec(c+dx))^2\tan^7(c+dx)\,dx$$
$$=\frac{840\,a^2\cos(dx+c)^8\log(-\cos(dx+c))-1680\,ab\cos(dx+c)^7+1680\,ab\cos(dx+c)^5+420\,(3\,a^2-b^2)\,c}{}$$

input
```
integrate((a+b*sec(d*x+c))^2*tan(d*x+c)^7,x, algorithm="fricas")
```

output
```
1/840*(840*a^2*cos(d*x + c)^8*log(-cos(d*x + c)) - 1680*a*b*cos(d*x + c)^7
 + 1680*a*b*cos(d*x + c)^5 + 420*(3*a^2 - b^2)*cos(d*x + c)^6 - 1008*a*b*c
os(d*x + c)^3 - 630*(a^2 - b^2)*cos(d*x + c)^4 + 240*a*b*cos(d*x + c) + 14
0*(a^2 - 3*b^2)*cos(d*x + c)^2 + 105*b^2)/(d*cos(d*x + c)^8)
```

3.272.6 Sympy [A] (verification not implemented)

Time = 1.49 (sec) , antiderivative size = 252, normalized size of antiderivative = 1.69

$$\int (a+b\sec(c+dx))^2\tan^7(c+dx)\,dx$$

$$= \begin{cases} -\frac{a^2\log\left(\tan^2(c+dx)+1\right)}{2d} + \frac{a^2\tan^6(c+dx)}{6d} - \frac{a^2\tan^4(c+dx)}{4d} + \frac{a^2\tan^2(c+dx)}{2d} + \frac{2ab\tan^6(c+dx)\sec(c+dx)}{7d} - \frac{12ab\tan^4(c+dx)\sec} \\ x(a+b\sec(c))^2\tan^7(c) \end{cases}$$

input
```
integrate((a+b*sec(d*x+c))**2*tan(d*x+c)**7,x)
```

output
```
Piecewise((-a**2*log(tan(c + d*x)**2 + 1)/(2*d) + a**2*tan(c + d*x)**6/(6*
d) - a**2*tan(c + d*x)**4/(4*d) + a**2*tan(c + d*x)**2/(2*d) + 2*a*b*tan(c
 + d*x)**6*sec(c + d*x)/(7*d) - 12*a*b*tan(c + d*x)**4*sec(c + d*x)/(35*d)
 + 16*a*b*tan(c + d*x)**2*sec(c + d*x)/(35*d) - 32*a*b*sec(c + d*x)/(35*d)
 + b**2*tan(c + d*x)**6*sec(c + d*x)**2/(8*d) - b**2*tan(c + d*x)**4*sec(c
 + d*x)**2/(8*d) + b**2*tan(c + d*x)**2*sec(c + d*x)**2/(8*d) - b**2*sec(c
 + d*x)**2/(8*d), Ne(d, 0)), (x*(a + b*sec(c))**2*tan(c)**7, True))
```

3.272.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.93

$$\int (a+b\sec(c+dx))^2\tan^7(c+dx)\,dx$$

$$= \frac{840\,a^2\log\left(\cos(dx+c)\right) - \frac{1680\,ab\cos(dx+c)^7 - 1680\,ab\cos(dx+c)^5 - 420\left(3\,a^2-b^2\right)\cos(dx+c)^6 + 1008\,ab\cos(dx+c)^3 + 630\left(a^2-b^2\right)\cos(}{\cos(dx+c)^8}}{840\,d}$$

input
```
integrate((a+b*sec(d*x+c))^2*tan(d*x+c)^7,x, algorithm="maxima")
```

output
```
1/840*(840*a^2*log(cos(d*x + c)) - (1680*a*b*cos(d*x + c)^7 - 1680*a*b*cos
(d*x + c)^5 - 420*(3*a^2 - b^2)*cos(d*x + c)^6 + 1008*a*b*cos(d*x + c)^3 +
 630*(a^2 - b^2)*cos(d*x + c)^4 - 240*a*b*cos(d*x + c) - 140*(a^2 - 3*b^2)
*cos(d*x + c)^2 - 105*b^2)/cos(d*x + c)^8)/d
```

3.272.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 415 vs. $2(137) = 274$.

Time = 3.31 (sec) , antiderivative size = 415, normalized size of antiderivative = 2.79

$$\int (a + b\sec(c+dx))^2 \tan^7(c+dx)\,dx =$$

$$-\frac{840\,a^2 \log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right) - 840\,a^2 \log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right) + \frac{2283\,a^2+1536\,ab+\frac{19944\,a^2(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{12288\,a}{\cos(dx+c)}}{}$$

input
```
integrate((a+b*sec(d*x+c))^2*tan(d*x+c)^7,x, algorithm="giac")
```

output
```
-1/840*(840*a^2*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)) - 840
*a^2*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)) + (2283*a^2 + 15
36*a*b + 19944*a^2*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 12288*a*b*(cos(
d*x + c) - 1)/(cos(d*x + c) + 1) + 77364*a^2*(cos(d*x + c) - 1)^2/(cos(d*x
 + c) + 1)^2 + 43008*a*b*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 17544
8*a^2*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 + 86016*a*b*(cos(d*x + c)
- 1)^3/(cos(d*x + c) + 1)^3 + 231490*a^2*(cos(d*x + c) - 1)^4/(cos(d*x + c
) + 1)^4 + 53760*a*b*(cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4 - 26880*b^2
*(cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4 + 175448*a^2*(cos(d*x + c) - 1)
^5/(cos(d*x + c) + 1)^5 + 77364*a^2*(cos(d*x + c) - 1)^6/(cos(d*x + c) + 1
)^6 + 19944*a^2*(cos(d*x + c) - 1)^7/(cos(d*x + c) + 1)^7 + 2283*a^2*(cos(
d*x + c) - 1)^8/(cos(d*x + c) + 1)^8)/((cos(d*x + c) - 1)/(cos(d*x + c) +
1) + 1)^8)/d
```

3.272.9 Mupad [B] (verification not implemented)

Time = 17.89 (sec) , antiderivative size = 280, normalized size of antiderivative = 1.88

$$\int (a + b\sec(c+dx))^2 \tan^7(c+dx)\,dx =$$

$$-\frac{\frac{64\,a\,b}{35}+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4\left(16\,a^2+\frac{256\,b\,a}{5}\right)-\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2\left(2\,a^2+\frac{512\,b\,a}{35}\right)-\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6\left(\frac{170\,a^2}{3}+\frac{512\,b\,a}{5}\right)-}{d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{16}-8\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{14}+28\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{12}-56\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}+\right.}$$

$$-\frac{2\,a^2\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2\right)}{d}$$

input
```
int(tan(c + d*x)^7*(a + b/cos(c + d*x))^2,x)
```

output
```
- ((64*a*b)/35 + tan(c/2 + (d*x)/2)^4*((256*a*b)/5 + 16*a^2) - tan(c/2 + (
d*x)/2)^2*((512*a*b)/35 + 2*a^2) - tan(c/2 + (d*x)/2)^6*((512*a*b)/5 + (17
0*a^2)/3) - (170*a^2*tan(c/2 + (d*x)/2)^10)/3 + 16*a^2*tan(c/2 + (d*x)/2)^
12 - 2*a^2*tan(c/2 + (d*x)/2)^14 + tan(c/2 + (d*x)/2)^8*(64*a*b + (256*a^2
)/3 - 32*b^2))/(d*(28*tan(c/2 + (d*x)/2)^4 - 8*tan(c/2 + (d*x)/2)^2 - 56*t
an(c/2 + (d*x)/2)^6 + 70*tan(c/2 + (d*x)/2)^8 - 56*tan(c/2 + (d*x)/2)^10 +
 28*tan(c/2 + (d*x)/2)^12 - 8*tan(c/2 + (d*x)/2)^14 + tan(c/2 + (d*x)/2)^1
6 + 1)) - (2*a^2*atanh(tan(c/2 + (d*x)/2)^2))/d
```

3.273 $\int (a+b\sec(c+dx))^2\tan^5(c+dx)\,dx$

3.273.1 Optimal result 1992
3.273.2 Mathematica [A] (verified) 1992
3.273.3 Rubi [A] (verified) 1993
3.273.4 Maple [A] (verified) 1994
3.273.5 Fricas [A] (verification not implemented) 1995
3.273.6 Sympy [A] (verification not implemented) 1995
3.273.7 Maxima [A] (verification not implemented) 1996
3.273.8 Giac [B] (verification not implemented) 1996
3.273.9 Mupad [B] (verification not implemented) 1997

3.273.1 Optimal result

Integrand size = 21, antiderivative size = 115

$$\begin{aligned}\int (a+b\sec(c+dx))^2\tan^5(c+dx)\,dx = &-\frac{a^2\log(\cos(c+dx))}{d}\\ &+\frac{2ab\sec(c+dx)}{d}-\frac{a^2\sec^2(c+dx)}{d}\\ &-\frac{4ab\sec^3(c+dx)}{3d}+\frac{a^2\sec^4(c+dx)}{4d}\\ &+\frac{2ab\sec^5(c+dx)}{5d}+\frac{b^2\tan^6(c+dx)}{6d}\end{aligned}$$

output
```
-a^2*ln(cos(d*x+c))/d+2*a*b*sec(d*x+c)/d-a^2*sec(d*x+c)^2/d-4/3*a*b*sec(d*
x+c)^3/d+1/4*a^2*sec(d*x+c)^4/d+2/5*a*b*sec(d*x+c)^5/d+1/6*b^2*tan(d*x+c)^
6/d
```

3.273.2 Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.91

$$\begin{aligned}&\int (a+b\sec(c+dx))^2\tan^5(c+dx)\,dx\\ &=\frac{-60a^2\log(\cos(c+dx))+120ab\sec(c+dx)+30(-2a^2+b^2)\sec^2(c+dx)-80ab\sec^3(c+dx)+15(a^2-}{60d}\end{aligned}$$

input
```
Integrate[(a + b*Sec[c + d*x])^2*Tan[c + d*x]^5,x]
```

output
```
(-60*a^2*Log[Cos[c + d*x]] + 120*a*b*Sec[c + d*x] + 30*(-2*a^2 + b^2)*Sec[
c + d*x]^2 - 80*a*b*Sec[c + d*x]^3 + 15*(a^2 - 2*b^2)*Sec[c + d*x]^4 + 24*
a*b*Sec[c + d*x]^5 + 10*b^2*Sec[c + d*x]^6)/(60*d)
```

3.273.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.16, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3042, 25, 4373, 522, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^5(c+dx)(a+b\sec(c+dx))^2\,dx$$

$$\downarrow 3042$$

$$\int -\cot\left(c+dx+\frac{\pi}{2}\right)^5\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^2 dx$$

$$\downarrow 25$$

$$-\int \cot\left(\frac{1}{2}(2c+\pi)+dx\right)^5\left(a+b\csc\left(\frac{1}{2}(2c+\pi)+dx\right)\right)^2 dx$$

$$\downarrow 4373$$

$$\frac{\int \frac{\cos(c+dx)(a+b\sec(c+dx))^2\left(b^2-b^2\sec^2(c+dx)\right)^2}{b} d(b\sec(c+dx))}{b^4 d}$$

$$\downarrow 522$$

$$\frac{\int \left(b^5\sec^5(c+dx)+2ab^4\sec^4(c+dx)+b^3\left(a^2-2b^2\right)\sec^3(c+dx)-4ab^4\sec^2(c+dx)-b^3\left(2a^2-b^2\right)\sec(c+dx)\right.}{b^4 d}$$

$$\downarrow 2009$$

$$\frac{a^2b^4\log(b\sec(c+dx))+\frac{1}{4}b^4\left(a^2-2b^2\right)\sec^4(c+dx)-\frac{1}{2}b^4\left(2a^2-b^2\right)\sec^2(c+dx)+\frac{2}{5}ab^5\sec^5(c+dx)-\frac{4}{3}ab^5\sec^3}{b^4 d}$$

input
```
Int[(a + b*Sec[c + d*x])^2*Tan[c + d*x]^5,x]
```

output
```
(a^2*b^4*Log[b*Sec[c + d*x]] + 2*a*b^5*Sec[c + d*x] - (b^4*(2*a^2 - b^2)*S
ec[c + d*x]^2)/2 - (4*a*b^5*Sec[c + d*x]^3)/3 + (b^4*(a^2 - 2*b^2)*Sec[c +
 d*x]^4)/4 + (2*a*b^5*Sec[c + d*x]^5)/5 + (b^6*Sec[c + d*x]^6)/6)/(b^4*d)
```

3.273.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 522
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_.
), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(c + d*x)^n*(a + b*x^2)^p, x],
x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4373
```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Simp[-(-1)^((m - 1)/2)/(d*b^(m - 1))   Subst[Int[(b^2 - x^
2)^((m - 1)/2)*((a + x)^n/x), x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
 d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

3.273.4 Maple [A] (verified)

Time = 2.28 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.81

method	result
parts	$\frac{a^2\left(\frac{\tan(dx+c)^4}{4}-\frac{\tan(dx+c)^2}{2}+\frac{\ln\left(1+\tan(dx+c)^2\right)}{2}\right)}{d}+\frac{b^2\tan(dx+c)^6}{6d}+\frac{2ab\left(\frac{\sec(dx+c)^5}{5}-\frac{2\sec(dx+c)^3}{3}+\sec(dx+c)\right)}{d}$
derivativedivides	$\frac{\frac{b^2\sec(dx+c)^6}{6}+\frac{2ab\sec(dx+c)^5}{5}+\frac{a^2\sec(dx+c)^4}{4}-\frac{b^2\sec(dx+c)^4}{2}-\frac{4ab\sec(dx+c)^3}{3}-a^2\sec(dx+c)^2+\frac{\sec(dx+c)^2b^2}{2}+2ab\sec(dx$
default	$\frac{\frac{b^2\sec(dx+c)^6}{6}+\frac{2ab\sec(dx+c)^5}{5}+\frac{a^2\sec(dx+c)^4}{4}-\frac{b^2\sec(dx+c)^4}{2}-\frac{4ab\sec(dx+c)^3}{3}-a^2\sec(dx+c)^2+\frac{\sec(dx+c)^2b^2}{2}+2ab\sec(dx-$
risch	$ia^2x+\frac{2ia^2c}{d}+\frac{4ab\,e^{11i(dx+c)}-4a^2e^{10i(dx+c)}+2b^2e^{10i(dx+c)}+\frac{28ab\,e^{9i(dx+c)}}{3}-12a^2e^{8i(dx+c)}+\frac{104ab\,e^{7i(dx+c)}}{5}-16$

input
```
int((a+b*sec(d*x+c))^2*tan(d*x+c)^5,x,method=_RETURNVERBOSE)
```

output
```
a^2/d*(1/4*tan(d*x+c)^4-1/2*tan(d*x+c)^2+1/2*ln(1+tan(d*x+c)^2))+1/6*b^2*t
an(d*x+c)^6/d+2*a*b/d*(1/5*sec(d*x+c)^5-2/3*sec(d*x+c)^3+sec(d*x+c))
```

3.273.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.00

$$\int (a + b\sec(c + dx))^2 \tan^5(c + dx)\, dx =$$

$$-\frac{60\,a^2\cos(dx+c)^6\log(-\cos(dx+c)) - 120\,ab\cos(dx+c)^5 + 80\,ab\cos(dx+c)^3 + 30\,(2\,a^2 - b^2)\cos(}{60\,d\cos(dx+c)^6}$$

input
```
integrate((a+b*sec(d*x+c))^2*tan(d*x+c)^5,x, algorithm="fricas")
```

output
```
-1/60*(60*a^2*cos(d*x + c)^6*log(-cos(d*x + c)) - 120*a*b*cos(d*x + c)^5 +
 80*a*b*cos(d*x + c)^3 + 30*(2*a^2 - b^2)*cos(d*x + c)^4 - 24*a*b*cos(d*x
+ c) - 15*(a^2 - 2*b^2)*cos(d*x + c)^2 - 10*b^2)/(d*cos(d*x + c)^6)
```

3.273.6 Sympy [A] (verification not implemented)

Time = 0.98 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.64

$$\int (a + b\sec(c + dx))^2 \tan^5(c + dx)\, dx$$

$$= \begin{cases} \frac{a^2\log(\tan^2(c+dx)+1)}{2d} + \frac{a^2\tan^4(c+dx)}{4d} - \frac{a^2\tan^2(c+dx)}{2d} + \frac{2ab\tan^4(c+dx)\sec(c+dx)}{5d} - \frac{8ab\tan^2(c+dx)\sec(c+dx)}{15d} + \frac{16ab\sec(c+} \\ x(a + b\sec(c))^2\tan^5(c) \end{cases}$$

input
```
integrate((a+b*sec(d*x+c))**2*tan(d*x+c)**5,x)
```

output
```
Piecewise((a**2*log(tan(c + d*x)**2 + 1)/(2*d) + a**2*tan(c + d*x)**4/(4*d
) - a**2*tan(c + d*x)**2/(2*d) + 2*a*b*tan(c + d*x)**4*sec(c + d*x)/(5*d)
- 8*a*b*tan(c + d*x)**2*sec(c + d*x)/(15*d) + 16*a*b*sec(c + d*x)/(15*d) +
 b**2*tan(c + d*x)**4*sec(c + d*x)**2/(6*d) - b**2*tan(c + d*x)**2*sec(c +
 d*x)**2/(6*d) + b**2*sec(c + d*x)**2/(6*d), Ne(d, 0)), (x*(a + b*sec(c))*
*2*tan(c)**5, True))
```

3.273.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.94

$$\int (a+b\sec(c+dx))^2\tan^5(c+dx)\,dx =$$

$$-\frac{60\,a^2\log\left(\cos\left(dx+c\right)\right)-\frac{120\,ab\cos(dx+c)^5-80\,ab\cos(dx+c)^3-30\left(2\,a^2-b^2\right)\cos(dx+c)^4+24\,ab\cos(dx+c)+15\left(a^2-2\,b^2\right)\cos(dx+c)}{\cos(dx+c)^6}}{60\,d}$$

input
```
integrate((a+b*sec(d*x+c))^2*tan(d*x+c)^5,x, algorithm="maxima")
```

output
```
-1/60*(60*a^2*log(cos(d*x + c)) - (120*a*b*cos(d*x + c)^5 - 80*a*b*cos(d*x
 + c)^3 - 30*(2*a^2 - b^2)*cos(d*x + c)^4 + 24*a*b*cos(d*x + c) + 15*(a^2
- 2*b^2)*cos(d*x + c)^2 + 10*b^2)/cos(d*x + c)^6)/d
```

3.273.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 341 vs. 2(107) = 214.

Time = 1.69 (sec) , antiderivative size = 341, normalized size of antiderivative = 2.97

$$\int (a+b\sec(c+dx))^2\tan^5(c+dx)\,dx$$

$$=\frac{60\,a^2\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right)-60\,a^2\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right)+\frac{147\,a^2+128\,ab+\frac{1002\,a^2(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{768\,ab(\cos(dx+c)-}{\cos(dx+c)+1}}{}$$

input
```
integrate((a+b*sec(d*x+c))^2*tan(d*x+c)^5,x, algorithm="giac")
```

output
```
1/60*(60*a^2*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)) - 60*a^2
*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)) + (147*a^2 + 128*a*b
 + 1002*a^2*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 768*a*b*(cos(d*x + c)
- 1)/(cos(d*x + c) + 1) + 2925*a^2*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)
^2 + 1920*a*b*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 4140*a^2*(cos(d*
x + c) - 1)^3/(cos(d*x + c) + 1)^3 + 1280*a*b*(cos(d*x + c) - 1)^3/(cos(d*
x + c) + 1)^3 - 640*b^2*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 + 2925*a
^2*(cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4 + 1002*a^2*(cos(d*x + c) - 1)
^5/(cos(d*x + c) + 1)^5 + 147*a^2*(cos(d*x + c) - 1)^6/(cos(d*x + c) + 1)^
6)/((cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)^6)/d
```

3.273.9 Mupad [B] (verification not implemented)

Time = 17.83 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.87

$$\int (a+b\sec(c+dx))^2\tan^5(c+dx)\,dx = \frac{2a^2\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2\right)}{d} + \frac{\frac{32ab}{15}+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4\left(12a^2+32ba\right)-\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2\left(2a^2+\frac{64ba}{5}\right)+12a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^8-2a^2\tan\left(\frac{c}{2}+\right.}{d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{12}-6\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}+15\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^8-20\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6+15\tan\left(\frac{c}{2}\right.\right.}$$

input
```
int(tan(c + d*x)^5*(a + b/cos(c + d*x))^2,x)
```

output
```
(2*a^2*atanh(tan(c/2 + (d*x)/2)^2))/d + ((32*a*b)/15 + tan(c/2 + (d*x)/2)^
4*(32*a*b + 12*a^2) - tan(c/2 + (d*x)/2)^2*((64*a*b)/5 + 2*a^2) + 12*a^2*t
an(c/2 + (d*x)/2)^8 - 2*a^2*tan(c/2 + (d*x)/2)^10 - tan(c/2 + (d*x)/2)^6*(
(64*a*b)/3 + 20*a^2 - (32*b^2)/3))/(d*(15*tan(c/2 + (d*x)/2)^4 - 6*tan(c/2
 + (d*x)/2)^2 - 20*tan(c/2 + (d*x)/2)^6 + 15*tan(c/2 + (d*x)/2)^8 - 6*tan(
c/2 + (d*x)/2)^10 + tan(c/2 + (d*x)/2)^12 + 1))
```

3.274 $\int (a + b\sec(c + dx))^2 \tan^3(c + dx)\, dx$

3.274.1 Optimal result 1998
3.274.2 Mathematica [A] (verified) 1998
3.274.3 Rubi [A] (verified) 1999
3.274.4 Maple [A] (verified) 2000
3.274.5 Fricas [A] (verification not implemented) 2001
3.274.6 Sympy [A] (verification not implemented) 2001
3.274.7 Maxima [A] (verification not implemented) 2002
3.274.8 Giac [B] (verification not implemented) 2002
3.274.9 Mupad [B] (verification not implemented) 2003

3.274.1 Optimal result

Integrand size = 21, antiderivative size = 87

$$\int (a + b\sec(c + dx))^2 \tan^3(c + dx)\, dx = \frac{a^2 \log(\cos(c + dx))}{d} - \frac{2ab\sec(c + dx)}{d} + \frac{(a^2 - b^2)\sec^2(c + dx)}{2d} + \frac{2ab\sec^3(c + dx)}{3d} + \frac{b^2\sec^4(c + dx)}{4d}$$

output
```
a^2*ln(cos(d*x+c))/d-2*a*b*sec(d*x+c)/d+1/2*(a^2-b^2)*sec(d*x+c)^2/d+2/3*a
*b*sec(d*x+c)^3/d+1/4*b^2*sec(d*x+c)^4/d
```

3.274.2 Mathematica [A] (verified)

Time = 0.38 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.85

$$\int (a + b\sec(c + dx))^2 \tan^3(c + dx)\, dx = \frac{12a^2 \log(\cos(c + dx)) - 24ab\sec(c + dx) + 6(a^2 - b^2)\sec^2(c + dx) + 8ab\sec^3(c + dx) + 3b^2\sec^4(c + dx)}{12d}$$

input
```
Integrate[(a + b*Sec[c + d*x])^2*Tan[c + d*x]^3,x]
```

output
```
(12*a^2*Log[Cos[c + d*x]] - 24*a*b*Sec[c + d*x] + 6*(a^2 - b^2)*Sec[c + d*
x]^2 + 8*a*b*Sec[c + d*x]^3 + 3*b^2*Sec[c + d*x]^4)/(12*d)
```

3.274.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.06, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3042, 25, 4373, 522, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^3(c+dx)(a+b\sec(c+dx))^2\,dx$$

$$\downarrow 3042$$

$$\int -\cot\left(c+dx+\frac{\pi}{2}\right)^3\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^2 dx$$

$$\downarrow 25$$

$$-\int \cot\left(\frac{1}{2}(2c+\pi)+dx\right)^3\left(a+b\csc\left(\frac{1}{2}(2c+\pi)+dx\right)\right)^2 dx$$

$$\downarrow 4373$$

$$-\frac{\int \frac{\cos(c+dx)(a+b\sec(c+dx))^2\left(b^2-b^2\sec^2(c+dx)\right)}{b}\,d(b\sec(c+dx))}{b^2 d}$$

$$\downarrow 522$$

$$-\frac{\int \left(-b^3\sec^3(c+dx)-2ab^2\sec^2(c+dx)-b\left(a^2-b^2\right)\sec(c+dx)+2ab^2+a^2b\cos(c+dx)\right)d(b\sec(c+dx))}{b^2 d}$$

$$\downarrow 2009$$

$$-\frac{-\frac{1}{2}b^2\left(a^2-b^2\right)\sec^2(c+dx)+a^2b^2\log(b\sec(c+dx))-\frac{2}{3}ab^3\sec^3(c+dx)+2ab^3\sec(c+dx)-\frac{1}{4}b^4\sec^4(c+dx)}{b^2 d}$$

input
```
Int[(a + b*Sec[c + d*x])^2*Tan[c + d*x]^3,x]
```

output
```
-((a^2*b^2*Log[b*Sec[c + d*x]] + 2*a*b^3*Sec[c + d*x] - (b^2*(a^2 - b^2)*S
ec[c + d*x]^2)/2 - (2*a*b^3*Sec[c + d*x]^3)/3 - (b^4*Sec[c + d*x]^4)/4)/(b
^2*d))
```

3.274.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 522
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_.
), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(c + d*x)^n*(a + b*x^2)^p, x],
x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4373
```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Simp[-(-1)^((m - 1)/2)/(d*b^(m - 1))   Subst[Int[(b^2 - x^
2)^((m - 1)/2)*((a + x)^n/x), x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
 d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

3.274.4 Maple [A] (verified)

Time = 1.51 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.86

method	result
parts	$\frac{a^2\left(\frac{\tan(dx+c)^2}{2}-\frac{\ln\left(1+\tan(dx+c)^2\right)}{2}\right)}{d}+\frac{b^2\tan(dx+c)^4}{4d}+\frac{2ab\left(\frac{\sec(dx+c)^3}{3}-\sec(dx+c)\right)}{d}$
derivativedivides	$\frac{\frac{b^2\sec(dx+c)^4}{4}+\frac{2ab\sec(dx+c)^3}{3}+\frac{a^2\sec(dx+c)^2}{2}-\frac{\sec(dx+c)^2b^2}{2}-2ab\sec(dx+c)-a^2\ln(\sec(dx+c))}{d}$
default	$\frac{\frac{b^2\sec(dx+c)^4}{4}+\frac{2ab\sec(dx+c)^3}{3}+\frac{a^2\sec(dx+c)^2}{2}-\frac{\sec(dx+c)^2b^2}{2}-2ab\sec(dx+c)-a^2\ln(\sec(dx+c))}{d}$
risch	$-ia^2x-\frac{2ia^2c}{d}-\frac{2\left(6ab\,e^{7i(dx+c)}-3a^2e^{6i(dx+c)}+3b^2e^{6i(dx+c)}+10ab\,e^{5i(dx+c)}-6a^2e^{4i(dx+c)}+10ab\,e^{3i(dx+c)}-3a^2\right.}{3d\left(e^{2i(dx+c)}+1\right)^4}$

input
```
int((a+b*sec(d*x+c))^2*tan(d*x+c)^3,x,method=_RETURNVERBOSE)
```

output
```
a^2/d*(1/2*tan(d*x+c)^2-1/2*ln(1+tan(d*x+c)^2))+1/4*b^2*tan(d*x+c)^4/d+2*a
*b/d*(1/3*sec(d*x+c)^3-sec(d*x+c))
```

3.274.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.94

$$\int (a + b\sec(c + dx))^2 \tan^3(c + dx)\, dx$$
$$= \frac{12\, a^2 \cos(dx + c)^4 \log(-\cos(dx + c)) - 24\, ab \cos(dx + c)^3 + 8\, ab \cos(dx + c) + 6\,(a^2 - b^2)\cos(dx + c)^2}{12\, d \cos(dx + c)^4}$$

input
```
integrate((a+b*sec(d*x+c))^2*tan(d*x+c)^3,x, algorithm="fricas")
```

output
```
1/12*(12*a^2*cos(d*x + c)^4*log(-cos(d*x + c)) - 24*a*b*cos(d*x + c)^3 + 8
*a*b*cos(d*x + c) + 6*(a^2 - b^2)*cos(d*x + c)^2 + 3*b^2)/(d*cos(d*x + c)^
4)
```

3.274.6 Sympy [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.45

$$\int (a + b\sec(c + dx))^2 \tan^3(c + dx)\, dx$$
$$= \begin{cases} -\frac{a^2 \log(\tan^2(c+dx)+1)}{2d} + \frac{a^2 \tan^2(c+dx)}{2d} + \frac{2ab \tan^2(c+dx)\sec(c+dx)}{3d} - \frac{4ab\sec(c+dx)}{3d} + \frac{b^2 \tan^2(c+dx)\sec^2(c+dx)}{4d} - \frac{b^2 \sec^2(c} \\ x(a + b\sec(c))^2 \tan^3(c) \end{cases}$$

input
```
integrate((a+b*sec(d*x+c))**2*tan(d*x+c)**3,x)
```

output
```
Piecewise((-a**2*log(tan(c + d*x)**2 + 1)/(2*d) + a**2*tan(c + d*x)**2/(2*
d) + 2*a*b*tan(c + d*x)**2*sec(c + d*x)/(3*d) - 4*a*b*sec(c + d*x)/(3*d) +
 b**2*tan(c + d*x)**2*sec(c + d*x)**2/(4*d) - b**2*sec(c + d*x)**2/(4*d),
Ne(d, 0)), (x*(a + b*sec(c))**2*tan(c)**3, True))
```

3.274.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.86

$$\int (a+b\sec(c+dx))^2\tan^3(c+dx)\,dx = \frac{12a^2\log(\cos(dx+c)) - \frac{24ab\cos(dx+c)^3 - 8ab\cos(dx+c) - 6(a^2-b^2)\cos(dx+c)^2 - 3b^2}{\cos(dx+c)^4}}{12d}$$

input
```
integrate((a+b*sec(d*x+c))^2*tan(d*x+c)^3,x, algorithm="maxima")
```

output
```
1/12*(12*a^2*log(cos(d*x + c)) - (24*a*b*cos(d*x + c)^3 - 8*a*b*cos(d*x +
c) - 6*(a^2 - b^2)*cos(d*x + c)^2 - 3*b^2)/cos(d*x + c)^4)/d
```

3.274.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 267 vs. 2(81) = 162.

Time = 0.74 (sec) , antiderivative size = 267, normalized size of antiderivative = 3.07

$$\int (a+b\sec(c+dx))^2\tan^3(c+dx)\,dx = -\frac{12a^2\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right) - 12a^2\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right) + \frac{25a^2+32ab+\frac{124a^2(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{128ab(\cos(dx+c)-}{\cos(dx+c)+1}}{12d}$$

input
```
integrate((a+b*sec(d*x+c))^2*tan(d*x+c)^3,x, algorithm="giac")
```

output
```
-1/12*(12*a^2*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)) - 12*a^
2*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)) + (25*a^2 + 32*a*b
+ 124*a^2*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 128*a*b*(cos(d*x + c) -
1)/(cos(d*x + c) + 1) + 198*a^2*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2
+ 96*a*b*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 48*b^2*(cos(d*x + c)
- 1)^2/(cos(d*x + c) + 1)^2 + 124*a^2*(cos(d*x + c) - 1)^3/(cos(d*x + c) +
 1)^3 + 25*a^2*(cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4)/((cos(d*x + c) -
 1)/(cos(d*x + c) + 1) + 1)^4)/d
```

3.274.9 Mupad [B] (verification not implemented)

Time = 16.45 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.74

$$\int (a+b\sec(c+dx))^2\tan^3(c+dx)\,dx =$$
$$-\frac{\frac{8ab}{3}-\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2\left(2a^2+\frac{32ba}{3}\right)-2a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4(4a^2+8ab-4b^2)}{d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^8-4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6+6\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4-4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+1\right)}$$
$$-\frac{2a^2\operatorname{atanh}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2\right)}{d}$$

input
```
int(tan(c + d*x)^3*(a + b/cos(c + d*x))^2,x)
```

output
```
- ((8*a*b)/3 - tan(c/2 + (d*x)/2)^2*((32*a*b)/3 + 2*a^2) - 2*a^2*tan(c/2 +
 (d*x)/2)^6 + tan(c/2 + (d*x)/2)^4*(8*a*b + 4*a^2 - 4*b^2))/(d*(6*tan(c/2
+ (d*x)/2)^4 - 4*tan(c/2 + (d*x)/2)^2 - 4*tan(c/2 + (d*x)/2)^6 + tan(c/2 +
 (d*x)/2)^8 + 1)) - (2*a^2*atanh(tan(c/2 + (d*x)/2)^2))/d
```

3.275 $\int (a + b\sec(c + dx))^2 \tan(c + dx)\, dx$

3.275.1 Optimal result 2004
3.275.2 Mathematica [A] (verified) 2004
3.275.3 Rubi [A] (verified) 2005
3.275.4 Maple [A] (verified) 2006
3.275.5 Fricas [A] (verification not implemented) 2007
3.275.6 Sympy [A] (verification not implemented) 2007
3.275.7 Maxima [A] (verification not implemented) 2007
3.275.8 Giac [B] (verification not implemented) 2008
3.275.9 Mupad [B] (verification not implemented) 2008

3.275.1 Optimal result

Integrand size = 19, antiderivative size = 47

$$\int (a + b\sec(c + dx))^2 \tan(c + dx)\, dx = -\frac{a^2 \log(\cos(c + dx))}{d} + \frac{2ab\sec(c + dx)}{d} + \frac{b^2 \sec^2(c + dx)}{2d}$$

output
```
-a^2*ln(cos(d*x+c))/d+2*a*b*sec(d*x+c)/d+1/2*b^2*sec(d*x+c)^2/d
```

3.275.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.89

$$\int (a + b\sec(c + dx))^2 \tan(c + dx)\, dx = \frac{-2a^2 \log(\cos(c + dx)) + 4ab\sec(c + dx) + b^2 \sec^2(c + dx)}{2d}$$

input
```
Integrate[(a + b*Sec[c + d*x])^2*Tan[c + d*x],x]
```

output
```
(-2*a^2*Log[Cos[c + d*x]] + 4*a*b*Sec[c + d*x] + b^2*Sec[c + d*x]^2)/(2*d)
```

3.275.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.91, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {3042, 25, 4373, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan(c+dx)(a+b\sec(c+dx))^2\,dx$$

$$\downarrow 3042$$

$$\int -\cot\left(c+dx+\frac{\pi}{2}\right)\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^2 dx$$

$$\downarrow 25$$

$$-\int \cot\left(\frac{1}{2}(2c+\pi)+dx\right)\left(a+b\csc\left(\frac{1}{2}(2c+\pi)+dx\right)\right)^2 dx$$

$$\downarrow 4373$$

$$\frac{\int \frac{\cos(c+dx)(a+b\sec(c+dx))^2}{b}\,d(b\sec(c+dx))}{d}$$

$$\downarrow 49$$

$$\frac{\int \left(\frac{\cos(c+dx)a^2}{b}+2a+b\sec(c+dx)\right)d(b\sec(c+dx))}{d}$$

$$\downarrow 2009$$

$$\frac{a^2\log(b\sec(c+dx))+2ab\sec(c+dx)+\frac{1}{2}b^2\sec^2(c+dx)}{d}$$

input
```
Int[(a + b*Sec[c + d*x])^2*Tan[c + d*x],x]
```

output
```
(a^2*Log[b*Sec[c + d*x]] + 2*a*b*Sec[c + d*x] + (b^2*Sec[c + d*x]^2)/2)/d
```

3.275.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4373
```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Simp[-(-1)^((m - 1)/2)/(d*b^(m - 1))   Subst[Int[(b^2 - x^
2)^((m - 1)/2)*((a + x)^n/x), x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
 d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

3.275.4 Maple [A] (verified)

Time = 0.72 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.85

method	result	size
derivativedivides	$\frac{\frac{\sec(dx+c)^2 b^2}{2}+2ab\sec(dx+c)+a^2\ln(\sec(dx+c))}{d}$	40
default	$\frac{\frac{\sec(dx+c)^2 b^2}{2}+2ab\sec(dx+c)+a^2\ln(\sec(dx+c))}{d}$	40
parts	$\frac{a^2\ln\left(1+\tan(dx+c)^2\right)}{2d}+\frac{b^2\sec(dx+c)^2}{2d}+\frac{2ab\sec(dx+c)}{d}$	50
risch	$ia^2x+\frac{2ia^2c}{d}+\frac{2b\left(2a\,\mathrm{e}^{3i(dx+c)}+b\,\mathrm{e}^{2i(dx+c)}+2\,\mathrm{e}^{i(dx+c)}a\right)}{d\left(\mathrm{e}^{2i(dx+c)}+1\right)^2}-\frac{a^2\ln\left(\mathrm{e}^{2i(dx+c)}+1\right)}{d}$	94

input
```
int((a+b*sec(d*x+c))^2*tan(d*x+c),x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/2*sec(d*x+c)^2*b^2+2*a*b*sec(d*x+c)+a^2*ln(sec(d*x+c)))
```

3.275.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.09

$$\int (a + b\sec(c + dx))^2 \tan(c + dx)\, dx$$
$$= -\frac{2\,a^2 \cos(dx + c)^2 \log(-\cos(dx + c)) - 4\,ab\cos(dx + c) - b^2}{2\,d\cos(dx + c)^2}$$

input
```
integrate((a+b*sec(d*x+c))^2*tan(d*x+c),x, algorithm="fricas")
```

output
```
-1/2*(2*a^2*cos(d*x + c)^2*log(-cos(d*x + c)) - 4*a*b*cos(d*x + c) - b^2)/
(d*cos(d*x + c)^2)
```

3.275.6 Sympy [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.28

$$\int (a + b\sec(c + dx))^2 \tan(c + dx)\, dx$$
$$= \begin{cases} \frac{a^2 \log(\tan^2(c+dx)+1)}{2d} + \frac{2ab\sec(c+dx)}{d} + \frac{b^2\sec^2(c+dx)}{2d} & \text{for } d \neq 0 \\ x(a + b\sec(c))^2 \tan(c) & \text{otherwise} \end{cases}$$

input
```
integrate((a+b*sec(d*x+c))**2*tan(d*x+c),x)
```

output
```
Piecewise((a**2*log(tan(c + d*x)**2 + 1)/(2*d) + 2*a*b*sec(c + d*x)/d + b*
*2*sec(c + d*x)**2/(2*d), Ne(d, 0)), (x*(a + b*sec(c))**2*tan(c), True))
```

3.275.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.89

$$\int (a + b\sec(c + dx))^2 \tan(c + dx)\, dx = -\frac{2\,a^2 \log(\cos(dx + c)) - \frac{4\,ab\cos(dx+c)+b^2}{\cos(dx+c)^2}}{2\,d}$$

input
```
integrate((a+b*sec(d*x+c))^2*tan(d*x+c),x, algorithm="maxima")
```

output
```
-1/2*(2*a^2*log(cos(d*x + c)) - (4*a*b*cos(d*x + c) + b^2)/cos(d*x + c)^2)
/d
```

3.275.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 191 vs. $2(45) = 90$.

Time = 0.36 (sec) , antiderivative size = 191, normalized size of antiderivative = 4.06

$$\int (a + b\sec(c + dx))^2 \tan(c + dx)\, dx$$
$$= \frac{2\,a^2 \log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right) - 2\,a^2 \log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right) + \frac{3\,a^2+8\,ab+\frac{6\,a^2(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{8\,ab(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{4\,b^2(\cos(}{\left(\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right)^2}}{2\,d}$$

input
```
integrate((a+b*sec(d*x+c))^2*tan(d*x+c),x, algorithm="giac")
```

output
```
1/2*(2*a^2*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)) - 2*a^2*lo
g(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)) + (3*a^2 + 8*a*b + 6*a^
2*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 8*a*b*(cos(d*x + c) - 1)/(cos(d*
x + c) + 1) - 4*b^2*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 3*a^2*(cos(d*x
 + c) - 1)^2/(cos(d*x + c) + 1)^2)/((cos(d*x + c) - 1)/(cos(d*x + c) + 1)
+ 1)^2)/d
```

3.275.9 Mupad [B] (verification not implemented)

Time = 13.20 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.72

$$\int (a + b\sec(c + dx))^2 \tan(c + dx)\, dx = \frac{4\,a\,b - \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2 (4\,a\,b - 2\,b^2)}{d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4 - 2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2 + 1\right)} + \frac{2\,a^2\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2\right)}{d}$$

input
```
int(tan(c + d*x)*(a + b/cos(c + d*x))^2,x)
```

output
```
(4*a*b - tan(c/2 + (d*x)/2)^2*(4*a*b - 2*b^2))/(d*(tan(c/2 + (d*x)/2)^4 -
2*tan(c/2 + (d*x)/2)^2 + 1)) + (2*a^2*atanh(tan(c/2 + (d*x)/2)^2))/d
```

3.276 $\int \cot(c+dx)(a+b\sec(c+dx))^2\,dx$

3.276.1 Optimal result 2010
3.276.2 Mathematica [A] (verified) 2010
3.276.3 Rubi [A] (verified) 2011
3.276.4 Maple [A] (verified) 2013
3.276.5 Fricas [A] (verification not implemented) 2013
3.276.6 Sympy [F] 2013
3.276.7 Maxima [A] (verification not implemented) 2014
3.276.8 Giac [A] (verification not implemented) 2014
3.276.9 Mupad [B] (verification not implemented) 2015

3.276.1 Optimal result

Integrand size = 19, antiderivative size = 61

$$\int \cot(c+dx)(a+b\sec(c+dx))^2\,dx = \frac{a^2\log(\cos(c+dx))}{d} + \frac{(a+b)^2\log(1-\sec(c+dx))}{2d} + \frac{(a-b)^2\log(1+\sec(c+dx))}{2d}$$

output
```
a^2*ln(cos(d*x+c))/d+1/2*(a+b)^2*ln(1-sec(d*x+c))/d+1/2*(a-b)^2*ln(1+sec(d
*x+c))/d
```

3.276.2 Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.89

$$\int \cot(c+dx)(a+b\sec(c+dx))^2\,dx = \frac{2a^2\log(\cos(c+dx)) + (a+b)^2\log(1-\sec(c+dx)) + (a-b)^2\log(1+\sec(c+dx))}{2d}$$

input
```
Integrate[Cot[c + d*x]*(a + b*Sec[c + d*x])^2,x]
```

output
```
(2*a^2*Log[Cos[c + d*x]] + (a + b)^2*Log[1 - Sec[c + d*x]] + (a - b)^2*Log
[1 + Sec[c + d*x]])/(2*d)
```

3.276.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.15, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {3042, 25, 4373, 524, 219, 240}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot(c+dx)(a+b\sec(c+dx))^2\,dx$$

$$\downarrow 3042$$

$$\int -\frac{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^2}{\cot\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\downarrow 25$$

$$-\int \frac{\left(a+b\csc\left(\frac{1}{2}(2c+\pi)+dx\right)\right)^2}{\cot\left(\frac{1}{2}(2c+\pi)+dx\right)}dx$$

$$\downarrow 4373$$

$$-\frac{b^2\int \frac{\cos(c+dx)(a+b\sec(c+dx))^2}{b(b^2-b^2\sec^2(c+dx))}d(b\sec(c+dx))}{d}$$

$$\downarrow 524$$

$$-\frac{b^2\left(\frac{(a^2+b^2)\int \frac{b\sec(c+dx)}{b^2-b^2\sec^2(c+dx)}d(b\sec(c+dx))}{b^2}+2a\int \frac{1}{b^2-b^2\sec^2(c+dx)}d(b\sec(c+dx))+\frac{a^2\log(b\sec(c+dx))}{b^2}\right)}{d}$$

$$\downarrow 219$$

$$-\frac{b^2\left(\frac{(a^2+b^2)\int \frac{b\sec(c+dx)}{b^2-b^2\sec^2(c+dx)}d(b\sec(c+dx))}{b^2}+\frac{a^2\log(b\sec(c+dx))}{b^2}+\frac{2a\operatorname{arctanh}(\sec(c+dx))}{b}\right)}{d}$$

$$\downarrow 240$$

$$-\frac{b^2\left(-\frac{(a^2+b^2)\log(b^2-b^2\sec^2(c+dx))}{2b^2}+\frac{a^2\log(b\sec(c+dx))}{b^2}+\frac{2a\operatorname{arctanh}(\sec(c+dx))}{b}\right)}{d}$$

input
```
Int[Cot[c + d*x]*(a + b*Sec[c + d*x])^2,x]
```

output
```
-((b^2*((2*a*ArcTanh[Sec[c + d*x]])/b + (a^2*Log[b*Sec[c + d*x]])/b^2 - ((
a^2 + b^2)*Log[b^2 - b^2*Sec[c + d*x]^2])/(2*b^2)))/d)
```

3.276.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 240
```
Int[(x_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[Log[RemoveContent[a + b*x
^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]
```

rule 524
```
Int[((c_) + (d_.)*(x_))^2/((x_)*((a_) + (b_.)*(x_)^2)), x_Symbol] :> Simp[(
c^2/a)*Log[x], x] + (-Simp[(b*c^2 - a*d^2)/a   Int[x/(a + b*x^2), x], x] +
Simp[2*c*d   Int[1/(a + b*x^2), x], x]) /; FreeQ[{a, b, c, d}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4373
```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Simp[-(-1)^((m - 1)/2)/(d*b^(m - 1))   Subst[Int[(b^2 - x^
2)^((m - 1)/2)*((a + x)^n/x), x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
 d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

3.276.4 Maple [A] (verified)

Time = 0.69 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.79

method	result
derivativedivides	$\frac{a^2 \ln(\sin(dx+c))+2ab\ln(-\cot(dx+c)+\csc(dx+c))+b^2\ln(\tan(dx+c))}{d}$
default	$\frac{a^2 \ln(\sin(dx+c))+2ab\ln(-\cot(dx+c)+\csc(dx+c))+b^2\ln(\tan(dx+c))}{d}$
risch	$-ia^2x-\frac{2ia^2c}{d}+\frac{a^2\ln(e^{i(dx+c)}-1)}{d}+\frac{2\ln(e^{i(dx+c)}-1)ab}{d}+\frac{\ln(e^{i(dx+c)}-1)b^2}{d}+\frac{a^2\ln(e^{i(dx+c)}+1)}{d}-\frac{2\ln($

input
```
int(cot(d*x+c)*(a+b*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^2*ln(sin(d*x+c))+2*a*b*ln(-cot(d*x+c)+csc(d*x+c))+b^2*ln(tan(d*x+c)
))
```

3.276.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.11

$$\int \cot(c+dx)(a+b\sec(c+dx))^2\,dx =$$
$$-\frac{2\,b^2\log\left(-\cos\left(dx+c\right)\right)-\left(a^2-2\,ab+b^2\right)\log\left(\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)-\left(a^2+2\,ab+b^2\right)\log\left(-\frac{1}{2}\,\cos\left(dx+\right.\right.}{2\,d}$$

input
```
integrate(cot(d*x+c)*(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

output
```
-1/2*(2*b^2*log(-cos(d*x + c)) - (a^2 - 2*a*b + b^2)*log(1/2*cos(d*x + c)
+ 1/2) - (a^2 + 2*a*b + b^2)*log(-1/2*cos(d*x + c) + 1/2))/d
```

3.276.6 Sympy [F]

$$\int \cot(c+dx)(a+b\sec(c+dx))^2\,dx = \int (a+b\sec\left(c+dx\right))^2\cot\left(c+dx\right)\,dx$$

input
```
integrate(cot(d*x+c)*(a+b*sec(d*x+c))**2,x)
```

output
```
Integral((a + b*sec(c + d*x))**2*cot(c + d*x), x)
```

3.276.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.02

$$\int \cot(c+dx)(a+b\sec(c+dx))^2\,dx = -\frac{2b^2\log(\cos(dx+c)) - (a^2-2ab+b^2)\log(\cos(dx+c)+1) - (a^2+2ab+b^2)\log(\cos(dx+c)-1)}{2d}$$

input
```
integrate(cot(d*x+c)*(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

output
```
-1/2*(2*b^2*log(cos(d*x + c)) - (a^2 - 2*a*b + b^2)*log(cos(d*x + c) + 1)
- (a^2 + 2*a*b + b^2)*log(cos(d*x + c) - 1))/d
```

3.276.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.66

$$\int \cot(c+dx)(a+b\sec(c+dx))^2\,dx = -\frac{2a^2\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right) + 2b^2\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right) - (a^2+2ab+b^2)\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)}{2d}$$

input
```
integrate(cot(d*x+c)*(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

output
```
-1/2*(2*a^2*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)) + 2*b^2*l
og(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)) - (a^2 + 2*a*b + b^2)*
log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1)))/d
```

3.276.9 Mupad [B] (verification not implemented)

Time = 14.38 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.57

$$\int \cot(c+dx)(a+b\sec(c+dx))^2\,dx = \frac{a^2\,\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{d} + \frac{b^2\,\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{d} - \frac{a^2\,\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+1\right)}{d} - \frac{b^2\,\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2-1\right)}{d} + \frac{2\,a\,b\,\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{d}$$

input
```
int(cot(c + d*x)*(a + b/cos(c + d*x))^2,x)
```

output
```
(a^2*log(tan(c/2 + (d*x)/2)))/d + (b^2*log(tan(c/2 + (d*x)/2)))/d - (a^2*l
og(tan(c/2 + (d*x)/2)^2 + 1))/d - (b^2*log(tan(c/2 + (d*x)/2)^2 - 1))/d +
(2*a*b*log(tan(c/2 + (d*x)/2)))/d
```

3.277 $\int \cot^3(c+dx)(a+b\sec(c+dx))^2\,dx$

3.277.1 Optimal result 2016
3.277.2 Mathematica [A] (verified) 2016
3.277.3 Rubi [A] (verified) 2017
3.277.4 Maple [A] (verified) 2019
3.277.5 Fricas [A] (verification not implemented) 2019
3.277.6 Sympy [F] 2020
3.277.7 Maxima [A] (verification not implemented) 2020
3.277.8 Giac [B] (verification not implemented) 2020
3.277.9 Mupad [B] (verification not implemented) 2021

3.277.1 Optimal result

Integrand size = 21, antiderivative size = 92

$$\int \cot^3(c+dx)(a+b\sec(c+dx))^2\,dx = -\frac{a^2\log(\cos(c+dx))}{d} - \frac{a(a+b)\log(1-\sec(c+dx))}{2d} - \frac{a(a-b)\log(1+\sec(c+dx))}{2d} - \frac{\cot^2(c+dx)\left(a^2+b^2+2ab\sec(c+dx)\right)}{2d}$$

output
```
-a^2*ln(cos(d*x+c))/d-1/2*a*(a+b)*ln(1-sec(d*x+c))/d-1/2*a*(a-b)*ln(1+sec(
d*x+c))/d-1/2*cot(d*x+c)^2*(a^2+b^2+2*a*b*sec(d*x+c))/d
```

3.277.2 Mathematica [A] (verified)

Time = 0.56 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.97

$$\int \cot^3(c+dx)(a+b\sec(c+dx))^2\,dx = -\frac{4a^2\log(\cos(c+dx)) + 2a(a+b)\log(1-\sec(c+dx)) + 2a(a-b)\log(1+\sec(c+dx)) + \frac{(a+b)^2}{-1+\sec(c+dx)} -}{4d}$$

input
```
Integrate[Cot[c + d*x]^3*(a + b*Sec[c + d*x])^2,x]
```

output
```
-1/4*(4*a^2*Log[Cos[c + d*x]] + 2*a*(a + b)*Log[1 - Sec[c + d*x]] + 2*a*(a
 - b)*Log[1 + Sec[c + d*x]] + (a + b)^2/(-1 + Sec[c + d*x]) - (a - b)^2/(1
 + Sec[c + d*x]))/d
```

3.277.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.26, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 25, 4373, 532, 27, 523, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^3(c+dx)(a+b\sec(c+dx))^2\,dx$$

$$\downarrow 3042$$

$$\int -\frac{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^2}{\cot\left(c+dx+\frac{\pi}{2}\right)^3}dx$$

$$\downarrow 25$$

$$-\int \frac{\left(a+b\csc\left(\frac{1}{2}(2c+\pi)+dx\right)\right)^2}{\cot\left(\frac{1}{2}(2c+\pi)+dx\right)^3}dx$$

$$\downarrow 4373$$

$$\frac{b^4\int \frac{\cos(c+dx)(a+b\sec(c+dx))^2}{b(b^2-b^2\sec^2(c+dx))^2}d(b\sec(c+dx))}{d}$$

$$\downarrow 532$$

$$\frac{b^4\left(\frac{a^2+2ab\sec(c+dx)+b^2}{2b^2(b^2-b^2\sec^2(c+dx))}-\frac{\int -\frac{2a\cos(c+dx)(a+b\sec(c+dx))}{b\left(b^2-b^2\sec^2(c+dx)\right)}d(b\sec(c+dx))}{2b^2}\right)}{d}$$

$$\downarrow 27$$

$$\frac{b^4\left(\frac{a\int \frac{\cos(c+dx)(a+b\sec(c+dx))}{b\left(b^2-b^2\sec^2(c+dx)\right)}d(b\sec(c+dx))}{b^2}+\frac{a^2+2ab\sec(c+dx)+b^2}{2b^2(b^2-b^2\sec^2(c+dx))}\right)}{d}$$

$$\downarrow 523$$

$$\frac{b^4\left(\frac{a\int\left(\frac{b-a}{2b^2(\sec(c+dx)b+b)}+\frac{a\cos(c+dx)}{b^3}+\frac{a+b}{2b^2(b-b\sec(c+dx))}\right)d(b\sec(c+dx))}{b^2}+\frac{a^2+2ab\sec(c+dx)+b^2}{2b^2(b^2-b^2\sec^2(c+dx))}\right)}{d}$$

$$\downarrow 2009$$

$$\frac{b^4\left(\frac{a^2+2ab\sec(c+dx)+b^2}{2b^2(b^2-b^2\sec^2(c+dx))}+\frac{a\left(\frac{a\log(b\sec(c+dx))}{b^2}-\frac{(a+b)\log(b-b\sec(c+dx))}{2b^2}-\frac{(a-b)\log(b\sec(c+dx)+b)}{2b^2}\right)}{b^2}\right)}{d}$$

input
```
Int[Cot[c + d*x]^3*(a + b*Sec[c + d*x])^2,x]
```

output
```
(b^4*((a*((a*Log[b*Sec[c + d*x]])/b^2 - ((a + b)*Log[b - b*Sec[c + d*x]])/
(2*b^2) - ((a - b)*Log[b + b*Sec[c + d*x]])/(2*b^2)))/b^2 + (a^2 + b^2 + 2
*a*b*Sec[c + d*x])/(2*b^2*(b^2 - b^2*Sec[c + d*x]^2))))/d
```

3.277.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 523
```
Int[((x_)^(m_.)*((c_) + (d_.)*(x_)))/((a_) + (b_.)*(x_)^2), x_Symbol] :> In
t[ExpandIntegrand[x^m*((c + d*x)/(a + b*x^2)), x], x] /; FreeQ[{a, b, c, d}
, x] && IntegerQ[m]
```

rule 532
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> With[{Qx = PolynomialQuotient[x^m*(c + d*x)^n, a + b*x^2, x], e = Coe
ff[PolynomialRemainder[x^m*(c + d*x)^n, a + b*x^2, x], x, 0], f = Coeff[Pol
ynomialRemainder[x^m*(c + d*x)^n, a + b*x^2, x], x, 1]}, Simp[(a*f - b*e*x)
*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   Int[x^m
*(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*(Qx/x^m) + e*((2*p + 3)/x^m),
x], x], x]] /; FreeQ[{a, b, c, d}, x] && IGtQ[n, 0] && ILtQ[m, 0] && LtQ[p,
 -1] && IntegerQ[2*p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4373
```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Simp[-(-1)^((m - 1)/2)/(d*b^(m - 1))   Subst[Int[(b^2 - x^
2)^((m - 1)/2)*((a + x)^n/x), x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
 d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

3.277.4 Maple [A] (verified)

Time = 0.75 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.00

method	result
derivativedivides	$\frac{a^2\left(-\frac{\cot(dx+c)^2}{2}-\ln(\sin(dx+c))\right)+2ab\left(-\frac{\cos(dx+c)^3}{2\sin(dx+c)^2}-\frac{\cos(dx+c)}{2}-\frac{\ln(-\cot(dx+c)+\csc(dx+c))}{2}\right)-\frac{b^2}{2\sin(dx+c)^2}}{d}$
default	$\frac{a^2\left(-\frac{\cot(dx+c)^2}{2}-\ln(\sin(dx+c))\right)+2ab\left(-\frac{\cos(dx+c)^3}{2\sin(dx+c)^2}-\frac{\cos(dx+c)}{2}-\frac{\ln(-\cot(dx+c)+\csc(dx+c))}{2}\right)-\frac{b^2}{2\sin(dx+c)^2}}{d}$
risch	$ia^2x+\frac{2ia^2c}{d}+\frac{2ab\,e^{3i(dx+c)}+2a^2e^{2i(dx+c)}+2b^2e^{2i(dx+c)}+2ab\,e^{i(dx+c)}}{d\left(e^{2i(dx+c)}-1\right)^2}-\frac{a^2\ln\left(e^{i(dx+c)}-1\right)}{d}-\frac{\ln\left(e^{i(dx+c)}-1\right)}{d}$

input
```
int(cot(d*x+c)^3*(a+b*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^2*(-1/2*cot(d*x+c)^2-ln(sin(d*x+c)))+2*a*b*(-1/2/sin(d*x+c)^2*cos(d
*x+c)^3-1/2*cos(d*x+c)-1/2*ln(-cot(d*x+c)+csc(d*x+c)))-1/2*b^2/sin(d*x+c)^
2)
```

3.277.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.23

$$\int \cot^3(c+dx)(a+b\sec(c+dx))^2\,dx$$
$$=\frac{2\,ab\cos(dx+c)+a^2+b^2-\left((a^2-ab)\cos(dx+c)^2-a^2+ab\right)\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)-\left((a^2+ab)\cos\right.}{2\left(d\cos(dx+c)^2-d\right)}$$

input
```
integrate(cot(d*x+c)^3*(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

output
```
1/2*(2*a*b*cos(d*x + c) + a^2 + b^2 - ((a^2 - a*b)*cos(d*x + c)^2 - a^2 +
a*b)*log(1/2*cos(d*x + c) + 1/2) - ((a^2 + a*b)*cos(d*x + c)^2 - a^2 - a*b
)*log(-1/2*cos(d*x + c) + 1/2))/(d*cos(d*x + c)^2 - d)
```

3.277.6 Sympy [F]

$$\int \cot^3(c+dx)(a+b\sec(c+dx))^2\,dx = \int (a+b\sec(c+dx))^2\cot^3(c+dx)\,dx$$

input
```
integrate(cot(d*x+c)**3*(a+b*sec(d*x+c))**2,x)
```

output
```
Integral((a + b*sec(c + d*x))**2*cot(c + d*x)**3, x)
```

3.277.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.78

$$\int \cot^3(c+dx)(a+b\sec(c+dx))^2\,dx$$
$$= -\frac{(a^2-ab)\log(\cos(dx+c)+1)+(a^2+ab)\log(\cos(dx+c)-1)-\frac{2\,ab\cos(dx+c)+a^2+b^2}{\cos(dx+c)^2-1}}{2\,d}$$

input
```
integrate(cot(d*x+c)^3*(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

output
```
-1/2*((a^2 - a*b)*log(cos(d*x + c) + 1) + (a^2 + a*b)*log(cos(d*x + c) - 1
) - (2*a*b*cos(d*x + c) + a^2 + b^2)/(cos(d*x + c)^2 - 1))/d
```

3.277.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 209 vs. 2(86) = 172.

Time = 0.33 (sec) , antiderivative size = 209, normalized size of antiderivative = 2.27

$$\int \cot^3(c+dx)(a+b\sec(c+dx))^2\,dx$$
$$= \frac{8\,a^2\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right)+\frac{a^2(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{2\,ab(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{b^2(\cos(dx+c)-1)}{\cos(dx+c)+1}-4\,(a^2+ab)\log\left(\frac{|-\cos(dx+}{|\cos(dx+c}\right.}{8\,d}$$

input
```
integrate(cot(d*x+c)^3*(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

output
```
1/8*(8*a^2*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)) + a^2*(cos
(d*x + c) - 1)/(cos(d*x + c) + 1) - 2*a*b*(cos(d*x + c) - 1)/(cos(d*x + c)
 + 1) + b^2*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 4*(a^2 + a*b)*log(abs(
-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1)) + (a^2 + 2*a*b + b^2 + 4*a^2*(co
s(d*x + c) - 1)/(cos(d*x + c) + 1) + 4*a*b*(cos(d*x + c) - 1)/(cos(d*x + c
) + 1))*(cos(d*x + c) + 1)/(cos(d*x + c) - 1))/d
```

3.277.9 Mupad [B] (verification not implemented)

Time = 14.27 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.07

$$\int \cot^3(c+dx)(a+b\sec(c+dx))^2\,dx = \frac{a^2\,\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+1\right)}{d} - \frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2(a-b)^2}{8\,d} - \frac{\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)\,(a^2+b\,a)}{d} - \frac{\cot\left(\frac{c}{2}+\frac{dx}{2}\right)^2\left(\frac{a^2}{8}+\frac{a\,b}{4}+\frac{b^2}{8}\right)}{d}$$

input
```
int(cot(c + d*x)^3*(a + b/cos(c + d*x))^2,x)
```

output
```
(a^2*log(tan(c/2 + (d*x)/2)^2 + 1))/d - (tan(c/2 + (d*x)/2)^2*(a - b)^2)/(
8*d) - (log(tan(c/2 + (d*x)/2))*(a*b + a^2))/d - (cot(c/2 + (d*x)/2)^2*((a
*b)/4 + a^2/8 + b^2/8))/d
```

3.278 $\int \cot^5(c+dx)(a+b\sec(c+dx))^2\,dx$

3.278.1 Optimal result . . . 2022
3.278.2 Mathematica [A] (verified) . . . 2023
3.278.3 Rubi [A] (verified) . . . 2023
3.278.4 Maple [A] (verified) . . . 2026
3.278.5 Fricas [A] (verification not implemented) . . . 2026
3.278.6 Sympy [F] . . . 2027
3.278.7 Maxima [A] (verification not implemented) . . . 2027
3.278.8 Giac [B] (verification not implemented) . . . 2028
3.278.9 Mupad [B] (verification not implemented) . . . 2028

3.278.1 Optimal result

Integrand size = 21, antiderivative size = 126

$$\begin{aligned}\int \cot^5(c+dx)(a+b\sec(c+dx))^2\,dx = {}& \frac{a^2\log(\cos(c+dx))}{d}\\ &+\frac{a(4a+3b)\log(1-\sec(c+dx))}{8d}\\ &+\frac{a(4a-3b)\log(1+\sec(c+dx))}{8d}\\ &+\frac{a\cot^2(c+dx)(2a+3b\sec(c+dx))}{4d}\\ &-\frac{\cot^4(c+dx)\left(a^2+b^2+2ab\sec(c+dx)\right)}{4d}\end{aligned}$$

output

```
a^2*ln(cos(d*x+c))/d+1/8*a*(4*a+3*b)*ln(1-sec(d*x+c))/d+1/8*a*(4*a-3*b)*ln
(1+sec(d*x+c))/d+1/4*a*cot(d*x+c)^2*(2*a+3*b*sec(d*x+c))/d-1/4*cot(d*x+c)^
4*(a^2+b^2+2*a*b*sec(d*x+c))/d
```

3.278.2 Mathematica [A] (verified)

Time = 3.25 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.10

$$\int \cot^5(c+dx)(a+b\sec(c+dx))^2\,dx$$
$$= \frac{16a^2\log(\cos(c+dx)) + 2a(4a+3b)\log(1-\sec(c+dx)) + 2a(4a-3b)\log(1+\sec(c+dx)) - \frac{(a+b)^2}{(-1+\sec(c+d}}{16d}$$

input
```
Integrate[Cot[c + d*x]^5*(a + b*Sec[c + d*x])^2,x]
```

output
```
(16*a^2*Log[Cos[c + d*x]] + 2*a*(4*a + 3*b)*Log[1 - Sec[c + d*x]] + 2*a*(4
*a - 3*b)*Log[1 + Sec[c + d*x]] - (a + b)^2/(-1 + Sec[c + d*x])^2 + ((a +
b)*(5*a + b))/(-1 + Sec[c + d*x]) - (a - b)^2/(1 + Sec[c + d*x])^2 - ((a -
 b)*(5*a - b))/(1 + Sec[c + d*x]))/(16*d)
```

3.278.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.38, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3042, 25, 4373, 532, 27, 532, 25, 523, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^5(c+dx)(a+b\sec(c+dx))^2\,dx$$
$$\downarrow 3042$$
$$\int -\frac{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^2}{\cot\left(c+dx+\frac{\pi}{2}\right)^5}dx$$
$$\downarrow 25$$
$$-\int \frac{\left(a+b\csc\left(\frac{1}{2}(2c+\pi)+dx\right)\right)^2}{\cot\left(\frac{1}{2}(2c+\pi)+dx\right)^5}dx$$
$$\downarrow 4373$$
$$-\frac{b^6\int \frac{\cos(c+dx)(a+b\sec(c+dx))^2}{b(b^2-b^2\sec^2(c+dx))^3}d(b\sec(c+dx))}{d}$$

$$\downarrow 532$$

$$-\frac{b^6\left(\frac{a^2+2ab\sec(c+dx)+b^2}{4b^2\left(b^2-b^2\sec^2(c+dx)\right)^2}-\frac{\int -\frac{2a\cos(c+dx)(2a+3b\sec(c+dx))}{b\left(b^2-b^2\sec^2(c+dx)\right)^2}d(b\sec(c+dx))}{4b^2}\right)}{d}$$

$$\downarrow 27$$

$$-\frac{b^6\left(\frac{a\int\frac{\cos(c+dx)(2a+3b\sec(c+dx))}{b\left(b^2-b^2\sec^2(c+dx)\right)^2}d(b\sec(c+dx))}{2b^2}+\frac{a^2+2ab\sec(c+dx)+b^2}{4b^2\left(b^2-b^2\sec^2(c+dx)\right)^2}\right)}{d}$$

$$\downarrow 532$$

$$-\frac{b^6\left(\frac{a\left(\frac{2a+3b\sec(c+dx)}{2b^2\left(b^2-b^2\sec^2(c+dx)\right)}-\frac{\int -\frac{\cos(c+dx)(4a+3b\sec(c+dx))}{b\left(b^2-b^2\sec^2(c+dx)\right)}d(b\sec(c+dx))}{2b^2}\right)}{2b^2}+\frac{a^2+2ab\sec(c+dx)+b^2}{4b^2\left(b^2-b^2\sec^2(c+dx)\right)^2}\right)}{d}$$

$$\downarrow 25$$

$$-\frac{b^6\left(\frac{a\left(\frac{\int\frac{\cos(c+dx)(4a+3b\sec(c+dx))}{b\left(b^2-b^2\sec^2(c+dx)\right)}d(b\sec(c+dx))}{2b^2}+\frac{2a+3b\sec(c+dx)}{2b^2\left(b^2-b^2\sec^2(c+dx)\right)}\right)}{2b^2}+\frac{a^2+2ab\sec(c+dx)+b^2}{4b^2\left(b^2-b^2\sec^2(c+dx)\right)^2}\right)}{d}$$

$$\downarrow 523$$

$$-\frac{b^6\left(\frac{a\left(\frac{\int\left(\frac{3b-4a}{2b^2(\sec(c+dx)b+b)}+\frac{4a\cos(c+dx)}{b^3}+\frac{4a+3b}{2b^2(b-b\sec(c+dx))}\right)d(b\sec(c+dx))}{2b^2}+\frac{2a+3b\sec(c+dx)}{2b^2\left(b^2-b^2\sec^2(c+dx)\right)}\right)}{2b^2}+\frac{a^2+2ab\sec(c+dx)+b^2}{4b^2\left(b^2-b^2\sec^2(c+dx)\right)^2}\right)}{d}$$

$$\downarrow 2009$$

$$-\frac{b^6\left(\frac{a^2+2ab\sec(c+dx)+b^2}{4b^2\left(b^2-b^2\sec^2(c+dx)\right)^2}+\frac{a\left(\frac{2a+3b\sec(c+dx)}{2b^2\left(b^2-b^2\sec^2(c+dx)\right)}+\frac{\frac{4a\log(b\sec(c+dx))}{b^2}-\frac{(4a+3b)\log(b-b\sec(c+dx))}{2b^2}-\frac{(4a-3b)\log(b\sec(c+dx)+b)}{2b^2}}{2b^2}\right)}{2b^2}\right)}{d}$$

input
```
Int[Cot[c + d*x]^5*(a + b*Sec[c + d*x])^2,x]
```

output
```
-((b^6*((a^2 + b^2 + 2*a*b*Sec[c + d*x])/(4*b^2*(b^2 - b^2*Sec[c + d*x]^2)
^2) + (a*(((4*a*Log[b*Sec[c + d*x]])/b^2 - ((4*a + 3*b)*Log[b - b*Sec[c +
d*x]])/(2*b^2) - ((4*a - 3*b)*Log[b + b*Sec[c + d*x]])/(2*b^2))/(2*b^2) +
(2*a + 3*b*Sec[c + d*x])/(2*b^2*(b^2 - b^2*Sec[c + d*x]^2))))/(2*b^2)))/d)
```

3.278.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 523
```
Int[((x_)^(m_.)*((c_) + (d_.)*(x_)))/((a_) + (b_.)*(x_)^2), x_Symbol] :> In
t[ExpandIntegrand[x^m*((c + d*x)/(a + b*x^2)), x], x] /; FreeQ[{a, b, c, d}
, x] && IntegerQ[m]
```

rule 532
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> With[{Qx = PolynomialQuotient[x^m*(c + d*x)^n, a + b*x^2, x], e = Coe
ff[PolynomialRemainder[x^m*(c + d*x)^n, a + b*x^2, x], x, 0], f = Coeff[Pol
ynomialRemainder[x^m*(c + d*x)^n, a + b*x^2, x], x, 1]}, Simp[(a*f - b*e*x)
*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   Int[x^m
*(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*(Qx/x^m) + e*((2*p + 3)/x^m),
x], x], x]] /; FreeQ[{a, b, c, d}, x] && IGtQ[n, 0] && ILtQ[m, 0] && LtQ[p,
 -1] && IntegerQ[2*p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4373
```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Simp[-(-1)^((m - 1)/2)/(d*b^(m - 1))   Subst[Int[(b^2 - x^
2)^((m - 1)/2)*((a + x)^n/x), x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
 d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

3.278.4 Maple [A] (verified)

Time = 1.14 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.08

method	result
derivativedivides	$\frac{a^2\left(-\frac{\cot(dx+c)^4}{4}+\frac{\cot(dx+c)^2}{2}+\ln(\sin(dx+c))\right)+2ab\left(-\frac{\cos(dx+c)^5}{4\sin(dx+c)^4}+\frac{\cos(dx+c)^5}{8\sin(dx+c)^2}+\frac{\cos(dx+c)^3}{8}+\frac{3\cos(dx+c)}{8}+\frac{3\ln(-\cot($
default	$\frac{a^2\left(-\frac{\cot(dx+c)^4}{4}+\frac{\cot(dx+c)^2}{2}+\ln(\sin(dx+c))\right)+2ab\left(-\frac{\cos(dx+c)^5}{4\sin(dx+c)^4}+\frac{\cos(dx+c)^5}{8\sin(dx+c)^2}+\frac{\cos(dx+c)^3}{8}+\frac{3\cos(dx+c)}{8}+\frac{3\ln(-\cot($
risch	$-ia^2x-\frac{2ia^2c}{d}-\frac{5ab\,e^{7i(dx+c)}+8a^2e^{6i(dx+c)}+4b^2e^{6i(dx+c)}+3ab\,e^{5i(dx+c)}-8a^2e^{4i(dx+c)}+3ab\,e^{3i(dx+c)}+8a^2e^{2i(}}{2d\left(e^{2i(dx+c)}-1\right)^4}$

input
```
int(cot(d*x+c)^5*(a+b*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^2*(-1/4*cot(d*x+c)^4+1/2*cot(d*x+c)^2+ln(sin(d*x+c)))+2*a*b*(-1/4/s
in(d*x+c)^4*cos(d*x+c)^5+1/8/sin(d*x+c)^2*cos(d*x+c)^5+1/8*cos(d*x+c)^3+3/
8*cos(d*x+c)+3/8*ln(-cot(d*x+c)+csc(d*x+c)))-1/4*b^2/sin(d*x+c)^4*cos(d*x+
c)^4)
```

3.278.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.61

$$\int \cot^5(c+dx)(a+b\sec(c+dx))^2\,dx =$$

$$-\frac{10\,ab\cos(dx+c)^3-6\,ab\cos(dx+c)+4\left(2\,a^2+b^2\right)\cos(dx+c)^2-6\,a^2-2\,b^2-\left(\left(4\,a^2-3\,ab\right)\cos(dx\right.}$$

input
```
integrate(cot(d*x+c)^5*(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

output
```
-1/8*(10*a*b*cos(d*x + c)^3 - 6*a*b*cos(d*x + c) + 4*(2*a^2 + b^2)*cos(d*x
 + c)^2 - 6*a^2 - 2*b^2 - ((4*a^2 - 3*a*b)*cos(d*x + c)^4 - 2*(4*a^2 - 3*a
*b)*cos(d*x + c)^2 + 4*a^2 - 3*a*b)*log(1/2*cos(d*x + c) + 1/2) - ((4*a^2
+ 3*a*b)*cos(d*x + c)^4 - 2*(4*a^2 + 3*a*b)*cos(d*x + c)^2 + 4*a^2 + 3*a*b
)*log(-1/2*cos(d*x + c) + 1/2))/(d*cos(d*x + c)^4 - 2*d*cos(d*x + c)^2 + d
)
```

3.278.6 Sympy [F]

$$\int \cot^5(c+dx)(a+b\sec(c+dx))^2\,dx = \int (a+b\sec(c+dx))^2\cot^5(c+dx)\,dx$$

input
```
integrate(cot(d*x+c)**5*(a+b*sec(d*x+c))**2,x)
```

output
```
Integral((a + b*sec(c + d*x))**2*cot(c + d*x)**5, x)
```

3.278.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.97

$$\int \cot^5(c+dx)(a+b\sec(c+dx))^2\,dx$$
$$= \frac{(4a^2-3ab)\log(\cos(dx+c)+1)+(4a^2+3ab)\log(\cos(dx+c)-1)-\frac{2\left(5ab\cos(dx+c)^3-3ab\cos(dx+c)+2(2a}{\cos(dx+c)^4-2\cos(dx}}{8d}$$

input
```
integrate(cot(d*x+c)^5*(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

output
```
1/8*((4*a^2 - 3*a*b)*log(cos(d*x + c) + 1) + (4*a^2 + 3*a*b)*log(cos(d*x +
 c) - 1) - 2*(5*a*b*cos(d*x + c)^3 - 3*a*b*cos(d*x + c) + 2*(2*a^2 + b^2)*
cos(d*x + c)^2 - 3*a^2 - b^2)/(cos(d*x + c)^4 - 2*cos(d*x + c)^2 + 1))/d
```

3.278.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 360 vs. $2(118) = 236$.

Time = 0.37 (sec) , antiderivative size = 360, normalized size of antiderivative = 2.86

$$\int \cot^5(c+dx)(a+b\sec(c+dx))^2\,dx =$$

$$-\frac{64\,a^2\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right)+\frac{12\,a^2(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{16\,ab(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{4\,b^2(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{a^2(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}-$$

input
```
integrate(cot(d*x+c)^5*(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

output
```
-1/64*(64*a^2*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)) + 12*a^
2*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 16*a*b*(cos(d*x + c) - 1)/(cos(d
*x + c) + 1) + 4*b^2*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + a^2*(cos(d*x
+ c) - 1)^2/(cos(d*x + c) + 1)^2 - 2*a*b*(cos(d*x + c) - 1)^2/(cos(d*x + c
) + 1)^2 + b^2*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 8*(4*a^2 + 3*a*
b)*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1)) + (a^2 + 2*a*b + b^2
+ 12*a^2*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 16*a*b*(cos(d*x + c) - 1)
/(cos(d*x + c) + 1) + 4*b^2*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 48*a^2
*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 36*a*b*(cos(d*x + c) - 1)^2/(
cos(d*x + c) + 1)^2)*(cos(d*x + c) + 1)^2/(cos(d*x + c) - 1)^2)/d
```

3.278.9 Mupad [B] (verification not implemented)

Time = 14.34 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.30

$$\begin{aligned}
&\int \cot^5(c+dx)(a+b\sec(c+dx))^2\,dx \\
&= \frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2\left(\frac{5\,a^2}{32}-\frac{3\,a\,b}{16}+\frac{b^2}{32}+\frac{(a-b)^2}{32}\right)}{d}-\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4(a-b)^2}{64\,d} \\
&\quad+\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)\left(a^2+\frac{3\,b\,a}{4}\right)}{d}-\frac{a^2\,\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+1\right)}{d} \\
&\quad-\frac{\cot\left(\frac{c}{2}+\frac{dx}{2}\right)^4\left(\frac{a\,b}{2}+\frac{a^2}{4}+\frac{b^2}{4}-\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2(3\,a^2+4\,a\,b+b^2)\right)}{16\,d}
\end{aligned}$$

input
```
int(cot(c + d*x)^5*(a + b/cos(c + d*x))^2,x)
```

output

```
(tan(c/2 + (d*x)/2)^2*((5*a^2)/32 - (3*a*b)/16 + b^2/32 + (a - b)^2/32))/d
 - (tan(c/2 + (d*x)/2)^4*(a - b)^2)/(64*d) + (log(tan(c/2 + (d*x)/2))*((3*
a*b)/4 + a^2))/d - (a^2*log(tan(c/2 + (d*x)/2)^2 + 1))/d - (cot(c/2 + (d*x
)/2)^4*((a*b)/2 + a^2/4 + b^2/4 - tan(c/2 + (d*x)/2)^2*(4*a*b + 3*a^2 + b^
2)))/(16*d)
```

3.279 $\int (a+b\sec(c+dx))^2\tan^6(c+dx)\,dx$

3.279.1 Optimal result 2030
3.279.2 Mathematica [A] (verified) 2030
3.279.3 Rubi [A] (verified) 2031
3.279.4 Maple [A] (verified) 2032
3.279.5 Fricas [A] (verification not implemented) 2033
3.279.6 Sympy [F] 2033
3.279.7 Maxima [A] (verification not implemented) 2033
3.279.8 Giac [A] (verification not implemented) 2034
3.279.9 Mupad [B] (verification not implemented) 2035

3.279.1 Optimal result

Integrand size = 21, antiderivative size = 157

$$\int (a+b\sec(c+dx))^2\tan^6(c+dx)\,dx = -a^2x - \frac{5ab\operatorname{arctanh}(\sin(c+dx))}{8d} + \frac{a^2\tan(c+dx)}{d} + \frac{5ab\sec(c+dx)\tan(c+dx)}{8d} - \frac{a^2\tan^3(c+dx)}{3d} - \frac{5ab\sec(c+dx)\tan^3(c+dx)}{12d} + \frac{a^2\tan^5(c+dx)}{5d} + \frac{ab\sec(c+dx)\tan^5(c+dx)}{3d} + \frac{b^2\tan^7(c+dx)}{7d}$$

output
```
-a^2*x-5/8*a*b*arctanh(sin(d*x+c))/d+a^2*tan(d*x+c)/d+5/8*a*b*sec(d*x+c)*t
an(d*x+c)/d-1/3*a^2*tan(d*x+c)^3/d-5/12*a*b*sec(d*x+c)*tan(d*x+c)^3/d+1/5*
a^2*tan(d*x+c)^5/d+1/3*a*b*sec(d*x+c)*tan(d*x+c)^5/d+1/7*b^2*tan(d*x+c)^7/
d
```

3.279.2 Mathematica [A] (verified)

Time = 1.26 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.84

$$\int (a+b\sec(c+dx))^2\tan^6(c+dx)\,dx$$
$$= \frac{-840a^2\arctan(\tan(c+dx)) - 525ab\operatorname{arctanh}(\sin(c+dx)) + \tan(c+dx)\,(175ab(-1+7\cos(2(c+dx)))\,s$$

input
```
Integrate[(a + b*Sec[c + d*x])^2*Tan[c + d*x]^6,x]
```

output
```
(-840*a^2*ArcTan[Tan[c + d*x]] - 525*a*b*ArcTanh[Sin[c + d*x]] + Tan[c + d
*x]*(175*a*b*(-1 + 7*Cos[2*(c + d*x)])*Sec[c + d*x]^5 + 105*a*b*Sec[c + d*
x]*(-5 + 16*Tan[c + d*x]^4) + 8*(105*a^2 - 35*a^2*Tan[c + d*x]^2 + 21*a^2*
Tan[c + d*x]^4 + 15*b^2*Tan[c + d*x]^6)))/(840*d)
```

3.279.3 Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3042, 4374, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^6(c+dx)(a+b\sec(c+dx))^2\,dx$$

$$\downarrow 3042$$

$$\int \cot\left(c+dx+\frac{\pi}{2}\right)^6\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^2 dx$$

$$\downarrow 4374$$

$$\int \left(a^2\tan^6(c+dx)+2ab\tan^6(c+dx)\sec(c+dx)+b^2\tan^6(c+dx)\sec^2(c+dx)\right)dx$$

$$\downarrow 2009$$

$$\frac{a^2\tan^5(c+dx)}{5d}-\frac{a^2\tan^3(c+dx)}{3d}+\frac{a^2\tan(c+dx)}{d}-a^2x-\frac{5ab\operatorname{arctanh}(\sin(c+dx))}{8d}+\frac{ab\tan^5(c+dx)\sec(c+dx)}{3d}-\frac{5ab\tan^3(c+dx)\sec(c+dx)}{12d}+\frac{5ab\tan(c+dx)\sec(c+dx)}{8d}+\frac{b^2\tan^7(c+dx)}{7d}$$

input
```
Int[(a + b*Sec[c + d*x])^2*Tan[c + d*x]^6,x]
```

output
```
-(a^2*x) - (5*a*b*ArcTanh[Sin[c + d*x]])/(8*d) + (a^2*Tan[c + d*x])/d + (5
*a*b*Sec[c + d*x]*Tan[c + d*x])/(8*d) - (a^2*Tan[c + d*x]^3)/(3*d) - (5*a*
b*Sec[c + d*x]*Tan[c + d*x]^3)/(12*d) + (a^2*Tan[c + d*x]^5)/(5*d) + (a*b*
Sec[c + d*x]*Tan[c + d*x]^5)/(3*d) + (b^2*Tan[c + d*x]^7)/(7*d)
```

3.279.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4374
```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

3.279.4 Maple [A] (verified)

Time = 3.66 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.06

method	result
parts	$\frac{a^2\left(\frac{\tan(dx+c)^5}{5}-\frac{\tan(dx+c)^3}{3}+\tan(dx+c)-\arctan(\tan(dx+c))\right)}{d}+\frac{b^2\tan(dx+c)^7}{7d}+\frac{2ab\left(\frac{\sin(dx+c)^7}{6\cos(dx+c)^6}-\frac{\sin(dx+c)^7}{24\cos(dx+c)^4}$
derivativedivides	$\frac{a^2\left(\frac{\tan(dx+c)^5}{5}-\frac{\tan(dx+c)^3}{3}+\tan(dx+c)-dx-c\right)+2ab\left(\frac{\sin(dx+c)^7}{6\cos(dx+c)^6}-\frac{\sin(dx+c)^7}{24\cos(dx+c)^4}+\frac{\sin(dx+c)^7}{16\cos(dx+c)^2}+\frac{\sin(dx+c)^5}{16}+\frac{5\sin(d}{4}$
default	$\frac{a^2\left(\frac{\tan(dx+c)^5}{5}-\frac{\tan(dx+c)^3}{3}+\tan(dx+c)-dx-c\right)+2ab\left(\frac{\sin(dx+c)^7}{6\cos(dx+c)^6}-\frac{\sin(dx+c)^7}{24\cos(dx+c)^4}+\frac{\sin(dx+c)^7}{16\cos(dx+c)^2}+\frac{\sin(dx+c)^5}{16}+\frac{5\sin(d}{4}$
risch	$-a^2x-\frac{i\left(1155ab\,e^{13i(dx+c)}-2520a^2e^{12i(dx+c)}+840b^2e^{12i(dx+c)}+980ab\,e^{11i(dx+c)}-10080a^2e^{10i(dx+c)}+2975ab\,e^{9i}\right.$

input
```
int((a+b*sec(d*x+c))^2*tan(d*x+c)^6,x,method=_RETURNVERBOSE)
```

output
```
a^2/d*(1/5*tan(d*x+c)^5-1/3*tan(d*x+c)^3+tan(d*x+c)-arctan(tan(d*x+c)))+1/
7*b^2*tan(d*x+c)^7/d+2*a*b/d*(1/6*sin(d*x+c)^7/cos(d*x+c)^6-1/24*sin(d*x+c
)^7/cos(d*x+c)^4+1/16*sin(d*x+c)^7/cos(d*x+c)^2+1/16*sin(d*x+c)^5+5/48*sin
(d*x+c)^3+5/16*sin(d*x+c)-5/16*ln(sec(d*x+c)+tan(d*x+c)))
```

3.279.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.17

$$\int (a+b\sec(c+dx))^2\tan^6(c+dx)\,dx =$$

$$-\frac{1680\,a^2 dx\cos(dx+c)^7 + 525\,ab\cos(dx+c)^7\log(\sin(dx+c)+1) - 525\,ab\cos(dx+c)^7\log(-\sin(dx$$

input
```
integrate((a+b*sec(d*x+c))^2*tan(d*x+c)^6,x, algorithm="fricas")
```

output
```
-1/1680*(1680*a^2*d*x*cos(d*x + c)^7 + 525*a*b*cos(d*x + c)^7*log(sin(d*x
+ c) + 1) - 525*a*b*cos(d*x + c)^7*log(-sin(d*x + c) + 1) - 2*(1155*a*b*co
s(d*x + c)^5 + 8*(161*a^2 - 15*b^2)*cos(d*x + c)^6 - 910*a*b*cos(d*x + c)^
3 - 8*(77*a^2 - 45*b^2)*cos(d*x + c)^4 + 280*a*b*cos(d*x + c) + 24*(7*a^2
- 15*b^2)*cos(d*x + c)^2 + 120*b^2)*sin(d*x + c))/(d*cos(d*x + c)^7)
```

3.279.6 Sympy [F]

$$\int (a+b\sec(c+dx))^2\tan^6(c+dx)\,dx = \int (a+b\sec(c+dx))^2\tan^6(c+dx)\,dx$$

input
```
integrate((a+b*sec(d*x+c))**2*tan(d*x+c)**6,x)
```

output
```
Integral((a + b*sec(c + d*x))**2*tan(c + d*x)**6, x)
```

3.279.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.96

$$\int (a+b\sec(c+dx))^2\tan^6(c+dx)\,dx$$

$$= \frac{240\,b^2\tan(dx+c)^7 + 112\left(3\tan(dx+c)^5 - 5\tan(dx+c)^3 - 15\,dx - 15\,c + 15\tan(dx+c)\right)a^2 - 35\,ab}{1680\,d}$$

input
```
integrate((a+b*sec(d*x+c))^2*tan(d*x+c)^6,x, algorithm="maxima")
```

output
```
1/1680*(240*b^2*tan(d*x + c)^7 + 112*(3*tan(d*x + c)^5 - 5*tan(d*x + c)^3
- 15*d*x - 15*c + 15*tan(d*x + c))*a^2 - 35*a*b*(2*(33*sin(d*x + c)^5 - 40
*sin(d*x + c)^3 + 15*sin(d*x + c))/(sin(d*x + c)^6 - 3*sin(d*x + c)^4 + 3*
sin(d*x + c)^2 - 1) + 15*log(sin(d*x + c) + 1) - 15*log(sin(d*x + c) - 1))
)/d
```

3.279.8 Giac [A] (verification not implemented)

Time = 2.30 (sec) , antiderivative size = 282, normalized size of antiderivative = 1.80

$$\int (a + b\sec(c + dx))^2 \tan^6(c + dx)\,dx =$$

$$840\,(dx + c)a^2 + 525\,ab\log\left(\left|\tan\left(\tfrac{1}{2}\,dx + \tfrac{1}{2}\,c\right) + 1\right|\right) - 525\,ab\log\left(\left|\tan\left(\tfrac{1}{2}\,dx + \tfrac{1}{2}\,c\right) - 1\right|\right) + \frac{2\left(840\,a^2\tan\left(\frac{1}{2}\right.\right.}{}$$

input
```
integrate((a+b*sec(d*x+c))^2*tan(d*x+c)^6,x, algorithm="giac")
```

output
```
-1/840*(840*(d*x + c)*a^2 + 525*a*b*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 5
25*a*b*log(abs(tan(1/2*d*x + 1/2*c) - 1)) + 2*(840*a^2*tan(1/2*d*x + 1/2*c
)^13 - 525*a*b*tan(1/2*d*x + 1/2*c)^13 - 6160*a^2*tan(1/2*d*x + 1/2*c)^11
+ 3500*a*b*tan(1/2*d*x + 1/2*c)^11 + 19768*a^2*tan(1/2*d*x + 1/2*c)^9 - 99
05*a*b*tan(1/2*d*x + 1/2*c)^9 - 28896*a^2*tan(1/2*d*x + 1/2*c)^7 + 7680*b^
2*tan(1/2*d*x + 1/2*c)^7 + 19768*a^2*tan(1/2*d*x + 1/2*c)^5 + 9905*a*b*tan
(1/2*d*x + 1/2*c)^5 - 6160*a^2*tan(1/2*d*x + 1/2*c)^3 - 3500*a*b*tan(1/2*d
*x + 1/2*c)^3 + 840*a^2*tan(1/2*d*x + 1/2*c) + 525*a*b*tan(1/2*d*x + 1/2*c
))/(tan(1/2*d*x + 1/2*c)^2 - 1)^7)/d
```

3.279.9 Mupad [B] (verification not implemented)

Time = 15.42 (sec) , antiderivative size = 403, normalized size of antiderivative = 2.57

$$\int (a+b\sec(c+dx))^2\tan^6(c+dx)\,dx$$

$$=\frac{\left(\frac{5ab}{4}-2a^2\right)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{13}+\left(\frac{44a^2}{3}-\frac{25ab}{3}\right)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{11}+\left(\frac{283ab}{12}-\frac{706a^2}{15}\right)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^9+\left(\frac{344a^2}{5}-\right.}{d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{14}-7\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{12}+21\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}-35\right.}$$

$$-\frac{2a^2\operatorname{atan}\left(\frac{64a^6\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{64a^6+25a^4b^2}+\frac{25a^4b^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{64a^6+25a^4b^2}\right)}{d}$$

$$-\frac{5ab\operatorname{atanh}\left(\frac{40a^5b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{40a^5b+\frac{125a^3b^3}{8}}+\frac{125a^3b^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{8\left(40a^5b+\frac{125a^3b^3}{8}\right)}\right)}{4d}$$

input
```
int(tan(c + d*x)^6*(a + b/cos(c + d*x))^2,x)
```

output
```
(tan(c/2 + (d*x)/2)^7*((344*a^2)/5 - (128*b^2)/7) + tan(c/2 + (d*x)/2)^13*
((5*a*b)/4 - 2*a^2) + tan(c/2 + (d*x)/2)^3*((25*a*b)/3 + (44*a^2)/3) - tan
(c/2 + (d*x)/2)^11*((25*a*b)/3 - (44*a^2)/3) - tan(c/2 + (d*x)/2)^5*((283*
a*b)/12 + (706*a^2)/15) + tan(c/2 + (d*x)/2)^9*((283*a*b)/12 - (706*a^2)/1
5) - tan(c/2 + (d*x)/2)*((5*a*b)/4 + 2*a^2))/(d*(7*tan(c/2 + (d*x)/2)^2 -
21*tan(c/2 + (d*x)/2)^4 + 35*tan(c/2 + (d*x)/2)^6 - 35*tan(c/2 + (d*x)/2)^
8 + 21*tan(c/2 + (d*x)/2)^10 - 7*tan(c/2 + (d*x)/2)^12 + tan(c/2 + (d*x)/2
)^14 - 1)) - (2*a^2*atan((64*a^6*tan(c/2 + (d*x)/2))/(64*a^6 + 25*a^4*b^2)
 + (25*a^4*b^2*tan(c/2 + (d*x)/2))/(64*a^6 + 25*a^4*b^2)))/d - (5*a*b*atan
h((40*a^5*b*tan(c/2 + (d*x)/2))/(40*a^5*b + (125*a^3*b^3)/8) + (125*a^3*b^
3*tan(c/2 + (d*x)/2))/(8*(40*a^5*b + (125*a^3*b^3)/8))))/(4*d)
```

3.280 $\int (a + b\sec(c + dx))^2 \tan^4(c + dx)\, dx$

3.280.1 Optimal result 2036
3.280.2 Mathematica [A] (verified) 2036
3.280.3 Rubi [A] (verified) 2037
3.280.4 Maple [A] (verified) 2038
3.280.5 Fricas [A] (verification not implemented) 2039
3.280.6 Sympy [F] 2039
3.280.7 Maxima [A] (verification not implemented) 2039
3.280.8 Giac [B] (verification not implemented) 2040
3.280.9 Mupad [B] (verification not implemented) 2041

3.280.1 Optimal result

Integrand size = 21, antiderivative size = 116

$$\int (a + b\sec(c + dx))^2 \tan^4(c + dx)\, dx = a^2 x + \frac{3ab\operatorname{arctanh}(\sin(c + dx))}{4d} - \frac{a^2 \tan(c + dx)}{d} - \frac{3ab\sec(c + dx)\tan(c + dx)}{4d} + \frac{a^2 \tan^3(c + dx)}{3d} + \frac{ab\sec(c + dx)\tan^3(c + dx)}{2d} + \frac{b^2 \tan^5(c + dx)}{5d}$$

output

```
a^2*x+3/4*a*b*arctanh(sin(d*x+c))/d-a^2*tan(d*x+c)/d-3/4*a*b*sec(d*x+c)*ta
n(d*x+c)/d+1/3*a^2*tan(d*x+c)^3/d+1/2*a*b*sec(d*x+c)*tan(d*x+c)^3/d+1/5*b^
2*tan(d*x+c)^5/d
```

3.280.2 Mathematica [A] (verified)

Time = 0.62 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.92

$$\int (a + b\sec(c + dx))^2 \tan^4(c + dx)\, dx = \frac{60a^2 \arctan(\tan(c + dx)) + 45ab\operatorname{arctanh}(\sin(c + dx)) + \tan(c + dx)\left(-90ab\sec^3(c + dx) + 15ab\sec(c + \right.}{60d}$$

input

```
Integrate[(a + b*Sec[c + d*x])^2*Tan[c + d*x]^4,x]
```

output
```
(60*a^2*ArcTan[Tan[c + d*x]] + 45*a*b*ArcTanh[Sin[c + d*x]] + Tan[c + d*x]
*(-90*a*b*Sec[c + d*x]^3 + 15*a*b*Sec[c + d*x]*(3 + 8*Tan[c + d*x]^2) + 4*
(-15*a^2 + 5*a^2*Tan[c + d*x]^2 + 3*b^2*Tan[c + d*x]^4)))/(60*d)
```

3.280.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3042, 4374, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^4(c+dx)(a+b\sec(c+dx))^2\,dx$$

$$\downarrow 3042$$

$$\int \cot\left(c+dx+\frac{\pi}{2}\right)^4\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^2 dx$$

$$\downarrow 4374$$

$$\int \left(a^2\tan^4(c+dx)+2ab\tan^4(c+dx)\sec(c+dx)+b^2\tan^4(c+dx)\sec^2(c+dx)\right)dx$$

$$\downarrow 2009$$

$$\frac{a^2\tan^3(c+dx)}{3d}-\frac{a^2\tan(c+dx)}{d}+a^2x+\frac{3ab\text{arctanh}(\sin(c+dx))}{4d}+\frac{ab\tan^3(c+dx)\sec(c+dx)}{2d}-\frac{3ab\tan(c+dx)\sec(c+dx)}{4d}+\frac{b^2\tan^5(c+dx)}{5d}$$

input
```
Int[(a + b*Sec[c + d*x])^2*Tan[c + d*x]^4,x]
```

output
```
a^2*x + (3*a*b*ArcTanh[Sin[c + d*x]])/(4*d) - (a^2*Tan[c + d*x])/d - (3*a*
b*Sec[c + d*x]*Tan[c + d*x])/(4*d) + (a^2*Tan[c + d*x]^3)/(3*d) + (a*b*Sec
[c + d*x]*Tan[c + d*x]^3)/(2*d) + (b^2*Tan[c + d*x]^5)/(5*d)
```

3.280.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4374
```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

3.280.4 Maple [A] (verified)

Time = 2.36 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.11

method	result
derivativedivides	$\frac{a^2\left(\frac{\tan(dx+c)^3}{3}-\tan(dx+c)+dx+c\right)+2ab\left(\frac{\sin(dx+c)^5}{4\cos(dx+c)^4}-\frac{\sin(dx+c)^5}{8\cos(dx+c)^2}-\frac{\sin(dx+c)^3}{8}-\frac{3\sin(dx+c)}{8}+\frac{3\ln(\sec(dx+c)+\tan(dx+}{8}}{d}$
default	$\frac{a^2\left(\frac{\tan(dx+c)^3}{3}-\tan(dx+c)+dx+c\right)+2ab\left(\frac{\sin(dx+c)^5}{4\cos(dx+c)^4}-\frac{\sin(dx+c)^5}{8\cos(dx+c)^2}-\frac{\sin(dx+c)^3}{8}-\frac{3\sin(dx+c)}{8}+\frac{3\ln(\sec(dx+c)+\tan(dx+}{8}}{d}$
parts	$\frac{a^2\left(\frac{\tan(dx+c)^3}{3}-\tan(dx+c)+\arctan(\tan(dx+c))\right)}{d}+\frac{b^2\tan(dx+c)^5}{5d}+\frac{2ab\left(\frac{\sin(dx+c)^5}{4\cos(dx+c)^4}-\frac{\sin(dx+c)^5}{8\cos(dx+c)^2}-\frac{\sin(dx+c)^3}{8}-\right.}{d}$
risch	$a^2x+\frac{i\left(75ab\,e^{9i(dx+c)}-120a^2e^{8i(dx+c)}+60b^2e^{8i(dx+c)}+30ab\,e^{7i(dx+c)}-360a^2e^{6i(dx+c)}-440a^2e^{4i(dx+c)}+120b^2e^{4i(}{30d\left(e^{2i(dx+c)}+1\right)^5}$

input
```
int((a+b*sec(d*x+c))^2*tan(d*x+c)^4,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^2*(1/3*tan(d*x+c)^3-tan(d*x+c)+d*x+c)+2*a*b*(1/4*sin(d*x+c)^5/cos(d
*x+c)^4-1/8*sin(d*x+c)^5/cos(d*x+c)^2-1/8*sin(d*x+c)^3-3/8*sin(d*x+c)+3/8*
ln(sec(d*x+c)+tan(d*x+c)))+1/5*b^2*sin(d*x+c)^5/cos(d*x+c)^5)
```

3.280.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.30

$$\int (a + b\sec(c + dx))^2 \tan^4(c + dx)\, dx$$
$$= \frac{120\, a^2 dx \cos(dx + c)^5 + 45\, ab \cos(dx + c)^5 \log(\sin(dx + c) + 1) - 45\, ab \cos(dx + c)^5 \log(-\sin(dx + c)$$

input
```
integrate((a+b*sec(d*x+c))^2*tan(d*x+c)^4,x, algorithm="fricas")
```

output
```
1/120*(120*a^2*d*x*cos(d*x + c)^5 + 45*a*b*cos(d*x + c)^5*log(sin(d*x + c)
 + 1) - 45*a*b*cos(d*x + c)^5*log(-sin(d*x + c) + 1) - 2*(75*a*b*cos(d*x +
 c)^3 + 4*(20*a^2 - 3*b^2)*cos(d*x + c)^4 - 30*a*b*cos(d*x + c) - 4*(5*a^2
 - 6*b^2)*cos(d*x + c)^2 - 12*b^2)*sin(d*x + c))/(d*cos(d*x + c)^5)
```

3.280.6 Sympy [F]

$$\int (a + b\sec(c + dx))^2 \tan^4(c + dx)\, dx = \int (a + b\sec(c + dx))^2 \tan^4(c + dx)\, dx$$

input
```
integrate((a+b*sec(d*x+c))**2*tan(d*x+c)**4,x)
```

output
```
Integral((a + b*sec(c + d*x))**2*tan(c + d*x)**4, x)
```

3.280.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.02

$$\int (a + b\sec(c + dx))^2 \tan^4(c + dx)\, dx$$
$$= \frac{24\, b^2 \tan(dx + c)^5 + 40\left(\tan(dx + c)^3 + 3\, dx + 3\, c - 3\, \tan(dx + c)\right) a^2 + 15\, ab\left(\frac{2\left(5\sin(dx+c)^3 - 3\sin(dx+c)\right)}{\sin(dx+c)^4 - 2\sin(dx+c)^2 + 1}\right.}{120\, d}$$

input
```
integrate((a+b*sec(d*x+c))^2*tan(d*x+c)^4,x, algorithm="maxima")
```

output
```
1/120*(24*b^2*tan(d*x + c)^5 + 40*(tan(d*x + c)^3 + 3*d*x + 3*c - 3*tan(d*
x + c))*a^2 + 15*a*b*(2*(5*sin(d*x + c)^3 - 3*sin(d*x + c))/(sin(d*x + c)^
4 - 2*sin(d*x + c)^2 + 1) + 3*log(sin(d*x + c) + 1) - 3*log(sin(d*x + c) -
 1)))/d
```

3.280.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 220 vs. $2(106) = 212$.

Time $= 1.04$ (sec) , antiderivative size $= 220$, normalized size of antiderivative $= 1.90$

$$\int (a + b\sec(c + dx))^2 \tan^4(c + dx)\, dx$$

$$= \frac{60\,(dx + c)a^2 + 45\,ab\log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) + 1\right|\right) - 45\,ab\log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) - 1\right|\right) + \frac{2\left(60\,a^2\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)\right.}{}}{}$$

input
```
integrate((a+b*sec(d*x+c))^2*tan(d*x+c)^4,x, algorithm="giac")
```

output
```
1/60*(60*(d*x + c)*a^2 + 45*a*b*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 45*a*
b*log(abs(tan(1/2*d*x + 1/2*c) - 1)) + 2*(60*a^2*tan(1/2*d*x + 1/2*c)^9 -
45*a*b*tan(1/2*d*x + 1/2*c)^9 - 320*a^2*tan(1/2*d*x + 1/2*c)^7 + 210*a*b*t
an(1/2*d*x + 1/2*c)^7 + 520*a^2*tan(1/2*d*x + 1/2*c)^5 - 192*b^2*tan(1/2*d
*x + 1/2*c)^5 - 320*a^2*tan(1/2*d*x + 1/2*c)^3 - 210*a*b*tan(1/2*d*x + 1/2
*c)^3 + 60*a^2*tan(1/2*d*x + 1/2*c) + 45*a*b*tan(1/2*d*x + 1/2*c))/(tan(1/
2*d*x + 1/2*c)^2 - 1)^5)/d
```

3.280.9 Mupad [B] (verification not implemented)

Time = 15.41 (sec) , antiderivative size = 332, normalized size of antiderivative = 2.86

$$\int (a+b\sec(c+dx))^2\tan^4(c+dx)\,dx$$

$$= \frac{\left(2a^2-\frac{3ab}{2}\right)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^9+\left(7ab-\frac{32a^2}{3}\right)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^7+\left(\frac{52a^2}{3}-\frac{32b^2}{5}\right)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5+\left(-\frac{32a^2}{3}-7b\right.}{d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}-5\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^8+10\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6-10\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4+5\right.}$$

$$+\frac{2a^2\operatorname{atan}\left(\frac{64a^6\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{64a^6+36a^4b^2}+\frac{36a^4b^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{64a^6+36a^4b^2}\right)}{d}$$

$$+\frac{3ab\operatorname{atanh}\left(\frac{48a^5b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{48a^5b+27a^3b^3}+\frac{27a^3b^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{48a^5b+27a^3b^3}\right)}{2d}$$

input
```
int(tan(c + d*x)^4*(a + b/cos(c + d*x))^2,x)
```

output
```
(tan(c/2 + (d*x)/2)^5*((52*a^2)/3 - (32*b^2)/5) - tan(c/2 + (d*x)/2)^9*((3
*a*b)/2 - 2*a^2) - tan(c/2 + (d*x)/2)^3*(7*a*b + (32*a^2)/3) + tan(c/2 + (
d*x)/2)^7*(7*a*b - (32*a^2)/3) + tan(c/2 + (d*x)/2)*((3*a*b)/2 + 2*a^2))/(
d*(5*tan(c/2 + (d*x)/2)^2 - 10*tan(c/2 + (d*x)/2)^4 + 10*tan(c/2 + (d*x)/2
)^6 - 5*tan(c/2 + (d*x)/2)^8 + tan(c/2 + (d*x)/2)^10 - 1)) + (2*a^2*atan((
64*a^6*tan(c/2 + (d*x)/2))/(64*a^6 + 36*a^4*b^2) + (36*a^4*b^2*tan(c/2 + (
d*x)/2))/(64*a^6 + 36*a^4*b^2)))/d + (3*a*b*atanh((48*a^5*b*tan(c/2 + (d*x
)/2))/(48*a^5*b + 27*a^3*b^3) + (27*a^3*b^3*tan(c/2 + (d*x)/2))/(48*a^5*b
+ 27*a^3*b^3)))/(2*d)
```

3.281 $\int (a + b\sec(c + dx))^2 \tan^2(c + dx)\, dx$

3.281.1 Optimal result 2042
3.281.2 Mathematica [A] (verified) 2042
3.281.3 Rubi [A] (verified) 2043
3.281.4 Maple [A] (verified) 2044
3.281.5 Fricas [A] (verification not implemented) 2044
3.281.6 Sympy [F] 2045
3.281.7 Maxima [A] (verification not implemented) 2045
3.281.8 Giac [B] (verification not implemented) 2045
3.281.9 Mupad [B] (verification not implemented) 2046

3.281.1 Optimal result

Integrand size = 21, antiderivative size = 70

$$\int (a + b\sec(c + dx))^2 \tan^2(c + dx)\, dx = -a^2 x - \frac{ab\operatorname{arctanh}(\sin(c + dx))}{d} + \frac{a^2 \tan(c + dx)}{d} + \frac{ab\sec(c + dx)\tan(c + dx)}{d} + \frac{b^2 \tan^3(c + dx)}{3d}$$

output
```
-a^2*x-a*b*arctanh(sin(d*x+c))/d+a^2*tan(d*x+c)/d+a*b*sec(d*x+c)*tan(d*x+c
)/d+1/3*b^2*tan(d*x+c)^3/d
```

3.281.2 Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.94

$$\int (a + b\sec(c + dx))^2 \tan^2(c + dx)\, dx = \frac{-3a^2 \arctan(\tan(c + dx)) - 3ab\operatorname{arctanh}(\sin(c + dx)) + \tan(c + dx)\,(3a^2 + 3ab\sec(c + dx) + b^2 \tan^2(c +$$

input
```
Integrate[(a + b*Sec[c + d*x])^2*Tan[c + d*x]^2,x]
```

output
```
(-3*a^2*ArcTan[Tan[c + d*x]] - 3*a*b*ArcTanh[Sin[c + d*x]] + Tan[c + d*x]*
(3*a^2 + 3*a*b*Sec[c + d*x] + b^2*Tan[c + d*x]^2))/(3*d)
```

3.281.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3042, 4374, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^2(c+dx)(a+b\sec(c+dx))^2\,dx$$

$$\downarrow 3042$$

$$\int \cot\left(c+dx+\frac{\pi}{2}\right)^2\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^2 dx$$

$$\downarrow 4374$$

$$\int \left(a^2\tan^2(c+dx)+2ab\tan^2(c+dx)\sec(c+dx)+b^2\tan^2(c+dx)\sec^2(c+dx)\right)dx$$

$$\downarrow 2009$$

$$\frac{a^2\tan(c+dx)}{d}-a^2x-\frac{ab\operatorname{arctanh}(\sin(c+dx))}{d}+\frac{ab\tan(c+dx)\sec(c+dx)}{d}+\frac{b^2\tan^3(c+dx)}{3d}$$

input
```
Int[(a + b*Sec[c + d*x])^2*Tan[c + d*x]^2,x]
```

output
```
-(a^2*x) - (a*b*ArcTanh[Sin[c + d*x]])/d + (a^2*Tan[c + d*x])/d + (a*b*Sec
[c + d*x]*Tan[c + d*x])/d + (b^2*Tan[c + d*x]^3)/(3*d)
```

3.281.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4374
```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

3.281.4 Maple [A] (verified)

Time = 1.27 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.30

method	result
parts	$\frac{a^2(\tan(dx+c)-\arctan(\tan(dx+c)))}{d}+\frac{b^2\tan(dx+c)^3}{3d}+\frac{2ab\left(\frac{\sin(dx+c)^3}{2\cos(dx+c)^2}+\frac{\sin(dx+c)}{2}-\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)}{d}$
derivativedivides	$\frac{a^2(\tan(dx+c)-dx-c)+2ab\left(\frac{\sin(dx+c)^3}{2\cos(dx+c)^2}+\frac{\sin(dx+c)}{2}-\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+\frac{b^2\sin(dx+c)^3}{3\cos(dx+c)^3}}{d}$
default	$\frac{a^2(\tan(dx+c)-dx-c)+2ab\left(\frac{\sin(dx+c)^3}{2\cos(dx+c)^2}+\frac{\sin(dx+c)}{2}-\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+\frac{b^2\sin(dx+c)^3}{3\cos(dx+c)^3}}{d}$
risch	$-a^2x-\frac{2i\left(3ab\,e^{5i(dx+c)}-3a^2e^{4i(dx+c)}+3b^2e^{4i(dx+c)}-6a^2e^{2i(dx+c)}-3ab\,e^{i(dx+c)}-3a^2+b^2\right)}{3d\left(e^{2i(dx+c)}+1\right)^3}-\frac{ab\ln\left(e^{i(dx+c)}+i\right)}{d}$

input
```
int((a+b*sec(d*x+c))^2*tan(d*x+c)^2,x,method=_RETURNVERBOSE)
```

output
```
a^2/d*(tan(d*x+c)-arctan(tan(d*x+c)))+1/3*b^2*tan(d*x+c)^3/d+2*a*b/d*(1/2*
sin(d*x+c)^3/cos(d*x+c)^2+1/2*sin(d*x+c)-1/2*ln(sec(d*x+c)+tan(d*x+c)))
```

3.281.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.64

$$\int (a+b\sec(c+dx))^2\tan^2(c+dx)\,dx =$$
$$-\frac{6\,a^2dx\cos(dx+c)^3+3\,ab\cos(dx+c)^3\log(\sin(dx+c)+1)-3\,ab\cos(dx+c)^3\log(-\sin(dx+c)+}{6\,d\cos(dx+c)^3}$$

input
```
integrate((a+b*sec(d*x+c))^2*tan(d*x+c)^2,x, algorithm="fricas")
```

output
```
-1/6*(6*a^2*d*x*cos(d*x + c)^3 + 3*a*b*cos(d*x + c)^3*log(sin(d*x + c) + 1
) - 3*a*b*cos(d*x + c)^3*log(-sin(d*x + c) + 1) - 2*(3*a*b*cos(d*x + c) +
(3*a^2 - b^2)*cos(d*x + c)^2 + b^2)*sin(d*x + c))/(d*cos(d*x + c)^3)
```

3.281.6 Sympy [F]

$$\int (a+b\sec(c+dx))^2\tan^2(c+dx)\,dx = \int (a+b\sec(c+dx))^2\tan^2(c+dx)\,dx$$

input
```
integrate((a+b*sec(d*x+c))**2*tan(d*x+c)**2,x)
```

output
```
Integral((a + b*sec(c + d*x))**2*tan(c + d*x)**2, x)
```

3.281.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.17

$$\int (a+b\sec(c+dx))^2\tan^2(c+dx)\,dx$$
$$= \frac{2\,b^2\tan(dx+c)^3 - 6\,(dx+c-\tan(dx+c))a^2 - 3\,ab\left(\frac{2\sin(dx+c)}{\sin(dx+c)^2-1} + \log(\sin(dx+c)+1) - \log(\sin(dx+c)-1)\right)}{6\,d}$$

input
```
integrate((a+b*sec(d*x+c))^2*tan(d*x+c)^2,x, algorithm="maxima")
```

output
```
1/6*(2*b^2*tan(d*x + c)^3 - 6*(d*x + c - tan(d*x + c))*a^2 - 3*a*b*(2*sin(
d*x + c)/(sin(d*x + c)^2 - 1) + log(sin(d*x + c) + 1) - log(sin(d*x + c) -
 1)))/d
```

3.281.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 158 vs. 2(68) = 136.

Time = 0.57 (sec) , antiderivative size = 158, normalized size of antiderivative = 2.26

$$\int (a+b\sec(c+dx))^2\tan^2(c+dx)\,dx =$$
$$-\frac{3\,(dx+c)a^2 + 3\,ab\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right) - 3\,ab\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right) + \frac{2\left(3\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5 -\right.}{3\,d}$$

input
```
integrate((a+b*sec(d*x+c))^2*tan(d*x+c)^2,x, algorithm="giac")
```

output
```
-1/3*(3*(d*x + c)*a^2 + 3*a*b*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 3*a*b*l
og(abs(tan(1/2*d*x + 1/2*c) - 1)) + 2*(3*a^2*tan(1/2*d*x + 1/2*c)^5 - 3*a*
b*tan(1/2*d*x + 1/2*c)^5 - 6*a^2*tan(1/2*d*x + 1/2*c)^3 + 4*b^2*tan(1/2*d*
x + 1/2*c)^3 + 3*a^2*tan(1/2*d*x + 1/2*c) + 3*a*b*tan(1/2*d*x + 1/2*c))/(t
an(1/2*d*x + 1/2*c)^2 - 1)^3)/d
```

3.281.9 Mupad [B] (verification not implemented)

Time = 14.66 (sec) , antiderivative size = 227, normalized size of antiderivative = 3.24

$$\int (a+b\sec(c+dx))^2\tan^2(c+dx)\,dx =$$

$$-\frac{\frac{b^2\sin(3c+3dx)}{12}-\frac{b^2\sin(c+dx)}{4}-\frac{a^2\sin(3c+3dx)}{4}-\frac{a^2\sin(c+dx)}{4}+\frac{3a^2\cos(c+dx)\operatorname{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{dx}{2}\right)}{\cos\left(\frac{c}{2}+\frac{dx}{2}\right)}\right)}{2}+\frac{a^2\operatorname{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{dx}{2}\right)}{\cos\left(\frac{c}{2}+\frac{dx}{2}\right)}\right.}{2}}{d\left(\frac{3\cos(c+dx)}{4}+\frac{\cos(}{}\right.}$$

input
```
int(tan(c + d*x)^2*(a + b/cos(c + d*x))^2,x)
```

output
```
-((b^2*sin(3*c + 3*d*x))/12 - (b^2*sin(c + d*x))/4 - (a^2*sin(3*c + 3*d*x)
)/4 - (a^2*sin(c + d*x))/4 + (3*a^2*cos(c + d*x)*atan(sin(c/2 + (d*x)/2)/c
os(c/2 + (d*x)/2)))/2 + (a^2*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*c
os(3*c + 3*d*x))/2 - (a*b*sin(2*c + 2*d*x))/2 + (3*a*b*cos(c + d*x)*atanh(
sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/2 + (a*b*atanh(sin(c/2 + (d*x)/2)/
cos(c/2 + (d*x)/2))*cos(3*c + 3*d*x))/2)/(d*((3*cos(c + d*x))/4 + cos(3*c
+ 3*d*x)/4))
```

3.282 $\int \cot^2(c+dx)(a+b\sec(c+dx))^2\,dx$

3.282.1 Optimal result 2047
3.282.2 Mathematica [A] (verified) 2047
3.282.3 Rubi [A] (verified) 2048
3.282.4 Maple [C] (verified) 2049
3.282.5 Fricas [A] (verification not implemented) 2049
3.282.6 Sympy [F] 2049
3.282.7 Maxima [A] (verification not implemented) 2050
3.282.8 Giac [A] (verification not implemented) 2050
3.282.9 Mupad [B] (verification not implemented) 2050

3.282.1 Optimal result

Integrand size = 21, antiderivative size = 48

$$\int \cot^2(c+dx)(a+b\sec(c+dx))^2\,dx = -a^2x - \frac{a^2\cot(c+dx)}{d} - \frac{b^2\cot(c+dx)}{d} - \frac{2ab\csc(c+dx)}{d}$$

output
```
-a^2*x-a^2*cot(d*x+c)/d-b^2*cot(d*x+c)/d-2*a*b*csc(d*x+c)/d
```

3.282.2 Mathematica [A] (verified)

Time = 0.62 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.81

$$\int \cot^2(c+dx)(a+b\sec(c+dx))^2\,dx = -\frac{(a^2+b^2)\cot(c+dx) + a(a(c+dx)+2b\csc(c+dx))}{d}$$

input
```
Integrate[Cot[c + d*x]^2*(a + b*Sec[c + d*x])^2,x]
```

output
```
-(((a^2 + b^2)*Cot[c + d*x] + a*(a*(c + d*x) + 2*b*Csc[c + d*x]))/d)
```

3.282.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3042, 4374, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^2(c+dx)(a+b\sec(c+dx))^2\,dx$$

$$\downarrow 3042$$

$$\int \frac{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^2}{\cot\left(c+dx+\frac{\pi}{2}\right)^2}dx$$

$$\downarrow 4374$$

$$\int \left(a^2\cot^2(c+dx)+2ab\cot(c+dx)\csc(c+dx)+b^2\csc^2(c+dx)\right)dx$$

$$\downarrow 2009$$

$$-\frac{a^2\cot(c+dx)}{d}+a^2(-x)-\frac{2ab\csc(c+dx)}{d}-\frac{b^2\cot(c+dx)}{d}$$

input
```
Int[Cot[c + d*x]^2*(a + b*Sec[c + d*x])^2,x]
```

output
```
-(a^2*x) - (a^2*Cot[c + d*x])/d - (b^2*Cot[c + d*x])/d - (2*a*b*Csc[c + d*
x])/d
```

3.282.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4374
```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

3.282.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.13 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.98

method	result	size
risch	$-a^2x-\frac{2i\left(2ab\,e^{i(dx+c)}+a^2+b^2\right)}{d\left(e^{2i(dx+c)}-1\right)}$	47
derivativedivides	$\frac{a^2(-\cot(dx+c)-dx-c)-\frac{2ab}{\sin(dx+c)}-b^2\cot(dx+c)}{d}$	49
default	$\frac{a^2(-\cot(dx+c)-dx-c)-\frac{2ab}{\sin(dx+c)}-b^2\cot(dx+c)}{d}$	49

input
```
int(cot(d*x+c)^2*(a+b*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
-a^2*x-2*I*(2*a*b*exp(I*(d*x+c))+a^2+b^2)/d/(exp(2*I*(d*x+c))-1)
```

3.282.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.92

$$\int \cot^2(c+dx)(a+b\sec(c+dx))^2\,dx = -\frac{a^2 dx\sin(dx+c)+2\,ab+(a^2+b^2)\cos(dx+c)}{d\sin(dx+c)}$$

input
```
integrate(cot(d*x+c)^2*(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

output
```
-(a^2*d*x*sin(d*x + c) + 2*a*b + (a^2 + b^2)*cos(d*x + c))/(d*sin(d*x + c)
)
```

3.282.6 Sympy [F]

$$\int \cot^2(c+dx)(a+b\sec(c+dx))^2\,dx = \int (a+b\sec(c+dx))^2\cot^2(c+dx)\,dx$$

input
```
integrate(cot(d*x+c)**2*(a+b*sec(d*x+c))**2,x)
```

output
```
Integral((a + b*sec(c + d*x))**2*cot(c + d*x)**2, x)
```

3.282.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.98

$$\int \cot^2(c+dx)(a+b\sec(c+dx))^2\,dx = -\frac{\left(dx+c+\frac{1}{\tan(dx+c)}\right)a^2+\frac{2\,ab}{\sin(dx+c)}+\frac{b^2}{\tan(dx+c)}}{d}$$

input
```
integrate(cot(d*x+c)^2*(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

output
```
-((d*x + c + 1/tan(d*x + c))*a^2 + 2*a*b/sin(d*x + c) + b^2/tan(d*x + c))/
d
```

3.282.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.67

$$\int \cot^2(c+dx)(a+b\sec(c+dx))^2\,dx =$$
$$-\frac{2\,(dx+c)a^2-a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+2\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+\frac{a^2+2\,ab+b^2}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}}{2\,d}$$

input
```
integrate(cot(d*x+c)^2*(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

output
```
-1/2*(2*(d*x + c)*a^2 - a^2*tan(1/2*d*x + 1/2*c) + 2*a*b*tan(1/2*d*x + 1/2
*c) - b^2*tan(1/2*d*x + 1/2*c) + (a^2 + 2*a*b + b^2)/tan(1/2*d*x + 1/2*c))
/d
```

3.282.9 Mupad [B] (verification not implemented)

Time = 14.14 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.21

$$\int \cot^2(c+dx)(a+b\sec(c+dx))^2\,dx = \frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\,(a-b)^2}{2\,d}-\frac{\frac{a^2}{2}+a\,b+\frac{b^2}{2}}{d\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}-a^2\,x$$

input
```
int(cot(c + d*x)^2*(a + b/cos(c + d*x))^2,x)
```

output
```
(tan(c/2 + (d*x)/2)*(a - b)^2)/(2*d) - (a*b + a^2/2 + b^2/2)/(d*tan(c/2 +
(d*x)/2)) - a^2*x
```

3.283 $\int \cot^4(c+dx)(a+b\sec(c+dx))^2\,dx$

3.283.1 Optimal result 2051
3.283.2 Mathematica [C] (verified) 2051
3.283.3 Rubi [A] (verified) 2052
3.283.4 Maple [A] (verified) 2053
3.283.5 Fricas [A] (verification not implemented) 2054
3.283.6 Sympy [F] 2054
3.283.7 Maxima [A] (verification not implemented) 2054
3.283.8 Giac [B] (verification not implemented) 2055
3.283.9 Mupad [B] (verification not implemented) 2055

3.283.1 Optimal result

Integrand size = 21, antiderivative size = 85

$$\int \cot^4(c+dx)(a+b\sec(c+dx))^2\,dx = a^2x + \frac{a^2\cot(c+dx)}{d} - \frac{a^2\cot^3(c+dx)}{3d} - \frac{b^2\cot^3(c+dx)}{3d} + \frac{2ab\csc(c+dx)}{d} - \frac{2ab\csc^3(c+dx)}{3d}$$

output
```
a^2*x+a^2*cot(d*x+c)/d-1/3*a^2*cot(d*x+c)^3/d-1/3*b^2*cot(d*x+c)^3/d+2*a*b
*csc(d*x+c)/d-2/3*a*b*csc(d*x+c)^3/d
```

3.283.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.26 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.82

$$\int \cot^4(c+dx)(a+b\sec(c+dx))^2\,dx = -\frac{b(b\cot^3(c+dx)+2a\csc(c+dx)(-3+\csc^2(c+dx)))+a^2\cot^3(c+dx)\,\text{Hypergeometric2F1}\left(-\frac{3}{2},1,-\right.}{3d}$$

input
```
Integrate[Cot[c + d*x]^4*(a + b*Sec[c + d*x])^2,x]
```

output
```
-1/3*(b*(b*Cot[c + d*x]^3 + 2*a*Csc[c + d*x]*(-3 + Csc[c + d*x]^2)) + a^2*
Cot[c + d*x]^3*Hypergeometric2F1[-3/2, 1, -1/2, -Tan[c + d*x]^2])/d
```

3.283.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3042, 4374, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^4(c+dx)(a+b\sec(c+dx))^2\,dx$$

$$\downarrow 3042$$

$$\int \frac{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^2}{\cot\left(c+dx+\frac{\pi}{2}\right)^4}dx$$

$$\downarrow 4374$$

$$\int \left(a^2\cot^4(c+dx)+2ab\cot^3(c+dx)\csc(c+dx)+b^2\cot^2(c+dx)\csc^2(c+dx)\right)dx$$

$$\downarrow 2009$$

$$-\frac{a^2\cot^3(c+dx)}{3d}+\frac{a^2\cot(c+dx)}{d}+a^2x-\frac{2ab\csc^3(c+dx)}{3d}+\frac{2ab\csc(c+dx)}{d}-\frac{b^2\cot^3(c+dx)}{3d}$$

input
```
Int[Cot[c + d*x]^4*(a + b*Sec[c + d*x])^2,x]
```

output
```
a^2*x + (a^2*Cot[c + d*x])/d - (a^2*Cot[c + d*x]^3)/(3*d) - (b^2*Cot[c + d
*x]^3)/(3*d) + (2*a*b*Csc[c + d*x])/d - (2*a*b*Csc[c + d*x]^3)/(3*d)
```

3.283.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4374
```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

3.283.4 Maple [A] (verified)

Time = 1.32 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.31

method	result
derivativedivides	$\frac{a^2\left(-\frac{\cot(dx+c)^3}{3}+\cot(dx+c)+dx+c\right)+2ab\left(-\frac{\cos(dx+c)^4}{3\sin(dx+c)^3}+\frac{\cos(dx+c)^4}{3\sin(dx+c)}+\frac{\left(2+\cos(dx+c)^2\right)\sin(dx+c)}{3}\right)-\frac{b^2\cos(dx+c)^3}{3\sin(dx+c)^3}}{d}$
default	$\frac{a^2\left(-\frac{\cot(dx+c)^3}{3}+\cot(dx+c)+dx+c\right)+2ab\left(-\frac{\cos(dx+c)^4}{3\sin(dx+c)^3}+\frac{\cos(dx+c)^4}{3\sin(dx+c)}+\frac{\left(2+\cos(dx+c)^2\right)\sin(dx+c)}{3}\right)-\frac{b^2\cos(dx+c)^3}{3\sin(dx+c)^3}}{d}$
risch	$a^2x+\frac{2i\left(6ab\,\mathrm{e}^{5i(dx+c)}+6a^2\mathrm{e}^{4i(dx+c)}+3b^2\mathrm{e}^{4i(dx+c)}-4ab\,\mathrm{e}^{3i(dx+c)}-6a^2\mathrm{e}^{2i(dx+c)}+6ab\,\mathrm{e}^{i(dx+c)}+4a^2+b^2\right)}{3d\left(\mathrm{e}^{2i(dx+c)}-1\right)^3}$

input
```
int(cot(d*x+c)^4*(a+b*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^2*(-1/3*cot(d*x+c)^3+cot(d*x+c)+d*x+c)+2*a*b*(-1/3/sin(d*x+c)^3*cos
(d*x+c)^4+1/3/sin(d*x+c)*cos(d*x+c)^4+1/3*(2+cos(d*x+c)^2)*sin(d*x+c))-1/3
*b^2/sin(d*x+c)^3*cos(d*x+c)^3)
```

3.283.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.20

$$\int \cot^4(c+dx)(a+b\sec(c+dx))^2\,dx$$
$$= \frac{6\,ab\cos(dx+c)^2 + (4\,a^2+b^2)\cos(dx+c)^3 - 3\,a^2\cos(dx+c) - 4\,ab + 3\left(a^2 dx\cos(dx+c)^2 - a^2 dx\right)\sin}{3\left(d\cos(dx+c)^2 - d\right)\sin(dx+c)}$$

input
```
integrate(cot(d*x+c)^4*(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

output
```
1/3*(6*a*b*cos(d*x + c)^2 + (4*a^2 + b^2)*cos(d*x + c)^3 - 3*a^2*cos(d*x +
 c) - 4*a*b + 3*(a^2*d*x*cos(d*x + c)^2 - a^2*d*x)*sin(d*x + c))/((d*cos(d
*x + c)^2 - d)*sin(d*x + c))
```

3.283.6 Sympy [F]

$$\int \cot^4(c+dx)(a+b\sec(c+dx))^2\,dx = \int (a+b\sec(c+dx))^2\cot^4(c+dx)\,dx$$

input
```
integrate(cot(d*x+c)**4*(a+b*sec(d*x+c))**2,x)
```

output
```
Integral((a + b*sec(c + d*x))**2*cot(c + d*x)**4, x)
```

3.283.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.89

$$\int \cot^4(c+dx)(a+b\sec(c+dx))^2\,dx$$
$$= \frac{\left(3\,dx + 3\,c + \frac{3\tan(dx+c)^2-1}{\tan(dx+c)^3}\right)a^2 + \frac{2\left(3\sin(dx+c)^2-1\right)ab}{\sin(dx+c)^3} - \frac{b^2}{\tan(dx+c)^3}}{3\,d}$$

input
```
integrate(cot(d*x+c)^4*(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

output
```
1/3*((3*d*x + 3*c + (3*tan(d*x + c)^2 - 1)/tan(d*x + c)^3)*a^2 + 2*(3*sin(
d*x + c)^2 - 1)*a*b/sin(d*x + c)^3 - b^2/tan(d*x + c)^3)/d
```

3.283.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 176 vs. $2(79) = 158$.

Time $= 0.32$ (sec) , antiderivative size $= 176$, normalized size of antiderivative $= 2.07$

$$\int \cot^4(c+dx)(a+b\sec(c+dx))^2\,dx$$

$$= \frac{a^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3 - 2ab\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3 + b^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3 + 24(dx+c)a^2 - 15a^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right.}{}$$

input
```
integrate(cot(d*x+c)^4*(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

output
```
1/24*(a^2*tan(1/2*d*x + 1/2*c)^3 - 2*a*b*tan(1/2*d*x + 1/2*c)^3 + b^2*tan(
1/2*d*x + 1/2*c)^3 + 24*(d*x + c)*a^2 - 15*a^2*tan(1/2*d*x + 1/2*c) + 18*a
*b*tan(1/2*d*x + 1/2*c) - 3*b^2*tan(1/2*d*x + 1/2*c) + (15*a^2*tan(1/2*d*x
 + 1/2*c)^2 + 18*a*b*tan(1/2*d*x + 1/2*c)^2 + 3*b^2*tan(1/2*d*x + 1/2*c)^2
 - a^2 - 2*a*b - b^2)/tan(1/2*d*x + 1/2*c)^3)/d
```

3.283.9 Mupad [B] (verification not implemented)

Time $= 14.00$ (sec) , antiderivative size $= 118$, normalized size of antiderivative $= 1.39$

$$\int \cot^4(c+dx)(a+b\sec(c+dx))^2\,dx$$

$$= a^2 x + \frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3(a-b)^2}{24d} - \frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\left(\frac{a(a-b)}{2}+\frac{(a-b)^2}{8}\right)}{d} - \frac{\cot\left(\frac{c}{2}+\frac{dx}{2}\right)^3\left(\frac{2ab}{3}+\frac{a^2}{3}+\frac{b^2}{3}-\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2(5a^2+6ab+b^2)\right)}{8d}$$

input
```
int(cot(c + d*x)^4*(a + b/cos(c + d*x))^2,x)
```

output
```
a^2*x + (tan(c/2 + (d*x)/2)^3*(a - b)^2)/(24*d) - (tan(c/2 + (d*x)/2)*((a*
(a - b))/2 + (a - b)^2/8))/d - (cot(c/2 + (d*x)/2)^3*((2*a*b)/3 + a^2/3 +
b^2/3 - tan(c/2 + (d*x)/2)^2*(6*a*b + 5*a^2 + b^2)))/(8*d)
```

3.284 $\int \cot^6(c+dx)(a+b\sec(c+dx))^2\,dx$

3.284.1 Optimal result . . . 2056
3.284.2 Mathematica [C] (verified) . . . 2056
3.284.3 Rubi [A] (verified) . . . 2057
3.284.4 Maple [A] (verified) . . . 2058
3.284.5 Fricas [A] (verification not implemented) . . . 2059
3.284.6 Sympy [F] . . . 2059
3.284.7 Maxima [A] (verification not implemented) . . . 2059
3.284.8 Giac [B] (verification not implemented) . . . 2060
3.284.9 Mupad [B] (verification not implemented) . . . 2061

3.284.1 Optimal result

Integrand size = 21, antiderivative size = 122

$$\int \cot^6(c+dx)(a+b\sec(c+dx))^2\,dx = -a^2x - \frac{a^2\cot(c+dx)}{d} + \frac{a^2\cot^3(c+dx)}{3d} - \frac{a^2\cot^5(c+dx)}{5d} - \frac{b^2\cot^5(c+dx)}{5d} - \frac{2ab\csc(c+dx)}{d} + \frac{4ab\csc^3(c+dx)}{3d} - \frac{2ab\csc^5(c+dx)}{5d}$$

output

```
-a^2*x-a^2*cot(d*x+c)/d+1/3*a^2*cot(d*x+c)^3/d-1/5*a^2*cot(d*x+c)^5/d-1/5*
b^2*cot(d*x+c)^5/d-2*a*b*csc(d*x+c)/d+4/3*a*b*csc(d*x+c)^3/d-2/5*a*b*csc(d
*x+c)^5/d
```

3.284.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.38 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.69

$$\int \cot^6(c+dx)(a+b\sec(c+dx))^2\,dx = -\frac{b(3b\cot^5(c+dx)+2a\csc(c+dx)\,(15-10\csc^2(c+dx)+3\csc^4(c+dx)))+3a^2\cot^5(c+dx)\,\text{Hyperge}}{15d}$$

input
```
Integrate[Cot[c + d*x]^6*(a + b*Sec[c + d*x])^2,x]
```

output
```
-1/15*(b*(3*b*Cot[c + d*x]^5 + 2*a*Csc[c + d*x]*(15 - 10*Csc[c + d*x]^2 +
3*Csc[c + d*x]^4)) + 3*a^2*Cot[c + d*x]^5*Hypergeometric2F1[-5/2, 1, -3/2,
 -Tan[c + d*x]^2])/d
```

3.284.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3042, 4374, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^6(c+dx)(a+b\sec(c+dx))^2\,dx$$

$$\downarrow 3042$$

$$\int \frac{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^2}{\cot\left(c+dx+\frac{\pi}{2}\right)^6}dx$$

$$\downarrow 4374$$

$$\int \left(a^2\cot^6(c+dx)+2ab\cot^5(c+dx)\csc(c+dx)+b^2\cot^4(c+dx)\csc^2(c+dx)\right)dx$$

$$\downarrow 2009$$

$$-\frac{a^2\cot^5(c+dx)}{5d}+\frac{a^2\cot^3(c+dx)}{3d}-\frac{a^2\cot(c+dx)}{d}-a^2x-\frac{2ab\csc^5(c+dx)}{5d}+\frac{4ab\csc^3(c+dx)}{3d}-\frac{2ab\csc(c+dx)}{d}-\frac{b^2\cot^5(c+dx)}{5d}$$

input
```
Int[Cot[c + d*x]^6*(a + b*Sec[c + d*x])^2,x]
```

output
```
-(a^2*x) - (a^2*Cot[c + d*x])/d + (a^2*Cot[c + d*x]^3)/(3*d) - (a^2*Cot[c
+ d*x]^5)/(5*d) - (b^2*Cot[c + d*x]^5)/(5*d) - (2*a*b*Csc[c + d*x])/d + (4
*a*b*Csc[c + d*x]^3)/(3*d) - (2*a*b*Csc[c + d*x]^5)/(5*d)
```

3.284.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4374
```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

3.284.4 Maple [A] (verified)

Time = 2.06 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.26

method	result
derivativedivides	$\frac{a^2\left(-\frac{\cot(dx+c)^5}{5}+\frac{\cot(dx+c)^3}{3}-\cot(dx+c)-dx-c\right)+2ab\left(-\frac{\cos(dx+c)^6}{5\sin(dx+c)^5}+\frac{\cos(dx+c)^6}{15\sin(dx+c)^3}-\frac{\cos(dx+c)^6}{5\sin(dx+c)}-\frac{\left(\frac{8}{3}+\cos(dx+c)^4+\frac{4}{3}\right.}{d}$
default	$\frac{a^2\left(-\frac{\cot(dx+c)^5}{5}+\frac{\cot(dx+c)^3}{3}-\cot(dx+c)-dx-c\right)+2ab\left(-\frac{\cos(dx+c)^6}{5\sin(dx+c)^5}+\frac{\cos(dx+c)^6}{15\sin(dx+c)^3}-\frac{\cos(dx+c)^6}{5\sin(dx+c)}-\frac{\left(\frac{8}{3}+\cos(dx+c)^4+\frac{4}{3}\right.}{d}$
risch	$-a^2x-\frac{2i\left(30ab\,e^{9i(dx+c)}+45a^2e^{8i(dx+c)}+15b^2e^{8i(dx+c)}-40ab\,e^{7i(dx+c)}-90a^2e^{6i(dx+c)}+116ab\,e^{5i(dx+c)}+140a^2e^{4}\right.}{15d\left(e^{2i(dx+c)}-1\right)^5}$

input
```
int(cot(d*x+c)^6*(a+b*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^2*(-1/5*cot(d*x+c)^5+1/3*cot(d*x+c)^3-cot(d*x+c)-d*x-c)+2*a*b*(-1/5
/sin(d*x+c)^5*cos(d*x+c)^6+1/15/sin(d*x+c)^3*cos(d*x+c)^6-1/5/sin(d*x+c)*c
os(d*x+c)^6-1/5*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c))-1/5*b^2/si
n(d*x+c)^5*cos(d*x+c)^5)
```

3.284.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.25

$$\int \cot^6(c+dx)(a+b\sec(c+dx))^2\,dx =$$

$$-\frac{30\,ab\cos(dx+c)^4+(23\,a^2+3\,b^2)\cos(dx+c)^5-35\,a^2\cos(dx+c)^3-40\,ab\cos(dx+c)^2+15\,a^2\cos(}{15\left(d\cos(dx+c)^4-2\,d\cos(dx+c)^2\right.}$$

input
```
integrate(cot(d*x+c)^6*(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

output
```
-1/15*(30*a*b*cos(d*x + c)^4 + (23*a^2 + 3*b^2)*cos(d*x + c)^5 - 35*a^2*co
s(d*x + c)^3 - 40*a*b*cos(d*x + c)^2 + 15*a^2*cos(d*x + c) + 16*a*b + 15*(
a^2*d*x*cos(d*x + c)^4 - 2*a^2*d*x*cos(d*x + c)^2 + a^2*d*x)*sin(d*x + c))
/((d*cos(d*x + c)^4 - 2*d*cos(d*x + c)^2 + d)*sin(d*x + c))
```

3.284.6 Sympy [F]

$$\int \cot^6(c+dx)(a+b\sec(c+dx))^2\,dx = \int (a+b\sec(c+dx))^2\cot^6(c+dx)\,dx$$

input
```
integrate(cot(d*x+c)**6*(a+b*sec(d*x+c))**2,x)
```

output
```
Integral((a + b*sec(c + d*x))**2*cot(c + d*x)**6, x)
```

3.284.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.79

$$\int \cot^6(c+dx)(a+b\sec(c+dx))^2\,dx =$$

$$-\frac{\left(15\,dx+15\,c+\frac{15\tan(dx+c)^4-5\tan(dx+c)^2+3}{\tan(dx+c)^5}\right)a^2+\frac{2\left(15\sin(dx+c)^4-10\sin(dx+c)^2+3\right)ab}{\sin(dx+c)^5}+\frac{3\,b^2}{\tan(dx+c)^5}}{15\,d}$$

input
```
integrate(cot(d*x+c)^6*(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

output
```
-1/15*((15*d*x + 15*c + (15*tan(d*x + c)^4 - 5*tan(d*x + c)^2 + 3)/tan(d*x
 + c)^5)*a^2 + 2*(15*sin(d*x + c)^4 - 10*sin(d*x + c)^2 + 3)*a*b/sin(d*x +
 c)^5 + 3*b^2/tan(d*x + c)^5)/d
```

3.284.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 273 vs. $2(112) = 224$.

Time $= 0.36$ (sec) , antiderivative size $= 273$, normalized size of antiderivative $= 2.24$

$$\int \cot^6(c+dx)(a+b\sec(c+dx))^2\,dx$$

$$= \frac{3\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5 - 6\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5 + 3\,b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5 - 35\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3 + 50\,ab\tan$$

input
```
integrate(cot(d*x+c)^6*(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

output
```
1/480*(3*a^2*tan(1/2*d*x + 1/2*c)^5 - 6*a*b*tan(1/2*d*x + 1/2*c)^5 + 3*b^2
*tan(1/2*d*x + 1/2*c)^5 - 35*a^2*tan(1/2*d*x + 1/2*c)^3 + 50*a*b*tan(1/2*d
*x + 1/2*c)^3 - 15*b^2*tan(1/2*d*x + 1/2*c)^3 - 480*(d*x + c)*a^2 + 330*a^
2*tan(1/2*d*x + 1/2*c) - 300*a*b*tan(1/2*d*x + 1/2*c) + 30*b^2*tan(1/2*d*x
 + 1/2*c) - (330*a^2*tan(1/2*d*x + 1/2*c)^4 + 300*a*b*tan(1/2*d*x + 1/2*c)
^4 + 30*b^2*tan(1/2*d*x + 1/2*c)^4 - 35*a^2*tan(1/2*d*x + 1/2*c)^2 - 50*a*
b*tan(1/2*d*x + 1/2*c)^2 - 15*b^2*tan(1/2*d*x + 1/2*c)^2 + 3*a^2 + 6*a*b +
 3*b^2)/tan(1/2*d*x + 1/2*c)^5)/d
```

3.284.9 Mupad [B] (verification not implemented)

Time = 13.70 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.57

$$\int \cot^6(c+dx)(a+b\sec(c+dx))^2\,dx$$

$$= \frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5(a-b)^2}{160\,d} - a^2x - \frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3\left(\frac{a^2}{16}-\frac{ab}{12}+\frac{b^2}{48}+\frac{(a-b)^2}{96}\right)}{d}$$
$$- \frac{\frac{2ab}{5}+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4(22a^2+20ab+2b^2)+\frac{a^2}{5}+\frac{b^2}{5}-\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2\left(\frac{7a^2}{3}+\frac{10ab}{3}+b^2\right)}{32\,d\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5}$$
$$+ \frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\left(\frac{21a^2}{32}-\frac{9ab}{16}+\frac{b^2}{32}+\frac{(a-b)^2}{32}\right)}{d}$$

input
```
int(cot(c + d*x)^6*(a + b/cos(c + d*x))^2,x)
```

output
```
(tan(c/2 + (d*x)/2)^5*(a - b)^2)/(160*d) - a^2*x - (tan(c/2 + (d*x)/2)^3*(
a^2/16 - (a*b)/12 + b^2/48 + (a - b)^2/96))/d - ((2*a*b)/5 + tan(c/2 + (d*
x)/2)^4*(20*a*b + 22*a^2 + 2*b^2) + a^2/5 + b^2/5 - tan(c/2 + (d*x)/2)^2*(
(10*a*b)/3 + (7*a^2)/3 + b^2))/(32*d*tan(c/2 + (d*x)/2)^5) + (tan(c/2 + (d
*x)/2)*((21*a^2)/32 - (9*a*b)/16 + b^2/32 + (a - b)^2/32))/d
```

3.285 $\int \cot^8(c+dx)(a+b\sec(c+dx))^2\,dx$

3.285.1 Optimal result . . . 2062
3.285.2 Mathematica [C] (verified) . . . 2063
3.285.3 Rubi [A] (verified) . . . 2063
3.285.4 Maple [A] (verified) . . . 2064
3.285.5 Fricas [A] (verification not implemented) . . . 2065
3.285.6 Sympy [F(-1)] . . . 2065
3.285.7 Maxima [A] (verification not implemented) . . . 2066
3.285.8 Giac [B] (verification not implemented) . . . 2066
3.285.9 Mupad [B] (verification not implemented) . . . 2067

3.285.1 Optimal result

Integrand size = 21, antiderivative size = 153

$$\begin{aligned}\int \cot^8(c+dx)(a+b\sec(c+dx))^2\,dx = {} & a^2x+\frac{a^2\cot(c+dx)}{d}\\ & -\frac{a^2\cot^3(c+dx)}{3d}+\frac{a^2\cot^5(c+dx)}{5d}\\ & -\frac{a^2\cot^7(c+dx)}{7d}-\frac{b^2\cot^7(c+dx)}{7d}\\ & +\frac{2ab\csc(c+dx)}{d}-\frac{2ab\csc^3(c+dx)}{d}\\ & +\frac{6ab\csc^5(c+dx)}{5d}-\frac{2ab\csc^7(c+dx)}{7d}\end{aligned}$$

output

```
a^2*x+a^2*cot(d*x+c)/d-1/3*a^2*cot(d*x+c)^3/d+1/5*a^2*cot(d*x+c)^5/d-1/7*a
^2*cot(d*x+c)^7/d-1/7*b^2*cot(d*x+c)^7/d+2*a*b*csc(d*x+c)/d-2*a*b*csc(d*x+
c)^3/d+6/5*a*b*csc(d*x+c)^5/d-2/7*a*b*csc(d*x+c)^7/d
```

3.285.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.40 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.61

$$\int \cot^8(c+dx)(a+b\sec(c+dx))^2\,dx$$
$$= \frac{b(-5b\cot^7(c+dx)+2a\csc(c+dx)\,(35-35\csc^2(c+dx)+21\csc^4(c+dx)-5\csc^6(c+dx)))-5a^2\cot^7}{35d}$$

input
```
Integrate[Cot[c + d*x]^8*(a + b*Sec[c + d*x])^2,x]
```

output
```
(b*(-5*b*Cot[c + d*x]^7 + 2*a*Csc[c + d*x]*(35 - 35*Csc[c + d*x]^2 + 21*Cs
c[c + d*x]^4 - 5*Csc[c + d*x]^6)) - 5*a^2*Cot[c + d*x]^7*Hypergeometric2F1
[-7/2, 1, -5/2, -Tan[c + d*x]^2])/(35*d)
```

3.285.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3042, 4374, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^8(c+dx)(a+b\sec(c+dx))^2\,dx$$
$$\downarrow 3042$$
$$\int \frac{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^2}{\cot\left(c+dx+\frac{\pi}{2}\right)^8}dx$$
$$\downarrow 4374$$
$$\int \left(a^2\cot^8(c+dx)+2ab\cot^7(c+dx)\csc(c+dx)+b^2\cot^6(c+dx)\csc^2(c+dx)\right)dx$$
$$\downarrow 2009$$
$$-\frac{a^2\cot^7(c+dx)}{7d}+\frac{a^2\cot^5(c+dx)}{5d}-\frac{a^2\cot^3(c+dx)}{3d}+\frac{a^2\cot(c+dx)}{d}+a^2x-\frac{2ab\csc^7(c+dx)}{7d}+$$
$$\frac{6ab\csc^5(c+dx)}{5d}-\frac{2ab\csc^3(c+dx)}{d}+\frac{2ab\csc(c+dx)}{d}-\frac{b^2\cot^7(c+dx)}{7d}$$

input
```
Int[Cot[c + d*x]^8*(a + b*Sec[c + d*x])^2,x]
```

output
```
a^2*x + (a^2*Cot[c + d*x])/d - (a^2*Cot[c + d*x]^3)/(3*d) + (a^2*Cot[c + d
*x]^5)/(5*d) - (a^2*Cot[c + d*x]^7)/(7*d) - (b^2*Cot[c + d*x]^7)/(7*d) + (
2*a*b*Csc[c + d*x])/d - (2*a*b*Csc[c + d*x]^3)/d + (6*a*b*Csc[c + d*x]^5)/
(5*d) - (2*a*b*Csc[c + d*x]^7)/(7*d)
```

3.285.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4374
```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

3.285.4 Maple [A] (verified)

Time = 3.22 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.22

method	result
derivativedivides	$\frac{a^2\left(-\frac{\cot(dx+c)^7}{7}+\frac{\cot(dx+c)^5}{5}-\frac{\cot(dx+c)^3}{3}+\cot(dx+c)+dx+c\right)+2ab\left(-\frac{\cos(dx+c)^8}{7\sin(dx+c)^7}+\frac{\cos(dx+c)^8}{35\sin(dx+c)^5}-\frac{\cos(dx+c)^8}{35\sin(dx+c)^3}+\frac{\cos}{7\si}$
default	$\frac{a^2\left(-\frac{\cot(dx+c)^7}{7}+\frac{\cot(dx+c)^5}{5}-\frac{\cot(dx+c)^3}{3}+\cot(dx+c)+dx+c\right)+2ab\left(-\frac{\cos(dx+c)^8}{7\sin(dx+c)^7}+\frac{\cos(dx+c)^8}{35\sin(dx+c)^5}-\frac{\cos(dx+c)^8}{35\sin(dx+c)^3}+\frac{\cos}{7\si}$
risch	$a^2x+\frac{2i(210ab\,e^{13i(dx+c)}+420a^2e^{12i(dx+c)}+105b^2e^{12i(dx+c)}-420ab\,e^{11i(dx+c)}-1260a^2e^{10i(dx+c)}+1806ab\,e^{9i(dx+c}$

input
```
int(cot(d*x+c)^8*(a+b*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^2*(-1/7*cot(d*x+c)^7+1/5*cot(d*x+c)^5-1/3*cot(d*x+c)^3+cot(d*x+c)+d
*x+c)+2*a*b*(-1/7/sin(d*x+c)^7*cos(d*x+c)^8+1/35/sin(d*x+c)^5*cos(d*x+c)^8
-1/35/sin(d*x+c)^3*cos(d*x+c)^8+1/7/sin(d*x+c)*cos(d*x+c)^8+1/7*(16/5+cos(
d*x+c)^6+6/5*cos(d*x+c)^4+8/5*cos(d*x+c)^2)*sin(d*x+c))-1/7*b^2/sin(d*x+c)
^7*cos(d*x+c)^7)
```

3.285.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.35

$$\int \cot^8(c+dx)(a+b\sec(c+dx))^2\,dx$$
$$= \frac{210\,ab\cos(dx+c)^6 + (176\,a^2+15\,b^2)\cos(dx+c)^7 - 406\,a^2\cos(dx+c)^5 - 420\,ab\cos(dx+c)^4 + 350\,a^2}{105\,(d\cos(}$$

input
```
integrate(cot(d*x+c)^8*(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

output
```
1/105*(210*a*b*cos(d*x + c)^6 + (176*a^2 + 15*b^2)*cos(d*x + c)^7 - 406*a^
2*cos(d*x + c)^5 - 420*a*b*cos(d*x + c)^4 + 350*a^2*cos(d*x + c)^3 + 336*a
*b*cos(d*x + c)^2 - 105*a^2*cos(d*x + c) - 96*a*b + 105*(a^2*d*x*cos(d*x +
 c)^6 - 3*a^2*d*x*cos(d*x + c)^4 + 3*a^2*d*x*cos(d*x + c)^2 - a^2*d*x)*sin
(d*x + c))/((d*cos(d*x + c)^6 - 3*d*cos(d*x + c)^4 + 3*d*cos(d*x + c)^2 -
d)*sin(d*x + c))
```

3.285.6 Sympy [F(-1)]

Timed out.

$$\int \cot^8(c+dx)(a+b\sec(c+dx))^2\,dx = \text{Timed out}$$

input
```
integrate(cot(d*x+c)**8*(a+b*sec(d*x+c))**2,x)
```

output
```
Timed out
```

3.285.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.76

$$\int \cot^8(c+dx)(a+b\sec(c+dx))^2\,dx$$

$$= \frac{\left(105\,dx + 105\,c + \frac{105\tan(dx+c)^6 - 35\tan(dx+c)^4 + 21\tan(dx+c)^2 - 15}{\tan(dx+c)^7}\right)a^2 + \frac{6\left(35\sin(dx+c)^6 - 35\sin(dx+c)^4 + 21\sin(dx+c)^2 - 5\right)a}{\sin(dx+c)^7}}{105\,d}$$

input
```
integrate(cot(d*x+c)^8*(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

output
```
1/105*((105*d*x + 105*c + (105*tan(d*x + c)^6 - 35*tan(d*x + c)^4 + 21*tan
(d*x + c)^2 - 15)/tan(d*x + c)^7)*a^2 + 6*(35*sin(d*x + c)^6 - 35*sin(d*x
+ c)^4 + 21*sin(d*x + c)^2 - 5)*a*b/sin(d*x + c)^7 - 15*b^2/tan(d*x + c)^7
)/d
```

3.285.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 366 vs. 2(141) = 282.

Time = 0.40 (sec) , antiderivative size = 366, normalized size of antiderivative = 2.39

$$\int \cot^8(c+dx)(a+b\sec(c+dx))^2\,dx$$

$$= \frac{15\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7 - 30\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7 + 15\,b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7 - 189\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5 + 294}{}$$

input
```
integrate(cot(d*x+c)^8*(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

output
```
1/13440*(15*a^2*tan(1/2*d*x + 1/2*c)^7 - 30*a*b*tan(1/2*d*x + 1/2*c)^7 + 1
5*b^2*tan(1/2*d*x + 1/2*c)^7 - 189*a^2*tan(1/2*d*x + 1/2*c)^5 + 294*a*b*ta
n(1/2*d*x + 1/2*c)^5 - 105*b^2*tan(1/2*d*x + 1/2*c)^5 + 1295*a^2*tan(1/2*d
*x + 1/2*c)^3 - 1470*a*b*tan(1/2*d*x + 1/2*c)^3 + 315*b^2*tan(1/2*d*x + 1/
2*c)^3 + 13440*(d*x + c)*a^2 - 9765*a^2*tan(1/2*d*x + 1/2*c) + 7350*a*b*ta
n(1/2*d*x + 1/2*c) - 525*b^2*tan(1/2*d*x + 1/2*c) + (9765*a^2*tan(1/2*d*x
+ 1/2*c)^6 + 7350*a*b*tan(1/2*d*x + 1/2*c)^6 + 525*b^2*tan(1/2*d*x + 1/2*c
)^6 - 1295*a^2*tan(1/2*d*x + 1/2*c)^4 - 1470*a*b*tan(1/2*d*x + 1/2*c)^4 -
315*b^2*tan(1/2*d*x + 1/2*c)^4 + 189*a^2*tan(1/2*d*x + 1/2*c)^2 + 294*a*b*
tan(1/2*d*x + 1/2*c)^2 + 105*b^2*tan(1/2*d*x + 1/2*c)^2 - 15*a^2 - 30*a*b
- 15*b^2)/tan(1/2*d*x + 1/2*c)^7)/d
```

3.285.9 Mupad [B] (verification not implemented)

Time = 13.75 (sec) , antiderivative size = 258, normalized size of antiderivative = 1.69

$$\int \cot^8(c+dx)(a+b\sec(c+dx))^2\,dx = a^2\,x + \frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^7(a-b)^2}{896\,d} + \frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3\left(\frac{3a^2}{32}-\frac{5ab}{48}+\frac{b^2}{48}+\frac{(a-b)^2}{384}\right)}{d} - \frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5\left(\frac{a^2}{80}-\frac{3ab}{160}+\frac{b^2}{160}+\frac{(a-b)^2}{640}\right)}{d} - \frac{\frac{2ab}{7}+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4\left(\frac{37a^2}{3}+14ab+3b^2\right)-\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6(93a^2+70ab+5b^2)+\frac{a^2}{7}+\frac{b^2}{7}-\tan\left(\frac{c}{2}+\right.}{128\,d\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^7} - \frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\left(\frac{23a^2}{32}-\frac{17ab}{32}+\frac{b^2}{32}+\frac{(a-b)^2}{128}\right)}{d}$$

input
```
int(cot(c + d*x)^8*(a + b/cos(c + d*x))^2,x)
```

output
```
a^2*x + (tan(c/2 + (d*x)/2)^7*(a - b)^2)/(896*d) + (tan(c/2 + (d*x)/2)^3*(
(3*a^2)/32 - (5*a*b)/48 + b^2/48 + (a - b)^2/384))/d - (tan(c/2 + (d*x)/2)
^5*(a^2/80 - (3*a*b)/160 + b^2/160 + (a - b)^2/640))/d - ((2*a*b)/7 + tan(
c/2 + (d*x)/2)^4*(14*a*b + (37*a^2)/3 + 3*b^2) - tan(c/2 + (d*x)/2)^6*(70*
a*b + 93*a^2 + 5*b^2) + a^2/7 + b^2/7 - tan(c/2 + (d*x)/2)^2*((14*a*b)/5 +
 (9*a^2)/5 + b^2))/(128*d*tan(c/2 + (d*x)/2)^7) - (tan(c/2 + (d*x)/2)*((23
*a^2)/32 - (17*a*b)/32 + b^2/32 + (a - b)^2/128))/d
```

3.286 $\int \frac{\tan^9(c+dx)}{a+b\sec(c+dx)}\,dx$

3.286.1 Optimal result 2068
3.286.2 Mathematica [A] (verified) 2069
3.286.3 Rubi [A] (verified) 2069
3.286.4 Maple [A] (verified) 2071
3.286.5 Fricas [A] (verification not implemented) 2071
3.286.6 Sympy [F] 2072
3.286.7 Maxima [A] (verification not implemented) 2072
3.286.8 Giac [B] (verification not implemented) 2073
3.286.9 Mupad [B] (verification not implemented) 2074

3.286.1 Optimal result

Integrand size = 21, antiderivative size = 250

$$\begin{aligned}\int \frac{\tan^9(c+dx)}{a+b\sec(c+dx)}\,dx = &-\frac{\log(\cos(c+dx))}{ad} - \frac{(a^2-b^2)^4\log(a+b\sec(c+dx))}{ab^8d}\\ &+\frac{(a^6-4a^4b^2+6a^2b^4-4b^6)\sec(c+dx)}{b^7d}\\ &-\frac{a(a^4-4a^2b^2+6b^4)\sec^2(c+dx)}{2b^6d}\\ &+\frac{(a^4-4a^2b^2+6b^4)\sec^3(c+dx)}{3b^5d} - \frac{a(a^2-4b^2)\sec^4(c+dx)}{4b^4d}\\ &+\frac{(a^2-4b^2)\sec^5(c+dx)}{5b^3d} - \frac{a\sec^6(c+dx)}{6b^2d} + \frac{\sec^7(c+dx)}{7bd}\end{aligned}$$

output

```
-ln(cos(d*x+c))/a/d-(a^2-b^2)^4*ln(a+b*sec(d*x+c))/a/b^8/d+(a^6-4*a^4*b^2+
6*a^2*b^4-4*b^6)*sec(d*x+c)/b^7/d-1/2*a*(a^4-4*a^2*b^2+6*b^4)*sec(d*x+c)^2
/b^6/d+1/3*(a^4-4*a^2*b^2+6*b^4)*sec(d*x+c)^3/b^5/d-1/4*a*(a^2-4*b^2)*sec(
d*x+c)^4/b^4/d+1/5*(a^2-4*b^2)*sec(d*x+c)^5/b^3/d-1/6*a*sec(d*x+c)^6/b^2/d
+1/7*sec(d*x+c)^7/b/d
```

3.286.2 Mathematica [A] (verified)

Time = 3.91 (sec) , antiderivative size = 228, normalized size of antiderivative = 0.91

$$\int \frac{\tan^9(c+dx)}{a+b\sec(c+dx)}\,dx$$
$$= \frac{-\frac{b^8\log(\cos(c+dx))}{a} - \frac{(a^2-b^2)^4\log(a+b\sec(c+dx))}{a} + b(a^6-4a^4b^2+6a^2b^4-4b^6)\sec(c+dx) - \frac{1}{2}ab^2(a^4-4a^2b^2+6$$

input
```
Integrate[Tan[c + d*x]^9/(a + b*Sec[c + d*x]),x]
```

output
```
(-((b^8*Log[Cos[c + d*x]])/a) - ((a^2 - b^2)^4*Log[a + b*Sec[c + d*x]])/a
+ b*(a^6 - 4*a^4*b^2 + 6*a^2*b^4 - 4*b^6)*Sec[c + d*x] - (a*b^2*(a^4 - 4*a
^2*b^2 + 6*b^4)*Sec[c + d*x]^2)/2 + (b^3*(a^4 - 4*a^2*b^2 + 6*b^4)*Sec[c +
 d*x]^3)/3 - (a*b^4*(a^2 - 4*b^2)*Sec[c + d*x]^4)/4 + (b^5*(a^2 - 4*b^2)*S
ec[c + d*x]^5)/5 - (a*b^6*Sec[c + d*x]^6)/6 + (b^7*Sec[c + d*x]^7)/7)/(b^8
*d)
```

3.286.3 Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 229, normalized size of antiderivative = 0.92, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3042, 25, 4373, 522, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^9(c+dx)}{a+b\sec(c+dx)}\,dx$$
$$\downarrow 3042$$
$$\int -\frac{\cot\left(c+dx+\frac{\pi}{2}\right)^9}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx$$
$$\downarrow 25$$
$$-\int \frac{\cot\left(\frac{1}{2}(2c+\pi)+dx\right)^9}{a+b\csc\left(\frac{1}{2}(2c+\pi)+dx\right)}dx$$
$$\downarrow 4373$$

$$\frac{\int \frac{\cos(c+dx)\left(b^2-b^2\sec^2(c+dx)\right)^4}{b(a+b\sec(c+dx))}d(b\sec(c+dx))}{b^8 d}$$

$$\downarrow 522$$

$$\int \left(\frac{\cos(c+dx)b^7}{a}+\sec^6(c+dx)b^6-a\sec^5(c+dx)b^5+\left(a^2-4b^2\right)\sec^4(c+dx)b^4-a\left(a^2-4b^2\right)\sec^3(c+dx)b^3+\left(a^4\right.\right.$$

$$\downarrow 2009$$

$$-\frac{\left(a^2-b^2\right)^4\log(a+b\sec(c+dx))}{a}+\frac{1}{5}b^5\left(a^2-4b^2\right)\sec^5(c+dx)-\frac{1}{4}ab^4\left(a^2-4b^2\right)\sec^4(c+dx)-\frac{1}{2}ab^2\left(a^4-4a^2b^2+6b^4\right)s$$

input
```
Int[Tan[c + d*x]^9/(a + b*Sec[c + d*x]),x]
```

output
```
((b^8*Log[b*Sec[c + d*x]])/a - ((a^2 - b^2)^4*Log[a + b*Sec[c + d*x]])/a +
 b*(a^6 - 4*a^4*b^2 + 6*a^2*b^4 - 4*b^6)*Sec[c + d*x] - (a*b^2*(a^4 - 4*a^
2*b^2 + 6*b^4)*Sec[c + d*x]^2)/2 + (b^3*(a^4 - 4*a^2*b^2 + 6*b^4)*Sec[c +
d*x]^3)/3 - (a*b^4*(a^2 - 4*b^2)*Sec[c + d*x]^4)/4 + (b^5*(a^2 - 4*b^2)*Se
c[c + d*x]^5)/5 - (a*b^6*Sec[c + d*x]^6)/6 + (b^7*Sec[c + d*x]^7)/7)/(b^8*
d)
```

3.286.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 522
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_.
), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(c + d*x)^n*(a + b*x^2)^p, x],
x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4373
```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Simp[-(-1)^((m - 1)/2)/(d*b^(m - 1))   Subst[Int[(b^2 - x^
2)^((m - 1)/2)*((a + x)^n/x), x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
 d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

3.286.4 Maple [A] (verified)

Time = 2.18 (sec) , antiderivative size = 273, normalized size of antiderivative = 1.09

method	result
derivativedivides	$\frac{\left(-a^8+4a^6b^2-6a^4b^4+4a^2b^6-b^8\right)\ln(b+a\cos(dx+c))}{b^8a}-\frac{a}{6b^2\cos(dx+c)^6}-\frac{-a^2+4b^2}{5b^3\cos(dx+c)^5}-\frac{-a^4+4a^2b^2-6b^4}{3b^5\cos(dx+c)^3}-\frac{-a^6+4a^4b^2-6a^2b^4+}{b^7\cos(dx+c)}$ / d
default	$\frac{\left(-a^8+4a^6b^2-6a^4b^4+4a^2b^6-b^8\right)\ln(b+a\cos(dx+c))}{b^8a}-\frac{a}{6b^2\cos(dx+c)^6}-\frac{-a^2+4b^2}{5b^3\cos(dx+c)^5}-\frac{-a^4+4a^2b^2-6b^4}{3b^5\cos(dx+c)^3}-\frac{-a^6+4a^4b^2-6a^2b^4+}{b^7\cos(dx+c)}$ / d
risch	Expression too large to display

input
```
int(tan(d*x+c)^9/(a+b*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*((-a^8+4*a^6*b^2-6*a^4*b^4+4*a^2*b^6-b^8)/b^8/a*ln(b+a*cos(d*x+c))-1/6
/b^2*a/cos(d*x+c)^6-1/5*(-a^2+4*b^2)/b^3/cos(d*x+c)^5-1/3*(-a^4+4*a^2*b^2-
6*b^4)/b^5/cos(d*x+c)^3-(-a^6+4*a^4*b^2-6*a^2*b^4+4*b^6)/b^7/cos(d*x+c)-1/
4*(a^2-4*b^2)/b^4*a/cos(d*x+c)^4-1/2*(a^4-4*a^2*b^2+6*b^4)/b^6*a/cos(d*x+c
)^2+(a^6-4*a^4*b^2+6*a^2*b^4-4*b^6)/b^8*a*ln(cos(d*x+c))+1/7/b/cos(d*x+c)^
7)
```

3.286.5 Fricas [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 293, normalized size of antiderivative = 1.17

$$\int \frac{\tan^9(c+dx)}{a+b\sec(c+dx)}\,dx =$$
$$-\frac{70\,a^2b^6\cos(dx+c)+420\left(a^8-4\,a^6b^2+6\,a^4b^4-4\,a^2b^6+b^8\right)\cos(dx+c)^7\log(a\cos(dx+c)+b)-42}{}$$

input
```
integrate(tan(d*x+c)^9/(a+b*sec(d*x+c)),x, algorithm="fricas")
```

output
```
-1/420*(70*a^2*b^6*cos(d*x + c) + 420*(a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2
*b^6 + b^8)*cos(d*x + c)^7*log(a*cos(d*x + c) + b) - 420*(a^8 - 4*a^6*b^2
+ 6*a^4*b^4 - 4*a^2*b^6)*cos(d*x + c)^7*log(-cos(d*x + c)) - 60*a*b^7 - 42
0*(a^7*b - 4*a^5*b^3 + 6*a^3*b^5 - 4*a*b^7)*cos(d*x + c)^6 + 210*(a^6*b^2
- 4*a^4*b^4 + 6*a^2*b^6)*cos(d*x + c)^5 - 140*(a^5*b^3 - 4*a^3*b^5 + 6*a*b
^7)*cos(d*x + c)^4 + 105*(a^4*b^4 - 4*a^2*b^6)*cos(d*x + c)^3 - 84*(a^3*b^
5 - 4*a*b^7)*cos(d*x + c)^2)/(a*b^8*d*cos(d*x + c)^7)
```

3.286.6 Sympy [F]

$$\int \frac{\tan^9(c+dx)}{a+b\sec(c+dx)}\,dx = \int \frac{\tan^9(c+dx)}{a+b\sec(c+dx)}\,dx$$

input
```
integrate(tan(d*x+c)**9/(a+b*sec(d*x+c)),x)
```

output
```
Integral(tan(c + d*x)**9/(a + b*sec(c + d*x)), x)
```

3.286.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 268, normalized size of antiderivative = 1.07

$$\int \frac{\tan^9(c+dx)}{a+b\sec(c+dx)}\,dx$$
$$= \frac{\frac{420\,(a^7-4\,a^5b^2+6\,a^3b^4-4\,ab^6)\log(\cos(dx+c))}{b^8} - \frac{420\,(a^8-4\,a^6b^2+6\,a^4b^4-4\,a^2b^6+b^8)\log(a\cos(dx+c)+b)}{ab^8} - \frac{70\,ab^5\cos(dx+c)-420\,(a^6-4\,a^4}{}}{}$$

input
```
integrate(tan(d*x+c)^9/(a+b*sec(d*x+c)),x, algorithm="maxima")
```

output
```
1/420*(420*(a^7 - 4*a^5*b^2 + 6*a^3*b^4 - 4*a*b^6)*log(cos(d*x + c))/b^8 -
 420*(a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8)*log(a*cos(d*x + c) +
b)/(a*b^8) - (70*a*b^5*cos(d*x + c) - 420*(a^6 - 4*a^4*b^2 + 6*a^2*b^4 - 4
*b^6)*cos(d*x + c)^6 - 60*b^6 + 210*(a^5*b - 4*a^3*b^3 + 6*a*b^5)*cos(d*x
+ c)^5 - 140*(a^4*b^2 - 4*a^2*b^4 + 6*b^6)*cos(d*x + c)^4 + 105*(a^3*b^3 -
 4*a*b^5)*cos(d*x + c)^3 - 84*(a^2*b^4 - 4*b^6)*cos(d*x + c)^2)/(b^7*cos(d
*x + c)^7))/d
```

3.286.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1768 vs. 2(238) = 476.

Time = 5.76 (sec) , antiderivative size = 1768, normalized size of antiderivative = 7.07

$$\int \frac{\tan^9(c+dx)}{a+b\sec(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate(tan(d*x+c)^9/(a+b*sec(d*x+c)),x, algorithm="giac")
```

output
```
-1/420*(210*(a^7 - 4*a^5*b^2 + 6*a^3*b^4 - 4*a*b^6)*log(abs(a + b - 2*b*(c
os(d*x + c) - 1)/(cos(d*x + c) + 1) - a*(cos(d*x + c) - 1)^2/(cos(d*x + c)
 + 1)^2 + b*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2))/b^8 - 420*(a^7 - 4
*a^5*b^2 + 6*a^3*b^4 - 4*a*b^6)*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c)
+ 1) - 1))/b^8 - 210*(a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + 2*b^8)*log
(abs(2*b + 2*a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 2*b*(cos(d*x + c) -
 1)/(cos(d*x + c) + 1) - 2*abs(a))/abs(2*b + 2*a*(cos(d*x + c) - 1)/(cos(d
*x + c) + 1) - 2*b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 2*abs(a)))/(b^8
*abs(a)) + (1089*a^7 - 840*a^6*b - 4356*a^5*b^2 + 3080*a^4*b^3 + 6534*a^3*
b^4 - 4088*a^2*b^5 - 4356*a*b^6 + 2232*b^7 + 7623*a^7*(cos(d*x + c) - 1)/(
cos(d*x + c) + 1) - 5040*a^6*b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 313
32*a^5*b^2*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 19040*a^4*b^3*(cos(d*x
+ c) - 1)/(cos(d*x + c) + 1) + 48258*a^3*b^4*(cos(d*x + c) - 1)/(cos(d*x +
 c) + 1) - 26096*a^2*b^5*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 33012*a*b
^6*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 14784*b^7*(cos(d*x + c) - 1)/(c
os(d*x + c) + 1) + 22869*a^7*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 1
2600*a^6*b*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 95676*a^5*b^2*(cos(
d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 47880*a^4*b^3*(cos(d*x + c) - 1)^2/
(cos(d*x + c) + 1)^2 + 151494*a^3*b^4*(cos(d*x + c) - 1)^2/(cos(d*x + c) +
 1)^2 - 67368*a^2*b^5*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 10743...
```

3.286.9 Mupad [B] (verification not implemented)

Time = 15.15 (sec) , antiderivative size = 631, normalized size of antiderivative = 2.52

$$\int \frac{\tan^9(c+dx)}{a+b\sec(c+dx)}\,dx = \frac{\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+1\right)}{a\,d}$$
$$-\frac{2\left(105a^6-385a^4b^2+511a^2b^4-279b^6\right)}{105b^7}+\frac{2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{12}\left(a^6+a^5b-3a^4b^2-3a^3b^3+3a^2b^4+3ab^5-b^6\right)}{b^7}-\frac{2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}\left(6a^6+5a^5b$$

$$-\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2-1\right)\left(-a^7+4a^5b^2-6a^3b^4+4ab^6\right)}{b^8\,d}$$
$$-\frac{\ln\left(a+b-a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2\right)\left(a^2-b^2\right)^4}{a\,b^8\,d}$$

input
```
int(tan(c + d*x)^9/(a + b/cos(c + d*x)),x)
```

output
```
log(tan(c/2 + (d*x)/2)^2 + 1)/(a*d) - ((2*(105*a^6 - 279*b^6 + 511*a^2*b^4
 - 385*a^4*b^2))/(105*b^7) + (2*tan(c/2 + (d*x)/2)^12*(3*a*b^5 + a^5*b + a
^6 - b^6 + 3*a^2*b^4 - 3*a^3*b^3 - 3*a^4*b^2))/b^7 - (2*tan(c/2 + (d*x)/2)
^10*(19*a*b^5 + 5*a^5*b + 6*a^6 - 8*b^6 + 22*a^2*b^4 - 17*a^3*b^3 - 20*a^4
*b^2))/b^7 - (4*tan(c/2 + (d*x)/2)^6*(71*a*b^5 + 15*a^5*b + 30*a^6 - 96*b^
6 + 154*a^2*b^4 - 54*a^3*b^3 - 112*a^4*b^2))/(3*b^7) + (2*tan(c/2 + (d*x)/
2)^8*(142*a*b^5 + 30*a^5*b + 45*a^6 - 87*b^6 + 203*a^2*b^4 - 108*a^3*b^3 -
 161*a^4*b^2))/(3*b^7) + (2*tan(c/2 + (d*x)/2)^4*(95*a*b^5 + 25*a^5*b + 75
*a^6 - 239*b^6 + 401*a^2*b^4 - 85*a^3*b^3 - 285*a^4*b^2))/(5*b^7) - (2*tan
(c/2 + (d*x)/2)^2*(45*a*b^5 + 15*a^5*b + 90*a^6 - 264*b^6 + 466*a^2*b^4 -
45*a^3*b^3 - 340*a^4*b^2))/(15*b^7))/(d*(7*tan(c/2 + (d*x)/2)^2 - 21*tan(c
/2 + (d*x)/2)^4 + 35*tan(c/2 + (d*x)/2)^6 - 35*tan(c/2 + (d*x)/2)^8 + 21*t
an(c/2 + (d*x)/2)^10 - 7*tan(c/2 + (d*x)/2)^12 + tan(c/2 + (d*x)/2)^14 - 1
)) - (log(tan(c/2 + (d*x)/2)^2 - 1)*(4*a*b^6 - a^7 - 6*a^3*b^4 + 4*a^5*b^2
))/(b^8*d) - (log(a + b - a*tan(c/2 + (d*x)/2)^2 + b*tan(c/2 + (d*x)/2)^2)
*(a^2 - b^2)^4)/(a*b^8*d)
```

3.287 $\int \frac{\tan^7(c+dx)}{a+b\sec(c+dx)}\,dx$

3.287.1 Optimal result . . . 2075
3.287.2 Mathematica [A] (verified) . . . 2075
3.287.3 Rubi [A] (verified) . . . 2076
3.287.4 Maple [A] (verified) . . . 2077
3.287.5 Fricas [A] (verification not implemented) . . . 2078
3.287.6 Sympy [F] . . . 2078
3.287.7 Maxima [A] (verification not implemented) . . . 2079
3.287.8 Giac [B] (verification not implemented) . . . 2079
3.287.9 Mupad [B] (verification not implemented) . . . 2080

3.287.1 Optimal result

Integrand size = 21, antiderivative size = 170

$$\int \frac{\tan^7(c+dx)}{a+b\sec(c+dx)}\,dx = \frac{\log(\cos(c+dx))}{ad} - \frac{\left(a^2-b^2\right)^3\log(a+b\sec(c+dx))}{ab^6d} + \frac{\left(a^4-3a^2b^2+3b^4\right)\sec(c+dx)}{b^5d} - \frac{a\left(a^2-3b^2\right)\sec^2(c+dx)}{2b^4d} + \frac{\left(a^2-3b^2\right)\sec^3(c+dx)}{3b^3d} - \frac{a\sec^4(c+dx)}{4b^2d} + \frac{\sec^5(c+dx)}{5bd}$$

output
```
ln(cos(d*x+c))/a/d-(a^2-b^2)^3*ln(a+b*sec(d*x+c))/a/b^6/d+(a^4-3*a^2*b^2+3
*b^4)*sec(d*x+c)/b^5/d-1/2*a*(a^2-3*b^2)*sec(d*x+c)^2/b^4/d+1/3*(a^2-3*b^2
)*sec(d*x+c)^3/b^3/d-1/4*a*sec(d*x+c)^4/b^2/d+1/5*sec(d*x+c)^5/b/d
```

3.287.2 Mathematica [A] (verified)

Time = 6.23 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.92

$$\int \frac{\tan^7(c+dx)}{a+b\sec(c+dx)}\,dx = -\frac{-\frac{b^6\log(\cos(c+dx))}{a} + \frac{\left(a^2-b^2\right)^3\log(a+b\sec(c+dx))}{a} - b\left(a^4-3a^2b^2+3b^4\right)\sec(c+dx) + \frac{1}{2}ab^2\left(a^2-3b^2\right)\sec^2(c+d}{b^6d}$$

input
```
Integrate[Tan[c + d*x]^7/(a + b*Sec[c + d*x]),x]
```

output
```
-((-((b^6*Log[Cos[c + d*x]])/a) + ((a^2 - b^2)^3*Log[a + b*Sec[c + d*x]])/
a - b*(a^4 - 3*a^2*b^2 + 3*b^4)*Sec[c + d*x] + (a*b^2*(a^2 - 3*b^2)*Sec[c
+ d*x]^2)/2 - (b^3*(a^2 - 3*b^2)*Sec[c + d*x]^3)/3 + (a*b^4*Sec[c + d*x]^4
)/4 - (b^5*Sec[c + d*x]^5)/5)/(b^6*d))
```

3.287.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 157, normalized size of antiderivative = 0.92, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3042, 25, 4373, 522, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^7(c+dx)}{a+b\sec(c+dx)}\,dx$$

$$\downarrow 3042$$

$$\int -\frac{\cot\left(c+dx+\frac{\pi}{2}\right)^7}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\downarrow 25$$

$$-\int \frac{\cot\left(\frac{1}{2}(2c+\pi)+dx\right)^7}{a+b\csc\left(\frac{1}{2}(2c+\pi)+dx\right)}dx$$

$$\downarrow 4373$$

$$-\frac{\int \frac{\cos(c+dx)\left(b^2-b^2\sec^2(c+dx)\right)^3}{b(a+b\sec(c+dx))}d(b\sec(c+dx))}{b^6 d}$$

$$\downarrow 522$$

$$-\frac{\int \left(\frac{\cos(c+dx)b^5}{a} - \sec^4(c+dx)b^4 + a\sec^3(c+dx)b^3 - \left(a^2-3b^2\right)\sec^2(c+dx)b^2 + a\left(a^2-3b^2\right)\sec(c+dx)b - a^4\right.}{b^6 d}$$

$$\downarrow 2009$$

$$-\frac{\frac{1}{2}ab^2\left(a^2-3b^2\right)\sec^2(c+dx) + \frac{\left(a^2-b^2\right)^3\log(a+b\sec(c+dx))}{a} - \frac{1}{3}b^3\left(a^2-3b^2\right)\sec^3(c+dx) - b\left(a^4-3a^2b^2+3b^4\right)\sec(c}{b^6 d}$$

input
```
Int[Tan[c + d*x]^7/(a + b*Sec[c + d*x]),x]
```

output
```
-(((b^6*Log[b*Sec[c + d*x]])/a + ((a^2 - b^2)^3*Log[a + b*Sec[c + d*x]])/a
 - b*(a^4 - 3*a^2*b^2 + 3*b^4)*Sec[c + d*x] + (a*b^2*(a^2 - 3*b^2)*Sec[c +
 d*x]^2)/2 - (b^3*(a^2 - 3*b^2)*Sec[c + d*x]^3)/3 + (a*b^4*Sec[c + d*x]^4)
/4 - (b^5*Sec[c + d*x]^5)/5)/(b^6*d))
```

3.287.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 522
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_.
), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(c + d*x)^n*(a + b*x^2)^p, x],
x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4373
```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Simp[-(-1)^((m - 1)/2)/(d*b^(m - 1))   Subst[Int[(b^2 - x^
2)^((m - 1)/2)*((a + x)^n/x), x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
 d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

3.287.4 Maple [A] (verified)

Time = 1.68 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.08

method	result
derivativedivides	$\frac{-\frac{a}{4b^2\cos(dx+c)^4}-\frac{-a^2+3b^2}{3b^3\cos(dx+c)^3}-\frac{-a^4+3a^2b^2-3b^4}{b^5\cos(dx+c)}-\frac{\left(a^2-3b^2\right)a}{2b^4\cos(dx+c)^2}+\frac{\left(a^4-3a^2b^2+3b^4\right)a\ln(\cos(dx+c))}{b^6}+\frac{1}{5b\cos(dx+c)^5}+\frac{(-}{d}$
default	$\frac{-\frac{a}{4b^2\cos(dx+c)^4}-\frac{-a^2+3b^2}{3b^3\cos(dx+c)^3}-\frac{-a^4+3a^2b^2-3b^4}{b^5\cos(dx+c)}-\frac{\left(a^2-3b^2\right)a}{2b^4\cos(dx+c)^2}+\frac{\left(a^4-3a^2b^2+3b^4\right)a\ln(\cos(dx+c))}{b^6}+\frac{1}{5b\cos(dx+c)^5}+\frac{(-}{d}$
risch	$-\frac{ix}{a}-\frac{2ic}{ad}+\frac{2a^4e^{9i(dx+c)}-6a^2b^2e^{9i(dx+c)}+6b^4e^{9i(dx+c)}-2a^3b\,e^{8i(dx+c)}+6a\,b^3e^{8i(dx+c)}+8a^4e^{7i(dx+c)}-\frac{64a^2b^2e^{7i}}{3}}{}$

input
```
int(tan(d*x+c)^7/(a+b*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(-1/4/b^2*a/cos(d*x+c)^4-1/3*(-a^2+3*b^2)/b^3/cos(d*x+c)^3-(-a^4+3*a^2
*b^2-3*b^4)/b^5/cos(d*x+c)-1/2*(a^2-3*b^2)/b^4*a/cos(d*x+c)^2+(a^4-3*a^2*b
^2+3*b^4)/b^6*a*ln(cos(d*x+c))+1/5/b/cos(d*x+c)^5+(-a^6+3*a^4*b^2-3*a^2*b^
4+b^6)/b^6/a*ln(b+a*cos(d*x+c)))
```

3.287.5 Fricas [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.21

$$\int \frac{\tan^7(c+dx)}{a+b\sec(c+dx)}\,dx =$$
$$-\frac{15\,a^2b^4\cos(dx+c)+60\,(a^6-3\,a^4b^2+3\,a^2b^4-b^6)\cos(dx+c)^5\log(a\cos(dx+c)+b)-60\,(a^6-3\,a^4$$

input
```
integrate(tan(d*x+c)^7/(a+b*sec(d*x+c)),x, algorithm="fricas")
```

output
```
-1/60*(15*a^2*b^4*cos(d*x + c) + 60*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*co
s(d*x + c)^5*log(a*cos(d*x + c) + b) - 60*(a^6 - 3*a^4*b^2 + 3*a^2*b^4)*co
s(d*x + c)^5*log(-cos(d*x + c)) - 12*a*b^5 - 60*(a^5*b - 3*a^3*b^3 + 3*a*b
^5)*cos(d*x + c)^4 + 30*(a^4*b^2 - 3*a^2*b^4)*cos(d*x + c)^3 - 20*(a^3*b^3
 - 3*a*b^5)*cos(d*x + c)^2)/(a*b^6*d*cos(d*x + c)^5)
```

3.287.6 Sympy [F]

$$\int \frac{\tan^7(c+dx)}{a+b\sec(c+dx)}\,dx = \int \frac{\tan^7(c+dx)}{a+b\sec(c+dx)}\,dx$$

input
```
integrate(tan(d*x+c)**7/(a+b*sec(d*x+c)),x)
```

output
```
Integral(tan(c + d*x)**7/(a + b*sec(c + d*x)), x)
```

3.287.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.08

$$\int \frac{\tan^7(c+dx)}{a+b\sec(c+dx)}\,dx$$
$$= \frac{\frac{60\,(a^5-3\,a^3b^2+3\,ab^4)\log(\cos(dx+c))}{b^6} - \frac{60\,(a^6-3\,a^4b^2+3\,a^2b^4-b^6)\log(a\cos(dx+c)+b)}{ab^6} - \frac{15\,ab^3\cos(dx+c)-60\,(a^4-3\,a^2b^2+3\,b^4)\cos(dx+c)}{}}{60\,d}$$

input
```
integrate(tan(d*x+c)^7/(a+b*sec(d*x+c)),x, algorithm="maxima")
```

output
```
1/60*(60*(a^5 - 3*a^3*b^2 + 3*a*b^4)*log(cos(d*x + c))/b^6 - 60*(a^6 - 3*a
^4*b^2 + 3*a^2*b^4 - b^6)*log(a*cos(d*x + c) + b)/(a*b^6) - (15*a*b^3*cos(
d*x + c) - 60*(a^4 - 3*a^2*b^2 + 3*b^4)*cos(d*x + c)^4 - 12*b^4 + 30*(a^3*
b - 3*a*b^3)*cos(d*x + c)^3 - 20*(a^2*b^2 - 3*b^4)*cos(d*x + c)^2)/(b^5*co
s(d*x + c)^5))/d
```

3.287.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1052 vs. 2(162) = 324.

Time = 3.41 (sec) , antiderivative size = 1052, normalized size of antiderivative = 6.19

$$\int \frac{\tan^7(c+dx)}{a+b\sec(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate(tan(d*x+c)^7/(a+b*sec(d*x+c)),x, algorithm="giac")
```

output

```
-1/60*(30*(a^5 - 3*a^3*b^2 + 3*a*b^4)*log(abs(a + b - 2*b*(cos(d*x + c) -
1)/(cos(d*x + c) + 1) - a*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + b*(c
os(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2))/b^6 - 60*(a^5 - 3*a^3*b^2 + 3*a*
b^4)*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1))/b^6 - 30*(a^6 -
3*a^4*b^2 + 3*a^2*b^4 - 2*b^6)*log(abs(2*b + 2*a*(cos(d*x + c) - 1)/(cos(d
*x + c) + 1) - 2*b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 2*abs(a))/abs(2
*b + 2*a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 2*b*(cos(d*x + c) - 1)/(c
os(d*x + c) + 1) + 2*abs(a)))/(b^6*abs(a)) + (137*a^5 - 120*a^4*b - 411*a^
3*b^2 + 320*a^2*b^3 + 411*a*b^4 - 264*b^5 + 685*a^5*(cos(d*x + c) - 1)/(co
s(d*x + c) + 1) - 480*a^4*b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 2175*a
^3*b^2*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1360*a^2*b^3*(cos(d*x + c)
- 1)/(cos(d*x + c) + 1) + 2295*a*b^4*(cos(d*x + c) - 1)/(cos(d*x + c) + 1)
 - 1200*b^5*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1370*a^5*(cos(d*x + c)
 - 1)^2/(cos(d*x + c) + 1)^2 - 720*a^4*b*(cos(d*x + c) - 1)^2/(cos(d*x + c
) + 1)^2 - 4470*a^3*b^2*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 2000*a
^2*b^3*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 5070*a*b^4*(cos(d*x + c
) - 1)^2/(cos(d*x + c) + 1)^2 - 1920*b^5*(cos(d*x + c) - 1)^2/(cos(d*x + c
) + 1)^2 + 1370*a^5*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 - 480*a^4*b*
(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 - 4470*a^3*b^2*(cos(d*x + c) - 1
)^3/(cos(d*x + c) + 1)^3 + 1200*a^2*b^3*(cos(d*x + c) - 1)^3/(cos(d*x +...
```

3.287.9 Mupad [B] (verification not implemented)

Time = 15.60 (sec) , antiderivative size = 395, normalized size of antiderivative = 2.32

$$\int \frac{\tan^7(c+dx)}{a+b\sec(c+dx)}\,dx = \frac{a\,\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2-1\right)\,\left(a^4-3\,a^2\,b^2+3\,b^4\right)}{b^6\,d}$$
$$-\frac{\frac{2\,\left(15\,a^4-40\,a^2\,b^2+33\,b^4\right)}{15\,b^5}+\frac{2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^8\,\left(a^4+a^3\,b-2\,a^2\,b^2-2\,a\,b^3+b^4\right)}{b^5}-\frac{2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6\,\left(4\,a^4+3\,a^3\,b-10\,a^2\,b^2-8\,a\,b^3+6\,b^4\right)}{b^5}-\frac{2\tan}{}}{d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}-5\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^8+10\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6-10\right.}$$
$$-\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+1\right)}{a\,d}-\frac{\ln\left(a+b-a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2\right)\,\left(a^2-b^2\right)^3}{a\,b^6\,d}$$

input

```
int(tan(c + d*x)^7/(a + b/cos(c + d*x)),x)
```

output

```
(a*log(tan(c/2 + (d*x)/2)^2 - 1)*(a^4 + 3*b^4 - 3*a^2*b^2))/(b^6*d) - ((2*
(15*a^4 + 33*b^4 - 40*a^2*b^2))/(15*b^5) + (2*tan(c/2 + (d*x)/2)^8*(a^3*b
- 2*a*b^3 + a^4 + b^4 - 2*a^2*b^2))/b^5 - (2*tan(c/2 + (d*x)/2)^6*(3*a^3*b
 - 8*a*b^3 + 4*a^4 + 6*b^4 - 10*a^2*b^2))/b^5 - (2*tan(c/2 + (d*x)/2)^2*(3
*a^3*b - 6*a*b^3 + 12*a^4 + 30*b^4 - 34*a^2*b^2))/(3*b^5) + (2*tan(c/2 + (
d*x)/2)^4*(9*a^3*b - 24*a*b^3 + 18*a^4 + 48*b^4 - 50*a^2*b^2))/(3*b^5))/(d
*(5*tan(c/2 + (d*x)/2)^2 - 10*tan(c/2 + (d*x)/2)^4 + 10*tan(c/2 + (d*x)/2)
^6 - 5*tan(c/2 + (d*x)/2)^8 + tan(c/2 + (d*x)/2)^10 - 1)) - log(tan(c/2 +
(d*x)/2)^2 + 1)/(a*d) - (log(a + b - a*tan(c/2 + (d*x)/2)^2 + b*tan(c/2 +
(d*x)/2)^2)*(a^2 - b^2)^3)/(a*b^6*d)
```

3.288 $\int \frac{\tan^5(c+dx)}{a+b\sec(c+dx)}\,dx$

3.288.1 Optimal result . . . 2082
3.288.2 Mathematica [A] (verified) . . . 2082
3.288.3 Rubi [A] (verified) . . . 2083
3.288.4 Maple [A] (verified) . . . 2084
3.288.5 Fricas [A] (verification not implemented) . . . 2085
3.288.6 Sympy [F] . . . 2085
3.288.7 Maxima [A] (verification not implemented) . . . 2085
3.288.8 Giac [B] (verification not implemented) . . . 2086
3.288.9 Mupad [B] (verification not implemented) . . . 2087

3.288.1 Optimal result

Integrand size = 21, antiderivative size = 108

$$\int \frac{\tan^5(c+dx)}{a+b\sec(c+dx)}\,dx = -\frac{\log(\cos(c+dx))}{ad} - \frac{\left(a^2-b^2\right)^2\log(a+b\sec(c+dx))}{ab^4d} + \frac{\left(a^2-2b^2\right)\sec(c+dx)}{b^3d} - \frac{a\sec^2(c+dx)}{2b^2d} + \frac{\sec^3(c+dx)}{3bd}$$

output
```
-ln(cos(d*x+c))/a/d-(a^2-b^2)^2*ln(a+b*sec(d*x+c))/a/b^4/d+(a^2-2*b^2)*sec
(d*x+c)/b^3/d-1/2*a*sec(d*x+c)^2/b^2/d+1/3*sec(d*x+c)^3/b/d
```

3.288.2 Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.91

$$\int \frac{\tan^5(c+dx)}{a+b\sec(c+dx)}\,dx = \frac{-\frac{b^4\log(\cos(c+dx))}{a} - \frac{\left(a^2-b^2\right)^2\log(a+b\sec(c+dx))}{a} + b\left(a^2-2b^2\right)\sec(c+dx) - \frac{1}{2}ab^2\sec^2(c+dx) + \frac{1}{3}b^3\sec^3(c+dx)}{b^4d}$$

input
```
Integrate[Tan[c + d*x]^5/(a + b*Sec[c + d*x]),x]
```

output
```
(-((b^4*Log[Cos[c + d*x]])/a) - ((a^2 - b^2)^2*Log[a + b*Sec[c + d*x]])/a
+ b*(a^2 - 2*b^2)*Sec[c + d*x] - (a*b^2*Sec[c + d*x]^2)/2 + (b^3*Sec[c + d
*x]^3)/3)/(b^4*d)
```

3.288.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.92, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3042, 25, 4373, 522, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^5(c+dx)}{a+b\sec(c+dx)}\,dx$$

$$\downarrow 3042$$

$$\int -\frac{\cot\left(c+dx+\frac{\pi}{2}\right)^5}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\downarrow 25$$

$$-\int \frac{\cot\left(\frac{1}{2}(2c+\pi)+dx\right)^5}{a+b\csc\left(\frac{1}{2}(2c+\pi)+dx\right)}dx$$

$$\downarrow 4373$$

$$\frac{\int \frac{\cos(c+dx)\left(b^2-b^2\sec^2(c+dx)\right)^2}{b(a+b\sec(c+dx))}d(b\sec(c+dx))}{b^4 d}$$

$$\downarrow 522$$

$$\frac{\int \left(\frac{\cos(c+dx)b^3}{a}+\sec^2(c+dx)b^2-a\sec(c+dx)b+a^2\left(1-\frac{2b^2}{a^2}\right)-\frac{\left(a^2-b^2\right)^2}{a(a+b\sec(c+dx))}\right)d(b\sec(c+dx))}{b^4 d}$$

$$\downarrow 2009$$

$$\frac{b\left(a^2-2b^2\right)\sec(c+dx)-\frac{\left(a^2-b^2\right)^2\log(a+b\sec(c+dx))}{a}+\frac{b^4\log(b\sec(c+dx))}{a}-\frac{1}{2}ab^2\sec^2(c+dx)+\frac{1}{3}b^3\sec^3(c+dx)}{b^4 d}$$

input
```
Int[Tan[c + d*x]^5/(a + b*Sec[c + d*x]),x]
```

output
```
((b^4*Log[b*Sec[c + d*x]])/a - ((a^2 - b^2)^2*Log[a + b*Sec[c + d*x]])/a +
 b*(a^2 - 2*b^2)*Sec[c + d*x] - (a*b^2*Sec[c + d*x]^2)/2 + (b^3*Sec[c + d*
x]^3)/3)/(b^4*d)
```

3.288.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 522
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_.
), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(c + d*x)^n*(a + b*x^2)^p, x],
x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4373
```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Simp[-(-1)^((m - 1)/2)/(d*b^(m - 1))   Subst[Int[(b^2 - x^
2)^((m - 1)/2)*((a + x)^n/x), x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
 d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

3.288.4 Maple [A] (verified)

Time = 1.24 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.06

method	result
derivativedivides	$\frac{\left(-a^4+2a^2b^2-b^4\right)\ln(b+a\cos(dx+c))}{b^4a}-\frac{a}{2b^2\cos(dx+c)^2}-\frac{-a^2+2b^2}{b^3\cos(dx+c)}+\frac{\left(a^2-2b^2\right)a\ln(\cos(dx+c))}{b^4}+\frac{1}{3b\cos(dx+c)^3}$
default	$\frac{\left(-a^4+2a^2b^2-b^4\right)\ln(b+a\cos(dx+c))}{b^4a}-\frac{a}{2b^2\cos(dx+c)^2}-\frac{-a^2+2b^2}{b^3\cos(dx+c)}+\frac{\left(a^2-2b^2\right)a\ln(\cos(dx+c))}{b^4}+\frac{1}{3b\cos(dx+c)^3}$
risch	$\frac{ix}{a}+\frac{2ic}{ad}+\frac{2a^2e^{5i(dx+c)}-4b^2e^{5i(dx+c)}-2abe^{4i(dx+c)}+4a^2e^{3i(dx+c)}-\frac{16b^2e^{3i(dx+c)}}{3}-2abe^{2i(dx+c)}+2a^2e^{i(dx+c)}-4}{db^3\left(e^{2i(dx+c)}+1\right)^3}$

input
```
int(tan(d*x+c)^5/(a+b*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*((-a^4+2*a^2*b^2-b^4)/b^4/a*ln(b+a*cos(d*x+c))-1/2/b^2*a/cos(d*x+c)^2-
(-a^2+2*b^2)/b^3/cos(d*x+c)+(a^2-2*b^2)/b^4*a*ln(cos(d*x+c))+1/3/b/cos(d*x
+c)^3)
```

3.288.5 Fricas [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.19

$$\int \frac{\tan^5(c+dx)}{a+b\sec(c+dx)}\,dx =$$
$$-\frac{3\,a^2b^2\cos(dx+c)+6\left(a^4-2\,a^2b^2+b^4\right)\cos(dx+c)^3\log\left(a\cos(dx+c)+b\right)-6\left(a^4-2\,a^2b^2\right)\cos(dx+$$
$$6\,ab^4d\cos(dx+c)^3$$

input
```
integrate(tan(d*x+c)^5/(a+b*sec(d*x+c)),x, algorithm="fricas")
```

output
```
-1/6*(3*a^2*b^2*cos(d*x + c) + 6*(a^4 - 2*a^2*b^2 + b^4)*cos(d*x + c)^3*lo
g(a*cos(d*x + c) + b) - 6*(a^4 - 2*a^2*b^2)*cos(d*x + c)^3*log(-cos(d*x +
c)) - 2*a*b^3 - 6*(a^3*b - 2*a*b^3)*cos(d*x + c)^2)/(a*b^4*d*cos(d*x + c)^
3)
```

3.288.6 Sympy [F]

$$\int \frac{\tan^5(c+dx)}{a+b\sec(c+dx)}\,dx = \int \frac{\tan^5(c+dx)}{a+b\sec(c+dx)}\,dx$$

input
```
integrate(tan(d*x+c)**5/(a+b*sec(d*x+c)),x)
```

output
```
Integral(tan(c + d*x)**5/(a + b*sec(c + d*x)), x)
```

3.288.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.02

$$\int \frac{\tan^5(c+dx)}{a+b\sec(c+dx)}\,dx$$
$$= \frac{\frac{6\left(a^3-2\,ab^2\right)\log(\cos(dx+c))}{b^4} - \frac{6\left(a^4-2\,a^2b^2+b^4\right)\log(a\cos(dx+c)+b)}{ab^4} - \frac{3\,ab\cos(dx+c)-6\left(a^2-2\,b^2\right)\cos(dx+c)^2-2\,b^2}{b^3\cos(dx+c)^3}}{6\,d}$$

input
```
integrate(tan(d*x+c)^5/(a+b*sec(d*x+c)),x, algorithm="maxima")
```

output
```
1/6*(6*(a^3 - 2*a*b^2)*log(cos(d*x + c))/b^4 - 6*(a^4 - 2*a^2*b^2 + b^4)*l
og(a*cos(d*x + c) + b)/(a*b^4) - (3*a*b*cos(d*x + c) - 6*(a^2 - 2*b^2)*cos
(d*x + c)^2 - 2*b^2)/(b^3*cos(d*x + c)^3))/d
```

3.288.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 560 vs. $2(104) = 208$.

Time $= 1.53$ (sec) , antiderivative size $= 560$, normalized size of antiderivative $= 5.19$

$$\int \frac{\tan^5(c+dx)}{a+b\sec(c+dx)}\,dx =$$

$$\frac{3\left(a^3-2ab^2\right)\log\left(\left|a+b-\frac{2b(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{a(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}+\frac{b(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}\right|\right)}{b^4} - \frac{6\left(a^3-2ab^2\right)\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right)}{b^4} - \frac{3\left(a^4-2a^2b^2+\right.}{}$$

$$-$$

input
```
integrate(tan(d*x+c)^5/(a+b*sec(d*x+c)),x, algorithm="giac")
```

output
```
-1/6*(3*(a^3 - 2*a*b^2)*log(abs(a + b - 2*b*(cos(d*x + c) - 1)/(cos(d*x +
c) + 1) - a*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + b*(cos(d*x + c) -
1)^2/(cos(d*x + c) + 1)^2))/b^4 - 6*(a^3 - 2*a*b^2)*log(abs(-(cos(d*x + c)
 - 1)/(cos(d*x + c) + 1) - 1))/b^4 - 3*(a^4 - 2*a^2*b^2 + 2*b^4)*log(abs(2
*b + 2*a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 2*b*(cos(d*x + c) - 1)/(c
os(d*x + c) + 1) - 2*abs(a))/abs(2*b + 2*a*(cos(d*x + c) - 1)/(cos(d*x + c
) + 1) - 2*b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 2*abs(a)))/(b^4*abs(a
)) + (11*a^3 - 12*a^2*b - 22*a*b^2 + 20*b^3 + 33*a^3*(cos(d*x + c) - 1)/(c
os(d*x + c) + 1) - 24*a^2*b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 78*a*b
^2*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 48*b^3*(cos(d*x + c) - 1)/(cos(
d*x + c) + 1) + 33*a^3*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 12*a^2*
b*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 78*a*b^2*(cos(d*x + c) - 1)^
2/(cos(d*x + c) + 1)^2 + 12*b^3*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2
+ 11*a^3*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 - 22*a*b^2*(cos(d*x + c
) - 1)^3/(cos(d*x + c) + 1)^3)/(b^4*((cos(d*x + c) - 1)/(cos(d*x + c) + 1)
 + 1)^3))/d
```

3.288.9 Mupad [B] (verification not implemented)

Time = 15.08 (sec) , antiderivative size = 227, normalized size of antiderivative = 2.10

$$\int \frac{\tan^5(c+dx)}{a+b\sec(c+dx)}\,dx = \frac{\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+1\right)}{a\,d} - \frac{\frac{2\,(3\,a^2-5\,b^2)}{3\,b^3} - \frac{2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2\,(2\,a^2+a\,b-4\,b^2)}{b^3} + \frac{2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4\,(a^2+a\,b-b^2)}{b^3}}{d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6 - 3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4 + 3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2 - 1\right)} + \frac{a\,\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2-1\right)\,(a^2-2\,b^2)}{b^4\,d} - \frac{\ln\left(a+b-a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2\right)\,(a^2-b^2)^2}{a\,b^4\,d}$$

input
```
int(tan(c + d*x)^5/(a + b/cos(c + d*x)),x)
```

output
```
log(tan(c/2 + (d*x)/2)^2 + 1)/(a*d) - ((2*(3*a^2 - 5*b^2))/(3*b^3) - (2*ta
n(c/2 + (d*x)/2)^2*(a*b + 2*a^2 - 4*b^2))/b^3 + (2*tan(c/2 + (d*x)/2)^4*(a
*b + a^2 - b^2))/b^3)/(d*(3*tan(c/2 + (d*x)/2)^2 - 3*tan(c/2 + (d*x)/2)^4
+ tan(c/2 + (d*x)/2)^6 - 1)) + (a*log(tan(c/2 + (d*x)/2)^2 - 1)*(a^2 - 2*b
^2))/(b^4*d) - (log(a + b - a*tan(c/2 + (d*x)/2)^2 + b*tan(c/2 + (d*x)/2)^
2)*(a^2 - b^2)^2)/(a*b^4*d)
```

3.289 $\int \frac{\tan^3(c+dx)}{a+b\sec(c+dx)}\,dx$

3.289.1 Optimal result 2088
3.289.2 Mathematica [A] (verified) 2088
3.289.3 Rubi [A] (verified) 2089
3.289.4 Maple [A] (verified) 2090
3.289.5 Fricas [A] (verification not implemented) 2091
3.289.6 Sympy [F] 2091
3.289.7 Maxima [A] (verification not implemented) 2091
3.289.8 Giac [B] (verification not implemented) 2092
3.289.9 Mupad [B] (verification not implemented) 2092

3.289.1 Optimal result

Integrand size = 21, antiderivative size = 59

$$\int \frac{\tan^3(c+dx)}{a+b\sec(c+dx)}\,dx = \frac{\log(\cos(c+dx))}{ad} - \frac{(a^2-b^2)\log(a+b\sec(c+dx))}{ab^2d} + \frac{\sec(c+dx)}{bd}$$

output
```
ln(cos(d*x+c))/a/d-(a^2-b^2)*ln(a+b*sec(d*x+c))/a/b^2/d+sec(d*x+c)/b/d
```

3.289.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.88

$$\int \frac{\tan^3(c+dx)}{a+b\sec(c+dx)}\,dx = \frac{b^2\log(\cos(c+dx)) + (-a^2+b^2)\log(a+b\sec(c+dx)) + ab\sec(c+dx)}{ab^2d}$$

input
```
Integrate[Tan[c + d*x]^3/(a + b*Sec[c + d*x]),x]
```

output
```
(b^2*Log[Cos[c + d*x]] + (-a^2 + b^2)*Log[a + b*Sec[c + d*x]] + a*b*Sec[c
+ d*x])/(a*b^2*d)
```

3.289.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.98, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3042, 25, 4373, 522, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^3(c+dx)}{a+b\sec(c+dx)}\,dx$$

$$\downarrow 3042$$

$$\int -\frac{\cot\left(c+dx+\frac{\pi}{2}\right)^3}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\downarrow 25$$

$$-\int \frac{\cot\left(\frac{1}{2}(2c+\pi)+dx\right)^3}{a+b\csc\left(\frac{1}{2}(2c+\pi)+dx\right)}dx$$

$$\downarrow 4373$$

$$-\frac{\int \frac{\cos(c+dx)\left(b^2-b^2\sec^2(c+dx)\right)}{b(a+b\sec(c+dx))}d(b\sec(c+dx))}{b^2d}$$

$$\downarrow 522$$

$$-\frac{\int \left(\frac{a^2-b^2}{a(a+b\sec(c+dx))}+\frac{b\cos(c+dx)}{a}-1\right)d(b\sec(c+dx))}{b^2d}$$

$$\downarrow 2009$$

$$-\frac{\frac{\left(a^2-b^2\right)\log(a+b\sec(c+dx))}{a}+\frac{b^2\log(b\sec(c+dx))}{a}-b\sec(c+dx)}{b^2d}$$

input
```
Int[Tan[c + d*x]^3/(a + b*Sec[c + d*x]),x]
```

output
```
-(((b^2*Log[b*Sec[c + d*x]])/a + ((a^2 - b^2)*Log[a + b*Sec[c + d*x]])/a -
 b*Sec[c + d*x])/(b^2*d))
```

3.289.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 522
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_.
), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(c + d*x)^n*(a + b*x^2)^p, x],
x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4373
```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Simp[-(-1)^((m - 1)/2)/(d*b^(m - 1))   Subst[Int[(b^2 - x^
2)^((m - 1)/2)*((a + x)^n/x), x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
 d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

3.289.4 Maple [A] (verified)

Time = 0.72 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.97

method	result
derivativedivides	$\frac{\frac{a\ln(\cos(dx+c))}{b^2}+\frac{1}{b\cos(dx+c)}+\frac{\left(-a^2+b^2\right)\ln(b+a\cos(dx+c))}{b^2a}}{d}$
default	$\frac{\frac{a\ln(\cos(dx+c))}{b^2}+\frac{1}{b\cos(dx+c)}+\frac{\left(-a^2+b^2\right)\ln(b+a\cos(dx+c))}{b^2a}}{d}$
risch	$-\frac{ix}{a}-\frac{2ic}{ad}+\frac{2e^{i(dx+c)}}{db\left(e^{2i(dx+c)}+1\right)}-\frac{a\ln\left(e^{2i(dx+c)}+\frac{2be^{i(dx+c)}}{a}+1\right)}{b^2d}+\frac{\ln\left(e^{2i(dx+c)}+\frac{2be^{i(dx+c)}}{a}+1\right)}{ad}+\frac{a\ln\left(e^{2i($

input
```
int(tan(d*x+c)^3/(a+b*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/b^2*a*ln(cos(d*x+c))+1/b/cos(d*x+c)+(-a^2+b^2)/b^2/a*ln(b+a*cos(d*x
+c)))
```

3.289.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.17

$$\int \frac{\tan^3(c+dx)}{a+b\sec(c+dx)}\,dx = \frac{a^2\cos(dx+c)\log(-\cos(dx+c)) - (a^2-b^2)\cos(dx+c)\log(a\cos(dx+c)+b) + ab}{ab^2d\cos(dx+c)}$$

input
```
integrate(tan(d*x+c)^3/(a+b*sec(d*x+c)),x, algorithm="fricas")
```

output
```
(a^2*cos(d*x + c)*log(-cos(d*x + c)) - (a^2 - b^2)*cos(d*x + c)*log(a*cos(
d*x + c) + b) + a*b)/(a*b^2*d*cos(d*x + c))
```

3.289.6 Sympy [F]

$$\int \frac{\tan^3(c+dx)}{a+b\sec(c+dx)}\,dx = \int \frac{\tan^3(c+dx)}{a+b\sec(c+dx)}\,dx$$

input
```
integrate(tan(d*x+c)**3/(a+b*sec(d*x+c)),x)
```

output
```
Integral(tan(c + d*x)**3/(a + b*sec(c + d*x)), x)
```

3.289.7 Maxima [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.97

$$\int \frac{\tan^3(c+dx)}{a+b\sec(c+dx)}\,dx = \frac{\frac{a\log(\cos(dx+c))}{b^2} - \frac{(a^2-b^2)\log(a\cos(dx+c)+b)}{ab^2} + \frac{1}{b\cos(dx+c)}}{d}$$

input
```
integrate(tan(d*x+c)^3/(a+b*sec(d*x+c)),x, algorithm="maxima")
```

output
```
(a*log(cos(d*x + c))/b^2 - (a^2 - b^2)*log(a*cos(d*x + c) + b)/(a*b^2) + 1
/(b*cos(d*x + c)))/d
```

3.289.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 289 vs. $2(59) = 118$.

Time = 0.61 (sec) , antiderivative size = 289, normalized size of antiderivative = 4.90

$$\int \frac{\tan^3(c+dx)}{a+b\sec(c+dx)}\,dx = \frac{a\log\left(\left|a+b-\frac{2\,b(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{a(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}+\frac{b(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}\right|\right)}{b^2} - \frac{2\,a\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right)}{b^2} - \frac{\left(a^2-2\,b^2\right)\log\left(\left|2\,b+\frac{2\,a(\cos(dx+c)-1)}{\cos(dx+c)+1}\right.\right.}{b^2|a|}$$

$$\left.\left.\left|2\,b+\frac{2\,a(\cos(dx+c)-1)}{\cos(dx+c)+1}\right.\right.$$

$$\Bigg)\Bigg)$$

$$\Bigg)\Bigg)$$

$$\Bigg)\Bigg)

output
```
(a*log(tan(c/2 + (d*x)/2)^2 - 1))/(b^2*d) - 2/(b*d*(tan(c/2 + (d*x)/2)^2 -
 1)) - log(tan(c/2 + (d*x)/2)^2 + 1)/(a*d) - (log(a + b - a*tan(c/2 + (d*x
)/2)^2 + b*tan(c/2 + (d*x)/2)^2)*(a/b^2 - 1/a))/d
```

3.290 $\int \frac{\tan(c+dx)}{a+b\sec(c+dx)}\,dx$

3.290.1 Optimal result 2094
3.290.2 Mathematica [A] (verified) 2094
3.290.3 Rubi [A] (verified) 2095
3.290.4 Maple [A] (verified) 2096
3.290.5 Fricas [A] (verification not implemented) 2097
3.290.6 Sympy [B] (verification not implemented) 2097
3.290.7 Maxima [A] (verification not implemented) 2098
3.290.8 Giac [B] (verification not implemented) 2098
3.290.9 Mupad [B] (verification not implemented) 2098

3.290.1 Optimal result

Integrand size = 19, antiderivative size = 35

$$\int \frac{\tan(c+dx)}{a+b\sec(c+dx)}\,dx = -\frac{\log(\cos(c+dx))}{ad} - \frac{\log(a+b\sec(c+dx))}{ad}$$

output
```
-ln(cos(d*x+c))/a/d-ln(a+b*sec(d*x+c))/a/d
```

3.290.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.54

$$\int \frac{\tan(c+dx)}{a+b\sec(c+dx)}\,dx = -\frac{\log(b+a\cos(c+dx))}{ad}$$

input
```
Integrate[Tan[c + d*x]/(a + b*Sec[c + d*x]),x]
```

output
```
-(Log[b + a*Cos[c + d*x]]/(a*d))
```

3.290.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.97, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {3042, 25, 4373, 47, 14, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan(c+dx)}{a+b\sec(c+dx)}\,dx$$

$$\downarrow 3042$$

$$\int -\frac{\cot\left(c+dx+\frac{\pi}{2}\right)}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\downarrow 25$$

$$-\int \frac{\cot\left(\frac{1}{2}(2c+\pi)+dx\right)}{a+b\csc\left(\frac{1}{2}(2c+\pi)+dx\right)}dx$$

$$\downarrow 4373$$

$$\frac{\int \frac{\cos(c+dx)}{b(a+b\sec(c+dx))}d(b\sec(c+dx))}{d}$$

$$\downarrow 47$$

$$\frac{\frac{\int \frac{\cos(c+dx)}{b}d(b\sec(c+dx))}{a} - \frac{\int \frac{1}{a+b\sec(c+dx)}d(b\sec(c+dx))}{a}}{d}$$

$$\downarrow 14$$

$$\frac{\frac{\log(b\sec(c+dx))}{a} - \frac{\int \frac{1}{a+b\sec(c+dx)}d(b\sec(c+dx))}{a}}{d}$$

$$\downarrow 16$$

$$\frac{\frac{\log(b\sec(c+dx))}{a} - \frac{\log(a+b\sec(c+dx))}{a}}{d}$$

input
```
Int[Tan[c + d*x]/(a + b*Sec[c + d*x]),x]
```

output
```
(Log[b*Sec[c + d*x]]/a - Log[a + b*Sec[c + d*x]]/a)/d
```

3.290.3.1 Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4373
```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Simp[-(-1)^((m - 1)/2)/(d*b^(m - 1))   Subst[Int[(b^2 - x^
2)^((m - 1)/2)*((a + x)^n/x), x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
 d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

3.290.4 Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.94

method	result	size
derivativedivides	$\frac{-\frac{\ln(a+b\sec(dx+c))}{a}+\frac{\ln(\sec(dx+c))}{a}}{d}$	33
default	$\frac{-\frac{\ln(a+b\sec(dx+c))}{a}+\frac{\ln(\sec(dx+c))}{a}}{d}$	33
risch	$\frac{ix}{a}+\frac{2ic}{ad}-\frac{\ln\left(e^{2i(dx+c)}+\frac{2b e^{i(dx+c)}}{a}+1\right)}{ad}$	54

input
```
int(tan(d*x+c)/(a+b*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(-1/a*ln(a+b*sec(d*x+c))+1/a*ln(sec(d*x+c)))
```

3.290.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.54

$$\int \frac{\tan(c+dx)}{a+b\sec(c+dx)}\,dx = -\frac{\log\left(a\cos\left(dx+c\right)+b\right)}{ad}$$

input
```
integrate(tan(d*x+c)/(a+b*sec(d*x+c)),x, algorithm="fricas")
```

output
```
-log(a*cos(d*x + c) + b)/(a*d)
```

3.290.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 82 vs. 2(27) = 54.

Time = 2.92 (sec) , antiderivative size = 82, normalized size of antiderivative = 2.34

$$\int \frac{\tan(c+dx)}{a+b\sec(c+dx)}\,dx = \begin{cases} \frac{\tilde{\infty} x \tan\left(c\right)}{\sec\left(c\right)} & \text{for } a = 0 \wedge b = 0 \wedge d = 0 \\ -\frac{1}{bd\sec\left(c+dx\right)} & \text{for } a = 0 \\ \frac{\log\left(\tan^2\left(c+dx\right)+1\right)}{2ad} & \text{for } b = 0 \\ \frac{x\tan\left(c\right)}{a+b\sec\left(c\right)} & \text{for } d = 0 \\ -\frac{\log\left(\frac{a}{b}+\sec\left(c+dx\right)\right)}{ad} + \frac{\log\left(\tan^2\left(c+dx\right)+1\right)}{2ad} & \text{otherwise} \end{cases}$$

input
```
integrate(tan(d*x+c)/(a+b*sec(d*x+c)),x)
```

output
```
Piecewise((zoo*x*tan(c)/sec(c), Eq(a, 0) & Eq(b, 0) & Eq(d, 0)), (-1/(b*d*
sec(c + d*x)), Eq(a, 0)), (log(tan(c + d*x)**2 + 1)/(2*a*d), Eq(b, 0)), (x
*tan(c)/(a + b*sec(c)), Eq(d, 0)), (-log(a/b + sec(c + d*x))/(a*d) + log(t
an(c + d*x)**2 + 1)/(2*a*d), True))
```

3.290.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.54

$$\int \frac{\tan(c+dx)}{a+b\sec(c+dx)}\,dx = -\frac{\log\left(a\cos\left(dx+c\right)+b\right)}{ad}$$

input
```
integrate(tan(d*x+c)/(a+b*sec(d*x+c)),x, algorithm="maxima")
```

output
```
-log(a*cos(d*x + c) + b)/(a*d)
```

3.290.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 114 vs. 2(35) = 70.

Time = 0.35 (sec) , antiderivative size = 114, normalized size of antiderivative = 3.26

$$\int \frac{\tan(c+dx)}{a+b\sec(c+dx)}\,dx = \frac{\log\left(\left|\frac{2\,b+\frac{2\,a(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{2\,b(\cos(dx+c)-1)}{\cos(dx+c)+1}-2\,|a|}{2\,b+\frac{2\,a(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{2\,b(\cos(dx+c)-1)}{\cos(dx+c)+1}+2\,|a|}\right|\right)}{d|a|}$$

input
```
integrate(tan(d*x+c)/(a+b*sec(d*x+c)),x, algorithm="giac")
```

output
```
log(abs(2*b + 2*a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 2*b*(cos(d*x + c
) - 1)/(cos(d*x + c) + 1) - 2*abs(a))/abs(2*b + 2*a*(cos(d*x + c) - 1)/(co
s(d*x + c) + 1) - 2*b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 2*abs(a)))/(
d*abs(a))
```

3.290.9 Mupad [B] (verification not implemented)

Time = 14.70 (sec) , antiderivative size = 71, normalized size of antiderivative = 2.03

$$\int \frac{\tan(c+dx)}{a+b\sec(c+dx)}\,dx = \frac{\operatorname{atan}\left(\frac{a\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{a\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,1\mathrm{i}+b\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,1\mathrm{i}+b\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,1\mathrm{i}}\right)\,2\mathrm{i}}{a\,d}$$

input
```
int(tan(c + d*x)/(a + b/cos(c + d*x)),x)
```

output
```
(atan((a*sin(c/2 + (d*x)/2)^2)/(a*cos(c/2 + (d*x)/2)^2*1i + b*cos(c/2 + (d
*x)/2)^2*1i + b*sin(c/2 + (d*x)/2)^2*1i))*2i)/(a*d)
```

3.291 $\int \frac{\cot(c+dx)}{a+b\sec(c+dx)}\,dx$

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3.291.9 Mupad [B] (verification not implemented) 2103

3.291.1 Optimal result

Integrand size = 19, antiderivative size = 94

$$\int \frac{\cot(c+dx)}{a+b\sec(c+dx)}\,dx = \frac{\log(\cos(c+dx))}{ad} + \frac{\log(1-\sec(c+dx))}{2(a+b)d} + \frac{\log(1+\sec(c+dx))}{2(a-b)d} - \frac{b^2\log(a+b\sec(c+dx))}{a\left(a^2-b^2\right)d}$$

output
```
ln(cos(d*x+c))/a/d+1/2*ln(1-sec(d*x+c))/(a+b)/d+1/2*ln(1+sec(d*x+c))/(a-b)
/d-b^2*ln(a+b*sec(d*x+c))/a/(a^2-b^2)/d
```

3.291.2 Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.87

$$\int \frac{\cot(c+dx)}{a+b\sec(c+dx)}\,dx = \frac{\frac{2\log(\cos(c+dx))}{a} + \frac{\log(1-\sec(c+dx))}{a+b} + \frac{\log(1+\sec(c+dx))}{a-b} - \frac{2b^2\log(a+b\sec(c+dx))}{a^3-ab^2}}{2d}$$

input
```
Integrate[Cot[c + d*x]/(a + b*Sec[c + d*x]),x]
```

output
```
((2*Log[Cos[c + d*x]])/a + Log[1 - Sec[c + d*x]]/(a + b) + Log[1 + Sec[c +
 d*x]]/(a - b) - (2*b^2*Log[a + b*Sec[c + d*x]])/(a^3 - a*b^2))/(2*d)
```

3.291.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.06, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {3042, 25, 4373, 615, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot(c+dx)}{a+b\sec(c+dx)}\,dx$$

$$\downarrow 3042$$

$$\int -\frac{1}{\cot\left(c+dx+\frac{\pi}{2}\right)\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)}dx$$

$$\downarrow 25$$

$$-\int \frac{1}{\cot\left(\frac{1}{2}(2c+\pi)+dx\right)\left(a+b\csc\left(\frac{1}{2}(2c+\pi)+dx\right)\right)}dx$$

$$\downarrow 4373$$

$$-\frac{b^2\int \frac{\cos(c+dx)}{b(a+b\sec(c+dx))(b^2-b^2\sec^2(c+dx))}d(b\sec(c+dx))}{d}$$

$$\downarrow 615$$

$$-\frac{b^2\int \left(\frac{\cos(c+dx)}{ab^3}+\frac{1}{2b^2(a+b)(b-b\sec(c+dx))}+\frac{1}{a(a-b)(a+b)(a+b\sec(c+dx))}-\frac{1}{2(a-b)b^2(\sec(c+dx)b+b)}\right)d(b\sec(c+dx))}{d}$$

$$\downarrow 2009$$

$$-\frac{b^2\left(\frac{\log(a+b\sec(c+dx))}{a(a^2-b^2)}+\frac{\log(b\sec(c+dx))}{ab^2}-\frac{\log(b-b\sec(c+dx))}{2b^2(a+b)}-\frac{\log(b\sec(c+dx)+b)}{2b^2(a-b)}\right)}{d}$$

input
```
Int[Cot[c + d*x]/(a + b*Sec[c + d*x]),x]
```

output
```
-((b^2*(Log[b*Sec[c + d*x]]/(a*b^2) - Log[b - b*Sec[c + d*x]]/(2*b^2*(a +
b)) + Log[a + b*Sec[c + d*x]]/(a*(a^2 - b^2)) - Log[b + b*Sec[c + d*x]]/(2
*(a - b)*b^2)))/d)
```

3.291.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 615
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_),
 x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(c + d*x)^n*(a + b*x^2)^p, x], x]
 /; FreeQ[{a, b, c, d, e, m, n}, x] && ILtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4373
```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Simp[-(-1)^((m - 1)/2)/(d*b^(m - 1))   Subst[Int[(b^2 - x^
2)^((m - 1)/2)*((a + x)^n/x), x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
 d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

3.291.4 Maple [A] (verified)

Time = 0.62 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.80

method	result
derivativedivides	$\frac{-\frac{b^2\ln(b+a\cos(dx+c))}{(a+b)(a-b)a}+\frac{\ln(\cos(dx+c)-1)}{2a+2b}+\frac{\ln(\cos(dx+c)+1)}{2a-2b}}{d}$
default	$\frac{-\frac{b^2\ln(b+a\cos(dx+c))}{(a+b)(a-b)a}+\frac{\ln(\cos(dx+c)-1)}{2a+2b}+\frac{\ln(\cos(dx+c)+1)}{2a-2b}}{d}$
risch	$\frac{ix}{a}-\frac{ix}{a-b}-\frac{ic}{d(a-b)}-\frac{ix}{a+b}-\frac{ic}{d(a+b)}+\frac{2ib^2x}{a(a^2-b^2)}+\frac{2ib^2c}{da(a^2-b^2)}+\frac{\ln(e^{i(dx+c)}+1)}{d(a-b)}+\frac{\ln(e^{i(dx+c)}-1)}{d(a+b)}-\frac{b}{}$

input
```
int(cot(d*x+c)/(a+b*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(-b^2/(a+b)/(a-b)/a*ln(b+a*cos(d*x+c))+1/(2*a+2*b)*ln(cos(d*x+c)-1)+1/
(2*a-2*b)*ln(cos(d*x+c)+1))
```

3.291.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.80

$$\int \frac{\cot(c+dx)}{a+b\sec(c+dx)}\,dx = -\frac{2\,b^2\log\left(a\cos\left(dx+c\right)+b\right)-\left(a^2+ab\right)\log\left(\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)-\left(a^2-ab\right)\log\left(-\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)}{2\left(a^3-ab^2\right)d}$$

input
```
integrate(cot(d*x+c)/(a+b*sec(d*x+c)),x, algorithm="fricas")
```

output
```
-1/2*(2*b^2*log(a*cos(d*x + c) + b) - (a^2 + a*b)*log(1/2*cos(d*x + c) + 1
/2) - (a^2 - a*b)*log(-1/2*cos(d*x + c) + 1/2))/((a^3 - a*b^2)*d)
```

3.291.6 Sympy [F]

$$\int \frac{\cot(c+dx)}{a+b\sec(c+dx)}\,dx = \int \frac{\cot\left(c+dx\right)}{a+b\sec\left(c+dx\right)}\,dx$$

input
```
integrate(cot(d*x+c)/(a+b*sec(d*x+c)),x)
```

output
```
Integral(cot(c + d*x)/(a + b*sec(c + d*x)), x)
```

3.291.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.72

$$\int \frac{\cot(c+dx)}{a+b\sec(c+dx)}\,dx = -\frac{\frac{2\,b^2\log(a\cos(dx+c)+b)}{a^3-ab^2}-\frac{\log(\cos(dx+c)+1)}{a-b}-\frac{\log(\cos(dx+c)-1)}{a+b}}{2\,d}$$

input
```
integrate(cot(d*x+c)/(a+b*sec(d*x+c)),x, algorithm="maxima")
```

output
```
-1/2*(2*b^2*log(a*cos(d*x + c) + b)/(a^3 - a*b^2) - log(cos(d*x + c) + 1)/
(a - b) - log(cos(d*x + c) - 1)/(a + b))/d
```

3.291.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 257 vs. $2(90) = 180$.

Time $= 0.31$ (sec) , antiderivative size $= 257$, normalized size of antiderivative $= 2.73$

$$\int \frac{\cot(c+dx)}{a+b\sec(c+dx)}\,dx = -\frac{\frac{a\log\left(\left|-a-b+\frac{2\,b(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{a(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}-\frac{b(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}\right|\right)}{a^2-b^2} - \frac{(a^2-2\,b^2)\log\left(\left|\frac{-2\,b-\frac{2\,a(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{2\,b(\cos(dx+c)-1)}{\cos(dx+c)+1}-2\,|a|}{-2\,b-\frac{2\,a(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{2\,b(\cos(dx+c)-1)}{\cos(dx+c)+1}+2\,|a|}\right|\right)}{(a^2-b^2)|a|}}{2\,d}$$

input
```
integrate(cot(d*x+c)/(a+b*sec(d*x+c)),x, algorithm="giac")
```

output
```
-1/2*(a*log(abs(-a - b + 2*b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + a*(co
s(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - b*(cos(d*x + c) - 1)^2/(cos(d*x +
 c) + 1)^2))/(a^2 - b^2) - (a^2 - 2*b^2)*log(abs(-2*b - 2*a*(cos(d*x + c)
- 1)/(cos(d*x + c) + 1) + 2*b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 2*ab
s(a))/abs(-2*b - 2*a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 2*b*(cos(d*x
+ c) - 1)/(cos(d*x + c) + 1) + 2*abs(a)))/((a^2 - b^2)*abs(a)) - log(abs(-
cos(d*x + c) + 1)/abs(cos(d*x + c) + 1))/(a + b))/d
```

3.291.9 Mupad [B] (verification not implemented)

Time $= 14.75$ (sec) , antiderivative size $= 93$, normalized size of antiderivative $= 0.99$

$$\int \frac{\cot(c+dx)}{a+b\sec(c+dx)}\,dx = \frac{\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{d\,(a+b)} - \frac{\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+1\right)}{a\,d} + \frac{b^2\ln\left(a+b-a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2\right)}{d\,(a\,b^2-a^3)}$$

input
```
int(cot(c + d*x)/(a + b/cos(c + d*x)),x)
```

output
```
log(tan(c/2 + (d*x)/2))/(d*(a + b)) - log(tan(c/2 + (d*x)/2)^2 + 1)/(a*d)
+ (b^2*log(a + b - a*tan(c/2 + (d*x)/2)^2 + b*tan(c/2 + (d*x)/2)^2))/(d*(a
*b^2 - a^3))
```

3.292 $\int \frac{\cot^3(c+dx)}{a+b\sec(c+dx)}\,dx$

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3.292.2 Mathematica [A] (verified) . . . 2104
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3.292.8 Giac [B] (verification not implemented) . . . 2108
3.292.9 Mupad [B] (verification not implemented) . . . 2109

3.292.1 Optimal result

Integrand size = 21, antiderivative size = 157

$$\int \frac{\cot^3(c+dx)}{a+b\sec(c+dx)}\,dx = -\frac{\log(\cos(c+dx))}{ad} - \frac{(2a+3b)\log(1-\sec(c+dx))}{4(a+b)^2 d} - \frac{(2a-3b)\log(1+\sec(c+dx))}{4(a-b)^2 d} - \frac{b^4\log(a+b\sec(c+dx))}{a\left(a^2-b^2\right)^2 d} + \frac{1}{4(a+b)d(1-\sec(c+dx))} + \frac{1}{4(a-b)d(1+\sec(c+dx))}$$

output
```
-ln(cos(d*x+c))/a/d-1/4*(2*a+3*b)*ln(1-sec(d*x+c))/(a+b)^2/d-1/4*(2*a-3*b)
*ln(1+sec(d*x+c))/(a-b)^2/d-b^4*ln(a+b*sec(d*x+c))/a/(a^2-b^2)^2/d+1/4/(a+
b)/d/(1-sec(d*x+c))+1/4/(a-b)/d/(1+sec(d*x+c))
```

3.292.2 Mathematica [A] (verified)

Time = 0.98 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.96

$$\int \frac{\cot^3(c+dx)}{a+b\sec(c+dx)}\,dx = \frac{b^4\left(-\frac{4\log(\cos(c+dx))}{ab^4} - \frac{(2a+3b)\log(1-\sec(c+dx))}{b^4(a+b)^2} - \frac{(2a-3b)\log(1+\sec(c+dx))}{(a-b)^2b^4} - \frac{4\log(a+b\sec(c+dx))}{a(a-b)^2(a+b)^2} - \frac{1}{b^4(a+b)(-1+\sec(c+dx))}\right.}{4d}$$

input
```
Integrate[Cot[c + d*x]^3/(a + b*Sec[c + d*x]),x]
```

output
```
(b^4*((-4*Log[Cos[c + d*x]])/(a*b^4) - ((2*a + 3*b)*Log[1 - Sec[c + d*x]])
/(b^4*(a + b)^2) - ((2*a - 3*b)*Log[1 + Sec[c + d*x]])/((a - b)^2*b^4) - (
4*Log[a + b*Sec[c + d*x]])/(a*(a - b)^2*(a + b)^2) - 1/(b^4*(a + b)*(-1 +
Sec[c + d*x])) + 1/((a - b)*b^4*(1 + Sec[c + d*x])))/(4*d)
```

3.292.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.05, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3042, 25, 4373, 615, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^3(c+dx)}{a+b\sec(c+dx)}\,dx$$

$$\downarrow 3042$$

$$\int -\frac{1}{\cot\left(c+dx+\frac{\pi}{2}\right)^3\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)}\,dx$$

$$\downarrow 25$$

$$-\int \frac{1}{\cot\left(\frac{1}{2}(2c+\pi)+dx\right)^3\left(a+b\csc\left(\frac{1}{2}(2c+\pi)+dx\right)\right)}\,dx$$

$$\downarrow 4373$$

$$\frac{b^4\int \frac{\cos(c+dx)}{b(a+b\sec(c+dx))(b^2-b^2\sec^2(c+dx))^2}\,d(b\sec(c+dx))}{d}$$

$$\downarrow 615$$

$$\frac{b^4\int\left(\frac{3b-2a}{4(a-b)^2b^4(\sec(c+dx)b+b)}+\frac{\cos(c+dx)}{ab^5}+\frac{2a+3b}{4b^4(a+b)^2(b-b\sec(c+dx))}-\frac{1}{a(a-b)^2(a+b)^2(a+b\sec(c+dx))}+\frac{1}{4b^3(a+b)(b-b\sec(c+dx))}\right.}{d}$$

$$\downarrow 2009$$

$$\frac{b^4\left(-\frac{\log(a+b\sec(c+dx))}{a(a^2-b^2)^2}+\frac{\log(b\sec(c+dx))}{ab^4}-\frac{(2a+3b)\log(b-b\sec(c+dx))}{4b^4(a+b)^2}-\frac{(2a-3b)\log(b\sec(c+dx)+b)}{4b^4(a-b)^2}+\frac{1}{4b^3(a+b)(b-b\sec(c+dx))}+\right.}{d}$$

input
```
Int[Cot[c + d*x]^3/(a + b*Sec[c + d*x]),x]
```

output
```
(b^4*(Log[b*Sec[c + d*x]]/(a*b^4) - ((2*a + 3*b)*Log[b - b*Sec[c + d*x]])/
(4*b^4*(a + b)^2) - Log[a + b*Sec[c + d*x]]/(a*(a^2 - b^2)^2) - ((2*a - 3*
b)*Log[b + b*Sec[c + d*x]])/(4*(a - b)^2*b^4) + 1/(4*b^3*(a + b)*(b - b*Se
c[c + d*x])) + 1/(4*(a - b)*b^3*(b + b*Sec[c + d*x])))/d
```

3.292.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 615
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_),
 x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(c + d*x)^n*(a + b*x^2)^p, x], x]
 /; FreeQ[{a, b, c, d, e, m, n}, x] && ILtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4373
```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Simp[-(-1)^((m - 1)/2)/(d*b^(m - 1))   Subst[Int[(b^2 - x^
2)^((m - 1)/2)*((a + x)^n/x), x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
 d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

3.292.4 Maple [A] (verified)

Time = 0.94 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.80

method	result
derivativedivides	$\frac{-\frac{b^4\ln(b+a\cos(dx+c))}{(a+b)^2(a-b)^2a}-\frac{1}{(4a-4b)(\cos(dx+c)+1)}+\frac{(-2a+3b)\ln(\cos(dx+c)+1)}{4(a-b)^2}+\frac{1}{(4a+4b)(\cos(dx+c)-1)}+\frac{(-2a-3b)\ln(\cos(dx+c)-1}{4(a+b)^2}}{d}$
default	$\frac{-\frac{b^4\ln(b+a\cos(dx+c))}{(a+b)^2(a-b)^2a}-\frac{1}{(4a-4b)(\cos(dx+c)+1)}+\frac{(-2a+3b)\ln(\cos(dx+c)+1)}{4(a-b)^2}+\frac{1}{(4a+4b)(\cos(dx+c)-1)}+\frac{(-2a-3b)\ln(\cos(dx+c)-1}{4(a+b)^2}}{d}$
risch	$-\frac{3ibc}{2d(a^2-2ab+b^2)}+\frac{2ib^4x}{a(a^4-2a^2b^2+b^4)}+\frac{iac}{d(a^2-2ab+b^2)}+\frac{iax}{a^2+2ab+b^2}+\frac{3ibc}{2d(a^2+2ab+b^2)}-\frac{3ibx}{2(a^2-2ab+b^2)}-$

input
```
int(cot(d*x+c)^3/(a+b*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(-b^4/(a+b)^2/(a-b)^2/a*ln(b+a*cos(d*x+c))-1/(4*a-4*b)/(cos(d*x+c)+1)+
1/4/(a-b)^2*(-2*a+3*b)*ln(cos(d*x+c)+1)+1/(4*a+4*b)/(cos(d*x+c)-1)+1/4/(a+
b)^2*(-2*a-3*b)*ln(cos(d*x+c)-1))
```

3.292.5 Fricas [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 263, normalized size of antiderivative = 1.68

$$\int \frac{\cot^3(c+dx)}{a+b\sec(c+dx)}\,dx$$
$$= \frac{2\,a^4 - 2\,a^2b^2 - 2\,(a^3b - ab^3)\cos(dx+c) - 4\,(b^4\cos(dx+c)^2 - b^4)\log(a\cos(dx+c)+b) + (2\,a^4 + a^3b$$

input
```
integrate(cot(d*x+c)^3/(a+b*sec(d*x+c)),x, algorithm="fricas")
```

output
```
1/4*(2*a^4 - 2*a^2*b^2 - 2*(a^3*b - a*b^3)*cos(d*x + c) - 4*(b^4*cos(d*x +
 c)^2 - b^4)*log(a*cos(d*x + c) + b) + (2*a^4 + a^3*b - 4*a^2*b^2 - 3*a*b^
3 - (2*a^4 + a^3*b - 4*a^2*b^2 - 3*a*b^3)*cos(d*x + c)^2)*log(1/2*cos(d*x
+ c) + 1/2) + (2*a^4 - a^3*b - 4*a^2*b^2 + 3*a*b^3 - (2*a^4 - a^3*b - 4*a^
2*b^2 + 3*a*b^3)*cos(d*x + c)^2)*log(-1/2*cos(d*x + c) + 1/2))/((a^5 - 2*a
^3*b^2 + a*b^4)*d*cos(d*x + c)^2 - (a^5 - 2*a^3*b^2 + a*b^4)*d)
```

3.292.6 Sympy [F]

$$\int \frac{\cot^3(c+dx)}{a+b\sec(c+dx)}\,dx = \int \frac{\cot^3(c+dx)}{a+b\sec(c+dx)}\,dx$$

input
```
integrate(cot(d*x+c)**3/(a+b*sec(d*x+c)),x)
```

output
```
Integral(cot(c + d*x)**3/(a + b*sec(c + d*x)), x)
```

3.292.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.92

$$\int \frac{\cot^3(c+dx)}{a+b\sec(c+dx)}\,dx = -\frac{\frac{4b^4\log(a\cos(dx+c)+b)}{a^5-2a^3b^2+ab^4}+\frac{(2a-3b)\log(\cos(dx+c)+1)}{a^2-2ab+b^2}+\frac{(2a+3b)\log(\cos(dx+c)-1)}{a^2+2ab+b^2}+\frac{2(b\cos(dx+c)-a)}{(a^2-b^2)\cos(dx+c)^2-a^2+b^2}}{4d}$$

input
```
integrate(cot(d*x+c)^3/(a+b*sec(d*x+c)),x, algorithm="maxima")
```

output
```
-1/4*(4*b^4*log(a*cos(d*x + c) + b)/(a^5 - 2*a^3*b^2 + a*b^4) + (2*a - 3*b
)*log(cos(d*x + c) + 1)/(a^2 - 2*a*b + b^2) + (2*a + 3*b)*log(cos(d*x + c)
 - 1)/(a^2 + 2*a*b + b^2) + 2*(b*cos(d*x + c) - a)/((a^2 - b^2)*cos(d*x +
c)^2 - a^2 + b^2))/d
```

3.292.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 403 vs. 2(147) = 294.

Time = 0.35 (sec) , antiderivative size = 403, normalized size of antiderivative = 2.57

$$\int \frac{\cot^3(c+dx)}{a+b\sec(c+dx)}\,dx = -\frac{\frac{2(2a+3b)\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)}{a^2+2ab+b^2}-\frac{4(a^3-2ab^2)\log\left(\left|a+b-\frac{2b(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{a(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}+\frac{b(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}\right|\right)}{a^4-2a^2b^2+b^4}-\frac{\left(a+b+\frac{4a(\cos(dx+c)}{\cos(dx+c)+}\right.}{(a^2+}}{8}$$

input
```
integrate(cot(d*x+c)^3/(a+b*sec(d*x+c)),x, algorithm="giac")
```

output
```
-1/8*(2*(2*a + 3*b)*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1))/(a^2
 + 2*a*b + b^2) - 4*(a^3 - 2*a*b^2)*log(abs(a + b - 2*b*(cos(d*x + c) - 1)
/(cos(d*x + c) + 1) - a*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + b*(cos
(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2))/(a^4 - 2*a^2*b^2 + b^4) - (a + b +
 4*a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 6*b*(cos(d*x + c) - 1)/(cos(d
*x + c) + 1))*(cos(d*x + c) + 1)/((a^2 + 2*a*b + b^2)*(cos(d*x + c) - 1))
- 4*(a^4 - 2*a^2*b^2 + 2*b^4)*log(abs(2*b + 2*a*(cos(d*x + c) - 1)/(cos(d*
x + c) + 1) - 2*b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 2*abs(a))/abs(2*
b + 2*a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 2*b*(cos(d*x + c) - 1)/(co
s(d*x + c) + 1) + 2*abs(a)))/((a^4 - 2*a^2*b^2 + b^4)*abs(a)) - (cos(d*x +
 c) - 1)/((a - b)*(cos(d*x + c) + 1)))/d
```

3.292.9 Mupad [B] (verification not implemented)

Time = 14.73 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.11

$$\int \frac{\cot^3(c+dx)}{a+b\sec(c+dx)}\,dx = \frac{\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+1\right)}{a\,d} - \frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2}{2\,d\,(4\,a-4\,b)} - \frac{\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)\,(2\,a+3\,b)}{d\,(2\,a^2+4\,a\,b+2\,b^2)} - \frac{a-b}{2\,d\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2\,(a+b)\,(4\,a-4\,b)} - \frac{b^4\,\ln\left(a+b-a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2\right)}{a\,d\,(a^2-b^2)^2}$$

input
```
int(cot(c + d*x)^3/(a + b/cos(c + d*x)),x)
```

output
```
log(tan(c/2 + (d*x)/2)^2 + 1)/(a*d) - tan(c/2 + (d*x)/2)^2/(2*d*(4*a - 4*b
)) - (log(tan(c/2 + (d*x)/2))*(2*a + 3*b))/(d*(4*a*b + 2*a^2 + 2*b^2)) - (
a - b)/(2*d*tan(c/2 + (d*x)/2)^2*(a + b)*(4*a - 4*b)) - (b^4*log(a + b - a
*tan(c/2 + (d*x)/2)^2 + b*tan(c/2 + (d*x)/2)^2))/(a*d*(a^2 - b^2)^2)
```

3.293 $\int \frac{\cot^5(c+dx)}{a+b\sec(c+dx)}\,dx$

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3.293.1 Optimal result

Integrand size = 21, antiderivative size = 234

$$\begin{aligned}\int \frac{\cot^5(c+dx)}{a+b\sec(c+dx)}\,dx = {}& \frac{\log(\cos(c+dx))}{ad} + \frac{(8a^2+21ab+15b^2)\log(1-\sec(c+dx))}{16(a+b)^3d} \\ & + \frac{(8a^2-21ab+15b^2)\log(1+\sec(c+dx))}{16(a-b)^3d} \\ & - \frac{b^6\log(a+b\sec(c+dx))}{a\left(a^2-b^2\right)^3 d} - \frac{1}{16(a+b)d(1-\sec(c+dx))^2} \\ & - \frac{5a+7b}{16(a+b)^2d(1-\sec(c+dx))} - \frac{1}{16(a-b)d(1+\sec(c+dx))^2} \\ & - \frac{5a-7b}{16(a-b)^2d(1+\sec(c+dx))}\end{aligned}$$

output
```
ln(cos(d*x+c))/a/d+1/16*(8*a^2+21*a*b+15*b^2)*ln(1-sec(d*x+c))/(a+b)^3/d+1
/16*(8*a^2-21*a*b+15*b^2)*ln(1+sec(d*x+c))/(a-b)^3/d-b^6*ln(a+b*sec(d*x+c)
)/a/(a^2-b^2)^3/d-1/16/(a+b)/d/(1-sec(d*x+c))^2+1/16*(-5*a-7*b)/(a+b)^2/d/
(1-sec(d*x+c))-1/16/(a-b)/d/(1+sec(d*x+c))^2+1/16*(-5*a+7*b)/(a-b)^2/d/(1+
sec(d*x+c))
```

3.293.2 Mathematica [A] (verified)

Time = 4.50 (sec) , antiderivative size = 220, normalized size of antiderivative = 0.94

$$\int \frac{\cot^5(c+dx)}{a+b\sec(c+dx)}\,dx = -\frac{b^6\left(-\frac{16\log(\cos(c+dx))}{ab^6} - \frac{(8a^2+21ab+15b^2)\log(1-\sec(c+dx))}{b^6(a+b)^3} - \frac{(8a^2-21ab+15b^2)\log(1+\sec(c+dx))}{(a-b)^3b^6} + \frac{16\log(a+b\sec(c+dx))}{a(a-b)^3(a+b)^3} + \frac{}{}\right.}{16d}$$

input
```
Integrate[Cot[c + d*x]^5/(a + b*Sec[c + d*x]),x]
```

output
```
-1/16*(b^6*((-16*Log[Cos[c + d*x]])/(a*b^6) - ((8*a^2 + 21*a*b + 15*b^2)*L
og[1 - Sec[c + d*x]])/(b^6*(a + b)^3) - ((8*a^2 - 21*a*b + 15*b^2)*Log[1 +
 Sec[c + d*x]])/((a - b)^3*b^6) + (16*Log[a + b*Sec[c + d*x]])/(a*(a - b)^
3*(a + b)^3) + 1/(b^6*(a + b)*(-1 + Sec[c + d*x])^2) + (-5*a - 7*b)/(b^6*(
a + b)^2*(-1 + Sec[c + d*x])) + 1/((a - b)*b^6*(1 + Sec[c + d*x])^2) + (5*
a - 7*b)/((a - b)^2*b^6*(1 + Sec[c + d*x]))))/d
```

3.293.3 Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 246, normalized size of antiderivative = 1.05, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3042, 25, 4373, 615, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^5(c+dx)}{a+b\sec(c+dx)}\,dx$$
$$\downarrow 3042$$
$$\int -\frac{1}{\cot\left(c+dx+\frac{\pi}{2}\right)^5\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)}dx$$
$$\downarrow 25$$
$$-\int \frac{1}{\cot\left(\frac{1}{2}(2c+\pi)+dx\right)^5\left(a+b\csc\left(\frac{1}{2}(2c+\pi)+dx\right)\right)}dx$$
$$\downarrow 4373$$

$$-\frac{b^6 \int \frac{\cos(c+dx)}{b(a+b\sec(c+dx))(b^2-b^2\sec^2(c+dx))^3} d(b\sec(c+dx))}{d}$$

$$\downarrow 615$$

$$-\frac{b^6 \int \left(\frac{7b-5a}{16(a-b)^2b^5(\sec(c+dx)b+b)^2} + \frac{\cos(c+dx)}{ab^7} + \frac{8a^2+21ba+15b^2}{16b^6(a+b)^3(b-b\sec(c+dx))} + \frac{1}{a(a-b)^3(a+b)^3(a+b\sec(c+dx))} + \frac{8a^2-21ba+15}{16b^6(b-a)^3(\sec(c+}\right.}{d}$$

$$\downarrow 2009$$

$$-\frac{b^6 \left(\frac{\log(a+b\sec(c+dx))}{a(a^2-b^2)^3} - \frac{(8a^2+21ab+15b^2)\log(b-b\sec(c+dx))}{16b^6(a+b)^3} - \frac{(8a^2-21ab+15b^2)\log(b\sec(c+dx)+b)}{16b^6(a-b)^3} + \frac{\log(b\sec(c+dx))}{ab^6} + \frac{}{16b^5(a-}\right.}{d}$$

input
```
Int[Cot[c + d*x]^5/(a + b*Sec[c + d*x]),x]
```

output
```
-((b^6*(Log[b*Sec[c + d*x]]/(a*b^6) - ((8*a^2 + 21*a*b + 15*b^2)*Log[b - b
*Sec[c + d*x]])/(16*b^6*(a + b)^3) + Log[a + b*Sec[c + d*x]]/(a*(a^2 - b^2
)^3) - ((8*a^2 - 21*a*b + 15*b^2)*Log[b + b*Sec[c + d*x]])/(16*(a - b)^3*b
^6) + 1/(16*b^4*(a + b)*(b - b*Sec[c + d*x])^2) + (5*a + 7*b)/(16*b^5*(a +
 b)^2*(b - b*Sec[c + d*x])) + 1/(16*(a - b)*b^4*(b + b*Sec[c + d*x])^2) +
(5*a - 7*b)/(16*(a - b)^2*b^5*(b + b*Sec[c + d*x])))))/d)
```

3.293.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 615
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_),
 x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(c + d*x)^n*(a + b*x^2)^p, x], x]
 /; FreeQ[{a, b, c, d, e, m, n}, x] && ILtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4373
```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Simp[-(-1)^((m - 1)/2)/(d*b^(m - 1))   Subst[Int[(b^2 - x^
2)^((m - 1)/2)*((a + x)^n/x), x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
 d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

3.293.4 Maple [A] (verified)

Time = 1.22 (sec) , antiderivative size = 193, normalized size of antiderivative = 0.82

method	result
derivativedivides	$\frac{-\frac{b^6\ln(b+a\cos(dx+c))}{(a+b)^3(a-b)^3a}-\frac{1}{2(8a+8b)(\cos(dx+c)-1)^2}-\frac{7a+9b}{16(a+b)^2(\cos(dx+c)-1)}+\frac{\left(8a^2+21ab+15b^2\right)\ln(\cos(dx+c)-1)}{16(a+b)^3}-\frac{1}{2(8a-8b)(\cos}{d}$
default	$\frac{-\frac{b^6\ln(b+a\cos(dx+c))}{(a+b)^3(a-b)^3a}-\frac{1}{2(8a+8b)(\cos(dx+c)-1)^2}-\frac{7a+9b}{16(a+b)^2(\cos(dx+c)-1)}+\frac{\left(8a^2+21ab+15b^2\right)\ln(\cos(dx+c)-1)}{16(a+b)^3}-\frac{1}{2(8a-8b)(\cos}{d}$
risch	$\frac{ix}{a}-\frac{ia^2c}{d(a^3-3a^2b+3ab^2-b^3)}+\frac{2ib^6c}{da(a^6-3a^4b^2+3a^2b^4-b^6)}+\frac{21iabc}{8d(a^3-3a^2b+3ab^2-b^3)}-\frac{ia^2x}{a^3-3a^2b+3ab^2-b^3}+\frac{}{a}$

input
```
int(cot(d*x+c)^5/(a+b*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(-b^6/(a+b)^3/(a-b)^3/a*ln(b+a*cos(d*x+c))-1/2/(8*a+8*b)/(cos(d*x+c)-1
)^2-1/16*(7*a+9*b)/(a+b)^2/(cos(d*x+c)-1)+1/16*(8*a^2+21*a*b+15*b^2)/(a+b)
^3*ln(cos(d*x+c)-1)-1/2/(8*a-8*b)/(cos(d*x+c)+1)^2-1/16*(-7*a+9*b)/(a-b)^2
/(cos(d*x+c)+1)+1/16*(8*a^2-21*a*b+15*b^2)/(a-b)^3*ln(cos(d*x+c)+1))
```

3.293.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 576 vs. $2(218) = 436$.

Time = 0.43 (sec) , antiderivative size = 576, normalized size of antiderivative = 2.46

$$\int \frac{\cot^5(c+dx)}{a+b\sec(c+dx)}\,dx$$
$$= \frac{12\,a^6-32\,a^4b^2+20\,a^2b^4+2\left(5\,a^5b-14\,a^3b^3+9\,ab^5\right)\cos(dx+c)^3-8\left(2\,a^6-5\,a^4b^2+3\,a^2b^4\right)\cos(dx+c}{}$$

input
```
integrate(cot(d*x+c)^5/(a+b*sec(d*x+c)),x, algorithm="fricas")
```

output
```
1/16*(12*a^6 - 32*a^4*b^2 + 20*a^2*b^4 + 2*(5*a^5*b - 14*a^3*b^3 + 9*a*b^5
)*cos(d*x + c)^3 - 8*(2*a^6 - 5*a^4*b^2 + 3*a^2*b^4)*cos(d*x + c)^2 - 2*(3
*a^5*b - 10*a^3*b^3 + 7*a*b^5)*cos(d*x + c) - 16*(b^6*cos(d*x + c)^4 - 2*b
^6*cos(d*x + c)^2 + b^6)*log(a*cos(d*x + c) + b) + (8*a^6 + 3*a^5*b - 24*a
^4*b^2 - 10*a^3*b^3 + 24*a^2*b^4 + 15*a*b^5 + (8*a^6 + 3*a^5*b - 24*a^4*b^
2 - 10*a^3*b^3 + 24*a^2*b^4 + 15*a*b^5)*cos(d*x + c)^4 - 2*(8*a^6 + 3*a^5*
b - 24*a^4*b^2 - 10*a^3*b^3 + 24*a^2*b^4 + 15*a*b^5)*cos(d*x + c)^2)*log(1
/2*cos(d*x + c) + 1/2) + (8*a^6 - 3*a^5*b - 24*a^4*b^2 + 10*a^3*b^3 + 24*a
^2*b^4 - 15*a*b^5 + (8*a^6 - 3*a^5*b - 24*a^4*b^2 + 10*a^3*b^3 + 24*a^2*b^
4 - 15*a*b^5)*cos(d*x + c)^4 - 2*(8*a^6 - 3*a^5*b - 24*a^4*b^2 + 10*a^3*b^
3 + 24*a^2*b^4 - 15*a*b^5)*cos(d*x + c)^2)*log(-1/2*cos(d*x + c) + 1/2))/(
(a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6)*d*cos(d*x + c)^4 - 2*(a^7 - 3*a^5*b^
2 + 3*a^3*b^4 - a*b^6)*d*cos(d*x + c)^2 + (a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a
*b^6)*d)
```

3.293.6 Sympy [F]

$$\int \frac{\cot^5(c+dx)}{a+b\sec(c+dx)}\,dx = \int \frac{\cot^5(c+dx)}{a+b\sec(c+dx)}\,dx$$

input
```
integrate(cot(d*x+c)**5/(a+b*sec(d*x+c)),x)
```

output
```
Integral(cot(c + d*x)**5/(a + b*sec(c + d*x)), x)
```

3.293.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 289, normalized size of antiderivative = 1.24

$$\int \frac{\cot^5(c+dx)}{a+b\sec(c+dx)}\,dx =$$

$$-\frac{\frac{16\,b^6\log(a\cos(dx+c)+b)}{a^7-3\,a^5b^2+3\,a^3b^4-ab^6} - \frac{(8\,a^2-21\,ab+15\,b^2)\log(\cos(dx+c)+1)}{a^3-3\,a^2b+3\,ab^2-b^3} - \frac{(8\,a^2+21\,ab+15\,b^2)\log(\cos(dx+c)-1)}{a^3+3\,a^2b+3\,ab^2+b^3} - \frac{2\left((5\,a^2b-9\,b^3)\cos(dx+c\right.}{(a^4-2\,a^2b^2+b^4}}{16\,d}$$

input
```
integrate(cot(d*x+c)^5/(a+b*sec(d*x+c)),x, algorithm="maxima")
```

output
```
-1/16*(16*b^6*log(a*cos(d*x + c) + b)/(a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6
) - (8*a^2 - 21*a*b + 15*b^2)*log(cos(d*x + c) + 1)/(a^3 - 3*a^2*b + 3*a*b
^2 - b^3) - (8*a^2 + 21*a*b + 15*b^2)*log(cos(d*x + c) - 1)/(a^3 + 3*a^2*b
 + 3*a*b^2 + b^3) - 2*((5*a^2*b - 9*b^3)*cos(d*x + c)^3 + 6*a^3 - 10*a*b^2
 - 4*(2*a^3 - 3*a*b^2)*cos(d*x + c)^2 - (3*a^2*b - 7*b^3)*cos(d*x + c))/((
a^4 - 2*a^2*b^2 + b^4)*cos(d*x + c)^4 + a^4 - 2*a^2*b^2 + b^4 - 2*(a^4 - 2
*a^2*b^2 + b^4)*cos(d*x + c)^2))/d
```

3.293.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 649 vs. $2(218) = 436$.

Time $= 0.39$ (sec) , antiderivative size $= 649$, normalized size of antiderivative $= 2.77$

$$\int \frac{\cot^5(c+dx)}{a+b\sec(c+dx)}\,dx$$

$$= \frac{4\left(8a^2+21ab+15b^2\right)\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)}{a^3+3a^2b+3ab^2+b^3} - \frac{32\left(a^5-3a^3b^2+3ab^4\right)\log\left(\left|a+b-\frac{2b(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{a(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}+\frac{b(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}\right|\right)}{a^6-3a^4b^2+3a^2b^4-b^6} - \frac{12a}{c}$$

input
```
integrate(cot(d*x+c)^5/(a+b*sec(d*x+c)),x, algorithm="giac")
```

output
```
1/64*(4*(8*a^2 + 21*a*b + 15*b^2)*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x +
 c) + 1))/(a^3 + 3*a^2*b + 3*a*b^2 + b^3) - 32*(a^5 - 3*a^3*b^2 + 3*a*b^4)
*log(abs(a + b - 2*b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - a*(cos(d*x +
c) - 1)^2/(cos(d*x + c) + 1)^2 + b*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)
^2))/(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6) - (12*a*(cos(d*x + c) - 1)/(cos(d
*x + c) + 1) - 16*b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + a*(cos(d*x + c
) - 1)^2/(cos(d*x + c) + 1)^2 - b*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^
2)/(a^2 - 2*a*b + b^2) - (a^2 + 2*a*b + b^2 + 12*a^2*(cos(d*x + c) - 1)/(c
os(d*x + c) + 1) + 28*a*b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 16*b^2*(
cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 48*a^2*(cos(d*x + c) - 1)^2/(cos(d*
x + c) + 1)^2 + 126*a*b*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 90*b^2
*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2)*(cos(d*x + c) + 1)^2/((a^3 + 3
*a^2*b + 3*a*b^2 + b^3)*(cos(d*x + c) - 1)^2) - 32*(a^6 - 3*a^4*b^2 + 3*a^
2*b^4 - 2*b^6)*log(abs(2*b + 2*a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 2
*b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 2*abs(a))/abs(2*b + 2*a*(cos(d*
x + c) - 1)/(cos(d*x + c) + 1) - 2*b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1)
 + 2*abs(a)))/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*abs(a)))/d
```

3.293.9 Mupad [B] (verification not implemented)

Time = 15.27 (sec) , antiderivative size = 290, normalized size of antiderivative = 1.24

$$\int \frac{\cot^5(c+dx)}{a+b\sec(c+dx)}\,dx = \frac{\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)\left(8a^2+21ab+15b^2\right)}{d\left(8a^3+24a^2b+24ab^2+8b^3\right)} - \frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4}{4d\left(16a-16b\right)} - \frac{\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+1\right)}{ad} - \frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2\left(\frac{16b}{(16a-16b)^2}-\frac{3}{16a-16b}\right)}{d} - \frac{\frac{a^2-2ab+b^2}{4(a+b)}+\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2\left(-3a^3+2a^2b+5ab^2-4b^3\right)}{(a+b)^2}}{d\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4\left(16a^2-32ab+16b^2\right)} - \frac{b^6\ln\left(a+b-a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2\right)}{ad\left(a^2-b^2\right)^3}$$

input
```
int(cot(c + d*x)^5/(a + b/cos(c + d*x)),x)
```

output
```
(log(tan(c/2 + (d*x)/2))*(21*a*b + 8*a^2 + 15*b^2))/(d*(24*a*b^2 + 24*a^2*
b + 8*a^3 + 8*b^3)) - tan(c/2 + (d*x)/2)^4/(4*d*(16*a - 16*b)) - log(tan(c
/2 + (d*x)/2)^2 + 1)/(a*d) - (tan(c/2 + (d*x)/2)^2*((16*b)/(16*a - 16*b)^2
 - 3/(16*a - 16*b)))/d - ((a^2 - 2*a*b + b^2)/(4*(a + b)) + (tan(c/2 + (d*
x)/2)^2*(5*a*b^2 + 2*a^2*b - 3*a^3 - 4*b^3))/(a + b)^2)/(d*tan(c/2 + (d*x)
/2)^4*(16*a^2 - 32*a*b + 16*b^2)) - (b^6*log(a + b - a*tan(c/2 + (d*x)/2)^
2 + b*tan(c/2 + (d*x)/2)^2))/(a*d*(a^2 - b^2)^3)
```

3.294 $\int \frac{\tan^6(c+dx)}{a+b\sec(c+dx)}\,dx$

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3.294.1 Optimal result

Integrand size = 21, antiderivative size = 198

$$\begin{aligned}\int \frac{\tan^6(c+dx)}{a+b\sec(c+dx)}\,dx = &-\frac{x}{a}+\frac{(8a^4-20a^2b^2+15b^4)\operatorname{arctanh}(\sin(c+dx))}{8b^5d}\\ &-\frac{2(a-b)^{5/2}(a+b)^{5/2}\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{ab^5d}\\ &-\frac{a(a^2-2b^2)\tan(c+dx)}{b^4d}+\frac{(4a^2-7b^2)\sec(c+dx)\tan(c+dx)}{8b^3d}\\ &-\frac{a\tan^3(c+dx)}{3b^2d}+\frac{\sec(c+dx)\tan^3(c+dx)}{4bd}\end{aligned}$$

output

```
-x/a+1/8*(8*a^4-20*a^2*b^2+15*b^4)*arctanh(sin(d*x+c))/b^5/d-2*(a-b)^(5/2)
*(a+b)^(5/2)*arctanh((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))/a/b^5/d-a
*(a^2-2*b^2)*tan(d*x+c)/b^4/d+1/8*(4*a^2-7*b^2)*sec(d*x+c)*tan(d*x+c)/b^3/
d-1/3*a*tan(d*x+c)^3/b^2/d+1/4*sec(d*x+c)*tan(d*x+c)^3/b/d
```

3.294.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 907 vs. $2(198) = 396$.

Time $= 6.53$ (sec) , antiderivative size $= 907$, normalized size of antiderivative $= 4.58$

$$\begin{aligned}
\int \frac{\tan^6(c+dx)}{a+b\sec(c+dx)}\,dx &= -\frac{(c+dx)(b+a\cos(c+dx))\sec(c+dx)}{ad(a+b\sec(c+dx))} \\
&- \frac{2(-a^2+b^2)^3 \operatorname{arctanh}\left(\frac{(-a+b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)(b+a\cos(c+dx))\sec(c+dx)}{ab^5\sqrt{a^2-b^2}d(a+b\sec(c+dx))} \\
&+ \frac{(-8a^4+20a^2b^2-15b^4)(b+a\cos(c+dx))\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)\sec(c+dx)}{8b^5d(a+b\sec(c+dx))} \\
&+ \frac{(8a^4-20a^2b^2+15b^4)(b+a\cos(c+dx))\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)\sec(c+dx)}{8b^5d(a+b\sec(c+dx))} \\
&+ \frac{(b+a\cos(c+dx))\sec(c+dx)}{16bd(a+b\sec(c+dx))\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^4} \\
&+ \frac{(12a^2-4ab-27b^2)(b+a\cos(c+dx))\sec(c+dx)}{48b^3d(a+b\sec(c+dx))\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^2} \\
&- \frac{a(b+a\cos(c+dx))\sec(c+dx)\sin\left(\frac{1}{2}(c+dx)\right)}{6b^2d(a+b\sec(c+dx))\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^3} \\
&- \frac{(b+a\cos(c+dx))\sec(c+dx)}{16bd(a+b\sec(c+dx))\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^4} \\
&- \frac{a(b+a\cos(c+dx))\sec(c+dx)\sin\left(\frac{1}{2}(c+dx)\right)}{6b^2d(a+b\sec(c+dx))\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^3} \\
&+ \frac{(-12a^2+4ab+27b^2)(b+a\cos(c+dx))\sec(c+dx)}{48b^3d(a+b\sec(c+dx))\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^2} \\
&+ \frac{(b+a\cos(c+dx))\sec(c+dx)\left(-3a^3\sin\left(\frac{1}{2}(c+dx)\right)+7ab^2\sin\left(\frac{1}{2}(c+dx)\right)\right)}{3b^4d(a+b\sec(c+dx))\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)} \\
&+ \frac{(b+a\cos(c+dx))\sec(c+dx)\left(-3a^3\sin\left(\frac{1}{2}(c+dx)\right)+7ab^2\sin\left(\frac{1}{2}(c+dx)\right)\right)}{3b^4d(a+b\sec(c+dx))\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)}
\end{aligned}$$

input
```
Integrate[Tan[c + d*x]^6/(a + b*Sec[c + d*x]),x]
```

output

```
-(((c + d*x)*(b + a*Cos[c + d*x])*Sec[c + d*x])/(a*d*(a + b*Sec[c + d*x]))
) - (2*(-a^2 + b^2)^3*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]]
*(b + a*Cos[c + d*x])*Sec[c + d*x])/(a*b^5*Sqrt[a^2 - b^2]*d*(a + b*Sec[c
 + d*x])) + ((-8*a^4 + 20*a^2*b^2 - 15*b^4)*(b + a*Cos[c + d*x])*Log[Cos[(c
 + d*x)/2] - Sin[(c + d*x)/2]]*Sec[c + d*x])/(8*b^5*d*(a + b*Sec[c + d*x])
) + ((8*a^4 - 20*a^2*b^2 + 15*b^4)*(b + a*Cos[c + d*x])*Log[Cos[(c + d*x)/
2] + Sin[(c + d*x)/2]]*Sec[c + d*x])/(8*b^5*d*(a + b*Sec[c + d*x])) + ((b
 + a*Cos[c + d*x])*Sec[c + d*x])/(16*b*d*(a + b*Sec[c + d*x])*(Cos[(c + d*x
)/2] - Sin[(c + d*x)/2])^4) + ((12*a^2 - 4*a*b - 27*b^2)*(b + a*Cos[c + d*
x])*Sec[c + d*x])/(48*b^3*d*(a + b*Sec[c + d*x])*(Cos[(c + d*x)/2] - Sin[(
c + d*x)/2])^2) - (a*(b + a*Cos[c + d*x])*Sec[c + d*x]*Sin[(c + d*x)/2])/(
6*b^2*d*(a + b*Sec[c + d*x])*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^3) - ((
b + a*Cos[c + d*x])*Sec[c + d*x])/(16*b*d*(a + b*Sec[c + d*x])*(Cos[(c + d
*x)/2] + Sin[(c + d*x)/2])^4) - (a*(b + a*Cos[c + d*x])*Sec[c + d*x]*Sin[(
c + d*x)/2])/(6*b^2*d*(a + b*Sec[c + d*x])*(Cos[(c + d*x)/2] + Sin[(c + d*
x)/2])^3) + ((-12*a^2 + 4*a*b + 27*b^2)*(b + a*Cos[c + d*x])*Sec[c + d*x])
/(48*b^3*d*(a + b*Sec[c + d*x])*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2) +
 ((b + a*Cos[c + d*x])*Sec[c + d*x]*(-3*a^3*Sin[(c + d*x)/2] + 7*a*b^2*Sin
[(c + d*x)/2]))/(3*b^4*d*(a + b*Sec[c + d*x])*(Cos[(c + d*x)/2] - Sin[(c +
 d*x)/2])) + ((b + a*Cos[c + d*x])*Sec[c + d*x]*(-3*a^3*Sin[(c + d*x)/2...
```

3.294.3 Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 271, normalized size of antiderivative = 1.37, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3042, 4386, 3042, 3376, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^6(c+dx)}{a+b\sec(c+dx)}\,dx$$

$$\downarrow 3042$$

$$\int \frac{\cot\left(c+dx+\frac{\pi}{2}\right)^6}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\downarrow 4386$$

$$\int \frac{\sin(c+dx)\tan^5(c+dx)}{a\cos(c+dx)+b}dx$$

$$\downarrow 3042$$

$$\int \frac{\cos\left(c+dx+\frac{\pi}{2}\right)^6}{\sin\left(c+dx+\frac{\pi}{2}\right)^5\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+b\right)}dx$$

$$\downarrow 3376$$

$$\int\left(\frac{\left(3ab^2-a^3\right)\sec^2(c+dx)}{b^4}-\frac{\left(a^2-b^2\right)^3}{ab^5(a\cos(c+dx)+b)}+\frac{\left(a^2-3b^2\right)\sec^3(c+dx)}{b^3}+\frac{\left(a^4-3a^2b^2+3b^4\right)\sec(c+dx)}{b^5}\right.$$

$$\downarrow 2009$$

$$\frac{\left(a^2-3b^2\right)\operatorname{arctanh}(\sin(c+dx))}{2b^3d}-\frac{a\left(a^2-3b^2\right)\tan(c+dx)}{b^4d}+\frac{\left(a^2-3b^2\right)\tan(c+dx)\sec(c+dx)}{2b^3d}+$$
$$\frac{\left(a^4-3a^2b^2+3b^4\right)\operatorname{arctanh}(\sin(c+dx))}{b^5d}-\frac{2(a-b)^{5/2}(a+b)^{5/2}\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{ab^5d}-$$
$$\frac{a\tan^3(c+dx)}{3b^2d}-\frac{a\tan(c+dx)}{b^2d}-\frac{x}{a}+\frac{3\operatorname{arctanh}(\sin(c+dx))}{8bd}+\frac{\tan(c+dx)\sec^3(c+dx)}{4bd}+$$
$$\frac{3\tan(c+dx)\sec(c+dx)}{8bd}$$

input
```
Int[Tan[c + d*x]^6/(a + b*Sec[c + d*x]),x]
```

output
```
-(x/a) + (3*ArcTanh[Sin[c + d*x]])/(8*b*d) + ((a^2 - 3*b^2)*ArcTanh[Sin[c
+ d*x]])/(2*b^3*d) + ((a^4 - 3*a^2*b^2 + 3*b^4)*ArcTanh[Sin[c + d*x]])/(b^
5*d) - (2*(a - b)^(5/2)*(a + b)^(5/2)*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2
])/Sqrt[a + b]])/(a*b^5*d) - (a*Tan[c + d*x])/(b^2*d) - (a*(a^2 - 3*b^2)*T
an[c + d*x])/(b^4*d) + (3*Sec[c + d*x]*Tan[c + d*x])/(8*b*d) + ((a^2 - 3*b
^2)*Sec[c + d*x]*Tan[c + d*x])/(2*b^3*d) + (Sec[c + d*x]^3*Tan[c + d*x])/(
4*b*d) - (a*Tan[c + d*x]^3)/(3*b^2*d)
```

3.294.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3376
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig[(d*sin[
e + f*x])^n*(a + b*sin[e + f*x])^m*(1 - sin[e + f*x]^2)^(p/2), x], x] /; Fr
eeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && IntegersQ[m, 2*n, p/2] && (
LtQ[m, -1] || (EqQ[m, -1] && GtQ[p, 0]))
```

rule 4386
```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Int[Cos[c + d*x]^m*((b + a*Sin[c + d*x])^n/Sin[c + d*x]^(m
 + n)), x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[n] &&
 IntegerQ[m] && (IntegerQ[m/2] || LeQ[m, 1])
```

3.294.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 410 vs. $2(181) = 362$.

Time = 1.54 (sec) , antiderivative size = 411, normalized size of antiderivative = 2.08

method	result
derivativedivides	$-\frac{2\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a}-\frac{2(a-b)\left(a^5+a^4b-2a^3b^2-2a^2b^3+ab^4+b^5\right)\operatorname{arctanh}\left(\frac{(a-b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a-b)(a+b)}}\right)}{b^5a\sqrt{(a-b)(a+b)}}+\frac{1}{4b\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^4}-\frac{}{6b^2}$
default	$-\frac{2\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a}-\frac{2(a-b)\left(a^5+a^4b-2a^3b^2-2a^2b^3+ab^4+b^5\right)\operatorname{arctanh}\left(\frac{(a-b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a-b)(a+b)}}\right)}{b^5a\sqrt{(a-b)(a+b)}}+\frac{1}{4b\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^4}-\frac{}{6b^2}$
risch	$-\frac{x}{a}-\frac{i\left(12a^2be^{7i(dx+c)}-27b^3e^{7i(dx+c)}+24a^3e^{6i(dx+c)}-72ab^2e^{6i(dx+c)}+12a^2be^{5i(dx+c)}-3b^3e^{5i(dx+c)}+72a^3e^{4i(dx}{}$

input
```
int(tan(d*x+c)^6/(a+b*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(-2/a*arctan(tan(1/2*d*x+1/2*c))-2/b^5*(a-b)*(a^5+a^4*b-2*a^3*b^2-2*a^
2*b^3+a*b^4+b^5)/a/((a-b)*(a+b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((
a-b)*(a+b))^(1/2))+1/4/b/(tan(1/2*d*x+1/2*c)-1)^4-1/6*(-2*a-3*b)/b^2/(tan(
1/2*d*x+1/2*c)-1)^3-1/8*(-4*a^2-4*a*b+5*b^2)/b^3/(tan(1/2*d*x+1/2*c)-1)^2+
1/8/b^5*(-8*a^4+20*a^2*b^2-15*b^4)*ln(tan(1/2*d*x+1/2*c)-1)-1/8*(-8*a^3-4*
a^2*b+16*a*b^2+7*b^3)/b^4/(tan(1/2*d*x+1/2*c)-1)-1/4/b/(tan(1/2*d*x+1/2*c)
+1)^4-1/6*(-2*a-3*b)/b^2/(tan(1/2*d*x+1/2*c)+1)^3-1/8*(4*a^2+4*a*b-5*b^2)/
b^3/(tan(1/2*d*x+1/2*c)+1)^2+1/8*(8*a^4-20*a^2*b^2+15*b^4)/b^5*ln(tan(1/2*
d*x+1/2*c)+1)-1/8*(-8*a^3-4*a^2*b+16*a*b^2+7*b^3)/b^4/(tan(1/2*d*x+1/2*c)+
1))
```

3.294.5 Fricas [A] (verification not implemented)

Time = 0.61 (sec) , antiderivative size = 603, normalized size of antiderivative = 3.05

$$\int \frac{\tan^6(c+dx)}{a+b\sec(c+dx)}\,dx$$

$$= \left[-\frac{48\,b^5 dx\cos(dx+c)^4 - 24\,(a^4-2\,a^2b^2+b^4)\sqrt{a^2-b^2}\cos(dx+c)^4\log\left(\frac{2\,ab\cos(dx+c)-(a^2-2\,b^2)\cos(dx+c)^2-2}{a^2\cos(dx+c)^2-}\right.\right.$$

$$\left. -\frac{48\,b^5 dx\cos(dx+c)^4 + 48\,(a^4-2\,a^2b^2+b^4)\sqrt{-a^2+b^2}\arctan\left(-\frac{\sqrt{-a^2+b^2}(b\cos(dx+c)+a)}{(a^2-b^2)\sin(dx+c)}\right)\cos(dx+c)^4 -}{}\right.$$

input
```
integrate(tan(d*x+c)^6/(a+b*sec(d*x+c)),x, algorithm="fricas")
```

output
```
[-1/48*(48*b^5*d*x*cos(d*x + c)^4 - 24*(a^4 - 2*a^2*b^2 + b^4)*sqrt(a^2 -
b^2)*cos(d*x + c)^4*log((2*a*b*cos(d*x + c) - (a^2 - 2*b^2)*cos(d*x + c)^2
 - 2*sqrt(a^2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x + c) + 2*a^2 - b^2)/(a^2
*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + b^2)) - 3*(8*a^5 - 20*a^3*b^2 + 15*
a*b^4)*cos(d*x + c)^4*log(sin(d*x + c) + 1) + 3*(8*a^5 - 20*a^3*b^2 + 15*a
*b^4)*cos(d*x + c)^4*log(-sin(d*x + c) + 1) + 2*(8*a^2*b^3*cos(d*x + c) -
6*a*b^4 + 8*(3*a^4*b - 7*a^2*b^3)*cos(d*x + c)^3 - 3*(4*a^3*b^2 - 9*a*b^4)
*cos(d*x + c)^2)*sin(d*x + c))/(a*b^5*d*cos(d*x + c)^4), -1/48*(48*b^5*d*x
*cos(d*x + c)^4 + 48*(a^4 - 2*a^2*b^2 + b^4)*sqrt(-a^2 + b^2)*arctan(-sqrt
(-a^2 + b^2)*(b*cos(d*x + c) + a)/((a^2 - b^2)*sin(d*x + c)))*cos(d*x + c)
^4 - 3*(8*a^5 - 20*a^3*b^2 + 15*a*b^4)*cos(d*x + c)^4*log(sin(d*x + c) + 1
) + 3*(8*a^5 - 20*a^3*b^2 + 15*a*b^4)*cos(d*x + c)^4*log(-sin(d*x + c) + 1
) + 2*(8*a^2*b^3*cos(d*x + c) - 6*a*b^4 + 8*(3*a^4*b - 7*a^2*b^3)*cos(d*x
+ c)^3 - 3*(4*a^3*b^2 - 9*a*b^4)*cos(d*x + c)^2)*sin(d*x + c))/(a*b^5*d*co
s(d*x + c)^4)]
```

3.294.6 Sympy [F]

$$\int \frac{\tan^6(c+dx)}{a+b\sec(c+dx)}\,dx = \int \frac{\tan^6(c+dx)}{a+b\sec(c+dx)}\,dx$$

input
```
integrate(tan(d*x+c)**6/(a+b*sec(d*x+c)),x)
```

output
```
Integral(tan(c + d*x)**6/(a + b*sec(c + d*x)), x)
```

3.294.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\tan^6(c+dx)}{a+b\sec(c+dx)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(tan(d*x+c)^6/(a+b*sec(d*x+c)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` f
or more de
```

3.294.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 746 vs. $2(181) = 362$.

Time = 2.11 (sec) , antiderivative size = 746, normalized size of antiderivative = 3.77

$$\int \frac{\tan^6(c+dx)}{a+b\sec(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate(tan(d*x+c)^6/(a+b*sec(d*x+c)),x, algorithm="giac")
```

output
```
-1/24*(24*((a^4 + a^3*b - 2*a^2*b^2 - 2*a*b^3 + b^4)*sqrt(-a^2 + b^2)*abs(
a)*abs(-a + b)*abs(b) + (a^5*b + a^4*b^2 - 2*a^3*b^3 - 2*a^2*b^4 + a*b^5 +
 2*b^6)*sqrt(-a^2 + b^2)*abs(-a + b))*(pi*floor(1/2*(d*x + c)/pi + 1/2) +
arctan(tan(1/2*d*x + 1/2*c)/sqrt(-(b^6 + sqrt(b^12 + (a*b^5 + b^6)*(a*b^5
- b^6)))/(a*b^5 - b^6))))/((a*b^4 - b^5)*a^2*b^2 + (a*b^6 - b^7)*abs(a)*ab
s(b)) + 24*(a^6*b - 3*a^4*b^3 + 3*a^2*b^5 + a*b^6 - 2*b^7 - a^5*abs(a)*abs
(b) + 3*a^3*b^2*abs(a)*abs(b) - 3*a*b^4*abs(a)*abs(b) + b^5*abs(a)*abs(b))
*(pi*floor(1/2*(d*x + c)/pi + 1/2) + arctan(tan(1/2*d*x + 1/2*c)/sqrt(-(b^
6 - sqrt(b^12 + (a*b^5 + b^6)*(a*b^5 - b^6)))/(a*b^5 - b^6))))/(a^2*b^6 -
b^6*abs(a)*abs(b)) - 3*(8*a^4 - 20*a^2*b^2 + 15*b^4)*log(abs(tan(1/2*d*x +
 1/2*c) + 1))/b^5 + 3*(8*a^4 - 20*a^2*b^2 + 15*b^4)*log(abs(tan(1/2*d*x +
1/2*c) - 1))/b^5 - 2*(24*a^3*tan(1/2*d*x + 1/2*c)^7 + 12*a^2*b*tan(1/2*d*x
 + 1/2*c)^7 - 48*a*b^2*tan(1/2*d*x + 1/2*c)^7 - 21*b^3*tan(1/2*d*x + 1/2*c
)^7 - 72*a^3*tan(1/2*d*x + 1/2*c)^5 - 12*a^2*b*tan(1/2*d*x + 1/2*c)^5 + 17
6*a*b^2*tan(1/2*d*x + 1/2*c)^5 + 45*b^3*tan(1/2*d*x + 1/2*c)^5 + 72*a^3*ta
n(1/2*d*x + 1/2*c)^3 - 12*a^2*b*tan(1/2*d*x + 1/2*c)^3 - 176*a*b^2*tan(1/2
*d*x + 1/2*c)^3 + 45*b^3*tan(1/2*d*x + 1/2*c)^3 - 24*a^3*tan(1/2*d*x + 1/2
*c) + 12*a^2*b*tan(1/2*d*x + 1/2*c) + 48*a*b^2*tan(1/2*d*x + 1/2*c) - 21*b
^3*tan(1/2*d*x + 1/2*c))/((tan(1/2*d*x + 1/2*c)^2 - 1)^4*b^4))/d
```

3.294.9 Mupad [B] (verification not implemented)

Time = 16.74 (sec) , antiderivative size = 9148, normalized size of antiderivative = 46.20

$$\int \frac{\tan^6(c+dx)}{a+b\sec(c+dx)}\,dx = \text{Too large to display}$$

input
```
int(tan(c + d*x)^6/(a + b/cos(c + d*x)),x)
```

output

```
((tan(c/2 + (d*x)/2)*(16*a*b^2 + 4*a^2*b - 8*a^3 - 7*b^3))/(4*b^4) - (tan(
c/2 + (d*x)/2)^7*(16*a*b^2 - 4*a^2*b - 8*a^3 + 7*b^3))/(4*b^4) - (tan(c/2
+ (d*x)/2)^3*(176*a*b^2 + 12*a^2*b - 72*a^3 - 45*b^3))/(12*b^4) + (tan(c/2
 + (d*x)/2)^5*(176*a*b^2 - 12*a^2*b - 72*a^3 + 45*b^3))/(12*b^4))/(d*(6*ta
n(c/2 + (d*x)/2)^4 - 4*tan(c/2 + (d*x)/2)^2 - 4*tan(c/2 + (d*x)/2)^6 + tan
(c/2 + (d*x)/2)^8 + 1)) + (2*atan((((((((((128*(192*a^2*b^22 - 256*a^3*b^2
1 - 568*a^4*b^20 + 1016*a^5*b^19 + 280*a^6*b^18 - 1176*a^7*b^17 + 288*a^8*
b^16 + 416*a^9*b^15 - 192*a^10*b^14))/b^16 - (tan(c/2 + (d*x)/2)*(128*a^2*
b^23 - 384*a^3*b^22 + 512*a^4*b^21 - 512*a^5*b^20 + 384*a^6*b^19 - 128*a^7
*b^18)*128i)/(a*b^16))*1i)/a - (128*tan(c/2 + (d*x)/2)*(128*b^23 - 384*a*b
^22 - 322*a^2*b^21 + 1222*a^3*b^20 + 903*a^4*b^19 - 3047*a^5*b^18 + 755*a^
6*b^17 + 905*a^7*b^16 + 120*a^8*b^15 + 1000*a^9*b^14 - 1792*a^10*b^13 - 51
2*a^11*b^12 + 1472*a^12*b^11 - 192*a^13*b^10 - 384*a^14*b^9 + 128*a^15*b^8
))/b^16)*1i)/a - (128*(576*a*b^21 - 192*b^22 + 1043*a^2*b^20 - 2996*a^3*b^
19 - 3575*a^4*b^18 + 8886*a^5*b^17 + 7376*a^6*b^16 - 18310*a^7*b^15 - 7672
*a^8*b^14 + 24883*a^9*b^13 + 2308*a^10*b^12 - 21295*a^11*b^11 + 2736*a^12*
b^10 + 11096*a^13*b^9 - 3080*a^14*b^8 - 3256*a^15*b^7 + 1248*a^16*b^6 + 41
6*a^17*b^5 - 192*a^18*b^4))/b^16)*1i)/a - (128*tan(c/2 + (d*x)/2)*(1414*a*
b^20 - 64*a^20*b + 64*a^21 - 514*b^21 + 684*a^2*b^19 - 3084*a^3*b^18 - 434
0*a^4*b^17 + 6000*a^5*b^16 + 15860*a^6*b^15 - 14740*a^7*b^14 - 27983*a^...
```

3.295 $\int \frac{\tan^4(c+dx)}{a+b\sec(c+dx)}\,dx$

3.295.1 Optimal result . . . 2126
3.295.2 Mathematica [B] (verified) . . . 2126
3.295.3 Rubi [A] (verified) . . . 2127
3.295.4 Maple [B] (verified) . . . 2130
3.295.5 Fricas [A] (verification not implemented) . . . 2131
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3.295.8 Giac [B] (verification not implemented) . . . 2132
3.295.9 Mupad [B] (verification not implemented) . . . 2133

3.295.1 Optimal result

Integrand size = 21, antiderivative size = 126

$$\int \frac{\tan^4(c+dx)}{a+b\sec(c+dx)}\,dx = \frac{x}{a} + \frac{(2a^2-3b^2)\operatorname{arctanh}(\sin(c+dx))}{2b^3d} - \frac{2(a-b)^{3/2}(a+b)^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{ab^3d} - \frac{a\tan(c+dx)}{b^2d} + \frac{\sec(c+dx)\tan(c+dx)}{2bd}$$

output
```
x/a+1/2*(2*a^2-3*b^2)*arctanh(sin(d*x+c))/b^3/d-2*(a-b)^(3/2)*(a+b)^(3/2)*
arctanh((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))/a/b^3/d-a*tan(d*x+c)/b
^2/d+1/2*sec(d*x+c)*tan(d*x+c)/b/d
```

3.295.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 287 vs. 2(126) = 252.

Time = 1.91 (sec) , antiderivative size = 287, normalized size of antiderivative = 2.28

$$\int \frac{\tan^4(c+dx)}{a+b\sec(c+dx)}\,dx = \frac{(b+a\cos(c+dx))\sec(c+dx)\left(\frac{4c}{a}+\frac{4dx}{a}+\frac{8(a^2-b^2)^{3/2}\operatorname{arctanh}\left(\frac{(-a+b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{ab^3}-\frac{4a^2\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}\right.\right.\right.}{b^3}}{}$$

input
```
Integrate[Tan[c + d*x]^4/(a + b*Sec[c + d*x]),x]
```

output
```
((b + a*Cos[c + d*x])*Sec[c + d*x]*((4*c)/a + (4*d*x)/a + (8*(a^2 - b^2)^(
3/2)*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a*b^3) - (4*a^
2*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]])/b^3 + (6*Log[Cos[(c + d*x)/2]
 - Sin[(c + d*x)/2]])/b + (4*a^2*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]])/
b^3 - (6*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]])/b + 1/(b*(Cos[(c + d*x)
/2] - Sin[(c + d*x)/2])^2) - 1/(b*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2)
 - (4*a*Tan[c + d*x])/b^2))/(4*d*(a + b*Sec[c + d*x]))
```

3.295.3 Rubi [A] (verified)

Time = 0.79 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.16, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.524$, Rules used = {3042, 4386, 3042, 3372, 25, 3042, 3536, 3042, 3138, 221, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^4(c+dx)}{a+b\sec(c+dx)}\,dx$$

$$\downarrow 3042$$

$$\int \frac{\cot\left(c+dx+\frac{\pi}{2}\right)^4}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}\,dx$$

$$\downarrow 4386$$

$$\int \frac{\sin(c+dx)\tan^3(c+dx)}{a\cos(c+dx)+b}\,dx$$

$$\downarrow 3042$$

$$\int \frac{\cos\left(c+dx+\frac{\pi}{2}\right)^4}{\sin\left(c+dx+\frac{\pi}{2}\right)^3\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+b\right)}\,dx$$

$$\downarrow 3372$$

$$-\frac{\int -\frac{\left(2a^2+b\cos(c+dx)a-3b^2+2b^2\cos^2(c+dx)\right)\sec(c+dx)}{b+a\cos(c+dx)}\,dx}{2b^2} - \frac{a\tan(c+dx)}{b^2 d} + \frac{\tan(c+dx)\sec(c+dx)}{2bd}$$

$$\downarrow 25$$

$$\frac{\int \frac{\left(2a^2+b\cos(c+dx)a-3b^2+2b^2\cos^2(c+dx)\right)\sec(c+dx)}{b+a\cos(c+dx)}dx}{2b^2} - \frac{a\tan(c+dx)}{b^2d} + \frac{\tan(c+dx)\sec(c+dx)}{2bd}$$

$$\downarrow 3042$$

$$\frac{\int \frac{2a^2+b\sin\left(c+dx+\frac{\pi}{2}\right)a-3b^2+2b^2\sin\left(c+dx+\frac{\pi}{2}\right)^2}{\sin\left(c+dx+\frac{\pi}{2}\right)\left(b+a\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{2b^2} - \frac{a\tan(c+dx)}{b^2d} + \frac{\tan(c+dx)\sec(c+dx)}{2bd}$$

$$\downarrow 3536$$

$$\frac{-\frac{2\left(a^2-b^2\right)^2\int\frac{1}{b+a\cos(c+dx)}dx}{ab} + \frac{\left(2a^2-3b^2\right)\int\sec(c+dx)dx}{b} + \frac{2b^2x}{a}}{2b^2} - \frac{a\tan(c+dx)}{b^2d} + \frac{\tan(c+dx)\sec(c+dx)}{2bd}$$

$$\downarrow 3042$$

$$\frac{-\frac{2\left(a^2-b^2\right)^2\int\frac{1}{b+a\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{ab} + \frac{\left(2a^2-3b^2\right)\int\csc\left(c+dx+\frac{\pi}{2}\right)dx}{b} + \frac{2b^2x}{a}}{2b^2} - \frac{a\tan(c+dx)}{b^2d} + \frac{\tan(c+dx)\sec(c+dx)}{2bd}$$

$$\downarrow 3138$$

$$\frac{-\frac{4\left(a^2-b^2\right)^2\int\frac{1}{-\left((a-b)\tan^2\left(\frac{1}{2}(c+dx)\right)\right)+a+b}d\tan\left(\frac{1}{2}(c+dx)\right)}{abd} + \frac{\left(2a^2-3b^2\right)\int\csc\left(c+dx+\frac{\pi}{2}\right)dx}{b} + \frac{2b^2x}{a}}{2b^2} - \frac{a\tan(c+dx)}{b^2d} + \frac{\tan(c+dx)\sec(c+dx)}{2bd}$$

$$\downarrow 221$$

$$\frac{\frac{\left(2a^2-3b^2\right)\int\csc\left(c+dx+\frac{\pi}{2}\right)dx}{b} - \frac{4\left(a^2-b^2\right)^2\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{abd\sqrt{a-b}\sqrt{a+b}} + \frac{2b^2x}{a}}{2b^2} - \frac{a\tan(c+dx)}{b^2d} + \frac{\tan(c+dx)\sec(c+dx)}{2bd}$$

$$\downarrow 4257$$

$$\frac{\frac{\left(2a^2-3b^2\right)\operatorname{arctanh}(\sin(c+dx))}{bd} - \frac{4\left(a^2-b^2\right)^2\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{abd\sqrt{a-b}\sqrt{a+b}} + \frac{2b^2x}{a}}{2b^2} - \frac{a\tan(c+dx)}{b^2d} + \frac{\tan(c+dx)\sec(c+dx)}{2bd}$$

input
```
Int[Tan[c + d*x]^4/(a + b*Sec[c + d*x]),x]
```

output
```
((2*b^2*x)/a + ((2*a^2 - 3*b^2)*ArcTanh[Sin[c + d*x]])/(b*d) - (4*(a^2 - b
^2)^2*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a*Sqrt[a - b]*
b*Sqrt[a + b]*d))/(2*b^2) - (a*Tan[c + d*x])/(b^2*d) + (Sec[c + d*x]*Tan[c
 + d*x])/(2*b*d)
```

3.295.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

rule 3372
```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[Cos[e + f*x]*(a + b*
Sin[e + f*x])^(m + 1)*((d*Sin[e + f*x])^(n + 1)/(a*d*f*(n + 1))), x] + (-Si
mp[b*(m + n + 2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*((d*Sin[e + f*x]
)^(n + 2)/(a^2*d^2*f*(n + 1)*(n + 2))), x] - Simp[1/(a^2*d^2*(n + 1)*(n + 2
))   Int[(a + b*Sin[e + f*x])^m*(d*Sin[e + f*x])^(n + 2)*Simp[a^2*n*(n + 2)
 - b^2*(m + n + 2)*(m + n + 3) + a*b*m*Sin[e + f*x] - (a^2*(n + 1)*(n + 2)
- b^2*(m + n + 2)*(m + n + 4))*Sin[e + f*x]^2, x], x], x]) /; FreeQ[{a, b,
d, e, f, m}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || IntegersQ[2*m, 2*n])
 &&  !m < -1 && LtQ[n, -1] && (LtQ[n, -2] || EqQ[m + n + 4, 0])
```

rule 3536
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)])), x_Symbol] :> Simp[C*(x/(b*d)), x] + (Simp[(A*b^2 - a*b*B + a^2*C)
/(b*(b*c - a*d))   Int[1/(a + b*Sin[e + f*x]), x], x] - Simp[(c^2*C - B*c*d
 + A*d^2)/(d*(b*c - a*d))   Int[1/(c + d*Sin[e + f*x]), x], x]) /; FreeQ[{a
, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] &&
NeQ[c^2 - d^2, 0]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 4386
```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Int[Cos[c + d*x]^m*((b + a*Sin[c + d*x])^n/Sin[c + d*x]^(m
 + n)), x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[n] &&
 IntegerQ[m] && (IntegerQ[m/2] || LeQ[m, 1])
```

3.295.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 233 vs. $2(113) = 226$.

Time = 0.88 (sec) , antiderivative size = 234, normalized size of antiderivative = 1.86

method	result
derivativedivides	$\frac{-\frac{2(a-b)\left(a^3+a^2b-ab^2-b^3\right)\operatorname{arctanh}\left(\frac{(a-b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a-b)(a+b)}}\right)}{b^3a\sqrt{(a-b)(a+b)}}-\frac{1}{2b\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^2}-\frac{-2a-b}{2b^2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}+\frac{\left(2a^2-3b^2\right)\ln\left(\tan\left($
default	$\frac{-\frac{2(a-b)\left(a^3+a^2b-ab^2-b^3\right)\operatorname{arctanh}\left(\frac{(a-b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a-b)(a+b)}}\right)}{b^3a\sqrt{(a-b)(a+b)}}-\frac{1}{2b\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^2}-\frac{-2a-b}{2b^2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}+\frac{\left(2a^2-3b^2\right)\ln\left(\tan\left($
risch	$\frac{x}{a}-\frac{i\left(b\,\mathrm{e}^{3i(dx+c)}+2a\,\mathrm{e}^{2i(dx+c)}-b\,\mathrm{e}^{i(dx+c)}+2a\right)}{db^2\left(\mathrm{e}^{2i(dx+c)}+1\right)^2}+\frac{\ln\left(\mathrm{e}^{i(dx+c)}+i\right)a^2}{db^3}-\frac{3\ln\left(\mathrm{e}^{i(dx+c)}+i\right)}{2db}-\frac{\ln\left(\mathrm{e}^{i(dx+c)}-i\right)a^2}{db^3}+$

input
```
int(tan(d*x+c)^4/(a+b*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(-2/b^3*(a-b)*(a^3+a^2*b-a*b^2-b^3)/a/((a-b)*(a+b))^(1/2)*arctanh((a-b
)*tan(1/2*d*x+1/2*c)/((a-b)*(a+b))^(1/2))-1/2/b/(tan(1/2*d*x+1/2*c)+1)^2-1
/2*(-2*a-b)/b^2/(tan(1/2*d*x+1/2*c)+1)+1/2*(2*a^2-3*b^2)/b^3*ln(tan(1/2*d*
x+1/2*c)+1)+2/a*arctan(tan(1/2*d*x+1/2*c))+1/2/b/(tan(1/2*d*x+1/2*c)-1)^2-
1/2*(-2*a-b)/b^2/(tan(1/2*d*x+1/2*c)-1)+1/2/b^3*(-2*a^2+3*b^2)*ln(tan(1/2*
d*x+1/2*c)-1))
```

3.295.5 Fricas [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 444, normalized size of antiderivative = 3.52

$$\int \frac{\tan^4(c+dx)}{a+b\sec(c+dx)}\,dx$$
$$= \left[\frac{4\,b^3 dx\cos(dx+c)^2 - 2\,(a^2-b^2)^{\frac{3}{2}}\cos(dx+c)^2\log\left(\frac{2\,ab\cos(dx+c)-(a^2-2\,b^2)\cos(dx+c)^2+2\sqrt{a^2-b^2}(b\cos(dx+c)+a)\sin}{a^2\cos(dx+c)^2+2\,ab\cos(dx+c)+b^2}\right.}{}\right.$$

input
```
integrate(tan(d*x+c)^4/(a+b*sec(d*x+c)),x, algorithm="fricas")
```

output
```
[1/4*(4*b^3*d*x*cos(d*x + c)^2 - 2*(a^2 - b^2)^(3/2)*cos(d*x + c)^2*log((2
*a*b*cos(d*x + c) - (a^2 - 2*b^2)*cos(d*x + c)^2 + 2*sqrt(a^2 - b^2)*(b*co
s(d*x + c) + a)*sin(d*x + c) + 2*a^2 - b^2)/(a^2*cos(d*x + c)^2 + 2*a*b*co
s(d*x + c) + b^2)) + (2*a^3 - 3*a*b^2)*cos(d*x + c)^2*log(sin(d*x + c) + 1
) - (2*a^3 - 3*a*b^2)*cos(d*x + c)^2*log(-sin(d*x + c) + 1) - 2*(2*a^2*b*c
os(d*x + c) - a*b^2)*sin(d*x + c))/(a*b^3*d*cos(d*x + c)^2), 1/4*(4*b^3*d*
x*cos(d*x + c)^2 - 4*(a^2 - b^2)*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)
*(b*cos(d*x + c) + a)/((a^2 - b^2)*sin(d*x + c)))*cos(d*x + c)^2 + (2*a^3
- 3*a*b^2)*cos(d*x + c)^2*log(sin(d*x + c) + 1) - (2*a^3 - 3*a*b^2)*cos(d*
x + c)^2*log(-sin(d*x + c) + 1) - 2*(2*a^2*b*cos(d*x + c) - a*b^2)*sin(d*x
 + c))/(a*b^3*d*cos(d*x + c)^2)]
```

3.295.6 Sympy [F]

$$\int \frac{\tan^4(c+dx)}{a+b\sec(c+dx)}\,dx = \int \frac{\tan^4(c+dx)}{a+b\sec(c+dx)}\,dx$$

input
```
integrate(tan(d*x+c)**4/(a+b*sec(d*x+c)),x)
```

output
```
Integral(tan(c + d*x)**4/(a + b*sec(c + d*x)), x)
```

3.295.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\tan^4(c+dx)}{a+b\sec(c+dx)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(tan(d*x+c)^4/(a+b*sec(d*x+c)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` f
or more de
```

3.295.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 476 vs. $2(113) = 226$.

Time $= 0.90$ (sec) , antiderivative size $= 476$, normalized size of antiderivative $= 3.78$

$$\int \frac{\tan^4(c+dx)}{a+b\sec(c+dx)}\,dx =$$

$$\frac{2\left(\left(a^2+ab-b^2\right)\sqrt{-a^2+b^2}|a||-a+b||b|+\left(a^3b+a^2b^2-ab^3-2\,b^4\right)\sqrt{-a^2+b^2}|-a+b|\right)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor+\arctan\left(\frac{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{-\frac{b^4+\sqrt{b^8+\left(ab^3+b^4\right)\left(ab^3-b^4\right)}}{ab^3-b^4}}}\right)\right)}{\left(ab^2-b^3\right)a^2b^2+\left(ab^4-b^5\right)|a||b|}$$

input
```
integrate(tan(d*x+c)^4/(a+b*sec(d*x+c)),x, algorithm="giac")
```

output
```
-1/2*(2*((a^2 + a*b - b^2)*sqrt(-a^2 + b^2)*abs(a)*abs(-a + b)*abs(b) + (a
^3*b + a^2*b^2 - a*b^3 - 2*b^4)*sqrt(-a^2 + b^2)*abs(-a + b))*(pi*floor(1/
2*(d*x + c)/pi + 1/2) + arctan(tan(1/2*d*x + 1/2*c)/sqrt(-(b^4 + sqrt(b^8
+ (a*b^3 + b^4)*(a*b^3 - b^4)))/(a*b^3 - b^4))))/((a*b^2 - b^3)*a^2*b^2 +
(a*b^4 - b^5)*abs(a)*abs(b)) + 2*(a^4*b - 2*a^2*b^3 - a*b^4 + 2*b^5 - a^3*
abs(a)*abs(b) + 2*a*b^2*abs(a)*abs(b) - b^3*abs(a)*abs(b))*(pi*floor(1/2*(
d*x + c)/pi + 1/2) + arctan(tan(1/2*d*x + 1/2*c)/sqrt(-(b^4 - sqrt(b^8 + (
a*b^3 + b^4)*(a*b^3 - b^4)))/(a*b^3 - b^4))))/(a^2*b^4 - b^4*abs(a)*abs(b)
) - (2*a^2 - 3*b^2)*log(abs(tan(1/2*d*x + 1/2*c) + 1))/b^3 + (2*a^2 - 3*b^
2)*log(abs(tan(1/2*d*x + 1/2*c) - 1))/b^3 - 2*(2*a*tan(1/2*d*x + 1/2*c)^3
+ b*tan(1/2*d*x + 1/2*c)^3 - 2*a*tan(1/2*d*x + 1/2*c) + b*tan(1/2*d*x + 1/
2*c))/((tan(1/2*d*x + 1/2*c)^2 - 1)^2*b^2))/d
```

3.295.9 Mupad [B] (verification not implemented)

Time = 16.06 (sec) , antiderivative size = 6062, normalized size of antiderivative = 48.11

$$\int \frac{\tan^4(c+dx)}{a+b\sec(c+dx)}\,dx = \text{Too large to display}$$

input
```
int(tan(c + d*x)^4/(a + b/cos(c + d*x)),x)
```

output

```
(atan((sin(c/2 + (d*x)/2)*1i)/cos(c/2 + (d*x)/2))*cos(c/2 + (d*x)/2)^4*3i)
/(b*d*(cos(c/2 + (d*x)/2)^4 + sin(c/2 + (d*x)/2)^4 - 2*cos(c/2 + (d*x)/2)^
2*sin(c/2 + (d*x)/2)^2)) + (atan((sin(c/2 + (d*x)/2)*1i)/cos(c/2 + (d*x)/2
))*sin(c/2 + (d*x)/2)^4*3i)/(b*d*(cos(c/2 + (d*x)/2)^4 + sin(c/2 + (d*x)/2
)^4 - 2*cos(c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2)^2)) + (cos(c/2 + (d*x)/2)*
sin(c/2 + (d*x)/2)^3)/(b*d*(cos(c/2 + (d*x)/2)^4 + sin(c/2 + (d*x)/2)^4 -
2*cos(c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2)^2)) + (cos(c/2 + (d*x)/2)^3*sin(
c/2 + (d*x)/2))/(b*d*(cos(c/2 + (d*x)/2)^4 + sin(c/2 + (d*x)/2)^4 - 2*cos(
c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2)^2)) + (2*atan((4*a^13*sin(c/2 + (d*x)/
2) + 4*a^12*b*sin(c/2 + (d*x)/2) + 12*a^2*b^11*sin(c/2 + (d*x)/2) + 12*a^3
*b^10*sin(c/2 + (d*x)/2) + 15*a^4*b^9*sin(c/2 + (d*x)/2) + 15*a^5*b^8*sin(
c/2 + (d*x)/2) - 59*a^6*b^7*sin(c/2 + (d*x)/2) - 59*a^7*b^6*sin(c/2 + (d*x
)/2) + 57*a^8*b^5*sin(c/2 + (d*x)/2) + 57*a^9*b^4*sin(c/2 + (d*x)/2) - 24*
a^10*b^3*sin(c/2 + (d*x)/2) - 24*a^11*b^2*sin(c/2 + (d*x)/2))/(a*cos(c/2 +
 (d*x)/2)*(12*a*b^11 + 4*a^11*b + 4*a^12 + 12*a^2*b^10 + 15*a^3*b^9 + 15*a
^4*b^8 - 59*a^5*b^7 - 59*a^6*b^6 + 57*a^7*b^5 + 57*a^8*b^4 - 24*a^9*b^3 -
24*a^10*b^2)))*cos(c/2 + (d*x)/2)^4)/(a*d*(cos(c/2 + (d*x)/2)^4 + sin(c/2
+ (d*x)/2)^4 - 2*cos(c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2)^2)) + (2*atan((4*
a^13*sin(c/2 + (d*x)/2) + 4*a^12*b*sin(c/2 + (d*x)/2) + 12*a^2*b^11*sin(c/
2 + (d*x)/2) + 12*a^3*b^10*sin(c/2 + (d*x)/2) + 15*a^4*b^9*sin(c/2 + (d...
```

3.296 $\int \frac{\tan^2(c+dx)}{a+b\sec(c+dx)}\,dx$

3.296.1 Optimal result . . . 2135
3.296.2 Mathematica [A] (verified) . . . 2135
3.296.3 Rubi [A] (verified) . . . 2136
3.296.4 Maple [A] (verified) . . . 2139
3.296.5 Fricas [A] (verification not implemented) . . . 2139
3.296.6 Sympy [F] . . . 2140
3.296.7 Maxima [F(-2)] . . . 2140
3.296.8 Giac [B] (verification not implemented) . . . 2141
3.296.9 Mupad [B] (verification not implemented) . . . 2141

3.296.1 Optimal result

Integrand size = 21, antiderivative size = 76

$$\int \frac{\tan^2(c+dx)}{a+b\sec(c+dx)}\,dx = -\frac{x}{a}+\frac{\operatorname{arctanh}(\sin(c+dx))}{bd} - \frac{2\sqrt{a-b}\sqrt{a+b}\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{abd}$$

output
```
-x/a+arctanh(sin(d*x+c))/b/d-2*arctanh((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b
)^(1/2))*(a-b)^(1/2)*(a+b)^(1/2)/a/b/d
```

3.296.2 Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.51

$$\int \frac{\tan^2(c+dx)}{a+b\sec(c+dx)}\,dx = -\frac{bc+bdx-2\sqrt{a^2-b^2}\operatorname{arctanh}\left(\frac{(-a+b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)+a\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)-a\log(}{abd}$$

input
```
Integrate[Tan[c + d*x]^2/(a + b*Sec[c + d*x]),x]
```

output
```
-((b*c + b*d*x - 2*Sqrt[a^2 - b^2]*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqr
t[a^2 - b^2]] + a*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - a*Log[Cos[(c
+ d*x)/2] + Sin[(c + d*x)/2]])/(a*b*d))
```

3.296.3 Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.16, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.619$, Rules used = {3042, 4382, 3042, 4539, 25, 3042, 4257, 4407, 3042, 4318, 3042, 3138, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^2(c+dx)}{a+b\sec(c+dx)}\,dx$$

$$\downarrow 3042$$

$$\int \frac{\cot\left(c+dx+\frac{\pi}{2}\right)^2}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\downarrow 4382$$

$$\int \frac{\sec^2(c+dx)-1}{a+b\sec(c+dx)}dx$$

$$\downarrow 3042$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^2-1}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\downarrow 4539$$

$$\frac{\int -\frac{b+a\sec(c+dx)}{a+b\sec(c+dx)}dx}{b}+\frac{\int \sec(c+dx)dx}{b}$$

$$\downarrow 25$$

$$\frac{\int \sec(c+dx)dx}{b}-\frac{\int \frac{b+a\sec(c+dx)}{a+b\sec(c+dx)}dx}{b}$$

$$\downarrow 3042$$

$$\frac{\int \csc\left(c+dx+\frac{\pi}{2}\right)dx}{b}-\frac{\int \frac{b+a\csc\left(c+dx+\frac{\pi}{2}\right)}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{b}$$

$$\downarrow 4257$$

$$\frac{\operatorname{arctanh}(\sin(c+dx))}{bd}-\frac{\int \frac{b+a\csc\left(c+dx+\frac{\pi}{2}\right)}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{b}$$

$$\downarrow 4407$$

$$\frac{\operatorname{arctanh}(\sin(c+dx))}{bd} - \frac{\frac{(a^2-b^2)\int \frac{\sec(c+dx)}{a+b\sec(c+dx)}dx}{a} + \frac{bx}{a}}{b}$$

$$\downarrow 3042$$

$$\frac{\operatorname{arctanh}(\sin(c+dx))}{bd} - \frac{\frac{(a^2-b^2)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a} + \frac{bx}{a}}{b}$$

$$\downarrow 4318$$

$$\frac{\operatorname{arctanh}(\sin(c+dx))}{bd} - \frac{\frac{(a^2-b^2)\int \frac{1}{\frac{a\cos(c+dx)}{b}+1}dx}{ab} + \frac{bx}{a}}{b}$$

$$\downarrow 3042$$

$$\frac{\operatorname{arctanh}(\sin(c+dx))}{bd} - \frac{\frac{(a^2-b^2)\int \frac{1}{\frac{a\sin\left(c+dx+\frac{\pi}{2}\right)}{b}+1}dx}{ab} + \frac{bx}{a}}{b}$$

$$\downarrow 3138$$

$$\frac{\operatorname{arctanh}(\sin(c+dx))}{bd} - \frac{\frac{2(a^2-b^2)\int \frac{1}{\left(1-\frac{a}{b}\right)\tan^2\left(\frac{1}{2}(c+dx)\right)+\frac{a+b}{b}}d\tan\left(\frac{1}{2}(c+dx)\right)}{abd} + \frac{bx}{a}}{b}$$

$$\downarrow 221$$

$$\frac{\operatorname{arctanh}(\sin(c+dx))}{bd} - \frac{\frac{2(a^2-b^2)\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{ad\sqrt{a-b}\sqrt{a+b}} + \frac{bx}{a}}{b}$$

input
```
Int[Tan[c + d*x]^2/(a + b*Sec[c + d*x]),x]
```

output
```
ArcTanh[Sin[c + d*x]]/(b*d) - ((b*x)/a + (2*(a^2 - b^2)*ArcTanh[(Sqrt[a -
b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a*Sqrt[a - b]*Sqrt[a + b]*d))/b
```

3.296.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 4318
```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Simp[1/b   Int[1/(1 + (a/b)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4382
```
Int[cot[(c_.) + (d_.)*(x_)]^2*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_),
x_Symbol] :> Int[(-1 + Csc[c + d*x]^2)*(a + b*Csc[c + d*x])^n, x] /; FreeQ[
{a, b, c, d, n}, x] && NeQ[a^2 - b^2, 0]
```

rule 4407
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[c*(x/a), x] - Simp[(b*c - a*d)/a   Int[Csc[e + f*
x]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c
- a*d, 0]
```

rule 4539
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))/(csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)), x_Symbol] :> Simp[C/b   Int[Csc[e + f*x], x], x] + Simp[1/b   In
t[(A*b - a*C*Csc[e + f*x])/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e,
f, A, C}, x]
```

3.296.4 Maple [A] (verified)

Time = 0.81 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.42

method	result
derivativedivides	$\frac{\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{b}-\frac{2(a-b)(a+b)\operatorname{arctanh}\left(\frac{(a-b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a-b)(a+b)}}\right)}{ba\sqrt{(a-b)(a+b)}}-\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{b}-\frac{2\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a}}{d}$
default	$\frac{\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{b}-\frac{2(a-b)(a+b)\operatorname{arctanh}\left(\frac{(a-b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a-b)(a+b)}}\right)}{ba\sqrt{(a-b)(a+b)}}-\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{b}-\frac{2\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a}}{d}$
risch	$-\frac{x}{a}-\frac{\sqrt{a^2-b^2}\,\ln\left(e^{i(dx+c)}+\frac{b+i\sqrt{a^2-b^2}}{a}\right)}{dba}+\frac{\sqrt{a^2-b^2}\,\ln\left(e^{i(dx+c)}-\frac{i\sqrt{a^2-b^2}-b}{a}\right)}{dba}+\frac{\ln\left(e^{i(dx+c)}+i\right)}{db}-\frac{\ln\left(e^{i(dx}\right.}{db}$

input
```
int(tan(d*x+c)^2/(a+b*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/b*ln(tan(1/2*d*x+1/2*c)+1)-2/b*(a-b)*(a+b)/a/((a-b)*(a+b))^(1/2)*ar
ctanh((a-b)*tan(1/2*d*x+1/2*c)/((a-b)*(a+b))^(1/2))-1/b*ln(tan(1/2*d*x+1/2
*c)-1)-2/a*arctan(tan(1/2*d*x+1/2*c)))
```

3.296.5 Fricas [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 253, normalized size of antiderivative = 3.33

$$\int \frac{\tan^2(c+dx)}{a+b\sec(c+dx)}\,dx$$
$$= \left[-\frac{2\,bdx - a\log\left(\sin\left(dx+c\right)+1\right)+a\log\left(-\sin\left(dx+c\right)+1\right)-\sqrt{a^2-b^2}\log\left(\frac{2\,ab\cos(dx+c)-(a^2-2\,b^2)\cos(dx+}{a^2\cos(dx}\right.\right.$$
$$\left.\left.-\frac{2\,bdx - a\log\left(\sin\left(dx+c\right)+1\right)+a\log\left(-\sin\left(dx+c\right)+1\right)+2\sqrt{-a^2+b^2}\arctan\left(-\frac{\sqrt{-a^2+b^2}(b\cos(dx+c)+}{(a^2-b^2)\sin(dx+c)}\right.\right.\right.$$

input
```
integrate(tan(d*x+c)^2/(a+b*sec(d*x+c)),x, algorithm="fricas")
```

output
```
[-1/2*(2*b*d*x - a*log(sin(d*x + c) + 1) + a*log(-sin(d*x + c) + 1) - sqrt
(a^2 - b^2)*log((2*a*b*cos(d*x + c) - (a^2 - 2*b^2)*cos(d*x + c)^2 - 2*sqr
t(a^2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x + c) + 2*a^2 - b^2)/(a^2*cos(d*x
 + c)^2 + 2*a*b*cos(d*x + c) + b^2)))/(a*b*d), -1/2*(2*b*d*x - a*log(sin(d
*x + c) + 1) + a*log(-sin(d*x + c) + 1) + 2*sqrt(-a^2 + b^2)*arctan(-sqrt(
-a^2 + b^2)*(b*cos(d*x + c) + a)/((a^2 - b^2)*sin(d*x + c))))/(a*b*d)]
```

3.296.6 Sympy [F]

$$\int \frac{\tan^2(c+dx)}{a+b\sec(c+dx)}\,dx = \int \frac{\tan^2(c+dx)}{a+b\sec(c+dx)}\,dx$$

input
```
integrate(tan(d*x+c)**2/(a+b*sec(d*x+c)),x)
```

output
```
Integral(tan(c + d*x)**2/(a + b*sec(c + d*x)), x)
```

3.296.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\tan^2(c+dx)}{a+b\sec(c+dx)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(tan(d*x+c)^2/(a+b*sec(d*x+c)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` f
or more de
```

3.296.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 140 vs. $2(67) = 134$.

Time = 0.49 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.84

$$\int \frac{\tan^2(c+dx)}{a+b\sec(c+dx)}\,dx =$$

$$-\frac{\frac{dx+c}{a} - \frac{\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{b} + \frac{\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{b} + \frac{2\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(-2\,a+2\,b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b\tan\left(\frac{1}{2}\,dx+\right.\right.\right.}{\sqrt{-a^2+b^2}}}{\sqrt{-a^2+b^2}ab}}{d}$$

input
```
integrate(tan(d*x+c)^2/(a+b*sec(d*x+c)),x, algorithm="giac")
```

output
```
-((d*x + c)/a - log(abs(tan(1/2*d*x + 1/2*c) + 1))/b + log(abs(tan(1/2*d*x
 + 1/2*c) - 1))/b + 2*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) +
arctan(-(a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(-a^2 + b^2)
))*(a^2 - b^2)/(sqrt(-a^2 + b^2)*a*b))/d
```

3.296.9 Mupad [B] (verification not implemented)

Time = 14.41 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.59

$$\int \frac{\tan^2(c+dx)}{a+b\sec(c+dx)}\,dx = \frac{2\,\operatorname{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{b\,d} - \frac{2\,\operatorname{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{a\,d} - \frac{2\,\operatorname{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\sqrt{a^2-b^2}}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)(a+b)}\right)\sqrt{a^2-b^2}}{a\,b\,d}$$

input
```
int(tan(c + d*x)^2/(a + b/cos(c + d*x)),x)
```

output
```
(2*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/(b*d) - (2*atan(sin(c/2 +
 (d*x)/2)/cos(c/2 + (d*x)/2)))/(a*d) - (2*atanh((sin(c/2 + (d*x)/2)*(a^2 -
 b^2)^(1/2))/(cos(c/2 + (d*x)/2)*(a + b)))*(a^2 - b^2)^(1/2))/(a*b*d)
```

3.297 $\int \frac{\cot^2(c+dx)}{a+b\sec(c+dx)}\,dx$

3.297.1 Optimal result . . . 2142
3.297.2 Mathematica [A] (verified) . . . 2142
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3.297.1 Optimal result

Integrand size = 21, antiderivative size = 106

$$\int \frac{\cot^2(c+dx)}{a+b\sec(c+dx)}\,dx = -\frac{x}{a} - \frac{2b^3 \operatorname{arctanh}\left(\frac{\sqrt{a^2-b^2}\tan\left(\frac{1}{2}(c+dx)\right)}{a+b}\right)}{a\left(a^2-b^2\right)^{3/2}d} - \frac{a\cot(c+dx)}{\left(a^2-b^2\right)d} + \frac{b\csc(c+dx)}{\left(a^2-b^2\right)d}$$

output
```
-x/a-2*b^3*arctanh((a^2-b^2)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b))/a/(a^2-b^2)^(
3/2)/d-a*cot(d*x+c)/(a^2-b^2)/d+b*csc(d*x+c)/(a^2-b^2)/d
```

3.297.2 Mathematica [A] (verified)

Time = 0.61 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.39

$$\int \frac{\cot^2(c+dx)}{a+b\sec(c+dx)}\,dx = -\frac{\csc\left(\frac{1}{2}(c+dx)\right)\sec\left(\frac{1}{2}(c+dx)\right)\left(-2b^3\operatorname{arctanh}\left(\frac{(-a+b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)\sin(c+dx) + \sqrt{a^2-b^2}(-ab+a^2\cos$$
$$2a(a-b)(a+b)\sqrt{a^2-b^2}d$$

input
```
Integrate[Cot[c + d*x]^2/(a + b*Sec[c + d*x]),x]
```

output
```
-1/2*(Csc[(c + d*x)/2]*Sec[(c + d*x)/2]*(-2*b^3*ArcTanh[((-a + b)*Tan[(c +
 d*x)/2])/Sqrt[a^2 - b^2]]*Sin[c + d*x] + Sqrt[a^2 - b^2]*(-(a*b) + a^2*Co
s[c + d*x] + (a^2 - b^2)*(c + d*x)*Sin[c + d*x])))/(a*(a - b)*(a + b)*Sqrt
[a^2 - b^2]*d)
```

3.297.3 Rubi [A] (verified)

Time = 0.74 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.18, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.762$, Rules used = {3042, 4386, 3042, 25, 3381, 25, 3042, 25, 3086, 24, 3214, 3042, 3138, 221, 3954, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^2(c+dx)}{a+b\sec(c+dx)}\,dx$$

$$\downarrow 3042$$

$$\int \frac{1}{\cot\left(c+dx+\frac{\pi}{2}\right)^2\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)}dx$$

$$\downarrow 4386$$

$$\int \frac{\cos(c+dx)\cot^2(c+dx)}{a\cos(c+dx)+b}dx$$

$$\downarrow 3042$$

$$\int -\frac{\sin\left(c+dx-\frac{\pi}{2}\right)^3}{\cos\left(c+dx-\frac{\pi}{2}\right)^2\left(b-a\sin\left(c+dx-\frac{\pi}{2}\right)\right)}dx$$

$$\downarrow 25$$

$$-\int \frac{\sin\left(\frac{1}{2}(2c-\pi)+dx\right)^3}{\cos\left(\frac{1}{2}(2c-\pi)+dx\right)^2\left(b-a\sin\left(\frac{1}{2}(2c-\pi)+dx\right)\right)}dx$$

$$\downarrow 3381$$

$$-\frac{b^2\int -\frac{\cos(c+dx)}{b+a\cos(c+dx)}dx}{a^2-b^2}+\frac{a\int\cot^2(c+dx)dx}{a^2-b^2}+\frac{b\int -\cot(c+dx)\csc(c+dx)dx}{a^2-b^2}$$

$$\downarrow 25$$

$$\frac{b^2\int \frac{\cos(c+dx)}{b+a\cos(c+dx)}dx}{a^2-b^2}+\frac{a\int\cot^2(c+dx)dx}{a^2-b^2}-\frac{b\int\cot(c+dx)\csc(c+dx)dx}{a^2-b^2}$$

$$\downarrow 3042$$

$$\frac{b^2 \int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)}{b+a\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{a^2-b^2} + \frac{a\int \tan\left(c+dx+\frac{\pi}{2}\right)^2 dx}{a^2-b^2} - \frac{b\int -\sec\left(c+dx-\frac{\pi}{2}\right)\tan\left(c+dx-\frac{\pi}{2}\right)dx}{a^2-b^2}$$

$$\downarrow 25$$

$$\frac{b^2 \int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)}{b+a\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{a^2-b^2} + \frac{a\int \tan\left(c+dx+\frac{\pi}{2}\right)^2 dx}{a^2-b^2} + \frac{b\int \sec\left(\frac{1}{2}(2c-\pi)+dx\right)\tan\left(\frac{1}{2}(2c-\pi)+dx\right)dx}{a^2-b^2}$$

$$\downarrow 3086$$

$$\frac{b^2 \int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)}{b+a\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{a^2-b^2} + \frac{a\int \tan\left(c+dx+\frac{\pi}{2}\right)^2 dx}{a^2-b^2} + \frac{b\int 1 d\csc(c+dx)}{d\left(a^2-b^2\right)}$$

$$\downarrow 24$$

$$\frac{b^2 \int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)}{b+a\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{a^2-b^2} + \frac{a\int \tan\left(c+dx+\frac{\pi}{2}\right)^2 dx}{a^2-b^2} + \frac{b\csc(c+dx)}{d\left(a^2-b^2\right)}$$

$$\downarrow 3214$$

$$\frac{b^2\left(\frac{x}{a} - \frac{b\int \frac{1}{b+a\cos(c+dx)}dx}{a}\right)}{a^2-b^2} + \frac{a\int \tan\left(c+dx+\frac{\pi}{2}\right)^2 dx}{a^2-b^2} + \frac{b\csc(c+dx)}{d\left(a^2-b^2\right)}$$

$$\downarrow 3042$$

$$\frac{b^2\left(\frac{x}{a} - \frac{b\int \frac{1}{b+a\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{a}\right)}{a^2-b^2} + \frac{a\int \tan\left(c+dx+\frac{\pi}{2}\right)^2 dx}{a^2-b^2} + \frac{b\csc(c+dx)}{d\left(a^2-b^2\right)}$$

$$\downarrow 3138$$

$$\frac{b^2\left(\frac{x}{a} - \frac{2b\int \frac{1}{-\left((a-b)\tan^2\left(\frac{1}{2}(c+dx)\right)\right)+a+b}d\tan\left(\frac{1}{2}(c+dx)\right)}{ad}\right)}{a^2-b^2} + \frac{a\int \tan\left(c+dx+\frac{\pi}{2}\right)^2 dx}{a^2-b^2} + \frac{b\csc(c+dx)}{d\left(a^2-b^2\right)}$$

$$\downarrow 221$$

$$\frac{a\int \tan\left(c+dx+\frac{\pi}{2}\right)^2 dx}{a^2-b^2} + \frac{b^2\left(\frac{x}{a} - \frac{2b\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{ad\sqrt{a-b}\sqrt{a+b}}\right)}{a^2-b^2} + \frac{b\csc(c+dx)}{d\left(a^2-b^2\right)}$$

$$\downarrow 3954$$

$$\frac{a\left(-\int 1dx-\frac{\cot(c+dx)}{d}\right)}{a^2-b^2}+\frac{b^2\left(\frac{x}{a}-\frac{2b\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{ad\sqrt{a-b}\sqrt{a+b}}\right)}{a^2-b^2}+\frac{b\csc(c+dx)}{d\left(a^2-b^2\right)}$$

$$\downarrow 24$$

$$\frac{b^2\left(\frac{x}{a}-\frac{2b\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{ad\sqrt{a-b}\sqrt{a+b}}\right)}{a^2-b^2}+\frac{a\left(-\frac{\cot(c+dx)}{d}-x\right)}{a^2-b^2}+\frac{b\csc(c+dx)}{d\left(a^2-b^2\right)}$$

input
```
Int[Cot[c + d*x]^2/(a + b*Sec[c + d*x]),x]
```

output
```
(b^2*(x/a - (2*b*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a*S
qrt[a - b]*Sqrt[a + b]*d)))/(a^2 - b^2) + (a*(-x - Cot[c + d*x]/d))/(a^2 -
 b^2) + (b*Csc[c + d*x])/((a^2 - b^2)*d)
```

3.297.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3086
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[a/f   Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2
), x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2
] &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

rule 3214
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Simp[(b*c - a*d)/d   Int[1/(c + d
*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

rule 3381
```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[a*(d^2/(a^2
- b^2))   Int[(g*Cos[e + f*x])^p*(d*Sin[e + f*x])^(n - 2), x], x] + (-Simp[
b*(d/(a^2 - b^2))   Int[(g*Cos[e + f*x])^p*(d*Sin[e + f*x])^(n - 1), x], x]
 - Simp[a^2*(d^2/(g^2*(a^2 - b^2)))   Int[(g*Cos[e + f*x])^(p + 2)*((d*Sin[
e + f*x])^(n - 2)/(a + b*Sin[e + f*x])), x], x]) /; FreeQ[{a, b, d, e, f, g
}, x] && NeQ[a^2 - b^2, 0] && IntegersQ[2*n, 2*p] && LtQ[p, -1] && GtQ[n, 1
]
```

rule 3954
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Simp[b^2   Int[(b*Tan[c + d*x])^(n - 2), x]
, x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

rule 4386
```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Int[Cos[c + d*x]^m*((b + a*Sin[c + d*x])^n/Sin[c + d*x]^(m
 + n)), x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[n] &&
 IntegerQ[m] && (IntegerQ[m/2] || LeQ[m, 1])
```

3.297.4 Maple [A] (verified)

Time = 0.76 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.08

method	result
derivativedivides	$\frac{\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2a-2b}-\frac{2\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a}-\frac{2b^3\operatorname{arctanh}\left(\frac{(a-b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a-b)(a+b)}}\right)}{(a-b)(a+b)a\sqrt{(a-b)(a+b)}}-\frac{1}{2(a+b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}}{d}$
default	$\frac{\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2a-2b}-\frac{2\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a}-\frac{2b^3\operatorname{arctanh}\left(\frac{(a-b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a-b)(a+b)}}\right)}{(a-b)(a+b)a\sqrt{(a-b)(a+b)}}-\frac{1}{2(a+b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}}{d}$
risch	$-\frac{x}{a}-\frac{2i\left(-b\,e^{i(dx+c)}+a\right)}{d(a^2-b^2)\left(e^{2i(dx+c)}-1\right)}-\frac{b^3\ln\left(e^{i(dx+c)}+\frac{ia^2-ib^2+b\sqrt{a^2-b^2}}{\sqrt{a^2-b^2}\,a}\right)}{\sqrt{a^2-b^2}\,(a+b)(a-b)da}+\frac{b^3\ln\left(e^{i(dx+c)}+\frac{-ia^2+ib^2+b\sqrt{a^2-b^2}}{a\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}\,(a+b)(a-b)da}$

input
```
int(cot(d*x+c)^2/(a+b*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/2*tan(1/2*d*x+1/2*c)/(a-b)-2/a*arctan(tan(1/2*d*x+1/2*c))-2/(a-b)/(
a+b)*b^3/a/((a-b)*(a+b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a-b)*(a+
b))^(1/2))-1/2/(a+b)/tan(1/2*d*x+1/2*c))
```

3.297.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 362, normalized size of antiderivative = 3.42

$$\int \frac{\cot^2(c+dx)}{a+b\sec(c+dx)}\,dx$$
$$= \left[-\frac{\sqrt{a^2-b^2}b^3\log\left(\frac{2ab\cos(dx+c)-(a^2-2b^2)\cos(dx+c)^2+2\sqrt{a^2-b^2}(b\cos(dx+c)+a)\sin(dx+c)+2a^2-b^2}{a^2\cos(dx+c)^2+2ab\cos(dx+c)+b^2}\right)\sin(dx+c)-2a^3b}{2(a^5-2a^3b^2+ab^4)d\sin(dx+}\right.$$
$$\left.-\frac{\sqrt{-a^2+b^2}b^3\arctan\left(-\frac{\sqrt{-a^2+b^2}(b\cos(dx+c)+a)}{(a^2-b^2)\sin(dx+c)}\right)\sin(dx+c)-a^3b+ab^3+(a^4-2a^2b^2+b^4)dx\sin(dx+}{(a^5-2a^3b^2+ab^4)d\sin(dx+c)}\right]$$

input
```
integrate(cot(d*x+c)^2/(a+b*sec(d*x+c)),x, algorithm="fricas")
```

output
```
[-1/2*(sqrt(a^2 - b^2)*b^3*log((2*a*b*cos(d*x + c) - (a^2 - 2*b^2)*cos(d*x
 + c)^2 + 2*sqrt(a^2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x + c) + 2*a^2 - b^
2)/(a^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + b^2))*sin(d*x + c) - 2*a^3*b
 + 2*a*b^3 + 2*(a^4 - 2*a^2*b^2 + b^4)*d*x*sin(d*x + c) + 2*(a^4 - a^2*b^2
)*cos(d*x + c))/((a^5 - 2*a^3*b^2 + a*b^4)*d*sin(d*x + c)), -(sqrt(-a^2 +
b^2)*b^3*arctan(-sqrt(-a^2 + b^2)*(b*cos(d*x + c) + a)/((a^2 - b^2)*sin(d*
x + c)))*sin(d*x + c) - a^3*b + a*b^3 + (a^4 - 2*a^2*b^2 + b^4)*d*x*sin(d*
x + c) + (a^4 - a^2*b^2)*cos(d*x + c))/((a^5 - 2*a^3*b^2 + a*b^4)*d*sin(d*
x + c))]
```

3.297.6 Sympy [F]

$$\int \frac{\cot^2(c+dx)}{a+b\sec(c+dx)}\,dx = \int \frac{\cot^2(c+dx)}{a+b\sec(c+dx)}\,dx$$

input
```
integrate(cot(d*x+c)**2/(a+b*sec(d*x+c)),x)
```

output
```
Integral(cot(c + d*x)**2/(a + b*sec(c + d*x)), x)
```

3.297.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\cot^2(c+dx)}{a+b\sec(c+dx)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(cot(d*x+c)^2/(a+b*sec(d*x+c)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` f
or more de
```

3.297.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 582 vs. $2(101) = 202$.

Time = 0.38 (sec) , antiderivative size = 582, normalized size of antiderivative = 5.49

$$\int \frac{\cot^2(c+dx)}{a+b\sec(c+dx)}\,dx =$$

$$-\frac{2\left(a^5-a^4b-2a^3b^2+3a^2b^3+ab^4-2b^5-a^2\left|-a^3+ab^2\right|+ab\left|-a^3+ab^2\right|+b^2\left|-a^3+ab^2\right|\right)\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor+\arctan\left(\frac{\tan\left(\frac{1}{2}dx\right.}{\sqrt{-\frac{a^2b-b^3+\sqrt{\left(a^3+a^2b-ab^2-b^3\right)}}{a^3-a^2b}}}\right.\right.}{a^2b\left|-a^3+ab^2\right|-b^3\left|-a^3+ab^2\right|+\left(a^3-ab^2\right)^2}$$

input
```
integrate(cot(d*x+c)^2/(a+b*sec(d*x+c)),x, algorithm="giac")
```

output
```
-1/2*(2*(a^5 - a^4*b - 2*a^3*b^2 + 3*a^2*b^3 + a*b^4 - 2*b^5 - a^2*abs(-a^
3 + a*b^2) + a*b*abs(-a^3 + a*b^2) + b^2*abs(-a^3 + a*b^2))*(pi*floor(1/2*
(d*x + c)/pi + 1/2) + arctan(tan(1/2*d*x + 1/2*c)/sqrt(-(a^2*b - b^3 + sqr
t((a^3 + a^2*b - a*b^2 - b^3)*(a^3 - a^2*b - a*b^2 + b^3) + (a^2*b - b^3)^
2))/(a^3 - a^2*b - a*b^2 + b^3))))/(a^2*b*abs(-a^3 + a*b^2) - b^3*abs(-a^3
 + a*b^2) + (a^3 - a*b^2)^2) + 2*((a^2 - a*b - b^2)*sqrt(-a^2 + b^2)*abs(-
a^3 + a*b^2)*abs(-a + b) + (a^5 - a^4*b - 2*a^3*b^2 + 3*a^2*b^3 + a*b^4 -
2*b^5)*sqrt(-a^2 + b^2)*abs(-a + b))*(pi*floor(1/2*(d*x + c)/pi + 1/2) + a
rctan(tan(1/2*d*x + 1/2*c)/sqrt(-(a^2*b - b^3 - sqrt((a^3 + a^2*b - a*b^2
- b^3)*(a^3 - a^2*b - a*b^2 + b^3) + (a^2*b - b^3)^2))/(a^3 - a^2*b - a*b^
2 + b^3))))/((a^3 - a*b^2)^2*(a^2 - 2*a*b + b^2) - (a^4*b - 2*a^3*b^2 + 2*
a*b^4 - b^5)*abs(-a^3 + a*b^2)) - tan(1/2*d*x + 1/2*c)/(a - b) + 1/((a + b
)*tan(1/2*d*x + 1/2*c)))/d
```

3.297.9 Mupad [B] (verification not implemented)

Time = 16.55 (sec) , antiderivative size = 1002, normalized size of antiderivative = 9.45

$$\int \frac{\cot^2(c+dx)}{a+b\sec(c+dx)}\,dx =$$

$$-\frac{1\mathrm{i}\cos(c+dx)\,a^6-a^5\,b\,1\mathrm{i}-2\mathrm{i}\cos(c+dx)\,a^4\,b^2+a^3\,b^3\,2\mathrm{i}+1\mathrm{i}\cos(c+dx)\,a^2\,b^4-a\,b^5\,1\mathrm{i}}{1\mathrm{i}\,d\sin(c+dx)\,a^7-3\mathrm{i}\,d\sin(c+dx)\,a^5\,b^2+3\mathrm{i}\,d\sin(c+dx)\,a^3\,b^4-1\mathrm{i}\,d\sin(c+dx)\,a\,b^6}$$

$$+\frac{-a^6\operatorname{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{dx}{2}\right)}{\cos\left(\frac{c}{2}+\frac{dx}{2}\right)}\right)2\mathrm{i}+b^6\operatorname{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{dx}{2}\right)}{\cos\left(\frac{c}{2}+\frac{dx}{2}\right)}\right)2\mathrm{i}-a^2\,b^4\operatorname{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{dx}{2}\right)}{\cos\left(\frac{c}{2}+\frac{dx}{2}\right)}\right)6\mathrm{i}+a^4\,b^2\operatorname{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{dx}{2}\right)}{\cos\left(\frac{c}{2}+\frac{dx}{2}\right)}\right.}$$

input
```
int(cot(c + d*x)^2/(a + b/cos(c + d*x)),x)
```

output
```
(b^6*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*2i - a^6*atan(sin(c/2 + (
d*x)/2)/cos(c/2 + (d*x)/2))*2i - a^2*b^4*atan(sin(c/2 + (d*x)/2)/cos(c/2 +
 (d*x)/2))*6i + a^4*b^2*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*6i + b
^3*atanh((2*b^7*sin(c/2 + (d*x)/2)*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)^(3/
2) - a^13*sin(c/2 + (d*x)/2)*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)^(1/2) + 2
*b^13*sin(c/2 + (d*x)/2)*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)^(1/2) - 9*a^2
*b^11*sin(c/2 + (d*x)/2)*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)^(1/2) + 3*a^3
*b^10*sin(c/2 + (d*x)/2)*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)^(1/2) + 18*a^
4*b^9*sin(c/2 + (d*x)/2)*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)^(1/2) - 12*a^
5*b^8*sin(c/2 + (d*x)/2)*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)^(1/2) - 21*a^
6*b^7*sin(c/2 + (d*x)/2)*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)^(1/2) + 19*a^
7*b^6*sin(c/2 + (d*x)/2)*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)^(1/2) + 15*a^
8*b^5*sin(c/2 + (d*x)/2)*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)^(1/2) - 15*a^
9*b^4*sin(c/2 + (d*x)/2)*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)^(1/2) - 6*a^1
0*b^3*sin(c/2 + (d*x)/2)*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)^(1/2) + 6*a^1
1*b^2*sin(c/2 + (d*x)/2)*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)^(1/2) + a^12*
b*sin(c/2 + (d*x)/2)*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)^(1/2))/(cos(c/2 +
 (d*x)/2)*(a^16 - 3*a^2*b^14 + 18*a^4*b^12 - 46*a^6*b^10 + 65*a^8*b^8 - 55
*a^10*b^6 + 28*a^12*b^4 - 8*a^14*b^2)))*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2
)^(1/2)*2i)/(a^7*d*1i + a^3*b^4*d*3i - a^5*b^2*d*3i - a*b^6*d*1i) - (a^...
```

3.298 $\int \frac{\cot^4(c+dx)}{a+b\sec(c+dx)}\,dx$

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3.298.1 Optimal result

Integrand size = 21, antiderivative size = 177

$$\int \frac{\cot^4(c+dx)}{a+b\sec(c+dx)}\,dx = \frac{x}{a} - \frac{2b^5 \operatorname{arctanh}\left(\frac{\sqrt{a^2-b^2}\tan\left(\frac{1}{2}(c+dx)\right)}{a+b}\right)}{a\left(a^2-b^2\right)^{5/2}d} + \frac{a(a^2-2b^2)\cot(c+dx)}{\left(a^2-b^2\right)^2 d} - \frac{a\cot^3(c+dx)}{3\left(a^2-b^2\right)d} - \frac{b(a^2-2b^2)\csc(c+dx)}{\left(a^2-b^2\right)^2 d} + \frac{b\csc^3(c+dx)}{3\left(a^2-b^2\right)d}$$

output

```
x/a-2*b^5*arctanh((a^2-b^2)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b))/a/(a^2-b^2)^(5
/2)/d+a*(a^2-2*b^2)*cot(d*x+c)/(a^2-b^2)^2/d-1/3*a*cot(d*x+c)^3/(a^2-b^2)/
d-b*(a^2-2*b^2)*csc(d*x+c)/(a^2-b^2)^2/d+1/3*b*csc(d*x+c)^3/(a^2-b^2)/d
```

3.298.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 416 vs. $2(177) = 354$.

Time = 6.44 (sec) , antiderivative size = 416, normalized size of antiderivative = 2.35

$$\begin{aligned}\int \frac{\cot^4(c+dx)}{a+b\sec(c+dx)}\,dx &= \frac{(c+dx)(b+a\cos(c+dx))\sec(c+dx)}{ad(a+b\sec(c+dx))} \\ &+ \frac{2b^5\operatorname{arctanh}\left(\frac{(-a+b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)(b+a\cos(c+dx))\sec(c+dx)}{a\sqrt{a^2-b^2}\left(-a^2+b^2\right)^2 d(a+b\sec(c+dx))} \\ &+ \frac{\left(8a\cos\left(\frac{1}{2}(c+dx)\right)+11b\cos\left(\frac{1}{2}(c+dx)\right)\right)(b+a\cos(c+dx))\csc\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)}{12(a+b)^2 d(a+b\sec(c+dx))} \\ &- \frac{(b+a\cos(c+dx))\cot\left(\frac{1}{2}(c+dx)\right)\csc^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)}{24(a+b)d(a+b\sec(c+dx))} \\ &+ \frac{(b+a\cos(c+dx))\sec\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\left(-8a\sin\left(\frac{1}{2}(c+dx)\right)+11b\sin\left(\frac{1}{2}(c+dx)\right)\right)}{12(-a+b)^2 d(a+b\sec(c+dx))} \\ &- \frac{(b+a\cos(c+dx))\sec^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\tan\left(\frac{1}{2}(c+dx)\right)}{24(-a+b)d(a+b\sec(c+dx))}\end{aligned}$$

input
```
Integrate[Cot[c + d*x]^4/(a + b*Sec[c + d*x]),x]
```

output
```
((c + d*x)*(b + a*Cos[c + d*x])*Sec[c + d*x])/(a*d*(a + b*Sec[c + d*x])) +
 (2*b^5*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]]*(b + a*Cos[c
+ d*x])*Sec[c + d*x])/(a*Sqrt[a^2 - b^2]*(-a^2 + b^2)^2*d*(a + b*Sec[c + d
*x])) + ((8*a*Cos[(c + d*x)/2] + 11*b*Cos[(c + d*x)/2])*(b + a*Cos[c + d*x
])*Csc[(c + d*x)/2]*Sec[c + d*x])/(12*(a + b)^2*d*(a + b*Sec[c + d*x])) -
((b + a*Cos[c + d*x])*Cot[(c + d*x)/2]*Csc[(c + d*x)/2]^2*Sec[c + d*x])/(2
4*(a + b)*d*(a + b*Sec[c + d*x])) + ((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]
*Sec[c + d*x]*(-8*a*Sin[(c + d*x)/2] + 11*b*Sin[(c + d*x)/2]))/(12*(-a + b
)^2*d*(a + b*Sec[c + d*x])) - ((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Sec
[c + d*x]*Tan[(c + d*x)/2])/(24*(-a + b)*d*(a + b*Sec[c + d*x]))
```

3.298.3 Rubi [A] (verified)

Time = 1.34 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.25, number of steps used = 26, number of rules used = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 1.190$, Rules used = {3042, 4386, 3042, 25, 3381, 25, 3042, 25, 3086, 2009, 3381, 25, 3042, 25, 3086, 24, 3214, 3042, 3138, 221, 3954, 24, 3042, 3954, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^4(c+dx)}{a+b\sec(c+dx)}\,dx$$

$$\downarrow 3042$$

$$\int \frac{1}{\cot\left(c+dx+\frac{\pi}{2}\right)^4\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)}dx$$

$$\downarrow 4386$$

$$\int \frac{\cos(c+dx)\cot^4(c+dx)}{a\cos(c+dx)+b}dx$$

$$\downarrow 3042$$

$$\int -\frac{\sin\left(c+dx-\frac{\pi}{2}\right)^5}{\cos\left(c+dx-\frac{\pi}{2}\right)^4\left(b-a\sin\left(c+dx-\frac{\pi}{2}\right)\right)}dx$$

$$\downarrow 25$$

$$-\int \frac{\sin\left(\frac{1}{2}(2c-\pi)+dx\right)^5}{\cos\left(\frac{1}{2}(2c-\pi)+dx\right)^4\left(b-a\sin\left(\frac{1}{2}(2c-\pi)+dx\right)\right)}dx$$

$$\downarrow 3381$$

$$\frac{a\int \cot^4(c+dx)dx}{a^2-b^2} - \frac{b^2\int -\frac{\cos(c+dx)\cot^2(c+dx)}{b+a\cos(c+dx)}dx}{a^2-b^2} + \frac{b\int -\cot^3(c+dx)\csc(c+dx)dx}{a^2-b^2}$$

$$\downarrow 25$$

$$\frac{a\int \cot^4(c+dx)dx}{a^2-b^2} + \frac{b^2\int \frac{\cos(c+dx)\cot^2(c+dx)}{b+a\cos(c+dx)}dx}{a^2-b^2} - \frac{b\int \cot^3(c+dx)\csc(c+dx)dx}{a^2-b^2}$$

$$\downarrow 3042$$

$$\frac{a\int \tan\left(c+dx+\frac{\pi}{2}\right)^4 dx}{a^2-b^2} + \frac{b^2\int -\frac{\sin\left(c+dx-\frac{\pi}{2}\right)^3}{\cos\left(c+dx-\frac{\pi}{2}\right)^2\left(b-a\sin\left(c+dx-\frac{\pi}{2}\right)\right)}dx}{a^2-b^2} - \frac{b\int -\sec\left(c+dx-\frac{\pi}{2}\right)\tan\left(c+dx-\frac{\pi}{2}\right)^3 dx}{a^2-b^2}$$

$$\downarrow 25$$

$$\frac{a\int \tan\left(c+dx+\frac{\pi}{2}\right)^4 dx}{a^2-b^2} - \frac{b^2\int \frac{\sin\left(\frac{1}{2}(2c-\pi)+dx\right)^3}{\cos\left(\frac{1}{2}(2c-\pi)+dx\right)^2\left(b-a\sin\left(\frac{1}{2}(2c-\pi)+dx\right)\right)}dx}{a^2-b^2} + \frac{b\int \sec\left(\frac{1}{2}(2c-\pi)+dx\right)\tan\left(\frac{1}{2}(2c-\pi)+dx\right)^3 dx}{a^2-b^2}$$

$$\downarrow 3086$$

$$\frac{a\int \tan\left(c+dx+\frac{\pi}{2}\right)^4 dx}{a^2-b^2} + \frac{b\int \left(\csc^2(c+dx)-1\right) d\csc(c+dx)}{d\left(a^2-b^2\right)} - \frac{b^2\int \frac{\sin\left(\frac{1}{2}(2c-\pi)+dx\right)^3}{\cos\left(\frac{1}{2}(2c-\pi)+dx\right)^2\left(b-a\sin\left(\frac{1}{2}(2c-\pi)+dx\right)\right)}dx}{a^2-b^2}$$

$$\downarrow 2009$$

$$\frac{a\int \tan\left(c+dx+\frac{\pi}{2}\right)^4 dx}{a^2-b^2} - \frac{b^2\int \frac{\sin\left(\frac{1}{2}(2c-\pi)+dx\right)^3}{\cos\left(\frac{1}{2}(2c-\pi)+dx\right)^2\left(b-a\sin\left(\frac{1}{2}(2c-\pi)+dx\right)\right)}dx}{a^2-b^2} + \frac{b\left(\frac{1}{3}\csc^3(c+dx)-\csc(c+dx)\right)}{d\left(a^2-b^2\right)}$$

$$\downarrow 3381$$

$$\frac{a\int \tan\left(c+dx+\frac{\pi}{2}\right)^4 dx}{a^2-b^2} - \frac{b^2\left(\frac{b^2\int -\frac{\cos(c+dx)}{b+a\cos(c+dx)}dx}{a^2-b^2} - \frac{a\int \cot^2(c+dx)dx}{a^2-b^2} - \frac{b\int -\cot(c+dx)\csc(c+dx)dx}{a^2-b^2}\right)}{a^2-b^2} + \frac{b\left(\frac{1}{3}\csc^3(c+dx)-\csc(c+dx)\right)}{d\left(a^2-b^2\right)}$$

$$\downarrow 25$$

$$\frac{a\int \tan\left(c+dx+\frac{\pi}{2}\right)^4 dx}{a^2-b^2} - \frac{b^2\left(-\frac{b^2\int \frac{\cos(c+dx)}{b+a\cos(c+dx)}dx}{a^2-b^2} - \frac{a\int \cot^2(c+dx)dx}{a^2-b^2} + \frac{b\int \cot(c+dx)\csc(c+dx)dx}{a^2-b^2}\right)}{a^2-b^2} + \frac{b\left(\frac{1}{3}\csc^3(c+dx)-\csc(c+dx)\right)}{d\left(a^2-b^2\right)}$$

$$\downarrow 3042$$

$$\frac{a\int \tan\left(c+dx+\frac{\pi}{2}\right)^4 dx}{a^2-b^2} - \frac{b^2\left(-\frac{b^2\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)}{b+a\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{a^2-b^2} - \frac{a\int \tan\left(c+dx+\frac{\pi}{2}\right)^2 dx}{a^2-b^2} + \frac{b\int -\sec\left(c+dx-\frac{\pi}{2}\right)\tan\left(c+dx-\frac{\pi}{2}\right)dx}{a^2-b^2}\right)}{a^2-b^2} + \frac{b\left(\frac{1}{3}\csc^3(c+dx)-\csc(c+dx)\right)}{d\left(a^2-b^2\right)}$$

$$\downarrow 25$$

$$\frac{a\int \tan\left(c+dx+\frac{\pi}{2}\right)^4 dx}{a^2-b^2} - \frac{b^2\left(-\frac{b^2\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)}{b+a\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{a^2-b^2} - \frac{a\int \tan\left(c+dx+\frac{\pi}{2}\right)^2 dx}{a^2-b^2} - \frac{b\int \sec\left(\frac{1}{2}(2c-\pi)+dx\right)\tan\left(\frac{1}{2}(2c-\pi)+dx\right)dx}{a^2-b^2}\right)}{a^2-b^2} + \frac{b\left(\frac{1}{3}\csc^3(c+dx)-\csc(c+dx)\right)}{d\left(a^2-b^2\right)}$$

$$\downarrow 3086$$

$$\frac{a\int \tan\left(c+dx+\frac{\pi}{2}\right)^4 dx}{a^2-b^2} - \frac{b^2\left(-\frac{b^2\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)}{b+a\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{a^2-b^2} - \frac{a\int \tan\left(c+dx+\frac{\pi}{2}\right)^2 dx}{a^2-b^2} - \frac{b\int 1d\csc(c+dx)}{d(a^2-b^2)}\right)}{a^2-b^2} + \frac{b\left(\frac{1}{3}\csc^3(c+dx)-\csc(c+dx)\right)}{d\left(a^2-b^2\right)}$$

$$\downarrow 24$$

$$\frac{a\int \tan\left(c+dx+\frac{\pi}{2}\right)^4 dx}{a^2-b^2} - \frac{b^2\left(-\frac{b^2\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)}{b+a\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{a^2-b^2} - \frac{a\int \tan\left(c+dx+\frac{\pi}{2}\right)^2 dx}{a^2-b^2} - \frac{b\csc(c+dx)}{d(a^2-b^2)}\right)}{a^2-b^2} + \frac{b\left(\frac{1}{3}\csc^3(c+dx)-\csc(c+dx)\right)}{d\left(a^2-b^2\right)}$$

$$\downarrow 3214$$

$$\frac{a\int \tan\left(c+dx+\frac{\pi}{2}\right)^4 dx}{a^2-b^2} - \frac{b^2\left(-\frac{b^2\left(\frac{x}{a}-\frac{b\int \frac{1}{b+a\cos(c+dx)}dx}{a}\right)}{a^2-b^2} - \frac{a\int \tan\left(c+dx+\frac{\pi}{2}\right)^2 dx}{a^2-b^2} - \frac{b\csc(c+dx)}{d(a^2-b^2)}\right)}{a^2-b^2} + \frac{b\left(\frac{1}{3}\csc^3(c+dx)-\csc(c+dx)\right)}{d\left(a^2-b^2\right)}$$

$$\downarrow 3042$$

$$\frac{a\int \tan\left(c+dx+\frac{\pi}{2}\right)^4 dx}{a^2-b^2} - \frac{b^2\left(-\frac{b^2\left(\frac{x}{a}-\frac{b\int \frac{1}{b+a\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{a}\right)}{a^2-b^2} - \frac{a\int \tan\left(c+dx+\frac{\pi}{2}\right)^2 dx}{a^2-b^2} - \frac{b\csc(c+dx)}{d(a^2-b^2)}\right)}{a^2-b^2} + \frac{b\left(\frac{1}{3}\csc^3(c+dx)-\csc(c+dx)\right)}{d\left(a^2-b^2\right)}$$

$$\downarrow 3138$$

$$\frac{a\int \tan\left(c+dx+\frac{\pi}{2}\right)^4 dx}{a^2-b^2} -$$

$$b^2\left(-\frac{b^2\left(\frac{x}{a}-\frac{2b\int \frac{1}{-\left((a-b)\tan^2\left(\frac{1}{2}(c+dx)\right)\right)+a+b}d\tan\left(\frac{1}{2}(c+dx)\right)}{ad}\right)}{a^2-b^2} - \frac{a\int \tan\left(c+dx+\frac{\pi}{2}\right)^2 dx}{a^2-b^2} - \frac{b\csc(c+dx)}{d(a^2-b^2)}\right)$$

$$\frac{}{a^2-b^2} +$$

$$\frac{b\left(\frac{1}{3}\csc^3(c+dx)-\csc(c+dx)\right)}{d\left(a^2-b^2\right)}$$

$$\downarrow 221$$

$$b^2\left(-\frac{a\int \tan\left(c+dx+\frac{\pi}{2}\right)^2 dx}{a^2-b^2} - \frac{b^2\left(\frac{x}{a}-\frac{2b\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{ad\sqrt{a-b}\sqrt{a+b}}\right)}{a^2-b^2} - \frac{b\csc(c+dx)}{d(a^2-b^2)}\right)$$

$$-\frac{}{a^2-b^2} +$$

$$\frac{a\int \tan\left(c+dx+\frac{\pi}{2}\right)^4 dx}{a^2-b^2} + \frac{b\left(\frac{1}{3}\csc^3(c+dx)-\csc(c+dx)\right)}{d\left(a^2-b^2\right)}$$

$$\downarrow 3954$$

$$b^2\left(-\frac{a\left(-\int 1dx-\frac{\cot(c+dx)}{d}\right)}{a^2-b^2} - \frac{b^2\left(\frac{x}{a}-\frac{2b\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{ad\sqrt{a-b}\sqrt{a+b}}\right)}{a^2-b^2} - \frac{b\csc(c+dx)}{d(a^2-b^2)}\right)$$

$$-\frac{}{a^2-b^2} +$$

$$\frac{a\left(-\int \cot^2(c+dx)dx-\frac{\cot^3(c+dx)}{3d}\right)}{a^2-b^2} + \frac{b\left(\frac{1}{3}\csc^3(c+dx)-\csc(c+dx)\right)}{d\left(a^2-b^2\right)}$$

$$\downarrow 24$$

$$\frac{a\left(-\int \cot^2(c+dx)dx-\frac{\cot^3(c+dx)}{3d}\right)}{a^2-b^2}-$$
$$b^2\left(-\frac{b^2\left(\frac{x}{a}-\frac{2b\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{ad\sqrt{a-b}\sqrt{a+b}}\right)}{a^2-b^2}-\frac{a\left(-\frac{\cot(c+dx)}{d}-x\right)}{a^2-b^2}-\frac{b\csc(c+dx)}{d(a^2-b^2)}\right)$$
$$\frac{a^2-b^2}{\frac{b\left(\frac{1}{3}\csc^3(c+dx)-\csc(c+dx)\right)}{d\left(a^2-b^2\right)}}+$$

$$\downarrow 3042$$

$$\frac{a\left(-\int \tan\left(c+dx+\frac{\pi}{2}\right)^2 dx-\frac{\cot^3(c+dx)}{3d}\right)}{a^2-b^2}-$$
$$b^2\left(-\frac{b^2\left(\frac{x}{a}-\frac{2b\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{ad\sqrt{a-b}\sqrt{a+b}}\right)}{a^2-b^2}-\frac{a\left(-\frac{\cot(c+dx)}{d}-x\right)}{a^2-b^2}-\frac{b\csc(c+dx)}{d(a^2-b^2)}\right)$$
$$\frac{a^2-b^2}{\frac{b\left(\frac{1}{3}\csc^3(c+dx)-\csc(c+dx)\right)}{d\left(a^2-b^2\right)}}+$$

$$\downarrow 3954$$

$$\frac{a\left(\int 1dx-\frac{\cot^3(c+dx)}{3d}+\frac{\cot(c+dx)}{d}\right)}{a^2-b^2}-$$
$$b^2\left(-\frac{b^2\left(\frac{x}{a}-\frac{2b\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{ad\sqrt{a-b}\sqrt{a+b}}\right)}{a^2-b^2}-\frac{a\left(-\frac{\cot(c+dx)}{d}-x\right)}{a^2-b^2}-\frac{b\csc(c+dx)}{d(a^2-b^2)}\right)$$
$$\frac{a^2-b^2}{\frac{b\left(\frac{1}{3}\csc^3(c+dx)-\csc(c+dx)\right)}{d\left(a^2-b^2\right)}}+$$

$$\downarrow 24$$

$$b^2\left(-\frac{b^2\left(\frac{x}{a}-\frac{2b\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{ad\sqrt{a-b}\sqrt{a+b}}\right)}{a^2-b^2}-\frac{a\left(-\frac{\cot(c+dx)}{d}-x\right)}{a^2-b^2}-\frac{b\csc(c+dx)}{d(a^2-b^2)}\right)$$
$$-\frac{}{a^2-b^2}+$$
$$\frac{a\left(-\frac{\cot^3(c+dx)}{3d}+\frac{\cot(c+dx)}{d}+x\right)}{a^2-b^2}+\frac{b\left(\frac{1}{3}\csc^3(c+dx)-\csc(c+dx)\right)}{d\left(a^2-b^2\right)}$$

input
```
Int[Cot[c + d*x]^4/(a + b*Sec[c + d*x]),x]
```

output
```
(a*(x + Cot[c + d*x]/d - Cot[c + d*x]^3/(3*d)))/(a^2 - b^2) - (b^2*(-((b^2
*(x/a - (2*b*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a*Sqrt[
a - b]*Sqrt[a + b]*d)))/(a^2 - b^2)) - (a*(-x - Cot[c + d*x]/d))/(a^2 - b^
2) - (b*Csc[c + d*x])/((a^2 - b^2)*d)))/(a^2 - b^2) + (b*(-Csc[c + d*x] +
Csc[c + d*x]^3/3))/((a^2 - b^2)*d)
```

3.298.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3086
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[a/f   Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2
), x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2
] &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

rule 3214
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Simp[(b*c - a*d)/d   Int[1/(c + d
*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

rule 3381
```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[a*(d^2/(a^2
- b^2))   Int[(g*Cos[e + f*x])^p*(d*Sin[e + f*x])^(n - 2), x], x] + (-Simp[
b*(d/(a^2 - b^2))   Int[(g*Cos[e + f*x])^p*(d*Sin[e + f*x])^(n - 1), x], x]
 - Simp[a^2*(d^2/(g^2*(a^2 - b^2)))   Int[(g*Cos[e + f*x])^(p + 2)*((d*Sin[
e + f*x])^(n - 2)/(a + b*Sin[e + f*x])), x], x]) /; FreeQ[{a, b, d, e, f, g
}, x] && NeQ[a^2 - b^2, 0] && IntegersQ[2*n, 2*p] && LtQ[p, -1] && GtQ[n, 1
]
```

rule 3954
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Simp[b^2   Int[(b*Tan[c + d*x])^(n - 2), x]
, x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

rule 4386
```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Int[Cos[c + d*x]^m*((b + a*Sin[c + d*x])^n/Sin[c + d*x]^(m
 + n)), x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[n] &&
 IntegerQ[m] && (IntegerQ[m/2] || LeQ[m, 1])
```

3.298.4 Maple [A] (verified)

Time = 0.90 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.04

method	result
derivativedivides	$\frac{\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3 a}{3}-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3 b}{3}-5\tan\left(\frac{dx}{2}+\frac{c}{2}\right)a+7\tan\left(\frac{dx}{2}+\frac{c}{2}\right)b}{8(a-b)^2}+\frac{2\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a}-\frac{1}{24(a+b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}-\frac{-5a}{8(a+b)^2\,\text{ta}}}{d}$
default	$\frac{\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3 a}{3}-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3 b}{3}-5\tan\left(\frac{dx}{2}+\frac{c}{2}\right)a+7\tan\left(\frac{dx}{2}+\frac{c}{2}\right)b}{8(a-b)^2}+\frac{2\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a}-\frac{1}{24(a+b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}-\frac{-5a}{8(a+b)^2\,\text{ta}}}{d}$
risch	$\frac{x}{a}-\frac{2i\left(3a^2b\,e^{5i(dx+c)}-6b^3e^{5i(dx+c)}-6a^3e^{4i(dx+c)}+9a\,b^2e^{4i(dx+c)}-2a^2b\,e^{3i(dx+c)}+8b^3e^{3i(dx+c)}+6a^3e^{2i(dx+c)}-12a\right.}{3(a^4-2a^2b^2+b^4)\left(e^{2i(dx+c)}-1\right)^3 d}$

input
```
int(cot(d*x+c)^4/(a+b*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/8/(a-b)^2*(1/3*tan(1/2*d*x+1/2*c)^3*a-1/3*tan(1/2*d*x+1/2*c)^3*b-5*
tan(1/2*d*x+1/2*c)*a+7*tan(1/2*d*x+1/2*c)*b)+2/a*arctan(tan(1/2*d*x+1/2*c)
)-1/24/(a+b)/tan(1/2*d*x+1/2*c)^3-1/8/(a+b)^2*(-5*a-7*b)/tan(1/2*d*x+1/2*c
)-2/(a+b)^2/(a-b)^2*b^5/a/((a-b)*(a+b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/
2*c)/((a-b)*(a+b))^(1/2)))
```

3.298.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 342 vs. $2(168) = 336$.

Time = 0.31 (sec) , antiderivative size = 742, normalized size of antiderivative = 4.19

$$\int \frac{\cot^4(c+dx)}{a+b\sec(c+dx)}\,dx$$
$$= \left[\frac{4\,a^5b-14\,a^3b^3+10\,ab^5+2\left(4\,a^6-11\,a^4b^2+7\,a^2b^4\right)\cos\left(dx+c\right)^3+3\left(b^5\cos\left(dx+c\right)^2-b^5\right)\sqrt{a^2-b^2}\,\text{lo}}{}\right.$$

input
```
integrate(cot(d*x+c)^4/(a+b*sec(d*x+c)),x, algorithm="fricas")
```

output
```
[1/6*(4*a^5*b - 14*a^3*b^3 + 10*a*b^5 + 2*(4*a^6 - 11*a^4*b^2 + 7*a^2*b^4)
*cos(d*x + c)^3 + 3*(b^5*cos(d*x + c)^2 - b^5)*sqrt(a^2 - b^2)*log((2*a*b*
cos(d*x + c) - (a^2 - 2*b^2)*cos(d*x + c)^2 - 2*sqrt(a^2 - b^2)*(b*cos(d*x
 + c) + a)*sin(d*x + c) + 2*a^2 - b^2)/(a^2*cos(d*x + c)^2 + 2*a*b*cos(d*x
 + c) + b^2))*sin(d*x + c) - 6*(a^5*b - 3*a^3*b^3 + 2*a*b^5)*cos(d*x + c)^
2 - 6*(a^6 - 3*a^4*b^2 + 2*a^2*b^4)*cos(d*x + c) + 6*((a^6 - 3*a^4*b^2 + 3
*a^2*b^4 - b^6)*d*x*cos(d*x + c)^2 - (a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*d
*x)*sin(d*x + c))/(((a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6)*d*cos(d*x + c)^2
 - (a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6)*d)*sin(d*x + c)), 1/3*(2*a^5*b -
7*a^3*b^3 + 5*a*b^5 + (4*a^6 - 11*a^4*b^2 + 7*a^2*b^4)*cos(d*x + c)^3 - 3*
(b^5*cos(d*x + c)^2 - b^5)*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*co
s(d*x + c) + a)/((a^2 - b^2)*sin(d*x + c)))*sin(d*x + c) - 3*(a^5*b - 3*a^
3*b^3 + 2*a*b^5)*cos(d*x + c)^2 - 3*(a^6 - 3*a^4*b^2 + 2*a^2*b^4)*cos(d*x
+ c) + 3*((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*d*x*cos(d*x + c)^2 - (a^6 -
3*a^4*b^2 + 3*a^2*b^4 - b^6)*d*x)*sin(d*x + c))/(((a^7 - 3*a^5*b^2 + 3*a^3
*b^4 - a*b^6)*d*cos(d*x + c)^2 - (a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6)*d)*
sin(d*x + c))]
```

3.298.6 Sympy [F]

$$\int \frac{\cot^4(c+dx)}{a+b\sec(c+dx)}\,dx = \int \frac{\cot^4(c+dx)}{a+b\sec(c+dx)}\,dx$$

input
```
integrate(cot(d*x+c)**4/(a+b*sec(d*x+c)),x)
```

output
```
Integral(cot(c + d*x)**4/(a + b*sec(c + d*x)), x)
```

3.298.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\cot^4(c+dx)}{a+b\sec(c+dx)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(cot(d*x+c)^4/(a+b*sec(d*x+c)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` f
or more de
```

3.298.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1073 vs. $2(168) = 336$.

Time $= 0.48$ (sec) , antiderivative size $= 1073$, normalized size of antiderivative $= 6.06$

$$\int \frac{\cot^4(c+dx)}{a+b\sec(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate(cot(d*x+c)^4/(a+b*sec(d*x+c)),x, algorithm="giac")
```

output
```
-1/24*(24*((a^4 - a^3*b - 2*a^2*b^2 + 2*a*b^3 + b^4)*sqrt(-a^2 + b^2)*abs(
a^5 - 2*a^3*b^2 + a*b^4)*abs(-a + b) - (a^9 - a^8*b - 4*a^7*b^2 + 4*a^6*b^
3 + 6*a^5*b^4 - 7*a^4*b^5 - 4*a^3*b^6 + 6*a^2*b^7 + a*b^8 - 2*b^9)*sqrt(-a
^2 + b^2)*abs(-a + b))*(pi*floor(1/2*(d*x + c)/pi + 1/2) + arctan(tan(1/2*
d*x + 1/2*c)/sqrt(-(a^4*b - 2*a^2*b^3 + b^5 + sqrt((a^5 + a^4*b - 2*a^3*b^
2 - 2*a^2*b^3 + a*b^4 + b^5)*(a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4
 - b^5) + (a^4*b - 2*a^2*b^3 + b^5)^2))/(a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^
3 + a*b^4 - b^5))))/((a^5 - 2*a^3*b^2 + a*b^4)^2*(a^2 - 2*a*b + b^2) + (a^
6*b - 2*a^5*b^2 - a^4*b^3 + 4*a^3*b^4 - a^2*b^5 - 2*a*b^6 + b^7)*abs(a^5 -
 2*a^3*b^2 + a*b^4)) + 24*(a^9 - a^8*b - 4*a^7*b^2 + 4*a^6*b^3 + 6*a^5*b^4
 - 7*a^4*b^5 - 4*a^3*b^6 + 6*a^2*b^7 + a*b^8 - 2*b^9 + a^4*abs(a^5 - 2*a^3
*b^2 + a*b^4) - a^3*b*abs(a^5 - 2*a^3*b^2 + a*b^4) - 2*a^2*b^2*abs(a^5 - 2
*a^3*b^2 + a*b^4) + 2*a*b^3*abs(a^5 - 2*a^3*b^2 + a*b^4) + b^4*abs(a^5 - 2
*a^3*b^2 + a*b^4))*(pi*floor(1/2*(d*x + c)/pi + 1/2) + arctan(tan(1/2*d*x
+ 1/2*c)/sqrt(-(a^4*b - 2*a^2*b^3 + b^5 - sqrt((a^5 + a^4*b - 2*a^3*b^2 -
2*a^2*b^3 + a*b^4 + b^5)*(a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^
5) + (a^4*b - 2*a^2*b^3 + b^5)^2))/(a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 +
a*b^4 - b^5))))/(a^4*b*abs(a^5 - 2*a^3*b^2 + a*b^4) - 2*a^2*b^3*abs(a^5 -
2*a^3*b^2 + a*b^4) + b^5*abs(a^5 - 2*a^3*b^2 + a*b^4) - (a^5 - 2*a^3*b^2 +
 a*b^4)^2) - (a^2*tan(1/2*d*x + 1/2*c)^3 - 2*a*b*tan(1/2*d*x + 1/2*c)^3...
```

3.298.9 Mupad [B] (verification not implemented)

Time = 23.38 (sec) , antiderivative size = 3859, normalized size of antiderivative = 21.80

$$\int \frac{\cot^4(c+dx)}{a+b\sec(c+dx)}\,dx = \text{Too large to display}$$

input
```
int(cot(c + d*x)^4/(a + b/cos(c + d*x)),x)
```

output
```
(a^10*((cos(3*c + 3*d*x)*4i)/3 - sin(c + d*x)*atan(sin(c/2 + (d*x)/2)/cos(
c/2 + (d*x)/2))*6i + atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*sin(3*c +
 3*d*x)*2i) + a*((b^9*8i)/3 - b^9*cos(2*c + 2*d*x)*4i) - a^7*((b^3*14i)/3
- b^3*cos(2*c + 2*d*x)*10i) + a^5*(b^5*10i - b^5*cos(2*c + 2*d*x)*18i) - a
^3*((b^7*26i)/3 - b^7*cos(2*c + 2*d*x)*14i) + a^9*((b*2i)/3 - b*cos(2*c +
2*d*x)*2i) + a^8*(b^2*cos(c + d*x)*1i - (b^2*cos(3*c + 3*d*x)*19i)/3 + b^2
*sin(c + d*x)*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*30i - b^2*atan(s
in(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*sin(3*c + 3*d*x)*10i) - a^2*(b^8*cos
(c + d*x)*1i - (b^8*cos(3*c + 3*d*x)*7i)/3 + b^8*sin(c + d*x)*atan(sin(c/2
 + (d*x)/2)/cos(c/2 + (d*x)/2))*30i - b^8*atan(sin(c/2 + (d*x)/2)/cos(c/2
+ (d*x)/2))*sin(3*c + 3*d*x)*10i) - a^6*(b^4*cos(c + d*x)*3i - b^4*cos(3*c
 + 3*d*x)*11i + b^4*sin(c + d*x)*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2
))*60i - b^4*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*sin(3*c + 3*d*x)*
20i) + a^4*(b^6*cos(c + d*x)*3i - (b^6*cos(3*c + 3*d*x)*25i)/3 + b^6*sin(c
 + d*x)*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*60i - b^6*atan(sin(c/2
 + (d*x)/2)/cos(c/2 + (d*x)/2))*sin(3*c + 3*d*x)*20i) + b^10*sin(c + d*x)*
atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*6i - b^10*atan(sin(c/2 + (d*x)
/2)/cos(c/2 + (d*x)/2))*sin(3*c + 3*d*x)*2i + b^5*atanh((2*b^11*sin(c/2 +
(d*x)/2)*(a^10 - b^10 + 5*a^2*b^8 - 10*a^4*b^6 + 10*a^6*b^4 - 5*a^8*b^2)^(
3/2) - a^21*sin(c/2 + (d*x)/2)*(a^10 - b^10 + 5*a^2*b^8 - 10*a^4*b^6 + ...
```

3.299 $\int \frac{\tan^9(c+dx)}{(a+b\sec(c+dx))^2}\,dx$

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3.299.1 Optimal result

Integrand size = 21, antiderivative size = 255

$$\begin{aligned}\int \frac{\tan^9(c+dx)}{(a+b\sec(c+dx))^2}\,dx = &-\frac{\log(\cos(c+dx))}{a^2 d}+\frac{\left(a^2-b^2\right)^3\left(7a^2+b^2\right)\log(a+b\sec(c+dx))}{a^2b^8d}\\ &-\frac{2a(3a^4-8a^2b^2+6b^4)\sec(c+dx)}{b^7d}\\ &+\frac{(5a^4-12a^2b^2+6b^4)\sec^2(c+dx)}{2b^6d}\\ &-\frac{4a(a^2-2b^2)\sec^3(c+dx)}{3b^5d}+\frac{(3a^2-4b^2)\sec^4(c+dx)}{4b^4d}\\ &-\frac{2a\sec^5(c+dx)}{5b^3d}+\frac{\sec^6(c+dx)}{6b^2d}+\frac{\left(a^2-b^2\right)^4}{ab^8d(a+b\sec(c+dx))}\end{aligned}$$

output

```
-ln(cos(d*x+c))/a^2/d+(a^2-b^2)^3*(7*a^2+b^2)*ln(a+b*sec(d*x+c))/a^2/b^8/d
-2*a*(3*a^4-8*a^2*b^2+6*b^4)*sec(d*x+c)/b^7/d+1/2*(5*a^4-12*a^2*b^2+6*b^4)
*sec(d*x+c)^2/b^6/d-4/3*a*(a^2-2*b^2)*sec(d*x+c)^3/b^5/d+1/4*(3*a^2-4*b^2)
*sec(d*x+c)^4/b^4/d-2/5*a*sec(d*x+c)^5/b^3/d+1/6*sec(d*x+c)^6/b^2/d+(a^2-b
^2)^4/a/b^8/d/(a+b*sec(d*x+c))
```

3.299.2 Mathematica [A] (verified)

Time = 0.91 (sec) , antiderivative size = 230, normalized size of antiderivative = 0.90

$$\int \frac{\tan^9(c+dx)}{(a+b\sec(c+dx))^2}\,dx$$
$$= \frac{-\frac{b^8\log(\cos(c+dx))}{a^2} + \frac{(a^2-b^2)^3(7a^2+b^2)\log(a+b\sec(c+dx))}{a^2} - 2ab(3a^4-8a^2b^2+6b^4)\sec(c+dx) + \frac{1}{2}b^2(5a^4-12a^2b^2$$

input
```
Integrate[Tan[c + d*x]^9/(a + b*Sec[c + d*x])^2,x]
```

output
```
(-((b^8*Log[Cos[c + d*x]])/a^2) + ((a^2 - b^2)^3*(7*a^2 + b^2)*Log[a + b*S
ec[c + d*x]])/a^2 - 2*a*b*(3*a^4 - 8*a^2*b^2 + 6*b^4)*Sec[c + d*x] + (b^2*
(5*a^4 - 12*a^2*b^2 + 6*b^4)*Sec[c + d*x]^2)/2 - (4*a*b^3*(a^2 - 2*b^2)*Se
c[c + d*x]^3)/3 + (b^4*(3*a^2 - 4*b^2)*Sec[c + d*x]^4)/4 - (2*a*b^5*Sec[c
+ d*x]^5)/5 + (b^6*Sec[c + d*x]^6)/6 + (a^2 - b^2)^4/(a*(a + b*Sec[c + d*x
])))/(b^8*d)
```

3.299.3 Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 231, normalized size of antiderivative = 0.91, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3042, 25, 4373, 522, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^9(c+dx)}{(a+b\sec(c+dx))^2}\,dx$$
$$\downarrow 3042$$
$$\int -\frac{\cot\left(c+dx+\frac{\pi}{2}\right)^9}{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^2}\,dx$$
$$\downarrow 25$$
$$-\int \frac{\cot\left(\frac{1}{2}(2c+\pi)+dx\right)^9}{\left(a+b\csc\left(\frac{1}{2}(2c+\pi)+dx\right)\right)^2}\,dx$$
$$\downarrow 4373$$

$$\frac{\int \frac{\cos(c+dx)\left(b^2-b^2\sec^2(c+dx)\right)^4}{b(a+b\sec(c+dx))^2}d(b\sec(c+dx))}{b^8 d}$$

$$\downarrow 522$$

$$\frac{\int\left(\frac{\cos(c+dx)b^7}{a^2}+\sec^5(c+dx)b^5-2a\sec^4(c+dx)b^4+\left(3a^2-4b^2\right)\sec^3(c+dx)b^3-4a\left(a^2-2b^2\right)\sec^2(c+dx)b^2+\right.}{b}$$

$$\downarrow 2009$$

$$\frac{b^8\log(b\sec(c+dx))}{a^2}+\frac{\left(a^2-b^2\right)^4}{a(a+b\sec(c+dx))}+\frac{\left(a^2-b^2\right)^3\left(7a^2+b^2\right)\log(a+b\sec(c+dx))}{a^2}+\frac{1}{4}b^4\left(3a^2-4b^2\right)\sec^4(c+dx)-\frac{4}{3}ab^3\left(a^2-2b^2\right.$$

input
```
Int[Tan[c + d*x]^9/(a + b*Sec[c + d*x])^2,x]
```

output
```
((b^8*Log[b*Sec[c + d*x]])/a^2 + ((a^2 - b^2)^3*(7*a^2 + b^2)*Log[a + b*Se
c[c + d*x]])/a^2 - 2*a*b*(3*a^4 - 8*a^2*b^2 + 6*b^4)*Sec[c + d*x] + (b^2*(
5*a^4 - 12*a^2*b^2 + 6*b^4)*Sec[c + d*x]^2)/2 - (4*a*b^3*(a^2 - 2*b^2)*Sec
[c + d*x]^3)/3 + (b^4*(3*a^2 - 4*b^2)*Sec[c + d*x]^4)/4 - (2*a*b^5*Sec[c +
 d*x]^5)/5 + (b^6*Sec[c + d*x]^6)/6 + (a^2 - b^2)^4/(a*(a + b*Sec[c + d*x]
)))/(b^8*d)
```

3.299.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 522
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_.
), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(c + d*x)^n*(a + b*x^2)^p, x],
x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4373
```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Simp[-(-1)^((m - 1)/2)/(d*b^(m - 1))   Subst[Int[(b^2 - x^
2)^((m - 1)/2)*((a + x)^n/x), x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
 d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

3.299.4 Maple [A] (verified)

Time = 3.60 (sec) , antiderivative size = 287, normalized size of antiderivative = 1.13

method	result
derivativedivides	$\frac{-\frac{a^8-4a^6b^2+6a^4b^4-4a^2b^6+b^8}{a^2b^7(b+a\cos(dx+c))}+\frac{\left(7a^8-20a^6b^2+18a^4b^4-4a^2b^6-b^8\right)\ln(b+a\cos(dx+c))}{b^8a^2}-\frac{-3a^2+4b^2}{4b^4\cos(dx+c)^4}-\frac{-5a^4+12a^2b^2-6b^4}{2b^6\cos(dx+c)^2}+}{d}$
default	$\frac{-\frac{a^8-4a^6b^2+6a^4b^4-4a^2b^6+b^8}{a^2b^7(b+a\cos(dx+c))}+\frac{\left(7a^8-20a^6b^2+18a^4b^4-4a^2b^6-b^8\right)\ln(b+a\cos(dx+c))}{b^8a^2}-\frac{-3a^2+4b^2}{4b^4\cos(dx+c)^4}-\frac{-5a^4+12a^2b^2-6b^4}{2b^6\cos(dx+c)^2}+}{d}$
risch	Expression too large to display

input
```
int(tan(d*x+c)^9/(a+b*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-(a^8-4*a^6*b^2+6*a^4*b^4-4*a^2*b^6+b^8)/a^2/b^7/(b+a*cos(d*x+c))+(7*
a^8-20*a^6*b^2+18*a^4*b^4-4*a^2*b^6-b^8)/b^8/a^2*ln(b+a*cos(d*x+c))-1/4*(-
3*a^2+4*b^2)/b^4/cos(d*x+c)^4-1/2*(-5*a^4+12*a^2*b^2-6*b^4)/b^6/cos(d*x+c)
^2+(-7*a^6+20*a^4*b^2-18*a^2*b^4+4*b^6)/b^8*ln(cos(d*x+c))+1/6/b^2/cos(d*x
+c)^6-2/5/b^3*a/cos(d*x+c)^5-4/3*a*(a^2-2*b^2)/b^5/cos(d*x+c)^3-2*a*(3*a^4
-8*a^2*b^2+6*b^4)/b^7/cos(d*x+c))
```

3.299.5 Fricas [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 423, normalized size of antiderivative = 1.66

$$\int \frac{\tan^9(c+dx)}{(a+b\sec(c+dx))^2}\,dx =$$
$$-\frac{14\,a^3b^6\cos(dx+c)-10\,a^2b^7+60\,(7\,a^8b-20\,a^6b^3+18\,a^4b^5-4\,a^2b^7+b^9)\cos(dx+c)^6+30\,(7\,a^7b^2-}$$

input
```
integrate(tan(d*x+c)^9/(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

output
```
-1/60*(14*a^3*b^6*cos(d*x + c) - 10*a^2*b^7 + 60*(7*a^8*b - 20*a^6*b^3 + 1
8*a^4*b^5 - 4*a^2*b^7 + b^9)*cos(d*x + c)^6 + 30*(7*a^7*b^2 - 20*a^5*b^4 +
 18*a^3*b^6)*cos(d*x + c)^5 - 10*(7*a^6*b^3 - 20*a^4*b^5 + 18*a^2*b^7)*cos
(d*x + c)^4 + 5*(7*a^5*b^4 - 20*a^3*b^6)*cos(d*x + c)^3 - 3*(7*a^4*b^5 - 2
0*a^2*b^7)*cos(d*x + c)^2 - 60*((7*a^9 - 20*a^7*b^2 + 18*a^5*b^4 - 4*a^3*b
^6 - a*b^8)*cos(d*x + c)^7 + (7*a^8*b - 20*a^6*b^3 + 18*a^4*b^5 - 4*a^2*b^
7 - b^9)*cos(d*x + c)^6)*log(a*cos(d*x + c) + b) + 60*((7*a^9 - 20*a^7*b^2
 + 18*a^5*b^4 - 4*a^3*b^6)*cos(d*x + c)^7 + (7*a^8*b - 20*a^6*b^3 + 18*a^4
*b^5 - 4*a^2*b^7)*cos(d*x + c)^6)*log(-cos(d*x + c)))/(a^3*b^8*d*cos(d*x +
 c)^7 + a^2*b^9*d*cos(d*x + c)^6)
```

3.299.6 Sympy [F]

$$\int \frac{\tan^9(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \int \frac{\tan^9(c+dx)}{(a+b\sec(c+dx))^2}\,dx$$

input
```
integrate(tan(d*x+c)**9/(a+b*sec(d*x+c))**2,x)
```

output
```
Integral(tan(c + d*x)**9/(a + b*sec(c + d*x))**2, x)
```

3.299.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 321, normalized size of antiderivative = 1.26

$$\int \frac{\tan^9(c+dx)}{(a+b\sec(c+dx))^2}\,dx =$$
$$-\frac{14\,a^3b^5\cos(dx+c) - 10\,a^2b^6 + 60\,(7\,a^8 - 20\,a^6b^2 + 18\,a^4b^4 - 4\,a^2b^6 + b^8)\cos(dx+c)^6 + 30\,(7\,a^7b - 20\,a^5b^3 + 18\,a^3b^5)\cos(dx+c)^5 - 10\,(7\,a^6b^2 - 20\,a^4$$
$$\overline{a^3b^7\cos(dx+c)^7 + a^2b^8\cos(dx+c)^6}$$

input
```
integrate(tan(d*x+c)^9/(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

output
```
-1/60*((14*a^3*b^5*cos(d*x + c) - 10*a^2*b^6 + 60*(7*a^8 - 20*a^6*b^2 + 18
*a^4*b^4 - 4*a^2*b^6 + b^8)*cos(d*x + c)^6 + 30*(7*a^7*b - 20*a^5*b^3 + 18
*a^3*b^5)*cos(d*x + c)^5 - 10*(7*a^6*b^2 - 20*a^4*b^4 + 18*a^2*b^6)*cos(d*
x + c)^4 + 5*(7*a^5*b^3 - 20*a^3*b^5)*cos(d*x + c)^3 - 3*(7*a^4*b^4 - 20*a
^2*b^6)*cos(d*x + c)^2)/(a^3*b^7*cos(d*x + c)^7 + a^2*b^8*cos(d*x + c)^6)
+ 60*(7*a^6 - 20*a^4*b^2 + 18*a^2*b^4 - 4*b^6)*log(cos(d*x + c))/b^8 - 60*
(7*a^8 - 20*a^6*b^2 + 18*a^4*b^4 - 4*a^2*b^6 - b^8)*log(a*cos(d*x + c) + b
)/(a^2*b^8))/d
```

3.299.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1696 vs. $2(245) = 490$.

Time $= 5.91$ (sec) , antiderivative size $= 1696$, normalized size of antiderivative $= 6.65$

$$\int \frac{\tan^9(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \text{Too large to display}$$

input
```
integrate(tan(d*x+c)^9/(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

output
```
1/60*(60*(7*a^9 - 7*a^8*b - 20*a^7*b^2 + 20*a^6*b^3 + 18*a^5*b^4 - 18*a^4*
b^5 - 4*a^3*b^6 + 4*a^2*b^7 - a*b^8 + b^9)*log(abs(a + b + a*(cos(d*x + c)
 - 1)/(cos(d*x + c) + 1) - b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1)))/(a^3*
b^8 - a^2*b^9) + 60*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1))/a
^2 - 60*(7*a^6 - 20*a^4*b^2 + 18*a^2*b^4 - 4*b^6)*log(abs(-(cos(d*x + c) -
 1)/(cos(d*x + c) + 1) - 1))/b^8 - 60*(7*a^9 + 9*a^8*b - 18*a^7*b^2 - 26*a
^6*b^3 + 12*a^5*b^4 + 24*a^4*b^5 + 2*a^3*b^6 - 6*a^2*b^7 - 3*a*b^8 - b^9 +
 7*a^9*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 7*a^8*b*(cos(d*x + c) - 1)/
(cos(d*x + c) + 1) - 20*a^7*b^2*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 20
*a^6*b^3*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 18*a^5*b^4*(cos(d*x + c)
- 1)/(cos(d*x + c) + 1) - 18*a^4*b^5*(cos(d*x + c) - 1)/(cos(d*x + c) + 1)
 - 4*a^3*b^6*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 4*a^2*b^7*(cos(d*x +
c) - 1)/(cos(d*x + c) + 1) - a*b^8*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) +
 b^9*(cos(d*x + c) - 1)/(cos(d*x + c) + 1))/((a + b + a*(cos(d*x + c) - 1)
/(cos(d*x + c) + 1) - b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1))*a^2*b^8) +
(1029*a^6 - 720*a^5*b - 2940*a^4*b^2 + 1760*a^3*b^3 + 2646*a^2*b^4 - 1168*
a*b^5 - 588*b^6 + 6174*a^6*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 3600*a^
5*b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 18240*a^4*b^2*(cos(d*x + c) -
1)/(cos(d*x + c) + 1) + 9120*a^3*b^3*(cos(d*x + c) - 1)/(cos(d*x + c) + 1)
 + 16956*a^2*b^4*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 6288*a*b^5*(co...
```

3.299.9 Mupad [B] (verification not implemented)

Time = 16.98 (sec) , antiderivative size = 760, normalized size of antiderivative = 2.98

$$\int \frac{\tan^9(c+dx)}{(a+b\sec(c+dx))^2}\,dx$$

$$= \frac{2\left(-105\,a^7-105\,a^6\,b+265\,a^5\,b^2+265\,a^4\,b^3-191\,a^3\,b^4-191\,a^2\,b^5+15\,a\,b^6+15\,b^7\right)}{15\,a\,b^7} - \frac{2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}\left(-42\,a^7-7\,a^6\,b+113\,a^5\,b^2+13\,a^4\,b^3-95\,a^3\,b^4\right.}{a\,b^7}$$

$$+\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+1\right)}{a^2\,d}-\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2-1\right)\left(7\,a^6-20\,a^4\,b^2+18\,a^2\,b^4-4\,b^6\right)}{b^8\,d}$$
$$+\frac{\ln\left(a+b-a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2\right)\left(a^2-b^2\right)^3\left(7\,a^2+b^2\right)}{a^2\,b^8\,d}$$

input
```
int(tan(c + d*x)^9/(a + b/cos(c + d*x))^2,x)
```

output
```
((2*(15*a*b^6 - 105*a^6*b - 105*a^7 + 15*b^7 - 191*a^2*b^5 - 191*a^3*b^4 +
 265*a^4*b^3 + 265*a^5*b^2))/(15*a*b^7) - (2*tan(c/2 + (d*x)/2)^10*(19*a*b
^6 - 7*a^6*b - 42*a^7 + 6*b^7 - 5*a^2*b^5 - 95*a^3*b^4 + 13*a^4*b^3 + 113*
a^5*b^2))/(a*b^7) - (4*tan(c/2 + (d*x)/2)^6*(7*a*b^6 - 105*a^6*b - 210*a^7
 + 30*b^7 - 145*a^2*b^5 - 362*a^3*b^4 + 244*a^4*b^3 + 523*a^5*b^2))/(3*a*b
^7) + (2*tan(c/2 + (d*x)/2)^8*(91*a*b^6 - 105*a^6*b - 315*a^7 + 45*b^7 - 9
9*a^2*b^5 - 613*a^3*b^4 + 223*a^4*b^3 + 809*a^5*b^2))/(3*a*b^7) + (2*tan(c
/2 + (d*x)/2)^4*(10*a*b^6 - 350*a^6*b - 525*a^7 + 75*b^7 - 598*a^2*b^5 - 8
62*a^3*b^4 + 860*a^4*b^3 + 1290*a^5*b^2))/(5*a*b^7) - (2*tan(c/2 + (d*x)/2
)^2*(45*a*b^6 - 525*a^6*b - 630*a^7 + 90*b^7 - 955*a^2*b^5 - 1067*a^3*b^4
 + 1325*a^4*b^3 + 1555*a^5*b^2))/(15*a*b^7) + (2*tan(c/2 + (d*x)/2)^12*(4*a
*b^6 - 7*a^7 + b^7 - 18*a^3*b^4 + 20*a^5*b^2))/(a*b^7))/(d*(a + b - tan(c/
2 + (d*x)/2)^14*(a - b) - tan(c/2 + (d*x)/2)^2*(7*a + 5*b) + tan(c/2 + (d*
x)/2)^12*(7*a - 5*b) + tan(c/2 + (d*x)/2)^4*(21*a + 9*b) - tan(c/2 + (d*x)
/2)^10*(21*a - 9*b) - tan(c/2 + (d*x)/2)^6*(35*a + 5*b) + tan(c/2 + (d*x)/
2)^8*(35*a - 5*b))) + log(tan(c/2 + (d*x)/2)^2 + 1)/(a^2*d) - (log(tan(c/2
 + (d*x)/2)^2 - 1)*(7*a^6 - 4*b^6 + 18*a^2*b^4 - 20*a^4*b^2))/(b^8*d) + (l
og(a + b - a*tan(c/2 + (d*x)/2)^2 + b*tan(c/2 + (d*x)/2)^2)*(a^2 - b^2)^3*
(7*a^2 + b^2))/(a^2*b^8*d)
```

3.300 $\int \frac{\tan^7(c+dx)}{(a+b\sec(c+dx))^2}\,dx$

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3.300.1 Optimal result

Integrand size = 21, antiderivative size = 179

$$\int \frac{\tan^7(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \frac{\log(\cos(c+dx))}{a^2 d} + \frac{(a^2-b^2)^2(5a^2+b^2)\log(a+b\sec(c+dx))}{a^2 b^6 d} - \frac{2a(2a^2-3b^2)\sec(c+dx)}{b^5 d} + \frac{3(a^2-b^2)\sec^2(c+dx)}{2b^4 d} - \frac{2a\sec^3(c+dx)}{3b^3 d} + \frac{\sec^4(c+dx)}{4b^2 d} + \frac{(a^2-b^2)^3}{ab^6 d(a+b\sec(c+dx))}$$

output

```
ln(cos(d*x+c))/a^2/d+(a^2-b^2)^2*(5*a^2+b^2)*ln(a+b*sec(d*x+c))/a^2/b^6/d-
2*a*(2*a^2-3*b^2)*sec(d*x+c)/b^5/d+3/2*(a^2-b^2)*sec(d*x+c)^2/b^4/d-2/3*a*
sec(d*x+c)^3/b^3/d+1/4*sec(d*x+c)^4/b^2/d+(a^2-b^2)^3/a/b^6/d/(a+b*sec(d*x
+c))
```

3.300.2 Mathematica [A] (verified)

Time = 1.51 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.91

$$\int \frac{\tan^7(c+dx)}{(a+b\sec(c+dx))^2}\,dx = -\frac{-\frac{b^6\log(\cos(c+dx))}{a^2} - \frac{(a^2-b^2)^2(5a^2+b^2)\log(a+b\sec(c+dx))}{a^2} + 2ab(2a^2-3b^2)\sec(c+dx) - \frac{3}{2}(a-b)b^2(a+b)\sec^2(c}{b^6 d}$$

input
```
Integrate[Tan[c + d*x]^7/(a + b*Sec[c + d*x])^2,x]
```

output
```
-((-((b^6*Log[Cos[c + d*x]])/a^2) - ((a^2 - b^2)^2*(5*a^2 + b^2)*Log[a + b
*Sec[c + d*x]])/a^2 + 2*a*b*(2*a^2 - 3*b^2)*Sec[c + d*x] - (3*(a - b)*b^2*
(a + b)*Sec[c + d*x]^2)/2 + (2*a*b^3*Sec[c + d*x]^3)/3 - (b^4*Sec[c + d*x]
^4)/4 - (a^2 - b^2)^3/(a*(a + b*Sec[c + d*x])))/(b^6*d))
```

3.300.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.92, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3042, 25, 4373, 522, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^7(c+dx)}{(a+b\sec(c+dx))^2}\,dx$$

$$\downarrow 3042$$

$$\int -\frac{\cot\left(c+dx+\frac{\pi}{2}\right)^7}{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^2}dx$$

$$\downarrow 25$$

$$-\int \frac{\cot\left(\frac{1}{2}(2c+\pi)+dx\right)^7}{\left(a+b\csc\left(\frac{1}{2}(2c+\pi)+dx\right)\right)^2}dx$$

$$\downarrow 4373$$

$$-\frac{\int \frac{\cos(c+dx)\left(b^2-b^2\sec^2(c+dx)\right)^3}{b(a+b\sec(c+dx))^2}d(b\sec(c+dx))}{b^6 d}$$

$$\downarrow 522$$

$$-\frac{\int \left(\frac{\cos(c+dx)b^5}{a^2} - \sec^3(c+dx)b^3 + 2a\sec^2(c+dx)b^2 - 3\left(a^2-b^2\right)\sec(c+dx)b + 2\left(2a^3-3ab^2\right) - \frac{\left(a^2-b^2\right)^2\left(5a^2+b^2\right.}{a^2(a+b\sec(c+dx)}\right.}{b^6 d}$$

$$\downarrow 2009$$

$$-\frac{\frac{b^6\log(b\sec(c+dx))}{a^2} - \frac{3}{2}b^2\left(a^2-b^2\right)\sec^2(c+dx) + 2ab\left(2a^2-3b^2\right)\sec(c+dx) - \frac{\left(a^2-b^2\right)^3}{a(a+b\sec(c+dx))} - \frac{\left(a^2-b^2\right)^2\left(5a^2+b^2\right)\log}{a^2}}{b^6 d}$$

input
```
Int[Tan[c + d*x]^7/(a + b*Sec[c + d*x])^2,x]
```

output
```
-(((b^6*Log[b*Sec[c + d*x]])/a^2 - ((a^2 - b^2)^2*(5*a^2 + b^2)*Log[a + b*
Sec[c + d*x]])/a^2 + 2*a*b*(2*a^2 - 3*b^2)*Sec[c + d*x] - (3*b^2*(a^2 - b^
2)*Sec[c + d*x]^2)/2 + (2*a*b^3*Sec[c + d*x]^3)/3 - (b^4*Sec[c + d*x]^4)/4
 - (a^2 - b^2)^3/(a*(a + b*Sec[c + d*x])))/(b^6*d))
```

3.300.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 522
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_.
), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(c + d*x)^n*(a + b*x^2)^p, x],
x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4373
```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Simp[-(-1)^((m - 1)/2)/(d*b^(m - 1))   Subst[Int[(b^2 - x^
2)^((m - 1)/2)*((a + x)^n/x), x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
 d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

3.300.4 Maple [A] (verified)

Time = 2.54 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.12

method	result
derivativedivides	$\frac{-\frac{a^6-3a^4b^2+3a^2b^4-b^6}{a^2b^5(b+a\cos(dx+c))}+\frac{\left(5a^6-9a^4b^2+3a^2b^4+b^6\right)\ln(b+a\cos(dx+c))}{b^6a^2}-\frac{-3a^2+3b^2}{2b^4\cos(dx+c)^2}+\frac{\left(-5a^4+9a^2b^2-3b^4\right)\ln(\cos(dx+c))}{b^6}+\frac{}{4b}}{d}$
default	$\frac{-\frac{a^6-3a^4b^2+3a^2b^4-b^6}{a^2b^5(b+a\cos(dx+c))}+\frac{\left(5a^6-9a^4b^2+3a^2b^4+b^6\right)\ln(b+a\cos(dx+c))}{b^6a^2}-\frac{-3a^2+3b^2}{2b^4\cos(dx+c)^2}+\frac{\left(-5a^4+9a^2b^2-3b^4\right)\ln(\cos(dx+c))}{b^6}+\frac{}{4b}}{d}$
risch	$-\frac{ix}{a^2}-\frac{2ic}{a^2d}-\frac{2(-12b^6e^{3i(dx+c)}-3b^6e^{i(dx+c)}+15a^6e^{i(dx+c)}+60a^6e^{3i(dx+c)}+60a^6e^{7i(dx+c)}+90a^6e^{5i(dx+c)}-3b^6e^{9}$

input
```
int(tan(d*x+c)^7/(a+b*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/a^2/b^5/(b+a*cos(d*x+c))+(5*a^6-9*a^4*
b^2+3*a^2*b^4+b^6)/b^6/a^2*ln(b+a*cos(d*x+c))-1/2*(-3*a^2+3*b^2)/b^4/cos(d
*x+c)^2+(-5*a^4+9*a^2*b^2-3*b^4)/b^6*ln(cos(d*x+c))+1/4/b^2/cos(d*x+c)^4-2
/3/b^3*a/cos(d*x+c)^3-2*a*(2*a^2-3*b^2)/b^5/cos(d*x+c))
```

3.300.5 Fricas [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 312, normalized size of antiderivative = 1.74

$$\int \frac{\tan^7(c+dx)}{(a+b\sec(c+dx))^2}\,dx =$$
$$-\frac{5\,a^3b^4\cos(dx+c)-3\,a^2b^5+12\left(5\,a^6b-9\,a^4b^3+3\,a^2b^5-b^7\right)\cos(dx+c)^4+6\left(5\,a^5b^2-9\,a^3b^4\right)\cos(dx$$

input
```
integrate(tan(d*x+c)^7/(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

output
```
-1/12*(5*a^3*b^4*cos(d*x + c) - 3*a^2*b^5 + 12*(5*a^6*b - 9*a^4*b^3 + 3*a^
2*b^5 - b^7)*cos(d*x + c)^4 + 6*(5*a^5*b^2 - 9*a^3*b^4)*cos(d*x + c)^3 - 2
*(5*a^4*b^3 - 9*a^2*b^5)*cos(d*x + c)^2 - 12*((5*a^7 - 9*a^5*b^2 + 3*a^3*b
^4 + a*b^6)*cos(d*x + c)^5 + (5*a^6*b - 9*a^4*b^3 + 3*a^2*b^5 + b^7)*cos(d
*x + c)^4)*log(a*cos(d*x + c) + b) + 12*((5*a^7 - 9*a^5*b^2 + 3*a^3*b^4)*c
os(d*x + c)^5 + (5*a^6*b - 9*a^4*b^3 + 3*a^2*b^5)*cos(d*x + c)^4)*log(-cos
(d*x + c)))/(a^3*b^6*d*cos(d*x + c)^5 + a^2*b^7*d*cos(d*x + c)^4)
```

3.300.6 Sympy [F]

$$\int \frac{\tan^7(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \int \frac{\tan^7(c+dx)}{(a+b\sec(c+dx))^2}\,dx$$

input
```
integrate(tan(d*x+c)**7/(a+b*sec(d*x+c))**2,x)
```

output
```
Integral(tan(c + d*x)**7/(a + b*sec(c + d*x))**2, x)
```

3.300.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.27

$$\int \frac{\tan^7(c+dx)}{(a+b\sec(c+dx))^2}\,dx = -\frac{\frac{5a^3b^3\cos(dx+c)-3a^2b^4+12\left(5a^6-9a^4b^2+3a^2b^4-b^6\right)\cos(dx+c)^4+6\left(5a^5b-9a^3b^3\right)\cos(dx+c)^3-2\left(5a^4b^2-9a^2b^4\right)\cos(dx+c)^2}{a^3b^5\cos(dx+c)^5+a^2b^6\cos(dx+c)^4}+\frac{12\left(5a^{\cdots}\right.}{12\,d}$$

input
```
integrate(tan(d*x+c)^7/(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

output
```
-1/12*((5*a^3*b^3*cos(d*x + c) - 3*a^2*b^4 + 12*(5*a^6 - 9*a^4*b^2 + 3*a^2
*b^4 - b^6)*cos(d*x + c)^4 + 6*(5*a^5*b - 9*a^3*b^3)*cos(d*x + c)^3 - 2*(5
*a^4*b^2 - 9*a^2*b^4)*cos(d*x + c)^2)/(a^3*b^5*cos(d*x + c)^5 + a^2*b^6*co
s(d*x + c)^4) + 12*(5*a^4 - 9*a^2*b^2 + 3*b^4)*log(cos(d*x + c))/b^6 - 12*
(5*a^6 - 9*a^4*b^2 + 3*a^2*b^4 + b^6)*log(a*cos(d*x + c) + b)/(a^2*b^6))/d
```

3.300.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1023 vs. 2(173) = 346.

Time = 3.43 (sec) , antiderivative size = 1023, normalized size of antiderivative = 5.72

$$\int \frac{\tan^7(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \text{Too large to display}$$

input
```
integrate(tan(d*x+c)^7/(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

output
```
1/12*(12*(5*a^7 - 5*a^6*b - 9*a^5*b^2 + 9*a^4*b^3 + 3*a^3*b^4 - 3*a^2*b^5
+ a*b^6 - b^7)*log(abs(a + b + a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - b
*(cos(d*x + c) - 1)/(cos(d*x + c) + 1)))/(a^3*b^6 - a^2*b^7) - 12*log(abs(
-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1))/a^2 - 12*(5*a^4 - 9*a^2*b^2 +
 3*b^4)*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1))/b^6 - 12*(5*a
^7 + 7*a^6*b - 7*a^5*b^2 - 13*a^4*b^3 - a^3*b^4 + 5*a^2*b^5 + 3*a*b^6 + b^
7 + 5*a^7*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 5*a^6*b*(cos(d*x + c) -
1)/(cos(d*x + c) + 1) - 9*a^5*b^2*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) +
9*a^4*b^3*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 3*a^3*b^4*(cos(d*x + c)
- 1)/(cos(d*x + c) + 1) - 3*a^2*b^5*(cos(d*x + c) - 1)/(cos(d*x + c) + 1)
+ a*b^6*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - b^7*(cos(d*x + c) - 1)/(co
s(d*x + c) + 1))/((a + b + a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - b*(co
s(d*x + c) - 1)/(cos(d*x + c) + 1))*a^2*b^6) + (125*a^4 - 96*a^3*b - 225*a
^2*b^2 + 128*a*b^3 + 75*b^4 + 500*a^4*(cos(d*x + c) - 1)/(cos(d*x + c) + 1
) - 288*a^3*b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 972*a^2*b^2*(cos(d*x
 + c) - 1)/(cos(d*x + c) + 1) + 416*a*b^3*(cos(d*x + c) - 1)/(cos(d*x + c)
 + 1) + 348*b^4*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 750*a^4*(cos(d*x +
 c) - 1)^2/(cos(d*x + c) + 1)^2 - 288*a^3*b*(cos(d*x + c) - 1)^2/(cos(d*x
+ c) + 1)^2 - 1494*a^2*b^2*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 384
*a*b^3*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 594*b^4*(cos(d*x + c...
```

3.300.9 Mupad [B] (verification not implemented)

Time = 15.73 (sec) , antiderivative size = 505, normalized size of antiderivative = 2.82

$$\int \frac{\tan^7(c+dx)}{(a+b\sec(c+dx))^2}\,dx$$

$$= \frac{2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6\left(20a^5+5a^4b-31a^3b^2-4a^2b^3+8ab^4+4b^5\right)}{ab^5} - \frac{2\left(15a^5+15a^4b-22a^3b^2-22a^2b^3+3ab^4+3b^5\right)}{3ab^5} + \frac{2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2(60a^5+4}{d\left((b-a)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}+(5a-3b)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^8+(2b-10\right.}$$

$$-\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+1\right)}{a^2d} - \frac{\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2-1\right)\left(5a^4-9a^2b^2+3b^4\right)}{b^6d}$$

$$+\frac{\ln\left(a+b-a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2\right)\left(a^2-b^2\right)^2\left(5a^2+b^2\right)}{a^2b^6d}$$

input
```
int(tan(c + d*x)^7/(a + b/cos(c + d*x))^2,x)
```

output

```
((2*tan(c/2 + (d*x)/2)^6*(8*a*b^4 + 5*a^4*b + 20*a^5 + 4*b^5 - 4*a^2*b^3 -
 31*a^3*b^2))/(a*b^5) - (2*(3*a*b^4 + 15*a^4*b + 15*a^5 + 3*b^5 - 22*a^2*b
^3 - 22*a^3*b^2))/(3*a*b^5) + (2*tan(c/2 + (d*x)/2)^2*(6*a*b^4 + 45*a^4*b
+ 60*a^5 + 12*b^5 - 66*a^2*b^3 - 83*a^3*b^2))/(3*a*b^5) - (2*tan(c/2 + (d*
x)/2)^4*(6*a*b^4 + 45*a^4*b + 90*a^5 + 18*b^5 - 56*a^2*b^3 - 127*a^3*b^2))
/(3*a*b^5) + (2*tan(c/2 + (d*x)/2)^8*(a - b)*(4*a*b^3 - 5*a^3*b - 5*a^4 +
b^4 + 4*a^2*b^2))/(a*b^5))/(d*(a + b - tan(c/2 + (d*x)/2)^10*(a - b) - tan
(c/2 + (d*x)/2)^2*(5*a + 3*b) + tan(c/2 + (d*x)/2)^4*(10*a + 2*b) + tan(c/
2 + (d*x)/2)^8*(5*a - 3*b) - tan(c/2 + (d*x)/2)^6*(10*a - 2*b))) - log(tan
(c/2 + (d*x)/2)^2 + 1)/(a^2*d) - (log(tan(c/2 + (d*x)/2)^2 - 1)*(5*a^4 + 3
*b^4 - 9*a^2*b^2))/(b^6*d) + (log(a + b - a*tan(c/2 + (d*x)/2)^2 + b*tan(c
/2 + (d*x)/2)^2)*(a^2 - b^2)^2*(5*a^2 + b^2))/(a^2*b^6*d)
```

3.301 $\int \frac{\tan^5(c+dx)}{(a+b\sec(c+dx))^2}\,dx$

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3.301.1 Optimal result

Integrand size = 21, antiderivative size = 121

$$\int \frac{\tan^5(c+dx)}{(a+b\sec(c+dx))^2}\,dx = -\frac{\log(\cos(c+dx))}{a^2 d} + \frac{(a^2-b^2)(3a^2+b^2)\log(a+b\sec(c+dx))}{a^2 b^4 d} - \frac{2a\sec(c+dx)}{b^3 d} + \frac{\sec^2(c+dx)}{2b^2 d} + \frac{(a^2-b^2)^2}{ab^4 d(a+b\sec(c+dx))}$$

output
```
-ln(cos(d*x+c))/a^2/d+(a^2-b^2)*(3*a^2+b^2)*ln(a+b*sec(d*x+c))/a^2/b^4/d-2
*a*sec(d*x+c)/b^3/d+1/2*sec(d*x+c)^2/b^2/d+(a^2-b^2)^2/a/b^4/d/(a+b*sec(d*
x+c))
```

3.301.2 Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.88

$$\int \frac{\tan^5(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \frac{-\frac{b^4\log(\cos(c+dx))}{a^2} + \frac{(a-b)(a+b)(3a^2+b^2)\log(a+b\sec(c+dx))}{a^2} - 2ab\sec(c+dx) + \frac{1}{2}b^2\sec^2(c+dx) + \frac{(a^2-b^2)^2}{a(a+b\sec(c+dx))}}{b^4 d}$$

input
```
Integrate[Tan[c + d*x]^5/(a + b*Sec[c + d*x])^2,x]
```

output
```
(-((b^4*Log[Cos[c + d*x]])/a^2) + ((a - b)*(a + b)*(3*a^2 + b^2)*Log[a + b
*Sec[c + d*x]])/a^2 - 2*a*b*Sec[c + d*x] + (b^2*Sec[c + d*x]^2)/2 + (a^2 -
 b^2)^2/(a*(a + b*Sec[c + d*x])))/(b^4*d)
```

3.301.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.90, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3042, 25, 4373, 522, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^5(c+dx)}{(a+b\sec(c+dx))^2}\,dx$$

$$\downarrow 3042$$

$$\int -\frac{\cot\left(c+dx+\frac{\pi}{2}\right)^5}{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^2}dx$$

$$\downarrow 25$$

$$-\int \frac{\cot\left(\frac{1}{2}(2c+\pi)+dx\right)^5}{\left(a+b\csc\left(\frac{1}{2}(2c+\pi)+dx\right)\right)^2}dx$$

$$\downarrow 4373$$

$$\frac{\int \frac{\cos(c+dx)\left(b^2-b^2\sec^2(c+dx)\right)^2}{b(a+b\sec(c+dx))^2}d(b\sec(c+dx))}{b^4 d}$$

$$\downarrow 522$$

$$\frac{\int \left(\frac{\cos(c+dx)b^3}{a^2}+\sec(c+dx)b-2a+\frac{\left(a^2-b^2\right)\left(3a^2+b^2\right)}{a^2(a+b\sec(c+dx))}-\frac{\left(a^2-b^2\right)^2}{a(a+b\sec(c+dx))^2}\right)d(b\sec(c+dx))}{b^4 d}$$

$$\downarrow 2009$$

$$\frac{\frac{b^4\log(b\sec(c+dx))}{a^2}+\frac{\left(a^2-b^2\right)^2}{a(a+b\sec(c+dx))}+\frac{\left(a^2-b^2\right)\left(3a^2+b^2\right)\log(a+b\sec(c+dx))}{a^2}-2ab\sec(c+dx)+\frac{1}{2}b^2\sec^2(c+dx)}{b^4 d}$$

input
```
Int[Tan[c + d*x]^5/(a + b*Sec[c + d*x])^2,x]
```

output
```
((b^4*Log[b*Sec[c + d*x]])/a^2 + ((a^2 - b^2)*(3*a^2 + b^2)*Log[a + b*Sec[
c + d*x]])/a^2 - 2*a*b*Sec[c + d*x] + (b^2*Sec[c + d*x]^2)/2 + (a^2 - b^2)
^2/(a*(a + b*Sec[c + d*x])))/(b^4*d)
```

3.301.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 522
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_.
), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(c + d*x)^n*(a + b*x^2)^p, x],
x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4373
```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Simp[-(-1)^((m - 1)/2)/(d*b^(m - 1))   Subst[Int[(b^2 - x^
2)^((m - 1)/2)*((a + x)^n/x), x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
 d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

3.301.4 Maple [A] (verified)

Time = 1.27 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.05

method	result
derivativedivides	$\frac{\left(-3a^2+2b^2\right)\ln(\cos(dx+c))}{b^4}+\frac{1}{2b^2\cos(dx+c)^2}-\frac{2a}{b^3\cos(dx+c)}-\frac{a^4-2a^2b^2+b^4}{a^2b^3(b+a\cos(dx+c))}+\frac{\left(3a^4-2a^2b^2-b^4\right)\ln(b+a\cos(dx+c))}{b^4a^2}}{d}$
default	$\frac{\left(-3a^2+2b^2\right)\ln(\cos(dx+c))}{b^4}+\frac{1}{2b^2\cos(dx+c)^2}-\frac{2a}{b^3\cos(dx+c)}-\frac{a^4-2a^2b^2+b^4}{a^2b^3(b+a\cos(dx+c))}+\frac{\left(3a^4-2a^2b^2-b^4\right)\ln(b+a\cos(dx+c))}{b^4a^2}}{d}$
risch	$\frac{ix}{a^2}+\frac{2ic}{a^2d}-\frac{2\left(3a^4e^{5i(dx+c)}-2a^2b^2e^{5i(dx+c)}+b^4e^{5i(dx+c)}+3a^3be^{4i(dx+c)}+6a^4e^{3i(dx+c)}-6a^2b^2e^{3i(dx+c)}+2b^4e^{3i(dx}{db^3\left(e^{2i(dx+c)}+1\right)^2a^2\left(ae^{2i(dx+c)}+2be^{i(dx+c)}\right.}$

input
```
int(tan(d*x+c)^5/(a+b*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*((-3*a^2+2*b^2)/b^4*ln(cos(d*x+c))+1/2/b^2/cos(d*x+c)^2-2/b^3*a/cos(d*
x+c)-(a^4-2*a^2*b^2+b^4)/a^2/b^3/(b+a*cos(d*x+c))+(3*a^4-2*a^2*b^2-b^4)/b^
4/a^2*ln(b+a*cos(d*x+c)))
```

3.301.5 Fricas [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.81

$$\int \frac{\tan^5(c+dx)}{(a+b\sec(c+dx))^2}\,dx =$$
$$-\frac{3\,a^3b^2\cos(dx+c) - a^2b^3 + 2\,(3\,a^4b - 2\,a^2b^3 + b^5)\cos(dx+c)^2 - 2\,((3\,a^5 - 2\,a^3b^2 - ab^4)\cos(dx+c)^3}{}$$

input
```
integrate(tan(d*x+c)^5/(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

output
```
-1/2*(3*a^3*b^2*cos(d*x + c) - a^2*b^3 + 2*(3*a^4*b - 2*a^2*b^3 + b^5)*cos
(d*x + c)^2 - 2*((3*a^5 - 2*a^3*b^2 - a*b^4)*cos(d*x + c)^3 + (3*a^4*b - 2
*a^2*b^3 - b^5)*cos(d*x + c)^2)*log(a*cos(d*x + c) + b) + 2*((3*a^5 - 2*a^
3*b^2)*cos(d*x + c)^3 + (3*a^4*b - 2*a^2*b^3)*cos(d*x + c)^2)*log(-cos(d*x
 + c)))/(a^3*b^4*d*cos(d*x + c)^3 + a^2*b^5*d*cos(d*x + c)^2)
```

3.301.6 Sympy [F]

$$\int \frac{\tan^5(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \int \frac{\tan^5(c+dx)}{(a+b\sec(c+dx))^2}\,dx$$

input
```
integrate(tan(d*x+c)**5/(a+b*sec(d*x+c))**2,x)
```

output
```
Integral(tan(c + d*x)**5/(a + b*sec(c + d*x))**2, x)
```

3.301.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.23

$$\int \frac{\tan^5(c+dx)}{(a+b\sec(c+dx))^2}\,dx = -\frac{\frac{3a^3b\cos(dx+c)-a^2b^2+2\left(3a^4-2a^2b^2+b^4\right)\cos(dx+c)^2}{a^3b^3\cos(dx+c)^3+a^2b^4\cos(dx+c)^2}+\frac{2\left(3a^2-2b^2\right)\log(\cos(dx+c))}{b^4}-\frac{2\left(3a^4-2a^2b^2-b^4\right)\log(a\cos(dx+c)+b)}{a^2b^4}}{2d}$$

input
```
integrate(tan(d*x+c)^5/(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

output
```
-1/2*((3*a^3*b*cos(d*x + c) - a^2*b^2 + 2*(3*a^4 - 2*a^2*b^2 + b^4)*cos(d*
x + c)^2)/(a^3*b^3*cos(d*x + c)^3 + a^2*b^4*cos(d*x + c)^2) + 2*(3*a^2 - 2
*b^2)*log(cos(d*x + c))/b^4 - 2*(3*a^4 - 2*a^2*b^2 - b^4)*log(a*cos(d*x +
c) + b)/(a^2*b^4))/d
```

3.301.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 568 vs. 2(119) = 238.

Time = 1.56 (sec) , antiderivative size = 568, normalized size of antiderivative = 4.69

$$\int \frac{\tan^5(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \frac{2\left(3a^5-3a^4b-2a^3b^2+2a^2b^3-ab^4+b^5\right)\log\left(\left|a+b+\frac{a(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{b(\cos(dx+c)-1)}{\cos(dx+c)+1}\right|\right)}{a^3b^4-a^2b^5}+\frac{2\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right)}{a^2}-\frac{2\left(3a^2-2b^2\right)\log\left(\left|-\frac{\cos}{\cos}\right.\right.}{b^4}$$

input
```
integrate(tan(d*x+c)^5/(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

output
```
1/2*(2*(3*a^5 - 3*a^4*b - 2*a^3*b^2 + 2*a^2*b^3 - a*b^4 + b^5)*log(abs(a +
 b + a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - b*(cos(d*x + c) - 1)/(cos(d
*x + c) + 1)))/(a^3*b^4 - a^2*b^5) + 2*log(abs(-(cos(d*x + c) - 1)/(cos(d*
x + c) + 1) + 1))/a^2 - 2*(3*a^2 - 2*b^2)*log(abs(-(cos(d*x + c) - 1)/(cos
(d*x + c) + 1) - 1))/b^4 + (9*a^2 - 8*a*b - 6*b^2 + 18*a^2*(cos(d*x + c) -
 1)/(cos(d*x + c) + 1) - 8*a*b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 16*
b^2*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 9*a^2*(cos(d*x + c) - 1)^2/(co
s(d*x + c) + 1)^2 - 6*b^2*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2)/(b^4*
((cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)^2) - 2*(3*a^5 + 5*a^4*b - 4*a^
2*b^3 - 3*a*b^4 - b^5 + 3*a^5*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 3*a^
4*b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 2*a^3*b^2*(cos(d*x + c) - 1)/(
cos(d*x + c) + 1) + 2*a^2*b^3*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - a*b^
4*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + b^5*(cos(d*x + c) - 1)/(cos(d*x
+ c) + 1))/((a + b + a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - b*(cos(d*x
+ c) - 1)/(cos(d*x + c) + 1))*a^2*b^4))/d
```

3.301.9 Mupad [B] (verification not implemented)

Time = 14.90 (sec) , antiderivative size = 286, normalized size of antiderivative = 2.36

$$\int \frac{\tan^5(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \frac{\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+1\right)}{a^2\,d}$$
$$-\frac{\frac{2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2\left(-6a^3-3a^2b+ab^2+2b^3\right)}{ab^3}-\frac{2\left(-3a^3-3a^2b+ab^2+b^3\right)}{ab^3}+\frac{2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4(a-b)\left(3a^2+3ab+b^2\right)}{ab^3}}{d\left((b-a)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6+(3a-b)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4+(-3a-b)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+a+b\right)}$$
$$-\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2-1\right)\left(3a^2-2b^2\right)}{b^4\,d}$$
$$-\frac{\ln\left(a+b-a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2\right)\left(-3a^4+2a^2b^2+b^4\right)}{a^2b^4\,d}$$

input
```
int(tan(c + d*x)^5/(a + b/cos(c + d*x))^2,x)
```

output

```
log(tan(c/2 + (d*x)/2)^2 + 1)/(a^2*d) - ((2*tan(c/2 + (d*x)/2)^2*(a*b^2 -
3*a^2*b - 6*a^3 + 2*b^3))/(a*b^3) - (2*(a*b^2 - 3*a^2*b - 3*a^3 + b^3))/(a
*b^3) + (2*tan(c/2 + (d*x)/2)^4*(a - b)*(3*a*b + 3*a^2 + b^2))/(a*b^3))/(d
*(a + b - tan(c/2 + (d*x)/2)^2*(3*a + b) - tan(c/2 + (d*x)/2)^6*(a - b) +
tan(c/2 + (d*x)/2)^4*(3*a - b))) - (log(tan(c/2 + (d*x)/2)^2 - 1)*(3*a^2 -
 2*b^2))/(b^4*d) - (log(a + b - a*tan(c/2 + (d*x)/2)^2 + b*tan(c/2 + (d*x)
/2)^2)*(b^4 - 3*a^4 + 2*a^2*b^2))/(a^2*b^4*d)
```

3.302 $\int \frac{\tan^3(c+dx)}{(a+b\sec(c+dx))^2}\,dx$

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3.302.9 Mupad [B] (verification not implemented) . . . 2189

3.302.1 Optimal result

Integrand size = 21, antiderivative size = 74

$$\int \frac{\tan^3(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \frac{\log(\cos(c+dx))}{a^2 d} + \frac{(a^2+b^2)\log(a+b\sec(c+dx))}{a^2 b^2 d} + \frac{a^2-b^2}{ab^2 d(a+b\sec(c+dx))}$$

output
```
ln(cos(d*x+c))/a^2/d+(a^2+b^2)*ln(a+b*sec(d*x+c))/a^2/b^2/d+(a^2-b^2)/a/b^
2/d/(a+b*sec(d*x+c))
```

3.302.2 Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.84

$$\int \frac{\tan^3(c+dx)}{(a+b\sec(c+dx))^2}\,dx = -\frac{\frac{b-\frac{b^3}{a^2}}{b+a\cos(c+dx)} + \log(\cos(c+dx)) - \frac{(a^2+b^2)\log(b+a\cos(c+dx))}{a^2}}{b^2 d}$$

input
```
Integrate[Tan[c + d*x]^3/(a + b*Sec[c + d*x])^2,x]
```

output
```
-(((b - b^3/a^2)/(b + a*Cos[c + d*x]) + Log[Cos[c + d*x]] - ((a^2 + b^2)*L
og[b + a*Cos[c + d*x]])/a^2)/(b^2*d))
```

3.302.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3042, 25, 4373, 522, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^3(c+dx)}{(a+b\sec(c+dx))^2}\,dx$$

$$\downarrow 3042$$

$$\int -\frac{\cot\left(c+dx+\frac{\pi}{2}\right)^3}{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^2}dx$$

$$\downarrow 25$$

$$-\int \frac{\cot\left(\frac{1}{2}(2c+\pi)+dx\right)^3}{\left(a+b\csc\left(\frac{1}{2}(2c+\pi)+dx\right)\right)^2}dx$$

$$\downarrow 4373$$

$$-\frac{\int \frac{\cos(c+dx)\left(b^2-b^2\sec^2(c+dx)\right)}{b(a+b\sec(c+dx))^2}d(b\sec(c+dx))}{b^2 d}$$

$$\downarrow 522$$

$$-\frac{\int \left(\frac{-a^2-b^2}{a^2(a+b\sec(c+dx))}+\frac{b\cos(c+dx)}{a^2}+\frac{a^2-b^2}{a(a+b\sec(c+dx))^2}\right)d(b\sec(c+dx))}{b^2 d}$$

$$\downarrow 2009$$

$$-\frac{-\frac{a^2-b^2}{a(a+b\sec(c+dx))}+\frac{b^2\log(b\sec(c+dx))}{a^2}-\frac{\left(a^2+b^2\right)\log(a+b\sec(c+dx))}{a^2}}{b^2 d}$$

input
```
Int[Tan[c + d*x]^3/(a + b*Sec[c + d*x])^2,x]
```

output
```
-(((b^2*Log[b*Sec[c + d*x]])/a^2 - ((a^2 + b^2)*Log[a + b*Sec[c + d*x]])/a
^2 - (a^2 - b^2)/(a*(a + b*Sec[c + d*x])))/(b^2*d))
```

3.302.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 522
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_.
), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(c + d*x)^n*(a + b*x^2)^p, x],
x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4373
```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Simp[-(-1)^((m - 1)/2)/(d*b^(m - 1))   Subst[Int[(b^2 - x^
2)^((m - 1)/2)*((a + x)^n/x), x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
 d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

3.302.4 Maple [A] (verified)

Time = 0.84 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.97

method	result
derivativedivides	$\frac{-\frac{a^2-b^2}{a^2b(b+a\cos(dx+c))}+\frac{\left(a^2+b^2\right)\ln(b+a\cos(dx+c))}{b^2a^2}-\frac{\ln(\cos(dx+c))}{b^2}}{d}$
default	$\frac{-\frac{a^2-b^2}{a^2b(b+a\cos(dx+c))}+\frac{\left(a^2+b^2\right)\ln(b+a\cos(dx+c))}{b^2a^2}-\frac{\ln(\cos(dx+c))}{b^2}}{d}$
risch	$-\frac{ix}{a^2}-\frac{2ic}{a^2d}-\frac{2\left(a^2-b^2\right)e^{i(dx+c)}}{a^2bd\left(a\,e^{2i(dx+c)}+2b\,e^{i(dx+c)}+a\right)}+\frac{\ln\left(e^{2i(dx+c)}+\frac{2b\,e^{i(dx+c)}}{a}+1\right)}{b^2d}+\frac{\ln\left(e^{2i(dx+c)}+\frac{2b\,e^{i(dx+c)}}{a}+\right.}{a^2d}$

input
```
int(tan(d*x+c)^3/(a+b*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-(a^2-b^2)/a^2/b/(b+a*cos(d*x+c))+(a^2+b^2)/b^2/a^2*ln(b+a*cos(d*x+c)
)-1/b^2*ln(cos(d*x+c)))
```

3.302.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.38

$$\int \frac{\tan^3(c+dx)}{(a+b\sec(c+dx))^2}\,dx = -\frac{a^2b-b^3-(a^2b+b^3+(a^3+ab^2)\cos(dx+c))\log(a\cos(dx+c)+b)+(a^3\cos(dx+c)+a^2b)\log(-c}{a^3b^2d\cos(dx+c)+a^2b^3d}$$

input
```
integrate(tan(d*x+c)^3/(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

output
```
-(a^2*b - b^3 - (a^2*b + b^3 + (a^3 + a*b^2)*cos(d*x + c))*log(a*cos(d*x +
 c) + b) + (a^3*cos(d*x + c) + a^2*b)*log(-cos(d*x + c)))/(a^3*b^2*d*cos(d
*x + c) + a^2*b^3*d)
```

3.302.6 Sympy [F]

$$\int \frac{\tan^3(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \int \frac{\tan^3(c+dx)}{(a+b\sec(c+dx))^2}\,dx$$

input
```
integrate(tan(d*x+c)**3/(a+b*sec(d*x+c))**2,x)
```

output
```
Integral(tan(c + d*x)**3/(a + b*sec(c + d*x))**2, x)
```

3.302.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.00

$$\int \frac{\tan^3(c+dx)}{(a+b\sec(c+dx))^2}\,dx = -\frac{\frac{a^2-b^2}{a^3b\cos(dx+c)+a^2b^2}+\frac{\log(\cos(dx+c))}{b^2}-\frac{(a^2+b^2)\log(a\cos(dx+c)+b)}{a^2b^2}}{d}$$

input
```
integrate(tan(d*x+c)^3/(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

output
```
-((a^2 - b^2)/(a^3*b*cos(d*x + c) + a^2*b^2) + log(cos(d*x + c))/b^2 - (a^
2 + b^2)*log(a*cos(d*x + c) + b)/(a^2*b^2))/d
```

3.302.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 313 vs. $2(74) = 148$.

Time = 0.65 (sec) , antiderivative size = 313, normalized size of antiderivative = 4.23

$$\int \frac{\tan^3(c+dx)}{(a+b\sec(c+dx))^2}\,dx$$

$$= \frac{\frac{(a^3-a^2b+ab^2-b^3)\log\left(\left|a+b+\frac{a(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{b(\cos(dx+c)-1)}{\cos(dx+c)+1}\right|\right)}{a^3b^2-a^2b^3} - \frac{\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right)}{a^2} - \frac{\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right)}{b^2} - \frac{a^3+3a^2b+3ab^2+}{}}{d}$$

input
```
integrate(tan(d*x+c)^3/(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

output
```
((a^3 - a^2*b + a*b^2 - b^3)*log(abs(a + b + a*(cos(d*x + c) - 1)/(cos(d*x
 + c) + 1) - b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1)))/(a^3*b^2 - a^2*b^3)
 - log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1))/a^2 - log(abs(-(co
s(d*x + c) - 1)/(cos(d*x + c) + 1) - 1))/b^2 - (a^3 + 3*a^2*b + 3*a*b^2 +
b^3 + a^3*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - a^2*b*(cos(d*x + c) - 1)
/(cos(d*x + c) + 1) + a*b^2*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - b^3*(c
os(d*x + c) - 1)/(cos(d*x + c) + 1))/((a + b + a*(cos(d*x + c) - 1)/(cos(d
*x + c) + 1) - b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1))*a^2*b^2))/d
```

3.302.9 Mupad [B] (verification not implemented)

Time = 14.31 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.68

$$\int \frac{\tan^3(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \frac{\ln\left(a+b-a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2\right)\left(\frac{1}{a^2}+\frac{1}{b^2}\right)}{d} - \frac{\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+1\right)}{a^2\,d} - \frac{\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2-1\right)}{b^2\,d} - \frac{2\,(a+b)}{a\,b\,d\left((b-a)\,\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+a+b\right)}$$

input
```
int(tan(c + d*x)^3/(a + b/cos(c + d*x))^2,x)
```

output
```
(log(a + b - a*tan(c/2 + (d*x)/2)^2 + b*tan(c/2 + (d*x)/2)^2)*(1/a^2 + 1/b
^2))/d - log(tan(c/2 + (d*x)/2)^2 + 1)/(a^2*d) - log(tan(c/2 + (d*x)/2)^2
- 1)/(b^2*d) - (2*(a + b))/(a*b*d*(a + b - tan(c/2 + (d*x)/2)^2*(a - b)))
```

3.303 $\int \frac{\tan(c+dx)}{(a+b\sec(c+dx))^2}\,dx$

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3.303.1 Optimal result

Integrand size = 19, antiderivative size = 54

$$\int \frac{\tan(c+dx)}{(a+b\sec(c+dx))^2}\,dx = -\frac{\log(\cos(c+dx))}{a^2 d} - \frac{\log(a+b\sec(c+dx))}{a^2 d} + \frac{1}{ad(a+b\sec(c+dx))}$$

output
```
-ln(cos(d*x+c))/a^2/d-ln(a+b*sec(d*x+c))/a^2/d+1/a/d/(a+b*sec(d*x+c))
```

3.303.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00

$$\int \frac{\tan(c+dx)}{(a+b\sec(c+dx))^2}\,dx = -\frac{b+b\log(b+a\cos(c+dx))+a\cos(c+dx)\log(b+a\cos(c+dx))}{a^2 d(b+a\cos(c+dx))}$$

input
```
Integrate[Tan[c + d*x]/(a + b*Sec[c + d*x])^2,x]
```

output
```
-((b + b*Log[b + a*Cos[c + d*x]] + a*Cos[c + d*x]*Log[b + a*Cos[c + d*x]])
/(a^2*d*(b + a*Cos[c + d*x])))
```

3.303.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.93, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {3042, 25, 4373, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan(c+dx)}{(a+b\sec(c+dx))^2}\,dx$$

$$\downarrow 3042$$

$$\int -\frac{\cot\left(c+dx+\frac{\pi}{2}\right)}{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^2}dx$$

$$\downarrow 25$$

$$-\int \frac{\cot\left(\frac{1}{2}(2c+\pi)+dx\right)}{\left(a+b\csc\left(\frac{1}{2}(2c+\pi)+dx\right)\right)^2}dx$$

$$\downarrow 4373$$

$$\frac{\int \frac{\cos(c+dx)}{b(a+b\sec(c+dx))^2}d(b\sec(c+dx))}{d}$$

$$\downarrow 54$$

$$\frac{\int \left(\frac{\cos(c+dx)}{a^2 b}-\frac{1}{a^2(a+b\sec(c+dx))}-\frac{1}{a(a+b\sec(c+dx))^2}\right)d(b\sec(c+dx))}{d}$$

$$\downarrow 2009$$

$$\frac{\frac{\log(b\sec(c+dx))}{a^2}-\frac{\log(a+b\sec(c+dx))}{a^2}+\frac{1}{a(a+b\sec(c+dx))}}{d}$$

input
```
Int[Tan[c + d*x]/(a + b*Sec[c + d*x])^2,x]
```

output
```
(Log[b*Sec[c + d*x]]/a^2 - Log[a + b*Sec[c + d*x]]/a^2 + 1/(a*(a + b*Sec[c
 + d*x])))/d
```

3.303.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4373
```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Simp[-(-1)^((m - 1)/2)/(d*b^(m - 1))   Subst[Int[(b^2 - x^
2)^((m - 1)/2)*((a + x)^n/x), x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
 d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

3.303.4 Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.91

method	result	size
derivativedivides	$\frac{-\frac{\ln(a+b\sec(dx+c))}{a^2}+\frac{1}{a(a+b\sec(dx+c))}+\frac{\ln(\sec(dx+c))}{a^2}}{d}$	49
default	$\frac{-\frac{\ln(a+b\sec(dx+c))}{a^2}+\frac{1}{a(a+b\sec(dx+c))}+\frac{\ln(\sec(dx+c))}{a^2}}{d}$	49
risch	$\frac{ix}{a^2}+\frac{2ic}{a^2d}-\frac{2b\,e^{i(dx+c)}}{a^2d\left(a\,e^{2i(dx+c)}+2b\,e^{i(dx+c)}+a\right)}-\frac{\ln\left(e^{2i(dx+c)}+\frac{2b\,e^{i(dx+c)}}{a}+1\right)}{a^2d}$	99

input
```
int(tan(d*x+c)/(a+b*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-1/a^2*ln(a+b*sec(d*x+c))+1/a/(a+b*sec(d*x+c))+1/a^2*ln(sec(d*x+c)))
```

3.303.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.85

$$\int \frac{\tan(c+dx)}{(a+b\sec(c+dx))^2}\,dx = -\frac{(a\cos(dx+c)+b)\log(a\cos(dx+c)+b)+b}{a^3 d\cos(dx+c)+a^2 bd}$$

input
```
integrate(tan(d*x+c)/(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

output
```
-((a*cos(d*x + c) + b)*log(a*cos(d*x + c) + b) + b)/(a^3*d*cos(d*x + c) +
a^2*b*d)
```

3.303.6 Sympy [F]

$$\int \frac{\tan(c+dx)}{(a+b\sec(c+dx))^2}\,dx$$

$$= \begin{cases} \frac{\tilde{\infty} x \tan(c)}{\sec^2(c)} \\ \frac{\log(\tan^2(c+dx)+1)}{2a^2 d} \\ -\frac{1}{2b^2 d\sec^2(c+dx)} \\ \frac{\int \frac{\tan(c+dx)}{\cos^2(c+dx)\sec^2(c+dx)-2\cos(c+dx)\sec(c+dx)+1}\,dx}{a^2} \\ \frac{x\tan(c)}{(a+b\sec(c))^2} \\ -\frac{2a\log\left(\frac{a}{b}+\sec(c+dx)\right)}{2a^3 d+2a^2 bd\sec(c+dx)} + \frac{a\log(\tan^2(c+dx)+1)}{2a^3 d+2a^2 bd\sec(c+dx)} + \frac{2a}{2a^3 d+2a^2 bd\sec(c+dx)} - \frac{2b\log\left(\frac{a}{b}+\sec(c+dx)\right)\sec(c+dx)}{2a^3 d+2a^2 bd\sec(c+dx)} + \frac{b\log(\tan^2(c+dx)} \end{cases}$$

input
```
integrate(tan(d*x+c)/(a+b*sec(d*x+c))**2,x)
```

output
```
Piecewise((zoo*x*tan(c)/sec(c)**2, Eq(a, 0) & Eq(b, 0) & Eq(d, 0)), (log(t
an(c + d*x)**2 + 1)/(2*a**2*d), Eq(b, 0)), (-1/(2*b**2*d*sec(c + d*x)**2),
 Eq(a, 0)), (Integral(tan(c + d*x)/(cos(c + d*x)**2*sec(c + d*x)**2 - 2*co
s(c + d*x)*sec(c + d*x) + 1), x)/a**2, Eq(b, -a*cos(c + d*x))), (x*tan(c)/
(a + b*sec(c))**2, Eq(d, 0)), (-2*a*log(a/b + sec(c + d*x))/(2*a**3*d + 2*
a**2*b*d*sec(c + d*x)) + a*log(tan(c + d*x)**2 + 1)/(2*a**3*d + 2*a**2*b*d
*sec(c + d*x)) + 2*a/(2*a**3*d + 2*a**2*b*d*sec(c + d*x)) - 2*b*log(a/b +
sec(c + d*x))*sec(c + d*x)/(2*a**3*d + 2*a**2*b*d*sec(c + d*x)) + b*log(ta
n(c + d*x)**2 + 1)*sec(c + d*x)/(2*a**3*d + 2*a**2*b*d*sec(c + d*x)), True
))
```

3.303.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.76

$$\int \frac{\tan(c+dx)}{(a+b\sec(c+dx))^2}\,dx = -\frac{\frac{b}{a^3\cos(dx+c)+a^2b}+\frac{\log(a\cos(dx+c)+b)}{a^2}}{d}$$

input
```
integrate(tan(d*x+c)/(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

output
```
-(b/(a^3*cos(d*x + c) + a^2*b) + log(a*cos(d*x + c) + b)/a^2)/d
```

3.303.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 238 vs. 2(54) = 108.

Time = 0.37 (sec) , antiderivative size = 238, normalized size of antiderivative = 4.41

$$\int \frac{\tan(c+dx)}{(a+b\sec(c+dx))^2}\,dx = -\frac{\frac{(a-b)\log\left(\left|a+b+\frac{a(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{b(\cos(dx+c)-1)}{\cos(dx+c)+1}\right|\right)}{a^3-a^2b} - \frac{a^2-2ab-b^2+\frac{a^2(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{2ab(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{b^2(\cos(dx+c)-1)}{\cos(dx+c)+1}}{(a^3-a^2b)\left(a+b+\frac{a(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{b(\cos(dx+c)-1)}{\cos(dx+c)+1}\right)} - \frac{\log\left(\left|-\frac{\cos(dx}{\cos(dx}\right.\right.}{a^2}}{d}$$

input
```
integrate(tan(d*x+c)/(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

output
```
-((a - b)*log(abs(a + b + a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - b*(cos
(d*x + c) - 1)/(cos(d*x + c) + 1)))/(a^3 - a^2*b) - (a^2 - 2*a*b - b^2 + a
^2*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 2*a*b*(cos(d*x + c) - 1)/(cos(d
*x + c) + 1) + b^2*(cos(d*x + c) - 1)/(cos(d*x + c) + 1))/((a^3 - a^2*b)*(
a + b + a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - b*(cos(d*x + c) - 1)/(co
s(d*x + c) + 1))) - log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1))/a
^2)/d
```

3.303.9 Mupad [B] (verification not implemented)

Time = 14.37 (sec) , antiderivative size = 257, normalized size of antiderivative = 4.76

$$\int \frac{\tan(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \frac{2\,\mathrm{atanh}\left(\frac{a}{2\left(\frac{a}{2}+b+\frac{a\cos(c+dx)}{2}\right)}-\frac{a\cos(c+dx)}{2\left(\frac{a}{2}+b+\frac{a\cos(c+dx)}{2}\right)}\right)}{a^2 d}$$

$$-\frac{b\left(a+a\cos(c+dx)-2\,a\,\mathrm{atanh}\left(\frac{a}{2\left(\frac{a}{2}+b+\frac{a\cos(c+dx)}{2}\right)}-\frac{a\cos(c+dx)}{2\left(\frac{a}{2}+b+\frac{a\cos(c+dx)}{2}\right)}\right)+2\,a\cos(c+dx)\,\mathrm{atanh}\left(\right.\right.}{a^2 d\,(a-b)\,(b+}$$

input
```
int(tan(c + d*x)/(a + b/cos(c + d*x))^2,x)
```

output
```
(2*atanh(a/(2*(a/2 + b + (a*cos(c + d*x))/2)) - (a*cos(c + d*x))/(2*(a/2 +
 b + (a*cos(c + d*x))/2))))/(a^2*d) - (b*(a + a*cos(c + d*x) - 2*a*atanh(a
/(2*(a/2 + b + (a*cos(c + d*x))/2)) - (a*cos(c + d*x))/(2*(a/2 + b + (a*co
s(c + d*x))/2))) + 2*a*cos(c + d*x)*atanh(a/(2*(a/2 + b + (a*cos(c + d*x))
/2)) - (a*cos(c + d*x))/(2*(a/2 + b + (a*cos(c + d*x))/2))) + (2*atanh(a/(
2*(a/2 + b + (a*cos(c + d*x))/2)) - (a*cos(c + d*x))/(2*(a/2 + b + (a*cos(
c + d*x))/2)))*(a^3*d - a^3*d*cos(c + d*x)))/(a^2*d)))/(a^2*d*(a - b)*(b +
 a*cos(c + d*x)))
```

3.304 $\int \frac{\cot(c+dx)}{(a+b\sec(c+dx))^2}\,dx$

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3.304.1 Optimal result

Integrand size = 19, antiderivative size = 138

$$\int \frac{\cot(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \frac{\log(\cos(c+dx))}{a^2 d} + \frac{\log(1-\sec(c+dx))}{2(a+b)^2 d} + \frac{\log(1+\sec(c+dx))}{2(a-b)^2 d} - \frac{b^2(3a^2-b^2)\log(a+b\sec(c+dx))}{a^2\left(a^2-b^2\right)^2 d} + \frac{b^2}{a\left(a^2-b^2\right) d(a+b\sec(c+dx))}$$

output
```
ln(cos(d*x+c))/a^2/d+1/2*ln(1-sec(d*x+c))/(a+b)^2/d+1/2*ln(1+sec(d*x+c))/(
a-b)^2/d-b^2*(3*a^2-b^2)*ln(a+b*sec(d*x+c))/a^2/(a^2-b^2)^2/d+b^2/a/(a^2-b
^2)/d/(a+b*sec(d*x+c))
```

3.304.2 Mathematica [A] (verified)

Time = 0.43 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.99

$$\int \frac{\cot(c+dx)}{(a+b\sec(c+dx))^2}\,dx = -\frac{b^2\left(-\frac{\log(\cos(c+dx))}{a^2b^2} - \frac{\log(1-\sec(c+dx))}{2b^2(a+b)^2} - \frac{\log(1+\sec(c+dx))}{2(a-b)^2b^2} + \frac{(3a^2-b^2)\log(a+b\sec(c+dx))}{a^2(a-b)^2(a+b)^2} - \frac{1}{a(a^2-b^2)(a+b\sec(c+dx))}\right)}{d}$$

input
```
Integrate[Cot[c + d*x]/(a + b*Sec[c + d*x])^2,x]
```

output
```
-((b^2*(-(Log[Cos[c + d*x]]/(a^2*b^2)) - Log[1 - Sec[c + d*x]]/(2*b^2*(a +
 b)^2) - Log[1 + Sec[c + d*x]]/(2*(a - b)^2*b^2) + ((3*a^2 - b^2)*Log[a +
b*Sec[c + d*x]])/(a^2*(a - b)^2*(a + b)^2) - 1/(a*(a^2 - b^2)*(a + b*Sec[c
 + d*x])))/d)
```

3.304.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.01, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {3042, 25, 4373, 615, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot(c+dx)}{(a+b\sec(c+dx))^2}\,dx$$

$$\downarrow 3042$$

$$\int -\frac{1}{\cot\left(c+dx+\frac{\pi}{2}\right)\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^2}dx$$

$$\downarrow 25$$

$$-\int \frac{1}{\cot\left(\frac{1}{2}(2c+\pi)+dx\right)\left(a+b\csc\left(\frac{1}{2}(2c+\pi)+dx\right)\right)^2}dx$$

$$\downarrow 4373$$

$$-\frac{b^2\int \frac{\cos(c+dx)}{b(a+b\sec(c+dx))^2(b^2-b^2\sec^2(c+dx))}d(b\sec(c+dx))}{d}$$

$$\downarrow 615$$

$$-\frac{b^2\int \left(\frac{3a^2-b^2}{a^2(a-b)^2(a+b)^2(a+b\sec(c+dx))}+\frac{\cos(c+dx)}{a^2b^3}+\frac{1}{2b^2(a+b)^2(b-b\sec(c+dx))}-\frac{1}{2(a-b)^2b^2(\sec(c+dx)b+b)}+\frac{1}{a(a-b)(a+b)(a+b\sec($$

$$\downarrow 2009$$

$$-\frac{b^2\left(-\frac{1}{a(a^2-b^2)(a+b\sec(c+dx))}+\frac{\log(b\sec(c+dx))}{a^2b^2}+\frac{(3a^2-b^2)\log(a+b\sec(c+dx))}{a^2(a^2-b^2)^2}-\frac{\log(b-b\sec(c+dx))}{2b^2(a+b)^2}-\frac{\log(b\sec(c+dx)+b)}{2b^2(a-b)^2}\right)}{d}$$

input
```
Int[Cot[c + d*x]/(a + b*Sec[c + d*x])^2,x]
```

output
```
-((b^2*(Log[b*Sec[c + d*x]]/(a^2*b^2) - Log[b - b*Sec[c + d*x]]/(2*b^2*(a
+ b)^2) + ((3*a^2 - b^2)*Log[a + b*Sec[c + d*x]])/(a^2*(a^2 - b^2)^2) - Lo
g[b + b*Sec[c + d*x]]/(2*(a - b)^2*b^2) - 1/(a*(a^2 - b^2)*(a + b*Sec[c +
d*x])))/d)
```

3.304.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 615
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_),
 x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(c + d*x)^n*(a + b*x^2)^p, x], x]
 /; FreeQ[{a, b, c, d, e, m, n}, x] && ILtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4373
```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Simp[-(-1)^((m - 1)/2)/(d*b^(m - 1))   Subst[Int[(b^2 - x^
2)^((m - 1)/2)*((a + x)^n/x), x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
 d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

3.304.4 Maple [A] (verified)

Time = 1.05 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.83

method	result
derivativedivides	$\frac{\frac{\ln(\cos(dx+c)+1)}{2(a-b)^2}+\frac{\ln(\cos(dx+c)-1)}{2(a+b)^2}-\frac{b^3}{a^2(a+b)(a-b)(b+a\cos(dx+c))}-\frac{b^2\left(3a^2-b^2\right)\ln(b+a\cos(dx+c))}{(a+b)^2(a-b)^2a^2}}{d}$
default	$\frac{\frac{\ln(\cos(dx+c)+1)}{2(a-b)^2}+\frac{\ln(\cos(dx+c)-1)}{2(a+b)^2}-\frac{b^3}{a^2(a+b)(a-b)(b+a\cos(dx+c))}-\frac{b^2\left(3a^2-b^2\right)\ln(b+a\cos(dx+c))}{(a+b)^2(a-b)^2a^2}}{d}$
risch	$\frac{ix}{a^2}-\frac{ix}{a^2+2ab+b^2}-\frac{ic}{d(a^2+2ab+b^2)}-\frac{ix}{a^2-2ab+b^2}-\frac{ic}{d(a^2-2ab+b^2)}+\frac{6ib^2x}{a^4-2a^2b^2+b^4}+\frac{6ib^2c}{d(a^4-2a^2b^2+b^4)}-$

input
```
int(cot(d*x+c)/(a+b*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/2/(a-b)^2*ln(cos(d*x+c)+1)+1/2/(a+b)^2*ln(cos(d*x+c)-1)-1/a^2*b^3/(
a+b)/(a-b)/(b+a*cos(d*x+c))-b^2*(3*a^2-b^2)/(a+b)^2/(a-b)^2/a^2*ln(b+a*cos
(d*x+c)))
```

3.304.5 Fricas [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 234, normalized size of antiderivative = 1.70

$$\int \frac{\cot(c+dx)}{(a+b\sec(c+dx))^2}\,dx =$$
$$-\frac{2\,a^2b^3-2\,b^5+2\,(3\,a^2b^3-b^5+(3\,a^3b^2-ab^4)\cos(dx+c))\log(a\cos(dx+c)+b)-(a^4b+2\,a^3b^2+a^2b}{2\,((a^7-2}$$

input
```
integrate(cot(d*x+c)/(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

output
```
-1/2*(2*a^2*b^3 - 2*b^5 + 2*(3*a^2*b^3 - b^5 + (3*a^3*b^2 - a*b^4)*cos(d*x
 + c))*log(a*cos(d*x + c) + b) - (a^4*b + 2*a^3*b^2 + a^2*b^3 + (a^5 + 2*a
^4*b + a^3*b^2)*cos(d*x + c))*log(1/2*cos(d*x + c) + 1/2) - (a^4*b - 2*a^3
*b^2 + a^2*b^3 + (a^5 - 2*a^4*b + a^3*b^2)*cos(d*x + c))*log(-1/2*cos(d*x
+ c) + 1/2))/((a^7 - 2*a^5*b^2 + a^3*b^4)*d*cos(d*x + c) + (a^6*b - 2*a^4*
b^3 + a^2*b^5)*d)
```

3.304.6 Sympy [F]

$$\int \frac{\cot(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \int \frac{\cot(c+dx)}{(a+b\sec(c+dx))^2}\,dx$$

input
```
integrate(cot(d*x+c)/(a+b*sec(d*x+c))**2,x)
```

output
```
Integral(cot(c + d*x)/(a + b*sec(c + d*x))**2, x)
```

3.304.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.03

$$\int \frac{\cot(c+dx)}{(a+b\sec(c+dx))^2}\,dx$$
$$= -\frac{\frac{2b^3}{a^4b-a^2b^3+(a^5-a^3b^2)\cos(dx+c)} + \frac{2\left(3a^2b^2-b^4\right)\log(a\cos(dx+c)+b)}{a^6-2a^4b^2+a^2b^4} - \frac{\log(\cos(dx+c)+1)}{a^2-2ab+b^2} - \frac{\log(\cos(dx+c)-1)}{a^2+2ab+b^2}}{2d}$$

input
```
integrate(cot(d*x+c)/(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

output
```
-1/2*(2*b^3/(a^4*b - a^2*b^3 + (a^5 - a^3*b^2)*cos(d*x + c)) + 2*(3*a^2*b^
2 - b^4)*log(a*cos(d*x + c) + b)/(a^6 - 2*a^4*b^2 + a^2*b^4) - log(cos(d*x
 + c) + 1)/(a^2 - 2*a*b + b^2) - log(cos(d*x + c) - 1)/(a^2 + 2*a*b + b^2)
)/d
```

3.304.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 303 vs. 2(134) = 268.

Time = 0.32 (sec) , antiderivative size = 303, normalized size of antiderivative = 2.20

$$\int \frac{\cot(c+dx)}{(a+b\sec(c+dx))^2}\,dx =$$
$$-\frac{\frac{2\left(3a^2b^2-b^4\right)\log\left(\left|-a-b-\frac{a(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{b(\cos(dx+c)-1)}{\cos(dx+c)+1}\right|\right)}{a^6-2a^4b^2+a^2b^4} - \frac{\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)}{a^2+2ab+b^2} - \frac{2\left(3a^2b^2+4ab^3+b^4+\frac{3a^2b^2(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{b^4(\cos}{\cos}\right.}{\left(a^5+a^4b-a^3b^2-a^2b^3\right)\left(a+b+\frac{a(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{b(\cos}{c}\right.}$$
$$2d$$

input
```
integrate(cot(d*x+c)/(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

output
```
-1/2*(2*(3*a^2*b^2 - b^4)*log(abs(-a - b - a*(cos(d*x + c) - 1)/(cos(d*x +
 c) + 1) + b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1)))/(a^6 - 2*a^4*b^2 + a^
2*b^4) - log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1))/(a^2 + 2*a*b +
b^2) - 2*(3*a^2*b^2 + 4*a*b^3 + b^4 + 3*a^2*b^2*(cos(d*x + c) - 1)/(cos(d*
x + c) + 1) - b^4*(cos(d*x + c) - 1)/(cos(d*x + c) + 1))/((a^5 + a^4*b - a
^3*b^2 - a^2*b^3)*(a + b + a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - b*(co
s(d*x + c) - 1)/(cos(d*x + c) + 1))) + 2*log(abs(-(cos(d*x + c) - 1)/(cos(
d*x + c) + 1) + 1))/a^2)/d
```

3.304.9 Mupad [B] (verification not implemented)

Time = 14.62 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.16

$$\int \frac{\cot(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \frac{\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{d\,(a^2+2\,a\,b+b^2)} - \frac{\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+1\right)}{a^2\,d} - \frac{b^2\,\ln\left(a+b-a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2\right)\,(3\,a^2-b^2)}{a^2\,d\,(a^2-b^2)^2} - \frac{2\,b^3}{a\,d\,(a+b)\,(a-b)^2\left((b-a)\,\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+a+b\right)}$$

input
```
int(cot(c + d*x)/(a + b/cos(c + d*x))^2,x)
```

output
```
log(tan(c/2 + (d*x)/2))/(d*(2*a*b + a^2 + b^2)) - log(tan(c/2 + (d*x)/2)^2
 + 1)/(a^2*d) - (b^2*log(a + b - a*tan(c/2 + (d*x)/2)^2 + b*tan(c/2 + (d*x
)/2)^2)*(3*a^2 - b^2))/(a^2*d*(a^2 - b^2)^2) - (2*b^3)/(a*d*(a + b)*(a - b
)^2*(a + b - tan(c/2 + (d*x)/2)^2*(a - b)))
```

3.305 $\int \frac{\cot^3(c+dx)}{(a+b\sec(c+dx))^2}\,dx$

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3.305.1 Optimal result

Integrand size = 21, antiderivative size = 197

$$\begin{aligned}\int \frac{\cot^3(c+dx)}{(a+b\sec(c+dx))^2}\,dx = &-\frac{\log(\cos(c+dx))}{a^2 d}-\frac{(a+2b)\log(1-\sec(c+dx))}{2(a+b)^3 d}\\ &-\frac{(a-2b)\log(1+\sec(c+dx))}{2(a-b)^3 d}\\ &-\frac{b^4(5a^2-b^2)\log(a+b\sec(c+dx))}{a^2\left(a^2-b^2\right)^3 d}\\ &+\frac{1}{4(a+b)^2 d(1-\sec(c+dx))}+\frac{1}{4(a-b)^2 d(1+\sec(c+dx))}\\ &+\frac{b^4}{a\left(a^2-b^2\right)^2 d(a+b\sec(c+dx))}\end{aligned}$$

output

```
-ln(cos(d*x+c))/a^2/d-1/2*(a+2*b)*ln(1-sec(d*x+c))/(a+b)^3/d-1/2*(a-2*b)*l
n(1+sec(d*x+c))/(a-b)^3/d-b^4*(5*a^2-b^2)*ln(a+b*sec(d*x+c))/a^2/(a^2-b^2)
^3/d+1/4/(a+b)^2/d/(1-sec(d*x+c))+1/4/(a-b)^2/d/(1+sec(d*x+c))+b^4/a/(a^2-
b^2)^2/d/(a+b*sec(d*x+c))
```

3.305.2 Mathematica [A] (verified)

Time = 1.77 (sec) , antiderivative size = 187, normalized size of antiderivative = 0.95

$$\int \frac{\cot^3(c+dx)}{(a+b\sec(c+dx))^2}\,dx$$
$$= \frac{b^4\left(-\frac{4\log(\cos(c+dx))}{a^2b^4} - \frac{2(a+2b)\log(1-\sec(c+dx))}{b^4(a+b)^3} - \frac{2(a-2b)\log(1+\sec(c+dx))}{(a-b)^3b^4} - \frac{4(5a^2-b^2)\log(a+b\sec(c+dx))}{a^2(a-b)^3(a+b)^3} - \frac{1}{b^4(a+b)^2(-1+se}\right.}{4d}$$

input
```
Integrate[Cot[c + d*x]^3/(a + b*Sec[c + d*x])^2,x]
```

output
```
(b^4*((-4*Log[Cos[c + d*x]])/(a^2*b^4) - (2*(a + 2*b)*Log[1 - Sec[c + d*x]
])/(b^4*(a + b)^3) - (2*(a - 2*b)*Log[1 + Sec[c + d*x]])/((a - b)^3*b^4) -
 (4*(5*a^2 - b^2)*Log[a + b*Sec[c + d*x]])/(a^2*(a - b)^3*(a + b)^3) - 1/(
b^4*(a + b)^2*(-1 + Sec[c + d*x])) + 1/((a - b)^2*b^4*(1 + Sec[c + d*x]))
 + 4/(a*(a - b)^2*(a + b)^2*(a + b*Sec[c + d*x])))/(4*d)
```

3.305.3 Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.01, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3042, 25, 4373, 615, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^3(c+dx)}{(a+b\sec(c+dx))^2}\,dx$$
$$\downarrow 3042$$
$$\int -\frac{1}{\cot\left(c+dx+\frac{\pi}{2}\right)^3\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^2}dx$$
$$\downarrow 25$$
$$-\int \frac{1}{\cot\left(\frac{1}{2}(2c+\pi)+dx\right)^3\left(a+b\csc\left(\frac{1}{2}(2c+\pi)+dx\right)\right)^2}dx$$
$$\downarrow 4373$$
$$\frac{b^4\int \frac{\cos(c+dx)}{b(a+b\sec(c+dx))^2(b^2-b^2\sec^2(c+dx))^2}d(b\sec(c+dx))}{d}$$

$$\downarrow 615$$

$$\frac{b^4 \int \left(\frac{2b-a}{2(a-b)^3 b^4 (\sec(c+dx)b+b)} + \frac{\cos(c+dx)}{a^2 b^5} + \frac{a+2b}{2b^4(a+b)^3(b-b\sec(c+dx))} + \frac{b^2-5a^2}{a^2(a-b)^3(a+b)^3(a+b\sec(c+dx))} + \frac{1}{4b^3(a+b)^2(b-b\sec(c+dx}\right.}{d}$$

$$\downarrow 2009$$

$$\frac{b^4 \left(\frac{\log(b\sec(c+dx))}{a^2 b^4} + \frac{1}{a(a^2-b^2)^2(a+b\sec(c+dx))} - \frac{(5a^2-b^2)\log(a+b\sec(c+dx))}{a^2(a^2-b^2)^3} - \frac{(a+2b)\log(b-b\sec(c+dx))}{2b^4(a+b)^3} - \frac{(a-2b)\log(b\sec(c+dx)+}{2b^4(a-b)^3}\right.}{d}$$

input
```
Int[Cot[c + d*x]^3/(a + b*Sec[c + d*x])^2,x]
```

output
```
(b^4*(Log[b*Sec[c + d*x]]/(a^2*b^4) - ((a + 2*b)*Log[b - b*Sec[c + d*x]])/
(2*b^4*(a + b)^3) - ((5*a^2 - b^2)*Log[a + b*Sec[c + d*x]])/(a^2*(a^2 - b^
2)^3) - ((a - 2*b)*Log[b + b*Sec[c + d*x]])/(2*(a - b)^3*b^4) + 1/(4*b^3*(
a + b)^2*(b - b*Sec[c + d*x])) + 1/(a*(a^2 - b^2)^2*(a + b*Sec[c + d*x]))
+ 1/(4*(a - b)^2*b^3*(b + b*Sec[c + d*x])))/d
```

3.305.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 615
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_),
 x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(c + d*x)^n*(a + b*x^2)^p, x], x]
 /; FreeQ[{a, b, c, d, e, m, n}, x] && ILtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4373
```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Simp[-(-1)^((m - 1)/2)/(d*b^(m - 1))   Subst[Int[(b^2 - x^
2)^((m - 1)/2)*((a + x)^n/x), x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
 d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

3.305.4 Maple [A] (verified)

Time = 1.47 (sec) , antiderivative size = 164, normalized size of antiderivative = 0.83

method	result
derivativedivides	$\frac{\frac{1}{4(a+b)^2(\cos(dx+c)-1)}+\frac{(-a-2b)\ln(\cos(dx+c)-1)}{2(a+b)^3}-\frac{1}{4(a-b)^2(\cos(dx+c)+1)}+\frac{(2b-a)\ln(\cos(dx+c)+1)}{2(a-b)^3}-\frac{b^5}{a^2(a+b)^2(a-b)^2(b+a\cos}}{d}$
default	$\frac{\frac{1}{4(a+b)^2(\cos(dx+c)-1)}+\frac{(-a-2b)\ln(\cos(dx+c)-1)}{2(a+b)^3}-\frac{1}{4(a-b)^2(\cos(dx+c)+1)}+\frac{(2b-a)\ln(\cos(dx+c)+1)}{2(a-b)^3}-\frac{b^5}{a^2(a+b)^2(a-b)^2(b+a\cos}}{d}$
risch	$-\frac{2ibx}{a^3-3a^2b+3ab^2-b^3}+\frac{10ib^4x}{a^6-3a^4b^2+3a^2b^4-b^6}-\frac{2ib^6c}{a^2d(a^6-3a^4b^2+3a^2b^4-b^6)}+\frac{iac}{d(a^3+3a^2b+3ab^2+b^3)}+\frac{}{d(a^6-}$

input
```
int(cot(d*x+c)^3/(a+b*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/4/(a+b)^2/(cos(d*x+c)-1)+1/2*(-a-2*b)/(a+b)^3*ln(cos(d*x+c)-1)-1/4/
(a-b)^2/(cos(d*x+c)+1)+1/2*(2*b-a)/(a-b)^3*ln(cos(d*x+c)+1)-b^5/a^2/(a+b)^
2/(a-b)^2/(b+a*cos(d*x+c))-b^4*(5*a^2-b^2)/(a+b)^3/(a-b)^3/a^2*ln(b+a*cos(
d*x+c)))
```

3.305.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 693 vs. $2(187) = 374$.

Time = 0.41 (sec) , antiderivative size = 693, normalized size of antiderivative = 3.52

$$\int \frac{\cot^3(c+dx)}{(a+b\sec(c+dx))^2}\,dx$$
$$= \frac{a^6b+a^2b^5-2b^7-2\left(a^6b-a^4b^3+a^2b^5-b^7\right)\cos(dx+c)^2+\left(a^7-2a^5b^2+a^3b^4\right)\cos(dx+c)+2\left(5a^2b^5\right.}{}$$

input
```
integrate(cot(d*x+c)^3/(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

output
```
1/2*(a^6*b + a^2*b^5 - 2*b^7 - 2*(a^6*b - a^4*b^3 + a^2*b^5 - b^7)*cos(d*x
 + c)^2 + (a^7 - 2*a^5*b^2 + a^3*b^4)*cos(d*x + c) + 2*(5*a^2*b^5 - b^7 -
(5*a^3*b^4 - a*b^6)*cos(d*x + c)^3 - (5*a^2*b^5 - b^7)*cos(d*x + c)^2 + (5
*a^3*b^4 - a*b^6)*cos(d*x + c))*log(a*cos(d*x + c) + b) + (a^6*b + a^5*b^2
 - 3*a^4*b^3 - 5*a^3*b^4 - 2*a^2*b^5 - (a^7 + a^6*b - 3*a^5*b^2 - 5*a^4*b^
3 - 2*a^3*b^4)*cos(d*x + c)^3 - (a^6*b + a^5*b^2 - 3*a^4*b^3 - 5*a^3*b^4 -
 2*a^2*b^5)*cos(d*x + c)^2 + (a^7 + a^6*b - 3*a^5*b^2 - 5*a^4*b^3 - 2*a^3*
b^4)*cos(d*x + c))*log(1/2*cos(d*x + c) + 1/2) + (a^6*b - a^5*b^2 - 3*a^4*
b^3 + 5*a^3*b^4 - 2*a^2*b^5 - (a^7 - a^6*b - 3*a^5*b^2 + 5*a^4*b^3 - 2*a^3
*b^4)*cos(d*x + c)^3 - (a^6*b - a^5*b^2 - 3*a^4*b^3 + 5*a^3*b^4 - 2*a^2*b^
5)*cos(d*x + c)^2 + (a^7 - a^6*b - 3*a^5*b^2 + 5*a^4*b^3 - 2*a^3*b^4)*cos(
d*x + c))*log(-1/2*cos(d*x + c) + 1/2))/((a^9 - 3*a^7*b^2 + 3*a^5*b^4 - a^
3*b^6)*d*cos(d*x + c)^3 + (a^8*b - 3*a^6*b^3 + 3*a^4*b^5 - a^2*b^7)*d*cos(
d*x + c)^2 - (a^9 - 3*a^7*b^2 + 3*a^5*b^4 - a^3*b^6)*d*cos(d*x + c) - (a^8
*b - 3*a^6*b^3 + 3*a^4*b^5 - a^2*b^7)*d)
```

3.305.6 Sympy [F]

$$\int \frac{\cot^3(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \int \frac{\cot^3(c+dx)}{(a+b\sec(c+dx))^2}\,dx$$

input
```
integrate(cot(d*x+c)**3/(a+b*sec(d*x+c))**2,x)
```

output
```
Integral(cot(c + d*x)**3/(a + b*sec(c + d*x))**2, x)
```

3.305.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 303, normalized size of antiderivative = 1.54

$$\int \frac{\cot^3(c+dx)}{(a+b\sec(c+dx))^2}\,dx = -\frac{\frac{2\,(5\,a^2b^4-b^6)\log(a\cos(dx+c)+b)}{a^8-3\,a^6b^2+3\,a^4b^4-a^2b^6} + \frac{(a-2\,b)\log(\cos(dx+c)+1)}{a^3-3\,a^2b+3\,ab^2-b^3} + \frac{(a+2\,b)\log(\cos(dx+c)-1)}{a^3+3\,a^2b+3\,ab^2+b^3} + \frac{a^4b+a^2b^3+2\,b^5}{a^6b-2\,a^4b^3+a^2b^5-(a^7-2\,a^5b^2+a^3b^4)\,\mathrm{co}}}{2\,d}$$

input
```
integrate(cot(d*x+c)^3/(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

output
```
-1/2*(2*(5*a^2*b^4 - b^6)*log(a*cos(d*x + c) + b)/(a^8 - 3*a^6*b^2 + 3*a^4
*b^4 - a^2*b^6) + (a - 2*b)*log(cos(d*x + c) + 1)/(a^3 - 3*a^2*b + 3*a*b^2
 - b^3) + (a + 2*b)*log(cos(d*x + c) - 1)/(a^3 + 3*a^2*b + 3*a*b^2 + b^3)
+ (a^4*b + a^2*b^3 + 2*b^5 - 2*(a^4*b + b^5)*cos(d*x + c)^2 + (a^5 - a^3*b
^2)*cos(d*x + c))/(a^6*b - 2*a^4*b^3 + a^2*b^5 - (a^7 - 2*a^5*b^2 + a^3*b^
4)*cos(d*x + c)^3 - (a^6*b - 2*a^4*b^3 + a^2*b^5)*cos(d*x + c)^2 + (a^7 -
2*a^5*b^2 + a^3*b^4)*cos(d*x + c)))/d
```

3.305.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 656 vs. $2(187) = 374$.

Time $= 0.38$ (sec) , antiderivative size $= 656$, normalized size of antiderivative $= 3.33$

$$\int \frac{\cot^3(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \frac{4\,(a+2\,b)\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)}{a^3+3\,a^2b+3\,ab^2+b^3} + \frac{8\,(5\,a^2b^4-b^6)\log\left(\left|-a-b-\frac{a(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{b(\cos(dx+c)-1)}{\cos(dx+c)+1}\right|\right)}{a^8-3\,a^6b^2+3\,a^4b^4-a^2b^6} - \frac{a^5-a^4b-a^3b^2+a^2b^3+\frac{3\,a^5(\cos(dx+c)-1)}{\cos(dx+c)+1}}{-}$$

input
```
integrate(cot(d*x+c)^3/(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

output
```
-1/8*(4*(a + 2*b)*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1))/(a^3 +
 3*a^2*b + 3*a*b^2 + b^3) + 8*(5*a^2*b^4 - b^6)*log(abs(-a - b - a*(cos(d*
x + c) - 1)/(cos(d*x + c) + 1) + b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1)))
/(a^8 - 3*a^6*b^2 + 3*a^4*b^4 - a^2*b^6) - (a^5 - a^4*b - a^3*b^2 + a^2*b^
3 + 3*a^5*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 3*a^4*b*(cos(d*x + c) -
1)/(cos(d*x + c) + 1) - 3*a^3*b^2*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) +
3*a^2*b^3*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 20*a*b^4*(cos(d*x + c) -
 1)/(cos(d*x + c) + 1) + 4*b^5*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 2*a
^5*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 4*a^4*b*(cos(d*x + c) - 1)^
2/(cos(d*x + c) + 1)^2 - 2*a^3*b^2*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)
^2 + 12*a^2*b^3*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 4*a*b^4*(cos(d
*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 4*b^5*(cos(d*x + c) - 1)^2/(cos(d*x
+ c) + 1)^2)/((a^6 - 2*a^4*b^2 + a^2*b^4)*(a*(cos(d*x + c) - 1)/(cos(d*x +
 c) + 1) + b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + a*(cos(d*x + c) - 1)^
2/(cos(d*x + c) + 1)^2 - b*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2)) - (
cos(d*x + c) - 1)/((a^2 - 2*a*b + b^2)*(cos(d*x + c) + 1)) - 8*log(abs(-(c
os(d*x + c) - 1)/(cos(d*x + c) + 1) + 1))/a^2)/d
```

3.305.9 Mupad [B] (verification not implemented)

Time = 14.96 (sec) , antiderivative size = 313, normalized size of antiderivative = 1.59

$$\int \frac{\cot^3(c+dx)}{(a+b\sec(c+dx))^2}\,dx$$

$$= \frac{\frac{a^2-2ab+b^2}{2(a+b)} - \frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2\left(a^5-4a^4b+6a^3b^2-4a^2b^3+ab^4-16b^5\right)}{2a(a+b)^2(a-b)}}{d\left(\left(4a^3-12a^2b+12ab^2-4b^3\right)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4+\left(-4a^3+4a^2b+4ab^2-4b^3\right)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2\right)}$$
$$- \frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2}{8d(a-b)^2} + \frac{\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+1\right)}{a^2d} - \frac{\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)(a+2b)}{d\left(a^3+3a^2b+3ab^2+b^3\right)}$$
$$- \frac{b^4\ln\left(a+b-a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2\right)(5a^2-b^2)}{a^2d(a^2-b^2)^3}$$

input
```
int(cot(c + d*x)^3/(a + b/cos(c + d*x))^2,x)
```

output
```
((a^2 - 2*a*b + b^2)/(2*(a + b)) - (tan(c/2 + (d*x)/2)^2*(a*b^4 - 4*a^4*b
+ a^5 - 16*b^5 - 4*a^2*b^3 + 6*a^3*b^2))/(2*a*(a + b)^2*(a - b)))/(d*(tan(
c/2 + (d*x)/2)^2*(4*a*b^2 + 4*a^2*b - 4*a^3 - 4*b^3) + tan(c/2 + (d*x)/2)^
4*(12*a*b^2 - 12*a^2*b + 4*a^3 - 4*b^3))) - tan(c/2 + (d*x)/2)^2/(8*d*(a -
 b)^2) + log(tan(c/2 + (d*x)/2)^2 + 1)/(a^2*d) - (log(tan(c/2 + (d*x)/2))*
(a + 2*b))/(d*(3*a*b^2 + 3*a^2*b + a^3 + b^3)) - (b^4*log(a + b - a*tan(c/
2 + (d*x)/2)^2 + b*tan(c/2 + (d*x)/2)^2)*(5*a^2 - b^2))/(a^2*d*(a^2 - b^2)
^3)
```

3.306 $\int \frac{\cot^5(c+dx)}{(a+b\sec(c+dx))^2}\,dx$

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3.306.2 Mathematica [A] (verified) . . . 2210
3.306.3 Rubi [A] (verified) . . . 2210
3.306.4 Maple [A] (verified) . . . 2212
3.306.5 Fricas [B] (verification not implemented) . . . 2212
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3.306.8 Giac [B] (verification not implemented) . . . 2214
3.306.9 Mupad [B] (verification not implemented) . . . 2215

3.306.1 Optimal result

Integrand size = 21, antiderivative size = 278

$$\begin{aligned}\int \frac{\cot^5(c+dx)}{(a+b\sec(c+dx))^2}\,dx = {}& \frac{\log(\cos(c+dx))}{a^2 d} + \frac{(4a^2+13ab+12b^2)\log(1-\sec(c+dx))}{8(a+b)^4 d} \\ & + \frac{(4a^2-13ab+12b^2)\log(1+\sec(c+dx))}{8(a-b)^4 d} \\ & - \frac{b^6(7a^2-b^2)\log(a+b\sec(c+dx))}{a^2\left(a^2-b^2\right)^4 d} \\ & - \frac{1}{16(a+b)^2 d(1-\sec(c+dx))^2} \\ & - \frac{5a+9b}{16(a+b)^3 d(1-\sec(c+dx))} \\ & - \frac{1}{16(a-b)^2 d(1+\sec(c+dx))^2} \\ & - \frac{5a-9b}{16(a-b)^3 d(1+\sec(c+dx))} \\ & + \frac{b^6}{a\left(a^2-b^2\right)^3 d(a+b\sec(c+dx))}\end{aligned}$$

output

```
ln(cos(d*x+c))/a^2/d+1/8*(4*a^2+13*a*b+12*b^2)*ln(1-sec(d*x+c))/(a+b)^4/d+
1/8*(4*a^2-13*a*b+12*b^2)*ln(1+sec(d*x+c))/(a-b)^4/d-b^6*(7*a^2-b^2)*ln(a+
b*sec(d*x+c))/a^2/(a^2-b^2)^4/d-1/16/(a+b)^2/d/(1-sec(d*x+c))^2+1/16*(-5*a
-9*b)/(a+b)^3/d/(1-sec(d*x+c))-1/16/(a-b)^2/d/(1+sec(d*x+c))^2+1/16*(-5*a+
9*b)/(a-b)^3/d/(1+sec(d*x+c))+b^6/a/(a^2-b^2)^3/d/(a+b*sec(d*x+c))
```

3.306.2 Mathematica [A] (verified)

Time = 6.45 (sec) , antiderivative size = 283, normalized size of antiderivative = 1.02

$$\int \frac{\cot^5(c+dx)}{(a+b\sec(c+dx))^2}\,dx = -\frac{b^6\left(-\frac{\log(\cos(c+dx))}{a^2b^6} - \frac{(4a^2+13ab+12b^2)\log(1-\sec(c+dx))}{8b^6(a+b)^4} - \frac{(4a^2-13ab+12b^2)\log(1+\sec(c+dx))}{8(a-b)^4b^6} + \frac{(7a^2-b^2)\log(a+b\sec(c+dx))}{a^2(a-b)^4(a+b)^4}\right.}{}$$

input
```
Integrate[Cot[c + d*x]^5/(a + b*Sec[c + d*x])^2,x]
```

output
```
-((b^6*(-(Log[Cos[c + d*x]]/(a^2*b^6)) - ((4*a^2 + 13*a*b + 12*b^2)*Log[1
 - Sec[c + d*x]])/(8*b^6*(a + b)^4) - ((4*a^2 - 13*a*b + 12*b^2)*Log[1 + Se
c[c + d*x]])/(8*(a - b)^4*b^6) + ((7*a^2 - b^2)*Log[a + b*Sec[c + d*x]])/(
a^2*(a - b)^4*(a + b)^4) + 1/(16*b^4*(a + b)^2*(b - b*Sec[c + d*x])^2) + (
5*a + 9*b)/(16*b^5*(a + b)^3*(b - b*Sec[c + d*x])) - 1/(a*(a - b)^3*(a + b
)^3*(a + b*Sec[c + d*x])) + 1/(16*(a - b)^2*b^4*(b + b*Sec[c + d*x])^2) +
(5*a - 9*b)/(16*(a - b)^3*b^5*(b + b*Sec[c + d*x]))))/d)
```

3.306.3 Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 285, normalized size of antiderivative = 1.03, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3042, 25, 4373, 615, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^5(c+dx)}{(a+b\sec(c+dx))^2}\,dx$$
$$\downarrow \text{3042}$$
$$\int -\frac{1}{\cot\left(c+dx+\frac{\pi}{2}\right)^5\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^2}dx$$
$$\downarrow \text{25}$$
$$-\int \frac{1}{\cot\left(\frac{1}{2}(2c+\pi)+dx\right)^5\left(a+b\csc\left(\frac{1}{2}(2c+\pi)+dx\right)\right)^2}dx$$
$$\downarrow \text{4373}$$

$$-\frac{b^6\int \frac{\cos(c+dx)}{b(a+b\sec(c+dx))^2(b^2-b^2\sec^2(c+dx))^3}d(b\sec(c+dx))}{d}$$

$$\downarrow 615$$

$$-\frac{b^6\int \left(\frac{9b-5a}{16(a-b)^3b^5(\sec(c+dx)b+b)^2}+\frac{\cos(c+dx)}{a^2b^7}+\frac{4a^2+13ba+12b^2}{8b^6(a+b)^4(b-b\sec(c+dx))}+\frac{7a^2-b^2}{a^2(a-b)^4(a+b)^4(a+b\sec(c+dx))}+\frac{-4a^2+13ba-12}{8(a-b)^4b^6(\sec(c+d}\right.}{}$$

$$\downarrow 2009$$

$$-\frac{b^6\left(\frac{\log(b\sec(c+dx))}{a^2b^6}-\frac{1}{a(a^2-b^2)^3(a+b\sec(c+dx))}+\frac{(7a^2-b^2)\log(a+b\sec(c+dx))}{a^2(a^2-b^2)^4}-\frac{(4a^2+13ab+12b^2)\log(b-b\sec(c+dx))}{8b^6(a+b)^4}-\frac{(4a^2-13a}{}\right.}{}$$

input
```
Int[Cot[c + d*x]^5/(a + b*Sec[c + d*x])^2,x]
```

output
```
-((b^6*(Log[b*Sec[c + d*x]]/(a^2*b^6) - ((4*a^2 + 13*a*b + 12*b^2)*Log[b -
 b*Sec[c + d*x]])/(8*b^6*(a + b)^4) + ((7*a^2 - b^2)*Log[a + b*Sec[c + d*x
]])/(a^2*(a^2 - b^2)^4) - ((4*a^2 - 13*a*b + 12*b^2)*Log[b + b*Sec[c + d*x
]])/(8*(a - b)^4*b^6) + 1/(16*b^4*(a + b)^2*(b - b*Sec[c + d*x])^2) + (5*a
 + 9*b)/(16*b^5*(a + b)^3*(b - b*Sec[c + d*x])) - 1/(a*(a^2 - b^2)^3*(a +
b*Sec[c + d*x])) + 1/(16*(a - b)^2*b^4*(b + b*Sec[c + d*x])^2) + (5*a - 9*
b)/(16*(a - b)^3*b^5*(b + b*Sec[c + d*x])))/d)
```

3.306.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 615
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_),
 x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(c + d*x)^n*(a + b*x^2)^p, x], x]
 /; FreeQ[{a, b, c, d, e, m, n}, x] && ILtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4373
```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Simp[-(-1)^((m - 1)/2)/(d*b^(m - 1))   Subst[Int[(b^2 - x^
2)^((m - 1)/2)*((a + x)^n/x), x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
 d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

3.306.4 Maple [A] (verified)

Time = 2.36 (sec) , antiderivative size = 230, normalized size of antiderivative = 0.83

method	result
derivativedivides	$\frac{-\frac{1}{16(a-b)^2(\cos(dx+c)+1)^2}-\frac{-7a+11b}{16(a-b)^3(\cos(dx+c)+1)}+\frac{\left(4a^2-13ab+12b^2\right)\ln(\cos(dx+c)+1)}{8(a-b)^4}-\frac{b^7}{a^2(a+b)^3(a-b)^3(b+a\cos(dx+c))}-\frac{b}{}}{d}$
default	$\frac{-\frac{1}{16(a-b)^2(\cos(dx+c)+1)^2}-\frac{-7a+11b}{16(a-b)^3(\cos(dx+c)+1)}+\frac{\left(4a^2-13ab+12b^2\right)\ln(\cos(dx+c)+1)}{8(a-b)^4}-\frac{b^7}{a^2(a+b)^3(a-b)^3(b+a\cos(dx+c))}-\frac{b}{}}{d}$
risch	Expression too large to display

input
```
int(cot(d*x+c)^5/(a+b*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-1/16/(a-b)^2/(cos(d*x+c)+1)^2-1/16*(-7*a+11*b)/(a-b)^3/(cos(d*x+c)+1
)+1/8*(4*a^2-13*a*b+12*b^2)/(a-b)^4*ln(cos(d*x+c)+1)-b^7/a^2/(a+b)^3/(a-b)
^3/(b+a*cos(d*x+c))-b^6*(7*a^2-b^2)/(a+b)^4/(a-b)^4/a^2*ln(b+a*cos(d*x+c))
-1/16/(a+b)^2/(cos(d*x+c)-1)^2-1/16*(7*a+11*b)/(a+b)^3/(cos(d*x+c)-1)+1/8*
(4*a^2+13*a*b+12*b^2)/(a+b)^4*ln(cos(d*x+c)-1))
```

3.306.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1378 vs. 2(262) = 524.

Time = 0.62 (sec) , antiderivative size = 1378, normalized size of antiderivative = 4.96

$$\int \frac{\cot^5(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \text{Too large to display}$$

input
```
integrate(cot(d*x+c)^5/(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

output
```
1/8*(6*a^8*b - 18*a^6*b^3 + 2*a^4*b^5 + 2*a^2*b^7 + 8*b^9 + 2*(5*a^8*b - 1
8*a^6*b^3 + 13*a^4*b^5 - 4*a^2*b^7 + 4*b^9)*cos(d*x + c)^4 - 2*(4*a^9 - 15
*a^7*b^2 + 18*a^5*b^4 - 7*a^3*b^6)*cos(d*x + c)^3 - 2*(7*a^8*b - 24*a^6*b^
3 + 11*a^4*b^5 - 2*a^2*b^7 + 8*b^9)*cos(d*x + c)^2 + 6*(a^9 - 4*a^7*b^2 +
5*a^5*b^4 - 2*a^3*b^6)*cos(d*x + c) - 8*(7*a^2*b^7 - b^9 + (7*a^3*b^6 - a*
b^8)*cos(d*x + c)^5 + (7*a^2*b^7 - b^9)*cos(d*x + c)^4 - 2*(7*a^3*b^6 - a*
b^8)*cos(d*x + c)^3 - 2*(7*a^2*b^7 - b^9)*cos(d*x + c)^2 + (7*a^3*b^6 - a*
b^8)*cos(d*x + c))*log(a*cos(d*x + c) + b) + (4*a^8*b + 3*a^7*b^2 - 16*a^6
*b^3 - 14*a^5*b^4 + 24*a^4*b^5 + 35*a^3*b^6 + 12*a^2*b^7 + (4*a^9 + 3*a^8*
b - 16*a^7*b^2 - 14*a^6*b^3 + 24*a^5*b^4 + 35*a^4*b^5 + 12*a^3*b^6)*cos(d*
x + c)^5 + (4*a^8*b + 3*a^7*b^2 - 16*a^6*b^3 - 14*a^5*b^4 + 24*a^4*b^5 + 3
5*a^3*b^6 + 12*a^2*b^7)*cos(d*x + c)^4 - 2*(4*a^9 + 3*a^8*b - 16*a^7*b^2 -
 14*a^6*b^3 + 24*a^5*b^4 + 35*a^4*b^5 + 12*a^3*b^6)*cos(d*x + c)^3 - 2*(4*
a^8*b + 3*a^7*b^2 - 16*a^6*b^3 - 14*a^5*b^4 + 24*a^4*b^5 + 35*a^3*b^6 + 12
*a^2*b^7)*cos(d*x + c)^2 + (4*a^9 + 3*a^8*b - 16*a^7*b^2 - 14*a^6*b^3 + 24
*a^5*b^4 + 35*a^4*b^5 + 12*a^3*b^6)*cos(d*x + c))*log(1/2*cos(d*x + c) + 1
/2) + (4*a^8*b - 3*a^7*b^2 - 16*a^6*b^3 + 14*a^5*b^4 + 24*a^4*b^5 - 35*a^3
*b^6 + 12*a^2*b^7 + (4*a^9 - 3*a^8*b - 16*a^7*b^2 + 14*a^6*b^3 + 24*a^5*b^
4 - 35*a^4*b^5 + 12*a^3*b^6)*cos(d*x + c)^5 + (4*a^8*b - 3*a^7*b^2 - 16*a^
6*b^3 + 14*a^5*b^4 + 24*a^4*b^5 - 35*a^3*b^6 + 12*a^2*b^7)*cos(d*x + c)...
```

3.306.6 Sympy [F]

$$\int \frac{\cot^5(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \int \frac{\cot^5(c+dx)}{(a+b\sec(c+dx))^2}\,dx$$

input
```
integrate(cot(d*x+c)**5/(a+b*sec(d*x+c))**2,x)
```

output
```
Integral(cot(c + d*x)**5/(a + b*sec(c + d*x))**2, x)
```

3.306.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 558 vs. $2(262) = 524$.

Time = 0.21 (sec) , antiderivative size = 558, normalized size of antiderivative = 2.01

$$\int \frac{\cot^5(c+dx)}{(a+b\sec(c+dx))^2}\,dx =$$

$$-\frac{8\,(7\,a^2b^6-b^8)\log(a\cos(dx+c)+b)}{a^{10}-4\,a^8b^2+6\,a^6b^4-4\,a^4b^6+a^2b^8} - \frac{(4\,a^2-13\,ab+12\,b^2)\log(\cos(dx+c)+1)}{a^4-4\,a^3b+6\,a^2b^2-4\,ab^3+b^4} - \frac{(4\,a^2+13\,ab+12\,b^2)\log(\cos(dx+c)-1)}{a^4+4\,a^3b+6\,a^2b^2+4\,ab^3+b^4} - \frac{}{a^8b-3\,a^6b^3+3\,a}$$

input
```
integrate(cot(d*x+c)^5/(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

output
```
-1/8*(8*(7*a^2*b^6 - b^8)*log(a*cos(d*x + c) + b)/(a^10 - 4*a^8*b^2 + 6*a^
6*b^4 - 4*a^4*b^6 + a^2*b^8) - (4*a^2 - 13*a*b + 12*b^2)*log(cos(d*x + c)
+ 1)/(a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4) - (4*a^2 + 13*a*b + 12*b^
2)*log(cos(d*x + c) - 1)/(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4) - 2*(
3*a^6*b - 6*a^4*b^3 - 5*a^2*b^5 - 4*b^7 + (5*a^6*b - 13*a^4*b^3 - 4*b^7)*c
os(d*x + c)^4 - (4*a^7 - 11*a^5*b^2 + 7*a^3*b^4)*cos(d*x + c)^3 - (7*a^6*b
 - 17*a^4*b^3 - 6*a^2*b^5 - 8*b^7)*cos(d*x + c)^2 + 3*(a^7 - 3*a^5*b^2 + 2
*a^3*b^4)*cos(d*x + c))/(a^8*b - 3*a^6*b^3 + 3*a^4*b^5 - a^2*b^7 + (a^9 -
3*a^7*b^2 + 3*a^5*b^4 - a^3*b^6)*cos(d*x + c)^5 + (a^8*b - 3*a^6*b^3 + 3*a
^4*b^5 - a^2*b^7)*cos(d*x + c)^4 - 2*(a^9 - 3*a^7*b^2 + 3*a^5*b^4 - a^3*b^
6)*cos(d*x + c)^3 - 2*(a^8*b - 3*a^6*b^3 + 3*a^4*b^5 - a^2*b^7)*cos(d*x +
c)^2 + (a^9 - 3*a^7*b^2 + 3*a^5*b^4 - a^3*b^6)*cos(d*x + c)))/d
```

3.306.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 795 vs. $2(262) = 524$.

Time = 0.43 (sec) , antiderivative size = 795, normalized size of antiderivative = 2.86

$$\int \frac{\cot^5(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \text{Too large to display}$$

input
```
integrate(cot(d*x+c)^5/(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

output

```
1/64*(8*(4*a^2 + 13*a*b + 12*b^2)*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x +
 c) + 1))/(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4) - 64*(7*a^2*b^6 - b^
8)*log(abs(-a - b - a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + b*(cos(d*x +
 c) - 1)/(cos(d*x + c) + 1)))/(a^10 - 4*a^8*b^2 + 6*a^6*b^4 - 4*a^4*b^6 +
a^2*b^8) - (12*a^2*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 32*a*b*(cos(d*x
 + c) - 1)/(cos(d*x + c) + 1) + 20*b^2*(cos(d*x + c) - 1)/(cos(d*x + c) +
1) + a^2*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 2*a*b*(cos(d*x + c) -
 1)^2/(cos(d*x + c) + 1)^2 + b^2*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2
)/(a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4) - (a^2 + 2*a*b + b^2 + 12*a^
2*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 32*a*b*(cos(d*x + c) - 1)/(cos(d
*x + c) + 1) + 20*b^2*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 48*a^2*(cos(
d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 156*a*b*(cos(d*x + c) - 1)^2/(cos(d
*x + c) + 1)^2 + 144*b^2*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2)*(cos(d
*x + c) + 1)^2/((a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*(cos(d*x + c)
- 1)^2) + 64*(7*a^3*b^6 + 5*a^2*b^7 - 3*a*b^8 - b^9 + 7*a^3*b^6*(cos(d*x +
 c) - 1)/(cos(d*x + c) + 1) - 7*a^2*b^7*(cos(d*x + c) - 1)/(cos(d*x + c) +
 1) - a*b^8*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + b^9*(cos(d*x + c) - 1)
/(cos(d*x + c) + 1))/((a^10 - 4*a^8*b^2 + 6*a^6*b^4 - 4*a^4*b^6 + a^2*b^8)
*(a + b + a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - b*(cos(d*x + c) - 1)/(
cos(d*x + c) + 1))) - 64*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1)...
```

3.306.9 Mupad [B] (verification not implemented)

Time = 15.76 (sec) , antiderivative size = 471, normalized size of antiderivative = 1.69

$$\int \frac{\cot^5(c+dx)}{(a+b\sec(c+dx))^2}\,dx$$

$$= \frac{\frac{a^3-3a^2b+3ab^2-b^3}{4(a+b)} + \frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2\left(-13a^4+20a^3b+18a^2b^2-44ab^3+19b^4\right)}{4(a+b)^2} + \frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4\left(3a^7-10a^6b+5a^5b^2+20a^4b^3-35a^3b^4+\right.}{a(a+b)^3(a-b)}}{d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6\left(16a^4-64a^3b+96a^2b^2-64ab^3+16b^4\right)-\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4\left(16a^4-32a^3b+32a\right.\right.}$$

$$-\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4}{64d(a-b)^2} - \frac{\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+1\right)}{a^2d} - \frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2\left(\frac{16a^2+32ab-48b^2}{512(a-b)^4}-\frac{7}{32(a-b)^2}\right)}{d}$$

$$+\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)\left(4a^2+13ab+12b^2\right)}{d\left(4a^4+16a^3b+24a^2b^2+16ab^3+4b^4\right)}$$

$$-\frac{b^6\ln\left(a+b-a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2\right)\left(7a^2-b^2\right)}{a^2d\left(a^2-b^2\right)^4}$$

input

```
int(cot(c + d*x)^5/(a + b/cos(c + d*x))^2,x)
```

output

```
((3*a*b^2 - 3*a^2*b + a^3 - b^3)/(4*(a + b)) + (tan(c/2 + (d*x)/2)^2*(20*a
^3*b - 44*a*b^3 - 13*a^4 + 19*b^4 + 18*a^2*b^2))/(4*(a + b)^2) + (tan(c/2
+ (d*x)/2)^4*(3*a^7 - 10*a^6*b - 5*a*b^6 + 32*b^7 + 22*a^2*b^5 - 35*a^3*b^
4 + 20*a^4*b^3 + 5*a^5*b^2))/(a*(a + b)^3*(a - b)))/(d*(tan(c/2 + (d*x)/2)
^6*(16*a^4 - 64*a^3*b - 64*a*b^3 + 16*b^4 + 96*a^2*b^2) - tan(c/2 + (d*x)/
2)^4*(32*a*b^3 - 32*a^3*b + 16*a^4 - 16*b^4))) - tan(c/2 + (d*x)/2)^4/(64*
d*(a - b)^2) - log(tan(c/2 + (d*x)/2)^2 + 1)/(a^2*d) - (tan(c/2 + (d*x)/2)
^2*((32*a*b + 16*a^2 - 48*b^2)/(512*(a - b)^4) - 7/(32*(a - b)^2)))/d + (l
og(tan(c/2 + (d*x)/2))*(13*a*b + 4*a^2 + 12*b^2))/(d*(16*a*b^3 + 16*a^3*b
+ 4*a^4 + 4*b^4 + 24*a^2*b^2)) - (b^6*log(a + b - a*tan(c/2 + (d*x)/2)^2 +
 b*tan(c/2 + (d*x)/2)^2)*(7*a^2 - b^2))/(a^2*d*(a^2 - b^2)^4)
```

3.307 $\int \frac{\tan^6(c+dx)}{(a+b\sec(c+dx))^2}\,dx$

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3.307.1 Optimal result

Integrand size = 21, antiderivative size = 200

$$\int \frac{\tan^6(c+dx)}{(a+b\sec(c+dx))^2}\,dx = -\frac{x}{a^2} - \frac{a(4a^2-5b^2)\operatorname{arctanh}(\sin(c+dx))}{b^5 d} + \frac{2(a-b)^{3/2}(a+b)^{3/2}\left(4a^2+b^2\right)\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^2 b^5 d} + \frac{\left(a^2-b^2\right)^2\sin(c+dx)}{ab^4 d(b+a\cos(c+dx))} + \frac{\left(3a^2-2b^2\right)\tan(c+dx)}{b^4 d} - \frac{a\sec(c+dx)\tan(c+dx)}{b^3 d} + \frac{\tan^3(c+dx)}{3b^2 d}$$

output

```
-x/a^2-a*(4*a^2-5*b^2)*arctanh(sin(d*x+c))/b^5/d+2*(a-b)^(3/2)*(a+b)^(3/2)
*(4*a^2+b^2)*arctanh((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))/a^2/b^5/d
+(a^2-b^2)^2*sin(d*x+c)/a/b^4/d/(b+a*cos(d*x+c))+(3*a^2-2*b^2)*tan(d*x+c)/
b^4/d-a*sec(d*x+c)*tan(d*x+c)/b^3/d+1/3*tan(d*x+c)^3/b^2/d
```

3.307.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 559 vs. $2(200) = 400$.

Time = 6.18 (sec) , antiderivative size = 559, normalized size of antiderivative = 2.80

$$\int \frac{\tan^6(c+dx)}{(a+b\sec(c+dx))^2}\,dx$$

$$= \frac{(b+a\cos(c+dx))\sec^2(c+dx)\left(-\frac{12(c+dx)(b+a\cos(c+dx))}{a^2} - \frac{24(a^2-b^2)^{3/2}(4a^2+b^2)\operatorname{arctanh}\left(\frac{(-a+b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)(b+a}{a^2b^5}\right.}{}$$

input
```
Integrate[Tan[c + d*x]^6/(a + b*Sec[c + d*x])^2,x]
```

output
```
((b + a*Cos[c + d*x])*Sec[c + d*x]^2*((-12*(c + d*x)*(b + a*Cos[c + d*x]))
/a^2 - (24*(a^2 - b^2)^(3/2)*(4*a^2 + b^2)*ArcTanh[((-a + b)*Tan[(c + d*x)
/2])/Sqrt[a^2 - b^2]]*(b + a*Cos[c + d*x]))/(a^2*b^5) + (12*(4*a^3 - 5*a*b
^2)*(b + a*Cos[c + d*x])*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]])/b^5 + (
12*a*(-4*a^2 + 5*b^2)*(b + a*Cos[c + d*x])*Log[Cos[(c + d*x)/2] + Sin[(c +
 d*x)/2]])/b^5 + ((-6*a + b)*(b + a*Cos[c + d*x]))/(b^3*(Cos[(c + d*x)/2]
- Sin[(c + d*x)/2])^2) + (2*(b + a*Cos[c + d*x])*Sin[(c + d*x)/2])/(b^2*(C
os[(c + d*x)/2] - Sin[(c + d*x)/2])^3) + (4*(9*a^2 - 7*b^2)*(b + a*Cos[c +
 d*x])*Sin[(c + d*x)/2])/(b^4*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])) + (2*
(b + a*Cos[c + d*x])*Sin[(c + d*x)/2])/(b^2*(Cos[(c + d*x)/2] + Sin[(c + d
*x)/2])^3) + ((6*a - b)*(b + a*Cos[c + d*x]))/(b^3*(Cos[(c + d*x)/2] + Sin
[(c + d*x)/2])^2) + (4*(9*a^2 - 7*b^2)*(b + a*Cos[c + d*x])*Sin[(c + d*x)/
2])/(b^4*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])) + (12*(a^2 - b^2)^2*Sin[c
+ d*x])/(a*b^4)))/(12*d*(a + b*Sec[c + d*x])^2)
```

3.307.3 Rubi [A] (verified)

Time = 0.78 (sec) , antiderivative size = 283, normalized size of antiderivative = 1.42, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3042, 4386, 3042, 3376, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^6(c+dx)}{(a+b\sec(c+dx))^2}\,dx$$

$$\downarrow 3042$$

$$\int \frac{\cot\left(c+dx+\frac{\pi}{2}\right)^6}{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^2}dx$$

$$\downarrow 4386$$

$$\int \frac{\sin^2(c+dx)\tan^4(c+dx)}{(a\cos(c+dx)+b)^2}dx$$

$$\downarrow 3042$$

$$\int \frac{\cos\left(c+dx+\frac{\pi}{2}\right)^6}{\sin\left(c+dx+\frac{\pi}{2}\right)^4\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+b\right)^2}dx$$

$$\downarrow 3376$$

$$\int \left(\frac{2\left(3ab^2-2a^3\right)\sec(c+dx)}{b^5}+\frac{\left(a^2-b^2\right)^3}{a^2b^4(a\cos(c+dx)+b)^2}-\frac{3\left(b^2-a^2\right)\sec^2(c+dx)}{b^4}-\frac{1}{a^2}+\frac{2\left(2a^6-3a^4b^2+b^6\right.}{a^2b^5(a\cos(c+dx)+}\right.$$

$$\downarrow 2009$$

$$-\frac{2(a-b)^{3/2}(a+b)^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^2b^3d}-\frac{2a\left(2a^2-3b^2\right)\operatorname{arctanh}(\sin(c+dx))}{b^5d}+$$
$$\frac{4(a-b)^{3/2}(a+b)^{3/2}\left(2a^2+b^2\right)\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^2b^5d}+\frac{3\left(a^2-b^2\right)\tan(c+dx)}{b^4d}+$$
$$\frac{\left(a^2-b^2\right)^2\sin(c+dx)}{ab^4d(a\cos(c+dx)+b)}-\frac{x}{a^2}-\frac{a\operatorname{arctanh}(\sin(c+dx))}{b^3d}-\frac{a\tan(c+dx)\sec(c+dx)}{b^3d}+$$
$$\frac{\tan^3(c+dx)}{3b^2d}+\frac{\tan(c+dx)}{b^2d}$$

input
```
Int[Tan[c + d*x]^6/(a + b*Sec[c + d*x])^2,x]
```

output
```
-(x/a^2) - (a*ArcTanh[Sin[c + d*x]])/(b^3*d) - (2*a*(2*a^2 - 3*b^2)*ArcTan
h[Sin[c + d*x]])/(b^5*d) - (2*(a - b)^(3/2)*(a + b)^(3/2)*ArcTanh[(Sqrt[a
- b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a^2*b^3*d) + (4*(a - b)^(3/2)*(a + b
)^(3/2)*(2*a^2 + b^2)*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])
/(a^2*b^5*d) + ((a^2 - b^2)^2*Sin[c + d*x])/(a*b^4*d*(b + a*Cos[c + d*x]))
 + Tan[c + d*x]/(b^2*d) + (3*(a^2 - b^2)*Tan[c + d*x])/(b^4*d) - (a*Sec[c
+ d*x]*Tan[c + d*x])/(b^3*d) + Tan[c + d*x]^3/(3*b^2*d)
```

3.307.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3376
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig[(d*sin[
e + f*x])^n*(a + b*sin[e + f*x])^m*(1 - sin[e + f*x]^2)^(p/2), x], x] /; Fr
eeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && IntegersQ[m, 2*n, p/2] && (
LtQ[m, -1] || (EqQ[m, -1] && GtQ[p, 0]))
```

rule 4386
```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Int[Cos[c + d*x]^m*((b + a*Sin[c + d*x])^n/Sin[c + d*x]^(m
 + n)), x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[n] &&
 IntegerQ[m] && (IntegerQ[m/2] || LeQ[m, 1])
```

3.307.4 Maple [A] (verified)

Time = 2.08 (sec) , antiderivative size = 365, normalized size of antiderivative = 1.82

method	result
derivativedivides	$-\frac{1}{3b^2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^3}-\frac{2a+b}{2b^3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}-\frac{3a^2+ab-2b^2}{b^4\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{a\left(4a^2-5b^2\right)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{b^5}-2\left(\frac{\left(a^5b-2a^3b^3+$
default	$-\frac{1}{3b^2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^3}-\frac{2a+b}{2b^3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}-\frac{3a^2+ab-2b^2}{b^4\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{a\left(4a^2-5b^2\right)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{b^5}-2\left(\frac{\left(a^5b-2a^3b^3+$
risch	$-\frac{x}{a^2}+\frac{2i\left(6a^4b\,e^{7i(dx+c)}-6a^2b^3e^{7i(dx+c)}+3b^5e^{7i(dx+c)}+12a^5e^{6i(dx+c)}-9a^3b^2e^{6i(dx+c)}+3a\,b^4e^{6i(dx+c)}+30a^4b\,e^{5i(}\right.$

input
```
int(tan(d*x+c)^6/(a+b*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-1/3/b^2/(tan(1/2*d*x+1/2*c)-1)^3-1/2*(2*a+b)/b^3/(tan(1/2*d*x+1/2*c)
-1)^2-(3*a^2+a*b-2*b^2)/b^4/(tan(1/2*d*x+1/2*c)-1)+a*(4*a^2-5*b^2)/b^5*ln(
tan(1/2*d*x+1/2*c)-1)-2/b^5/a^2*((a^5*b-2*a^3*b^3+a*b^5)*tan(1/2*d*x+1/2*c
)/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)-(4*a^6-7*a^4*b^2+2*a
^2*b^4+b^6)/((a-b)*(a+b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a-b)*(a
+b))^(1/2)))-2/a^2*arctan(tan(1/2*d*x+1/2*c))-1/3/b^2/(tan(1/2*d*x+1/2*c)+
1)^3-1/2*(-2*a-b)/b^3/(tan(1/2*d*x+1/2*c)+1)^2-(3*a^2+a*b-2*b^2)/b^4/(tan(
1/2*d*x+1/2*c)+1)-a*(4*a^2-5*b^2)/b^5*ln(tan(1/2*d*x+1/2*c)+1))
```

3.307.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 393 vs. $2(189) = 378$.

Time $= 0.56$ (sec) , antiderivative size $= 843$, normalized size of antiderivative $= 4.22$

$$\int \frac{\tan^6(c+dx)}{(a+b\sec(c+dx))^2}\,dx$$

$$= \left[-\frac{6\,ab^5 dx\cos(dx+c)^4 + 6\,b^6 dx\cos(dx+c)^3 + 3\left((4\,a^5 - 3\,a^3b^2 - ab^4)\cos(dx+c)^4 + (4\,a^4b - 3\,a^2b^3 -\right.\right.$$

$$\left. -\frac{6\,ab^5 dx\cos(dx+c)^4 + 6\,b^6 dx\cos(dx+c)^3 - 6\left((4\,a^5 - 3\,a^3b^2 - ab^4)\cos(dx+c)^4 + (4\,a^4b - 3\,a^2b^3 -\right.\right.$$

input
```
integrate(tan(d*x+c)^6/(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

output
```
[-1/6*(6*a*b^5*d*x*cos(d*x + c)^4 + 6*b^6*d*x*cos(d*x + c)^3 + 3*((4*a^5 -
 3*a^3*b^2 - a*b^4)*cos(d*x + c)^4 + (4*a^4*b - 3*a^2*b^3 - b^5)*cos(d*x +
 c)^3)*sqrt(a^2 - b^2)*log((2*a*b*cos(d*x + c) - (a^2 - 2*b^2)*cos(d*x + c
)^2 - 2*sqrt(a^2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x + c) + 2*a^2 - b^2)/(
a^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + b^2)) + 3*((4*a^6 - 5*a^4*b^2)*c
os(d*x + c)^4 + (4*a^5*b - 5*a^3*b^3)*cos(d*x + c)^3)*log(sin(d*x + c) + 1
) - 3*((4*a^6 - 5*a^4*b^2)*cos(d*x + c)^4 + (4*a^5*b - 5*a^3*b^3)*cos(d*x
+ c)^3)*log(-sin(d*x + c) + 1) + 2*(2*a^3*b^3*cos(d*x + c) - a^2*b^4 - (12
*a^5*b - 13*a^3*b^3 + 3*a*b^5)*cos(d*x + c)^3 - (6*a^4*b^2 - 7*a^2*b^4)*co
s(d*x + c)^2)*sin(d*x + c))/(a^3*b^5*d*cos(d*x + c)^4 + a^2*b^6*d*cos(d*x
+ c)^3), -1/6*(6*a*b^5*d*x*cos(d*x + c)^4 + 6*b^6*d*x*cos(d*x + c)^3 - 6*(
(4*a^5 - 3*a^3*b^2 - a*b^4)*cos(d*x + c)^4 + (4*a^4*b - 3*a^2*b^3 - b^5)*c
os(d*x + c)^3)*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*cos(d*x + c) +
 a)/((a^2 - b^2)*sin(d*x + c))) + 3*((4*a^6 - 5*a^4*b^2)*cos(d*x + c)^4 +
(4*a^5*b - 5*a^3*b^3)*cos(d*x + c)^3)*log(sin(d*x + c) + 1) - 3*((4*a^6 -
5*a^4*b^2)*cos(d*x + c)^4 + (4*a^5*b - 5*a^3*b^3)*cos(d*x + c)^3)*log(-sin
(d*x + c) + 1) + 2*(2*a^3*b^3*cos(d*x + c) - a^2*b^4 - (12*a^5*b - 13*a^3*
b^3 + 3*a*b^5)*cos(d*x + c)^3 - (6*a^4*b^2 - 7*a^2*b^4)*cos(d*x + c)^2)*si
n(d*x + c))/(a^3*b^5*d*cos(d*x + c)^4 + a^2*b^6*d*cos(d*x + c)^3)]
```

3.307.6 Sympy [F]

$$\int \frac{\tan^6(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \int \frac{\tan^6(c+dx)}{(a+b\sec(c+dx))^2}\,dx$$

input
```
integrate(tan(d*x+c)**6/(a+b*sec(d*x+c))**2,x)
```

output
```
Integral(tan(c + d*x)**6/(a + b*sec(c + d*x))**2, x)
```

3.307.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\tan^6(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(tan(d*x+c)^6/(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` f
or more de
```

3.307.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 411 vs. $2(189) = 378$.

Time $= 2.11$ (sec) , antiderivative size $= 411$, normalized size of antiderivative $= 2.06$

$$\int \frac{\tan^6(c+dx)}{(a+b\sec(c+dx))^2}\,dx =$$

$$\frac{\frac{3\,(dx+c)}{a^2}+\frac{3\,(4\,a^3-5\,ab^2)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{b^5}-\frac{3\,(4\,a^3-5\,ab^2)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{b^5}+\frac{6\,(a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,a^2b^2\tan\left(\frac{1}{2}\,dx-\right.}{\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-b\tan\left(\frac{1}{2}\,dx\right.}$$

input
```
integrate(tan(d*x+c)^6/(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

output
```
-1/3*(3*(d*x + c)/a^2 + 3*(4*a^3 - 5*a*b^2)*log(abs(tan(1/2*d*x + 1/2*c) +
 1))/b^5 - 3*(4*a^3 - 5*a*b^2)*log(abs(tan(1/2*d*x + 1/2*c) - 1))/b^5 + 6*
(a^4*tan(1/2*d*x + 1/2*c) - 2*a^2*b^2*tan(1/2*d*x + 1/2*c) + b^4*tan(1/2*d
*x + 1/2*c))/((a*tan(1/2*d*x + 1/2*c)^2 - b*tan(1/2*d*x + 1/2*c)^2 - a - b
)*a*b^4) - 6*(4*a^6 - 7*a^4*b^2 + 2*a^2*b^4 + b^6)*(pi*floor(1/2*(d*x + c)
/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d
*x + 1/2*c))/sqrt(-a^2 + b^2)))/(sqrt(-a^2 + b^2)*a^2*b^5) + 2*(9*a^2*tan(
1/2*d*x + 1/2*c)^5 + 3*a*b*tan(1/2*d*x + 1/2*c)^5 - 6*b^2*tan(1/2*d*x + 1/
2*c)^5 - 18*a^2*tan(1/2*d*x + 1/2*c)^3 + 16*b^2*tan(1/2*d*x + 1/2*c)^3 + 9
*a^2*tan(1/2*d*x + 1/2*c) - 3*a*b*tan(1/2*d*x + 1/2*c) - 6*b^2*tan(1/2*d*x
 + 1/2*c))/((tan(1/2*d*x + 1/2*c)^2 - 1)^3*b^4))/d
```

3.307.9 Mupad [B] (verification not implemented)

Time = 17.23 (sec) , antiderivative size = 9452, normalized size of antiderivative = 47.26

$$\int \frac{\tan^6(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \text{Too large to display}$$

input
```
int(tan(c + d*x)^6/(a + b/cos(c + d*x))^2,x)
```

output
```
((2*tan(c/2 + (d*x)/2)^5*(10*a*b^3 - 6*a^3*b + 36*a^4 + 9*b^4 - 37*a^2*b^2
))/(3*a*b^4) - (2*tan(c/2 + (d*x)/2)^3*(6*a^3*b - 10*a*b^3 + 36*a^4 + 9*b^
4 - 37*a^2*b^2))/(3*a*b^4) + (2*tan(c/2 + (d*x)/2)*(2*a^3*b - 2*a*b^3 + 4*
a^4 + b^4 - 5*a^2*b^2))/(a*b^4) + (2*tan(c/2 + (d*x)/2)^7*(a - b)*(3*a*b^2
 - 2*a^2*b - 4*a^3 + b^3))/(a*b^4))/(d*(a + b + tan(c/2 + (d*x)/2)^8*(a -
b) - tan(c/2 + (d*x)/2)^2*(4*a + 2*b) - tan(c/2 + (d*x)/2)^6*(4*a - 2*b) +
 6*a*tan(c/2 + (d*x)/2)^4)) - (2*atan((((((((((8192*(9*a^5*b^21 - 3*a^4*b^
22 - 13*a^6*b^20 + 6*a^7*b^19 + 25*a^8*b^18 - 41*a^9*b^17 + 3*a^10*b^16 +
26*a^11*b^15 - 12*a^12*b^14))/(a^3*b^16) - (tan(c/2 + (d*x)/2)*(2*a^6*b^23
 - 6*a^7*b^22 + 8*a^8*b^21 - 8*a^9*b^20 + 6*a^10*b^19 - 2*a^11*b^18)*8192i
)/(a^6*b^16))*1i)/a^2 - (8192*tan(c/2 + (d*x)/2)*(2*a^2*b^23 - 6*a^3*b^22
+ 12*a^4*b^21 - 12*a^5*b^20 - 8*a^6*b^19 + 12*a^7*b^18 - 60*a^8*b^17 + 160
*a^9*b^16 - 60*a^10*b^15 - 100*a^11*b^14 + 82*a^12*b^13 - 118*a^13*b^12 +
128*a^14*b^11 + 32*a^15*b^10 - 96*a^16*b^9 + 32*a^17*b^8))/(a^4*b^16))*1i)
/a^2 + (8192*(4*a*b^21 - 3*b^22 - 8*a^2*b^20 + 16*a^3*b^19 + 20*a^4*b^18 -
 26*a^5*b^17 + 74*a^6*b^16 - 280*a^7*b^15 + 192*a^8*b^14 + 332*a^9*b^13 -
1088*a^10*b^12 + 1040*a^11*b^11 + 1129*a^12*b^10 - 2366*a^13*b^9 + 20*a^14
*b^8 + 1696*a^15*b^7 - 528*a^16*b^6 - 416*a^17*b^5 + 192*a^18*b^4))/(a^3*b
^16))*1i)/a^2 - (8192*tan(c/2 + (d*x)/2)*(a*b^20 - 256*a^20*b + 256*a^21 -
 b^21 - 4*a^2*b^19 + 4*a^3*b^18 - 40*a^4*b^17 + 140*a^5*b^16 - 250*a^6*...
```

3.308 $\int \frac{\tan^4(c+dx)}{(a+b\sec(c+dx))^2}\,dx$

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3.308.1 Optimal result

Integrand size = 21, antiderivative size = 150

$$\int \frac{\tan^4(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \frac{x}{a^2} - \frac{2a\operatorname{arctanh}(\sin(c+dx))}{b^3 d} + \frac{2\sqrt{a-b}\sqrt{a+b}(2a^2+b^2)\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^2 b^3 d} + \frac{(2a^2-b^2)\sin(c+dx)}{ab^2 d(b+a\cos(c+dx))} + \frac{\tan(c+dx)}{bd(b+a\cos(c+dx))}$$

output
```
x/a^2-2*a*arctanh(sin(d*x+c))/b^3/d+(2*a^2-b^2)*sin(d*x+c)/a/b^2/d/(b+a*co
s(d*x+c))+2*(2*a^2+b^2)*arctanh((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2)
)*(a-b)^(1/2)*(a+b)^(1/2)/a^2/b^3/d+tan(d*x+c)/b/d/(b+a*cos(d*x+c))
```

3.308.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 327 vs. 2(150) = 300.

Time = 1.53 (sec) , antiderivative size = 327, normalized size of antiderivative = 2.18

$$\int \frac{\tan^4(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \frac{(b+a\cos(c+dx))\sec^2(c+dx)\left(\frac{(c+dx)(b+a\cos(c+dx))}{a^2} + \frac{2\left(-2a^4+a^2b^2+b^4\right)\operatorname{arctanh}\left(\frac{(-a+b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)(b+a\cos(c+dx)}{a^2b^3\sqrt{a^2-b^2}}\right.$$

input
```
Integrate[Tan[c + d*x]^4/(a + b*Sec[c + d*x])^2,x]
```

output
```
((b + a*Cos[c + d*x])*Sec[c + d*x]^2*(((c + d*x)*(b + a*Cos[c + d*x]))/a^2
 + (2*(-2*a^4 + a^2*b^2 + b^4)*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqrt[a^
2 - b^2]]*(b + a*Cos[c + d*x]))/(a^2*b^3*Sqrt[a^2 - b^2]) + (2*a*(b + a*Co
s[c + d*x])*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]])/b^3 - (2*a*(b + a*Co
s[c + d*x])*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]])/b^3 + ((b + a*Cos[c
 + d*x])*Sin[(c + d*x)/2])/(b^2*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])) + ((
b + a*Cos[c + d*x])*Sin[(c + d*x)/2])/(b^2*(Cos[(c + d*x)/2] + Sin[(c + d*
x)/2])) + ((a^2 - b^2)*Sin[c + d*x])/(a*b^2)))/(d*(a + b*Sec[c + d*x])^2)
```

3.308.3 Rubi [A] (verified)

Time = 0.82 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.16, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.524$, Rules used = {3042, 4386, 3042, 3369, 25, 3042, 3536, 3042, 3138, 221, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^4(c+dx)}{(a+b\sec(c+dx))^2}\,dx$$

$$\downarrow 3042$$

$$\int \frac{\cot\left(c+dx+\frac{\pi}{2}\right)^4}{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^2}dx$$

$$\downarrow 4386$$

$$\int \frac{\sin^2(c+dx)\tan^2(c+dx)}{(a\cos(c+dx)+b)^2}dx$$

$$\downarrow 3042$$

$$\int \frac{\cos\left(c+dx+\frac{\pi}{2}\right)^4}{\sin\left(c+dx+\frac{\pi}{2}\right)^2\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+b\right)^2}dx$$

$$\downarrow 3369$$

$$\frac{\int -\frac{\left(2a^2+b\cos(c+dx)a-b^2\cos^2(c+dx)\right)\sec(c+dx)}{b+a\cos(c+dx)}dx}{ab^2} + \frac{\left(2a^2-b^2\right)\sin(c+dx)}{ab^2d(a\cos(c+dx)+b)} + \frac{\tan(c+dx)}{bd(a\cos(c+dx)+b)}$$

$$\downarrow 25$$

$$-\frac{\int \frac{\left(2a^2+b\cos(c+dx)a-b^2\cos^2(c+dx)\right)\sec(c+dx)}{b+a\cos(c+dx)}dx}{ab^2}+\frac{\left(2a^2-b^2\right)\sin(c+dx)}{ab^2d(a\cos(c+dx)+b)}+\frac{\tan(c+dx)}{bd(a\cos(c+dx)+b)}$$

$$\downarrow 3042$$

$$-\frac{\int \frac{2a^2+b\sin\left(c+dx+\frac{\pi}{2}\right)a-b^2\sin\left(c+dx+\frac{\pi}{2}\right)^2}{\sin\left(c+dx+\frac{\pi}{2}\right)\left(b+a\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{ab^2}+\frac{\left(2a^2-b^2\right)\sin(c+dx)}{ab^2d(a\cos(c+dx)+b)}+\frac{\tan(c+dx)}{bd(a\cos(c+dx)+b)}$$

$$\downarrow 3536$$

$$-\frac{-\frac{\left(a^2-b^2\right)\left(2a^2+b^2\right)\int \frac{1}{b+a\cos(c+dx)}dx}{ab}+\frac{2a^2\int\sec(c+dx)dx}{b}-\frac{b^2x}{a}}{ab^2}+\frac{\left(2a^2-b^2\right)\sin(c+dx)}{ab^2d(a\cos(c+dx)+b)}+\frac{\tan(c+dx)}{bd(a\cos(c+dx)+b)}$$

$$\downarrow 3042$$

$$-\frac{-\frac{\left(a^2-b^2\right)\left(2a^2+b^2\right)\int \frac{1}{b+a\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{ab}+\frac{2a^2\int\csc\left(c+dx+\frac{\pi}{2}\right)dx}{b}-\frac{b^2x}{a}}{ab^2}+\frac{\left(2a^2-b^2\right)\sin(c+dx)}{ab^2d(a\cos(c+dx)+b)}+\frac{\tan(c+dx)}{bd(a\cos(c+dx)+b)}$$

$$\downarrow 3138$$

$$-\frac{-\frac{2\left(a^2-b^2\right)\left(2a^2+b^2\right)\int \frac{1}{-\left((a-b)\tan^2\left(\frac{1}{2}(c+dx)\right)\right)+a+b}d\tan\left(\frac{1}{2}(c+dx)\right)}{abd}+\frac{2a^2\int\csc\left(c+dx+\frac{\pi}{2}\right)dx}{b}-\frac{b^2x}{a}}{ab^2}+\frac{\left(2a^2-b^2\right)\sin(c+dx)}{ab^2d(a\cos(c+dx)+b)}+\frac{\tan(c+dx)}{bd(a\cos(c+dx)+b)}$$

$$\downarrow 221$$

$$-\frac{\frac{2a^2\int\csc\left(c+dx+\frac{\pi}{2}\right)dx}{b}-\frac{2\left(a^2-b^2\right)\left(2a^2+b^2\right)\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{abd\sqrt{a-b}\sqrt{a+b}}-\frac{b^2x}{a}}{ab^2}+\frac{\left(2a^2-b^2\right)\sin(c+dx)}{ab^2d(a\cos(c+dx)+b)}+\frac{\tan(c+dx)}{bd(a\cos(c+dx)+b)}$$

$$\downarrow 4257$$

$$-\frac{-\frac{2\left(a^2-b^2\right)\left(2a^2+b^2\right)\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{abd\sqrt{a-b}\sqrt{a+b}}+\frac{2a^2\operatorname{arctanh}(\sin(c+dx))}{bd}-\frac{b^2x}{a}}{ab^2}+\frac{\left(2a^2-b^2\right)\sin(c+dx)}{ab^2d(a\cos(c+dx)+b)}+\frac{\tan(c+dx)}{bd(a\cos(c+dx)+b)}$$

input
```
Int[Tan[c + d*x]^4/(a + b*Sec[c + d*x])^2,x]
```

output
```
-((-((b^2*x)/a) + (2*a^2*ArcTanh[Sin[c + d*x]])/(b*d) - (2*(a^2 - b^2)*(2*
a^2 + b^2)*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a*Sqrt[a
 - b]*b*Sqrt[a + b]*d))/(a*b^2)) + ((2*a^2 - b^2)*Sin[c + d*x])/(a*b^2*d*(b
 + a*Cos[c + d*x])) + Tan[c + d*x]/(b*d*(b + a*Cos[c + d*x]))
```

3.308.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

rule 3369
```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[Cos[e + f*x]*(d*Sin[
e + f*x])^(n + 1)*((a + b*Sin[e + f*x])^(m + 1)/(a*d*f*(n + 1))), x] + (-Si
mp[(a^2*(n + 1) - b^2*(m + n + 2))*Cos[e + f*x]*(d*Sin[e + f*x])^(n + 2)*((
a + b*Sin[e + f*x])^(m + 1)/(a^2*b*d^2*f*(n + 1)*(m + 1))), x] + Simp[1/(a^
2*b*d*(n + 1)*(m + 1))   Int[(d*Sin[e + f*x])^(n + 1)*(a + b*Sin[e + f*x])^
(m + 1)*Simp[a^2*(n + 1)*(n + 2) - b^2*(m + n + 2)*(m + n + 3) + a*b*(m + 1
)*Sin[e + f*x] - (a^2*(n + 1)*(n + 3) - b^2*(m + n + 2)*(m + n + 4))*Sin[e
+ f*x]^2, x], x], x]) /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] &&
IntegersQ[2*m, 2*n] && LtQ[m, -1] && LtQ[n, -1]
```

rule 3536
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)])), x_Symbol] :> Simp[C*(x/(b*d)), x] + (Simp[(A*b^2 - a*b*B + a^2*C)
/(b*(b*c - a*d))   Int[1/(a + b*Sin[e + f*x]), x], x] - Simp[(c^2*C - B*c*d
 + A*d^2)/(d*(b*c - a*d))   Int[1/(c + d*Sin[e + f*x]), x], x]) /; FreeQ[{a
, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] &&
NeQ[c^2 - d^2, 0]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 4386
```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Int[Cos[c + d*x]^m*((b + a*Sin[c + d*x])^n/Sin[c + d*x]^(m
 + n)), x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[n] &&
 IntegerQ[m] && (IntegerQ[m/2] || LeQ[m, 1])
```

3.308.4 Maple [A] (verified)

Time = 1.15 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.46

method	result
derivativedivides	$\frac{-\frac{1}{b^2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}-\frac{2a\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{b^3}+\frac{2\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a^2}-\frac{1}{b^2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{2a\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{b^3}-\frac{2\left(\frac{}{\tan}\right.}{d}$
default	$\frac{-\frac{1}{b^2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}-\frac{2a\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{b^3}+\frac{2\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a^2}-\frac{1}{b^2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{2a\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{b^3}-\frac{2\left(\frac{}{\tan}\right.}{d}$
risch	$\frac{x}{a^2}+\frac{2i\left(a^2b\,e^{3i(dx+c)}-b^3e^{3i(dx+c)}+2a^3e^{2i(dx+c)}-ab^2e^{2i(dx+c)}+3a^2be^{i(dx+c)}-b^3e^{i(dx+c)}+2a^3-ab^2\right)}{da^2b^2\left(ae^{2i(dx+c)}+2be^{i(dx+c)}+a\right)\left(e^{2i(dx+c)}+1\right)}+\frac{2a\ln\left(e^{i(}\right.}{b^3}$

input
```
int(tan(d*x+c)^4/(a+b*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-1/b^2/(tan(1/2*d*x+1/2*c)+1)-2*a/b^3*ln(tan(1/2*d*x+1/2*c)+1)+2/a^2*
arctan(tan(1/2*d*x+1/2*c))-1/b^2/(tan(1/2*d*x+1/2*c)-1)+2*a/b^3*ln(tan(1/2
*d*x+1/2*c)-1)-2/a^2/b^3*((a^3*b-a*b^3)*tan(1/2*d*x+1/2*c)/(tan(1/2*d*x+1/
2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)-(2*a^4-a^2*b^2-b^4)/((a-b)*(a+b))^(1/
2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a-b)*(a+b))^(1/2))))
```

3.308.5 Fricas [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 584, normalized size of antiderivative = 3.89

$$\int \frac{\tan^4(c+dx)}{(a+b\sec(c+dx))^2}\,dx$$
$$= \left[\frac{2\,ab^3 dx\cos(dx+c)^2 + 2\,b^4 dx\cos(dx+c) + \left((2\,a^3+ab^2)\cos(dx+c)^2 + (2\,a^2b+b^3)\cos(dx+c)\right)\sqrt{a^2}\right.$$

input
```
integrate(tan(d*x+c)^4/(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

output
```
[1/2*(2*a*b^3*d*x*cos(d*x + c)^2 + 2*b^4*d*x*cos(d*x + c) + ((2*a^3 + a*b^
2)*cos(d*x + c)^2 + (2*a^2*b + b^3)*cos(d*x + c))*sqrt(a^2 - b^2)*log((2*a
*b*cos(d*x + c) - (a^2 - 2*b^2)*cos(d*x + c)^2 + 2*sqrt(a^2 - b^2)*(b*cos(
d*x + c) + a)*sin(d*x + c) + 2*a^2 - b^2)/(a^2*cos(d*x + c)^2 + 2*a*b*cos(
d*x + c) + b^2)) - 2*(a^4*cos(d*x + c)^2 + a^3*b*cos(d*x + c))*log(sin(d*x
 + c) + 1) + 2*(a^4*cos(d*x + c)^2 + a^3*b*cos(d*x + c))*log(-sin(d*x + c)
 + 1) + 2*(a^2*b^2 + (2*a^3*b - a*b^3)*cos(d*x + c))*sin(d*x + c))/(a^3*b^
3*d*cos(d*x + c)^2 + a^2*b^4*d*cos(d*x + c)), (a*b^3*d*x*cos(d*x + c)^2 +
b^4*d*x*cos(d*x + c) + ((2*a^3 + a*b^2)*cos(d*x + c)^2 + (2*a^2*b + b^3)*c
os(d*x + c))*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*cos(d*x + c) + a
)/((a^2 - b^2)*sin(d*x + c))) - (a^4*cos(d*x + c)^2 + a^3*b*cos(d*x + c))*
log(sin(d*x + c) + 1) + (a^4*cos(d*x + c)^2 + a^3*b*cos(d*x + c))*log(-sin
(d*x + c) + 1) + (a^2*b^2 + (2*a^3*b - a*b^3)*cos(d*x + c))*sin(d*x + c))/
(a^3*b^3*d*cos(d*x + c)^2 + a^2*b^4*d*cos(d*x + c))]
```

3.308.6 Sympy [F]

$$\int \frac{\tan^4(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \int \frac{\tan^4(c+dx)}{(a+b\sec(c+dx))^2}\,dx$$

input
```
integrate(tan(d*x+c)**4/(a+b*sec(d*x+c))**2,x)
```

output
```
Integral(tan(c + d*x)**4/(a + b*sec(c + d*x))**2, x)
```

3.308.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\tan^4(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(tan(d*x+c)^4/(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` f
or more de
```

3.308.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 294 vs. $2(141) = 282$.

Time $= 0.94$ (sec) , antiderivative size $= 294$, normalized size of antiderivative $= 1.96$

$$\int \frac{\tan^4(c+dx)}{(a+b\sec(c+dx))^2}\,dx$$

$$= \frac{dx+c}{a^2} - \frac{2a\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)}{b^3} + \frac{2a\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)}{b^3} - \frac{2\left(2a^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3 - ab\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3 - b^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3 - 2c\right.}{\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4 - b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4 -}$$

input
```
integrate(tan(d*x+c)^4/(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

output
```
((d*x + c)/a^2 - 2*a*log(abs(tan(1/2*d*x + 1/2*c) + 1))/b^3 + 2*a*log(abs(
tan(1/2*d*x + 1/2*c) - 1))/b^3 - 2*(2*a^2*tan(1/2*d*x + 1/2*c)^3 - a*b*tan
(1/2*d*x + 1/2*c)^3 - b^2*tan(1/2*d*x + 1/2*c)^3 - 2*a^2*tan(1/2*d*x + 1/2
*c) - a*b*tan(1/2*d*x + 1/2*c) + b^2*tan(1/2*d*x + 1/2*c))/((a*tan(1/2*d*x
 + 1/2*c)^4 - b*tan(1/2*d*x + 1/2*c)^4 - 2*a*tan(1/2*d*x + 1/2*c)^2 + a +
b)*a*b^2) + 2*(2*a^4 - a^2*b^2 - b^4)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sg
n(-2*a + 2*b) + arctan(-(a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/
sqrt(-a^2 + b^2)))/(sqrt(-a^2 + b^2)*a^2*b^3))/d
```

3.308.9 Mupad [B] (verification not implemented)

Time = 16.19 (sec) , antiderivative size = 7044, normalized size of antiderivative = 46.96

$$\int \frac{\tan^4(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \text{Too large to display}$$

input
```
int(tan(c + d*x)^4/(a + b/cos(c + d*x))^2,x)
```

output
```
(2*atan((((((((((8192*(8*a^5*b^13 - 3*a^4*b^14 - 9*a^6*b^12 + 5*a^7*b^11 +
 6*a^8*b^10 - 13*a^9*b^9 + 6*a^10*b^8))/(a^3*b^8) - (tan(c/2 + (d*x)/2)*(2
*a^6*b^15 - 6*a^7*b^14 + 8*a^8*b^13 - 8*a^9*b^12 + 6*a^10*b^11 - 2*a^11*b^
10)*8192i)/(a^6*b^8))*1i)/a^2 - (8192*tan(c/2 + (d*x)/2)*(2*a^2*b^15 - 6*a
^3*b^14 + 10*a^4*b^13 - 10*a^5*b^12 + a^6*b^11 + 3*a^7*b^10 - 9*a^8*b^9 +
25*a^9*b^8 - 28*a^10*b^7 + 28*a^11*b^6 - 24*a^12*b^5 + 8*a^13*b^4))/(a^4*b
^8))*1i)/a^2 - (8192*(3*b^14 - 5*a*b^13 + 6*a^2*b^12 - 8*a^3*b^11 - 5*a^4*
b^10 + 11*a^5*b^9 - 18*a^6*b^8 + 40*a^7*b^7 - 34*a^8*b^6 + 14*a^9*b^5 + 24
*a^10*b^4 - 52*a^11*b^3 + 24*a^12*b^2))/(a^3*b^8))*1i)/a^2 + (8192*tan(c/2
 + (d*x)/2)*(16*a^12*b - a*b^12 - 16*a^13 + b^13 + 2*a^2*b^11 - 2*a^3*b^10
 + 5*a^4*b^9 - 21*a^5*b^8 + 44*a^6*b^7 - 44*a^7*b^6 + 12*a^8*b^5 + 4*a^9*b
^4 - 16*a^10*b^3 + 16*a^11*b^2))/(a^4*b^8))/a^2 - (((((((((8192*(8*a^5*b^13
 - 3*a^4*b^14 - 9*a^6*b^12 + 5*a^7*b^11 + 6*a^8*b^10 - 13*a^9*b^9 + 6*a^10
*b^8))/(a^3*b^8) + (tan(c/2 + (d*x)/2)*(2*a^6*b^15 - 6*a^7*b^14 + 8*a^8*b^
13 - 8*a^9*b^12 + 6*a^10*b^11 - 2*a^11*b^10)*8192i)/(a^6*b^8))*1i)/a^2 + (
8192*tan(c/2 + (d*x)/2)*(2*a^2*b^15 - 6*a^3*b^14 + 10*a^4*b^13 - 10*a^5*b^
12 + a^6*b^11 + 3*a^7*b^10 - 9*a^8*b^9 + 25*a^9*b^8 - 28*a^10*b^7 + 28*a^1
1*b^6 - 24*a^12*b^5 + 8*a^13*b^4))/(a^4*b^8))*1i)/a^2 - (8192*(3*b^14 - 5*
a*b^13 + 6*a^2*b^12 - 8*a^3*b^11 - 5*a^4*b^10 + 11*a^5*b^9 - 18*a^6*b^8 +
40*a^7*b^7 - 34*a^8*b^6 + 14*a^9*b^5 + 24*a^10*b^4 - 52*a^11*b^3 + 24*a...
```

3.309 $\int \frac{\tan^2(c+dx)}{(a+b\sec(c+dx))^2}\,dx$

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3.309.1 Optimal result

Integrand size = 21, antiderivative size = 85

$$\int \frac{\tan^2(c+dx)}{(a+b\sec(c+dx))^2}\,dx = -\frac{x}{a^2} + \frac{2b\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^2\sqrt{a-b}\sqrt{a+b}d} + \frac{\tan(c+dx)}{ad(a+b\sec(c+dx))}$$

output
```
-x/a^2+2*b*arctanh((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))/a^2/d/(a-b)
^(1/2)/(a+b)^(1/2)+tan(d*x+c)/a/d/(a+b*sec(d*x+c))
```

3.309.2 Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.94

$$\int \frac{\tan^2(c+dx)}{(a+b\sec(c+dx))^2}\,dx = -\frac{c+dx+\frac{2b\operatorname{arctanh}\left(\frac{(-a+b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}} - \frac{a\sin(c+dx)}{b+a\cos(c+dx)}}{a^2d}$$

input
```
Integrate[Tan[c + d*x]^2/(a + b*Sec[c + d*x])^2,x]
```

output
```
-((c + d*x + (2*b*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/Sq
rt[a^2 - b^2] - (a*Sin[c + d*x])/(b + a*Cos[c + d*x]))/(a^2*d))
```

3.309.3 Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.06, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$, Rules used = {3042, 4382, 3042, 4549, 27, 3042, 4270, 3042, 3138, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^2(c+dx)}{(a+b\sec(c+dx))^2}\,dx$$

$$\downarrow 3042$$

$$\int \frac{\cot\left(c+dx+\frac{\pi}{2}\right)^2}{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^2}dx$$

$$\downarrow 4382$$

$$\int \frac{\sec^2(c+dx)-1}{(a+b\sec(c+dx))^2}dx$$

$$\downarrow 3042$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^2-1}{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^2}dx$$

$$\downarrow 4549$$

$$\frac{\tan(c+dx)}{ad(a+b\sec(c+dx))} - \frac{\int \frac{a^2-b^2}{a+b\sec(c+dx)}dx}{a\left(a^2-b^2\right)}$$

$$\downarrow 27$$

$$\frac{\tan(c+dx)}{ad(a+b\sec(c+dx))} - \frac{\int \frac{1}{a+b\sec(c+dx)}dx}{a}$$

$$\downarrow 3042$$

$$\frac{\tan(c+dx)}{ad(a+b\sec(c+dx))} - \frac{\int \frac{1}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a}$$

$$\downarrow 4270$$

$$\frac{\tan(c+dx)}{ad(a+b\sec(c+dx))} - \frac{\frac{x}{a} - \frac{\int \frac{1}{\frac{a\cos(c+dx)}{b}+1}dx}{a}}{a}$$

$$\downarrow 3042$$

$$\frac{\tan(c+dx)}{ad(a+b\sec(c+dx))} - \frac{\frac{x}{a} - \frac{\int \frac{1}{a\sin\left(c+dx+\frac{\pi}{2}\right)+1}dx}{b}}{a}$$

$$\downarrow 3138$$

$$\frac{\tan(c+dx)}{ad(a+b\sec(c+dx))} - \frac{\frac{x}{a} - \frac{2\int \frac{1}{\left(1-\frac{a}{b}\right)\tan^2\left(\frac{1}{2}(c+dx)\right)+\frac{a+b}{b}}d\tan\left(\frac{1}{2}(c+dx)\right)}{ad}}{a}$$

$$\downarrow 221$$

$$\frac{\tan(c+dx)}{ad(a+b\sec(c+dx))} - \frac{\frac{x}{a} - \frac{2b\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{ad\sqrt{a-b}\sqrt{a+b}}}{a}$$

input
```
Int[Tan[c + d*x]^2/(a + b*Sec[c + d*x])^2,x]
```

output
```
-((x/a - (2*b*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a*Sqrt
[a - b]*Sqrt[a + b]*d))/a) + Tan[c + d*x]/(a*d*(a + b*Sec[c + d*x]))
```

3.309.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

rule 4270
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(-1), x_Symbol] :> Simp[x/a, x]
- Simp[1/a   Int[1/(1 + (a/b)*Sin[c + d*x]), x], x] /; FreeQ[{a, b, c, d},
x] && NeQ[a^2 - b^2, 0]
```

rule 4382
```
Int[cot[(c_.) + (d_.)*(x_)]^2*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_),
x_Symbol] :> Int[(-1 + Csc[c + d*x]^2)*(a + b*Csc[c + d*x])^n, x] /; FreeQ[
{a, b, c, d, n}, x] && NeQ[a^2 - b^2, 0]
```

rule 4549
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_))^(m_), x_Symbol] :> Simp[(A*b^2 + a^2*C)*Cot[e + f*x]*((a + b*Csc[
e + f*x])^(m + 1)/(a*f*(m + 1)*(a^2 - b^2))), x] + Simp[1/(a*(m + 1)*(a^2 -
 b^2))   Int[(a + b*Csc[e + f*x])^(m + 1)*Simp[A*(a^2 - b^2)*(m + 1) - a*b*
(A + C)*(m + 1)*Csc[e + f*x] + (A*b^2 + a^2*C)*(m + 2)*Csc[e + f*x]^2, x],
x], x] /; FreeQ[{a, b, e, f, A, C}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[2*m
] && LtQ[m, -1]
```

3.309.4 Maple [A] (verified)

Time = 0.79 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.35

method	result
derivativedivides	$\frac{-\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2 a-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2 b-a-b}+\frac{2b\,\operatorname{arctanh}\left(\frac{(a-b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a-b)(a+b)}}\right)}{\sqrt{(a-b)(a+b)}}}{a^2}-\frac{2\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a^2}}{d}$
default	$\frac{-\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2 a-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2 b-a-b}+\frac{2b\,\operatorname{arctanh}\left(\frac{(a-b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a-b)(a+b)}}\right)}{\sqrt{(a-b)(a+b)}}}{a^2}-\frac{2\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a^2}}{d}$
risch	$-\frac{x}{a^2}+\frac{2i\left(b\,\mathrm{e}^{i(dx+c)}+a\right)}{a^2 d\left(a\,\mathrm{e}^{2i(dx+c)}+2b\,\mathrm{e}^{i(dx+c)}+a\right)}-\frac{b\ln\left(\mathrm{e}^{i(dx+c)}+\frac{-ia^2+ib^2+b\sqrt{a^2-b^2}}{a\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}\,d\,a^2}+\frac{b\ln\left(\mathrm{e}^{i(dx+c)}+\frac{ia^2-ib^2+b\sqrt{a^2-}}{\sqrt{a^2-b^2}\,a}\right.}{\sqrt{a^2-b^2}\,d\,a^2}$

input
```
int(tan(d*x+c)^2/(a+b*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(2/a^2*(-a*tan(1/2*d*x+1/2*c)/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*
c)^2*b-a-b)+b/((a-b)*(a+b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a-b)*
(a+b))^(1/2)))-2/a^2*arctan(tan(1/2*d*x+1/2*c)))
```

3.309.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 162 vs. $2(76) = 152$.

Time = 0.30 (sec) , antiderivative size = 383, normalized size of antiderivative = 4.51

$$\int \frac{\tan^2(c+dx)}{(a+b\sec(c+dx))^2}\,dx$$

$$= \left[-\frac{2\left(a^3-ab^2\right)dx\cos\left(dx+c\right)+2\left(a^2b-b^3\right)dx-\left(ab\cos\left(dx+c\right)+b^2\right)\sqrt{a^2-b^2}\log\left(\frac{2\,ab\cos(dx+c)-\left(a^2-2\,b^2\right)}{2\left(\left(a^5-a^3b^2\right)d\cos\left(dx+c\right)+\left(a^4b-\right.\right.\right.\right.}{}\right.$$

$$\left. -\frac{\left(a^3-ab^2\right)dx\cos\left(dx+c\right)+\left(a^2b-b^3\right)dx-\left(ab\cos\left(dx+c\right)+b^2\right)\sqrt{-a^2+b^2}\arctan\left(-\frac{\sqrt{-a^2+b^2}(b\cos(dx+c}{(a^2-b^2)\sin(dx+c}\right.\right.}{\left(a^5-a^3b^2\right)d\cos\left(dx+c\right)+\left(a^4b-a^2b^3\right)d}$$

input
```
integrate(tan(d*x+c)^2/(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

output
```
[-1/2*(2*(a^3 - a*b^2)*d*x*cos(d*x + c) + 2*(a^2*b - b^3)*d*x - (a*b*cos(d
*x + c) + b^2)*sqrt(a^2 - b^2)*log((2*a*b*cos(d*x + c) - (a^2 - 2*b^2)*cos
(d*x + c)^2 + 2*sqrt(a^2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x + c) + 2*a^2
- b^2)/(a^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + b^2)) - 2*(a^3 - a*b^2)*
sin(d*x + c))/((a^5 - a^3*b^2)*d*cos(d*x + c) + (a^4*b - a^2*b^3)*d), -((a
^3 - a*b^2)*d*x*cos(d*x + c) + (a^2*b - b^3)*d*x - (a*b*cos(d*x + c) + b^2
)*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*cos(d*x + c) + a)/((a^2 - b
^2)*sin(d*x + c))) - (a^3 - a*b^2)*sin(d*x + c))/((a^5 - a^3*b^2)*d*cos(d*
x + c) + (a^4*b - a^2*b^3)*d)]
```

3.309.6 Sympy [F]

$$\int \frac{\tan^2(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \int \frac{\tan^2\left(c+dx\right)}{\left(a+b\sec\left(c+dx\right)\right)^2}\,dx$$

input
```
integrate(tan(d*x+c)**2/(a+b*sec(d*x+c))**2,x)
```

output
```
Integral(tan(c + d*x)**2/(a + b*sec(c + d*x))**2, x)
```

3.309.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\tan^2(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(tan(d*x+c)^2/(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` f
or more de
```

3.309.8 Giac [A] (verification not implemented)

Time = 0.54 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.69

$$\int \frac{\tan^2(c+dx)}{(a+b\sec(c+dx))^2}\,dx$$
$$= \frac{\frac{2\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(-2a+2b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\sqrt{-a^2+b^2}}\right)\right)b}{\sqrt{-a^2+b^2}a^2} - \frac{dx+c}{a^2} - \frac{2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-a-b\right)a}}{d}$$

input
```
integrate(tan(d*x+c)^2/(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

output
```
(2*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*
d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(-a^2 + b^2)))*b/(sqrt(-a^2 + b
^2)*a^2) - (d*x + c)/a^2 - 2*tan(1/2*d*x + 1/2*c)/((a*tan(1/2*d*x + 1/2*c)
^2 - b*tan(1/2*d*x + 1/2*c)^2 - a - b)*a))/d
```

3.309.9 Mupad [B] (verification not implemented)

Time = 14.69 (sec) , antiderivative size = 551, normalized size of antiderivative = 6.48

$$\int \frac{\tan^2(c+dx)}{(a+b\sec(c+dx))^2}\,dx = -\frac{2\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{dx}{2}\right)}{\cos\left(\frac{c}{2}+\frac{dx}{2}\right)}\right)}{a^2\,d}$$
$$-\frac{b^2\left(a\,\sin(c+dx)+2\,\mathrm{atanh}\left(\frac{2\,b^3\,\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\,(a^2-b^2)^{3/2}-a^5\,\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\,\sqrt{a^2-b^2}+2\,b^5\,\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\,\sqrt{a^2-b^2}-3\,a^2\,b^3\,\sin\left(\frac{c}{2}+\right.\right.\right.}{\left.\left.\left.\cos\left(\frac{c}{2}+\frac{dx}{2}\right)\,(a\,b^2-a^3)\,(b\,(a^2-b^2)+a\,b^2-\right.\right.\right.}$$

input
```
int(tan(c + d*x)^2/(a + b/cos(c + d*x))^2,x)
```

output
```
- (2*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/(a^2*d) - (b^2*(a*sin(c
+ d*x) + 2*atanh((2*b^3*sin(c/2 + (d*x)/2)*(a^2 - b^2)^(3/2) - a^5*sin(c/2
 + (d*x)/2)*(a^2 - b^2)^(1/2) + 2*b^5*sin(c/2 + (d*x)/2)*(a^2 - b^2)^(1/2)
 - 3*a^2*b^3*sin(c/2 + (d*x)/2)*(a^2 - b^2)^(1/2) + a^3*b^2*sin(c/2 + (d*x
)/2)*(a^2 - b^2)^(1/2) + a^4*b*sin(c/2 + (d*x)/2)*(a^2 - b^2)^(1/2))/(cos(
c/2 + (d*x)/2)*(a*b^2 - a^3)*(b*(a^2 - b^2) + a*b^2 - a^2*b - a^3 + b^3)))
*(a^2 - b^2)^(1/2)) - a^3*sin(c + d*x) + 2*a*b*cos(c + d*x)*atanh((2*b^3*s
in(c/2 + (d*x)/2)*(a^2 - b^2)^(3/2) - a^5*sin(c/2 + (d*x)/2)*(a^2 - b^2)^(
1/2) + 2*b^5*sin(c/2 + (d*x)/2)*(a^2 - b^2)^(1/2) - 3*a^2*b^3*sin(c/2 + (d
*x)/2)*(a^2 - b^2)^(1/2) + a^3*b^2*sin(c/2 + (d*x)/2)*(a^2 - b^2)^(1/2) +
a^4*b*sin(c/2 + (d*x)/2)*(a^2 - b^2)^(1/2))/(cos(c/2 + (d*x)/2)*(a*b^2 - a
^3)*(b*(a^2 - b^2) + a*b^2 - a^2*b - a^3 + b^3)))*(a^2 - b^2)^(1/2))/(a^2*
d*(a^2 - b^2)*(b + a*cos(c + d*x)))
```

3.310 $\int \frac{\cot^2(c+dx)}{(a+b\sec(c+dx))^2}\,dx$

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3.310.1 Optimal result

Integrand size = 21, antiderivative size = 227

$$\begin{aligned}\int \frac{\cot^2(c+dx)}{(a+b\sec(c+dx))^2}\,dx = &-\frac{x}{a^2}-\frac{2b^5\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^2(a-b)^{5/2}(a+b)^{5/2}d}\\ &-\frac{4b^3(2a^2-b^2)\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^2(a-b)^{5/2}(a+b)^{5/2}d}\\ &-\frac{\sin(c+dx)}{2(a+b)^2d(1-\cos(c+dx))}+\frac{\sin(c+dx)}{2(a-b)^2d(1+\cos(c+dx))}\\ &+\frac{b^4\sin(c+dx)}{a\left(a^2-b^2\right)^2d(b+a\cos(c+dx))}\end{aligned}$$

output

```
-x/a^2-2*b^5*arctanh((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))/a^2/(a-b)
^(5/2)/(a+b)^(5/2)/d-4*b^3*(2*a^2-b^2)*arctanh((a-b)^(1/2)*tan(1/2*d*x+1/2
*c)/(a+b)^(1/2))/a^2/(a-b)^(5/2)/(a+b)^(5/2)/d-1/2*sin(d*x+c)/(a+b)^2/d/(1
-cos(d*x+c))+1/2*sin(d*x+c)/(a-b)^2/d/(1+cos(d*x+c))+b^4*sin(d*x+c)/a/(a^2
-b^2)^2/d/(b+a*cos(d*x+c))
```

3.310.2 Mathematica [A] (verified)

Time = 1.62 (sec) , antiderivative size = 209, normalized size of antiderivative = 0.92

$$\int \frac{\cot^2(c+dx)}{(a+b\sec(c+dx))^2}\,dx$$

$$= \frac{(b+a\cos(c+dx))\sec^2(c+dx)\left(-\frac{2(c+dx)(b+a\cos(c+dx))}{a^2} - \frac{4b^3(-4a^2+b^2)\operatorname{arctanh}\left(\frac{(-a+b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)(b+a\cos(c+dx)}{a^2(a^2-b^2)^{5/2}}\right.}{2d(a+b\sec(c+dx))^2}$$

input
```
Integrate[Cot[c + d*x]^2/(a + b*Sec[c + d*x])^2,x]
```

output
```
((b + a*Cos[c + d*x])*Sec[c + d*x]^2*((-2*(c + d*x)*(b + a*Cos[c + d*x]))/
a^2 - (4*b^3*(-4*a^2 + b^2)*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqrt[a^2 -
 b^2]]*(b + a*Cos[c + d*x]))/(a^2*(a^2 - b^2)^(5/2)) - ((b + a*Cos[c + d*x
])*Cot[(c + d*x)/2])/(a + b)^2 + (2*b^4*Sin[c + d*x])/(a*(a - b)^2*(a + b)
^2) + ((b + a*Cos[c + d*x])*Tan[(c + d*x)/2])/(a - b)^2))/(2*d*(a + b*Sec[
c + d*x])^2)
```

3.310.3 Rubi [A] (verified)

Time = 0.71 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3042, 4386, 3042, 3376, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^2(c+dx)}{(a+b\sec(c+dx))^2}\,dx$$

$$\downarrow 3042$$

$$\int \frac{1}{\cot\left(c+dx+\frac{\pi}{2}\right)^2\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^2}\,dx$$

$$\downarrow 4386$$

$$\int \frac{\cos^2(c+dx)\cot^2(c+dx)}{(a\cos(c+dx)+b)^2}\,dx$$

$$\downarrow 3042$$

$$\int \frac{\sin\left(c+dx-\frac{\pi}{2}\right)^4}{\cos\left(c+dx-\frac{\pi}{2}\right)^2\left(b-a\sin\left(c+dx-\frac{\pi}{2}\right)\right)^2}dx$$

$$\downarrow 3376$$

$$\int\left(\frac{b^4}{a^2\left(a^2-b^2\right)(-a\cos(c+dx)-b)^2}+\frac{2\left(2a^2b^3-b^5\right)}{a^2\left(a^2-b^2\right)^2(-a\cos(c+dx)-b)}-\frac{1}{a^2}-\frac{1}{2(a-b)^2(-\cos(c+dx)-1)}\right.$$

$$\downarrow 2009$$

$$-\frac{2b^5\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^2d(a-b)^{5/2}(a+b)^{5/2}}-\frac{4b^3\left(2a^2-b^2\right)\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^2d(a-b)^{5/2}(a+b)^{5/2}}+$$
$$\frac{b^4\sin(c+dx)}{ad\left(a^2-b^2\right)^2(a\cos(c+dx)+b)}-\frac{x}{a^2}-\frac{\sin(c+dx)}{2d(a+b)^2(1-\cos(c+dx))}+\frac{\sin(c+dx)}{2d(a-b)^2(\cos(c+dx)+1)}$$

input
```
Int[Cot[c + d*x]^2/(a + b*Sec[c + d*x])^2,x]
```

output
```
-(x/a^2) - (2*b^5*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a^
2*(a - b)^(5/2)*(a + b)^(5/2)*d) - (4*b^3*(2*a^2 - b^2)*ArcTanh[(Sqrt[a -
b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a^2*(a - b)^(5/2)*(a + b)^(5/2)*d) - S
in[c + d*x]/(2*(a + b)^2*d*(1 - Cos[c + d*x])) + Sin[c + d*x]/(2*(a - b)^2
*d*(1 + Cos[c + d*x])) + (b^4*Sin[c + d*x])/(a*(a^2 - b^2)^2*d*(b + a*Cos[
c + d*x]))
```

3.310.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3376
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig[(d*sin[
e + f*x])^n*(a + b*sin[e + f*x])^m*(1 - sin[e + f*x]^2)^(p/2), x], x] /; Fr
eeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && IntegersQ[m, 2*n, p/2] && (
LtQ[m, -1] || (EqQ[m, -1] && GtQ[p, 0]))
```

rule 4386

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Int[Cos[c + d*x]^m*((b + a*Sin[c + d*x])^n/Sin[c + d*x]^(m
 + n)), x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[n] &&
 IntegerQ[m] && (IntegerQ[m/2] || LeQ[m, 1])
```

3.310.4 Maple [A] (verified)

Time = 1.44 (sec) , antiderivative size = 184, normalized size of antiderivative = 0.81

method	result
derivativedivides	$\frac{\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2a^2-4ab+2b^2}-\frac{1}{2(a+b)^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}+\frac{2b^3\left(-\frac{ab\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2a-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2b-a-b}-\frac{\left(4a^2-b^2\right)\operatorname{arctanh}\left(\frac{(a-b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a-b)(a+b)}}\right)}{\sqrt{(a-b)(a+b)}}\right)}{(a-b)^2(a+b)^2a^2}}{d}$
default	$\frac{\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2a^2-4ab+2b^2}-\frac{1}{2(a+b)^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}+\frac{2b^3\left(-\frac{ab\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2a-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2b-a-b}-\frac{\left(4a^2-b^2\right)\operatorname{arctanh}\left(\frac{(a-b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a-b)(a+b)}}\right)}{\sqrt{(a-b)(a+b)}}\right)}{(a-b)^2(a+b)^2a^2}}{d}$
risch	$-\frac{x}{a^2}-\frac{2i\left(-2a^4b\,e^{3i(dx+c)}-b^5e^{3i(dx+c)}+a^5e^{2i(dx+c)}-3a^3b^2e^{2i(dx+c)}-a\,b^4e^{2i(dx+c)}+2a^2b^3e^{i(dx+c)}+b^5e^{i(dx+c)}+a^5\right.}{\left(a\,e^{2i(dx+c)}+2b\,e^{i(dx+c)}+a\right)(a^2-b^2)^2a^2\left(e^{2i(dx+c)}-1\right)d}$

input

```
int(cot(d*x+c)^2/(a+b*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output

```
1/d*(1/2*tan(1/2*d*x+1/2*c)/(a^2-2*a*b+b^2)-1/2/(a+b)^2/tan(1/2*d*x+1/2*c)
+2*b^3/(a-b)^2/(a+b)^2/a^2*(-a*b*tan(1/2*d*x+1/2*c)/(tan(1/2*d*x+1/2*c)^2*
a-tan(1/2*d*x+1/2*c)^2*b-a-b)-(4*a^2-b^2)/((a-b)*(a+b))^(1/2)*arctanh((a-b
)*tan(1/2*d*x+1/2*c)/((a-b)*(a+b))^(1/2)))-2/a^2*arctan(tan(1/2*d*x+1/2*c)
))
```

3.310.5 Fricas [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 705, normalized size of antiderivative = 3.11

$$\int \frac{\cot^2(c+dx)}{(a+b\sec(c+dx))^2}\,dx$$
$$= \left[\frac{4\,a^5b^2-2\,a^3b^4-2\,ab^6-(4\,a^2b^4-b^6+(4\,a^3b^3-ab^5)\cos(dx+c))\sqrt{a^2-b^2}\log\left(\frac{2\,ab\cos(dx+c)-(a^2-2\,b^2)\cos(}{a^2\,co}\right.\right.$$

input
```
integrate(cot(d*x+c)^2/(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

output
```
[1/2*(4*a^5*b^2 - 2*a^3*b^4 - 2*a*b^6 - (4*a^2*b^4 - b^6 + (4*a^3*b^3 - a*
b^5)*cos(d*x + c))*sqrt(a^2 - b^2)*log((2*a*b*cos(d*x + c) - (a^2 - 2*b^2)
*cos(d*x + c)^2 + 2*sqrt(a^2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x + c) + 2*
a^2 - b^2)/(a^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + b^2))*sin(d*x + c) -
 2*(a^7 - a*b^6)*cos(d*x + c)^2 + 2*(a^6*b - 2*a^4*b^3 + a^2*b^5)*cos(d*x
+ c) - 2*((a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6)*d*x*cos(d*x + c) + (a^6*b
- 3*a^4*b^3 + 3*a^2*b^5 - b^7)*d*x)*sin(d*x + c))/(((a^9 - 3*a^7*b^2 + 3*a
^5*b^4 - a^3*b^6)*d*cos(d*x + c) + (a^8*b - 3*a^6*b^3 + 3*a^4*b^5 - a^2*b^
7)*d)*sin(d*x + c)), (2*a^5*b^2 - a^3*b^4 - a*b^6 - (4*a^2*b^4 - b^6 + (4*
a^3*b^3 - a*b^5)*cos(d*x + c))*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(
b*cos(d*x + c) + a)/((a^2 - b^2)*sin(d*x + c)))*sin(d*x + c) - (a^7 - a*b^
6)*cos(d*x + c)^2 + (a^6*b - 2*a^4*b^3 + a^2*b^5)*cos(d*x + c) - ((a^7 - 3
*a^5*b^2 + 3*a^3*b^4 - a*b^6)*d*x*cos(d*x + c) + (a^6*b - 3*a^4*b^3 + 3*a^
2*b^5 - b^7)*d*x)*sin(d*x + c))/(((a^9 - 3*a^7*b^2 + 3*a^5*b^4 - a^3*b^6)*
d*cos(d*x + c) + (a^8*b - 3*a^6*b^3 + 3*a^4*b^5 - a^2*b^7)*d)*sin(d*x + c)
)]
```

3.310.6 Sympy [F]

$$\int \frac{\cot^2(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \int \frac{\cot^2(c+dx)}{(a+b\sec(c+dx))^2}\,dx$$

input
```
integrate(cot(d*x+c)**2/(a+b*sec(d*x+c))**2,x)
```

output
```
Integral(cot(c + d*x)**2/(a + b*sec(c + d*x))**2, x)
```

3.310.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\cot^2(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(cot(d*x+c)^2/(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` f
or more de
```

3.310.8 Giac [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 332, normalized size of antiderivative = 1.46

$$\int \frac{\cot^2(c+dx)}{(a+b\sec(c+dx))^2}\,dx = -\frac{\frac{4\left(4a^2b^3-b^5\right)\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(-2a+2b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\sqrt{-a^2+b^2}}\right)\right)}{\left(a^6-2a^4b^2+a^2b^4\right)\sqrt{-a^2+b^2}} - \frac{\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{a^2-2ab+b^2} + \frac{a^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-3a^3}{(a^5-2}}{2d}$$

input
```
integrate(cot(d*x+c)^2/(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

output
```
-1/2*(4*(4*a^2*b^3 - b^5)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b
) + arctan(-(a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(-a^2 +
b^2)))/((a^6 - 2*a^4*b^2 + a^2*b^4)*sqrt(-a^2 + b^2)) - tan(1/2*d*x + 1/2*
c)/(a^2 - 2*a*b + b^2) + (a^4*tan(1/2*d*x + 1/2*c)^2 - 3*a^3*b*tan(1/2*d*x
 + 1/2*c)^2 + 3*a^2*b^2*tan(1/2*d*x + 1/2*c)^2 - a*b^3*tan(1/2*d*x + 1/2*c
)^2 + 4*b^4*tan(1/2*d*x + 1/2*c)^2 - a^4 + a^3*b + a^2*b^2 - a*b^3)/((a^5
- 2*a^3*b^2 + a*b^4)*(a*tan(1/2*d*x + 1/2*c)^3 - b*tan(1/2*d*x + 1/2*c)^3
- a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))) + 2*(d*x + c)/a^2)/d
```

3.310.9 Mupad [B] (verification not implemented)

Time = 18.96 (sec) , antiderivative size = 6093, normalized size of antiderivative = 26.84

$$\int \frac{\cot^2(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \text{Too large to display}$$

input
```
int(cot(c + d*x)^2/(a + b/cos(c + d*x))^2,x)
```

output
```
((a^2 - 2*a*b + b^2)/(a + b) - (tan(c/2 + (d*x)/2)^2*(a^4 - 3*a^3*b - a*b^
3 + 4*b^4 + 3*a^2*b^2))/(a*(a + b)^2))/(d*(tan(c/2 + (d*x)/2)^3*(6*a*b^2 -
 6*a^2*b + 2*a^3 - 2*b^3) + tan(c/2 + (d*x)/2)*(2*a*b^2 + 2*a^2*b - 2*a^3
 - 2*b^3))) - (2*atan(-((tan(c/2 + (d*x)/2)*(32*a^26 - 96*a^25*b - 64*a^3*b
^23 + 128*a^4*b^22 + 672*a^5*b^21 - 1376*a^6*b^20 - 3008*a^7*b^19 + 6528*a
^8*b^18 + 7072*a^9*b^17 - 17632*a^10*b^16 - 8480*a^11*b^15 + 29600*a^12*b^
14 + 2176*a^13*b^13 - 31744*a^14*b^12 + 8224*a^15*b^11 + 21344*a^16*b^10 -
 12992*a^17*b^9 - 8128*a^18*b^8 + 9568*a^19*b^7 + 992*a^20*b^6 - 4000*a^21
*b^5 + 480*a^22*b^4 + 928*a^23*b^3 - 224*a^24*b^2) - ((32*a^28 - 32*a^27*b
 + 32*a^6*b^22 - 416*a^8*b^20 + 224*a^9*b^19 + 2080*a^10*b^18 - 1824*a^11*
b^17 - 5472*a^12*b^16 + 6528*a^13*b^15 + 8256*a^14*b^14 - 13440*a^15*b^13
 - 6720*a^16*b^12 + 17472*a^17*b^11 + 1344*a^18*b^10 - 14784*a^19*b^9 + 288
0*a^20*b^8 + 8064*a^21*b^7 - 3168*a^22*b^6 - 2688*a^23*b^5 + 1504*a^24*b^4
 + 480*a^25*b^3 - 352*a^26*b^2 - (tan(c/2 + (d*x)/2)*(128*a^8*b^22 - 64*a^
7*b^23 - 64*a^29*b + 576*a^9*b^21 - 1280*a^10*b^20 - 2240*a^11*b^19 + 5760
*a^12*b^18 + 4800*a^13*b^17 - 15360*a^14*b^16 - 5760*a^15*b^15 + 26880*a^1
6*b^14 + 2688*a^17*b^13 - 32256*a^18*b^12 + 2688*a^19*b^11 + 26880*a^20*b^
10 - 5760*a^21*b^9 - 15360*a^22*b^8 + 4800*a^23*b^7 + 5760*a^24*b^6 - 2240
*a^25*b^5 - 1280*a^26*b^4 + 576*a^27*b^3 + 128*a^28*b^2)*1i)/a^2)*1i)/a^2)
/a^2 + (tan(c/2 + (d*x)/2)*(32*a^26 - 96*a^25*b - 64*a^3*b^23 + 128*a^4...
```

3.311 $\int \frac{\cot^4(c+dx)}{(a+b\sec(c+dx))^2}\,dx$

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3.311.1 Optimal result

Integrand size = 21, antiderivative size = 360

$$\begin{aligned}\int \frac{\cot^4(c+dx)}{(a+b\sec(c+dx))^2}\,dx = {}& \frac{x}{a^2} - \frac{2b^7 \operatorname{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^2(a-b)^{7/2}(a+b)^{7/2}d} \\ & - \frac{4b^5(3a^2-b^2)\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^2(a-b)^{7/2}(a+b)^{7/2}d} \\ & - \frac{\sin(c+dx)}{12(a+b)^2 d(1-\cos(c+dx))^2} \\ & - \frac{\sin(c+dx)}{12(a+b)^2 d(1-\cos(c+dx))} \\ & + \frac{(3a+5b)\sin(c+dx)}{4(a+b)^3 d(1-\cos(c+dx))} + \frac{\sin(c+dx)}{12(a-b)^2 d(1+\cos(c+dx))^2} \\ & - \frac{(3a-5b)\sin(c+dx)}{4(a-b)^3 d(1+\cos(c+dx))} + \frac{\sin(c+dx)}{12(a-b)^2 d(1+\cos(c+dx))} \\ & + \frac{b^6\sin(c+dx)}{a\left(a^2-b^2\right)^3 d(b+a\cos(c+dx))}\end{aligned}$$

output

```
x/a^2-2*b^7*arctanh((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))/a^2/(a-b)^
(7/2)/(a+b)^(7/2)/d-4*b^5*(3*a^2-b^2)*arctanh((a-b)^(1/2)*tan(1/2*d*x+1/2*
c)/(a+b)^(1/2))/a^2/(a-b)^(7/2)/(a+b)^(7/2)/d-1/12*sin(d*x+c)/(a+b)^2/d/(1
-cos(d*x+c))^2-1/12*sin(d*x+c)/(a+b)^2/d/(1-cos(d*x+c))+1/4*(3*a+5*b)*sin(
d*x+c)/(a+b)^3/d/(1-cos(d*x+c))+1/12*sin(d*x+c)/(a-b)^2/d/(1+cos(d*x+c))^2
-1/4*(3*a-5*b)*sin(d*x+c)/(a-b)^3/d/(1+cos(d*x+c))+1/12*sin(d*x+c)/(a-b)^2
/d/(1+cos(d*x+c))+b^6*sin(d*x+c)/a/(a^2-b^2)^3/d/(b+a*cos(d*x+c))
```

3.311.2 Mathematica [A] (verified)

Time = 2.11 (sec) , antiderivative size = 303, normalized size of antiderivative = 0.84

$$\int \frac{\cot^4(c+dx)}{(a+b\sec(c+dx))^2}\,dx$$

$$= \frac{(b+a\cos(c+dx))\sec^2(c+dx)\left(\frac{24(c+dx)(b+a\cos(c+dx))}{a^2} - \frac{48b^5(-6a^2+b^2)\operatorname{arctanh}\left(\frac{(-a+b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)(b+a\cos(c+dx}{a^2(a^2-b^2)^{7/2}}\right.}{}$$

input
```
Integrate[Cot[c + d*x]^4/(a + b*Sec[c + d*x])^2,x]
```

output
```
((b + a*Cos[c + d*x])*Sec[c + d*x]^2*((24*(c + d*x)*(b + a*Cos[c + d*x]))/
a^2 - (48*b^5*(-6*a^2 + b^2)*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqrt[a^2
- b^2]]*(b + a*Cos[c + d*x]))/(a^2*(a^2 - b^2)^(7/2)) + (4*(4*a + 7*b)*(b
+ a*Cos[c + d*x])*Cot[(c + d*x)/2])/(a + b)^3 - ((b + a*Cos[c + d*x])*Cot[
(c + d*x)/2]*Csc[(c + d*x)/2]^2)/(a + b)^2 + (24*b^6*Sin[c + d*x])/(a*(a -
 b)^3*(a + b)^3) + (4*(-4*a + 7*b)*(b + a*Cos[c + d*x])*Tan[(c + d*x)/2])/
(a - b)^3 + ((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/(a
- b)^2))/(24*d*(a + b*Sec[c + d*x])^2)
```

3.311.3 Rubi [A] (verified)

Time = 0.91 (sec) , antiderivative size = 360, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3042, 4386, 3042, 3376, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^4(c+dx)}{(a+b\sec(c+dx))^2}\,dx$$

$$\downarrow 3042$$

$$\int \frac{1}{\cot\left(c+dx+\frac{\pi}{2}\right)^4\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^2}dx$$

$$\downarrow 4386$$

$$\int \frac{\cos^2(c+dx)\cot^4(c+dx)}{(a\cos(c+dx)+b)^2}dx$$

$$\downarrow 3042$$

$$\int \frac{\sin\left(c+dx-\frac{\pi}{2}\right)^6}{\cos\left(c+dx-\frac{\pi}{2}\right)^4\left(b-a\sin\left(c+dx-\frac{\pi}{2}\right)\right)^2}dx$$

$$\downarrow 3376$$

$$\int \left(\frac{b^6}{a^2\left(a^2-b^2\right)^2(-a\cos(c+dx)-b)^2}+\frac{2b^5\left(3a^2-b^2\right)}{a^2\left(a^2-b^2\right)^3(-a\cos(c+dx)-b)}+\frac{1}{a^2}+\frac{3a-5b}{4(a-b)^3(-\cos(c+dx)-1}\right.$$

$$\downarrow 2009$$

$$-\frac{2b^7\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^2d(a-b)^{7/2}(a+b)^{7/2}}-\frac{4b^5\left(3a^2-b^2\right)\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^2d(a-b)^{7/2}(a+b)^{7/2}}+$$
$$\frac{b^6\sin(c+dx)}{ad\left(a^2-b^2\right)^3(a\cos(c+dx)+b)}+\frac{x}{a^2}+\frac{(3a+5b)\sin(c+dx)}{4d(a+b)^3(1-\cos(c+dx))}-$$
$$\frac{\sin(c+dx)}{12d(a+b)^2(1-\cos(c+dx))}+\frac{\sin(c+dx)}{12d(a-b)^2(\cos(c+dx)+1)}-\frac{(3a-5b)\sin(c+dx)}{4d(a-b)^3(\cos(c+dx)+1)}-$$
$$\frac{\sin(c+dx)}{12d(a+b)^2(1-\cos(c+dx))^2}+\frac{\sin(c+dx)}{12d(a-b)^2(\cos(c+dx)+1)^2}$$

input
```
Int[Cot[c + d*x]^4/(a + b*Sec[c + d*x])^2,x]
```

output
```
x/a^2 - (2*b^7*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a^2*(
a - b)^(7/2)*(a + b)^(7/2)*d) - (4*b^5*(3*a^2 - b^2)*ArcTanh[(Sqrt[a - b]*
Tan[(c + d*x)/2])/Sqrt[a + b]])/(a^2*(a - b)^(7/2)*(a + b)^(7/2)*d) - Sin[
c + d*x]/(12*(a + b)^2*d*(1 - Cos[c + d*x])^2) - Sin[c + d*x]/(12*(a + b)^
2*d*(1 - Cos[c + d*x])) + ((3*a + 5*b)*Sin[c + d*x])/(4*(a + b)^3*d*(1 - C
os[c + d*x])) + Sin[c + d*x]/(12*(a - b)^2*d*(1 + Cos[c + d*x])^2) - ((3*a
 - 5*b)*Sin[c + d*x])/(4*(a - b)^3*d*(1 + Cos[c + d*x])) + Sin[c + d*x]/(1
2*(a - b)^2*d*(1 + Cos[c + d*x])) + (b^6*Sin[c + d*x])/(a*(a^2 - b^2)^3*d*
(b + a*Cos[c + d*x]))
```

3.311.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3376
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig[(d*sin[
e + f*x])^n*(a + b*sin[e + f*x])^m*(1 - sin[e + f*x]^2)^(p/2), x], x] /; Fr
eeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && IntegersQ[m, 2*n, p/2] && (
LtQ[m, -1] || (EqQ[m, -1] && GtQ[p, 0]))
```

rule 4386
```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Int[Cos[c + d*x]^m*((b + a*Sin[c + d*x])^n/Sin[c + d*x]^(m
 + n)), x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[n] &&
 IntegerQ[m] && (IntegerQ[m/2] || LeQ[m, 1])
```

3.311.4 Maple [A] (verified)

Time = 1.82 (sec) , antiderivative size = 260, normalized size of antiderivative = 0.72

method	result
derivativedivides	$\frac{\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3 a}{3}-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3 b}{3}-5\tan\left(\frac{dx}{2}+\frac{c}{2}\right)a+9\tan\left(\frac{dx}{2}+\frac{c}{2}\right)b}{8\left(a^2-2ab+b^2\right)(a-b)}-\frac{1}{24(a+b)^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}-\frac{-5a-9b}{8(a+b)^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}+\frac{2\arctan\left(\text{ta}}{d}$
default	$\frac{\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3 a}{3}-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3 b}{3}-5\tan\left(\frac{dx}{2}+\frac{c}{2}\right)a+9\tan\left(\frac{dx}{2}+\frac{c}{2}\right)b}{8\left(a^2-2ab+b^2\right)(a-b)}-\frac{1}{24(a+b)^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}-\frac{-5a-9b}{8(a+b)^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}+\frac{2\arctan\left(\text{ta}}{d}$
risch	$\frac{x}{a^2}+\frac{2i\left(-3a\,b^6-7b^4a^3+17a^5b^2\mathrm{e}^{4i(dx+c)}-53a^3b^4\mathrm{e}^{4i(dx+c)}-9a\,b^6\mathrm{e}^{4i(dx+c)}-14a^6b\,\mathrm{e}^{3i(dx+c)}+26a^4b^3\mathrm{e}^{3i(dx+c)}+24a^2b\right.}{}$

input
```
int(cot(d*x+c)^4/(a+b*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/8/(a^2-2*a*b+b^2)/(a-b)*(1/3*tan(1/2*d*x+1/2*c)^3*a-1/3*tan(1/2*d*x
+1/2*c)^3*b-5*tan(1/2*d*x+1/2*c)*a+9*tan(1/2*d*x+1/2*c)*b)-1/24/(a+b)^2/ta
n(1/2*d*x+1/2*c)^3-1/8/(a+b)^3*(-5*a-9*b)/tan(1/2*d*x+1/2*c)+2/a^2*arctan(
tan(1/2*d*x+1/2*c))+2*b^5/(a+b)^3/(a-b)^3/a^2*(-a*b*tan(1/2*d*x+1/2*c)/(ta
n(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)-(6*a^2-b^2)/((a-b)*(a+b))
^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a-b)*(a+b))^(1/2))))
```

3.311.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 711 vs. $2(324) = 648$.

Time = 0.36 (sec) , antiderivative size = 1481, normalized size of antiderivative = 4.11

$$\int \frac{\cot^4(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \text{Too large to display}$$

input
```
integrate(cot(d*x+c)^4/(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

output
```
[1/6*(8*a^7*b^2 - 40*a^5*b^4 + 26*a^3*b^6 + 6*a*b^8 + 2*(4*a^9 - 13*a^7*b^
2 + 2*a^5*b^4 + 4*a^3*b^6 + 3*a*b^8)*cos(d*x + c)^4 - 2*(2*a^8*b - 11*a^6*
b^3 + 16*a^4*b^5 - 7*a^2*b^7)*cos(d*x + c)^3 - 3*(6*a^2*b^6 - b^8 - (6*a^3
*b^5 - a*b^7)*cos(d*x + c)^3 - (6*a^2*b^6 - b^8)*cos(d*x + c)^2 + (6*a^3*b
^5 - a*b^7)*cos(d*x + c))*sqrt(a^2 - b^2)*log((2*a*b*cos(d*x + c) - (a^2 -
 2*b^2)*cos(d*x + c)^2 - 2*sqrt(a^2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x +
c) + 2*a^2 - b^2)/(a^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + b^2))*sin(d*x
 + c) - 6*(a^9 - 2*a^7*b^2 - 7*a^5*b^4 + 6*a^3*b^6 + 2*a*b^8)*cos(d*x + c)
^2 + 2*(a^8*b - 8*a^6*b^3 + 13*a^4*b^5 - 6*a^2*b^7)*cos(d*x + c) + 6*((a^9
 - 4*a^7*b^2 + 6*a^5*b^4 - 4*a^3*b^6 + a*b^8)*d*x*cos(d*x + c)^3 + (a^8*b
- 4*a^6*b^3 + 6*a^4*b^5 - 4*a^2*b^7 + b^9)*d*x*cos(d*x + c)^2 - (a^9 - 4*a
^7*b^2 + 6*a^5*b^4 - 4*a^3*b^6 + a*b^8)*d*x*cos(d*x + c) - (a^8*b - 4*a^6*
b^3 + 6*a^4*b^5 - 4*a^2*b^7 + b^9)*d*x)*sin(d*x + c))/(((a^11 - 4*a^9*b^2
+ 6*a^7*b^4 - 4*a^5*b^6 + a^3*b^8)*d*cos(d*x + c)^3 + (a^10*b - 4*a^8*b^3
+ 6*a^6*b^5 - 4*a^4*b^7 + a^2*b^9)*d*cos(d*x + c)^2 - (a^11 - 4*a^9*b^2 +
6*a^7*b^4 - 4*a^5*b^6 + a^3*b^8)*d*cos(d*x + c) - (a^10*b - 4*a^8*b^3 + 6*
a^6*b^5 - 4*a^4*b^7 + a^2*b^9)*d)*sin(d*x + c)), 1/3*(4*a^7*b^2 - 20*a^5*b
^4 + 13*a^3*b^6 + 3*a*b^8 + (4*a^9 - 13*a^7*b^2 + 2*a^5*b^4 + 4*a^3*b^6 +
3*a*b^8)*cos(d*x + c)^4 - (2*a^8*b - 11*a^6*b^3 + 16*a^4*b^5 - 7*a^2*b^7)*
cos(d*x + c)^3 + 3*(6*a^2*b^6 - b^8 - (6*a^3*b^5 - a*b^7)*cos(d*x + c)^...
```

3.311.6 Sympy [F]

$$\int \frac{\cot^4(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \int \frac{\cot^4(c+dx)}{(a+b\sec(c+dx))^2}\,dx$$

input
```
integrate(cot(d*x+c)**4/(a+b*sec(d*x+c))**2,x)
```

output
```
Integral(cot(c + d*x)**4/(a + b*sec(c + d*x))**2, x)
```

3.311.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\cot^4(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(cot(d*x+c)^4/(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` f
or more de
```

3.311.8 Giac [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 487, normalized size of antiderivative = 1.35

$$\int \frac{\cot^4(c+dx)}{(a+b\sec(c+dx))^2}\,dx =$$

$$-\frac{48\,b^6\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\left(a^7-3\,a^5b^2+3\,a^3b^4-ab^6\right)\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-a-b\right)} - \frac{48\left(6\,a^2b^5-b^7\right)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(2\,a-2\,b)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\right.\right.\right.}{\left(a^8-3\,a^6b^2+3\,a^4b^4-a^2b^6\right)\sqrt{-a^2+b^2}}$$

input
```
integrate(cot(d*x+c)^4/(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

output
```
-1/24*(48*b^6*tan(1/2*d*x + 1/2*c)/((a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6)*
(a*tan(1/2*d*x + 1/2*c)^2 - b*tan(1/2*d*x + 1/2*c)^2 - a - b)) - 48*(6*a^2
*b^5 - b^7)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(2*a - 2*b) + arctan((a*t
an(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(-a^2 + b^2)))/((a^8 - 3
*a^6*b^2 + 3*a^4*b^4 - a^2*b^6)*sqrt(-a^2 + b^2)) - (a^4*tan(1/2*d*x + 1/2
*c)^3 - 4*a^3*b*tan(1/2*d*x + 1/2*c)^3 + 6*a^2*b^2*tan(1/2*d*x + 1/2*c)^3
- 4*a*b^3*tan(1/2*d*x + 1/2*c)^3 + b^4*tan(1/2*d*x + 1/2*c)^3 - 15*a^4*tan
(1/2*d*x + 1/2*c) + 72*a^3*b*tan(1/2*d*x + 1/2*c) - 126*a^2*b^2*tan(1/2*d*
x + 1/2*c) + 96*a*b^3*tan(1/2*d*x + 1/2*c) - 27*b^4*tan(1/2*d*x + 1/2*c))/
(a^6 - 6*a^5*b + 15*a^4*b^2 - 20*a^3*b^3 + 15*a^2*b^4 - 6*a*b^5 + b^6) - 2
4*(d*x + c)/a^2 - (15*a*tan(1/2*d*x + 1/2*c)^2 + 27*b*tan(1/2*d*x + 1/2*c)
^2 - a - b)/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*tan(1/2*d*x + 1/2*c)^3))/d
```

3.311.9 Mupad [B] (verification not implemented)

Time = 19.43 (sec) , antiderivative size = 8348, normalized size of antiderivative = 23.19

$$\int \frac{\cot^4(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \text{Too large to display}$$

input
```
int(cot(c + d*x)^4/(a + b/cos(c + d*x))^2,x)
```

output

```
tan(c/2 + (d*x)/2)^3/(24*d*(a - b)^2) + ((3*a*b^2 - 3*a^2*b + a^3 - b^3)/(
3*(a + b)) + (2*tan(c/2 + (d*x)/2)^2*(11*a^3*b - 31*a*b^3 - 8*a^4 + 13*b^4
 + 15*a^2*b^2))/(3*(a + b)^2) - (tan(c/2 + (d*x)/2)^4*(11*a^5*b - 9*a*b^5
- 5*a^6 + 16*b^6 + 31*a^2*b^4 - 34*a^3*b^3 + 6*a^4*b^2))/(a*(a + b)^3))/(d
*(tan(c/2 + (d*x)/2)^5*(8*a^4 - 32*a^3*b - 32*a*b^3 + 8*b^4 + 48*a^2*b^2)
- tan(c/2 + (d*x)/2)^3*(16*a*b^3 - 16*a^3*b + 8*a^4 - 8*b^4))) + (tan(c/2
+ (d*x)/2)*((16*a*b + 8*a^2 - 24*b^2)/(64*(a - b)^4) - 3/(4*(a - b)^2)))/d
 + (2*atan(((tan(c/2 + (d*x)/2)*(32*a^36 - 96*a^35*b + 64*a^3*b^33 - 128*a
^4*b^32 - 1056*a^5*b^31 + 2080*a^6*b^30 + 7680*a^7*b^29 - 15360*a^8*b^28 -
 31360*a^9*b^27 + 67200*a^10*b^26 + 77760*a^11*b^25 - 194240*a^12*b^24 - 1
14240*a^13*b^23 + 393792*a^14*b^22 + 68096*a^15*b^21 - 580608*a^16*b^20 +
96000*a^17*b^19 + 636160*a^18*b^18 - 300960*a^19*b^17 - 522720*a^20*b^16 +
 412640*a^21*b^15 + 319520*a^22*b^14 - 373632*a^23*b^13 - 138880*a^24*b^12
 + 243456*a^25*b^11 + 36096*a^26*b^10 - 116480*a^27*b^9 + 40320*a^29*b^7 -
 4480*a^30*b^6 - 9600*a^31*b^5 + 1920*a^32*b^4 + 1408*a^33*b^3 - 384*a^34*
b^2) - ((32*a^38 - 32*a^37*b - 32*a^6*b^32 - 32*a^7*b^31 + 640*a^8*b^30 -
4992*a^10*b^28 + 2624*a^11*b^27 + 21504*a^12*b^26 - 19872*a^13*b^25 - 5792
0*a^14*b^24 + 77472*a^15*b^23 + 100992*a^16*b^22 - 195008*a^17*b^21 - 1070
08*a^18*b^20 + 344960*a^19*b^19 + 39424*a^20*b^18 - 446688*a^21*b^17 + 760
32*a^22*b^16 + 431904*a^23*b^15 - 161920*a^24*b^14 - 313984*a^25*b^13 +...
```

3.312 $\int \frac{(e\tan(c+dx))^{5/2}}{a+b\sec(c+dx)}\,dx$

3.312.1 Optimal result 2256
3.312.2 Mathematica [C] (warning: unable to verify) 2257
3.312.3 Rubi [F] 2258
3.312.4 Maple [B] (warning: unable to verify) 2267
3.312.5 Fricas [F(-1)] 2268
3.312.6 Sympy [F] 2269
3.312.7 Maxima [F] 2269
3.312.8 Giac [F] 2269
3.312.9 Mupad [F(-1)] 2270

3.312.1 Optimal result

Integrand size = 25, antiderivative size = 761

$$\int \frac{(e\tan(c+dx))^{5/2}}{a+b\sec(c+dx)}\,dx = \frac{ae^{5/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}b^2d}$$
$$-\frac{(a^2-b^2)\,e^{5/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}ab^2d}$$
$$-\frac{ae^{5/2}\arctan\left(1+\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}b^2d}+\frac{(a^2-b^2)\,e^{5/2}\arctan\left(1+\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}ab^2d}$$
$$-\frac{ae^{5/2}\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}b^2d}$$
$$+\frac{(a^2-b^2)\,e^{5/2}\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}ab^2d}$$
$$+\frac{ae^{5/2}\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)+\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}b^2d}$$
$$-\frac{(a^2-b^2)\,e^{5/2}\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)+\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}ab^2d}$$
$$+\frac{2\sqrt{2}\sqrt{a-b}\sqrt{a+b}e^2\sqrt{\cos(c+dx)}\,\text{EllipticPi}\left(-\frac{\sqrt{a-b}}{\sqrt{a+b}},\arcsin\left(\frac{\sqrt{\sin(c+dx)}}{\sqrt{1+\cos(c+dx)}}\right),-1\right)\sqrt{e\tan(c+dx)}}{abd\sqrt{\sin(c+dx)}}$$
$$-\frac{2\sqrt{2}\sqrt{a-b}\sqrt{a+b}e^2\sqrt{\cos(c+dx)}\,\text{EllipticPi}\left(\frac{\sqrt{a-b}}{\sqrt{a+b}},\arcsin\left(\frac{\sqrt{\sin(c+dx)}}{\sqrt{1+\cos(c+dx)}}\right),-1\right)\sqrt{e\tan(c+dx)}}{abd\sqrt{\sin(c+dx)}}$$
$$-\frac{2e^2\cos(c+dx)E\left(c-\frac{\pi}{4}+dx\middle|2\right)\sqrt{e\tan(c+dx)}}{bd\sqrt{\sin(2c+2dx)}}+\frac{2e\cos(c+dx)(e\tan(c+dx))^{3/2}}{bd}$$

output
```
1/2*a*e^(5/2)*arctan(1-2^(1/2)*(e*tan(d*x+c))^(1/2)/e^(1/2))/b^2/d*2^(1/2)
-1/2*(a^2-b^2)*e^(5/2)*arctan(1-2^(1/2)*(e*tan(d*x+c))^(1/2)/e^(1/2))/a/b^
2/d*2^(1/2)-1/2*a*e^(5/2)*arctan(1+2^(1/2)*(e*tan(d*x+c))^(1/2)/e^(1/2))/b
^2/d*2^(1/2)+1/2*(a^2-b^2)*e^(5/2)*arctan(1+2^(1/2)*(e*tan(d*x+c))^(1/2)/e
^(1/2))/a/b^2/d*2^(1/2)-1/4*a*e^(5/2)*ln(e^(1/2)-2^(1/2)*(e*tan(d*x+c))^(1
/2)+e^(1/2)*tan(d*x+c))/b^2/d*2^(1/2)+1/4*(a^2-b^2)*e^(5/2)*ln(e^(1/2)-2^(
1/2)*(e*tan(d*x+c))^(1/2)+e^(1/2)*tan(d*x+c))/a/b^2/d*2^(1/2)+1/4*a*e^(5/2
)*ln(e^(1/2)+2^(1/2)*(e*tan(d*x+c))^(1/2)+e^(1/2)*tan(d*x+c))/b^2/d*2^(1/2
)-1/4*(a^2-b^2)*e^(5/2)*ln(e^(1/2)+2^(1/2)*(e*tan(d*x+c))^(1/2)+e^(1/2)*ta
n(d*x+c))/a/b^2/d*2^(1/2)+2*e^2*EllipticPi(sin(d*x+c)^(1/2)/(1+cos(d*x+c))
^(1/2),-(a-b)^(1/2)/(a+b)^(1/2),I)*2^(1/2)*(a-b)^(1/2)*(a+b)^(1/2)*cos(d*x
+c)^(1/2)*(e*tan(d*x+c))^(1/2)/a/b/d/sin(d*x+c)^(1/2)-2*e^2*EllipticPi(sin
(d*x+c)^(1/2)/(1+cos(d*x+c))^(1/2),(a-b)^(1/2)/(a+b)^(1/2),I)*2^(1/2)*(a-b
)^(1/2)*(a+b)^(1/2)*cos(d*x+c)^(1/2)*(e*tan(d*x+c))^(1/2)/a/b/d/sin(d*x+c)
^(1/2)+2*e^2*cos(d*x+c)*(sin(c+1/4*Pi+d*x)^2)^(1/2)/sin(c+1/4*Pi+d*x)*Elli
pticE(cos(c+1/4*Pi+d*x),2^(1/2))*(e*tan(d*x+c))^(1/2)/b/d/sin(2*d*x+2*c)^(
1/2)+2*e*cos(d*x+c)*(e*tan(d*x+c))^(3/2)/b/d
```

3.312.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 52.97 (sec) , antiderivative size = 1846, normalized size of antiderivative = 2.43

$$\int \frac{(e\tan(c+dx))^{5/2}}{a+b\sec(c+dx)}\,dx = \text{Too large to display}$$

input
```
Integrate[(e*Tan[c + d*x])^(5/2)/(a + b*Sec[c + d*x]),x]
```

output

```
(2*(b + a*Cos[c + d*x])*Cot[c + d*x]*(e*Tan[c + d*x])^(5/2))/(b*d*(a + b*S
ec[c + d*x])) - ((b + a*Cos[c + d*x])*Sec[c + d*x]*(e*Tan[c + d*x])^(5/2)*
((4*a*Sec[c + d*x]^2*((-2*ArcTan[1 - (Sqrt[2]*Sqrt[b]*Sqrt[Tan[c + d*x]])/
(-a^2 + b^2)^(1/4)] + 2*ArcTan[1 + (Sqrt[2]*Sqrt[b]*Sqrt[Tan[c + d*x]])/(-
a^2 + b^2)^(1/4)] + Log[Sqrt[-a^2 + b^2] - Sqrt[2]*Sqrt[b]*(-a^2 + b^2)^(1
/4)*Sqrt[Tan[c + d*x]] + b*Tan[c + d*x]] - Log[Sqrt[-a^2 + b^2] + Sqrt[2]*
Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Tan[c + d*x]] + b*Tan[c + d*x]])/(4*Sqrt[2
]*Sqrt[b]*(-a^2 + b^2)^(1/4)) + (a*AppellF1[3/4, 1/2, 1, 7/4, -Tan[c + d*x
]^2, (b^2*Tan[c + d*x]^2)/(a^2 - b^2)]*Tan[c + d*x]^(3/2))/(3*a^2 - 3*b^2)
)*(a + b*Sqrt[1 + Tan[c + d*x]^2]))/((b + a*Cos[c + d*x])*(1 + Tan[c + d*x
]^2)^(3/2)) - (b*Sec[c + d*x]*(6*Sqrt[2]*(a^2 - b^2)*ArcTan[1 - Sqrt[2]*Sq
rt[Tan[c + d*x]]] - 6*Sqrt[2]*a^2*ArcTan[1 + Sqrt[2]*Sqrt[Tan[c + d*x]]] +
 6*Sqrt[2]*b^2*ArcTan[1 + Sqrt[2]*Sqrt[Tan[c + d*x]]] - (6 + 6*I)*Sqrt[b]*
(a^2 - b^2)^(3/4)*ArcTan[1 - ((1 + I)*Sqrt[b]*Sqrt[Tan[c + d*x]])/(a^2 - b
^2)^(1/4)] + (6 + 6*I)*Sqrt[b]*(a^2 - b^2)^(3/4)*ArcTan[1 + ((1 + I)*Sqrt[
b]*Sqrt[Tan[c + d*x]])/(a^2 - b^2)^(1/4)] - 3*Sqrt[2]*a^2*Log[1 - Sqrt[2]*
Sqrt[Tan[c + d*x]] + Tan[c + d*x]] + 3*Sqrt[2]*b^2*Log[1 - Sqrt[2]*Sqrt[Ta
n[c + d*x]] + Tan[c + d*x]] + 3*Sqrt[2]*a^2*Log[1 + Sqrt[2]*Sqrt[Tan[c + d
*x]] + Tan[c + d*x]] - 3*Sqrt[2]*b^2*Log[1 + Sqrt[2]*Sqrt[Tan[c + d*x]] +
Tan[c + d*x]] + (3 + 3*I)*Sqrt[b]*(a^2 - b^2)^(3/4)*Log[Sqrt[a^2 - b^2]...
```

3.312.3 Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e\tan(c+dx))^{5/2}}{a+b\sec(c+dx)}\,dx$$

$$\downarrow 3042$$

$$\int \frac{\left(-e\cot\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\downarrow 4379$$

$$\frac{e^2(a^2-b^2)\int \frac{\sqrt{e\tan(c+dx)}}{a+b\sec(c+dx)}dx}{b^2} - \frac{e^2\int(a-b\sec(c+dx))\sqrt{e\tan(c+dx)}dx}{b^2}$$

$$\downarrow 3042$$

$$\frac{e^2(a^2-b^2)\int \frac{\sqrt{-e\cot\left(c+dx+\frac{\pi}{2}\right)}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{b^2} - \frac{e^2\int\sqrt{-e\cot\left(c+dx+\frac{\pi}{2}\right)}\left(a-b\csc\left(c+dx+\frac{\pi}{2}\right)\right)dx}{b^2}$$

$$\downarrow 4372$$

$$\frac{e^2(a^2-b^2)\int \frac{\sqrt{-e\cot(c+dx+\frac{\pi}{2})}}{a+b\csc(c+dx+\frac{\pi}{2})}dx}{b^2}-\frac{e^2\left(a\int\sqrt{e\tan(c+dx)}dx-b\int\sec(c+dx)\sqrt{e\tan(c+dx)}dx\right)}{b^2}$$

$$\downarrow 3042$$

$$\frac{e^2(a^2-b^2)\int \frac{\sqrt{-e\cot(c+dx+\frac{\pi}{2})}}{a+b\csc(c+dx+\frac{\pi}{2})}dx}{b^2}-\frac{e^2\left(a\int\sqrt{e\tan(c+dx)}dx-b\int\sec(c+dx)\sqrt{e\tan(c+dx)}dx\right)}{b^2}$$

$$\downarrow 3093$$

$$\frac{e^2(a^2-b^2)\int \frac{\sqrt{-e\cot(c+dx+\frac{\pi}{2})}}{a+b\csc(c+dx+\frac{\pi}{2})}dx}{b^2}-\frac{e^2\left(a\int\sqrt{e\tan(c+dx)}dx-b\left(\frac{2\cos(c+dx)(e\tan(c+dx))^{3/2}}{de}-2\int\cos(c+dx)\sqrt{e\tan(c+dx)}dx\right)\right)}{b^2}$$

$$\downarrow 3042$$

$$\frac{e^2(a^2-b^2)\int \frac{\sqrt{-e\cot(c+dx+\frac{\pi}{2})}}{a+b\csc(c+dx+\frac{\pi}{2})}dx}{b^2}-\frac{e^2\left(a\int\sqrt{e\tan(c+dx)}dx-b\left(\frac{2\cos(c+dx)(e\tan(c+dx))^{3/2}}{de}-2\int\frac{\sqrt{e\tan(c+dx)}}{\sec(c+dx)}dx\right)\right)}{b^2}$$

$$\downarrow 3095$$

$$\frac{e^2(a^2-b^2)\int \frac{\sqrt{-e\cot(c+dx+\frac{\pi}{2})}}{a+b\csc(c+dx+\frac{\pi}{2})}dx}{b^2}-\frac{e^2\left(a\int\sqrt{e\tan(c+dx)}dx-b\left(\frac{2\cos(c+dx)(e\tan(c+dx))^{3/2}}{de}-\frac{2\sqrt{\cos(c+dx)}\sqrt{e\tan(c+dx)}\int\sqrt{\cos(c+dx)}\sqrt{\sin(c+dx)}dx}{\sqrt{\sin(c+dx)}}\right)\right)}{b^2}$$

$$\downarrow 3042$$

$$\frac{e^2(a^2-b^2)\int \frac{\sqrt{-e\cot(c+dx+\frac{\pi}{2})}}{a+b\csc(c+dx+\frac{\pi}{2})}dx}{b^2}-\frac{e^2\left(a\int\sqrt{e\tan(c+dx)}dx-b\left(\frac{2\cos(c+dx)(e\tan(c+dx))^{3/2}}{de}-\frac{2\sqrt{\cos(c+dx)}\sqrt{e\tan(c+dx)}\int\sqrt{\cos(c+dx)}\sqrt{\sin(c+dx)}dx}{\sqrt{\sin(c+dx)}}\right)\right)}{b^2}$$

$$\downarrow 3052$$

$$\frac{e^2\left(a^2-b^2\right)\int \frac{\sqrt{-e\cot\left(c+dx+\frac{\pi}{2}\right)}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{b^2} -$$
$$\frac{e^2\left(a\int \sqrt{e\tan(c+dx)}dx - b\left(\frac{2\cos(c+dx)(e\tan(c+dx))^{3/2}}{de} - \frac{2\cos(c+dx)\sqrt{e\tan(c+dx)}\int \sqrt{\sin(2c+2dx)}dx}{\sqrt{\sin(2c+2dx)}}\right)\right)}{b^2}$$

$$\downarrow 3042$$

$$\frac{e^2\left(a^2-b^2\right)\int \frac{\sqrt{-e\cot\left(c+dx+\frac{\pi}{2}\right)}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{b^2} -$$
$$\frac{e^2\left(a\int \sqrt{e\tan(c+dx)}dx - b\left(\frac{2\cos(c+dx)(e\tan(c+dx))^{3/2}}{de} - \frac{2\cos(c+dx)\sqrt{e\tan(c+dx)}\int \sqrt{\sin(2c+2dx)}dx}{\sqrt{\sin(2c+2dx)}}\right)\right)}{b^2}$$

$$\downarrow 3119$$

$$\frac{e^2\left(a^2-b^2\right)\int \frac{\sqrt{-e\cot\left(c+dx+\frac{\pi}{2}\right)}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{b^2} -$$
$$\frac{e^2\left(a\int \sqrt{e\tan(c+dx)}dx - b\left(\frac{2\cos(c+dx)(e\tan(c+dx))^{3/2}}{de} - \frac{2\cos(c+dx)E\left(c+dx-\frac{\pi}{4}\middle|2\right)\sqrt{e\tan(c+dx)}}{d\sqrt{\sin(2c+2dx)}}\right)\right)}{b^2}$$

$$\downarrow 3957$$

$$\frac{e^2\left(a^2-b^2\right)\int \frac{\sqrt{-e\cot\left(c+dx+\frac{\pi}{2}\right)}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{b^2} -$$
$$\frac{e^2\left(\frac{ae\int \frac{\sqrt{e\tan(c+dx)}}{\tan^2(c+dx)e^2+e^2}d(e\tan(c+dx))}{d} - b\left(\frac{2\cos(c+dx)(e\tan(c+dx))^{3/2}}{de} - \frac{2\cos(c+dx)E\left(c+dx-\frac{\pi}{4}\middle|2\right)\sqrt{e\tan(c+dx)}}{d\sqrt{\sin(2c+2dx)}}\right)\right)}{b^2}$$

$$\downarrow 266$$

$$\frac{e^2\left(a^2-b^2\right)\int \frac{\sqrt{-e\cot\left(c+dx+\frac{\pi}{2}\right)}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{b^2} -$$
$$\frac{e^2\left(\frac{2ae\int \frac{e^2\tan^2(c+dx)}{e^4\tan^4(c+dx)+e^2}d\sqrt{e\tan(c+dx)}}{d} - b\left(\frac{2\cos(c+dx)(e\tan(c+dx))^{3/2}}{de} - \frac{2\cos(c+dx)E\left(c+dx-\frac{\pi}{4}\middle|2\right)\sqrt{e\tan(c+dx)}}{d\sqrt{\sin(2c+2dx)}}\right)\right)}{b^2}$$

$$\downarrow 826$$

$$\frac{e^2\left(a^2-b^2\right)\int \frac{\sqrt{-e\cot\left(c+dx+\frac{\pi}{2}\right)}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{b^2} -$$
$$\frac{e^2\left(\frac{2ae\left(\frac{1}{2}\int \frac{e^2\tan^2(c+dx)+e}{e^4\tan^4(c+dx)+e^2}d\sqrt{e\tan(c+dx)} - \frac{1}{2}\int \frac{e-e^2\tan^2(c+dx)}{e^4\tan^4(c+dx)+e^2}d\sqrt{e\tan(c+dx)}\right)}{d} - b\left(\frac{2\cos(c+dx)(e\tan(c+dx))^{3/2}}{de} - \frac{2\cos(c+dx)E(c}{d\sqrt{}}\right.\right.}{b^2}$$

$$\downarrow 1476$$

$$\frac{e^2\left(a^2-b^2\right)\int \frac{\sqrt{-e\cot\left(c+dx+\frac{\pi}{2}\right)}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{b^2}-$$

$$e^2\left(\frac{2ae\left(\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{e^2\tan^2(c+dx)-\sqrt{2}e^{3/2}\tan(c+dx)+e}d\sqrt{e\tan(c+dx)}+\frac{1}{2}\int \frac{1}{e^2\tan^2(c+dx)+\sqrt{2}e^{3/2}\tan(c+dx)+e}d\sqrt{e\tan(c+dx)}\right)-\frac{1}{2}\int \frac{e-e^2\tan^2(c+d}{e^4\tan^4(c+dx)+}\right.}{d}\right.$$

$$\left.\left.\right)\right)$$

$$b^2$$

$$\downarrow 1082$$

$$\frac{e^2\left(a^2-b^2\right)\int \frac{\sqrt{-e\cot\left(c+dx+\frac{\pi}{2}\right)}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{b^2}-$$

$$e^2\left(\frac{2ae\left(\frac{1}{2}\left(\frac{\int \frac{1}{-e^2\tan^2(c+dx)-1}d\left(1-\sqrt{2}\sqrt{e}\tan(c+dx)\right)}{\sqrt{2}\sqrt{e}}-\frac{\int \frac{1}{-e^2\tan^2(c+dx)-1}d\left(\sqrt{2}\sqrt{e}\tan(c+dx)+1\right)}{\sqrt{2}\sqrt{e}}\right)-\frac{1}{2}\int \frac{e-e^2\tan^2(c+dx)}{e^4\tan^4(c+dx)+e^2}d\sqrt{e\tan(c+dx)}\right)}{d}-\right.$$

$$\left.\right)$$

$$b^2$$

$$\downarrow 217$$

$$\frac{e^2\left(a^2-b^2\right)\int \frac{\sqrt{-e\cot\left(c+dx+\frac{\pi}{2}\right)}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{b^2}-$$

$$e^2\left(\frac{2ae\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{e}\tan(c+dx)+1\right)}{\sqrt{2}\sqrt{e}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{e}\tan(c+dx)\right)}{\sqrt{2}\sqrt{e}}\right)-\frac{1}{2}\int \frac{e-e^2\tan^2(c+dx)}{e^4\tan^4(c+dx)+e^2}d\sqrt{e\tan(c+dx)}\right)}{d}-b\left(\frac{2\cos(c+dx)(e\tan(c+dx))^3}{de}\right.\right.$$

$$\left.\left.\right)\right)$$

$$b^2$$

$$\downarrow 1479$$

$$\frac{e^2\left(a^2-b^2\right)\int \frac{\sqrt{-e\cot\left(c+dx+\frac{\pi}{2}\right)}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{b^2}-$$

$$e^2\left(\frac{2ae\left(\frac{1}{2}\left(\frac{\int -\frac{\sqrt{2}\sqrt{e}-2\sqrt{e\tan(c+dx)}}{e^2\tan^2(c+dx)-\sqrt{2}e^{3/2}\tan(c+dx)+e}d\sqrt{e\tan(c+dx)}}{2\sqrt{2}\sqrt{e}}+\frac{\int -\frac{\sqrt{2}\left(\sqrt{e}+\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{e^2\tan^2(c+dx)+\sqrt{2}e^{3/2}\tan(c+dx)+e}d\sqrt{e\tan(c+dx)}}{2\sqrt{2}\sqrt{e}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{e}\tan(c+\right.}{\sqrt{2}\sqrt{e}}\right.\right.}{d}\right.$$

$$\left.\left.\right)\right)$$

$$b^2$$

$$\downarrow 25$$

$$\frac{e^2\left(a^2-b^2\right)\int\frac{\sqrt{-e\cot\left(c+dx+\frac{\pi}{2}\right)}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{b^2}-$$

$$e^2\left(\frac{2ae\left(\frac{1}{2}\left(-\frac{\int\frac{\sqrt{2}\sqrt{e}-2\sqrt{e\tan(c+dx)}}{e^2\tan^2(c+dx)-\sqrt{2}e^{3/2}\tan(c+dx)+e}d\sqrt{e\tan(c+dx)}}{2\sqrt{2}\sqrt{e}}-\frac{\int\frac{\sqrt{2}\left(\sqrt{e}+\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{e^2\tan^2(c+dx)+\sqrt{2}e^{3/2}\tan(c+dx)+e}d\sqrt{e\tan(c+dx)}}{2\sqrt{2}\sqrt{e}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{e}\tan(c+d\right.\right.\right.}{\sqrt{2}\sqrt{e}}}{d}\right.$$

$$\Big\downarrow 27$$

$$\frac{e^2\left(a^2-b^2\right)\int\frac{\sqrt{-e\cot\left(c+dx+\frac{\pi}{2}\right)}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{b^2}-$$

$$e^2\left(\frac{2ae\left(\frac{1}{2}\left(-\frac{\int\frac{\sqrt{2}\sqrt{e}-2\sqrt{e\tan(c+dx)}}{e^2\tan^2(c+dx)-\sqrt{2}e^{3/2}\tan(c+dx)+e}d\sqrt{e\tan(c+dx)}}{2\sqrt{2}\sqrt{e}}-\frac{\int\frac{\sqrt{e}+\sqrt{2}\sqrt{e\tan(c+dx)}}{e^2\tan^2(c+dx)+\sqrt{2}e^{3/2}\tan(c+dx)+e}d\sqrt{e\tan(c+dx)}}{2\sqrt{e}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{e}\tan(c+d\right.\right.\right.}{\sqrt{2}\sqrt{e}}}{d}\right.$$

$$\Big\downarrow 1103$$

$$\frac{e^2\left(a^2-b^2\right)\int\frac{\sqrt{-e\cot\left(c+dx+\frac{\pi}{2}\right)}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{b^2}-$$

$$e^2\left(\frac{2ae\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{e}\tan(c+dx)+1\right)}{\sqrt{2}\sqrt{e}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{e}\tan(c+dx)\right)}{\sqrt{2}\sqrt{e}}\right)+\frac{1}{2}\left(\frac{\log\left(-\sqrt{2}e^{3/2}\tan(c+dx)+e^2\tan^2(c+dx)+e\right)}{2\sqrt{2}\sqrt{e}}-\frac{\log\left(\sqrt{2}e^{3/2}\tan(c+dx)+e^2\tan\right.\right.\right.}{2\sqrt{2}\sqrt{e}}}{d}\right.$$

$$\Big\downarrow 4378$$

$$\frac{e^2\left(a^2-b^2\right)\left(\frac{\int\sqrt{e\tan(c+dx)}dx}{a}-\frac{b\int\frac{\sqrt{e\tan(c+dx)}}{b+a\cos(c+dx)}dx}{a}\right)}{b^2}-$$

$$e^2\left(\frac{2ae\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{e}\tan(c+dx)+1\right)}{\sqrt{2}\sqrt{e}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{e}\tan(c+dx)\right)}{\sqrt{2}\sqrt{e}}\right)+\frac{1}{2}\left(\frac{\log\left(-\sqrt{2}e^{3/2}\tan(c+dx)+e^2\tan^2(c+dx)+e\right)}{2\sqrt{2}\sqrt{e}}-\frac{\log\left(\sqrt{2}e^{3/2}\tan(c+dx)+e^2\tan\right.\right.\right.}{2\sqrt{2}\sqrt{e}}}{d}\right.$$

$$\Big\downarrow 3042$$

$$\frac{e^2\left(a^2-b^2\right)\left(\frac{\int\sqrt{e\tan(c+dx)}dx}{a}-\frac{b\int\frac{\sqrt{-e\cot\left(c+dx+\frac{\pi}{2}\right)}}{b+a\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{a}\right)}{b^2}-$$
$$e^2\left(\frac{2ae\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{e}\tan(c+dx)+1\right)}{\sqrt{2}\sqrt{e}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{e}\tan(c+dx)\right)}{\sqrt{2}\sqrt{e}}\right)+\frac{1}{2}\left(\frac{\log\left(-\sqrt{2}e^{3/2}\tan(c+dx)+e^2\tan^2(c+dx)+e\right)}{2\sqrt{2}\sqrt{e}}-\frac{\log\left(\sqrt{2}e^{3/2}\tan(c+dx)+e^2\tan\right.\right.\right.}{2\sqrt{2}\sqrt{e}}$$
$$\frac{d}{b^2}$$

$$\downarrow 3212$$

$$\frac{e^2\left(a^2-b^2\right)\left(\frac{\int\sqrt{e\tan(c+dx)}dx}{a}-\frac{b\sqrt{e\tan(c+dx)}\sqrt{e\cot(c+dx)}\int\frac{1}{(b+a\cos(c+dx))\sqrt{e\cot(c+dx)}}dx}{a}\right)}{b^2}-$$
$$e^2\left(\frac{2ae\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{e}\tan(c+dx)+1\right)}{\sqrt{2}\sqrt{e}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{e}\tan(c+dx)\right)}{\sqrt{2}\sqrt{e}}\right)+\frac{1}{2}\left(\frac{\log\left(-\sqrt{2}e^{3/2}\tan(c+dx)+e^2\tan^2(c+dx)+e\right)}{2\sqrt{2}\sqrt{e}}-\frac{\log\left(\sqrt{2}e^{3/2}\tan(c+dx)+e^2\tan\right.\right.\right.}{2\sqrt{2}\sqrt{e}}$$
$$\frac{d}{b^2}$$

$$\downarrow 3042$$

$$\frac{e^2\left(a^2-b^2\right)\left(\frac{\int\sqrt{e\tan(c+dx)}dx}{a}-\frac{b\sqrt{e\tan(c+dx)}\sqrt{e\cot(c+dx)}\int\frac{1}{\left(b-a\sin\left(c+dx-\frac{\pi}{2}\right)\right)\sqrt{-e\tan\left(c+dx-\frac{\pi}{2}\right)}}dx}{a}\right)}{b^2}-$$
$$e^2\left(\frac{2ae\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{e}\tan(c+dx)+1\right)}{\sqrt{2}\sqrt{e}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{e}\tan(c+dx)\right)}{\sqrt{2}\sqrt{e}}\right)+\frac{1}{2}\left(\frac{\log\left(-\sqrt{2}e^{3/2}\tan(c+dx)+e^2\tan^2(c+dx)+e\right)}{2\sqrt{2}\sqrt{e}}-\frac{\log\left(\sqrt{2}e^{3/2}\tan(c+dx)+e^2\tan\right.\right.\right.}{2\sqrt{2}\sqrt{e}}$$
$$\frac{d}{b^2}$$

$$\downarrow 3209$$

$$\frac{e^2\left(a^2-b^2\right)\left(\frac{\int\sqrt{e\tan(c+dx)}dx}{a}-\frac{b\sqrt{-\cos(c+dx)}\sqrt{e\tan(c+dx)}\int\frac{\sqrt{\sin(c+dx)}}{\sqrt{-\cos(c+dx)}(b+a\cos(c+dx))}dx}{a\sqrt{\sin(c+dx)}}\right)}{b^2}-$$
$$e^2\left(\frac{2ae\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{e}\tan(c+dx)+1\right)}{\sqrt{2}\sqrt{e}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{e}\tan(c+dx)\right)}{\sqrt{2}\sqrt{e}}\right)+\frac{1}{2}\left(\frac{\log\left(-\sqrt{2}e^{3/2}\tan(c+dx)+e^2\tan^2(c+dx)+e\right)}{2\sqrt{2}\sqrt{e}}-\frac{\log\left(\sqrt{2}e^{3/2}\tan(c+dx)+e^2\tan\right.\right.\right.}{2\sqrt{2}\sqrt{e}}$$
$$\frac{d}{b^2}$$

$$\downarrow 3042$$

$$\frac{e^2\left(a^2-b^2\right)\left(\frac{\int\sqrt{e\tan(c+dx)}dx}{a}-\frac{b\sqrt{-\cos(c+dx)}\sqrt{e\tan(c+dx)}\int\frac{\sqrt{-\cos\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{-\sin\left(c+dx+\frac{\pi}{2}\right)}\left(b+a\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a\sqrt{\sin(c+dx)}}\right)}{b^2}-$$

$$e^2\left(\frac{2ae\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{e}\tan(c+dx)+1\right)}{\sqrt{2}\sqrt{e}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{e}\tan(c+dx)\right)}{\sqrt{2}\sqrt{e}}\right)+\frac{1}{2}\left(\frac{\log\left(-\sqrt{2}e^{3/2}\tan(c+dx)+e^2\tan^2(c+dx)+e\right)}{2\sqrt{2}\sqrt{e}}-\frac{\log\left(\sqrt{2}e^{3/2}\tan(c+dx)+e^2\tan\right.\right.\right.}{2\sqrt{2}\sqrt{e}}\right.$$

$$\left.\left.\left.

3.312.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3052
```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Simp[Sqrt[a*Sin[e + f*x]]*(Sqrt[b*Cos[e + f*x]]/Sqrt[Sin[2*e
 + 2*f*x]])   Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

rule 3093
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[a^2*(a*Sec[e + f*x])^(m - 2)*((b*Tan[e + f*x])^(n +
1)/(b*f*(m + n - 1))), x] + Simp[a^2*((m - 2)/(m + n - 1))   Int[(a*Sec[e +
 f*x])^(m - 2)*(b*Tan[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (
GtQ[m, 1] || (EqQ[m, 1] && EqQ[n, 1/2])) && NeQ[m + n - 1, 0] && IntegersQ[
2*m, 2*n]
```

rule 3095
```
Int[Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]]/sec[(e_.) + (f_.)*(x_)], x_Symbol]
:> Simp[Sqrt[Cos[e + f*x]]*(Sqrt[b*Tan[e + f*x]]/Sqrt[Sin[e + f*x]])   Int[
Sqrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]], x], x] /; FreeQ[{b, e, f}, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3209
```
Int[1/(((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(g_)*tan[(e_.) + (f_.)*(
x_)]]), x_Symbol] :> Simp[Sqrt[Sin[e + f*x]]/(Sqrt[Cos[e + f*x]]*Sqrt[g*Tan
[e + f*x]])   Int[Sqrt[Cos[e + f*x]]/(Sqrt[Sin[e + f*x]]*(a + b*Sin[e + f*x
])), x], x] /; FreeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

rule 3212
```
Int[(cot[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Simp[g^(2*IntPart[p])*(g*Cot[e + f*x])^FracPart[p]
*(g*Tan[e + f*x])^FracPart[p]   Int[(a + b*Sin[e + f*x])^m/(g*Tan[e + f*x])
^p, x], x] /; FreeQ[{a, b, e, f, g, m, p}, x] &&  !IntegerQ[p]
```

rule 3385
```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/(Sqrt[(d_)*sin[(e_.) + (f_.)*(x_)]]
*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Simp[Sqrt[Sin[e + f*
x]]/Sqrt[d*Sin[e + f*x]]   Int[Sqrt[g*Cos[e + f*x]]/(Sqrt[Sin[e + f*x]]*(a
+ b*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e, f, g}, x] && NeQ[a^2 - b^2
, 0]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

rule 4372
```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(e*Cot[c + d*x])^m, x], x] + Simp[b   Int[
(e*Cot[c + d*x])^m*Csc[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, m}, x]
```

rule 4378
```
Int[Sqrt[cot[(c_.) + (d_.)*(x_)]*(e_.)]/(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a
_)), x_Symbol] :> Simp[1/a   Int[Sqrt[e*Cot[c + d*x]], x], x] - Simp[b/a
Int[Sqrt[e*Cot[c + d*x]]/(b + a*Sin[c + d*x]), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && NeQ[a^2 - b^2, 0]
```

rule 4379
```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)/(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[-e^2/b^2   Int[(e*Cot[c + d*x])^(m - 2)*(a - b*Csc[
c + d*x]), x], x] + Simp[e^2*((a^2 - b^2)/b^2)   Int[(e*Cot[c + d*x])^(m -
2)/(a + b*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b
^2, 0] && IGtQ[m - 1/2, 0]
```

3.312.4 Maple [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2966 vs. $2(646) = 1292$.

Time = 3.69 (sec) , antiderivative size = 2967, normalized size of antiderivative = 3.90

method	result	size
default	Expression too large to display	2967

input
```
int((e*tan(d*x+c))^(5/2)/(a+b*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*2^(1/2)/b/((a^2-b^2)^(1/2)-a+b)/((a^2-b^2)^(1/2)+a-b)/a*(a-b)*e^2*(e*t
an(d*x+c))^(1/2)*(-(csc(d*x+c)-cot(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c)+
1)^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)*EllipticPi((csc(d*x+c)-cot(d*x+c)+1
)^(1/2),1/2+1/2*I,1/2*2^(1/2))*b^2*cos(d*x+c)-(csc(d*x+c)-cot(d*x+c)+1)^(1
/2)*(cot(d*x+c)-csc(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)*Elliptic
Pi((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2-1/2*I,1/2*2^(1/2))*b^2*cos(d*x+c)+I
*(csc(d*x+c)-cot(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c)+1)^(1/2)*(cot(d*x+
c)-csc(d*x+c))^(1/2)*EllipticPi((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2-1/2*I,
1/2*2^(1/2))*b^2-I*(csc(d*x+c)-cot(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c)+
1)^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)*EllipticPi((csc(d*x+c)-cot(d*x+c)+1
)^(1/2),1/2+1/2*I,1/2*2^(1/2))*b^2+(csc(d*x+c)-cot(d*x+c)+1)^(1/2)*(cot(d*
x+c)-csc(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)*EllipticPi((csc(d*x
+c)-cot(d*x+c)+1)^(1/2),-(a-b)/(-a+b+((a-b)*(a+b))^(1/2)),1/2*2^(1/2))*b^2
*cos(d*x+c)-2*2^(1/2)*a*b+I*(csc(d*x+c)-cot(d*x+c)+1)^(1/2)*(cot(d*x+c)-cs
c(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)*EllipticPi((csc(d*x+c)-cot
(d*x+c)+1)^(1/2),1/2-1/2*I,1/2*2^(1/2))*b^2*cos(d*x+c)+(a^2-b^2)^(1/2)*(cs
c(d*x+c)-cot(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c)+1)^(1/2)*(cot(d*x+c)-c
sc(d*x+c))^(1/2)*EllipticPi((csc(d*x+c)-cot(d*x+c)+1)^(1/2),(a-b)/(a-b+((a
-b)*(a+b))^(1/2)),1/2*2^(1/2))*a+(a^2-b^2)^(1/2)*(csc(d*x+c)-cot(d*x+c)+1)
^(1/2)*(cot(d*x+c)-csc(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)*El...
```

3.312.5 Fricas [F(-1)]

Timed out.

$$\int \frac{(e\tan(c+dx))^{5/2}}{a+b\sec(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((e*tan(d*x+c))^(5/2)/(a+b*sec(d*x+c)),x, algorithm="fricas")
```

output
```
Timed out
```

3.312.6 Sympy [F]

$$\int \frac{(e\tan(c+dx))^{5/2}}{a+b\sec(c+dx)}\,dx = \int \frac{(e\tan(c+dx))^{\frac{5}{2}}}{a+b\sec(c+dx)}\,dx$$

input
```
integrate((e*tan(d*x+c))**(5/2)/(a+b*sec(d*x+c)),x)
```

output
```
Integral((e*tan(c + d*x))**(5/2)/(a + b*sec(c + d*x)), x)
```

3.312.7 Maxima [F]

$$\int \frac{(e\tan(c+dx))^{5/2}}{a+b\sec(c+dx)}\,dx = \int \frac{(e\tan(dx+c))^{\frac{5}{2}}}{b\sec(dx+c)+a}\,dx$$

input
```
integrate((e*tan(d*x+c))^(5/2)/(a+b*sec(d*x+c)),x, algorithm="maxima")
```

output
```
integrate((e*tan(d*x + c))^(5/2)/(b*sec(d*x + c) + a), x)
```

3.312.8 Giac [F]

$$\int \frac{(e\tan(c+dx))^{5/2}}{a+b\sec(c+dx)}\,dx = \int \frac{(e\tan(dx+c))^{\frac{5}{2}}}{b\sec(dx+c)+a}\,dx$$

input
```
integrate((e*tan(d*x+c))^(5/2)/(a+b*sec(d*x+c)),x, algorithm="giac")
```

output
```
integrate((e*tan(d*x + c))^(5/2)/(b*sec(d*x + c) + a), x)
```

3.312.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(e\tan(c+dx))^{5/2}}{a+b\sec(c+dx)}\,dx = \int \frac{\cos(c+d\,x)\,(e\tan(c+d\,x))^{5/2}}{b+a\,\cos(c+d\,x)}\,dx$$

input
```
int((e*tan(c + d*x))^(5/2)/(a + b/cos(c + d*x)),x)
```

output
```
int((cos(c + d*x)*(e*tan(c + d*x))^(5/2))/(b + a*cos(c + d*x)), x)
```

3.313 $\int \frac{(e\tan(c+dx))^{3/2}}{a+b\sec(c+dx)}\,dx$

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3.313.1 Optimal result

Integrand size = 25, antiderivative size = 740

$$\int \frac{(e\tan(c+dx))^{3/2}}{a+b\sec(c+dx)}\,dx = \frac{ae^{3/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}b^2d}$$
$$-\frac{(a^2-b^2)\,e^{3/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}ab^2d}$$
$$-\frac{ae^{3/2}\arctan\left(1+\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}b^2d}+\frac{(a^2-b^2)\,e^{3/2}\arctan\left(1+\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}ab^2d}$$
$$+\frac{ae^{3/2}\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}b^2d}$$
$$-\frac{(a^2-b^2)\,e^{3/2}\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}ab^2d}$$
$$-\frac{ae^{3/2}\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)+\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}b^2d}$$
$$+\frac{(a^2-b^2)\,e^{3/2}\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)+\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}ab^2d}$$
$$-\frac{2\sqrt{2}\sqrt{a^2-b^2}e^2\,\text{EllipticPi}\left(\frac{b}{a-\sqrt{a^2-b^2}},\arcsin\left(\frac{\sqrt{-\cos(c+dx)}}{\sqrt{1+\sin(c+dx)}}\right),-1\right)\sqrt{\sin(c+dx)}}{abd\sqrt{-\cos(c+dx)}\sqrt{e\tan(c+dx)}}$$
$$+\frac{2\sqrt{2}\sqrt{a^2-b^2}e^2\,\text{EllipticPi}\left(\frac{b}{a+\sqrt{a^2-b^2}},\arcsin\left(\frac{\sqrt{-\cos(c+dx)}}{\sqrt{1+\sin(c+dx)}}\right),-1\right)\sqrt{\sin(c+dx)}}{abd\sqrt{-\cos(c+dx)}\sqrt{e\tan(c+dx)}}$$
$$+\frac{e^2\,\text{EllipticF}\left(c-\frac{\pi}{4}+dx,2\right)\sec(c+dx)\sqrt{\sin(2c+2dx)}}{bd\sqrt{e\tan(c+dx)}}$$

output
```
1/2*a*e^(3/2)*arctan(1-2^(1/2)*(e*tan(d*x+c))^(1/2)/e^(1/2))/b^2/d*2^(1/2)
-1/2*(a^2-b^2)*e^(3/2)*arctan(1-2^(1/2)*(e*tan(d*x+c))^(1/2)/e^(1/2))/a/b^
2/d*2^(1/2)-1/2*a*e^(3/2)*arctan(1+2^(1/2)*(e*tan(d*x+c))^(1/2)/e^(1/2))/b
^2/d*2^(1/2)+1/2*(a^2-b^2)*e^(3/2)*arctan(1+2^(1/2)*(e*tan(d*x+c))^(1/2)/e
^(1/2))/a/b^2/d*2^(1/2)+1/4*a*e^(3/2)*ln(e^(1/2)-2^(1/2)*(e*tan(d*x+c))^(1
/2)+e^(1/2)*tan(d*x+c))/b^2/d*2^(1/2)-1/4*(a^2-b^2)*e^(3/2)*ln(e^(1/2)-2^(
1/2)*(e*tan(d*x+c))^(1/2)+e^(1/2)*tan(d*x+c))/a/b^2/d*2^(1/2)-1/4*a*e^(3/2
)*ln(e^(1/2)+2^(1/2)*(e*tan(d*x+c))^(1/2)+e^(1/2)*tan(d*x+c))/b^2/d*2^(1/2
)+1/4*(a^2-b^2)*e^(3/2)*ln(e^(1/2)+2^(1/2)*(e*tan(d*x+c))^(1/2)+e^(1/2)*ta
n(d*x+c))/a/b^2/d*2^(1/2)-2*e^2*EllipticPi((-cos(d*x+c))^(1/2)/(1+sin(d*x+
c))^(1/2),b/(a-(a^2-b^2)^(1/2)),I)*2^(1/2)*(a^2-b^2)^(1/2)*sin(d*x+c)^(1/2
)/a/b/d/(-cos(d*x+c))^(1/2)/(e*tan(d*x+c))^(1/2)+2*e^2*EllipticPi((-cos(d*
x+c))^(1/2)/(1+sin(d*x+c))^(1/2),b/(a+(a^2-b^2)^(1/2)),I)*2^(1/2)*(a^2-b^2
)^(1/2)*sin(d*x+c)^(1/2)/a/b/d/(-cos(d*x+c))^(1/2)/(e*tan(d*x+c))^(1/2)-e^
2*(sin(c+1/4*Pi+d*x)^2)^(1/2)/sin(c+1/4*Pi+d*x)*EllipticF(cos(c+1/4*Pi+d*x
),2^(1/2))*sec(d*x+c)*sin(2*d*x+2*c)^(1/2)/b/d/(e*tan(d*x+c))^(1/2)
```

3.313.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 21.45 (sec) , antiderivative size = 548, normalized size of antiderivative = 0.74

$$\int \frac{(e\tan(c+dx))^{3/2}}{a+b\sec(c+dx)}\,dx =$$

$$-\frac{\cos(c+dx)\left(a+b\sqrt{\sec^2(c+dx)}\right)(e\tan(c+dx))^{3/2}\left(-5(a^2-b^2)\left(2\sqrt{2}\sqrt{b}\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right.\right.\right.}{}$$

input
```
Integrate[(e*Tan[c + d*x])^(3/2)/(a + b*Sec[c + d*x]),x]
```

output

```
-1/20*(Cos[c + d*x]*(a + b*Sqrt[Sec[c + d*x]^2])*(e*Tan[c + d*x])^(3/2)*(-
5*(a^2 - b^2)*(2*Sqrt[2]*Sqrt[b]*ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]] -
2*Sqrt[2]*Sqrt[b]*ArcTan[1 + Sqrt[2]*Sqrt[Tan[c + d*x]]] - (2 - 2*I)*(a^2
- b^2)^(1/4)*ArcTan[1 - ((1 + I)*Sqrt[b]*Sqrt[Tan[c + d*x]])/(a^2 - b^2)^(
1/4)] + (2 - 2*I)*(a^2 - b^2)^(1/4)*ArcTan[1 + ((1 + I)*Sqrt[b]*Sqrt[Tan[c
 + d*x]])/(a^2 - b^2)^(1/4)] + Sqrt[2]*Sqrt[b]*Log[1 - Sqrt[2]*Sqrt[Tan[c
+ d*x]] + Tan[c + d*x]] - Sqrt[2]*Sqrt[b]*Log[1 + Sqrt[2]*Sqrt[Tan[c + d*x
]] + Tan[c + d*x]] - (1 - I)*(a^2 - b^2)^(1/4)*Log[Sqrt[a^2 - b^2] - (1 +
I)*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Tan[c + d*x]] + I*b*Tan[c + d*x]] + (1 -
 I)*(a^2 - b^2)^(1/4)*Log[Sqrt[a^2 - b^2] + (1 + I)*Sqrt[b]*(a^2 - b^2)^(1
/4)*Sqrt[Tan[c + d*x]] + I*b*Tan[c + d*x]]) + 8*a*b^(3/2)*AppellF1[5/4, 1/
2, 1, 9/4, -Tan[c + d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2 - b^2)]*Tan[c + d*x]
^(5/2)))/(a*Sqrt[b]*(a^2 - b^2)*d*(b + a*Cos[c + d*x])*Tan[c + d*x]^(3/2))
```

3.313.3 Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e\tan(c+dx))^{3/2}}{a+b\sec(c+dx)}\,dx$$

$$\downarrow 3042$$

$$\int \frac{\left(-e\cot\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\downarrow 4379$$

$$\frac{e^2\left(a^2-b^2\right)\int \frac{1}{(a+b\sec(c+dx))\sqrt{e\tan(c+dx)}}dx}{b^2} - \frac{e^2\int \frac{a-b\sec(c+dx)}{\sqrt{e\tan(c+dx)}}dx}{b^2}$$

$$\downarrow 3042$$

$$\frac{e^2\left(a^2-b^2\right)\int \frac{1}{\sqrt{-e\cot\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{b^2} - \frac{e^2\int \frac{a-b\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{-e\cot\left(c+dx+\frac{\pi}{2}\right)}}dx}{b^2}$$

$$\downarrow 4372$$

$$\frac{e^2\left(a^2-b^2\right)\int \frac{1}{\sqrt{-e\cot\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{b^2} - \frac{e^2\left(a\int \frac{1}{\sqrt{e\tan(c+dx)}}dx - b\int \frac{\sec(c+dx)}{\sqrt{e\tan(c+dx)}}dx\right)}{b^2}$$

$$\downarrow 3042$$

$$\frac{e^2\left(a^2-b^2\right)\int \frac{1}{\sqrt{-e\cot\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{b^2}-\frac{e^2\left(a\int \frac{1}{\sqrt{e\tan(c+dx)}}dx-b\int \frac{\sec(c+dx)}{\sqrt{e\tan(c+dx)}}dx\right)}{b^2}$$

$$\downarrow 3094$$

$$\frac{e^2\left(a^2-b^2\right)\int \frac{1}{\sqrt{-e\cot\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{b^2}-\frac{e^2\left(a\int \frac{1}{\sqrt{e\tan(c+dx)}}dx-\frac{b\sqrt{\sin(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{\sin(c+dx)}}dx}{\sqrt{\cos(c+dx)}\sqrt{e\tan(c+dx)}}\right)}{b^2}$$

$$\downarrow 3042$$

$$\frac{e^2\left(a^2-b^2\right)\int \frac{1}{\sqrt{-e\cot\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{b^2}-\frac{e^2\left(a\int \frac{1}{\sqrt{e\tan(c+dx)}}dx-\frac{b\sqrt{\sin(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{\sin(c+dx)}}dx}{\sqrt{\cos(c+dx)}\sqrt{e\tan(c+dx)}}\right)}{b^2}$$

$$\downarrow 3053$$

$$\frac{e^2\left(a^2-b^2\right)\int \frac{1}{\sqrt{-e\cot\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{b^2}-\frac{e^2\left(a\int \frac{1}{\sqrt{e\tan(c+dx)}}dx-\frac{b\sqrt{\sin(2c+2dx)}\sec(c+dx)\int \frac{1}{\sqrt{\sin(2c+2dx)}}dx}{\sqrt{e\tan(c+dx)}}\right)}{b^2}$$

$$\downarrow 3042$$

$$\frac{e^2\left(a^2-b^2\right)\int \frac{1}{\sqrt{-e\cot\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{b^2}-\frac{e^2\left(a\int \frac{1}{\sqrt{e\tan(c+dx)}}dx-\frac{b\sqrt{\sin(2c+2dx)}\sec(c+dx)\int \frac{1}{\sqrt{\sin(2c+2dx)}}dx}{\sqrt{e\tan(c+dx)}}\right)}{b^2}$$

$$\downarrow 3120$$

$$\frac{e^2\left(a^2-b^2\right)\int \frac{1}{\sqrt{-e\cot\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{b^2}-\frac{e^2\left(a\int \frac{1}{\sqrt{e\tan(c+dx)}}dx-\frac{b\sqrt{\sin(2c+2dx)}\sec(c+dx)\,\text{EllipticF}\left(c+dx-\frac{\pi}{4},2\right)}{d\sqrt{e\tan(c+dx)}}\right)}{b^2}$$

$$\downarrow 3957$$

$$\frac{e^2\left(a^2-b^2\right)\int\frac{1}{\sqrt{-e\cot\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{b^2}-\frac{e^2\left(\frac{ae\int\frac{1}{\sqrt{e\tan(c+dx)}\left(\tan^2(c+dx)e^2+e^2\right)}d(e\tan(c+dx))}{d}-\frac{b\sqrt{\sin(2c+2dx)}\sec(c+dx)\,\text{EllipticF}\left(c+dx-\frac{\pi}{4},2\right)}{d\sqrt{e\tan(c+dx)}}\right)}{b^2}$$

$$\downarrow 266$$

$$\frac{e^2\left(a^2-b^2\right)\int\frac{1}{\sqrt{-e\cot\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{b^2}-\frac{e^2\left(\frac{2ae\int\frac{1}{e^4\tan^4(c+dx)+e^2}d\sqrt{e\tan(c+dx)}}{d}-\frac{b\sqrt{\sin(2c+2dx)}\sec(c+dx)\,\text{EllipticF}\left(c+dx-\frac{\pi}{4},2\right)}{d\sqrt{e\tan(c+dx)}}\right)}{b^2}$$

$$\downarrow 755$$

$$\frac{e^2\left(a^2-b^2\right)\int\frac{1}{\sqrt{-e\cot\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{b^2}-\frac{e^2\left(\frac{2ae\left(\frac{\int\frac{e-e^2\tan^2(c+dx)}{e^4\tan^4(c+dx)+e^2}d\sqrt{e\tan(c+dx)}}{2e}+\frac{\int\frac{e^2\tan^2(c+dx)+e}{e^4\tan^4(c+dx)+e^2}d\sqrt{e\tan(c+dx)}}{2e}\right)}{d}-\frac{b\sqrt{\sin(2c+2dx)}\sec(c+dx)\,\text{EllipticF}\left(c+dx-\frac{\pi}{4},2\right)}{d\sqrt{e\tan(c+dx)}}\right)}{b^2}$$

$$\downarrow 1476$$

$$\frac{e^2\left(a^2-b^2\right)\int\frac{1}{\sqrt{-e\cot\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{b^2}-\frac{e^2\left(\frac{2ae\left(\frac{\frac{1}{2}\int\frac{1}{e^2\tan^2(c+dx)-\sqrt{2}e^{3/2}\tan(c+dx)+e}d\sqrt{e\tan(c+dx)}+\frac{1}{2}\int\frac{1}{e^2\tan^2(c+dx)+\sqrt{2}e^{3/2}\tan(c+dx)+e}d\sqrt{e\tan(c+dx)}}{2e}+\frac{\int\frac{e-e^2\tan^2(c+dx)}{e^4\tan^4(c+dx)+e^2}d\sqrt{e\tan(c}}{2e}\right)}{d}\right)}{b^2}$$

$$\downarrow 1082$$

$$\frac{e^2\left(a^2-b^2\right)\int\frac{1}{\sqrt{-e\cot\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{b^2}-\frac{e^2\left(\frac{2ae\left(\frac{\int\frac{1}{-e^2\tan^2(c+dx)-1}d\left(1-\sqrt{2}\sqrt{e}\tan(c+dx)\right)}{\sqrt{2}\sqrt{e}}-\frac{\int\frac{1}{-e^2\tan^2(c+dx)-1}d\left(\sqrt{2}\sqrt{e}\tan(c+dx)+1\right)}{\sqrt{2}\sqrt{e}}}{2e}+\frac{\int\frac{e-e^2\tan^2(c+dx)}{e^4\tan^4(c+dx)+e^2}d\sqrt{e\tan(c+dx)}}{2e}\right)}{d}-\frac{b\sqrt{\sin(2c}}{}\right)}{b^2}$$

$$\downarrow 217$$

$$\frac{e^2\left(a^2-b^2\right)\int \frac{1}{\sqrt{-e\cot\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{b^2} -$$

$$e^2\left(\frac{2ae\left(\frac{\int \frac{e-e^2\tan^2(c+dx)}{e^4\tan^4(c+dx)+e^2}d\sqrt{e\tan(c+dx)}}{2e}+\frac{\frac{\arctan\left(\sqrt{2}\sqrt{e}\tan(c+dx)+1\right)}{\sqrt{2}\sqrt{e}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{e}\tan(c+dx)\right)}{\sqrt{2}\sqrt{e}}}{2e}\right)}{d}-\frac{b\sqrt{\sin(2c+2dx)}\sec(c+dx)\,\text{EllipticF}\left(c+\right.}{d\sqrt{e\tan(c+dx)}}\right.$$

$$b^2$$

$$\downarrow 1479$$

$$\frac{e^2\left(a^2-b^2\right)\int \frac{1}{\sqrt{-e\cot\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{b^2} -$$

$$e^2\left(\frac{2ae\left(\frac{-\frac{\int -\frac{\sqrt{2}\sqrt{e}-2\sqrt{e\tan(c+dx)}}{e^2\tan^2(c+dx)-\sqrt{2}e^{3/2}\tan(c+dx)+e}d\sqrt{e\tan(c+dx)}}{2\sqrt{2}\sqrt{e}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{e}+\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{e^2\tan^2(c+dx)+\sqrt{2}e^{3/2}\tan(c+dx)+e}d\sqrt{e\tan(c+dx)}}{2\sqrt{2}\sqrt{e}}}{2e}+\frac{\arctan\left(\sqrt{2}\sqrt{e}\tan(c+dx)+1\right)}{\sqrt{2}\sqrt{e}}\right.}{d}\right.$$

$$b^2$$

$$\downarrow 25$$

$$\frac{e^2\left(a^2-b^2\right)\int \frac{1}{\sqrt{-e\cot\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{b^2} -$$

$$e^2\left(\frac{2ae\left(\frac{\int \frac{\sqrt{2}\sqrt{e}-2\sqrt{e\tan(c+dx)}}{e^2\tan^2(c+dx)-\sqrt{2}e^{3/2}\tan(c+dx)+e}d\sqrt{e\tan(c+dx)}}{2\sqrt{2}\sqrt{e}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{e}+\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{e^2\tan^2(c+dx)+\sqrt{2}e^{3/2}\tan(c+dx)+e}d\sqrt{e\tan(c+dx)}}{2\sqrt{2}\sqrt{e}}}{2e}+\frac{\arctan\left(\sqrt{2}\sqrt{e}\tan(c+dx)+1\right)}{\sqrt{2}\sqrt{e}}-\frac{\text{arc}}{2e}\right.}{d}\right.$$

$$b^2$$

$$\downarrow 27$$

$$\frac{e^2\left(a^2-b^2\right)\int \frac{1}{\sqrt{-e\cot\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{b^2} -$$

$$e^2\left(\frac{2ae\left(\frac{\int \frac{\sqrt{2}\sqrt{e}-2\sqrt{e\tan(c+dx)}}{e^2\tan^2(c+dx)-\sqrt{2}e^{3/2}\tan(c+dx)+e}d\sqrt{e\tan(c+dx)}}{2\sqrt{2}\sqrt{e}}+\frac{\int \frac{\sqrt{e}+\sqrt{2}\sqrt{e\tan(c+dx)}}{e^2\tan^2(c+dx)+\sqrt{2}e^{3/2}\tan(c+dx)+e}d\sqrt{e\tan(c+dx)}}{2\sqrt{e}}}{2e}+\frac{\frac{\arctan\left(\sqrt{2}\sqrt{e}\tan(c+dx)+1\right)}{\sqrt{2}\sqrt{e}}-\frac{\text{arc}}{}}{2e}\right)}{d}\right)$$

$$\frac{}{b^2}$$

$$\downarrow 1103$$

$$\frac{e^2\left(a^2-b^2\right)\int \frac{1}{\sqrt{-e\cot\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{b^2} -$$

$$e^2\left(\frac{2ae\left(\frac{\frac{\arctan\left(\sqrt{2}\sqrt{e}\tan(c+dx)+1\right)}{\sqrt{2}\sqrt{e}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{e}\tan(c+dx)\right)}{\sqrt{2}\sqrt{e}}}{2e}+\frac{\frac{\log\left(\sqrt{2}e^{3/2}\tan(c+dx)+e^2\tan^2(c+dx)+e\right)}{2\sqrt{2}\sqrt{e}}-\frac{\log\left(-\sqrt{2}e^{3/2}\tan(c+dx)+e^2\tan^2(c+dx)+\right.}{2\sqrt{2}\sqrt{e}}}{2e}\right)}{d}\right)$$

$$\frac{}{b^2}$$

$$\downarrow 4380$$

$$\frac{e^2\left(a^2-b^2\right)\left(\frac{\int \frac{1}{\sqrt{e\tan(c+dx)}}dx}{a}-\frac{b\int \frac{1}{(b+a\cos(c+dx))\sqrt{e\tan(c+dx)}}dx}{a}\right)}{b^2} -$$

$$e^2\left(\frac{2ae\left(\frac{\frac{\arctan\left(\sqrt{2}\sqrt{e}\tan(c+dx)+1\right)}{\sqrt{2}\sqrt{e}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{e}\tan(c+dx)\right)}{\sqrt{2}\sqrt{e}}}{2e}+\frac{\frac{\log\left(\sqrt{2}e^{3/2}\tan(c+dx)+e^2\tan^2(c+dx)+e\right)}{2\sqrt{2}\sqrt{e}}-\frac{\log\left(-\sqrt{2}e^{3/2}\tan(c+dx)+e^2\tan^2(c+dx)+\right.}{2\sqrt{2}\sqrt{e}}}{2e}\right)}{d}\right)$$

$$\frac{}{b^2}$$

$$\downarrow 3042$$

$$\frac{e^2\left(a^2-b^2\right)\left(\frac{\int \frac{1}{\sqrt{e\tan(c+dx)}}dx}{a}-\frac{b\int \frac{1}{\sqrt{-e\cot\left(c+dx+\frac{\pi}{2}\right)}\left(b+a\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a}\right)}{b^2} -$$

$$e^2\left(\frac{2ae\left(\frac{\frac{\arctan\left(\sqrt{2}\sqrt{e}\tan(c+dx)+1\right)}{\sqrt{2}\sqrt{e}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{e}\tan(c+dx)\right)}{\sqrt{2}\sqrt{e}}}{2e}+\frac{\frac{\log\left(\sqrt{2}e^{3/2}\tan(c+dx)+e^2\tan^2(c+dx)+e\right)}{2\sqrt{2}\sqrt{e}}-\frac{\log\left(-\sqrt{2}e^{3/2}\tan(c+dx)+e^2\tan^2(c+dx)+\right.}{2\sqrt{2}\sqrt{e}}}{2e}\right)}{d}\right)$$

$$\frac{}{b^2}$$

3.313. $\int \frac{(e\tan(c+dx))^{3/2}}{a+b\sec(c+dx)}dx$

$$\downarrow 3212$$

$$\frac{e^2\left(a^2-b^2\right)\left(\frac{\int \frac{1}{\sqrt{e\tan(c+dx)}}dx}{a}-\frac{b\int \frac{\sqrt{e\cot(c+dx)}}{b+a\cos(c+dx)}dx}{a\sqrt{e\tan(c+dx)}\sqrt{e\cot(c+dx)}}\right)}{b^2}-$$

$$e^2\left(\frac{2ae\left(\frac{\frac{\arctan\left(\sqrt{2}\sqrt{e}\tan(c+dx)+1\right)}{\sqrt{2}\sqrt{e}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{e}\tan(c+dx)\right)}{\sqrt{2}\sqrt{e}}}{2e}+\frac{\frac{\log\left(\sqrt{2}e^{3/2}\tan(c+dx)+e^2\tan^2(c+dx)+e\right)}{2\sqrt{2}\sqrt{e}}-\frac{\log\left(-\sqrt{2}e^{3/2}\tan(c+dx)+e^2\tan^2(c+dx)+\right.}{2\sqrt{2}\sqrt{e}}}{2e}\right)}{d}\right.$$

$$\left.\frac{}{}\right)$$

$$b^2$$

$$\downarrow 3042$$

$$\frac{e^2\left(a^2-b^2\right)\left(\frac{\int \frac{1}{\sqrt{e\tan(c+dx)}}dx}{a}-\frac{b\int \frac{\sqrt{-e\tan\left(c+dx-\frac{\pi}{2}\right)}}{b-a\sin\left(c+dx-\frac{\pi}{2}\right)}dx}{a\sqrt{e\tan(c+dx)}\sqrt{e\cot(c+dx)}}\right)}{b^2}-$$

$$e^2\left(\frac{2ae\left(\frac{\frac{\arctan\left(\sqrt{2}\sqrt{e}\tan(c+dx)+1\right)}{\sqrt{2}\sqrt{e}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{e}\tan(c+dx)\right)}{\sqrt{2}\sqrt{e}}}{2e}+\frac{\frac{\log\left(\sqrt{2}e^{3/2}\tan(c+dx)+e^2\tan^2(c+dx)+e\right)}{2\sqrt{2}\sqrt{e}}-\frac{\log\left(-\sqrt{2}e^{3/2}\tan(c+dx)+e^2\tan^2(c+dx)+\right.}{2\sqrt{2}\sqrt{e}}}{2e}\right)}{d}\right.$$

$$\left.\frac{}{}\right)$$

$$b^2$$

$$\downarrow 3208$$

$$\frac{e^2\left(a^2-b^2\right)\left(\frac{\int \frac{1}{\sqrt{e\tan(c+dx)}}dx}{a}-\frac{b\sqrt{\sin(c+dx)}\int \frac{\sqrt{-\cos(c+dx)}}{(b+a\cos(c+dx))\sqrt{\sin(c+dx)}}dx}{a\sqrt{-\cos(c+dx)}\sqrt{e\tan(c+dx)}}\right)}{b^2}-$$

$$e^2\left(\frac{2ae\left(\frac{\frac{\arctan\left(\sqrt{2}\sqrt{e}\tan(c+dx)+1\right)}{\sqrt{2}\sqrt{e}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{e}\tan(c+dx)\right)}{\sqrt{2}\sqrt{e}}}{2e}+\frac{\frac{\log\left(\sqrt{2}e^{3/2}\tan(c+dx)+e^2\tan^2(c+dx)+e\right)}{2\sqrt{2}\sqrt{e}}-\frac{\log\left(-\sqrt{2}e^{3/2}\tan(c+dx)+e^2\tan^2(c+dx)+\right.}{2\sqrt{2}\sqrt{e}}}{2e}\right)}{d}\right.$$

$$\left.\frac{}{}\right)$$

$$b^2$$

$$\downarrow 3042$$

$$\frac{e^2\left(a^2-b^2\right)\left(\frac{\int \frac{1}{\sqrt{e\tan(c+dx)}}dx}{a}-\frac{b\sqrt{\sin(c+dx)}\int \frac{\sqrt{\sin\left(c+dx-\frac{\pi}{2}\right)}}{\sqrt{\cos\left(c+dx-\frac{\pi}{2}\right)}\left(b-a\sin\left(c+dx-\frac{\pi}{2}\right)\right)}dx}{a\sqrt{-\cos(c+dx)}\sqrt{e\tan(c+dx)}}\right)}{b^2}-$$

$$e^2\left(\frac{2ae\left(\frac{\frac{\arctan\left(\sqrt{2}\sqrt{e}\tan(c+dx)+1\right)}{\sqrt{2}\sqrt{e}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{e}\tan(c+dx)\right)}{\sqrt{2}\sqrt{e}}}{2e}+\frac{\frac{\log\left(\sqrt{2}e^{3/2}\tan(c+dx)+e^2\tan^2(c+dx)+e\right)}{2\sqrt{2}\sqrt{e}}-\frac{\log\left(-\sqrt{2}e^{3/2}\tan(c+dx)+e^2\tan^2(c+dx)+\right.}{2\sqrt{2}\sqrt{e}}}{2e}\right)}{d}\right.$$

$$\left.\frac{}{b^2}\right)$$

$$\downarrow 3386$$

$$e^2\left(a^2-b^2\right)\left(\frac{\int \frac{1}{\sqrt{e\tan(c+dx)}}dx}{a}-\frac{b\sqrt{\sin(c+dx)}\left(\frac{2\sqrt{2}\left(1-\frac{a}{\sqrt{a^2-b^2}}\right)\int -\frac{1}{\sqrt{1-\frac{\cos^2(c+dx)}{(\sin(c+dx)+1)^2}}\left(a-\sqrt{a^2-b^2}+\frac{b\cos(c+dx)}{\sin(c+dx)+1}\right)}d\frac{\sqrt{-\cos(c+dx)}}{\sqrt{\sin(c+dx)+1}}}{d}+\right.}{a\sqrt{-\cos(c+dx)}\sqrt{e\tan(c+}}\right.$$

$$\left.\frac{}{b^2}\right)$$

$$e^2\left(\frac{2ae\left(\frac{\frac{\arctan\left(\sqrt{2}\sqrt{e}\tan(c+dx)+1\right)}{\sqrt{2}\sqrt{e}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{e}\tan(c+dx)\right)}{\sqrt{2}\sqrt{e}}}{2e}+\frac{\frac{\log\left(\sqrt{2}e^{3/2}\tan(c+dx)+e^2\tan^2(c+dx)+e\right)}{2\sqrt{2}\sqrt{e}}-\frac{\log\left(-\sqrt{2}e^{3/2}\tan(c+dx)+e^2\tan^2(c+dx)+\right.}{2\sqrt{2}\sqrt{e}}}{2e}\right)}{d}\right.$$

$$\left.\frac{}{b^2}\right)$$

$$\downarrow 25$$

$$e^2\left(a^2-b^2\right)\left(\frac{\int \frac{1}{\sqrt{e\tan(c+dx)}}dx}{a}-\frac{b\sqrt{\sin(c+dx)}\left(-\frac{2\sqrt{2}\left(1-\frac{a}{\sqrt{a^2-b^2}}\right)\int \frac{1}{\sqrt{1-\frac{\cos^2(c+dx)}{(\sin(c+dx)+1)^2}}\left(a-\sqrt{a^2-b^2}+\frac{b\cos(c+dx)}{\sin(c+dx)+1}\right)}d\frac{\sqrt{-\cos(c+dx)}}{\sqrt{\sin(c+dx)+1}}}{d}-\frac{2\sqrt{2}(}{}\right.}{a\sqrt{-\cos(c+dx)}\sqrt{e\tan(c+d}}\right.$$

$$\left.\frac{}{b^2}\right)$$

$$\frac{e^2\left(2ae\left(\frac{\arctan\left(\sqrt{2}\sqrt{e}\tan(c+dx)+1\right)}{\sqrt{2}\sqrt{e}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{e}\tan(c+dx)\right)}{\sqrt{2}\sqrt{e}}\right)}{2e}+\frac{\log\left(\sqrt{2}e^{3/2}\tan(c+dx)+e^2\tan^2(c+dx)+e\right)}{2\sqrt{2}\sqrt{e}}-\frac{\log\left(-\sqrt{2}e^{3/2}\tan(c+dx)+e^2\tan^2(c+dx)+\right.}{2\sqrt{2}\sqrt{e}}$$

$$\left.\frac{}{d}\right)$$

$$b^2$$

$$\downarrow 1537$$

$$e^2\left(a^2-b^2\right)\left(\frac{\int \frac{1}{\sqrt{e\tan(c+dx)}}dx}{a}-\frac{b\sqrt{\sin(c+dx)}\left(-\frac{2\sqrt{2}\left(1-\frac{a}{\sqrt{a^2-b^2}}\right)\int \frac{1}{\sqrt{1-\frac{\cos(c+dx)}{\sin(c+dx)+1}}\sqrt{\frac{\cos(c+dx)}{\sin(c+dx)+1}+1}\left(a-\sqrt{a^2-b^2}+\frac{b\cos(c+dx)}{\sin(c+dx)+1}\right)}d\frac{\sqrt{-\cos}}{\sqrt{\sin(c-}}\right.}{d}\right.}{a\sqrt{-\cos(c+dx}}\right.$$

$$\left.\frac{}{b^2}\right)$$

$$\frac{e^2\left(2ae\left(\frac{\arctan\left(\sqrt{2}\sqrt{e}\tan(c+dx)+1\right)}{\sqrt{2}\sqrt{e}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{e}\tan(c+dx)\right)}{\sqrt{2}\sqrt{e}}\right)}{2e}+\frac{\log\left(\sqrt{2}e^{3/2}\tan(c+dx)+e^2\tan^2(c+dx)+e\right)}{2\sqrt{2}\sqrt{e}}-\frac{\log\left(-\sqrt{2}e^{3/2}\tan(c+dx)+e^2\tan^2(c+dx)+\right.}{2\sqrt{2}\sqrt{e}}$$

$$\left.\frac{}{d}\right)$$

$$b^2$$

$$\downarrow 412$$

$$\frac{e^2\left(a^2-b^2\right)\left(\frac{\int \frac{1}{\sqrt{e\tan(c+dx)}}dx}{a}-\frac{b\sqrt{\sin(c+dx)}\left(-\frac{2\sqrt{2}\left(1-\frac{a}{\sqrt{a^2-b^2}}\right)\text{EllipticPi}\left(\frac{b}{a-\sqrt{a^2-b^2}},\arcsin\left(\frac{\sqrt{-\cos(c+dx)}}{\sqrt{\sin(c+dx)+1}}\right),-1\right)}{d\left(a-\sqrt{a^2-b^2}\right)}-\frac{2\sqrt{2}\left(\frac{a}{\sqrt{a^2-b^2}}+1\right)\text{E}}{a\sqrt{-\cos(c+dx)}\sqrt{e\tan(c+dx)}}\right.}{b^2}}{e^2\left(\frac{2ae\left(\frac{\frac{\arctan\left(\sqrt{2}\sqrt{e}\tan(c+dx)+1\right)}{\sqrt{2}\sqrt{e}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{e}\tan(c+dx)\right)}{\sqrt{2}\sqrt{e}}}{2e}+\frac{\frac{\log\left(\sqrt{2}e^{3/2}\tan(c+dx)+e^2\tan^2(c+dx)+e\right)}{2\sqrt{2}\sqrt{e}}-\frac{\log\left(-\sqrt{2}e^{3/2}\tan(c+dx)+e^2\tan^2(c+dx)+\right.}{2\sqrt{2}\sqrt{e}}}{2e}\right.}{d}}{b^2}}$$

input
```
Int[(e*Tan[c + d*x])^(3/2)/(a + b*Sec[c + d*x]),x]
```

output
```
$Aborted
```

3.313.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 1537
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[(-a)*c, 2]}, Simp[Sqrt[-c]   Int[1/((d + e*x^2)*Sqrt[q + c*x^2]*Sqr
t[q - c*x^2]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] &
& GtQ[a, 0] && LtQ[c, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3053
```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Simp[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]])   Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f
}, x]
```

rule 3094
```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]], x_Symbol]
:> Simp[Sqrt[Sin[e + f*x]]/(Sqrt[Cos[e + f*x]]*Sqrt[b*Tan[e + f*x]])   Int[
1/(Sqrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]]), x], x] /; FreeQ[{b, e, f}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3208
```
Int[Sqrt[(g_.)*tan[(e_.) + (f_.)*(x_)]]/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_
)]), x_Symbol] :> Simp[Sqrt[Cos[e + f*x]]*(Sqrt[g*Tan[e + f*x]]/Sqrt[Sin[e
+ f*x]])   Int[Sqrt[Sin[e + f*x]]/(Sqrt[Cos[e + f*x]]*(a + b*Sin[e + f*x]))
, x], x] /; FreeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

rule 3212
```
Int[(cot[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Simp[g^(2*IntPart[p])*(g*Cot[e + f*x])^FracPart[p]
*(g*Tan[e + f*x])^FracPart[p]   Int[(a + b*Sin[e + f*x])^m/(g*Tan[e + f*x])
^p, x], x] /; FreeQ[{a, b, e, f, g, m, p}, x] &&  !IntegerQ[p]
```

rule 3386
```
Int[Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]/(Sqrt[cos[(e_.) + (f_.)*(x_)]]*((a_
) + (b_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> With[{q = Rt[-a^2 + b^2,
2]}, Simp[2*Sqrt[2]*d*((b + q)/(f*q))   Subst[Int[1/((d*(b + q) + a*x^2)*Sq
rt[1 - x^4/d^2]), x], x, Sqrt[d*Sin[e + f*x]]/Sqrt[1 + Cos[e + f*x]]], x] -
 Simp[2*Sqrt[2]*d*((b - q)/(f*q))   Subst[Int[1/((d*(b - q) + a*x^2)*Sqrt[1
 - x^4/d^2]), x], x, Sqrt[d*Sin[e + f*x]]/Sqrt[1 + Cos[e + f*x]]], x]] /; F
reeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

rule 4372
```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(e*Cot[c + d*x])^m, x], x] + Simp[b   Int[
(e*Cot[c + d*x])^m*Csc[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, m}, x]
```

rule 4379
```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)/(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[-e^2/b^2   Int[(e*Cot[c + d*x])^(m - 2)*(a - b*Csc[
c + d*x]), x], x] + Simp[e^2*((a^2 - b^2)/b^2)   Int[(e*Cot[c + d*x])^(m -
2)/(a + b*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b
^2, 0] && IGtQ[m - 1/2, 0]
```

rule 4380
```
Int[1/(Sqrt[cot[(c_.) + (d_.)*(x_)]*(e_.)]*(csc[(c_.) + (d_.)*(x_)]*(b_.) +
 (a_))), x_Symbol] :> Simp[1/a   Int[1/Sqrt[e*Cot[c + d*x]], x], x] - Simp[
b/a   Int[1/(Sqrt[e*Cot[c + d*x]]*(b + a*Sin[c + d*x])), x], x] /; FreeQ[{a
, b, c, d, e}, x] && NeQ[a^2 - b^2, 0]
```

3.313.4 Maple [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1650 vs. $2(636) = 1272$.

Time = 3.36 (sec) , antiderivative size = 1651, normalized size of antiderivative = 2.23

method	result	size
default	Expression too large to display	1651

input
```
int((e*tan(d*x+c))^(3/2)/(a+b*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/2/d*2^(1/2)/((a^2-b^2)^(1/2)-a+b)/((a^2-b^2)^(1/2)+a-b)/(a^2-b^2)^(1/2)/
a*(2*I*EllipticPi((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2-1/2*I,1/2*2^(1/2))*(
a^2-b^2)^(1/2)*a*b-2*I*EllipticPi((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2+1/2*
I,1/2*2^(1/2))*(a^2-b^2)^(1/2)*a*b-EllipticPi((csc(d*x+c)-cot(d*x+c)+1)^(1
/2),1/2-1/2*I,1/2*2^(1/2))*(a^2-b^2)^(1/2)*b^2-EllipticPi((csc(d*x+c)-cot(
d*x+c)+1)^(1/2),1/2+1/2*I,1/2*2^(1/2))*(a^2-b^2)^(1/2)*a^2-EllipticPi((csc
(d*x+c)-cot(d*x+c)+1)^(1/2),1/2+1/2*I,1/2*2^(1/2))*(a^2-b^2)^(1/2)*b^2-2*E
llipticPi((csc(d*x+c)-cot(d*x+c)+1)^(1/2),(a-b)/(a-b+((a-b)*(a+b))^(1/2)),
1/2*2^(1/2))*(a^2-b^2)^(1/2)*a^2+2*EllipticPi((csc(d*x+c)-cot(d*x+c)+1)^(1
/2),(a-b)/(a-b+((a-b)*(a+b))^(1/2)),1/2*2^(1/2))*(a^2-b^2)^(1/2)*b^2-2*Ell
ipticPi((csc(d*x+c)-cot(d*x+c)+1)^(1/2),(a-b)/(a-b+((a-b)*(a+b))^(1/2)),1/
2*2^(1/2))*a^2*b-2*EllipticPi((csc(d*x+c)-cot(d*x+c)+1)^(1/2),(a-b)/(a-b+(
(a-b)*(a+b))^(1/2)),1/2*2^(1/2))*a*b^2-2*EllipticPi((csc(d*x+c)-cot(d*x+c)
+1)^(1/2),-(a-b)/(-a+b+((a-b)*(a+b))^(1/2)),1/2*2^(1/2))*(a^2-b^2)^(1/2)*a
^2+2*EllipticPi((csc(d*x+c)-cot(d*x+c)+1)^(1/2),-(a-b)/(-a+b+((a-b)*(a+b))
^(1/2)),1/2*2^(1/2))*(a^2-b^2)^(1/2)*b^2+2*EllipticPi((csc(d*x+c)-cot(d*x+
c)+1)^(1/2),-(a-b)/(-a+b+((a-b)*(a+b))^(1/2)),1/2*2^(1/2))*a^2*b+2*Ellipti
cPi((csc(d*x+c)-cot(d*x+c)+1)^(1/2),-(a-b)/(-a+b+((a-b)*(a+b))^(1/2)),1/2*
2^(1/2))*a*b^2-I*EllipticPi((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2+1/2*I,1/2*
2^(1/2))*(a^2-b^2)^(3/2)+I*EllipticPi((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1...
```

3.313.5 Fricas [F(-1)]

Timed out.

$$\int \frac{(e\tan(c+dx))^{3/2}}{a+b\sec(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((e*tan(d*x+c))^(3/2)/(a+b*sec(d*x+c)),x, algorithm="fricas")
```

output
```
Timed out
```

3.313.6 Sympy [F]

$$\int \frac{(e\tan(c+dx))^{3/2}}{a+b\sec(c+dx)}\,dx = \int \frac{(e\tan(c+dx))^{\frac{3}{2}}}{a+b\sec(c+dx)}\,dx$$

input
```
integrate((e*tan(d*x+c))**(3/2)/(a+b*sec(d*x+c)),x)
```

output
```
Integral((e*tan(c + d*x))**(3/2)/(a + b*sec(c + d*x)), x)
```

3.313.7 Maxima [F]

$$\int \frac{(e\tan(c+dx))^{3/2}}{a+b\sec(c+dx)}\,dx = \int \frac{(e\tan(dx+c))^{\frac{3}{2}}}{b\sec(dx+c)+a}\,dx$$

input
```
integrate((e*tan(d*x+c))^(3/2)/(a+b*sec(d*x+c)),x, algorithm="maxima")
```

output
```
integrate((e*tan(d*x + c))^(3/2)/(b*sec(d*x + c) + a), x)
```

3.313.8 Giac [F]

$$\int \frac{(e\tan(c+dx))^{3/2}}{a+b\sec(c+dx)}\,dx = \int \frac{(e\tan(dx+c))^{\frac{3}{2}}}{b\sec(dx+c)+a}\,dx$$

input
```
integrate((e*tan(d*x+c))^(3/2)/(a+b*sec(d*x+c)),x, algorithm="giac")
```

output
```
integrate((e*tan(d*x + c))^(3/2)/(b*sec(d*x + c) + a), x)
```

3.313.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(e\tan(c+dx))^{3/2}}{a+b\sec(c+dx)}\,dx = \int \frac{\cos(c+d\,x)\,(e\tan(c+d\,x))^{3/2}}{b+a\,\cos(c+d\,x)}\,dx$$

input
```
int((e*tan(c + d*x))^(3/2)/(a + b/cos(c + d*x)),x)
```

output
```
int((cos(c + d*x)*(e*tan(c + d*x))^(3/2))/(b + a*cos(c + d*x)), x)
```

3.314 $\int \frac{\sqrt{e\tan(c+dx)}}{a+b\sec(c+dx)}\,dx$

3.314.1 Optimal result 2289
3.314.2 Mathematica [C] (warning: unable to verify) 2290
3.314.3 Rubi [A] (warning: unable to verify) 2291
3.314.4 Maple [B] (warning: unable to verify) 2298
3.314.5 Fricas [F(-1)] 2299
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3.314.7 Maxima [F] 2300
3.314.8 Giac [F] 2300
3.314.9 Mupad [F(-1)] 2301

3.314.1 Optimal result

Integrand size = 25, antiderivative size = 415

$$
\begin{aligned}
&\int \frac{\sqrt{e\tan(c+dx)}}{a+b\sec(c+dx)}\,dx \\
&= -\frac{\sqrt{e}\arctan\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}ad}+\frac{\sqrt{e}\arctan\left(1+\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}ad} \\
&\quad+\frac{\sqrt{e}\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}ad} \\
&\quad-\frac{\sqrt{e}\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)+\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}ad} \\
&\quad+\frac{2\sqrt{2}b\sqrt{\cos(c+dx)}\,\text{EllipticPi}\left(-\frac{\sqrt{a-b}}{\sqrt{a+b}},\arcsin\left(\frac{\sqrt{\sin(c+dx)}}{\sqrt{1+\cos(c+dx)}}\right),-1\right)\sqrt{e\tan(c+dx)}}{a\sqrt{a-b}\sqrt{a+b}d\sqrt{\sin(c+dx)}} \\
&\quad-\frac{2\sqrt{2}b\sqrt{\cos(c+dx)}\,\text{EllipticPi}\left(\frac{\sqrt{a-b}}{\sqrt{a+b}},\arcsin\left(\frac{\sqrt{\sin(c+dx)}}{\sqrt{1+\cos(c+dx)}}\right),-1\right)\sqrt{e\tan(c+dx)}}{a\sqrt{a-b}\sqrt{a+b}d\sqrt{\sin(c+dx)}}
\end{aligned}
$$

output
```
-1/2*arctan(1-2^(1/2)*(e*tan(d*x+c))^(1/2)/e^(1/2))*e^(1/2)/a/d*2^(1/2)+1/
2*arctan(1+2^(1/2)*(e*tan(d*x+c))^(1/2)/e^(1/2))*e^(1/2)/a/d*2^(1/2)+1/4*l
n(e^(1/2)-2^(1/2)*(e*tan(d*x+c))^(1/2)+e^(1/2)*tan(d*x+c))*e^(1/2)/a/d*2^(
1/2)-1/4*ln(e^(1/2)+2^(1/2)*(e*tan(d*x+c))^(1/2)+e^(1/2)*tan(d*x+c))*e^(1/
2)/a/d*2^(1/2)+2*b*EllipticPi(sin(d*x+c)^(1/2)/(1+cos(d*x+c))^(1/2),-(a-b)
^(1/2)/(a+b)^(1/2),I)*2^(1/2)*cos(d*x+c)^(1/2)*(e*tan(d*x+c))^(1/2)/a/d/(a
-b)^(1/2)/(a+b)^(1/2)/sin(d*x+c)^(1/2)-2*b*EllipticPi(sin(d*x+c)^(1/2)/(1+
cos(d*x+c))^(1/2),(a-b)^(1/2)/(a+b)^(1/2),I)*2^(1/2)*cos(d*x+c)^(1/2)*(e*t
an(d*x+c))^(1/2)/a/d/(a-b)^(1/2)/(a+b)^(1/2)/sin(d*x+c)^(1/2)
```

3.314.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 5.98 (sec) , antiderivative size = 646, normalized size of antiderivative = 1.56

$$\int \frac{\sqrt{e\tan(c+dx)}}{a+b\sec(c+dx)}\,dx =$$

$$-\frac{\cos(c+dx)\left(a+b\sqrt{\sec^2(c+dx)}\right)\sqrt{e\tan(c+dx)}\left(6\sqrt{2}(a^2-b^2)\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)-6\sqrt{}\right.}{}$$

input
```
Integrate[Sqrt[e*Tan[c + d*x]]/(a + b*Sec[c + d*x]),x]
```

output
```
-1/12*(Cos[c + d*x]*(a + b*Sqrt[Sec[c + d*x]^2])*Sqrt[e*Tan[c + d*x]]*(6*S
qrt[2]*(a^2 - b^2)*ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]] - 6*Sqrt[2]*a^2*
ArcTan[1 + Sqrt[2]*Sqrt[Tan[c + d*x]]] + 6*Sqrt[2]*b^2*ArcTan[1 + Sqrt[2]*
Sqrt[Tan[c + d*x]]] - (6 + 6*I)*Sqrt[b]*(a^2 - b^2)^(3/4)*ArcTan[1 - ((1 +
 I)*Sqrt[b]*Sqrt[Tan[c + d*x]])/(a^2 - b^2)^(1/4)] + (6 + 6*I)*Sqrt[b]*(a^
2 - b^2)^(3/4)*ArcTan[1 + ((1 + I)*Sqrt[b]*Sqrt[Tan[c + d*x]])/(a^2 - b^2)
^(1/4)] - 3*Sqrt[2]*a^2*Log[1 - Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]]
 + 3*Sqrt[2]*b^2*Log[1 - Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]] + 3*Sq
rt[2]*a^2*Log[1 + Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]] - 3*Sqrt[2]*b
^2*Log[1 + Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]] + (3 + 3*I)*Sqrt[b]*
(a^2 - b^2)^(3/4)*Log[Sqrt[a^2 - b^2] - (1 + I)*Sqrt[b]*(a^2 - b^2)^(1/4)*
Sqrt[Tan[c + d*x]] + I*b*Tan[c + d*x]] - (3 + 3*I)*Sqrt[b]*(a^2 - b^2)^(3/
4)*Log[Sqrt[a^2 - b^2] + (1 + I)*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Tan[c + d*
x]] + I*b*Tan[c + d*x]] + 8*a*b*AppellF1[3/4, 1/2, 1, 7/4, -Tan[c + d*x]^2
, (b^2*Tan[c + d*x]^2)/(a^2 - b^2)]*Tan[c + d*x]^(3/2)))/(a*(a^2 - b^2)*d*
(b + a*Cos[c + d*x])*Sqrt[Tan[c + d*x]])
```

3.314.3 Rubi [A] (warning: unable to verify)

Time = 1.35 (sec) , antiderivative size = 352, normalized size of antiderivative = 0.85, number of steps used = 24, number of rules used = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.920$, Rules used = {3042, 4378, 3042, 3212, 3042, 3209, 3042, 3385, 3042, 3384, 993, 1537, 412, 3957, 266, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{e\tan(c+dx)}}{a+b\sec(c+dx)}\,dx$$

$$\downarrow 3042$$

$$\int \frac{\sqrt{-e\cot\left(c+dx+\frac{\pi}{2}\right)}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\downarrow 4378$$

$$\frac{\int \sqrt{e\tan(c+dx)}dx}{a} - \frac{b\int \frac{\sqrt{e\tan(c+dx)}}{b+a\cos(c+dx)}dx}{a}$$

$$\downarrow 3042$$

$$\frac{\int \sqrt{e\tan(c+dx)}dx}{a} - \frac{b\int \frac{\sqrt{-e\cot\left(c+dx+\frac{\pi}{2}\right)}}{b+a\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{a}$$

$$\downarrow 3212$$

$$\frac{\int \sqrt{e\tan(c+dx)}dx}{a} - \frac{b\sqrt{e\tan(c+dx)}\sqrt{e\cot(c+dx)}\int \frac{1}{(b+a\cos(c+dx))\sqrt{e\cot(c+dx)}}dx}{a}$$

$$\downarrow 3042$$

$$\frac{\int \sqrt{e\tan(c+dx)}dx}{a} - \frac{b\sqrt{e\tan(c+dx)}\sqrt{e\cot(c+dx)}\int \frac{1}{(b-a\sin(c+dx-\frac{\pi}{2}))\sqrt{-e\tan(c+dx-\frac{\pi}{2})}}dx}{a}$$

$$\downarrow 3209$$

$$\frac{\int \sqrt{e\tan(c+dx)}dx}{a} - \frac{b\sqrt{-\cos(c+dx)}\sqrt{e\tan(c+dx)}\int \frac{\sqrt{\sin(c+dx)}}{\sqrt{-\cos(c+dx)}(b+a\cos(c+dx))}dx}{a\sqrt{\sin(c+dx)}}$$

$$\downarrow 3042$$

$$\frac{\int \sqrt{e\tan(c+dx)}dx}{a} - \frac{b\sqrt{-\cos(c+dx)}\sqrt{e\tan(c+dx)}\int \frac{\sqrt{-\cos\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{-\sin\left(c+dx+\frac{\pi}{2}\right)}\left(b+a\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a\sqrt{\sin(c+dx)}}$$

$$\downarrow 3385$$

$$\frac{\int \sqrt{e\tan(c+dx)}dx}{a} - \frac{b\sqrt{\cos(c+dx)}\sqrt{e\tan(c+dx)}\int \frac{\sqrt{\sin(c+dx)}}{\sqrt{\cos(c+dx)}(b+a\cos(c+dx))}dx}{a\sqrt{\sin(c+dx)}}$$

$$\downarrow 3042$$

$$\frac{\int \sqrt{e\tan(c+dx)}dx}{a} - \frac{b\sqrt{\cos(c+dx)}\sqrt{e\tan(c+dx)}\int \frac{\sqrt{-\cos\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(b+a\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a\sqrt{\sin(c+dx)}}$$

$$\downarrow 3384$$

$$\frac{\int \sqrt{e\tan(c+dx)}dx}{a} - \frac{4\sqrt{2}b\sqrt{\cos(c+dx)}\sqrt{e\tan(c+dx)}\int \frac{\sin(c+dx)}{(\cos(c+dx)+1)\sqrt{1-\frac{\sin^2(c+dx)}{(\cos(c+dx)+1)^2}}\left(-\frac{(a-b)\sin^2(c+dx)}{(\cos(c+dx)+1)^2}+a+b\right)}d\frac{\sqrt{\sin(c+dx)}}{\sqrt{\cos(c+dx)+1}}}{ad\sqrt{\sin(c+dx)}}$$

$$\downarrow 993$$

$$\frac{\int \sqrt{e\tan(c+dx)}dx}{a} - \frac{4\sqrt{2}b\sqrt{\cos(c+dx)}\sqrt{e\tan(c+dx)}\left(\frac{\int \frac{1}{\left(\sqrt{a+b}-\frac{\sqrt{a-b}\sin(c+dx)}{\cos(c+dx)+1}\right)\sqrt{1-\frac{\sin^2(c+dx)}{(\cos(c+dx)+1)^2}}}d\frac{\sqrt{\sin(c+dx)}}{\sqrt{\cos(c+dx)+1}}}{2\sqrt{a-b}} - \frac{\int \frac{1}{\left(\frac{\sqrt{a-b}\sin(c+dx)}{\cos(c+dx)+1}+\sqrt{a+b}\right)\sqrt{1-}}}{2\sqrt{a}}\right.}{ad\sqrt{\sin(c+dx)}}$$

$$\downarrow 1537$$

$$\frac{\int \sqrt{e\tan(c+dx)}dx}{a} - \frac{4\sqrt{2}b\sqrt{\cos(c+dx)}\sqrt{e\tan(c+dx)}\left(\frac{\int \frac{1}{\sqrt{1-\frac{\sin(c+dx)}{\cos(c+dx)+1}}\sqrt{\frac{\sin(c+dx)}{\cos(c+dx)+1}+1}\left(\sqrt{a+b}-\frac{\sqrt{a-b}\sin(c+dx)}{\cos(c+dx)+1}\right)}d\frac{\sqrt{\sin(c+dx)}}{\sqrt{\cos(c+dx)+1}}}{2\sqrt{a-b}} - \frac{\int \frac{1}{\sqrt{1-\frac{\sin(c+dx)}{\cos(c+dx)+}}}}{}\right.}{ad\sqrt{\sin(c+dx)}}$$

$$\downarrow 412$$

$$\frac{\frac{\int \sqrt{e\tan(c+dx)}dx}{a} -}{4\sqrt{2}b\sqrt{\cos(c+dx)}\sqrt{e\tan(c+dx)}\left(\frac{\text{EllipticPi}\left(\frac{\sqrt{a-b}}{\sqrt{a+b}},\arcsin\left(\frac{\sqrt{\sin(c+dx)}}{\sqrt{\cos(c+dx)+1}}\right),-1\right)}{2\sqrt{a-b}\sqrt{a+b}} - \frac{\text{EllipticPi}\left(-\frac{\sqrt{a-b}}{\sqrt{a+b}},\arcsin\left(\frac{\sqrt{\sin(c+dx)}}{\sqrt{\cos(c+dx)+1}}\right),-1\right)}{2\sqrt{a-b}\sqrt{a+b}}\right.}$$
$$ad\sqrt{\sin(c+dx)}$$

$$\downarrow 3957$$

$$\frac{\frac{e\int \frac{\sqrt{e\tan(c+dx)}}{\tan^2(c+dx)e^2+e^2}d(e\tan(c+dx))}{ad} -}{4\sqrt{2}b\sqrt{\cos(c+dx)}\sqrt{e\tan(c+dx)}\left(\frac{\text{EllipticPi}\left(\frac{\sqrt{a-b}}{\sqrt{a+b}},\arcsin\left(\frac{\sqrt{\sin(c+dx)}}{\sqrt{\cos(c+dx)+1}}\right),-1\right)}{2\sqrt{a-b}\sqrt{a+b}} - \frac{\text{EllipticPi}\left(-\frac{\sqrt{a-b}}{\sqrt{a+b}},\arcsin\left(\frac{\sqrt{\sin(c+dx)}}{\sqrt{\cos(c+dx)+1}}\right),-1\right)}{2\sqrt{a-b}\sqrt{a+b}}\right.}$$
$$ad\sqrt{\sin(c+dx)}$$

$$\downarrow 266$$

$$\frac{\frac{2e\int \frac{e^2\tan^2(c+dx)}{e^4\tan^4(c+dx)+e^2}d\sqrt{e\tan(c+dx)}}{ad} -}{4\sqrt{2}b\sqrt{\cos(c+dx)}\sqrt{e\tan(c+dx)}\left(\frac{\text{EllipticPi}\left(\frac{\sqrt{a-b}}{\sqrt{a+b}},\arcsin\left(\frac{\sqrt{\sin(c+dx)}}{\sqrt{\cos(c+dx)+1}}\right),-1\right)}{2\sqrt{a-b}\sqrt{a+b}} - \frac{\text{EllipticPi}\left(-\frac{\sqrt{a-b}}{\sqrt{a+b}},\arcsin\left(\frac{\sqrt{\sin(c+dx)}}{\sqrt{\cos(c+dx)+1}}\right),-1\right)}{2\sqrt{a-b}\sqrt{a+b}}\right.}$$
$$ad\sqrt{\sin(c+dx)}$$

$$\downarrow 826$$

$$\frac{\frac{2e\left(\frac{1}{2}\int \frac{e^2\tan^2(c+dx)+e}{e^4\tan^4(c+dx)+e^2}d\sqrt{e\tan(c+dx)} - \frac{1}{2}\int \frac{e-e^2\tan^2(c+dx)}{e^4\tan^4(c+dx)+e^2}d\sqrt{e\tan(c+dx)}\right)}{ad} -}{4\sqrt{2}b\sqrt{\cos(c+dx)}\sqrt{e\tan(c+dx)}\left(\frac{\text{EllipticPi}\left(\frac{\sqrt{a-b}}{\sqrt{a+b}},\arcsin\left(\frac{\sqrt{\sin(c+dx)}}{\sqrt{\cos(c+dx)+1}}\right),-1\right)}{2\sqrt{a-b}\sqrt{a+b}} - \frac{\text{EllipticPi}\left(-\frac{\sqrt{a-b}}{\sqrt{a+b}},\arcsin\left(\frac{\sqrt{\sin(c+dx)}}{\sqrt{\cos(c+dx)+1}}\right),-1\right)}{2\sqrt{a-b}\sqrt{a+b}}\right.}$$
$$ad\sqrt{\sin(c+dx)}$$

$$\downarrow 1476$$

$$\frac{2e\left(\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{e^2\tan^2(c+dx)-\sqrt{2}e^{3/2}\tan(c+dx)+e}d\sqrt{e\tan(c+dx)} + \frac{1}{2}\int \frac{1}{e^2\tan^2(c+dx)+\sqrt{2}e^{3/2}\tan(c+dx)+e}d\sqrt{e\tan(c+dx)}\right) -\right.}{4\sqrt{2}b\sqrt{\cos(c+dx)}\sqrt{e\tan(c+dx)}\left(\frac{\text{EllipticPi}\left(\frac{\sqrt{a-b}}{\sqrt{a+b}},\arcsin\left(\frac{\sqrt{\sin(c+dx)}}{\sqrt{\cos(c+dx)+1}}\right),-1\right)}{2\sqrt{a-b}\sqrt{a+b}} - \frac{\text{EllipticPi}\left(-\frac{\sqrt{a-b}}{\sqrt{a+b}},\arcsin\left(\frac{\sqrt{\sin(c+dx)}}{\sqrt{\cos(c+dx)+1}}\right),-1\right)}{2\sqrt{a-b}\sqrt{a+b}}\right.}$$
$$ad\sqrt{\sin(c+dx)}$$

$$\downarrow 1082$$

$$\frac{2e\left(\frac{1}{2}\left(\frac{\int \frac{1}{-e^2\tan^2(c+dx)-1}d\left(1-\sqrt{2}\sqrt{e}\tan(c+dx)\right)}{\sqrt{2}\sqrt{e}}-\frac{\int \frac{1}{-e^2\tan^2(c+dx)-1}d\left(\sqrt{2}\sqrt{e}\tan(c+dx)+1\right)}{\sqrt{2}\sqrt{e}}\right)-\frac{1}{2}\int \frac{e-e^2\tan^2(c+dx)}{e^4\tan^4(c+dx)+e^2}d\sqrt{e\tan(c}}{4\sqrt{2}b\sqrt{\cos(c+dx)}\sqrt{e\tan(c+dx)}\left(\frac{\text{EllipticPi}\left(\frac{\sqrt{a-b}}{\sqrt{a+b}},\arcsin\left(\frac{\sqrt{\sin(c+dx)}}{\sqrt{\cos(c+dx)+1}}\right),-1\right)}{2\sqrt{a-b}\sqrt{a+b}}-\frac{\text{EllipticPi}\left(-\frac{\sqrt{a-b}}{\sqrt{a+b}},\arcsin\left(\frac{\sqrt{\sin(c+dx)}}{\sqrt{\cos(c+dx)+1}}\right),-1\right)}{2\sqrt{a-b}\sqrt{a+b}}\right)ad\sqrt{\sin(c+dx)}}$$

$$\downarrow 217$$

$$\frac{2e\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{e}\tan(c+dx)+1\right)}{\sqrt{2}\sqrt{e}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{e}\tan(c+dx)\right)}{\sqrt{2}\sqrt{e}}\right)-\frac{1}{2}\int \frac{e-e^2\tan^2(c+dx)}{e^4\tan^4(c+dx)+e^2}d\sqrt{e\tan(c+dx)}\right)}{4\sqrt{2}b\sqrt{\cos(c+dx)}\sqrt{e\tan(c+dx)}\left(\frac{\text{EllipticPi}\left(\frac{\sqrt{a-b}}{\sqrt{a+b}},\arcsin\left(\frac{\sqrt{\sin(c+dx)}}{\sqrt{\cos(c+dx)+1}}\right),-1\right)}{2\sqrt{a-b}\sqrt{a+b}}-\frac{\text{EllipticPi}\left(-\frac{\sqrt{a-b}}{\sqrt{a+b}},\arcsin\left(\frac{\sqrt{\sin(c+dx)}}{\sqrt{\cos(c+dx)+1}}\right),-1\right)}{2\sqrt{a-b}\sqrt{a+b}}\right)ad\sqrt{\sin(c+dx)}}-$$

$$\downarrow 1479$$

$$\frac{2e\left(\frac{1}{2}\left(\frac{\int -\frac{\sqrt{2}\sqrt{e}-2\sqrt{e\tan(c+dx)}}{e^2\tan^2(c+dx)-\sqrt{2}e^{3/2}\tan(c+dx)+e}d\sqrt{e\tan(c+dx)}}{2\sqrt{2}\sqrt{e}}+\frac{\int -\frac{\sqrt{2}\left(\sqrt{e}+\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{e^2\tan^2(c+dx)+\sqrt{2}e^{3/2}\tan(c+dx)+e}d\sqrt{e\tan(c+dx)}}{2\sqrt{2}\sqrt{e}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{}\right.}{\sqrt{}}\right.}{4\sqrt{2}b\sqrt{\cos(c+dx)}\sqrt{e\tan(c+dx)}\left(\frac{\text{EllipticPi}\left(\frac{\sqrt{a-b}}{\sqrt{a+b}},\arcsin\left(\frac{\sqrt{\sin(c+dx)}}{\sqrt{\cos(c+dx)+1}}\right),-1\right)}{2\sqrt{a-b}\sqrt{a+b}}-\frac{\text{EllipticPi}\left(-\frac{\sqrt{a-b}}{\sqrt{a+b}},\arcsin\left(\frac{\sqrt{\sin(c+dx)}}{\sqrt{\cos(c+dx)+1}}\right),-1\right)}{2\sqrt{a-b}\sqrt{a+b}}\right)ad\sqrt{\sin(c+dx)}}$$

$$\downarrow 25$$

$$\frac{2e\left(\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}\sqrt{e}-2\sqrt{e\tan(c+dx)}}{e^2\tan^2(c+dx)-\sqrt{2}e^{3/2}\tan(c+dx)+e}d\sqrt{e\tan(c+dx)}}{2\sqrt{2}\sqrt{e}}-\frac{\int \frac{\sqrt{2}\left(\sqrt{e}+\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{e^2\tan^2(c+dx)+\sqrt{2}e^{3/2}\tan(c+dx)+e}d\sqrt{e\tan(c+dx)}}{2\sqrt{2}\sqrt{e}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{e}\right.}{\sqrt{2}}\right.}{4\sqrt{2}b\sqrt{\cos(c+dx)}\sqrt{e\tan(c+dx)}\left(\frac{\text{EllipticPi}\left(\frac{\sqrt{a-b}}{\sqrt{a+b}},\arcsin\left(\frac{\sqrt{\sin(c+dx)}}{\sqrt{\cos(c+dx)+1}}\right),-1\right)}{2\sqrt{a-b}\sqrt{a+b}}-\frac{\text{EllipticPi}\left(-\frac{\sqrt{a-b}}{\sqrt{a+b}},\arcsin\left(\frac{\sqrt{\sin(c+dx)}}{\sqrt{\cos(c+dx)+1}}\right),-1\right)}{2\sqrt{a-b}\sqrt{a+b}}\right)ad\sqrt{\sin(c+dx)}}$$

$$\downarrow 27$$

$$\frac{2e\left(\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}\sqrt{e}-2\sqrt{e\tan(c+dx)}}{e^2\tan^2(c+dx)-\sqrt{2}e^{3/2}\tan(c+dx)+e}d\sqrt{e\tan(c+dx)}}{2\sqrt{2}\sqrt{e}}-\frac{\int \frac{\sqrt{e}+\sqrt{2}\sqrt{e\tan(c+dx)}}{e^2\tan^2(c+dx)+\sqrt{2}e^{3/2}\tan(c+dx)+e}d\sqrt{e\tan(c+dx)}}{2\sqrt{e}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{e}t\right.}{\sqrt{2}}\right.}{4\sqrt{2}b\sqrt{\cos(c+dx)}\sqrt{e\tan(c+dx)}\left(\frac{\text{EllipticPi}\left(\frac{\sqrt{a-b}}{\sqrt{a+b}},\arcsin\left(\frac{\sqrt{\sin(c+dx)}}{\sqrt{\cos(c+dx)+1}}\right),-1\right)}{2\sqrt{a-b}\sqrt{a+b}}-\frac{\text{EllipticPi}\left(-\frac{\sqrt{a-b}}{\sqrt{a+b}},\arcsin\left(\frac{\sqrt{\sin(c+dx)}}{\sqrt{\cos(c+dx)+1}}\right),-1\right.}{2\sqrt{a-b}\sqrt{a+b}}\right.}{ad\sqrt{\sin(c+dx)}}$$

$$\downarrow 1103$$

$$\frac{2e\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{e}\tan(c+dx)+1\right)}{\sqrt{2}\sqrt{e}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{e}\tan(c+dx)\right)}{\sqrt{2}\sqrt{e}}\right)+\frac{1}{2}\left(\frac{\log\left(-\sqrt{2}e^{3/2}\tan(c+dx)+e^2\tan^2(c+dx)+e\right)}{2\sqrt{2}\sqrt{e}}-\frac{\log\left(\sqrt{2}e^{3/2}\right.}{\sqrt{2}}\right.}{4\sqrt{2}b\sqrt{\cos(c+dx)}\sqrt{e\tan(c+dx)}\left(\frac{\text{EllipticPi}\left(\frac{\sqrt{a-b}}{\sqrt{a+b}},\arcsin\left(\frac{\sqrt{\sin(c+dx)}}{\sqrt{\cos(c+dx)+1}}\right),-1\right)}{2\sqrt{a-b}\sqrt{a+b}}-\frac{\text{EllipticPi}\left(-\frac{\sqrt{a-b}}{\sqrt{a+b}},\arcsin\left(\frac{\sqrt{\sin(c+dx)}}{\sqrt{\cos(c+dx)+1}}\right),-1\right.}{2\sqrt{a-b}\sqrt{a+b}}\right.}{ad\sqrt{\sin(c+dx)}}$$

input
```
Int[Sqrt[e*Tan[c + d*x]]/(a + b*Sec[c + d*x]),x]
```

output
```
(2*e*((-(ArcTan[1 - Sqrt[2]*Sqrt[e]*Tan[c + d*x]]/(Sqrt[2]*Sqrt[e])) + Arc
Tan[1 + Sqrt[2]*Sqrt[e]*Tan[c + d*x]]/(Sqrt[2]*Sqrt[e]))/2 + (Log[e - Sqrt
[2]*e^(3/2)*Tan[c + d*x] + e^2*Tan[c + d*x]^2]/(2*Sqrt[2]*Sqrt[e]) - Log[e
 + Sqrt[2]*e^(3/2)*Tan[c + d*x] + e^2*Tan[c + d*x]^2]/(2*Sqrt[2]*Sqrt[e]))
/2))/(a*d) - (4*Sqrt[2]*b*Sqrt[Cos[c + d*x]]*(-1/2*EllipticPi[-(Sqrt[a - b
]/Sqrt[a + b]), ArcSin[Sqrt[Sin[c + d*x]]/Sqrt[1 + Cos[c + d*x]]], -1]/(Sq
rt[a - b]*Sqrt[a + b]) + EllipticPi[Sqrt[a - b]/Sqrt[a + b], ArcSin[Sqrt[S
in[c + d*x]]/Sqrt[1 + Cos[c + d*x]]], -1]/(2*Sqrt[a - b]*Sqrt[a + b]))*Sqr
t[e*Tan[c + d*x]])/(a*d*Sqrt[Sin[c + d*x]])
```

3.314.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 993
```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-a/b, 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*
b)   Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Simp[s/(2*b)   Int[1/((r
 - s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 1537
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[(-a)*c, 2]}, Simp[Sqrt[-c]   Int[1/((d + e*x^2)*Sqrt[q + c*x^2]*Sqr
t[q - c*x^2]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] &
& GtQ[a, 0] && LtQ[c, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3209
```
Int[1/(((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(g_)*tan[(e_.) + (f_.)*(
x_)]]), x_Symbol] :> Simp[Sqrt[Sin[e + f*x]]/(Sqrt[Cos[e + f*x]]*Sqrt[g*Tan
[e + f*x]])   Int[Sqrt[Cos[e + f*x]]/(Sqrt[Sin[e + f*x]]*(a + b*Sin[e + f*x
])), x], x] /; FreeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

rule 3212
```
Int[(cot[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Simp[g^(2*IntPart[p])*(g*Cot[e + f*x])^FracPart[p]
*(g*Tan[e + f*x])^FracPart[p]   Int[(a + b*Sin[e + f*x])^m/(g*Tan[e + f*x])
^p, x], x] /; FreeQ[{a, b, e, f, g, m, p}, x] &&  !IntegerQ[p]
```

rule 3384
```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/(Sqrt[sin[(e_.) + (f_.)*(x_)]]*((a_
) + (b_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Simp[-4*Sqrt[2]*(g/f)   S
ubst[Int[x^2/(((a + b)*g^2 + (a - b)*x^4)*Sqrt[1 - x^4/g^2]), x], x, Sqrt[g
*Cos[e + f*x]]/Sqrt[1 + Sin[e + f*x]]], x] /; FreeQ[{a, b, e, f, g}, x] &&
NeQ[a^2 - b^2, 0]
```

rule 3385
```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/(Sqrt[(d_)*sin[(e_.) + (f_.)*(x_)]]
*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Simp[Sqrt[Sin[e + f*
x]]/Sqrt[d*Sin[e + f*x]]   Int[Sqrt[g*Cos[e + f*x]]/(Sqrt[Sin[e + f*x]]*(a
+ b*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e, f, g}, x] && NeQ[a^2 - b^2
, 0]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

rule 4378
```
Int[Sqrt[cot[(c_.) + (d_.)*(x_)]*(e_.)]/(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a
_)), x_Symbol] :> Simp[1/a   Int[Sqrt[e*Cot[c + d*x]], x], x] - Simp[b/a
Int[Sqrt[e*Cot[c + d*x]]/(b + a*Sin[c + d*x]), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && NeQ[a^2 - b^2, 0]
```

3.314.4 Maple [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 819 vs. $2(328) = 656$.

Time = 2.54 (sec) , antiderivative size = 820, normalized size of antiderivative = 1.98

method	result	size
default	Expression too large to display	820

input
```
int((e*tan(d*x+c))^(1/2)/(a+b*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*2^(1/2)*b/((a^2-b^2)^(1/2)-a+b)/((a^2-b^2)^(1/2)+a-b)/a*(I*EllipticPi(
(csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2-1/2*I,1/2*2^(1/2))*a-I*EllipticPi((csc
(d*x+c)-cot(d*x+c)+1)^(1/2),1/2-1/2*I,1/2*2^(1/2))*b-I*EllipticPi((csc(d*x
+c)-cot(d*x+c)+1)^(1/2),1/2+1/2*I,1/2*2^(1/2))*a+I*EllipticPi((csc(d*x+c)-
cot(d*x+c)+1)^(1/2),1/2+1/2*I,1/2*2^(1/2))*b-EllipticPi((csc(d*x+c)-cot(d*
x+c)+1)^(1/2),1/2-1/2*I,1/2*2^(1/2))*a+EllipticPi((csc(d*x+c)-cot(d*x+c)+1
)^(1/2),1/2-1/2*I,1/2*2^(1/2))*b-EllipticPi((csc(d*x+c)-cot(d*x+c)+1)^(1/2
),1/2+1/2*I,1/2*2^(1/2))*a+EllipticPi((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2+
1/2*I,1/2*2^(1/2))*b-(a^2-b^2)^(1/2)*EllipticPi((csc(d*x+c)-cot(d*x+c)+1)^
(1/2),(a-b)/(a-b+((a-b)*(a+b))^(1/2)),1/2*2^(1/2))+(a^2-b^2)^(1/2)*Ellipti
cPi((csc(d*x+c)-cot(d*x+c)+1)^(1/2),-(a-b)/(-a+b+((a-b)*(a+b))^(1/2)),1/2*
2^(1/2))+EllipticPi((csc(d*x+c)-cot(d*x+c)+1)^(1/2),(a-b)/(a-b+((a-b)*(a+b
))^(1/2)),1/2*2^(1/2))*a-b*EllipticPi((csc(d*x+c)-cot(d*x+c)+1)^(1/2),(a-b
)/(a-b+((a-b)*(a+b))^(1/2)),1/2*2^(1/2))+EllipticPi((csc(d*x+c)-cot(d*x+c)
+1)^(1/2),-(a-b)/(-a+b+((a-b)*(a+b))^(1/2)),1/2*2^(1/2))*a-b*EllipticPi((c
sc(d*x+c)-cot(d*x+c)+1)^(1/2),-(a-b)/(-a+b+((a-b)*(a+b))^(1/2)),1/2*2^(1/2
)))*(cot(d*x+c)-csc(d*x+c))^(1/2)*(2-2*csc(d*x+c)+2*cot(d*x+c))^(1/2)*(csc
(d*x+c)-cot(d*x+c)+1)^(1/2)*(-e/((1-cos(d*x+c))^2*csc(d*x+c)^2-1)*(-cot(d*
x+c)+csc(d*x+c)))^(1/2)*((1-cos(d*x+c))^2*csc(d*x+c)^2-1)/((1-cos(d*x+c))^
3*csc(d*x+c)^3+cot(d*x+c)-csc(d*x+c))^(1/2)/((1-cos(d*x+c))*((1-cos(d*x...
```

3.314.5 Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{e\tan(c+dx)}}{a+b\sec(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((e*tan(d*x+c))^(1/2)/(a+b*sec(d*x+c)),x, algorithm="fricas")
```

output
```
Timed out
```

3.314.6 Sympy [F]

$$\int \frac{\sqrt{e\tan(c+dx)}}{a+b\sec(c+dx)}\,dx = \int \frac{\sqrt{e\tan(c+dx)}}{a+b\sec(c+dx)}\,dx$$

input
```
integrate((e*tan(d*x+c))**(1/2)/(a+b*sec(d*x+c)),x)
```

output
```
Integral(sqrt(e*tan(c + d*x))/(a + b*sec(c + d*x)), x)
```

3.314.7 Maxima [F]

$$\int \frac{\sqrt{e\tan(c+dx)}}{a+b\sec(c+dx)}\,dx = \int \frac{\sqrt{e\tan(dx+c)}}{b\sec(dx+c)+a}\,dx$$

input
```
integrate((e*tan(d*x+c))^(1/2)/(a+b*sec(d*x+c)),x, algorithm="maxima")
```

output
```
integrate(sqrt(e*tan(d*x + c))/(b*sec(d*x + c) + a), x)
```

3.314.8 Giac [F]

$$\int \frac{\sqrt{e\tan(c+dx)}}{a+b\sec(c+dx)}\,dx = \int \frac{\sqrt{e\tan(dx+c)}}{b\sec(dx+c)+a}\,dx$$

input
```
integrate((e*tan(d*x+c))^(1/2)/(a+b*sec(d*x+c)),x, algorithm="giac")
```

output
```
integrate(sqrt(e*tan(d*x + c))/(b*sec(d*x + c) + a), x)
```

3.314.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{e\tan(c+dx)}}{a+b\sec(c+dx)}\,dx = \int \frac{\cos(c+d\,x)\,\sqrt{e\tan(c+d\,x)}}{b+a\,\cos(c+d\,x)}\,dx$$

input
```
int((e*tan(c + d*x))^(1/2)/(a + b/cos(c + d*x)),x)
```

output
```
int((cos(c + d*x)*(e*tan(c + d*x))^(1/2))/(b + a*cos(c + d*x)), x)
```

3.315 $\int \frac{1}{(a+b\sec(c+dx))\sqrt{e\tan(c+dx)}}\,dx$

3.315.1 Optimal result 2302
3.315.2 Mathematica [C] (warning: unable to verify) 2303
3.315.3 Rubi [A] (warning: unable to verify) 2304
3.315.4 Maple [B] (warning: unable to verify) 2311
3.315.5 Fricas [F(-1)] 2312
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3.315.7 Maxima [F] 2312
3.315.8 Giac [F] 2313
3.315.9 Mupad [F(-1)] 2313

3.315.1 Optimal result

Integrand size = 25, antiderivative size = 422

$$\begin{aligned}
&\int \frac{1}{(a+b\sec(c+dx))\sqrt{e\tan(c+dx)}}\,dx \\
&= -\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}ad\sqrt{e}} + \frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}ad\sqrt{e}} \\
&\quad - \frac{\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}ad\sqrt{e}} \\
&\quad + \frac{\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)+\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}ad\sqrt{e}} \\
&\quad - \frac{2\sqrt{2}b\,\text{EllipticPi}\left(\frac{b}{a-\sqrt{a^2-b^2}},\arcsin\left(\frac{\sqrt{-\cos(c+dx)}}{\sqrt{1+\sin(c+dx)}}\right),-1\right)\sqrt{\sin(c+dx)}}{a\sqrt{a^2-b^2}d\sqrt{-\cos(c+dx)}\sqrt{e\tan(c+dx)}} \\
&\quad + \frac{2\sqrt{2}b\,\text{EllipticPi}\left(\frac{b}{a+\sqrt{a^2-b^2}},\arcsin\left(\frac{\sqrt{-\cos(c+dx)}}{\sqrt{1+\sin(c+dx)}}\right),-1\right)\sqrt{\sin(c+dx)}}{a\sqrt{a^2-b^2}d\sqrt{-\cos(c+dx)}\sqrt{e\tan(c+dx)}}
\end{aligned}$$

output
```
-1/2*arctan(1-2^(1/2)*(e*tan(d*x+c))^(1/2)/e^(1/2))/a/d*2^(1/2)/e^(1/2)+1/
2*arctan(1+2^(1/2)*(e*tan(d*x+c))^(1/2)/e^(1/2))/a/d*2^(1/2)/e^(1/2)-1/4*l
n(e^(1/2)-2^(1/2)*(e*tan(d*x+c))^(1/2)+e^(1/2)*tan(d*x+c))/a/d*2^(1/2)/e^(
1/2)+1/4*ln(e^(1/2)+2^(1/2)*(e*tan(d*x+c))^(1/2)+e^(1/2)*tan(d*x+c))/a/d*2
^(1/2)/e^(1/2)-2*b*EllipticPi((-cos(d*x+c))^(1/2)/(1+sin(d*x+c))^(1/2),b/(
a-(a^2-b^2)^(1/2)),I)*2^(1/2)*sin(d*x+c)^(1/2)/a/d/(a^2-b^2)^(1/2)/(-cos(d
*x+c))^(1/2)/(e*tan(d*x+c))^(1/2)+2*b*EllipticPi((-cos(d*x+c))^(1/2)/(1+si
n(d*x+c))^(1/2),b/(a+(a^2-b^2)^(1/2)),I)*2^(1/2)*sin(d*x+c)^(1/2)/a/d/(a^2
-b^2)^(1/2)/(-cos(d*x+c))^(1/2)/(e*tan(d*x+c))^(1/2)
```

3.315.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 38.30 (sec) , antiderivative size = 1475, normalized size of antiderivative = 3.50

$$\int \frac{1}{(a+b\sec(c+dx))\sqrt{e\tan(c+dx)}}\,dx = \text{Too large to display}$$

input
```
Integrate[1/((a + b*Sec[c + d*x])*Sqrt[e*Tan[c + d*x]]),x]
```

output
```
((b + a*Cos[c + d*x])*Sec[c + d*x]*Sqrt[Tan[c + d*x]]*((2*Sec[c + d*x]^3*(
a + b*Sqrt[1 + Tan[c + d*x]^2])*(((-1/8 + I/8)*a*(2*ArcTan[1 - ((1 + I)*Sq
rt[b]*Sqrt[Tan[c + d*x]])/(a^2 - b^2)^(1/4)] - 2*ArcTan[1 + ((1 + I)*Sqrt[
b]*Sqrt[Tan[c + d*x]])/(a^2 - b^2)^(1/4)] + Log[Sqrt[a^2 - b^2] - (1 + I)*
Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Tan[c + d*x]] + I*b*Tan[c + d*x]] - Log[Sqr
t[a^2 - b^2] + (1 + I)*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Tan[c + d*x]] + I*b*
Tan[c + d*x]]))/(Sqrt[b]*(a^2 - b^2)^(3/4)) + (5*b*(-a^2 + b^2)*AppellF1[1
/4, -1/2, 1, 5/4, -Tan[c + d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2 - b^2)]*Sqrt[
Tan[c + d*x]]*Sqrt[1 + Tan[c + d*x]^2])/((5*(a^2 - b^2)*AppellF1[1/4, -1/2
, 1, 5/4, -Tan[c + d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2 - b^2)] + 2*(2*b^2*Ap
pellF1[5/4, -1/2, 2, 9/4, -Tan[c + d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2 - b^2
)] + (a^2 - b^2)*AppellF1[5/4, 1/2, 1, 9/4, -Tan[c + d*x]^2, (b^2*Tan[c +
d*x]^2)/(a^2 - b^2)])*Tan[c + d*x]^2)*(a^2 - b^2*(1 + Tan[c + d*x]^2)))))/
((b + a*Cos[c + d*x])*(1 + Tan[c + d*x]^2)^2) + (Cos[2*(c + d*x)]*Sec[c +
d*x]^3*(a + b*Sqrt[1 + Tan[c + d*x]^2])*((-20*Sqrt[2]*ArcTan[1 - Sqrt[2]*S
qrt[Tan[c + d*x]]])/a + (20*Sqrt[2]*ArcTan[1 + Sqrt[2]*Sqrt[Tan[c + d*x]]]
)/a + ((10 - 10*I)*(a^2 - 2*b^2)*ArcTan[1 - ((1 + I)*Sqrt[b]*Sqrt[Tan[c +
d*x]])/(a^2 - b^2)^(1/4)])/(a*Sqrt[b]*(a^2 - b^2)^(3/4)) - ((10 - 10*I)*(a
^2 - 2*b^2)*ArcTan[1 + ((1 + I)*Sqrt[b]*Sqrt[Tan[c + d*x]])/(a^2 - b^2)^(1
/4)])/(a*Sqrt[b]*(a^2 - b^2)^(3/4)) - (10*Sqrt[2]*Log[1 - Sqrt[2]*Sqrt[...
```

3.315.3 Rubi [A] (warning: unable to verify)

Time = 1.24 (sec) , antiderivative size = 412, normalized size of antiderivative = 0.98, number of steps used = 22, number of rules used = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.840$, Rules used = {3042, 4380, 3042, 3212, 3042, 3208, 3042, 3386, 25, 1537, 412, 3957, 266, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{e\tan(c+dx)}(a+b\sec(c+dx))}\,dx$$

$$\downarrow 3042$$

$$\int \frac{1}{\sqrt{-e\cot\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)}\,dx$$

$$\downarrow 4380$$

$$\frac{\int \frac{1}{\sqrt{e\tan(c+dx)}}\,dx}{a} - \frac{b\int \frac{1}{(b+a\cos(c+dx))\sqrt{e\tan(c+dx)}}\,dx}{a}$$

$$\downarrow 3042$$

$$\frac{\int \frac{1}{\sqrt{e\tan(c+dx)}}\,dx}{a} - \frac{b\int \frac{1}{\sqrt{-e\cot\left(c+dx+\frac{\pi}{2}\right)}\left(b+a\sin\left(c+dx+\frac{\pi}{2}\right)\right)}\,dx}{a}$$

$$\downarrow 3212$$

$$\frac{\int \frac{1}{\sqrt{e\tan(c+dx)}}\,dx}{a} - \frac{b\int \frac{\sqrt{e\cot(c+dx)}}{b+a\cos(c+dx)}\,dx}{a\sqrt{e\tan(c+dx)}\sqrt{e\cot(c+dx)}}$$

$$\downarrow 3042$$

$$\frac{\int \frac{1}{\sqrt{e\tan(c+dx)}}\,dx}{a} - \frac{b\int \frac{\sqrt{-e\tan\left(c+dx-\frac{\pi}{2}\right)}}{b-a\sin\left(c+dx-\frac{\pi}{2}\right)}\,dx}{a\sqrt{e\tan(c+dx)}\sqrt{e\cot(c+dx)}}$$

$$\downarrow 3208$$

$$\frac{\int \frac{1}{\sqrt{e\tan(c+dx)}}\,dx}{a} - \frac{b\sqrt{\sin(c+dx)}\int \frac{\sqrt{-\cos(c+dx)}}{(b+a\cos(c+dx))\sqrt{\sin(c+dx)}}\,dx}{a\sqrt{-\cos(c+dx)}\sqrt{e\tan(c+dx)}}$$

$$\downarrow 3042$$

$$\frac{\int \frac{1}{\sqrt{e\tan(c+dx)}}dx}{a} - \frac{b\sqrt{\sin(c+dx)}\int \frac{\sqrt{\sin\left(c+dx-\frac{\pi}{2}\right)}}{\sqrt{\cos\left(c+dx-\frac{\pi}{2}\right)}\left(b-a\sin\left(c+dx-\frac{\pi}{2}\right)\right)}dx}{a\sqrt{-\cos(c+dx)}\sqrt{e\tan(c+dx)}}$$

$$\downarrow 3386$$

$$\frac{\int \frac{1}{\sqrt{e\tan(c+dx)}}dx}{a} - b\sqrt{\sin(c+dx)}\left(\frac{2\sqrt{2}\left(1-\frac{a}{\sqrt{a^2-b^2}}\right)\int -\frac{1}{\sqrt{1-\frac{\cos^2(c+dx)}{(\sin(c+dx)+1)^2}}\left(a-\sqrt{a^2-b^2}+\frac{b\cos(c+dx)}{\sin(c+dx)+1}\right)}d\frac{\sqrt{-\cos(c+dx)}}{\sqrt{\sin(c+dx)+1}}}{d} + \frac{2\sqrt{2}\left(\frac{a}{\sqrt{a^2-b^2}}+1\right)\int -\frac{}{\sqrt{1-\frac{\cos^2}{(\sin(c+}}}\right.$$

$$a\sqrt{-\cos(c+dx)}\sqrt{e\tan(c+dx)}$$

$$\downarrow 25$$

$$\frac{\int \frac{1}{\sqrt{e\tan(c+dx)}}dx}{a} - b\sqrt{\sin(c+dx)}\left(-\frac{2\sqrt{2}\left(1-\frac{a}{\sqrt{a^2-b^2}}\right)\int \frac{1}{\sqrt{1-\frac{\cos^2(c+dx)}{(\sin(c+dx)+1)^2}}\left(a-\sqrt{a^2-b^2}+\frac{b\cos(c+dx)}{\sin(c+dx)+1}\right)}d\frac{\sqrt{-\cos(c+dx)}}{\sqrt{\sin(c+dx)+1}}}{d} - \frac{2\sqrt{2}\left(\frac{a}{\sqrt{a^2-b^2}}+1\right)\int \frac{}{\sqrt{1-\frac{\cos^2(}{(\sin(c+}}}\right.$$

$$a\sqrt{-\cos(c+dx)}\sqrt{e\tan(c+dx)}$$

$$\downarrow 1537$$

$$\frac{\int \frac{1}{\sqrt{e\tan(c+dx)}}dx}{a} - b\sqrt{\sin(c+dx)}\left(-\frac{2\sqrt{2}\left(1-\frac{a}{\sqrt{a^2-b^2}}\right)\int \frac{1}{\sqrt{1-\frac{\cos(c+dx)}{\sin(c+dx)+1}}\sqrt{\frac{\cos(c+dx)}{\sin(c+dx)+1}+1}\left(a-\sqrt{a^2-b^2}+\frac{b\cos(c+dx)}{\sin(c+dx)+1}\right)}d\frac{\sqrt{-\cos(c+dx)}}{\sqrt{\sin(c+dx)+1}}}{d} - \frac{2\sqrt{2}\left(\frac{a}{\sqrt{a^2-b^2}}+1\right)}{}\right.$$

$$a\sqrt{-\cos(c+dx)}\sqrt{e\tan(c+dx)}$$

$$\downarrow 412$$

$$\frac{\int \frac{1}{\sqrt{e\tan(c+dx)}}dx}{a} - b\sqrt{\sin(c+dx)}\left(-\frac{2\sqrt{2}\left(1-\frac{a}{\sqrt{a^2-b^2}}\right)\text{EllipticPi}\left(\frac{b}{a-\sqrt{a^2-b^2}},\arcsin\left(\frac{\sqrt{-\cos(c+dx)}}{\sqrt{\sin(c+dx)+1}}\right),-1\right)}{d\left(a-\sqrt{a^2-b^2}\right)} - \frac{2\sqrt{2}\left(\frac{a}{\sqrt{a^2-b^2}}+1\right)\text{EllipticPi}\left(\frac{b}{a+\sqrt{a^2-b^2}},\text{arc}\right.}{d\left(\sqrt{a^2-b^2}+a\right)}\right)$$

$$a\sqrt{-\cos(c+dx)}\sqrt{e\tan(c+dx)}$$

$$\downarrow 3957$$

3.315. $\int \frac{1}{(a+b\sec(c+dx))\sqrt{e\tan(c+dx)}}dx$

$$\frac{\frac{e\int \frac{1}{\sqrt{e\tan(c+dx)}(\tan^2(c+dx)e^2+e^2)}d(e\tan(c+dx))}{ad}-}{b\sqrt{\sin(c+dx)}\left(-\frac{2\sqrt{2}\left(1-\frac{a}{\sqrt{a^2-b^2}}\right)\text{EllipticPi}\left(\frac{b}{a-\sqrt{a^2-b^2}},\arcsin\left(\frac{\sqrt{-\cos(c+dx)}}{\sqrt{\sin(c+dx)+1}}\right),-1\right)}{d\left(a-\sqrt{a^2-b^2}\right)}-\frac{2\sqrt{2}\left(\frac{a}{\sqrt{a^2-b^2}}+1\right)\text{EllipticPi}\left(\frac{b}{a+\sqrt{a^2-b^2}},\arc\right.}{d\left(\sqrt{a^2-b^2}+a\right)}\right.}{a\sqrt{-\cos(c+dx)}\sqrt{e\tan(c+dx)}}$$

$$\downarrow 266$$

$$\frac{\frac{2e\int \frac{1}{e^4\tan^4(c+dx)+e^2}d\sqrt{e\tan(c+dx)}}{ad}-}{b\sqrt{\sin(c+dx)}\left(-\frac{2\sqrt{2}\left(1-\frac{a}{\sqrt{a^2-b^2}}\right)\text{EllipticPi}\left(\frac{b}{a-\sqrt{a^2-b^2}},\arcsin\left(\frac{\sqrt{-\cos(c+dx)}}{\sqrt{\sin(c+dx)+1}}\right),-1\right)}{d\left(a-\sqrt{a^2-b^2}\right)}-\frac{2\sqrt{2}\left(\frac{a}{\sqrt{a^2-b^2}}+1\right)\text{EllipticPi}\left(\frac{b}{a+\sqrt{a^2-b^2}},\arc\right.}{d\left(\sqrt{a^2-b^2}+a\right)}\right.}{a\sqrt{-\cos(c+dx)}\sqrt{e\tan(c+dx)}}$$

$$\downarrow 755$$

$$\frac{2e\left(\frac{\int \frac{e-e^2\tan^2(c+dx)}{e^4\tan^4(c+dx)+e^2}d\sqrt{e\tan(c+dx)}}{2e}+\frac{\int \frac{e^2\tan^2(c+dx)+e}{e^4\tan^4(c+dx)+e^2}d\sqrt{e\tan(c+dx)}}{2e}\right)-}{b\sqrt{\sin(c+dx)}\left(-\frac{2\sqrt{2}\left(1-\frac{a}{\sqrt{a^2-b^2}}\right)\text{EllipticPi}\left(\frac{b}{a-\sqrt{a^2-b^2}},\arcsin\left(\frac{\sqrt{-\cos(c+dx)}}{\sqrt{\sin(c+dx)+1}}\right),-1\right)}{d\left(a-\sqrt{a^2-b^2}\right)}-\frac{2\sqrt{2}\left(\frac{a}{\sqrt{a^2-b^2}}+1\right)\text{EllipticPi}\left(\frac{b}{a+\sqrt{a^2-b^2}},\arc\right.}{d\left(\sqrt{a^2-b^2}+a\right)}\right.}{a\sqrt{-\cos(c+dx)}\sqrt{e\tan(c+dx)}}$$

$$\downarrow 1476$$

$$\frac{2e\left(\frac{\frac{1}{2}\int \frac{1}{e^2\tan^2(c+dx)-\sqrt{2}e^{3/2}\tan(c+dx)+e}d\sqrt{e\tan(c+dx)}+\frac{1}{2}\int \frac{1}{e^2\tan^2(c+dx)+\sqrt{2}e^{3/2}\tan(c+dx)+e}d\sqrt{e\tan(c+dx)}}{2e}+\frac{\int \frac{e-e^2\tan^2(c+dx)}{e^4\tan^4(c+dx)+e^2}d\sqrt{e\tan(c+dx)}}{2e}\right)}{b\sqrt{\sin(c+dx)}\left(-\frac{2\sqrt{2}\left(1-\frac{a}{\sqrt{a^2-b^2}}\right)\text{EllipticPi}\left(\frac{b}{a-\sqrt{a^2-b^2}},\arcsin\left(\frac{\sqrt{-\cos(c+dx)}}{\sqrt{\sin(c+dx)+1}}\right),-1\right)}{d\left(a-\sqrt{a^2-b^2}\right)}-\frac{2\sqrt{2}\left(\frac{a}{\sqrt{a^2-b^2}}+1\right)\text{EllipticPi}\left(\frac{b}{a+\sqrt{a^2-b^2}},\arc\right.}{d\left(\sqrt{a^2-b^2}+a\right)}\right.}{a\sqrt{-\cos(c+dx)}\sqrt{e\tan(c+dx)}}$$

$$\downarrow 1082$$

$$\frac{2e\left(\frac{\frac{\int \frac{1}{-e^2\tan^2(c+dx)-1}d\left(1-\sqrt{2}\sqrt{e}\tan(c+dx)\right)}{\sqrt{2}\sqrt{e}}-\frac{\int \frac{1}{-e^2\tan^2(c+dx)-1}d\left(\sqrt{2}\sqrt{e}\tan(c+dx)+1\right)}{\sqrt{2}\sqrt{e}}}{2e}+\frac{\int \frac{e-e^2\tan^2(c+dx)}{e^4\tan^4(c+dx)+e^2}d\sqrt{e\tan(c+dx)}}{2e}\right)-}{b\sqrt{\sin(c+dx)}\left(-\frac{2\sqrt{2}\left(1-\frac{a}{\sqrt{a^2-b^2}}\right)\text{EllipticPi}\left(\frac{b}{a-\sqrt{a^2-b^2}},\arcsin\left(\frac{\sqrt{-\cos(c+dx)}}{\sqrt{\sin(c+dx)+1}}\right),-1\right)}{d\left(a-\sqrt{a^2-b^2}\right)}-\frac{2\sqrt{2}\left(\frac{a}{\sqrt{a^2-b^2}}+1\right)\text{EllipticPi}\left(\frac{b}{a+\sqrt{a^2-b^2}},\arc\right.}{d\left(\sqrt{a^2-b^2}+a\right)}\right.}{a\sqrt{-\cos(c+dx)}\sqrt{e\tan(c+dx)}}$$

$$\downarrow 217$$

$$\frac{2e\left(\frac{\int \frac{e-e^2\tan^2(c+dx)}{e^4\tan^4(c+dx)+e^2}\,d\sqrt{e\tan(c+dx)}}{2e}+\frac{\frac{\arctan\left(\sqrt{2}\sqrt{e}\tan(c+dx)+1\right)}{\sqrt{2}\sqrt{e}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{e}\tan(c+dx)\right)}{\sqrt{2}\sqrt{e}}}{2e}\right)}{ad}-$$

$$\frac{b\sqrt{\sin(c+dx)}\left(-\frac{2\sqrt{2}\left(1-\frac{a}{\sqrt{a^2-b^2}}\right)\text{EllipticPi}\left(\frac{b}{a-\sqrt{a^2-b^2}},\arcsin\left(\frac{\sqrt{-\cos(c+dx)}}{\sqrt{\sin(c+dx)+1}}\right),-1\right)}{d\left(a-\sqrt{a^2-b^2}\right)}-\frac{2\sqrt{2}\left(\frac{a}{\sqrt{a^2-b^2}}+1\right)\text{EllipticPi}\left(\frac{b}{a+\sqrt{a^2-b^2}},\arc\right.}{d\left(\sqrt{a^2-b^2}+a\right)}}{a\sqrt{-\cos(c+dx)}\sqrt{e\tan(c+dx)}}$$

$$\downarrow 1479$$

$$\frac{2e\left(\frac{-\frac{\int -\frac{\sqrt{2}\sqrt{e}-2\sqrt{e\tan(c+dx)}}{e^2\tan^2(c+dx)-\sqrt{2}e^{3/2}\tan(c+dx)+e}\,d\sqrt{e\tan(c+dx)}}{2\sqrt{2}\sqrt{e}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{e}+\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{e^2\tan^2(c+dx)+\sqrt{2}e^{3/2}\tan(c+dx)+e}\,d\sqrt{e\tan(c+dx)}}{2\sqrt{2}\sqrt{e}}}{2e}+\frac{\frac{\arctan\left(\sqrt{2}\sqrt{e}\tan(c+dx)+1\right)}{\sqrt{2}\sqrt{e}}-\frac{a}{2e}}\right.}{ad}$$

$$\frac{b\sqrt{\sin(c+dx)}\left(-\frac{2\sqrt{2}\left(1-\frac{a}{\sqrt{a^2-b^2}}\right)\text{EllipticPi}\left(\frac{b}{a-\sqrt{a^2-b^2}},\arcsin\left(\frac{\sqrt{-\cos(c+dx)}}{\sqrt{\sin(c+dx)+1}}\right),-1\right)}{d\left(a-\sqrt{a^2-b^2}\right)}-\frac{2\sqrt{2}\left(\frac{a}{\sqrt{a^2-b^2}}+1\right)\text{EllipticPi}\left(\frac{b}{a+\sqrt{a^2-b^2}},\arc\right.}{d\left(\sqrt{a^2-b^2}+a\right)}}{a\sqrt{-\cos(c+dx)}\sqrt{e\tan(c+dx)}}$$

$$\downarrow 25$$

$$\frac{2e\left(\frac{\int \frac{\sqrt{2}\sqrt{e}-2\sqrt{e\tan(c+dx)}}{e^2\tan^2(c+dx)-\sqrt{2}e^{3/2}\tan(c+dx)+e}\,d\sqrt{e\tan(c+dx)}}{2\sqrt{2}\sqrt{e}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{e}+\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{e^2\tan^2(c+dx)+\sqrt{2}e^{3/2}\tan(c+dx)+e}\,d\sqrt{e\tan(c+dx)}}{2\sqrt{2}\sqrt{e}}}{2e}+\frac{\frac{\arctan\left(\sqrt{2}\sqrt{e}\tan(c+dx)+1\right)}{\sqrt{2}\sqrt{e}}-\frac{\arctan}{2e}}\right.}{ad}$$

$$\frac{b\sqrt{\sin(c+dx)}\left(-\frac{2\sqrt{2}\left(1-\frac{a}{\sqrt{a^2-b^2}}\right)\text{EllipticPi}\left(\frac{b}{a-\sqrt{a^2-b^2}},\arcsin\left(\frac{\sqrt{-\cos(c+dx)}}{\sqrt{\sin(c+dx)+1}}\right),-1\right)}{d\left(a-\sqrt{a^2-b^2}\right)}-\frac{2\sqrt{2}\left(\frac{a}{\sqrt{a^2-b^2}}+1\right)\text{EllipticPi}\left(\frac{b}{a+\sqrt{a^2-b^2}},\arc\right.}{d\left(\sqrt{a^2-b^2}+a\right)}}{a\sqrt{-\cos(c+dx)}\sqrt{e\tan(c+dx)}}$$

$$\downarrow 27$$

$$\frac{2e\left(\frac{\int \frac{\sqrt{2}\sqrt{e}-2\sqrt{e\tan(c+dx)}}{e^2\tan^2(c+dx)-\sqrt{2}e^{3/2}\tan(c+dx)+e}\,d\sqrt{e\tan(c+dx)}}{2\sqrt{2}\sqrt{e}}+\frac{\int \frac{\sqrt{e}+\sqrt{2}\sqrt{e\tan(c+dx)}}{e^2\tan^2(c+dx)+\sqrt{2}e^{3/2}\tan(c+dx)+e}\,d\sqrt{e\tan(c+dx)}}{2\sqrt{e}}}{2e}+\frac{\frac{\arctan\left(\sqrt{2}\sqrt{e}\tan(c+dx)+1\right)}{\sqrt{2}\sqrt{e}}-\frac{\arctan}{2e}}\right.}{ad}$$

$$\frac{b\sqrt{\sin(c+dx)}\left(-\frac{2\sqrt{2}\left(1-\frac{a}{\sqrt{a^2-b^2}}\right)\text{EllipticPi}\left(\frac{b}{a-\sqrt{a^2-b^2}},\arcsin\left(\frac{\sqrt{-\cos(c+dx)}}{\sqrt{\sin(c+dx)+1}}\right),-1\right)}{d\left(a-\sqrt{a^2-b^2}\right)}-\frac{2\sqrt{2}\left(\frac{a}{\sqrt{a^2-b^2}}+1\right)\text{EllipticPi}\left(\frac{b}{a+\sqrt{a^2-b^2}},\arc\right.}{d\left(\sqrt{a^2-b^2}+a\right)}}{a\sqrt{-\cos(c+dx)}\sqrt{e\tan(c+dx)}}$$

$$\downarrow 1103$$

$$2e\left(\frac{\frac{\arctan\left(\sqrt{2}\sqrt{e}\tan(c+dx)+1\right)}{\sqrt{2}\sqrt{e}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{e}\tan(c+dx)\right)}{\sqrt{2}\sqrt{e}}}{2e}+\frac{\frac{\log\left(\sqrt{2}e^{3/2}\tan(c+dx)+e^2\tan^2(c+dx)+e\right)}{2\sqrt{2}\sqrt{e}}-\frac{\log\left(-\sqrt{2}e^{3/2}\tan(c+dx)+e^2\tan^2(c+dx)+e\right)}{2\sqrt{2}\sqrt{e}}}{2e}\right.$$

$$\left.\frac{ad}{b\sqrt{\sin(c+dx)}\left(-\frac{2\sqrt{2}\left(1-\frac{a}{\sqrt{a^2-b^2}}\right)\text{EllipticPi}\left(\frac{b}{a-\sqrt{a^2-b^2}},\arcsin\left(\frac{\sqrt{-\cos(c+dx)}}{\sqrt{\sin(c+dx)+1}}\right),-1\right)}{d\left(a-\sqrt{a^2-b^2}\right)}-\frac{2\sqrt{2}\left(\frac{a}{\sqrt{a^2-b^2}}+1\right)\text{EllipticPi}\left(\frac{b}{a+\sqrt{a^2-b^2}},\arc\right.}{d\left(\sqrt{a^2-b^2}+a\right)}\right.}{a\sqrt{-\cos(c+dx)}\sqrt{e\tan(c+dx)}}$$

input
```
Int[1/((a + b*Sec[c + d*x])*Sqrt[e*Tan[c + d*x]]),x]
```

output
```
(2*e*((-(ArcTan[1 - Sqrt[2]*Sqrt[e]*Tan[c + d*x]]/(Sqrt[2]*Sqrt[e])) + Arc
Tan[1 + Sqrt[2]*Sqrt[e]*Tan[c + d*x]]/(Sqrt[2]*Sqrt[e]))/(2*e) + (-1/2*Log
[e - Sqrt[2]*e^(3/2)*Tan[c + d*x] + e^2*Tan[c + d*x]^2]/(Sqrt[2]*Sqrt[e])
+ Log[e + Sqrt[2]*e^(3/2)*Tan[c + d*x] + e^2*Tan[c + d*x]^2]/(2*Sqrt[2]*Sq
rt[e]))/(2*e)))/(a*d) - (b*((-2*Sqrt[2]*(1 - a/Sqrt[a^2 - b^2])*EllipticPi
[b/(a - Sqrt[a^2 - b^2]), ArcSin[Sqrt[-Cos[c + d*x]]/Sqrt[1 + Sin[c + d*x]
]], -1])/((a - Sqrt[a^2 - b^2])*d) - (2*Sqrt[2]*(1 + a/Sqrt[a^2 - b^2])*El
lipticPi[b/(a + Sqrt[a^2 - b^2]), ArcSin[Sqrt[-Cos[c + d*x]]/Sqrt[1 + Sin[
c + d*x]]], -1])/((a + Sqrt[a^2 - b^2])*d))*Sqrt[Sin[c + d*x]])/(a*Sqrt[-C
os[c + d*x]]*Sqrt[e*Tan[c + d*x]])
```

3.315.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

3.315. $\int \frac{1}{(a+b\sec(c+dx))\sqrt{e\tan(c+dx)}}\,dx$

rule 1537
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[(-a)*c, 2]}, Simp[Sqrt[-c]   Int[1/((d + e*x^2)*Sqrt[q + c*x^2]*Sqr
t[q - c*x^2]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] &
& GtQ[a, 0] && LtQ[c, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3208
```
Int[Sqrt[(g_.)*tan[(e_.) + (f_.)*(x_)]]/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_
)]), x_Symbol] :> Simp[Sqrt[Cos[e + f*x]]*(Sqrt[g*Tan[e + f*x]]/Sqrt[Sin[e
+ f*x]])   Int[Sqrt[Sin[e + f*x]]/(Sqrt[Cos[e + f*x]]*(a + b*Sin[e + f*x]))
, x], x] /; FreeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

rule 3212
```
Int[(cot[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Simp[g^(2*IntPart[p])*(g*Cot[e + f*x])^FracPart[p]
*(g*Tan[e + f*x])^FracPart[p]   Int[(a + b*Sin[e + f*x])^m/(g*Tan[e + f*x])
^p, x], x] /; FreeQ[{a, b, e, f, g, m, p}, x] &&  !IntegerQ[p]
```

rule 3386
```
Int[Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]/(Sqrt[cos[(e_.) + (f_.)*(x_)]]*((a_
) + (b_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> With[{q = Rt[-a^2 + b^2,
2]}, Simp[2*Sqrt[2]*d*((b + q)/(f*q))   Subst[Int[1/((d*(b + q) + a*x^2)*Sq
rt[1 - x^4/d^2]), x], x, Sqrt[d*Sin[e + f*x]]/Sqrt[1 + Cos[e + f*x]]], x] -
 Simp[2*Sqrt[2]*d*((b - q)/(f*q))   Subst[Int[1/((d*(b - q) + a*x^2)*Sqrt[1
 - x^4/d^2]), x], x, Sqrt[d*Sin[e + f*x]]/Sqrt[1 + Cos[e + f*x]]], x]] /; F
reeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

rule 4380
```
Int[1/(Sqrt[cot[(c_.) + (d_.)*(x_)]*(e_.)]*(csc[(c_.) + (d_.)*(x_)]*(b_.) +
 (a_))), x_Symbol] :> Simp[1/a   Int[1/Sqrt[e*Cot[c + d*x]], x], x] - Simp[
b/a   Int[1/(Sqrt[e*Cot[c + d*x]]*(b + a*Sin[c + d*x])), x], x] /; FreeQ[{a
, b, c, d, e}, x] && NeQ[a^2 - b^2, 0]
```

3.315.4 Maple [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1918 vs. 2(343) = 686.

Time = 3.58 (sec) , antiderivative size = 1919, normalized size of antiderivative = 4.55

method	result	size
default	Expression too large to display	1919

input
```
int(1/(a+b*sec(d*x+c))/(e*tan(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2/d*2^(1/2)/((a^2-b^2)^(1/2)-a+b)/((a^2-b^2)^(1/2)+a-b)/(a^2-b^2)^(1/2)/
(a-b)/a*(3*I*EllipticPi((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2-1/2*I,1/2*2^(1
/2))*(a^2-b^2)^(1/2)*a^2*b-3*I*EllipticPi((csc(d*x+c)-cot(d*x+c)+1)^(1/2),
1/2-1/2*I,1/2*2^(1/2))*(a^2-b^2)^(1/2)*a*b^2-3*I*EllipticPi((csc(d*x+c)-co
t(d*x+c)+1)^(1/2),1/2+1/2*I,1/2*2^(1/2))*(a^2-b^2)^(1/2)*a^2*b+3*I*Ellipti
cPi((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2+1/2*I,1/2*2^(1/2))*(a^2-b^2)^(1/2)
*a*b^2-3*EllipticPi((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2-1/2*I,1/2*2^(1/2))
*(a^2-b^2)^(1/2)*a*b^2+3*EllipticPi((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2+1/
2*I,1/2*2^(1/2))*(a^2-b^2)^(1/2)*a^2*b-3*EllipticPi((csc(d*x+c)-cot(d*x+c)
+1)^(1/2),1/2+1/2*I,1/2*2^(1/2))*(a^2-b^2)^(1/2)*a*b^2+2*(a^2-b^2)^(1/2)*E
llipticPi((csc(d*x+c)-cot(d*x+c)+1)^(1/2),(a-b)/(a-b+((a-b)*(a+b))^(1/2)),
1/2*2^(1/2))*a*b^2+2*(a^2-b^2)^(1/2)*EllipticPi((csc(d*x+c)-cot(d*x+c)+1)^
(1/2),-(a-b)/(-a+b+((a-b)*(a+b))^(1/2)),1/2*2^(1/2))*a*b^2-I*EllipticPi((c
sc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2-1/2*I,1/2*2^(1/2))*(a^2-b^2)^(3/2)*b-I*E
llipticPi((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2-1/2*I,1/2*2^(1/2))*(a^2-b^2)
^(1/2)*a^3-I*EllipticPi((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2+1/2*I,1/2*2^(1
/2))*(a^2-b^2)^(3/2)*a-I*EllipticPi((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2+1/
2*I,1/2*2^(1/2))*(a^2-b^2)^(1/2)*b^3+I*EllipticPi((csc(d*x+c)-cot(d*x+c)+1
)^(1/2),1/2-1/2*I,1/2*2^(1/2))*(a^2-b^2)^(3/2)*a+I*EllipticPi((csc(d*x+c)-
cot(d*x+c)+1)^(1/2),1/2-1/2*I,1/2*2^(1/2))*(a^2-b^2)^(1/2)*b^3+I*Ellipt...
```

3.315.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\sec(c+dx))\sqrt{e\tan(c+dx)}}\,dx = \text{Timed out}$$

input
```
integrate(1/(a+b*sec(d*x+c))/(e*tan(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.315.6 Sympy [F]

$$\int \frac{1}{(a+b\sec(c+dx))\sqrt{e\tan(c+dx)}}\,dx = \int \frac{1}{\sqrt{e\tan(c+dx)}\,(a+b\sec(c+dx))}\,dx$$

input
```
integrate(1/(a+b*sec(d*x+c))/(e*tan(d*x+c))**(1/2),x)
```

output
```
Integral(1/(sqrt(e*tan(c + d*x))*(a + b*sec(c + d*x))), x)
```

3.315.7 Maxima [F]

$$\int \frac{1}{(a+b\sec(c+dx))\sqrt{e\tan(c+dx)}}\,dx = \int \frac{1}{(b\sec(dx+c)+a)\sqrt{e\tan(dx+c)}}\,dx$$

input
```
integrate(1/(a+b*sec(d*x+c))/(e*tan(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*sec(d*x + c) + a)*sqrt(e*tan(d*x + c))), x)
```

3.315.8 Giac [F]

$$\int \frac{1}{(a+b\sec(c+dx))\sqrt{e\tan(c+dx)}}\,dx = \int \frac{1}{(b\sec(dx+c)+a)\sqrt{e\tan(dx+c)}}\,dx$$

input
```
integrate(1/(a+b*sec(d*x+c))/(e*tan(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(1/((b*sec(d*x + c) + a)*sqrt(e*tan(d*x + c))), x)
```

3.315.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\sec(c+dx))\sqrt{e\tan(c+dx)}}\,dx = \int \frac{\cos(c+dx)}{\sqrt{e\tan(c+dx)}\,(b+a\cos(c+dx))}\,dx$$

input
```
int(1/((e*tan(c + d*x))^(1/2)*(a + b/cos(c + d*x))),x)
```

output
```
int(cos(c + d*x)/((e*tan(c + d*x))^(1/2)*(b + a*cos(c + d*x))), x)
```

3.316 $\int \frac{1}{(a+b\sec(c+dx))(e\tan(c+dx))^{3/2}}\,dx$

3.316.1 Optimal result . . . 2314
3.316.2 Mathematica [C] (warning: unable to verify) . . . 2315
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3.316.7 Maxima [F] . . . 2327
3.316.8 Giac [F] . . . 2327
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3.316.1 Optimal result

Integrand size = 25, antiderivative size = 863

$$\int \frac{1}{(a+b\sec(c+dx))(e\tan(c+dx))^{3/2}}\,dx = \frac{a\arctan\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,(a^2-b^2)\,de^{3/2}}$$
$$-\frac{b^2\arctan\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}a\,(a^2-b^2)\,de^{3/2}}-\frac{a\arctan\left(1+\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,(a^2-b^2)\,de^{3/2}}$$
$$+\frac{b^2\arctan\left(1+\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}a\,(a^2-b^2)\,de^{3/2}}-\frac{a\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}\,(a^2-b^2)\,de^{3/2}}$$
$$+\frac{b^2\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}a\,(a^2-b^2)\,de^{3/2}}$$
$$+\frac{a\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)+\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}\,(a^2-b^2)\,de^{3/2}}$$
$$-\frac{b^2\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)+\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}a\,(a^2-b^2)\,de^{3/2}}-\frac{2(a-b\sec(c+dx))}{(a^2-b^2)\,de\sqrt{e\tan(c+dx)}}$$
$$+\frac{2\sqrt{2}b^3\sqrt{\cos(c+dx)}\,\text{EllipticPi}\left(-\frac{\sqrt{a-b}}{\sqrt{a+b}},\arcsin\left(\frac{\sqrt{\sin(c+dx)}}{\sqrt{1+\cos(c+dx)}}\right),-1\right)\sqrt{e\tan(c+dx)}}{a(a-b)^{3/2}(a+b)^{3/2}de^2\sqrt{\sin(c+dx)}}$$
$$-\frac{2\sqrt{2}b^3\sqrt{\cos(c+dx)}\,\text{EllipticPi}\left(\frac{\sqrt{a-b}}{\sqrt{a+b}},\arcsin\left(\frac{\sqrt{\sin(c+dx)}}{\sqrt{1+\cos(c+dx)}}\right),-1\right)\sqrt{e\tan(c+dx)}}{a(a-b)^{3/2}(a+b)^{3/2}de^2\sqrt{\sin(c+dx)}}$$
$$+\frac{2b\cos(c+dx)E\left(c-\frac{\pi}{4}+dx\middle|2\right)\sqrt{e\tan(c+dx)}}{(a^2-b^2)\,de^2\sqrt{\sin(2c+2dx)}}-\frac{2b\cos(c+dx)(e\tan(c+dx))^{3/2}}{(a^2-b^2)\,de^3}$$

output
```
1/2*a*arctan(1-2^(1/2)*(e*tan(d*x+c))^(1/2)/e^(1/2))/(a^2-b^2)/d/e^(3/2)*2
^(1/2)-1/2*b^2*arctan(1-2^(1/2)*(e*tan(d*x+c))^(1/2)/e^(1/2))/a/(a^2-b^2)/
d/e^(3/2)*2^(1/2)-1/2*a*arctan(1+2^(1/2)*(e*tan(d*x+c))^(1/2)/e^(1/2))/(a^
2-b^2)/d/e^(3/2)*2^(1/2)+1/2*b^2*arctan(1+2^(1/2)*(e*tan(d*x+c))^(1/2)/e^(
1/2))/a/(a^2-b^2)/d/e^(3/2)*2^(1/2)-1/4*a*ln(e^(1/2)-2^(1/2)*(e*tan(d*x+c)
)^(1/2)+e^(1/2)*tan(d*x+c))/(a^2-b^2)/d/e^(3/2)*2^(1/2)+1/4*b^2*ln(e^(1/2)
-2^(1/2)*(e*tan(d*x+c))^(1/2)+e^(1/2)*tan(d*x+c))/a/(a^2-b^2)/d/e^(3/2)*2^
(1/2)+1/4*a*ln(e^(1/2)+2^(1/2)*(e*tan(d*x+c))^(1/2)+e^(1/2)*tan(d*x+c))/(a
^2-b^2)/d/e^(3/2)*2^(1/2)-1/4*b^2*ln(e^(1/2)+2^(1/2)*(e*tan(d*x+c))^(1/2)+
e^(1/2)*tan(d*x+c))/a/(a^2-b^2)/d/e^(3/2)*2^(1/2)-2*(a-b*sec(d*x+c))/(a^2-
b^2)/d/e/(e*tan(d*x+c))^(1/2)+2*b^3*EllipticPi(sin(d*x+c)^(1/2)/(1+cos(d*x
+c))^(1/2),-(a-b)^(1/2)/(a+b)^(1/2),I)*2^(1/2)*cos(d*x+c)^(1/2)*(e*tan(d*x
+c))^(1/2)/a/(a-b)^(3/2)/(a+b)^(3/2)/d/e^2/sin(d*x+c)^(1/2)-2*b^3*Elliptic
Pi(sin(d*x+c)^(1/2)/(1+cos(d*x+c))^(1/2),(a-b)^(1/2)/(a+b)^(1/2),I)*2^(1/2
)*cos(d*x+c)^(1/2)*(e*tan(d*x+c))^(1/2)/a/(a-b)^(3/2)/(a+b)^(3/2)/d/e^2/si
n(d*x+c)^(1/2)-2*b*cos(d*x+c)*(sin(c+1/4*Pi+d*x)^2)^(1/2)/sin(c+1/4*Pi+d*x
)*EllipticE(cos(c+1/4*Pi+d*x),2^(1/2))*(e*tan(d*x+c))^(1/2)/(a^2-b^2)/d/e^
2/sin(2*d*x+2*c)^(1/2)-2*b*cos(d*x+c)*(e*tan(d*x+c))^(3/2)/(a^2-b^2)/d/e^3
```

3.316.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 23.90 (sec) , antiderivative size = 1571, normalized size of antiderivative = 1.82

$$\int \frac{1}{(a+b\sec(c+dx))(e\tan(c+dx))^{3/2}}\,dx = \text{Too large to display}$$

input
```
Integrate[1/((a + b*Sec[c + d*x])*(e*Tan[c + d*x])^(3/2)),x]
```

output

```
((b + a*Cos[c + d*x])*Sec[c + d*x]*((-2*(b - a*Cos[c + d*x])*Csc[c + d*x])
/(-a^2 + b^2) + (2*b*Sin[c + d*x])/(-a^2 + b^2))*Tan[c + d*x]^2)/(d*(a + b
*Sec[c + d*x])*(e*Tan[c + d*x])^(3/2)) + ((b + a*Cos[c + d*x])*Sec[c + d*x
]*Tan[c + d*x]^(3/2)*(-1/12*((-a^2 + 3*b^2)*Sec[c + d*x]*(6*Sqrt[2]*(a^2 -
 b^2)*ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]] - 6*Sqrt[2]*a^2*ArcTan[1 + Sq
rt[2]*Sqrt[Tan[c + d*x]]] + 6*Sqrt[2]*b^2*ArcTan[1 + Sqrt[2]*Sqrt[Tan[c +
d*x]]] - (6 + 6*I)*Sqrt[b]*(a^2 - b^2)^(3/4)*ArcTan[1 - ((1 + I)*Sqrt[b]*S
qrt[Tan[c + d*x]])/(a^2 - b^2)^(1/4)] + (6 + 6*I)*Sqrt[b]*(a^2 - b^2)^(3/4
)*ArcTan[1 + ((1 + I)*Sqrt[b]*Sqrt[Tan[c + d*x]])/(a^2 - b^2)^(1/4)] - 3*S
qrt[2]*a^2*Log[1 - Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]] + 3*Sqrt[2]*
b^2*Log[1 - Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]] + 3*Sqrt[2]*a^2*Log
[1 + Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]] - 3*Sqrt[2]*b^2*Log[1 + Sq
rt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]] + (3 + 3*I)*Sqrt[b]*(a^2 - b^2)^(
3/4)*Log[Sqrt[a^2 - b^2] - (1 + I)*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Tan[c +
d*x]] + I*b*Tan[c + d*x]] - (3 + 3*I)*Sqrt[b]*(a^2 - b^2)^(3/4)*Log[Sqrt[a
^2 - b^2] + (1 + I)*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Tan[c + d*x]] + I*b*Tan
[c + d*x]] + 8*a*b*AppellF1[3/4, 1/2, 1, 7/4, -Tan[c + d*x]^2, (b^2*Tan[c
+ d*x]^2)/(a^2 - b^2)]*Tan[c + d*x]^(3/2))*(a + b*Sqrt[1 + Tan[c + d*x]^2]
))/((a^3 - a*b^2)*(b + a*Cos[c + d*x])*(1 + Tan[c + d*x]^2)) + (b*Cos[2*(c
 + d*x)]*Sec[c + d*x]^2*(-84*Sqrt[2]*b*ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + ...
```

3.316.3 Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(e\tan(c+dx))^{3/2}(a+b\sec(c+dx))}\,dx$$

$$\downarrow 3042$$

$$\int \frac{1}{\left(-e\cot\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)}dx$$

$$\downarrow 4381$$

$$\frac{b^2\int\frac{\sqrt{e\tan(c+dx)}}{a+b\sec(c+dx)}dx}{e^2\left(a^2-b^2\right)}+\frac{\int\frac{a-b\sec(c+dx)}{(e\tan(c+dx))^{3/2}}dx}{a^2-b^2}$$

$$\downarrow 3042$$

$$\frac{b^2\int\frac{\sqrt{-e\cot\left(c+dx+\frac{\pi}{2}\right)}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{e^2\left(a^2-b^2\right)}+\frac{\int\frac{a-b\csc\left(c+dx+\frac{\pi}{2}\right)}{\left(-e\cot\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{a^2-b^2}$$

$$\downarrow 4370$$

$$\frac{b^2 \int \frac{\sqrt{-e \cot(c+dx+\frac{\pi}{2})}}{a+b \csc(c+dx+\frac{\pi}{2})} dx}{e^2 (a^2-b^2)} + \frac{\frac{2 \int -\frac{1}{2}(a+b \sec(c+dx)) \sqrt{e \tan(c+dx)} dx}{e^2} - \frac{2(a-b \sec(c+dx))}{de \sqrt{e \tan(c+dx)}}}{a^2-b^2}$$

$$\downarrow 27$$

$$\frac{b^2 \int \frac{\sqrt{-e \cot(c+dx+\frac{\pi}{2})}}{a+b \csc(c+dx+\frac{\pi}{2})} dx}{e^2 (a^2-b^2)} + \frac{-\frac{\int (a+b \sec(c+dx)) \sqrt{e \tan(c+dx)} dx}{e^2} - \frac{2(a-b \sec(c+dx))}{de \sqrt{e \tan(c+dx)}}}{a^2-b^2}$$

$$\downarrow 3042$$

$$\frac{b^2 \int \frac{\sqrt{-e \cot(c+dx+\frac{\pi}{2})}}{a+b \csc(c+dx+\frac{\pi}{2})} dx}{e^2 (a^2-b^2)} + \frac{-\frac{\int \sqrt{-e \cot(c+dx+\frac{\pi}{2})} (a+b \csc(c+dx+\frac{\pi}{2})) dx}{e^2} - \frac{2(a-b \sec(c+dx))}{de \sqrt{e \tan(c+dx)}}}{a^2-b^2}$$

$$\downarrow 4372$$

$$\frac{b^2 \int \frac{\sqrt{-e \cot(c+dx+\frac{\pi}{2})}}{a+b \csc(c+dx+\frac{\pi}{2})} dx}{e^2 (a^2-b^2)} + \frac{-\frac{a \int \sqrt{e \tan(c+dx)} dx + b \int \sec(c+dx) \sqrt{e \tan(c+dx)} dx}{e^2} - \frac{2(a-b \sec(c+dx))}{de \sqrt{e \tan(c+dx)}}}{a^2-b^2}$$

$$\downarrow 3042$$

$$\frac{b^2 \int \frac{\sqrt{-e \cot(c+dx+\frac{\pi}{2})}}{a+b \csc(c+dx+\frac{\pi}{2})} dx}{e^2 (a^2-b^2)} + \frac{-\frac{a \int \sqrt{e \tan(c+dx)} dx + b \int \sec(c+dx) \sqrt{e \tan(c+dx)} dx}{e^2} - \frac{2(a-b \sec(c+dx))}{de \sqrt{e \tan(c+dx)}}}{a^2-b^2}$$

$$\downarrow 3093$$

$$\frac{b^2 \int \frac{\sqrt{-e \cot(c+dx+\frac{\pi}{2})}}{a+b \csc(c+dx+\frac{\pi}{2})} dx}{e^2 (a^2-b^2)} + \frac{-\frac{a \int \sqrt{e \tan(c+dx)} dx + b \left(\frac{2 \cos(c+dx) (e \tan(c+dx))^{3/2}}{de} - 2 \int \cos(c+dx) \sqrt{e \tan(c+dx)} dx \right)}{e^2} - \frac{2(a-b \sec(c+dx))}{de \sqrt{e \tan(c+dx)}}}{a^2-b^2}$$

$$\downarrow 3042$$

$$\frac{b^2 \int \frac{\sqrt{-e \cot(c+dx+\frac{\pi}{2})}}{a+b \csc(c+dx+\frac{\pi}{2})} dx}{e^2 (a^2-b^2)} + \frac{-\frac{a \int \sqrt{e \tan(c+dx)} dx + b \left(\frac{2 \cos(c+dx) (e \tan(c+dx))^{3/2}}{de} - 2 \int \frac{\sqrt{e \tan(c+dx)}}{\sec(c+dx)} dx \right)}{e^2} - \frac{2(a-b \sec(c+dx))}{de \sqrt{e \tan(c+dx)}}}{a^2-b^2}$$

$$\downarrow 3095$$

$$\frac{b^2\int\frac{\sqrt{-e\cot\left(c+dx+\frac{\pi}{2}\right)}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{e^2\left(a^2-b^2\right)}+\frac{-\frac{a\int\sqrt{e\tan(c+dx)}dx+b\left(\frac{2\cos(c+dx)(e\tan(c+dx))^{3/2}}{de}-\frac{2\sqrt{\cos(c+dx)}\sqrt{e\tan(c+dx)}\int\sqrt{\cos(c+dx)}\sqrt{\sin(c+dx)}dx}{\sqrt{\sin(c+dx)}}\right)}{e^2}-\frac{2(a-b\sec(c+dx))}{de\sqrt{e\tan(c+dx)}}}{a^2-b^2}$$

$$\downarrow 3042$$

$$\frac{b^2\int\frac{\sqrt{-e\cot\left(c+dx+\frac{\pi}{2}\right)}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{e^2\left(a^2-b^2\right)}+\frac{-\frac{a\int\sqrt{e\tan(c+dx)}dx+b\left(\frac{2\cos(c+dx)(e\tan(c+dx))^{3/2}}{de}-\frac{2\sqrt{\cos(c+dx)}\sqrt{e\tan(c+dx)}\int\sqrt{\cos(c+dx)}\sqrt{\sin(c+dx)}dx}{\sqrt{\sin(c+dx)}}\right)}{e^2}-\frac{2(a-b\sec(c+dx))}{de\sqrt{e\tan(c+dx)}}}{a^2-b^2}$$

$$\downarrow 3052$$

$$\frac{b^2\int\frac{\sqrt{-e\cot\left(c+dx+\frac{\pi}{2}\right)}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{e^2\left(a^2-b^2\right)}+\frac{-\frac{a\int\sqrt{e\tan(c+dx)}dx+b\left(\frac{2\cos(c+dx)(e\tan(c+dx))^{3/2}}{de}-\frac{2\cos(c+dx)\sqrt{e\tan(c+dx)}\int\sqrt{\sin(2c+2dx)}dx}{\sqrt{\sin(2c+2dx)}}\right)}{e^2}-\frac{2(a-b\sec(c+dx))}{de\sqrt{e\tan(c+dx)}}}{a^2-b^2}$$

$$\downarrow 3042$$

$$\frac{b^2\int\frac{\sqrt{-e\cot\left(c+dx+\frac{\pi}{2}\right)}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{e^2\left(a^2-b^2\right)}+\frac{-\frac{a\int\sqrt{e\tan(c+dx)}dx+b\left(\frac{2\cos(c+dx)(e\tan(c+dx))^{3/2}}{de}-\frac{2\cos(c+dx)\sqrt{e\tan(c+dx)}\int\sqrt{\sin(2c+2dx)}dx}{\sqrt{\sin(2c+2dx)}}\right)}{e^2}-\frac{2(a-b\sec(c+dx))}{de\sqrt{e\tan(c+dx)}}}{a^2-b^2}$$

$$\downarrow 3119$$

$$\frac{b^2\int\frac{\sqrt{-e\cot\left(c+dx+\frac{\pi}{2}\right)}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{e^2\left(a^2-b^2\right)}+\frac{-\frac{a\int\sqrt{e\tan(c+dx)}dx+b\left(\frac{2\cos(c+dx)(e\tan(c+dx))^{3/2}}{de}-\frac{2\cos(c+dx)E\left(c+dx-\frac{\pi}{4}\middle|2\right)\sqrt{e\tan(c+dx)}}{d\sqrt{\sin(2c+2dx)}}\right)}{e^2}-\frac{2(a-b\sec(c+dx))}{de\sqrt{e\tan(c+dx)}}}{a^2-b^2}$$

$$\downarrow 3957$$

$$\frac{b^2\int\frac{\sqrt{-e\cot\left(c+dx+\frac{\pi}{2}\right)}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{e^2\left(a^2-b^2\right)}+$$
$$-\frac{\frac{ae\int\frac{\sqrt{e\tan(c+dx)}}{\tan^2(c+dx)e^2+e^2}d(e\tan(c+dx))}{d}+b\left(\frac{2\cos(c+dx)(e\tan(c+dx))^{3/2}}{de}-\frac{2\cos(c+dx)E\left(c+dx-\frac{\pi}{4}\middle|2\right)\sqrt{e\tan(c+dx)}}{d\sqrt{\sin(2c+2dx)}}\right)}{e^2}-\frac{2(a-b\sec(c+dx))}{de\sqrt{e\tan(c+dx)}}$$
$$a^2-b^2$$

$$\downarrow 266$$

$$\frac{b^2\int\frac{\sqrt{-e\cot\left(c+dx+\frac{\pi}{2}\right)}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{e^2\left(a^2-b^2\right)}+$$
$$-\frac{\frac{2ae\int\frac{e^2\tan^2(c+dx)}{e^4\tan^4(c+dx)+e^2}d\sqrt{e\tan(c+dx)}}{d}+b\left(\frac{2\cos(c+dx)(e\tan(c+dx))^{3/2}}{de}-\frac{2\cos(c+dx)E\left(c+dx-\frac{\pi}{4}\middle|2\right)\sqrt{e\tan(c+dx)}}{d\sqrt{\sin(2c+2dx)}}\right)}{e^2}-\frac{2(a-b\sec(c+dx))}{de\sqrt{e\tan(c+dx)}}$$
$$a^2-b^2$$

$$\downarrow 826$$

$$\frac{b^2\int\frac{\sqrt{-e\cot\left(c+dx+\frac{\pi}{2}\right)}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{e^2\left(a^2-b^2\right)}+$$
$$-\frac{\frac{2ae\left(\frac{1}{2}\int\frac{e^2\tan^2(c+dx)+e}{e^4\tan^4(c+dx)+e^2}d\sqrt{e\tan(c+dx)}-\frac{1}{2}\int\frac{e-e^2\tan^2(c+dx)}{e^4\tan^4(c+dx)+e^2}d\sqrt{e\tan(c+dx)}\right)}{d}+b\left(\frac{2\cos(c+dx)(e\tan(c+dx))^{3/2}}{de}-\frac{2\cos(c+dx)E\left(c+dx-\frac{\pi}{4}\middle|2\right)\sqrt{e\tan(c+dx)}}{d\sqrt{\sin(2c+2dx)}}\right)}{e^2}$$
$$a^2-b^2$$

$$\downarrow 1476$$

$$\frac{b^2\int\frac{\sqrt{-e\cot\left(c+dx+\frac{\pi}{2}\right)}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{e^2\left(a^2-b^2\right)}+$$
$$-\frac{\frac{2ae\left(\frac{1}{2}\left(\frac{1}{2}\int\frac{1}{e^2\tan^2(c+dx)-\sqrt{2}e^{3/2}\tan(c+dx)+e}d\sqrt{e\tan(c+dx)}+\frac{1}{2}\int\frac{1}{e^2\tan^2(c+dx)+\sqrt{2}e^{3/2}\tan(c+dx)+e}d\sqrt{e\tan(c+dx)}\right)-\frac{1}{2}\int\frac{e-e^2\tan^2(c+dx)}{e^4\tan^4(c+dx)+e^2}d\sqrt{e\tan}\right.}{d}}{e^2}$$
$$a^2-b^2$$

$$\downarrow 1082$$

$$\frac{b^2\int\frac{\sqrt{-e\cot\left(c+dx+\frac{\pi}{2}\right)}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{e^2\left(a^2-b^2\right)}+$$
$$-\frac{\frac{2ae\left(\frac{1}{2}\left(\frac{\int\frac{1}{-e^2\tan^2(c+dx)-1}d\left(1-\sqrt{2}\sqrt{e}\tan(c+dx)\right)}{\sqrt{2}\sqrt{e}}-\frac{\int\frac{1}{-e^2\tan^2(c+dx)-1}d\left(\sqrt{2}\sqrt{e}\tan(c+dx)+1\right)}{\sqrt{2}\sqrt{e}}\right)-\frac{1}{2}\int\frac{e-e^2\tan^2(c+dx)}{e^4\tan^4(c+dx)+e^2}d\sqrt{e\tan(c+dx)}\right)}{d}+b\left(\frac{2\cos(c+}{e^2}\right.}$$
$$a^2-b^2$$

$$\downarrow 217$$

$$-\frac{2ae\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{e}\tan(c+dx)+1\right)}{\sqrt{2}\sqrt{e}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{e}\tan(c+dx)\right)}{\sqrt{2}\sqrt{e}}\right)-\frac{1}{2}\int\frac{e-e^2\tan^2(c+dx)}{e^4\tan^4(c+dx)+e^2}d\sqrt{e\tan(c+dx)}\right)}{d}+b\left(\frac{2\cos(c+dx)(e\tan(c+dx))^{3/2}}{de}-\frac{2\cos(c+d}{e^2}\right)}{a^2-b^2}$$

$$\frac{b^2\int\frac{\sqrt{-e\cot\left(c+dx+\frac{\pi}{2}\right)}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{e^2\left(a^2-b^2\right)}$$

$$\downarrow 1479$$

$$-\frac{2ae\left(\frac{1}{2}\left(\frac{\int-\frac{\sqrt{2}\sqrt{e}-2\sqrt{e\tan(c+dx)}}{e^2\tan^2(c+dx)-\sqrt{2}e^{3/2}\tan(c+dx)+e}d\sqrt{e\tan(c+dx)}}{2\sqrt{2}\sqrt{e}}+\frac{\int-\frac{\sqrt{2}\left(\sqrt{e}+\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{e^2\tan^2(c+dx)+\sqrt{2}e^{3/2}\tan(c+dx)+e}d\sqrt{e\tan(c+dx)}}{2\sqrt{2}\sqrt{e}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{e}\tan(c+dx)+\right.}{\sqrt{2}\sqrt{e}}\right.\right.}{d}}{e^2}}{a^2-b^2}$$

$$\frac{b^2\int\frac{\sqrt{-e\cot\left(c+dx+\frac{\pi}{2}\right)}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{e^2\left(a^2-b^2\right)}$$

$$\downarrow 25$$

$$-\frac{2ae\left(\frac{1}{2}\left(-\frac{\int\frac{\sqrt{2}\sqrt{e}-2\sqrt{e\tan(c+dx)}}{e^2\tan^2(c+dx)-\sqrt{2}e^{3/2}\tan(c+dx)+e}d\sqrt{e\tan(c+dx)}}{2\sqrt{2}\sqrt{e}}-\frac{\int\frac{\sqrt{2}\left(\sqrt{e}+\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{e^2\tan^2(c+dx)+\sqrt{2}e^{3/2}\tan(c+dx)+e}d\sqrt{e\tan(c+dx)}}{2\sqrt{2}\sqrt{e}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{e}\tan(c+dx)+1\right)}{\sqrt{2}\sqrt{e}}\right.\right.}{d}}{e^2}}{a^2-b^2}$$

$$\frac{b^2\int\frac{\sqrt{-e\cot\left(c+dx+\frac{\pi}{2}\right)}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{e^2\left(a^2-b^2\right)}$$

$$\downarrow 27$$

$$-\frac{2ae\left(\frac{1}{2}\left(-\frac{\int\frac{\sqrt{2}\sqrt{e}-2\sqrt{e\tan(c+dx)}}{e^2\tan^2(c+dx)-\sqrt{2}e^{3/2}\tan(c+dx)+e}d\sqrt{e\tan(c+dx)}}{2\sqrt{2}\sqrt{e}}-\frac{\int\frac{\sqrt{e}+\sqrt{2}\sqrt{e\tan(c+dx)}}{e^2\tan^2(c+dx)+\sqrt{2}e^{3/2}\tan(c+dx)+e}d\sqrt{e\tan(c+dx)}}{2\sqrt{e}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{e}\tan(c+dx)+1\right)}{\sqrt{2}\sqrt{e}}\right.\right.}{d}}{e^2}}{a^2-b^2}$$

$$\frac{b^2\int\frac{\sqrt{-e\cot\left(c+dx+\frac{\pi}{2}\right)}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{e^2\left(a^2-b^2\right)}$$

$$\downarrow 1103$$

$$\frac{b^2 \int \frac{\sqrt{-e \cot\left(c+dx+\frac{\pi}{2}\right)}}{a+b \csc\left(c+dx+\frac{\pi}{2}\right)} dx}{e^2 \left(a^2-b^2\right)} + \frac{2ae \left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{e}\tan(c+dx)+1\right)}{\sqrt{2}\sqrt{e}} - \frac{\arctan\left(1-\sqrt{2}\sqrt{e}\tan(c+dx)\right)}{\sqrt{2}\sqrt{e}}\right)+\frac{1}{2}\left(\frac{\log\left(-\sqrt{2}e^{3/2}\tan(c+dx)+e^2\tan^2(c+dx)+e\right)}{2\sqrt{2}\sqrt{e}} - \frac{\log\left(\sqrt{2}e^{3/2}\tan(c+dx)+e^2\tan^2(c+\right.\right.\right.}{d}$$

$$- \frac{e^2}{a^2-b^2}$$

$$\downarrow 4378$$

$$\frac{b^2\left(\frac{\int \sqrt{e\tan(c+dx)}dx}{a} - \frac{b\int \frac{\sqrt{e\tan(c+dx)}}{b+a\cos(c+dx)}dx}{a}\right)}{e^2\left(a^2-b^2\right)} + \frac{2ae \left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{e}\tan(c+dx)+1\right)}{\sqrt{2}\sqrt{e}} - \frac{\arctan\left(1-\sqrt{2}\sqrt{e}\tan(c+dx)\right)}{\sqrt{2}\sqrt{e}}\right)+\frac{1}{2}\left(\frac{\log\left(-\sqrt{2}e^{3/2}\tan(c+dx)+e^2\tan^2(c+dx)+e\right)}{2\sqrt{2}\sqrt{e}} - \frac{\log\left(\sqrt{2}e^{3/2}\tan(c+dx)+e^2\tan^2(c+\right.\right.\right.}{d}$$

$$- \frac{e^2}{a^2-b^2}$$

$$\downarrow 3042$$

$$\frac{b^2\left(\frac{\int \sqrt{e\tan(c+dx)}dx}{a} - \frac{b\int \frac{\sqrt{-e\cot\left(c+dx+\frac{\pi}{2}\right)}}{b+a\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{a}\right)}{e^2\left(a^2-b^2\right)} + \frac{2ae \left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{e}\tan(c+dx)+1\right)}{\sqrt{2}\sqrt{e}} - \frac{\arctan\left(1-\sqrt{2}\sqrt{e}\tan(c+dx)\right)}{\sqrt{2}\sqrt{e}}\right)+\frac{1}{2}\left(\frac{\log\left(-\sqrt{2}e^{3/2}\tan(c+dx)+e^2\tan^2(c+dx)+e\right)}{2\sqrt{2}\sqrt{e}} - \frac{\log\left(\sqrt{2}e^{3/2}\tan(c+dx)+e^2\tan^2(c+\right.\right.\right.}{d}$$

$$- \frac{e^2}{a^2-b^2}$$

$$\downarrow 3212$$

$$\frac{b^2\left(\frac{\int \sqrt{e\tan(c+dx)}dx}{a} - \frac{b\sqrt{e\tan(c+dx)}\sqrt{e\cot(c+dx)}\int \frac{1}{(b+a\cos(c+dx))\sqrt{e\cot(c+dx)}}dx}{a}\right)}{e^2\left(a^2-b^2\right)} + \frac{2ae \left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{e}\tan(c+dx)+1\right)}{\sqrt{2}\sqrt{e}} - \frac{\arctan\left(1-\sqrt{2}\sqrt{e}\tan(c+dx)\right)}{\sqrt{2}\sqrt{e}}\right)+\frac{1}{2}\left(\frac{\log\left(-\sqrt{2}e^{3/2}\tan(c+dx)+e^2\tan^2(c+dx)+e\right)}{2\sqrt{2}\sqrt{e}} - \frac{\log\left(\sqrt{2}e^{3/2}\tan(c+dx)+e^2\tan^2(c+\right.\right.\right.}{d}$$

$$- \frac{e^2}{a^2-b^2}$$

$$\downarrow 3042$$

$$\frac{b^2\left(\frac{\int\sqrt{e\tan(c+dx)}dx}{a}-\frac{b\sqrt{e\tan(c+dx)}\sqrt{e\cot(c+dx)}\int\frac{1}{\left(b-a\sin\left(c+dx-\frac{\pi}{2}\right)\right)\sqrt{-e\tan\left(c+dx-\frac{\pi}{2}\right)}}dx}{a}\right)}{e^2\left(a^2-b^2\right)}+$$
$$-\frac{\frac{2ae\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{e}\tan(c+dx)+1\right)}{\sqrt{2}\sqrt{e}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{e}\tan(c+dx)\right)}{\sqrt{2}\sqrt{e}}\right)+\frac{1}{2}\left(\frac{\log\left(-\sqrt{2}e^{3/2}\tan(c+dx)+e^2\tan^2(c+dx)+e\right)}{2\sqrt{2}\sqrt{e}}-\frac{\log\left(\sqrt{2}e^{3/2}\tan(c+dx)+e^2\tan^2(c+\right.\right.}{d}}{e^2}}{a^2-b^2}$$

$$\downarrow 3209$$

$$\frac{b^2\left(\frac{\int\sqrt{e\tan(c+dx)}dx}{a}-\frac{b\sqrt{-\cos(c+dx)}\sqrt{e\tan(c+dx)}\int\frac{\sqrt{\sin(c+dx)}}{\sqrt{-\cos(c+dx)}(b+a\cos(c+dx))}dx}{a\sqrt{\sin(c+dx)}}\right)}{e^2\left(a^2-b^2\right)}+$$
$$-\frac{\frac{2ae\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{e}\tan(c+dx)+1\right)}{\sqrt{2}\sqrt{e}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{e}\tan(c+dx)\right)}{\sqrt{2}\sqrt{e}}\right)+\frac{1}{2}\left(\frac{\log\left(-\sqrt{2}e^{3/2}\tan(c+dx)+e^2\tan^2(c+dx)+e\right)}{2\sqrt{2}\sqrt{e}}-\frac{\log\left(\sqrt{2}e^{3/2}\tan(c+dx)+e^2\tan^2(c+\right.\right.}{d}}{e^2}}{a^2-b^2}$$

input
```
Int[1/((a + b*Sec[c + d*x])*(e*Tan[c + d*x])^(3/2)),x]
```

output
```
$Aborted
```

3.316.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3052
```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Simp[Sqrt[a*Sin[e + f*x]]*(Sqrt[b*Cos[e + f*x]]/Sqrt[Sin[2*e
 + 2*f*x]])   Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

rule 3093
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[a^2*(a*Sec[e + f*x])^(m - 2)*((b*Tan[e + f*x])^(n +
1)/(b*f*(m + n - 1))), x] + Simp[a^2*((m - 2)/(m + n - 1))   Int[(a*Sec[e +
 f*x])^(m - 2)*(b*Tan[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (
GtQ[m, 1] || (EqQ[m, 1] && EqQ[n, 1/2])) && NeQ[m + n - 1, 0] && IntegersQ[
2*m, 2*n]
```

rule 3095
```
Int[Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]]/sec[(e_.) + (f_.)*(x_)], x_Symbol]
:> Simp[Sqrt[Cos[e + f*x]]*(Sqrt[b*Tan[e + f*x]]/Sqrt[Sin[e + f*x]])   Int[
Sqrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]], x], x] /; FreeQ[{b, e, f}, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3209
```
Int[1/(((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(g_)*tan[(e_.) + (f_.)*(
x_)]]), x_Symbol] :> Simp[Sqrt[Sin[e + f*x]]/(Sqrt[Cos[e + f*x]]*Sqrt[g*Tan
[e + f*x]])   Int[Sqrt[Cos[e + f*x]]/(Sqrt[Sin[e + f*x]]*(a + b*Sin[e + f*x
])), x], x] /; FreeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

rule 3212
```
Int[(cot[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Simp[g^(2*IntPart[p])*(g*Cot[e + f*x])^FracPart[p]
*(g*Tan[e + f*x])^FracPart[p]   Int[(a + b*Sin[e + f*x])^m/(g*Tan[e + f*x])
^p, x], x] /; FreeQ[{a, b, e, f, g, m, p}, x] &&  !IntegerQ[p]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

rule 4370
```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(-(e*Cot[c + d*x])^(m + 1))*((a + b*Csc[c + d*x])/(
d*e*(m + 1))), x] - Simp[1/(e^2*(m + 1))   Int[(e*Cot[c + d*x])^(m + 2)*(a*
(m + 1) + b*(m + 2)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && L
tQ[m, -1]
```

rule 4372
```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(e*Cot[c + d*x])^m, x], x] + Simp[b   Int[
(e*Cot[c + d*x])^m*Csc[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, m}, x]
```

rule 4378
```
Int[Sqrt[cot[(c_.) + (d_.)*(x_)]*(e_.)]/(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a
_)), x_Symbol] :> Simp[1/a   Int[Sqrt[e*Cot[c + d*x]], x], x] - Simp[b/a
Int[Sqrt[e*Cot[c + d*x]]/(b + a*Sin[c + d*x]), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && NeQ[a^2 - b^2, 0]
```

rule 4381
```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)/(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[1/(a^2 - b^2)   Int[(e*Cot[c + d*x])^m*(a - b*Csc[c
 + d*x]), x], x] + Simp[b^2/(e^2*(a^2 - b^2))   Int[(e*Cot[c + d*x])^(m + 2
)/(a + b*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^
2, 0] && ILtQ[m + 1/2, 0]
```

3.316.4 Maple [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 4015 vs. $2(746) = 1492$.

Time = 3.27 (sec) , antiderivative size = 4016, normalized size of antiderivative = 4.65

method	result	size
default	Expression too large to display	4016

input
```
int(1/(a+b*sec(d*x+c))/(e*tan(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/d*2^(1/2)*b/((a^2-b^2)^(1/2)-a+b)/((a^2-b^2)^(1/2)+a-b)/(a-b)/a/(a+b)*(
1-cos(d*x+c))*(-((1-cos(d*x+c))*((1-cos(d*x+c))^2*csc(d*x+c)^2-1)*csc(d*x+
c))^(1/2)*(csc(d*x+c)-cot(d*x+c)+1)^(1/2)*(2-2*csc(d*x+c)+2*cot(d*x+c))^(1
/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)*EllipticPi((csc(d*x+c)-cot(d*x+c)+1)^(1/
2),1/2-1/2*I,1/2*2^(1/2))*a^3-2*((1-cos(d*x+c))^3*csc(d*x+c)^3+cot(d*x+c)-
csc(d*x+c))^(1/2)*a*b^2*(1-cos(d*x+c))^2*csc(d*x+c)^2+4*((1-cos(d*x+c))^3*
csc(d*x+c)^3+cot(d*x+c)-csc(d*x+c))^(1/2)*a^2*b*(1-cos(d*x+c))^2*csc(d*x+c
)^2+((1-cos(d*x+c))*((1-cos(d*x+c))^2*csc(d*x+c)^2-1)*csc(d*x+c))^(1/2)*(c
sc(d*x+c)-cot(d*x+c)+1)^(1/2)*(2-2*csc(d*x+c)+2*cot(d*x+c))^(1/2)*(cot(d*x
+c)-csc(d*x+c))^(1/2)*(a^2-b^2)^(1/2)*EllipticPi((csc(d*x+c)-cot(d*x+c)+1)
^(1/2),(a-b)/(a-b+((a-b)*(a+b))^(1/2)),1/2*2^(1/2))*b^2-((1-cos(d*x+c))*((
1-cos(d*x+c))^2*csc(d*x+c)^2-1)*csc(d*x+c))^(1/2)*(csc(d*x+c)-cot(d*x+c)+1
)^(1/2)*(2-2*csc(d*x+c)+2*cot(d*x+c))^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)*
(a^2-b^2)^(1/2)*EllipticPi((csc(d*x+c)-cot(d*x+c)+1)^(1/2),-(a-b)/(-a+b+((
a-b)*(a+b))^(1/2)),1/2*2^(1/2))*b^2-((1-cos(d*x+c))*((1-cos(d*x+c))^2*csc(
d*x+c)^2-1)*csc(d*x+c))^(1/2)*(csc(d*x+c)-cot(d*x+c)+1)^(1/2)*(2-2*csc(d*x
+c)+2*cot(d*x+c))^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)*EllipticPi((csc(d*x+
c)-cot(d*x+c)+1)^(1/2),(a-b)/(a-b+((a-b)*(a+b))^(1/2)),1/2*2^(1/2))*a*b^2+
((1-cos(d*x+c))*((1-cos(d*x+c))^2*csc(d*x+c)^2-1)*csc(d*x+c))^(1/2)*(csc(d
*x+c)-cot(d*x+c)+1)^(1/2)*(2-2*csc(d*x+c)+2*cot(d*x+c))^(1/2)*(cot(d*x+...
```

3.316.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\sec(c+dx))(e\tan(c+dx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/(a+b*sec(d*x+c))/(e*tan(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.316.6 Sympy [F]

$$\int \frac{1}{(a+b\sec(c+dx))(e\tan(c+dx))^{3/2}}\,dx = \int \frac{1}{(e\tan(c+dx))^{\frac{3}{2}}\,(a+b\sec(c+dx))}\,dx$$

input
```
integrate(1/(a+b*sec(d*x+c))/(e*tan(d*x+c))**(3/2),x)
```

output
```
Integral(1/((e*tan(c + d*x))**(3/2)*(a + b*sec(c + d*x))), x)
```

3.316.7 Maxima [F]

$$\int \frac{1}{(a+b\sec(c+dx))(e\tan(c+dx))^{3/2}}\,dx = \int \frac{1}{(b\sec(dx+c)+a)\,(e\tan(dx+c))^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a+b*sec(d*x+c))/(e*tan(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*sec(d*x + c) + a)*(e*tan(d*x + c))^(3/2)), x)
```

3.316.8 Giac [F]

$$\int \frac{1}{(a+b\sec(c+dx))(e\tan(c+dx))^{3/2}}\,dx = \int \frac{1}{(b\sec(dx+c)+a)\,(e\tan(dx+c))^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a+b*sec(d*x+c))/(e*tan(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
integrate(1/((b*sec(d*x + c) + a)*(e*tan(d*x + c))^(3/2)), x)
```

3.316.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\sec(c+dx))(e\tan(c+dx))^{3/2}}\,dx = \int \frac{\cos(c+dx)}{(e\tan(c+dx))^{3/2}\,(b+a\cos(c+dx))}\,dx$$

input
```
int(1/((e*tan(c + d*x))^(3/2)*(a + b/cos(c + d*x))),x)
```

output
```
int(cos(c + d*x)/((e*tan(c + d*x))^(3/2)*(b + a*cos(c + d*x))), x)
```

3.317 $\int \frac{1}{(a+b\sec(c+dx))(e\tan(c+dx))^{5/2}}\,dx$

3.317.1 Optimal result 2330
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3.317.9 Mupad [F(-1)] 2343

3.317.1 Optimal result

Integrand size = 25, antiderivative size = 836

$$\int \frac{1}{(a+b\sec(c+dx))(e\tan(c+dx))^{5/2}}\,dx = \frac{a\arctan\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,(a^2-b^2)\,de^{5/2}}$$
$$-\frac{b^2\arctan\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}a\,(a^2-b^2)\,de^{5/2}}$$
$$-\frac{a\arctan\left(1+\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,(a^2-b^2)\,de^{5/2}}+\frac{b^2\arctan\left(1+\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}a\,(a^2-b^2)\,de^{5/2}}$$
$$+\frac{a\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}\,(a^2-b^2)\,de^{5/2}}$$
$$-\frac{b^2\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}a\,(a^2-b^2)\,de^{5/2}}$$
$$-\frac{a\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)+\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}\,(a^2-b^2)\,de^{5/2}}$$
$$+\frac{b^2\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)+\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}a\,(a^2-b^2)\,de^{5/2}}$$
$$-\frac{2(a-b\sec(c+dx))}{3\,(a^2-b^2)\,de(e\tan(c+dx))^{3/2}}$$
$$-\frac{2\sqrt{2}b^3\,\text{EllipticPi}\left(\frac{b}{a-\sqrt{a^2-b^2}},\arcsin\left(\frac{\sqrt{-\cos(c+dx)}}{\sqrt{1+\sin(c+dx)}}\right),-1\right)\sqrt{\sin(c+dx)}}{a\,(a^2-b^2)^{3/2}\,de^2\sqrt{-\cos(c+dx)}\sqrt{e\tan(c+dx)}}$$
$$+\frac{2\sqrt{2}b^3\,\text{EllipticPi}\left(\frac{b}{a+\sqrt{a^2-b^2}},\arcsin\left(\frac{\sqrt{-\cos(c+dx)}}{\sqrt{1+\sin(c+dx)}}\right),-1\right)\sqrt{\sin(c+dx)}}{a\,(a^2-b^2)^{3/2}\,de^2\sqrt{-\cos(c+dx)}\sqrt{e\tan(c+dx)}}$$
$$+\frac{b\,\text{EllipticF}\left(c-\frac{\pi}{4}+dx,2\right)\sec(c+dx)\sqrt{\sin(2c+2dx)}}{3\,(a^2-b^2)\,de^2\sqrt{e\tan(c+dx)}}$$

output
```
1/2*a*arctan(1-2^(1/2)*(e*tan(d*x+c))^(1/2)/e^(1/2))/(a^2-b^2)/d/e^(5/2)*2
^(1/2)-1/2*b^2*arctan(1-2^(1/2)*(e*tan(d*x+c))^(1/2)/e^(1/2))/a/(a^2-b^2)/
d/e^(5/2)*2^(1/2)-1/2*a*arctan(1+2^(1/2)*(e*tan(d*x+c))^(1/2)/e^(1/2))/(a^
2-b^2)/d/e^(5/2)*2^(1/2)+1/2*b^2*arctan(1+2^(1/2)*(e*tan(d*x+c))^(1/2)/e^(
1/2))/a/(a^2-b^2)/d/e^(5/2)*2^(1/2)+1/4*a*ln(e^(1/2)-2^(1/2)*(e*tan(d*x+c)
)^(1/2)+e^(1/2)*tan(d*x+c))/(a^2-b^2)/d/e^(5/2)*2^(1/2)-1/4*b^2*ln(e^(1/2)
-2^(1/2)*(e*tan(d*x+c))^(1/2)+e^(1/2)*tan(d*x+c))/a/(a^2-b^2)/d/e^(5/2)*2^
(1/2)-1/4*a*ln(e^(1/2)+2^(1/2)*(e*tan(d*x+c))^(1/2)+e^(1/2)*tan(d*x+c))/(a
^2-b^2)/d/e^(5/2)*2^(1/2)+1/4*b^2*ln(e^(1/2)+2^(1/2)*(e*tan(d*x+c))^(1/2)+
e^(1/2)*tan(d*x+c))/a/(a^2-b^2)/d/e^(5/2)*2^(1/2)-2*b^3*EllipticPi((-cos(d
*x+c))^(1/2)/(1+sin(d*x+c))^(1/2),b/(a-(a^2-b^2)^(1/2)),I)*2^(1/2)*sin(d*x
+c)^(1/2)/a/(a^2-b^2)^(3/2)/d/e^2/(-cos(d*x+c))^(1/2)/(e*tan(d*x+c))^(1/2)
+2*b^3*EllipticPi((-cos(d*x+c))^(1/2)/(1+sin(d*x+c))^(1/2),b/(a+(a^2-b^2)^
(1/2)),I)*2^(1/2)*sin(d*x+c)^(1/2)/a/(a^2-b^2)^(3/2)/d/e^2/(-cos(d*x+c))^(
1/2)/(e*tan(d*x+c))^(1/2)-1/3*b*(sin(c+1/4*Pi+d*x)^2)^(1/2)/sin(c+1/4*Pi+d
*x)*EllipticF(cos(c+1/4*Pi+d*x),2^(1/2))*sec(d*x+c)*sin(2*d*x+2*c)^(1/2)/(
a^2-b^2)/d/e^2/(e*tan(d*x+c))^(1/2)-2/3*(a-b*sec(d*x+c))/(a^2-b^2)/d/e/(e*
tan(d*x+c))^(3/2)
```

3.317.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 33.62 (sec) , antiderivative size = 2169, normalized size of antiderivative = 2.59

$$\int \frac{1}{(a+b\sec(c+dx))(e\tan(c+dx))^{5/2}}\,dx = \text{Result too large to show}$$

input
```
Integrate[1/((a + b*Sec[c + d*x])*(e*Tan[c + d*x])^(5/2)),x]
```

output

```
((b + a*Cos[c + d*x])*((2*a)/(3*(a^2 - b^2)) - (2*(-a + b*Cos[c + d*x])*Cs
c[c + d*x]^2)/(3*(-a^2 + b^2)))*Sec[c + d*x]*Tan[c + d*x]^3)/(d*(a + b*Sec
[c + d*x])*(e*Tan[c + d*x])^(5/2)) - ((b + a*Cos[c + d*x])*Sec[c + d*x]*Ta
n[c + d*x]^(5/2)*((2*(3*a^2 - 5*b^2)*Sec[c + d*x]^3*(a + b*Sqrt[1 + Tan[c
+ d*x]^2])*(((-1/8 + I/8)*a*(2*ArcTan[1 - ((1 + I)*Sqrt[b]*Sqrt[Tan[c + d*
x]])/(a^2 - b^2)^(1/4)] - 2*ArcTan[1 + ((1 + I)*Sqrt[b]*Sqrt[Tan[c + d*x]]
)/(a^2 - b^2)^(1/4)] + Log[Sqrt[a^2 - b^2] - (1 + I)*Sqrt[b]*(a^2 - b^2)^(
1/4)*Sqrt[Tan[c + d*x]] + I*b*Tan[c + d*x]] - Log[Sqrt[a^2 - b^2] + (1 + I
)*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Tan[c + d*x]] + I*b*Tan[c + d*x]]))/(Sqrt
[b]*(a^2 - b^2)^(3/4)) + (5*b*(-a^2 + b^2)*AppellF1[1/4, -1/2, 1, 5/4, -Ta
n[c + d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2 - b^2)]*Sqrt[Tan[c + d*x]]*Sqrt[1
+ Tan[c + d*x]^2])/((5*(a^2 - b^2)*AppellF1[1/4, -1/2, 1, 5/4, -Tan[c + d*
x]^2, (b^2*Tan[c + d*x]^2)/(a^2 - b^2)] + 2*(2*b^2*AppellF1[5/4, -1/2, 2,
9/4, -Tan[c + d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2 - b^2)] + (a^2 - b^2)*Appe
llF1[5/4, 1/2, 1, 9/4, -Tan[c + d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2 - b^2)])
*Tan[c + d*x]^2)*(a^2 - b^2*(1 + Tan[c + d*x]^2)))))/((b + a*Cos[c + d*x])
*(1 + Tan[c + d*x]^2)^2) + (8*a*b*Sec[c + d*x]^2*(a + b*Sqrt[1 + Tan[c + d
*x]^2])*((Sqrt[b]*(-2*ArcTan[1 - (Sqrt[2]*Sqrt[b]*Sqrt[Tan[c + d*x]])/(-a^
2 + b^2)^(1/4)] + 2*ArcTan[1 + (Sqrt[2]*Sqrt[b]*Sqrt[Tan[c + d*x]])/(-a^2
+ b^2)^(1/4)] - Log[Sqrt[-a^2 + b^2] - Sqrt[2]*Sqrt[b]*(-a^2 + b^2)^(1/...
```

3.317.3 Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(e\tan(c+dx))^{5/2}(a+b\sec(c+dx))}\,dx$$

$$\downarrow 3042$$

$$\int \frac{1}{\left(-e\cot\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)}\,dx$$

$$\downarrow 4381$$

$$\frac{b^2\int \frac{1}{(a+b\sec(c+dx))\sqrt{e\tan(c+dx)}}\,dx}{e^2\left(a^2-b^2\right)} + \frac{\int \frac{a-b\sec(c+dx)}{(e\tan(c+dx))^{5/2}}\,dx}{a^2-b^2}$$

$$\downarrow 3042$$

$$\frac{b^2\int \frac{1}{\sqrt{-e\cot\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)}\,dx}{e^2\left(a^2-b^2\right)} + \frac{\int \frac{a-b\csc\left(c+dx+\frac{\pi}{2}\right)}{\left(-e\cot\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}\,dx}{a^2-b^2}$$

$$\downarrow 4370$$

$$\frac{b^2\int\frac{1}{\sqrt{-e\cot\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{e^2\left(a^2-b^2\right)}+\frac{\frac{2\int-\frac{3a-b\sec(c+dx)}{2\sqrt{e\tan(c+dx)}}dx}{3e^2}-\frac{2(a-b\sec(c+dx))}{3de(e\tan(c+dx))^{3/2}}}{a^2-b^2}$$

$$\downarrow 27$$

$$\frac{b^2\int\frac{1}{\sqrt{-e\cot\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{e^2\left(a^2-b^2\right)}+\frac{-\frac{\int\frac{3a-b\sec(c+dx)}{\sqrt{e\tan(c+dx)}}dx}{3e^2}-\frac{2(a-b\sec(c+dx))}{3de(e\tan(c+dx))^{3/2}}}{a^2-b^2}$$

$$\downarrow 3042$$

$$\frac{b^2\int\frac{1}{\sqrt{-e\cot\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{e^2\left(a^2-b^2\right)}+\frac{-\frac{\int\frac{3a-b\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{-e\cot\left(c+dx+\frac{\pi}{2}\right)}}dx}{3e^2}-\frac{2(a-b\sec(c+dx))}{3de(e\tan(c+dx))^{3/2}}}{a^2-b^2}$$

$$\downarrow 4372$$

$$\frac{b^2\int\frac{1}{\sqrt{-e\cot\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{e^2\left(a^2-b^2\right)}+\frac{-\frac{3a\int\frac{1}{\sqrt{e\tan(c+dx)}}dx-b\int\frac{\sec(c+dx)}{\sqrt{e\tan(c+dx)}}dx}{3e^2}-\frac{2(a-b\sec(c+dx))}{3de(e\tan(c+dx))^{3/2}}}{a^2-b^2}$$

$$\downarrow 3042$$

$$\frac{b^2\int\frac{1}{\sqrt{-e\cot\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{e^2\left(a^2-b^2\right)}+\frac{-\frac{3a\int\frac{1}{\sqrt{e\tan(c+dx)}}dx-b\int\frac{\sec(c+dx)}{\sqrt{e\tan(c+dx)}}dx}{3e^2}-\frac{2(a-b\sec(c+dx))}{3de(e\tan(c+dx))^{3/2}}}{a^2-b^2}$$

$$\downarrow 3094$$

$$\frac{b^2\int\frac{1}{\sqrt{-e\cot\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{e^2\left(a^2-b^2\right)}+\frac{-\frac{3a\int\frac{1}{\sqrt{e\tan(c+dx)}}dx-\frac{b\sqrt{\sin(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}\sqrt{\sin(c+dx)}}dx}{\sqrt{\cos(c+dx)}\sqrt{e\tan(c+dx)}}}{3e^2}-\frac{2(a-b\sec(c+dx))}{3de(e\tan(c+dx))^{3/2}}}{a^2-b^2}$$

$$\downarrow 3042$$

$$\frac{b^2\int\frac{1}{\sqrt{-e\cot\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{e^2\left(a^2-b^2\right)}+\frac{-\frac{3a\int\frac{1}{\sqrt{e\tan(c+dx)}}dx-\frac{b\sqrt{\sin(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}\sqrt{\sin(c+dx)}}dx}{\sqrt{\cos(c+dx)}\sqrt{e\tan(c+dx)}}}{3e^2}-\frac{2(a-b\sec(c+dx))}{3de(e\tan(c+dx))^{3/2}}}{a^2-b^2}$$

$$\downarrow 3053$$

$$\frac{\frac{b^2\int\frac{1}{\sqrt{-e\cot\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{e^2\left(a^2-b^2\right)}+\frac{-\frac{3a\int\frac{1}{\sqrt{e\tan(c+dx)}}dx-\frac{b\sqrt{\sin(2c+2dx)}\sec(c+dx)\int\frac{1}{\sqrt{\sin(2c+2dx)}}dx}{\sqrt{e\tan(c+dx)}}}{3e^2}-\frac{2(a-b\sec(c+dx))}{3de(e\tan(c+dx))^{3/2}}}{a^2-b^2}}{a^2-b^2}$$

$$\downarrow 3042$$

$$\frac{\frac{b^2\int\frac{1}{\sqrt{-e\cot\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{e^2\left(a^2-b^2\right)}+\frac{-\frac{3a\int\frac{1}{\sqrt{e\tan(c+dx)}}dx-\frac{b\sqrt{\sin(2c+2dx)}\sec(c+dx)\int\frac{1}{\sqrt{\sin(2c+2dx)}}dx}{\sqrt{e\tan(c+dx)}}}{3e^2}-\frac{2(a-b\sec(c+dx))}{3de(e\tan(c+dx))^{3/2}}}{a^2-b^2}}{a^2-b^2}$$

$$\downarrow 3120$$

$$\frac{\frac{b^2\int\frac{1}{\sqrt{-e\cot\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{e^2\left(a^2-b^2\right)}+\frac{-\frac{3a\int\frac{1}{\sqrt{e\tan(c+dx)}}dx-\frac{b\sqrt{\sin(2c+2dx)}\sec(c+dx)\,\text{EllipticF}\left(c+dx-\frac{\pi}{4},2\right)}{d\sqrt{e\tan(c+dx)}}}{3e^2}-\frac{2(a-b\sec(c+dx))}{3de(e\tan(c+dx))^{3/2}}}{a^2-b^2}}{a^2-b^2}$$

$$\downarrow 3957$$

$$\frac{\frac{b^2\int\frac{1}{\sqrt{-e\cot\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{e^2\left(a^2-b^2\right)}+\frac{-\frac{\frac{3ae\int\frac{1}{\sqrt{e\tan(c+dx)}\left(\tan^2(c+dx)e^2+e^2\right)}d(e\tan(c+dx))}{d}-\frac{b\sqrt{\sin(2c+2dx)}\sec(c+dx)\,\text{EllipticF}\left(c+dx-\frac{\pi}{4},2\right)}{d\sqrt{e\tan(c+dx)}}}{3e^2}-\frac{2(a-b\sec(c+dx))}{3de(e\tan(c+dx))^{3/2}}}{a^2-b^2}}{a^2-b^2}$$

$$\downarrow 266$$

$$\frac{\frac{b^2\int\frac{1}{\sqrt{-e\cot\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{e^2\left(a^2-b^2\right)}+\frac{-\frac{\frac{6ae\int\frac{1}{e^4\tan^4(c+dx)+e^2}d\sqrt{e\tan(c+dx)}}{d}-\frac{b\sqrt{\sin(2c+2dx)}\sec(c+dx)\,\text{EllipticF}\left(c+dx-\frac{\pi}{4},2\right)}{d\sqrt{e\tan(c+dx)}}}{3e^2}-\frac{2(a-b\sec(c+dx))}{3de(e\tan(c+dx))^{3/2}}}{a^2-b^2}}{a^2-b^2}$$

$$\downarrow 755$$

$$\frac{b^2 \int \frac{1}{\sqrt{-e \cot\left(c+dx+\frac{\pi}{2}\right)}\left(a+b \csc\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{e^2\left(a^2-b^2\right)}+$$

$$\frac{6ae\left(\frac{\int \frac{e-e^2 \tan^2(c+dx)}{e^4 \tan^4(c+dx)+e^2}d\sqrt{e \tan(c+dx)}}{2e}+\frac{\int \frac{e^2 \tan^2(c+dx)+e}{e^4 \tan^4(c+dx)+e^2}d\sqrt{e \tan(c+dx)}}{2e}\right)}{d}-\frac{b\sqrt{\sin(2c+2dx)}\sec(c+dx)\,\text{EllipticF}\left(c+dx-\frac{\pi}{4},2\right)}{d\sqrt{e \tan(c+dx)}}$$

$$-\frac{3e^2}{a^2-b^2}-\frac{2(a-b \sec(c+dx)}{3de(e \tan(c+dx))^3}$$

$$\downarrow 1476$$

$$\frac{b^2 \int \frac{1}{\sqrt{-e \cot\left(c+dx+\frac{\pi}{2}\right)}\left(a+b \csc\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{e^2\left(a^2-b^2\right)}+$$

$$\frac{6ae\left(\frac{\frac{1}{2}\int \frac{1}{e^2 \tan^2(c+dx)-\sqrt{2}e^{3/2}\tan(c+dx)+e}d\sqrt{e \tan(c+dx)}+\frac{1}{2}\int \frac{1}{e^2 \tan^2(c+dx)+\sqrt{2}e^{3/2}\tan(c+dx)+e}d\sqrt{e \tan(c+dx)}}{2e}+\frac{\int \frac{e-e^2 \tan^2(c+dx)}{e^4 \tan^4(c+dx)+e^2}d\sqrt{e \tan(c+dx)}}{2e}\right)}{d}$$

$$-\frac{3e^2}{a^2-b^2}$$

$$\downarrow 1082$$

$$\frac{b^2 \int \frac{1}{\sqrt{-e \cot\left(c+dx+\frac{\pi}{2}\right)}\left(a+b \csc\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{e^2\left(a^2-b^2\right)}+$$

$$\frac{6ae\left(\frac{\int \frac{1}{-e^2 \tan^2(c+dx)-1}d\left(1-\sqrt{2}\sqrt{e}\tan(c+dx)\right)}{\sqrt{2}\sqrt{e}}-\frac{\int \frac{1}{-e^2 \tan^2(c+dx)-1}d\left(\sqrt{2}\sqrt{e}\tan(c+dx)+1\right)}{\sqrt{2}\sqrt{e}}}{2e}+\frac{\int \frac{e-e^2 \tan^2(c+dx)}{e^4 \tan^4(c+dx)+e^2}d\sqrt{e \tan(c+dx)}}{2e}\right)}{d}$$

$$-\frac{3e^2}{a^2-b^2}$$

$$\downarrow 217$$

$$\frac{6ae\left(\frac{\int \frac{e-e^2 \tan^2(c+dx)}{e^4 \tan^4(c+dx)+e^2}d\sqrt{e \tan(c+dx)}}{2e}+\frac{\frac{\arctan\left(\sqrt{2}\sqrt{e}\tan(c+dx)+1\right)}{\sqrt{2}\sqrt{e}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{e}\tan(c+dx)\right)}{\sqrt{2}\sqrt{e}}}{2e}\right)}{d}-\frac{b\sqrt{\sin(2c+2dx)}\sec(c+dx)\,\text{EllipticF}\left(c+dx-\frac{\pi}{4},2\right)}{d\sqrt{e \tan(c+dx)}}$$

$$-\frac{3e^2}{a^2-b^2}$$

$$\frac{b^2 \int \frac{1}{\sqrt{-e \cot\left(c+dx+\frac{\pi}{2}\right)}\left(a+b \csc\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{e^2\left(a^2-b^2\right)}$$

$$\downarrow 1479$$

$$-\frac{6ae\left(\frac{-\frac{\int -\frac{\sqrt{2}\sqrt{e}-2\sqrt{e\tan(c+dx)}}{e^2\tan^2(c+dx)-\sqrt{2}e^{3/2}\tan(c+dx)+e}d\sqrt{e\tan(c+dx)}}{2\sqrt{2}\sqrt{e}}}{2e}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{e}+\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{e^2\tan^2(c+dx)+\sqrt{2}e^{3/2}\tan(c+dx)+e}d\sqrt{e\tan(c+dx)}}{2\sqrt{2}\sqrt{e}}}{2e}+\frac{\arctan\left(\sqrt{2}\sqrt{e}\tan(c+dx)+1\right)}{\sqrt{2}\sqrt{e}}-\frac{\text{ar}}{2e}\right)}{d}}{3e^2}}{a^2-b^2}$$

$$\frac{b^2\int\frac{1}{\sqrt{-e\cot\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{e^2\left(a^2-b^2\right)}$$

$$\downarrow 25$$

$$-\frac{6ae\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt{e}-2\sqrt{e\tan(c+dx)}}{e^2\tan^2(c+dx)-\sqrt{2}e^{3/2}\tan(c+dx)+e}d\sqrt{e\tan(c+dx)}}{2\sqrt{2}\sqrt{e}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{e}+\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{e^2\tan^2(c+dx)+\sqrt{2}e^{3/2}\tan(c+dx)+e}d\sqrt{e\tan(c+dx)}}{2\sqrt{2}\sqrt{e}}}{2e}+\frac{\arctan\left(\sqrt{2}\sqrt{e}\tan(c+dx)+1\right)}{\sqrt{2}\sqrt{e}}-\frac{\arctan(}{2e}\right)}{d}}{3e^2}}{a^2-b^2}$$

$$\frac{b^2\int\frac{1}{\sqrt{-e\cot\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{e^2\left(a^2-b^2\right)}$$

$$\downarrow 27$$

$$-\frac{6ae\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt{e}-2\sqrt{e\tan(c+dx)}}{e^2\tan^2(c+dx)-\sqrt{2}e^{3/2}\tan(c+dx)+e}d\sqrt{e\tan(c+dx)}}{2\sqrt{2}\sqrt{e}}+\frac{\int \frac{\sqrt{e}+\sqrt{2}\sqrt{e\tan(c+dx)}}{e^2\tan^2(c+dx)+\sqrt{2}e^{3/2}\tan(c+dx)+e}d\sqrt{e\tan(c+dx)}}{2\sqrt{e}}}{2e}+\frac{\arctan\left(\sqrt{2}\sqrt{e}\tan(c+dx)+1\right)}{\sqrt{2}\sqrt{e}}-\frac{\arctan(}{2e}\right)}{d}}{3e^2}}{a^2-b^2}$$

$$\frac{b^2\int\frac{1}{\sqrt{-e\cot\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{e^2\left(a^2-b^2\right)}$$

$$\downarrow 1103$$

$$\frac{b^2\int\frac{1}{\sqrt{-e\cot\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{e^2\left(a^2-b^2\right)}+$$

$$-\frac{6ae\left(\frac{\frac{\arctan\left(\sqrt{2}\sqrt{e}\tan(c+dx)+1\right)}{\sqrt{2}\sqrt{e}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{e}\tan(c+dx)\right)}{\sqrt{2}\sqrt{e}}}{2e}+\frac{\frac{\log\left(\sqrt{2}e^{3/2}\tan(c+dx)+e^2\tan^2(c+dx)+e\right)}{2\sqrt{2}\sqrt{e}}-\frac{\log\left(-\sqrt{2}e^{3/2}\tan(c+dx)+e^2\tan^2(c+dx)+e\right)}{2\sqrt{2}\sqrt{e}}}{2e}\right)}{d}}{3e^2}}{a^2-b^2}$$

$$\downarrow 4380$$

$$\frac{b^2\left(\frac{\int \frac{1}{\sqrt{e\tan(c+dx)}}dx}{a}-\frac{b\int \frac{1}{(b+a\cos(c+dx))\sqrt{e\tan(c+dx)}}dx}{a}\right)}{e^2\left(a^2-b^2\right)}+$$

$$6ae\left(\frac{\frac{\arctan\left(\sqrt{2}\sqrt{e}\tan(c+dx)+1\right)}{\sqrt{2}\sqrt{e}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{e}\tan(c+dx)\right)}{\sqrt{2}\sqrt{e}}}{2e}+\frac{\frac{\log\left(\sqrt{2}e^{3/2}\tan(c+dx)+e^2\tan^2(c+dx)+e\right)}{2\sqrt{2}\sqrt{e}}-\frac{\log\left(-\sqrt{2}e^{3/2}\tan(c+dx)+e^2\tan^2(c+dx)+e\right)}{2\sqrt{2}\sqrt{e}}}{2e}\right)$$

$$-\frac{\frac{d}{3e^2}}{a^2-b^2}$$

$$\downarrow 3042$$

$$\frac{b^2\left(\frac{\int \frac{1}{\sqrt{e\tan(c+dx)}}dx}{a}-\frac{b\int \frac{1}{\sqrt{-e\cot\left(c+dx+\frac{\pi}{2}\right)}\left(b+a\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a}\right)}{e^2\left(a^2-b^2\right)}+$$

$$6ae\left(\frac{\frac{\arctan\left(\sqrt{2}\sqrt{e}\tan(c+dx)+1\right)}{\sqrt{2}\sqrt{e}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{e}\tan(c+dx)\right)}{\sqrt{2}\sqrt{e}}}{2e}+\frac{\frac{\log\left(\sqrt{2}e^{3/2}\tan(c+dx)+e^2\tan^2(c+dx)+e\right)}{2\sqrt{2}\sqrt{e}}-\frac{\log\left(-\sqrt{2}e^{3/2}\tan(c+dx)+e^2\tan^2(c+dx)+e\right)}{2\sqrt{2}\sqrt{e}}}{2e}\right)$$

$$-\frac{\frac{d}{3e^2}}{a^2-b^2}$$

$$\downarrow 3212$$

$$\frac{b^2\left(\frac{\int \frac{1}{\sqrt{e\tan(c+dx)}}dx}{a}-\frac{b\int \frac{\sqrt{e\cot(c+dx)}}{b+a\cos(c+dx)}dx}{a\sqrt{e\tan(c+dx)}\sqrt{e\cot(c+dx)}}\right)}{e^2\left(a^2-b^2\right)}+$$

$$6ae\left(\frac{\frac{\arctan\left(\sqrt{2}\sqrt{e}\tan(c+dx)+1\right)}{\sqrt{2}\sqrt{e}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{e}\tan(c+dx)\right)}{\sqrt{2}\sqrt{e}}}{2e}+\frac{\frac{\log\left(\sqrt{2}e^{3/2}\tan(c+dx)+e^2\tan^2(c+dx)+e\right)}{2\sqrt{2}\sqrt{e}}-\frac{\log\left(-\sqrt{2}e^{3/2}\tan(c+dx)+e^2\tan^2(c+dx)+e\right)}{2\sqrt{2}\sqrt{e}}}{2e}\right)$$

$$-\frac{\frac{d}{3e^2}}{a^2-b^2}$$

$$\downarrow 3042$$

$$\frac{b^2\left(\frac{\int \frac{1}{\sqrt{e\tan(c+dx)}}dx}{a}-\frac{b\int \frac{\sqrt{-e\tan\left(c+dx-\frac{\pi}{2}\right)}}{b-a\sin\left(c+dx-\frac{\pi}{2}\right)}dx}{a\sqrt{e\tan(c+dx)}\sqrt{e\cot(c+dx)}}\right)}{e^2\left(a^2-b^2\right)}+$$

$$6ae\left(\frac{\frac{\arctan\left(\sqrt{2}\sqrt{e}\tan(c+dx)+1\right)}{\sqrt{2}\sqrt{e}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{e}\tan(c+dx)\right)}{\sqrt{2}\sqrt{e}}}{2e}+\frac{\frac{\log\left(\sqrt{2}e^{3/2}\tan(c+dx)+e^2\tan^2(c+dx)+e\right)}{2\sqrt{2}\sqrt{e}}-\frac{\log\left(-\sqrt{2}e^{3/2}\tan(c+dx)+e^2\tan^2(c+dx)+e\right)}{2\sqrt{2}\sqrt{e}}}{2e}\right)$$

$$-\frac{\frac{d}{3e^2}}{a^2-b^2}$$

$$\downarrow 3208$$

$$\frac{b^2\left(\frac{\int \frac{1}{\sqrt{e\tan(c+dx)}}dx}{a}-\frac{b\sqrt{\sin(c+dx)}\int \frac{\sqrt{-\cos(c+dx)}}{(b+a\cos(c+dx))\sqrt{\sin(c+dx)}}dx}{a\sqrt{-\cos(c+dx)}\sqrt{e\tan(c+dx)}}\right)}{e^2\left(a^2-b^2\right)}+$$

$$\frac{6ae\left(\frac{\arctan\left(\sqrt{2}\sqrt{e}\tan(c+dx)+1\right)}{\sqrt{2}\sqrt{e}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{e}\tan(c+dx)\right)}{\sqrt{2}\sqrt{e}}+\frac{\log\left(\sqrt{2}e^{3/2}\tan(c+dx)+e^2\tan^2(c+dx)+e\right)}{2\sqrt{2}\sqrt{e}}-\frac{\log\left(-\sqrt{2}e^{3/2}\tan(c+dx)+e^2\tan^2(c+dx)+e\right)}{2\sqrt{2}\sqrt{e}}\right)}{2e}$$

$$-\frac{\frac{}{d}}{3e^2}$$

$$a^2-b^2$$

$$\downarrow 3042$$

$$\frac{b^2\left(\frac{\int \frac{1}{\sqrt{e\tan(c+dx)}}dx}{a}-\frac{b\sqrt{\sin(c+dx)}\int \frac{\sqrt{\sin\left(c+dx-\frac{\pi}{2}\right)}}{\sqrt{\cos\left(c+dx-\frac{\pi}{2}\right)}\left(b-a\sin\left(c+dx-\frac{\pi}{2}\right)\right)}dx}{a\sqrt{-\cos(c+dx)}\sqrt{e\tan(c+dx)}}\right)}{e^2\left(a^2-b^2\right)}+$$

$$\frac{6ae\left(\frac{\arctan\left(\sqrt{2}\sqrt{e}\tan(c+dx)+1\right)}{\sqrt{2}\sqrt{e}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{e}\tan(c+dx)\right)}{\sqrt{2}\sqrt{e}}+\frac{\log\left(\sqrt{2}e^{3/2}\tan(c+dx)+e^2\tan^2(c+dx)+e\right)}{2\sqrt{2}\sqrt{e}}-\frac{\log\left(-\sqrt{2}e^{3/2}\tan(c+dx)+e^2\tan^2(c+dx)+e\right)}{2\sqrt{2}\sqrt{e}}\right)}{2e}$$

$$-\frac{\frac{}{d}}{3e^2}$$

$$a^2-b^2$$

$$\downarrow 3386$$

$$b^2\left(\frac{\int \frac{1}{\sqrt{e\tan(c+dx)}}dx}{a}-\frac{b\sqrt{\sin(c+dx)}\left(\frac{2\sqrt{2}\left(1-\frac{a}{\sqrt{a^2-b^2}}\right)\int -\frac{1}{\sqrt{1-\frac{\cos^2(c+dx)}{(\sin(c+dx)+1)^2}}\left(a-\sqrt{a^2-b^2}+\frac{b\cos(c+dx)}{\sin(c+dx)+1}\right)}d\frac{\sqrt{-\cos(c+dx)}}{\sqrt{\sin(c+dx)+1}}}{d}+\frac{2\sqrt{2}\left(\frac{a}{\sqrt{a^2-b^2}}+1\right)}{}\right.}{a\sqrt{-\cos(c+dx)}\sqrt{e\tan(c+dx)}}\right)$$

$$e^2\left(a^2-b^2\right)$$

$$\frac{6ae\left(\frac{\arctan\left(\sqrt{2}\sqrt{e}\tan(c+dx)+1\right)}{\sqrt{2}\sqrt{e}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{e}\tan(c+dx)\right)}{\sqrt{2}\sqrt{e}}+\frac{\log\left(\sqrt{2}e^{3/2}\tan(c+dx)+e^2\tan^2(c+dx)+e\right)}{2\sqrt{2}\sqrt{e}}-\frac{\log\left(-\sqrt{2}e^{3/2}\tan(c+dx)+e^2\tan^2(c+dx)+e\right)}{2\sqrt{2}\sqrt{e}}\right)}{2e}$$

$$-\frac{\frac{}{d}}{3e^2}$$

$$a^2-b^2$$

input
```
Int[1/((a + b*Sec[c + d*x])*(e*Tan[c + d*x])^(5/2)),x]
```

output
```
$Aborted
```

3.317.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3053
```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Simp[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]])   Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f
}, x]
```

rule 3094
```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]], x_Symbol]
:> Simp[Sqrt[Sin[e + f*x]]/(Sqrt[Cos[e + f*x]]*Sqrt[b*Tan[e + f*x]])   Int[
1/(Sqrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]]), x], x] /; FreeQ[{b, e, f}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3208
```
Int[Sqrt[(g_.)*tan[(e_.) + (f_.)*(x_)]]/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_
)]), x_Symbol] :> Simp[Sqrt[Cos[e + f*x]]*(Sqrt[g*Tan[e + f*x]]/Sqrt[Sin[e
+ f*x]])   Int[Sqrt[Sin[e + f*x]]/(Sqrt[Cos[e + f*x]]*(a + b*Sin[e + f*x]))
, x], x] /; FreeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

rule 3212
```
Int[(cot[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Simp[g^(2*IntPart[p])*(g*Cot[e + f*x])^FracPart[p]
*(g*Tan[e + f*x])^FracPart[p]   Int[(a + b*Sin[e + f*x])^m/(g*Tan[e + f*x])
^p, x], x] /; FreeQ[{a, b, e, f, g, m, p}, x] &&  !IntegerQ[p]
```

rule 3386
```
Int[Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]/(Sqrt[cos[(e_.) + (f_.)*(x_)]]*((a_
) + (b_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> With[{q = Rt[-a^2 + b^2,
2]}, Simp[2*Sqrt[2]*d*((b + q)/(f*q))   Subst[Int[1/((d*(b + q) + a*x^2)*Sq
rt[1 - x^4/d^2]), x], x, Sqrt[d*Sin[e + f*x]]/Sqrt[1 + Cos[e + f*x]]], x] -
 Simp[2*Sqrt[2]*d*((b - q)/(f*q))   Subst[Int[1/((d*(b - q) + a*x^2)*Sqrt[1
 - x^4/d^2]), x], x, Sqrt[d*Sin[e + f*x]]/Sqrt[1 + Cos[e + f*x]]], x]] /; F
reeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

rule 4370
```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(-(e*Cot[c + d*x])^(m + 1))*((a + b*Csc[c + d*x])/(
d*e*(m + 1))), x] - Simp[1/(e^2*(m + 1))   Int[(e*Cot[c + d*x])^(m + 2)*(a*
(m + 1) + b*(m + 2)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && L
tQ[m, -1]
```

rule 4372
```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(e*Cot[c + d*x])^m, x], x] + Simp[b   Int[
(e*Cot[c + d*x])^m*Csc[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, m}, x]
```

rule 4380
```
Int[1/(Sqrt[cot[(c_.) + (d_.)*(x_)]*(e_.)]*(csc[(c_.) + (d_.)*(x_)]*(b_.) +
 (a_))), x_Symbol] :> Simp[1/a   Int[1/Sqrt[e*Cot[c + d*x]], x], x] - Simp[
b/a   Int[1/(Sqrt[e*Cot[c + d*x]]*(b + a*Sin[c + d*x])), x], x] /; FreeQ[{a
, b, c, d, e}, x] && NeQ[a^2 - b^2, 0]
```

rule 4381
```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)/(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[1/(a^2 - b^2)   Int[(e*Cot[c + d*x])^m*(a - b*Csc[c
 + d*x]), x], x] + Simp[b^2/(e^2*(a^2 - b^2))   Int[(e*Cot[c + d*x])^(m + 2
)/(a + b*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^
2, 0] && ILtQ[m + 1/2, 0]
```

3.317.4 Maple [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 7580 vs. $2(725) = 1450$.

Time = 4.79 (sec) , antiderivative size = 7581, normalized size of antiderivative = 9.07

method	result	size
default	Expression too large to display	7581

input
```
int(1/(a+b*sec(d*x+c))/(e*tan(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

3.317.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\sec(c+dx))(e\tan(c+dx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/(a+b*sec(d*x+c))/(e*tan(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.317.6 Sympy [F]

$$\int \frac{1}{(a+b\sec(c+dx))(e\tan(c+dx))^{5/2}}\,dx = \int \frac{1}{(e\tan(c+dx))^{\frac{5}{2}}\,(a+b\sec(c+dx))}\,dx$$

input
```
integrate(1/(a+b*sec(d*x+c))/(e*tan(d*x+c))**(5/2),x)
```

output
```
Integral(1/((e*tan(c + d*x))**(5/2)*(a + b*sec(c + d*x))), x)
```

3.317.7 Maxima [F]

$$\int \frac{1}{(a+b\sec(c+dx))(e\tan(c+dx))^{5/2}}\,dx = \int \frac{1}{(b\sec(dx+c)+a)\,(e\tan(dx+c))^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(a+b*sec(d*x+c))/(e*tan(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*sec(d*x + c) + a)*(e*tan(d*x + c))^(5/2)), x)
```

3.317.8 Giac [F]

$$\int \frac{1}{(a+b\sec(c+dx))(e\tan(c+dx))^{5/2}}\,dx = \int \frac{1}{(b\sec(dx+c)+a)\,(e\tan(dx+c))^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(a+b*sec(d*x+c))/(e*tan(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
integrate(1/((b*sec(d*x + c) + a)*(e*tan(d*x + c))^(5/2)), x)
```

3.317.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\sec(c+dx))(e\tan(c+dx))^{5/2}}\,dx = \int \frac{\cos(c+d\,x)}{(e\tan(c+d\,x))^{5/2}\,(b+a\,\cos(c+d\,x))}\,dx$$

input
```
int(1/((e*tan(c + d*x))^(5/2)*(a + b/cos(c + d*x))),x)
```

output
```
int(cos(c + d*x)/((e*tan(c + d*x))^(5/2)*(b + a*cos(c + d*x))), x)
```

3.318 $\int \sqrt{a+b\sec(c+dx)}\tan^5(c+dx)\,dx$

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3.318.2 Mathematica [A] (verified) . . . 2345
3.318.3 Rubi [A] (warning: unable to verify) . . . 2345
3.318.4 Maple [B] (verified) . . . 2347
3.318.5 Fricas [A] (verification not implemented) . . . 2348
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3.318.8 Giac [B] (verification not implemented) . . . 2350
3.318.9 Mupad [F(-1)] . . . 2350

3.318.1 Optimal result

Integrand size = 23, antiderivative size = 169

$$\begin{aligned}\int \sqrt{a+b\sec(c+dx)}\tan^5(c+dx)\,dx = &-\frac{2\sqrt{a}\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a}}\right)}{d}\\ &+\frac{2\sqrt{a+b\sec(c+dx)}}{d}\\ &-\frac{2a(a^2-2b^2)\left(a+b\sec(c+dx)\right)^{3/2}}{3b^4d}\\ &+\frac{2(3a^2-2b^2)\left(a+b\sec(c+dx)\right)^{5/2}}{5b^4d}\\ &-\frac{6a\left(a+b\sec(c+dx)\right)^{7/2}}{7b^4d}\\ &+\frac{2\left(a+b\sec(c+dx)\right)^{9/2}}{9b^4d}\end{aligned}$$

output
```
-2/3*a*(a^2-2*b^2)*(a+b*sec(d*x+c))^(3/2)/b^4/d+2/5*(3*a^2-2*b^2)*(a+b*sec
(d*x+c))^(5/2)/b^4/d-6/7*a*(a+b*sec(d*x+c))^(7/2)/b^4/d+2/9*(a+b*sec(d*x+c
))^(9/2)/b^4/d-2*arctanh((a+b*sec(d*x+c))^(1/2)/a^(1/2))*a^(1/2)/d+2*(a+b*
sec(d*x+c))^(1/2)/d
```

3.318.2 Mathematica [A] (verified)

Time = 1.91 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.90

$$\int \sqrt{a+b\sec(c+dx)}\tan^5(c+dx)\,dx$$
$$= \frac{-2\sqrt{a}b^4\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a}}\right) + 2b^4\sqrt{a+b\sec(c+dx)} - \frac{2}{3}a(a^2-2b^2)\,(a+b\sec(c+dx))^{3/2} + \frac{2}{5}(3a^2 -}{b^4 d}$$

input
```
Integrate[Sqrt[a + b*Sec[c + d*x]]*Tan[c + d*x]^5,x]
```

output
```
(-2*Sqrt[a]*b^4*ArcTanh[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a]] + 2*b^4*Sqrt[a +
 b*Sec[c + d*x]] - (2*a*(a^2 - 2*b^2)*(a + b*Sec[c + d*x])^(3/2))/3 + (2*(
3*a^2 - 2*b^2)*(a + b*Sec[c + d*x])^(5/2))/5 - (6*a*(a + b*Sec[c + d*x])^(
7/2))/7 + (2*(a + b*Sec[c + d*x])^(9/2))/9)/(b^4*d)
```

3.318.3 Rubi [A] (warning: unable to verify)

Time = 0.36 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.78, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {3042, 25, 4373, 517, 25, 1584, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^5(c+dx)\sqrt{a+b\sec(c+dx)}\,dx$$
$$\downarrow 3042$$
$$\int -\cot\left(c+dx+\frac{\pi}{2}\right)^5\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}\,dx$$
$$\downarrow 25$$
$$-\int \cot\left(\frac{1}{2}(2c+\pi)+dx\right)^5\sqrt{a+b\csc\left(\frac{1}{2}(2c+\pi)+dx\right)}\,dx$$
$$\downarrow 4373$$
$$\frac{\int \frac{\cos(c+dx)\sqrt{a+b\sec(c+dx)}\left(b^2-b^2\sec^2(c+dx)\right)^2}{b}\,d(b\sec(c+dx))}{b^4 d}$$
$$\downarrow 517$$

$$\frac{2\int -\frac{b^2\sec^2(c+dx)\left(b^4\sec^4(c+dx)-2ab^2\sec^2(c+dx)+a^2-b^2\right)^2}{a-b^2\sec^2(c+dx)}d\sqrt{a+b\sec(c+dx)}}{b^4d}$$

$$\downarrow 25$$

$$-\frac{2\int \frac{b^2\sec^2(c+dx)\left(b^4\sec^4(c+dx)-2ab^2\sec^2(c+dx)+a^2-b^2\right)^2}{a-b^2\sec^2(c+dx)}d\sqrt{a+b\sec(c+dx)}}{b^4d}$$

$$\downarrow 1584$$

$$-\frac{2\int \left(-b^8\sec^8(c+dx)+3ab^6\sec^6(c+dx)-b^4\left(3a^2-2b^2\right)\sec^4(c+dx)+ab^2\left(a^2-2b^2\right)\sec^2(c+dx)-b^4+\frac{}{a-b}\right.}{b^4d}$$

$$\downarrow 2009$$

$$\frac{2\left(\frac{1}{5}b^5\left(3a^2-2b^2\right)\sec^5(c+dx)-\frac{1}{3}ab^3\left(a^2-2b^2\right)\sec^3(c+dx)-\sqrt{a}b^4\text{arctanh}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a}}\right)-\frac{3}{7}ab^7\sec^7(c+dx\right.}{b^4d}$$

input
```
Int[Sqrt[a + b*Sec[c + d*x]]*Tan[c + d*x]^5,x]
```

output
```
(2*(-(Sqrt[a]*b^4*ArcTanh[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a]]) + b^5*Sec[c +
 d*x] - (a*b^3*(a^2 - 2*b^2)*Sec[c + d*x]^3)/3 + (b^5*(3*a^2 - 2*b^2)*Sec[
c + d*x]^5)/5 - (3*a*b^7*Sec[c + d*x]^7)/7 + (b^9*Sec[c + d*x]^9)/9))/(b^4
*d)
```

3.318.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 517
```
Int[((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.),
 x_Symbol] :> Simp[2*(e^m/d^(m + 2*p + 1))   Subst[Int[x^(2*n + 1)*(-c + x^
2)^m*(b*c^2 + a*d^2 - 2*b*c*x^2 + b*x^4)^p, x], x, Sqrt[c + d*x]], x] /; Fr
eeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && ILtQ[m, 0] && IntegerQ[n + 1/2]
```

rule 1584
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4373
```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Simp[-(-1)^((m - 1)/2)/(d*b^(m - 1))   Subst[Int[(b^2 - x^
2)^((m - 1)/2)*((a + x)^n/x), x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
 d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

3.318.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 882 vs. $2(145) = 290$.

Time = 16.53 (sec) , antiderivative size = 883, normalized size of antiderivative = 5.22

method	result	size
default	Expression too large to display	883

input
```
int((a+b*sec(d*x+c))^(1/2)*tan(d*x+c)^5,x,method=_RETURNVERBOSE)
```

output
```
-1/315/d/b^4*(a+b*sec(d*x+c))^(1/2)/(cos(d*x+c)+1)/((b+a*cos(d*x+c))*cos(d
*x+c)/(cos(d*x+c)+1)^2)^(1/2)*(315*cos(d*x+c)*ln(4*cos(d*x+c)*((b+a*cos(d*
x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*a^(1/2)+4*a*cos(d*x+c)+4*a^(1/2)*
((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)+2*b)*a^(1/2)*b^4+32*(
(b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*a^4*cos(d*x+c)-168*((b
+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*a^2*b^2*cos(d*x+c)-630*c
os(d*x+c)*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*b^4+32*((b+
a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*a^4-16*((b+a*cos(d*x+c))*
cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*a^3*b-168*((b+a*cos(d*x+c))*cos(d*x+c)/
(cos(d*x+c)+1)^2)^(1/2)*a^2*b^2+84*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c
)+1)^2)^(1/2)*a*b^3-630*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/
2)*b^4-16*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*a^3*b*sec(d
*x+c)+12*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*a^2*b^2*sec(
d*x+c)+84*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*a*b^3*sec(d
*x+c)+252*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*b^4*sec(d*x
+c)+12*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*a^2*b^2*sec(d*
x+c)^2-10*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*a*b^3*sec(d
*x+c)^2+252*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*b^4*sec(d
*x+c)^2-10*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*a*b^3*sec(
d*x+c)^3-70*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*b^4*se...
```

3.318.5 Fricas [A] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 425, normalized size of antiderivative = 2.51

$$\int \sqrt{a+b\sec(c+dx)}\tan^5(c+dx)\,dx$$

$$= \left[\frac{315\sqrt{a}b^4\cos(dx+c)^4\log\left(-8a^2\cos(dx+c)^2-8ab\cos(dx+c)-b^2+4\left(2a\cos(dx+c)^2+b\cos(dx\right.\right.\right.$$

input
```
integrate((a+b*sec(d*x+c))^(1/2)*tan(d*x+c)^5,x, algorithm="fricas")
```

output
```
[1/630*(315*sqrt(a)*b^4*cos(d*x + c)^4*log(-8*a^2*cos(d*x + c)^2 - 8*a*b*c
os(d*x + c) - b^2 + 4*(2*a*cos(d*x + c)^2 + b*cos(d*x + c))*sqrt(a)*sqrt((
a*cos(d*x + c) + b)/cos(d*x + c))) + 4*(5*a*b^3*cos(d*x + c) - (16*a^4 - 8
4*a^2*b^2 - 315*b^4)*cos(d*x + c)^4 + 35*b^4 + 2*(4*a^3*b - 21*a*b^3)*cos(
d*x + c)^3 - 6*(a^2*b^2 + 21*b^4)*cos(d*x + c)^2)*sqrt((a*cos(d*x + c) + b
)/cos(d*x + c)))/(b^4*d*cos(d*x + c)^4), 1/315*(315*sqrt(-a)*b^4*arctan(2*
sqrt(-a)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c))*cos(d*x + c)/(2*a*cos(d*x
 + c) + b))*cos(d*x + c)^4 + 2*(5*a*b^3*cos(d*x + c) - (16*a^4 - 84*a^2*b^
2 - 315*b^4)*cos(d*x + c)^4 + 35*b^4 + 2*(4*a^3*b - 21*a*b^3)*cos(d*x + c)
^3 - 6*(a^2*b^2 + 21*b^4)*cos(d*x + c)^2)*sqrt((a*cos(d*x + c) + b)/cos(d*
x + c)))/(b^4*d*cos(d*x + c)^4)]
```

3.318.6 Sympy [F]

$$\int \sqrt{a+b\sec(c+dx)}\tan^5(c+dx)\,dx = \int \sqrt{a+b\sec(c+dx)}\tan^5(c+dx)\,dx$$

input
```
integrate((a+b*sec(d*x+c))**(1/2)*tan(d*x+c)**5,x)
```

output
```
Integral(sqrt(a + b*sec(c + d*x))*tan(c + d*x)**5, x)
```

3.318.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.13

$$\int \sqrt{a+b\sec(c+dx)}\tan^5(c+dx)\,dx$$

$$= \frac{315\sqrt{a}\log\left(\frac{\sqrt{a+\frac{b}{\cos(dx+c)}}-\sqrt{a}}{\sqrt{a+\frac{b}{\cos(dx+c)}}+\sqrt{a}}\right)+630\sqrt{a+\frac{b}{\cos(dx+c)}}+\frac{70\left(a+\frac{b}{\cos(dx+c)}\right)^{\frac{9}{2}}}{b^4}-\frac{270\left(a+\frac{b}{\cos(dx+c)}\right)^{\frac{7}{2}}a}{b^4}+\frac{378\left(a+\frac{b}{\cos(dx+c)}\right)}{b^4}}{315\,d}$$

input
```
integrate((a+b*sec(d*x+c))^(1/2)*tan(d*x+c)^5,x, algorithm="maxima")
```

output
```
1/315*(315*sqrt(a)*log((sqrt(a + b/cos(d*x + c)) - sqrt(a))/(sqrt(a + b/co
s(d*x + c)) + sqrt(a))) + 630*sqrt(a + b/cos(d*x + c)) + 70*(a + b/cos(d*x
 + c))^(9/2)/b^4 - 270*(a + b/cos(d*x + c))^(7/2)*a/b^4 + 378*(a + b/cos(d
*x + c))^(5/2)*a^2/b^4 - 210*(a + b/cos(d*x + c))^(3/2)*a^3/b^4 - 252*(a +
 b/cos(d*x + c))^(5/2)/b^2 + 420*(a + b/cos(d*x + c))^(3/2)*a/b^2)/d
```

3.318.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 966 vs. 2(145) = 290.

Time = 1.76 (sec) , antiderivative size = 966, normalized size of antiderivative = 5.72

$$\int \sqrt{a+b\sec(c+dx)}\tan^5(c+dx)\,dx = \text{Too large to display}$$

input
```
integrate((a+b*sec(d*x+c))^(1/2)*tan(d*x+c)^5,x, algorithm="giac")
```

output
```
2/315*(315*a*arctan(-1/2*(sqrt(a - b)*tan(1/2*d*x + 1/2*c)^2 - sqrt(a*tan(
1/2*d*x + 1/2*c)^4 - b*tan(1/2*d*x + 1/2*c)^4 - 2*a*tan(1/2*d*x + 1/2*c)^2
 + a + b) + sqrt(a - b))/sqrt(-a))/sqrt(-a) - 2*(315*(sqrt(a - b)*tan(1/2*
d*x + 1/2*c)^2 - sqrt(a*tan(1/2*d*x + 1/2*c)^4 - b*tan(1/2*d*x + 1/2*c)^4
 - 2*a*tan(1/2*d*x + 1/2*c)^2 + a + b))^8*a - 3150*(sqrt(a - b)*tan(1/2*d*x
 + 1/2*c)^2 - sqrt(a*tan(1/2*d*x + 1/2*c)^4 - b*tan(1/2*d*x + 1/2*c)^4 - 2
*a*tan(1/2*d*x + 1/2*c)^2 + a + b))^7*sqrt(a - b)*a + 210*(sqrt(a - b)*tan
(1/2*d*x + 1/2*c)^2 - sqrt(a*tan(1/2*d*x + 1/2*c)^4 - b*tan(1/2*d*x + 1/2*
c)^4 - 2*a*tan(1/2*d*x + 1/2*c)^2 + a + b))^6*(39*a^2 - 5*a*b - 32*b^2) -
630*(sqrt(a - b)*tan(1/2*d*x + 1/2*c)^2 - sqrt(a*tan(1/2*d*x + 1/2*c)^4 -
b*tan(1/2*d*x + 1/2*c)^4 - 2*a*tan(1/2*d*x + 1/2*c)^2 + a + b))^5*(9*a^2 +
 15*a*b - 16*b^2)*sqrt(a - b) - 252*(25*a^3 - 37*a^2*b + 80*a*b^2 - 72*b^3
)*(sqrt(a - b)*tan(1/2*d*x + 1/2*c)^2 - sqrt(a*tan(1/2*d*x + 1/2*c)^4 - b*
tan(1/2*d*x + 1/2*c)^4 - 2*a*tan(1/2*d*x + 1/2*c)^2 + a + b))^4 + 945*a^5
+ 3864*a^4*b + 2562*a^3*b^2 + 2448*a^2*b^3 - 1083*a*b^4 + 224*b^5 + 42*(25
5*a^3 + 2*a^2*b + 655*a*b^2 - 288*b^3)*(sqrt(a - b)*tan(1/2*d*x + 1/2*c)^2
 - sqrt(a*tan(1/2*d*x + 1/2*c)^4 - b*tan(1/2*d*x + 1/2*c)^4 - 2*a*tan(1/2*
d*x + 1/2*c)^2 + a + b))^3*sqrt(a - b) - 18*(175*a^4 - 483*a^3*b + 1113*a^
2*b^2 - 773*a*b^3 + 448*b^4)*(sqrt(a - b)*tan(1/2*d*x + 1/2*c)^2 - sqrt(a*
tan(1/2*d*x + 1/2*c)^4 - b*tan(1/2*d*x + 1/2*c)^4 - 2*a*tan(1/2*d*x + 1...
```

3.318.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+b\sec(c+dx)}\tan^5(c+dx)\,dx = \int \tan(c+d\,x)^5\sqrt{a+\frac{b}{\cos(c+d\,x)}}\,dx$$

input
```
int(tan(c + d*x)^5*(a + b/cos(c + d*x))^(1/2),x)
```

output
```
int(tan(c + d*x)^5*(a + b/cos(c + d*x))^(1/2), x)
```

3.319 $\int \sqrt{a+b\sec(c+dx)}\tan^3(c+dx)\,dx$

3.319.1 Optimal result . . . 2352
3.319.2 Mathematica [A] (verified) . . . 2352
3.319.3 Rubi [A] (warning: unable to verify) . . . 2353
3.319.4 Maple [B] (verified) . . . 2354
3.319.5 Fricas [A] (verification not implemented) . . . 2355
3.319.6 Sympy [F] . . . 2356
3.319.7 Maxima [A] (verification not implemented) . . . 2356
3.319.8 Giac [B] (verification not implemented) . . . 2356
3.319.9 Mupad [F(-1)] . . . 2357

3.319.1 Optimal result

Integrand size = 23, antiderivative size = 100

$$\int \sqrt{a+b\sec(c+dx)}\tan^3(c+dx)\,dx = \frac{2\sqrt{a}\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a}}\right)}{d} - \frac{2\sqrt{a+b\sec(c+dx)}}{d} - \frac{2a(a+b\sec(c+dx))^{3/2}}{3b^2d} + \frac{2(a+b\sec(c+dx))^{5/2}}{5b^2d}$$

output
```
-2/3*a*(a+b*sec(d*x+c))^(3/2)/b^2/d+2/5*(a+b*sec(d*x+c))^(5/2)/b^2/d+2*arc
tanh((a+b*sec(d*x+c))^(1/2)/a^(1/2))*a^(1/2)/d-2*(a+b*sec(d*x+c))^(1/2)/d
```

3.319.2 Mathematica [A] (verified)

Time = 0.70 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.87

$$\int \sqrt{a+b\sec(c+dx)}\tan^3(c+dx)\,dx = \frac{2\left(15\sqrt{a}\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a}}\right) + \frac{\sqrt{a+b\sec(c+dx)}\left(-2a^2-15b^2+ab\sec(c+dx)+3b^2\sec^2(c+dx)\right)}{b^2}\right)}{15d}$$

input
```
Integrate[Sqrt[a + b*Sec[c + d*x]]*Tan[c + d*x]^3,x]
```

output
```
(2*(15*Sqrt[a]*ArcTanh[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a]] + (Sqrt[a + b*Sec
[c + d*x]]*(-2*a^2 - 15*b^2 + a*b*Sec[c + d*x] + 3*b^2*Sec[c + d*x]^2))/b^
2))/(15*d)
```

3.319.3 Rubi [A] (warning: unable to verify)

Time = 0.30 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.81, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3042, 25, 4373, 517, 1584, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^3(c+dx)\sqrt{a+b\sec(c+dx)}\,dx$$

$$\downarrow 3042$$

$$\int -\cot\left(c+dx+\frac{\pi}{2}\right)^3\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\downarrow 25$$

$$-\int \cot\left(\frac{1}{2}(2c+\pi)+dx\right)^3\sqrt{a+b\csc\left(\frac{1}{2}(2c+\pi)+dx\right)}dx$$

$$\downarrow 4373$$

$$-\frac{\int \frac{\cos(c+dx)\sqrt{a+b\sec(c+dx)}\left(b^2-b^2\sec^2(c+dx)\right)}{b}d(b\sec(c+dx))}{b^2d}$$

$$\downarrow 517$$

$$-\frac{2\int \frac{b^2\sec^2(c+dx)\left(b^4\sec^4(c+dx)-2ab^2\sec^2(c+dx)+a^2-b^2\right)}{a-b^2\sec^2(c+dx)}d\sqrt{a+b\sec(c+dx)}}{b^2d}$$

$$\downarrow 1584$$

$$-\frac{2\int \left(-b^4\sec^4(c+dx)+ab^2\sec^2(c+dx)+b^2-\frac{ab^2}{a-b^2\sec^2(c+dx)}\right)d\sqrt{a+b\sec(c+dx)}}{b^2d}$$

$$\downarrow 2009$$

$$-\frac{2\left(-\sqrt{a}b^2\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a}}\right)+\frac{1}{3}ab^3\sec^3(c+dx)-\frac{1}{5}b^5\sec^5(c+dx)+b^3\sec(c+dx)\right)}{b^2d}$$

input
```
Int[Sqrt[a + b*Sec[c + d*x]]*Tan[c + d*x]^3,x]
```

output
```
(-2*(-(Sqrt[a]*b^2*ArcTanh[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a]]) + b^3*Sec[c
+ d*x] + (a*b^3*Sec[c + d*x]^3)/3 - (b^5*Sec[c + d*x]^5)/5))/(b^2*d)
```

3.319.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 517
```
Int[((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.),
 x_Symbol] :> Simp[2*(e^m/d^(m + 2*p + 1))   Subst[Int[x^(2*n + 1)*(-c + x^
2)^m*(b*c^2 + a*d^2 - 2*b*c*x^2 + b*x^4)^p, x], x, Sqrt[c + d*x]], x] /; Fr
eeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && ILtQ[m, 0] && IntegerQ[n + 1/2]
```

rule 1584
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4373
```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Simp[-(-1)^((m - 1)/2)/(d*b^(m - 1))   Subst[Int[(b^2 - x^
2)^((m - 1)/2)*((a + x)^n/x), x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
 d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

3.319.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 464 vs. $2(84) = 168$.

Time = 10.67 (sec) , antiderivative size = 465, normalized size of antiderivative = 4.65

method	result
default	$\frac{\sqrt{a+b\sec(dx+c)}\left(15\sqrt{a}\,\cos(dx+c)\ln\left(4\cos(dx+c)\sqrt{\frac{(b+a\cos(dx+c))\cos(dx+c)}{(\cos(dx+c)+1)^2}}\,\sqrt{a}+4a\cos(dx+c)+4\sqrt{a}\sqrt{\frac{(b+a\cos(dx+c))\cos(dx+c)}{(\cos(dx+c)+1)^2}}\right.\right.$

input
```
int((a+b*sec(d*x+c))^(1/2)*tan(d*x+c)^3,x,method=_RETURNVERBOSE)
```

output
```
1/15/d/b^2*(a+b*sec(d*x+c))^(1/2)/(cos(d*x+c)+1)/((b+a*cos(d*x+c))*cos(d*x
+c)/(cos(d*x+c)+1)^2)^(1/2)*(15*a^(1/2)*cos(d*x+c)*ln(4*cos(d*x+c)*((b+a*c
os(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*a^(1/2)+4*a*cos(d*x+c)+4*a^(
1/2)*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)+2*b)*b^2-4*((b+a
*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*a^2*cos(d*x+c)-30*cos(d*x+
c)*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*b^2-4*((b+a*cos(d*
x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*a^2+2*((b+a*cos(d*x+c))*cos(d*x+c
)/(cos(d*x+c)+1)^2)^(1/2)*a*b-30*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+
1)^2)^(1/2)*b^2+2*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*a*b
*sec(d*x+c)+6*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*b^2*sec
(d*x+c)+6*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*b^2*sec(d*x
+c)^2)
```

3.319.5 Fricas [A] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 311, normalized size of antiderivative = 3.11

$$\int \sqrt{a+b\sec(c+dx)}\tan^3(c+dx)\,dx$$

$$= \left[\frac{15\sqrt{a}b^2\cos(dx+c)^2\log\left(-8a^2\cos(dx+c)^2-8ab\cos(dx+c)-b^2-4\left(2a\cos(dx+c)^2+b\cos(dx+\right.\right.}{30b^2d\cos(dx+} \right.$$

$$\left. -\frac{15\sqrt{-a}b^2\arctan\left(\frac{2\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+b}{\cos(dx+c)}}\cos(dx+c)}{2a\cos(dx+c)+b}\right)\cos(dx+c)^2-2\left(ab\cos(dx+c)-(2a^2+15b^2)\cos(dx\right.}{15b^2d\cos(dx+c)^2} \right]$$

input
```
integrate((a+b*sec(d*x+c))^(1/2)*tan(d*x+c)^3,x, algorithm="fricas")
```

output
```
[1/30*(15*sqrt(a)*b^2*cos(d*x + c)^2*log(-8*a^2*cos(d*x + c)^2 - 8*a*b*cos
(d*x + c) - b^2 - 4*(2*a*cos(d*x + c)^2 + b*cos(d*x + c))*sqrt(a)*sqrt((a*
cos(d*x + c) + b)/cos(d*x + c))) + 4*(a*b*cos(d*x + c) - (2*a^2 + 15*b^2)*
cos(d*x + c)^2 + 3*b^2)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c)))/(b^2*d*co
s(d*x + c)^2), -1/15*(15*sqrt(-a)*b^2*arctan(2*sqrt(-a)*sqrt((a*cos(d*x +
c) + b)/cos(d*x + c))*cos(d*x + c)/(2*a*cos(d*x + c) + b))*cos(d*x + c)^2
- 2*(a*b*cos(d*x + c) - (2*a^2 + 15*b^2)*cos(d*x + c)^2 + 3*b^2)*sqrt((a*c
os(d*x + c) + b)/cos(d*x + c)))/(b^2*d*cos(d*x + c)^2)]
```

3.319.6 Sympy [F]

$$\int \sqrt{a+b\sec(c+dx)}\tan^3(c+dx)\,dx = \int \sqrt{a+b\sec(c+dx)}\tan^3(c+dx)\,dx$$

input
```
integrate((a+b*sec(d*x+c))**(1/2)*tan(d*x+c)**3,x)
```

output
```
Integral(sqrt(a + b*sec(c + d*x))*tan(c + d*x)**3, x)
```

3.319.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.08

$$\int \sqrt{a+b\sec(c+dx)}\tan^3(c+dx)\,dx$$
$$= -\frac{15\sqrt{a}\log\left(\frac{\sqrt{a+\frac{b}{\cos(dx+c)}}-\sqrt{a}}{\sqrt{a+\frac{b}{\cos(dx+c)}}+\sqrt{a}}\right)+30\sqrt{a+\frac{b}{\cos(dx+c)}}-\frac{6\left(a+\frac{b}{\cos(dx+c)}\right)^{\frac{5}{2}}}{b^2}+\frac{10\left(a+\frac{b}{\cos(dx+c)}\right)^{\frac{3}{2}}a}{b^2}}{15\,d}$$

input
```
integrate((a+b*sec(d*x+c))^(1/2)*tan(d*x+c)^3,x, algorithm="maxima")
```

output
```
-1/15*(15*sqrt(a)*log((sqrt(a + b/cos(d*x + c)) - sqrt(a))/(sqrt(a + b/cos
(d*x + c)) + sqrt(a))) + 30*sqrt(a + b/cos(d*x + c)) - 6*(a + b/cos(d*x +
c))^(5/2)/b^2 + 10*(a + b/cos(d*x + c))^(3/2)*a/b^2)/d
```

3.319.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 539 vs. 2(84) = 168.

Time = 0.70 (sec) , antiderivative size = 539, normalized size of antiderivative = 5.39

$$\int \sqrt{a+b\sec(c+dx)}\tan^3(c+dx)\,dx =$$
$$-\frac{2\left(\frac{15\,a\arctan\left(-\frac{\sqrt{a-b}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-\sqrt{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a+b}+\sqrt{a-b}}{2\sqrt{-a}}\right)}{\sqrt{-a}}-\frac{2\left(15\left(\sqrt{a-b}\tan\left(\frac{1}{2}\,dx\right.\right.\right.}$$

input
```
integrate((a+b*sec(d*x+c))^(1/2)*tan(d*x+c)^3,x, algorithm="giac")
```

output
```
-2/15*(15*a*arctan(-1/2*(sqrt(a - b)*tan(1/2*d*x + 1/2*c)^2 - sqrt(a*tan(1
/2*d*x + 1/2*c)^4 - b*tan(1/2*d*x + 1/2*c)^4 - 2*a*tan(1/2*d*x + 1/2*c)^2
+ a + b) + sqrt(a - b))/sqrt(-a))/sqrt(-a) - 2*(15*(sqrt(a - b)*tan(1/2*d*
x + 1/2*c)^2 - sqrt(a*tan(1/2*d*x + 1/2*c)^4 - b*tan(1/2*d*x + 1/2*c)^4 -
2*a*tan(1/2*d*x + 1/2*c)^2 + a + b))^4*a - 30*(sqrt(a - b)*tan(1/2*d*x + 1
/2*c)^2 - sqrt(a*tan(1/2*d*x + 1/2*c)^4 - b*tan(1/2*d*x + 1/2*c)^4 - 2*a*t
an(1/2*d*x + 1/2*c)^2 + a + b))^3*(a + 2*b)*sqrt(a - b) + 20*(sqrt(a - b)*
tan(1/2*d*x + 1/2*c)^2 - sqrt(a*tan(1/2*d*x + 1/2*c)^4 - b*tan(1/2*d*x + 1
/2*c)^4 - 2*a*tan(1/2*d*x + 1/2*c)^2 + a + b))^2*(4*a*b - 3*b^2) - 15*a^3
- 10*a^2*b - 35*a*b^2 + 12*b^3 + 10*(sqrt(a - b)*tan(1/2*d*x + 1/2*c)^2 -
sqrt(a*tan(1/2*d*x + 1/2*c)^4 - b*tan(1/2*d*x + 1/2*c)^4 - 2*a*tan(1/2*d*x
 + 1/2*c)^2 + a + b))*(3*a^2 - a*b + 6*b^2)*sqrt(a - b))/(sqrt(a - b)*tan(
1/2*d*x + 1/2*c)^2 - sqrt(a*tan(1/2*d*x + 1/2*c)^4 - b*tan(1/2*d*x + 1/2*c
)^4 - 2*a*tan(1/2*d*x + 1/2*c)^2 + a + b) - sqrt(a - b))^5)*sgn(cos(d*x +
c))/d
```

3.319.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+b\sec(c+dx)}\tan^3(c+dx)\,dx = \int \tan(c+d\,x)^3\sqrt{a+\frac{b}{\cos(c+d\,x)}}\,dx$$

input
```
int(tan(c + d*x)^3*(a + b/cos(c + d*x))^(1/2),x)
```

output
```
int(tan(c + d*x)^3*(a + b/cos(c + d*x))^(1/2), x)
```

3.320 $\int \sqrt{a+b\sec(c+dx)}\tan(c+dx)\,dx$

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3.320.1 Optimal result

Integrand size = 21, antiderivative size = 51

$$\int \sqrt{a+b\sec(c+dx)}\tan(c+dx)\,dx = -\frac{2\sqrt{a}\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a}}\right)}{d}+\frac{2\sqrt{a+b\sec(c+dx)}}{d}$$

output
```
-2*arctanh((a+b*sec(d*x+c))^(1/2)/a^(1/2))*a^(1/2)/d+2*(a+b*sec(d*x+c))^(1
/2)/d
```

3.320.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.96

$$\int \sqrt{a+b\sec(c+dx)}\tan(c+dx)\,dx = \frac{-2\sqrt{a}\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a}}\right)+2\sqrt{a+b\sec(c+dx)}}{d}$$

input
```
Integrate[Sqrt[a + b*Sec[c + d*x]]*Tan[c + d*x],x]
```

output
```
(-2*Sqrt[a]*ArcTanh[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a]] + 2*Sqrt[a + b*Sec[c
 + d*x]])/d
```

3.320.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.96, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 25, 4373, 60, 73, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan(c+dx)\sqrt{a+b\sec(c+dx)}\,dx$$

$$\downarrow 3042$$

$$\int -\cot\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\downarrow 25$$

$$-\int \cot\left(\frac{1}{2}(2c+\pi)+dx\right)\sqrt{a+b\csc\left(\frac{1}{2}(2c+\pi)+dx\right)}dx$$

$$\downarrow 4373$$

$$\frac{\int \frac{\cos(c+dx)\sqrt{a+b\sec(c+dx)}}{b}d(b\sec(c+dx))}{d}$$

$$\downarrow 60$$

$$\frac{a\int \frac{\cos(c+dx)}{b\sqrt{a+b\sec(c+dx)}}d(b\sec(c+dx)) + 2\sqrt{a+b\sec(c+dx)}}{d}$$

$$\downarrow 73$$

$$\frac{2a\int \frac{1}{b^2\sec^2(c+dx)-a}d\sqrt{a+b\sec(c+dx)} + 2\sqrt{a+b\sec(c+dx)}}{d}$$

$$\downarrow 220$$

$$\frac{2\sqrt{a+b\sec(c+dx)} - 2\sqrt{a}\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a}}\right)}{d}$$

input
```
Int[Sqrt[a + b*Sec[c + d*x]]*Tan[c + d*x],x]
```

output
```
(-2*Sqrt[a]*ArcTanh[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a]] + 2*Sqrt[a + b*Sec[c
 + d*x]])/d
```

3.320.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4373
```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Simp[-(-1)^((m - 1)/2)/(d*b^(m - 1))   Subst[Int[(b^2 - x^
2)^((m - 1)/2)*((a + x)^n/x), x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
 d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

3.320.4 Maple [A] (verified)

Time = 0.66 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.82

method	result	size
derivativedivides	$\frac{2\sqrt{a+b\sec(dx+c)}-2\sqrt{a}\,\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec(dx+c)}}{\sqrt{a}}\right)}{d}$	42
default	$\frac{2\sqrt{a+b\sec(dx+c)}-2\sqrt{a}\,\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec(dx+c)}}{\sqrt{a}}\right)}{d}$	42

input
```
int((a+b*sec(d*x+c))^(1/2)*tan(d*x+c),x,method=_RETURNVERBOSE)
```

output
```
1/d*(2*(a+b*sec(d*x+c))^(1/2)-2*a^(1/2)*arctanh((a+b*sec(d*x+c))^(1/2)/a^(
1/2)))
```

3.320.5 Fricas [A] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 192, normalized size of antiderivative = 3.76

$$\int \sqrt{a+b\sec(c+dx)}\tan(c+dx)\,dx$$

$$= \left[\frac{\sqrt{a}\log\left(-8\,a^2\cos(dx+c)^2-8\,ab\cos(dx+c)-b^2+4\left(2\,a\cos(dx+c)^2+b\cos(dx+c)\right)\sqrt{a}\sqrt{\frac{a\cos(dx+c)+b}{\cos(dx+c)}}\right.}{2\,d}\right.$$

input
```
integrate((a+b*sec(d*x+c))^(1/2)*tan(d*x+c),x, algorithm="fricas")
```

output
```
[1/2*(sqrt(a)*log(-8*a^2*cos(d*x + c)^2 - 8*a*b*cos(d*x + c) - b^2 + 4*(2*
a*cos(d*x + c)^2 + b*cos(d*x + c))*sqrt(a)*sqrt((a*cos(d*x + c) + b)/cos(d
*x + c))) + 4*sqrt((a*cos(d*x + c) + b)/cos(d*x + c)))/d, (sqrt(-a)*arctan
(2*sqrt(-a)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c))*cos(d*x + c)/(2*a*cos(
d*x + c) + b)) + 2*sqrt((a*cos(d*x + c) + b)/cos(d*x + c)))/d]
```

3.320.6 Sympy [F]

$$\int \sqrt{a+b\sec(c+dx)}\tan(c+dx)\,dx = \int \sqrt{a+b\sec(c+dx)}\tan(c+dx)\,dx$$

input
```
integrate((a+b*sec(d*x+c))**(1/2)*tan(d*x+c),x)
```

output
```
Integral(sqrt(a + b*sec(c + d*x))*tan(c + d*x), x)
```

3.320.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.31

$$\int \sqrt{a+b\sec(c+dx)}\tan(c+dx)\,dx = \frac{\sqrt{a}\log\left(\frac{\sqrt{a+\frac{b}{\cos(dx+c)}}-\sqrt{a}}{\sqrt{a+\frac{b}{\cos(dx+c)}}+\sqrt{a}}\right)+2\sqrt{a+\frac{b}{\cos(dx+c)}}}{d}$$

input
```
integrate((a+b*sec(d*x+c))^(1/2)*tan(d*x+c),x, algorithm="maxima")
```

output
```
(sqrt(a)*log((sqrt(a + b/cos(d*x + c)) - sqrt(a))/(sqrt(a + b/cos(d*x + c)
) + sqrt(a))) + 2*sqrt(a + b/cos(d*x + c)))/d
```

3.320.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 185 vs. 2(43) = 86.

Time = 0.40 (sec) , antiderivative size = 185, normalized size of antiderivative = 3.63

$$\int \sqrt{a+b\sec(c+dx)}\tan(c+dx)\,dx$$
$$= \frac{2\left(\frac{a\arctan\left(-\frac{\sqrt{a-b}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4-2a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a+b}+\sqrt{a-b}}{2\sqrt{-a}}\right)}{\sqrt{-a}} - \frac{}{\sqrt{a-b}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-\sqrt{a\,t}}}{d}$$

input
```
integrate((a+b*sec(d*x+c))^(1/2)*tan(d*x+c),x, algorithm="giac")
```

output
```
2*(a*arctan(-1/2*(sqrt(a - b)*tan(1/2*d*x + 1/2*c)^2 - sqrt(a*tan(1/2*d*x
+ 1/2*c)^4 - b*tan(1/2*d*x + 1/2*c)^4 - 2*a*tan(1/2*d*x + 1/2*c)^2 + a + b
) + sqrt(a - b))/sqrt(-a))/sqrt(-a) - 2*b/(sqrt(a - b)*tan(1/2*d*x + 1/2*c
)^2 - sqrt(a*tan(1/2*d*x + 1/2*c)^4 - b*tan(1/2*d*x + 1/2*c)^4 - 2*a*tan(1
/2*d*x + 1/2*c)^2 + a + b) - sqrt(a - b)))*sgn(cos(d*x + c))/d
```

3.320.9 Mupad [B] (verification not implemented)

Time = 13.72 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.92

$$\int \sqrt{a+b\sec(c+dx)}\tan(c+dx)\,dx = \frac{2\sqrt{a+\frac{b}{\cos(c+dx)}}}{d} - \frac{2\sqrt{a}\,\operatorname{atanh}\left(\frac{\sqrt{a+\frac{b}{\cos(c+dx)}}}{\sqrt{a}}\right)}{d}$$

input
```
int(tan(c + d*x)*(a + b/cos(c + d*x))^(1/2),x)
```

output
```
(2*(a + b/cos(c + d*x))^(1/2))/d - (2*a^(1/2)*atanh((a + b/cos(c + d*x))^(
1/2)/a^(1/2)))/d
```

3.321 $\int \cot(c+dx)\sqrt{a+b\sec(c+dx)}\,dx$

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3.321.1 Optimal result

Integrand size = 21, antiderivative size = 106

$$\int \cot(c+dx)\sqrt{a+b\sec(c+dx)}\,dx = \frac{2\sqrt{a}\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a}}\right)}{d} - \frac{\sqrt{a-b}\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a-b}}\right)}{d} - \frac{\sqrt{a+b}\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)}{d}$$

output
```
2*arctanh((a+b*sec(d*x+c))^(1/2)/a^(1/2))*a^(1/2)/d-arctanh((a+b*sec(d*x+c
))^(1/2)/(a-b)^(1/2))*(a-b)^(1/2)/d-arctanh((a+b*sec(d*x+c))^(1/2)/(a+b)^(
1/2))*(a+b)^(1/2)/d
```

3.321.2 Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.94

$$\int \cot(c+dx)\sqrt{a+b\sec(c+dx)}\,dx = -\frac{-2\sqrt{a}\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a}}\right) + \sqrt{a-b}\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a-b}}\right) + \sqrt{a+b}\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)}{d}$$

input
```
Integrate[Cot[c + d*x]*Sqrt[a + b*Sec[c + d*x]],x]
```

output
```
-((-2*Sqrt[a]*ArcTanh[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a]] + Sqrt[a - b]*ArcT
anh[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a - b]] + Sqrt[a + b]*ArcTanh[Sqrt[a + b
*Sec[c + d*x]]/Sqrt[a + b]])/d)
```

3.321.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.11, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 25, 4373, 561, 1610, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot(c+dx)\sqrt{a+b\sec(c+dx)}\,dx$$

$$\downarrow 3042$$

$$\int -\frac{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}{\cot\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\downarrow 25$$

$$-\int \frac{\sqrt{a+b\csc\left(\frac{1}{2}(2c+\pi)+dx\right)}}{\cot\left(\frac{1}{2}(2c+\pi)+dx\right)}dx$$

$$\downarrow 4373$$

$$-\frac{b^2\int \frac{\cos(c+dx)\sqrt{a+b\sec(c+dx)}}{b(b^2-b^2\sec^2(c+dx))}d(b\sec(c+dx))}{d}$$

$$\downarrow 561$$

$$-\frac{2b^2\int \frac{b^2\sec^2(c+dx)}{(a-b^2\sec^2(c+dx))(b^4\sec^4(c+dx)-2ab^2\sec^2(c+dx)+a^2-b^2)}d\sqrt{a+b\sec(c+dx)}}{d}$$

$$\downarrow 1610$$

$$-\frac{2b^2\int \left(-\frac{a}{b^2(a-b^2\sec^2(c+dx))}+\frac{a+b}{2b^2(-b^2\sec^2(c+dx)+a+b)}+\frac{b-a}{2b^2(b^2\sec^2(c+dx)-a+b)}\right)d\sqrt{a+b\sec(c+dx)}}{d}$$

$$\downarrow 2009$$

$$-\frac{2b^2\left(-\frac{\sqrt{a}\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a}}\right)}{b^2}+\frac{\sqrt{a-b}\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a-b}}\right)}{2b^2}+\frac{\sqrt{a+b}\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)}{2b^2}\right)}{d}$$

input
```
Int[Cot[c + d*x]*Sqrt[a + b*Sec[c + d*x]],x]
```

output
```
(-2*b^2*(-((Sqrt[a]*ArcTanh[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a]])/b^2) + (Sqr
t[a - b]*ArcTanh[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a - b]])/(2*b^2) + (Sqrt[a
+ b]*ArcTanh[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]])/(2*b^2)))/d
```

3.321.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 561
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> With[{k = Denominator[n]}, Simp[k/d   Subst[Int[x^(k*(n + 1) - 1)*(-c
/d + x^k/d)^m*Simp[(b*c^2 + a*d^2)/d^2 - 2*b*c*(x^k/d^2) + b*(x^(2*k)/d^2),
 x]^p, x], x, (c + d*x)^(1/k)], x]] /; FreeQ[{a, b, c, d, m, p}, x] && Frac
tionQ[n] && IntegerQ[p] && IntegerQ[m]
```

rule 1610
```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.))/((a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*((d + e*x^2)^q/(a
+ b*x^2 + c*x^4)), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4
*a*c, 0] && IntegerQ[q] && IntegerQ[m]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4373
```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Simp[-(-1)^((m - 1)/2)/(d*b^(m - 1))   Subst[Int[(b^2 - x^
2)^((m - 1)/2)*((a + x)^n/x), x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
 d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

3.321.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 542 vs. $2(88) = 176$.

Time = 2.24 (sec) , antiderivative size = 543, normalized size of antiderivative = 5.12

method	result
default	$-\frac{\sqrt{a+b\sec(dx+c)}\left(\sqrt{a+b}\,\ln\left(-\frac{2\left(2\cos(dx+c)\sqrt{\frac{(b+a\cos(dx+c))\cos(dx+c)}{(\cos(dx+c)+1)^2}}\,\sqrt{a+b}+2a\cos(dx+c)+\cos(dx+c)b+2\sqrt{a+b}\sqrt{\frac{(b+a\cos(dx+c))\cos}{(\cos(dx+c)+1}}\right.\right.\right.}{\cos(dx+c)-1}$

input
```
int(cot(d*x+c)*(a+b*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2/d/(a-b)^(1/2)*(a+b*sec(d*x+c))^(1/2)*((a+b)^(1/2)*ln(-2*(2*cos(d*x+c)
*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*(a+b)^(1/2)+2*a*cos(
d*x+c)+cos(d*x+c)*b+2*(a+b)^(1/2)*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)
+1)^2)^(1/2)+b)/(cos(d*x+c)-1))*(a-b)^(1/2)-2*a^(1/2)*ln(4*cos(d*x+c)*((b+
a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*a^(1/2)+4*a*cos(d*x+c)+4*
a^(1/2)*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)+2*b)*(a-b)^(1
/2)-a*ln(1/(a-b)^(1/2)*(2*(a-b)^(1/2)*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*
x+c)+1)^2)^(1/2)*cos(d*x+c)+2*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^
2)^(1/2)*(a-b)^(1/2)-2*a*cos(d*x+c)+cos(d*x+c)*b-b)/(cos(d*x+c)+1))+ln(1/(
a-b)^(1/2)*(2*(a-b)^(1/2)*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(
1/2)*cos(d*x+c)+2*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*(a-
b)^(1/2)-2*a*cos(d*x+c)+cos(d*x+c)*b-b)/(cos(d*x+c)+1))*b)*cos(d*x+c)/(cos
(d*x+c)+1)/((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)
```

3.321.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 190 vs. $2(88) = 176$.

Time = 0.60 (sec) , antiderivative size = 2132, normalized size of antiderivative = 20.11

$$\int \cot(c+dx)\sqrt{a+b\sec(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate(cot(d*x+c)*(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
[1/4*(2*sqrt(a)*log(-8*a^2*cos(d*x + c)^2 - 8*a*b*cos(d*x + c) - b^2 - 4*(
2*a*cos(d*x + c)^2 + b*cos(d*x + c))*sqrt(a)*sqrt((a*cos(d*x + c) + b)/cos
(d*x + c))) + sqrt(a - b)*log(-((8*a^2 - 8*a*b + b^2)*cos(d*x + c)^2 + b^2
 - 4*((2*a - b)*cos(d*x + c)^2 + b*cos(d*x + c))*sqrt(a - b)*sqrt((a*cos(d
*x + c) + b)/cos(d*x + c)) + 2*(4*a*b - 3*b^2)*cos(d*x + c))/(cos(d*x + c)
^2 + 2*cos(d*x + c) + 1)) + sqrt(a + b)*log(-((8*a^2 + 8*a*b + b^2)*cos(d*
x + c)^2 + b^2 - 4*((2*a + b)*cos(d*x + c)^2 + b*cos(d*x + c))*sqrt(a + b)
*sqrt((a*cos(d*x + c) + b)/cos(d*x + c)) + 2*(4*a*b + 3*b^2)*cos(d*x + c))
/(cos(d*x + c)^2 - 2*cos(d*x + c) + 1)))/d, 1/4*(2*sqrt(-a - b)*arctan(2*s
qrt(-a - b)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c))*cos(d*x + c)/((2*a + b
)*cos(d*x + c) + b)) + 2*sqrt(a)*log(-8*a^2*cos(d*x + c)^2 - 8*a*b*cos(d*x
 + c) - b^2 - 4*(2*a*cos(d*x + c)^2 + b*cos(d*x + c))*sqrt(a)*sqrt((a*cos(
d*x + c) + b)/cos(d*x + c))) + sqrt(a - b)*log(-((8*a^2 - 8*a*b + b^2)*cos
(d*x + c)^2 + b^2 - 4*((2*a - b)*cos(d*x + c)^2 + b*cos(d*x + c))*sqrt(a -
 b)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c)) + 2*(4*a*b - 3*b^2)*cos(d*x +
c))/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)))/d, -1/4*(2*sqrt(-a + b)*arctan
(-2*sqrt(-a + b)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c))*cos(d*x + c)/((2*
a - b)*cos(d*x + c) + b)) - 2*sqrt(a)*log(-8*a^2*cos(d*x + c)^2 - 8*a*b*co
s(d*x + c) - b^2 - 4*(2*a*cos(d*x + c)^2 + b*cos(d*x + c))*sqrt(a)*sqrt((a
*cos(d*x + c) + b)/cos(d*x + c))) - sqrt(a + b)*log(-((8*a^2 + 8*a*b + ...
```

3.321.6 Sympy [F]

$$\int \cot(c+dx)\sqrt{a+b\sec(c+dx)}\,dx = \int \sqrt{a+b\sec(c+dx)}\cot(c+dx)\,dx$$

input
```
integrate(cot(d*x+c)*(a+b*sec(d*x+c))**(1/2),x)
```

output
```
Integral(sqrt(a + b*sec(c + d*x))*cot(c + d*x), x)
```

3.321.7 Maxima [F]

$$\int \cot(c+dx)\sqrt{a+b\sec(c+dx)}\,dx = \int \sqrt{b\sec(dx+c)+a}\cot(dx+c)\,dx$$

input
```
integrate(cot(d*x+c)*(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*sec(d*x + c) + a)*cot(d*x + c), x)
```

3.321.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 293 vs. $2(88) = 176$.

Time $= 0.63$ (sec) , antiderivative size $= 293$, normalized size of antiderivative $= 2.76$

$$\int \cot(c+dx)\sqrt{a+b\sec(c+dx)}\,dx =$$

$$-\left(\frac{4\,a\arctan\left(-\frac{\sqrt{a-b}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-\sqrt{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a+b}+\sqrt{a-b}}{2\sqrt{-a}}\right)}{\sqrt{-a}} - \frac{2\,(a+b)\arctan\left(-\frac{\sqrt{a-b}\tan}{}\right.}{}\right.$$

input
```
integrate(cot(d*x+c)*(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
-1/2*(4*a*arctan(-1/2*(sqrt(a - b)*tan(1/2*d*x + 1/2*c)^2 - sqrt(a*tan(1/2
*d*x + 1/2*c)^4 - b*tan(1/2*d*x + 1/2*c)^4 - 2*a*tan(1/2*d*x + 1/2*c)^2 +
a + b) + sqrt(a - b))/sqrt(-a))/sqrt(-a) - 2*(a + b)*arctan(-(sqrt(a - b)*
tan(1/2*d*x + 1/2*c)^2 - sqrt(a*tan(1/2*d*x + 1/2*c)^4 - b*tan(1/2*d*x + 1
/2*c)^4 - 2*a*tan(1/2*d*x + 1/2*c)^2 + a + b))/sqrt(-a - b))/sqrt(-a - b)
+ sqrt(a - b)*log(abs(-(sqrt(a - b)*tan(1/2*d*x + 1/2*c)^2 - sqrt(a*tan(1/
2*d*x + 1/2*c)^4 - b*tan(1/2*d*x + 1/2*c)^4 - 2*a*tan(1/2*d*x + 1/2*c)^2 +
 a + b))*(a - b) + sqrt(a - b)*a)))*sgn(cos(d*x + c))/d
```

3.321.9 Mupad [F(-1)]

Timed out.

$$\int \cot(c+dx)\sqrt{a+b\sec(c+dx)}\,dx = \int \cot(c+d\,x)\,\sqrt{a+\frac{b}{\cos(c+d\,x)}}\,dx$$

input
```
int(cot(c + d*x)*(a + b/cos(c + d*x))^(1/2),x)
```

output
```
int(cot(c + d*x)*(a + b/cos(c + d*x))^(1/2), x)
```

3.322 $\int \cot^3(c+dx)\sqrt{a+b\sec(c+dx)}\,dx$

3.322.1 Optimal result 2371
3.322.2 Mathematica [A] (verified) 2372
3.322.3 Rubi [A] (warning: unable to verify) 2372
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3.322.9 Mupad [F(-1)] 2380

3.322.1 Optimal result

Integrand size = 23, antiderivative size = 215

$$\begin{aligned}\int \cot^3(c+dx)\sqrt{a+b\sec(c+dx)}\,dx = &-\frac{2\sqrt{a}\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a}}\right)}{d}\\ &+\frac{a\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a-b}}\right)}{\sqrt{a-b}d}\\ &-\frac{3b\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a-b}}\right)}{4\sqrt{a-b}d}\\ &+\frac{a\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)}{\sqrt{a+b}d}\\ &+\frac{3b\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)}{4\sqrt{a+b}d}\\ &-\frac{\cot^2(c+dx)\sqrt{a+b\sec(c+dx)}}{2d}\end{aligned}$$

output

```
-2*arctanh((a+b*sec(d*x+c))^(1/2)/a^(1/2))*a^(1/2)/d+a*arctanh((a+b*sec(d*
x+c))^(1/2)/(a-b)^(1/2))/d/(a-b)^(1/2)-3/4*b*arctanh((a+b*sec(d*x+c))^(1/2
)/(a-b)^(1/2))/d/(a-b)^(1/2)+a*arctanh((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2))
/d/(a+b)^(1/2)+3/4*b*arctanh((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2))/d/(a+b)^(
1/2)-1/2*cot(d*x+c)^2*(a+b*sec(d*x+c))^(1/2)/d
```

3.322.2 Mathematica [A] (verified)

Time = 2.45 (sec) , antiderivative size = 212, normalized size of antiderivative = 0.99

$$\int \cot^3(c+dx)\sqrt{a+b\sec(c+dx)}\,dx$$
$$= \frac{-\frac{b\arctan\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{-a+b}}\right)}{\sqrt{-a+b}} - 8\sqrt{a}\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a}}\right) + \frac{4\sqrt{-(a-b)^2}\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a-b}}\right)}{\sqrt{-a+b}} + \frac{4a\operatorname{arctanh}\left(\frac{\sqrt{a+b}}{\sqrt{a+b}}\right)}{\sqrt{a+b}}}{4d}$$

input
```
Integrate[Cot[c + d*x]^3*Sqrt[a + b*Sec[c + d*x]],x]
```

output
```
(-((b*ArcTan[Sqrt[a + b*Sec[c + d*x]]/Sqrt[-a + b]])/Sqrt[-a + b]) - 8*Sqr
t[a]*ArcTanh[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a]] + (4*Sqrt[-(a - b)^2]*ArcTa
nh[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a - b]])/Sqrt[-a + b] + (4*a*ArcTanh[Sqrt
[a + b*Sec[c + d*x]]/Sqrt[a + b]])/Sqrt[a + b] + (3*b*ArcTanh[Sqrt[a + b*S
ec[c + d*x]]/Sqrt[a + b]])/Sqrt[a + b] - 2*Cot[c + d*x]^2*Sqrt[a + b*Sec[c
 + d*x]])/(4*d)
```

3.322.3 Rubi [A] (warning: unable to verify)

Time = 0.55 (sec) , antiderivative size = 268, normalized size of antiderivative = 1.25, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.565$, Rules used = {3042, 25, 4373, 561, 25, 1652, 25, 1484, 1492, 27, 1406, 220, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^3(c+dx)\sqrt{a+b\sec(c+dx)}\,dx$$
$$\downarrow 3042$$
$$\int -\frac{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}{\cot\left(c+dx+\frac{\pi}{2}\right)^3}dx$$
$$\downarrow 25$$
$$-\int \frac{\sqrt{a+b\csc\left(\frac{1}{2}(2c+\pi)+dx\right)}}{\cot\left(\frac{1}{2}(2c+\pi)+dx\right)^3}dx$$
$$\downarrow 4373$$

$$\frac{b^4 \int \frac{\cos(c+dx)\sqrt{a+b\sec(c+dx)}}{b(b^2-b^2\sec^2(c+dx))^2} d(b\sec(c+dx))}{d}$$

$$\downarrow 561$$

$$\frac{2b^4 \int -\frac{b^2\sec^2(c+dx)}{(a-b^2\sec^2(c+dx))(b^4\sec^4(c+dx)-2ab^2\sec^2(c+dx)+a^2-b^2)^2} d\sqrt{a+b\sec(c+dx)}}{d}$$

$$\downarrow 25$$

$$-\frac{2b^4 \int \frac{b^2\sec^2(c+dx)}{(a-b^2\sec^2(c+dx))(b^4\sec^4(c+dx)-2ab^2\sec^2(c+dx)+a^2-b^2)^2} d\sqrt{a+b\sec(c+dx)}}{d}$$

$$\downarrow 1652$$

$$2b^4\left(\frac{\int -\frac{a^2-b^2\sec^2(c+dx)a-b^2}{\left(b^4\sec^4(c+dx)-2ab^2\sec^2(c+dx)+a^2-b^2\right)^2} d\sqrt{a+b\sec(c+dx)}}{b^2} + \frac{a\int \frac{1}{\left(a-b^2\sec^2(c+dx)\right)\left(b^4\sec^4(c+dx)-2ab^2\sec^2(c+dx)+a^2-b^2\right)} d\sqrt{a+b\sec(}}{b^2}\right.$$

$$d$$

$$\downarrow 25$$

$$2b^4\left(\frac{a\int \frac{1}{\left(a-b^2\sec^2(c+dx)\right)\left(b^4\sec^4(c+dx)-2ab^2\sec^2(c+dx)+a^2-b^2\right)} d\sqrt{a+b\sec(c+dx)}}{b^2} - \frac{\int \frac{a^2-b^2\sec^2(c+dx)a-b^2}{\left(b^4\sec^4(c+dx)-2ab^2\sec^2(c+dx)+a^2-b^2\right)^2} d\sqrt{a+b\sec(c-}}{b^2}\right.$$

$$d$$

$$\downarrow 1484$$

$$2b^4\left(\frac{a\int \left(\frac{1}{2b^2\left(-b^2\sec^2(c+dx)+a+b\right)} - \frac{1}{2b^2\left(b^2\sec^2(c+dx)-a+b\right)} - \frac{1}{b^2\left(a-b^2\sec^2(c+dx)\right)}\right) d\sqrt{a+b\sec(c+dx)}}{b^2} - \frac{\int \frac{a^2-b^2\sec^2(c+dx)a-b^2}{\left(b^4\sec^4(c+dx)-2ab^2\sec^2(c+dx)+a^2}}{b^2}\right.$$

$$d$$

$$\downarrow 1492$$

$$2b^4\left(\frac{a\int \left(\frac{1}{2b^2\left(-b^2\sec^2(c+dx)+a+b\right)} - \frac{1}{2b^2\left(b^2\sec^2(c+dx)-a+b\right)} - \frac{1}{b^2\left(a-b^2\sec^2(c+dx)\right)}\right) d\sqrt{a+b\sec(c+dx)}}{b^2} - \frac{\sqrt{a+b\sec(c+dx)}}{4\left(a^2-2ab^2\sec^2(c+dx)+b^4\sec^4(c+dx)-\right.}\right.$$

$$d$$

$$\downarrow 27$$

$$2b^4\left(\frac{a\int\left(\frac{1}{2b^2\left(-b^2\sec^2(c+dx)+a+b\right)}-\frac{1}{2b^2\left(b^2\sec^2(c+dx)-a+b\right)}-\frac{1}{b^2\left(a-b^2\sec^2(c+dx)\right)}\right)d\sqrt{a+b\sec(c+dx)}}{b^2}-\frac{\frac{3}{4}\int\frac{1}{b^4\sec^4(c+dx)-2ab^2\sec^2(c+dx)+a^2}}{d}\right.$$

$$\downarrow 1406$$

$$2b^4\left(\frac{a\int\left(\frac{1}{2b^2\left(-b^2\sec^2(c+dx)+a+b\right)}-\frac{1}{2b^2\left(b^2\sec^2(c+dx)-a+b\right)}-\frac{1}{b^2\left(a-b^2\sec^2(c+dx)\right)}\right)d\sqrt{a+b\sec(c+dx)}}{b^2}-\frac{\frac{3}{4}\left(\frac{\int\frac{1}{b^2\sec^2(c+dx)-a-b}d\sqrt{a+b\sec(c+dx)}}{2b}\right.}{d}\right.$$

$$\downarrow 220$$

$$2b^4\left(\frac{a\int\left(\frac{1}{2b^2\left(-b^2\sec^2(c+dx)+a+b\right)}-\frac{1}{2b^2\left(b^2\sec^2(c+dx)-a+b\right)}-\frac{1}{b^2\left(a-b^2\sec^2(c+dx)\right)}\right)d\sqrt{a+b\sec(c+dx)}}{b^2}-\frac{\frac{\sqrt{a+b\sec(c+dx)}}{4\left(a^2-2ab^2\sec^2(c+dx)+b^4\sec^4(c+dx)-}\right.}{d}\right.$$

$$\downarrow 2009$$

$$2b^4\left(\frac{a\left(-\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}b^2}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a-b}}\right)}{2b^2\sqrt{a-b}}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)}{2b^2\sqrt{a+b}}\right)}{b^2}-\frac{\frac{\sqrt{a+b\sec(c+dx)}}{4\left(a^2-2ab^2\sec^2(c+dx)+b^4\sec^4(c+dx)-}\right.}{d}\right.$$

input
```
Int[Cot[c + d*x]^3*Sqrt[a + b*Sec[c + d*x]],x]
```

output
```
(2*b^4*((a*(-(ArcTanh[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a]]/(Sqrt[a]*b^2)) + A
rcTanh[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a - b]]/(2*Sqrt[a - b]*b^2) + ArcTanh
[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]]/(2*b^2*Sqrt[a + b])))/b^2 - ((3*(Ar
cTanh[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a - b]]/(2*Sqrt[a - b]*b) - ArcTanh[Sq
rt[a + b*Sec[c + d*x]]/Sqrt[a + b]]/(2*b*Sqrt[a + b])))/4 + Sqrt[a + b*Sec
[c + d*x]]/(4*(a^2 - b^2 - 2*a*b^2*Sec[c + d*x]^2 + b^4*Sec[c + d*x]^4)))/
b^2))/d
```

3.322.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 561
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> With[{k = Denominator[n]}, Simp[k/d   Subst[Int[x^(k*(n + 1) - 1)*(-c
/d + x^k/d)^m*Simp[(b*c^2 + a*d^2)/d^2 - 2*b*c*(x^k/d^2) + b*(x^(2*k)/d^2),
 x]^p, x], x, (c + d*x)^(1/k)], x]] /; FreeQ[{a, b, c, d, m, p}, x] && Frac
tionQ[n] && IntegerQ[p] && IntegerQ[m]
```

rule 1406
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[b^
2 - 4*a*c, 2]}, Simp[c/q   Int[1/(b/2 - q/2 + c*x^2), x], x] - Simp[c/q   I
nt[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c
, 0] && PosQ[b^2 - 4*a*c]
```

rule 1484
```
Int[((d_) + (e_.)*(x_)^2)^(q_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symb
ol] :> Int[ExpandIntegrand[(d + e*x^2)^q/(a + b*x^2 + c*x^4), x], x] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2,
 0] && IntegerQ[q]
```

rule 1492
```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*((a + b*x^2 +
 c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2
 - 4*a*c))   Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p +
7)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
 b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
 LtQ[p, -1] && IntegerQ[2*p]
```

rule 1652
```
Int[(((f_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_))/((d_) +
 (e_.)*(x_)^2), x_Symbol] :> Simp[f^2/(c*d^2 - b*d*e + a*e^2)   Int[(f*x)^(
m - 2)*(a*e + c*d*x^2)*(a + b*x^2 + c*x^4)^p, x], x] - Simp[d*e*(f^2/(c*d^2
 - b*d*e + a*e^2))   Int[(f*x)^(m - 2)*((a + b*x^2 + c*x^4)^(p + 1)/(d + e*
x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ
[p, -1] && GtQ[m, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4373
```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Simp[-(-1)^((m - 1)/2)/(d*b^(m - 1))   Subst[Int[(b^2 - x^
2)^((m - 1)/2)*((a + x)^n/x), x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
 d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

3.322.4 Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 2071 vs. $2(177) = 354$.

Time = 2.20 (sec) , antiderivative size = 2072, normalized size of antiderivative = 9.64

method	result	size
default	Expression too large to display	2072

input
```
int(cot(d*x+c)^3*(a+b*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/8/d/(a-b)^(3/2)/(a+b)*(-((1-cos(d*x+c))^4*a*csc(d*x+c)^4-(1-cos(d*x+c))^
4*b*csc(d*x+c)^4-2*a*(1-cos(d*x+c))^2*csc(d*x+c)^2+a+b)^(1/2)*(a-b)^(3/2)*
a*(1-cos(d*x+c))^4*csc(d*x+c)^4+((1-cos(d*x+c))^4*a*csc(d*x+c)^4-(1-cos(d*
x+c))^4*b*csc(d*x+c)^4-2*a*(1-cos(d*x+c))^2*csc(d*x+c)^2+a+b)^(1/2)*(a-b)^
(3/2)*b*(1-cos(d*x+c))^4*csc(d*x+c)^4+8*a^(3/2)*ln(2*(-2*a*(1-cos(d*x+c))^
2*csc(d*x+c)^2+b*(1-cos(d*x+c))^2*csc(d*x+c)^2+2*a^(1/2)*((1-cos(d*x+c))^4
*a*csc(d*x+c)^4-(1-cos(d*x+c))^4*b*csc(d*x+c)^4-2*a*(1-cos(d*x+c))^2*csc(d
*x+c)^2+a+b)^(1/2)+2*a+b)/((1-cos(d*x+c))^2*csc(d*x+c)^2+1))*(1-cos(d*x+c)
)^2*(a-b)^(3/2)*csc(d*x+c)^2+8*a^(1/2)*ln(2*(-2*a*(1-cos(d*x+c))^2*csc(d*x
+c)^2+b*(1-cos(d*x+c))^2*csc(d*x+c)^2+2*a^(1/2)*((1-cos(d*x+c))^4*a*csc(d*
x+c)^4-(1-cos(d*x+c))^4*b*csc(d*x+c)^4-2*a*(1-cos(d*x+c))^2*csc(d*x+c)^2+a
+b)^(1/2)+2*a+b)/((1-cos(d*x+c))^2*csc(d*x+c)^2+1))*(1-cos(d*x+c))^2*(a-b)
^(3/2)*b*csc(d*x+c)^2-4*(a+b)^(1/2)*ln(2*(-a*(1-cos(d*x+c))^2*csc(d*x+c)^2
+(a+b)^(1/2)*((1-cos(d*x+c))^4*a*csc(d*x+c)^4-(1-cos(d*x+c))^4*b*csc(d*x+c
)^4-2*a*(1-cos(d*x+c))^2*csc(d*x+c)^2+a+b)^(1/2)+a+b)/(1-cos(d*x+c))^2*sin
(d*x+c)^2)*(1-cos(d*x+c))^2*(a-b)^(3/2)*a*csc(d*x+c)^2-3*(a+b)^(1/2)*ln(2*
(-a*(1-cos(d*x+c))^2*csc(d*x+c)^2+(a+b)^(1/2)*((1-cos(d*x+c))^4*a*csc(d*x+
c)^4-(1-cos(d*x+c))^4*b*csc(d*x+c)^4-2*a*(1-cos(d*x+c))^2*csc(d*x+c)^2+a+b
)^(1/2)+a+b)/(1-cos(d*x+c))^2*sin(d*x+c)^2)*(1-cos(d*x+c))^2*(a-b)^(3/2)*b
*csc(d*x+c)^2+((1-cos(d*x+c))^4*a*csc(d*x+c)^4-(1-cos(d*x+c))^4*b*csc(d...
```

3.322.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 364 vs. $2(177) = 354$.

Time $= 1.81$ (sec) , antiderivative size $= 3523$, normalized size of antiderivative $= 16.39$

$$\int \cot^3(c+dx)\sqrt{a+b\sec(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate(cot(d*x+c)^3*(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
[1/16*(8*(a^2 - b^2)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c))*cos(d*x + c)^
2 + 8*((a^2 - b^2)*cos(d*x + c)^2 - a^2 + b^2)*sqrt(a)*log(-8*a^2*cos(d*x
+ c)^2 - 8*a*b*cos(d*x + c) - b^2 + 4*(2*a*cos(d*x + c)^2 + b*cos(d*x + c)
)*sqrt(a)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c))) - ((4*a^2 + a*b - 3*b^2
)*cos(d*x + c)^2 - 4*a^2 - a*b + 3*b^2)*sqrt(a - b)*log(-((8*a^2 - 8*a*b +
 b^2)*cos(d*x + c)^2 + b^2 - 4*((2*a - b)*cos(d*x + c)^2 + b*cos(d*x + c))
*sqrt(a - b)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c)) + 2*(4*a*b - 3*b^2)*c
os(d*x + c))/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) + ((4*a^2 - a*b - 3*b^
2)*cos(d*x + c)^2 - 4*a^2 + a*b + 3*b^2)*sqrt(a + b)*log(-((8*a^2 + 8*a*b
+ b^2)*cos(d*x + c)^2 + b^2 + 4*((2*a + b)*cos(d*x + c)^2 + b*cos(d*x + c)
)*sqrt(a + b)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c)) + 2*(4*a*b + 3*b^2)*
cos(d*x + c))/(cos(d*x + c)^2 - 2*cos(d*x + c) + 1)))/((a^2 - b^2)*d*cos(d
*x + c)^2 - (a^2 - b^2)*d), 1/16*(8*(a^2 - b^2)*sqrt((a*cos(d*x + c) + b)/
cos(d*x + c))*cos(d*x + c)^2 - 2*((4*a^2 - a*b - 3*b^2)*cos(d*x + c)^2 - 4
*a^2 + a*b + 3*b^2)*sqrt(-a - b)*arctan(2*sqrt(-a - b)*sqrt((a*cos(d*x + c
) + b)/cos(d*x + c))*cos(d*x + c)/((2*a + b)*cos(d*x + c) + b)) + 8*((a^2
- b^2)*cos(d*x + c)^2 - a^2 + b^2)*sqrt(a)*log(-8*a^2*cos(d*x + c)^2 - 8*a
*b*cos(d*x + c) - b^2 + 4*(2*a*cos(d*x + c)^2 + b*cos(d*x + c))*sqrt(a)*sq
rt((a*cos(d*x + c) + b)/cos(d*x + c))) - ((4*a^2 + a*b - 3*b^2)*cos(d*x +
c)^2 - 4*a^2 - a*b + 3*b^2)*sqrt(a - b)*log(-((8*a^2 - 8*a*b + b^2)*cos...
```

3.322.6 Sympy [F]

$$\int \cot^3(c+dx)\sqrt{a+b\sec(c+dx)}\,dx = \int \sqrt{a+b\sec(c+dx)}\cot^3(c+dx)\,dx$$

input
```
integrate(cot(d*x+c)**3*(a+b*sec(d*x+c))**(1/2),x)
```

output
```
Integral(sqrt(a + b*sec(c + d*x))*cot(c + d*x)**3, x)
```

3.322.7 Maxima [F]

$$\int \cot^3(c+dx)\sqrt{a+b\sec(c+dx)}\,dx = \int \sqrt{b\sec(dx+c)+a}\cot(dx+c)^3\,dx$$

input
```
integrate(cot(d*x+c)^3*(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*sec(d*x + c) + a)*cot(d*x + c)^3, x)
```

3.322.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 514 vs. $2(177) = 354$.

Time $= 0.84$ (sec) , antiderivative size $= 514$, normalized size of antiderivative $= 2.39$

$$\int \cot^3(c+dx)\sqrt{a+b\sec(c+dx)}\,dx$$
$$= \left(\frac{16\,a\arctan\left(-\frac{\sqrt{a-b}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-\sqrt{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a+b+\sqrt{a-b}}}{2\sqrt{-a}}\right)}{\sqrt{-a}} - \frac{2\,(4\,a+3\,b)\arctan\left(-\frac{\sqrt{a-b}\,\mathrm{ta}}{}\right.}{}$$

input
```
integrate(cot(d*x+c)^3*(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
1/8*(16*a*arctan(-1/2*(sqrt(a - b)*tan(1/2*d*x + 1/2*c)^2 - sqrt(a*tan(1/2
*d*x + 1/2*c)^4 - b*tan(1/2*d*x + 1/2*c)^4 - 2*a*tan(1/2*d*x + 1/2*c)^2 +
a + b) + sqrt(a - b))/sqrt(-a))/sqrt(-a) - 2*(4*a + 3*b)*arctan(-(sqrt(a -
 b)*tan(1/2*d*x + 1/2*c)^2 - sqrt(a*tan(1/2*d*x + 1/2*c)^4 - b*tan(1/2*d*x
 + 1/2*c)^4 - 2*a*tan(1/2*d*x + 1/2*c)^2 + a + b))/sqrt(-a - b))/sqrt(-a -
 b) + (4*a - 3*b)*log(abs((sqrt(a - b)*tan(1/2*d*x + 1/2*c)^2 - sqrt(a*tan
(1/2*d*x + 1/2*c)^4 - b*tan(1/2*d*x + 1/2*c)^4 - 2*a*tan(1/2*d*x + 1/2*c)^
2 + a + b))*(a - b) - sqrt(a - b)*a))/sqrt(a - b) + sqrt(a*tan(1/2*d*x + 1
/2*c)^4 - b*tan(1/2*d*x + 1/2*c)^4 - 2*a*tan(1/2*d*x + 1/2*c)^2 + a + b) -
 2*((sqrt(a - b)*tan(1/2*d*x + 1/2*c)^2 - sqrt(a*tan(1/2*d*x + 1/2*c)^4 -
b*tan(1/2*d*x + 1/2*c)^4 - 2*a*tan(1/2*d*x + 1/2*c)^2 + a + b))*a - (a + b
)*sqrt(a - b))/((sqrt(a - b)*tan(1/2*d*x + 1/2*c)^2 - sqrt(a*tan(1/2*d*x +
 1/2*c)^4 - b*tan(1/2*d*x + 1/2*c)^4 - 2*a*tan(1/2*d*x + 1/2*c)^2 + a + b)
)^2 - a - b))*sgn(cos(d*x + c))/d
```

3.322.9 Mupad [F(-1)]

Timed out.

$$\int \cot^3(c+dx)\sqrt{a+b\sec(c+dx)}\,dx = \int \cot(c+d\,x)^3\sqrt{a+\frac{b}{\cos(c+d\,x)}}\,dx$$

input
```
int(cot(c + d*x)^3*(a + b/cos(c + d*x))^(1/2),x)
```

output
```
int(cot(c + d*x)^3*(a + b/cos(c + d*x))^(1/2), x)
```

3.323 $\int \sqrt{a+b\sec(c+dx)}\tan^2(c+dx)\,dx$

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3.323.1 Optimal result

Integrand size = 23, antiderivative size = 344

$$\int \sqrt{a+b\sec(c+dx)}\tan^2(c+dx)\,dx =$$
$$-\frac{2a(a-b)\sqrt{a+b}\cot(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{3b^2d}$$
$$-\frac{2\sqrt{a+b}(a+2b)\cot(c+dx)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{3bd}$$
$$+\frac{2\sqrt{a+b}\cot(c+dx)\,\text{EllipticPi}\left(\frac{a+b}{a},\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{d}$$
$$+\frac{2\sqrt{a+b\sec(c+dx)}\tan(c+dx)}{3d}$$

output

```
-2/3*a*(a-b)*cot(d*x+c)*EllipticE((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),((a+b
)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+
c))/(a-b))^(1/2)/b^2/d-2/3*(a+2*b)*cot(d*x+c)*EllipticF((a+b*sec(d*x+c))^(
1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(d*x+c))/(a+b))
^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/b/d+2*cot(d*x+c)*EllipticPi((a+b*se
c(d*x+c))^(1/2)/(a+b)^(1/2),(a+b)/a,((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1
-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/d+2/3*(a+b*sec(d
*x+c))^(1/2)*tan(d*x+c)/d
```

3.323.2 Mathematica [A] (verified)

Time = 11.57 (sec) , antiderivative size = 376, normalized size of antiderivative = 1.09

$$\int \sqrt{a+b\sec(c+dx)}\tan^2(c+dx)\,dx =$$

$$-\frac{2\cos^2\left(\frac{1}{2}(c+dx)\right)\sqrt{a+b\sec(c+dx)}\left(2a(a+b)\sqrt{\frac{\cos(c+dx)}{1+\cos(c+dx)}}\sqrt{\frac{b+a\cos(c+dx)}{(a+b)(1+\cos(c+dx))}}E\left(\arcsin\left(\tan\left(\frac{1}{2}(c+d\right.\right.\right.}$$

$$+\frac{\sqrt{a+b\sec(c+dx)}\left(\frac{2a\sin(c+dx)}{3b}+\frac{2}{3}\tan(c+dx)\right)}{d}$$

input
```
Integrate[Sqrt[a + b*Sec[c + d*x]]*Tan[c + d*x]^2,x]
```

output
```
(-2*Cos[(c + d*x)/2]^2*Sqrt[a + b*Sec[c + d*x]]*(2*a*(a + b)*Sqrt[Cos[c +
d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d
*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] - 4*(2*a - b)*
b*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)
*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]
 + 12*a*b*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/
((a + b)*(1 + Cos[c + d*x]))]*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (a
- b)/(a + b)] + a*Cos[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan
[(c + d*x)/2]))/(3*b*d*(b + a*Cos[c + d*x])) + (Sqrt[a + b*Sec[c + d*x]]*(
(2*a*Sin[c + d*x])/(3*b) + (2*Tan[c + d*x])/3))/d
```

3.323.3 Rubi [A] (verified)

Time = 1.25 (sec) , antiderivative size = 345, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.565$, Rules used = {3042, 4382, 3042, 4545, 27, 3042, 4546, 3042, 4409, 3042, 4271, 4319, 4492}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^2(c+dx)\sqrt{a+b\sec(c+dx)}\,dx$$

$$\downarrow 3042$$

$$\int \cot\left(c+dx+\frac{\pi}{2}\right)^2\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\downarrow 4382$$

$$\int \left(\sec^2(c+dx)-1\right)\sqrt{a+b\sec(c+dx)}dx$$

$$\downarrow 3042$$

$$\int \left(\csc\left(c+dx+\frac{\pi}{2}\right)^2-1\right)\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\downarrow 4545$$

$$\frac{2}{3}\int -\frac{-a\sec^2(c+dx)+2b\sec(c+dx)+3a}{2\sqrt{a+b\sec(c+dx)}}dx+\frac{2\tan(c+dx)\sqrt{a+b\sec(c+dx)}}{3d}$$

$$\downarrow 27$$

$$\frac{2\tan(c+dx)\sqrt{a+b\sec(c+dx)}}{3d}-\frac{1}{3}\int \frac{-a\sec^2(c+dx)+2b\sec(c+dx)+3a}{\sqrt{a+b\sec(c+dx)}}dx$$

$$\downarrow 3042$$

$$\frac{2\tan(c+dx)\sqrt{a+b\sec(c+dx)}}{3d}-\frac{1}{3}\int \frac{-a\csc\left(c+dx+\frac{\pi}{2}\right)^2+2b\csc\left(c+dx+\frac{\pi}{2}\right)+3a}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\downarrow 4546$$

$$\frac{1}{3}\left(a\int \frac{\sec(c+dx)(\sec(c+dx)+1)}{\sqrt{a+b\sec(c+dx)}}dx-\int \frac{3a+(a+2b)\sec(c+dx)}{\sqrt{a+b\sec(c+dx)}}dx\right)+\frac{2\tan(c+dx)\sqrt{a+b\sec(c+dx)}}{3d}$$

$$\downarrow 3042$$

$$\frac{1}{3}\left(a\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)+1\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-\int \frac{3a+(a+2b)\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx\right)+\frac{2\tan(c+dx)\sqrt{a+b\sec(c+dx)}}{3d}$$

$$\downarrow 4409$$

$$\frac{1}{3}\left(a\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)+1\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-3a\int \frac{1}{\sqrt{a+b\sec(c+dx)}}dx-(a+2b)\int \frac{\sec(c+dx)}{\sqrt{a+b\sec(c+dx)}}dx\right)+\frac{2\tan(c+dx)\sqrt{a+b\sec(c+dx)}}{3d}$$

$$\downarrow 3042$$

$$\frac{1}{3}\left(-3a\int\frac{1}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-(a+2b)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+a\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+\right.\right.\right.}{\sqrt{a+b\csc\left(c+dx\right.}}$$

$$\frac{2\tan(c+dx)\sqrt{a+b\sec(c+dx)}}{3d}$$

$$\downarrow 4271$$

$$\frac{1}{3}\left(-(a+2b)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+a\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)+1\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{6\sqrt{a+b}\cot(c+\right.\right.$$

$$\frac{2\tan(c+dx)\sqrt{a+b\sec(c+dx)}}{3d}$$

$$\downarrow 4319$$

$$\frac{1}{3}\left(a\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)+1\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{2\sqrt{a+b}(a+2b)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1}{a-b}}\right.$$

$$\frac{2\tan(c+dx)\sqrt{a+b\sec(c+dx)}}{3d}$$

$$\downarrow 4492$$

$$\frac{1}{3}\left(-\frac{2a(a-b)\sqrt{a+b}\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}E\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{b^2d}-\frac{2\sqrt{a+b}(a\right.$$

$$\frac{2\tan(c+dx)\sqrt{a+b\sec(c+dx)}}{3d}$$

input
```
Int[Sqrt[a + b*Sec[c + d*x]]*Tan[c + d*x]^2,x]
```

output
```
((-2*a*(a - b)*Sqrt[a + b]*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c
+ d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)
]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(b^2*d) - (2*Sqrt[a + b]*(a + 2
*b)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (
a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c
+ d*x]))/(a - b))])/(b*d) + (6*Sqrt[a + b]*Cot[c + d*x]*EllipticPi[(a + b)
/a, ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b
*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/d)/
3 + (2*Sqrt[a + b*Sec[c + d*x]]*Tan[c + d*x])/(3*d)
```

3.323. $\int \sqrt{a+b\sec(c+dx)}\tan^2(c+dx)\,dx$

3.323.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4271
```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[2*(Rt[a
 + b, 2]/(a*d*Cot[c + d*x]))*Sqrt[b*((1 - Csc[c + d*x])/(a + b))]*Sqrt[(-b)
*((1 + Csc[c + d*x])/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[
c + d*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

rule 4319
```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*(Rt[a + b, 2]/(b*f*Cot[e + f*x]))*Sqrt[(b*(1 - Csc[e + f*
x]))/(a + b)]*Sqrt[(-b)*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt
[a + b*Csc[e + f*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4382
```
Int[cot[(c_.) + (d_.)*(x_)]^2*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_),
x_Symbol] :> Int[(-1 + Csc[c + d*x]^2)*(a + b*Csc[c + d*x])^n, x] /; FreeQ[
{a, b, c, d, n}, x] && NeQ[a^2 - b^2, 0]
```

rule 4409
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Simp[c   Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] +
Simp[d   Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

rule 4492
```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(A*b - a*B)*Rt[a
 + b*(B/A), 2]*Sqrt[b*((1 - Csc[e + f*x])/(a + b))]*(Sqrt[(-b)*((1 + Csc[e
+ f*x])/(a - b))]/(b^2*f*Cot[e + f*x]))*EllipticE[ArcSin[Sqrt[a + b*Csc[e +
 f*x]]/Rt[a + b*(B/A), 2]], (a*A + b*B)/(a*A - b*B)], x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

rule 4545
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_))^(m_.), x_Symbol] :> Simp[(-C)*Cot[e + f*x]*((a + b*Csc[e + f*x])^
m/(f*(m + 1))), x] + Simp[1/(m + 1)   Int[(a + b*Csc[e + f*x])^(m - 1)*Simp
[a*A*(m + 1) + (A*b*(m + 1) + b*C*m)*Csc[e + f*x] + a*C*m*Csc[e + f*x]^2, x
], x], x] /; FreeQ[{a, b, e, f, A, C}, x] && NeQ[a^2 - b^2, 0] && IGtQ[2*m,
 0]
```

rule 4546
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Int[(A + (B - C
)*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]], x] + Simp[C   Int[Csc[e + f*x]*((
1 + Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]]), x], x] /; FreeQ[{a, b, e, f, A
, B, C}, x] && NeQ[a^2 - b^2, 0]
```

3.323.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1379 vs. $2(309) = 618$.

Time = 11.95 (sec) , antiderivative size = 1380, normalized size of antiderivative = 4.01

method	result	size
default	Expression too large to display	1380

input
```
int((a+b*sec(d*x+c))^(1/2)*tan(d*x+c)^2,x,method=_RETURNVERBOSE)
```

output

```
2/3/d/b*(a+b*sec(d*x+c))^(1/2)/(b+a*cos(d*x+c))/(cos(d*x+c)+1)*(-4*Ellipti
cF(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(c
os(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*a*b*cos(d*x+c)^2+2*E
llipticF(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+
c))/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*b^2*cos(d*x+c)
^2+EllipticE(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))*(1/(a+b)*(b+a*cos(
d*x+c))/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*a^2*cos(d*
x+c)^2+EllipticE(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))*(1/(a+b)*(b+a*
cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*a*b*co
s(d*x+c)^2+6*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(
cos(d*x+c)+1))^(1/2)*EllipticPi(cot(d*x+c)-csc(d*x+c),-1,((a-b)/(a+b))^(1/
2))*a*b*cos(d*x+c)^2-8*EllipticF(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2)
)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+
1))^(1/2)*a*b*cos(d*x+c)+4*EllipticF(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(
1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x
+c)+1))^(1/2)*b^2*cos(d*x+c)+2*EllipticE(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b
))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos
(d*x+c)+1))^(1/2)*a^2*cos(d*x+c)+2*EllipticE(cot(d*x+c)-csc(d*x+c),((a-b)/
(a+b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/
(cos(d*x+c)+1))^(1/2)*a*b*cos(d*x+c)+12*(cos(d*x+c)/(cos(d*x+c)+1))^(1/...
```

3.323.5 Fricas [F]

$$\int \sqrt{a+b\sec(c+dx)}\tan^2(c+dx)\,dx = \int \sqrt{b\sec\left(dx+c\right)+a}\tan\left(dx+c\right)^2\,dx$$

input

```
integrate((a+b*sec(d*x+c))^(1/2)*tan(d*x+c)^2,x, algorithm="fricas")
```

output

```
integral(sqrt(b*sec(d*x + c) + a)*tan(d*x + c)^2, x)
```

3.323.6 Sympy [F]

$$\int \sqrt{a+b\sec(c+dx)}\tan^2(c+dx)\,dx = \int \sqrt{a+b\sec(c+dx)}\tan^2(c+dx)\,dx$$

input
```
integrate((a+b*sec(d*x+c))**(1/2)*tan(d*x+c)**2,x)
```

output
```
Integral(sqrt(a + b*sec(c + d*x))*tan(c + d*x)**2, x)
```

3.323.7 Maxima [F]

$$\int \sqrt{a+b\sec(c+dx)}\tan^2(c+dx)\,dx = \int \sqrt{b\sec(dx+c)+a}\tan(dx+c)^2\,dx$$

input
```
integrate((a+b*sec(d*x+c))^(1/2)*tan(d*x+c)^2,x, algorithm="maxima")
```

output
```
integrate(sqrt(b*sec(d*x + c) + a)*tan(d*x + c)^2, x)
```

3.323.8 Giac [F]

$$\int \sqrt{a+b\sec(c+dx)}\tan^2(c+dx)\,dx = \int \sqrt{b\sec(dx+c)+a}\tan(dx+c)^2\,dx$$

input
```
integrate((a+b*sec(d*x+c))^(1/2)*tan(d*x+c)^2,x, algorithm="giac")
```

output
```
integrate(sqrt(b*sec(d*x + c) + a)*tan(d*x + c)^2, x)
```

3.323.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+b\sec(c+dx)}\tan^2(c+dx)\,dx = \int \tan(c+d\,x)^2\sqrt{a+\frac{b}{\cos(c+d\,x)}}\,dx$$

input
```
int(tan(c + d*x)^2*(a + b/cos(c + d*x))^(1/2),x)
```

output
```
int(tan(c + d*x)^2*(a + b/cos(c + d*x))^(1/2), x)
```

3.324 $\int \sqrt{a+b\sec(c+dx)}\,dx$

3.324.1 Optimal result . . . 2390
3.324.2 Mathematica [A] (verified) . . . 2390
3.324.3 Rubi [A] (verified) . . . 2391
3.324.4 Maple [A] (verified) . . . 2392
3.324.5 Fricas [F] . . . 2392
3.324.6 Sympy [F] . . . 2393
3.324.7 Maxima [F] . . . 2393
3.324.8 Giac [F] . . . 2393
3.324.9 Mupad [F(-1)] . . . 2394

3.324.1 Optimal result

Integrand size = 14, antiderivative size = 125

$$\int \sqrt{a+b\sec(c+dx)}\,dx = -\frac{2\cot(c+dx)\,\text{EllipticPi}\left(\frac{a}{a+b},\arcsin\left(\frac{\sqrt{a+b}}{\sqrt{a+b\sec(c+dx)}}\right),\frac{a-b}{a+b}\right)\sqrt{-\frac{b(1-\sec(c+dx))}{a+b\sec(c+dx)}}\sqrt{\frac{b(1+\sec(c+dx))}{a+b\sec(c+dx)}}(a+b\sec(c+$$
$$\sqrt{a+b}d$$

output
```
-2*cot(d*x+c)*EllipticPi((a+b)^(1/2)/(a+b*sec(d*x+c))^(1/2),a/(a+b),((a-b)
/(a+b))^(1/2))*(a+b*sec(d*x+c))*(-b*(1-sec(d*x+c))/(a+b*sec(d*x+c)))^(1/2)
*(b*(1+sec(d*x+c))/(a+b*sec(d*x+c)))^(1/2)/d/(a+b)^(1/2)
```

3.324.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.21

$$\int \sqrt{a+b\sec(c+dx)}\,dx$$
$$= \frac{4\cos^2\left(\frac{1}{2}(c+dx)\right)\sqrt{\frac{\cos(c+dx)}{1+\cos(c+dx)}}\sqrt{\frac{b+a\cos(c+dx)}{(a+b)(1+\cos(c+dx))}}\left((-a+b)\,\text{EllipticF}\left(\arcsin\left(\tan\left(\frac{1}{2}(c+dx)\right)\right),\frac{a-b}{a+b}\right)+2c\right.}{d(b+a\cos(c+dx))}$$

input
```
Integrate[Sqrt[a + b*Sec[c + d*x]],x]
```

output
```
(4*Cos[(c + d*x)/2]^2*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Co
s[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*((-a + b)*EllipticF[ArcSin[Tan[(
c + d*x)/2]], (a - b)/(a + b)] + 2*a*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2
]], (a - b)/(a + b)])*Sqrt[a + b*Sec[c + d*x]])/(d*(b + a*Cos[c + d*x]))
```

3.324.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3042, 4267}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a+b\sec(c+dx)}\,dx$$

$$\downarrow 3042$$

$$\int \sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}\,dx$$

$$\downarrow 4267$$

$$-\frac{2\cot(c+dx)\sqrt{-\frac{b(1-\sec(c+dx))}{a+b\sec(c+dx)}}\sqrt{\frac{b(\sec(c+dx)+1)}{a+b\sec(c+dx)}}(a+b\sec(c+dx))\,\text{EllipticPi}\left(\frac{a}{a+b},\arcsin\left(\frac{\sqrt{a+b}}{\sqrt{a+b\sec(c+dx)}}\right),\frac{a-b}{a+b}\right)}{d\sqrt{a+b}}$$

input
```
Int[Sqrt[a + b*Sec[c + d*x]],x]
```

output
```
(-2*Cot[c + d*x]*EllipticPi[a/(a + b), ArcSin[Sqrt[a + b]/Sqrt[a + b*Sec[c
 + d*x]]], (a - b)/(a + b)]*Sqrt[-((b*(1 - Sec[c + d*x]))/(a + b*Sec[c + d
*x]))]*Sqrt[(b*(1 + Sec[c + d*x]))/(a + b*Sec[c + d*x])]*(a + b*Sec[c + d*
x]))/(Sqrt[a + b]*d)
```

3.324.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4267
```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[2*((a + b
*Csc[c + d*x])/(d*Rt[a + b, 2]*Cot[c + d*x]))*Sqrt[b*((1 + Csc[c + d*x])/(a
 + b*Csc[c + d*x]))]*Sqrt[(-b)*((1 - Csc[c + d*x])/(a + b*Csc[c + d*x]))]*E
llipticPi[a/(a + b), ArcSin[Rt[a + b, 2]/Sqrt[a + b*Csc[c + d*x]]], (a - b)
/(a + b)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0]
```

3.324.4 Maple [A] (verified)

Time = 6.57 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.46

method	result
default	$\frac{2(\cos(dx+c)+1)\left(\text{EllipticF}\left(\cot(dx+c)-\csc(dx+c),\sqrt{\frac{a-b}{a+b}}\right)a-\text{EllipticF}\left(\cot(dx+c)-\csc(dx+c),\sqrt{\frac{a-b}{a+b}}\right)b-2a\,\text{EllipticPi}\left(\cot(dx+c)-\right.\right.}{d(b+a\cos(dx+c))}$

input
```
int((a+b*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/d*(cos(d*x+c)+1)*(EllipticF(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))*a
-EllipticF(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))*b-2*a*EllipticPi(cot
(d*x+c)-csc(d*x+c),-1,((a-b)/(a+b))^(1/2)))*(1/(a+b)*(b+a*cos(d*x+c))/(cos
(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(a+b*sec(d*x+c))^(1/2)
/(b+a*cos(d*x+c))
```

3.324.5 Fricas [F]

$$\int \sqrt{a+b\sec(c+dx)}\,dx = \int \sqrt{b\sec(dx+c)+a}\,dx$$

input
```
integrate((a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b*sec(d*x + c) + a), x)
```

3.324.6 Sympy [F]

$$\int \sqrt{a+b\sec(c+dx)}\,dx = \int \sqrt{a+b\sec(c+dx)}\,dx$$

input
```
integrate((a+b*sec(d*x+c))**(1/2),x)
```

output
```
Integral(sqrt(a + b*sec(c + d*x)), x)
```

3.324.7 Maxima [F]

$$\int \sqrt{a+b\sec(c+dx)}\,dx = \int \sqrt{b\sec(dx+c)+a}\,dx$$

input
```
integrate((a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*sec(d*x + c) + a), x)
```

3.324.8 Giac [F]

$$\int \sqrt{a+b\sec(c+dx)}\,dx = \int \sqrt{b\sec(dx+c)+a}\,dx$$

input
```
integrate((a+b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*sec(d*x + c) + a), x)
```

3.324.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+b\sec(c+dx)}\,dx = \int \sqrt{a+\frac{b}{\cos(c+d\,x)}}\,dx$$

input
```
int((a + b/cos(c + d*x))^(1/2),x)
```

output
```
int((a + b/cos(c + d*x))^(1/2), x)
```

3.325 $\int \cot^2(c+dx)\sqrt{a+b\sec(c+dx)}\,dx$

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3.325.1 Optimal result

Integrand size = 23, antiderivative size = 246

$$\int \cot^2(c+dx)\sqrt{a+b\sec(c+dx)}\,dx$$
$$= \frac{\sqrt{a+b}\cot(c+dx)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{d}$$
$$-\frac{\cot(c+dx)\sqrt{a+b\sec(c+dx)}}{d}$$
$$+\frac{2\cot(c+dx)\,\text{EllipticPi}\left(\frac{a}{a+b},\arcsin\left(\frac{\sqrt{a+b}}{\sqrt{a+b\sec(c+dx)}}\right),\frac{a-b}{a+b}\right)\sqrt{-\frac{b(1-\sec(c+dx))}{a+b\sec(c+dx)}}\sqrt{\frac{b(1+\sec(c+dx))}{a+b\sec(c+dx)}}(a+b\sec(c+$$
$$\sqrt{a+b}d$$

output

```
cot(d*x+c)*EllipticF((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/2
))*(a+b)^(1/2)*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1
/2)/d+2*cot(d*x+c)*EllipticPi((a+b)^(1/2)/(a+b*sec(d*x+c))^(1/2),a/(a+b),(
(a-b)/(a+b))^(1/2))*(a+b*sec(d*x+c))*(-b*(1-sec(d*x+c))/(a+b*sec(d*x+c)))^
(1/2)*(b*(1+sec(d*x+c))/(a+b*sec(d*x+c)))^(1/2)/d/(a+b)^(1/2)-cot(d*x+c)*(
a+b*sec(d*x+c))^(1/2)/d
```

3.325.2 Mathematica [A] (verified)

Time = 3.61 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.63

$$\int \cot^2(c+dx)\sqrt{a+b\sec(c+dx)}\,dx$$

$$= \frac{\sqrt{a+b\sec(c+dx)}\left(-\cot(c+dx) - \frac{2\cos^2\left(\frac{1}{2}(c+dx)\right)\left((-2a+b)\,\text{EllipticF}\left(\arcsin\left(\tan\left(\frac{1}{2}(c+dx)\right)\right),\frac{a-b}{a+b}\right)+4a\,\text{EllipticPi}\left(-1,\arcsin\right.\right.\right.}{b+a\cos(c+dx)}}{d}$$

input
```
Integrate[Cot[c + d*x]^2*Sqrt[a + b*Sec[c + d*x]],x]
```

output
```
(Sqrt[a + b*Sec[c + d*x]]*(-Cot[c + d*x] - (2*Cos[(c + d*x)/2]^2*((-2*a +
b)*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] + 4*a*EllipticPi[-
1, ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)])*Sqrt[(1 + Sec[c + d*x])^(-1
)]*Sqrt[(a + b*Sec[c + d*x])/((a + b)*(1 + Sec[c + d*x]))])/(b + a*Cos[c +
 d*x])))/d
```

3.325.3 Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 246, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3042, 4384, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^2(c+dx)\sqrt{a+b\sec(c+dx)}\,dx$$

$$\downarrow 3042$$

$$\int \frac{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}{\cot\left(c+dx+\frac{\pi}{2}\right)^2}dx$$

$$\downarrow 4384$$

$$\int \left(\csc^2(c+dx)\sqrt{a+b\sec(c+dx)} - \sqrt{a+b\sec(c+dx)}\right)dx$$

$$\downarrow 2009$$

$$\frac{\sqrt{a+b}\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{d}+$$
$$\frac{2\cot(c+dx)\sqrt{-\frac{b(1-\sec(c+dx))}{a+b\sec(c+dx)}}\sqrt{\frac{b(\sec(c+dx)+1)}{a+b\sec(c+dx)}}(a+b\sec(c+dx))\,\text{EllipticPi}\left(\frac{a}{a+b},\arcsin\left(\frac{\sqrt{a+b}}{\sqrt{a+b\sec(c+dx)}}\right),\frac{a-b}{a+b}\right)}{d\sqrt{a+b}}-$$
$$\frac{\cot(c+dx)\sqrt{a+b\sec(c+dx)}}{d}$$

input
```
Int[Cot[c + d*x]^2*Sqrt[a + b*Sec[c + d*x]],x]
```

output
```
(Sqrt[a + b]*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a
 + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1
 + Sec[c + d*x]))/(a - b))])/d - (Cot[c + d*x]*Sqrt[a + b*Sec[c + d*x]])/d
 + (2*Cot[c + d*x]*EllipticPi[a/(a + b), ArcSin[Sqrt[a + b]/Sqrt[a + b*Sec
[c + d*x]]], (a - b)/(a + b)]*Sqrt[-((b*(1 - Sec[c + d*x]))/(a + b*Sec[c +
 d*x]))]*Sqrt[(b*(1 + Sec[c + d*x]))/(a + b*Sec[c + d*x])]*(a + b*Sec[c +
d*x]))/(Sqrt[a + b]*d)
```

3.325.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4384
```
Int[cot[(c_.) + (d_.)*(x_)]^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_
), x_Symbol] :> Int[ExpandIntegrand[(a + b*Csc[c + d*x])^n, (-1 + Sec[c + d
*x]^2)^(-m/2), x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[a^2 - b^2, 0] &&
ILtQ[m/2, 0] && IntegerQ[n - 1/2] && EqQ[m, -2]
```

3.325.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 545 vs. $2(226) = 452$.

Time = 5.81 (sec) , antiderivative size = 546, normalized size of antiderivative = 2.22

method	result
default	$\frac{\sqrt{a+b\sec(dx+c)}\left(4\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,\text{EllipticPi}\left(\cot(dx+c)-\csc(dx+c),-1,\sqrt{\frac{a-b}{a+b}}\right)\sqrt{\frac{b+a\cos(dx+c)}{(a+b)(\cos(dx+c)+1)}}\,a\cos(dx+c)-2\,\text{EllipticF}\left(\cot\right.\right.$

input
```
int(cot(d*x+c)^2*(a+b*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/d*(a+b*sec(d*x+c))^(1/2)/(b+a*cos(d*x+c))*(4*(cos(d*x+c)/(cos(d*x+c)+1))
^(1/2)*EllipticPi(cot(d*x+c)-csc(d*x+c),-1,((a-b)/(a+b))^(1/2))*(1/(a+b)*(
b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a*cos(d*x+c)-2*EllipticF(cot(d*x+c)-
csc(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)
*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a*cos(d*x+c)+EllipticF(cot(d*x+c)-
csc(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)
*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*b*cos(d*x+c)+4*(cos(d*x+c)/(cos(d*
x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticPi(
cot(d*x+c)-csc(d*x+c),-1,((a-b)/(a+b))^(1/2))*a-2*EllipticF(cot(d*x+c)-csc
(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b
+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a+EllipticF(cot(d*x+c)-csc(d*x+c),((a
-b)/(a+b))^(1/2))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+
c))/(cos(d*x+c)+1))^(1/2)*b-a*cos(d*x+c)*cot(d*x+c)-cot(d*x+c)*b)
```

3.325.5 Fricas [F]

$$\int \cot^2(c+dx)\sqrt{a+b\sec(c+dx)}\,dx = \int \sqrt{b\sec\left(dx+c\right)+a}\cot\left(dx+c\right)^2\,dx$$

input
```
integrate(cot(d*x+c)^2*(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b*sec(d*x + c) + a)*cot(d*x + c)^2, x)
```

3.325.6 Sympy [F]

$$\int \cot^2(c+dx)\sqrt{a+b\sec(c+dx)}\,dx = \int \sqrt{a+b\sec(c+dx)}\cot^2(c+dx)\,dx$$

input
```
integrate(cot(d*x+c)**2*(a+b*sec(d*x+c))**(1/2),x)
```

output
```
Integral(sqrt(a + b*sec(c + d*x))*cot(c + d*x)**2, x)
```

3.325.7 Maxima [F]

$$\int \cot^2(c+dx)\sqrt{a+b\sec(c+dx)}\,dx = \int \sqrt{b\sec(dx+c)+a}\cot(dx+c)^2\,dx$$

input
```
integrate(cot(d*x+c)^2*(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*sec(d*x + c) + a)*cot(d*x + c)^2, x)
```

3.325.8 Giac [F]

$$\int \cot^2(c+dx)\sqrt{a+b\sec(c+dx)}\,dx = \int \sqrt{b\sec(dx+c)+a}\cot(dx+c)^2\,dx$$

input
```
integrate(cot(d*x+c)^2*(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*sec(d*x + c) + a)*cot(d*x + c)^2, x)
```

3.325.9 Mupad [F(-1)]

Timed out.

$$\int \cot^2(c+dx)\sqrt{a+b\sec(c+dx)}\,dx = \int \cot(c+d\,x)^2\sqrt{a+\frac{b}{\cos(c+d\,x)}}\,dx$$

input
```
int(cot(c + d*x)^2*(a + b/cos(c + d*x))^(1/2),x)
```

output
```
int(cot(c + d*x)^2*(a + b/cos(c + d*x))^(1/2), x)
```

3.326 $\int \frac{\tan^5(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$

3.326.1 Optimal result 2401
3.326.2 Mathematica [A] (verified) 2401
3.326.3 Rubi [A] (warning: unable to verify) 2402
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3.326.7 Maxima [A] (verification not implemented) 2406
3.326.8 Giac [B] (verification not implemented) 2406
3.326.9 Mupad [F(-1)] 2407

3.326.1 Optimal result

Integrand size = 23, antiderivative size = 148

$$\int \frac{\tan^5(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = -\frac{2\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}d} - \frac{2a(a^2-2b^2)\sqrt{a+b\sec(c+dx)}}{b^4d} + \frac{2(3a^2-2b^2)(a+b\sec(c+dx))^{3/2}}{3b^4d} - \frac{6a(a+b\sec(c+dx))^{5/2}}{5b^4d} + \frac{2(a+b\sec(c+dx))^{7/2}}{7b^4d}$$

output
```
2/3*(3*a^2-2*b^2)*(a+b*sec(d*x+c))^(3/2)/b^4/d-6/5*a*(a+b*sec(d*x+c))^(5/2
)/b^4/d+2/7*(a+b*sec(d*x+c))^(7/2)/b^4/d-2*arctanh((a+b*sec(d*x+c))^(1/2)/
a^(1/2))/d/a^(1/2)-2*a*(a^2-2*b^2)*(a+b*sec(d*x+c))^(1/2)/b^4/d
```

3.326.2 Mathematica [A] (verified)

Time = 0.87 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.89

$$\int \frac{\tan^5(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = \frac{-\frac{2b^4\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}} - 2a(a^2-2b^2)\sqrt{a+b\sec(c+dx)} + \frac{2}{3}(3a^2-2b^2)(a+b\sec(c+dx))^{3/2} - \frac{6}{5}a(}{b^4d}$$

input
```
Integrate[Tan[c + d*x]^5/Sqrt[a + b*Sec[c + d*x]],x]
```

output
```
((-2*b^4*ArcTanh[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a]])/Sqrt[a] - 2*a*(a^2 - 2
*b^2)*Sqrt[a + b*Sec[c + d*x]] + (2*(3*a^2 - 2*b^2)*(a + b*Sec[c + d*x])^(
3/2))/3 - (6*a*(a + b*Sec[c + d*x])^(5/2))/5 + (2*(a + b*Sec[c + d*x])^(7/
2))/7)/(b^4*d)
```

3.326.3 Rubi [A] (warning: unable to verify)

Time = 0.34 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.83, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {3042, 25, 4373, 517, 25, 1467, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^5(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$$

$$\downarrow 3042$$

$$\int -\frac{\cot\left(c+dx+\frac{\pi}{2}\right)^5}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\downarrow 25$$

$$-\int \frac{\cot\left(\frac{1}{2}(2c+\pi)+dx\right)^5}{\sqrt{a+b\csc\left(\frac{1}{2}(2c+\pi)+dx\right)}}dx$$

$$\downarrow 4373$$

$$\frac{\int \frac{\cos(c+dx)\left(b^2-b^2\sec^2(c+dx)\right)^2}{b\sqrt{a+b\sec(c+dx)}}d(b\sec(c+dx))}{b^4d}$$

$$\downarrow 517$$

$$\frac{2\int -\frac{\left(b^4\sec^4(c+dx)-2ab^2\sec^2(c+dx)+a^2-b^2\right)^2}{a-b^2\sec^2(c+dx)}d\sqrt{a+b\sec(c+dx)}}{b^4d}$$

$$\downarrow 25$$

$$-\frac{2\int \frac{\left(b^4\sec^4(c+dx)-2ab^2\sec^2(c+dx)+a^2-b^2\right)^2}{a-b^2\sec^2(c+dx)}d\sqrt{a+b\sec(c+dx)}}{b^4d}$$

$$\downarrow 1467$$

$$-\frac{2\int\left(-b^6\sec^6(c+dx)+3ab^4\sec^4(c+dx)-b^2\left(3a^2-2b^2\right)\sec^2(c+dx)+a^3-2ab^2+\frac{b^4}{a-b^2\sec^2(c+dx)}\right)d\sqrt{a+b\s}}{b^4d}$$

$$\downarrow 2009$$

$$\frac{2\left(-a\left(a^2-2b^2\right)\sqrt{a+b\sec(c+dx)}+\frac{1}{3}b^3\left(3a^2-2b^2\right)\sec^3(c+dx)-\frac{b^4\text{arctanh}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}}-\frac{3}{5}ab^5\sec^5(c+dx\right.}{b^4d}$$

input
```
Int[Tan[c + d*x]^5/Sqrt[a + b*Sec[c + d*x]],x]
```

output
```
(2*(-((b^4*ArcTanh[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a]])/Sqrt[a]) + (b^3*(3*a
^2 - 2*b^2)*Sec[c + d*x]^3)/3 - (3*a*b^5*Sec[c + d*x]^5)/5 + (b^7*Sec[c +
d*x]^7)/7 - a*(a^2 - 2*b^2)*Sqrt[a + b*Sec[c + d*x]]))/(b^4*d)
```

3.326.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 517
```
Int[((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.),
 x_Symbol] :> Simp[2*(e^m/d^(m + 2*p + 1))   Subst[Int[x^(2*n + 1)*(-c + x^
2)^m*(b*c^2 + a*d^2 - 2*b*c*x^2 + b*x^4)^p, x], x, Sqrt[c + d*x]], x] /; Fr
eeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && ILtQ[m, 0] && IntegerQ[n + 1/2]
```

rule 1467
```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4373
```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Simp[-(-1)^((m - 1)/2)/(d*b^(m - 1))   Subst[Int[(b^2 - x^
2)^((m - 1)/2)*((a + x)^n/x), x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
 d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

3.326.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 645 vs. $2(128) = 256$.

Time = 16.65 (sec) , antiderivative size = 646, normalized size of antiderivative = 4.36

method	result
default	$-\frac{\sqrt{a+b\sec(dx+c)}\left(105\cos(dx+c)\ln\left(4\cos(dx+c)\sqrt{\frac{(b+a\cos(dx+c))\cos(dx+c)}{(\cos(dx+c)+1)^2}}\sqrt{a}+4a\cos(dx+c)+4\sqrt{a}\sqrt{\frac{(b+a\cos(dx+c))\cos(dx+c)}{(\cos(dx+c)+1)^2}}\right.\right.$

input
```
int(tan(d*x+c)^5/(a+b*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/105/d/a/b^4*(a+b*sec(d*x+c))^(1/2)/(cos(d*x+c)+1)/((b+a*cos(d*x+c))*cos
(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*(105*cos(d*x+c)*ln(4*cos(d*x+c)*((b+a*cos(
d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*a^(1/2)+4*a*cos(d*x+c)+4*a^(1/2
)*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)+2*b)*a^(1/2)*b^4+96
*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*a^4*cos(d*x+c)-280*(
(b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*a^2*b^2*cos(d*x+c)+96*
((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*a^4-48*((b+a*cos(d*x+
c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*a^3*b-280*((b+a*cos(d*x+c))*cos(d*x
+c)/(cos(d*x+c)+1)^2)^(1/2)*a^2*b^2+140*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(
d*x+c)+1)^2)^(1/2)*a*b^3-48*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)
^(1/2)*a^3*b*sec(d*x+c)+36*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^
(1/2)*a^2*b^2*sec(d*x+c)+140*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2
)^(1/2)*a*b^3*sec(d*x+c)+36*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)
^(1/2)*a^2*b^2*sec(d*x+c)^2-30*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)
^2)^(1/2)*a*b^3*sec(d*x+c)^2-30*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1
)^2)^(1/2)*a*b^3*sec(d*x+c)^3)
```

3.326.5 Fricas [A] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 381, normalized size of antiderivative = 2.57

$$\int \frac{\tan^5(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$$

$$= \left[\frac{105\sqrt{a}b^4\cos(dx+c)^3\log\left(-8a^2\cos(dx+c)^2-8ab\cos(dx+c)-b^2+4\left(2a\cos(dx+c)^2+b\cos(dx\right.\right.}{}\right.$$

input
```
integrate(tan(d*x+c)^5/(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
[1/210*(105*sqrt(a)*b^4*cos(d*x + c)^3*log(-8*a^2*cos(d*x + c)^2 - 8*a*b*c
os(d*x + c) - b^2 + 4*(2*a*cos(d*x + c)^2 + b*cos(d*x + c))*sqrt(a)*sqrt((
a*cos(d*x + c) + b)/cos(d*x + c))) - 4*(18*a^2*b^2*cos(d*x + c) - 15*a*b^3
 + 4*(12*a^4 - 35*a^2*b^2)*cos(d*x + c)^3 - 2*(12*a^3*b - 35*a*b^3)*cos(d*
x + c)^2)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c)))/(a*b^4*d*cos(d*x + c)^3
), 1/105*(105*sqrt(-a)*b^4*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + b)/cos
(d*x + c))*cos(d*x + c)/(2*a*cos(d*x + c) + b))*cos(d*x + c)^3 - 2*(18*a^2
*b^2*cos(d*x + c) - 15*a*b^3 + 4*(12*a^4 - 35*a^2*b^2)*cos(d*x + c)^3 - 2*
(12*a^3*b - 35*a*b^3)*cos(d*x + c)^2)*sqrt((a*cos(d*x + c) + b)/cos(d*x +
c)))/(a*b^4*d*cos(d*x + c)^3)]
```

3.326.6 Sympy [F]

$$\int \frac{\tan^5(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{\tan^5(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$$

input
```
integrate(tan(d*x+c)**5/(a+b*sec(d*x+c))**(1/2),x)
```

output
```
Integral(tan(c + d*x)**5/sqrt(a + b*sec(c + d*x)), x)
```

3.326.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.18

$$\int \frac{\tan^5(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$$
$$= \frac{\frac{105\log\left(\frac{\sqrt{a+\frac{b}{\cos(dx+c)}}-\sqrt{a}}{\sqrt{a+\frac{b}{\cos(dx+c)}}+\sqrt{a}}\right)}{\sqrt{a}} + \frac{30\left(a+\frac{b}{\cos(dx+c)}\right)^{\frac{7}{2}}}{b^4} - \frac{126\left(a+\frac{b}{\cos(dx+c)}\right)^{\frac{5}{2}}a}{b^4} + \frac{210\left(a+\frac{b}{\cos(dx+c)}\right)^{\frac{3}{2}}a^2}{b^4} - \frac{210\sqrt{a+\frac{b}{\cos(dx+c)}}a^3}{b^4} - \frac{14}{}}{105\,d}$$

input
```
integrate(tan(d*x+c)^5/(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
1/105*(105*log((sqrt(a + b/cos(d*x + c)) - sqrt(a))/(sqrt(a + b/cos(d*x +
c)) + sqrt(a)))/sqrt(a) + 30*(a + b/cos(d*x + c))^(7/2)/b^4 - 126*(a + b/c
os(d*x + c))^(5/2)*a/b^4 + 210*(a + b/cos(d*x + c))^(3/2)*a^2/b^4 - 210*sq
rt(a + b/cos(d*x + c))*a^3/b^4 - 140*(a + b/cos(d*x + c))^(3/2)/b^2 + 420*
sqrt(a + b/cos(d*x + c))*a/b^2)/d
```

3.326.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 699 vs. 2(128) = 256.

Time = 1.81 (sec) , antiderivative size = 699, normalized size of antiderivative = 4.72

$$\int \frac{\tan^5(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$$
$$= 2\left(\frac{105\arctan\left(-\frac{\sqrt{a-b}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4-2a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a+b}+\sqrt{a-b}}{2\sqrt{-a}}\right)}{\sqrt{-a}} - \frac{2\left(105\left(\sqrt{a-b}\tan\left(\frac{1}{2}dx+\right.\right.\right.}{}$$

input
```
integrate(tan(d*x+c)^5/(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
2/105*(105*arctan(-1/2*(sqrt(a - b)*tan(1/2*d*x + 1/2*c)^2 - sqrt(a*tan(1/
2*d*x + 1/2*c)^4 - b*tan(1/2*d*x + 1/2*c)^4 - 2*a*tan(1/2*d*x + 1/2*c)^2 +
 a + b) + sqrt(a - b))/sqrt(-a))/sqrt(-a) - 2*(105*(sqrt(a - b)*tan(1/2*d*
x + 1/2*c)^2 - sqrt(a*tan(1/2*d*x + 1/2*c)^4 - b*tan(1/2*d*x + 1/2*c)^4 -
2*a*tan(1/2*d*x + 1/2*c)^2 + a + b))^6 - 840*(sqrt(a - b)*tan(1/2*d*x + 1/
2*c)^2 - sqrt(a*tan(1/2*d*x + 1/2*c)^4 - b*tan(1/2*d*x + 1/2*c)^4 - 2*a*ta
n(1/2*d*x + 1/2*c)^2 + a + b))^5*sqrt(a - b) + 35*(sqrt(a - b)*tan(1/2*d*x
 + 1/2*c)^2 - sqrt(a*tan(1/2*d*x + 1/2*c)^4 - b*tan(1/2*d*x + 1/2*c)^4 - 2
*a*tan(1/2*d*x + 1/2*c)^2 + a + b))^4*(27*a - 23*b) + 280*(sqrt(a - b)*tan
(1/2*d*x + 1/2*c)^2 - sqrt(a*tan(1/2*d*x + 1/2*c)^4 - b*tan(1/2*d*x + 1/2*
c)^4 - 2*a*tan(1/2*d*x + 1/2*c)^2 + a + b))^3*(3*a + 4*b)*sqrt(a - b) - 21
*(sqrt(a - b)*tan(1/2*d*x + 1/2*c)^2 - sqrt(a*tan(1/2*d*x + 1/2*c)^4 - b*t
an(1/2*d*x + 1/2*c)^4 - 2*a*tan(1/2*d*x + 1/2*c)^2 + a + b))^2*(65*a^2 - 2
*a*b - 15*b^2) + 315*a^3 + 707*a^2*b - 7*a*b^2 - 55*b^3 - 56*(sqrt(a - b)*
tan(1/2*d*x + 1/2*c)^2 - sqrt(a*tan(1/2*d*x + 1/2*c)^4 - b*tan(1/2*d*x + 1
/2*c)^4 - 2*a*tan(1/2*d*x + 1/2*c)^2 + a + b))*(19*a*b + 5*b^2)*sqrt(a - b
))/(sqrt(a - b)*tan(1/2*d*x + 1/2*c)^2 - sqrt(a*tan(1/2*d*x + 1/2*c)^4 - b
*tan(1/2*d*x + 1/2*c)^4 - 2*a*tan(1/2*d*x + 1/2*c)^2 + a + b) - sqrt(a - b
))^7)/(d*sgn(cos(d*x + c)))
```

3.326.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\tan^5(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{\tan(c+d\,x)^5}{\sqrt{a+\frac{b}{\cos(c+d\,x)}}}\,dx$$

input
```
int(tan(c + d*x)^5/(a + b/cos(c + d*x))^(1/2),x)
```

output
```
int(tan(c + d*x)^5/(a + b/cos(c + d*x))^(1/2), x)
```

3.327 $\int \frac{\tan^3(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$

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3.327.1 Optimal result

Integrand size = 23, antiderivative size = 79

$$\int \frac{\tan^3(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = \frac{2\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}d} - \frac{2a\sqrt{a+b\sec(c+dx)}}{b^2 d} + \frac{2(a+b\sec(c+dx))^{3/2}}{3b^2 d}$$

output
```
2/3*(a+b*sec(d*x+c))^(3/2)/b^2/d+2*arctanh((a+b*sec(d*x+c))^(1/2)/a^(1/2))
/d/a^(1/2)-2*a*(a+b*sec(d*x+c))^(1/2)/b^2/d
```

3.327.2 Mathematica [A] (verified)

Time = 0.41 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.84

$$\int \frac{\tan^3(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = \frac{2\left(\frac{3\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}} + \frac{(-2a+b\sec(c+dx))\sqrt{a+b\sec(c+dx)}}{b^2}\right)}{3d}$$

input
```
Integrate[Tan[c + d*x]^3/Sqrt[a + b*Sec[c + d*x]],x]
```

output
```
(2*((3*ArcTanh[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a]])/Sqrt[a] + ((-2*a + b*Sec
[c + d*x])*Sqrt[a + b*Sec[c + d*x]])/b^2))/(3*d)
```

3.327.3 Rubi [A] (warning: unable to verify)

Time = 0.30 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.90, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3042, 25, 4373, 517, 1467, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^3(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$$

$$\downarrow 3042$$

$$\int -\frac{\cot\left(c+dx+\frac{\pi}{2}\right)^3}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\downarrow 25$$

$$-\int \frac{\cot\left(\frac{1}{2}(2c+\pi)+dx\right)^3}{\sqrt{a+b\csc\left(\frac{1}{2}(2c+\pi)+dx\right)}}dx$$

$$\downarrow 4373$$

$$-\frac{\int \frac{\cos(c+dx)\left(b^2-b^2\sec^2(c+dx)\right)}{b\sqrt{a+b\sec(c+dx)}}d(b\sec(c+dx))}{b^2 d}$$

$$\downarrow 517$$

$$-\frac{2\int \frac{b^4\sec^4(c+dx)-2ab^2\sec^2(c+dx)+a^2-b^2}{a-b^2\sec^2(c+dx)}d\sqrt{a+b\sec(c+dx)}}{b^2 d}$$

$$\downarrow 1467$$

$$-\frac{2\int \left(-\sec^2(c+dx)b^2-\frac{b^2}{a-b^2\sec^2(c+dx)}+a\right)d\sqrt{a+b\sec(c+dx)}}{b^2 d}$$

$$\downarrow 2009$$

$$-\frac{2\left(-\frac{b^2\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}}+a\sqrt{a+b\sec(c+dx)}-\frac{1}{3}b^3\sec^3(c+dx)\right)}{b^2 d}$$

input
```
Int[Tan[c + d*x]^3/Sqrt[a + b*Sec[c + d*x]],x]
```

output
```
(-2*(-((b^2*ArcTanh[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a]])/Sqrt[a]) - (b^3*Sec
[c + d*x]^3)/3 + a*Sqrt[a + b*Sec[c + d*x]]))/(b^2*d)
```

3.327.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 517
```
Int[((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.),
 x_Symbol] :> Simp[2*(e^m/d^(m + 2*p + 1))   Subst[Int[x^(2*n + 1)*(-c + x^
2)^m*(b*c^2 + a*d^2 - 2*b*c*x^2 + b*x^4)^p, x], x, Sqrt[c + d*x]], x] /; Fr
eeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && ILtQ[m, 0] && IntegerQ[n + 1/2]
```

rule 1467
```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4373
```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Simp[-(-1)^((m - 1)/2)/(d*b^(m - 1))   Subst[Int[(b^2 - x^
2)^((m - 1)/2)*((a + x)^n/x), x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
 d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

3.327.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 311 vs. 2(67) = 134.

Time = 11.29 (sec) , antiderivative size = 312, normalized size of antiderivative = 3.95

method	result
default	$-\frac{\sqrt{a+b\sec(dx+c)}\left(-3\sqrt{a}\,\cos(dx+c)\ln\left(4\cos(dx+c)\sqrt{\frac{(b+a\cos(dx+c))\cos(dx+c)}{(\cos(dx+c)+1)^2}}\,\sqrt{a}+4a\cos(dx+c)+4\sqrt{a}\,\sqrt{\frac{(b+a\cos(dx+c))\cos(dx+}{(\cos(dx+c)+1)^2}}\right.\right.}{3da\,b^2(}$

input
```
int(tan(d*x+c)^3/(a+b*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3/d/a/b^2*(a+b*sec(d*x+c))^(1/2)/(cos(d*x+c)+1)/((b+a*cos(d*x+c))*cos(d
*x+c)/(cos(d*x+c)+1)^2)^(1/2)*(-3*a^(1/2)*cos(d*x+c)*ln(4*cos(d*x+c)*((b+a
*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*a^(1/2)+4*a*cos(d*x+c)+4*a
^(1/2)*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)+2*b)*b^2+4*((b
+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*a^2*cos(d*x+c)+4*((b+a*c
os(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*a^2-2*((b+a*cos(d*x+c))*cos(
d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*a*b-2*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x
+c)+1)^2)^(1/2)*a*b*sec(d*x+c))
```

3.327.5 Fricas [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 273, normalized size of antiderivative = 3.46

$$\int \frac{\tan^3(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$$

$$= \left[\frac{3\sqrt{a}b^2\cos(dx+c)\log\left(-8a^2\cos(dx+c)^2-8ab\cos(dx+c)-b^2-4\left(2a\cos(dx+c)^2+b\cos(dx+\right.\right.}{6ab^2d\cos(dx+c)}$$

$$\left.\left.-\frac{3\sqrt{-a}b^2\arctan\left(\frac{2\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+b}{\cos(dx+c)}}\cos(dx+c)}{2a\cos(dx+c)+b}\right)\cos(dx+c)+2\left(2a^2\cos(dx+c)-ab\right)\sqrt{\frac{a\cos(dx+c)+b}{\cos(dx+c)}}}{3ab^2d\cos(dx+c)}\right]$$

input
```
integrate(tan(d*x+c)^3/(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
[1/6*(3*sqrt(a)*b^2*cos(d*x + c)*log(-8*a^2*cos(d*x + c)^2 - 8*a*b*cos(d*x
 + c) - b^2 - 4*(2*a*cos(d*x + c)^2 + b*cos(d*x + c))*sqrt(a)*sqrt((a*cos(
d*x + c) + b)/cos(d*x + c))) - 4*(2*a^2*cos(d*x + c) - a*b)*sqrt((a*cos(d*
x + c) + b)/cos(d*x + c)))/(a*b^2*d*cos(d*x + c)), -1/3*(3*sqrt(-a)*b^2*ar
ctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c))*cos(d*x + c)/(2*a*
cos(d*x + c) + b))*cos(d*x + c) + 2*(2*a^2*cos(d*x + c) - a*b)*sqrt((a*cos
(d*x + c) + b)/cos(d*x + c)))/(a*b^2*d*cos(d*x + c))]
```

3.327.6 Sympy [F]

$$\int \frac{\tan^3(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{\tan^3(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$$

input
```
integrate(tan(d*x+c)**3/(a+b*sec(d*x+c))**(1/2),x)
```

output
```
Integral(tan(c + d*x)**3/sqrt(a + b*sec(c + d*x)), x)
```

3.327.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.16

$$\int \frac{\tan^3(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = -\frac{\frac{3\log\left(\frac{\sqrt{a+\frac{b}{\cos(dx+c)}}-\sqrt{a}}{\sqrt{a+\frac{b}{\cos(dx+c)}}+\sqrt{a}}\right)}{\sqrt{a}} - \frac{2\left(a+\frac{b}{\cos(dx+c)}\right)^{\frac{3}{2}}}{b^2} + \frac{6\sqrt{a+\frac{b}{\cos(dx+c)}}a}{b^2}}{3\,d}$$

input
```
integrate(tan(d*x+c)^3/(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
-1/3*(3*log((sqrt(a + b/cos(d*x + c)) - sqrt(a))/(sqrt(a + b/cos(d*x + c))
 + sqrt(a)))/sqrt(a) - 2*(a + b/cos(d*x + c))^(3/2)/b^2 + 6*sqrt(a + b/cos
(d*x + c))*a/b^2)/d
```

3.327.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 265 vs. $2(67) = 134$.

Time $= 0.82$ (sec) , antiderivative size $= 265$, normalized size of antiderivative $= 3.35$

$$\int \frac{\tan^3(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = -\frac{2\left(\frac{3\arctan\left(-\frac{\sqrt{a-b}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4-2a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a+b+\sqrt{a-b}}}{2\sqrt{-a}}\right)}{\sqrt{-a}} - \frac{2\left(3\left(\sqrt{a-b}\tan\left(\frac{1}{2}dx+\frac{1}{2}\right.\right.\right.}{\left(\sqrt{a-b}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right.\right.}\right)}{3\,d\,\mathrm{sgn}\left(\cos\left(dx+c\right)\right)}$$

input
```
integrate(tan(d*x+c)^3/(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
-2/3*(3*arctan(-1/2*(sqrt(a - b)*tan(1/2*d*x + 1/2*c)^2 - sqrt(a*tan(1/2*d
*x + 1/2*c)^4 - b*tan(1/2*d*x + 1/2*c)^4 - 2*a*tan(1/2*d*x + 1/2*c)^2 + a
+ b) + sqrt(a - b))/sqrt(-a))/sqrt(-a) - 2*(3*(sqrt(a - b)*tan(1/2*d*x + 1
/2*c)^2 - sqrt(a*tan(1/2*d*x + 1/2*c)^4 - b*tan(1/2*d*x + 1/2*c)^4 - 2*a*t
an(1/2*d*x + 1/2*c)^2 + a + b))^2 - 3*a - b)/(sqrt(a - b)*tan(1/2*d*x + 1/
2*c)^2 - sqrt(a*tan(1/2*d*x + 1/2*c)^4 - b*tan(1/2*d*x + 1/2*c)^4 - 2*a*ta
n(1/2*d*x + 1/2*c)^2 + a + b) - sqrt(a - b))^3)/(d*sgn(cos(d*x + c)))
```

3.327.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\tan^3(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{\tan(c+d\,x)^3}{\sqrt{a+\frac{b}{\cos(c+d\,x)}}}\,dx$$

input
```
int(tan(c + d*x)^3/(a + b/cos(c + d*x))^(1/2),x)
```

output
```
int(tan(c + d*x)^3/(a + b/cos(c + d*x))^(1/2), x)
```

3.328 $\int \frac{\tan(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$

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3.328.1 Optimal result

Integrand size = 21, antiderivative size = 31

$$\int \frac{\tan(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = -\frac{2\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}d}$$

output
```
-2*arctanh((a+b*sec(d*x+c))^(1/2)/a^(1/2))/d/a^(1/2)
```

3.328.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$\int \frac{\tan(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = -\frac{2\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}d}$$

input
```
Integrate[Tan[c + d*x]/Sqrt[a + b*Sec[c + d*x]],x]
```

output
```
(-2*ArcTanh[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a]])/(Sqrt[a]*d)
```

3.328.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3042, 25, 4373, 73, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$$

$$\downarrow 3042$$

$$\int -\frac{\cot\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\downarrow 25$$

$$-\int \frac{\cot\left(\frac{1}{2}(2c+\pi)+dx\right)}{\sqrt{a+b\csc\left(\frac{1}{2}(2c+\pi)+dx\right)}}dx$$

$$\downarrow 4373$$

$$\frac{\int \frac{\cos(c+dx)}{b\sqrt{a+b\sec(c+dx)}}d(b\sec(c+dx))}{d}$$

$$\downarrow 73$$

$$\frac{2\int \frac{1}{b^2\sec^2(c+dx)-a}d\sqrt{a+b\sec(c+dx)}}{d}$$

$$\downarrow 220$$

$$-\frac{2\text{arctanh}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}d}$$

input
```
Int[Tan[c + d*x]/Sqrt[a + b*Sec[c + d*x]],x]
```

output
```
(-2*ArcTanh[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a]])/(Sqrt[a]*d)
```

3.328.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4373
```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Simp[-(-1)^((m - 1)/2)/(d*b^(m - 1))   Subst[Int[(b^2 - x^
2)^((m - 1)/2)*((a + x)^n/x), x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
 d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

3.328.4 Maple [A] (verified)

Time = 0.74 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.84

method	result	size
derivativedivides	$-\frac{2\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec(dx+c)}}{\sqrt{a}}\right)}{d\sqrt{a}}$	26
default	$-\frac{2\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec(dx+c)}}{\sqrt{a}}\right)}{d\sqrt{a}}$	26

input
```
int(tan(d*x+c)/(a+b*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2*arctanh((a+b*sec(d*x+c))^(1/2)/a^(1/2))/d/a^(1/2)
```

3.328.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 60 vs. $2(25) = 50$.

Time = 0.39 (sec) , antiderivative size = 145, normalized size of antiderivative = 4.68

$$\int \frac{\tan(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$$

$$= \left[\frac{\log\left(-8\,a^2\cos(dx+c)^2 - 8\,ab\cos(dx+c) - b^2 + 4\left(2\,a\cos(dx+c)^2 + b\cos(dx+c)\right)\sqrt{a}\sqrt{\frac{a\cos(dx+c)+b}{\cos(dx+c)}}\right.}{2\sqrt{a}d}\right.$$

input
```
integrate(tan(d*x+c)/(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
[1/2*log(-8*a^2*cos(d*x + c)^2 - 8*a*b*cos(d*x + c) - b^2 + 4*(2*a*cos(d*x
 + c)^2 + b*cos(d*x + c))*sqrt(a)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c)))
/(sqrt(a)*d), sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + b)/cos(d*x
 + c))*cos(d*x + c)/(2*a*cos(d*x + c) + b))/(a*d)]
```

3.328.6 Sympy [F]

$$\int \frac{\tan(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{\tan(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$$

input
```
integrate(tan(d*x+c)/(a+b*sec(d*x+c))**(1/2),x)
```

output
```
Integral(tan(c + d*x)/sqrt(a + b*sec(c + d*x)), x)
```

3.328.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.58

$$\int \frac{\tan(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = \frac{\log\left(\frac{\sqrt{a+\frac{b}{\cos(dx+c)}}-\sqrt{a}}{\sqrt{a+\frac{b}{\cos(dx+c)}}+\sqrt{a}}\right)}{\sqrt{a}d}$$

input
```
integrate(tan(d*x+c)/(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
log((sqrt(a + b/cos(d*x + c)) - sqrt(a))/(sqrt(a + b/cos(d*x + c)) + sqrt(
a)))/(sqrt(a)*d)
```

3.328.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 102 vs. $2(25) = 50$.

Time = 0.49 (sec) , antiderivative size = 102, normalized size of antiderivative = 3.29

$$\int \frac{\tan(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = \frac{2\arctan\left(-\frac{\sqrt{a-b}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4-2a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a+b}+\sqrt{a-b}}{2\sqrt{-a}}\right)}{\sqrt{-a}d\,\mathrm{sgn}\left(\cos\left(dx+c\right)\right)}$$

input
```
integrate(tan(d*x+c)/(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
2*arctan(-1/2*(sqrt(a - b)*tan(1/2*d*x + 1/2*c)^2 - sqrt(a*tan(1/2*d*x + 1
/2*c)^4 - b*tan(1/2*d*x + 1/2*c)^4 - 2*a*tan(1/2*d*x + 1/2*c)^2 + a + b) +
 sqrt(a - b))/sqrt(-a))/(sqrt(-a)*d*sgn(cos(d*x + c)))
```

3.328.9 Mupad [B] (verification not implemented)

Time = 14.77 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.87

$$\int \frac{\tan(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = -\frac{2\,\mathrm{atanh}\left(\frac{\sqrt{a+\frac{b}{\cos(c+d\,x)}}}{\sqrt{a}}\right)}{\sqrt{a}\,d}$$

input
```
int(tan(c + d*x)/(a + b/cos(c + d*x))^(1/2),x)
```

output
```
-(2*atanh((a + b/cos(c + d*x))^(1/2)/a^(1/2)))/(a^(1/2)*d)
```

3.329 $\int \frac{\cot(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$

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3.329.8 Giac [F(-2)] . . . 2424
3.329.9 Mupad [F(-1)] . . . 2424

3.329.1 Optimal result

Integrand size = 21, antiderivative size = 106

$$\int \frac{\cot(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = \frac{2\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}d} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a-b}}\right)}{\sqrt{a-b}d} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)}{\sqrt{a+b}d}$$

output
```
2*arctanh((a+b*sec(d*x+c))^(1/2)/a^(1/2))/d/a^(1/2)-arctanh((a+b*sec(d*x+c
))^(1/2)/(a-b)^(1/2))/d/(a-b)^(1/2)-arctanh((a+b*sec(d*x+c))^(1/2)/(a+b)^(
1/2))/d/(a+b)^(1/2)
```

3.329.2 Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.94

$$\int \frac{\cot(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = -\frac{-\frac{2\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}} + \frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a-b}}\right)}{\sqrt{a-b}} + \frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)}{\sqrt{a+b}}}{d}$$

input
```
Integrate[Cot[c + d*x]/Sqrt[a + b*Sec[c + d*x]],x]
```

output
```
-(((-2*ArcTanh[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a]])/Sqrt[a] + ArcTanh[Sqrt[a
 + b*Sec[c + d*x]]/Sqrt[a - b]]/Sqrt[a - b] + ArcTanh[Sqrt[a + b*Sec[c + d
*x]]/Sqrt[a + b]]/Sqrt[a + b])/d)
```

3.329.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.11, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 25, 4373, 561, 1484, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$$

$$\downarrow 3042$$

$$\int -\frac{1}{\cot\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\downarrow 25$$

$$-\int \frac{1}{\cot\left(\frac{1}{2}(2c+\pi)+dx\right)\sqrt{a+b\csc\left(\frac{1}{2}(2c+\pi)+dx\right)}}dx$$

$$\downarrow 4373$$

$$-\frac{b^2\int \frac{\cos(c+dx)}{b\sqrt{a+b\sec(c+dx)}(b^2-b^2\sec^2(c+dx))}d(b\sec(c+dx))}{d}$$

$$\downarrow 561$$

$$-\frac{2b^2\int \frac{1}{(a-b^2\sec^2(c+dx))(b^4\sec^4(c+dx)-2ab^2\sec^2(c+dx)+a^2-b^2)}d\sqrt{a+b\sec(c+dx)}}{d}$$

$$\downarrow 1484$$

$$-\frac{2b^2\int \left(\frac{1}{2b^2(-b^2\sec^2(c+dx)+a+b)}-\frac{1}{2b^2(b^2\sec^2(c+dx)-a+b)}-\frac{1}{b^2(a-b^2\sec^2(c+dx))}\right)d\sqrt{a+b\sec(c+dx)}}{d}$$

$$\downarrow 2009$$

$$-\frac{2b^2\left(-\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}b^2}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a-b}}\right)}{2b^2\sqrt{a-b}}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)}{2b^2\sqrt{a+b}}\right)}{d}$$

input
```
Int[Cot[c + d*x]/Sqrt[a + b*Sec[c + d*x]],x]
```

output
```
(-2*b^2*(-(ArcTanh[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a]]/(Sqrt[a]*b^2)) + ArcT
anh[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a - b]]/(2*Sqrt[a - b]*b^2) + ArcTanh[Sq
rt[a + b*Sec[c + d*x]]/Sqrt[a + b]]/(2*b^2*Sqrt[a + b])))/d
```

3.329.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 561
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> With[{k = Denominator[n]}, Simp[k/d   Subst[Int[x^(k*(n + 1) - 1)*(-c
/d + x^k/d)^m*Simp[(b*c^2 + a*d^2)/d^2 - 2*b*c*(x^k/d^2) + b*(x^(2*k)/d^2),
 x]^p, x], x, (c + d*x)^(1/k)], x]] /; FreeQ[{a, b, c, d, m, p}, x] && Frac
tionQ[n] && IntegerQ[p] && IntegerQ[m]
```

rule 1484
```
Int[((d_) + (e_.)*(x_)^2)^(q_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symb
ol] :> Int[ExpandIntegrand[(d + e*x^2)^q/(a + b*x^2 + c*x^4), x], x] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2,
 0] && IntegerQ[q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4373
```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Simp[-(-1)^((m - 1)/2)/(d*b^(m - 1))   Subst[Int[(b^2 - x^
2)^((m - 1)/2)*((a + x)^n/x), x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
 d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

3.329.4 Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 657 vs. $2(88) = 176$.

Time = 2.29 (sec) , antiderivative size = 658, normalized size of antiderivative = 6.21

method	result
default	$-\frac{\sqrt{a+b\sec(dx+c)}\left(-2a^{\frac{3}{2}}(a-b)^{\frac{3}{2}}\ln\left(4\cos(dx+c)\sqrt{\frac{(b+a\cos(dx+c))\cos(dx+c)}{(\cos(dx+c)+1)^2}}\sqrt{a}+4a\cos(dx+c)+4\sqrt{a}\sqrt{\frac{(b+a\cos(dx+c))\cos(dx+c)}{(\cos(dx+c)+1)^2}}-\right.\right.}{}$

input
```
int(cot(d*x+c)/(a+b*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2/d/(a-b)^(3/2)/(a+b)/a*(a+b*sec(d*x+c))^(1/2)*(-2*a^(3/2)*(a-b)^(3/2)*
ln(4*cos(d*x+c)*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*a^(1/
2)+4*a*cos(d*x+c)+4*a^(1/2)*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)
^(1/2)+2*b)+(a+b)^(1/2)*(a-b)^(3/2)*ln(-2*(2*cos(d*x+c)*((b+a*cos(d*x+c))*
cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*(a+b)^(1/2)+2*a*cos(d*x+c)+cos(d*x+c)*b
+2*(a+b)^(1/2)*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)+b)/(co
s(d*x+c)-1))*a-2*a^(1/2)*(a-b)^(3/2)*ln(4*cos(d*x+c)*((b+a*cos(d*x+c))*cos
(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*a^(1/2)+4*a*cos(d*x+c)+4*a^(1/2)*((b+a*cos
(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)+2*b)*b-a^3*ln(1/(a-b)^(1/2)*(2
*(a-b)^(1/2)*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*cos(d*x+
c)+2*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*(a-b)^(1/2)-2*a*
cos(d*x+c)+cos(d*x+c)*b-b)/(cos(d*x+c)+1))+ln(1/(a-b)^(1/2)*(2*(a-b)^(1/2)
*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*cos(d*x+c)+2*((b+a*c
os(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*(a-b)^(1/2)-2*a*cos(d*x+c)+c
os(d*x+c)*b-b)/(cos(d*x+c)+1))*a*b^2)*cos(d*x+c)/(cos(d*x+c)+1)/((b+a*cos(
d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)
```

3.329.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 226 vs. $2(88) = 176$.

Time = 1.42 (sec) , antiderivative size = 2420, normalized size of antiderivative = 22.83

$$\int \frac{\cot(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = \text{Too large to display}$$

input
```
integrate(cot(d*x+c)/(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
[1/4*(2*(a^2 - b^2)*sqrt(a)*log(-8*a^2*cos(d*x + c)^2 - 8*a*b*cos(d*x + c)
 - b^2 - 4*(2*a*cos(d*x + c)^2 + b*cos(d*x + c))*sqrt(a)*sqrt((a*cos(d*x +
 c) + b)/cos(d*x + c))) + (a^2 + a*b)*sqrt(a - b)*log(-((8*a^2 - 8*a*b + b
^2)*cos(d*x + c)^2 + b^2 - 4*((2*a - b)*cos(d*x + c)^2 + b*cos(d*x + c))*s
qrt(a - b)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c)) + 2*(4*a*b - 3*b^2)*cos
(d*x + c))/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) + (a^2 - a*b)*sqrt(a + b
)*log(-((8*a^2 + 8*a*b + b^2)*cos(d*x + c)^2 + b^2 - 4*((2*a + b)*cos(d*x
+ c)^2 + b*cos(d*x + c))*sqrt(a + b)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c
)) + 2*(4*a*b + 3*b^2)*cos(d*x + c))/(cos(d*x + c)^2 - 2*cos(d*x + c) + 1)
))/((a^3 - a*b^2)*d), -1/4*(4*(a^2 - b^2)*sqrt(-a)*arctan(2*sqrt(-a)*sqrt(
(a*cos(d*x + c) + b)/cos(d*x + c))*cos(d*x + c)/(2*a*cos(d*x + c) + b)) -
(a^2 + a*b)*sqrt(a - b)*log(-((8*a^2 - 8*a*b + b^2)*cos(d*x + c)^2 + b^2 -
 4*((2*a - b)*cos(d*x + c)^2 + b*cos(d*x + c))*sqrt(a - b)*sqrt((a*cos(d*x
 + c) + b)/cos(d*x + c)) + 2*(4*a*b - 3*b^2)*cos(d*x + c))/(cos(d*x + c)^2
 + 2*cos(d*x + c) + 1)) - (a^2 - a*b)*sqrt(a + b)*log(-((8*a^2 + 8*a*b + b
^2)*cos(d*x + c)^2 + b^2 - 4*((2*a + b)*cos(d*x + c)^2 + b*cos(d*x + c))*s
qrt(a + b)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c)) + 2*(4*a*b + 3*b^2)*cos
(d*x + c))/(cos(d*x + c)^2 - 2*cos(d*x + c) + 1)))/((a^3 - a*b^2)*d), -1/4
*(2*(a^2 + a*b)*sqrt(-a + b)*arctan(-2*sqrt(-a + b)*sqrt((a*cos(d*x + c) +
 b)/cos(d*x + c))*cos(d*x + c)/((2*a - b)*cos(d*x + c) + b)) - 2*(a^2 -...
```

3.329.6 Sympy [F]

$$\int \frac{\cot(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{\cot(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$$

input
```
integrate(cot(d*x+c)/(a+b*sec(d*x+c))**(1/2),x)
```

output
```
Integral(cot(c + d*x)/sqrt(a + b*sec(c + d*x)), x)
```

3.329.7 Maxima [F]

$$\int \frac{\cot(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{\cot(dx+c)}{\sqrt{b\sec(dx+c)+a}}\,dx$$

input
```
integrate(cot(d*x+c)/(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(cot(d*x + c)/sqrt(b*sec(d*x + c) + a), x)
```

3.329.8 Giac [F(-2)]

Exception generated.

$$\int \frac{\cot(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(cot(d*x+c)/(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.329.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cot(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{\cot(c+d\,x)}{\sqrt{a+\frac{b}{\cos(c+d\,x)}}}\,dx$$

input
```
int(cot(c + d*x)/(a + b/cos(c + d*x))^(1/2),x)
```

output
```
int(cot(c + d*x)/(a + b/cos(c + d*x))^(1/2), x)
```

3.330 $\int \frac{\cot^3(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$

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3.330.1 Optimal result

Integrand size = 23, antiderivative size = 260

$$\int \frac{\cot^3(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = -\frac{2\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}d} + \frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a-b}}\right)}{\sqrt{a-b}d} - \frac{b\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a-b}}\right)}{4(a-b)^{3/2}d} + \frac{b\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)}{4(a+b)^{3/2}d} + \frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)}{\sqrt{a+b}d} + \frac{\sqrt{a+b\sec(c+dx)}}{4(a+b)d(1-\sec(c+dx))} + \frac{\sqrt{a+b\sec(c+dx)}}{4(a-b)d(1+\sec(c+dx))}$$

output

```
-1/4*b*arctanh((a+b*sec(d*x+c))^(1/2)/(a-b)^(1/2))/(a-b)^(3/2)/d+1/4*b*arc
tanh((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2))/(a+b)^(3/2)/d-2*arctanh((a+b*sec(
d*x+c))^(1/2)/a^(1/2))/d/a^(1/2)+arctanh((a+b*sec(d*x+c))^(1/2)/(a-b)^(1/2
))/d/(a-b)^(1/2)+arctanh((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2))/d/(a+b)^(1/2)
+1/4*(a+b*sec(d*x+c))^(1/2)/(a+b)/d/(1-sec(d*x+c))+1/4*(a+b*sec(d*x+c))^(1
/2)/(a-b)/d/(1+sec(d*x+c))
```

3.330.2 Mathematica [A] (verified)

Time = 2.38 (sec) , antiderivative size = 280, normalized size of antiderivative = 1.08

$$\int \frac{\cot^3(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$$

$$= \frac{-\frac{b^2\arctan\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{-a+b}}\right)}{(-a+b)^{3/2}} - \frac{8b\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}} + \frac{4b\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a-b}}\right)}{\sqrt{a-b}} + \frac{b^2\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)}{(a+b)^{3/2}} - }{4bd}$$

input
```
Integrate[Cot[c + d*x]^3/Sqrt[a + b*Sec[c + d*x]],x]
```

output
```
(-((b^2*ArcTan[Sqrt[a + b*Sec[c + d*x]]/Sqrt[-a + b]])/(-a + b)^(3/2)) - (
8*b*ArcTanh[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a]])/Sqrt[a] + (4*b*ArcTanh[Sqrt
[a + b*Sec[c + d*x]]/Sqrt[a - b]])/Sqrt[a - b] + (b^2*ArcTanh[Sqrt[a + b*S
ec[c + d*x]]/Sqrt[a + b]])/(a + b)^(3/2) - (4*a*ArcTanh[Sqrt[a + b*Sec[c +
 d*x]]/Sqrt[a + b]])/Sqrt[a + b] + 4*Sqrt[a + b]*ArcTanh[Sqrt[a + b*Sec[c
+ d*x]]/Sqrt[a + b]] - (b*Sqrt[a + b*Sec[c + d*x]])/((a + b)*(-1 + Sec[c +
 d*x])) + (b*Sqrt[a + b*Sec[c + d*x]])/((a - b)*(1 + Sec[c + d*x])))/(4*b*
d)
```

3.330.3 Rubi [A] (warning: unable to verify)

Time = 0.47 (sec) , antiderivative size = 272, normalized size of antiderivative = 1.05, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {3042, 25, 4373, 561, 25, 1567, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^3(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$$

$$\downarrow 3042$$

$$\int -\frac{1}{\cot\left(c+dx+\frac{\pi}{2}\right)^3\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}\,dx$$

$$\downarrow 25$$

$$-\int \frac{1}{\cot\left(\frac{1}{2}(2c+\pi)+dx\right)^3\sqrt{a+b\csc\left(\frac{1}{2}(2c+\pi)+dx\right)}}\,dx$$

$$\downarrow 4373$$

$$\frac{b^4 \int \frac{\cos(c+dx)}{b\sqrt{a+b\sec(c+dx)}(b^2-b^2\sec^2(c+dx))^2} d(b\sec(c+dx))}{d}$$

$$\downarrow 561$$

$$\frac{2b^4 \int -\frac{1}{(a-b^2\sec^2(c+dx))(b^4\sec^4(c+dx)-2ab^2\sec^2(c+dx)+a^2-b^2)^2} d\sqrt{a+b\sec(c+dx)}}{d}$$

$$\downarrow 25$$

$$-\frac{2b^4 \int \frac{1}{(a-b^2\sec^2(c+dx))(b^4\sec^4(c+dx)-2ab^2\sec^2(c+dx)+a^2-b^2)^2} d\sqrt{a+b\sec(c+dx)}}{d}$$

$$\downarrow 1567$$

$$-\frac{2b^4 \int \left(-\frac{1}{2b^4(-b^2\sec^2(c+dx)+a+b)} + \frac{1}{2b^4(b^2\sec^2(c+dx)-a+b)} - \frac{1}{4b^3(-b^2\sec^2(c+dx)+a+b)^2} + \frac{1}{4b^3(b^2\sec^2(c+dx)-a+b)^2} + \frac{1}{b^4(a-b^2}\right.}{d}$$

$$\downarrow 2009$$

$$\frac{2b^4\left(-\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}b^4} + \frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a-b}}\right)}{2b^4\sqrt{a-b}} + \frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)}{2b^4\sqrt{a+b}} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a-b}}\right)}{8b^3(a-b)^{3/2}} + \frac{\text{arc}}{}\right.}{d}$$

input
```
Int[Cot[c + d*x]^3/Sqrt[a + b*Sec[c + d*x]],x]
```

output
```
(2*b^4*(-(ArcTanh[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a]]/(Sqrt[a]*b^4)) + ArcTa
nh[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a - b]]/(2*Sqrt[a - b]*b^4) - ArcTanh[Sqr
t[a + b*Sec[c + d*x]]/Sqrt[a - b]]/(8*(a - b)^(3/2)*b^3) + ArcTanh[Sqrt[a
+ b*Sec[c + d*x]]/Sqrt[a + b]]/(8*b^3*(a + b)^(3/2)) + ArcTanh[Sqrt[a + b*
Sec[c + d*x]]/Sqrt[a + b]]/(2*b^4*Sqrt[a + b]) - Sec[c + d*x]/(8*(a - b)*b
^2*(a - b - b^2*Sec[c + d*x]^2)) + Sec[c + d*x]/(8*b^2*(a + b)*(a + b - b^
2*Sec[c + d*x]^2))))/d
```

3.330.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 561
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> With[{k = Denominator[n]}, Simp[k/d   Subst[Int[x^(k*(n + 1) - 1)*(-c
/d + x^k/d)^m*Simp[(b*c^2 + a*d^2)/d^2 - 2*b*c*(x^k/d^2) + b*(x^(2*k)/d^2),
 x]^p, x], x, (c + d*x)^(1/k)], x]] /; FreeQ[{a, b, c, d, m, p}, x] && Frac
tionQ[n] && IntegerQ[p] && IntegerQ[m]
```

rule 1567
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x], x]
 /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b^2 - 4*a*c, 0] && ((IntegerQ[p]
 && IntegerQ[q]) || IGtQ[p, 0] || IGtQ[q, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4373
```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Simp[-(-1)^((m - 1)/2)/(d*b^(m - 1))   Subst[Int[(b^2 - x^
2)^((m - 1)/2)*((a + x)^n/x), x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
 d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

3.330.4 Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 3106 vs. $2(218) = 436$.

Time = 2.01 (sec) , antiderivative size = 3107, normalized size of antiderivative = 11.95

method	result	size
default	Expression too large to display	3107

input
```
int(cot(d*x+c)^3/(a+b*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/8/d/(a+b)^2/(a-b)^(5/2)/a*((a*(1-cos(d*x+c))^2*csc(d*x+c)^2-b*(1-cos(d*x
+c))^2*csc(d*x+c)^2-a-b)/((1-cos(d*x+c))^2*csc(d*x+c)^2-1))^(1/2)*((1-cos(
d*x+c))^2*csc(d*x+c)^2-1)*(8*ln(2*(-2*a*(1-cos(d*x+c))^2*csc(d*x+c)^2+b*(1
-cos(d*x+c))^2*csc(d*x+c)^2+2*a^(1/2)*((1-cos(d*x+c))^4*a*csc(d*x+c)^4-(1-
cos(d*x+c))^4*b*csc(d*x+c)^4-2*a*(1-cos(d*x+c))^2*csc(d*x+c)^2+a+b)^(1/2)+
2*a+b)/((1-cos(d*x+c))^2*csc(d*x+c)^2+1))*a^(7/2)*(a-b)^(3/2)*(1-cos(d*x+c
))^2*csc(d*x+c)^2+8*ln(2*(-2*a*(1-cos(d*x+c))^2*csc(d*x+c)^2+b*(1-cos(d*x+
c))^2*csc(d*x+c)^2+2*a^(1/2)*((1-cos(d*x+c))^4*a*csc(d*x+c)^4-(1-cos(d*x+c
))^4*b*csc(d*x+c)^4-2*a*(1-cos(d*x+c))^2*csc(d*x+c)^2+a+b)^(1/2)+2*a+b)/((
1-cos(d*x+c))^2*csc(d*x+c)^2+1))*a^(5/2)*(a-b)^(3/2)*b*(1-cos(d*x+c))^2*cs
c(d*x+c)^2-8*ln(2*(-2*a*(1-cos(d*x+c))^2*csc(d*x+c)^2+b*(1-cos(d*x+c))^2*c
sc(d*x+c)^2+2*a^(1/2)*((1-cos(d*x+c))^4*a*csc(d*x+c)^4-(1-cos(d*x+c))^4*b*
csc(d*x+c)^4-2*a*(1-cos(d*x+c))^2*csc(d*x+c)^2+a+b)^(1/2)+2*a+b)/((1-cos(d
*x+c))^2*csc(d*x+c)^2+1))*a^(3/2)*(a-b)^(3/2)*b^2*(1-cos(d*x+c))^2*csc(d*x
+c)^2-(a-b)^(3/2)*((1-cos(d*x+c))^4*a*csc(d*x+c)^4-(1-cos(d*x+c))^4*b*csc(
d*x+c)^4-2*a*(1-cos(d*x+c))^2*csc(d*x+c)^2+a+b)^(1/2)*a^3*(1-cos(d*x+c))^4
*csc(d*x+c)^4+2*(a-b)^(3/2)*((1-cos(d*x+c))^4*a*csc(d*x+c)^4-(1-cos(d*x+c)
)^4*b*csc(d*x+c)^4-2*a*(1-cos(d*x+c))^2*csc(d*x+c)^2+a+b)^(1/2)*a^2*b*(1-c
os(d*x+c))^4*csc(d*x+c)^4-(a-b)^(3/2)*((1-cos(d*x+c))^4*a*csc(d*x+c)^4-(1-
cos(d*x+c))^4*b*csc(d*x+c)^4-2*a*(1-cos(d*x+c))^2*csc(d*x+c)^2+a+b)^(1/...
```

3.330.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 466 vs. $2(216) = 432$.

Time = 25.89 (sec) , antiderivative size = 4336, normalized size of antiderivative = 16.68

$$\int \frac{\cot^3(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = \text{Too large to display}$$

input
```
integrate(cot(d*x+c)^3/(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
[-1/16*(8*(a^4 - 2*a^2*b^2 + b^4 - (a^4 - 2*a^2*b^2 + b^4)*cos(d*x + c)^2)
*sqrt(a)*log(-8*a^2*cos(d*x + c)^2 - 8*a*b*cos(d*x + c) - b^2 + 4*(2*a*cos
(d*x + c)^2 + b*cos(d*x + c))*sqrt(a)*sqrt((a*cos(d*x + c) + b)/cos(d*x +
c))) + (4*a^4 + 3*a^3*b - 6*a^2*b^2 - 5*a*b^3 - (4*a^4 + 3*a^3*b - 6*a^2*b
^2 - 5*a*b^3)*cos(d*x + c)^2)*sqrt(a - b)*log(-((8*a^2 - 8*a*b + b^2)*cos(
d*x + c)^2 + b^2 + 4*((2*a - b)*cos(d*x + c)^2 + b*cos(d*x + c))*sqrt(a -
b)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c)) + 2*(4*a*b - 3*b^2)*cos(d*x + c
))/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) + (4*a^4 - 3*a^3*b - 6*a^2*b^2 +
 5*a*b^3 - (4*a^4 - 3*a^3*b - 6*a^2*b^2 + 5*a*b^3)*cos(d*x + c)^2)*sqrt(a
+ b)*log(-((8*a^2 + 8*a*b + b^2)*cos(d*x + c)^2 + b^2 + 4*((2*a + b)*cos(d
*x + c)^2 + b*cos(d*x + c))*sqrt(a + b)*sqrt((a*cos(d*x + c) + b)/cos(d*x
+ c)) + 2*(4*a*b + 3*b^2)*cos(d*x + c))/(cos(d*x + c)^2 - 2*cos(d*x + c) +
 1)) - 8*((a^4 - a^2*b^2)*cos(d*x + c)^2 - (a^3*b - a*b^3)*cos(d*x + c))*s
qrt((a*cos(d*x + c) + b)/cos(d*x + c)))/((a^5 - 2*a^3*b^2 + a*b^4)*d*cos(d
*x + c)^2 - (a^5 - 2*a^3*b^2 + a*b^4)*d), -1/16*(16*(a^4 - 2*a^2*b^2 + b^4
 - (a^4 - 2*a^2*b^2 + b^4)*cos(d*x + c)^2)*sqrt(-a)*arctan(2*sqrt(-a)*sqrt
((a*cos(d*x + c) + b)/cos(d*x + c))*cos(d*x + c)/(2*a*cos(d*x + c) + b)) +
 (4*a^4 + 3*a^3*b - 6*a^2*b^2 - 5*a*b^3 - (4*a^4 + 3*a^3*b - 6*a^2*b^2 - 5
*a*b^3)*cos(d*x + c)^2)*sqrt(a - b)*log(-((8*a^2 - 8*a*b + b^2)*cos(d*x +
c)^2 + b^2 + 4*((2*a - b)*cos(d*x + c)^2 + b*cos(d*x + c))*sqrt(a - b)*...
```

3.330.6 Sympy [F]

$$\int \frac{\cot^3(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{\cot^3(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$$

input
```
integrate(cot(d*x+c)**3/(a+b*sec(d*x+c))**(1/2),x)
```

output
```
Integral(cot(c + d*x)**3/sqrt(a + b*sec(c + d*x)), x)
```

3.330.7 Maxima [F]

$$\int \frac{\cot^3(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{\cot\left(dx+c\right)^3}{\sqrt{b\sec\left(dx+c\right)+a}}\,dx$$

input
```
integrate(cot(d*x+c)^3/(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(cot(d*x + c)^3/sqrt(b*sec(d*x + c) + a), x)
```

3.330.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 533 vs. $2(216) = 432$.

Time $= 0.92$ (sec) , antiderivative size $= 533$, normalized size of antiderivative $= 2.05$

$$\int \frac{\cot^3(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$$

$$= \frac{16\arctan\left(-\frac{\sqrt{a-b}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-\sqrt{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a+b}+\sqrt{a-b}}{2\sqrt{-a}}\right)}{\sqrt{-a}} - \frac{2\left(4\,a+5\,b\right)\arctan\left(-\frac{\sqrt{a-b}\tan\left(\frac{1}{2}\right.\right.}{}$$

input
```
integrate(cot(d*x+c)^3/(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
1/8*(16*arctan(-1/2*(sqrt(a - b)*tan(1/2*d*x + 1/2*c)^2 - sqrt(a*tan(1/2*d
*x + 1/2*c)^4 - b*tan(1/2*d*x + 1/2*c)^4 - 2*a*tan(1/2*d*x + 1/2*c)^2 + a
+ b) + sqrt(a - b))/sqrt(-a))/sqrt(-a) - 2*(4*a + 5*b)*arctan(-(sqrt(a - b
)*tan(1/2*d*x + 1/2*c)^2 - sqrt(a*tan(1/2*d*x + 1/2*c)^4 - b*tan(1/2*d*x +
 1/2*c)^4 - 2*a*tan(1/2*d*x + 1/2*c)^2 + a + b))/sqrt(-a - b))/((a + b)*sq
rt(-a - b)) + (4*a - 5*b)*log(abs((sqrt(a - b)*tan(1/2*d*x + 1/2*c)^2 - sq
rt(a*tan(1/2*d*x + 1/2*c)^4 - b*tan(1/2*d*x + 1/2*c)^4 - 2*a*tan(1/2*d*x +
 1/2*c)^2 + a + b))*(a - b) - sqrt(a - b)*a))/(a - b)^(3/2) + sqrt(a*tan(1
/2*d*x + 1/2*c)^4 - b*tan(1/2*d*x + 1/2*c)^4 - 2*a*tan(1/2*d*x + 1/2*c)^2
+ a + b)/(a - b) - 2*((sqrt(a - b)*tan(1/2*d*x + 1/2*c)^2 - sqrt(a*tan(1/2
*d*x + 1/2*c)^4 - b*tan(1/2*d*x + 1/2*c)^4 - 2*a*tan(1/2*d*x + 1/2*c)^2 +
a + b))*a - (a + b)*sqrt(a - b))/(((sqrt(a - b)*tan(1/2*d*x + 1/2*c)^2 - s
qrt(a*tan(1/2*d*x + 1/2*c)^4 - b*tan(1/2*d*x + 1/2*c)^4 - 2*a*tan(1/2*d*x
+ 1/2*c)^2 + a + b))^2 - a - b)*(a + b)))/(d*sgn(cos(d*x + c)))
```

3.330.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cot^3(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{\cot(c+d\,x)^3}{\sqrt{a+\frac{b}{\cos(c+d\,x)}}}\,dx$$

input
```
int(cot(c + d*x)^3/(a + b/cos(c + d*x))^(1/2),x)
```

output
```
int(cot(c + d*x)^3/(a + b/cos(c + d*x))^(1/2), x)
```

3.331 $\int \frac{\tan^4(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$

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3.331.1 Optimal result

Integrand size = 23, antiderivative size = 404

$$\int \frac{\tan^4(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx =$$
$$-\frac{2\sqrt{a+b}\cot(c+dx)\,\text{EllipticPi}\left(\frac{a+b}{a},\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{ad}$$
$$-\frac{2(a-b)\sqrt{a+b}(8a^2-21b^2)\cot(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{-\frac{b(-1+\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+}}{15b^4d}$$
$$+\frac{2\sqrt{a+b}(-8a^2+2ab+21b^2)\cot(c+dx)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{-\frac{b(-1+\sec(c+dx))}{a+b}}\sqrt{\frac{b(}{}}$$
$$-\frac{8a\sqrt{a+b\sec(c+dx)}\tan(c+dx)}{15b^2d}+\frac{2\sec(c+dx)\sqrt{a+b\sec(c+dx)}\tan(c+dx)}{5bd}$$

output

```
-2*cot(d*x+c)*EllipticPi((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),(a+b)/a,((a+b)
/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c
))/(a-b))^(1/2)/a/d-2/15*(a-b)*(8*a^2-21*b^2)*cot(d*x+c)*EllipticE((a+b*se
c(d*x+c))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(-b*(-1+sec(d
*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/b^4/d+2/15*(-8*a^2+2*a
*b+21*b^2)*cot(d*x+c)*EllipticF((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),((a+b)/
(a-b))^(1/2))*(a+b)^(1/2)*(-b*(-1+sec(d*x+c))/(a+b))^(1/2)*(b*(1+sec(d*x+c
))/(-a+b))^(1/2)/b^3/d-8/15*a*(a+b*sec(d*x+c))^(1/2)*tan(d*x+c)/b^2/d+2/5*
sec(d*x+c)*(a+b*sec(d*x+c))^(1/2)*tan(d*x+c)/b/d
```

3.331.2 Mathematica [A] (verified)

Time = 15.80 (sec) , antiderivative size = 446, normalized size of antiderivative = 1.10

$$\int \frac{\tan^4(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$$

$$= \frac{2\sqrt{\sec(c+dx)}\left(\frac{\sqrt{\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)}\left(-2(8a^3+8a^2b-21ab^2-21b^3)E\left(\arcsin\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)\middle|\frac{a-b}{a+b}\right)\sqrt{\frac{1}{1+\sec(c+dx)}}\sqrt{\frac{a+b\sec(c+\right.\right.\right.}{(a+b)(1+\sec(c}$$

input
```
Integrate[Tan[c + d*x]^4/Sqrt[a + b*Sec[c + d*x]],x]
```

output
```
(2*Sqrt[Sec[c + d*x]]*((Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(-2*(8*a^3 +
 8*a^2*b - 21*a*b^2 - 21*b^3)*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/
(a + b)]*Sqrt[(1 + Sec[c + d*x])^(-1)]*Sqrt[(a + b*Sec[c + d*x])/((a + b)*
(1 + Sec[c + d*x]))] + 4*b*(4*a^2 + a*b - 18*b^2)*EllipticF[ArcSin[Tan[(c
 + d*x)/2]], (a - b)/(a + b)]*Sqrt[(1 + Sec[c + d*x])^(-1)]*Sqrt[(a + b*Sec
[c + d*x])/((a + b)*(1 + Sec[c + d*x]))] + 60*b^3*EllipticPi[-1, ArcSin[Ta
n[(c + d*x)/2]], (a - b)/(a + b)]*Sqrt[(1 + Sec[c + d*x])^(-1)]*Sqrt[(a +
b*Sec[c + d*x])/((a + b)*(1 + Sec[c + d*x]))] + ((8*a^2 - 21*b^2)*(b + a*C
os[c + d*x])*Sec[(c + d*x)/2]^3*(Sin[(c + d*x)/2] - Sin[(3*(c + d*x))/2]))
/2))/Sqrt[Sec[(c + d*x)/2]^2] + (b + a*Cos[c + d*x])*Sqrt[Sec[c + d*x]]*((
8*a^2 - 21*b^2)*Sin[c + d*x] + b*(-4*a + 3*b*Sec[c + d*x])*Tan[c + d*x])))
/(15*b^3*d*Sqrt[a + b*Sec[c + d*x]])
```

3.331.3 Rubi [A] (verified)

Time = 1.23 (sec) , antiderivative size = 610, normalized size of antiderivative = 1.51, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3042, 4383, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^4(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$$

$$\downarrow 3042$$

$$\int \frac{\cot\left(c+dx+\frac{\pi}{2}\right)^4}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\downarrow 4383$$

$$\int \left(\frac{\sec^4(c+dx)}{\sqrt{a+b\sec(c+dx)}}-\frac{2\sec^2(c+dx)}{\sqrt{a+b\sec(c+dx)}}+\frac{1}{\sqrt{a+b\sec(c+dx)}}\right)dx$$

$$\downarrow 2009$$

$$-\frac{2(a-b)\sqrt{a+b}\left(8a^2+9b^2\right)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}E\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{15b^4d}-$$
$$\frac{2\sqrt{a+b}\left(8a^2-2ab+9b^2\right)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{15b^3d}+$$
$$\frac{4(a-b)\sqrt{a+b}\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}E\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{b^2d}+$$
$$\frac{4\sqrt{a+b}\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{bd}-$$
$$\frac{2\sqrt{a+b}\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\,\text{EllipticPi}\left(\frac{a+b}{a},\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{ad}-$$
$$\frac{8a\tan(c+dx)\sqrt{a+b\sec(c+dx)}}{15b^2d}+\frac{2\tan(c+dx)\sec(c+dx)\sqrt{a+b\sec(c+dx)}}{5bd}$$

input
```
Int[Tan[c + d*x]^4/Sqrt[a + b*Sec[c + d*x]],x]
```

output
```
(4*(a - b)*Sqrt[a + b]*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d*
x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sq
rt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(b^2*d) - (2*(a - b)*Sqrt[a + b]*(8
*a^2 + 9*b^2)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[
a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(
1 + Sec[c + d*x]))/(a - b))])/(15*b^4*d) + (4*Sqrt[a + b]*Cot[c + d*x]*Ell
ipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt
[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/
(b*d) - (2*Sqrt[a + b]*(8*a^2 - 2*a*b + 9*b^2)*Cot[c + d*x]*EllipticF[ArcS
in[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Se
c[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(15*b^3*d)
- (2*Sqrt[a + b]*Cot[c + d*x]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Sec[
c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a +
b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(a*d) - (8*a*Sqrt[a + b*Sec[c
 + d*x]]*Tan[c + d*x])/(15*b^2*d) + (2*Sec[c + d*x]*Sqrt[a + b*Sec[c + d*x
]]*Tan[c + d*x])/(5*b*d)
```

3.331.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4383
```
Int[cot[(c_.) + (d_.)*(x_)]^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_
), x_Symbol] :> Int[ExpandIntegrand[(a + b*Csc[c + d*x])^n, (-1 + Csc[c + d
*x]^2)^(m/2), x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[a^2 - b^2, 0] && I
GtQ[m/2, 0] && IntegerQ[n - 1/2]
```

3.331.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2242 vs. $2(365) = 730$.

Time = 15.87 (sec) , antiderivative size = 2243, normalized size of antiderivative = 5.55

method	result	size
default	Expression too large to display	2243

input
```
int(tan(d*x+c)^4/(a+b*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/15/d/b^3*(a+b*sec(d*x+c))^(1/2)/(b+a*cos(d*x+c))/(cos(d*x+c)+1)*(-a*b^2*
sin(d*x+c)+4*a^2*b*sin(d*x+c)+8*EllipticE(cot(d*x+c)-csc(d*x+c),((a-b)/(a+
b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(co
s(d*x+c)+1))^(1/2)*a^2*b-21*EllipticE(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^
(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*
x+c)+1))^(1/2)*a*b^2+8*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(co
s(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticE(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b
))^(1/2))*a^3*cos(d*x+c)^2-21*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1
/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticE(cot(d*x+c)-csc(d*x+c),((a-
b)/(a+b))^(1/2))*b^3*cos(d*x+c)^2-8*EllipticF(cot(d*x+c)-csc(d*x+c),((a-b)
/(a+b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)
/(cos(d*x+c)+1))^(1/2)*a^2*b-2*EllipticF(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b
))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos
(d*x+c)+1))^(1/2)*a*b^2+72*EllipticF(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(
1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x
+c)+1))^(1/2)*b^3*cos(d*x+c)+16*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^
(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticE(cot(d*x+c)-csc(d*x+c),((
a-b)/(a+b))^(1/2))*a^3*cos(d*x+c)+36*EllipticF(cot(d*x+c)-csc(d*x+c),((a-b
)/(a+b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c
)/(cos(d*x+c)+1))^(1/2)*b^3*cos(d*x+c)^2-42*(1/(a+b)*(b+a*cos(d*x+c))/(...
```

3.331.5 Fricas [F]

$$\int \frac{\tan^4(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{\tan\left(dx+c\right)^4}{\sqrt{b\sec\left(dx+c\right)+a}}\,dx$$

input
```
integrate(tan(d*x+c)^4/(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
integral(tan(d*x + c)^4/sqrt(b*sec(d*x + c) + a), x)
```

3.331.6 Sympy [F]

$$\int \frac{\tan^4(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{\tan^4(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$$

input
```
integrate(tan(d*x+c)**4/(a+b*sec(d*x+c))**(1/2),x)
```

output
```
Integral(tan(c + d*x)**4/sqrt(a + b*sec(c + d*x)), x)
```

3.331.7 Maxima [F]

$$\int \frac{\tan^4(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{\tan(dx+c)^4}{\sqrt{b\sec(dx+c)+a}}\,dx$$

input
```
integrate(tan(d*x+c)^4/(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(tan(d*x + c)^4/sqrt(b*sec(d*x + c) + a), x)
```

3.331.8 Giac [F]

$$\int \frac{\tan^4(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{\tan(dx+c)^4}{\sqrt{b\sec(dx+c)+a}}\,dx$$

input
```
integrate(tan(d*x+c)^4/(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(tan(d*x + c)^4/sqrt(b*sec(d*x + c) + a), x)
```

3.331.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\tan^4(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{\tan(c+d\,x)^4}{\sqrt{a+\frac{b}{\cos(c+d\,x)}}}\,dx$$

input
```
int(tan(c + d*x)^4/(a + b/cos(c + d*x))^(1/2),x)
```

output
```
int(tan(c + d*x)^4/(a + b/cos(c + d*x))^(1/2), x)
```

3.332 $\int \frac{\tan^2(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$

3.332.1 Optimal result . . . 2440
3.332.2 Mathematica [A] (warning: unable to verify) . . . 2441
3.332.3 Rubi [A] (verified) . . . 2441
3.332.4 Maple [B] (verified) . . . 2444
3.332.5 Fricas [F] . . . 2445
3.332.6 Sympy [F] . . . 2446
3.332.7 Maxima [F] . . . 2446
3.332.8 Giac [F] . . . 2446
3.332.9 Mupad [F(-1)] . . . 2447

3.332.1 Optimal result

Integrand size = 23, antiderivative size = 310

$$\int \frac{\tan^2(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx =$$
$$-\frac{2(a-b)\sqrt{a+b}\cot(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)\middle|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{b^2d}$$
$$-\frac{2\sqrt{a+b}\cot(c+dx)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{bd}$$
$$+\frac{2\sqrt{a+b}\cot(c+dx)\,\text{EllipticPi}\left(\frac{a+b}{a},\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{ad}$$

output

```
-2*(a-b)*cot(d*x+c)*EllipticE((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),((a+b)/(a
-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/
(a-b))^(1/2)/b^2/d-2*cot(d*x+c)*EllipticF((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/
2),((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+
sec(d*x+c))/(a-b))^(1/2)/b/d+2*cot(d*x+c)*EllipticPi((a+b*sec(d*x+c))^(1/2
)/(a+b)^(1/2),(a+b)/a,((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(d*x+c))/(
a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/a/d
```

3.332.2 Mathematica [A] (warning: unable to verify)

Time = 12.55 (sec) , antiderivative size = 382, normalized size of antiderivative = 1.23

$$\int \frac{\tan^2(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$$
$$= \frac{-\cos^2\left(\frac{1}{2}(c+dx)\right)\sqrt{\cos(c+dx)\sec^4\left(\frac{1}{2}(c+dx)\right)}\sqrt{\sec(c+dx)}\sqrt{1+\sec(c+dx)}\left(2(a+b)\sqrt{\frac{b+a\cos(c+d}{(a+b)(1+\cos(c+}\right.}{}$$

input
```
Integrate[Tan[c + d*x]^2/Sqrt[a + b*Sec[c + d*x]],x]
```

output
```
(-(Cos[(c + d*x)/2]^2*Sqrt[Cos[c + d*x]*Sec[(c + d*x)/2]^4]*Sqrt[Sec[c + d
*x]]*Sqrt[1 + Sec[c + d*x]]*(2*(a + b)*Sqrt[(b + a*Cos[c + d*x])/((a + b)*
(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]
 - 4*b*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[Ar
cSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] + 4*b*Sqrt[(b + a*Cos[c + d*x])/(
(a + b)*(1 + Cos[c + d*x]))]*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (a -
 b)/(a + b)] + a*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*Si
n[(3*(c + d*x))/2] - a*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Tan[(c + d*x)
/2] + 2*b*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Tan[(c + d*x)/2])) + 2*(b
+ a*Cos[c + d*x])*Tan[c + d*x])/(b*d*Sqrt[a + b*Sec[c + d*x]])
```

3.332.3 Rubi [A] (verified)

Time = 1.01 (sec) , antiderivative size = 310, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.435$, Rules used = {3042, 4382, 3042, 4547, 3042, 4409, 3042, 4271, 4319, 4492}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^2(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$$
$$\downarrow 3042$$
$$\int \frac{\cot\left(c+dx+\frac{\pi}{2}\right)^2}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx$$
$$\downarrow 4382$$

$$\int \frac{\sec^2(c+dx)-1}{\sqrt{a+b\sec(c+dx)}}dx$$

$$\downarrow 3042$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^2-1}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\downarrow 4547$$

$$\int \frac{-\sec(c+dx)-1}{\sqrt{a+b\sec(c+dx)}}dx+\int \frac{\sec(c+dx)(\sec(c+dx)+1)}{\sqrt{a+b\sec(c+dx)}}dx$$

$$\downarrow 3042$$

$$\int \frac{-\csc\left(c+dx+\frac{\pi}{2}\right)-1}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)+1\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\downarrow 4409$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)+1\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-\int \frac{1}{\sqrt{a+b\sec(c+dx)}}dx-\int \frac{\sec(c+dx)}{\sqrt{a+b\sec(c+dx)}}dx$$

$$\downarrow 3042$$

$$-\int \frac{1}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)+1\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\downarrow 4271$$

$$-\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)+1\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sqrt{a+b}\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\operatorname{EllipticPi}\left(\frac{a+b}{a},\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{ad}$$

$$\downarrow 4319$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)+1\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{2\sqrt{a+b}\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{bd}+\frac{2\sqrt{a+b}\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\operatorname{EllipticPi}\left(\frac{a+b}{a},\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{ad}$$

$$\downarrow 4492$$

$$-\frac{2(a-b)\sqrt{a+b}\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}E\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{b^2d}-$$
$$\frac{2\sqrt{a+b}\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{bd}+$$
$$\frac{2\sqrt{a+b}\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\,\text{EllipticPi}\left(\frac{a+b}{a},\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{ad}$$

input
```
Int[Tan[c + d*x]^2/Sqrt[a + b*Sec[c + d*x]],x]
```

output
```
(-2*(a - b)*Sqrt[a + b]*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d
*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*S
qrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(b^2*d) - (2*Sqrt[a + b]*Cot[c + d
*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b
)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a -
 b))])/(b*d) + (2*Sqrt[a + b]*Cot[c + d*x]*EllipticPi[(a + b)/a, ArcSin[Sq
rt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c +
 d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(a*d)
```

3.332.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4271
```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[2*(Rt[a
 + b, 2]/(a*d*Cot[c + d*x]))*Sqrt[b*((1 - Csc[c + d*x])/(a + b))]*Sqrt[(-b)
*((1 + Csc[c + d*x])/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[
c + d*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

rule 4319
```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*(Rt[a + b, 2]/(b*f*Cot[e + f*x]))*Sqrt[(b*(1 - Csc[e + f*
x]))/(a + b)]*Sqrt[(-b)*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt
[a + b*Csc[e + f*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4382
```
Int[cot[(c_.) + (d_.)*(x_)]^2*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_),
x_Symbol] :> Int[(-1 + Csc[c + d*x]^2)*(a + b*Csc[c + d*x])^n, x] /; FreeQ[
{a, b, c, d, n}, x] && NeQ[a^2 - b^2, 0]
```

rule 4409
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Simp[c   Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] +
Simp[d   Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

rule 4492
```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(A*b - a*B)*Rt[a
 + b*(B/A), 2]*Sqrt[b*((1 - Csc[e + f*x])/(a + b))]*(Sqrt[(-b)*((1 + Csc[e
+ f*x])/(a - b))]/(b^2*f*Cot[e + f*x]))*EllipticE[ArcSin[Sqrt[a + b*Csc[e +
 f*x]]/Rt[a + b*(B/A), 2]], (a*A + b*B)/(a*A - b*B)], x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

rule 4547
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))/Sqrt[csc[(e_.) + (f_.)*(x_)]*
(b_.) + (a_)], x_Symbol] :> Int[(A - C*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x
]], x] + Simp[C   Int[Csc[e + f*x]*((1 + Csc[e + f*x])/Sqrt[a + b*Csc[e + f
*x]]), x], x] /; FreeQ[{a, b, e, f, A, C}, x] && NeQ[a^2 - b^2, 0]
```

3.332.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1068 vs. $2(283) = 566$.

Time = 10.92 (sec) , antiderivative size = 1069, normalized size of antiderivative = 3.45

method	result	size
default	Expression too large to display	1069

input
```
int(tan(d*x+c)^2/(a+b*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/d/b*(2*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(
d*x+c)+1))^(1/2)*EllipticPi(cot(d*x+c)-csc(d*x+c),-1,((a-b)/(a+b))^(1/2))*
b*cos(d*x+c)^2+EllipticE(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d
*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2
)*a*cos(d*x+c)^2+EllipticE(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))*(cos
(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1
/2)*b*cos(d*x+c)^2-2*EllipticF(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))*
(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1)
)^(1/2)*b*cos(d*x+c)^2+4*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(
cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticPi(cot(d*x+c)-csc(d*x+c),-1,((a-b
)/(a+b))^(1/2))*b*cos(d*x+c)+2*EllipticE(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b
))^(1/2))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos
(d*x+c)+1))^(1/2)*a*cos(d*x+c)+2*EllipticE(cot(d*x+c)-csc(d*x+c),((a-b)/(a
+b))^(1/2))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(c
os(d*x+c)+1))^(1/2)*b*cos(d*x+c)-4*EllipticF(cot(d*x+c)-csc(d*x+c),((a-b)/
(a+b))^(1/2))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/
(cos(d*x+c)+1))^(1/2)*b*cos(d*x+c)+2*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)
+1))^(1/2)*EllipticPi(cot(d*x+c)-csc(d*x+c),-1,((a-b)/(a+b))^(1/2))*(cos(d
*x+c)/(cos(d*x+c)+1))^(1/2)*b+EllipticE(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b)
)^(1/2))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(c...
```

3.332.5 Fricas [F]

$$\int \frac{\tan^2(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{\tan\left(dx+c\right)^2}{\sqrt{b\sec\left(dx+c\right)+a}}\,dx$$

input
```
integrate(tan(d*x+c)^2/(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
integral(tan(d*x + c)^2/sqrt(b*sec(d*x + c) + a), x)
```

3.332.6 Sympy [F]

$$\int \frac{\tan^2(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{\tan^2(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$$

input
```
integrate(tan(d*x+c)**2/(a+b*sec(d*x+c))**(1/2),x)
```

output
```
Integral(tan(c + d*x)**2/sqrt(a + b*sec(c + d*x)), x)
```

3.332.7 Maxima [F]

$$\int \frac{\tan^2(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{\tan(dx+c)^2}{\sqrt{b\sec(dx+c)+a}}\,dx$$

input
```
integrate(tan(d*x+c)^2/(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(tan(d*x + c)^2/sqrt(b*sec(d*x + c) + a), x)
```

3.332.8 Giac [F]

$$\int \frac{\tan^2(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{\tan(dx+c)^2}{\sqrt{b\sec(dx+c)+a}}\,dx$$

input
```
integrate(tan(d*x+c)^2/(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(tan(d*x + c)^2/sqrt(b*sec(d*x + c) + a), x)
```

3.332.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\tan^2(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{\tan(c+d\,x)^2}{\sqrt{a+\frac{b}{\cos(c+d\,x)}}}\,dx$$

input
```
int(tan(c + d*x)^2/(a + b/cos(c + d*x))^(1/2),x)
```

output
```
int(tan(c + d*x)^2/(a + b/cos(c + d*x))^(1/2), x)
```

3.333 $\int \frac{1}{\sqrt{a+b\sec(c+dx)}}\,dx$

3.333.1 Optimal result 2448
3.333.2 Mathematica [A] (verified) 2448
3.333.3 Rubi [A] (verified) 2449
3.333.4 Maple [A] (verified) 2450
3.333.5 Fricas [F] 2450
3.333.6 Sympy [F] 2451
3.333.7 Maxima [F] 2451
3.333.8 Giac [F] 2451
3.333.9 Mupad [F(-1)] 2452

3.333.1 Optimal result

Integrand size = 14, antiderivative size = 106

$$\int \frac{1}{\sqrt{a+b\sec(c+dx)}}\,dx = -\frac{2\sqrt{a+b}\cot(c+dx)\,\text{EllipticPi}\left(\frac{a+b}{a},\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{ad}$$

output
```
-2*cot(d*x+c)*EllipticPi((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),(a+b)/a,((a+b)
/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c
))/(a-b))^(1/2)/a/d
```

3.333.2 Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.30

$$\int \frac{1}{\sqrt{a+b\sec(c+dx)}}\,dx = -\frac{4\cos^2\left(\frac{1}{2}(c+dx)\right)\sqrt{\frac{\cos(c+dx)}{1+\cos(c+dx)}}\sqrt{\frac{b+a\cos(c+dx)}{(a+b)(1+\cos(c+dx))}}\left(\text{EllipticF}\left(\arcsin\left(\tan\left(\frac{1}{2}(c+dx)\right)\right),\frac{a-b}{a+b}\right)-2\,\text{Ellipti}\right.}{d\sqrt{a+b\sec(c+dx)}}$$

input
```
Integrate[1/Sqrt[a + b*Sec[c + d*x]],x]
```

output
```
(-4*Cos[(c + d*x)/2]^2*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*C
os[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*(EllipticF[ArcSin[Tan[(c + d*x)
/2]], (a - b)/(a + b)] - 2*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (a - b
)/(a + b)])*Sec[c + d*x])/(d*Sqrt[a + b*Sec[c + d*x]])
```

3.333.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3042, 4271}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a+b\sec(c+dx)}}\,dx$$

$$\downarrow 3042$$

$$\int \frac{1}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\downarrow 4271$$

$$-\frac{2\sqrt{a+b}\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\operatorname{EllipticPi}\left(\frac{a+b}{a},\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{ad}$$

input
```
Int[1/Sqrt[a + b*Sec[c + d*x]],x]
```

output
```
(-2*Sqrt[a + b]*Cot[c + d*x]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Sec[c
 + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b
)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(a*d)
```

3.333.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4271
```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[2*(Rt[a
 + b, 2]/(a*d*Cot[c + d*x]))*Sqrt[b*((1 - Csc[c + d*x])/(a + b))]*Sqrt[(-b)
*((1 + Csc[c + d*x])/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[
c + d*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

3.333.4 Maple [A] (verified)

Time = 5.44 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.39

method	result
default	$\frac{2(\cos(dx+c)+1)\left(\text{EllipticF}\left(\cot(dx+c)-\csc(dx+c),\sqrt{\frac{a-b}{a+b}}\right)-2\,\text{EllipticPi}\left(\cot(dx+c)-\csc(dx+c),-1,\sqrt{\frac{a-b}{a+b}}\right)\right)\sqrt{\frac{b+a\cos(dx+c)}{(a+b)(\cos(dx+c)+1)}}}{d(b+a\cos(dx+c))}$

input
```
int(1/(a+b*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/d*(cos(d*x+c)+1)*(EllipticF(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))-2
*EllipticPi(cot(d*x+c)-csc(d*x+c),-1,((a-b)/(a+b))^(1/2)))*(1/(a+b)*(b+a*c
os(d*x+c))/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(a+b*se
c(d*x+c))^(1/2)/(b+a*cos(d*x+c))
```

3.333.5 Fricas [F]

$$\int \frac{1}{\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{1}{\sqrt{b\sec(dx+c)+a}}\,dx$$

input
```
integrate(1/(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
integral(1/sqrt(b*sec(d*x + c) + a), x)
```

3.333.6 Sympy [F]

$$\int \frac{1}{\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{1}{\sqrt{a+b\sec(c+dx)}}\,dx$$

input
```
integrate(1/(a+b*sec(d*x+c))**(1/2),x)
```

output
```
Integral(1/sqrt(a + b*sec(c + d*x)), x)
```

3.333.7 Maxima [F]

$$\int \frac{1}{\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{1}{\sqrt{b\sec(dx+c)+a}}\,dx$$

input
```
integrate(1/(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(b*sec(d*x + c) + a), x)
```

3.333.8 Giac [F]

$$\int \frac{1}{\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{1}{\sqrt{b\sec(dx+c)+a}}\,dx$$

input
```
integrate(1/(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(b*sec(d*x + c) + a), x)
```

3.333.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{1}{\sqrt{a+\frac{b}{\cos(c+dx)}}}\,dx$$

input
```
int(1/(a + b/cos(c + d*x))^(1/2),x)
```

output
```
int(1/(a + b/cos(c + d*x))^(1/2), x)
```

3.334 $\int \frac{\cot^2(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$

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3.334.1 Optimal result

Integrand size = 23, antiderivative size = 361

$$\begin{aligned}
&\int \frac{\cot^2(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx \\
&= \frac{\cot(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)\middle|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{\sqrt{a+b}d} \\
&\quad - \frac{\cot(c+dx)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{\sqrt{a+b}d} \\
&\quad + \frac{2\sqrt{a+b}\cot(c+dx)\,\text{EllipticPi}\left(\frac{a+b}{a},\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{ad} \\
&\quad - \frac{\cot(c+dx)}{d\sqrt{a+b\sec(c+dx)}} + \frac{b^2\tan(c+dx)}{(a^2-b^2)\,d\sqrt{a+b\sec(c+dx)}}
\end{aligned}$$

output

```
cot(d*x+c)*EllipticE((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/2
))*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/d/(a+b)^
(1/2)-cot(d*x+c)*EllipticF((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),((a+b)/(a-b)
)^(1/2))*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/d/
(a+b)^(1/2)+2*cot(d*x+c)*EllipticPi((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),(a+
b)/a,((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(
1+sec(d*x+c))/(a-b))^(1/2)/a/d-cot(d*x+c)/d/(a+b*sec(d*x+c))^(1/2)+b^2*tan
(d*x+c)/(a^2-b^2)/d/(a+b*sec(d*x+c))^(1/2)
```

3.334.2 Mathematica [A] (verified)

Time = 13.15 (sec) , antiderivative size = 366, normalized size of antiderivative = 1.01

$$\int \frac{\cot^2(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$$
$$= \frac{\left(-8b(a+b)\sqrt{\frac{\cos(c+dx)}{1+\cos(c+dx)}}\sqrt{\frac{b+a\cos(c+dx)}{(a+b)(1+\cos(c+dx))}}\cot\left(\frac{1}{2}(c+dx)\right)E\left(\arcsin\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)|\frac{a-b}{a+b}\right)-8(2a^2-a$$

input
```
Integrate[Cot[c + d*x]^2/Sqrt[a + b*Sec[c + d*x]],x]
```

output
```
((-8*b*(a + b)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d
*x])/((a + b)*(1 + Cos[c + d*x]))]*Cot[(c + d*x)/2]*EllipticE[ArcSin[Tan[(
c + d*x)/2]], (a - b)/(a + b)] - 8*(2*a^2 - a*b - 3*b^2)*Sqrt[Cos[c + d*x]
/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x])
)]*Cot[(c + d*x)/2]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] +
 4*(a - b)*Csc[c + d*x]^2*(2*Cos[c + d*x]*(b + a*Cos[c + d*x]) + 32*(a + b
)*Cos[(c + d*x)/2]^3*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos
[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticPi[-1, ArcSin[Tan[(c + d*
x)/2]], (a - b)/(a + b)]*Sin[(c + d*x)/2]))*Tan[c + d*x])/(8*(-a^2 + b^2)*
d*Sqrt[a + b*Sec[c + d*x]])
```

3.334.3 Rubi [A] (verified)

Time = 0.76 (sec) , antiderivative size = 361, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3042, 4384, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^2(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$$
$$\downarrow 3042$$
$$\int \frac{1}{\cot\left(c+dx+\frac{\pi}{2}\right)^2\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx$$
$$\downarrow 4384$$

$$\int \left(\frac{\csc^2(c+dx)}{\sqrt{a+b\sec(c+dx)}} - \frac{1}{\sqrt{a+b\sec(c+dx)}} \right) dx$$

$$\downarrow 2009$$

$$\frac{b^2 \tan(c+dx)}{d\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}} - \frac{\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right), \frac{a+b}{a-b}\right)}{d\sqrt{a+b}} + \frac{\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\,E\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{d\sqrt{a+b}} + \frac{2\sqrt{a+b}\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\,\text{EllipticPi}\left(\frac{a+b}{a}, \arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right), \frac{a+b}{a-b}\right)}{ad} - \frac{\cot(c+dx)}{d\sqrt{a+b\sec(c+dx)}}$$

input
```
Int[Cot[c + d*x]^2/Sqrt[a + b*Sec[c + d*x]],x]
```

output
```
(Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a +
 b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d
*x]))/(a - b))])/(Sqrt[a + b]*d) - (Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a +
 b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x])
)/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(Sqrt[a + b]*d) + (2*S
qrt[a + b]*Cot[c + d*x]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Sec[c + d*
x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sq
rt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(a*d) - Cot[c + d*x]/(d*Sqrt[a + b*
Sec[c + d*x]]) + (b^2*Tan[c + d*x])/((a^2 - b^2)*d*Sqrt[a + b*Sec[c + d*x]
])
```

3.334.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4384
```
Int[cot[(c_.) + (d_.)*(x_)]^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_
), x_Symbol] :> Int[ExpandIntegrand[(a + b*Csc[c + d*x])^n, (-1 + Sec[c + d
*x]^2)^(-m/2), x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[a^2 - b^2, 0] &&
ILtQ[m/2, 0] && IntegerQ[n - 1/2] && EqQ[m, -2]
```

3.334.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1373 vs. 2(330) = 660.

Time = 7.10 (sec) , antiderivative size = 1374, normalized size of antiderivative = 3.81

method	result	size
default	Expression too large to display	1374

input
```
int(cot(d*x+c)^2/(a+b*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2/d/(a-b)/(a+b)*((a*(1-cos(d*x+c))^2*csc(d*x+c)^2-b*(1-cos(d*x+c))^2*cs
c(d*x+c)^2-a-b)/((1-cos(d*x+c))^2*csc(d*x+c)^2-1))^(1/2)*((1-cos(d*x+c))^2
*csc(d*x+c)^2-1)*(-4*(-(1-cos(d*x+c))^2*csc(d*x+c)^2+1)^(1/2)*(-(a*(1-cos(
d*x+c))^2*csc(d*x+c)^2-b*(1-cos(d*x+c))^2*csc(d*x+c)^2-a-b)/(a+b))^(1/2)*E
llipticF(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))*a^2*(-cot(d*x+c)+csc(d
*x+c))+2*(-(1-cos(d*x+c))^2*csc(d*x+c)^2+1)^(1/2)*(-(a*(1-cos(d*x+c))^2*cs
c(d*x+c)^2-b*(1-cos(d*x+c))^2*csc(d*x+c)^2-a-b)/(a+b))^(1/2)*EllipticF(cot
(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))*a*b*(-cot(d*x+c)+csc(d*x+c))+6*(-(
1-cos(d*x+c))^2*csc(d*x+c)^2+1)^(1/2)*(-(a*(1-cos(d*x+c))^2*csc(d*x+c)^2-b
*(1-cos(d*x+c))^2*csc(d*x+c)^2-a-b)/(a+b))^(1/2)*EllipticF(cot(d*x+c)-csc(
d*x+c),((a-b)/(a+b))^(1/2))*b^2*(-cot(d*x+c)+csc(d*x+c))-2*(-(1-cos(d*x+c)
)^2*csc(d*x+c)^2+1)^(1/2)*(-(a*(1-cos(d*x+c))^2*csc(d*x+c)^2-b*(1-cos(d*x+
c))^2*csc(d*x+c)^2-a-b)/(a+b))^(1/2)*EllipticE(cot(d*x+c)-csc(d*x+c),((a-b
)/(a+b))^(1/2))*a*b*(-cot(d*x+c)+csc(d*x+c))-2*(-(1-cos(d*x+c))^2*csc(d*x+
c)^2+1)^(1/2)*(-(a*(1-cos(d*x+c))^2*csc(d*x+c)^2-b*(1-cos(d*x+c))^2*csc(d*
x+c)^2-a-b)/(a+b))^(1/2)*EllipticE(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/
2))*b^2*(-cot(d*x+c)+csc(d*x+c))+8*(-(1-cos(d*x+c))^2*csc(d*x+c)^2+1)^(1/2
)*(-(a*(1-cos(d*x+c))^2*csc(d*x+c)^2-b*(1-cos(d*x+c))^2*csc(d*x+c)^2-a-b)/
(a+b))^(1/2)*EllipticPi(cot(d*x+c)-csc(d*x+c),-1,((a-b)/(a+b))^(1/2))*a^2*
(-cot(d*x+c)+csc(d*x+c))-8*(-(1-cos(d*x+c))^2*csc(d*x+c)^2+1)^(1/2)*(-(...
```

3.334.5 Fricas [F]

$$\int \frac{\cot^2(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{\cot\left(dx+c\right)^2}{\sqrt{b\sec\left(dx+c\right)+a}}\,dx$$

input
```
integrate(cot(d*x+c)^2/(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
integral(cot(d*x + c)^2/sqrt(b*sec(d*x + c) + a), x)
```

3.334.6 Sympy [F]

$$\int \frac{\cot^2(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{\cot^2\left(c+dx\right)}{\sqrt{a+b\sec\left(c+dx\right)}}\,dx$$

input
```
integrate(cot(d*x+c)**2/(a+b*sec(d*x+c))**(1/2),x)
```

output
```
Integral(cot(c + d*x)**2/sqrt(a + b*sec(c + d*x)), x)
```

3.334.7 Maxima [F]

$$\int \frac{\cot^2(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{\cot\left(dx+c\right)^2}{\sqrt{b\sec\left(dx+c\right)+a}}\,dx$$

input
```
integrate(cot(d*x+c)^2/(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(cot(d*x + c)^2/sqrt(b*sec(d*x + c) + a), x)
```

3.334.8 Giac [F]

$$\int \frac{\cot^2(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{\cot\left(dx+c\right)^2}{\sqrt{b\sec\left(dx+c\right)+a}}\,dx$$

input
```
integrate(cot(d*x+c)^2/(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(cot(d*x + c)^2/sqrt(b*sec(d*x + c) + a), x)
```

3.334.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cot^2(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{\cot(c+d\,x)^2}{\sqrt{a+\frac{b}{\cos(c+d\,x)}}}\,dx$$

input
```
int(cot(c + d*x)^2/(a + b/cos(c + d*x))^(1/2),x)
```

output
```
int(cot(c + d*x)^2/(a + b/cos(c + d*x))^(1/2), x)
```

3.335 $\int \frac{\tan^5(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx$

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3.335.8 Giac [F] . . . 2464
3.335.9 Mupad [F(-1)] . . . 2464

3.335.1 Optimal result

Integrand size = 23, antiderivative size = 148

$$\int \frac{\tan^5(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = -\frac{2\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a}}\right)}{a^{3/2}d} + \frac{2(a^2-b^2)^2}{ab^4d\sqrt{a+b\sec(c+dx)}} + \frac{2(3a^2-2b^2)\sqrt{a+b\sec(c+dx)}}{b^4d} - \frac{2a(a+b\sec(c+dx))^{3/2}}{b^4d} + \frac{2(a+b\sec(c+dx))^{5/2}}{5b^4d}$$

output
```
-2*arctanh((a+b*sec(d*x+c))^(1/2)/a^(1/2))/a^(3/2)/d-2*a*(a+b*sec(d*x+c))^
(3/2)/b^4/d+2/5*(a+b*sec(d*x+c))^(5/2)/b^4/d+2*(a^2-b^2)^2/a/b^4/d/(a+b*se
c(d*x+c))^(1/2)+2*(3*a^2-2*b^2)*(a+b*sec(d*x+c))^(1/2)/b^4/d
```

3.335.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.56 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.79

$$\int \frac{\tan^5(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = \frac{2\left(5b^4\,\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},1+\frac{b\sec(c+dx)}{a}\right)+a(4a(4a^2-5b^2)+2b(4$$
$$5ab^4d\sqrt{a+b\sec(c+$$

input
```
Integrate[Tan[c + d*x]^5/(a + b*Sec[c + d*x])^(3/2),x]
```

output
```
(2*(5*b^4*Hypergeometric2F1[-1/2, 1, 1/2, 1 + (b*Sec[c + d*x])/a] + a*(4*a
*(4*a^2 - 5*b^2) + 2*b*(4*a^2 - 5*b^2)*Sec[c + d*x] - 2*a*b^2*Sec[c + d*x]
^2 + b^3*Sec[c + d*x]^3)))/(5*a*b^4*d*Sqrt[a + b*Sec[c + d*x]])
```

3.335.3 Rubi [A] (warning: unable to verify)

Time = 0.37 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.80, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {3042, 25, 4373, 517, 25, 1584, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^5(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx$$

$$\downarrow 3042$$

$$\int -\frac{\cot\left(c+dx+\frac{\pi}{2}\right)^5}{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx$$

$$\downarrow 25$$

$$-\int \frac{\cot\left(\frac{1}{2}(2c+\pi)+dx\right)^5}{\left(a+b\csc\left(\frac{1}{2}(2c+\pi)+dx\right)\right)^{3/2}}dx$$

$$\downarrow 4373$$

$$\frac{\int \frac{\cos(c+dx)\left(b^2-b^2\sec^2(c+dx)\right)^2}{b(a+b\sec(c+dx))^{3/2}}d(b\sec(c+dx))}{b^4 d}$$

$$\downarrow 517$$

$$\frac{2\int -\frac{\cos^2(c+dx)\left(b^4\sec^4(c+dx)-2ab^2\sec^2(c+dx)+a^2-b^2\right)^2}{b^2(a-b^2\sec^2(c+dx))}d\sqrt{a+b\sec(c+dx)}}{b^4 d}$$

$$\downarrow 25$$

$$-\frac{2\int \frac{\cos^2(c+dx)\left(b^4\sec^4(c+dx)-2ab^2\sec^2(c+dx)+a^2-b^2\right)^2}{b^2(a-b^2\sec^2(c+dx))}d\sqrt{a+b\sec(c+dx)}}{b^4 d}$$

$$\downarrow 1584$$

$$-\frac{2\int\left(-\sec^4(c+dx)b^4+\frac{b^4}{a(a-b^2\sec^2(c+dx))}+3a\sec^2(c+dx)b^2-3a^2\left(1-\frac{2b^2}{3a^2}\right)+\frac{\left(a^2-b^2\right)^2\cos^2(c+dx)}{ab^2}\right)d\sqrt{a+b\sec}}{b^4d}$$

$$\downarrow 2009$$

$$\frac{2\left(-\frac{b^4\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a}}\right)}{a^{3/2}}+\frac{\left(a^2-b^2\right)^2\cos(c+dx)}{ab}+\left(3a^2-2b^2\right)\sqrt{a+b\sec(c+dx)}-ab^3\sec^3(c+dx)+\frac{1}{5}b^5\sec^5(\right.}{b^4d}$$

input
```
Int[Tan[c + d*x]^5/(a + b*Sec[c + d*x])^(3/2),x]
```

output
```
(2*(-((b^4*ArcTanh[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a]])/a^(3/2)) + ((a^2 - b
^2)^2*Cos[c + d*x])/(a*b) - a*b^3*Sec[c + d*x]^3 + (b^5*Sec[c + d*x]^5)/5
+ (3*a^2 - 2*b^2)*Sqrt[a + b*Sec[c + d*x]]))/(b^4*d)
```

3.335.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 517
```
Int[((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.),
 x_Symbol] :> Simp[2*(e^m/d^(m + 2*p + 1))   Subst[Int[x^(2*n + 1)*(-c + x^
2)^m*(b*c^2 + a*d^2 - 2*b*c*x^2 + b*x^4)^p, x], x, Sqrt[c + d*x]], x] /; Fr
eeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && ILtQ[m, 0] && IntegerQ[n + 1/2]
```

rule 1584
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4373
```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Simp[-(-1)^((m - 1)/2)/(d*b^(m - 1))   Subst[Int[(b^2 - x^
2)^((m - 1)/2)*((a + x)^n/x), x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
 d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

3.335.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1233 vs. 2(132) = 264.

Time = 15.60 (sec) , antiderivative size = 1234, normalized size of antiderivative = 8.34

method	result	size
default	Expression too large to display	1234

input
```
int(tan(d*x+c)^5/(a+b*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/5/d/a^2/b^4*(a+b*sec(d*x+c))^(1/2)/((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*
x+c)+1)^2)^(1/2)/(b+a*cos(d*x+c))^2/(cos(d*x+c)+1)*(5*cos(d*x+c)^3*ln(4*co
s(d*x+c)*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*a^(1/2)+4*a*
cos(d*x+c)+4*a^(1/2)*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)+
2*b)*a^(5/2)*b^4+10*cos(d*x+c)^2*ln(4*cos(d*x+c)*((b+a*cos(d*x+c))*cos(d*x
+c)/(cos(d*x+c)+1)^2)^(1/2)*a^(1/2)+4*a*cos(d*x+c)+4*a^(1/2)*((b+a*cos(d*x
+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)+2*b)*a^(3/2)*b^5-32*((b+a*cos(d*x+
c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*a^6*cos(d*x+c)^3+40*cos(d*x+c)^3*a^
4*b^2*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)-10*cos(d*x+c)^3
*a^2*b^4*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)+5*cos(d*x+c)
*ln(4*cos(d*x+c)*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*a^(1
/2)+4*a*cos(d*x+c)+4*a^(1/2)*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2
)^(1/2)+2*b)*a^(1/2)*b^6-32*cos(d*x+c)^2*((b+a*cos(d*x+c))*cos(d*x+c)/(cos
(d*x+c)+1)^2)^(1/2)*a^6-48*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^
(1/2)*a^5*b*cos(d*x+c)^2+40*cos(d*x+c)^2*((b+a*cos(d*x+c))*cos(d*x+c)/(cos
(d*x+c)+1)^2)^(1/2)*a^4*b^2+60*cos(d*x+c)^2*a^3*b^3*((b+a*cos(d*x+c))*cos(
d*x+c)/(cos(d*x+c)+1)^2)^(1/2)-10*cos(d*x+c)^2*a^2*b^4*((b+a*cos(d*x+c))*c
os(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)-10*cos(d*x+c)^2*a*b^5*((b+a*cos(d*x+c))*
cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)-48*cos(d*x+c)*((b+a*cos(d*x+c))*cos(d*x
+c)/(cos(d*x+c)+1)^2)^(1/2)*a^5*b-12*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(...
```

3.335.5 Fricas [A] (verification not implemented)

Time = 0.50 (sec) , antiderivative size = 467, normalized size of antiderivative = 3.16

$$\int \frac{\tan^5(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = \left[\frac{5\left(ab^4\cos(dx+c)^3+b^5\cos(dx+c)^2\right)\sqrt{a}\log\left(-8\,a^2\cos(dx+c)^2-8\,ab\cos\right.}{}\right.$$

input
```
integrate(tan(d*x+c)^5/(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
[1/10*(5*(a*b^4*cos(d*x + c)^3 + b^5*cos(d*x + c)^2)*sqrt(a)*log(-8*a^2*co
s(d*x + c)^2 - 8*a*b*cos(d*x + c) - b^2 + 4*(2*a*cos(d*x + c)^2 + b*cos(d*
x + c))*sqrt(a)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c))) - 4*(2*a^3*b^2*co
s(d*x + c) - a^2*b^3 - (16*a^5 - 20*a^3*b^2 + 5*a*b^4)*cos(d*x + c)^3 - 2*
(4*a^4*b - 5*a^2*b^3)*cos(d*x + c)^2)*sqrt((a*cos(d*x + c) + b)/cos(d*x +
c)))/(a^3*b^4*d*cos(d*x + c)^3 + a^2*b^5*d*cos(d*x + c)^2), 1/5*(5*(a*b^4*
cos(d*x + c)^3 + b^5*cos(d*x + c)^2)*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*co
s(d*x + c) + b)/cos(d*x + c))*cos(d*x + c)/(2*a*cos(d*x + c) + b)) - 2*(2*
a^3*b^2*cos(d*x + c) - a^2*b^3 - (16*a^5 - 20*a^3*b^2 + 5*a*b^4)*cos(d*x +
 c)^3 - 2*(4*a^4*b - 5*a^2*b^3)*cos(d*x + c)^2)*sqrt((a*cos(d*x + c) + b)/
cos(d*x + c)))/(a^3*b^4*d*cos(d*x + c)^3 + a^2*b^5*d*cos(d*x + c)^2)]
```

3.335.6 Sympy [F]

$$\int \frac{\tan^5(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{\tan^5(c+dx)}{(a+b\sec(c+dx))^{\frac{3}{2}}}\,dx$$

input
```
integrate(tan(d*x+c)**5/(a+b*sec(d*x+c))**(3/2),x)
```

output
```
Integral(tan(c + d*x)**5/(a + b*sec(c + d*x))**(3/2), x)
```

3.335.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.31

$$\int \frac{\tan^5(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = \frac{\frac{5\log\left(\frac{\sqrt{a+\frac{b}{\cos(dx+c)}}-\sqrt{a}}{\sqrt{a+\frac{b}{\cos(dx+c)}}+\sqrt{a}}\right)}{a^{\frac{3}{2}}} + \frac{10}{\sqrt{a+\frac{b}{\cos(dx+c)}}a} + \frac{2\left(a+\frac{b}{\cos(dx+c)}\right)^{\frac{5}{2}}}{b^4} - \frac{10\left(a+\frac{b}{\cos(dx+c)}\right)^{\frac{3}{2}}a}{b^4} + \frac{30}{}}{5\,d}$$

input
```
integrate(tan(d*x+c)^5/(a+b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
1/5*(5*log((sqrt(a + b/cos(d*x + c)) - sqrt(a))/(sqrt(a + b/cos(d*x + c))
+ sqrt(a)))/a^(3/2) + 10/(sqrt(a + b/cos(d*x + c))*a) + 2*(a + b/cos(d*x +
 c))^(5/2)/b^4 - 10*(a + b/cos(d*x + c))^(3/2)*a/b^4 + 30*sqrt(a + b/cos(d
*x + c))*a^2/b^4 + 10*a^3/(sqrt(a + b/cos(d*x + c))*b^4) - 20*sqrt(a + b/c
os(d*x + c))/b^2 - 20*a/(sqrt(a + b/cos(d*x + c))*b^2))/d
```

3.335.8 Giac [F]

$$\int \frac{\tan^5(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{\tan(dx+c)^5}{(b\sec(dx+c)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(tan(d*x+c)^5/(a+b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
integrate(tan(d*x + c)^5/(b*sec(d*x + c) + a)^(3/2), x)
```

3.335.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\tan^5(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{\tan(c+d\,x)^5}{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

input
```
int(tan(c + d*x)^5/(a + b/cos(c + d*x))^(3/2),x)
```

output
```
int(tan(c + d*x)^5/(a + b/cos(c + d*x))^(3/2), x)
```

3.336 $\int \frac{\tan^3(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx$

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3.336.1 Optimal result

Integrand size = 23, antiderivative size = 88

$$\int \frac{\tan^3(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = \frac{2\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a}}\right)}{a^{3/2}d} + \frac{2(a^2-b^2)}{ab^2d\sqrt{a+b\sec(c+dx)}} + \frac{2\sqrt{a+b\sec(c+dx)}}{b^2d}$$

output
```
2*arctanh((a+b*sec(d*x+c))^(1/2)/a^(1/2))/a^(3/2)/d+2*(a^2-b^2)/a/b^2/d/(a
+b*sec(d*x+c))^(1/2)+2*(a+b*sec(d*x+c))^(1/2)/b^2/d
```

3.336.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.26 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.75

$$\int \frac{\tan^3(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = \frac{2\left(-b^2\,\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},1+\frac{b\sec(c+dx)}{a}\right)+a(2a+b\sec(c+dx))\right)}{ab^2d\sqrt{a+b\sec(c+dx)}}$$

input
```
Integrate[Tan[c + d*x]^3/(a + b*Sec[c + d*x])^(3/2),x]
```

output
```
(2*(-(b^2*Hypergeometric2F1[-1/2, 1, 1/2, 1 + (b*Sec[c + d*x])/a]) + a*(2*
a + b*Sec[c + d*x])))/(a*b^2*d*Sqrt[a + b*Sec[c + d*x]])
```

3.336.3 Rubi [A] (warning: unable to verify)

Time = 0.33 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.90, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3042, 25, 4373, 517, 1584, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^3(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx$$

$$\downarrow 3042$$

$$\int -\frac{\cot\left(c+dx+\frac{\pi}{2}\right)^3}{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx$$

$$\downarrow 25$$

$$-\int \frac{\cot\left(\frac{1}{2}(2c+\pi)+dx\right)^3}{\left(a+b\csc\left(\frac{1}{2}(2c+\pi)+dx\right)\right)^{3/2}}dx$$

$$\downarrow 4373$$

$$-\frac{\int \frac{\cos(c+dx)\left(b^2-b^2\sec^2(c+dx)\right)}{b(a+b\sec(c+dx))^{3/2}}d(b\sec(c+dx))}{b^2d}$$

$$\downarrow 517$$

$$-\frac{2\int \frac{\cos^2(c+dx)\left(b^4\sec^4(c+dx)-2ab^2\sec^2(c+dx)+a^2-b^2\right)}{b^2\left(a-b^2\sec^2(c+dx)\right)}d\sqrt{a+b\sec(c+dx)}}{b^2d}$$

$$\downarrow 1584$$

$$-\frac{2\int \left(-\frac{b^2}{a\left(a-b^2\sec^2(c+dx)\right)}-1+\frac{\left(a^2-b^2\right)\cos^2(c+dx)}{ab^2}\right)d\sqrt{a+b\sec(c+dx)}}{b^2d}$$

$$\downarrow 2009$$

$$-\frac{2\left(-\frac{b^2\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a}}\right)}{a^{3/2}}-\frac{\left(a^2-b^2\right)\cos(c+dx)}{ab}-\sqrt{a+b\sec(c+dx)}\right)}{b^2d}$$

input
```
Int[Tan[c + d*x]^3/(a + b*Sec[c + d*x])^(3/2),x]
```

output
```
(-2*(-((b^2*ArcTanh[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a]])/a^(3/2)) - ((a^2 -
b^2)*Cos[c + d*x])/(a*b) - Sqrt[a + b*Sec[c + d*x]]))/(b^2*d)
```

3.336.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 517
```
Int[((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.),
 x_Symbol] :> Simp[2*(e^m/d^(m + 2*p + 1))   Subst[Int[x^(2*n + 1)*(-c + x^
2)^m*(b*c^2 + a*d^2 - 2*b*c*x^2 + b*x^4)^p, x], x, Sqrt[c + d*x]], x] /; Fr
eeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && ILtQ[m, 0] && IntegerQ[n + 1/2]
```

rule 1584
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4373
```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Simp[-(-1)^((m - 1)/2)/(d*b^(m - 1))   Subst[Int[(b^2 - x^
2)^((m - 1)/2)*((a + x)^n/x), x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
 d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

3.336.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 730 vs. $2(78) = 156$.

Time = 9.56 (sec) , antiderivative size = 731, normalized size of antiderivative = 8.31

method	result
default	$\frac{\left(\cos(dx+c)^3 a^{\frac{5}{2}} \ln\left(4\cos(dx+c)\sqrt{\frac{(b+a\cos(dx+c))\cos(dx+c)}{(\cos(dx+c)+1)^2}}\sqrt{a}+4a\cos(dx+c)+4\sqrt{a}\sqrt{\frac{(b+a\cos(dx+c))\cos(dx+c)}{(\cos(dx+c)+1)^2}}+2b\right)b^2+2\cos(dx+c$

input
```
int(tan(d*x+c)^3/(a+b*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/d/a^2/b^2*(cos(d*x+c)^3*a^(5/2)*ln(4*cos(d*x+c)*((b+a*cos(d*x+c))*cos(d*
x+c)/(cos(d*x+c)+1)^2)^(1/2)*a^(1/2)+4*a*cos(d*x+c)+4*a^(1/2)*((b+a*cos(d*
x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)+2*b)*b^2+2*cos(d*x+c)^2*a^(3/2)*l
n(4*cos(d*x+c)*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*a^(1/2
)+4*a*cos(d*x+c)+4*a^(1/2)*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^
(1/2)+2*b)*b^3+2*sin(d*x+c)^2*cos(d*x+c)*((b+a*cos(d*x+c))*cos(d*x+c)/(cos
(d*x+c)+1)^2)^(1/2)*a^2*b^2+4*cos(d*x+c)^3*((b+a*cos(d*x+c))*cos(d*x+c)/(c
os(d*x+c)+1)^2)^(1/2)*a^4+2*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)
^(1/2)*a^2*b^2*sin(d*x+c)^2+4*cos(d*x+c)^2*((b+a*cos(d*x+c))*cos(d*x+c)/(c
os(d*x+c)+1)^2)^(1/2)*a^4+6*cos(d*x+c)^2*((b+a*cos(d*x+c))*cos(d*x+c)/(cos
(d*x+c)+1)^2)^(1/2)*a^3*b-2*cos(d*x+c)^2*((b+a*cos(d*x+c))*cos(d*x+c)/(cos
(d*x+c)+1)^2)^(1/2)*a*b^3+cos(d*x+c)*ln(4*cos(d*x+c)*((b+a*cos(d*x+c))*cos
(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*a^(1/2)+4*a*cos(d*x+c)+4*a^(1/2)*((b+a*cos
(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)+2*b)*a^(1/2)*b^4+6*cos(d*x+c)*
((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*a^3*b-2*cos(d*x+c)*((
b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*a*b^3)*(a+b*sec(d*x+c))
^(1/2)/(b+a*cos(d*x+c))^2/((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(
1/2)/(cos(d*x+c)+1)
```

3.336.5 Fricas [A] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 317, normalized size of antiderivative = 3.60

$$\int \frac{\tan^3(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = \left[\frac{(ab^2\cos(dx+c)+b^3)\sqrt{a}\log\left(-8\,a^2\cos(dx+c)^2-8\,ab\cos(dx+c)-b^2-\right.\right.$$

$$\left.\left.-\frac{(ab^2\cos(dx+c)+b^3)\sqrt{-a}\arctan\left(\frac{2\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+b}{\cos(dx+c)}}\cos(dx+c)}{2\,a\cos(dx+c)+b}\right)-2\left(a^2b+(2\,a^3-ab^2)\cos(dx+c)\right)\sqrt{\frac{a\cos}{co}}}{a^3b^2d\cos(dx+c)+a^2b^3d}\right.\right.$$

input
```
integrate(tan(d*x+c)^3/(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
[1/2*((a*b^2*cos(d*x + c) + b^3)*sqrt(a)*log(-8*a^2*cos(d*x + c)^2 - 8*a*b
*cos(d*x + c) - b^2 - 4*(2*a*cos(d*x + c)^2 + b*cos(d*x + c))*sqrt(a)*sqrt
((a*cos(d*x + c) + b)/cos(d*x + c))) + 4*(a^2*b + (2*a^3 - a*b^2)*cos(d*x
+ c))*sqrt((a*cos(d*x + c) + b)/cos(d*x + c)))/(a^3*b^2*d*cos(d*x + c) + a
^2*b^3*d), -((a*b^2*cos(d*x + c) + b^3)*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a
*cos(d*x + c) + b)/cos(d*x + c))*cos(d*x + c)/(2*a*cos(d*x + c) + b)) - 2*
(a^2*b + (2*a^3 - a*b^2)*cos(d*x + c))*sqrt((a*cos(d*x + c) + b)/cos(d*x +
 c)))/(a^3*b^2*d*cos(d*x + c) + a^2*b^3*d)]
```

3.336.6 Sympy [F]

$$\int \frac{\tan^3(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{\tan^3(c+dx)}{(a+b\sec(c+dx))^{\frac{3}{2}}}\,dx$$

input
```
integrate(tan(d*x+c)**3/(a+b*sec(d*x+c))**(3/2),x)
```

output
```
Integral(tan(c + d*x)**3/(a + b*sec(c + d*x))**(3/2), x)
```

3.336.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.25

$$\int \frac{\tan^3(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = -\frac{\frac{\log\left(\frac{\sqrt{a+\frac{b}{\cos(dx+c)}}-\sqrt{a}}{\sqrt{a+\frac{b}{\cos(dx+c)}}+\sqrt{a}}\right)}{a^{\frac{3}{2}}}+\frac{2}{\sqrt{a+\frac{b}{\cos(dx+c)}}a}-\frac{2\sqrt{a+\frac{b}{\cos(dx+c)}}}{b^2}-\frac{2a}{\sqrt{a+\frac{b}{\cos(dx+c)}}b^2}}{d}$$

input
```
integrate(tan(d*x+c)^3/(a+b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
-(log((sqrt(a + b/cos(d*x + c)) - sqrt(a))/(sqrt(a + b/cos(d*x + c)) + sqr
t(a)))/a^(3/2) + 2/(sqrt(a + b/cos(d*x + c))*a) - 2*sqrt(a + b/cos(d*x + c
))/b^2 - 2*a/(sqrt(a + b/cos(d*x + c))*b^2))/d
```

3.336.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 258 vs. 2(78) = 156.

Time = 1.10 (sec) , antiderivative size = 258, normalized size of antiderivative = 2.93

$$\int \frac{\tan^3(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = -\frac{2\left(\frac{\left(2a^3\mathrm{sgn}(\cos(dx+c))-a^2b\mathrm{sgn}(\cos(dx+c))-ab^2\mathrm{sgn}(\cos(dx+c))\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2}{a^2b^2}-\frac{2a^3\mathrm{sgn}(\cos(dx+c))+a^2b\mathrm{sgn}(\cos(dx+c))-ab^2\mathrm{sgn}(\cos(dx+c))}{a^2b^2}}{\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4-2a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a+b}}+\frac{\arctan}{\qquad}\right)}{d}$$

input
```
integrate(tan(d*x+c)^3/(a+b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
-2*(((2*a^3*sgn(cos(d*x + c)) - a^2*b*sgn(cos(d*x + c)) - a*b^2*sgn(cos(d*
x + c)))*tan(1/2*d*x + 1/2*c)^2/(a^2*b^2) - (2*a^3*sgn(cos(d*x + c)) + a^2
*b*sgn(cos(d*x + c)) - a*b^2*sgn(cos(d*x + c)))/(a^2*b^2))/sqrt(a*tan(1/2*
d*x + 1/2*c)^4 - b*tan(1/2*d*x + 1/2*c)^4 - 2*a*tan(1/2*d*x + 1/2*c)^2 + a
 + b) + arctan(-1/2*(sqrt(a - b)*tan(1/2*d*x + 1/2*c)^2 - sqrt(a*tan(1/2*d
*x + 1/2*c)^4 - b*tan(1/2*d*x + 1/2*c)^4 - 2*a*tan(1/2*d*x + 1/2*c)^2 + a
+ b) + sqrt(a - b))/sqrt(-a))/(sqrt(-a)*a*sgn(cos(d*x + c))))/d
```

3.336.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\tan^3(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{\tan(c+d\,x)^3}{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

input
```
int(tan(c + d*x)^3/(a + b/cos(c + d*x))^(3/2),x)
```

output
```
int(tan(c + d*x)^3/(a + b/cos(c + d*x))^(3/2), x)
```

3.337 $\int \frac{\tan(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx$

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3.337.7 Maxima [A] (verification not implemented) . . . 2476
3.337.8 Giac [B] (verification not implemented) . . . 2476
3.337.9 Mupad [B] (verification not implemented) . . . 2477

3.337.1 Optimal result

Integrand size = 21, antiderivative size = 54

$$\int \frac{\tan(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = -\frac{2\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a}}\right)}{a^{3/2}d} + \frac{2}{ad\sqrt{a+b\sec(c+dx)}}$$

output
```
-2*arctanh((a+b*sec(d*x+c))^(1/2)/a^(1/2))/a^(3/2)/d+2/a/d/(a+b*sec(d*x+c)
)^(1/2)
```

3.337.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.10 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.80

$$\int \frac{\tan(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = \frac{2\,\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},1+\frac{b\sec(c+dx)}{a}\right)}{ad\sqrt{a+b\sec(c+dx)}}$$

input
```
Integrate[Tan[c + d*x]/(a + b*Sec[c + d*x])^(3/2),x]
```

output
```
(2*Hypergeometric2F1[-1/2, 1, 1/2, 1 + (b*Sec[c + d*x])/a])/(a*d*Sqrt[a +
b*Sec[c + d*x]])
```

3.337.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.96, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 25, 4373, 61, 73, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx$$

$$\downarrow 3042$$

$$\int -\frac{\cot\left(c+dx+\frac{\pi}{2}\right)}{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx$$

$$\downarrow 25$$

$$-\int \frac{\cot\left(\frac{1}{2}(2c+\pi)+dx\right)}{\left(a+b\csc\left(\frac{1}{2}(2c+\pi)+dx\right)\right)^{3/2}}dx$$

$$\downarrow 4373$$

$$\frac{\int \frac{\cos(c+dx)}{b(a+b\sec(c+dx))^{3/2}}d(b\sec(c+dx))}{d}$$

$$\downarrow 61$$

$$\frac{\frac{\int \frac{\cos(c+dx)}{b\sqrt{a+b\sec(c+dx)}}d(b\sec(c+dx))}{a}+\frac{2}{a\sqrt{a+b\sec(c+dx)}}}{d}$$

$$\downarrow 73$$

$$\frac{\frac{2\int \frac{1}{b^2\sec^2(c+dx)-a}d\sqrt{a+b\sec(c+dx)}}{a}+\frac{2}{a\sqrt{a+b\sec(c+dx)}}}{d}$$

$$\downarrow 220$$

$$\frac{\frac{2}{a\sqrt{a+b\sec(c+dx)}}-\frac{2\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a}}\right)}{a^{3/2}}}{d}$$

input
```
Int[Tan[c + d*x]/(a + b*Sec[c + d*x])^(3/2),x]
```

output
```
((-2*ArcTanh[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a]])/a^(3/2) + 2/(a*Sqrt[a + b*
Sec[c + d*x]]))/d
```

3.337.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4373
```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Simp[-(-1)^((m - 1)/2)/(d*b^(m - 1))   Subst[Int[(b^2 - x^
2)^((m - 1)/2)*((a + x)^n/x), x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
 d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

3.337.4 Maple [A] (verified)

Time = 0.79 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.83

method	result	size
derivativedivides	$\frac{-\frac{2\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec(dx+c)}}{\sqrt{a}}\right)}{a^{\frac{3}{2}}}+\frac{2}{a\sqrt{a+b\sec(dx+c)}}}{d}$	45
default	$\frac{-\frac{2\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec(dx+c)}}{\sqrt{a}}\right)}{a^{\frac{3}{2}}}+\frac{2}{a\sqrt{a+b\sec(dx+c)}}}{d}$	45

input
```
int(tan(d*x+c)/(a+b*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/d*(-2/a^(3/2)*arctanh((a+b*sec(d*x+c))^(1/2)/a^(1/2))+2/a/(a+b*sec(d*x+c
))^(1/2))
```

3.337.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 116 vs. $2(46) = 92$.

Time = 0.42 (sec) , antiderivative size = 260, normalized size of antiderivative = 4.81

$$\int \frac{\tan(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = \left[\frac{4\,a\sqrt{\frac{a\cos(dx+c)+b}{\cos(dx+c)}}\cos(dx+c)+(a\cos(dx+c)+b)\sqrt{a}\log\left(-8\,a^2\cos(dx+\right.}{2\,(a^3d\cos}\right.$$

input
```
integrate(tan(d*x+c)/(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
[1/2*(4*a*sqrt((a*cos(d*x + c) + b)/cos(d*x + c))*cos(d*x + c) + (a*cos(d*
x + c) + b)*sqrt(a)*log(-8*a^2*cos(d*x + c)^2 - 8*a*b*cos(d*x + c) - b^2 +
 4*(2*a*cos(d*x + c)^2 + b*cos(d*x + c))*sqrt(a)*sqrt((a*cos(d*x + c) + b)
/cos(d*x + c))))/(a^3*d*cos(d*x + c) + a^2*b*d), ((a*cos(d*x + c) + b)*sqr
t(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c))*cos(d*x +
c)/(2*a*cos(d*x + c) + b)) + 2*a*sqrt((a*cos(d*x + c) + b)/cos(d*x + c))*c
os(d*x + c))/(a^3*d*cos(d*x + c) + a^2*b*d)]
```

3.337.6 Sympy [F]

$$\int \frac{\tan(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{\tan\left(c+dx\right)}{\left(a+b\sec\left(c+dx\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(tan(d*x+c)/(a+b*sec(d*x+c))**(3/2),x)
```

output
```
Integral(tan(c + d*x)/(a + b*sec(c + d*x))**(3/2), x)
```

3.337.7 Maxima [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.30

$$\int \frac{\tan(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = \frac{\frac{\log\left(\frac{\sqrt{a+\frac{b}{\cos(dx+c)}}-\sqrt{a}}{\sqrt{a+\frac{b}{\cos(dx+c)}}+\sqrt{a}}\right)}{a^{\frac{3}{2}}} + \frac{2}{\sqrt{a+\frac{b}{\cos(dx+c)}}a}}{d}$$

input
```
integrate(tan(d*x+c)/(a+b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
(log((sqrt(a + b/cos(d*x + c)) - sqrt(a))/(sqrt(a + b/cos(d*x + c)) + sqrt
(a)))/a^(3/2) + 2/(sqrt(a + b/cos(d*x + c))*a))/d
```

3.337.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 214 vs. 2(46) = 92.

Time = 0.78 (sec) , antiderivative size = 214, normalized size of antiderivative = 3.96

$$\int \frac{\tan(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = 2\left(\frac{\arctan\left(-\frac{\sqrt{a-b}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-\sqrt{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a+b}+\sqrt{a-}}{2\sqrt{-a}}\right)}{\sqrt{-a}a\operatorname{sgn}(\cos(dx+c))}\right.$$

input
```
integrate(tan(d*x+c)/(a+b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
2*(arctan(-1/2*(sqrt(a - b)*tan(1/2*d*x + 1/2*c)^2 - sqrt(a*tan(1/2*d*x +
1/2*c)^4 - b*tan(1/2*d*x + 1/2*c)^4 - 2*a*tan(1/2*d*x + 1/2*c)^2 + a + b)
+ sqrt(a - b))/sqrt(-a))/(sqrt(-a)*a*sgn(cos(d*x + c))) + 2*b/(((sqrt(a -
b)*tan(1/2*d*x + 1/2*c)^2 - sqrt(a*tan(1/2*d*x + 1/2*c)^4 - b*tan(1/2*d*x
+ 1/2*c)^4 - 2*a*tan(1/2*d*x + 1/2*c)^2 + a + b))*sqrt(a - b) - a - b)*sqr
t(a - b)*a*sgn(cos(d*x + c))))/d
```

3.337.9 Mupad [B] (verification not implemented)

Time = 15.59 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.93

$$\int \frac{\tan(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = \frac{2}{a\,d\,\sqrt{a+\frac{b}{\cos(c+d\,x)}}} - \frac{2\,\mathrm{atanh}\left(\frac{\sqrt{a+\frac{b}{\cos(c+d\,x)}}}{\sqrt{a}}\right)}{a^{3/2}\,d}$$

input
```
int(tan(c + d*x)/(a + b/cos(c + d*x))^(3/2),x)
```

output
```
2/(a*d*(a + b/cos(c + d*x))^(1/2)) - (2*atanh((a + b/cos(c + d*x))^(1/2)/a
^(1/2)))/(a^(3/2)*d)
```

3.338 $\int \frac{\cot(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx$

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3.338.2 Mathematica [C] (verified) 2478
3.338.3 Rubi [A] (warning: unable to verify) 2479
3.338.4 Maple [B] (warning: unable to verify) 2481
3.338.5 Fricas [B] (verification not implemented) 2482
3.338.6 Sympy [F] 2482
3.338.7 Maxima [F] 2483
3.338.8 Giac [F(-2)] 2483
3.338.9 Mupad [F(-1)] 2483

3.338.1 Optimal result

Integrand size = 21, antiderivative size = 142

$$\int \frac{\cot(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = \frac{2\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a}}\right)}{a^{3/2}d} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a-b}}\right)}{(a-b)^{3/2}d} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)}{(a+b)^{3/2}d} + \frac{2b^2}{a\left(a^2-b^2\right)d\sqrt{a+b\sec(c+dx)}}$$

output
```
2*arctanh((a+b*sec(d*x+c))^(1/2)/a^(1/2))/a^(3/2)/d-arctanh((a+b*sec(d*x+c
))^(1/2)/(a-b)^(1/2))/(a-b)^(3/2)/d-arctanh((a+b*sec(d*x+c))^(1/2)/(a+b)^(
1/2))/(a+b)^(3/2)/d+2*b^2/a/(a^2-b^2)/d/(a+b*sec(d*x+c))^(1/2)
```

3.338.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.67 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.49

$$\int \frac{\cot(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = -\frac{-\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a-b}}\right)}{\sqrt{a-b}} + \frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)}{\sqrt{a+b}} - \frac{a\,\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},\frac{a+b\sec(c+dx)}{a-b}\right)}{(a-b)\sqrt{a+b\sec(c+dx)}} + \frac{a\,\text{Hypergeometric2F1}(}{(a+b)\sqrt{a}}}{bd}$$

input
```
Integrate[Cot[c + d*x]/(a + b*Sec[c + d*x])^(3/2),x]
```

output
```
-((-(ArcTanh[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a - b]]/Sqrt[a - b]) + ArcTanh[
Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]]/Sqrt[a + b] - (a*Hypergeometric2F1[-
1/2, 1, 1/2, (a + b*Sec[c + d*x])/(a - b)])/((a - b)*Sqrt[a + b*Sec[c + d*
x]]) + (a*Hypergeometric2F1[-1/2, 1, 1/2, (a + b*Sec[c + d*x])/(a + b)])/(
(a + b)*Sqrt[a + b*Sec[c + d*x]]) + (2*b*Hypergeometric2F1[-1/2, 1, 1/2, 1
 + (b*Sec[c + d*x])/a])/(a*Sqrt[a + b*Sec[c + d*x]]))/(b*d))
```

3.338.3 Rubi [A] (warning: unable to verify)

Time = 0.42 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.01, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 25, 4373, 561, 1610, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx$$

$$\downarrow 3042$$

$$\int -\frac{1}{\cot\left(c+dx+\frac{\pi}{2}\right)\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx$$

$$\downarrow 25$$

$$-\int \frac{1}{\cot\left(\frac{1}{2}(2c+\pi)+dx\right)\left(a+b\csc\left(\frac{1}{2}(2c+\pi)+dx\right)\right)^{3/2}}dx$$

$$\downarrow 4373$$

$$-\frac{b^2\int \frac{\cos(c+dx)}{b(a+b\sec(c+dx))^{3/2}(b^2-b^2\sec^2(c+dx))}d(b\sec(c+dx))}{d}$$

$$\downarrow 561$$

$$-\frac{2b^2\int \frac{\cos^2(c+dx)}{b^2(a-b^2\sec^2(c+dx))(b^4\sec^4(c+dx)-2ab^2\sec^2(c+dx)+a^2-b^2)}d\sqrt{a+b\sec(c+dx)}}{d}$$

$$\downarrow 1610$$

$$-\frac{2b^2\int \left(\frac{\cos^2(c+dx)}{ab^2(a^2-b^2)}-\frac{1}{ab^2(a-b^2\sec^2(c+dx))}+\frac{1}{2(a-b)b^2(-b^2\sec^2(c+dx)+a-b)}+\frac{1}{2b^2(a+b)(-b^2\sec^2(c+dx)+a+b)}\right)d\sqrt{a+b\sec(c}}{d}$$

$$\downarrow 2009$$

$$-\frac{2b^2\left(-\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a}}\right)}{a^{3/2}b^2}-\frac{\cos(c+dx)}{ab(a^2-b^2)}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a-b}}\right)}{2b^2(a-b)^{3/2}}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)}{2b^2(a+b)^{3/2}}\right)}{d}$$

input
```
Int[Cot[c + d*x]/(a + b*Sec[c + d*x])^(3/2),x]
```

output
```
(-2*b^2*(-(ArcTanh[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a]]/(a^(3/2)*b^2)) + ArcT
anh[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a - b]]/(2*(a - b)^(3/2)*b^2) + ArcTanh[
Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]]/(2*b^2*(a + b)^(3/2)) - Cos[c + d*x]
/(a*b*(a^2 - b^2))))/d
```

3.338.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 561
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> With[{k = Denominator[n]}, Simp[k/d   Subst[Int[x^(k*(n + 1) - 1)*(-c
/d + x^k/d)^m*Simp[(b*c^2 + a*d^2)/d^2 - 2*b*c*(x^k/d^2) + b*(x^(2*k)/d^2),
 x]^p, x], x, (c + d*x)^(1/k)], x]] /; FreeQ[{a, b, c, d, m, p}, x] && Frac
tionQ[n] && IntegerQ[p] && IntegerQ[m]
```

rule 1610
```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.))/((a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*((d + e*x^2)^q/(a
+ b*x^2 + c*x^4)), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4
*a*c, 0] && IntegerQ[q] && IntegerQ[m]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4373
```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Simp[-(-1)^((m - 1)/2)/(d*b^(m - 1))   Subst[Int[(b^2 - x^
2)^((m - 1)/2)*((a + x)^n/x), x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
 d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

3.338.4 Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 2691 vs. 2(122) = 244.

Time = 2.22 (sec) , antiderivative size = 2692, normalized size of antiderivative = 18.96

method	result	size
default	Expression too large to display	2692

input
```
int(cot(d*x+c)/(a+b*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2/d/(a-b)^(5/2)/a^2/(a+b)^2*(-2*(a-b)^(3/2)*ln(4*cos(d*x+c)*((b+a*cos(d
*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*a^(1/2)+4*a*cos(d*x+c)+4*a^(1/2)
*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)+2*b)*cos(d*x+c)*a^(9
/2)-2*(a-b)^(3/2)*a^(7/2)*ln(4*cos(d*x+c)*((b+a*cos(d*x+c))*cos(d*x+c)/(co
s(d*x+c)+1)^2)^(1/2)*a^(1/2)+4*a*cos(d*x+c)+4*a^(1/2)*((b+a*cos(d*x+c))*co
s(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)+2*b)*b*cos(d*x+c)-2*(a-b)^(3/2)*a^(7/2)*l
n(4*cos(d*x+c)*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*a^(1/2
)+4*a*cos(d*x+c)+4*a^(1/2)*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^
(1/2)+2*b)*b+2*(a-b)^(3/2)*ln(4*cos(d*x+c)*((b+a*cos(d*x+c))*cos(d*x+c)/(c
os(d*x+c)+1)^2)^(1/2)*a^(1/2)+4*a*cos(d*x+c)+4*a^(1/2)*((b+a*cos(d*x+c))*c
os(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)+2*b)*cos(d*x+c)*a^(5/2)*b^2-2*(a-b)^(3/2
)*ln(4*cos(d*x+c)*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*a^(
1/2)+4*a*cos(d*x+c)+4*a^(1/2)*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^
2)^(1/2)+2*b)*a^(5/2)*b^2+2*(a-b)^(3/2)*a^(3/2)*ln(4*cos(d*x+c)*((b+a*cos(
d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*a^(1/2)+4*a*cos(d*x+c)+4*a^(1/2
)*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)+2*b)*b^3*cos(d*x+c)
+(a+b)^(1/2)*(a-b)^(3/2)*ln(-2*(2*cos(d*x+c)*((b+a*cos(d*x+c))*cos(d*x+c)/
(cos(d*x+c)+1)^2)^(1/2)*(a+b)^(1/2)+2*a*cos(d*x+c)+cos(d*x+c)*b+2*(a+b)^(1
/2)*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)+b)/(cos(d*x+c)-1)
)*cos(d*x+c)*a^4-(a+b)^(1/2)*(a-b)^(3/2)*ln(-2*(2*cos(d*x+c)*((b+a*cos(...
```

3.338.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 414 vs. $2(122) = 244$.

Time = 24.54 (sec) , antiderivative size = 3924, normalized size of antiderivative = 27.63

$$\int \frac{\cot(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(cot(d*x+c)/(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
[1/4*(8*(a^3*b^2 - a*b^4)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c))*cos(d*x
+ c) + 2*(a^4*b - 2*a^2*b^3 + b^5 + (a^5 - 2*a^3*b^2 + a*b^4)*cos(d*x + c)
)*sqrt(a)*log(-8*a^2*cos(d*x + c)^2 - 8*a*b*cos(d*x + c) - b^2 - 4*(2*a*co
s(d*x + c)^2 + b*cos(d*x + c))*sqrt(a)*sqrt((a*cos(d*x + c) + b)/cos(d*x +
 c))) - (a^4*b + 2*a^3*b^2 + a^2*b^3 + (a^5 + 2*a^4*b + a^3*b^2)*cos(d*x +
 c))*sqrt(a - b)*log(-((8*a^2 - 8*a*b + b^2)*cos(d*x + c)^2 + b^2 + 4*((2*
a - b)*cos(d*x + c)^2 + b*cos(d*x + c))*sqrt(a - b)*sqrt((a*cos(d*x + c) +
 b)/cos(d*x + c)) + 2*(4*a*b - 3*b^2)*cos(d*x + c))/(cos(d*x + c)^2 + 2*co
s(d*x + c) + 1)) + (a^4*b - 2*a^3*b^2 + a^2*b^3 + (a^5 - 2*a^4*b + a^3*b^2
)*cos(d*x + c))*sqrt(a + b)*log(-((8*a^2 + 8*a*b + b^2)*cos(d*x + c)^2 + b
^2 - 4*((2*a + b)*cos(d*x + c)^2 + b*cos(d*x + c))*sqrt(a + b)*sqrt((a*cos
(d*x + c) + b)/cos(d*x + c)) + 2*(4*a*b + 3*b^2)*cos(d*x + c))/(cos(d*x +
c)^2 - 2*cos(d*x + c) + 1)))/((a^7 - 2*a^5*b^2 + a^3*b^4)*d*cos(d*x + c) +
 (a^6*b - 2*a^4*b^3 + a^2*b^5)*d), -1/4*(4*(a^4*b - 2*a^2*b^3 + b^5 + (a^5
 - 2*a^3*b^2 + a*b^4)*cos(d*x + c))*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos
(d*x + c) + b)/cos(d*x + c))*cos(d*x + c)/(2*a*cos(d*x + c) + b)) - 8*(a^3
*b^2 - a*b^4)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c))*cos(d*x + c) + (a^4*
b + 2*a^3*b^2 + a^2*b^3 + (a^5 + 2*a^4*b + a^3*b^2)*cos(d*x + c))*sqrt(a -
 b)*log(-((8*a^2 - 8*a*b + b^2)*cos(d*x + c)^2 + b^2 + 4*((2*a - b)*cos(d*
x + c)^2 + b*cos(d*x + c))*sqrt(a - b)*sqrt((a*cos(d*x + c) + b)/cos(d*...
```

3.338.6 Sympy [F]

$$\int \frac{\cot(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{\cot(c+dx)}{(a+b\sec(c+dx))^{\frac{3}{2}}}\,dx$$

input
```
integrate(cot(d*x+c)/(a+b*sec(d*x+c))**(3/2),x)
```

output
```
Integral(cot(c + d*x)/(a + b*sec(c + d*x))**(3/2), x)
```

3.338.7 Maxima [F]

$$\int \frac{\cot(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{\cot\left(dx+c\right)}{\left(b\sec\left(dx+c\right)+a\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(cot(d*x+c)/(a+b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate(cot(d*x + c)/(b*sec(d*x + c) + a)^(3/2), x)
```

3.338.8 Giac [F(-2)]

Exception generated.

$$\int \frac{\cot(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(cot(d*x+c)/(a+b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

3.338.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cot(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{\cot(c+d\,x)}{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

input
```
int(cot(c + d*x)/(a + b/cos(c + d*x))^(3/2),x)
```

output
```
int(cot(c + d*x)/(a + b/cos(c + d*x))^(3/2), x)
```

3.339 $\int \frac{\cot^3(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx$

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3.339.2 Mathematica [C] (verified) . . . 2485
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3.339.9 Mupad [F(-1)] . . . 2489

3.339.1 Optimal result

Integrand size = 23, antiderivative size = 236

$$\int \frac{\cot^3(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = -\frac{2\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a}}\right)}{a^{3/2}d} + \frac{(4a-7b)\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a-b}}\right)}{4(a-b)^{5/2}d} + \frac{(4a+7b)\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)}{4(a+b)^{5/2}d} + \frac{2b^4}{a\left(a^2-b^2\right)^2 d\sqrt{a+b\sec(c+dx)}} + \frac{\sqrt{a+b\sec(c+dx)}}{4(a+b)^2 d(1-\sec(c+dx))} + \frac{\sqrt{a+b\sec(c+dx)}}{4(a-b)^2 d(1+\sec(c+dx))}$$

output

```
-2*arctanh((a+b*sec(d*x+c))^(1/2)/a^(1/2))/a^(3/2)/d+1/4*(4*a-7*b)*arctanh
((a+b*sec(d*x+c))^(1/2)/(a-b)^(1/2))/(a-b)^(5/2)/d+1/4*(4*a+7*b)*arctanh((
a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2))/(a+b)^(5/2)/d+2*b^4/a/(a^2-b^2)^2/d/(a+
b*sec(d*x+c))^(1/2)+1/4*(a+b*sec(d*x+c))^(1/2)/(a+b)^2/d/(1-sec(d*x+c))+1/
4*(a+b*sec(d*x+c))^(1/2)/(a-b)^2/d/(1+sec(d*x+c))
```

3.339.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 1.09 (sec) , antiderivative size = 315, normalized size of antiderivative = 1.33

$$\int \frac{\cot^3(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = \frac{-\frac{2\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a-b}}\right)}{\sqrt{a-b}} + \frac{2\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)}{\sqrt{a+b}} - \frac{2a\,\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2}\right.}{(a-b)\sqrt{a+b\sec(c+}}$$

input
```
Integrate[Cot[c + d*x]^3/(a + b*Sec[c + d*x])^(3/2),x]
```

output
```
((-2*ArcTanh[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a - b]])/Sqrt[a - b] + (2*ArcTa
nh[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]])/Sqrt[a + b] - (2*a*Hypergeometri
c2F1[-1/2, 1, 1/2, (a + b*Sec[c + d*x])/(a - b)])/((a - b)*Sqrt[a + b*Sec[
c + d*x]]) + (2*a*Hypergeometric2F1[-1/2, 1, 1/2, (a + b*Sec[c + d*x])/(a
 + b)])/((a + b)*Sqrt[a + b*Sec[c + d*x]]) + (4*b*Hypergeometric2F1[-1/2, 1
, 1/2, 1 + (b*Sec[c + d*x])/a])/(a*Sqrt[a + b*Sec[c + d*x]]) + (b^2*Hyperg
eometric2F1[-1/2, 2, 1/2, (a + b*Sec[c + d*x])/(a - b)])/((a - b)^2*Sqrt[a
 + b*Sec[c + d*x]]) - (b^2*Hypergeometric2F1[-1/2, 2, 1/2, (a + b*Sec[c +
d*x])/(a + b)])/((a + b)^2*Sqrt[a + b*Sec[c + d*x]]))/(2*b*d)
```

3.339.3 Rubi [A] (warning: unable to verify)

Time = 0.59 (sec) , antiderivative size = 310, normalized size of antiderivative = 1.31, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {3042, 25, 4373, 561, 25, 1674, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^3(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx$$
$$\downarrow 3042$$
$$\int -\frac{1}{\cot\left(c+dx+\frac{\pi}{2}\right)^3\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx$$
$$\downarrow 25$$
$$-\int \frac{1}{\cot\left(\frac{1}{2}(2c+\pi)+dx\right)^3\left(a+b\csc\left(\frac{1}{2}(2c+\pi)+dx\right)\right)^{3/2}}dx$$

$$\downarrow 4373$$

$$\frac{b^4 \int \frac{\cos(c+dx)}{b(a+b\sec(c+dx))^{3/2}(b^2-b^2\sec^2(c+dx))^2}\, d(b\sec(c+dx))}{d}$$

$$\downarrow 561$$

$$\frac{2b^4 \int -\frac{\cos^2(c+dx)}{b^2(a-b^2\sec^2(c+dx))(b^4\sec^4(c+dx)-2ab^2\sec^2(c+dx)+a^2-b^2)^2}\, d\sqrt{a+b\sec(c+dx)}}{d}$$

$$\downarrow 25$$

$$-\frac{2b^4 \int \frac{\cos^2(c+dx)}{b^2(a-b^2\sec^2(c+dx))(b^4\sec^4(c+dx)-2ab^2\sec^2(c+dx)+a^2-b^2)^2}\, d\sqrt{a+b\sec(c+dx)}}{d}$$

$$\downarrow 1674$$

$$-\frac{2b^4 \int \left(\frac{\cos^2(c+dx)}{a(a-b)^2b^2(a+b)^2}+\frac{1}{ab^4(a-b^2\sec^2(c+dx))}+\frac{3b-2a}{4(a-b)^2b^4(-b^2\sec^2(c+dx)+a-b)}+\frac{-2a-3b}{4b^4(a+b)^2(-b^2\sec^2(c+dx)+a+b)}+\frac{}{4(a-b)b^3}\right.}{d}$$

$$\downarrow 2009$$

$$\frac{2b^4\left(-\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a}}\right)}{a^{3/2}b^4}+\frac{\cos(c+dx)}{ab(a^2-b^2)^2}+\frac{(2a-3b)\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a-b}}\right)}{4b^4(a-b)^{5/2}}+\frac{(2a+3b)\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)}{4b^4(a+b)^{5/2}}-\frac{\text{arc}}{}\right.}{d}$$

input

```
Int[Cot[c + d*x]^3/(a + b*Sec[c + d*x])^(3/2),x]
```

output

```
(2*b^4*(-(ArcTanh[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a]]/(a^(3/2)*b^4)) + ((2*a
 - 3*b)*ArcTanh[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a - b]])/(4*(a - b)^(5/2)*b^
4) - ArcTanh[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a - b]]/(8*(a - b)^(5/2)*b^3) +
 ArcTanh[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]]/(8*b^3*(a + b)^(5/2)) + ((2
*a + 3*b)*ArcTanh[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]])/(4*b^4*(a + b)^(5
/2)) + Cos[c + d*x]/(a*b*(a^2 - b^2)^2) - Sec[c + d*x]/(8*(a - b)^2*b^2*(a
 - b - b^2*Sec[c + d*x]^2)) + Sec[c + d*x]/(8*b^2*(a + b)^2*(a + b - b^2*S
ec[c + d*x]^2))))/d
```

3.339.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 561
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> With[{k = Denominator[n]}, Simp[k/d   Subst[Int[x^(k*(n + 1) - 1)*(-c
/d + x^k/d)^m*Simp[(b*c^2 + a*d^2)/d^2 - 2*b*c*(x^k/d^2) + b*(x^(2*k)/d^2),
 x]^p, x], x, (c + d*x)^(1/k)], x]] /; FreeQ[{a, b, c, d, m, p}, x] && Frac
tionQ[n] && IntegerQ[p] && IntegerQ[m]
```

rule 1674
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, p, q}, x] && N
eQ[b^2 - 4*a*c, 0] && (IGtQ[p, 0] || IGtQ[q, 0] || IntegersQ[m, q])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4373
```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Simp[-(-1)^((m - 1)/2)/(d*b^(m - 1))   Subst[Int[(b^2 - x^
2)^((m - 1)/2)*((a + x)^n/x), x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
 d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

3.339.4 Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 6366 vs. $2(204) = 408$.

Time = 2.34 (sec) , antiderivative size = 6367, normalized size of antiderivative = 26.98

method	result	size
default	Expression too large to display	6367

input
```
int(cot(d*x+c)^3/(a+b*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

3.339.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 936 vs. 2(202) = 404.

Time = 48.26 (sec) , antiderivative size = 8098, normalized size of antiderivative = 34.31

$$\int \frac{\cot^3(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(cot(d*x+c)^3/(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
Too large to include
```

3.339.6 Sympy [F]

$$\int \frac{\cot^3(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{\cot^3(c+dx)}{(a+b\sec(c+dx))^{\frac{3}{2}}}\,dx$$

input
```
integrate(cot(d*x+c)**3/(a+b*sec(d*x+c))**(3/2),x)
```

output
```
Integral(cot(c + d*x)**3/(a + b*sec(c + d*x))**(3/2), x)
```

3.339.7 Maxima [F(-1)]

Timed out.

$$\int \frac{\cot^3(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(cot(d*x+c)^3/(a+b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
Timed out
```

3.339.8 Giac [F(-2)]

Exception generated.

$$\int \frac{\cot^3(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(cot(d*x+c)^3/(a+b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{%%%{32,[2,6]%%%}+%%%{-32,[1,7]%%%},[6,1]%%%}+%%%{%%{[%%%{6
4,[2,6]%%
```

3.339.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cot^3(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{\cot(c+d\,x)^3}{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

input
```
int(cot(c + d*x)^3/(a + b/cos(c + d*x))^(3/2),x)
```

output
```
int(cot(c + d*x)^3/(a + b/cos(c + d*x))^(3/2), x)
```

3.340 $\int \frac{\tan^4(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx$

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3.340.2 Mathematica [A] (warning: unable to verify) . . . 2491
3.340.3 Rubi [A] (verified) . . . 2492
3.340.4 Maple [B] (verified) . . . 2495
3.340.5 Fricas [F] . . . 2495
3.340.6 Sympy [F] . . . 2496
3.340.7 Maxima [F(-1)] . . . 2496
3.340.8 Giac [F] . . . 2496
3.340.9 Mupad [F(-1)] . . . 2497

3.340.1 Optimal result

Integrand size = 23, antiderivative size = 530

$$\int \frac{\tan^4(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx =$$

$$-\frac{2\sqrt{a+b}\cot(c+dx)\,\text{EllipticPi}\left(\frac{a+b}{a},\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{a^2 d}$$

$$+\frac{2(8a^4-11a^2b^2+3b^4)\cot(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{-\frac{b(-1+\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{3ab^4\sqrt{a+b}d}$$

$$+\frac{2(2a+b)(4a^2+ab-3b^2)\cot(c+dx)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{-\frac{b(-1+\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec}{a}}}{3ab^3\sqrt{a+b}d}$$

$$-\frac{4a\tan(c+dx)}{(a^2-b^2)\,d\sqrt{a+b\sec(c+dx)}}+\frac{2b^2\tan(c+dx)}{a\,(a^2-b^2)\,d\sqrt{a+b\sec(c+dx)}}$$

$$-\frac{2a^2\sec(c+dx)\tan(c+dx)}{b\,(a^2-b^2)\,d\sqrt{a+b\sec(c+dx)}}+\frac{2(4a^2-b^2)\sqrt{a+b\sec(c+dx)}\tan(c+dx)}{3b^2\,(a^2-b^2)\,d}$$

output
```
-2*cot(d*x+c)*EllipticPi((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),(a+b)/a,((a+b)
/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c
))/(a-b))^(1/2)/a^2/d+2/3*(8*a^4-11*a^2*b^2+3*b^4)*cot(d*x+c)*EllipticE((a
+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/2))*(-b*(-1+sec(d*x+c))/
(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/a/b^4/d/(a+b)^(1/2)+2/3*(2*a+
b)*(4*a^2+a*b-3*b^2)*cot(d*x+c)*EllipticF((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/
2),((a+b)/(a-b))^(1/2))*(-b*(-1+sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c)
)/(a-b))^(1/2)/a/b^3/d/(a+b)^(1/2)-4*a*tan(d*x+c)/(a^2-b^2)/d/(a+b*sec(d*x
+c))^(1/2)+2*b^2*tan(d*x+c)/a/(a^2-b^2)/d/(a+b*sec(d*x+c))^(1/2)-2*a^2*sec
(d*x+c)*tan(d*x+c)/b/(a^2-b^2)/d/(a+b*sec(d*x+c))^(1/2)+2/3*(4*a^2-b^2)*(a
+b*sec(d*x+c))^(1/2)*tan(d*x+c)/b^2/(a^2-b^2)/d
```

3.340.2 Mathematica [A] (warning: unable to verify)

Time = 17.41 (sec) , antiderivative size = 531, normalized size of antiderivative = 1.00

$$\int \frac{\tan^4(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = \frac{2(b+a\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)\sqrt{\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)}\Big(2(8a^3+8a^2$$

$$+\frac{(b+a\cos(c+dx))^2\sec^2(c+dx)\left(\frac{2(-8a^2+3b^2)\sin(c+dx)}{3ab^3}-\frac{2(-a^2\sin(c+dx)+b^2\sin(c+dx))}{ab^2(b+a\cos(c+dx))}+\frac{2\tan(c+dx)}{3b^2}\right)}{d(a+b\sec(c+dx))^{3/2}}$$

input
```
Integrate[Tan[c + d*x]^4/(a + b*Sec[c + d*x])^(3/2),x]
```

output
```
(2*(b + a*Cos[c + d*x])*Sec[c + d*x]^(3/2)*Sqrt[Cos[(c + d*x)/2]^2*Sec[c +
 d*x]]*(2*(8*a^3 + 8*a^2*b - 3*a*b^2 - 3*b^3)*Sqrt[Cos[c + d*x]/(1 + Cos[c
 + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*Elliptic
E[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] - 4*a*b*(4*a + b)*Sqrt[Cos[c
+ d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c +
 d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] + 12*b^3*Sqr
t[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 +
 Cos[c + d*x]))]*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]
 + ((8*a^2 - 3*b^2)*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^3*(-Sin[(c + d*x
)/2] + Sin[(3*(c + d*x))/2]))/2))/(3*a*b^3*d*Sqrt[Sec[(c + d*x)/2]^2]*(a +
 b*Sec[c + d*x])^(3/2)) + ((b + a*Cos[c + d*x])^2*Sec[c + d*x]^2*((2*(-8*a
^2 + 3*b^2)*Sin[c + d*x])/(3*a*b^3) - (2*(-(a^2*Sin[c + d*x]) + b^2*Sin[c
+ d*x]))/(a*b^2*(b + a*Cos[c + d*x])) + (2*Tan[c + d*x])/(3*b^2)))/(d*(a +
 b*Sec[c + d*x])^(3/2))
```

3.340.3 Rubi [A] (verified)

Time = 1.90 (sec) , antiderivative size = 907, normalized size of antiderivative = 1.71, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3042, 4383, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^4(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx$$

$$\downarrow 3042$$

$$\int \frac{\cot\left(c+dx+\frac{\pi}{2}\right)^4}{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx$$

$$\downarrow 4383$$

$$\int \left(\frac{\sec^4(c+dx)}{(a+b\sec(c+dx))^{3/2}} - \frac{2\sec^2(c+dx)}{(a+b\sec(c+dx))^{3/2}} + \frac{1}{(a+b\sec(c+dx))^{3/2}}\right)dx$$

$$\downarrow 2009$$

$$-\frac{2\sec(c+dx)\tan(c+dx)a^2}{b\left(a^2-b^2\right)d\sqrt{a+b\sec(c+dx)}}-\frac{4\tan(c+dx)a}{\left(a^2-b^2\right)d\sqrt{a+b\sec(c+dx)}}+$$
$$\frac{2\left(8a^2-5b^2\right)\cot(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}a}{3b^4\sqrt{a+b}d}-$$
$$\frac{4\cot(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}a}{b^2\sqrt{a+b}d}+$$
$$\frac{2\left(4a^2-b^2\right)\sqrt{a+b\sec(c+dx)}\tan(c+dx)}{3b^2\left(a^2-b^2\right)d}+$$
$$\frac{2(2a+b)(4a+b)\cot(c+dx)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}}{3b^3\sqrt{a+b}d}-$$
$$\frac{4\cot(c+dx)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}}{b\sqrt{a+b}d}+$$
$$\frac{2b^2\tan(c+dx)}{\left(a^2-b^2\right)d\sqrt{a+b\sec(c+dx)}a}+$$
$$\frac{2\cot(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}}{\sqrt{a+b}da}-$$
$$\frac{2\cot(c+dx)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}}{\sqrt{a+b}da}-$$
$$\frac{2\sqrt{a+b}\cot(c+dx)\,\text{EllipticPi}\left(\frac{a+b}{a},\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}}{da^2}$$

input
```
Int[Tan[c + d*x]^4/(a + b*Sec[c + d*x])^(3/2),x]
```

output
```
(2*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a
 + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c +
 d*x]))/(a - b))])/(a*Sqrt[a + b]*d) - (4*a*Cot[c + d*x]*EllipticE[ArcSin[
Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c
 + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(b^2*Sqrt[a +
b]*d) + (2*a*(8*a^2 - 5*b^2)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[
c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a +
b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(3*b^4*Sqrt[a + b]*d) - (2*Co
t[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)
/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]
))/(a - b))])/(a*Sqrt[a + b]*d) - (4*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a
+ b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]
))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(b*Sqrt[a + b]*d) + (
2*(2*a + b)*(4*a + b)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x
]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqr
t[-((b*(1 + Sec[c + d*x]))/(a - b))])/(3*b^3*Sqrt[a + b]*d) - (2*Sqrt[a +
b]*Cot[c + d*x]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt
[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*
(1 + Sec[c + d*x]))/(a - b))])/(a^2*d) - (4*a*Tan[c + d*x])/((a^2 - b^2)*d
*Sqrt[a + b*Sec[c + d*x]]) + (2*b^2*Tan[c + d*x])/(a*(a^2 - b^2)*d*Sqrt...
```

3.340.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4383
```
Int[cot[(c_.) + (d_.)*(x_)]^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_
), x_Symbol] :> Int[ExpandIntegrand[(a + b*Csc[c + d*x])^n, (-1 + Csc[c + d
*x]^2)^(m/2), x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[a^2 - b^2, 0] && I
GtQ[m/2, 0] && IntegerQ[n - 1/2]
```

3.340.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1964 vs. 2(489) = 978.

Time = 14.84 (sec) , antiderivative size = 1965, normalized size of antiderivative = 3.71

method	result	size
default	Expression too large to display	1965

input
```
int(tan(d*x+c)^4/(a+b*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/3/d/a/b^3*(a+b*sec(d*x+c))^(1/2)/(b+a*cos(d*x+c))/(cos(d*x+c)+1)*(a*b^2*
sin(d*x+c)-4*a^2*b*sin(d*x+c)-3*b^3*cos(d*x+c)*sin(d*x+c)-8*EllipticE(cot(
d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+
c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*a^2*b+3*EllipticE(cot(d*x+c
)-csc(d*x+c),((a-b)/(a+b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1)
)^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*a*b^2-8*(1/(a+b)*(b+a*cos(d*x+c)
)/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticE(cot(d*
x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))*a^3*cos(d*x+c)^2+3*(1/(a+b)*(b+a*cos(
d*x+c))/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticE(
cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))*b^3*cos(d*x+c)^2+8*EllipticF(co
t(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*
x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*a^2*b+2*EllipticF(cot(d*x
+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+
1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*a*b^2-16*(1/(a+b)*(b+a*cos(d*x
+c))/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticE(cot
(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))*a^3*cos(d*x+c)+6*(1/(a+b)*(b+a*cos
(d*x+c))/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticE
(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))*b^3*cos(d*x+c)-6*(cos(d*x+c)/(
cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Ellip
ticPi(cot(d*x+c)-csc(d*x+c),-1,((a-b)/(a+b))^(1/2))*b^3*cos(d*x+c)^2-12...
```

3.340.5 Fricas [F]

$$\int \frac{\tan^4(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{\tan\left(dx+c\right)^4}{\left(b\sec\left(dx+c\right)+a\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(tan(d*x+c)^4/(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b*sec(d*x + c) + a)*tan(d*x + c)^4/(b^2*sec(d*x + c)^2 + 2*a
*b*sec(d*x + c) + a^2), x)
```

3.340.6 Sympy [F]

$$\int \frac{\tan^4(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{\tan^4(c+dx)}{(a+b\sec(c+dx))^{\frac{3}{2}}}\,dx$$

input
```
integrate(tan(d*x+c)**4/(a+b*sec(d*x+c))**(3/2),x)
```

output
```
Integral(tan(c + d*x)**4/(a + b*sec(c + d*x))**(3/2), x)
```

3.340.7 Maxima [F(-1)]

Timed out.

$$\int \frac{\tan^4(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(tan(d*x+c)^4/(a+b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
Timed out
```

3.340.8 Giac [F]

$$\int \frac{\tan^4(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{\tan(dx+c)^4}{(b\sec(dx+c)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(tan(d*x+c)^4/(a+b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
integrate(tan(d*x + c)^4/(b*sec(d*x + c) + a)^(3/2), x)
```

3.340.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\tan^4(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{\tan(c+d\,x)^4}{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

input
```
int(tan(c + d*x)^4/(a + b/cos(c + d*x))^(3/2),x)
```

output
```
int(tan(c + d*x)^4/(a + b/cos(c + d*x))^(3/2), x)
```

3.341 $\int \frac{\tan^2(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx$

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3.341.1 Optimal result

Integrand size = 23, antiderivative size = 344

$$\int \frac{\tan^2(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = \frac{2(a-b)\sqrt{a+b}\cot(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-}}{ab^2d}$$
$$+\frac{2\sqrt{a+b}\cot(c+dx)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{abd}$$
$$+\frac{2\sqrt{a+b}\cot(c+dx)\,\text{EllipticPi}\left(\frac{a+b}{a},\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{a^2d}$$
$$+\frac{2\tan(c+dx)}{ad\sqrt{a+b\sec(c+dx)}}$$

output

```
2*(a-b)*cot(d*x+c)*EllipticE((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),((a+b)/(a-
b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(
a-b))^(1/2)/a/b^2/d+2*cot(d*x+c)*EllipticF((a+b*sec(d*x+c))^(1/2)/(a+b)^(1
/2),((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1
+sec(d*x+c))/(a-b))^(1/2)/a/b/d+2*cot(d*x+c)*EllipticPi((a+b*sec(d*x+c))^(
1/2)/(a+b)^(1/2),(a+b)/a,((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(d*x+c)
)/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/a^2/d+2*tan(d*x+c)/a/d/(a+b
*sec(d*x+c))^(1/2)
```

3.341.2 Mathematica [A] (warning: unable to verify)

Time = 11.18 (sec) , antiderivative size = 329, normalized size of antiderivative = 0.96

$$\int \frac{\tan^2(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = \frac{(b+a\cos(c+dx))^2\sec^2(c+dx)\left(-\frac{2\sin(c+dx)}{ab}+\frac{2\sin(c+dx)}{a(b+a\cos(c+dx))}\right)}{d(a+b\sec(c+dx))^{3/2}} + \frac{4\sqrt{\frac{\cos(c+dx)}{(1+\cos(c+dx))^2}}(b+a\cos(c+dx))\sqrt{\sec(c+dx)}\left(\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\right)^{3/2}\left((a+b)E\left(\arcsin\left(\tan(\right.\right.\right.}$$

input
```
Integrate[Tan[c + d*x]^2/(a + b*Sec[c + d*x])^(3/2),x]
```

output
```
((b + a*Cos[c + d*x])^2*Sec[c + d*x]^2*((-2*Sin[c + d*x])/(a*b) + (2*Sin[c
 + d*x])/(a*(b + a*Cos[c + d*x]))))/(d*(a + b*Sec[c + d*x])^(3/2)) + (4*Sq
rt[Cos[c + d*x]/(1 + Cos[c + d*x])^2]*(b + a*Cos[c + d*x])*Sqrt[Sec[c + d*
x]]*(Cos[(c + d*x)/2]^2*Sec[c + d*x])^(3/2)*((a + b)*EllipticE[ArcSin[Tan[
(c + d*x)/2]], (a - b)/(a + b)]*Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2
]^2)/(a + b)] - 2*b*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (a - b)/(a +
b)]*Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)] + (b + a*Cos[c
 + d*x])*Sqrt[Cos[c + d*x]*Sec[(c + d*x)/2]^2]*Tan[(c + d*x)/2]))/(a*b*d*(
a + b*Sec[c + d*x])^(3/2))
```

3.341.3 Rubi [A] (verified)

Time = 1.38 (sec) , antiderivative size = 382, normalized size of antiderivative = 1.11, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.565$, Rules used = {3042, 4382, 3042, 4549, 27, 3042, 4547, 3042, 4409, 3042, 4271, 4319, 4492}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^2(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx$$
$$\downarrow 3042$$
$$\int \frac{\cot\left(c+dx+\frac{\pi}{2}\right)^2}{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx$$
$$\downarrow 4382$$

$$\int \frac{\sec^2(c+dx)-1}{(a+b\sec(c+dx))^{3/2}}dx$$

$$\downarrow 3042$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^2-1}{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx$$

$$\downarrow 4549$$

$$\frac{2\tan(c+dx)}{ad\sqrt{a+b\sec(c+dx)}}-\frac{2\int\frac{a^2-b^2+(a^2-b^2)\sec^2(c+dx)}{2\sqrt{a+b\sec(c+dx)}}dx}{a\left(a^2-b^2\right)}$$

$$\downarrow 27$$

$$\frac{2\tan(c+dx)}{ad\sqrt{a+b\sec(c+dx)}}-\frac{\int\frac{a^2-b^2+(a^2-b^2)\sec^2(c+dx)}{\sqrt{a+b\sec(c+dx)}}dx}{a\left(a^2-b^2\right)}$$

$$\downarrow 3042$$

$$\frac{2\tan(c+dx)}{ad\sqrt{a+b\sec(c+dx)}}-\frac{\int\frac{a^2-b^2+(a^2-b^2)\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a\left(a^2-b^2\right)}$$

$$\downarrow 4547$$

$$\frac{2\tan(c+dx)}{ad\sqrt{a+b\sec(c+dx)}}-\frac{\left(a^2-b^2\right)\int\frac{\sec(c+dx)(\sec(c+dx)+1)}{\sqrt{a+b\sec(c+dx)}}dx+\int\frac{a^2-b^2-\left(a^2-b^2\right)\sec(c+dx)}{\sqrt{a+b\sec(c+dx)}}dx}{a\left(a^2-b^2\right)}$$

$$\downarrow 3042$$

$$\frac{2\tan(c+dx)}{ad\sqrt{a+b\sec(c+dx)}}-\frac{\left(a^2-b^2\right)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)+1\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\int\frac{a^2-b^2+\left(b^2-a^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a\left(a^2-b^2\right)}$$

$$\downarrow 4409$$

$$\frac{2\tan(c+dx)}{ad\sqrt{a+b\sec(c+dx)}}-\frac{\left(a^2-b^2\right)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)+1\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\left(a^2-b^2\right)\int\frac{1}{\sqrt{a+b\sec(c+dx)}}dx-\left(a^2-b^2\right)\int\frac{\sec(c+dx)}{\sqrt{a+b\sec(c+dx)}}dx}{a\left(a^2-b^2\right)}$$

$$\downarrow 3042$$

$$\frac{2\tan(c+dx)}{ad\sqrt{a+b\sec(c+dx)}} - \frac{(a^2-b^2)\int \frac{1}{\sqrt{a+b\csc(c+dx+\frac{\pi}{2})}}dx - (a^2-b^2)\int \frac{\csc(c+dx+\frac{\pi}{2})}{\sqrt{a+b\csc(c+dx+\frac{\pi}{2})}}dx + (a^2-b^2)\int \frac{\csc(c+dx+\frac{\pi}{2})(\csc(c+dx+\frac{\pi}{2})+1)}{\sqrt{a+b\csc(c+dx+\frac{\pi}{2})}}dx}{a(a^2-b^2)}$$

$$\downarrow 4271$$

$$\frac{2\tan(c+dx)}{ad\sqrt{a+b\sec(c+dx)}} - \frac{-(a^2-b^2)\int \frac{\csc(c+dx+\frac{\pi}{2})}{\sqrt{a+b\csc(c+dx+\frac{\pi}{2})}}dx + (a^2-b^2)\int \frac{\csc(c+dx+\frac{\pi}{2})(\csc(c+dx+\frac{\pi}{2})+1)}{\sqrt{a+b\csc(c+dx+\frac{\pi}{2})}}dx - \frac{2\sqrt{a+b}(a^2-b^2)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}}{a(a^2-b^2)}$$

$$\downarrow 4319$$

$$\frac{2\tan(c+dx)}{ad\sqrt{a+b\sec(c+dx)}} - \frac{(a^2-b^2)\int \frac{\csc(c+dx+\frac{\pi}{2})(\csc(c+dx+\frac{\pi}{2})+1)}{\sqrt{a+b\csc(c+dx+\frac{\pi}{2})}}dx - \frac{2\sqrt{a+b}(a^2-b^2)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a}}\right.\right.}{bd}}{a(a^2-b^2)}$$

$$\downarrow 4492$$

$$\frac{2\tan(c+dx)}{ad\sqrt{a+b\sec(c+dx)}} - \frac{-\frac{2\sqrt{a+b}(a^2-b^2)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{bd} - \frac{2(a-b)\sqrt{a+b}(a^2-b^2)\cot(c+dx)\sqrt{}}{}}{}$$

input
```
Int[Tan[c + d*x]^2/(a + b*Sec[c + d*x])^(3/2),x]
```

output
```
-(((-2*(a - b)*Sqrt[a + b]*(a^2 - b^2)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[
a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*
x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(b^2*d) - (2*Sqrt[a
 + b]*(a^2 - b^2)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/S
qrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-(
(b*(1 + Sec[c + d*x]))/(a - b))])/(b*d) - (2*Sqrt[a + b]*(a^2 - b^2)*Cot[c
 + d*x]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]]
, (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec
[c + d*x]))/(a - b))])/(a*d))/(a*(a^2 - b^2))) + (2*Tan[c + d*x])/(a*d*Sqr
t[a + b*Sec[c + d*x]])
```

3.341. $\int \frac{\tan^2(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx$

3.341.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4271
```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[2*(Rt[a
 + b, 2]/(a*d*Cot[c + d*x]))*Sqrt[b*((1 - Csc[c + d*x])/(a + b))]*Sqrt[(-b)
*((1 + Csc[c + d*x])/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[
c + d*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

rule 4319
```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*(Rt[a + b, 2]/(b*f*Cot[e + f*x]))*Sqrt[(b*(1 - Csc[e + f*
x]))/(a + b)]*Sqrt[(-b)*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt
[a + b*Csc[e + f*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4382
```
Int[cot[(c_.) + (d_.)*(x_)]^2*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_),
x_Symbol] :> Int[(-1 + Csc[c + d*x]^2)*(a + b*Csc[c + d*x])^n, x] /; FreeQ[
{a, b, c, d, n}, x] && NeQ[a^2 - b^2, 0]
```

rule 4409
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Simp[c   Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] +
Simp[d   Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

rule 4492
```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(A*b - a*B)*Rt[a
 + b*(B/A), 2]*Sqrt[b*((1 - Csc[e + f*x])/(a + b))]*(Sqrt[(-b)*((1 + Csc[e
+ f*x])/(a - b))]/(b^2*f*Cot[e + f*x]))*EllipticE[ArcSin[Sqrt[a + b*Csc[e +
 f*x]]/Rt[a + b*(B/A), 2]], (a*A + b*B)/(a*A - b*B)], x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

rule 4547
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))/Sqrt[csc[(e_.) + (f_.)*(x_)]*
(b_.) + (a_)], x_Symbol] :> Int[(A - C*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x
]], x] + Simp[C   Int[Csc[e + f*x]*((1 + Csc[e + f*x])/Sqrt[a + b*Csc[e + f
*x]]), x], x] /; FreeQ[{a, b, e, f, A, C}, x] && NeQ[a^2 - b^2, 0]
```

rule 4549
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_))^(m_), x_Symbol] :> Simp[(A*b^2 + a^2*C)*Cot[e + f*x]*((a + b*Csc[
e + f*x])^(m + 1)/(a*f*(m + 1)*(a^2 - b^2))), x] + Simp[1/(a*(m + 1)*(a^2 -
 b^2))   Int[(a + b*Csc[e + f*x])^(m + 1)*Simp[A*(a^2 - b^2)*(m + 1) - a*b*
(A + C)*(m + 1)*Csc[e + f*x] + (A*b^2 + a^2*C)*(m + 2)*Csc[e + f*x]^2, x],
x], x] /; FreeQ[{a, b, e, f, A, C}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[2*m
] && LtQ[m, -1]
```

3.341.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 827 vs. $2(315) = 630$.

Time = 9.79 (sec) , antiderivative size = 828, normalized size of antiderivative = 2.41

method	result	size
default	Expression too large to display	828

input
```
int(tan(d*x+c)^2/(a+b*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/d/b/a*(EllipticE(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)
/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a*c
os(d*x+c)^2+EllipticE(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+
c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*b
*cos(d*x+c)^2-2*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c
)/(cos(d*x+c)+1))^(1/2)*EllipticPi(cot(d*x+c)-csc(d*x+c),-1,((a-b)/(a+b))^
(1/2))*b*cos(d*x+c)^2+2*EllipticE(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2
))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)
+1))^(1/2)*a*cos(d*x+c)+2*EllipticE(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1
/2))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+
c)+1))^(1/2)*b*cos(d*x+c)-4*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2
)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticPi(cot(d*x+c)-csc(d*x+c),-1,((
a-b)/(a+b))^(1/2))*b*cos(d*x+c)+EllipticE(cot(d*x+c)-csc(d*x+c),((a-b)/(a+
b))^(1/2))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(co
s(d*x+c)+1))^(1/2)*a+EllipticE(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))*
(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1)
)^(1/2)*b-2*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticPi(cot
(d*x+c)-csc(d*x+c),-1,((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/
2)*b+a*sin(d*x+c)*cos(d*x+c)-cos(d*x+c)*sin(d*x+c)*b)*(a+b*sec(d*x+c))^(1/
2)/(b+a*cos(d*x+c))/(cos(d*x+c)+1)
```

3.341.5 Fricas [F]

$$\int \frac{\tan^2(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{\tan\left(dx+c\right)^2}{\left(b\sec\left(dx+c\right)+a\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(tan(d*x+c)^2/(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b*sec(d*x + c) + a)*tan(d*x + c)^2/(b^2*sec(d*x + c)^2 + 2*a
*b*sec(d*x + c) + a^2), x)
```

3.341.6 Sympy [F]

$$\int \frac{\tan^2(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{\tan^2(c+dx)}{(a+b\sec(c+dx))^{\frac{3}{2}}}\,dx$$

input
```
integrate(tan(d*x+c)**2/(a+b*sec(d*x+c))**(3/2),x)
```

output
```
Integral(tan(c + d*x)**2/(a + b*sec(c + d*x))**(3/2), x)
```

3.341.7 Maxima [F]

$$\int \frac{\tan^2(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{\tan(dx+c)^2}{(b\sec(dx+c)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(tan(d*x+c)^2/(a+b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate(tan(d*x + c)^2/(b*sec(d*x + c) + a)^(3/2), x)
```

3.341.8 Giac [F]

$$\int \frac{\tan^2(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{\tan(dx+c)^2}{(b\sec(dx+c)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(tan(d*x+c)^2/(a+b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
integrate(tan(d*x + c)^2/(b*sec(d*x + c) + a)^(3/2), x)
```

3.341.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\tan^2(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{\tan(c+d\,x)^2}{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

input
```
int(tan(c + d*x)^2/(a + b/cos(c + d*x))^(3/2),x)
```

output
```
int(tan(c + d*x)^2/(a + b/cos(c + d*x))^(3/2), x)
```

3.342 $\int \frac{1}{(a+b\sec(c+dx))^{3/2}}\,dx$

3.342.1 Optimal result . . . 2507
3.342.2 Mathematica [B] (verified) . . . 2508
3.342.3 Rubi [A] (verified) . . . 2509
3.342.4 Maple [B] (warning: unable to verify) . . . 2512
3.342.5 Fricas [F] . . . 2513
3.342.6 Sympy [F] . . . 2514
3.342.7 Maxima [F] . . . 2514
3.342.8 Giac [F] . . . 2514
3.342.9 Mupad [F(-1)] . . . 2515

3.342.1 Optimal result

Integrand size = 14, antiderivative size = 347

$$\int \frac{1}{(a+b\sec(c+dx))^{3/2}}\,dx = \frac{2\cot(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{a\sqrt{a+b}d}$$
$$-\frac{2\cot(c+dx)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{a\sqrt{a+b}d}$$
$$-\frac{2\sqrt{a+b}\cot(c+dx)\,\text{EllipticPi}\left(\frac{a+b}{a},\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{a^2d}$$
$$+\frac{2b^2\tan(c+dx)}{a\left(a^2-b^2\right)d\sqrt{a+b\sec(c+dx)}}$$

output

```
2*cot(d*x+c)*EllipticE((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1
/2))*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/a/d/(a
+b)^(1/2)-2*cot(d*x+c)*EllipticF((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),((a+b)
/(a-b))^(1/2))*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1
/2)/a/d/(a+b)^(1/2)-2*cot(d*x+c)*EllipticPi((a+b*sec(d*x+c))^(1/2)/(a+b)^(
1/2),(a+b)/a,((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(d*x+c))/(a+b))^(1/
2)*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/a^2/d+2*b^2*tan(d*x+c)/a/(a^2-b^2)/d/(a
+b*sec(d*x+c))^(1/2)
```

3.342.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 972 vs. $2(347) = 694$.

Time = 6.08 (sec) , antiderivative size = 972, normalized size of antiderivative = 2.80

$$\int \frac{1}{(a+b\sec(c+dx))^{3/2}}\,dx = \frac{(b+a\cos(c+dx))^2\sec^2(c+dx)\left(\frac{2b\sin(c+dx)}{a(-a^2+b^2)}+\frac{2b^2\sin(c+dx)}{a(a^2-b^2)(b+a\cos(c+dx))}\right)}{d(a+b\sec(c+dx))^{3/2}}$$
$$+\frac{2(b+a\cos(c+dx))^{3/2}\sec^{\frac{3}{2}}(c+dx)\sqrt{\frac{a+b-a\tan^2\left(\frac{1}{2}(c+dx)\right)+b\tan^2\left(\frac{1}{2}(c+dx)\right)}{1+\tan^2\left(\frac{1}{2}(c+dx)\right)}}\left(ab\tan\left(\frac{1}{2}(c+dx)\right)+b^2\tan\left(\frac{1}{2}(c+\right.\right.}{}$$

input
```
Integrate[(a + b*Sec[c + d*x])^(-3/2),x]
```

output
```
((b + a*Cos[c + d*x])^2*Sec[c + d*x]^2*((2*b*Sin[c + d*x])/(a*(-a^2 + b^2)
) + (2*b^2*Sin[c + d*x])/(a*(a^2 - b^2)*(b + a*Cos[c + d*x])))/(d*(a + b*
Sec[c + d*x])^(3/2)) + (2*(b + a*Cos[c + d*x])^(3/2)*Sec[c + d*x]^(3/2)*Sq
rt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(1 + Tan[(c + d*x
)/2]^2)]*(a*b*Tan[(c + d*x)/2] + b^2*Tan[(c + d*x)/2] - 2*a*b*Tan[(c + d*x
)/2]^3 + a*b*Tan[(c + d*x)/2]^5 - b^2*Tan[(c + d*x)/2]^5 + 2*a^2*EllipticP
i[-1, ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]
^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] -
2*b^2*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sqrt[1 - T
an[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]
^2)/(a + b)] + 2*a^2*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (a - b)/(a +
 b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(
c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] - 2*b^2*EllipticPi[-1, ArcS
in[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c
+ d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a
 + b)] + b*(a + b)*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sq
rt[1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b - a*Tan[(c
 + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] - a*(a + b)*EllipticF[ArcSin
[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + Tan
[(c + d*x)/2]^2)*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2...
```

3.342.3 Rubi [A] (verified)

Time = 1.26 (sec) , antiderivative size = 376, normalized size of antiderivative = 1.08, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.786$, Rules used = {3042, 4272, 27, 3042, 4546, 3042, 4409, 3042, 4271, 4319, 4492}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+b\sec(c+dx))^{3/2}}\,dx$$

$$\downarrow 3042$$

$$\int \frac{1}{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx$$

$$\downarrow 4272$$

$$\frac{2b^2\tan(c+dx)}{ad\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}} - \frac{2\int -\frac{a^2-b\sec(c+dx)a-b^2-b^2\sec^2(c+dx)}{2\sqrt{a+b\sec(c+dx)}}dx}{a\left(a^2-b^2\right)}$$

$$\downarrow 27$$

$$\frac{\int \frac{a^2-b\sec(c+dx)a-b^2-b^2\sec^2(c+dx)}{\sqrt{a+b\sec(c+dx)}}dx}{a\left(a^2-b^2\right)} + \frac{2b^2\tan(c+dx)}{ad\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}$$

$$\downarrow 3042$$

$$\frac{\int \frac{a^2-b\csc\left(c+dx+\frac{\pi}{2}\right)a-b^2-b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a\left(a^2-b^2\right)} + \frac{2b^2\tan(c+dx)}{ad\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}$$

$$\downarrow 4546$$

$$\frac{\int \frac{a^2-b^2+\left(b^2-ab\right)\sec(c+dx)}{\sqrt{a+b\sec(c+dx)}}dx - b^2\int \frac{\sec(c+dx)(\sec(c+dx)+1)}{\sqrt{a+b\sec(c+dx)}}dx}{a\left(a^2-b^2\right)} + \frac{2b^2\tan(c+dx)}{ad\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}$$

$$\downarrow 3042$$

$$\frac{\int \frac{a^2-b^2+\left(b^2-ab\right)\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx - b^2\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)+1\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a\left(a^2-b^2\right)} + \frac{2b^2\tan(c+dx)}{ad\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}$$

$$\downarrow 4409$$

$$\frac{\left(a^2-b^2\right)\int \frac{1}{\sqrt{a+b\sec(c+dx)}}dx+b^2\left(-\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)+1\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx\right)-b(a-b)\int \frac{\sec(c+dx)}{\sqrt{a+b\sec(c+dx)}}dx}{a\left(a^2-b^2\right)}+$$
$$\frac{2b^2\tan(c+dx)}{ad\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}$$

$$\downarrow 3042$$

$$\frac{\left(a^2-b^2\right)\int \frac{1}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+b^2\left(-\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)+1\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx\right)-b(a-b)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a\left(a^2-b^2\right)}+$$
$$\frac{2b^2\tan(c+dx)}{ad\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}$$

$$\downarrow 4271$$

$$\frac{b^2\left(-\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)+1\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx\right)-b(a-b)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{2\sqrt{a+b}\left(a^2-b^2\right)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b}{}}}{a\left(a^2-b^2\right)}$$
$$\frac{2b^2\tan(c+dx)}{ad\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}$$

$$\downarrow 4319$$

$$\frac{b^2\left(-\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)+1\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx\right)-\frac{2\sqrt{a+b}\left(a^2-b^2\right)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\,\text{EllipticPi}\left(\frac{a+b}{a},\arcsin\left(\frac{\sqrt{a+}}{}\right)\right.}{ad}}{a\left(a^2-b^2\right)}$$
$$\frac{2b^2\tan(c+dx)}{ad\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}$$

$$\downarrow 4492$$

$$-\frac{2\sqrt{a+b}\left(a^2-b^2\right)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\,\text{EllipticPi}\left(\frac{a+b}{a},\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{ad}-\frac{2(a-b)\sqrt{a+b}\cot(c+dx)\sqrt{\frac{b}{}}}{}$$
$$\frac{2b^2\tan(c+dx)}{ad\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}$$

input
```
Int[(a + b*Sec[c + d*x])^(-3/2),x]
```

3.342. $\int \frac{1}{(a+b\sec(c+dx))^{3/2}}\,dx$

output
```
((2*(a - b)*Sqrt[a + b]*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d
*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*S
qrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/d - (2*(a - b)*Sqrt[a + b]*Cot[c +
 d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a -
 b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a
 - b))])/d - (2*Sqrt[a + b]*(a^2 - b^2)*Cot[c + d*x]*EllipticPi[(a + b)/a,
 ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1
 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(a*d))
/(a*(a^2 - b^2)) + (2*b^2*Tan[c + d*x])/(a*(a^2 - b^2)*d*Sqrt[a + b*Sec[c
+ d*x]])
```

3.342.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4271
```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[2*(Rt[a
 + b, 2]/(a*d*Cot[c + d*x]))*Sqrt[b*((1 - Csc[c + d*x])/(a + b))]*Sqrt[(-b)
*((1 + Csc[c + d*x])/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[
c + d*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

rule 4272
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Simp[b^2*Cot[
c + d*x]*((a + b*Csc[c + d*x])^(n + 1)/(a*d*(n + 1)*(a^2 - b^2))), x] + Sim
p[1/(a*(n + 1)*(a^2 - b^2))   Int[(a + b*Csc[c + d*x])^(n + 1)*Simp[(a^2 -
b^2)*(n + 1) - a*b*(n + 1)*Csc[c + d*x] + b^2*(n + 2)*Csc[c + d*x]^2, x],
], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && LtQ[n, -1] && Integ
erQ[2*n]
```

rule 4319
```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*(Rt[a + b, 2]/(b*f*Cot[e + f*x]))*Sqrt[(b*(1 - Csc[e + f*
x]))/(a + b)]*Sqrt[(-b)*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt
[a + b*Csc[e + f*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4409
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Simp[c   Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] +
Simp[d   Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

rule 4492
```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(A*b - a*B)*Rt[a
 + b*(B/A), 2]*Sqrt[b*((1 - Csc[e + f*x])/(a + b))]*(Sqrt[(-b)*((1 + Csc[e
+ f*x])/(a - b))]/(b^2*f*Cot[e + f*x]))*EllipticE[ArcSin[Sqrt[a + b*Csc[e +
 f*x]]/Rt[a + b*(B/A), 2]], (a*A + b*B)/(a*A - b*B)], x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

rule 4546
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Int[(A + (B - C
)*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]], x] + Simp[C   Int[Csc[e + f*x]*((
1 + Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]]), x], x] /; FreeQ[{a, b, e, f, A
, B, C}, x] && NeQ[a^2 - b^2, 0]
```

3.342.4 Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 1798 vs. $2(318) = 636$.

Time = 7.08 (sec) , antiderivative size = 1799, normalized size of antiderivative = 5.18

method	result	size
default	Expression too large to display	1799

input
```
int(1/(a+b*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/d/a/(a+b)/(a-b)*(-((a*(1-cos(d*x+c))^2*csc(d*x+c)^2-b*(1-cos(d*x+c))^2*c
sc(d*x+c)^2-a-b)*((1-cos(d*x+c))^2*csc(d*x+c)^2-1))^(1/2)*(-(1-cos(d*x+c))
^2*csc(d*x+c)^2+1)^(1/2)*(-(a*(1-cos(d*x+c))^2*csc(d*x+c)^2-b*(1-cos(d*x+c
))^2*csc(d*x+c)^2-a-b)/(a+b))^(1/2)*EllipticF(cot(d*x+c)-csc(d*x+c),((a-b)
/(a+b))^(1/2))*a^2-((a*(1-cos(d*x+c))^2*csc(d*x+c)^2-b*(1-cos(d*x+c))^2*cs
c(d*x+c)^2-a-b)*((1-cos(d*x+c))^2*csc(d*x+c)^2-1))^(1/2)*(-(1-cos(d*x+c))^
2*csc(d*x+c)^2+1)^(1/2)*(-(a*(1-cos(d*x+c))^2*csc(d*x+c)^2-b*(1-cos(d*x+c)
)^2*csc(d*x+c)^2-a-b)/(a+b))^(1/2)*EllipticF(cot(d*x+c)-csc(d*x+c),((a-b)/
(a+b))^(1/2))*a*b+((a*(1-cos(d*x+c))^2*csc(d*x+c)^2-b*(1-cos(d*x+c))^2*csc
(d*x+c)^2-a-b)*((1-cos(d*x+c))^2*csc(d*x+c)^2-1))^(1/2)*(-(1-cos(d*x+c))^2
*csc(d*x+c)^2+1)^(1/2)*(-(a*(1-cos(d*x+c))^2*csc(d*x+c)^2-b*(1-cos(d*x+c))
^2*csc(d*x+c)^2-a-b)/(a+b))^(1/2)*EllipticE(cot(d*x+c)-csc(d*x+c),((a-b)/(
a+b))^(1/2))*a*b+((a*(1-cos(d*x+c))^2*csc(d*x+c)^2-b*(1-cos(d*x+c))^2*csc(
d*x+c)^2-a-b)*((1-cos(d*x+c))^2*csc(d*x+c)^2-1))^(1/2)*(-(1-cos(d*x+c))^2*
csc(d*x+c)^2+1)^(1/2)*(-(a*(1-cos(d*x+c))^2*csc(d*x+c)^2-b*(1-cos(d*x+c))^
2*csc(d*x+c)^2-a-b)/(a+b))^(1/2)*EllipticE(cot(d*x+c)-csc(d*x+c),((a-b)/(a
+b))^(1/2))*b^2+2*((a*(1-cos(d*x+c))^2*csc(d*x+c)^2-b*(1-cos(d*x+c))^2*csc
(d*x+c)^2-a-b)*((1-cos(d*x+c))^2*csc(d*x+c)^2-1))^(1/2)*(-(1-cos(d*x+c))^2
*csc(d*x+c)^2+1)^(1/2)*(-(a*(1-cos(d*x+c))^2*csc(d*x+c)^2-b*(1-cos(d*x+c))
^2*csc(d*x+c)^2-a-b)/(a+b))^(1/2)*EllipticPi(cot(d*x+c)-csc(d*x+c),-1,(...
```

3.342.5 Fricas [F]

$$\int \frac{1}{(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{1}{(b\sec(dx+c)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b*sec(d*x + c) + a)/(b^2*sec(d*x + c)^2 + 2*a*b*sec(d*x + c)
 + a^2), x)
```

3.342.6 Sympy [F]

$$\int \frac{1}{(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{1}{(a+b\sec(c+dx))^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a+b*sec(d*x+c))**(3/2),x)
```

output
```
Integral((a + b*sec(c + d*x))**(-3/2), x)
```

3.342.7 Maxima [F]

$$\int \frac{1}{(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{1}{(b\sec(dx+c)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a+b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c) + a)^(-3/2), x)
```

3.342.8 Giac [F]

$$\int \frac{1}{(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{1}{(b\sec(dx+c)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a+b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c) + a)^(-3/2), x)
```

3.342.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{1}{\left(a+\frac{b}{\cos(c+dx)}\right)^{3/2}}\,dx$$

input
```
int(1/(a + b/cos(c + d*x))^(3/2),x)
```

output
```
int(1/(a + b/cos(c + d*x))^(3/2), x)
```

3.343 $\int \frac{\cot^2(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx$

3.343.1 Optimal result 2516
3.343.2 Mathematica [A] (verified) 2517
3.343.3 Rubi [A] (verified) 2517
3.343.4 Maple [B] (verified) 2520
3.343.5 Fricas [F(-1)] 2520
3.343.6 Sympy [F] 2521
3.343.7 Maxima [F] 2521
3.343.8 Giac [F] 2521
3.343.9 Mupad [F(-1)] 2522

3.343.1 Optimal result

Integrand size = 23, antiderivative size = 449

$$\int \frac{\cot^2(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = \frac{2\sqrt{a+b}\cot(c+dx)\,\text{EllipticPi}\left(\frac{a+b}{a},\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+}{a+b}}}{a^2 d}$$
$$+\frac{2(a^2+b^2)\cot(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{-\frac{b(-1+\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{a(a-b)(a+b)^{3/2}d}$$
$$-\frac{(a^2-ab+2b^2)\cot(c+dx)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{-\frac{b(-1+\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{a(a-b)(a+b)^{3/2}d}$$
$$-\frac{\cot(c+dx)}{d(a+b\sec(c+dx))^{3/2}}+\frac{b^2\tan(c+dx)}{(a^2-b^2)\,d(a+b\sec(c+dx))^{3/2}}$$
$$+\frac{2b^2(a^2+b^2)\tan(c+dx)}{a\left(a^2-b^2\right)^2 d\sqrt{a+b\sec(c+dx)}}$$

output

```
-cot(d*x+c)/d/(a+b*sec(d*x+c))^(3/2)+2*cot(d*x+c)*EllipticPi((a+b*sec(d*x+
c))^(1/2)/(a+b)^(1/2),(a+b)/a,((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(d
*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/a^2/d+2*(a^2+b^2)*cot(
d*x+c)*EllipticE((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/2))*(
-b*(-1+sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/a/(a-b)/(a
+b)^(3/2)/d-(a^2-a*b+2*b^2)*cot(d*x+c)*EllipticF((a+b*sec(d*x+c))^(1/2)/(a
+b)^(1/2),((a+b)/(a-b))^(1/2))*(-b*(-1+sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec
(d*x+c))/(a-b))^(1/2)/a/(a-b)/(a+b)^(3/2)/d+b^2*tan(d*x+c)/(a^2-b^2)/d/(a+
b*sec(d*x+c))^(3/2)+2*b^2*(a^2+b^2)*tan(d*x+c)/a/(a^2-b^2)^2/d/(a+b*sec(d*
x+c))^(1/2)
```

3.343.2 Mathematica [A] (verified)

Time = 10.81 (sec) , antiderivative size = 469, normalized size of antiderivative = 1.04

$$\int \frac{\cot^2(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = \frac{(b+a\cos(c+dx))\sec^2(c+dx)\left(-a(b+a\cos(c+dx))\left(-2ab+\left(a^2+b^2\right)\cos\right.\right.}{}$$

input
```
Integrate[Cot[c + d*x]^2/(a + b*Sec[c + d*x])^(3/2),x]
```

output
```
((b + a*Cos[c + d*x])*Sec[c + d*x]^2*(-(a*(b + a*Cos[c + d*x])*(-2*a*b + (
a^2 + b^2)*Cos[c + d*x])*Csc[c + d*x]) + 2*b^4*Sin[c + d*x] - 2*b*(a^2 + b
^2)*(b + a*Cos[c + d*x])*Sin[c + d*x] + 2*Cos[(c + d*x)/2]^2*(2*b*(a^3 + a
^2*b + a*b^2 + b^3)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[
c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]]
, (a - b)/(a + b)] + a*(2*a^3 - a^2*b - 6*a*b^2 - 3*b^3)*Sqrt[Cos[c + d*x]
/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x])
)]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] - 4*(a^2 - b^2)^2*
Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(
1 + Cos[c + d*x]))]*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (a - b)/(a +
b)] + b*(a^2 + b^2)*Cos[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*T
an[(c + d*x)/2])))/(a*(a^2 - b^2)^2*d*(a + b*Sec[c + d*x])^(3/2))
```

3.343.3 Rubi [A] (verified)

Time = 1.43 (sec) , antiderivative size = 664, normalized size of antiderivative = 1.48, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3042, 4384, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^2(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx$$

$$\downarrow 3042$$

$$\int \frac{1}{\cot\left(c+dx+\frac{\pi}{2}\right)^2\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx$$

$$\downarrow 4384$$

$$\int \left(\frac{\csc^2(c+dx)}{(a+b\sec(c+dx))^{3/2}} - \frac{1}{(a+b\sec(c+dx))^{3/2}} \right) dx$$

$$\Big\downarrow 2009$$

$$\frac{2\sqrt{a+b}\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\,\text{EllipticPi}\left(\frac{a+b}{a},\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{a^2 d} -$$
$$\frac{2b^2\tan(c+dx)}{ad\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}} + \frac{4ab^2\tan(c+dx)}{d\left(a^2-b^2\right)^2\sqrt{a+b\sec(c+dx)}} +$$
$$\frac{b^2\tan(c+dx)}{d\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}} +$$
$$\frac{2\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{ad\sqrt{a+b}} -$$
$$\frac{(3a-b)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{d(a-b)(a+b)^{3/2}} -$$
$$\frac{2\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\,E\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{ad\sqrt{a+b}} +$$
$$\frac{4a\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\,E\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{d(a-b)(a+b)^{3/2}} -$$
$$\frac{\cot(c+dx)}{d(a+b\sec(c+dx))^{3/2}}$$

input
```
Int[Cot[c + d*x]^2/(a + b*Sec[c + d*x])^(3/2),x]
```

output
```
(4*a*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]],
(a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c
 + d*x]))/(a - b))])/((a - b)*(a + b)^(3/2)*d) - (2*Cot[c + d*x]*EllipticE
[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1
 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(a*Sqr
t[a + b]*d) - ((3*a - b)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c +
d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*
Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/((a - b)*(a + b)^(3/2)*d) + (2*Co
t[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)
/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]
))/(a - b))])/(a*Sqrt[a + b]*d) + (2*Sqrt[a + b]*Cot[c + d*x]*EllipticPi[(
a + b)/a, ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*S
qrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))
])/(a^2*d) - Cot[c + d*x]/(d*(a + b*Sec[c + d*x])^(3/2)) + (b^2*Tan[c + d*
x])/((a^2 - b^2)*d*(a + b*Sec[c + d*x])^(3/2)) + (4*a*b^2*Tan[c + d*x])/((
a^2 - b^2)^2*d*Sqrt[a + b*Sec[c + d*x]]) - (2*b^2*Tan[c + d*x])/(a*(a^2 -
b^2)*d*Sqrt[a + b*Sec[c + d*x]])
```

3.343.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4384
```
Int[cot[(c_.) + (d_.)*(x_)]^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_
), x_Symbol] :> Int[ExpandIntegrand[(a + b*Csc[c + d*x])^n, (-1 + Sec[c + d
*x]^2)^(-m/2), x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[a^2 - b^2, 0] &&
ILtQ[m/2, 0] && IntegerQ[n - 1/2] && EqQ[m, -2]
```

3.343.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2061 vs. 2(416) = 832.

Time = 8.20 (sec) , antiderivative size = 2062, normalized size of antiderivative = 4.59

method	result	size
default	Expression too large to display	2062

input
```
int(cot(d*x+c)^2/(a+b*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/d/a/(a-b)^2/(a+b)^2*(a+b*sec(d*x+c))^(1/2)/(b+a*cos(d*x+c))*(-8*(cos(d*x
+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*
EllipticPi(cot(d*x+c)-csc(d*x+c),-1,((a-b)/(a+b))^(1/2))*a^2*b^2*cos(d*x+c
)-a^3*b*cot(d*x+c)+2*a^2*b^2*cot(d*x+c)-8*EllipticPi(cot(d*x+c)-csc(d*x+c)
,-1,((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*c
os(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^2*b^2+4*(cos(d*x+c)/(cos(d*x+c)+1))^(1/
2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticPi(cot(d*x+c)-c
sc(d*x+c),-1,((a-b)/(a+b))^(1/2))*b^4*cos(d*x+c)-2*(cos(d*x+c)/(cos(d*x+c)
+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE(cot(d
*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))*b^4*cos(d*x+c)-2*(cos(d*x+c)/(cos(d*
x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF(c
ot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))*a^4*cos(d*x+c)+4*(cos(d*x+c)/(co
s(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Ellipti
cPi(cot(d*x+c)-csc(d*x+c),-1,((a-b)/(a+b))^(1/2))*a^4*cos(d*x+c)-2*Ellipti
cF(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(cos(d*x+c)+1))^
(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^4+4*EllipticPi(cot
(d*x+c)-csc(d*x+c),-1,((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/
2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^4+4*EllipticPi(cot(d*
x+c)-csc(d*x+c),-1,((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*
(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*b^4-2*EllipticE(cot(d*x...
```

3.343.5 Fricas [F(-1)]

Timed out.

$$\int \frac{\cot^2(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(cot(d*x+c)^2/(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.343.6 Sympy [F]

$$\int \frac{\cot^2(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{\cot^2(c+dx)}{(a+b\sec(c+dx))^{\frac{3}{2}}}\,dx$$

input
```
integrate(cot(d*x+c)**2/(a+b*sec(d*x+c))**(3/2),x)
```

output
```
Integral(cot(c + d*x)**2/(a + b*sec(c + d*x))**(3/2), x)
```

3.343.7 Maxima [F]

$$\int \frac{\cot^2(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{\cot(dx+c)^2}{(b\sec(dx+c)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(cot(d*x+c)^2/(a+b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate(cot(d*x + c)^2/(b*sec(d*x + c) + a)^(3/2), x)
```

3.343.8 Giac [F]

$$\int \frac{\cot^2(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{\cot(dx+c)^2}{(b\sec(dx+c)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(cot(d*x+c)^2/(a+b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
integrate(cot(d*x + c)^2/(b*sec(d*x + c) + a)^(3/2), x)
```

3.343.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cot^2(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{\cot(c+d\,x)^2}{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

input
```
int(cot(c + d*x)^2/(a + b/cos(c + d*x))^(3/2),x)
```

output
```
int(cot(c + d*x)^2/(a + b/cos(c + d*x))^(3/2), x)
```

3.344 $\int (a+b\sec(e+fx))^3(d\tan(e+fx))^n\,dx$

3.344.1 Optimal result . . . 2523
3.344.2 Mathematica [A] (verified) . . . 2524
3.344.3 Rubi [A] (verified) . . . 2524
3.344.4 Maple [F] . . . 2526
3.344.5 Fricas [F] . . . 2526
3.344.6 Sympy [F] . . . 2526
3.344.7 Maxima [F] . . . 2527
3.344.8 Giac [F] . . . 2527
3.344.9 Mupad [F(-1)] . . . 2527

3.344.1 Optimal result

Integrand size = 23, antiderivative size = 245

$$\int (a+b\sec(e+fx))^3(d\tan(e+fx))^n\,dx = \frac{3ab^2(d\tan(e+fx))^{1+n}}{df(1+n)} + \frac{a^3\,\text{Hypergeometric2F1}\left(1,\frac{1+n}{2},\frac{3+n}{2},-\tan^2(e+fx)\right)(d\tan(e+fx))^{1+n}}{df(1+n)} + \frac{3a^2b\cos^2(e+fx)^{\frac{2+n}{2}}\,\text{Hypergeometric2F1}\left(\frac{1+n}{2},\frac{2+n}{2},\frac{3+n}{2},\sin^2(e+fx)\right)\sec(e+fx)(d\tan(e+fx))^{1+n}}{df(1+n)} + \frac{b^3\cos^2(e+fx)^{\frac{4+n}{2}}\,\text{Hypergeometric2F1}\left(\frac{1+n}{2},\frac{4+n}{2},\frac{3+n}{2},\sin^2(e+fx)\right)\sec^3(e+fx)(d\tan(e+fx))^{1+n}}{df(1+n)}$$

output

```
3*a*b^2*(d*tan(f*x+e))^(1+n)/d/f/(1+n)+a^3*hypergeom([1, 1/2+1/2*n],[3/2+1
/2*n],-tan(f*x+e)^2)*(d*tan(f*x+e))^(1+n)/d/f/(1+n)+3*a^2*b*(cos(f*x+e)^2)
^(1+1/2*n)*hypergeom([1+1/2*n, 1/2+1/2*n],[3/2+1/2*n],sin(f*x+e)^2)*sec(f*
x+e)*(d*tan(f*x+e))^(1+n)/d/f/(1+n)+b^3*(cos(f*x+e)^2)^(2+1/2*n)*hypergeom
([2+1/2*n, 1/2+1/2*n],[3/2+1/2*n],sin(f*x+e)^2)*sec(f*x+e)^3*(d*tan(f*x+e)
)^(1+n)/d/f/(1+n)
```

3.344.2 Mathematica [A] (verified)

Time = 3.16 (sec) , antiderivative size = 238, normalized size of antiderivative = 0.97

$$\int (a+b\sec(e+fx))^3(d\tan(e+fx))^n\,dx$$
$$= \frac{d(d\tan(e+fx))^{-1+n}\left(-\tan^2(e+fx)\right)^{-n/2}\left(9a^2b(1+n)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1-n}{2},\frac{3}{2},\sec^2(e+fx)\right)\sec\right.}{}$$

input
```
Integrate[(a + b*Sec[e + f*x])^3*(d*Tan[e + f*x])^n,x]
```

output
```
(d*(d*Tan[e + f*x])^(-1 + n)*(9*a^2*b*(1 + n)*Hypergeometric2F1[1/2, (1 -
n)/2, 3/2, Sec[e + f*x]^2]*Sec[e + f*x]*Sqrt[-Tan[e + f*x]^2] + b^3*(1 + n
)*Hypergeometric2F1[3/2, (1 - n)/2, 5/2, Sec[e + f*x]^2]*Sec[e + f*x]^3*Sq
rt[-Tan[e + f*x]^2] - 3*a^3*Hypergeometric2F1[1, (1 + n)/2, (3 + n)/2, -Ta
n[e + f*x]^2]*(-Tan[e + f*x]^2)^((2 + n)/2) + 9*a*b^2*(Sqrt[-Tan[e + f*x]^
2] - (-Tan[e + f*x]^2)^((2 + n)/2))))/(3*f*(1 + n)*(-Tan[e + f*x]^2)^(n/2)
)
```

3.344.3 Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3042, 4374, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+b\sec(e+fx))^3(d\tan(e+fx))^n\,dx$$
$$\downarrow 3042$$
$$\int \left(a+b\csc\left(e+fx+\frac{\pi}{2}\right)\right)^3\left(-d\cot\left(e+fx+\frac{\pi}{2}\right)\right)^n dx$$
$$\downarrow 4374$$
$$\int (a^3(d\tan(e+fx))^n+3a^2b\sec(e+fx)(d\tan(e+fx))^n+3ab^2\sec^2(e+fx)(d\tan(e+fx))^n+b^3\sec^3(e+fx)$$
$$\downarrow 2009$$

$$\frac{a^3(d\tan(e+fx))^{n+1}\,\text{Hypergeometric2F1}\left(1,\frac{n+1}{2},\frac{n+3}{2},-\tan^2(e+fx)\right)}{df(n+1)}+$$
$$\frac{3a^2b\sec(e+fx)\cos^2(e+fx)^{\frac{n+2}{2}}(d\tan(e+fx))^{n+1}\,\text{Hypergeometric2F1}\left(\frac{n+1}{2},\frac{n+2}{2},\frac{n+3}{2},\sin^2(e+fx)\right)}{df(n+1)}+$$
$$\frac{3ab^2(d\tan(e+fx))^{n+1}}{df(n+1)}+$$
$$\frac{b^3\sec^3(e+fx)\cos^2(e+fx)^{\frac{n+4}{2}}(d\tan(e+fx))^{n+1}\,\text{Hypergeometric2F1}\left(\frac{n+1}{2},\frac{n+4}{2},\frac{n+3}{2},\sin^2(e+fx)\right)}{df(n+1)}$$

input
```
Int[(a + b*Sec[e + f*x])^3*(d*Tan[e + f*x])^n,x]
```

output
```
(3*a*b^2*(d*Tan[e + f*x])^(1 + n))/(d*f*(1 + n)) + (a^3*Hypergeometric2F1[
1, (1 + n)/2, (3 + n)/2, -Tan[e + f*x]^2]*(d*Tan[e + f*x])^(1 + n))/(d*f*(
1 + n)) + (3*a^2*b*(Cos[e + f*x]^2)^((2 + n)/2)*Hypergeometric2F1[(1 + n)/
2, (2 + n)/2, (3 + n)/2, Sin[e + f*x]^2]*Sec[e + f*x]*(d*Tan[e + f*x])^(1
+ n))/(d*f*(1 + n)) + (b^3*(Cos[e + f*x]^2)^((4 + n)/2)*Hypergeometric2F1[
(1 + n)/2, (4 + n)/2, (3 + n)/2, Sin[e + f*x]^2]*Sec[e + f*x]^3*(d*Tan[e +
 f*x])^(1 + n))/(d*f*(1 + n))
```

3.344.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4374
```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

3.344.4 Maple [F]

$$\int (a + b \sec(fx + e))^3 (d \tan(fx + e))^n \, dx$$

input
```
int((a+b*sec(f*x+e))^3*(d*tan(f*x+e))^n,x)
```

output
```
int((a+b*sec(f*x+e))^3*(d*tan(f*x+e))^n,x)
```

3.344.5 Fricas [F]

$$\int (a + b \sec(e + fx))^3 (d \tan(e + fx))^n \, dx = \int (b \sec(fx + e) + a)^3 (d \tan(fx + e))^n \, dx$$

input
```
integrate((a+b*sec(f*x+e))^3*(d*tan(f*x+e))^n,x, algorithm="fricas")
```

output
```
integral((b^3*sec(f*x + e)^3 + 3*a*b^2*sec(f*x + e)^2 + 3*a^2*b*sec(f*x +
e) + a^3)*(d*tan(f*x + e))^n, x)
```

3.344.6 Sympy [F]

$$\int (a + b \sec(e + fx))^3 (d \tan(e + fx))^n \, dx = \int (d \tan(e + fx))^n (a + b \sec(e + fx))^3 \, dx$$

input
```
integrate((a+b*sec(f*x+e))**3*(d*tan(f*x+e))**n,x)
```

output
```
Integral((d*tan(e + f*x))**n*(a + b*sec(e + f*x))**3, x)
```

3.344.7 Maxima [F]

$$\int (a+b\sec(e+fx))^3(d\tan(e+fx))^n\,dx = \int (b\sec(fx+e)+a)^3(d\tan(fx+e))^n\,dx$$

input
```
integrate((a+b*sec(f*x+e))^3*(d*tan(f*x+e))^n,x, algorithm="maxima")
```

output
```
integrate((b*sec(f*x + e) + a)^3*(d*tan(f*x + e))^n, x)
```

3.344.8 Giac [F]

$$\int (a+b\sec(e+fx))^3(d\tan(e+fx))^n\,dx = \int (b\sec(fx+e)+a)^3(d\tan(fx+e))^n\,dx$$

input
```
integrate((a+b*sec(f*x+e))^3*(d*tan(f*x+e))^n,x, algorithm="giac")
```

output
```
integrate((b*sec(f*x + e) + a)^3*(d*tan(f*x + e))^n, x)
```

3.344.9 Mupad [F(-1)]

Timed out.

$$\int (a+b\sec(e+fx))^3(d\tan(e+fx))^n\,dx = \int (d\tan(e+f\,x))^n\left(a+\frac{b}{\cos(e+f\,x)}\right)^3 dx$$

input
```
int((d*tan(e + f*x))^n*(a + b/cos(e + f*x))^3,x)
```

output
```
int((d*tan(e + f*x))^n*(a + b/cos(e + f*x))^3, x)
```

3.345 $\int (a+b\sec(e+fx))^2(d\tan(e+fx))^n\,dx$

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3.345.2 Mathematica [A] (verified) . . . 2528
3.345.3 Rubi [A] (verified) . . . 2529
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3.345.7 Maxima [F] . . . 2531
3.345.8 Giac [F] . . . 2531
3.345.9 Mupad [F(-1)] . . . 2532

3.345.1 Optimal result

Integrand size = 23, antiderivative size = 160

$$\int (a+b\sec(e+fx))^2(d\tan(e+fx))^n\,dx = \frac{b^2(d\tan(e+fx))^{1+n}}{df(1+n)} + \frac{a^2\,\text{Hypergeometric2F1}\left(1,\frac{1+n}{2},\frac{3+n}{2},-\tan^2(e+fx)\right)(d\tan(e+fx))^{1+n}}{df(1+n)} + \frac{2ab\cos^2(e+fx)^{\frac{2+n}{2}}\,\text{Hypergeometric2F1}\left(\frac{1+n}{2},\frac{2+n}{2},\frac{3+n}{2},\sin^2(e+fx)\right)\sec(e+fx)(d\tan(e+fx))^{1+n}}{df(1+n)}$$

output
```
b^2*(d*tan(f*x+e))^(1+n)/d/f/(1+n)+a^2*hypergeom([1, 1/2+1/2*n],[3/2+1/2*n
],-tan(f*x+e)^2)*(d*tan(f*x+e))^(1+n)/d/f/(1+n)+2*a*b*(cos(f*x+e)^2)^(1+1/
2*n)*hypergeom([1+1/2*n, 1/2+1/2*n],[3/2+1/2*n],sin(f*x+e)^2)*sec(f*x+e)*(
d*tan(f*x+e))^(1+n)/d/f/(1+n)
```

3.345.2 Mathematica [A] (verified)

Time = 1.01 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.11

$$\int (a+b\sec(e+fx))^2(d\tan(e+fx))^n\,dx = \frac{d(d\tan(e+fx))^{-1+n}\left(-\tan^2(e+fx)\right)^{-n/2}\Big(2ab(1+n)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1-n}{2},\frac{3}{2},\sec^2(e+fx)\right)\sec($$

input
```
Integrate[(a + b*Sec[e + f*x])^2*(d*Tan[e + f*x])^n,x]
```

output
```
(d*(d*Tan[e + f*x])^(-1 + n)*(2*a*b*(1 + n)*Hypergeometric2F1[1/2, (1 - n)
/2, 3/2, Sec[e + f*x]^2]*Sec[e + f*x]*Sqrt[-Tan[e + f*x]^2] - a^2*Hypergeo
metric2F1[1, (1 + n)/2, (3 + n)/2, -Tan[e + f*x]^2]*(-Tan[e + f*x]^2)^((2
+ n)/2) + b^2*(Sqrt[-Tan[e + f*x]^2] - (-Tan[e + f*x]^2)^((2 + n)/2))))/(f
*(1 + n)*(-Tan[e + f*x]^2)^(n/2))
```

3.345.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3042, 4374, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+b\sec(e+fx))^2(d\tan(e+fx))^n\,dx$$

$$\downarrow 3042$$

$$\int \left(a+b\csc\left(e+fx+\frac{\pi}{2}\right)\right)^2\left(-d\cot\left(e+fx+\frac{\pi}{2}\right)\right)^n dx$$

$$\downarrow 4374$$

$$\int \left(a^2(d\tan(e+fx))^n+2ab\sec(e+fx)(d\tan(e+fx))^n+b^2\sec^2(e+fx)(d\tan(e+fx))^n\right)dx$$

$$\downarrow 2009$$

$$\frac{a^2(d\tan(e+fx))^{n+1}\,\text{Hypergeometric2F1}\left(1,\frac{n+1}{2},\frac{n+3}{2},-\tan^2(e+fx)\right)}{df(n+1)}+$$
$$\frac{2ab\sec(e+fx)\cos^2(e+fx)^{\frac{n+2}{2}}(d\tan(e+fx))^{n+1}\,\text{Hypergeometric2F1}\left(\frac{n+1}{2},\frac{n+2}{2},\frac{n+3}{2},\sin^2(e+fx)\right)}{df(n+1)}+$$
$$\frac{b^2(d\tan(e+fx))^{n+1}}{df(n+1)}$$

input
```
Int[(a + b*Sec[e + f*x])^2*(d*Tan[e + f*x])^n,x]
```

output
```
(b^2*(d*Tan[e + f*x])^(1 + n))/(d*f*(1 + n)) + (a^2*Hypergeometric2F1[1, (
1 + n)/2, (3 + n)/2, -Tan[e + f*x]^2]*(d*Tan[e + f*x])^(1 + n))/(d*f*(1 +
n)) + (2*a*b*(Cos[e + f*x]^2)^((2 + n)/2)*Hypergeometric2F1[(1 + n)/2, (2
+ n)/2, (3 + n)/2, Sin[e + f*x]^2]*Sec[e + f*x]*(d*Tan[e + f*x])^(1 + n))/
(d*f*(1 + n))
```

3.345.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4374
```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

3.345.4 Maple [F]

$$\int (a + b \sec(fx + e))^2 (d \tan(fx + e))^n \, dx$$

input
```
int((a+b*sec(f*x+e))^2*(d*tan(f*x+e))^n,x)
```

output
```
int((a+b*sec(f*x+e))^2*(d*tan(f*x+e))^n,x)
```

3.345.5 Fricas [F]

$$\int (a + b \sec(e + fx))^2 (d \tan(e + fx))^n \, dx = \int (b \sec(fx + e) + a)^2 (d \tan(fx + e))^n \, dx$$

input
```
integrate((a+b*sec(f*x+e))^2*(d*tan(f*x+e))^n,x, algorithm="fricas")
```

output
```
integral((b^2*sec(f*x + e)^2 + 2*a*b*sec(f*x + e) + a^2)*(d*tan(f*x + e))^
n, x)
```

3.345.6 Sympy [F]

$$\int (a+b\sec(e+fx))^2(d\tan(e+fx))^n\,dx = \int (d\tan(e+fx))^n\,(a+b\sec(e+fx))^2\,dx$$

input
```
integrate((a+b*sec(f*x+e))**2*(d*tan(f*x+e))**n,x)
```

output
```
Integral((d*tan(e + f*x))**n*(a + b*sec(e + f*x))**2, x)
```

3.345.7 Maxima [F]

$$\int (a+b\sec(e+fx))^2(d\tan(e+fx))^n\,dx = \int (b\sec(fx+e)+a)^2(d\tan(fx+e))^n\,dx$$

input
```
integrate((a+b*sec(f*x+e))^2*(d*tan(f*x+e))^n,x, algorithm="maxima")
```

output
```
integrate((b*sec(f*x + e) + a)^2*(d*tan(f*x + e))^n, x)
```

3.345.8 Giac [F]

$$\int (a+b\sec(e+fx))^2(d\tan(e+fx))^n\,dx = \int (b\sec(fx+e)+a)^2(d\tan(fx+e))^n\,dx$$

input
```
integrate((a+b*sec(f*x+e))^2*(d*tan(f*x+e))^n,x, algorithm="giac")
```

output
```
integrate((b*sec(f*x + e) + a)^2*(d*tan(f*x + e))^n, x)
```

3.345.9 Mupad [F(-1)]

Timed out.

$$\int (a+b\sec(e+fx))^2(d\tan(e+fx))^n\,dx = \int (d\tan(e+f\,x))^n\left(a+\frac{b}{\cos(e+f\,x)}\right)^2 dx$$

input
```
int((d*tan(e + f*x))^n*(a + b/cos(e + f*x))^2,x)
```

output
```
int((d*tan(e + f*x))^n*(a + b/cos(e + f*x))^2, x)
```

3.346 $\int (a+b\sec(e+fx))(d\tan(e+fx))^n\,dx$

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3.346.2 Mathematica [A] (verified) . . . 2533
3.346.3 Rubi [A] (verified) . . . 2534
3.346.4 Maple [F] . . . 2536
3.346.5 Fricas [F] . . . 2536
3.346.6 Sympy [F] . . . 2536
3.346.7 Maxima [F] . . . 2537
3.346.8 Giac [F] . . . 2537
3.346.9 Mupad [F(-1)] . . . 2537

3.346.1 Optimal result

Integrand size = 21, antiderivative size = 129

$$\int (a+b\sec(e+fx))(d\tan(e+fx))^n\,dx$$
$$= \frac{a\,\text{Hypergeometric2F1}\left(1,\frac{1+n}{2},\frac{3+n}{2},-\tan^2(e+fx)\right)(d\tan(e+fx))^{1+n}}{df(1+n)}$$
$$+\frac{b\cos^2(e+fx)^{\frac{2+n}{2}}\,\text{Hypergeometric2F1}\left(\frac{1+n}{2},\frac{2+n}{2},\frac{3+n}{2},\sin^2(e+fx)\right)\sec(e+fx)(d\tan(e+fx))^{1+n}}{df(1+n)}$$

output
```
a*hypergeom([1, 1/2+1/2*n],[3/2+1/2*n],-tan(f*x+e)^2)*(d*tan(f*x+e))^(1+n)
/d/f/(1+n)+b*(cos(f*x+e)^2)^(1+1/2*n)*hypergeom([1+1/2*n, 1/2+1/2*n],[3/2+
1/2*n],sin(f*x+e)^2)*sec(f*x+e)*(d*tan(f*x+e))^(1+n)/d/f/(1+n)
```

3.346.2 Mathematica [A] (verified)

Time = 0.58 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.82

$$\int (a+b\sec(e+fx))(d\tan(e+fx))^n\,dx$$
$$= \frac{(d\tan(e+fx))^n\left(\frac{a\,\text{Hypergeometric2F1}\left(1,\frac{1+n}{2},\frac{3+n}{2},-\tan^2(e+fx)\right)\tan(e+fx)}{1+n}+b\csc(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\right.\right.}{f}$$

input
```
Integrate[(a + b*Sec[e + f*x])*(d*Tan[e + f*x])^n,x]
```

output
```
((d*Tan[e + f*x])^n*((a*Hypergeometric2F1[1, (1 + n)/2, (3 + n)/2, -Tan[e
+ f*x]^2]*Tan[e + f*x])/(1 + n) + b*Csc[e + f*x]*Hypergeometric2F1[1/2, (1
 - n)/2, 3/2, Sec[e + f*x]^2]*(-Tan[e + f*x]^2)^((1 - n)/2)))/f
```

3.346.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 4372, 3042, 3097, 3957, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+b\sec(e+fx))(d\tan(e+fx))^n\,dx$$

$$\downarrow 3042$$

$$\int \left(a+b\csc\left(e+fx+\frac{\pi}{2}\right)\right)\left(-d\cot\left(e+fx+\frac{\pi}{2}\right)\right)^n dx$$

$$\downarrow 4372$$

$$a\int (d\tan(e+fx))^n dx + b\int \sec(e+fx)(d\tan(e+fx))^n dx$$

$$\downarrow 3042$$

$$a\int (d\tan(e+fx))^n dx + b\int \sec(e+fx)(d\tan(e+fx))^n dx$$

$$\downarrow 3097$$

$$a\int (d\tan(e+fx))^n dx + \frac{b\sec(e+fx)\cos^2(e+fx)^{\frac{n+2}{2}}(d\tan(e+fx))^{n+1}\,\text{Hypergeometric2F1}\left(\frac{n+1}{2},\frac{n+2}{2},\frac{n+3}{2},\sin^2(e+fx)\right)}{df(n+1)}$$

$$\downarrow 3957$$

$$\frac{ad\int \frac{(d\tan(e+fx))^n}{\tan^2(e+fx)d^2+d^2}d(d\tan(e+fx))}{f} + \frac{b\sec(e+fx)\cos^2(e+fx)^{\frac{n+2}{2}}(d\tan(e+fx))^{n+1}\,\text{Hypergeometric2F1}\left(\frac{n+1}{2},\frac{n+2}{2},\frac{n+3}{2},\sin^2(e+fx)\right)}{df(n+1)}$$

$$\downarrow 278$$

$$\frac{a(d\tan(e+fx))^{n+1}\,\text{Hypergeometric2F1}\left(1,\frac{n+1}{2},\frac{n+3}{2},-\tan^2(e+fx)\right)}{df(n+1)}+$$
$$\frac{b\sec(e+fx)\cos^2(e+fx)^{\frac{n+2}{2}}(d\tan(e+fx))^{n+1}\,\text{Hypergeometric2F1}\left(\frac{n+1}{2},\frac{n+2}{2},\frac{n+3}{2},\sin^2(e+fx)\right)}{df(n+1)}$$

input
```
Int[(a + b*Sec[e + f*x])*(d*Tan[e + f*x])^n,x]
```

output
```
(a*Hypergeometric2F1[1, (1 + n)/2, (3 + n)/2, -Tan[e + f*x]^2]*(d*Tan[e +
f*x])^(1 + n))/(d*f*(1 + n)) + (b*(Cos[e + f*x]^2)^((2 + n)/2)*Hypergeomet
ric2F1[(1 + n)/2, (2 + n)/2, (3 + n)/2, Sin[e + f*x]^2]*Sec[e + f*x]*(d*Ta
n[e + f*x])^(1 + n))/(d*f*(1 + n))
```

3.346.3.1 Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3097
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1)*((Cos[e
+ f*x]^2)^((m + n + 1)/2)/(b*f*(n + 1)))*Hypergeometric2F1[(n + 1)/2, (m +
n + 1)/2, (n + 3)/2, Sin[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] &&
  !IntegerQ[(n - 1)/2] &&  !IntegerQ[m/2]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

rule 4372
```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(e*Cot[c + d*x])^m, x], x] + Simp[b   Int[
(e*Cot[c + d*x])^m*Csc[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, m}, x]
```

3.346.4 Maple [F]

$$\int (a + b \sec(fx + e)) (d \tan(fx + e))^n \, dx$$

input
```
int((a+b*sec(f*x+e))*(d*tan(f*x+e))^n,x)
```

output
```
int((a+b*sec(f*x+e))*(d*tan(f*x+e))^n,x)
```

3.346.5 Fricas [F]

$$\int (a + b \sec(e + fx))(d \tan(e + fx))^n \, dx = \int (b \sec(fx + e) + a)(d \tan(fx + e))^n \, dx$$

input
```
integrate((a+b*sec(f*x+e))*(d*tan(f*x+e))^n,x, algorithm="fricas")
```

output
```
integral((b*sec(f*x + e) + a)*(d*tan(f*x + e))^n, x)
```

3.346.6 Sympy [F]

$$\int (a + b \sec(e + fx))(d \tan(e + fx))^n \, dx = \int (d \tan(e + fx))^n (a + b \sec(e + fx)) \, dx$$

input
```
integrate((a+b*sec(f*x+e))*(d*tan(f*x+e))**n,x)
```

output
```
Integral((d*tan(e + f*x))**n*(a + b*sec(e + f*x)), x)
```

3.346.7 Maxima [F]

$$\int (a+b\sec(e+fx))(d\tan(e+fx))^n\,dx = \int (b\sec(fx+e)+a)(d\tan(fx+e))^n\,dx$$

input
```
integrate((a+b*sec(f*x+e))*(d*tan(f*x+e))^n,x, algorithm="maxima")
```

output
```
integrate((b*sec(f*x + e) + a)*(d*tan(f*x + e))^n, x)
```

3.346.8 Giac [F]

$$\int (a+b\sec(e+fx))(d\tan(e+fx))^n\,dx = \int (b\sec(fx+e)+a)(d\tan(fx+e))^n\,dx$$

input
```
integrate((a+b*sec(f*x+e))*(d*tan(f*x+e))^n,x, algorithm="giac")
```

output
```
integrate((b*sec(f*x + e) + a)*(d*tan(f*x + e))^n, x)
```

3.346.9 Mupad [F(-1)]

Timed out.

$$\int (a+b\sec(e+fx))(d\tan(e+fx))^n\,dx = \int (d\tan(e+f\,x))^n\left(a+\frac{b}{\cos(e+f\,x)}\right)\,dx$$

input
```
int((d*tan(e + f*x))^n*(a + b/cos(e + f*x)),x)
```

output
```
int((d*tan(e + f*x))^n*(a + b/cos(e + f*x)), x)
```

3.347 $\int \frac{(d\tan(e+fx))^n}{a+b\sec(e+fx)}\,dx$

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3.347.2 Mathematica [B] (warning: unable to verify) . . . 2539
3.347.3 Rubi [F] . . . 2540
3.347.4 Maple [F] . . . 2540
3.347.5 Fricas [F] . . . 2541
3.347.6 Sympy [F] . . . 2541
3.347.7 Maxima [F] . . . 2541
3.347.8 Giac [F] . . . 2542
3.347.9 Mupad [F(-1)] . . . 2542

3.347.1 Optimal result

Integrand size = 23, antiderivative size = 266

$$\int \frac{(d\tan(e+fx))^n}{a+b\sec(e+fx)}\,dx$$
$$= \frac{d\,\text{AppellF1}\left(1-n,\frac{1-n}{2},\frac{1-n}{2},2-n,\frac{a+b}{a+b\sec(e+fx)},\frac{a-b}{a+b\sec(e+fx)}\right)\left(-\frac{b(1-\sec(e+fx))}{a+b\sec(e+fx)}\right)^{\frac{1-n}{2}}\left(\frac{b(1+\sec(e+fx))}{a+b\sec(e+fx)}\right)^{\frac{1-n}{2}}(d\ta}{af(1-n)}$$
$$-\frac{d\,\text{Hypergeometric2F1}\left(1,\frac{1+n}{2},\frac{3+n}{2},-\tan^2(e+fx)\right)(d\tan(e+fx))^{-1+n}\left(-\tan^2(e+fx)\right)^{\frac{1-n}{2}+\frac{1+n}{2}}}{af(1+n)}$$

output

```
d*AppellF1(1-n,1/2-1/2*n,1/2-1/2*n,2-n,(a-b)/(a+b*sec(f*x+e)),(a+b)/(a+b*s
ec(f*x+e)))*(-b*(1-sec(f*x+e))/(a+b*sec(f*x+e)))^(1/2-1/2*n)*(b*(1+sec(f*x
+e))/(a+b*sec(f*x+e)))^(1/2-1/2*n)*(d*tan(f*x+e))^(-1+n)/a/f/(1-n)+d*hyper
geom([1, 1/2+1/2*n],[3/2+1/2*n],-tan(f*x+e)^2)*(d*tan(f*x+e))^(-1+n)*tan(f
*x+e)^2/a/f/(1+n)
```

3.347.2 Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 786 vs. $2(266) = 532$.

Time = 4.22 (sec) , antiderivative size = 786, normalized size of antiderivative = 2.95

$$\int \frac{(d\tan(e+fx))^n}{a+b\sec(e+fx)}\,dx$$

$$= \frac{}{f(a+b\sec(e+fx))\left(\left((a+b)\,\text{AppellF1}\left(\frac{1+n}{2}, n, 1, \frac{3+n}{2}, \tan^2\left(\frac{1}{2}(e+fx)\right), -\tan^2\left(\frac{1}{2}(e+fx)\right)\right) - b\,\text{Appe}\right.}$$

input
```
Integrate[(d*Tan[e + f*x])^n/(a + b*Sec[e + f*x]),x]
```

output
```
(2*((a + b)*AppellF1[(1 + n)/2, n, 1, (3 + n)/2, Tan[(e + f*x)/2]^2, -Tan[
(e + f*x)/2]^2] - b*AppellF1[(1 + n)/2, n, 1, (3 + n)/2, Tan[(e + f*x)/2]^
2, ((a - b)*Tan[(e + f*x)/2]^2)/(a + b)])*Tan[(e + f*x)/2]*(d*Tan[e + f*x]
)^n)/(f*(a + b*Sec[e + f*x])*(((a + b)*AppellF1[(1 + n)/2, n, 1, (3 + n)/2
, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] - b*AppellF1[(1 + n)/2, n, 1, (
3 + n)/2, Tan[(e + f*x)/2]^2, ((a - b)*Tan[(e + f*x)/2]^2)/(a + b)])*Sec[(
e + f*x)/2]^2 - 16*n*((a + b)*AppellF1[(1 + n)/2, n, 1, (3 + n)/2, Tan[(e
+ f*x)/2]^2, -Tan[(e + f*x)/2]^2] - b*AppellF1[(1 + n)/2, n, 1, (3 + n)/2,
 Tan[(e + f*x)/2]^2, ((a - b)*Tan[(e + f*x)/2]^2)/(a + b)])*Cos[(e + f*x)/
2]*Csc[e + f*x]^3*Sec[e + f*x]*Sin[(e + f*x)/2]^5 + 2*n*((a + b)*AppellF1[
(1 + n)/2, n, 1, (3 + n)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] - b*A
ppellF1[(1 + n)/2, n, 1, (3 + n)/2, Tan[(e + f*x)/2]^2, ((a - b)*Tan[(e +
f*x)/2]^2)/(a + b)])*Csc[e + f*x]*Sec[e + f*x]*Tan[(e + f*x)/2] - (2*(1 +
n)*((a - b)*b*AppellF1[(3 + n)/2, n, 2, (5 + n)/2, Tan[(e + f*x)/2]^2, ((a
 - b)*Tan[(e + f*x)/2]^2)/(a + b)] + (a + b)^2*(AppellF1[(3 + n)/2, n, 2,
(5 + n)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] - n*AppellF1[(3 + n)/2
, 1 + n, 1, (5 + n)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]) + b*(a +
b)*n*AppellF1[(3 + n)/2, 1 + n, 1, (5 + n)/2, Tan[(e + f*x)/2]^2, ((a - b)
*Tan[(e + f*x)/2]^2)/(a + b)])*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2]^2)/((a
+ b)*(3 + n))))
```

3.347.3 Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d\tan(e+fx))^n}{a+b\sec(e+fx)}\,dx$$
$$\downarrow 3042$$
$$\int \frac{(-d\cot\left(e+fx+\frac{\pi}{2}\right))^n}{a+b\csc\left(e+fx+\frac{\pi}{2}\right)}\,dx$$
$$\downarrow 4387$$
$$\int \frac{(d\tan(e+fx))^n}{a+b\sec(e+fx)}\,dx$$

input
```
Int[(d*Tan[e + f*x])^n/(a + b*Sec[e + f*x]),x]
```

output
```
$Aborted
```

3.347.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4387
```
Int[((a_.) + csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_.)*(cot[(c_.) + (d_.)*(x_)]*
(e_.))^(m_.), x_Symbol] :> Unintegrable[(e*Cot[c + d*x])^m*(a + b*Csc[c + d
*x])^n, x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

3.347.4 Maple [F]

$$\int \frac{(d\tan\left(fx+e\right))^n}{a+b\sec\left(fx+e\right)}\,dx$$

input
```
int((d*tan(f*x+e))^n/(a+b*sec(f*x+e)),x)
```

output
```
int((d*tan(f*x+e))^n/(a+b*sec(f*x+e)),x)
```

3.347.5 Fricas [F]

$$\int \frac{(d\tan(e+fx))^n}{a+b\sec(e+fx)}\,dx = \int \frac{(d\tan(fx+e))^n}{b\sec(fx+e)+a}\,dx$$

input
```
integrate((d*tan(f*x+e))^n/(a+b*sec(f*x+e)),x, algorithm="fricas")
```

output
```
integral((d*tan(f*x + e))^n/(b*sec(f*x + e) + a), x)
```

3.347.6 Sympy [F]

$$\int \frac{(d\tan(e+fx))^n}{a+b\sec(e+fx)}\,dx = \int \frac{(d\tan(e+fx))^n}{a+b\sec(e+fx)}\,dx$$

input
```
integrate((d*tan(f*x+e))**n/(a+b*sec(f*x+e)),x)
```

output
```
Integral((d*tan(e + f*x))**n/(a + b*sec(e + f*x)), x)
```

3.347.7 Maxima [F]

$$\int \frac{(d\tan(e+fx))^n}{a+b\sec(e+fx)}\,dx = \int \frac{(d\tan(fx+e))^n}{b\sec(fx+e)+a}\,dx$$

input
```
integrate((d*tan(f*x+e))^n/(a+b*sec(f*x+e)),x, algorithm="maxima")
```

output
```
integrate((d*tan(f*x + e))^n/(b*sec(f*x + e) + a), x)
```

3.347.8 Giac [F]

$$\int \frac{(d\tan(e+fx))^n}{a+b\sec(e+fx)}\,dx = \int \frac{(d\tan(fx+e))^n}{b\sec(fx+e)+a}\,dx$$

input
```
integrate((d*tan(f*x+e))^n/(a+b*sec(f*x+e)),x, algorithm="giac")
```

output
```
integrate((d*tan(f*x + e))^n/(b*sec(f*x + e) + a), x)
```

3.347.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d\tan(e+fx))^n}{a+b\sec(e+fx)}\,dx = \int \frac{\cos(e+fx)\,(d\tan(e+fx))^n}{b+a\cos(e+fx)}\,dx$$

input
```
int((d*tan(e + f*x))^n/(a + b/cos(e + f*x)),x)
```

output
```
int((cos(e + f*x)*(d*tan(e + f*x))^n)/(b + a*cos(e + f*x)), x)
```

3.348 $\int (a+b\sec(c+dx))^{3/2}(e\tan(c+dx))^m\,dx$

3.348.1 Optimal result . . . 2543
3.348.2 Mathematica [N/A] . . . 2543
3.348.3 Rubi [N/A] . . . 2544
3.348.4 Maple [N/A] (verified) . . . 2545
3.348.5 Fricas [N/A] . . . 2545
3.348.6 Sympy [N/A] . . . 2545
3.348.7 Maxima [N/A] . . . 2546
3.348.8 Giac [N/A] . . . 2546
3.348.9 Mupad [N/A] . . . 2546

3.348.1 Optimal result

Integrand size = 25, antiderivative size = 25

$$\int (a+b\sec(c+dx))^{3/2}(e\tan(c+dx))^m\,dx = \text{Int}\left((a+b\sec(c+dx))^{3/2}(e\tan(c+dx))^m, x\right)$$

output
```
Unintegrable((a+b*sec(d*x+c))^(3/2)*(e*tan(d*x+c))^m,x)
```

3.348.2 Mathematica [N/A]

Not integrable

Time = 29.43 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int (a+b\sec(c+dx))^{3/2}(e\tan(c+dx))^m\,dx = \int (a+b\sec(c+dx))^{3/2}(e\tan(c+dx))^m\,dx$$

input
```
Integrate[(a + b*Sec[c + d*x])^(3/2)*(e*Tan[c + d*x])^m,x]
```

output
```
Integrate[(a + b*Sec[c + d*x])^(3/2)*(e*Tan[c + d*x])^m, x]
```

3.348.3 Rubi [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {3042, 4387}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+b\sec(c+dx))^{3/2}(e\tan(c+dx))^m\,dx$$

$$\downarrow 3042$$

$$\int \left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}\left(-e\cot\left(c+dx+\frac{\pi}{2}\right)\right)^m dx$$

$$\downarrow 4387$$

$$\int (a+b\sec(c+dx))^{3/2}(e\tan(c+dx))^m dx$$

input
```
Int[(a + b*Sec[c + d*x])^(3/2)*(e*Tan[c + d*x])^m,x]
```

output
```
$Aborted
```

3.348.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4387
```
Int[((a_.) + csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_.)*(cot[(c_.) + (d_.)*(x_)]*
(e_.))^(m_.), x_Symbol] :> Unintegrable[(e*Cot[c + d*x])^m*(a + b*Csc[c + d
*x])^n, x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

3.348.4 Maple [N/A] (verified)

Not integrable

Time = 1.18 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int (a+b\sec(dx+c))^{\frac{3}{2}}\,(e\tan(dx+c))^m\,dx$$

input
```
int((a+b*sec(d*x+c))^(3/2)*(e*tan(d*x+c))^m,x)
```

output
```
int((a+b*sec(d*x+c))^(3/2)*(e*tan(d*x+c))^m,x)
```

3.348.5 Fricas [N/A]

Not integrable

Time = 0.39 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int (a+b\sec(c+dx))^{3/2}(e\tan(c+dx))^m\,dx = \int (b\sec(dx+c)+a)^{\frac{3}{2}}(e\tan(dx+c))^m\,dx$$

input
```
integrate((a+b*sec(d*x+c))^(3/2)*(e*tan(d*x+c))^m,x, algorithm="fricas")
```

output
```
integral((b*sec(d*x + c) + a)^(3/2)*(e*tan(d*x + c))^m, x)
```

3.348.6 Sympy [N/A]

Not integrable

Time = 69.96 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.96

$$\int (a+b\sec(c+dx))^{3/2}(e\tan(c+dx))^m\,dx = \int (e\tan(c+dx))^m\,(a+b\sec(c+dx))^{\frac{3}{2}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))**(3/2)*(e*tan(d*x+c))**m,x)
```

output
```
Integral((e*tan(c + d*x))**m*(a + b*sec(c + d*x))**(3/2), x)
```

3.348.7 Maxima [N/A]

Not integrable

Time = 1.43 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int (a+b\sec(c+dx))^{3/2}(e\tan(c+dx))^m\,dx = \int (b\sec(dx+c)+a)^{\frac{3}{2}}(e\tan(dx+c))^m\,dx$$

input
```
integrate((a+b*sec(d*x+c))^(3/2)*(e*tan(d*x+c))^m,x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c) + a)^(3/2)*(e*tan(d*x + c))^m, x)
```

3.348.8 Giac [N/A]

Not integrable

Time = 1.87 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int (a+b\sec(c+dx))^{3/2}(e\tan(c+dx))^m\,dx = \int (b\sec(dx+c)+a)^{\frac{3}{2}}(e\tan(dx+c))^m\,dx$$

input
```
integrate((a+b*sec(d*x+c))^(3/2)*(e*tan(d*x+c))^m,x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c) + a)^(3/2)*(e*tan(d*x + c))^m, x)
```

3.348.9 Mupad [N/A]

Not integrable

Time = 19.00 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int (a+b\sec(c+dx))^{3/2}(e\tan(c+dx))^m\,dx = \int (e\tan(c+d\,x))^m\left(a+\frac{b}{\cos(c+d\,x)}\right)^{3/2}dx$$

input
```
int((e*tan(c + d*x))^m*(a + b/cos(c + d*x))^(3/2),x)
```

output
```
int((e*tan(c + d*x))^m*(a + b/cos(c + d*x))^(3/2), x)
```

3.349 $\int \sqrt{a+b\sec(c+dx)}(e\tan(c+dx))^m\,dx$

3.349.1 Optimal result 2547
3.349.2 Mathematica [N/A] 2547
3.349.3 Rubi [N/A] 2548
3.349.4 Maple [N/A] (verified) 2549
3.349.5 Fricas [N/A] 2549
3.349.6 Sympy [N/A] 2549
3.349.7 Maxima [N/A] 2550
3.349.8 Giac [N/A] 2550
3.349.9 Mupad [N/A] 2550

3.349.1 Optimal result

Integrand size = 25, antiderivative size = 25

$$\int \sqrt{a+b\sec(c+dx)}(e\tan(c+dx))^m\,dx = \text{Int}\left(\sqrt{a+b\sec(c+dx)}(e\tan(c+dx))^m, x\right)$$

output
```
Unintegrable((a+b*sec(d*x+c))^(1/2)*(e*tan(d*x+c))^m,x)
```

3.349.2 Mathematica [N/A]

Not integrable

Time = 0.89 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \sqrt{a+b\sec(c+dx)}(e\tan(c+dx))^m\,dx = \int \sqrt{a+b\sec(c+dx)}(e\tan(c+dx))^m\,dx$$

input
```
Integrate[Sqrt[a + b*Sec[c + d*x]]*(e*Tan[c + d*x])^m,x]
```

output
```
Integrate[Sqrt[a + b*Sec[c + d*x]]*(e*Tan[c + d*x])^m, x]
```

3.349.3 Rubi [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {3042, 4387}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a+b\sec(c+dx)}(e\tan(c+dx))^m\,dx$$

$$\downarrow 3042$$

$$\int \sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}\left(-e\cot\left(c+dx+\frac{\pi}{2}\right)\right)^m dx$$

$$\downarrow 4387$$

$$\int \sqrt{a+b\sec(c+dx)}(e\tan(c+dx))^m dx$$

input
```
Int[Sqrt[a + b*Sec[c + d*x]]*(e*Tan[c + d*x])^m,x]
```

output
```
$Aborted
```

3.349.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4387
```
Int[((a_.) + csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_.)*(cot[(c_.) + (d_.)*(x_)]*
(e_.))^(m_.), x_Symbol] :> Unintegrable[(e*Cot[c + d*x])^m*(a + b*Csc[c + d
*x])^n, x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

3.349.4 Maple [N/A] (verified)

Not integrable

Time = 1.10 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \sqrt{a+b\sec(dx+c)}\,(e\tan(dx+c))^m\,dx$$

input
```
int((a+b*sec(d*x+c))^(1/2)*(e*tan(d*x+c))^m,x)
```

output
```
int((a+b*sec(d*x+c))^(1/2)*(e*tan(d*x+c))^m,x)
```

3.349.5 Fricas [N/A]

Not integrable

Time = 0.38 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \sqrt{a+b\sec(c+dx)}(e\tan(c+dx))^m\,dx = \int \sqrt{b\sec(dx+c)+a}(e\tan(dx+c))^m\,dx$$

input
```
integrate((a+b*sec(d*x+c))^(1/2)*(e*tan(d*x+c))^m,x, algorithm="fricas")
```

output
```
integral(sqrt(b*sec(d*x + c) + a)*(e*tan(d*x + c))^m, x)
```

3.349.6 Sympy [N/A]

Not integrable

Time = 1.41 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.96

$$\int \sqrt{a+b\sec(c+dx)}(e\tan(c+dx))^m\,dx = \int (e\tan(c+dx))^m\sqrt{a+b\sec(c+dx)}\,dx$$

input
```
integrate((a+b*sec(d*x+c))**(1/2)*(e*tan(d*x+c))**m,x)
```

output
```
Integral((e*tan(c + d*x))**m*sqrt(a + b*sec(c + d*x)), x)
```

3.349.7 Maxima [N/A]

Not integrable

Time = 1.33 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \sqrt{a+b\sec(c+dx)}(e\tan(c+dx))^m\,dx = \int \sqrt{b\sec(dx+c)+a}(e\tan(dx+c))^m\,dx$$

input
```
integrate((a+b*sec(d*x+c))^(1/2)*(e*tan(d*x+c))^m,x, algorithm="maxima")
```

output
```
integrate(sqrt(b*sec(d*x + c) + a)*(e*tan(d*x + c))^m, x)
```

3.349.8 Giac [N/A]

Not integrable

Time = 1.04 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \sqrt{a+b\sec(c+dx)}(e\tan(c+dx))^m\,dx = \int \sqrt{b\sec(dx+c)+a}(e\tan(dx+c))^m\,dx$$

input
```
integrate((a+b*sec(d*x+c))^(1/2)*(e*tan(d*x+c))^m,x, algorithm="giac")
```

output
```
integrate(sqrt(b*sec(d*x + c) + a)*(e*tan(d*x + c))^m, x)
```

3.349.9 Mupad [N/A]

Not integrable

Time = 14.98 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \sqrt{a+b\sec(c+dx)}(e\tan(c+dx))^m\,dx = \int (e\tan(c+d\,x))^m\sqrt{a+\frac{b}{\cos(c+d\,x)}}\,dx$$

input
```
int((e*tan(c + d*x))^m*(a + b/cos(c + d*x))^(1/2),x)
```

output
```
int((e*tan(c + d*x))^m*(a + b/cos(c + d*x))^(1/2), x)
```

3.350 $\int \frac{(e\tan(c+dx))^m}{\sqrt{a+b\sec(c+dx)}}\,dx$

3.350.1 Optimal result 2551
3.350.2 Mathematica [N/A] 2551
3.350.3 Rubi [N/A] 2552
3.350.4 Maple [N/A] (verified) 2553
3.350.5 Fricas [N/A] 2553
3.350.6 Sympy [N/A] 2553
3.350.7 Maxima [N/A] 2554
3.350.8 Giac [N/A] 2554
3.350.9 Mupad [N/A] 2554

3.350.1 Optimal result

Integrand size = 25, antiderivative size = 25

$$\int \frac{(e\tan(c+dx))^m}{\sqrt{a+b\sec(c+dx)}}\,dx = \text{Int}\left(\frac{(e\tan(c+dx))^m}{\sqrt{a+b\sec(c+dx)}}, x\right)$$

output
```
Unintegrable((e*tan(d*x+c))^m/(a+b*sec(d*x+c))^(1/2),x)
```

3.350.2 Mathematica [N/A]

Not integrable

Time = 15.73 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{(e\tan(c+dx))^m}{\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{(e\tan(c+dx))^m}{\sqrt{a+b\sec(c+dx)}}\,dx$$

input
```
Integrate[(e*Tan[c + d*x])^m/Sqrt[a + b*Sec[c + d*x]],x]
```

output
```
Integrate[(e*Tan[c + d*x])^m/Sqrt[a + b*Sec[c + d*x]], x]
```

3.350.3 Rubi [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {3042, 4387}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e\tan(c+dx))^m}{\sqrt{a+b\sec(c+dx)}}\,dx$$
$$\downarrow 3042$$
$$\int \frac{(-e\cot\left(c+dx+\frac{\pi}{2}\right))^m}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx$$
$$\downarrow 4387$$
$$\int \frac{(e\tan(c+dx))^m}{\sqrt{a+b\sec(c+dx)}}dx$$

input
```
Int[(e*Tan[c + d*x])^m/Sqrt[a + b*Sec[c + d*x]],x]
```

output
```
$Aborted
```

3.350.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4387
```
Int[((a_.) + csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_.)*(cot[(c_.) + (d_.)*(x_)]*
(e_.))^(m_.), x_Symbol] :> Unintegrable[(e*Cot[c + d*x])^m*(a + b*Csc[c + d
*x])^n, x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

3.350.4 Maple [N/A] (verified)

Not integrable

Time = 1.15 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{(e\tan(dx+c))^m}{\sqrt{a+b\sec(dx+c)}}dx$$

input
```
int((e*tan(d*x+c))^m/(a+b*sec(d*x+c))^(1/2),x)
```

output
```
int((e*tan(d*x+c))^m/(a+b*sec(d*x+c))^(1/2),x)
```

3.350.5 Fricas [N/A]

Not integrable

Time = 0.38 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{(e\tan(c+dx))^m}{\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{(e\tan(dx+c))^m}{\sqrt{b\sec(dx+c)+a}}\,dx$$

input
```
integrate((e*tan(d*x+c))^m/(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
integral((e*tan(d*x + c))^m/sqrt(b*sec(d*x + c) + a), x)
```

3.350.6 Sympy [N/A]

Not integrable

Time = 0.72 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.96

$$\int \frac{(e\tan(c+dx))^m}{\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{(e\tan(c+dx))^m}{\sqrt{a+b\sec(c+dx)}}\,dx$$

input
```
integrate((e*tan(d*x+c))**m/(a+b*sec(d*x+c))**(1/2),x)
```

output
```
Integral((e*tan(c + d*x))**m/sqrt(a + b*sec(c + d*x)), x)
```

3.350.7 Maxima [N/A]

Not integrable

Time = 1.33 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{(e\tan(c+dx))^m}{\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{(e\tan(dx+c))^m}{\sqrt{b\sec(dx+c)+a}}\,dx$$

input
```
integrate((e*tan(d*x+c))^m/(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate((e*tan(d*x + c))^m/sqrt(b*sec(d*x + c) + a), x)
```

3.350.8 Giac [N/A]

Not integrable

Time = 0.84 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{(e\tan(c+dx))^m}{\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{(e\tan(dx+c))^m}{\sqrt{b\sec(dx+c)+a}}\,dx$$

input
```
integrate((e*tan(d*x+c))^m/(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate((e*tan(d*x + c))^m/sqrt(b*sec(d*x + c) + a), x)
```

3.350.9 Mupad [N/A]

Not integrable

Time = 20.96 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{(e\tan(c+dx))^m}{\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{(e\tan(c+d\,x))^m}{\sqrt{a+\frac{b}{\cos(c+d\,x)}}}\,dx$$

input
```
int((e*tan(c + d*x))^m/(a + b/cos(c + d*x))^(1/2),x)
```

output
```
int((e*tan(c + d*x))^m/(a + b/cos(c + d*x))^(1/2), x)
```

3.351 $\int \frac{(e\tan(c+dx))^m}{(a+b\sec(c+dx))^{3/2}}\,dx$

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3.351.7 Maxima [N/A] . . . 2558
3.351.8 Giac [N/A] . . . 2558
3.351.9 Mupad [N/A] . . . 2558

3.351.1 Optimal result

Integrand size = 25, antiderivative size = 25

$$\int \frac{(e\tan(c+dx))^m}{(a+b\sec(c+dx))^{3/2}}\,dx = \text{Int}\left(\frac{(e\tan(c+dx))^m}{(a+b\sec(c+dx))^{3/2}}, x\right)$$

output
```
Unintegrable((e*tan(d*x+c))^m/(a+b*sec(d*x+c))^(3/2),x)
```

3.351.2 Mathematica [N/A]

Not integrable

Time = 19.97 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{(e\tan(c+dx))^m}{(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{(e\tan(c+dx))^m}{(a+b\sec(c+dx))^{3/2}}\,dx$$

input
```
Integrate[(e*Tan[c + d*x])^m/(a + b*Sec[c + d*x])^(3/2),x]
```

output
```
Integrate[(e*Tan[c + d*x])^m/(a + b*Sec[c + d*x])^(3/2), x]
```

3.351.3 Rubi [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {3042, 4387}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e\tan(c+dx))^m}{(a+b\sec(c+dx))^{3/2}}\,dx$$
$$\downarrow 3042$$
$$\int \frac{\left(-e\cot\left(c+dx+\frac{\pi}{2}\right)\right)^m}{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx$$
$$\downarrow 4387$$
$$\int \frac{(e\tan(c+dx))^m}{(a+b\sec(c+dx))^{3/2}}dx$$

input
```
Int[(e*Tan[c + d*x])^m/(a + b*Sec[c + d*x])^(3/2),x]
```

output
```
$Aborted
```

3.351.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4387
```
Int[((a_.) + csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_.)*(cot[(c_.) + (d_.)*(x_)]*
(e_.))^(m_.), x_Symbol] :> Unintegrable[(e*Cot[c + d*x])^m*(a + b*Csc[c + d
*x])^n, x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

3.351.4 Maple [N/A] (verified)

Not integrable

Time = 1.13 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{(e\tan(dx+c))^m}{(a+b\sec(dx+c))^{\frac{3}{2}}}dx$$

input
```
int((e*tan(d*x+c))^m/(a+b*sec(d*x+c))^(3/2),x)
```

output
```
int((e*tan(d*x+c))^m/(a+b*sec(d*x+c))^(3/2),x)
```

3.351.5 Fricas [N/A]

Not integrable

Time = 0.54 (sec) , antiderivative size = 53, normalized size of antiderivative = 2.12

$$\int \frac{(e\tan(c+dx))^m}{(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{(e\tan(dx+c))^m}{(b\sec(dx+c)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*tan(d*x+c))^m/(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b*sec(d*x + c) + a)*(e*tan(d*x + c))^m/(b^2*sec(d*x + c)^2 +
 2*a*b*sec(d*x + c) + a^2), x)
```

3.351.6 Sympy [N/A]

Not integrable

Time = 4.32 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.96

$$\int \frac{(e\tan(c+dx))^m}{(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{(e\tan(c+dx))^m}{(a+b\sec(c+dx))^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*tan(d*x+c))**m/(a+b*sec(d*x+c))**(3/2),x)
```

output
```
Integral((e*tan(c + d*x))**m/(a + b*sec(c + d*x))**(3/2), x)
```

3.351.7 Maxima [N/A]

Not integrable

Time = 1.25 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{(e\tan(c+dx))^m}{(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{(e\tan(dx+c))^m}{(b\sec(dx+c)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*tan(d*x+c))^m/(a+b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate((e*tan(d*x + c))^m/(b*sec(d*x + c) + a)^(3/2), x)
```

3.351.8 Giac [N/A]

Not integrable

Time = 1.42 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{(e\tan(c+dx))^m}{(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{(e\tan(dx+c))^m}{(b\sec(dx+c)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*tan(d*x+c))^m/(a+b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
integrate((e*tan(d*x + c))^m/(b*sec(d*x + c) + a)^(3/2), x)
```

3.351.9 Mupad [N/A]

Not integrable

Time = 28.13 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{(e\tan(c+dx))^m}{(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{(e\tan(c+d\,x))^m}{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

input
```
int((e*tan(c + d*x))^m/(a + b/cos(c + d*x))^(3/2),x)
```

output
```
int((e*tan(c + d*x))^m/(a + b/cos(c + d*x))^(3/2), x)
```

3.352 $\int (a+b\sec(c+dx))^n(e\tan(c+dx))^m\,dx$

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3.352.8 Giac [N/A] 2562
3.352.9 Mupad [N/A] 2562

3.352.1 Optimal result

Integrand size = 23, antiderivative size = 23

$$\int (a+b\sec(c+dx))^n(e\tan(c+dx))^m\,dx = \text{Int}((a+b\sec(c+dx))^n(e\tan(c+dx))^m, x)$$

output
```
Unintegrable((a+b*sec(d*x+c))^n*(e*tan(d*x+c))^m,x)
```

3.352.2 Mathematica [N/A]

Not integrable

Time = 7.76 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int (a+b\sec(c+dx))^n(e\tan(c+dx))^m\,dx = \int (a+b\sec(c+dx))^n(e\tan(c+dx))^m\,dx$$

input
```
Integrate[(a + b*Sec[c + d*x])^n*(e*Tan[c + d*x])^m,x]
```

output
```
Integrate[(a + b*Sec[c + d*x])^n*(e*Tan[c + d*x])^m, x]
```

3.352.3 Rubi [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {3042, 4387}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (e\tan(c+dx))^m (a+b\sec(c+dx))^n\,dx$$

$$\downarrow 3042$$

$$\int \left(-e\cot\left(c+dx+\frac{\pi}{2}\right)\right)^m \left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^n dx$$

$$\downarrow 4387$$

$$\int (e\tan(c+dx))^m (a+b\sec(c+dx))^n dx$$

input
```
Int[(a + b*Sec[c + d*x])^n*(e*Tan[c + d*x])^m,x]
```

output
```
$Aborted
```

3.352.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4387
```
Int[((a_.) + csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_.)*(cot[(c_.) + (d_.)*(x_)]*
(e_.))^(m_.), x_Symbol] :> Unintegrable[(e*Cot[c + d*x])^m*(a + b*Csc[c + d
*x])^n, x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

3.352.4 Maple [N/A] (verified)

Not integrable

Time = 1.87 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int (a + b \sec (dx + c))^n (e \tan (dx + c))^m \, dx$$

input
```
int((a+b*sec(d*x+c))^n*(e*tan(d*x+c))^m,x)
```

output
```
int((a+b*sec(d*x+c))^n*(e*tan(d*x+c))^m,x)
```

3.352.5 Fricas [N/A]

Not integrable

Time = 0.46 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int (a + b \sec(c + dx))^n (e \tan(c + dx))^m \, dx = \int (b \sec (dx + c) + a)^n (e \tan (dx + c))^m \, dx$$

input
```
integrate((a+b*sec(d*x+c))^n*(e*tan(d*x+c))^m,x, algorithm="fricas")
```

output
```
integral((b*sec(d*x + c) + a)^n*(e*tan(d*x + c))^m, x)
```

3.352.6 Sympy [N/A]

Not integrable

Time = 33.98 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int (a + b \sec(c + dx))^n (e \tan(c + dx))^m \, dx = \int (e \tan (c + dx))^m (a + b \sec (c + dx))^n \, dx$$

input
```
integrate((a+b*sec(d*x+c))**n*(e*tan(d*x+c))**m,x)
```

output
```
Integral((e*tan(c + d*x))**m*(a + b*sec(c + d*x))**n, x)
```

3.352.7 Maxima [N/A]

Not integrable

Time = 1.89 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int (a+b\sec(c+dx))^n(e\tan(c+dx))^m\,dx = \int (b\sec(dx+c)+a)^n(e\tan(dx+c))^m\,dx$$

input
```
integrate((a+b*sec(d*x+c))^n*(e*tan(d*x+c))^m,x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c) + a)^n*(e*tan(d*x + c))^m, x)
```

3.352.8 Giac [N/A]

Not integrable

Time = 0.83 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int (a+b\sec(c+dx))^n(e\tan(c+dx))^m\,dx = \int (b\sec(dx+c)+a)^n(e\tan(dx+c))^m\,dx$$

input
```
integrate((a+b*sec(d*x+c))^n*(e*tan(d*x+c))^m,x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c) + a)^n*(e*tan(d*x + c))^m, x)
```

3.352.9 Mupad [N/A]

Not integrable

Time = 15.76 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.17

$$\int (a+b\sec(c+dx))^n(e\tan(c+dx))^m\,dx = \int (e\tan(c+d\,x))^m\left(a+\frac{b}{\cos(c+d\,x)}\right)^n dx$$

input
```
int((e*tan(c + d*x))^m*(a + b/cos(c + d*x))^n,x)
```

output
```
int((e*tan(c + d*x))^m*(a + b/cos(c + d*x))^n, x)
```

3.353 $\int (a + b\sec(c + dx))^n \tan^5(c + dx)\, dx$

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3.353.8 Giac [F] . . . 2567
3.353.9 Mupad [F(-1)] . . . 2567

3.353.1 Optimal result

Integrand size = 21, antiderivative size = 177

$$\begin{aligned}
&\int (a + b\sec(c + dx))^n \tan^5(c + dx)\, dx \\
&= -\frac{a(a^2 - 2b^2)\,(a + b\sec(c + dx))^{1+n}}{b^4 d(1 + n)} \\
&\quad - \frac{\text{Hypergeometric2F1}\left(1, 1 + n, 2 + n, 1 + \frac{b\sec(c+dx)}{a}\right)(a + b\sec(c + dx))^{1+n}}{ad(1 + n)} \\
&\quad + \frac{(3a^2 - 2b^2)\,(a + b\sec(c + dx))^{2+n}}{b^4 d(2 + n)} - \frac{3a(a + b\sec(c + dx))^{3+n}}{b^4 d(3 + n)} + \frac{(a + b\sec(c + dx))^{4+n}}{b^4 d(4 + n)}
\end{aligned}$$

output

```
-a*(a^2-2*b^2)*(a+b*sec(d*x+c))^(1+n)/b^4/d/(1+n)-hypergeom([1, 1+n],[2+n]
,1+b*sec(d*x+c)/a)*(a+b*sec(d*x+c))^(1+n)/a/d/(1+n)+(3*a^2-2*b^2)*(a+b*sec
(d*x+c))^(2+n)/b^4/d/(2+n)-3*a*(a+b*sec(d*x+c))^(3+n)/b^4/d/(3+n)+(a+b*sec
(d*x+c))^(4+n)/b^4/d/(4+n)
```

3.353.2 Mathematica [A] (verified)

Time = 1.37 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.77

$$\int (a+b\sec(c+dx))^n \tan^5(c+dx)\,dx$$

$$= \frac{(a+b\sec(c+dx))^{1+n}\left(-\frac{a^3-2ab^2}{1+n} - \frac{b^4\,\text{Hypergeometric2F1}\left(1,1+n,2+n,1+\frac{b\sec(c+dx)}{a}\right)}{a+an} + \frac{(3a^2-2b^2)(a+b\sec(c+dx))}{2+n} - \frac{3a(a+}{}\right.}{b^4 d}$$

input
```
Integrate[(a + b*Sec[c + d*x])^n*Tan[c + d*x]^5,x]
```

output
```
((a + b*Sec[c + d*x])^(1 + n)*(-((a^3 - 2*a*b^2)/(1 + n)) - (b^4*Hypergeom
etric2F1[1, 1 + n, 2 + n, 1 + (b*Sec[c + d*x])/a])/(a + a*n) + ((3*a^2 - 2
*b^2)*(a + b*Sec[c + d*x]))/(2 + n) - (3*a*(a + b*Sec[c + d*x])^2)/(3 + n)
 + (a + b*Sec[c + d*x])^3/(4 + n)))/(b^4*d)
```

3.353.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.90, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3042, 25, 4373, 522, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^5(c+dx)(a+b\sec(c+dx))^n\,dx$$

$$\downarrow 3042$$

$$\int -\cot\left(c+dx+\frac{\pi}{2}\right)^5\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^n dx$$

$$\downarrow 25$$

$$-\int \cot\left(\frac{1}{2}(2c+\pi)+dx\right)^5\left(a+b\csc\left(\frac{1}{2}(2c+\pi)+dx\right)\right)^n dx$$

$$\downarrow 4373$$

$$\frac{\int \frac{\cos(c+dx)(a+b\sec(c+dx))^n\left(b^2-b^2\sec^2(c+dx)\right)^2}{b}\,d(b\sec(c+dx))}{b^4 d}$$

$$\downarrow 522$$

$$\frac{\int \left(\left(2ab^2-a^3\right)(a+b\sec(c+dx))^n+b^3\cos(c+dx)(a+b\sec(c+dx))^n+\left(3a^2-2b^2\right)(a+b\sec(c+dx))^{n+1}-3a\right.}{b^4d}$$

$$\downarrow 2009$$

$$\frac{-\frac{a\left(a^2-2b^2\right)(a+b\sec(c+dx))^{n+1}}{n+1}+\frac{\left(3a^2-2b^2\right)(a+b\sec(c+dx))^{n+2}}{n+2}-\frac{b^4(a+b\sec(c+dx))^{n+1}\,\text{Hypergeometric2F1}\left(1,n+1,n+2,\frac{b\sec(c+dx)}{a}+1\right)}{a(n+1)}}{b^4d}$$

input
```
Int[(a + b*Sec[c + d*x])^n*Tan[c + d*x]^5,x]
```

output
```
(-((a*(a^2 - 2*b^2)*(a + b*Sec[c + d*x])^(1 + n))/(1 + n)) - (b^4*Hypergeo
metric2F1[1, 1 + n, 2 + n, 1 + (b*Sec[c + d*x])/a]*(a + b*Sec[c + d*x])^(1
 + n))/(a*(1 + n)) + ((3*a^2 - 2*b^2)*(a + b*Sec[c + d*x])^(2 + n))/(2 + n
) - (3*a*(a + b*Sec[c + d*x])^(3 + n))/(3 + n) + (a + b*Sec[c + d*x])^(4 +
 n)/(4 + n))/(b^4*d)
```

3.353.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 522
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_.
), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(c + d*x)^n*(a + b*x^2)^p, x],
x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4373
```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Simp[-(-1)^((m - 1)/2)/(d*b^(m - 1))   Subst[Int[(b^2 - x^
2)^((m - 1)/2)*((a + x)^n/x), x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
 d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

3.353.4 Maple [F]

$$\int (a + b \sec (dx + c))^n \tan (dx + c)^5 \, dx$$

input
```
int((a+b*sec(d*x+c))^n*tan(d*x+c)^5,x)
```

output
```
int((a+b*sec(d*x+c))^n*tan(d*x+c)^5,x)
```

3.353.5 Fricas [F]

$$\int (a + b \sec(c + dx))^n \tan^5(c + dx) \, dx = \int (b \sec (dx + c) + a)^n \tan (dx + c)^5 \, dx$$

input
```
integrate((a+b*sec(d*x+c))^n*tan(d*x+c)^5,x, algorithm="fricas")
```

output
```
integral((b*sec(d*x + c) + a)^n*tan(d*x + c)^5, x)
```

3.353.6 Sympy [F]

$$\int (a + b \sec(c + dx))^n \tan^5(c + dx) \, dx = \int (a + b \sec (c + dx))^n \tan^5 (c + dx) \, dx$$

input
```
integrate((a+b*sec(d*x+c))**n*tan(d*x+c)**5,x)
```

output
```
Integral((a + b*sec(c + d*x))**n*tan(c + d*x)**5, x)
```

3.353.7 Maxima [F]

$$\int (a+b\sec(c+dx))^n \tan^5(c+dx)\,dx = \int (b\sec(dx+c)+a)^n \tan(dx+c)^5\,dx$$

input
```
integrate((a+b*sec(d*x+c))^n*tan(d*x+c)^5,x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c) + a)^n*tan(d*x + c)^5, x)
```

3.353.8 Giac [F]

$$\int (a+b\sec(c+dx))^n \tan^5(c+dx)\,dx = \int (b\sec(dx+c)+a)^n \tan(dx+c)^5\,dx$$

input
```
integrate((a+b*sec(d*x+c))^n*tan(d*x+c)^5,x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c) + a)^n*tan(d*x + c)^5, x)
```

3.353.9 Mupad [F(-1)]

Timed out.

$$\int (a+b\sec(c+dx))^n \tan^5(c+dx)\,dx = \int \tan(c+d\,x)^5 \left(a+\frac{b}{\cos(c+d\,x)}\right)^n dx$$

input
```
int(tan(c + d*x)^5*(a + b/cos(c + d*x))^n,x)
```

output
```
int(tan(c + d*x)^5*(a + b/cos(c + d*x))^n, x)
```

3.354 $\int (a+b\sec(c+dx))^n \tan^3(c+dx)\,dx$

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3.354.1 Optimal result

Integrand size = 21, antiderivative size = 102

$$\begin{aligned}&\int (a+b\sec(c+dx))^n \tan^3(c+dx)\,dx \\ &= -\frac{a(a+b\sec(c+dx))^{1+n}}{b^2 d(1+n)} \\ &\quad + \frac{\text{Hypergeometric2F1}\left(1,1+n,2+n,1+\frac{b\sec(c+dx)}{a}\right)(a+b\sec(c+dx))^{1+n}}{ad(1+n)} \\ &\quad + \frac{(a+b\sec(c+dx))^{2+n}}{b^2 d(2+n)}\end{aligned}$$

output
```
-a*(a+b*sec(d*x+c))^(1+n)/b^2/d/(1+n)+hypergeom([1, 1+n],[2+n],1+b*sec(d*x
+c)/a)*(a+b*sec(d*x+c))^(1+n)/a/d/(1+n)+(a+b*sec(d*x+c))^(2+n)/b^2/d/(2+n)
```

3.354.2 Mathematica [A] (verified)

Time = 0.56 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.78

$$\begin{aligned}&\int (a+b\sec(c+dx))^n \tan^3(c+dx)\,dx \\ &= \frac{(a+b\sec(c+dx))^{1+n}\left(b^2(2+n)\,\text{Hypergeometric2F1}\left(1,1+n,2+n,1+\frac{b\sec(c+dx)}{a}\right)+a(-a+b(1+n)\right.}{ab^2 d(1+n)(2+n)}\end{aligned}$$

input
```
Integrate[(a + b*Sec[c + d*x])^n*Tan[c + d*x]^3,x]
```

output
```
((a + b*Sec[c + d*x])^(1 + n)*(b^2*(2 + n)*Hypergeometric2F1[1, 1 + n, 2 +
 n, 1 + (b*Sec[c + d*x])/a] + a*(-a + b*(1 + n)*Sec[c + d*x])))/(a*b^2*d*(
1 + n)*(2 + n))
```

3.354.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.97, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3042, 25, 4373, 522, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^3(c+dx)(a+b\sec(c+dx))^n\,dx$$

$$\downarrow 3042$$

$$\int -\cot\left(c+dx+\frac{\pi}{2}\right)^3\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^n dx$$

$$\downarrow 25$$

$$-\int \cot\left(\frac{1}{2}(2c+\pi)+dx\right)^3\left(a+b\csc\left(\frac{1}{2}(2c+\pi)+dx\right)\right)^n dx$$

$$\downarrow 4373$$

$$-\frac{\int \frac{\cos(c+dx)(a+b\sec(c+dx))^n\left(b^2-b^2\sec^2(c+dx)\right)}{b}d(b\sec(c+dx))}{b^2 d}$$

$$\downarrow 522$$

$$-\frac{\int \left(a(a+b\sec(c+dx))^n+b\cos(c+dx)(a+b\sec(c+dx))^n-(a+b\sec(c+dx))^{n+1}\right)d(b\sec(c+dx))}{b^2 d}$$

$$\downarrow 2009$$

$$-\frac{-\frac{b^2(a+b\sec(c+dx))^{n+1}\operatorname{Hypergeometric2F1}\left(1,n+1,n+2,\frac{b\sec(c+dx)}{a}+1\right)}{a(n+1)}+\frac{a(a+b\sec(c+dx))^{n+1}}{n+1}-\frac{(a+b\sec(c+dx))^{n+2}}{n+2}}{b^2 d}$$

input
```
Int[(a + b*Sec[c + d*x])^n*Tan[c + d*x]^3,x]
```

output
```
-(((a*(a + b*Sec[c + d*x])^(1 + n))/(1 + n) - (b^2*Hypergeometric2F1[1, 1
+ n, 2 + n, 1 + (b*Sec[c + d*x])/a]*(a + b*Sec[c + d*x])^(1 + n))/(a*(1 +
n)) - (a + b*Sec[c + d*x])^(2 + n)/(2 + n))/(b^2*d))
```

3.354.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 522
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_.
), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(c + d*x)^n*(a + b*x^2)^p, x],
x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4373
```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Simp[-(-1)^((m - 1)/2)/(d*b^(m - 1))   Subst[Int[(b^2 - x^
2)^((m - 1)/2)*((a + x)^n/x), x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
 d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

3.354.4 Maple [F]

$$\int (a + b \sec (dx + c))^n \tan (dx + c)^3 \, dx$$

input
```
int((a+b*sec(d*x+c))^n*tan(d*x+c)^3,x)
```

output
```
int((a+b*sec(d*x+c))^n*tan(d*x+c)^3,x)
```

3.354.5 Fricas [F]

$$\int (a + b\sec(c + dx))^n \tan^3(c + dx)\,dx = \int (b\sec(dx + c) + a)^n \tan(dx + c)^3\,dx$$

input
```
integrate((a+b*sec(d*x+c))^n*tan(d*x+c)^3,x, algorithm="fricas")
```

output
```
integral((b*sec(d*x + c) + a)^n*tan(d*x + c)^3, x)
```

3.354.6 Sympy [F]

$$\int (a + b\sec(c + dx))^n \tan^3(c + dx)\,dx = \int (a + b\sec(c + dx))^n \tan^3(c + dx)\,dx$$

input
```
integrate((a+b*sec(d*x+c))**n*tan(d*x+c)**3,x)
```

output
```
Integral((a + b*sec(c + d*x))**n*tan(c + d*x)**3, x)
```

3.354.7 Maxima [F]

$$\int (a + b\sec(c + dx))^n \tan^3(c + dx)\,dx = \int (b\sec(dx + c) + a)^n \tan(dx + c)^3\,dx$$

input
```
integrate((a+b*sec(d*x+c))^n*tan(d*x+c)^3,x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c) + a)^n*tan(d*x + c)^3, x)
```

3.354.8 Giac [F]

$$\int (a + b\sec(c + dx))^n \tan^3(c + dx)\,dx = \int (b\sec(dx + c) + a)^n \tan(dx + c)^3\,dx$$

input
```
integrate((a+b*sec(d*x+c))^n*tan(d*x+c)^3,x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c) + a)^n*tan(d*x + c)^3, x)
```

3.354.9 Mupad [F(-1)]

Timed out.

$$\int (a + b\sec(c + dx))^n \tan^3(c + dx)\,dx = \int \tan(c + d\,x)^3 \left(a + \frac{b}{\cos(c + d\,x)}\right)^n dx$$

input
```
int(tan(c + d*x)^3*(a + b/cos(c + d*x))^n,x)
```

output
```
int(tan(c + d*x)^3*(a + b/cos(c + d*x))^n, x)
```

3.355 $\int (a + b\sec(c + dx))^n \tan(c + dx)\, dx$

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3.355.8 Giac [F] . . . 2576
3.355.9 Mupad [F(-1)] . . . 2577

3.355.1 Optimal result

Integrand size = 19, antiderivative size = 48

$$\int (a + b\sec(c + dx))^n \tan(c + dx)\, dx = -\frac{\text{Hypergeometric2F1}\left(1, 1 + n, 2 + n, 1 + \frac{b\sec(c+dx)}{a}\right)(a + b\sec(c + dx))^{1+n}}{ad(1 + n)}$$

output
```
-hypergeom([1, 1+n],[2+n],1+b*sec(d*x+c)/a)*(a+b*sec(d*x+c))^(1+n)/a/d/(1+
n)
```

3.355.2 Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00

$$\int (a + b\sec(c + dx))^n \tan(c + dx)\, dx = -\frac{\text{Hypergeometric2F1}\left(1, 1 + n, 2 + n, 1 + \frac{b\sec(c+dx)}{a}\right)(a + b\sec(c + dx))^{1+n}}{ad(1 + n)}$$

input
```
Integrate[(a + b*Sec[c + d*x])^n*Tan[c + d*x],x]
```

output
```
-((Hypergeometric2F1[1, 1 + n, 2 + n, 1 + (b*Sec[c + d*x])/a]*(a + b*Sec[c
 + d*x])^(1 + n))/(a*d*(1 + n)))
```

3.355.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {3042, 25, 4373, 75}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan(c+dx)(a+b\sec(c+dx))^n\,dx$$

$$\downarrow 3042$$

$$\int -\cot\left(c+dx+\frac{\pi}{2}\right)\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^n dx$$

$$\downarrow 25$$

$$-\int \cot\left(\frac{1}{2}(2c+\pi)+dx\right)\left(a+b\csc\left(\frac{1}{2}(2c+\pi)+dx\right)\right)^n dx$$

$$\downarrow 4373$$

$$\frac{\int \frac{\cos(c+dx)(a+b\sec(c+dx))^n}{b} d(b\sec(c+dx))}{d}$$

$$\downarrow 75$$

$$-\frac{(a+b\sec(c+dx))^{n+1}\,\text{Hypergeometric2F1}\left(1,n+1,n+2,\frac{b\sec(c+dx)}{a}+1\right)}{ad(n+1)}$$

input
```
Int[(a + b*Sec[c + d*x])^n*Tan[c + d*x],x]
```

output
```
-((Hypergeometric2F1[1, 1 + n, 2 + n, 1 + (b*Sec[c + d*x])/a]*(a + b*Sec[c
 + d*x])^(1 + n))/(a*d*(1 + n)))
```

3.355.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 75
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4373
```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Simp[-(-1)^((m - 1)/2)/(d*b^(m - 1))   Subst[Int[(b^2 - x^
2)^((m - 1)/2)*((a + x)^n/x), x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
 d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

3.355.4 Maple [F]

$$\int (a + b \sec (dx + c))^n \tan (dx + c)\, dx$$

input
```
int((a+b*sec(d*x+c))^n*tan(d*x+c),x)
```

output
```
int((a+b*sec(d*x+c))^n*tan(d*x+c),x)
```

3.355.5 Fricas [F]

$$\int (a + b \sec(c + dx))^n \tan(c + dx)\, dx = \int (b \sec (dx + c) + a)^n \tan (dx + c)\ dx$$

input
```
integrate((a+b*sec(d*x+c))^n*tan(d*x+c),x, algorithm="fricas")
```

output
```
integral((b*sec(d*x + c) + a)^n*tan(d*x + c), x)
```

3.355.6 Sympy [F]

$$\int (a + b \sec(c + dx))^n \tan(c + dx)\, dx = \int (a + b \sec(c + dx))^n \tan(c + dx)\, dx$$

input
```
integrate((a+b*sec(d*x+c))**n*tan(d*x+c),x)
```

output
```
Integral((a + b*sec(c + d*x))**n*tan(c + d*x), x)
```

3.355.7 Maxima [F]

$$\int (a + b \sec(c + dx))^n \tan(c + dx)\, dx = \int (b \sec(dx + c) + a)^n \tan(dx + c)\, dx$$

input
```
integrate((a+b*sec(d*x+c))^n*tan(d*x+c),x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c) + a)^n*tan(d*x + c), x)
```

3.355.8 Giac [F]

$$\int (a + b \sec(c + dx))^n \tan(c + dx)\, dx = \int (b \sec(dx + c) + a)^n \tan(dx + c)\, dx$$

input
```
integrate((a+b*sec(d*x+c))^n*tan(d*x+c),x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c) + a)^n*tan(d*x + c), x)
```

3.355.9 Mupad [F(-1)]

Timed out.

$$\int (a + b \sec(c + dx))^n \tan(c + dx)\, dx = \int \tan(c + d\,x) \left(a + \frac{b}{\cos(c + d\,x)}\right)^n dx$$

input
```
int(tan(c + d*x)*(a + b/cos(c + d*x))^n,x)
```

output
```
int(tan(c + d*x)*(a + b/cos(c + d*x))^n, x)
```

3.356 $\int \cot(c+dx)(a+b\sec(c+dx))^n\,dx$

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3.356.5 Fricas [F] . . . 2581
3.356.6 Sympy [F] . . . 2581
3.356.7 Maxima [F] . . . 2582
3.356.8 Giac [F] . . . 2582
3.356.9 Mupad [F(-1)] . . . 2582

3.356.1 Optimal result

Integrand size = 19, antiderivative size = 162

$$\begin{aligned}
&\int \cot(c+dx)(a+b\sec(c+dx))^n\,dx \\
&= -\frac{\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{a+b\sec(c+dx)}{a-b}\right)(a+b\sec(c+dx))^{1+n}}{2(a-b)d(1+n)} \\
&\quad -\frac{\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{a+b\sec(c+dx)}{a+b}\right)(a+b\sec(c+dx))^{1+n}}{2(a+b)d(1+n)} \\
&\quad +\frac{\text{Hypergeometric2F1}\left(1,1+n,2+n,1+\frac{b\sec(c+dx)}{a}\right)(a+b\sec(c+dx))^{1+n}}{ad(1+n)}
\end{aligned}$$

output

```
-1/2*hypergeom([1, 1+n],[2+n],(a+b*sec(d*x+c))/(a-b))*(a+b*sec(d*x+c))^(1+
n)/(a-b)/d/(1+n)-1/2*hypergeom([1, 1+n],[2+n],(a+b*sec(d*x+c))/(a+b))*(a+b
*sec(d*x+c))^(1+n)/(a+b)/d/(1+n)+hypergeom([1, 1+n],[2+n],1+b*sec(d*x+c)/a
)*(a+b*sec(d*x+c))^(1+n)/a/d/(1+n)
```

3.356.2 Mathematica [A] (verified)

Time = 0.39 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.81

$$\int \cot(c+dx)(a+b\sec(c+dx))^n\,dx =$$
$$-\frac{\left(a(a+b)\,\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{a+b\sec(c+dx)}{a-b}\right)+(a-b)\left(a\,\text{Hypergeometric2F1}\left(1,1+n\right.\right.\right.}{2a(a-b)}$$

input
```
Integrate[Cot[c + d*x]*(a + b*Sec[c + d*x])^n,x]
```

output
```
-1/2*((a*(a + b)*Hypergeometric2F1[1, 1 + n, 2 + n, (a + b*Sec[c + d*x])/(
a - b)] + (a - b)*(a*Hypergeometric2F1[1, 1 + n, 2 + n, (a + b*Sec[c + d*x
])/(a + b)] - 2*(a + b)*Hypergeometric2F1[1, 1 + n, 2 + n, 1 + (b*Sec[c +
d*x])/a]))*(a + b*Sec[c + d*x])^(1 + n))/(a*(a - b)*(a + b)*d*(1 + n))
```

3.356.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.06, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {3042, 25, 4373, 615, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot(c+dx)(a+b\sec(c+dx))^n\,dx$$
$$\downarrow 3042$$
$$\int -\frac{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^n}{\cot\left(c+dx+\frac{\pi}{2}\right)}dx$$
$$\downarrow 25$$
$$-\int \frac{\left(a+b\csc\left(\frac{1}{2}(2c+\pi)+dx\right)\right)^n}{\cot\left(\frac{1}{2}(2c+\pi)+dx\right)}dx$$
$$\downarrow 4373$$
$$-\frac{b^2\int \frac{\cos(c+dx)(a+b\sec(c+dx))^n}{b(b^2-b^2\sec^2(c+dx))}d(b\sec(c+dx))}{d}$$
$$\downarrow 615$$

$$-\frac{b^2\int\left(\frac{\cos(c+dx)(a+b\sec(c+dx))^n}{b^3}-\frac{\sec(c+dx)(a+b\sec(c+dx))^n}{b(b^2\sec^2(c+dx)-b^2)}\right)d(b\sec(c+dx))}{d}$$

$$\downarrow \text{2009}$$

$$-\frac{b^2\left(\frac{(a+b\sec(c+dx))^{n+1}\,\text{Hypergeometric2F1}\left(1,n+1,n+2,\frac{a+b\sec(c+dx)}{a-b}\right)}{2b^2(n+1)(a-b)}+\frac{(a+b\sec(c+dx))^{n+1}\,\text{Hypergeometric2F1}\left(1,n+1,n+2,\frac{a+b\sec(c+\right.}{d}$$

input
```
Int[Cot[c + d*x]*(a + b*Sec[c + d*x])^n,x]
```

output
```
-((b^2*((Hypergeometric2F1[1, 1 + n, 2 + n, (a + b*Sec[c + d*x])/(a - b)]*
(a + b*Sec[c + d*x])^(1 + n))/(2*(a - b)*b^2*(1 + n)) + (Hypergeometric2F1
[1, 1 + n, 2 + n, (a + b*Sec[c + d*x])/(a + b)]*(a + b*Sec[c + d*x])^(1 +
n))/(2*b^2*(a + b)*(1 + n)) - (Hypergeometric2F1[1, 1 + n, 2 + n, 1 + (b*S
ec[c + d*x])/a]*(a + b*Sec[c + d*x])^(1 + n))/(a*b^2*(1 + n))))/d)
```

3.356.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 615
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_),
 x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(c + d*x)^n*(a + b*x^2)^p, x], x]
 /; FreeQ[{a, b, c, d, e, m, n}, x] && ILtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4373
```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Simp[-(-1)^((m - 1)/2)/(d*b^(m - 1))   Subst[Int[(b^2 - x^
2)^((m - 1)/2)*((a + x)^n/x), x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
 d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

3.356.4 Maple [F]

$$\int \cot(dx+c)\,(a+b\sec(dx+c))^n\,dx$$

input
```
int(cot(d*x+c)*(a+b*sec(d*x+c))^n,x)
```

output
```
int(cot(d*x+c)*(a+b*sec(d*x+c))^n,x)
```

3.356.5 Fricas [F]

$$\int \cot(c+dx)(a+b\sec(c+dx))^n\,dx = \int (b\sec(dx+c)+a)^n\cot(dx+c)\,dx$$

input
```
integrate(cot(d*x+c)*(a+b*sec(d*x+c))^n,x, algorithm="fricas")
```

output
```
integral((b*sec(d*x + c) + a)^n*cot(d*x + c), x)
```

3.356.6 Sympy [F]

$$\int \cot(c+dx)(a+b\sec(c+dx))^n\,dx = \int (a+b\sec(c+dx))^n\cot(c+dx)\,dx$$

input
```
integrate(cot(d*x+c)*(a+b*sec(d*x+c))**n,x)
```

output
```
Integral((a + b*sec(c + d*x))**n*cot(c + d*x), x)
```

3.356.7 Maxima [F]

$$\int \cot(c+dx)(a+b\sec(c+dx))^n\,dx = \int (b\sec(dx+c)+a)^n\cot(dx+c)\,dx$$

input
```
integrate(cot(d*x+c)*(a+b*sec(d*x+c))^n,x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c) + a)^n*cot(d*x + c), x)
```

3.356.8 Giac [F]

$$\int \cot(c+dx)(a+b\sec(c+dx))^n\,dx = \int (b\sec(dx+c)+a)^n\cot(dx+c)\,dx$$

input
```
integrate(cot(d*x+c)*(a+b*sec(d*x+c))^n,x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c) + a)^n*cot(d*x + c), x)
```

3.356.9 Mupad [F(-1)]

Timed out.

$$\int \cot(c+dx)(a+b\sec(c+dx))^n\,dx = \int \cot(c+d\,x)\left(a+\frac{b}{\cos(c+d\,x)}\right)^n dx$$

input
```
int(cot(c + d*x)*(a + b/cos(c + d*x))^n,x)
```

output
```
int(cot(c + d*x)*(a + b/cos(c + d*x))^n, x)
```

3.357 $\int \cot^3(c+dx)(a+b\sec(c+dx))^n\,dx$

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3.357.9 Mupad [F(-1)] . . . 2587

3.357.1 Optimal result

Integrand size = 21, antiderivative size = 279

$$\begin{aligned}
&\int \cot^3(c+dx)(a+b\sec(c+dx))^n\,dx \\
&= \frac{\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{a+b\sec(c+dx)}{a-b}\right)(a+b\sec(c+dx))^{1+n}}{2(a-b)d(1+n)} \\
&\quad + \frac{\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{a+b\sec(c+dx)}{a+b}\right)(a+b\sec(c+dx))^{1+n}}{2(a+b)d(1+n)} \\
&\quad - \frac{\text{Hypergeometric2F1}\left(1,1+n,2+n,1+\frac{b\sec(c+dx)}{a}\right)(a+b\sec(c+dx))^{1+n}}{ad(1+n)} \\
&\quad - \frac{b\,\text{Hypergeometric2F1}\left(2,1+n,2+n,\frac{a+b\sec(c+dx)}{a-b}\right)(a+b\sec(c+dx))^{1+n}}{4(a-b)^2d(1+n)} \\
&\quad + \frac{b\,\text{Hypergeometric2F1}\left(2,1+n,2+n,\frac{a+b\sec(c+dx)}{a+b}\right)(a+b\sec(c+dx))^{1+n}}{4(a+b)^2d(1+n)}
\end{aligned}$$

output

```
1/2*hypergeom([1, 1+n],[2+n],(a+b*sec(d*x+c))/(a-b))*(a+b*sec(d*x+c))^(1+n
)/(a-b)/d/(1+n)+1/2*hypergeom([1, 1+n],[2+n],(a+b*sec(d*x+c))/(a+b))*(a+b*
sec(d*x+c))^(1+n)/(a+b)/d/(1+n)-hypergeom([1, 1+n],[2+n],1+b*sec(d*x+c)/a)
*(a+b*sec(d*x+c))^(1+n)/a/d/(1+n)-1/4*b*hypergeom([2, 1+n],[2+n],(a+b*sec(
d*x+c))/(a-b))*(a+b*sec(d*x+c))^(1+n)/(a-b)^2/d/(1+n)+1/4*b*hypergeom([2,
1+n],[2+n],(a+b*sec(d*x+c))/(a+b))*(a+b*sec(d*x+c))^(1+n)/(a+b)^2/d/(1+n)
```

3.357.2 Mathematica [A] (verified)

Time = 0.71 (sec) , antiderivative size = 186, normalized size of antiderivative = 0.67

$$\int \cot^3(c+dx)(a+b\sec(c+dx))^n\,dx$$
$$= \left(\frac{2\,\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{a+b\sec(c+dx)}{a-b}\right)}{a-b} + \frac{2\,\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{a+b\sec(c+dx)}{a+b}\right)}{a+b} - \frac{4\,\text{Hypergeometric2F1}\left(1,1+n,2+\right.}{a}\right.$$

input
```
Integrate[Cot[c + d*x]^3*(a + b*Sec[c + d*x])^n,x]
```

output
```
(((2*Hypergeometric2F1[1, 1 + n, 2 + n, (a + b*Sec[c + d*x])/(a - b)])/(a
- b) + (2*Hypergeometric2F1[1, 1 + n, 2 + n, (a + b*Sec[c + d*x])/(a + b)]
)/(a + b) - (4*Hypergeometric2F1[1, 1 + n, 2 + n, 1 + (b*Sec[c + d*x])/a])
/a - (b*Hypergeometric2F1[2, 1 + n, 2 + n, (a + b*Sec[c + d*x])/(a - b)])/
(a - b)^2 + (b*Hypergeometric2F1[2, 1 + n, 2 + n, (a + b*Sec[c + d*x])/(a
+ b)])/(a + b)^2)*(a + b*Sec[c + d*x])^(1 + n))/(4*d*(1 + n))
```

3.357.3 Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 284, normalized size of antiderivative = 1.02, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3042, 25, 4373, 615, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^3(c+dx)(a+b\sec(c+dx))^n\,dx$$
$$\downarrow 3042$$
$$\int -\frac{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^n}{\cot\left(c+dx+\frac{\pi}{2}\right)^3}dx$$
$$\downarrow 25$$
$$-\int \frac{\left(a+b\csc\left(\frac{1}{2}(2c+\pi)+dx\right)\right)^n}{\cot\left(\frac{1}{2}(2c+\pi)+dx\right)^3}dx$$
$$\downarrow 4373$$

$$\frac{b^4 \int \frac{\cos(c+dx)(a+b\sec(c+dx))^n}{b(b^2-b^2\sec^2(c+dx))^2} d(b\sec(c+dx))}{d}$$

$$\downarrow 615$$

$$\frac{b^4 \int \left(\frac{\cos(c+dx)(a+b\sec(c+dx))^n}{b^5} - \frac{\sec(c+dx)(a+b\sec(c+dx))^n}{b^3(b^2\sec^2(c+dx)-b^2)} + \frac{(a+b\sec(c+dx))^n}{4b^3(b-b\sec(c+dx))^2} - \frac{(a+b\sec(c+dx))^n}{4b^3(\sec(c+dx)b+b)^2} \right) d(b\sec(c+dx))}{d}$$

$$\downarrow 2009$$

$$b^4 \left(\frac{(a+b\sec(c+dx))^{n+1}\,\text{Hypergeometric2F1}\left(1,n+1,n+2,\frac{a+b\sec(c+dx)}{a-b}\right)}{2b^4(n+1)(a-b)} + \frac{(a+b\sec(c+dx))^{n+1}\,\text{Hypergeometric2F1}\left(1,n+1,n+2,\frac{a+b\sec(c+dx}{a+b}\right.\right.$$

input
```
Int[Cot[c + d*x]^3*(a + b*Sec[c + d*x])^n,x]
```

output
```
(b^4*((Hypergeometric2F1[1, 1 + n, 2 + n, (a + b*Sec[c + d*x])/(a - b)]*(a
 + b*Sec[c + d*x])^(1 + n))/(2*(a - b)*b^4*(1 + n)) + (Hypergeometric2F1[1
, 1 + n, 2 + n, (a + b*Sec[c + d*x])/(a + b)]*(a + b*Sec[c + d*x])^(1 + n)
)/(2*b^4*(a + b)*(1 + n)) - (Hypergeometric2F1[1, 1 + n, 2 + n, 1 + (b*Sec
[c + d*x])/a]*(a + b*Sec[c + d*x])^(1 + n))/(a*b^4*(1 + n)) - (Hypergeomet
ric2F1[2, 1 + n, 2 + n, (a + b*Sec[c + d*x])/(a - b)]*(a + b*Sec[c + d*x])
^(1 + n))/(4*(a - b)^2*b^3*(1 + n)) + (Hypergeometric2F1[2, 1 + n, 2 + n,
(a + b*Sec[c + d*x])/(a + b)]*(a + b*Sec[c + d*x])^(1 + n))/(4*b^3*(a + b)
^2*(1 + n))))/d
```

3.357.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 615
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_),
 x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(c + d*x)^n*(a + b*x^2)^p, x], x]
 /; FreeQ[{a, b, c, d, e, m, n}, x] && ILtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4373
```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Simp[-(-1)^((m - 1)/2)/(d*b^(m - 1))   Subst[Int[(b^2 - x^
2)^((m - 1)/2)*((a + x)^n/x), x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
 d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

3.357.4 Maple [F]

$$\int \cot (dx+c)^3 (a+b\sec (dx+c))^n \, dx$$

input
```
int(cot(d*x+c)^3*(a+b*sec(d*x+c))^n,x)
```

output
```
int(cot(d*x+c)^3*(a+b*sec(d*x+c))^n,x)
```

3.357.5 Fricas [F]

$$\int \cot^3(c+dx)(a+b\sec(c+dx))^n \, dx = \int (b\sec (dx+c)+a)^n \cot (dx+c)^3 \, dx$$

input
```
integrate(cot(d*x+c)^3*(a+b*sec(d*x+c))^n,x, algorithm="fricas")
```

output
```
integral((b*sec(d*x + c) + a)^n*cot(d*x + c)^3, x)
```

3.357.6 Sympy [F]

$$\int \cot^3(c+dx)(a+b\sec(c+dx))^n \, dx = \int (a+b\sec (c+dx))^n \cot^3 (c+dx) \, dx$$

input
```
integrate(cot(d*x+c)**3*(a+b*sec(d*x+c))**n,x)
```

output
```
Integral((a + b*sec(c + d*x))**n*cot(c + d*x)**3, x)
```

3.357.7 Maxima [F]

$$\int \cot^3(c+dx)(a+b\sec(c+dx))^n\,dx = \int (b\sec(dx+c)+a)^n \cot(dx+c)^3\,dx$$

input
```
integrate(cot(d*x+c)^3*(a+b*sec(d*x+c))^n,x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c) + a)^n*cot(d*x + c)^3, x)
```

3.357.8 Giac [F]

$$\int \cot^3(c+dx)(a+b\sec(c+dx))^n\,dx = \int (b\sec(dx+c)+a)^n \cot(dx+c)^3\,dx$$

input
```
integrate(cot(d*x+c)^3*(a+b*sec(d*x+c))^n,x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c) + a)^n*cot(d*x + c)^3, x)
```

3.357.9 Mupad [F(-1)]

Timed out.

$$\int \cot^3(c+dx)(a+b\sec(c+dx))^n\,dx = \int \cot(c+d\,x)^3 \left(a+\frac{b}{\cos(c+d\,x)}\right)^n dx$$

input
```
int(cot(c + d*x)^3*(a + b/cos(c + d*x))^n,x)
```

output
```
int(cot(c + d*x)^3*(a + b/cos(c + d*x))^n, x)
```

3.358 $\int (a + b \sec(c + dx))^n \tan^4(c + dx)\, dx$

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3.358.2 Mathematica [N/A] . . . 2588
3.358.3 Rubi [N/A] . . . 2589
3.358.4 Maple [N/A] (verified) . . . 2590
3.358.5 Fricas [N/A] . . . 2590
3.358.6 Sympy [N/A] . . . 2590
3.358.7 Maxima [N/A] . . . 2591
3.358.8 Giac [N/A] . . . 2591
3.358.9 Mupad [N/A] . . . 2591

3.358.1 Optimal result

Integrand size = 21, antiderivative size = 21

$$\int (a + b \sec(c + dx))^n \tan^4(c + dx)\, dx = \text{Int}\big((a + b \sec(c + dx))^n \tan^4(c + dx), x\big)$$

output
```
Unintegrable((a+b*sec(d*x+c))^n*tan(d*x+c)^4,x)
```

3.358.2 Mathematica [N/A]

Not integrable

Time = 13.79 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int (a + b \sec(c + dx))^n \tan^4(c + dx)\, dx = \int (a + b \sec(c + dx))^n \tan^4(c + dx)\, dx$$

input
```
Integrate[(a + b*Sec[c + d*x])^n*Tan[c + d*x]^4,x]
```

output
```
Integrate[(a + b*Sec[c + d*x])^n*Tan[c + d*x]^4, x]
```

3.358.3 Rubi [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {3042, 4387}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^4(c+dx)(a+b\sec(c+dx))^n\,dx$$

$$\downarrow 3042$$

$$\int \cot\left(c+dx+\frac{\pi}{2}\right)^4\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^n dx$$

$$\downarrow 4387$$

$$\int \tan^4(c+dx)(a+b\sec(c+dx))^n dx$$

input
```
Int[(a + b*Sec[c + d*x])^n*Tan[c + d*x]^4,x]
```

output
```
$Aborted
```

3.358.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4387
```
Int[((a_.) + csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_.)*(cot[(c_.) + (d_.)*(x_)]*
(e_.))^(m_.), x_Symbol] :> Unintegrable[(e*Cot[c + d*x])^m*(a + b*Csc[c + d
*x])^n, x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

3.358.4 Maple [N/A] (verified)

Not integrable

Time = 1.38 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int (a + b \sec (dx + c))^n \tan (dx + c)^4 \, dx$$

input
```
int((a+b*sec(d*x+c))^n*tan(d*x+c)^4,x)
```

output
```
int((a+b*sec(d*x+c))^n*tan(d*x+c)^4,x)
```

3.358.5 Fricas [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int (a + b \sec(c + dx))^n \tan^4(c + dx) \, dx = \int (b \sec (dx + c) + a)^n \tan (dx + c)^4 \, dx$$

input
```
integrate((a+b*sec(d*x+c))^n*tan(d*x+c)^4,x, algorithm="fricas")
```

output
```
integral((b*sec(d*x + c) + a)^n*tan(d*x + c)^4, x)
```

3.358.6 Sympy [N/A]

Not integrable

Time = 29.68 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.95

$$\int (a + b \sec(c + dx))^n \tan^4(c + dx) \, dx = \int (a + b \sec (c + dx))^n \tan^4 (c + dx) \, dx$$

input
```
integrate((a+b*sec(d*x+c))**n*tan(d*x+c)**4,x)
```

output
```
Integral((a + b*sec(c + d*x))**n*tan(c + d*x)**4, x)
```

3.358.7 Maxima [N/A]

Not integrable

Time = 19.76 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int (a + b\sec(c + dx))^n \tan^4(c + dx)\,dx = \int (b\sec(dx + c) + a)^n \tan(dx + c)^4\,dx$$

input
```
integrate((a+b*sec(d*x+c))^n*tan(d*x+c)^4,x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c) + a)^n*tan(d*x + c)^4, x)
```

3.358.8 Giac [N/A]

Not integrable

Time = 1.55 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int (a + b\sec(c + dx))^n \tan^4(c + dx)\,dx = \int (b\sec(dx + c) + a)^n \tan(dx + c)^4\,dx$$

input
```
integrate((a+b*sec(d*x+c))^n*tan(d*x+c)^4,x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c) + a)^n*tan(d*x + c)^4, x)
```

3.358.9 Mupad [N/A]

Not integrable

Time = 17.99 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.19

$$\int (a + b\sec(c + dx))^n \tan^4(c + dx)\,dx = \int \tan(c + d\,x)^4 \left(a + \frac{b}{\cos(c + d\,x)}\right)^n dx$$

input
```
int(tan(c + d*x)^4*(a + b/cos(c + d*x))^n,x)
```

output
```
int(tan(c + d*x)^4*(a + b/cos(c + d*x))^n, x)
```

3.359 $\int (a+b\sec(c+dx))^n \tan^2(c+dx)\,dx$

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3.359.2 Mathematica [N/A] . . . 2593
3.359.3 Rubi [N/A] . . . 2593
3.359.4 Maple [N/A] (verified) . . . 2597
3.359.5 Fricas [N/A] . . . 2597
3.359.6 Sympy [N/A] . . . 2598
3.359.7 Maxima [N/A] . . . 2598
3.359.8 Giac [N/A] . . . 2598
3.359.9 Mupad [N/A] . . . 2599

3.359.1 Optimal result

Integrand size = 21, antiderivative size = 21

$$\int (a+b\sec(c+dx))^n \tan^2(c+dx)\,dx$$

$$= \frac{\sqrt{2}(a+b)\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{2},-1-n,\frac{3}{2},\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right)(a+b\sec(c+dx))^n\left(\frac{a+b\sec(c+dx)}{a+b}\right)}{bd\sqrt{1+\sec(c+dx)}}$$

$$-\frac{\sqrt{2}a\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{2},-n,\frac{3}{2},\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right)(a+b\sec(c+dx))^n\left(\frac{a+b\sec(c+dx)}{a+b}\right)^{-n}\tan(}{bd\sqrt{1+\sec(c+dx)}}$$

$$-\text{Int}((a+b\sec(c+dx))^n,x)$$

output
```
(a+b)*AppellF1(1/2,-1-n,1/2,3/2,b*(1-sec(d*x+c))/(a+b),1/2-1/2*sec(d*x+c))
*(a+b*sec(d*x+c))^n*2^(1/2)*tan(d*x+c)/b/d/(((a+b*sec(d*x+c))/(a+b))^n)/(1
+sec(d*x+c))^(1/2)-a*AppellF1(1/2,-n,1/2,3/2,b*(1-sec(d*x+c))/(a+b),1/2-1/
2*sec(d*x+c))*(a+b*sec(d*x+c))^n*2^(1/2)*tan(d*x+c)/b/d/(((a+b*sec(d*x+c))
/(a+b))^n)/(1+sec(d*x+c))^(1/2)-Unintegrable((a+b*sec(d*x+c))^n,x)
```

3.359.2 Mathematica [N/A]

Not integrable

Time = 12.26 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int (a+b\sec(c+dx))^n \tan^2(c+dx)\,dx = \int (a+b\sec(c+dx))^n \tan^2(c+dx)\,dx$$

input
```
Integrate[(a + b*Sec[c + d*x])^n*Tan[c + d*x]^2,x]
```

output
```
Integrate[(a + b*Sec[c + d*x])^n*Tan[c + d*x]^2, x]
```

3.359.3 Rubi [N/A]

Not integrable

Time = 0.96 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {3042, 4382, 3042, 4551, 25, 3042, 4321, 156, 155, 4412, 3042, 4273, 4321, 156, 155}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^2(c+dx)(a+b\sec(c+dx))^n\,dx$$

$$\downarrow 3042$$

$$\int \cot\left(c+dx+\frac{\pi}{2}\right)^2 \left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^n dx$$

$$\downarrow 4382$$

$$\int \left(\sec^2(c+dx)-1\right)(a+b\sec(c+dx))^n dx$$

$$\downarrow 3042$$

$$\int \left(\csc\left(c+dx+\frac{\pi}{2}\right)^2-1\right)\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^n dx$$

$$\downarrow 4551$$

$$\frac{\int -((b+a\sec(c+dx))(a+b\sec(c+dx))^n)\,dx}{b} + \frac{\int \sec(c+dx)(a+b\sec(c+dx))^{n+1} dx}{b}$$

$$\downarrow 25$$

$$\frac{\int \sec(c+dx)(a+b\sec(c+dx))^{n+1}dx}{b} - \frac{\int (b+a\sec(c+dx))(a+b\sec(c+dx))^n dx}{b}$$

$$\downarrow 3042$$

$$\frac{\int \csc\left(c+dx+\frac{\pi}{2}\right)\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{n+1}dx}{b} - \frac{\int \left(b+a\csc\left(c+dx+\frac{\pi}{2}\right)\right)\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^n dx}{b}$$

$$\downarrow 4321$$

$$-\frac{\int \left(b+a\csc\left(c+dx+\frac{\pi}{2}\right)\right)\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^n dx}{b} - \frac{\tan(c+dx)\int \frac{(a+b\sec(c+dx))^{n+1}}{\sqrt{1-\sec(c+dx)}\sqrt{\sec(c+dx)+1}}d\sec(c+dx)}{bd\sqrt{1-\sec(c+dx)}\sqrt{\sec(c+dx)+1}}$$

$$\downarrow 156$$

$$-\frac{\int \left(b+a\csc\left(c+dx+\frac{\pi}{2}\right)\right)\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^n dx}{b} - \frac{(a+b)\tan(c+dx)(a+b\sec(c+dx))^n\left(\frac{a+b\sec(c+dx)}{a+b}\right)^{-n}\int \frac{\left(\frac{a}{a+b}+\frac{b\sec(c+dx)}{a+b}\right)^{n+1}}{\sqrt{1-\sec(c+dx)}\sqrt{\sec(c+dx)+1}}d\sec(c+dx)}{bd\sqrt{1-\sec(c+dx)}\sqrt{\sec(c+dx)+1}}$$

$$\downarrow 155$$

$$\frac{\sqrt{2}(a+b)\tan(c+dx)(a+b\sec(c+dx))^n\left(\frac{a+b\sec(c+dx)}{a+b}\right)^{-n}\text{AppellF1}\left(\frac{1}{2},\frac{1}{2},-n-1,\frac{3}{2},\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec}{a-}\right.}{bd\sqrt{\sec(c+dx)+1}} - \frac{\int \left(b+a\csc\left(c+dx+\frac{\pi}{2}\right)\right)\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^n dx}{b}$$

$$\downarrow 4412$$

$$\frac{\sqrt{2}(a+b)\tan(c+dx)(a+b\sec(c+dx))^n\left(\frac{a+b\sec(c+dx)}{a+b}\right)^{-n}\text{AppellF1}\left(\frac{1}{2},\frac{1}{2},-n-1,\frac{3}{2},\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec}{a-}\right.}{bd\sqrt{\sec(c+dx)+1}} - \frac{b\int (a+b\sec(c+dx))^n dx + a\int \sec(c+dx)(a+b\sec(c+dx))^n dx}{b}$$

$$\downarrow 3042$$

$$\frac{\sqrt{2}(a+b)\tan(c+dx)(a+b\sec(c+dx))^n\left(\frac{a+b\sec(c+dx)}{a+b}\right)^{-n}\text{AppellF1}\left(\frac{1}{2},\frac{1}{2},-n-1,\frac{3}{2},\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec}{a-}\right.}{bd\sqrt{\sec(c+dx)+1}}$$

$$\frac{b\int\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^n dx+a\int\csc\left(c+dx+\frac{\pi}{2}\right)\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^n dx}{b}$$

$$\downarrow 4273$$

$$\frac{\sqrt{2}(a+b)\tan(c+dx)(a+b\sec(c+dx))^n\left(\frac{a+b\sec(c+dx)}{a+b}\right)^{-n}\text{AppellF1}\left(\frac{1}{2},\frac{1}{2},-n-1,\frac{3}{2},\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec}{a-}\right.}{bd\sqrt{\sec(c+dx)+1}}$$

$$\frac{a\int\csc\left(c+dx+\frac{\pi}{2}\right)\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^n dx+b\int(a+b\sec(c+dx))^n dx}{b}$$

$$\downarrow 4321$$

$$\frac{\sqrt{2}(a+b)\tan(c+dx)(a+b\sec(c+dx))^n\left(\frac{a+b\sec(c+dx)}{a+b}\right)^{-n}\text{AppellF1}\left(\frac{1}{2},\frac{1}{2},-n-1,\frac{3}{2},\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec}{a-}\right.}{bd\sqrt{\sec(c+dx)+1}}$$

$$\frac{b\int(a+b\sec(c+dx))^n dx-\frac{a\tan(c+dx)\int\frac{(a+b\sec(c+dx))^n}{\sqrt{1-\sec(c+dx)}\sqrt{\sec(c+dx)+1}}d\sec(c+dx)}{d\sqrt{1-\sec(c+dx)}\sqrt{\sec(c+dx)+1}}}{b}$$

$$\downarrow 156$$

$$\frac{\sqrt{2}(a+b)\tan(c+dx)(a+b\sec(c+dx))^n\left(\frac{a+b\sec(c+dx)}{a+b}\right)^{-n}\text{AppellF1}\left(\frac{1}{2},\frac{1}{2},-n-1,\frac{3}{2},\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec}{a-}\right.}{bd\sqrt{\sec(c+dx)+1}}$$

$$\frac{b\int(a+b\sec(c+dx))^n dx-\frac{a\tan(c+dx)(a+b\sec(c+dx))^n\left(\frac{a+b\sec(c+dx)}{a+b}\right)^{-n}\int\frac{\left(\frac{a}{a+b}+\frac{b\sec(c+dx)}{a+b}\right)^n}{\sqrt{1-\sec(c+dx)}\sqrt{\sec(c+dx)+1}}d\sec(c+dx)}{d\sqrt{1-\sec(c+dx)}\sqrt{\sec(c+dx)+1}}}{b}$$

$$\downarrow 155$$

$$\frac{\sqrt{2}(a+b)\tan(c+dx)(a+b\sec(c+dx))^n\left(\frac{a+b\sec(c+dx)}{a+b}\right)^{-n}\text{AppellF1}\left(\frac{1}{2},\frac{1}{2},-n-1,\frac{3}{2},\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec}{a-}\right.}{bd\sqrt{\sec(c+dx)+1}}$$

$$\frac{b\int(a+b\sec(c+dx))^n dx+\frac{\sqrt{2}a\tan(c+dx)(a+b\sec(c+dx))^n\left(\frac{a+b\sec(c+dx)}{a+b}\right)^{-n}\text{AppellF1}\left(\frac{1}{2},\frac{1}{2},-n,\frac{3}{2},\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right)}{d\sqrt{\sec(c+dx)+1}}}{b}$$

input
```
Int[(a + b*Sec[c + d*x])^n*Tan[c + d*x]^2,x]
```

output
```
$Aborted
```

3.359. $\int(a+b\sec(c+dx))^n\tan^2(c+dx)\,dx$

3.359.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 155
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*Simplify[b/(b*c - a*d)]^n*
Simplify[b/(b*e - a*f)]^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/
(b*c - a*d)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f,
 m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[Sim
plify[b/(b*c - a*d)], 0] && GtQ[Simplify[b/(b*e - a*f)], 0] &&  !(GtQ[Simpl
ify[d/(d*a - c*b)], 0] && GtQ[Simplify[d/(d*e - c*f)], 0] && SimplerQ[c + d
*x, a + b*x]) &&  !(GtQ[Simplify[f/(f*a - e*b)], 0] && GtQ[Simplify[f/(f*c
- e*d)], 0] && SimplerQ[e + f*x, a + b*x])
```

rule 156
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[(e + f*x)^FracPart[p]/(Simplify[b/(b*e - a*f)]^IntPart[p
]*(b*((e + f*x)/(b*e - a*f)))^FracPart[p])   Int[(a + b*x)^m*(c + d*x)^n*Si
mp[b*(e/(b*e - a*f)) + b*f*(x/(b*e - a*f)), x]^p, x], x] /; FreeQ[{a, b, c,
 d, e, f, m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] &
& GtQ[Simplify[b/(b*c - a*d)], 0] &&  !GtQ[Simplify[b/(b*e - a*f)], 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4273
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Unintegrable[
(a + b*Csc[c + d*x])^n, x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[a^2 - b^2, 0
] &&  !IntegerQ[2*n]
```

rule 4321
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Simp[Cot[e + f*x]/(f*Sqrt[1 + Csc[e + f*x]]*Sqrt[1 - Csc[e + f*x
]])   Subst[Int[(a + b*x)^m/(Sqrt[1 + x]*Sqrt[1 - x]), x], x, Csc[e + f*x]]
, x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^2, 0] &&  !IntegerQ[2*m]
```

rule 4382
```
Int[cot[(c_.) + (d_.)*(x_)]^2*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_),
x_Symbol] :> Int[(-1 + Csc[c + d*x]^2)*(a + b*Csc[c + d*x])^n, x] /; FreeQ[
{a, b, c, d, n}, x] && NeQ[a^2 - b^2, 0]
```

rule 4412
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_)), x_Symbol] :> Simp[c   Int[(a + b*Csc[e + f*x])^m, x], x] + Sim
p[d   Int[(a + b*Csc[e + f*x])^m*Csc[e + f*x], x], x] /; FreeQ[{a, b, c, d,
 e, f, m}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[2*m]
```

rule 4551
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_))^(m_), x_Symbol] :> Simp[1/b   Int[(a + b*Csc[e + f*x])^m*(A*b - a
*C*Csc[e + f*x]), x], x] + Simp[C/b   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])
^(m + 1), x], x] /; FreeQ[{a, b, e, f, A, C, m}, x] && NeQ[a^2 - b^2, 0] &&
  !IntegerQ[2*m]
```

3.359.4 Maple [N/A] (verified)

Not integrable

Time = 0.98 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int (a + b \sec (dx + c))^n \tan (dx + c)^2 \, dx$$

input
```
int((a+b*sec(d*x+c))^n*tan(d*x+c)^2,x)
```

output
```
int((a+b*sec(d*x+c))^n*tan(d*x+c)^2,x)
```

3.359.5 Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int (a + b \sec(c + dx))^n \tan^2(c + dx) \, dx = \int (b \sec (dx + c) + a)^n \tan (dx + c)^2 \, dx$$

input
```
integrate((a+b*sec(d*x+c))^n*tan(d*x+c)^2,x, algorithm="fricas")
```

output
```
integral((b*sec(d*x + c) + a)^n*tan(d*x + c)^2, x)
```

3.359.6 Sympy [N/A]

Not integrable

Time = 3.98 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.95

$$\int (a+b\sec(c+dx))^n \tan^2(c+dx)\,dx = \int (a+b\sec(c+dx))^n \tan^2(c+dx)\,dx$$

input
```
integrate((a+b*sec(d*x+c))**n*tan(d*x+c)**2,x)
```

output
```
Integral((a + b*sec(c + d*x))**n*tan(c + d*x)**2, x)
```

3.359.7 Maxima [N/A]

Not integrable

Time = 5.80 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int (a+b\sec(c+dx))^n \tan^2(c+dx)\,dx = \int (b\sec(dx+c)+a)^n \tan(dx+c)^2\,dx$$

input
```
integrate((a+b*sec(d*x+c))^n*tan(d*x+c)^2,x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c) + a)^n*tan(d*x + c)^2, x)
```

3.359.8 Giac [N/A]

Not integrable

Time = 0.71 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int (a+b\sec(c+dx))^n \tan^2(c+dx)\,dx = \int (b\sec(dx+c)+a)^n \tan(dx+c)^2\,dx$$

input
```
integrate((a+b*sec(d*x+c))^n*tan(d*x+c)^2,x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c) + a)^n*tan(d*x + c)^2, x)
```

3.359.9 Mupad [N/A]

Not integrable

Time = 15.07 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.19

$$\int (a + b \sec(c + dx))^n \tan^2(c + dx)\, dx = \int \tan(c + d\,x)^2 \left(a + \frac{b}{\cos(c + d\,x)}\right)^n dx$$

input
```
int(tan(c + d*x)^2*(a + b/cos(c + d*x))^n,x)
```

output
```
int(tan(c + d*x)^2*(a + b/cos(c + d*x))^n, x)
```

3.360 $\int \cot^2(c+dx)(a+b\sec(c+dx))^n\,dx$

3.360.1 Optimal result . . . 2600
3.360.2 Mathematica [N/A] . . . 2600
3.360.3 Rubi [N/A] . . . 2601
3.360.4 Maple [N/A] (verified) . . . 2602
3.360.5 Fricas [N/A] . . . 2602
3.360.6 Sympy [N/A] . . . 2602
3.360.7 Maxima [N/A] . . . 2603
3.360.8 Giac [N/A] . . . 2603
3.360.9 Mupad [N/A] . . . 2603

3.360.1 Optimal result

Integrand size = 21, antiderivative size = 21

$$\int \cot^2(c+dx)(a+b\sec(c+dx))^n\,dx = \text{Int}\left(\cot^2(c+dx)(a+b\sec(c+dx))^n, x\right)$$

output
```
Unintegrable(cot(d*x+c)^2*(a+b*sec(d*x+c))^n,x)
```

3.360.2 Mathematica [N/A]

Not integrable

Time = 8.65 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int \cot^2(c+dx)(a+b\sec(c+dx))^n\,dx = \int \cot^2(c+dx)(a+b\sec(c+dx))^n\,dx$$

input
```
Integrate[Cot[c + d*x]^2*(a + b*Sec[c + d*x])^n,x]
```

output
```
Integrate[Cot[c + d*x]^2*(a + b*Sec[c + d*x])^n, x]
```

3.360.3 Rubi [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {3042, 4387}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^2(c+dx)(a+b\sec(c+dx))^n\,dx$$

$$\downarrow 3042$$

$$\int \frac{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^n}{\cot\left(c+dx+\frac{\pi}{2}\right)^2}dx$$

$$\downarrow 4387$$

$$\int \cot^2(c+dx)(a+b\sec(c+dx))^n dx$$

input
```
Int[Cot[c + d*x]^2*(a + b*Sec[c + d*x])^n,x]
```

output
```
$Aborted
```

3.360.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4387
```
Int[((a_.) + csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_.)*(cot[(c_.) + (d_.)*(x_)]*
(e_.))^(m_.), x_Symbol] :> Unintegrable[(e*Cot[c + d*x])^m*(a + b*Csc[c + d
*x])^n, x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

3.360.4 Maple [N/A] (verified)

Not integrable

Time = 1.20 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \cot(dx+c)^2 (a+b\sec(dx+c))^n \, dx$$

input
```
int(cot(d*x+c)^2*(a+b*sec(d*x+c))^n,x)
```

output
```
int(cot(d*x+c)^2*(a+b*sec(d*x+c))^n,x)
```

3.360.5 Fricas [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int \cot^2(c+dx)(a+b\sec(c+dx))^n \, dx = \int (b\sec(dx+c)+a)^n \cot(dx+c)^2 \, dx$$

input
```
integrate(cot(d*x+c)^2*(a+b*sec(d*x+c))^n,x, algorithm="fricas")
```

output
```
integral((b*sec(d*x + c) + a)^n*cot(d*x + c)^2, x)
```

3.360.6 Sympy [N/A]

Not integrable

Time = 19.23 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.95

$$\int \cot^2(c+dx)(a+b\sec(c+dx))^n \, dx = \int (a+b\sec(c+dx))^n \cot^2(c+dx) \, dx$$

input
```
integrate(cot(d*x+c)**2*(a+b*sec(d*x+c))**n,x)
```

output
```
Integral((a + b*sec(c + d*x))**n*cot(c + d*x)**2, x)
```

3.360.7 Maxima [N/A]

Not integrable

Time = 5.85 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int \cot^2(c+dx)(a+b\sec(c+dx))^n\,dx = \int (b\sec(dx+c)+a)^n \cot(dx+c)^2\,dx$$

input
```
integrate(cot(d*x+c)^2*(a+b*sec(d*x+c))^n,x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c) + a)^n*cot(d*x + c)^2, x)
```

3.360.8 Giac [N/A]

Not integrable

Time = 0.57 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int \cot^2(c+dx)(a+b\sec(c+dx))^n\,dx = \int (b\sec(dx+c)+a)^n \cot(dx+c)^2\,dx$$

input
```
integrate(cot(d*x+c)^2*(a+b*sec(d*x+c))^n,x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c) + a)^n*cot(d*x + c)^2, x)
```

3.360.9 Mupad [N/A]

Not integrable

Time = 18.06 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.19

$$\int \cot^2(c+dx)(a+b\sec(c+dx))^n\,dx = \int \cot(c+d\,x)^2 \left(a+\frac{b}{\cos(c+d\,x)}\right)^n dx$$

input
```
int(cot(c + d*x)^2*(a + b/cos(c + d*x))^n,x)
```

output
```
int(cot(c + d*x)^2*(a + b/cos(c + d*x))^n, x)
```

3.361 $\int \cot^4(c+dx)(a+b\sec(c+dx))^n\,dx$

3.361.1 Optimal result . . . 2604
3.361.2 Mathematica [N/A] . . . 2604
3.361.3 Rubi [N/A] . . . 2605
3.361.4 Maple [N/A] (verified) . . . 2606
3.361.5 Fricas [N/A] . . . 2606
3.361.6 Sympy [F(-1)] . . . 2606
3.361.7 Maxima [N/A] . . . 2607
3.361.8 Giac [N/A] . . . 2607
3.361.9 Mupad [N/A] . . . 2607

3.361.1 Optimal result

Integrand size = 21, antiderivative size = 21

$$\int \cot^4(c+dx)(a+b\sec(c+dx))^n\,dx = \text{Int}\left(\cot^4(c+dx)(a+b\sec(c+dx))^n, x\right)$$

output
```
Unintegrable(cot(d*x+c)^4*(a+b*sec(d*x+c))^n,x)
```

3.361.2 Mathematica [N/A]

Not integrable

Time = 13.64 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int \cot^4(c+dx)(a+b\sec(c+dx))^n\,dx = \int \cot^4(c+dx)(a+b\sec(c+dx))^n\,dx$$

input
```
Integrate[Cot[c + d*x]^4*(a + b*Sec[c + d*x])^n,x]
```

output
```
Integrate[Cot[c + d*x]^4*(a + b*Sec[c + d*x])^n, x]
```

3.361.3 Rubi [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {3042, 4387}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^4(c+dx)(a+b\sec(c+dx))^n\,dx$$

$$\downarrow 3042$$

$$\int \frac{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^n}{\cot\left(c+dx+\frac{\pi}{2}\right)^4}dx$$

$$\downarrow 4387$$

$$\int \cot^4(c+dx)(a+b\sec(c+dx))^n dx$$

input
```
Int[Cot[c + d*x]^4*(a + b*Sec[c + d*x])^n,x]
```

output
```
$Aborted
```

3.361.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4387
```
Int[((a_.) + csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_.)*(cot[(c_.) + (d_.)*(x_)]*
(e_.))^(m_.), x_Symbol] :> Unintegrable[(e*Cot[c + d*x])^m*(a + b*Csc[c + d
*x])^n, x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

3.361.4 Maple [N/A] (verified)

Not integrable

Time = 1.17 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \cot(dx+c)^4 (a+b\sec(dx+c))^n \, dx$$

input
```
int(cot(d*x+c)^4*(a+b*sec(d*x+c))^n,x)
```

output
```
int(cot(d*x+c)^4*(a+b*sec(d*x+c))^n,x)
```

3.361.5 Fricas [N/A]

Not integrable

Time = 0.61 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int \cot^4(c+dx)(a+b\sec(c+dx))^n \, dx = \int (b\sec(dx+c)+a)^n \cot(dx+c)^4 \, dx$$

input
```
integrate(cot(d*x+c)^4*(a+b*sec(d*x+c))^n,x, algorithm="fricas")
```

output
```
integral((b*sec(d*x + c) + a)^n*cot(d*x + c)^4, x)
```

3.361.6 Sympy [F(-1)]

Timed out.

$$\int \cot^4(c+dx)(a+b\sec(c+dx))^n \, dx = \text{Timed out}$$

input
```
integrate(cot(d*x+c)**4*(a+b*sec(d*x+c))**n,x)
```

output
```
Timed out
```

3.361.7 Maxima [N/A]

Not integrable

Time = 18.68 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int \cot^4(c+dx)(a+b\sec(c+dx))^n\,dx = \int (b\sec(dx+c)+a)^n \cot(dx+c)^4\,dx$$

input
```
integrate(cot(d*x+c)^4*(a+b*sec(d*x+c))^n,x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c) + a)^n*cot(d*x + c)^4, x)
```

3.361.8 Giac [N/A]

Not integrable

Time = 0.68 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int \cot^4(c+dx)(a+b\sec(c+dx))^n\,dx = \int (b\sec(dx+c)+a)^n \cot(dx+c)^4\,dx$$

input
```
integrate(cot(d*x+c)^4*(a+b*sec(d*x+c))^n,x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c) + a)^n*cot(d*x + c)^4, x)
```

3.361.9 Mupad [N/A]

Not integrable

Time = 28.69 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.19

$$\int \cot^4(c+dx)(a+b\sec(c+dx))^n\,dx = \int \cot(c+d\,x)^4 \left(a+\frac{b}{\cos(c+d\,x)}\right)^n dx$$

input
```
int(cot(c + d*x)^4*(a + b/cos(c + d*x))^n,x)
```

output
```
int(cot(c + d*x)^4*(a + b/cos(c + d*x))^n, x)
```

3.362 $\int (a+b\sec(c+dx))^n \tan^{\frac{3}{2}}(c+dx)\,dx$

3.362.1 Optimal result 2608
3.362.2 Mathematica [N/A] 2608
3.362.3 Rubi [N/A] 2609
3.362.4 Maple [N/A] (verified) 2610
3.362.5 Fricas [N/A] 2610
3.362.6 Sympy [F(-1)] 2610
3.362.7 Maxima [N/A] 2611
3.362.8 Giac [N/A] 2611
3.362.9 Mupad [N/A] 2611

3.362.1 Optimal result

Integrand size = 23, antiderivative size = 23

$$\int (a+b\sec(c+dx))^n \tan^{\frac{3}{2}}(c+dx)\,dx = \text{Int}\left((a+b\sec(c+dx))^n \tan^{\frac{3}{2}}(c+dx), x\right)$$

output
```
Unintegrable((a+b*sec(d*x+c))^n*tan(d*x+c)^(3/2),x)
```

3.362.2 Mathematica [N/A]

Not integrable

Time = 7.60 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int (a+b\sec(c+dx))^n \tan^{\frac{3}{2}}(c+dx)\,dx = \int (a+b\sec(c+dx))^n \tan^{\frac{3}{2}}(c+dx)\,dx$$

input
```
Integrate[(a + b*Sec[c + d*x])^n*Tan[c + d*x]^(3/2),x]
```

output
```
Integrate[(a + b*Sec[c + d*x])^n*Tan[c + d*x]^(3/2), x]
```

3.362.3 Rubi [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {3042, 4387}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^n\,dx$$

$$\downarrow 3042$$

$$\int \left(-\cot\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^n dx$$

$$\downarrow 4387$$

$$\int \tan^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^n dx$$

input
```
Int[(a + b*Sec[c + d*x])^n*Tan[c + d*x]^(3/2),x]
```

output
```
$Aborted
```

3.362.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4387
```
Int[((a_.) + csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_.)*(cot[(c_.) + (d_.)*(x_)]*
(e_.))^(m_.), x_Symbol] :> Unintegrable[(e*Cot[c + d*x])^m*(a + b*Csc[c + d
*x])^n, x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

3.362.4 Maple [N/A] (verified)

Not integrable

Time = 1.15 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int (a + b \sec (dx + c))^n \tan (dx + c)^{\frac{3}{2}} \, dx$$

input
```
int((a+b*sec(d*x+c))^n*tan(d*x+c)^(3/2),x)
```

output
```
int((a+b*sec(d*x+c))^n*tan(d*x+c)^(3/2),x)
```

3.362.5 Fricas [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int (a + b \sec(c + dx))^n \tan^{\frac{3}{2}}(c + dx) \, dx = \int (b \sec (dx + c) + a)^n \tan (dx + c)^{\frac{3}{2}} \, dx$$

input
```
integrate((a+b*sec(d*x+c))^n*tan(d*x+c)^(3/2),x, algorithm="fricas")
```

output
```
integral((b*sec(d*x + c) + a)^n*tan(d*x + c)^(3/2), x)
```

3.362.6 Sympy [F(-1)]

Timed out.

$$\int (a + b \sec(c + dx))^n \tan^{\frac{3}{2}}(c + dx) \, dx = \text{Timed out}$$

input
```
integrate((a+b*sec(d*x+c))**n*tan(d*x+c)**(3/2),x)
```

output
```
Timed out
```

3.362.7 Maxima [N/A]

Not integrable

Time = 1.92 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int (a+b\sec(c+dx))^n \tan^{\frac{3}{2}}(c+dx)\,dx = \int (b\sec(dx+c)+a)^n \tan(dx+c)^{\frac{3}{2}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))^n*tan(d*x+c)^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c) + a)^n*tan(d*x + c)^(3/2), x)
```

3.362.8 Giac [N/A]

Not integrable

Time = 0.43 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int (a+b\sec(c+dx))^n \tan^{\frac{3}{2}}(c+dx)\,dx = \int (b\sec(dx+c)+a)^n \tan(dx+c)^{\frac{3}{2}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))^n*tan(d*x+c)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c) + a)^n*tan(d*x + c)^(3/2), x)
```

3.362.9 Mupad [N/A]

Not integrable

Time = 19.28 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int (a+b\sec(c+dx))^n \tan^{\frac{3}{2}}(c+dx)\,dx = \int \tan(c+d\,x)^{3/2}\left(a+\frac{b}{\cos(c+d\,x)}\right)^n dx$$

input
```
int(tan(c + d*x)^(3/2)*(a + b/cos(c + d*x))^n,x)
```

output
```
int(tan(c + d*x)^(3/2)*(a + b/cos(c + d*x))^n, x)
```

3.363 $\int (a+b\sec(c+dx))^n \sqrt{\tan(c+dx)}\,dx$

3.363.1 Optimal result . . . 2612
3.363.2 Mathematica [N/A] . . . 2612
3.363.3 Rubi [N/A] . . . 2613
3.363.4 Maple [N/A] (verified) . . . 2614
3.363.5 Fricas [N/A] . . . 2614
3.363.6 Sympy [N/A] . . . 2614
3.363.7 Maxima [N/A] . . . 2615
3.363.8 Giac [N/A] . . . 2615
3.363.9 Mupad [N/A] . . . 2615

3.363.1 Optimal result

Integrand size = 23, antiderivative size = 23

$$\int (a+b\sec(c+dx))^n \sqrt{\tan(c+dx)}\,dx = \text{Int}\left((a+b\sec(c+dx))^n \sqrt{\tan(c+dx)}, x\right)$$

output
```
Unintegrable((a+b*sec(d*x+c))^n*tan(d*x+c)^(1/2),x)
```

3.363.2 Mathematica [N/A]

Not integrable

Time = 10.14 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int (a+b\sec(c+dx))^n \sqrt{\tan(c+dx)}\,dx = \int (a+b\sec(c+dx))^n \sqrt{\tan(c+dx)}\,dx$$

input
```
Integrate[(a + b*Sec[c + d*x])^n*Sqrt[Tan[c + d*x]],x]
```

output
```
Integrate[(a + b*Sec[c + d*x])^n*Sqrt[Tan[c + d*x]], x]
```

3.363.3 Rubi [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {3042, 4387}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\tan(c+dx)}(a+b\sec(c+dx))^n\,dx$$

$$\downarrow 3042$$

$$\int \sqrt{-\cot\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^n dx$$

$$\downarrow 4387$$

$$\int \sqrt{\tan(c+dx)}(a+b\sec(c+dx))^n dx$$

input
```
Int[(a + b*Sec[c + d*x])^n*Sqrt[Tan[c + d*x]],x]
```

output
```
$Aborted
```

3.363.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4387
```
Int[((a_.) + csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_.)*(cot[(c_.) + (d_.)*(x_)]*
(e_.))^(m_.), x_Symbol] :> Unintegrable[(e*Cot[c + d*x])^m*(a + b*Csc[c + d
*x])^n, x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

3.363.4 Maple [N/A] (verified)

Not integrable

Time = 1.52 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int (a + b \sec (dx + c))^n \sqrt{\tan (dx + c)} dx$$

input
```
int((a+b*sec(d*x+c))^n*tan(d*x+c)^(1/2),x)
```

output
```
int((a+b*sec(d*x+c))^n*tan(d*x+c)^(1/2),x)
```

3.363.5 Fricas [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int (a + b \sec(c + dx))^n \sqrt{\tan(c + dx)}\, dx = \int (b \sec (dx + c) + a)^n \sqrt{\tan (dx + c)}\, dx$$

input
```
integrate((a+b*sec(d*x+c))^n*tan(d*x+c)^(1/2),x, algorithm="fricas")
```

output
```
integral((b*sec(d*x + c) + a)^n*sqrt(tan(d*x + c)), x)
```

3.363.6 Sympy [N/A]

Not integrable

Time = 6.02 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int (a + b \sec(c + dx))^n \sqrt{\tan(c + dx)}\, dx = \int (a + b \sec (c + dx))^n \sqrt{\tan (c + dx)}\, dx$$

input
```
integrate((a+b*sec(d*x+c))**n*tan(d*x+c)**(1/2),x)
```

output
```
Integral((a + b*sec(c + d*x))**n*sqrt(tan(c + d*x)), x)
```

3.363.7 Maxima [N/A]

Not integrable

Time = 1.75 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int (a+b\sec(c+dx))^n \sqrt{\tan(c+dx)}\,dx = \int (b\sec(dx+c)+a)^n \sqrt{\tan(dx+c)}\,dx$$

input
```
integrate((a+b*sec(d*x+c))^n*tan(d*x+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c) + a)^n*sqrt(tan(d*x + c)), x)
```

3.363.8 Giac [N/A]

Not integrable

Time = 0.58 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int (a+b\sec(c+dx))^n \sqrt{\tan(c+dx)}\,dx = \int (b\sec(dx+c)+a)^n \sqrt{\tan(dx+c)}\,dx$$

input
```
integrate((a+b*sec(d*x+c))^n*tan(d*x+c)^(1/2),x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c) + a)^n*sqrt(tan(d*x + c)), x)
```

3.363.9 Mupad [N/A]

Not integrable

Time = 14.74 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int (a+b\sec(c+dx))^n \sqrt{\tan(c+dx)}\,dx = \int \sqrt{\tan(c+d\,x)}\left(a+\frac{b}{\cos(c+d\,x)}\right)^n dx$$

input
```
int(tan(c + d*x)^(1/2)*(a + b/cos(c + d*x))^n,x)
```

output
```
int(tan(c + d*x)^(1/2)*(a + b/cos(c + d*x))^n, x)
```

3.364 $\int \frac{(a+b\sec(c+dx))^n}{\sqrt{\tan(c+dx)}}\,dx$

3.364.1 Optimal result 2616
3.364.2 Mathematica [N/A] 2616
3.364.3 Rubi [N/A] 2617
3.364.4 Maple [N/A] (verified) 2618
3.364.5 Fricas [N/A] 2618
3.364.6 Sympy [N/A] 2618
3.364.7 Maxima [N/A] 2619
3.364.8 Giac [N/A] 2619
3.364.9 Mupad [N/A] 2619

3.364.1 Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{(a+b\sec(c+dx))^n}{\sqrt{\tan(c+dx)}}\,dx = \text{Int}\left(\frac{(a+b\sec(c+dx))^n}{\sqrt{\tan(c+dx)}}, x\right)$$

output
```
Unintegrable((a+b*sec(d*x+c))^n/tan(d*x+c)^(1/2),x)
```

3.364.2 Mathematica [N/A]

Not integrable

Time = 23.22 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{(a+b\sec(c+dx))^n}{\sqrt{\tan(c+dx)}}\,dx = \int \frac{(a+b\sec(c+dx))^n}{\sqrt{\tan(c+dx)}}\,dx$$

input
```
Integrate[(a + b*Sec[c + d*x])^n/Sqrt[Tan[c + d*x]],x]
```

output
```
Integrate[(a + b*Sec[c + d*x])^n/Sqrt[Tan[c + d*x]], x]
```

3.364.3 Rubi [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {3042, 4387}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\sec(c+dx))^n}{\sqrt{\tan(c+dx)}}\,dx$$
$$\downarrow 3042$$
$$\int \frac{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^n}{\sqrt{-\cot\left(c+dx+\frac{\pi}{2}\right)}}dx$$
$$\downarrow 4387$$
$$\int \frac{(a+b\sec(c+dx))^n}{\sqrt{\tan(c+dx)}}dx$$

input
```
Int[(a + b*Sec[c + d*x])^n/Sqrt[Tan[c + d*x]],x]
```

output
```
$Aborted
```

3.364.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4387
```
Int[((a_.) + csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_.)*(cot[(c_.) + (d_.)*(x_)]*
(e_.))^(m_.), x_Symbol] :> Unintegrable[(e*Cot[c + d*x])^m*(a + b*Csc[c + d
*x])^n, x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

3.364.4 Maple [N/A] (verified)

Not integrable

Time = 1.33 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{(a+b\sec(dx+c))^n}{\sqrt{\tan(dx+c)}}dx$$

input
```
int((a+b*sec(d*x+c))^n/tan(d*x+c)^(1/2),x)
```

output
```
int((a+b*sec(d*x+c))^n/tan(d*x+c)^(1/2),x)
```

3.364.5 Fricas [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\sec(c+dx))^n}{\sqrt{\tan(c+dx)}}\,dx = \int \frac{(b\sec(dx+c)+a)^n}{\sqrt{\tan(dx+c)}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))^n/tan(d*x+c)^(1/2),x, algorithm="fricas")
```

output
```
integral((b*sec(d*x + c) + a)^n/sqrt(tan(d*x + c)), x)
```

3.364.6 Sympy [N/A]

Not integrable

Time = 2.49 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int \frac{(a+b\sec(c+dx))^n}{\sqrt{\tan(c+dx)}}\,dx = \int \frac{(a+b\sec(c+dx))^n}{\sqrt{\tan(c+dx)}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))**n/tan(d*x+c)**(1/2),x)
```

output
```
Integral((a + b*sec(c + d*x))**n/sqrt(tan(c + d*x)), x)
```

3.364.7 Maxima [N/A]

Not integrable

Time = 1.93 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\sec(c+dx))^n}{\sqrt{\tan(c+dx)}}\,dx = \int \frac{(b\sec(dx+c)+a)^n}{\sqrt{\tan(dx+c)}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))^n/tan(d*x+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c) + a)^n/sqrt(tan(d*x + c)), x)
```

3.364.8 Giac [N/A]

Not integrable

Time = 0.75 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\sec(c+dx))^n}{\sqrt{\tan(c+dx)}}\,dx = \int \frac{(b\sec(dx+c)+a)^n}{\sqrt{\tan(dx+c)}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))^n/tan(d*x+c)^(1/2),x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c) + a)^n/sqrt(tan(d*x + c)), x)
```

3.364.9 Mupad [N/A]

Not integrable

Time = 18.95 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{(a+b\sec(c+dx))^n}{\sqrt{\tan(c+dx)}}\,dx = \int \frac{\left(a+\frac{b}{\cos(c+d\,x)}\right)^n}{\sqrt{\tan(c+d\,x)}}\,dx$$

input
```
int((a + b/cos(c + d*x))^n/tan(c + d*x)^(1/2),x)
```

output
```
int((a + b/cos(c + d*x))^n/tan(c + d*x)^(1/2), x)
```

3.365 $\int \frac{(a+b\sec(c+dx))^n}{\tan^{\frac{3}{2}}(c+dx)}\,dx$

3.365.1 Optimal result 2620
3.365.2 Mathematica [N/A] 2620
3.365.3 Rubi [N/A] 2621
3.365.4 Maple [N/A] (verified) 2622
3.365.5 Fricas [N/A] 2622
3.365.6 Sympy [N/A] 2622
3.365.7 Maxima [N/A] 2623
3.365.8 Giac [N/A] 2623
3.365.9 Mupad [N/A] 2623

3.365.1 Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{(a+b\sec(c+dx))^n}{\tan^{\frac{3}{2}}(c+dx)}\,dx = \text{Int}\left(\frac{(a+b\sec(c+dx))^n}{\tan^{\frac{3}{2}}(c+dx)}, x\right)$$

output
```
Unintegrable((a+b*sec(d*x+c))^n/tan(d*x+c)^(3/2),x)
```

3.365.2 Mathematica [N/A]

Not integrable

Time = 10.64 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{(a+b\sec(c+dx))^n}{\tan^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{(a+b\sec(c+dx))^n}{\tan^{\frac{3}{2}}(c+dx)}\,dx$$

input
```
Integrate[(a + b*Sec[c + d*x])^n/Tan[c + d*x]^(3/2),x]
```

output
```
Integrate[(a + b*Sec[c + d*x])^n/Tan[c + d*x]^(3/2), x]
```

3.365.3 Rubi [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {3042, 4387}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\sec(c+dx))^n}{\tan^{\frac{3}{2}}(c+dx)}\,dx$$
$$\downarrow 3042$$
$$\int \frac{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^n}{\left(-\cot\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx$$
$$\downarrow 4387$$
$$\int \frac{(a+b\sec(c+dx))^n}{\tan^{\frac{3}{2}}(c+dx)}dx$$

input
```
Int[(a + b*Sec[c + d*x])^n/Tan[c + d*x]^(3/2),x]
```

output
```
$Aborted
```

3.365.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4387
```
Int[((a_.) + csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_.)*(cot[(c_.) + (d_.)*(x_)]*
(e_.))^(m_.), x_Symbol] :> Unintegrable[(e*Cot[c + d*x])^m*(a + b*Csc[c + d
*x])^n, x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

3.365.4 Maple [N/A] (verified)

Not integrable

Time = 1.12 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{(a+b\sec(dx+c))^n}{\tan(dx+c)^{\frac{3}{2}}}dx$$

input
```
int((a+b*sec(d*x+c))^n/tan(d*x+c)^(3/2),x)
```

output
```
int((a+b*sec(d*x+c))^n/tan(d*x+c)^(3/2),x)
```

3.365.5 Fricas [N/A]

Not integrable

Time = 0.44 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\sec(c+dx))^n}{\tan^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{(b\sec(dx+c)+a)^n}{\tan(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))^n/tan(d*x+c)^(3/2),x, algorithm="fricas")
```

output
```
integral((b*sec(d*x + c) + a)^n/tan(d*x + c)^(3/2), x)
```

3.365.6 Sympy [N/A]

Not integrable

Time = 52.60 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int \frac{(a+b\sec(c+dx))^n}{\tan^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{(a+b\sec(c+dx))^n}{\tan^{\frac{3}{2}}(c+dx)}\,dx$$

input
```
integrate((a+b*sec(d*x+c))**n/tan(d*x+c)**(3/2),x)
```

output
```
Integral((a + b*sec(c + d*x))**n/tan(c + d*x)**(3/2), x)
```

3.365.7 Maxima [N/A]

Not integrable

Time = 1.97 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\sec(c+dx))^n}{\tan^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{(b\sec(dx+c)+a)^n}{\tan(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))^n/tan(d*x+c)^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c) + a)^n/tan(d*x + c)^(3/2), x)
```

3.365.8 Giac [N/A]

Not integrable

Time = 0.56 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\sec(c+dx))^n}{\tan^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{(b\sec(dx+c)+a)^n}{\tan(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))^n/tan(d*x+c)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c) + a)^n/tan(d*x + c)^(3/2), x)
```

3.365.9 Mupad [N/A]

Not integrable

Time = 20.46 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{(a+b\sec(c+dx))^n}{\tan^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{\left(a+\frac{b}{\cos(c+d\,x)}\right)^n}{\tan(c+d\,x)^{3/2}}\,dx$$

input
```
int((a + b/cos(c + d*x))^n/tan(c + d*x)^(3/2),x)
```

output
```
int((a + b/cos(c + d*x))^n/tan(c + d*x)^(3/2), x)
```

CHAPTER 4

APPENDIX

4.1 Listing of Grading functions 2624

4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.1.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*          GradeAntiderivative[result,optimal] returns*)
```

```
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leafC
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A"," "}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A"," "}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal. $
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "<
            ,
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]
```

```
(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]
```

```
ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.1.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
```

```
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issues
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;
```

```
if ExpnType_result<=ExpnType_optimal then
    if debug then
         print("ExpnType_result<=ExpnType_optimal");
    fi;
    if is_contains_complex(result) then
        if is_contains_complex(optimal) then
            if debug then
                    print("both result and optimal complex");
            fi;
            if leaf_count_result<=2*leaf_count_optimal then
                return "A"," ";
            else
               return "B",cat("Both result and optimal contain complex but leaf count of
                               convert(leaf_count_result,string)," vs. $2 (",
                               convert(leaf_count_optimal,string)," ) = ",convert(2*leaf_
            end if
        else #result contains complex but optimal is not
            if debug then
                    print("result contains complex but optimal is not");
            fi;
            return "C","Result contains complex when optimal does not.";
        fi;
    else # result do not contain complex
         # this assumes optimal do not as well. No check is needed here.
        if debug then
               print("result do not contain complex, this assumes optimal do not as well"
        fi;
        if leaf_count_result<=2*leaf_count_optimal then
            if debug then
                print("leaf_count_result<=2*leaf_count_optimal");
            fi;
            return "A"," ";
        else
            if debug then
                print("leaf_count_result>2*leaf_count_optimal");
            fi;
            return "B",cat("Leaf count of result is larger than twice the leaf count of o
                           convert(leaf_count_result,string),"$ vs. $2(",
                           convert(leaf_count_optimal,string),")=",convert(2*leaf_cou
        fi;
    fi;
```

```
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
```

```
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
```

```
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.1.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#           Port of original Maple grading function by
#           Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#           added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
```

```
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
            return True
        else:
            return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
```

```
    return 1
  elif isinstance(expn,list):
    return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
  elif  is_sqrt(expn):
    if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
      return 1
    else:
      return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
  elif isinstance(expn,Pow):   #type(expn,'^')
    if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
      return expnType(expn.args[0])   #ExpnType(op(1,expn))
    elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
      if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
        return 1
      else:
        return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    else:
      return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnT
  elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'+') or type(expn,'*')
    m1 = expnType(expn.args[0])
    m2 = expnType(list(expn.args[1:]))
    return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
    return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
  elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
    m1 = max(map(expnType, list(expn.args)))
    return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
  elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
    m1 = max(map(expnType, list(expn.args)))
    return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
  elif is_appell_function(expn.func):
    m1 = max(map(expnType, list(expn.args)))
    return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
  elif isinstance(expn,RootSum):
    m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
    return max(7,m1)
  elif str(expn).find("Integral") != -1:
    m1 = max(map(expnType, list(expn.args)))
    return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
  else:
    return 9

#main function
```

```
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is large
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(le
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType_


    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)
```

```
    return grade, grade_annotation
```

4.1.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#              issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:    #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
```

```
            return False
    else:
        return False

def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
          'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
          'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
          'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
          'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m
```

```
def is_hypergeometric_function(func):
   return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
   return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath

def is_atom(expn):

   #debug=False
   if debug:
       print ("Enter is_atom, expn=",expn)

   if  not hasattr(expn, 'parent'):
      return False


   #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-
   try:
      if expn.parent() is SR:
         return expn.operator() is None
      if expn.parent() in (ZZ, QQ, AA, QQbar):
         return expn in expn.parent() # Should always return True
      if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
         return expn in expn.parent().base_ring() or expn in expn.parent().gens()

      return False

   except AttributeError as error:
      print("Exception,AttributeError in  is_atom")
      print ("cought exception" , type(error).__name__ )
      return False


def expnType(expn):

   if debug:
      print (">>>>>Enter expnType, expn=", expn)
      print (">>>>>is_atom(expn)=", is_atom(expn))

   if is_atom(expn):
      return 1
   elif type(expn)==list:   #isinstance(expn,list):
```

```
      return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
   elif is_sqrt(expn):
      if type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
         return 1
      else:
         return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
   elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
      if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
         return expnType(expn.operands()[0])   #expnType(expn.args[0])
      elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],Rational)
         if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
            return 1
         else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
      else:
         return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.
   elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinst
      m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
      m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
      return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
   elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
      return max(3,expnType(expn.operands()[0]))
   elif is_special_function(expn.operator()): #is_special_function(expn.func)
      m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
      return max(4,m1)   #max(4,m1)
   elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
      m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
      return max(5,m1)   #max(5,m1)
   elif is_appell_function(expn.operator()):
      m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
      return max(6,m1)   #max(6,m1)
   elif str(expn).find("Integral") != -1: #this will never happen, since it
            #is checked before calling the grading function that is passed.
            #but kept it here.
      m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
      return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
   else:
      return 9

#main function
def grade_antiderivative(result,optimal):
```

```
if debug:
    print ("Enter grade_antiderivative for sagemath")
    print("Enter grade_antiderivative, result=",result)
    print("Enter grade_antiderivative, optimal=",optimal)
    print("type(anti)=",type(result))
    print("type(optimal)=",type(optimal))

leaf_count_result  = tree_size(result) #leaf_count(result)
leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

#if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


expnType_result  = expnType(result)
expnType_optimal = expnType(optimal)

if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation =" "
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger t
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation =" "
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(leaf_
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_resu


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)
```

```
    return grade, grade_annotation
```